Title: Dual Gravity Approach to Near-Equilibrium Processes in Strongly Coupled Gauge Theories

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Abstract:

# Dual gravity approach to near-equilibrium processes in strongly coupled gauge theories

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***String Theory and Gauge Theory: Past, Present, and Future***
***Perimeter Institute workshop***

October 13, 2006

- Non-equilibrium regime of thermal gauge theories is of interest for RHIC and early universe physics

- This regime can be studied in perturbation theory, assuming the system is a weakly interacting one. However, this is often NOT the case. Nonperturbative approaches are needed.

- Lattice simulations cannot be used directly for real-time processes.

- Gauge theory/gravity duality CONJECTURE provides a theoretical tool to probe non-equilibrium, non-perturbative regime of SOME thermal gauge theories

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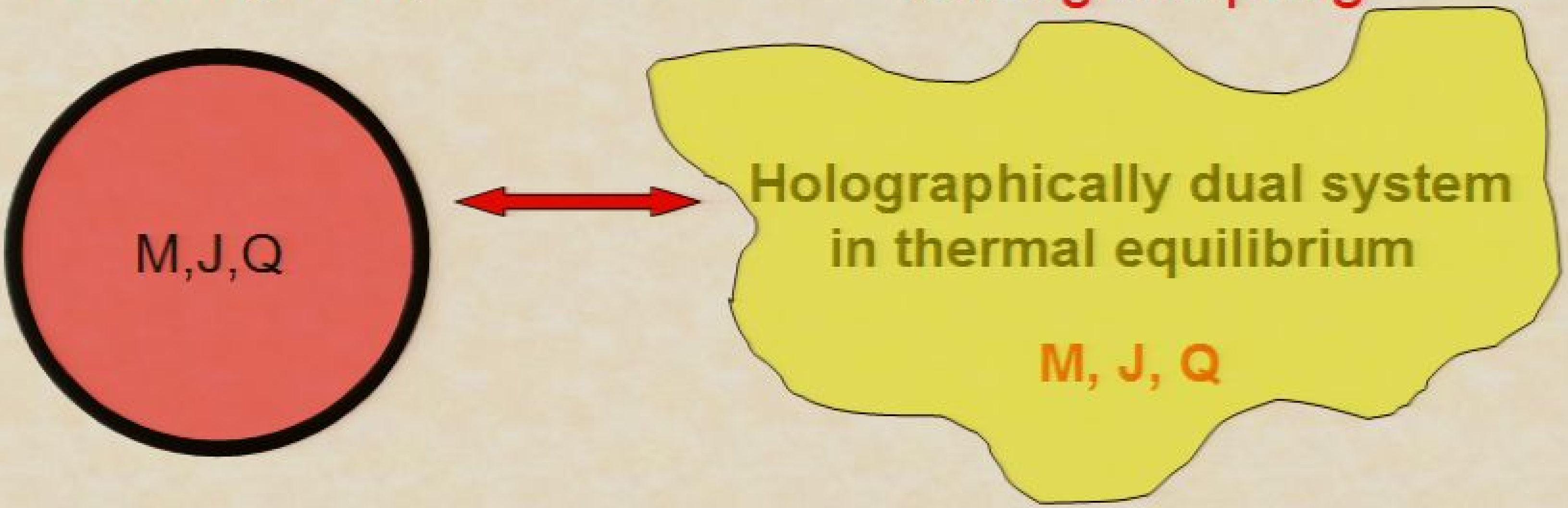
## 10-dim gravity

## 4-dim gauge theory – large N, strong coupling




$T_{\text{Hawking}}$ $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $\Longleftrightarrow$ ????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box"\ h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$ $\partial_t j^0 + \partial_i j^i = 0$

