Title: Magnons in Gauge Theories and String Theories

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Abstract:

Can $\mathcal{N}=4$ Super Yang-Mills be solved?

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- Magnons $\Rightarrow$ Only need to know the poles and zeros of the S-matrix

Outline

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Finite size effects and comparing small λ with large λ

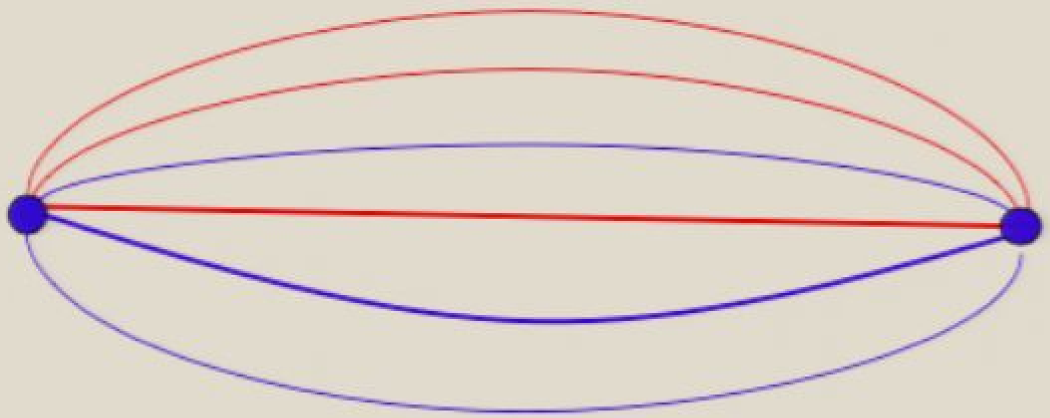
$SU(2)$ Sector of N=4

$$\mathcal{O}_L(x) = \mathrm{Tr}\,(\,ZZZ\ldots WW\ldots ZW\ldots WWZW\,)$$

$$Z = \phi_1 + i\,\phi_2$$
$$W = \phi_3 + i\,\phi_4$$

$$\boxed{\Delta_0 = L}$$

Tree level:



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Operator Mixing

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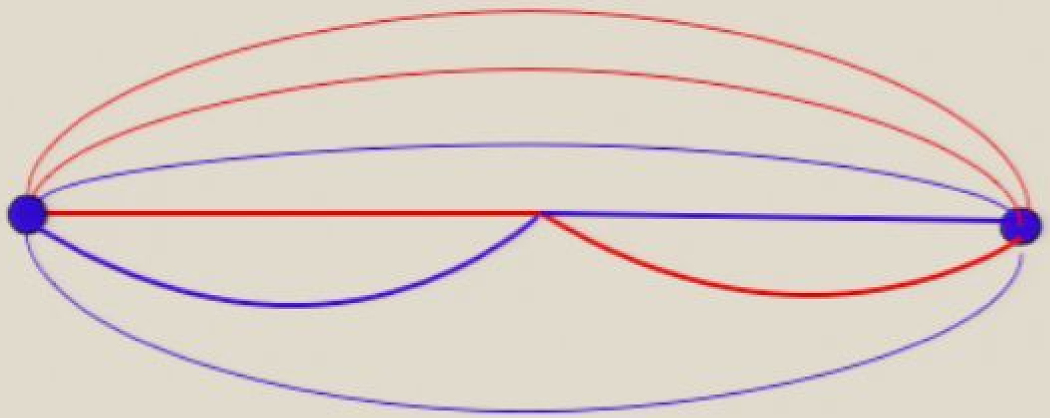

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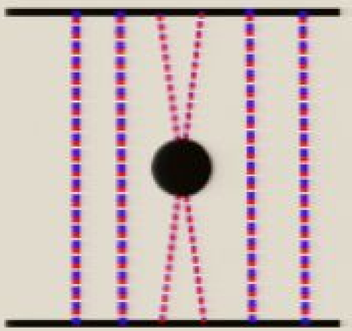

Planar:

$$H_2 = \frac{\lambda}{8\pi^2}\sum_{j=1}^{L}(1 - P_{j,j+1})$$

K. Zarembo, JM 2002

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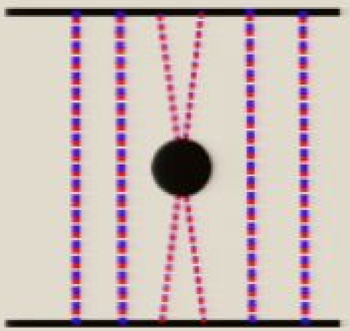

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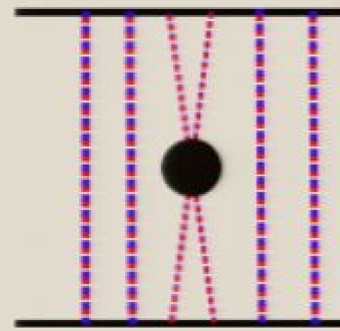

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$$g^2 = \frac{\lambda}{8\pi^2}$$

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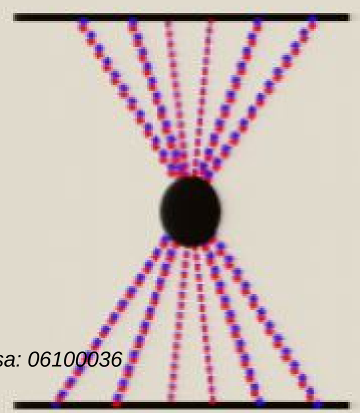


$$H = \sum_{n=1}^{\infty} g^{2n} H_{2n}$$

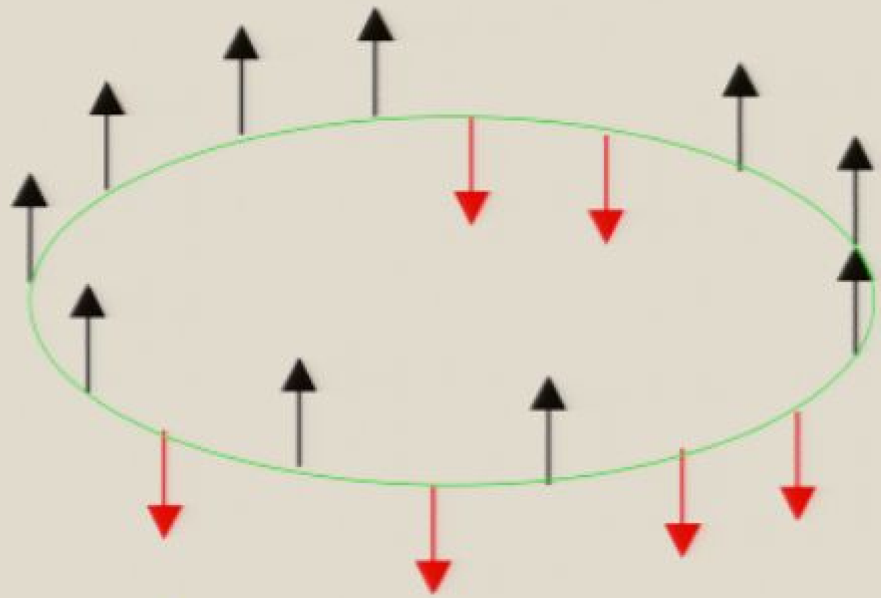
H_{2n} ranges over n sites

3-loops: Beisert, Kristjansen & Staudacher

5-loops: Beisert

Spin chains-Integrability

SU(2) Sector: Z, W Only



$J_1 \;\; W's$

$J_2 \;\; Z's$

$$H_2 = g^2 \sum_{j=1}^{L} \left(\frac{1}{2} - 2\vec{S}_j \cdot \vec{S}_{j+1} \right)$$

