Title: Recalling John in Supersymmetric Gauge Theories and Brane Constructions

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Abstract:

Recalling John in supersymmetric gauge theories and brane constructions

Akikazu Hashimoto

Perimeter Institute, October 2006

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FIRST YEAR GRADUATE STUDENTS (SEPTEMBER 1993)

Front row: l to r: Pan, Wei; Chen, Zhengdong; Balsano, Richard; Chiu, Weihsueh; Vinje, William; Chasheshkina, Ekaterina; Sauer, Karen; Hwang, Harold
Second row, l to r: Wiggins, Christopher; Harrison, Christopher; Stairs, Ingrid; Dundon, Christopher; Parikh, Maulik; Guerra, Erick; Hashimoto, Akikazu; Schnitzer, Mark
Third row, l to r: Bruder, Seth; Barnes, Christopher; Romalis, Mikhail; Brodie, John; Elowitz, Michael
Fourth row, l to r: Aksay, Emre; Carlson, Robert; Mail, Peter; Yildirim, Husnu; Rizzi, Anthony


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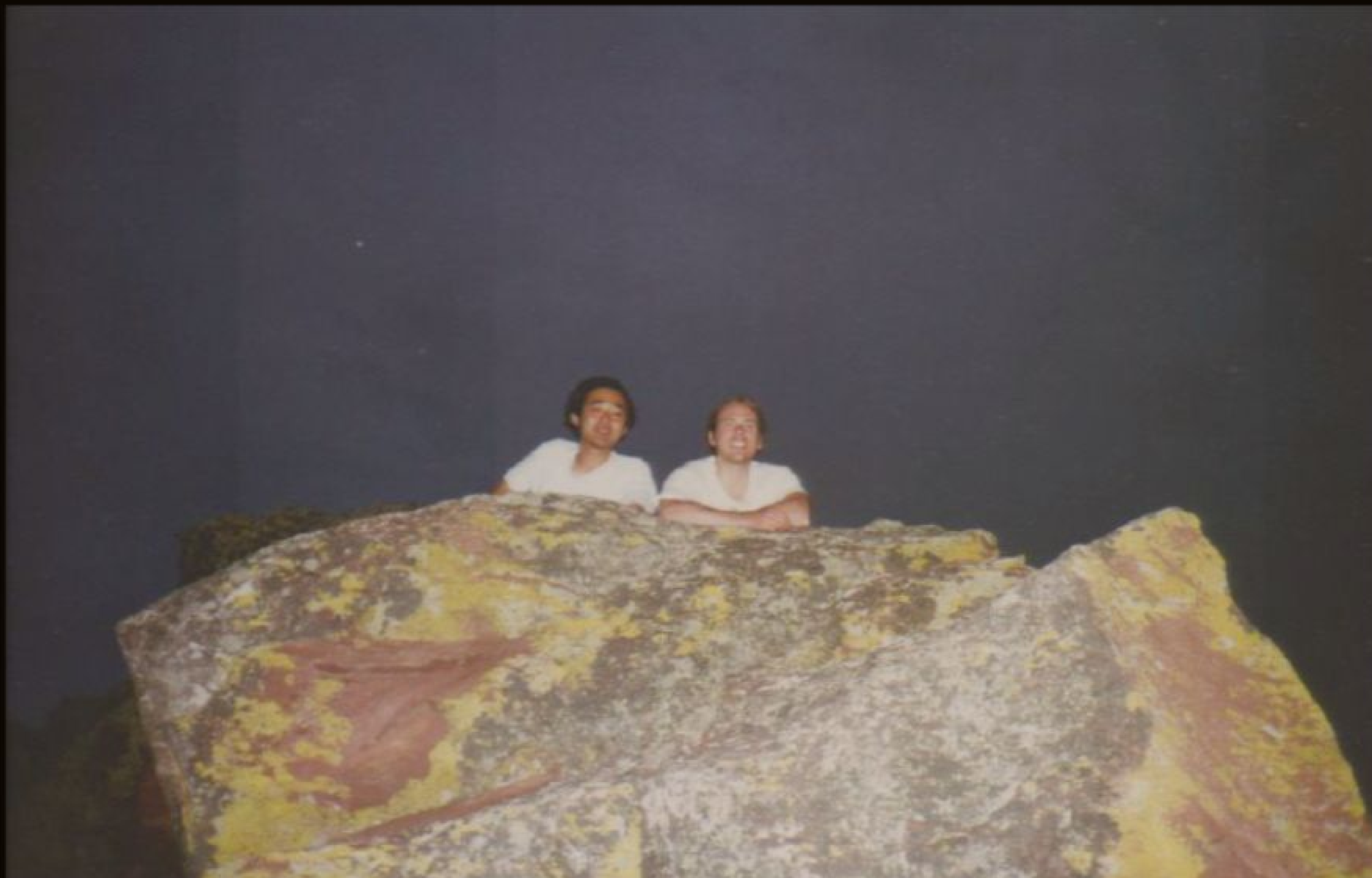