$$\partial_t j^0 = -D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

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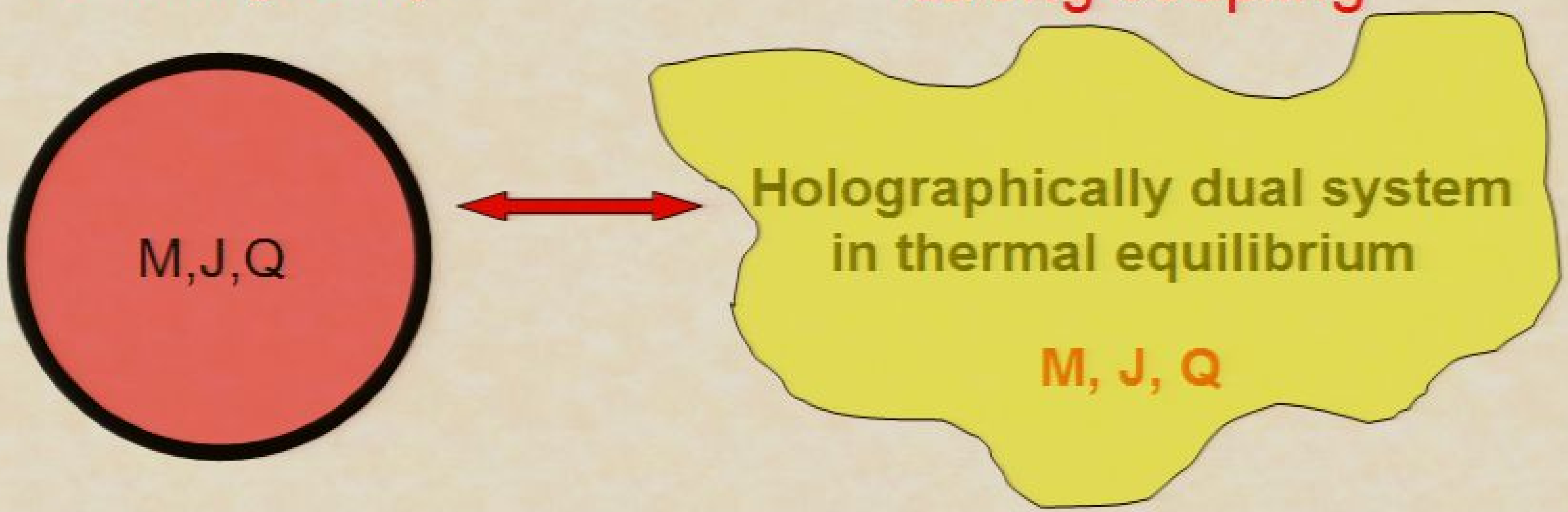


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# Transport (kinetic) coefficients

- Shear viscosity $\eta$
- Bulk viscosity $\zeta$
- Charge diffusion constant $D_Q$
- Thermal conductivity $\kappa_T$
- Electrical conductivity $\sigma$

Gauge/gravity dictionary determines correlators of gauge-invariant operators from gravity
(in the regime where gravity description is valid!)

e.g. for $N_c \to \infty, \quad g_{YM}^2 N_c \to \infty$ in $\mathcal{N}=4 \; SU(N_c)$ SYM

For example, one can compute the correlators such as

$\langle \mathrm{tr} F_{\mu\nu}^2(-\omega,-q) \;\; \mathrm{tr} F_{\mu\nu}^2(\omega,q) \rangle \qquad \langle T_{\mu\nu}(\omega,-q) \, T_{\sigma\rho}(\omega,q) \rangle$

in the limit $N_c \to \infty, \quad g_{YM}^2 N_c \to \infty$ in $\mathcal{N}=4 \; SU(N_c)$ SYM

by solving the equations describing fluctuations of the 10-dim gravity background involving AdS-Schwarzschild black hole

# Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[ T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using the gauge theory/gravity duality

# What is known?

✓ **Shear viscosity/entropy ratio:** $\frac{\eta}{s} = \frac{1}{4\pi}$

- in the limit $g^2 N = \infty \qquad N = \infty$
- universal for a large class of theories

