Title: Supersymmetry and Janus

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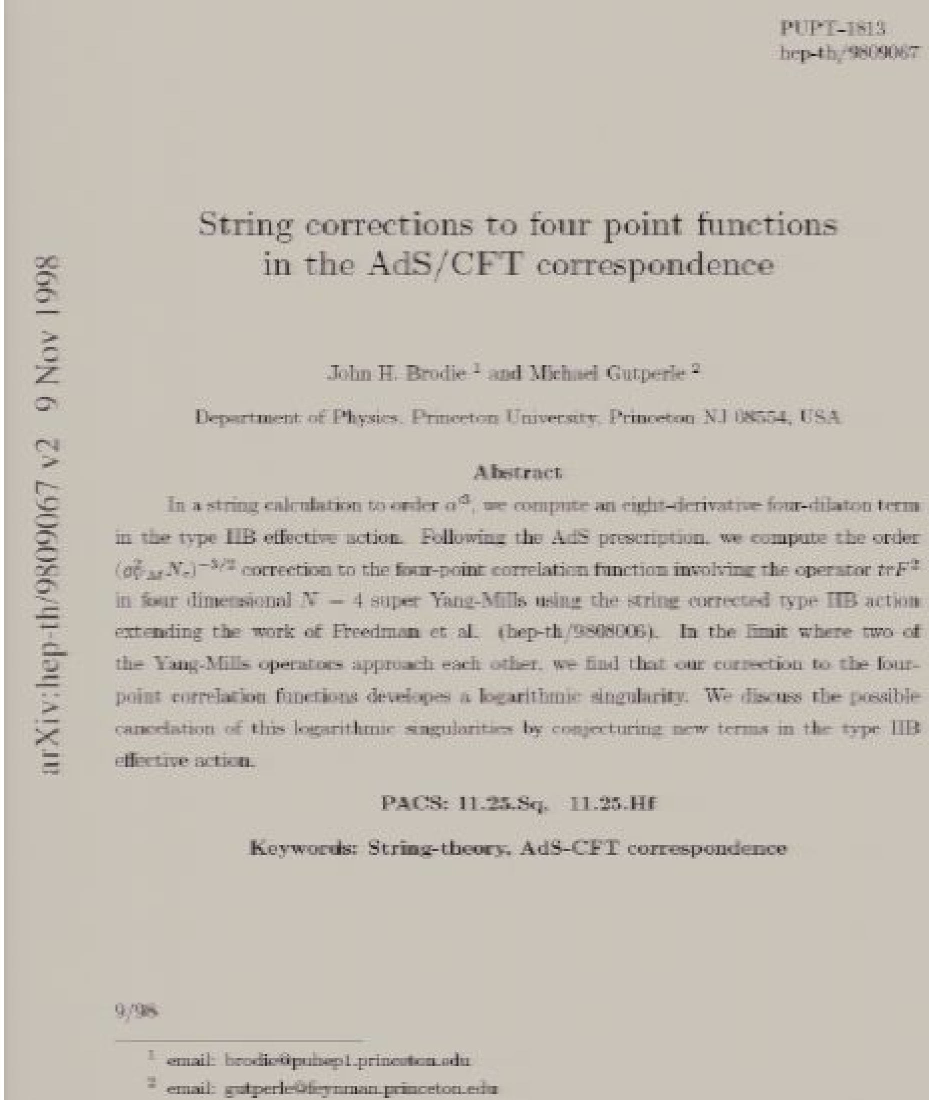
Abstract:



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String corrections to four point functions in the AdS/CFT correspondence

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Abstract

In a string calculation to order α'^3, we compute an eight-derivative four-dilaton term in the type IIB effective action. Following the AdS prescription, we compute the order $(g^2_{YM} N_c)^{-3/2}$ correction to the four-point correlation function involving the operator trF^2 in four dimensional $N = 4$ super Yang-Mills using the string corrected type IIB action extending the work of Freedman et al. (hep-th/9808006). In the limit where two of the Yang-Mills operators approach each other, we find that our correction to the four-point correlation functions developes a logarithmic singularity. We discuss the possible cancelation of this logarithmic singularities by conjecturing new terms in the type IIB effective action.

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Keywords: String-theory, AdS-CFT correspondence

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John and I worked on AdS/CFT in 1998 while we were both in Princeton

We finished the paper in Aspen in the summer

I will always cherish the memories of working with him, going on hikes and talking this summer.

