Title: String Theory and Gauge Theory: Past Present and Future Workshop 2

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Abstract:

Meta-stable Vacua in SQCD and MQCD

David Shih
Harvard University

K. Intriligator, N. Seiberg and DS
hep-th/0602239

I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and DS
hep-th/0608157

In memory of John Brodie

Field theory motivation

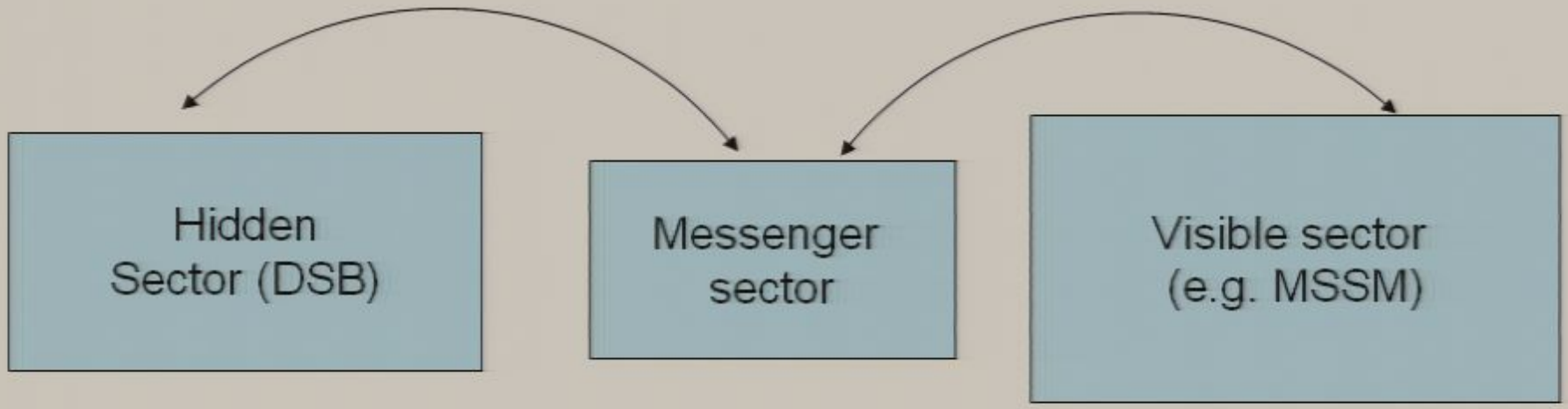
Dynamical SUSY breaking (DSB) is the idea that supersymmetry is spontaneously broken through non-perturbative effects in an asymptotically free gauge theory.

It provides a natural mechanism for generating large mass hierarchies (Witten). In fact, it is the only mechanism of natural SUSY breaking we know of.

As such, it is directly relevant to particle phenomenology and supersymmetric model building.

Field theory motivation (cont'd)

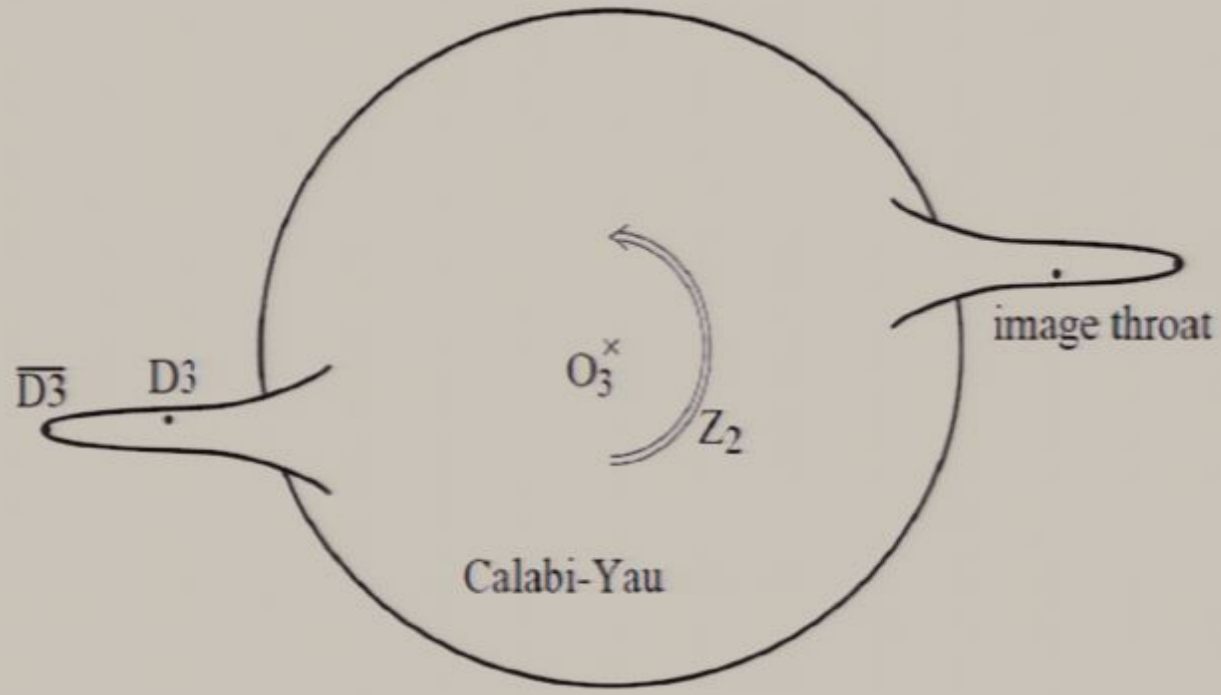
However, realistic SUSY models tend to be complicated constructions with many components.



Part of the complexity stems from the dearth of simple examples of DSB in field theory.

String theory motivation

Existing non-SUSY 4D string compactifications also tend to be quite complicated.



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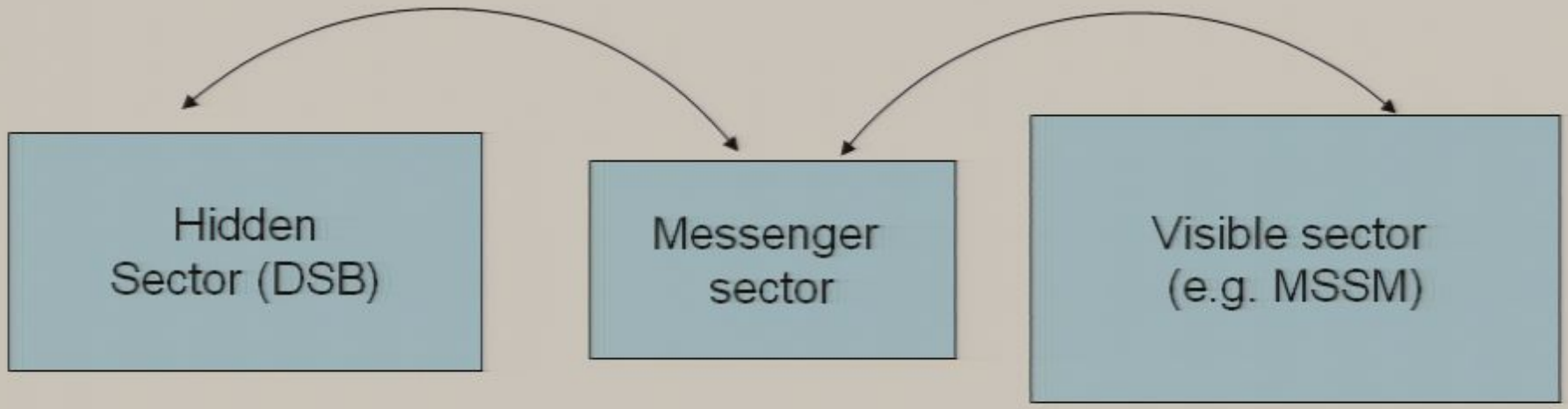

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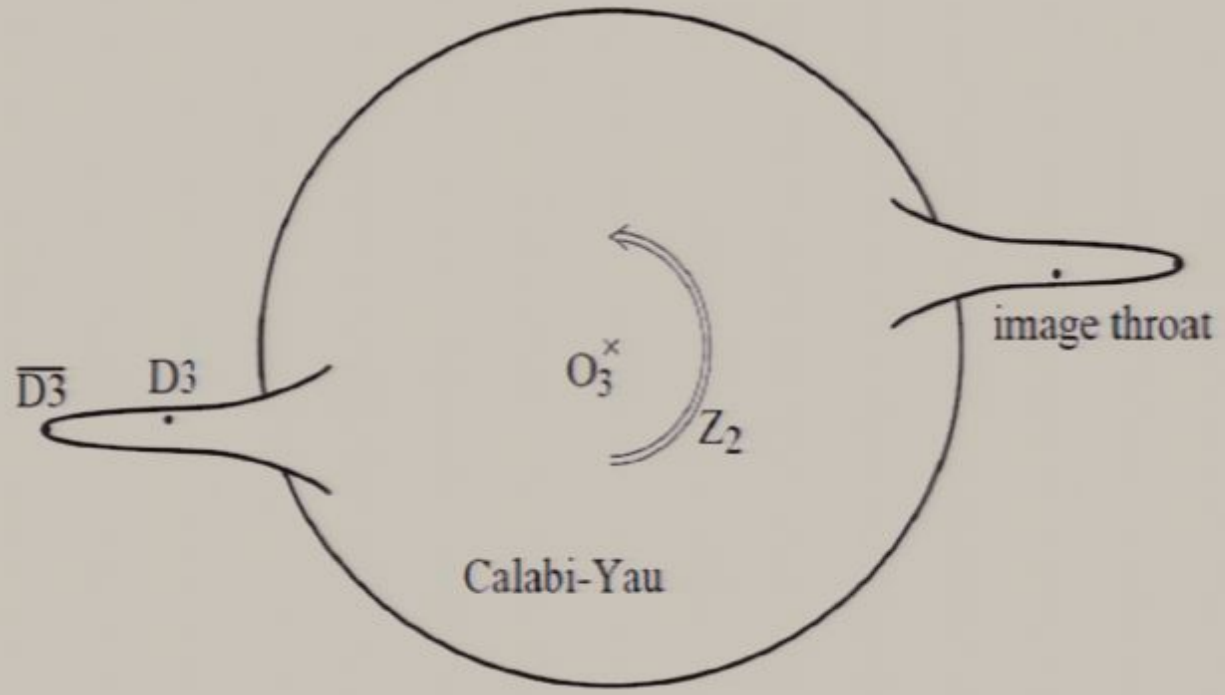