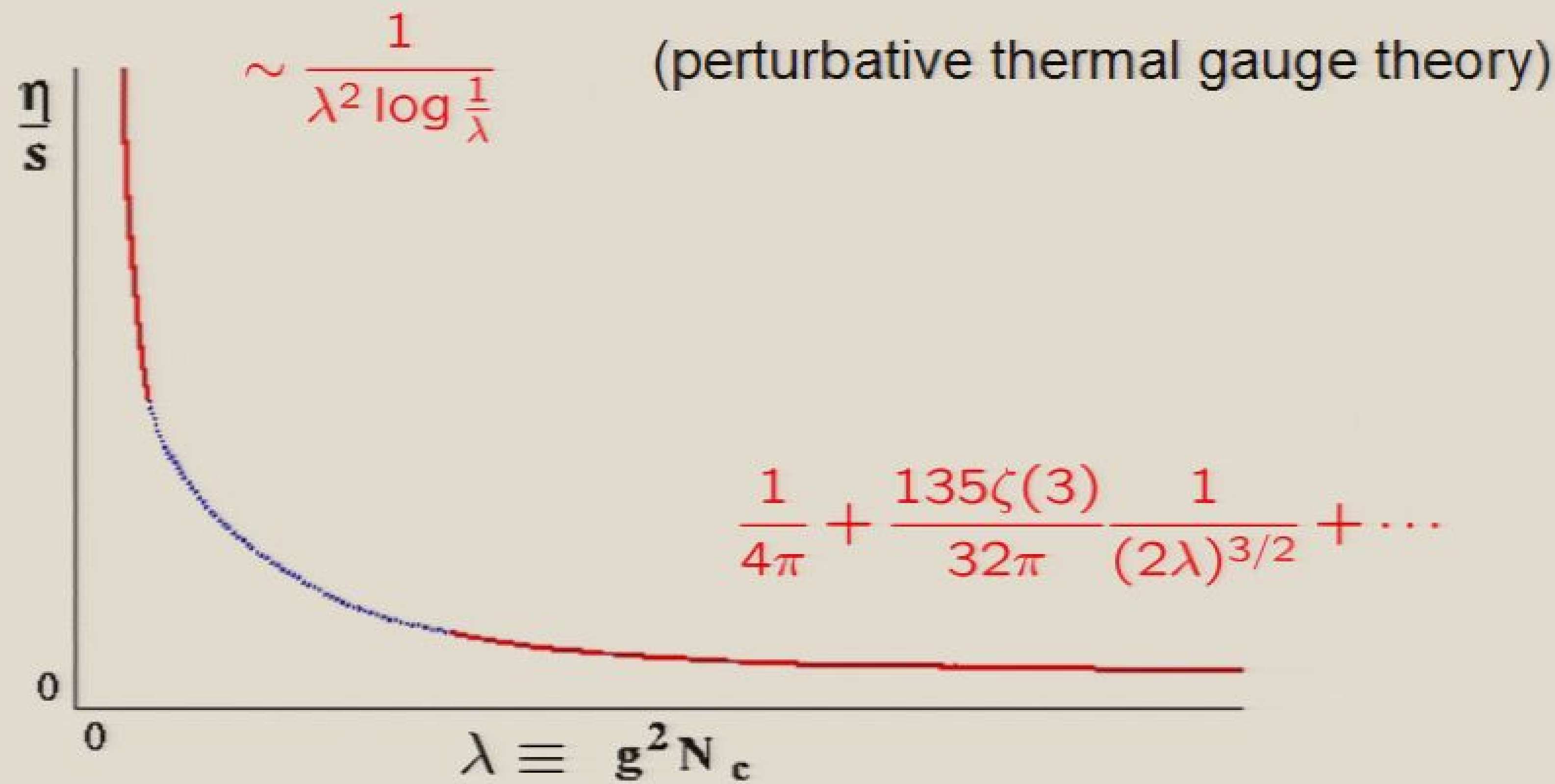
✓ **Bulk viscosity for non-conformal theories**

$$\frac{\zeta}{\eta} = -\kappa \left( v_s^2 - \frac{1}{3} \right)$$

- in the limit $g^2 N = \infty \quad N = \infty$
- model-dependent

✓ **R-charge diffusion constant for N=4 SYM:** $D_R = \frac{1}{2\pi T}$

# Shear viscosity in $\mathcal{N} = 4$ SYM




Correction to $1/4\pi$: A.Buchel, J.Liu, A.S., hep-th/0406264

# Universality of $\eta/s$

Theorem:

*For any thermal gauge theory (with zero chemical potential), the ratio of shear viscosity to entropy density is equal to* $1/4\pi$ ***in the regime described by a corresponding dual gravity theory***

(e.g. at $g_{YM}^2 N_c = \infty, N_c = \infty$ in $\mathcal{N} = 4$ SYM)

Remark:

String/Gravity dual to QCD (if it exists at all) is currently unknown.