Supersymmetry and Janus

Michael Gutperle (UCLA)

based on hep-th/0304129
with D. Bak (Seoul) and S. Hirano (NBI)

and on hep-th/0603013 and hep-th/0603012
with E. D'Hoker (UCLA) and J. Estes (UCLA)

– Typeset by FoilTEX –

Plan of the talk

- Introduction
- Janus solution and its relation to interface CFT
- Classification of Supersymmetric Interface Super Yang-Mills theories
- N=1 Supersymmetric Janus solution in ten dimensions
- Conclusions

Introduction

- The AdS/CFT correspondence relates string theory on a AdS space to a CFT on the boundary of AdS.
- The best studied example relates IIB string theory on $AdS_5 \times S_5$ and $\mathcal{N} = 4\ SU(N)$ Yang-Mills theory.
- The duality is expected to hold in less symmetric situations, corresponding to deformations of the original correspondence.

Janus solution and interface CFT

Janus solution is a simple dilatonic deformation of the $AdS_5 \times S_5$ background of IIB supergravity.

Named after the two faced Roman god of beginnings, endings, doors and string dualities.

The Janus solution is a deformation of $AdS_5 \times S_5$ where the dilaton is non constant and one parameterizes the noncompact space using AdS_4 slices

$$ds^2 = f(\mu)\left(d\mu^2 + ds^2_{AdS_4}\right) + ds^2_{S^5}$$

The five form and dilaton are given by

$$F_5 = 2f(\mu)^{\frac{5}{2}} d\mu \wedge \omega_{AdS_4} + 2\omega_{S^5}, \quad \phi = \phi(\mu)$$

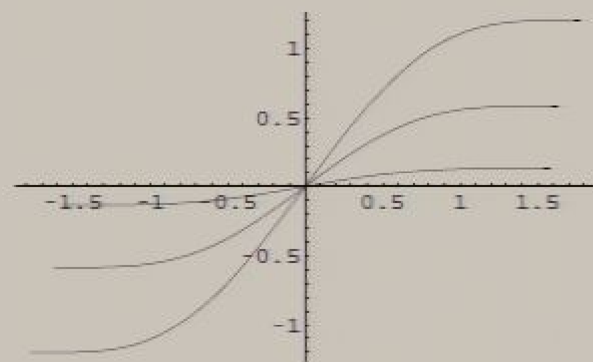
The undeformed AdS_5 is given by $f(\mu) = 1/\cos^2(\mu)$ and $\phi = const$. The coordinate μ ranges from $\mu \in [-\pi/2, \pi/2]$.

The equation of motion can be reduced to

$$f'f' = 4f^3 - 4f^2 + \frac{c_0^2}{6}\frac{1}{f}, \quad \phi'(\mu) = \frac{c_0}{f^{\frac{3}{2}}(\mu)}$$

For a nonzero c_0 the coordinate μ ranges from $\mu = [-\mu_0, \mu_0]$ with $\mu_0 > \pi/2$.

The dilaton ϕ approaches two constants values near $\mu = \pm\mu_0$



The Janus solution is 'fat' dilatonic domain wall with a AdS_4 world-volume.

All supersymmetries are broken, since the dilatino variation is always nonzero

$$\delta\lambda \;=\; iP_M\Gamma^M\mathcal{B}^{-1}\varepsilon^* - \frac{i}{24}\Gamma^{MNP}G_{MNP}\varepsilon$$

The solution is nevertheless stable against a large class of perturbations [Freedman et al. hep-th/0312055].

Near $\mu = \pm\mu_0$ the metric has the following asymptotic behavior in global coordinates for AdS_4

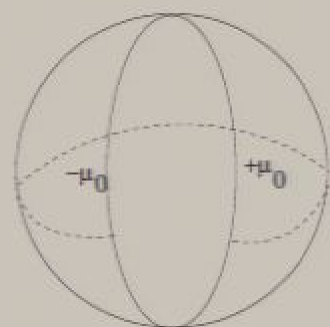
$$ds^2 \sim \frac{1}{(\mu \mp \mu_0)^2\cos^2\lambda}\big(\cos^2\lambda d\mu^2 - dt^2 + d\lambda^2 + \sin^2\lambda d\Omega^2_{S_2}\big)$$

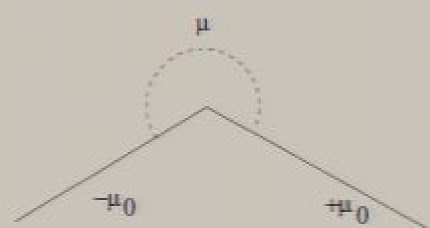
in Poincare coordinates for AdS_4

$$ds^2 \sim \frac{1}{(\mu \mp \mu_0)^2 z^2}(z^2 d\mu^2 - dt^2 + dx_1^2 + dx_2^2)$$

In global coordinates the boundary consists of two halves of S^3 at $\mu = \pm\mu_0$, joined at the pole of S_3 where $\lambda = \pi/2$.