Heisenberg Spin Chain

The Bethe Ansatz

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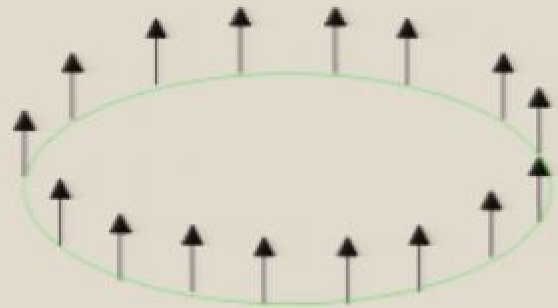
Ground state:  $|0\rangle$

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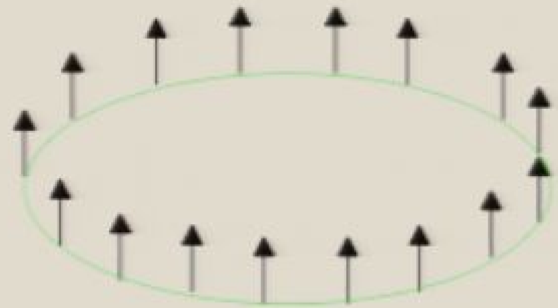

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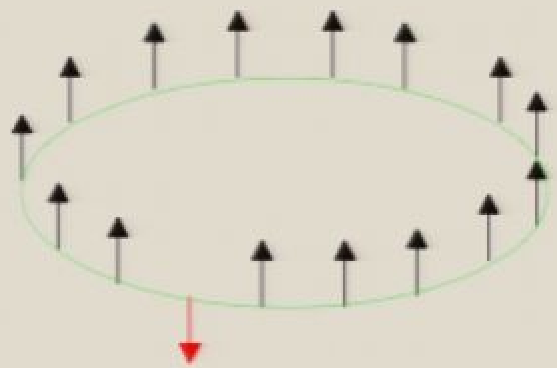

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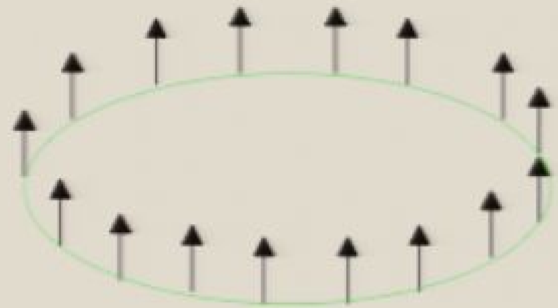

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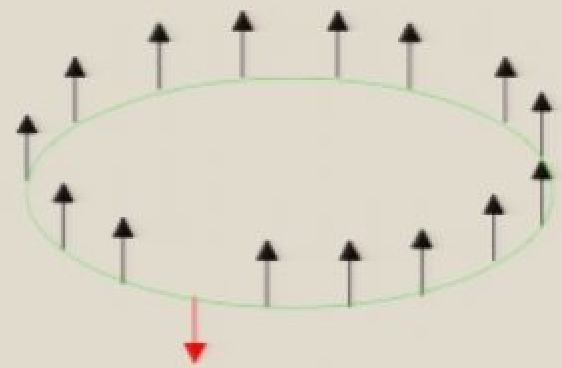

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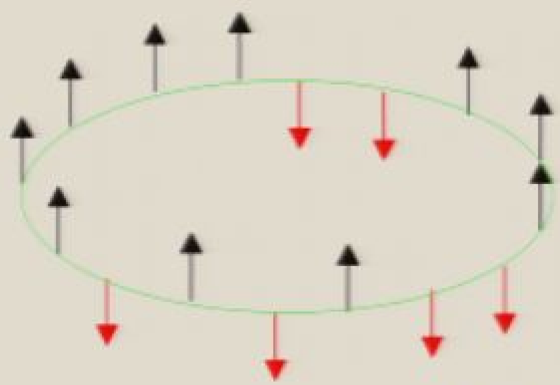
J_1 magnons:



$$|p_1 \dots p_{J_1}\rangle \neq \frac{1}{L^{J_1/2}} \prod_{j=1}^{J_1} \sum_{\ell_j=1}^{L} e^{ip_j \ell_j} |\ell_1 \dots \ell_{J_1}\rangle$$

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(asymptotic only)

Beisert,Dipple,Staudacher

$$x(u) = \frac{1}{2}\left(u + \sqrt{u^2 - 2g^2}\right)$$

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Trace condition: $\sum_j p_j = 2\pi n$

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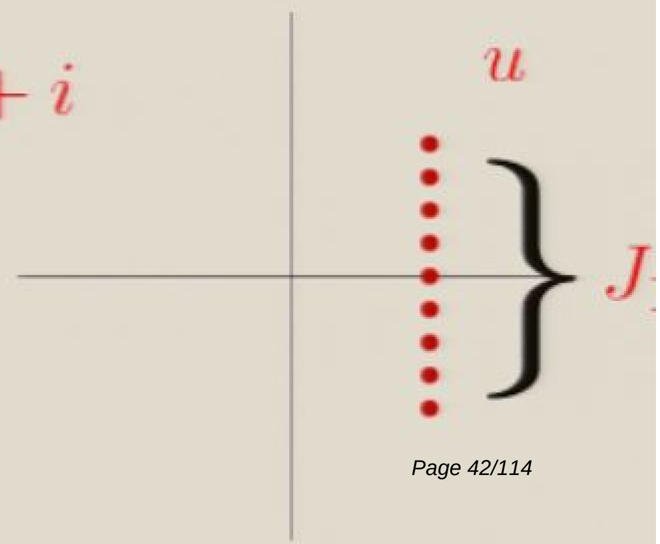

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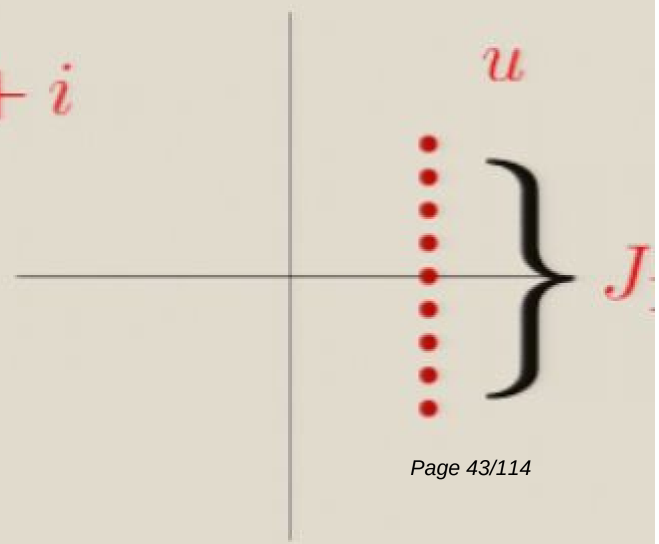