# Three roads to universality of $\eta/s$

- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

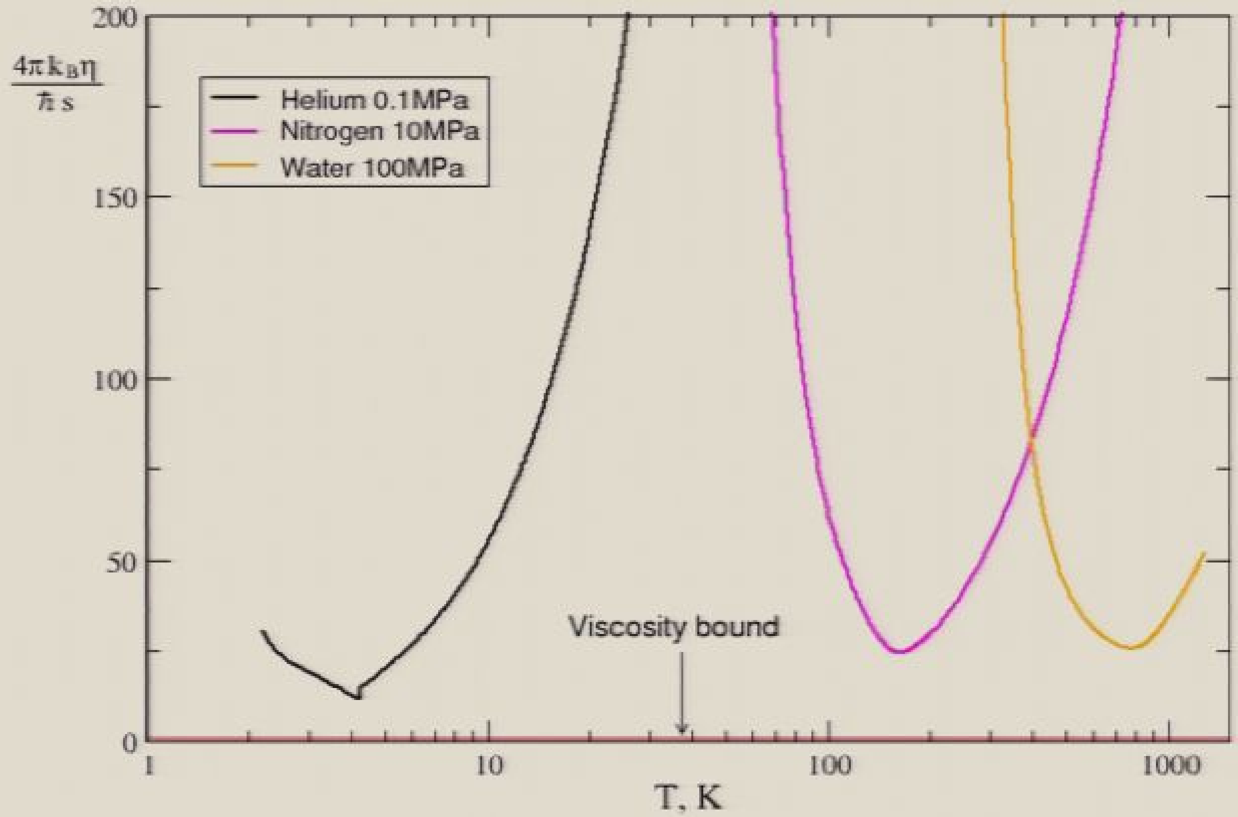
- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., to appear,
  P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

# A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$






P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

# Shear viscosity at non-zero chemical potential

$\mathcal{N} = 4$ SYM

$q_i \in U(1)^3 \subset SO(6)_R$ $\Longleftrightarrow$ Reissner-Nordstrom-AdS black hole with three R charges

$Z = \mathrm{tr}\, e^{-\beta H + \mu_i q_i}$

(see e.g. Yaffe, Yamada, hep-th/0602074)

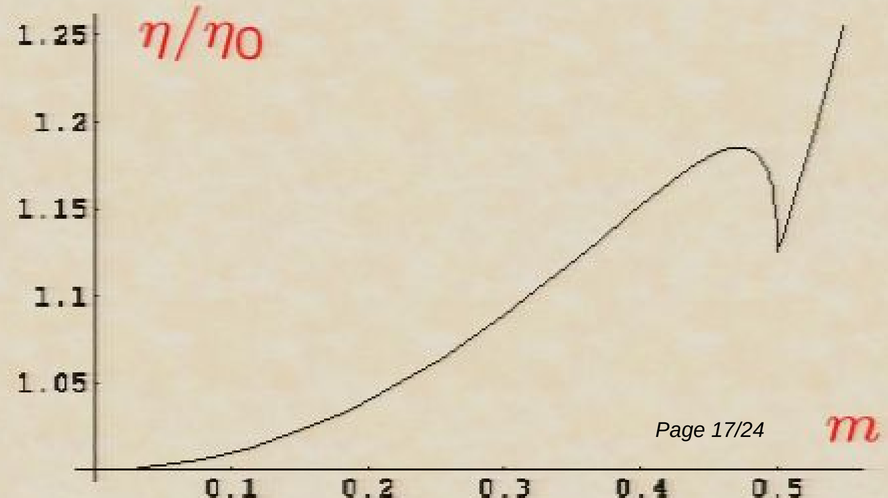
(Behrnd, Cvetic, Sabra, 1998)