In Poincare coordinates the spatial section of the boundary consists of two three dimensional half planes joined by a two dimensional interface.





The dilaton behaves as

$$\lim_{\mu \to \pm\mu_0} \phi(\mu) = \phi_\pm^{(0)} + \phi_\pm^{(1)} (\mu \mp \mu_0)^4 + \cdots$$

Interface conformal field theory

The standard field operator mapping of AdS/CFT relates the constant part of the dilaton $\phi \sim z^{\Delta-4}$ with the insertion of the operator dimension $\Delta = 4$ operator $\mathcal{L}' = trF^2 + \cdots$.

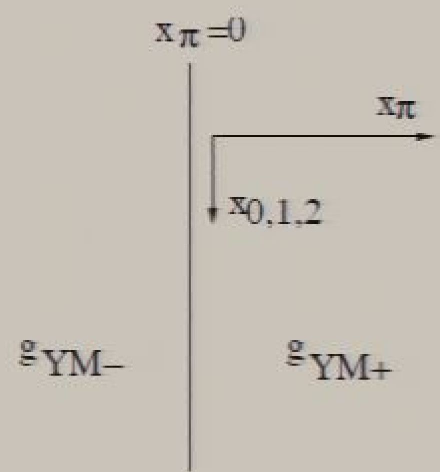
In Poincare coordinates the boundary corresponds to two half spaces. Since the action of SYM theory is given by

$$S = \int d^4x \frac{1}{g_{YM}^2} \mathcal{L}'$$

The dual of the Janus solution can be viewed as a theory where the YM coupling constant makes a jump across the interface

$$g_{YM}(x^\pi) = g_{YM}^{(0)}(1 + \epsilon\Theta(x^\pi))$$

where x^π is the coordinate normal to the three dimensional defect.



The position dependent gauge coupling breaks all supersymmetry, since the supersymmetry variation of the Lagrangian is

$$\delta\mathcal{L} = \frac{1}{g^2}\left(\partial_\kappa X^\kappa - (\partial_\kappa \bar{\zeta}) S^\kappa\right)$$

Nevertheless the theory inherits many properties of the original N=4 theory, and several checks of the correspondence have been performed [Freedman et al. hep-th/0407073].

There are no degrees of freedom localized at the interface $x^\pi = 0$. The presence of the interface breaks the conformal $SO(4,2)$ symmetry down to $SO(3,2)$ (just as in the case of a boundary CFT). The $SU(4)$ R-symmetry is unbroken by the interface CFT.

Classification of supersymmetric Interface CFTs

The original Janus solution breaks all supersymmetry. In Freedman et al. hep-th/0407073, it was shown that on the field theory side some supersymmetry can be restored by adding 'Interface counterterms'.

The theory was written in terms of $N = 1$ chiral and vector superfields. The interface terms break the R-symmetry to $SU(3)$ and there is one unbroken supersymmetry subject to the spinor projection

$$(1 + i\gamma^5\gamma^\pi)\epsilon = 0$$

The theory exhibits $N = 1$ interface susy, meaning 2 real unbroken supercharges.

In the following we want to answer the questions systematically

- Is the solution of Freedman et al. the only solution ?
- Can interface counterterms be added to get theories with more supersymmetry ?
- Can one classify all interface theories with supersymmetry ?

The action of $N = 4$ Super Yang-Mills is given by

$$\begin{aligned}\mathcal{L}_0 \;=\; & -\frac{1}{4g^2}\mathrm{tr}\left(F^{\mu\nu}F_{\mu\nu}\right) - \frac{1}{2g^2}\mathrm{tr}(D^\mu\phi^i D_\mu\phi^i) + \frac{1}{4g^2}\mathrm{tr}([\phi^i,\phi^j][\phi^i,\phi^j]) \\ & -\frac{i}{2g^2}\mathrm{tr}\left(\bar{\psi}\gamma^\mu D_\mu\psi\right) + \frac{i}{2g^2}\mathrm{tr}\left(D_\mu\bar{\psi}\gamma^\mu\psi\right) \\ & +\frac{1}{2g^2}\mathrm{tr}\left(\psi^t\mathcal{C}\rho^i[\phi^i,\psi] + \psi^\dagger\mathcal{C}(\rho^i)^*[\phi^i,\psi^*]\right)\end{aligned}$$