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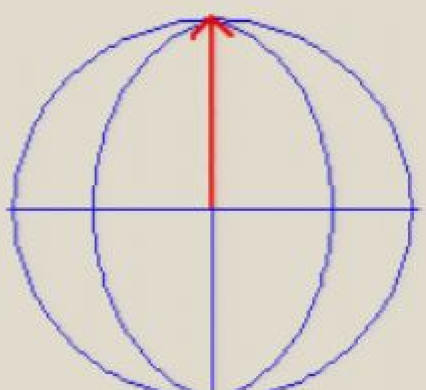

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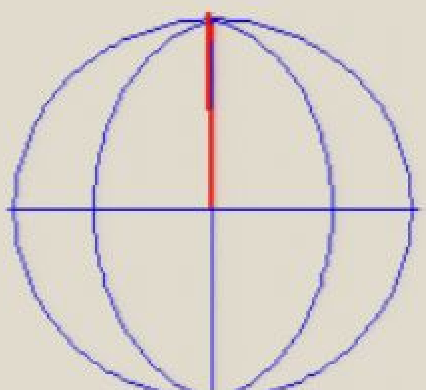

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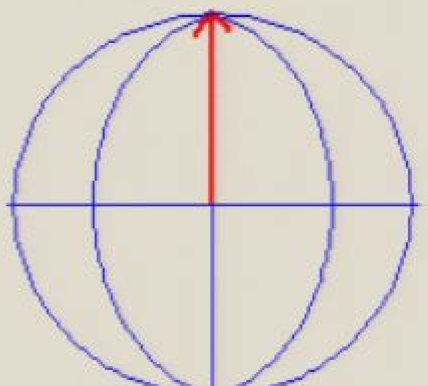

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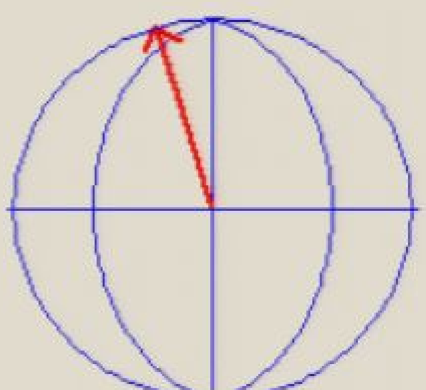

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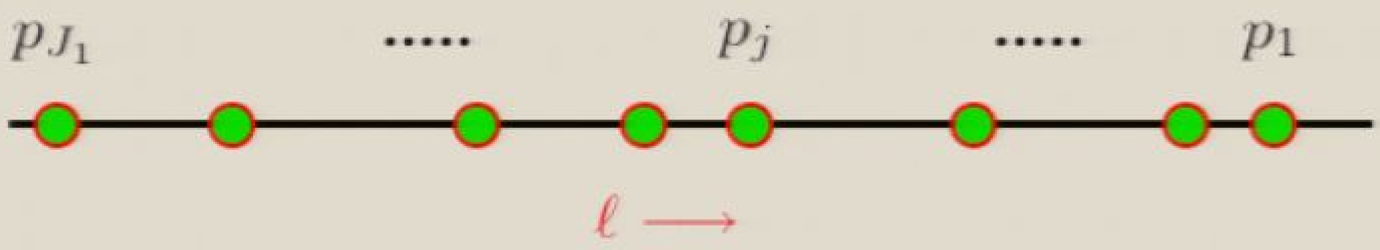

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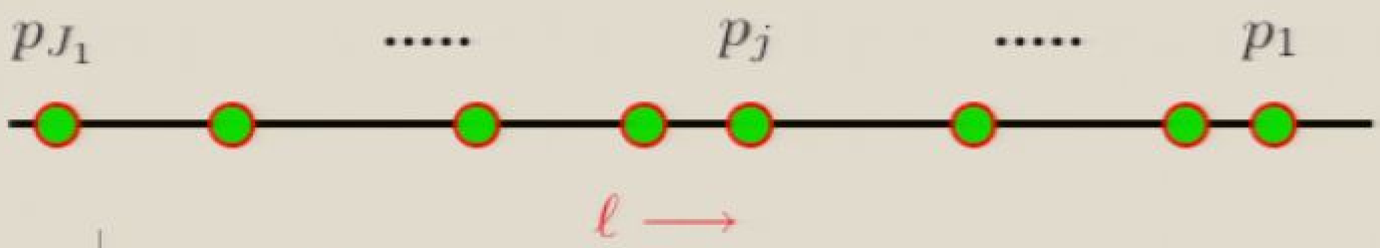

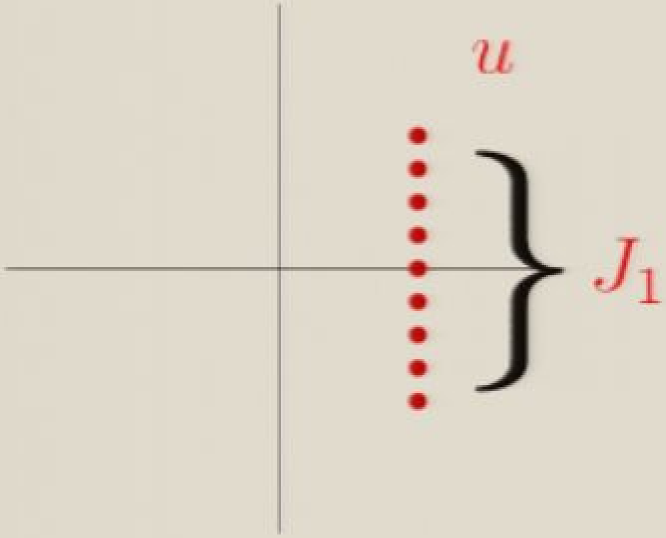

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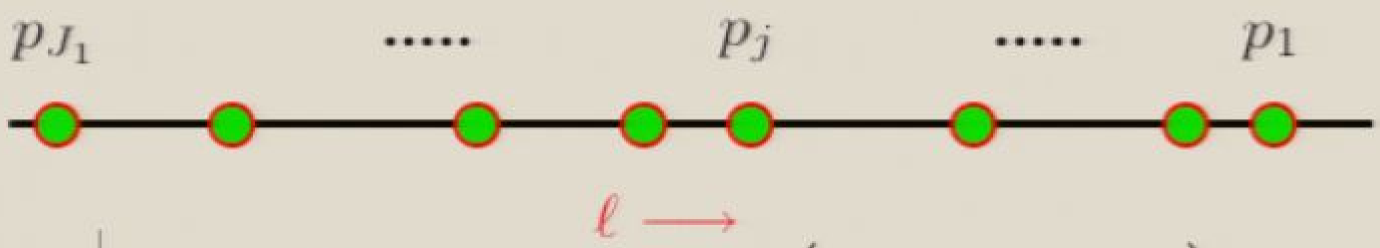