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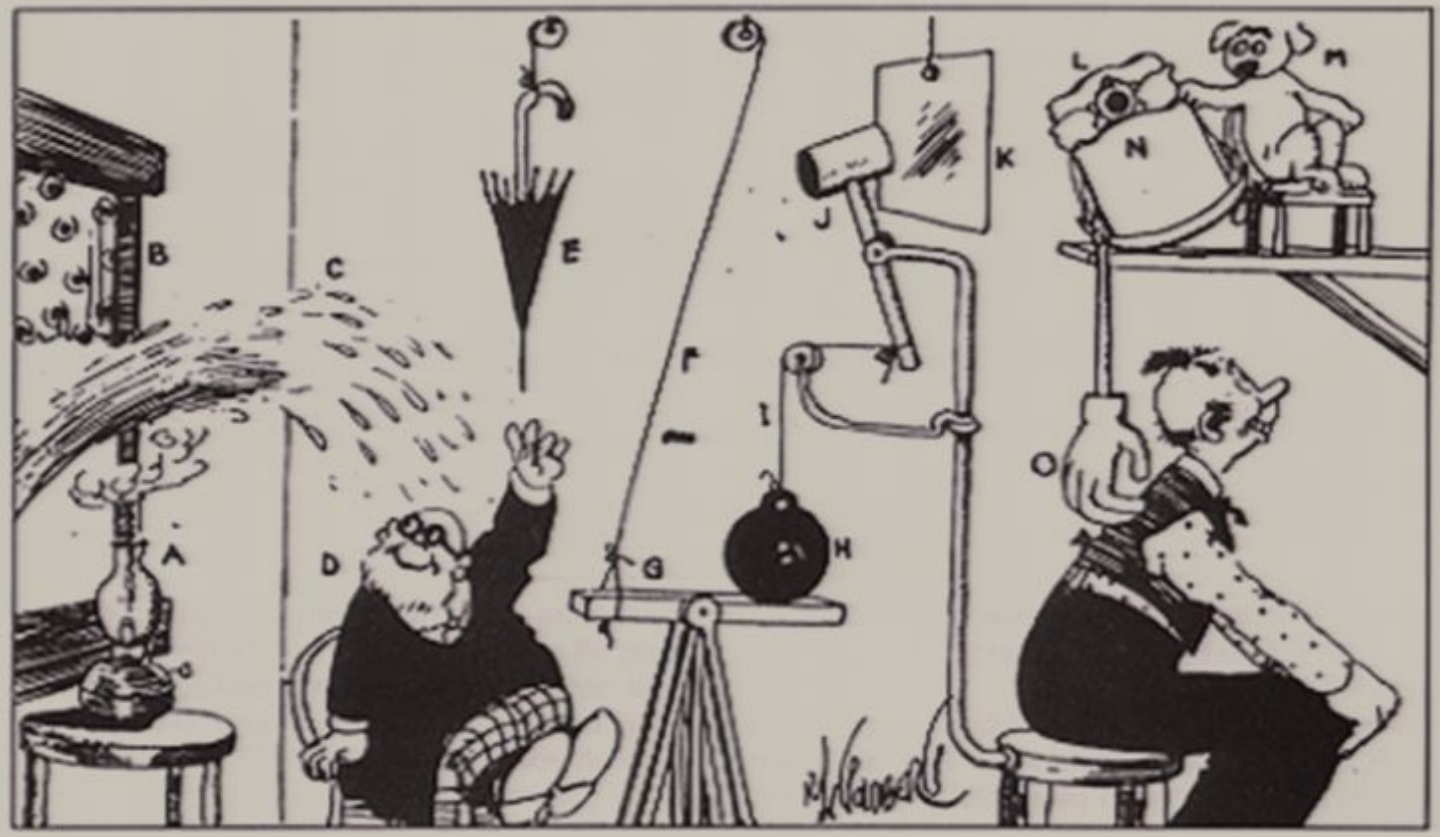
An Automatic Back Scratcher



Can these constructions be simplified??

DSB, simplified

Recently we found a particularly simple class of field theories that exhibit (meta-stable) DSB:

N=1 SQCD with massive flavors in the free-magnetic phase.

Thus, DSB is a more generic phenomenon than previously thought. Hopefully this will lead to much simpler low-energy models.

Can it also lead to simpler ~~SUSY~~ string compactifications?

Outline

- Part I: Meta-stable vacua in SQCD.
  - SUSY vacua of the electric theory.
  - Non-SUSY vacua of the magnetic theory.
  - Dynamical SUSY restoration in the magnetic theory.
  - Seiberg duality and meta-stable DSB.
- Part II: Meta-stable vacua in MQCD?
  - Embedding into IIA string theory using NS5, D4, D6 branes.
  - Electric and magnetic brane configurations.
  - SUSY-breaking state of the magnetic configuration.
  - An obstruction in the lift to M-theory...

Part I:
Meta-stable Vacua in SQCD

The "Electric" Theory

	$SU(N_c)$	$[SU(N_f)$	$SU(N_f)]$
Q	N_c	N_f	1
$\tilde{Q}$	$\overline{N}_c$	1	$\overline{N}_f$
$(M = Q\tilde{Q}$	1	N_f	$\overline{N}_f)$

- Beta function: $\beta_e = 3N_c - N_f$

Asymptotically free if $N_f < 3N_c$ (IR free if not)

- Superpotential: $W_{tree} = \mathrm{Tr}\, mQ\tilde{Q} = m_0 \mathrm{Tr}\, M$

At tree level, have a SUSY vacuum at $Q = \tilde{Q} = 0$

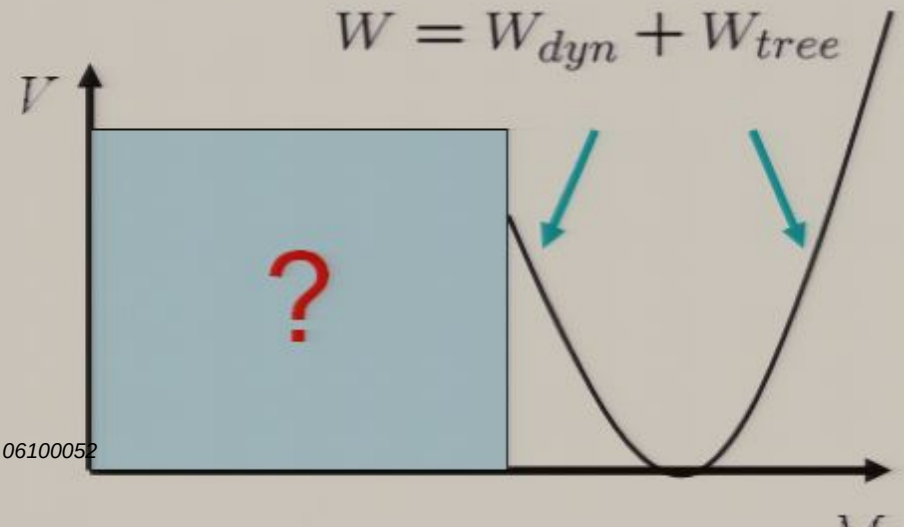
With N_f massive flavors, expect: $\mathrm{Tr}\,(-1)^F = N_c$ SUSY vacua

Massive N=1 SQCD (cont'd)

Indeed, nonperturbatively there are N_c SUSY vacua as a result of gaugino condensation in the low-energy $SU(N_c)$ SYM:

$$W_{eff} = N_c \Lambda_{eff}^3 = N_c \left(\Lambda^{3N_c - N_f} m_0^{N_f} \right)^{1/N_c}$$

$$\langle M \rangle = \frac{1}{N_f} \frac{\partial W_{eff}}{\partial m_0} = \left(\Lambda^{3N_c - N_f} m_0^{N_f - N_c} \right)^{1/N_c} \mathbf{1}_{N_f}$$



In addition, there are meta-stable non-SUSY vacua near the origin. To see these, we need to use Seiberg duality…

The "Magnetic" Theory

	$SU(N_f - N_c)$	$[SU(N_f)$	$SU(N_f)]$
q	$N_f - N_c$	N_f	1
$\tilde{q}$	$N_f - N_c$	1	$\overline{N}_f$
M	1	$\overline{N}_f$	N_f

- Beta function: $\beta_m = 2N_f - 3N_c$

Asymptotically free if $N_f > 3N_c/2$ (IR free if not)

- Superpotential: $W_{dual} = \frac{1}{\Lambda} \mathrm{Tr}\, q M \tilde{q} - m_0 \mathrm{Tr}\, M$

At tree-level, supersymmetry is broken:

$$F_M = \frac{1}{\Lambda} q\tilde{q} - m_0 \mathbf{1}_{N_f} \neq 0$$

Focus on the free-magnetic phase

What is the vacuum structure?

