Title: IR Free or Interacting? A Proposed Diagnostic

Date: Oct 14, 2006 09:45 AM

URL: http://pirsa.org/06100051

Abstract:

John's physics papers

- Susy gauge theories: 9605232, 9611197, **9802092**
- branes, strings, and connections with gauge theories: 9704043, 9705068, 9709228, 9711001, 9803140, 9809067, 0012068, 0101115, 0208191
- Giant gravitons and stringy quantum Hall fluid: 0010105, 0107178
- Brane inflation: 0301138

RG flows to the IR; where do they end?

- Some asymptotically free theories flow to IR free theories. E.g. QCD with just a few light flavors, flows to IR free pions.

- Others RG flow to interacting RG fixed points, e.g. QCD with many massless flavors (just barely asymptotically free). E.g. Banks-Zaks.

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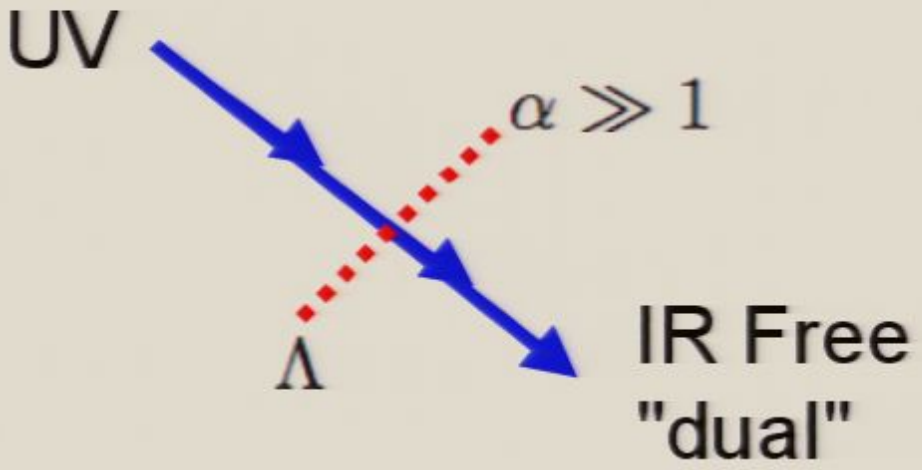
General intuition

Short RG flows:

$\beta(\alpha^*) = 0, \ \alpha^* < 1$

——— vs ———

Long RG flows:



Make this precise? = A longstanding goal.

One tool: 't Hooft anomaly matching

If there are unbroken chiral symmetries, 't Hooft anomaly matching constrains the IR spectrum.

A non-trivial matching, with an IR free spectrum, can be viewed as some evidence that the IR free scenario is correct.

Example: N=1 susy SU(2) with Q in **4**

(KI, N. Seiberg, S. Shenker, '94)

Scenario 1: IR free, with spectrum $X = Q^4$, satisfies very non-trivial 't Hooft matching. If correct, get dynamical susy breaking (by "confinement"), upon adding $W_{tree} = \lambda X$.

Scenario 2: Interacting SCFT at origin. Anomaly matching is a fluke. No DSB. $W_{tree} = \lambda X$ is irrelevant.

Which is correct? Still not known!

Known examples of highly non-trivial, but still misleading, anomaly matching

(KI, John Brodie, P. Cho '98)

SO(N) with matter S in two-index symmetric tensor. Form $O_n = \mathrm{Tr}(S^n)$, $n = 1..N$. These saturate the Tr R and Tr R^3 't Hooft anomalies. Highly non-trivial, for all N! (Also a Z_{2N+4} discrete anomaly matching satisfied.)

Suggests the theory is IR free. But, we show that it must instead be an interacting CFT.

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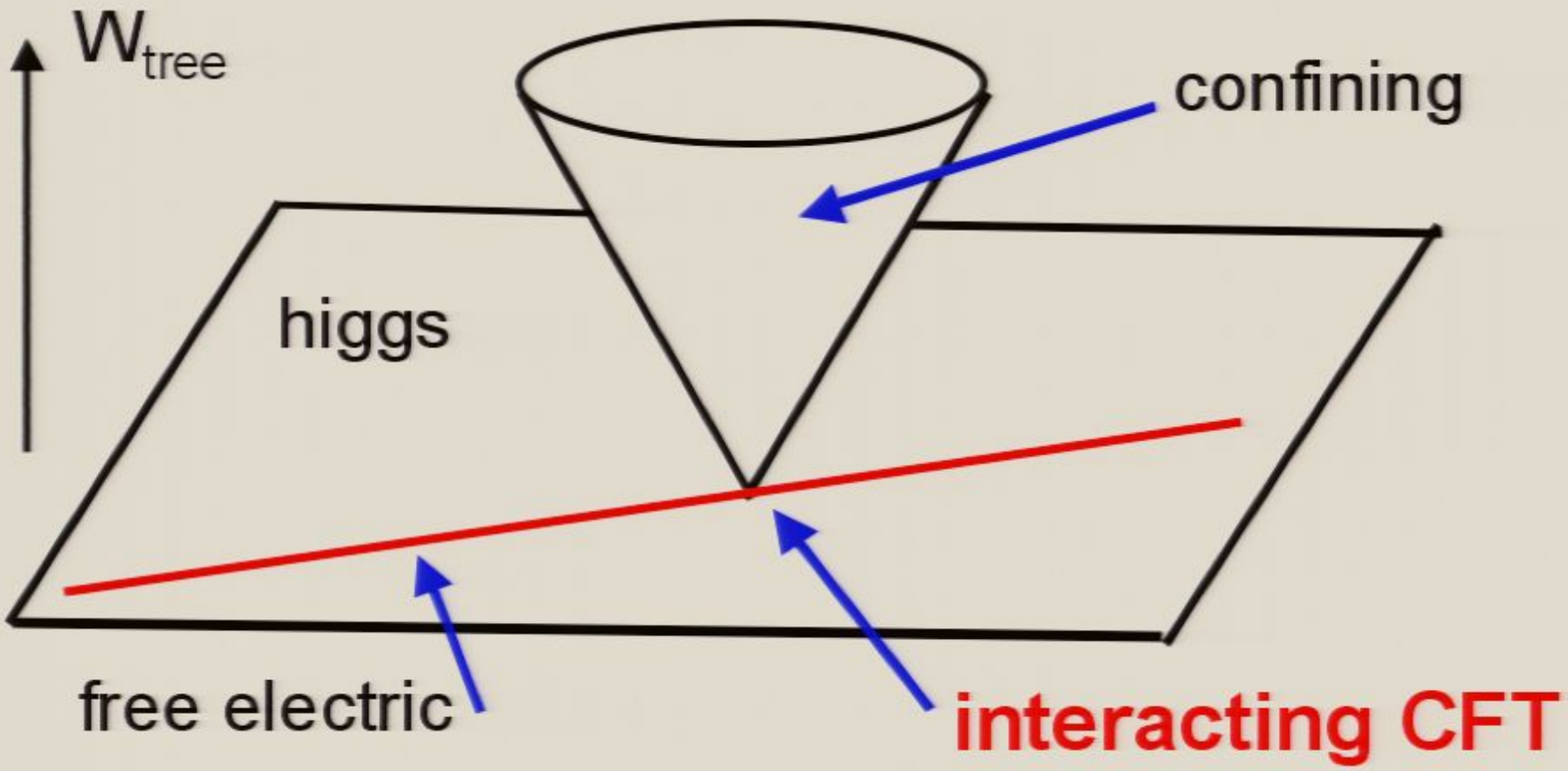

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Another IR diagnostic: a-function

$$a_{Cardy} \sim \int_{S^4} \langle T^\mu_\mu \rangle$$

Cardy's conjecture: $a_{IR} < a_{UV}$

$a_{IR} \geq 0$

Conjectured 4d analog of 2d Zamolodchikov's thm.

If true, could rule out incorrect IR scenarios. Many non-trivial checks of the conjecture in susy theories.