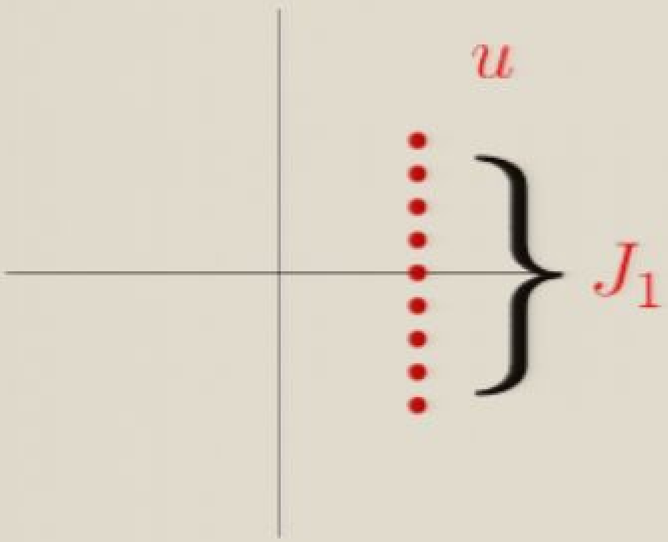

$$\psi \sim \exp\left(i \sum_j z_{j,j+1} Q_j \right) \qquad Q_j = \sum_{k=1}^{j} p_k$$

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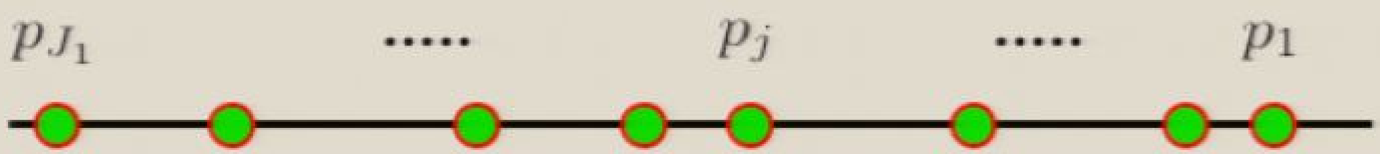
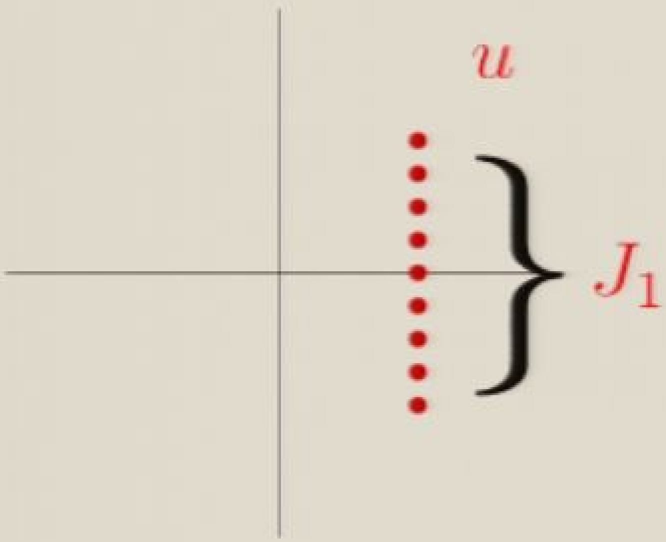


$\ell \longrightarrow$



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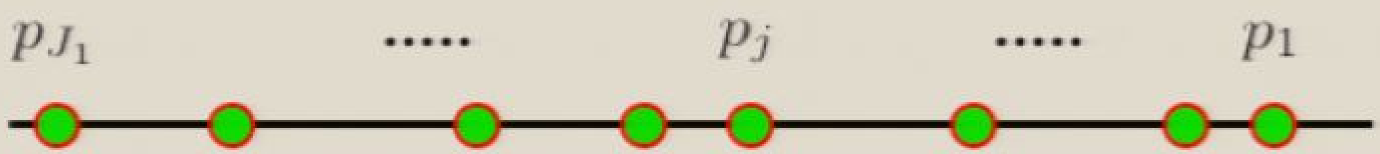

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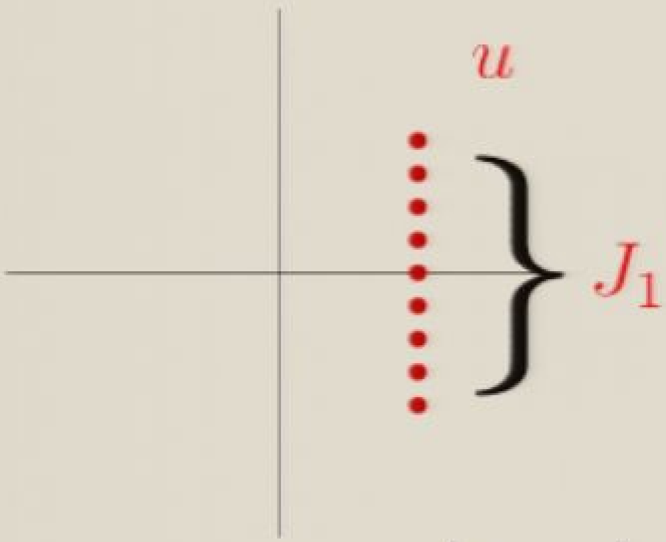

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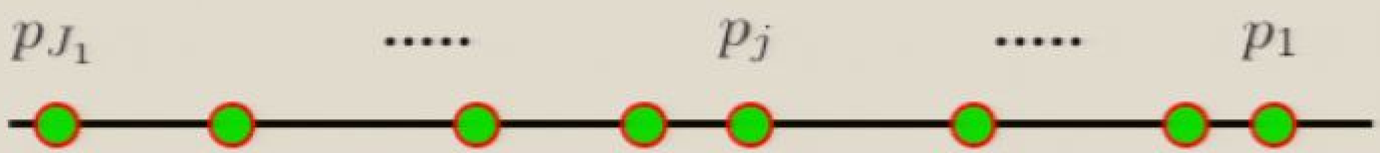

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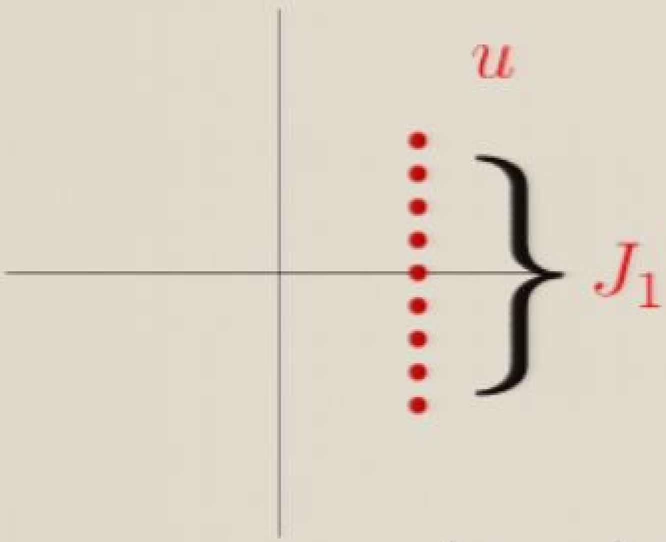