Let's focus now on $N_c < N_f \leq \frac{3}{2} N_c$ where theory is IR free.
Then the Kahler potential is smooth near the origin:

$$K = \frac{1}{\alpha \Lambda^2} \mathrm{Tr}\, M^\dagger M + \frac{1}{\beta} \mathrm{Tr}(q^\dagger q + \tilde{q}^\dagger \tilde{q}) + \dots$$

Using this, we can compute the scalar potential near the origin:

$$W_{dual} = \frac{1}{\Lambda} \mathrm{Tr}\, q M \tilde{q} + m_0 \mathrm{Tr}\, M \Rightarrow$$

$$V_{tree} = \frac{1}{\Lambda^2} \beta \left(|M\tilde{q}|^2 + |Mq|^2 \right) + \alpha |\tilde{q} q + m_0 A|^2$$

Non-SUSY vacua of the magnetic theory

$$V_{tree} = \frac{1}{\Lambda^2}\beta\left(|M\tilde{q}|^2 + |Mq|^2\right) + \alpha|\tilde{q}q + m_0\Lambda|^2$$

Classical vacua (up to global symmetries):

$$M = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \end{pmatrix}, \quad q = \begin{pmatrix} q_0 \\ 0 \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix}, \quad q_0\tilde{q}_0 = -m_0\Lambda$$

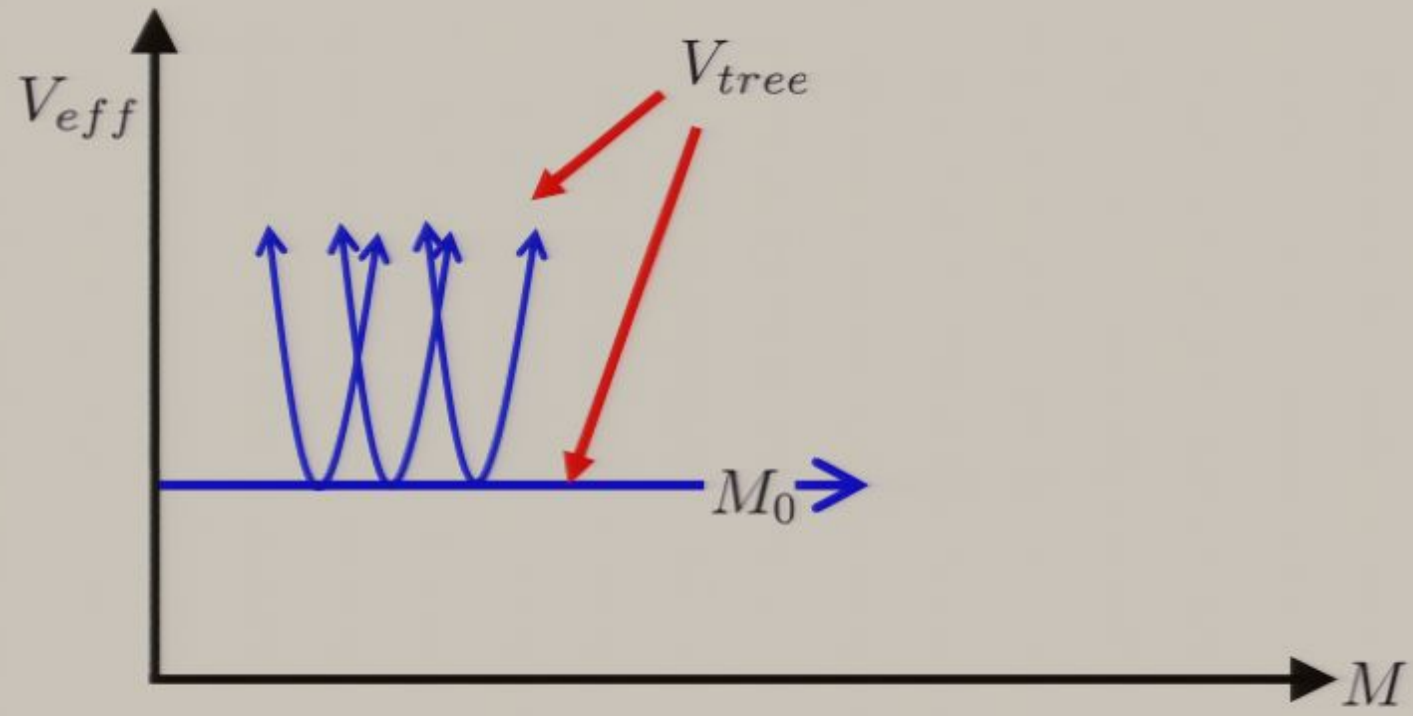
Arbitrary $N_c \times N_c$ matrix $(N_f - N_c) \times (N_f - N_c)$ matrices

$$V_{min} = N_c\alpha\,|m_0\Lambda|^2 \sim N_c m_0^2 \neq 0$$

(Note: $V_{origin} \sim N_f m_0^2 > V_{min}$)

Potential for the pseudo-moduli

M_0 and $(q_0, \tilde{q}_0)$ parameterize a pseudo-moduli space.



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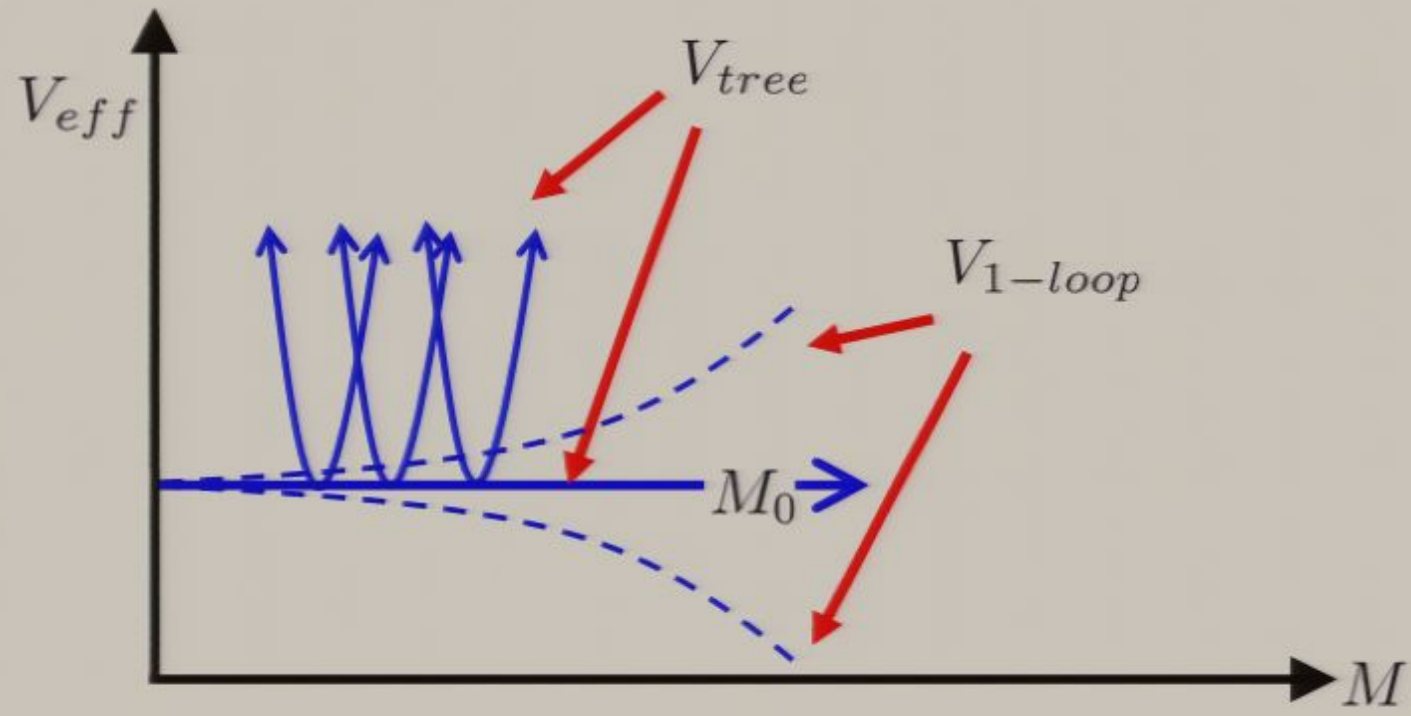


This space is lifted in perturbation theory, since SUSY is broken.

Potential for the pseudo-moduli, cont'd

In fact, expanding around the point of maximal unbroken global symmetry:

$$M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad q = \tilde{q} = \begin{pmatrix} \sqrt{-m_0 \Lambda}\, \mathbf{1}_{N_f - N_c} \\ 0 \end{pmatrix}$$

and using the Coleman-Weinberg formula, we find

$$V_{1-loop} \sim |m_0 \Lambda| \mathrm{Tr}\left[(N_f - N_c)\delta M^\dagger \delta M + N_c(\delta q + \delta q^\dagger)^2\right]$$

The mass-squareds are all positive!

For $N_c < N_f \leq \frac{3}{2} N_c$, the magnetic theory has local SUSY-breaking vacua!

Dynamical SUSY restoration

In addition, there are N_c SUSY vacua in the magnetic theory. These can be seen after integrating out the magnetic quarks and taking into account gaugino condensation in the effective $SU(N_f - N_c)$ SYM,

$$W_{eff} = (N_f - N_c)(\Lambda^{N_f - 3N_c} \det M)^{1/(N_f - N_c)} - m_0 \mathrm{Tr}\, M$$

$$\langle M \rangle = (\Lambda^{3N_c - N_f} m^{N_f - N_c})^{1/N_c}$$

So SUSY is non-perturbatively restored in the magnetic theory!

(Keep in mind: in free magnetic range, $N_f < 3N_c/2$, $W_{dyn} \sim M^{\#>3}$ so it is insignificant for the DSB vacua near the origin.)

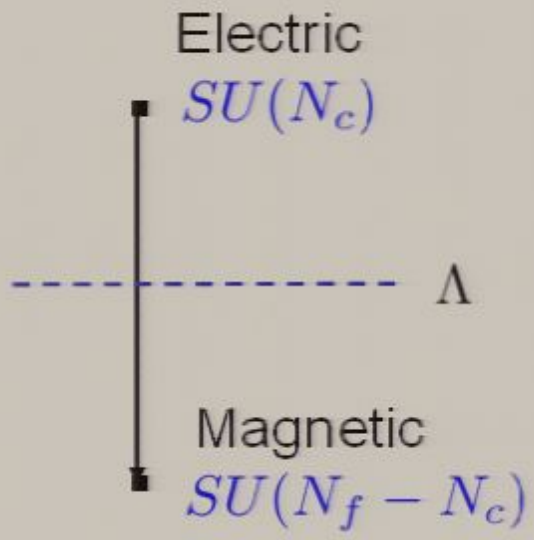
Seiberg Duality

What does this imply for the "electric" $SU(N_c)$ SQCD theory?

Seiberg duality: "Electric" and "magnetic" theories become the same theory in the IR.

$$N_c < N_f \leq \frac{3}{2} N_c$$

"Free magnetic phase"



Magnetic theory is a weakly-coupled description of the electric theory at energies $\ll \Lambda$!