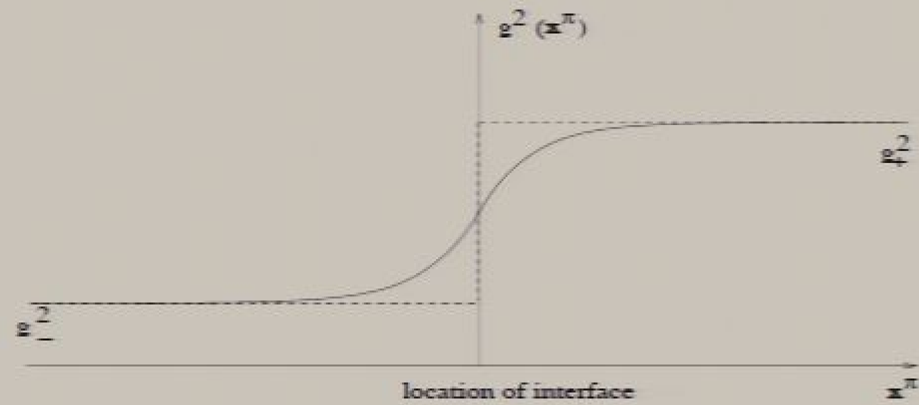
Where ρ^i, $i = 1,\cdots,6$ are the $SU(4)$ Clebsch-Gordan coefficients (SO(6)gamma matrices). The scalars ϕ^i and spinors ψ transform as a $\mathbf{6}$ and $\mathbf{4}$ of the SU(4) R-symmetry respectively. The theory has a $N = 4$ superconfomal invariance.

The supersymmetry transformations are given by

$$
\begin{aligned}
\delta_0 A_\mu &= i\bar{\psi}\gamma_\mu\zeta - i\bar{\zeta}\gamma_\mu\psi \\
\delta_0 \phi^i &= i\zeta^t \mathcal{C}\rho^i\psi + i\bar{\psi}\mathcal{B}(\rho^i)^*\zeta^* \\
\delta_0 \psi &= \frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\zeta + (D_\mu\phi^i)\gamma^\mu\mathcal{B}(\rho^i)^*\zeta^* - \frac{i}{2}[\phi^i,\phi^j]\rho^{ij}\zeta
\end{aligned}
$$

The original interface theory is given by a space dependent coupling constant $g_{YM}(x^\pi)$, which makes a discontinuous jump at $x^\pi = 0$. The interface counterterms are localized at $x^\pi = 0$. The interface will preserve $SO(2,3)$.

In order to avoid technical complications we introduce a smooth function $g_{YM}(x^\pi)$. This breaks the conformal symmetry.



One checks for the existence of 2+1 dim Poincare supersymmetry, which in the limit $g_{YM}(x^\pi) \to g_{YM}(1 + \Delta g_{YM}\Theta(x^\pi))$ to superconformal symmetry.

The Lagrangian is modified by terms proportional to $\partial_\pi g_{YM}$and $(\partial_\pi g_{YM})^2$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{interface}$$

where $\mathcal{L}_{interface} = 0$ for constant g_{YM} and contains all terms of scaling dimension 3 consistent with gauge symmetry and 2+1 dimensional Poincare invariance

In the superconformal limit $\partial_\pi g_{YM} \sim \delta(x^\pi)$ and the interface terms are localized at $x^\pi = 0$.

The supersymmetry transformations are modified

$$\delta\Phi = \delta_0\Phi + \delta_{interface}\Phi$$

The condition of interface supersymmetry then reads that the complete susy variation of the new Lagrangian vanishes

$$\delta\mathcal{L} = \delta_0\mathcal{L}_0 + \delta_0\mathcal{L}_{int} + \delta_{int}\mathcal{L}_0 + \delta_{int}\mathcal{L}_{int} = 0$$

The variation of the original action can be expressed in terms of the supercurrent and Noether variation of the original action.

$$-\frac{1}{g^2}(\partial_\pi \bar{\zeta})S^\pi + 2\frac{\partial_\pi g}{g^3}X^\pi + \delta_0 \mathcal{L}_{int} + \delta_{int}\mathcal{L}_0 + \delta_{int}\mathcal{L}_{int} = 0$$

up to total derivatives

The complete set of possible interface operators is given by

$$
\begin{aligned}
\mathcal{L}_\psi &= \frac{(\partial_\pi g)}{g^3}\mathrm{tr}\Big(y_1\bar\psi\gamma^\pi\psi + \frac{i}{4}y_2^{ij}\bar\psi\gamma^\pi\rho^{ij}\psi \\
&\quad -\frac{i}{2}y_3^{ijk}\left(\psi^t\mathcal{C}\rho^{ijk}\psi + \psi^\dagger\mathcal{C}(\rho^{ijk})^*\psi^*\right)\Big) \\
\mathcal{L}_\phi &= \frac{(\partial_\pi g)}{2g^3}\mathrm{tr}\left(z_1^{ij}\partial_\pi(\phi^i\phi^j) + 2z_2^{ij}\phi^{[i}D_\pi\phi^{j]} - iz_3^{ijk}\phi^i[\phi^j,\phi^k]\right) \\
\mathcal{L}_{\phi^2} &= \frac{(\partial_\pi g)^2}{2g^4}z_4^{ij}\,\mathrm{tr}\left(\phi^i\phi^j\right)
\end{aligned}
$$