We still have $\frac{\eta}{s} = \frac{1}{4\pi}$

J.Mas
D.Son, A.S.
O.Saremi
K.Maeda, M.Natsuume, T.Okamura

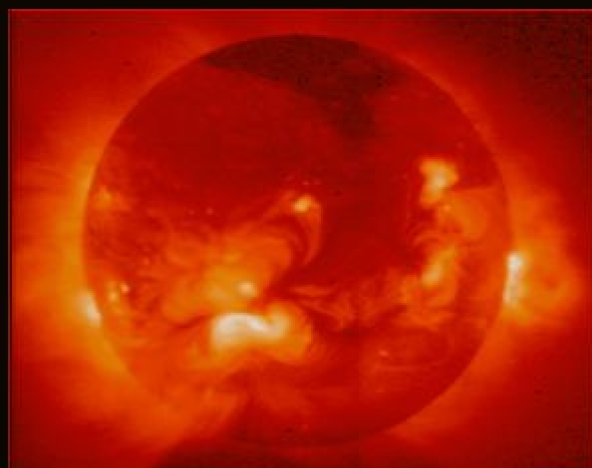
$$\eta = \pi N^2 T^3 \frac{m^2(1-\sqrt{1-4m^2}-m^2)^2}{(1-\sqrt{1-4m^2})^3}$$