Meta-stable DSB in SQCD

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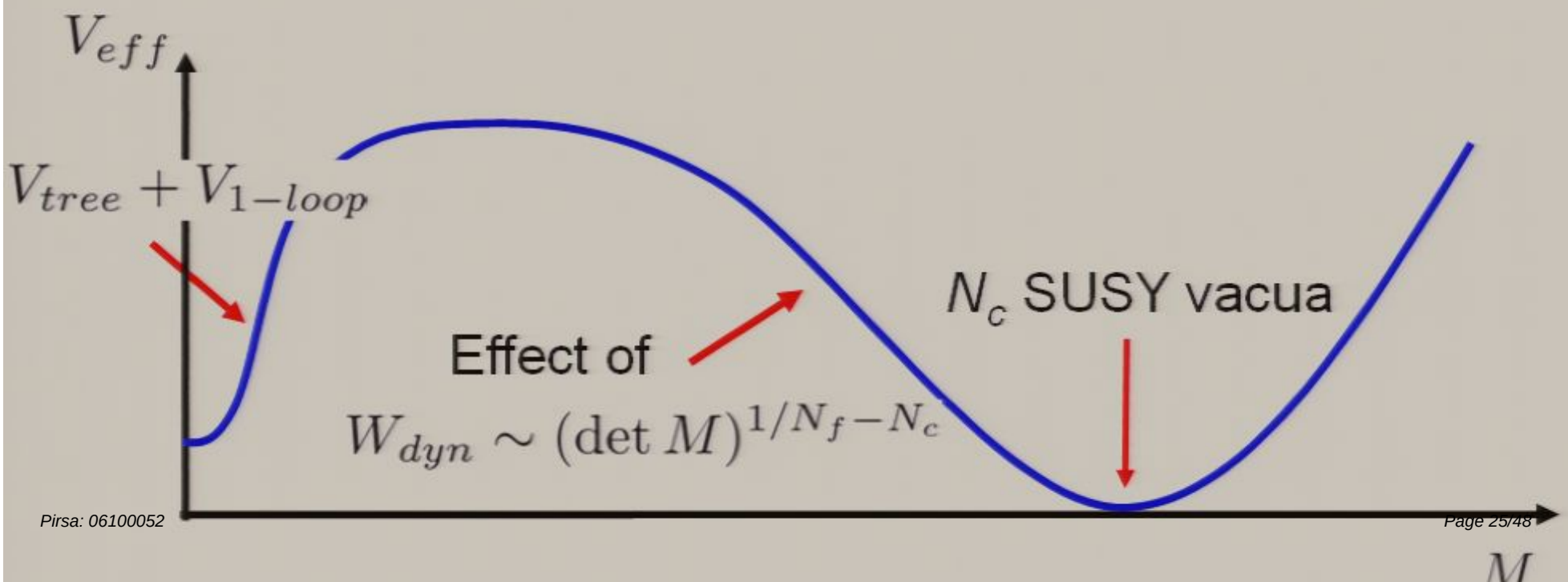

Part II:
Meta-stable Vacua in MQCD?

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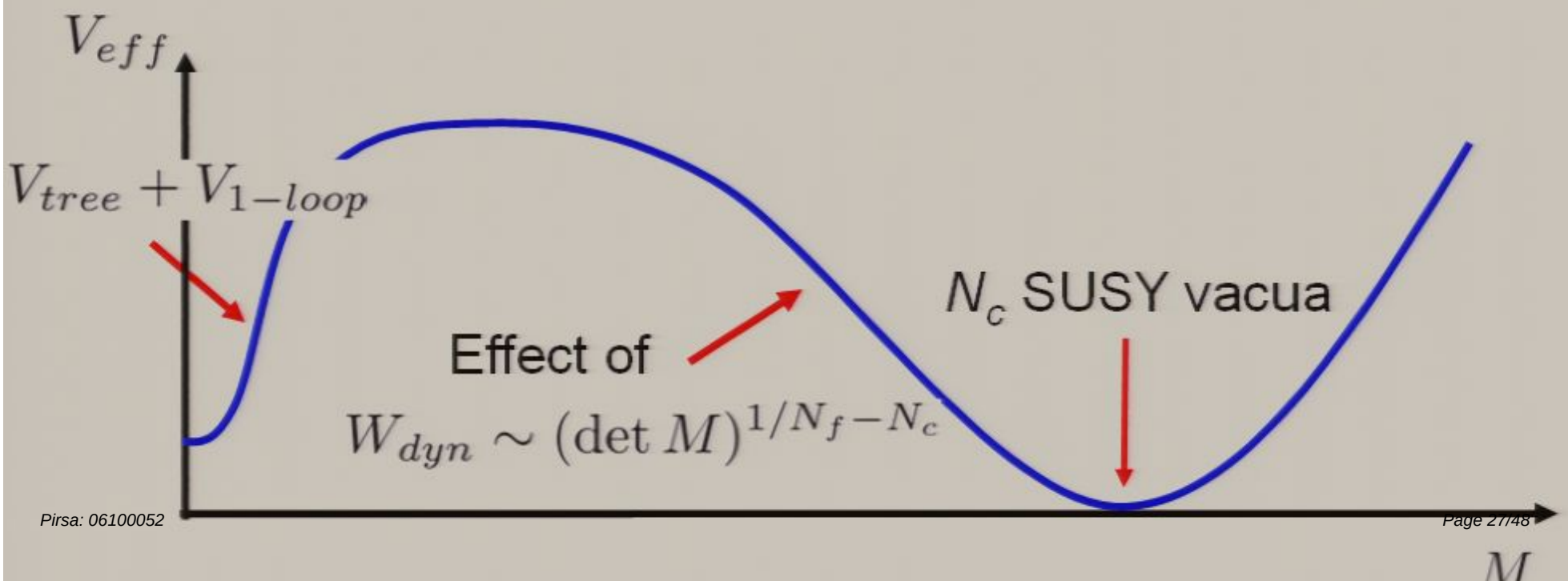
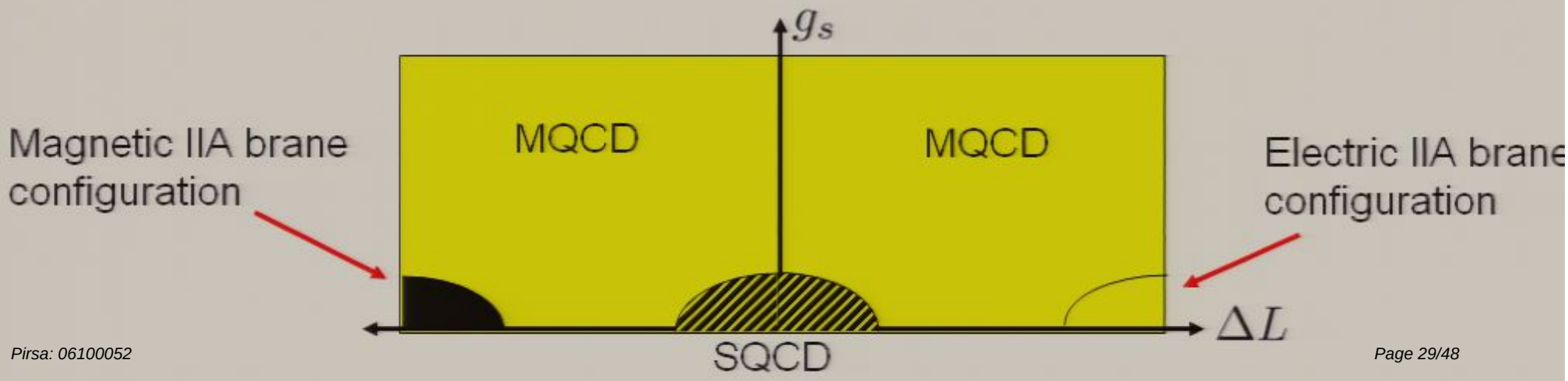


Part II:
Meta-stable Vacua in MQCD?

What is MQCD?

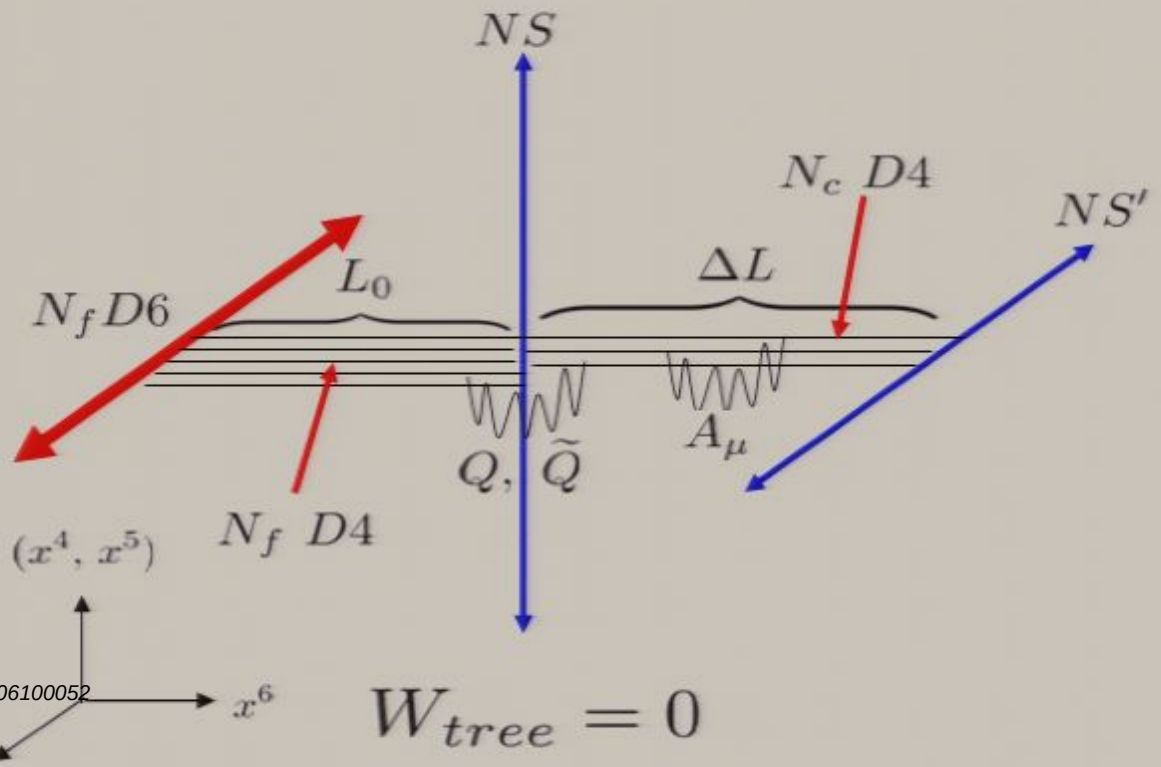
- It is a continuation of N=1 gauge theories into brane configurations of IIA string theory and M-theory.
- Typically there is a length scale ΔL which, along with g_s and ℓ_s, controls whether we are in the (non-overlapping) field theory, IIA, or M-theory regime.



