

Title: Chiral Symmetry Breaking from Intersecting Branes

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Abstract:

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Chiral Symmetry Breaking from Intersecting Branes

E. Antonyan, J. Harvey, S. Jensen, DK, hep-th/0604017

E. Antonyan, J. Harvey, DK, hep-th/0608149,0608177

and work in progress

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Introduction

Realizing field theories as low energy theories on branes in string theory has been a fruitful avenue of research in recent years. Sometimes, non-trivial field theory dynamics clarifies the behavior of branes in otherwise inaccessible regimes. In other cases it is the other way around.

Examples include:

- Seiberg duality.
- Seiberg-Witten theory.
- AdS/CFT.

In general, our understanding of the dynamics is enhanced by keeping in mind both points of view.

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Today I will discuss a system that follows the same pattern – a string model which realizes QCD as the low energy theory on a collection of D-branes. One of the interesting features of this model is that it embeds QCD in a wider class of theories which exhibit confinement and chiral symmetry breaking for all (generic) values of the parameters and can be solved in different limits using different techniques.

In particular, one can analyze the spectrum of mesons and their interactions, the behavior of the theory at finite temperature and density, and other properties. So far, this has been achieved in regimes in parameter space in which the theory is not QCD, but it is in the same universality class. We hope to eventually extend the results to the QCD regime.

Another nice aspect of this model is that it allows one to describe in a uniform way different looking systems in various dimensions that exhibit similar phenomena (J. Harvey's lecture).

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Pure Yang-Mills from D-branes

We start with a system of N_c $D4$ -branes in IIA string theory, which we will take to be stretched in the directions (01234). The low energy theory on the branes is $4 + 1$ dimensional supersymmetric $SU(N_c)$ Yang-Mills (YM) theory with 16 supercharges. The strength of interactions in this theory is determined by the 't Hooft coupling

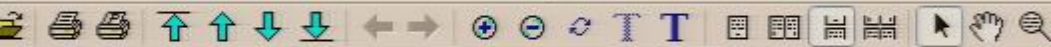
$$\lambda = g_s N_c l_s = g^2 N_c$$

where g is the five dimensional gauge coupling of the theory and g_s the string coupling.

We will study the dynamics in the limit

$$g_s \rightarrow 0, \quad N_c \rightarrow \infty$$

with $g_s N_c$ large but fixed.



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The coupling λ is a length scale. For energies $\ll \lambda$ the theory is weakly coupled. The coupling increases with the energy, and at $E \sim 1/\lambda$ the gauge theory description breaks down.

String theory resolves this problem by embedding the $4 + 1$ dimensional gauge theory in a $5 + 1$ dimensional CFT (the $(2, 0)$ theory on N_c $M5$ -branes) compactified on a circle. The transition between the $4 + 1$ and $5 + 1$ dimensional regimes occurs at distances of order λ .

To study QCD one needs to break SUSY and reduce from $4 + 1$ to $3 + 1$ dimensions. **E. Witten** proposed to do this by compactifying (say) x^4 :

$$x^4 \sim x^4 + 2\pi R_4$$

The fermions on the $D4$ -branes are taken to have *antiperiodic* boundary conditions around the circle. Thus, at energies $\ll 1/R_4$ they disappear from the spectrum. Scalars on the branes also get masses from loop effects and decouple at low energies.

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The only remaining massless degrees of freedom are the $SU(N_c)$ gauge fields. Their $3 + 1$ dimensional 't Hooft coupling is

$$\lambda_4 = \frac{\lambda}{2\pi R_4}$$

If λ_4 is small their dynamics is described by pure YM theory. This theory generates dynamically a scale

$$\Lambda_{QCD} \simeq \frac{1}{R_4} e^{-\frac{1}{\lambda_4}}$$

which is much smaller than other scales in the problem, such as $1/R_4$, $1/\lambda$. In this regime the theory is expected to have a discrete spectrum of glueballs with masses of order Λ_{QCD} , but its detailed structure is unknown.

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YM theory is hard, but here it is embedded in a larger set of theories labeled by λ_4 . YM corresponds to $\lambda_4 \ll 1$. In the opposite limit, $\lambda_4 \gg 1$, the dynamics simplifies. One can replace the $D4$ -branes by their near-horizon geometry,

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U)(dx^4)^2) \\ + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right)$$

where

$$R^3 = \pi \lambda, \\ f(U) = 1 - \left(\frac{U_{KK}}{U}\right)^3, \\ U_{KK} = \frac{4\pi \lambda}{9 R_4^2}$$

In this geometry, $U \geq U_{KK}$. This is related to confinement (the force between static quarks is linear).