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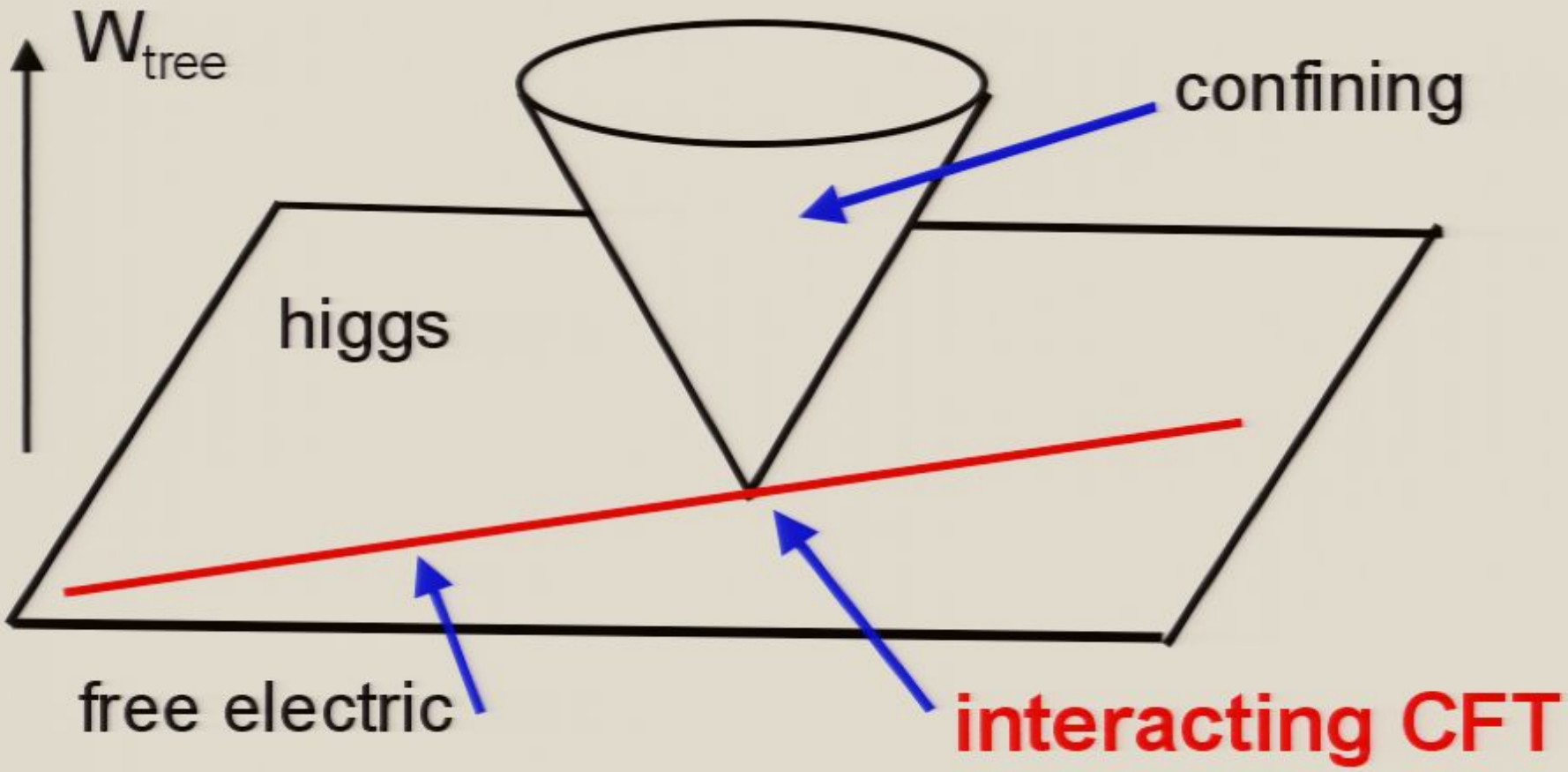

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I make another conjecture:

"Given two plausible IR scenarios, the correct one is that with larger a_{IR} ."

Motivation: $\Delta a \equiv a_{UV} - a_{IR}$ ~ RG flow length should be minimized.

A stronger conjecture: operators can only become IR free if that leads to a larger value for a_{IR} .

The conjectures work in every known example of susy gauge theories that I have checked.

Susy theories and a-maximization

$$a_{Cardy} = \frac{3}{32}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)$$

Anselmi, Freedman, Grisaru, Johansen '97

The correct R-symmetry is that which maximizes this function. Intriligator and Wecht '03

Almost proves Cardy's conjecture (for susy thys), but possibility of various IR free ops (more gen'ly, accidental symms) prevents a complete proof.

Unitarity: $\Delta(X) \geq 1$, with equality iff it's free.

Chiral X: $\Delta(X) = 3R(X)/2$ so $R(X) \geq 2/3$

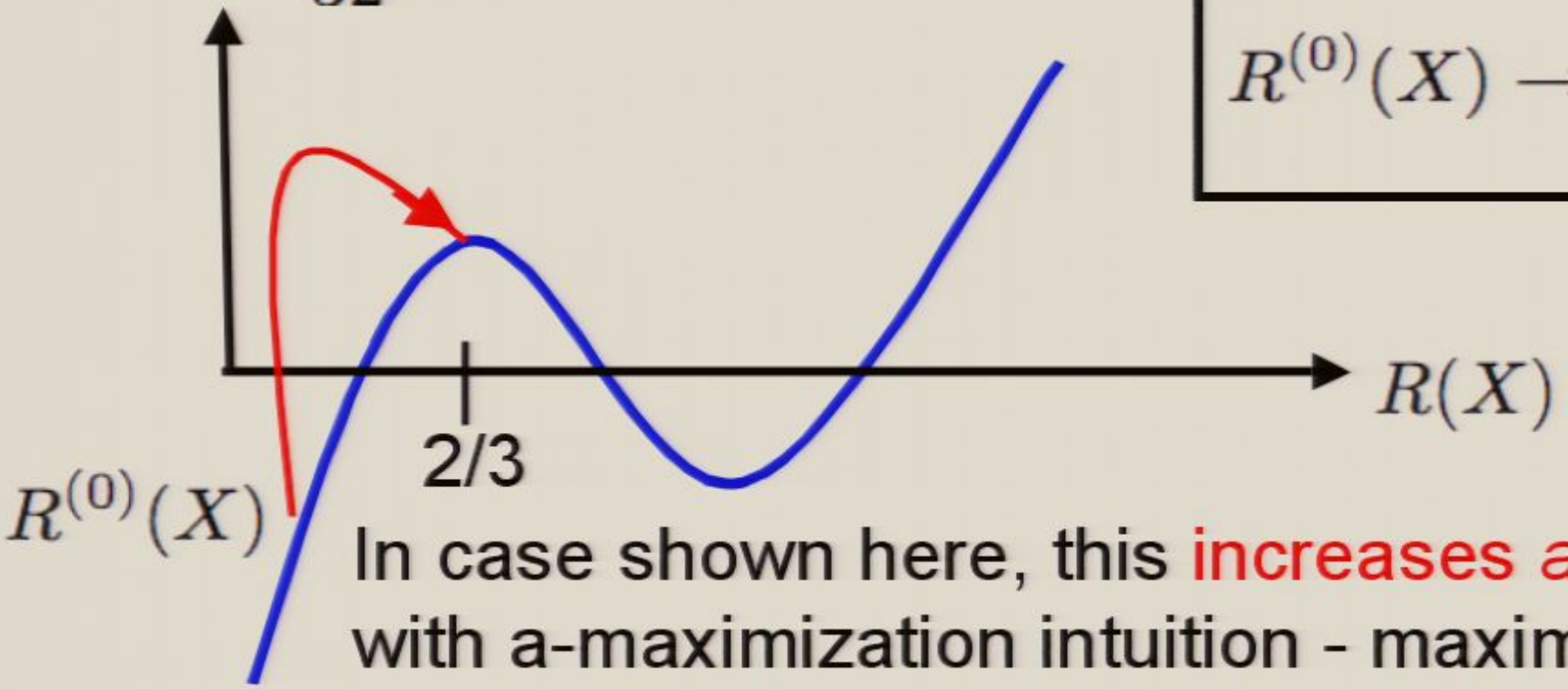
Effect of IR free chiral operator X on *a*