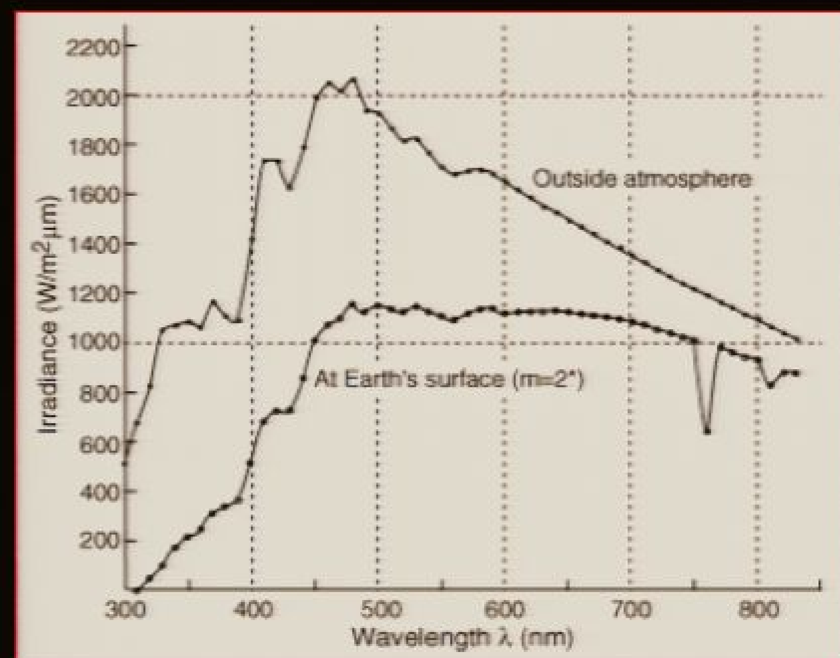
$m \equiv \mu/2\pi T$

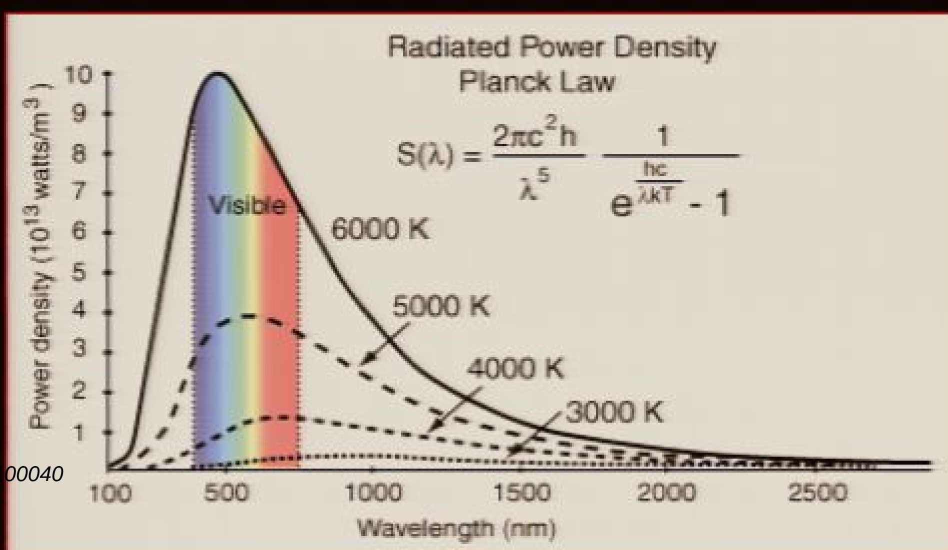


# Photon and dilepton emission from supersymmetric Yang-Mills plasma

S. Caron-Huot, P. Kovtun, G. Moore, A.S., L.G. Yaffe, hep-th/0607237





# Photon emission from SYM plasma

Photons interacting with matter: $e\, J_\mu^{\text{EM}}\, A^\mu$

To leading order in $e$

$$d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3}\,\frac{e^2}{2|k|}\,\eta^{\mu\nu} C^<_{\mu\nu}(k^0 = |k|)$$

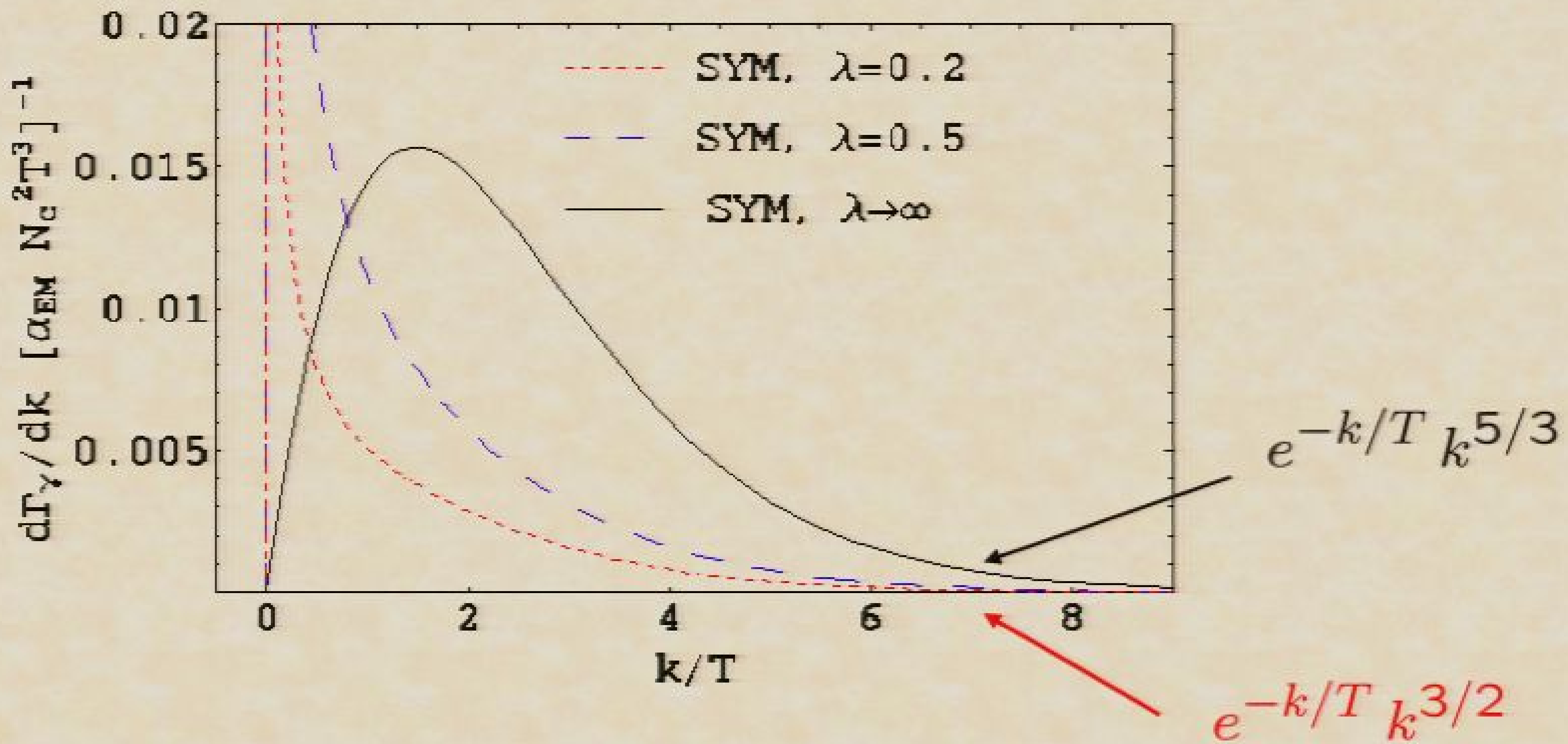
$$C^<_{\mu\nu} = \int d^4X e^{-iKX} \langle J_\mu^{\text{EM}}(0) J_\nu^{\text{EM}}(X)\rangle$$