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One can use supergravity to compute the spectrum of glueballs, by studying fluctuations about the solution. Can also compute the deconfinement temperature,

$$T_d \sim 1/R_4$$

and other finite temperature properties.

The theory described by supergravity is not YM, but it is obtained from it by varying the continuous parameter λ_4 . One does not expect to encounter a phase transition as λ_4 varies, so confinement should hold for all λ_4 . The details of the spectrum are expected to be different for small and large λ_4 .

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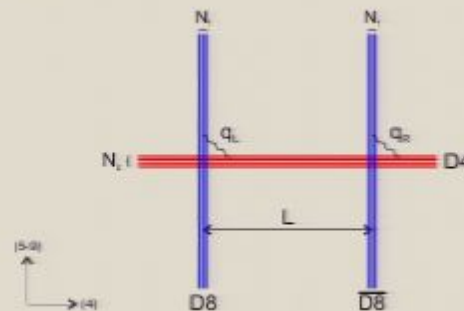
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So far we described pure YM using branes. In order to compare to experiment it is important to add light quarks. This should allow one to study chiral symmetry breaking and the spectrum and interactions of mesons.

One can proceed in a way proposed by [T. Sakai](#) and [S. Sugimoto](#). Add N_f $D8$ -branes and antibranes stretched in (012356789) which intersect the $D4$ -branes on $\mathbb{R}^{3,1}$.



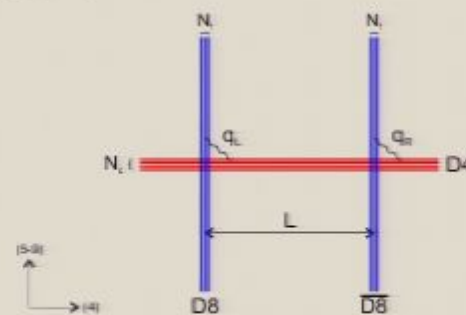
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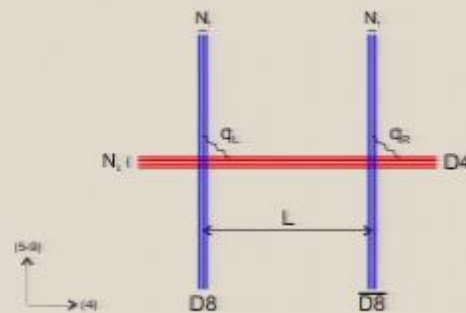
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The dynamics depends on two dimensionless parameters, λ/R_4 , λ/L . For $\lambda/R_4 \ll 1$, $L \sim R_4$, similar arguments to those above imply that the theory on the branes is $SU(N_c)$ QCD with N_f massless flavors. This theory is believed to confine and break chiral symmetry

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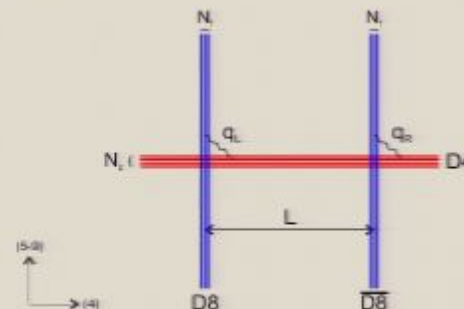
What happens when we vary $\frac{\lambda}{R_4}$, $\frac{\lambda}{L}$? It is natural to expect that there are again no phase transitions for finite values of these parameters. Thus, the theory should exhibit confinement and chiral symmetry breaking for all values of the couplings.