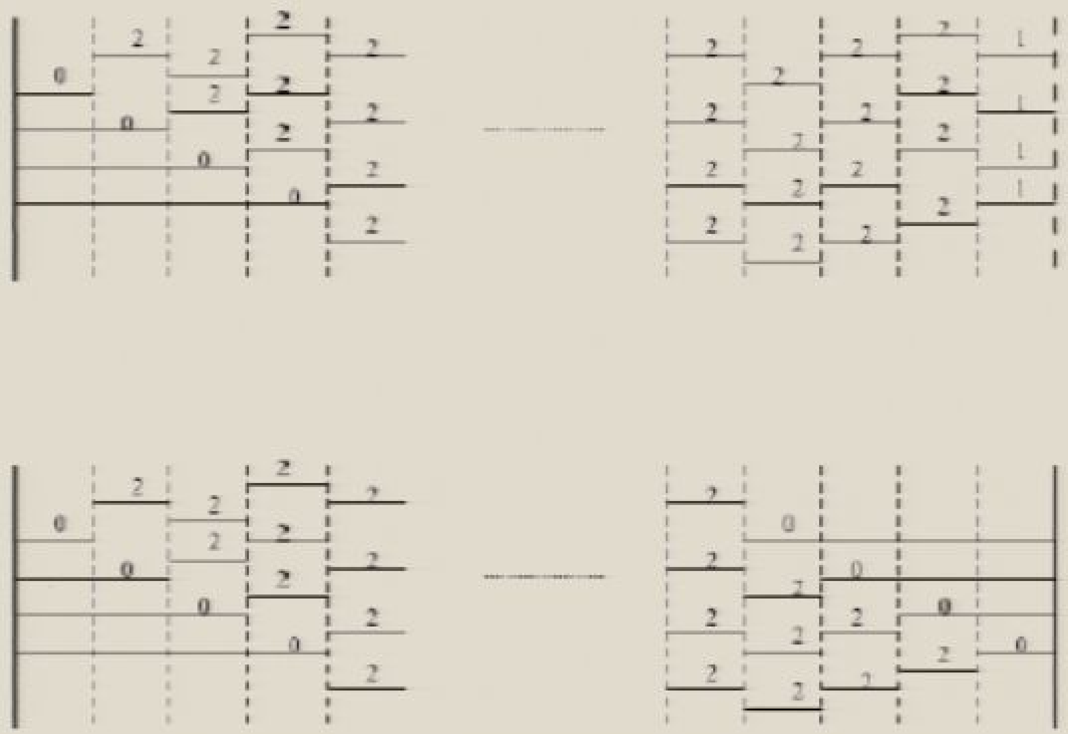


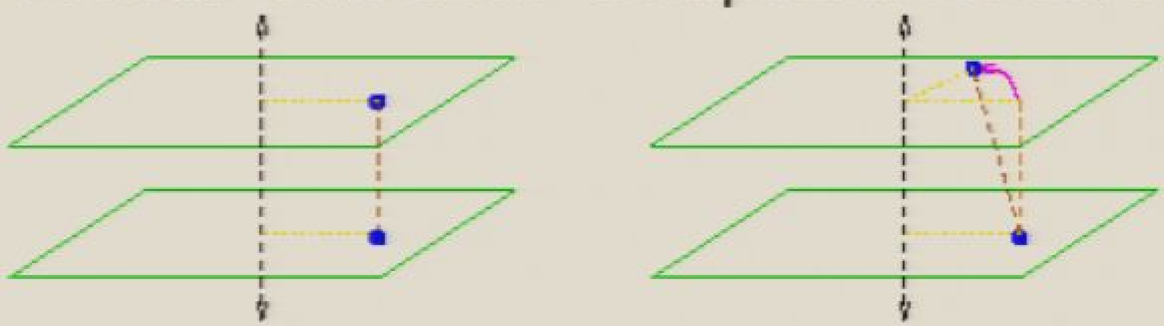
Fig. 2: The Higgs branch of $N = 1$ supersymmetric QCD versus the Higgs branch of $N = 2$ supersymmetric QCD. The numbers which are assigned to each D4-brane denote the number of chiral superfields which are associated with motion of the D4-brane along the branes at its ends. In the bottom picture, the $N = 2$ case, some of the branes, which carry no moduli, are not broken to avoid S-configurations. In the top picture, such a restriction does not exist for the NS' brane. Instead there are extra moduli as indicated in the figure.

The science title: **P**uff **F**ield **T**heory

Based on work with Ganor, Jue, Kim and Ndirango

See also Ganor, `hep-th/0609107`

Consider a Melvin Universe: Compactification with a twist



KK-reduction/T-duality and its U-dual gives rise to Melvin solution of SUGRA

- Flat space: $ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + dz^2$
- Twist: $ds^2 = -dt^2 + dr^2 + r^2(d\phi + \eta dz)^2 + dz^2$
- T-dualize

$$
\begin{aligned}
ds^2 &= -dt^2 + dr^2 + \frac{1}{1+\eta^2 r^2}(r^2 d\phi^2 + dz^2) \\
B &= \frac{\eta r^2}{1+\eta^2 r^2} d\phi \wedge dz \\
e^{2\phi} &= \frac{g^2}{1+\eta^2 r^2}
\end{aligned}
$$

Topology: $R^{1,3}$

Lots of interesting things happen if one adds a D3-brane and take the decoupling limit.

Type of Twist	Model