with respect to the SU(4) R-symmetry the operators transform as

$$
\begin{array}{llll}
y_1 : \mathbf{1} & z_1 : \mathbf{1}\oplus\mathbf{20'} & z_4 : \mathbf{1}\oplus\mathbf{20'} & \\
y_2 : \mathbf{15} & z_2 : \mathbf{15} & y_3 : \mathbf{10}\oplus\overline{\mathbf{10}} & z_3 : \mathbf{10}\oplus\overline{\mathbf{10}}
\end{array}
$$

The interface supersymmetry transformations are

$$\delta_{int}\phi^i = \delta_{int}A_\mu = 0, \quad \delta_{int}\psi = (\partial_\pi g)\chi^i\phi^i$$

with some arbitrary x^π dependent spinors χ^i

The vanishing of the variation can be decomposed by demanding that independent field combinations like $\phi^i\bar{\psi}$, $\phi^i\partial_\pi\bar{\psi}$, $F_{\mu\nu}\bar{\psi}$,, $D_\mu\phi^i\bar{\psi}$ and $[\phi^i, \phi^k]\bar{\psi}$.

The SO(2,1) Poincare symmetry can be used to further decompose the equations

for later convenience we defined

$$\begin{aligned} Y_2 &\equiv y_2^{ij}\rho^{ij} & Y_3 &\equiv -y_3^{ijk}\left(\rho^{ijk}\right)^* \\ Z_2 &\equiv z_2^{ij}\rho^{ij} & Z_3 &\equiv -z_3^{ijk}\left(\rho^{ijk}\right)^* \end{aligned} \qquad (1)$$

The equations for unbroken susy are given by

$$(1) \qquad \zeta = Y_3 \gamma^\pi \mathcal{B} \zeta^*$$

$$(2) \qquad Y_3 \rho^i \zeta = -(\rho^i)^* \gamma^\pi \mathcal{B} \zeta^* + z_1^{ij} (\rho^j)^* \gamma^\pi \mathcal{B} \zeta^*$$

$$(3) \qquad -\rho^{ij}\zeta + y_1 i \rho^{ij}\zeta + \rho^{ij}\frac{\partial_\pi \zeta}{\partial_\pi g} - \frac{1}{4} Y_2 \rho^{ij}\zeta + Y_3 (\rho^{ij})^* \gamma^\pi \mathcal{B}\zeta^* - 3 z_3^{ijk} (\rho^k)^* \gamma^\pi \mathcal{B} \zeta^*$$
$$+2 z_2^{ij}\zeta - (\rho^i)^* (z_1^{jk} + z_2^{jk}) \rho^k \zeta + (\rho^j)^* (z_1^{ik} + z_2^{ik}) \rho^k \zeta = 0$$

$$(4) \qquad \frac{\partial_\pi \zeta}{\partial_\pi g} = -i y_1 \zeta + \frac{1}{4} Y_2 \zeta$$

$$(5) \qquad \rho^i \frac{\partial_\pi \zeta}{\partial_\pi g} = i y_1 \rho^i \zeta + \frac{1}{4} Y_2^* \rho^i \zeta - z_2^{ij} \rho^j \zeta$$

$$(6) \qquad \chi^i = -(z_1^{ij} + z_2^{ij}) (\rho^j)^* \gamma^\pi \mathcal{B} \zeta^*$$

$$(7) \qquad (-i y_1 + \frac{1}{4} Y_2 - 1)(z_1^{ij} + z_2^{ij}) (\rho^j)^* \gamma^\pi \mathcal{B} \zeta^* - Y_3 (z_1^{ij} + z_2^{ij}) \rho^j \zeta$$

$$-z_4^{ij}(\rho^j)^*\gamma^\pi\mathcal{B}\zeta^* - (z_1^{ij}+z_2^{ij})(\rho^j)^*\frac{\partial_\pi\gamma^\pi\mathcal{B}\zeta^*}{\partial_\pi g} = 0$$

The $SU(4)$ symmetry SYM action and covariance of the interface Lagrangian allows to find the general solution to these complicated set of equations.