Anselmi, Erlich, Freedman, Johansen;
Kutasov, Parnachev, Sahakyan

$$a(R) = \frac{3}{32}(3(R-1)^3 - (R-1))$$

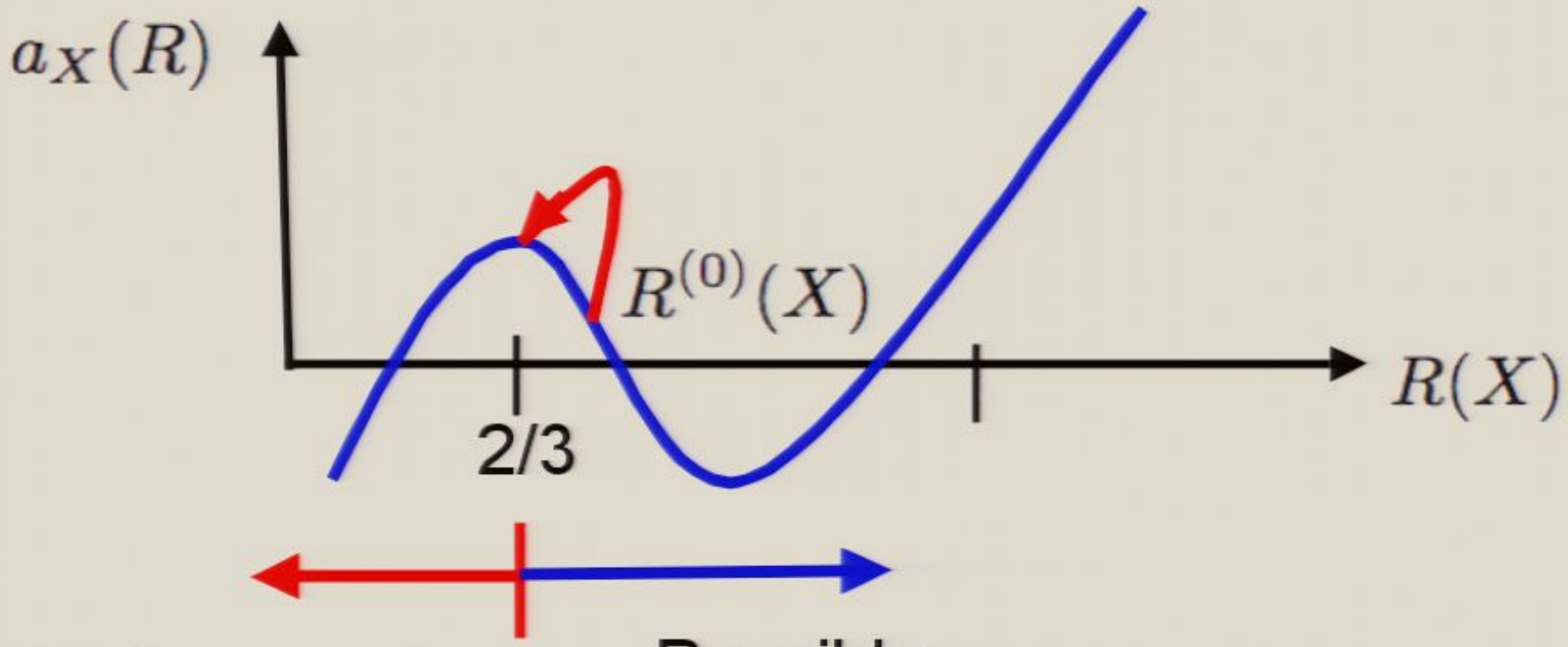
Replace *R(X)*:

$$R^{(0)}(X) \to 2/3$$



In case shown here, this increases *a*. Fits with a-maximization intuition - maximizing over a bigger space of possible R symms.

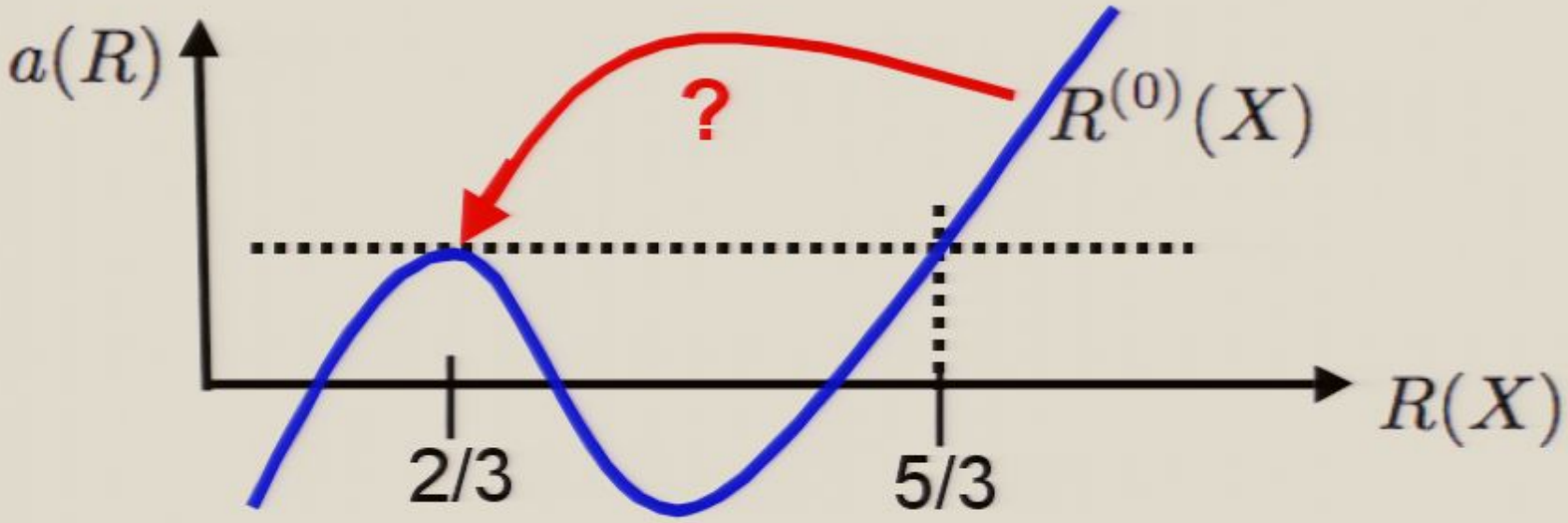
Chiral operators above the unitarity bound **can** also become IR free



Unitarity: **must** be IR free

Possibly IR free

IR free operator X with large $R^{(0)}$?



If so, accidental symmetry reduces a. Possible?

My conjecture: **No.** Only X with $R^{(0)}(X) < 5/3$ can become IR free. The weaker conjecture is that the full theory is IR free only if **total** value for a satisfies $a_{IR}^{free} > a_{IR}^{interacting}$