The "Electric" Brane Configuration

The classic example of this is the IIA brane construction of N=1 $SU(N_c)$ SQCD using NS5, D4 and D6 branes:

(Elitzur, Giveon & Kutasov)



$$W_{tree} = 0$$

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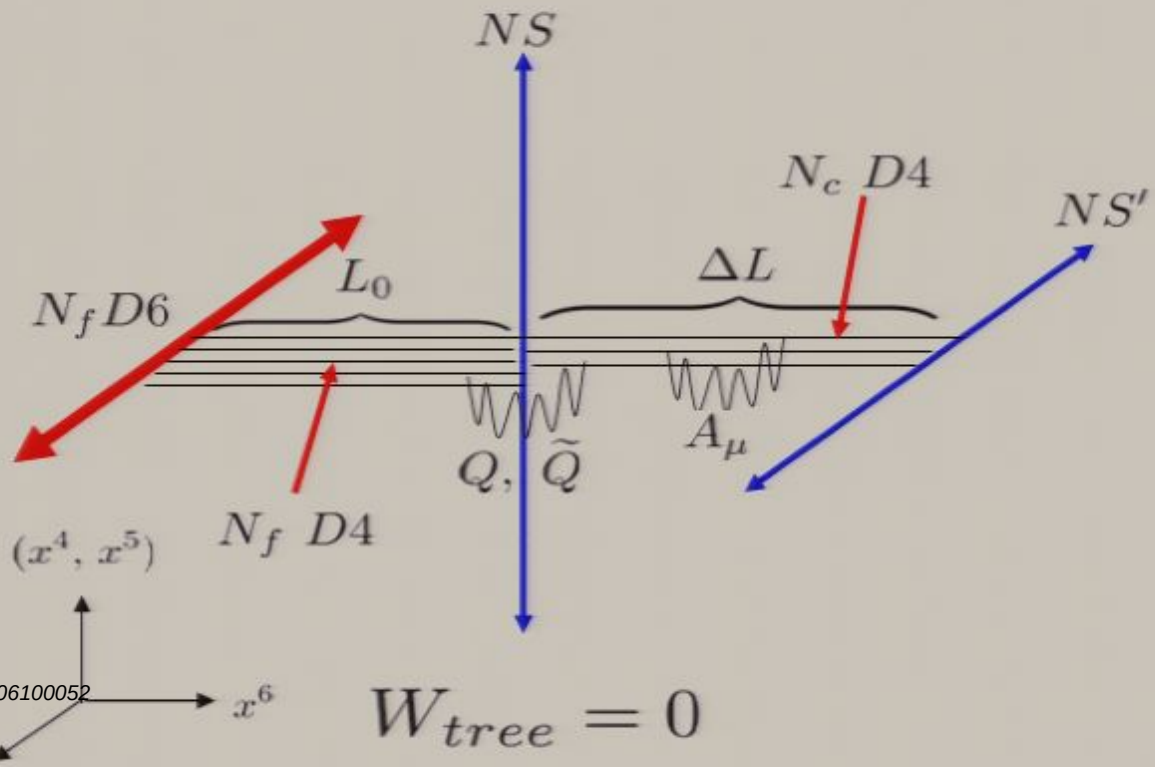


Field theory limit:

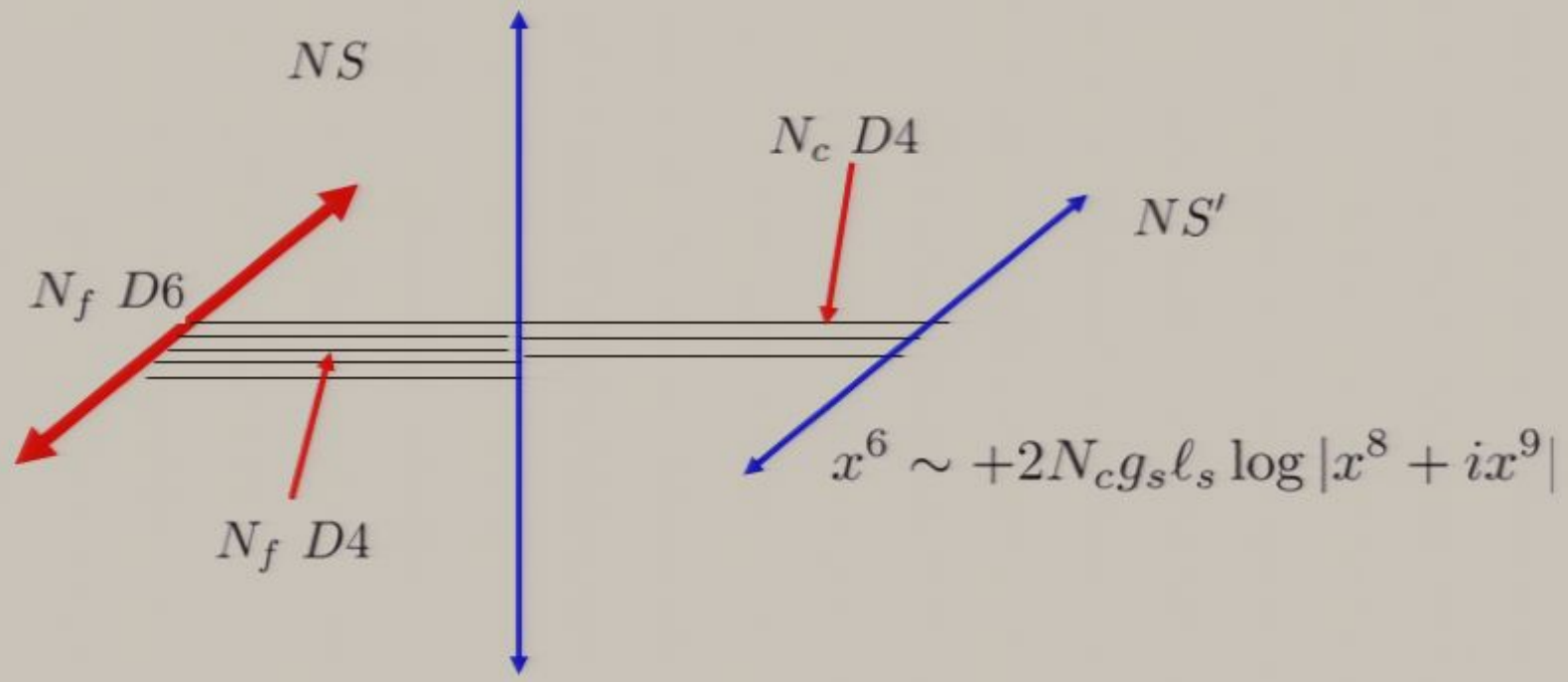
$$g_s \to 0, \quad \ell_s \to 0, \quad \Delta L \to 0$$

$$\frac{1}{g^2_{elec}} = \frac{|\Delta L|}{g_s \ell_s} \text{ fixed}$$

(In this limit, the brane description is not valid.)

Brane Bending

At $g_s \neq 0$, the NS5 branes bend due to their interactions with the D4 and D6 branes.