Melvin Twist: (z, ϕ)	Hashimoto-Thomas model • •
Melvin Shift Twist	Seiberg-Witten Model
Null Melvin Shift Twist	Aharony-Gomis-Mehen model
Null Melvin Twist	Dolan-Nappi model
Melvin Null Twist	Hashimoto-Sethi model
Melvin R Twist: (z)	Bergman-Ganor model •
Null Melvin R Twist	Ganor-Varadarajan model
R Melvin R Twist: $(\cdot)$	Lunin-Maldacena model •

Mostly non-local field theories: non-commutative gauge theories, dipole theories

Puff **F**ield **T**heory: a novel extension to this list

- $SO(d) \in SO(d,1)$
- $d = 3$ theory invariant under strong/weak coupling duality
- Simple SUGRA dual

- Consider a D0 in type IIA
- Lift to M-theory
- Twist
- Reduce back to IIA

$$ds^2 = -dt^2 + dr^2 + r^2(d\phi + \eta dz)^2 + d\vec{y}^2 + dz^2$$

$$z \sim z + g_s l_s = g_{YM0}^2 \alpha'^2, \qquad \eta = \frac{\Delta^3}{\alpha'^2}$$

$$\nu = g_{YM0}^2 \Delta^3 = \text{dimensionless, finite}$$

$$ds^2 = (1 + \eta^2 r^2)^{1/2} \left(-dt^2 + dr^2 + \frac{r^2}{1 + \eta^2 r^2} d\phi^2 + d\vec{y}^2 \right)$$

Easy to derive the SUGRA dual

Let's do the SUSY case of Melvin twists in two planes

Start with M-theory lift of D0-brane, twist:

$$ds_{11}^2 = -h^{-1}dt^2 + h(d\tilde{z} - v dt)^2 + d\rho^2 + \rho^2(ds_{B(2)}^2 + (d\phi + \eta d\tilde{z} + \mathcal{A})^2) + \sum_{i=1}^{5} dy_i^2$$

$$h(\rho, y) = 1 + \frac{gN\alpha'^{7/2}}{\rho^2 + \vec{y}^2}, \qquad v = h^{-1} \, .$$