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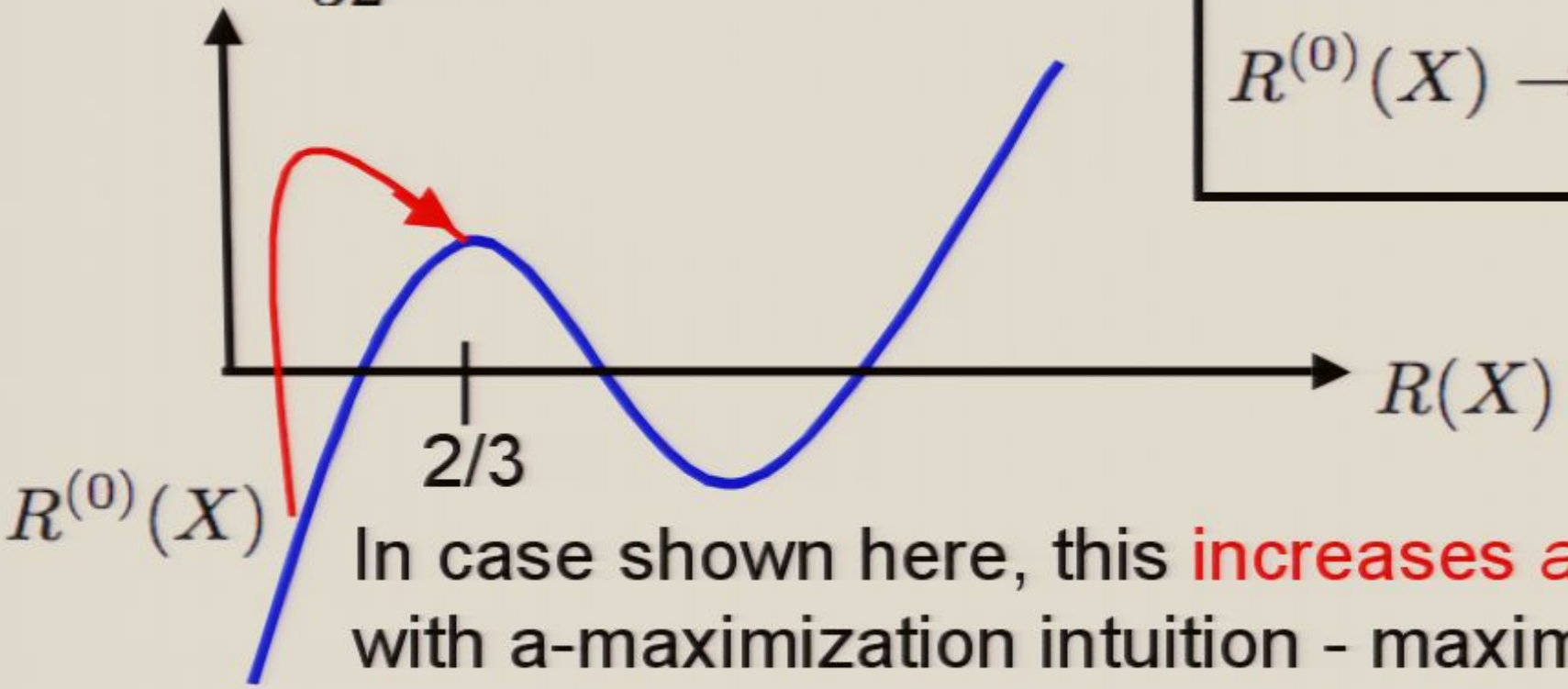

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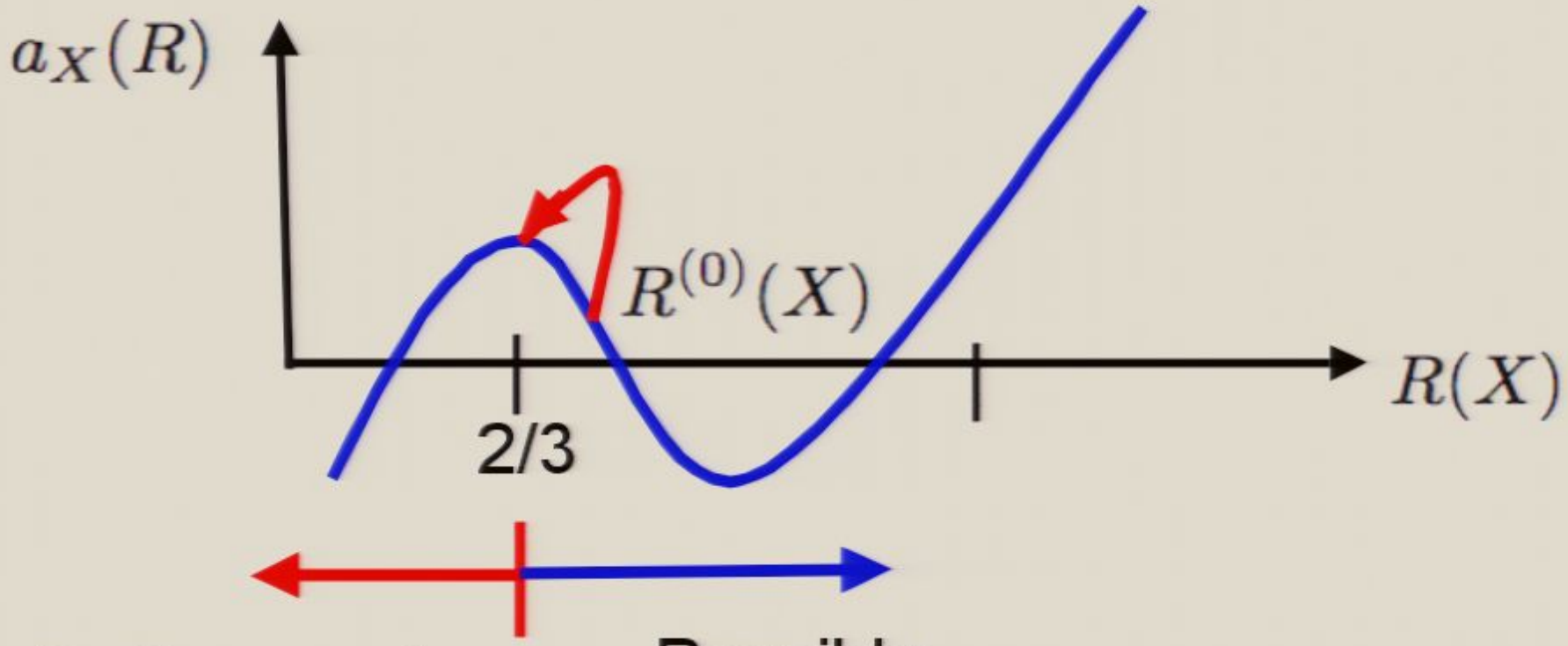

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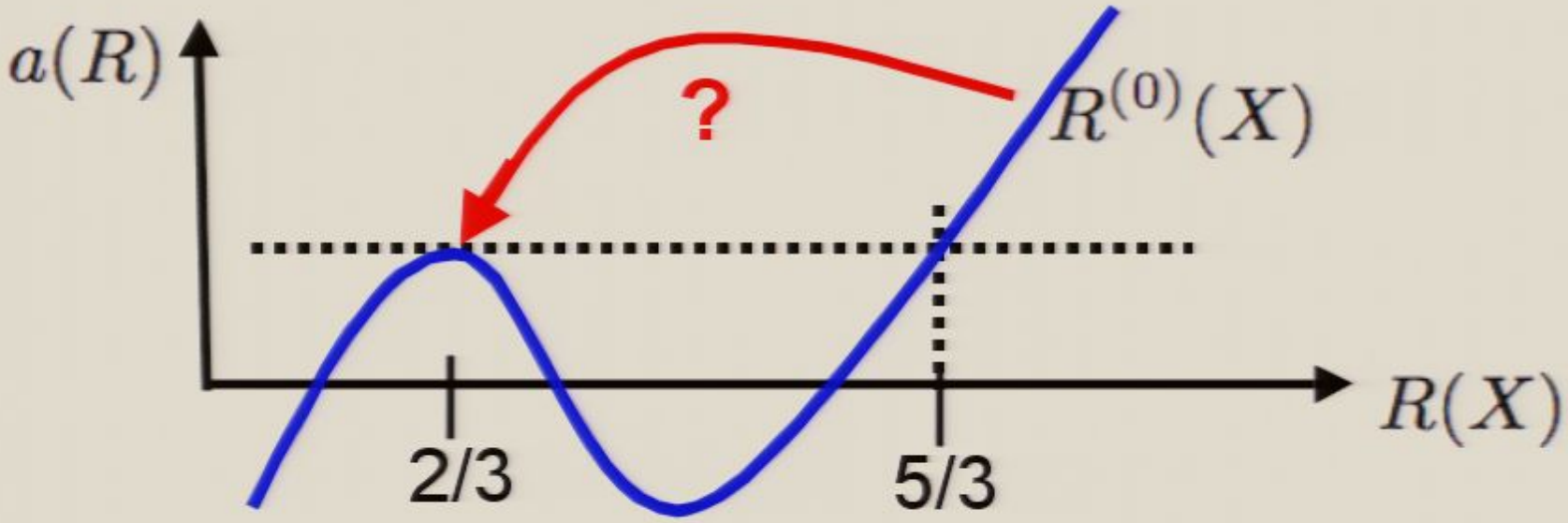