Mimic $J_\mu^{\text{EM}}$ by gauging global R-symmetry $U(1) \subset SU(4)$

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4\,\text{SYM}} + e\, J_\mu^3\, A^\mu - \frac{1}{4} F_{\mu\nu}^2$$

Need only to compute correlators of the R-currents $J_\mu^3$

# Photoproduction rate in SYM




(Normalized) photon production rate in SYM for various values of 't Hooft coupling

$$\frac{d\Gamma_\gamma}{dk\,\alpha_{em}N_c^2T^3} = n_B(k)\left(\frac{k}{4\pi T}\right)^2 \left|\,{}_2F_1\left(1-\frac{(1+i)k}{4\pi T}, 1+\frac{(1-i)k}{4\pi T}; 1-\frac{ik}{2\pi T}; -1\right)\right|^{-2}$$

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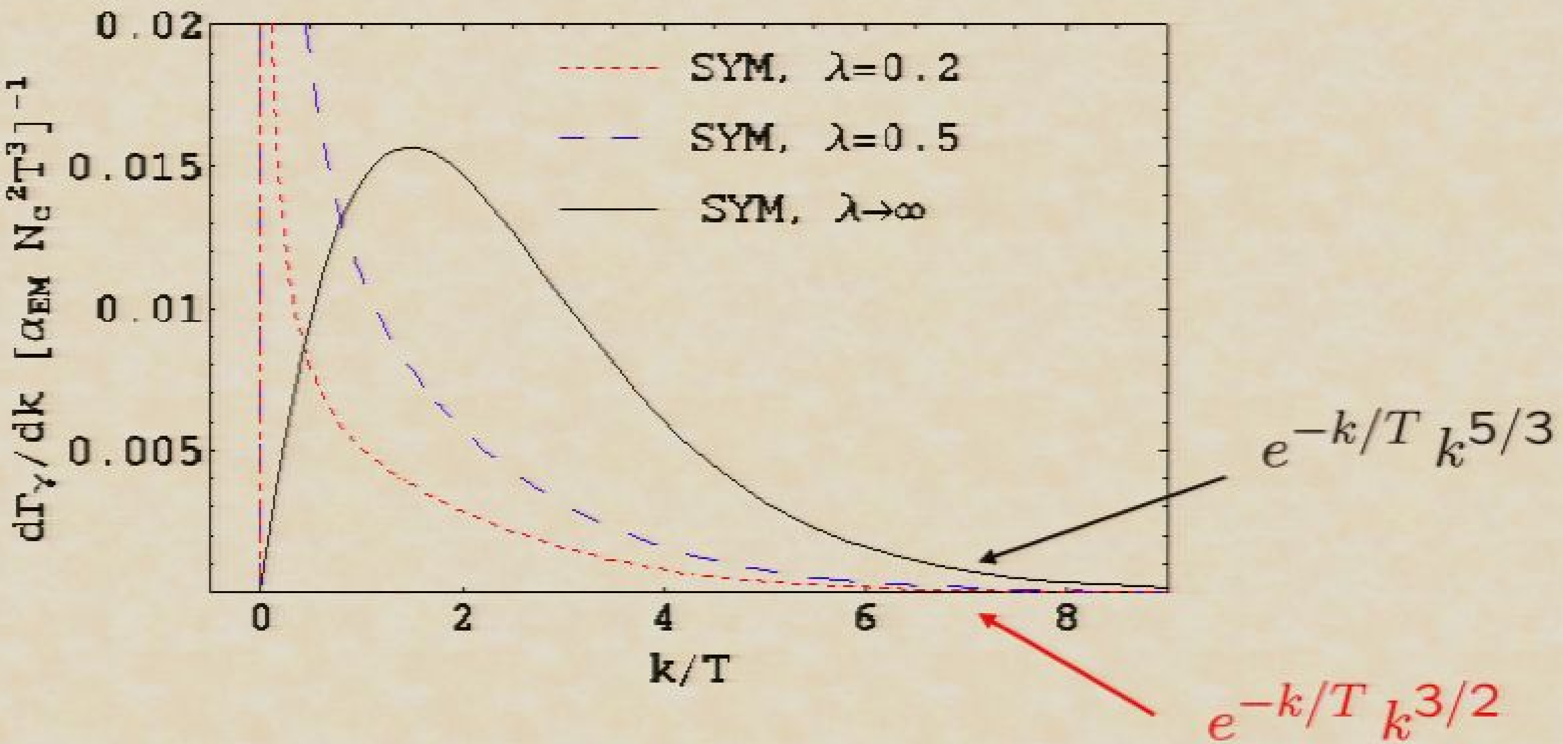