Reduce, Decouple: $U = r/\alpha' =$ fixed

In terms of scaled variables:

$$\frac{ds^2}{\alpha'} = \sqrt{H+\Delta^6 U^2}\Big(-H^{-1}dt^2 + dU^2 + U^2 ds^2_{B(2)} + U^2\left(d\phi + \mathcal{A} + \frac{\Delta^3}{H}dt\right)^2 + d\vec{Y}^2\Big)$$

$$\frac{A}{\alpha'^2} = \frac{1}{H+\Delta^6 U^2}(-dt + U^2\Delta^3 d\phi)$$

$$e^{\phi} = g^2_{YM}(H+\Delta^6 U^2)^{3/4}$$

$$U = \frac{\rho}{\alpha'}, \quad \vec{Y} = \frac{y}{\alpha'}, \quad H(U,\vec{Y}) = \alpha'^2 h(\rho,\vec{y}) = \frac{g^2_{YM0}N}{U^7}$$

Puff Quanutm Mechanics

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Puff Quanutm Mechanics

Straight forward to generalize to (3+1)-dimensions

$$\frac{ds^2}{\alpha'} = \sqrt{H + \Delta^6 U^2}\left(-H^{-1}dt^2 + \frac{1}{H+\Delta^6 U^2}d\vec{x}^2 + dU^2 + U^2 ds^2_{B(2)} + U^2\left(d\phi + \frac{\Delta^3}{H}dt\right)^2 + d\vec{Y}^2\right)$$

$$H = \frac{g^2_{YM3} N}{U^4}$$

Puff Field Theory

- Unbroken $SO(3) \in SO(3,1)$
- Constant dilaton and RR 5-form flux: S-dual

Thermodynamics of Puff Field Theory

- Easy: repeat the construction starting with non-extremal D0
- $S = N^2 V T^p$
- Same number of degrees of freedom as ordinary SYM

Microscopic formulation of Puff Field Theory

- What is the action?

Go back to Puff Quantum Mechanics

$$ds^2_{11} = -h^{-1}dt^2 + h(d\tilde{z} - vdt)^2 + d\rho^2$$
$$+\rho^2(ds^2_{B(2)} + (d\phi + \eta d\tilde{z} + \mathcal{A})^2) + \sum_{i=1}^{5} dy_i^2$$

If $g^2_{YM0}\Delta^3 = \tilde{R}\eta = -b/d =$ rational, there is an $SL(2,Z)$

$$\begin{pmatrix} d\phi \\ \frac{d\tilde{z}}{\tilde{R}} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d\phi \\ \frac{d\tilde{z}}{\tilde{R}} \end{pmatrix}$$

Then, the new IIA description is

$$\begin{aligned} ds^2 &= -h^{-1/2}dt^2 + h^{1/2}\left(d\rho^2 + \rho^2 ds^2_{B(2)} + \rho^2\left(\frac{d\phi}{d} + \mathcal{A}\right)^2 + d\vec{y}^2\right) \\ A &= -c\tilde{R}d\phi - v dt \\ e^{\phi} &= h^{3/4} \end{aligned}$$

Other than the seemingly innocent 1-form $c\tilde{R}d\phi$, this is just a Z_d orbifold of decoupled D0 $\longrightarrow$ local theory

$SL(2, Z)$ also acts on the rank of the gauge group $N \rightarrow d^2 N$, as well as coupling, etc.

Twisted Quiver Quantum Mechanics

- Such duality between local and non-local field is familiar from non-commutative gauge theories: Morita Equivalence

- Not to be confused with Seiberg-Witten correspondence

- What does $\frac{c}{d}(d\tilde{R})d\phi$ do to the quiver quantum mechanics? (In the case of non-commutative geometry, the analogue was 't Hooft non-Abelian flux AH and Ithzaki)