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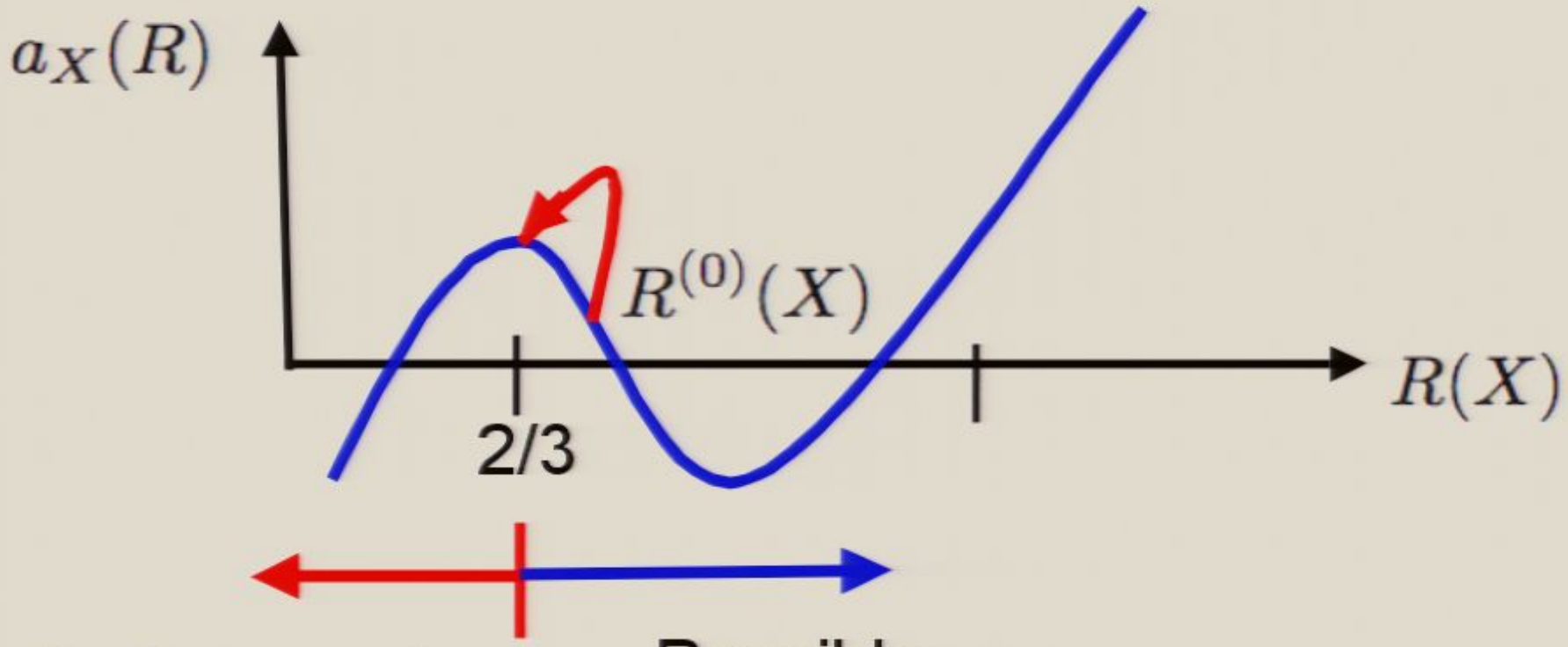

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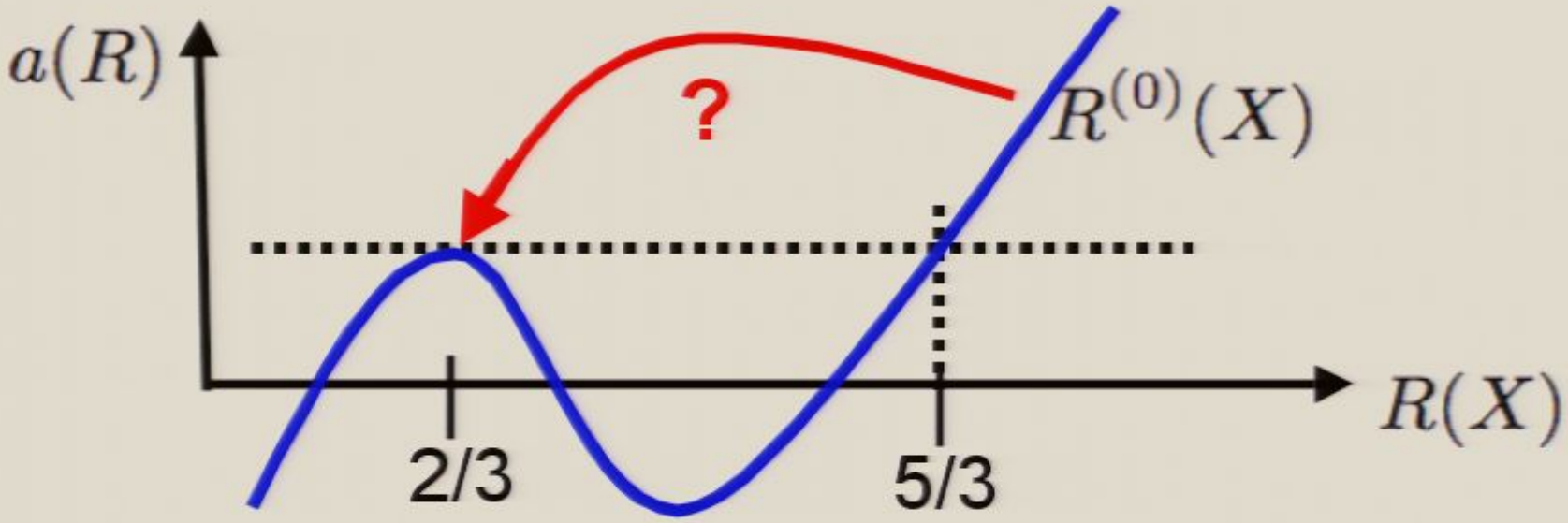

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Example SQCD

$U(1)_R$ determined to be: $R(Q) = (N_f - N_c)/N_f$

$R^{(0)}(M) = 2(N_f - N_c)/N_f$ ← Unitarity: M=IR free if $N_f \leq 3N_c/2$

$R^{(0)}(B) = N_c(N_f - N_c)/N_f$ ← Is B IR free or interacting?

$N_f = N_c + 1$ Seiberg: "both M and B are free."