$$x^6 \sim -(2N_c - N_f) g_s \ell_s \log|x^4 + ix^5|$$

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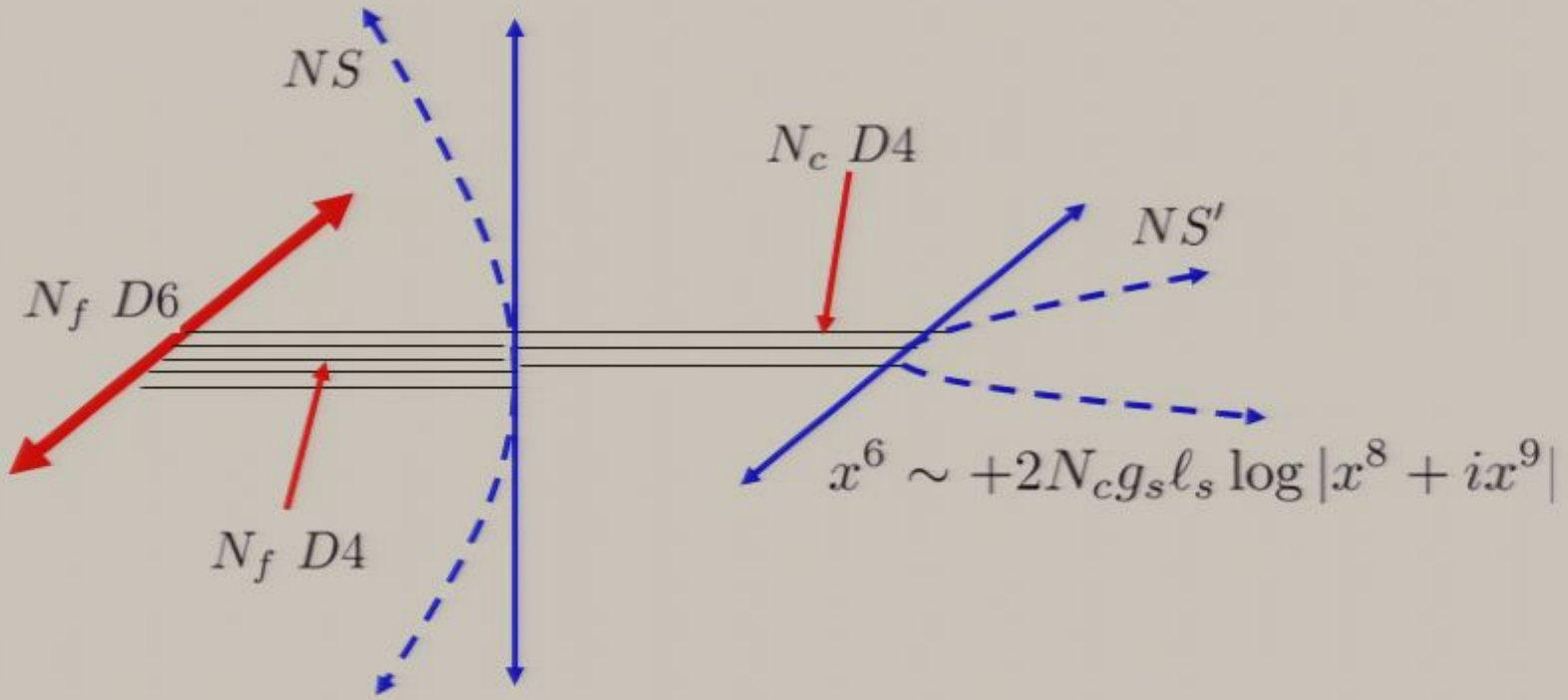
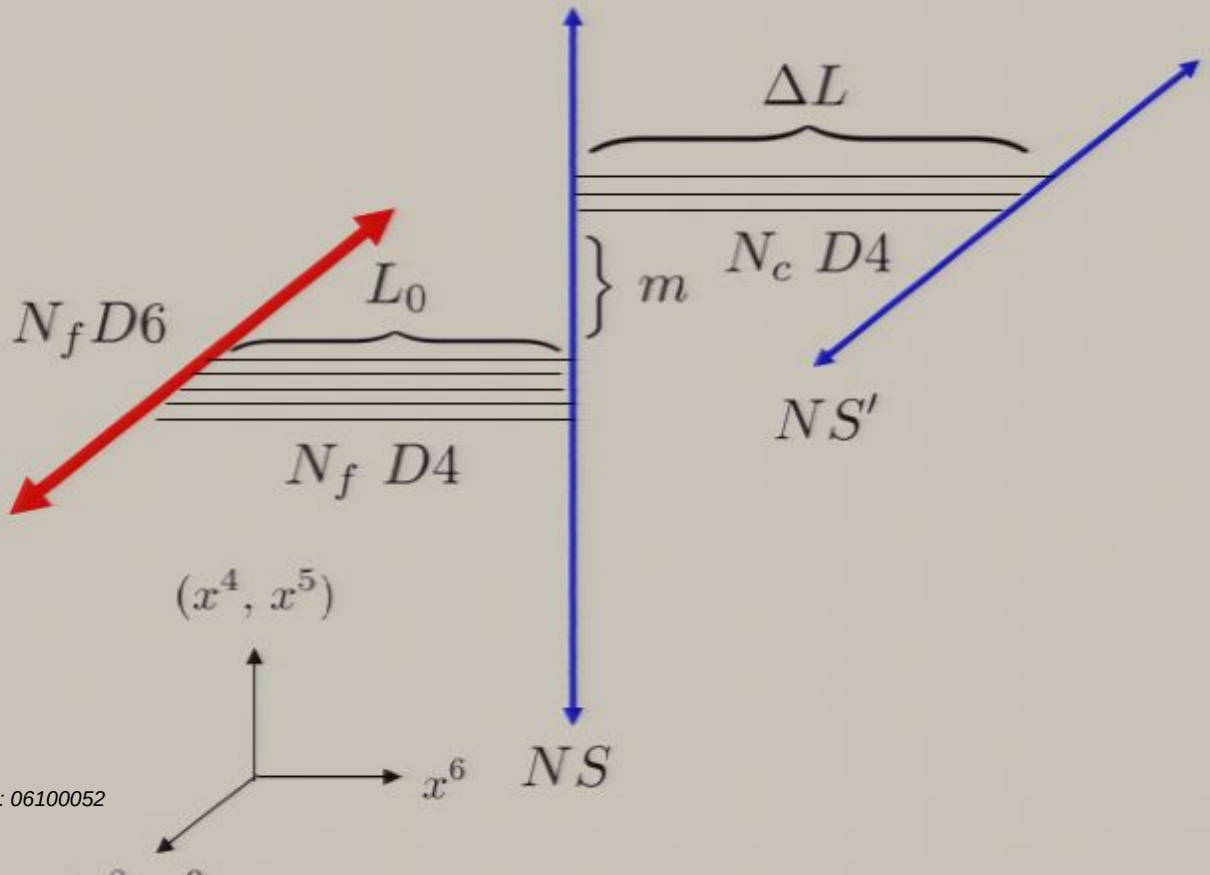


Mass deformation

Giving the electric quarks masses corresponds to moving the D6 branes in the $x^4 + ix^5$ direction. The resulting configuration is still supersymmetric.



$$W_{tree} = m\mathrm{Tr}\, Q\tilde{Q}$$

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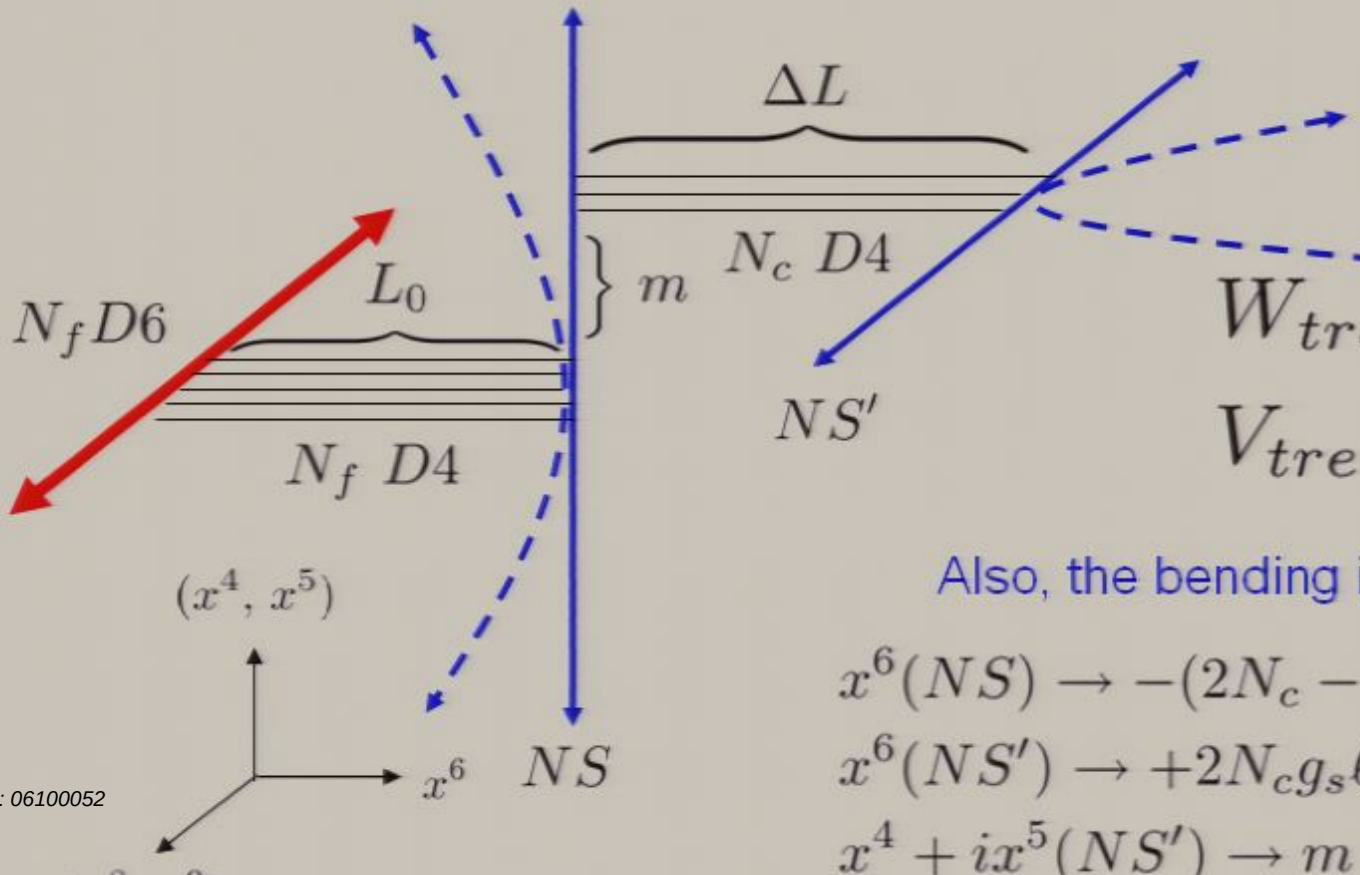

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$$V_{tree} = 0$$

Also, the bending is unchanged:

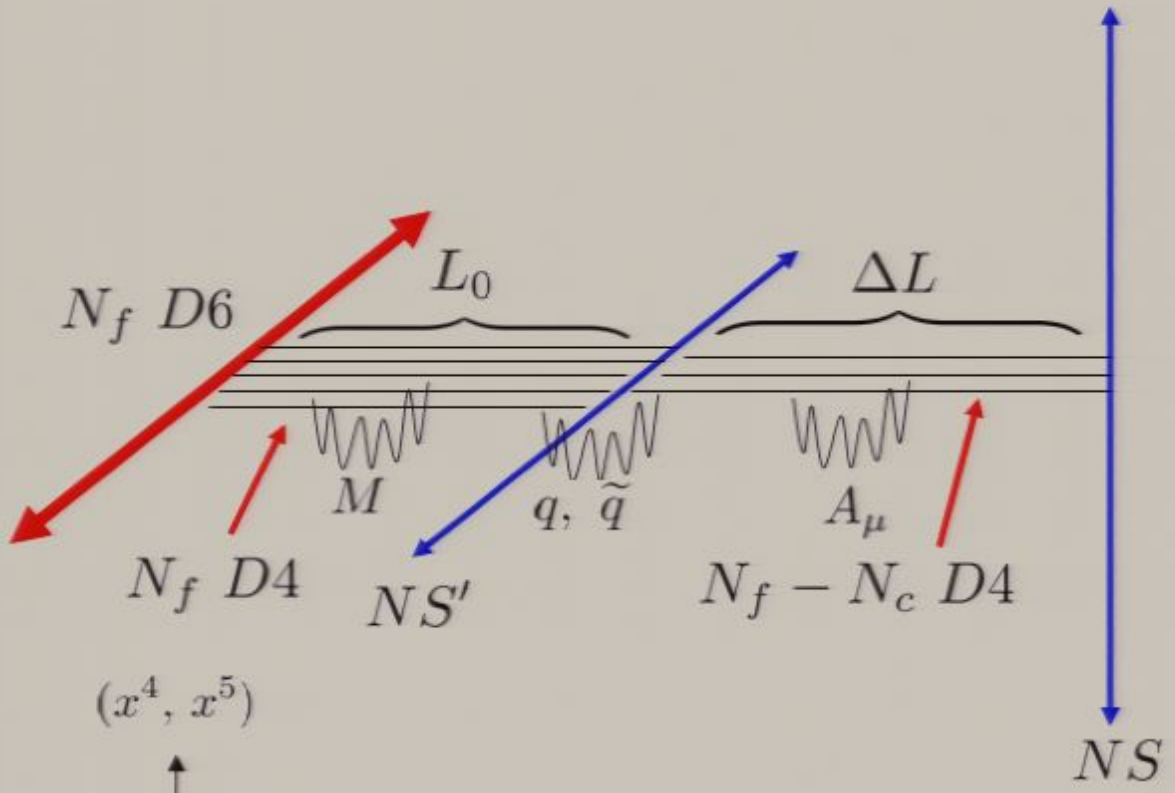
$$x^6(NS) \to -(2N_c - N_f)g_s\ell_s \log|x^4 + ix^5|$$