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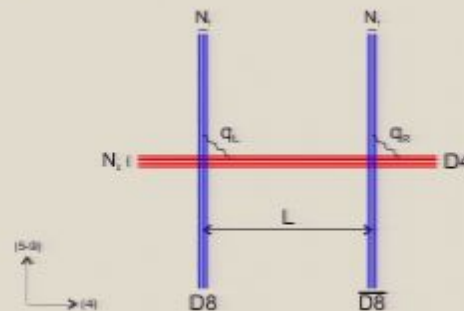
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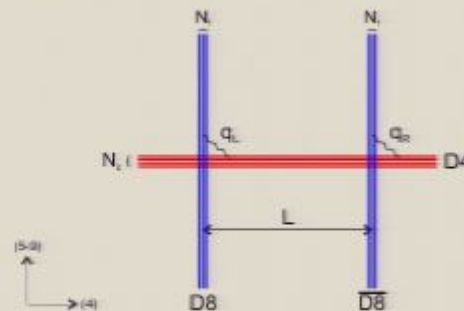
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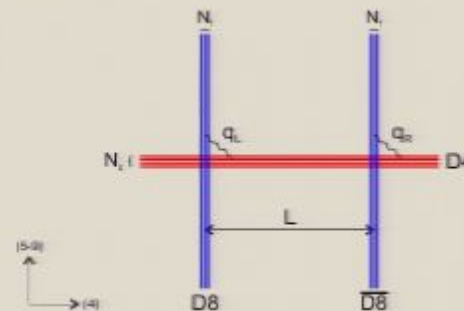
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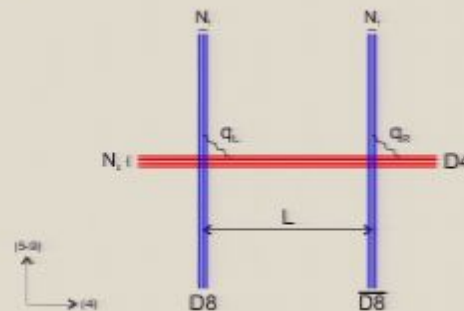
To test this picture, consider the extreme case, $R_4 = \infty$, in which the theory does not confine. The dynamics still depends non-trivially on λ/L , which can be thought of as the 4+1 dimensional 't Hooft coupling at the scale L .

DK: 10/13/2006

Adding Flavor

So far we described pure YM using branes. In order to compare to experiment it is important to add light quarks. This should allow one to study chiral symmetry breaking and the spectrum and interactions of mesons.

One can proceed in a way proposed by [T. Sakai](#) and [S. Sugimoto](#). Add N_f $D8$ -branes and antibranes stretched in (012356789) which intersect the $D4$ -branes on $\mathbb{R}^{3,1}$.



Now, in addition to the $SU(N_c)$ gauge field we have massless quarks in the fundamental of $SU(N_c)$, q_L, q_R , which are left and right-handed fermions living at the $4 - 8$ and $4 - \bar{8}$ intersections, respectively.

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An interesting limit is $R_4 \rightarrow \infty$ (decompactification of the extra dimension) in which the confinement scale Λ_{QCD} goes to zero. Question:

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DK 10/13/2006

To find the lowest energy configuration one needs to allow the x^4 of the eightbranes to be a function of U . The shape $U(x^4)$ is obtained by minimizing the energy of the brane (given by the DBI action)

$$E \sim \int dx^4 U^4 \sqrt{1 + \left(\frac{R}{U}\right)^3 U'^2}$$

Minimizing this one finds

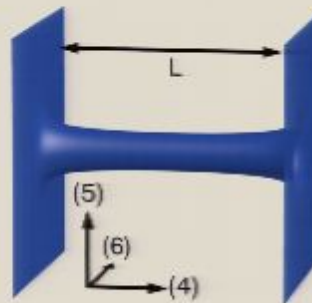
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where U_0 is related to L :

$$U_0 \sim \frac{\lambda}{L^2}$$

DK: 10/13/2006

Pictorially, this has the following form:



We see that the ground state of the system has the $D8$ -branes and antibranes connected by a “wormhole” into a single curved connected brane.

Comments:

- As L decreases, the width of the wormhole, U_0 , grows. This is reasonable since the force between q_L and q_R grows.
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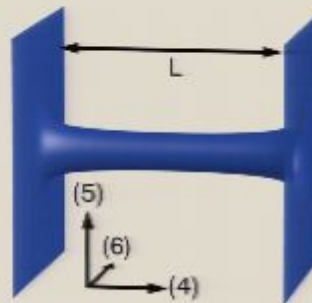
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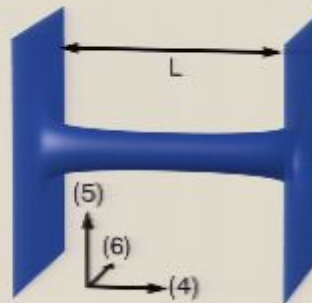
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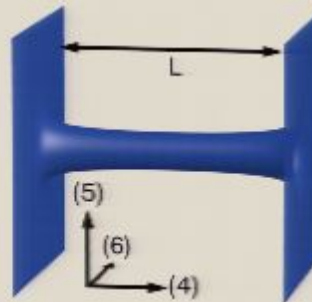
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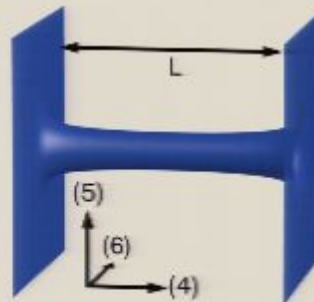
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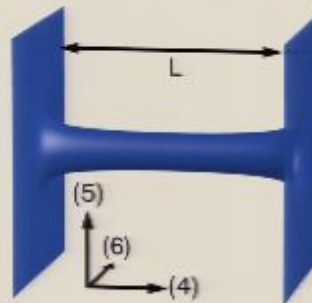
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Summary