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$$\Rightarrow \quad \frac{dj}{d\ell} = \sin^2 \frac{p}{2} \frac{J_1^2 - 4j^2}{J_1^2} = \sin^2 \frac{p}{2} \operatorname{sech}^2 \frac{\ell}{\Gamma} \qquad \Gamma = \frac{J_1}{2 \sin^2 \frac{p}{2}}$$

Comparison to Strings

Comparison to Strings

Hofman & Maldacena
Dorey
Chen, Dorey & Okamura
Arutyunov, Frolov & Zamaklar
Tirziu, Tseytlin & JM
Spradlin & Volovich
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EOM: $\partial_\sigma \psi = \tan^2\theta \, \partial_\sigma \varphi$

$$\mathcal{L} = \frac{\sqrt{1-w^2}\sqrt{\lambda}}{2\pi} \int d\varphi \, \sqrt{r^2 + r'^2} \qquad r \equiv \sin\theta, \qquad r' = \frac{dr}{d\varphi}$$

Generalization of Hofman & Maldacena

Minimize: $r = \dfrac{\sin\theta_0}{\cos\varphi}$ $\mathcal{L} = \dfrac{\sqrt{\lambda}}{\pi}\sqrt{1-w^2}\,\sin\dfrac{p}{2}$ $\sin\dfrac{p}{2} = \cos\theta_0$

$-\pi/2 + \theta_0 \le \varphi \le \pi/2 - \theta_0$

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$$E - J_2 = \sqrt{J_1^2 + 8g^2\sin^2\frac{p}{2}}$$

Same as spin-chain (gauge theory)

SU(2)xSU(2) Current Algebra

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$$g = \begin{pmatrix} Z & W \\ -\bar{W} & \bar{Z} \end{pmatrix}$$

$$Z = \sin\theta e^{i\varphi_1}$$

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$$j_{\alpha,R} = i g^{-1}\partial_\alpha g$$

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$$\text{Let} : d\ell = dE \quad j^{0,3}_{\substack{R \\ L}} = 1 - \frac{1}{1-k^2}(1 \pm w)\cos^2\theta$$

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$$q_R = \frac{J_R}{J_R + J_L}\frac{1}{1-k^2}$$

Giant Magnons in Finite Gap Equations

unimodularity

Monodromy Matrix: $\Omega(\mathrm{x})$ $\qquad \mathrm{Tr}\,\Omega(\mathrm{x}) = 2\cos P(\mathrm{x})$

spectral parameter

quasimomentum

Poles: $P(\mathrm{x}) = -\dfrac{E/4}{\mathrm{x} \pm \frac{g}{\sqrt{2}}} + \dots \qquad (\mathrm{x} \to \mp \dfrac{g}{\sqrt{2}})$

Giant magnons are condensates

Condensates:

$P(\mathrm{x}) \to P(\mathrm{x}) + 2\pi$

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Resolvent: $G(\mathrm{x}) = P(\mathrm{x}) + \dfrac{E/4}{\mathrm{x} + \frac{g}{\sqrt{2}}} + \dfrac{E/4}{\mathrm{x} - \frac{g}{\sqrt{2}}}$

Kazakov, Marshakov, Zarembo & JM

$$G(\mathrm{x}) = \sum_k \int_{\mathcal{B}_k} d\mathrm{x}' \frac{\rho(\mathrm{x}')}{\mathrm{x} - \mathrm{x}'} \qquad \rho(u) = -i$$

Condensate contour

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Single magnon only: Relax momentum condition

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BDS
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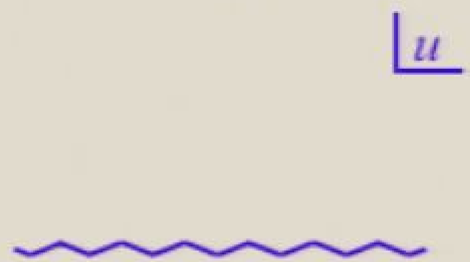

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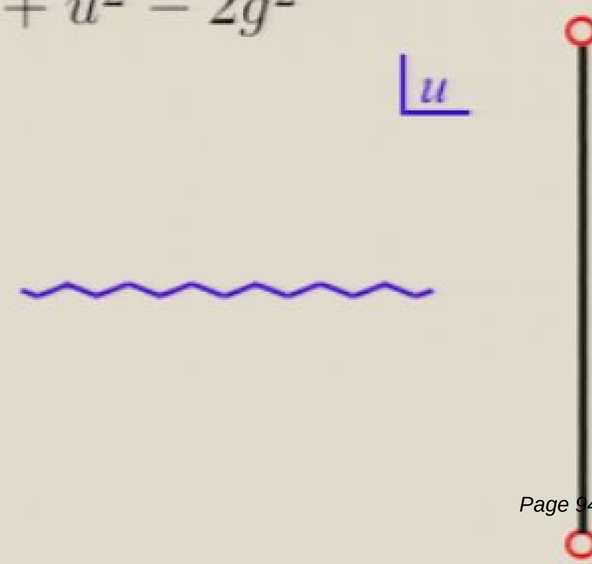

Decrease J_1
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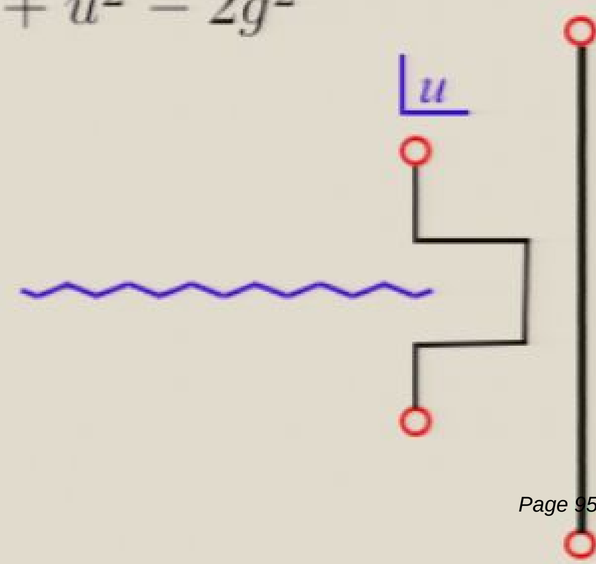

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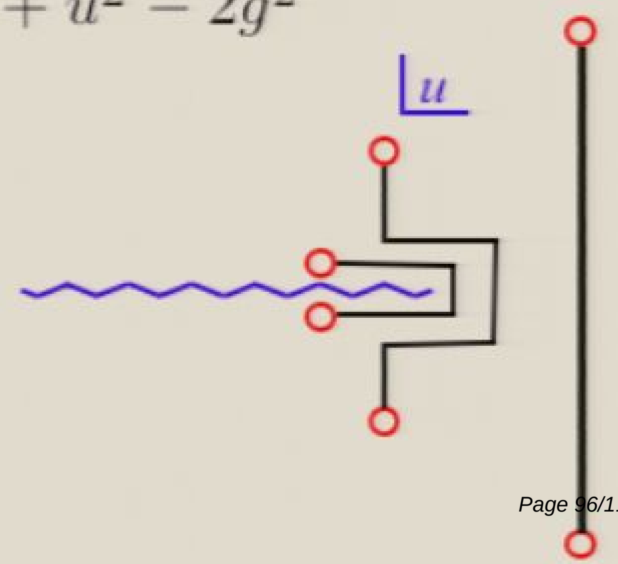