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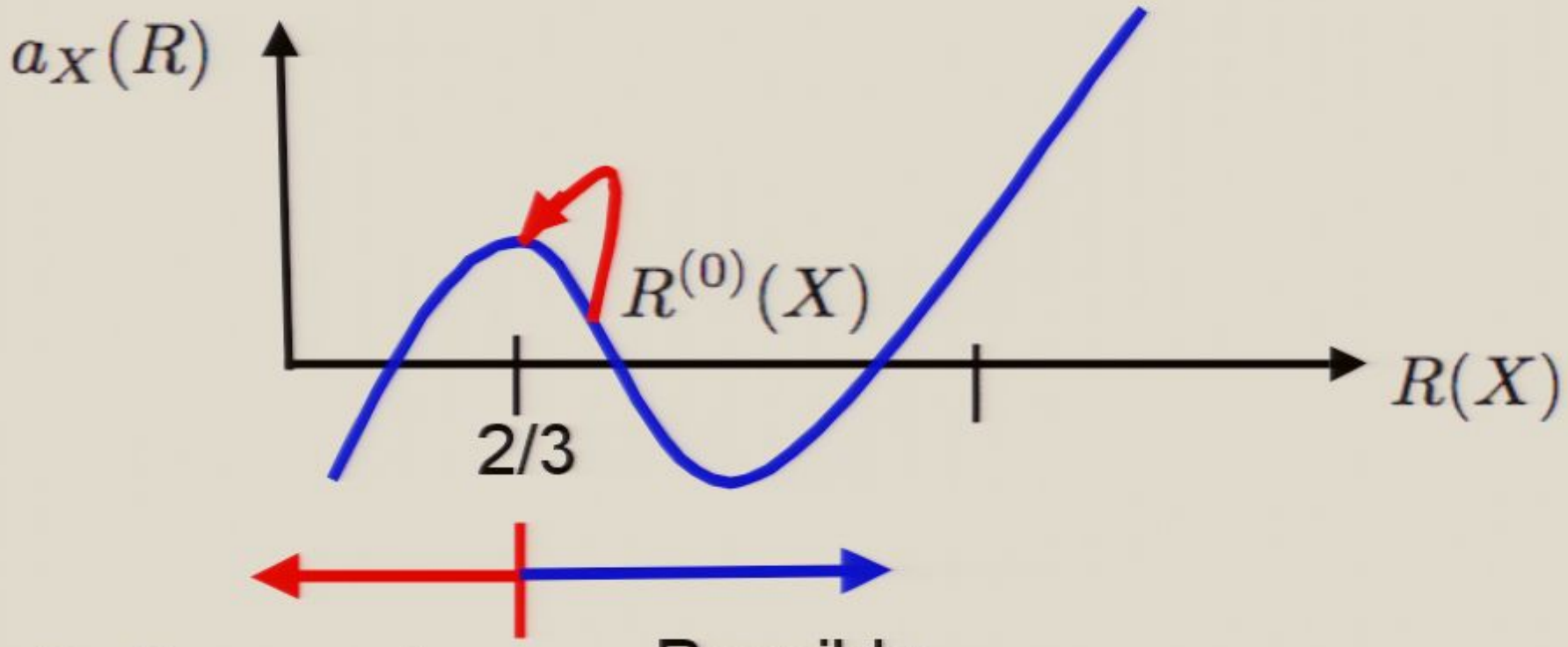

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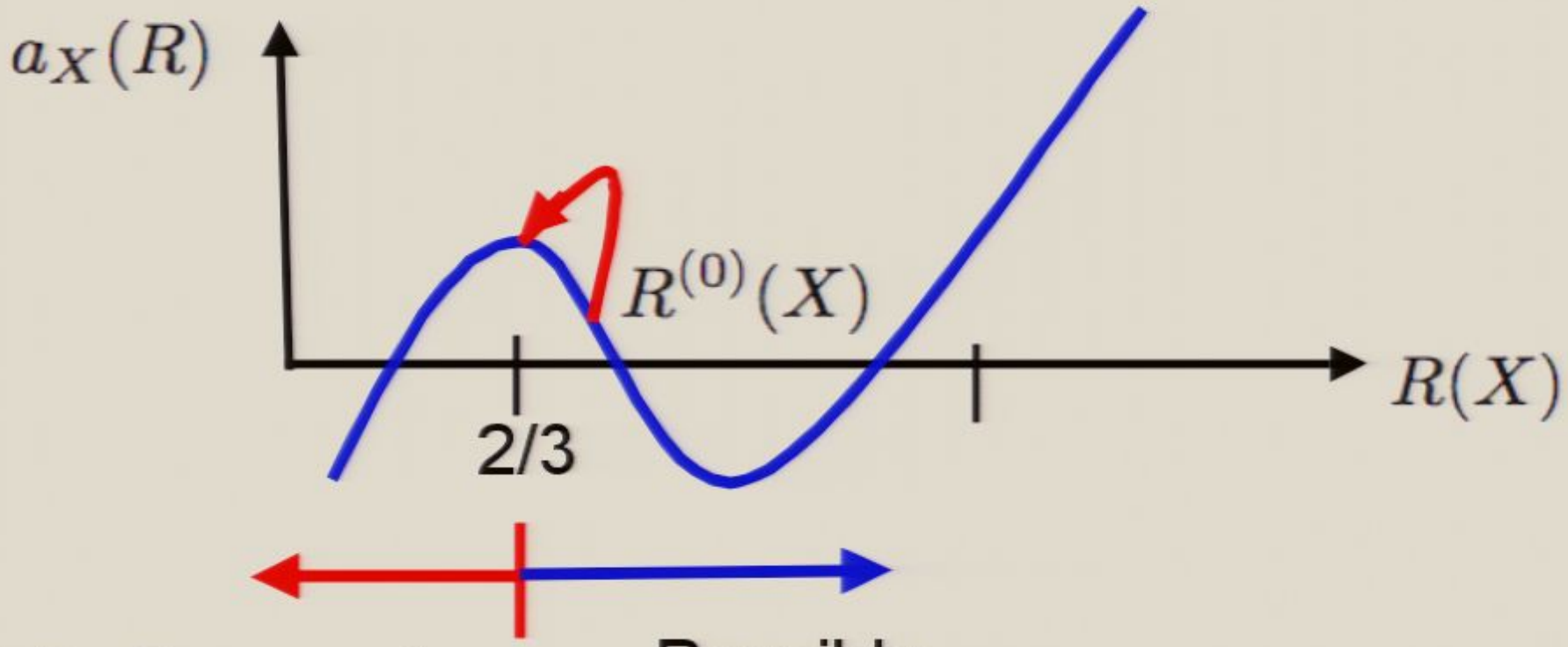

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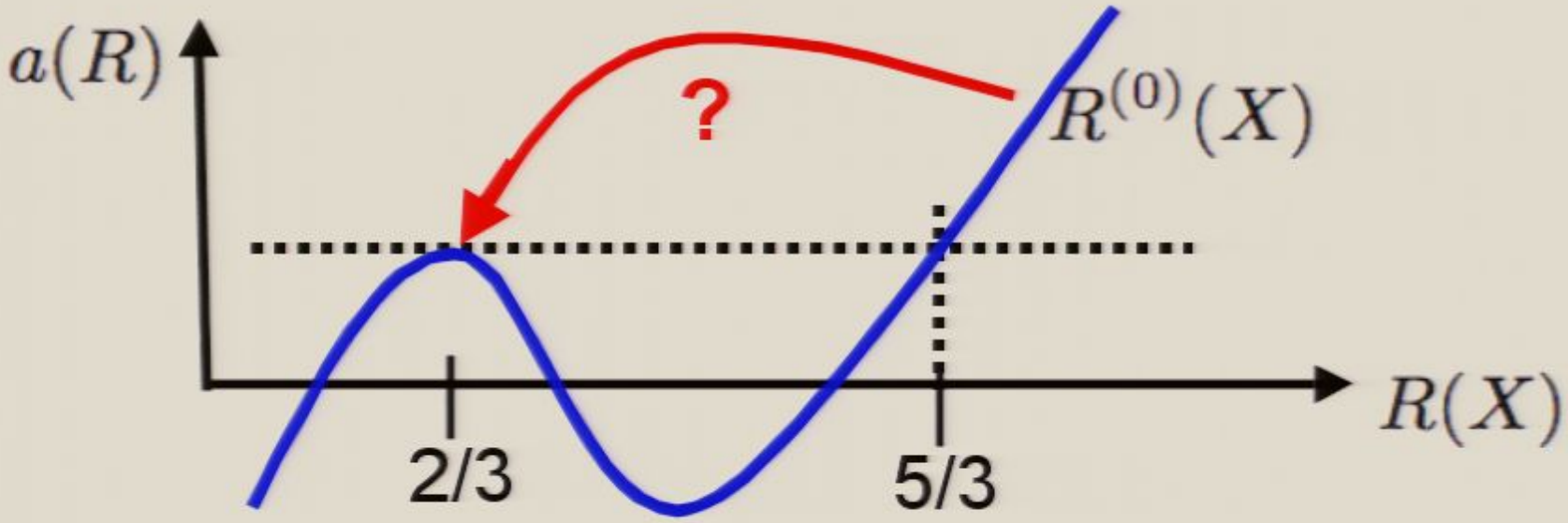