Firstly one can gauge away Y_2 and diagonalize Y_3

$$Y_2 = 0, \quad Y_3 = e^{i\theta}D_3 = e^{i\theta}\begin{pmatrix} 1 & & & \\ & a & & \\ & & b & \\ & & & c \end{pmatrix}$$

$$Z_2 \;\; = \;\; -4iy_1\left(I - 4\beta_1\otimes\beta_1^\dagger\right)$$

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$$Z_2 \;=\; -4iy_1\left(I - 4\beta_1 \otimes \beta_1^\dagger\right)$$

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\begin{aligned}
Z_3 &= 8Y_3 + 32iy_1\, e^{i\theta}\beta_1 \otimes \beta_1^t \\
z_4^{ij} &= -(z_1^{ik} + z_2^{ik})(z_1^{jk} + z_2^{jk})
\end{aligned}
$$

Where β is an eigenvector with unit eigenvalue of D_3. The number of linearly independent vectors β determines the number of unbroken supersymmetries.

The theory with $N = 1$ interface supersymmetry is given by choosing $a = b = c = 0$ and corresponds (after rescaling of fields) to the theory of Freedman et al.

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The theory with $N = 1$ interface supersymmetry is given by choosing $a = b = c = 0$ and corresponds (after rescaling of fields) to the theory of Freedman et al.

$$-z_4^{ij}(\rho^j)^*\gamma^\pi\mathcal{B}\zeta^* - (z_1^{ij}+z_2^{ij})(\rho^j)^*\frac{\partial_\pi\gamma^\pi\mathcal{B}\zeta^*}{\partial_\pi g} = 0$$

The $SU(4)$ symmetry SYM action and covariance of the interface Lagrangian allows to find the general solution to these complicated set of equations.

Firstly one can gauge away Y_2 and diagonalize Y_3

$$Y_2 = 0, \quad Y_3 = e^{i\theta}D_3 = e^{i\theta}\begin{pmatrix} 1 & & & \\ & a & & \\ & & b & \\ & & & c \end{pmatrix}$$

$$Z_2 \;=\; -4iy_1\left(I - 4\beta_1\otimes\beta_1^\dagger\right)$$

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The theory with $N = 1$ interface supersymmetry is given by choosing $a = b = c = 0$ and corresponds (after rescaling of fields) to the theory of Freedman et al.

There are theories with extended $N = 4$ and $N = 2$ interface supersymmetry where $y_1 = Y_2 = Z_2 = 0$ and D_3 given by

$$\begin{array}{llll} (\mathrm{I}) & b = c = 1 & e^{4i\theta} = 1 & SU(2) \times SU(2) \\ (\mathrm{II}) & b = c = 0 & \theta \text{ arbitrary} & SO(2) \times SU(2) \\ (\mathrm{III}) & b = c \neq 0, 1 & e^{4i\theta} = 1 & SO(2) \times SO(2) \end{array}$$

Theory (I) has $N = 4$ interface supersymmetry and (II,III) has $N = 2$ interface supersymmetry.

The theories with extended supersymmetry have a superconformal limit, a g_{YM} dependent rescaling of the scalar fields eliminates the $(\partial_\pi g_{YM})^2$ terms.

N=1 Supersymmetric Janus solution

The SYM analysis shows that there is a ten dimensional supersymmetric Janus solution which is dual to the supersymmetric interface theory of Freedman et al. The approach we are pursuing is to write down the most general ansatz consistent with the symmetries and solve the conditions for the existence of an unbroken supersymmetry.

The solution is of Janus type to guarantee the interface CFT structure.

The symmetry of the ansatz should respect the symmetries of the interface CFT

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The superpotential interface counterterms have dimension $\Delta = 3$ and are in the multiplet transforming as $\mathbf{10} \oplus \overline{\mathbf{10}}$ of SU(4). The operator field correspondence maps these operators to the second rank AST potential of IIB supergravity.

The $N = 1$ interface conformal field theory corresponds to one unbroken supersymmetry from the five dimensional supergravity point of view.

The $SU(3)$ symmetry can be implemented by noting that S_5 can be constructed as a $U(1)$ fibration over CP_2

$$ds^2_{S^5} = (d\beta + A_1)^2 + ds^2_{CP_2}$$

since $CP_2 = SU(3)/(SU(2) \times U(1))$ and $S_5 = SU(3)/SU(2)$.

The metric ansatz corresponds to Janus like slicing of AdS_5 and a squashing of the five sphere

$$ds^2 = f_4^2\left(d\mu^2 + ds^2_{AdS_4}\right) + f_1^2(d\beta + A_1)^2 + f_2^2 ds^2_{CP_2}$$

Where f_1, f_2, f_4 all depend on μ only.

The ansatz for the complex third rank AST is

$$G = a\, e^5 \wedge A_2 - ib\, e^4 \wedge A_2 + c\, e^5 \wedge \bar{A}_2 - id\, e^4 \wedge \bar{A}_2$$

Where a, b, c, d all depend only on μ

$$e^5 = f_1(\mu)(d\beta - A_1), \quad e^4 = f_4(\mu)d\mu$$

and $A_2 = dA_1$ is the Kaehler form on CP_2.