$$x^6(NS') \to +2N_c g_s \ell_s \log|x^8 + ix^9|$$

$$x^4 + ix^5(NS') \to m$$

Magnetic Brane Configuration

The "magnetic theory" has a similar IIA brane construction…



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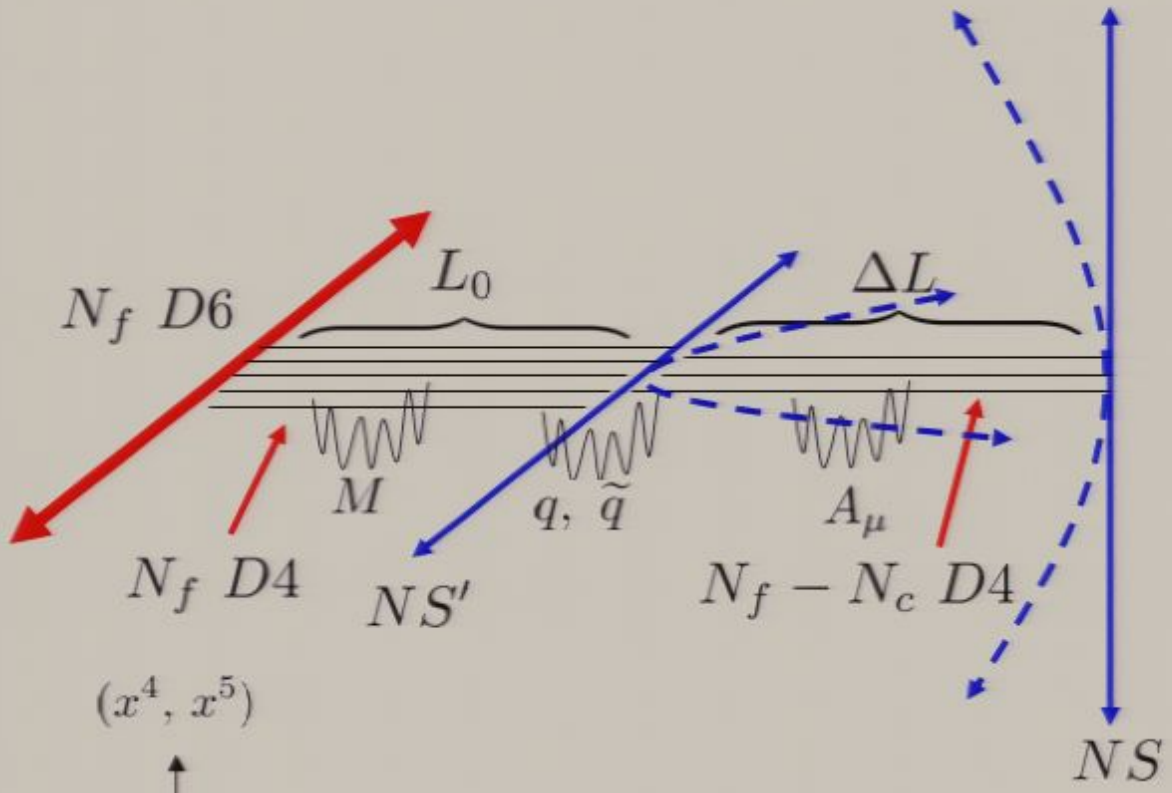

$$\frac{1}{g^2_{mag}} = \frac{|\Delta L|}{g_s \ell_s}$$

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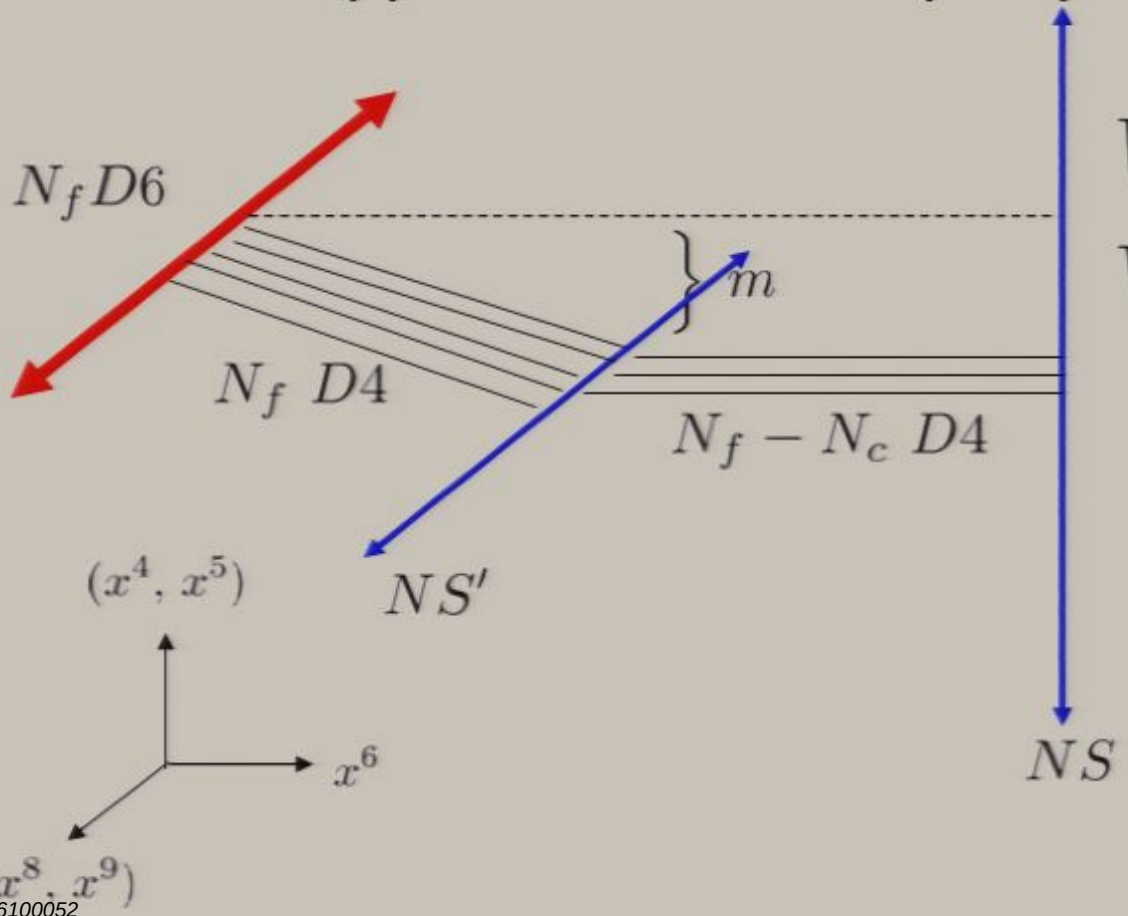
…with the same bending at infinity.

Magnetic mass deformation

In the magnetic brane configuration, the analogue of the mass deformation appears to break supersymmetry.



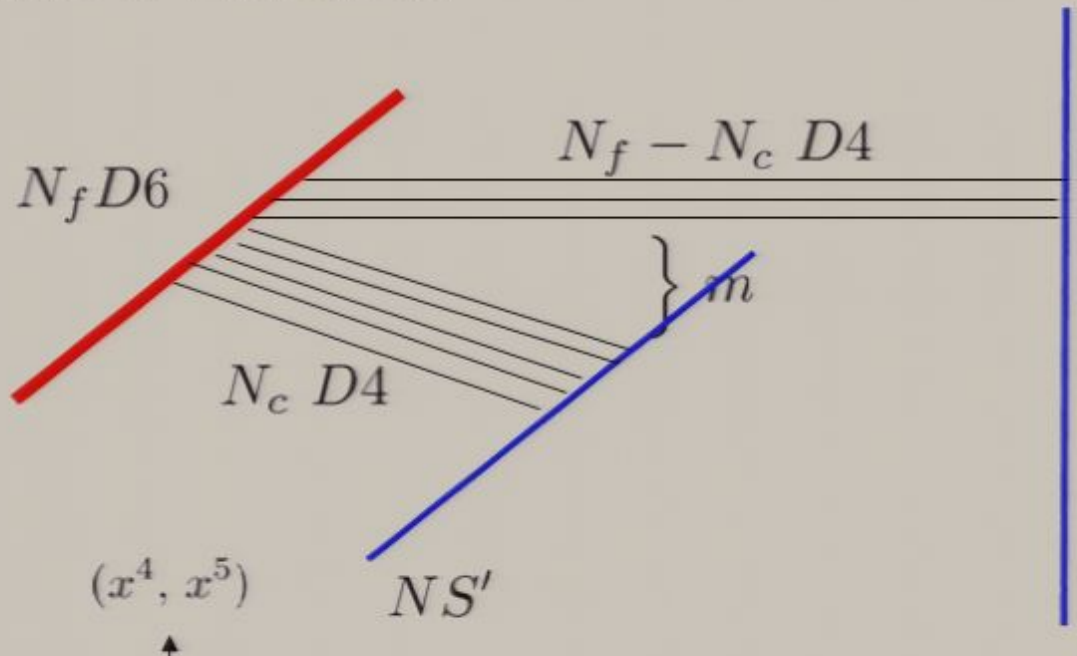
$$W_{tree} = Mq\tilde{q} + mM$$

$$V_{tree} \sim N_f m^2$$

This configuration corresponds to origin of field space in the magnetic theory.

Mass deformation (cont'd)

The system can lower its energy by snapping together $N_f - N_c$ of the D4 branes.



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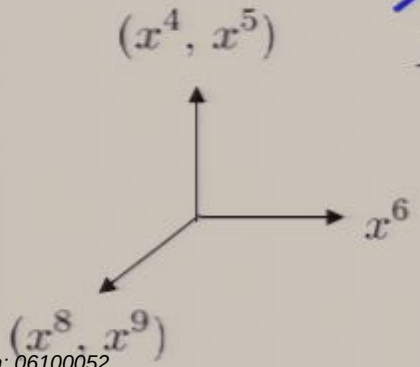

This provides a simple geometric realization of the SUSY-breaking vacuum of the magnetic theory. (Notice, however, that the NS' bending is different now...)