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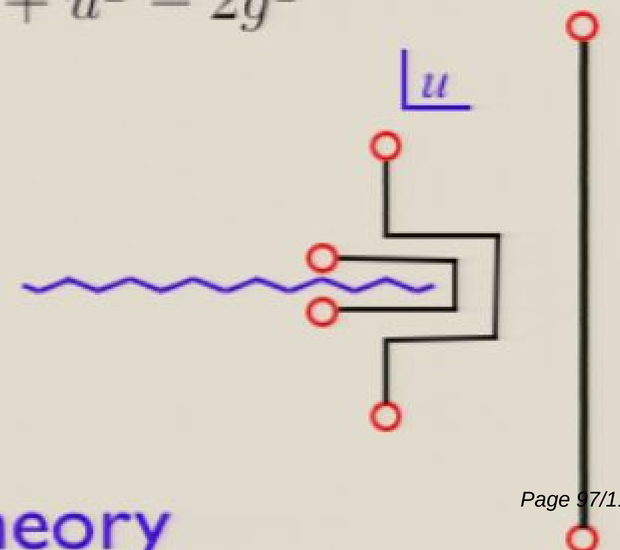
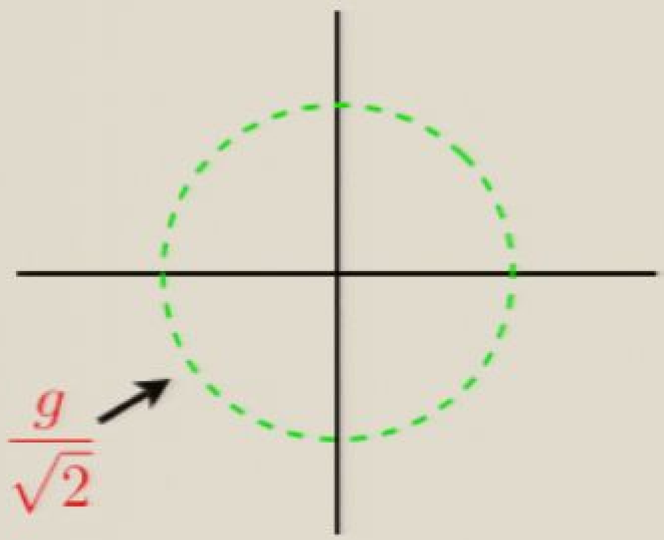


Same result as gauge theory

x plane



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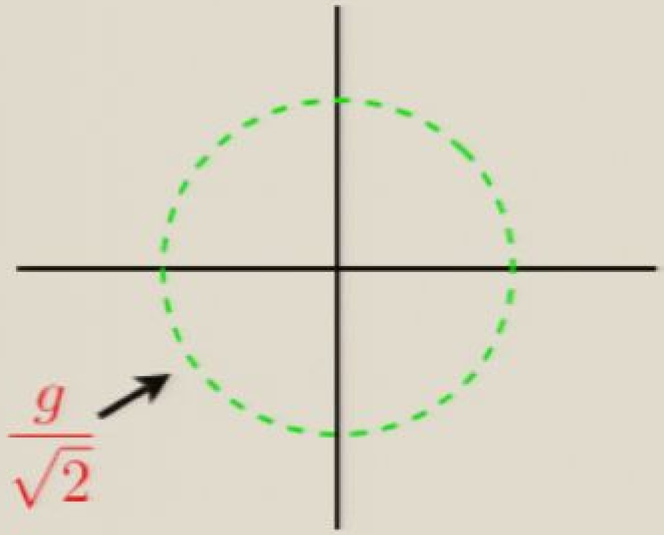

$$\lambda(u) = \frac{1}{2}\left(u + \sqrt{u^2 - 2g^2}\right)$$

u

$$x(u) = \frac{1}{2}\left(u + \sqrt{u^2 - 2g^2}\right)$$

$$u = x + \frac{g^2/2}{x}$$

$$x(u) = \frac{1}{2}\left(u + \sqrt{u^2 - 2g^2}\right)$$

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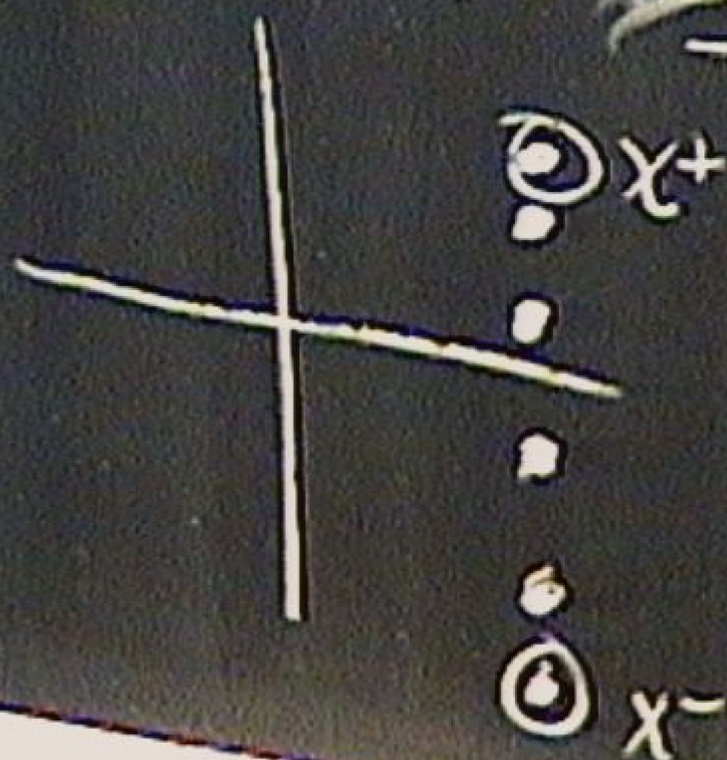
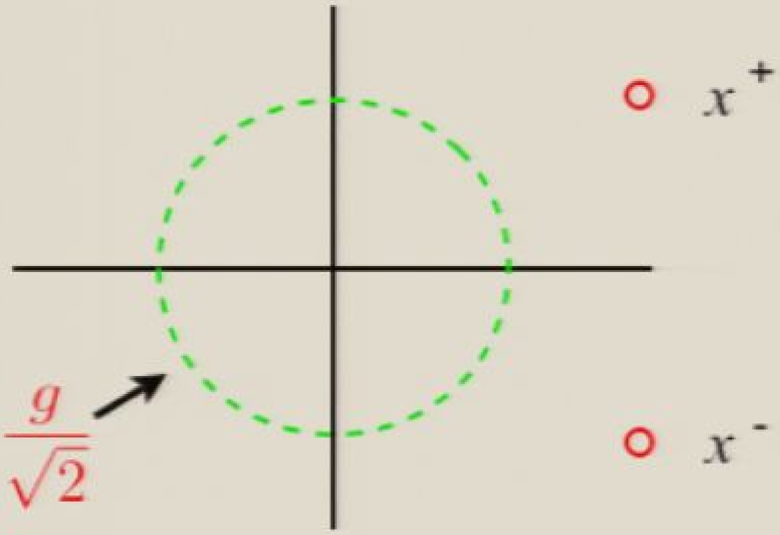
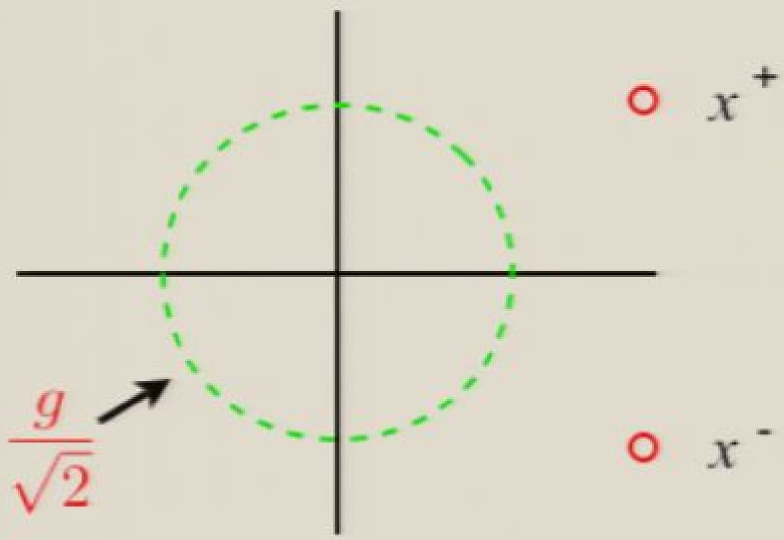