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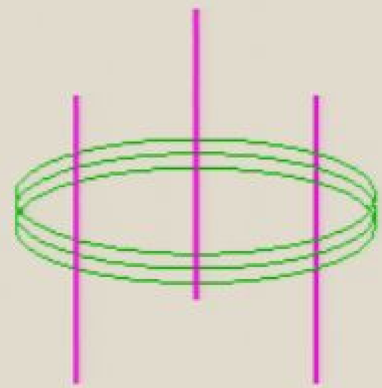
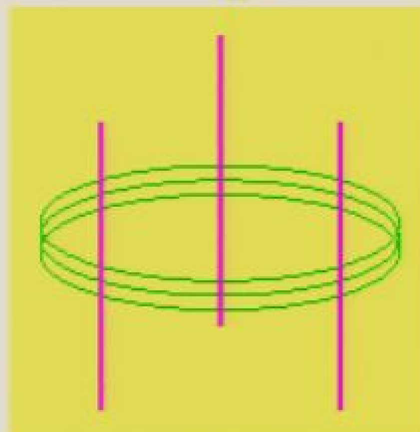
Strategy:

$$ds^2 = -h^{-1/2}dt^2 + h^{1/2}\left(d\rho^2 + \rho^2 ds^2_{B(2)} + \rho^2\left(\frac{d\phi}{d} + \mathcal{A}\right)^2 + d\vec{y}^2\right)$$

- Embed $\rho, B(2), \phi$ in Taub-NUT (can be removed later)
- Then, it is easier to visualize T-dualizing on ϕ
- Ignoring $\frac{c}{d}(d\tilde{R})d\phi$, the T-dual is decoupled dN D1 with d NS5 impurities



- In this picture $\frac{c}{d}(d\tilde{R})d\phi$ becomes the RR axion $\chi = \frac{c}{d}$
- 2 T-dualities ($\|$ NS5, $\perp$ D1) followed by S-duality maps this to dN D3-brane on T^2 with d D5 impurities with constant NSNS B-field $B = \frac{c}{d}$



- Simple NSNS background with D-branes: Seems definable microscopically (c.f. De Wolfe, Freedman, Ooguri)

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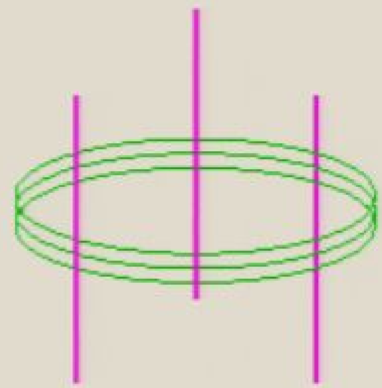

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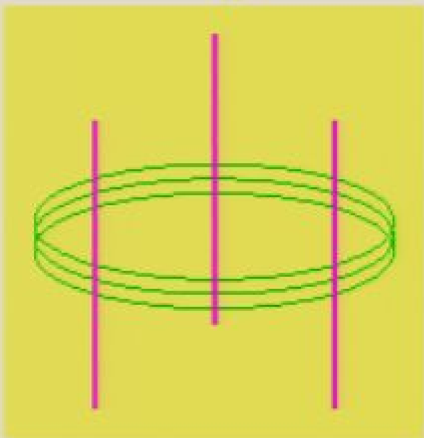

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Minor puzzle

- Axion χ is essentially the θ parameter of 1+1 SYM.
- Axion $\chi = \frac{c}{d}$ induces c F1 and D5-branes for every d D1 and NS5-branes, respectively. (c, d) F1/D1 system forms a threshold bound state. Also (c, d) D5/NS5 system.
- Does this mean that impurities see a binding potential?
- Puzzling because the collective coordinate of the impurity decouples in the zero slope limit.

- In this picture $\frac{c}{d}(d\tilde{R})d\phi$ becomes the RR axion $\chi = \frac{c}{d}$
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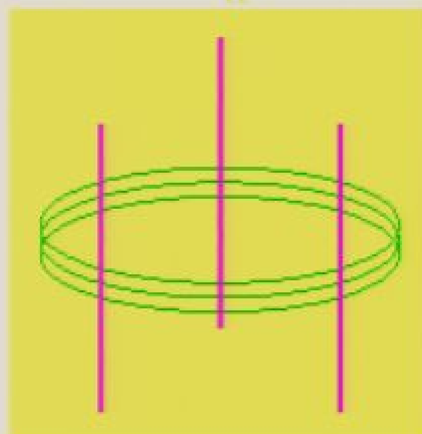

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