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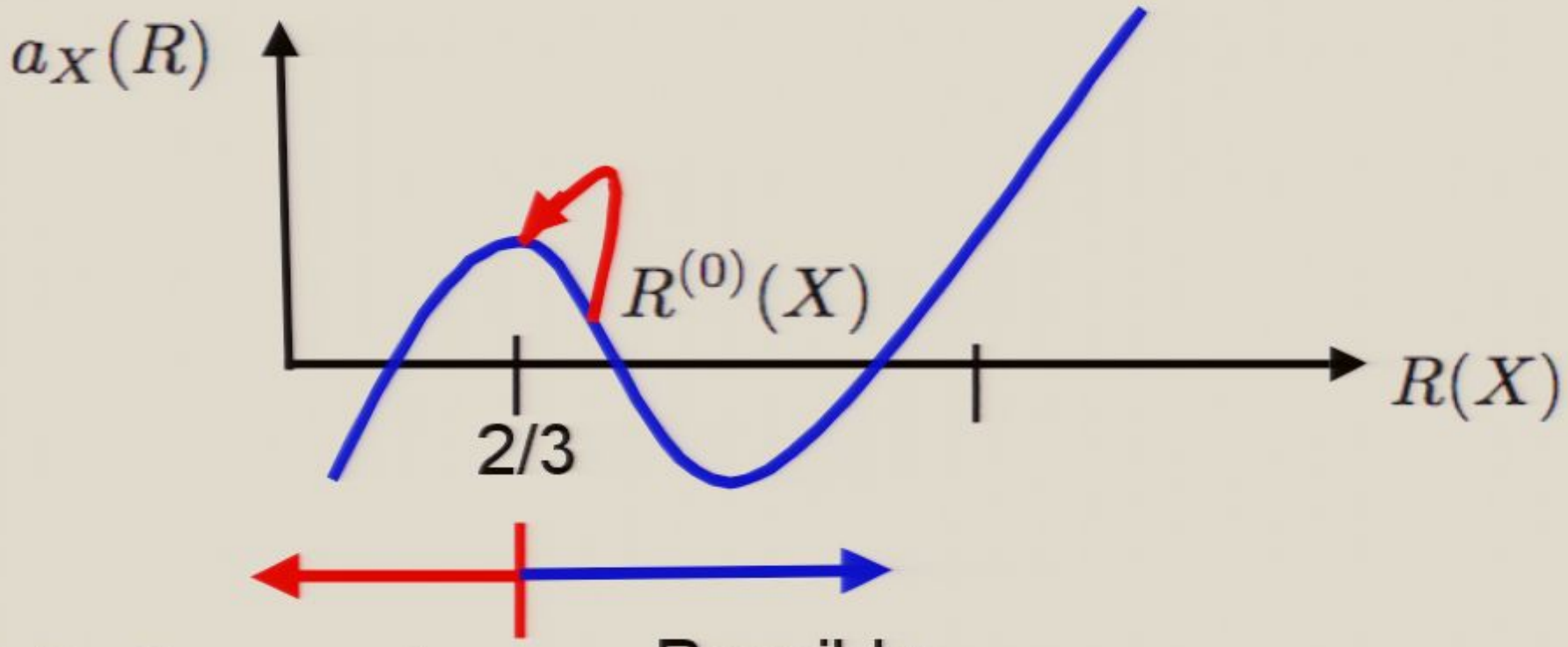

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Follows from $R^{(0)}(q) = N_c/N_f < 5/3$

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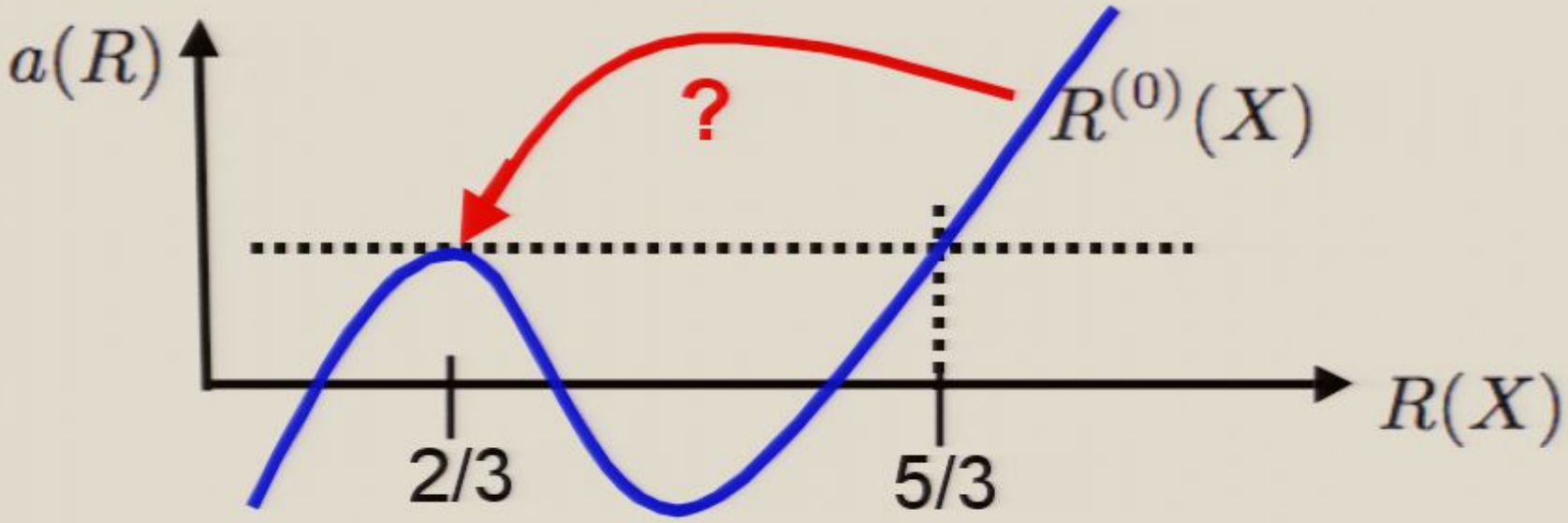

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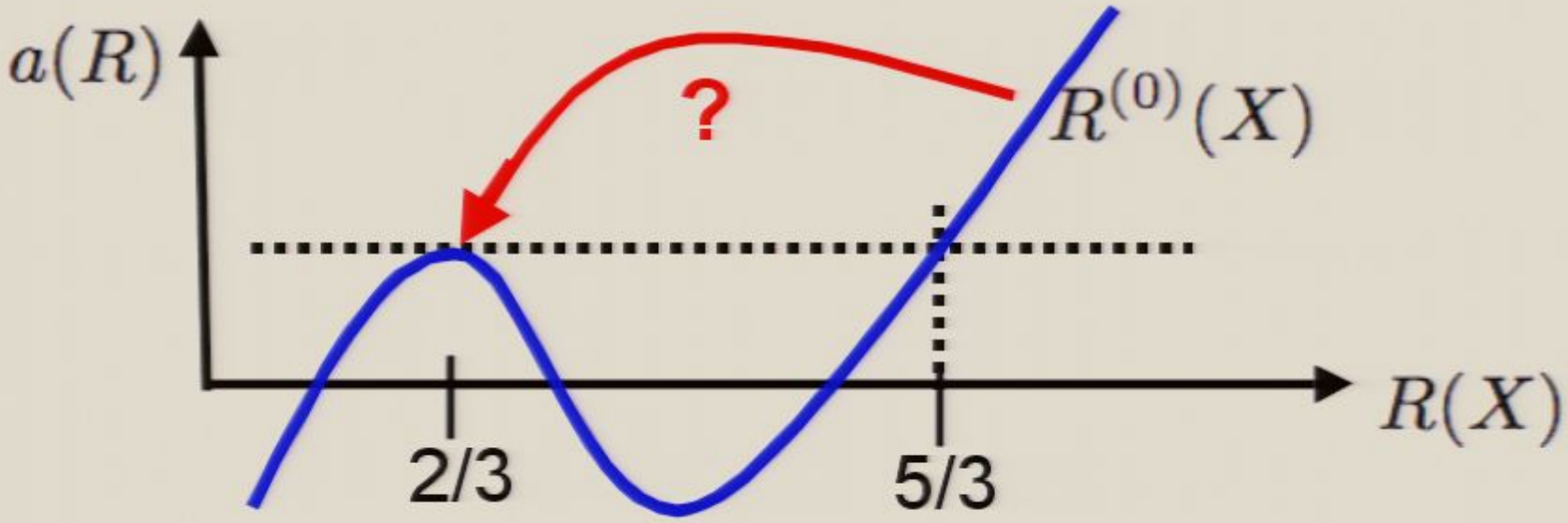