x plane



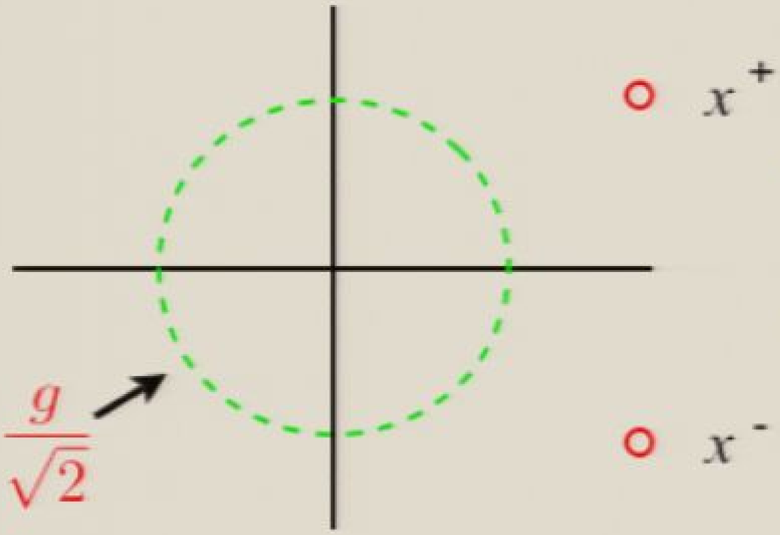
x plane



$$-i\,(x^+ - x^-) - i\left(\frac{g^2/2}{x^+} - \frac{g^2/2}{x^-}\right) = J_1$$

$$-i\,(x^+ - x^-) + i\left(\frac{g^2/2}{x^+} - \frac{g^2/2}{x^-}\right) = E - J_2$$

x plane



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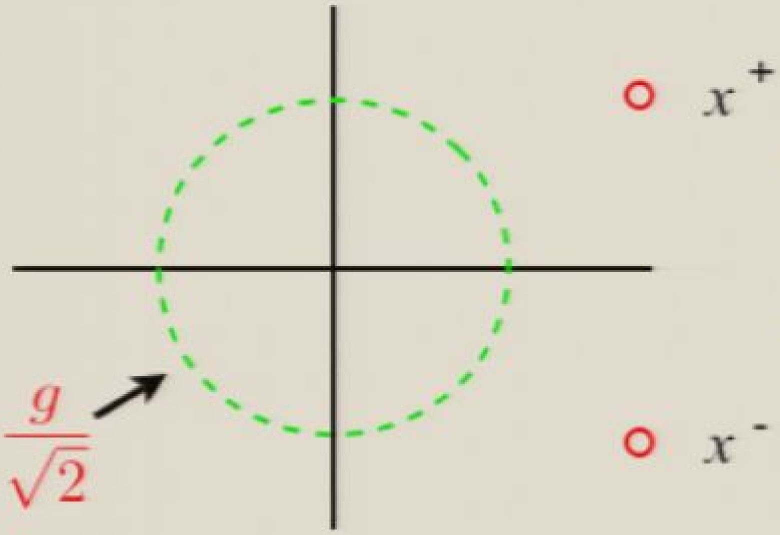

$$-i\left(x^+ - x^-\right) - i\left(\frac{g^2/2}{x^+} - \frac{g^2/2}{x^-}\right) = J_1$$

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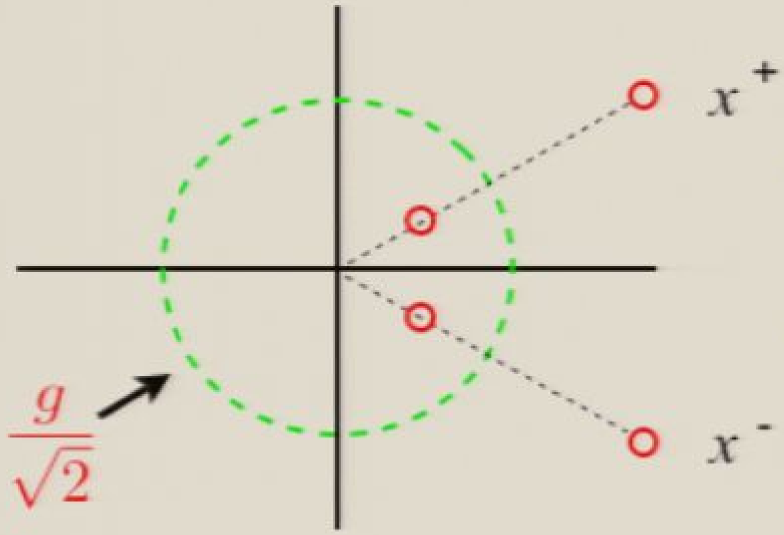
$$J_R = -i\left(x^+ - x^-\right)$$