The fermionic fields are the dilatino λ and the gravitino ψ_M, are complex Weyl spinors. The supersymmetry variations of the fermions are

$$\begin{aligned}\delta\lambda &= iP_M\Gamma^M\mathcal{B}^{-1}\varepsilon^* - \frac{i}{24}\Gamma^{MNP}G_{MNP}\varepsilon \\ \delta\psi_M &= D_\mu\varepsilon + \frac{i}{480}F_{(5)NPQRS}\Gamma^{NPQRS}\Gamma_M\varepsilon \\ &\quad + \frac{1}{96}(\Gamma_M{}^{NPQ}G_{NPQ} - 9\Gamma^{NP}G_{MNP})\mathcal{B}^{-1}\varepsilon^*\end{aligned}$$

The gravitino variation implies $\zeta_- = 0$.

The dilatino variation for ζ_+ implies

$$(c+d)(c^* - d^*) = f^4|B'|^2(f_4)^{-2}$$

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$$\begin{aligned} \frac{f_2'}{f_2} &= \frac{f_4^2}{2f^2B'}(ac+bd) \\ \frac{f_1 f_4}{f_2^2} - f_4 f_5 &= \frac{f_4^2}{2f^2B'}(ad+bc) \end{aligned}$$

The fiber component gives

$$\begin{aligned} \frac{f_1'}{f_1} &= \frac{f_4^2}{2f^2B'}(3ac-bd) \\ 3\frac{f_4}{f_1} - 2\frac{f_1 f_4}{f_2^2} - f_4 f_5 &= \frac{f_4^2}{2f^2B'}(3ad-bc) \end{aligned}$$

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The AdS_4 component gives

$$\frac{f_1^2 f_4^2}{f_2^4} - \left(\frac{f_2'}{f_2} + \frac{f_4'}{f_4}\right)^2 = 1$$

It can be shown that solutions to these first order equations are also solutions to the equations of motion.

A simple solution of this system of equation can be obtained by setting $a = 0$. All functions of the solution can then be obtained from a single function ψ satisfying

$$(\psi')^2 = \left(1 + \frac{C_2^2}{9\rho^8}\psi^6\right)^2 - \psi^2$$

where ρ and C_2 are integration constants.

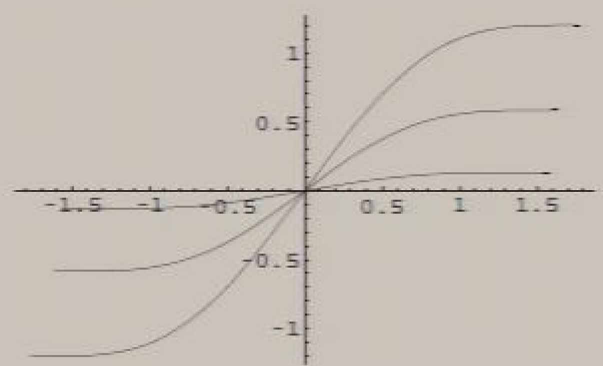
The relations for the metric functions f_1, f_2 and f_4 are

$$
\begin{aligned}
f_4^4 &= \frac{\rho^2}{\psi^4} + \frac{C_2^2}{9\rho^6}\psi^2 \\
f_2 &= \frac{\rho}{f_4 \psi}, \quad f_1 = \frac{\psi}{f_4}
\end{aligned}
$$

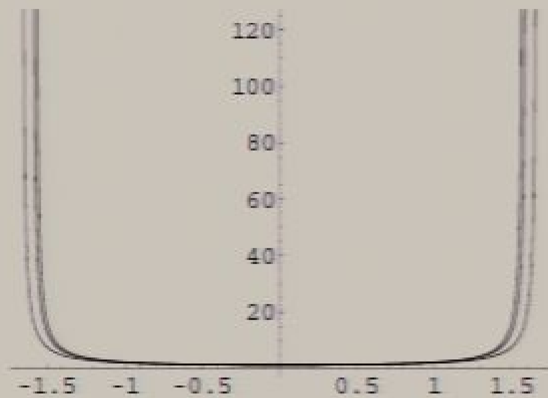
and similarly for the functions b, c, d of the AST.