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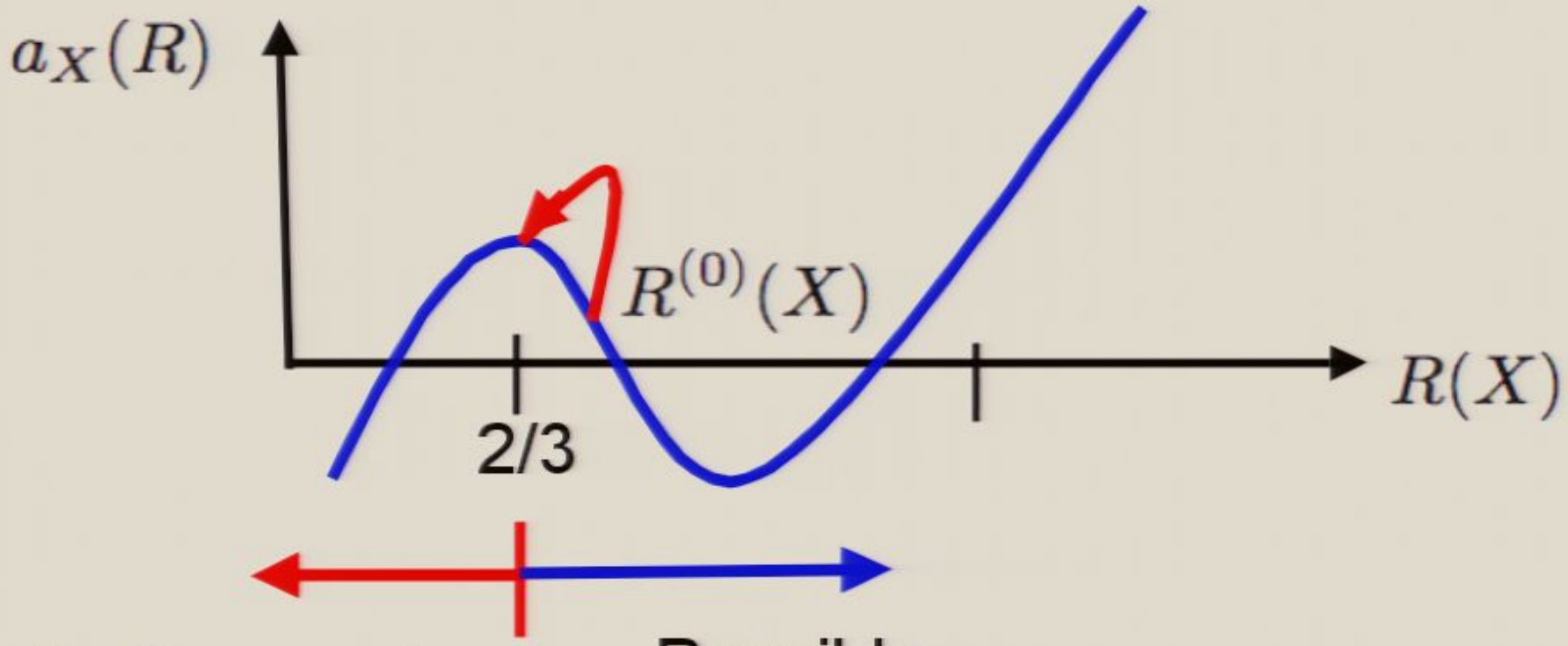

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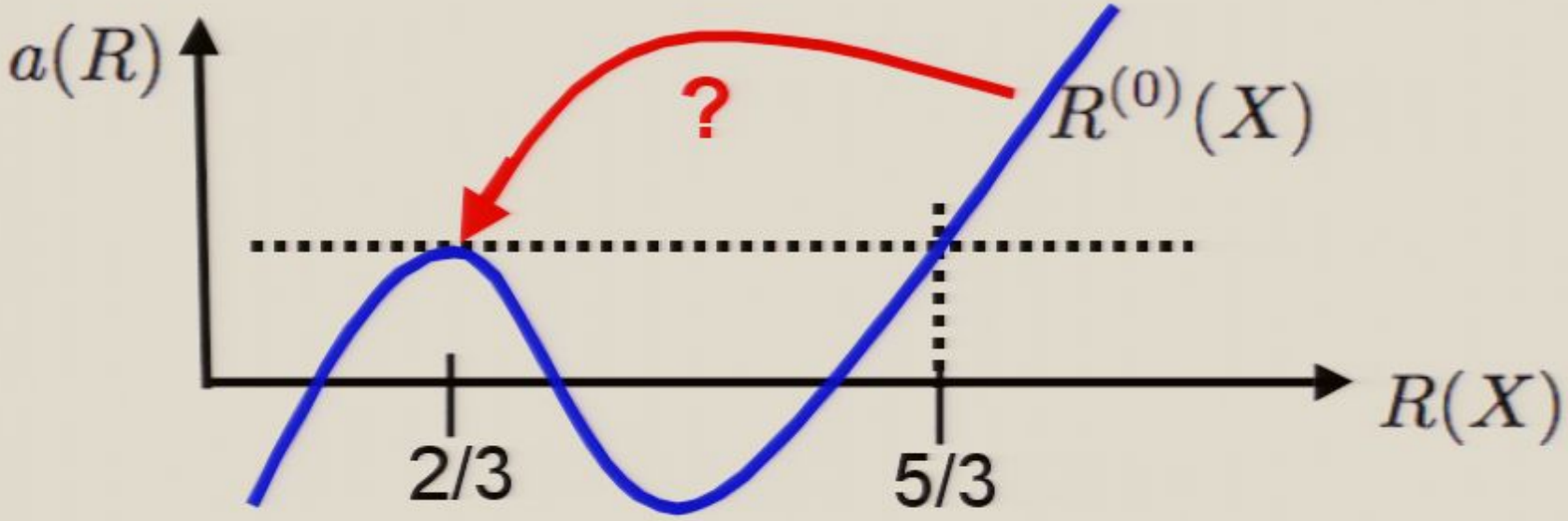

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Check diagnostic in other IR free cases

Many examples of theories have been argued to be IR free. The diagnostic checks if all have

$$a_{IR}^{free} > a_{IR}^{interacting}$$

The stronger conjecture is that all IR free ops X have $R^{(0)}(X) < 5/3$

This is indeed satisfied, for every example that I have checked.

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Also check known **non-free** examples

The conjectured diagnostic again works in every known case that I have checked. E.g. examples of **Brodie, Cho, KI:** $R(\mathrm{Tr}S^n) = 4n/(N+2)$ some exceed 5/3, so diagnostic says they're not all IR free. And for all N find:

$$a_{IR}^{interacting} > a_{IR}^{free}$$

Diagnostic correctly favors interacting over IR free scenario. This is the correct answer.

Back to SU(2) with Q in the 4

$$R(Q) = 3/5 \qquad \text{so} \qquad R^{(0)}(X = Q^4) = 12/5$$

Find $a_{IR}^{interacting} > a_{IR}^{free}$ since R(X)>5/3.

So, applied to this example, our conjectured diagnostic suggests that the correct IR phase is interacting (and the 't Hooft matching was a misleading fluke). If so, the theory does not yield DSB after all (unfortunately). Still, not a direct argument, so the jury is still out on this theory..

Conclude

- A speculative diagnostic for the IR phase: a plausible IR free phase is favored over an interacting phase if that has the larger conformal anomaly a. The stronger conjecture: each operator X can be IR free only if that increases a.

- Appears to work in every known (susy) example checked so far. (But doesn't give the answer we wanted, for DSB, in the SU(2) theory.)