Holomorphic M5 branes

But is this the IIA lift of the meta-stable vacuum of SQCD?

For that, we need to understand the $m \neq 0$ SUSY magnetic brane configuration. It exists due to non-perturbative effects in M-theory.

It can be thought of as a smooth M5 brane wrapping the holomorphic curve (Hori et al., Brandhuber et al.)

$$w = z, \qquad v = m\frac{z+z_0}{z}, \qquad y = \frac{(z+z_0)^{N_f}}{z^{N_f-N_c}}$$

where $z_0^{N_c} = m^{N_f-N_c}\Lambda^{3N_c-N_f}$ and

$$w = x^8 + ix^9, \qquad v = x^4 + ix^5, \qquad y = e^{(x^6 - L_0 + ix^{10})/2R}\left(\frac{r+x^6}{R}\right)^{N_f/2}$$

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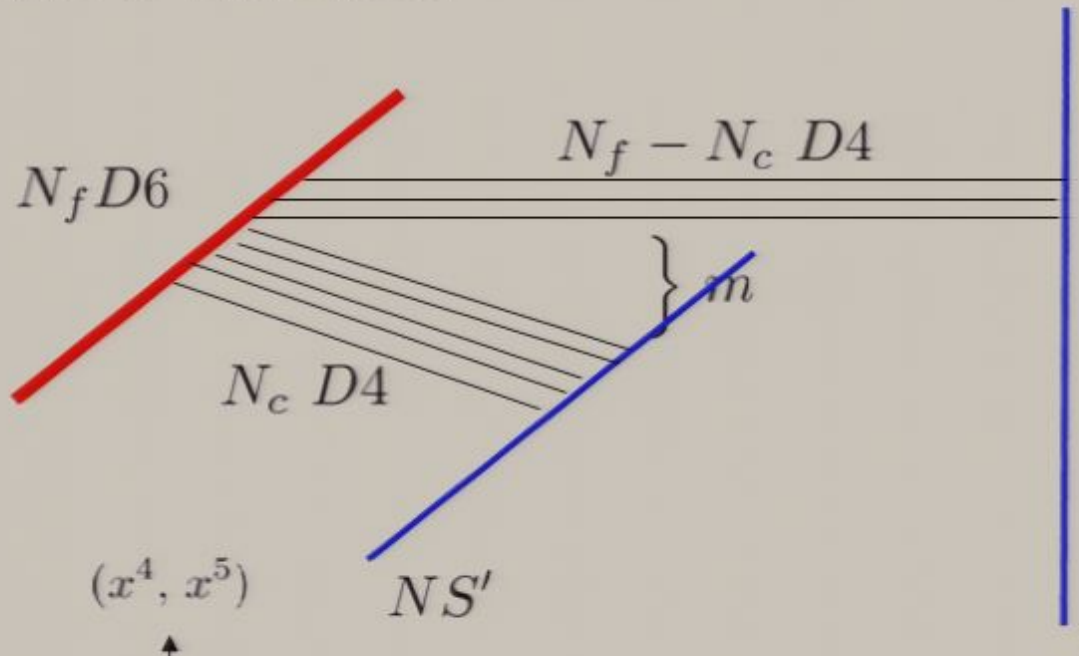

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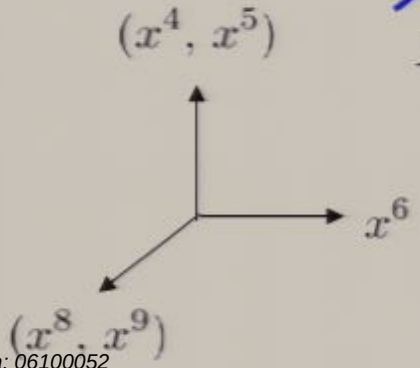

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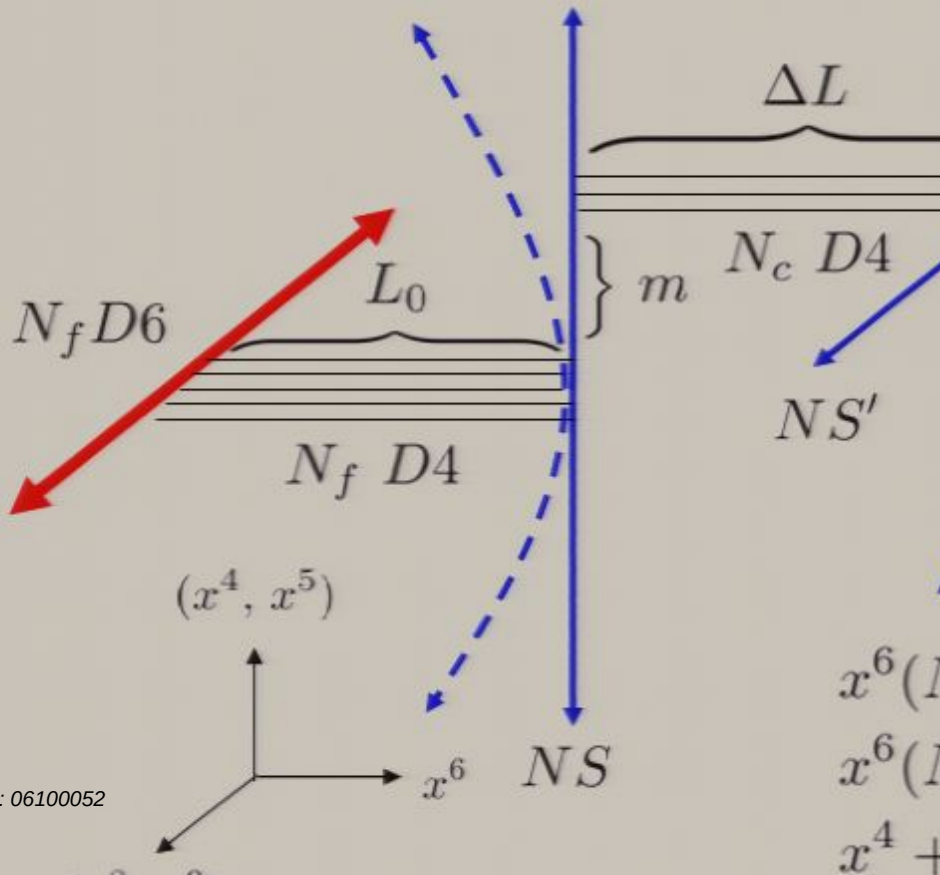

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where $z_0^{N_c} = m^{N_f-N_c}\Lambda^{3N_c-N_f}$ and

$$w = x^8 + ix^9, \qquad v = x^4 + ix^5, \qquad y = e^{(x^6-L_0+ix^{10})/2R}\left(\frac{r+x^6}{R}\right)^{N_f/2}$$

Meta-stable non-holomorphic curve?

If there is an MQCD lift of the SQCD meta-stable state, it must be a non-holomorphic, minimal-area surface in $(\mathrm{Taub}-\mathrm{NUT})\times\mathbf{R}^2$ with the same behavior at infinity as the SUSY curve.

In particular, it must become the NS' brane as $|x^8+ix^9|\to\infty$, with the bending we saw above:

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In particular, it must become the NS' brane as $|x^8+ix^9|\to\infty$, with the bending we saw above:

$$x^6 \to +2N_c g_s \ell_s \log|x^8+ix^9|$$
$$x^4+ix^5 \to m$$

No solution

In fact, with a plausible ansatz and a lot of calculation, one can show that there is no solution to the equations of motion that has this behavior at infinity.

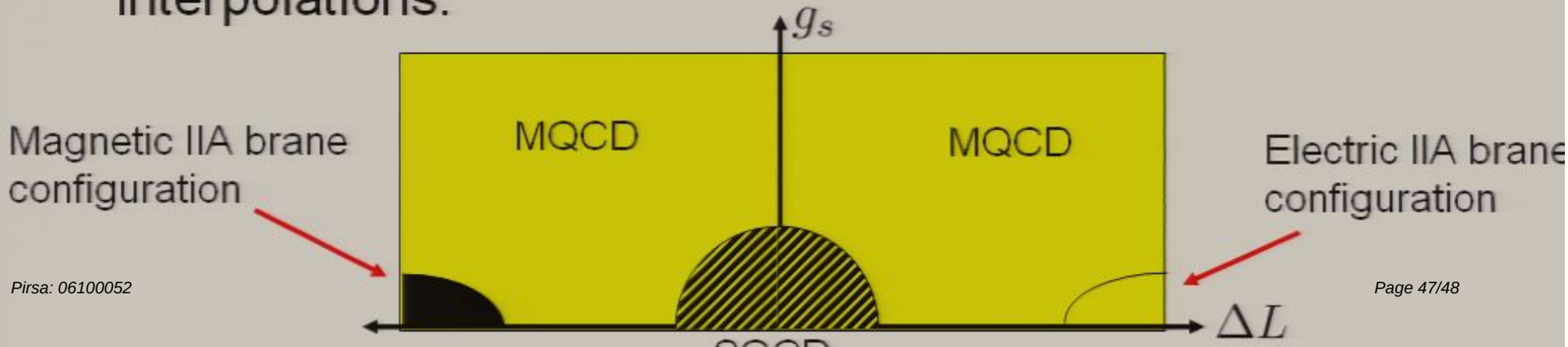
So we conclude that the meta-stable state of SQCD does not have a lift to MQCD!

(Indeed, this should have been expected all along from the bending of the non-SUSY IIA configuration.)

Interpretation

- Evidently, SQCD and MQCD are not in the same "universality class," as is widely believed. The former has meta-stable vacua which the latter does not.

- There is no contradiction, because MQCD and SQCD are related by interpolations in various parameters, and meta-stable states are generally not robust under such interpolations.



Open questions

- Do these conclusions still hold in the T-dual description, where the brane configuration is replaced with geometry? If so, what does this imply about "geometric engineering"?
- Is there a deeper reason why the meta-stable state of SQCD has the wrong brane bending? (cf. the meta-stable state of the KS geometry, which does not suffer from this problem…)
- Can we find any example where we have a controlled description of a meta-stable state in both field theory and string theory?