This solution is presumably closely related the ten dimensional lift of the supersymmetric Janus solution found in five dimensional $N = 2$ gauged supergravity [Clark and Karch, :hep-th/0506265]

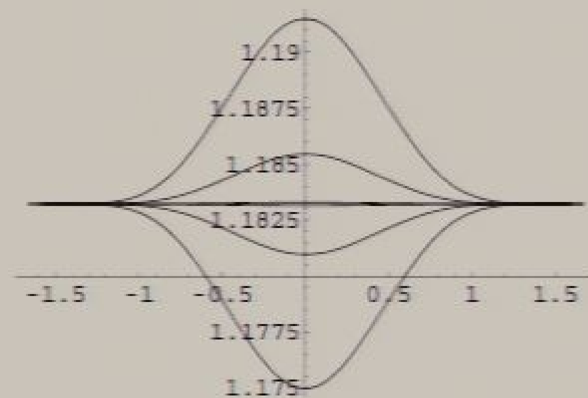
The equation can be easily solved numerically The dilaton is given by



the metric functions f_4 and f_1, f_2 are given by

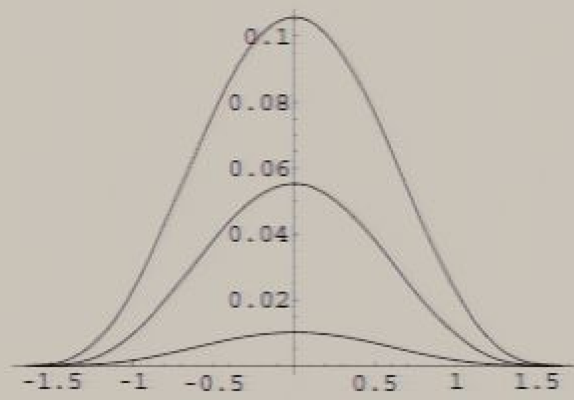


(a)



(b)

the functions c and d of the AST are given by



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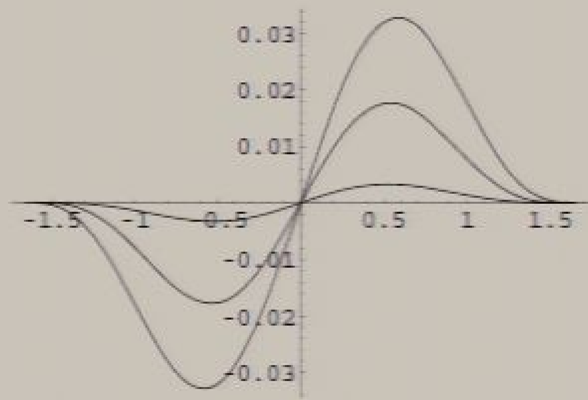

(b)

The AST field does indeed correspond to a interface counterterm localized on the defect:

For the Poincaré metric of Euclidean AdS_5

$$ds^2 = \frac{1}{z^2}\left(dz^2 + \sum_i dx_i^2\right)$$

Near the boundary of AdS_5, where $z \to 0$, a scalar field Φ_m of mass m behaves as

$$\Phi_m(z,x) \sim \phi_{non-norm}(x) z^{4-\Delta} + \phi_{norm}(x) z^{\Delta}$$

The metric of the Janus solution has a slightly more complicated asymptotic structure

$$ds^2 = \frac{1}{(\mu \mp \mu_0)^2 z^2}\left(dz^2 + \sum_{i=1}^{3} dx_i^2 + z^2 d\mu^2\right) + \cdots$$

the boundary is reached by $\epsilon^2 = (\mu \mp \mu_0)^2 z^2 \to 0$. The AST field satisfies

$$c(\mu) \;=\; const(\mu_0 \mp \mu)^3 + \cdots$$

Which corresponds to a dimension $\Delta = 3$ operator in the CFT. The non normalizable part, gives the operator insertion

$$\begin{aligned} c_{non-norm} &= \lim_{\epsilon \to 0} \epsilon^{\Delta - 4} c(\mu) \\ &= \lim_{\epsilon \to 0} \frac{const}{\epsilon}(\mu \mp \mu_0)^3 \end{aligned}$$

$$= \lim_{(\mu \mp \mu_0) z \to 0} const \frac{(\mu \mp \mu_0)^2}{z}$$

Which vanishes away from the interface where $z \neq 0$. I.e at the boundary $\mu \to \pm\mu_0$ away from the defect. There is no operator source.

At the interface $z \to 0$ the operator source blows up, indicating a delta function source localized at the interface.

Conclusions

- The Janus solution is an interesting deformation of $AdS_5 \times S_5$ solution leading to a dual interface CFT
- A complete classification of supersymmetric interface CFT's was given
- A g_{YM} dependent rescaling of the scalar fields allows a superconformal limit for all supersymmetric interface theories.
- The interface theory of Freedman et al is the only $N = 1$ (up to SU(4) rotations).
- Interface CFTs with $N = 2$ and $N = 4$ supersymmetry were found.