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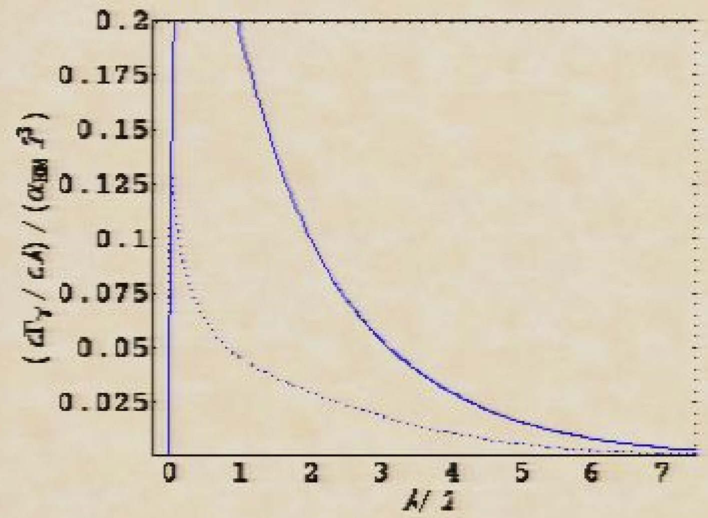
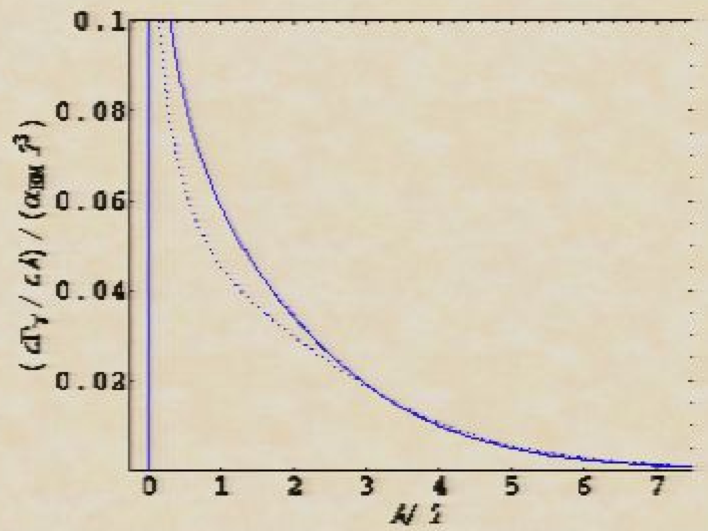
# How far is SYM from QCD?




pQCD (dotted line) vs
pSYM (solid line)
at equal coupling
(and $N_c$=3)




pQCD (dotted line) vs
pSYM (solid line)
at equal fermion thermal mass
(and $N_c$=3)

# Epilogue

- On the level of theoretical models, there exists a connection between near-equilibrium regime of certain strongly coupled thermal field theories and fluctuations of black holes
- This connection allows us to compute transport coefficients for these theories
- At the moment, this method is the only theoretical tool available to study the near-equilibrium regime of strongly coupled thermal field theories
- The result for the shear viscosity turns out to be universal for all such theories in the limit of infinitely strong coupling
- Prospects for experimental verification are not hopeless