The string embedding provides the following insights into QCD dynamics:

- QCD is a special case of a class of theories in which in general chiral symmetry breaking and confinement are distinct phenomena. One can have χ SB without confinement, but not the other way around.
- One can write a Lagrangian for $3 + 1$ dimensional fermions interacting with $4 + 1$ dimensional gauge fields, which exhibits χ SB at arbitrarily weak coupling. This generalizes models such as GN in $1 + 1$ dimensions.
- At strong coupling one can construct sugra models that confine, break chiral symmetry and have either the same or different deconfinement and chiral symmetry restoration temperatures, and study them in detail (spectrum, low energy interactions, etc).

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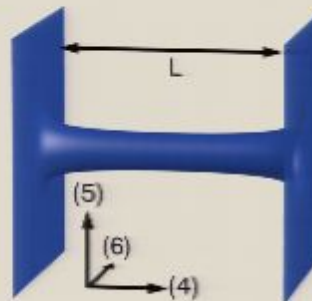
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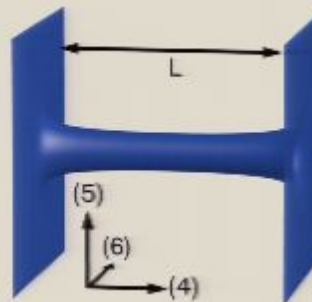
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One can use supergravity to compute the spectrum of glueballs, by studying fluctuations about the solution. Can also compute the deconfinement temperature,

$$T_d \sim 1/R_4$$

and other finite temperature properties.

The theory described by supergravity is not YM, but it is obtained from it by varying the continuous parameter λ_4 . One does not expect to encounter a phase transition as λ_4 varies, so confinement should hold for all λ_4 . The details of the spectrum are expected to be different for small and large λ_4 .

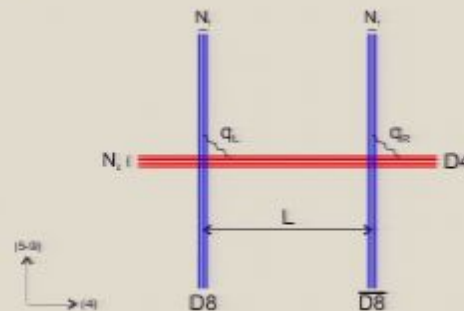
One can think of the parameter λ_4 as governing the strength of the confining force. When it is small, the theory is weakly coupled at the scale R_4 and vice-versa.

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Adding Flavor

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The dynamics depends on two dimensionless parameters, λ/R_4 , λ/L . For $\lambda/R_4 \ll 1$, $L \sim R_4$, similar arguments to those above imply that the theory on the branes is $SU(N_c)$ QCD with N_f massless flavors. This theory is believed to confine and break chiral symmetry

$$U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_{\text{diag}}$$

It is expected to develop a non-zero condensate

$$\langle q_L^\dagger \cdot q_R \rangle \neq 0$$

which leads to dynamical mass generation for the fermions q_L, q_R .

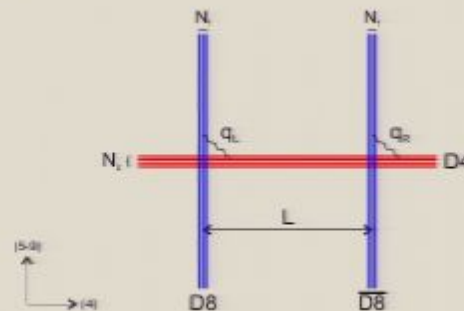
What happens when we vary $\frac{\lambda}{R_4}$, $\frac{\lambda}{L}$? It is natural to expect that there are again no phase transitions for finite values of these parameters. Thus, the theory should exhibit confinement and chiral symmetry breaking for all values of the couplings.

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i.e. a $4 + 1$ dimensional gauge theory with two codimension one defects containing left and right-handed fermions.