$$J_L = +i\left(\frac{g^2/2}{x^+} - \frac{g^2/2}{x^-}\right)$$

x plane



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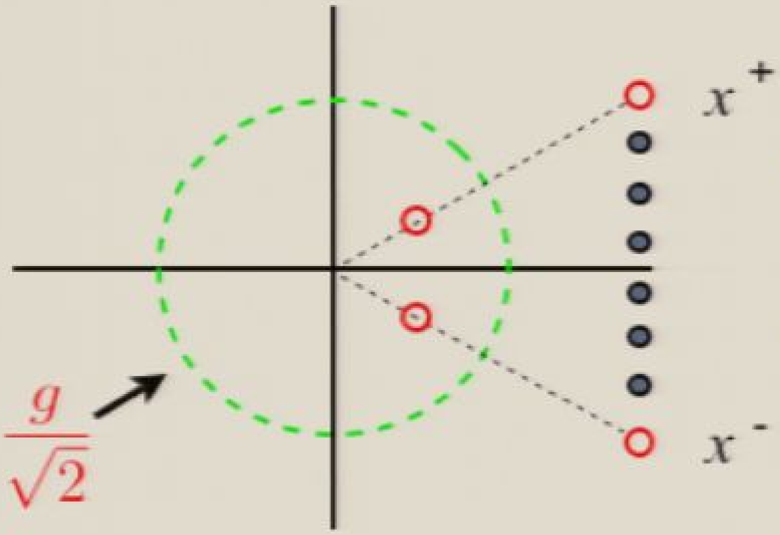

$$-i\left(x^{+}-x^{-}\right)-i\left(\frac{g^2/2}{x^+}-\frac{g^2/2}{x^-}\right)=J_1$$

$$-i\left(x^{+}-x^{-}\right)+i\left(\frac{g^2/2}{x^+}-\frac{g^2/2}{x^-}\right)=E-J_2$$

$$J_R=-i\left(x^{+}-x^{-}\right)$$

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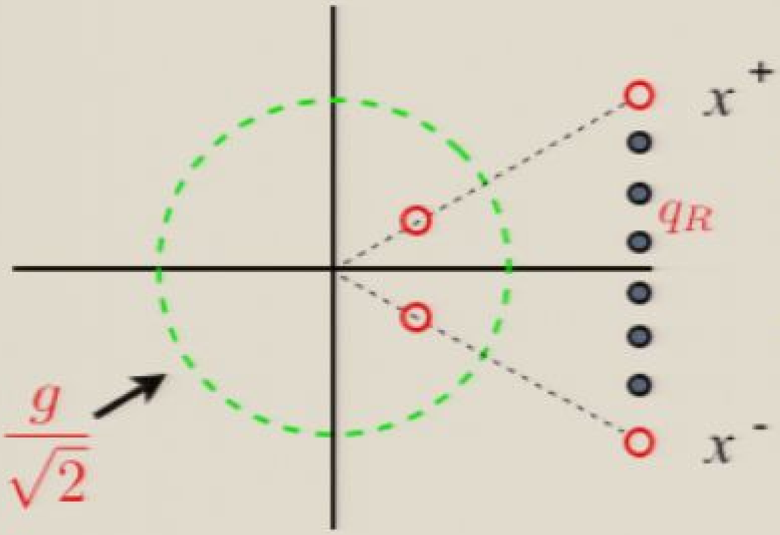

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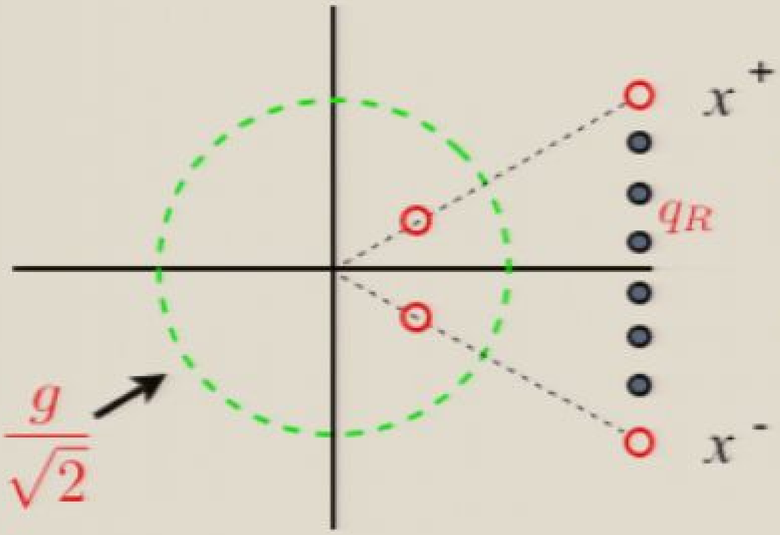
q_R is the down spin for each impurity

x plane



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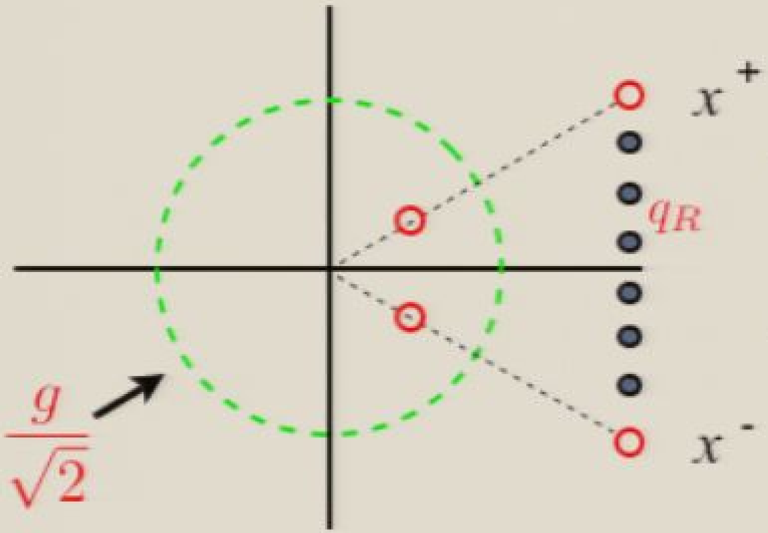
q_R is the down spin for each impurity

But q_R is not an integer??

x plane



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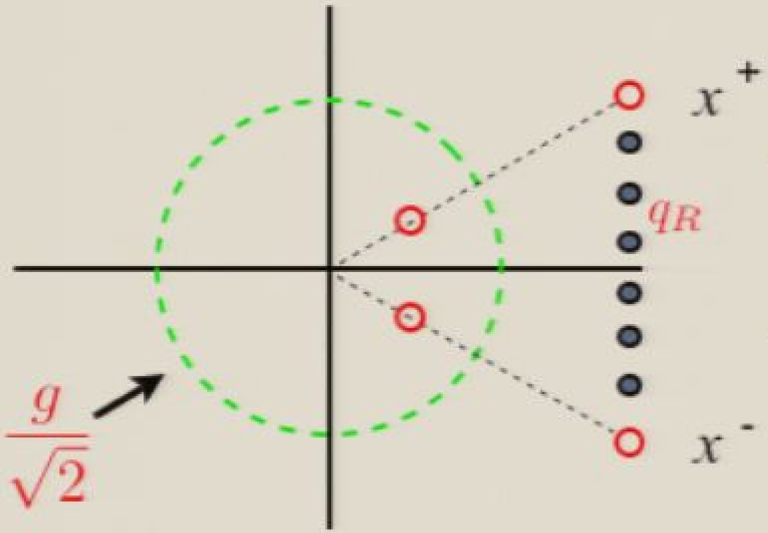

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q_R is the down spin for each impurity

But q_R is not an integer??

Conclusions

- Magnons are useful for comparing string and gauge theory
- Giant magnons in string theory have properties of being bound states
- q_R is in general noninteger. Not clear what to do with this.

The End/Fin