This description is in the spirit of the Nambu-Jona-Lasinio model of dynamical symmetry breaking. Here, the source of the attractive interaction between the left and right-handed fermions is exchange of $4 + 1$ dimensional gauge fields.



DK: 10/13/2006

The NJL model breaks chiral symmetry when the coupling exceeds a certain critical value. String theory makes the striking prediction that the above generalization of NJL **breaks chiral symmetry for arbitrarily weak coupling!**

DK 10/13/2006

One can use supergravity to compute the spectrum of glueballs, by studying fluctuations about the solution. Can also compute the deconfinement temperature,

$$T_d \sim 1/R_4$$

and other finite temperature properties.

The theory described by supergravity is not YM, but it is obtained from it by varying the continuous parameter λ_4 . One does not expect to encounter a phase transition as λ_4 varies, so confinement should hold for all λ_4 . The details of the spectrum are expected to be different for small and large λ_4 .

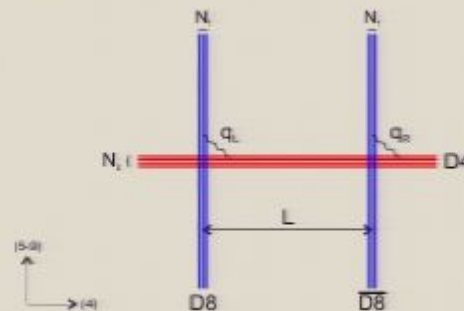
One can think of the parameter λ_4 as governing the strength of the confining force. When it is small, the theory is weakly coupled at the scale R_4 and vice-versa.

DK: 10/13/2006

Adding Flavor

So far we described pure YM using branes. In order to compare to experiment it is important to add light quarks. This should allow one to study chiral symmetry breaking and the spectrum and interactions of mesons.

One can proceed in a way proposed by [T. Sakai](#) and [S. Sugimoto](#). Add N_f $D8$ -branes and antibranes stretched in (012356789) which intersect the $D4$ -branes on $\mathbb{R}^{3,1}$.



Now, in addition to the $SU(N_c)$ gauge field we have massless quarks in the fundamental of $SU(N_c)$, q_L, q_R , which are left and right-handed fermions living at the $4 - 8$ and $4 - \bar{8}$ intersections, respectively.

DK 10/13/2006

The mass m_q is large in the supergravity regime.

- The symmetry breaking $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_{\text{diag}}$ has a nice geometric interpretation: due to brane recombination, one cannot perform independent $U(N_f)$ transformations on the $D8$ and $\bar{D}8$ -branes.
- One can study the spectrum of “mesons” in this strong coupling limit. While the mass of the fermion is $m_q \sim \lambda/L^2$, there are mesons with masses of order $1/L (\ll m_q)$ obtained from fluctuations of the DBI action. These are tightly bound states. From naived fermions.

This description is in the spirit of the Nambu-Jona-Lasinio model of dynamical symmetry breaking. Here, the source of the attractive interaction between the left and right-handed fermions is exchange of $4 + 1$ dimensional gauge fields.

DK 10/13/2006

As in the case without quarks, sugra describes a theory that is strongly coupled from the gauge theory point of view. For $\lambda \ll L$ the $4 + 1$ dimensional theory is weakly coupled and can be described by a Lagrangian of the form

$$L = -\frac{1}{4g^2}F_{MN}^2 + \delta(x^4 + \frac{L}{2})q_L^\dagger \bar{\sigma}^\mu (i\partial_\mu + A_\mu)q_L +$$

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Summary

The string embedding provides the following insights into QCD dynamics:

- QCD is a special case of a class of theories in which in general chiral symmetry breaking and confinement are distinct phenomena. One can have χ SB without confinement, but not the other way around.
- One can write a Lagrangian for $3 + 1$ dimensional fermions interacting with $4 + 1$ dimensional gauge fields, which exhibits χ SB at arbitrarily weak coupling. This generalizes models such as GN in $1 + 1$ dimensions.
- At strong coupling one can construct sugra models that confine, break chiral symmetry and have either the same or different deconfinement and chiral symmetry restoration temperatures, and study them in detail (spectrum, low energy interactions, etc).

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- The NJL model is extensively used to model hadrons. The string embedding goes a long way towards explaining why it is useful. Further work may allow one to considerably improve these descriptions.
- The two dimensionless parameters that naturally appear in the model (λ/R_4 and λ/L) govern the scales of confinement and chiral symmetry breaking. By tuning these parameters one can describe situations in which both phenomena occur in the supergravity regime, or ones in which one is seen in supergravity (vSB) and the other in field

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