Title: Why is general relativity a geometric theory?

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Abstract: <span>Bimetric theories of gravitation, whether empirically correct or not, are a reminder that a dynamical metric field need not have chrono-geometric significance: its null geodesics need not characterize the motion of light, nor need it be surveyed by physical rods and clocks. In standard GR, the chronometric nature of the metric field is a consequence of the strong equivalence principle, which is not a consequence of the Einstein field equations. It is argued that in understanding the special theory of relativity as the appropriate limit of general relativity, the interpretation of special relativity that best tallies with the above insight is the dynamical one defended by Pauli, JÃ¡nossy, Bell and others.</span>

# Why is general relativity theory geometric in nature? II The role of the strong equivalence principle

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*Physical Relativity*, OUP 2005
*Eur. J. Phys.* **22**, S85-S90 (2005) (special Einstein issue).

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# General Relativity

# What is the $g_{\mu\nu}$ field?

". . . from the point of view of the principle of general invariance we need not interpret $g_{\mu\nu}$ as a metric, nor $R_{\mu\nu}$ as a Ricci tensor. . . . [Einstein's field equations] do not rest on such an interpretaton; one can show that they are the only dynamical equations of second differential order for a symmetric tensor [ $g_{\mu\nu}$ ] that are in accord with the principle of general invariance as we have interpreted it. . . . As in all physical theories we will look for consequences of . . . [the field equations] that will lead us to associate with some observable element of the physical world."

James Anderson 1967

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"A strong burst of gravitational waves could come from the sky and knock down the rock of Gibraltar, precisely as a strong burst of electromagnetic radiation could. Why is the . . . [second] "matter" and the . . . [first] "space"? Why should we regard the . . . [first] burst as ontologically different from the second? Clearly the distinction can now be seen as ill-founded. ...

Physical reality is now described as a complex interacting ensemble of entities (fields), the location of which is only meaningful with respect to one another. The relation among dynamical entities of being contiguous . . . is the foundation of the spacetime structure. Among these various entities, there is one, the gravitational field, which interacts with every other one and thus determines the relative motion of the individual components of every object we want to use as rod or clock. Because of that, it admits a metrical interpretation."

Carlo Rovelli 1997

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- A form of "bimetric" theory: involves two "metric" fields
- The pure gravitational field has no chronometric significance

# TeVeS dynamics I

- Fundamental entities: a tensor field $g_{\mu\nu}$, a vector field $\mathcal{U}_\mu$, and two scalar fields $\phi$ and $\sigma$, as well as the ordinary matter fields.
- A second metric field, $\tilde{g}_{\mu\nu}$, is obtained from $g_{\mu\nu}$ by stretching it in the space-time directions orthogonal to $\mathcal{U}^\mu$ by a factor $e^{-2\phi}$, and shrinking it by the same factor in the direction parallel to $\mathcal{U}^\mu$ (so that it is not conformal to $g_{\mu\nu}$):

$$\tilde{g}_{\mu\nu} = e^{-2\phi}\left(g_{\mu\nu} + \mathcal{U}_\mu \mathcal{U}_\nu\right) - e^{2\phi}\mathcal{U}_\mu \mathcal{U}_\nu. \tag{1}$$

- The purely gravitational action essentially coincides with that of standard GR for $g_{\mu\nu}$:

$$S_{\mathrm{grav}} = \frac{1}{16\pi G}\int_\Omega R\sqrt{-g}\,\mathrm{d}^4x. \tag{2}$$

# TeVeS dynamics II

- The actions associated with the with the vector and scalar fields introduce a free dimensionless function $F$ and two dimensionless parameters and two fundamental constants. The basic equations of the theory are obtained from the total action

$$S = S_{\rm grav} + S_{\rm scalar} + S_{\rm vector} + S_{\rm matter} \tag{3}$$

by varying with respect to the fields $g^{\mu\nu}$, $\phi$, $\sigma$ and $\mathcal{U}_\mu$.

- the matter action associated with fields written generically as $f^\alpha$ is

$$S_{\rm matter} = \int_\Omega \mathcal{L}_{\rm matter}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha{}_{|\mu}, \cdots)\, {\rm d}^4 x, \tag{4}$$

where the covariant derivatives denoted by "$_|$" are defined relative to the connection compatible with $\tilde{g}_{\mu\nu}$. Note that the Lagrangian density $\mathcal{L}_{\rm matter}$ is to be understood as a multiple of $\sqrt{-\tilde{g}}$, which can be shown to be equal to $e^{2\phi}\sqrt{-g}$.

**The strong equivalence principle is defined relative to the $\tilde{\mathbf{g}}_{\mu\nu}$ field: ordinary rods and clocks survey $\tilde{\mathbf{g}}_{\mu\nu}$, not $\mathbf{g}_{\mu\nu}$.**

# TeVeS dynamics III

Analogue of Einstein's field equations:

$$G_{\mu\nu} = 8\pi G\tilde{T}_{\mu\nu} + \cdots ; \qquad \tilde{T}_{\mu\nu} \equiv \frac{-2}{\sqrt{-\tilde{g}}}\frac{\delta\mathcal{L}_{\text{matter}}}{\delta\tilde{g}^{\mu\nu}}.$$

Analogue of "response" equation:

$$\tilde{T}^{\mu}{}_{\nu|\mu} = 0.$$

## Conclusions:

- Material test bodies "follow" geodesics of $\tilde{g}_{\mu\nu}$.
- Light rays propagate on null geodesics of $\tilde{g}_{\mu\nu}$.
- Material rods and clocks survey $\tilde{g}_{\mu\nu}$.
- Gravitational waves propagate along null geodesics of $\tilde{g}_{\mu\nu}$.

So is the gravitational field in TeVeS geometrical?

# Operational aspects of the metric field in GR

- Geodesic motion of test bodies
- Light rays trace out null geodesics
- Rods and clocks survey the field: "chronometric significance"

# Strong equivalence principle

"... in any and every local Lorentz frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar special relativistic forms." *Misner, Thorne and Wheeler (1973), p. 386.*

"At each point of space-time it is possible to find a coordinate transformation such that the gravitational field variables can be eliminated from the field equations of matter." *Ohanian (1975).*

***Two components:***

I. *Universality.* All non-gravitational interactions pick out the same family of privileged local frames. (There is only one affine connection, a necessary condition for a "*geometrical*" theory like Einstein's.)

II. *Minimal coupling.* (E.g. The curvature tensor does not appear in the generalized Maxwell equations.)

# Special Relativity

# Ether wind experiments in the 19th century

## 1st order effects

explained by Fresnel drag coefficient
Lorentz' 1892 dynamical derivation

## 2nd order effects

Michelson-Morley experiment 1887
Deformation hypothesis of
FitzGerald (1889)
and Lorentz (1892, 1895)

M-M kinematics:

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\begin{aligned}
x' &= k\gamma(x - vt) \\
y' &= ky \\
z' &= kz \\
t' &= k\gamma(c/c')(t - vx/c^2)
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## Special case of M-M kinematics

in 1+1 dimensions

G. Bogoslovsky (1977)
T. Budden (1992)

Consistent with relativity principle and Einstein's light postulate.
For $n = 1/2$, get time dilation but no `twins' effect. HRB (1990)
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BUT:

"It seems to me ... that a physical theory can be satisfactory only when it builds up its structures from elementary foundations." (1908)

"... when we say we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question." (1919)

# Universal Time Dilation

## Einstein's 1905 discovery

$$x' = \gamma(x - vt)$$
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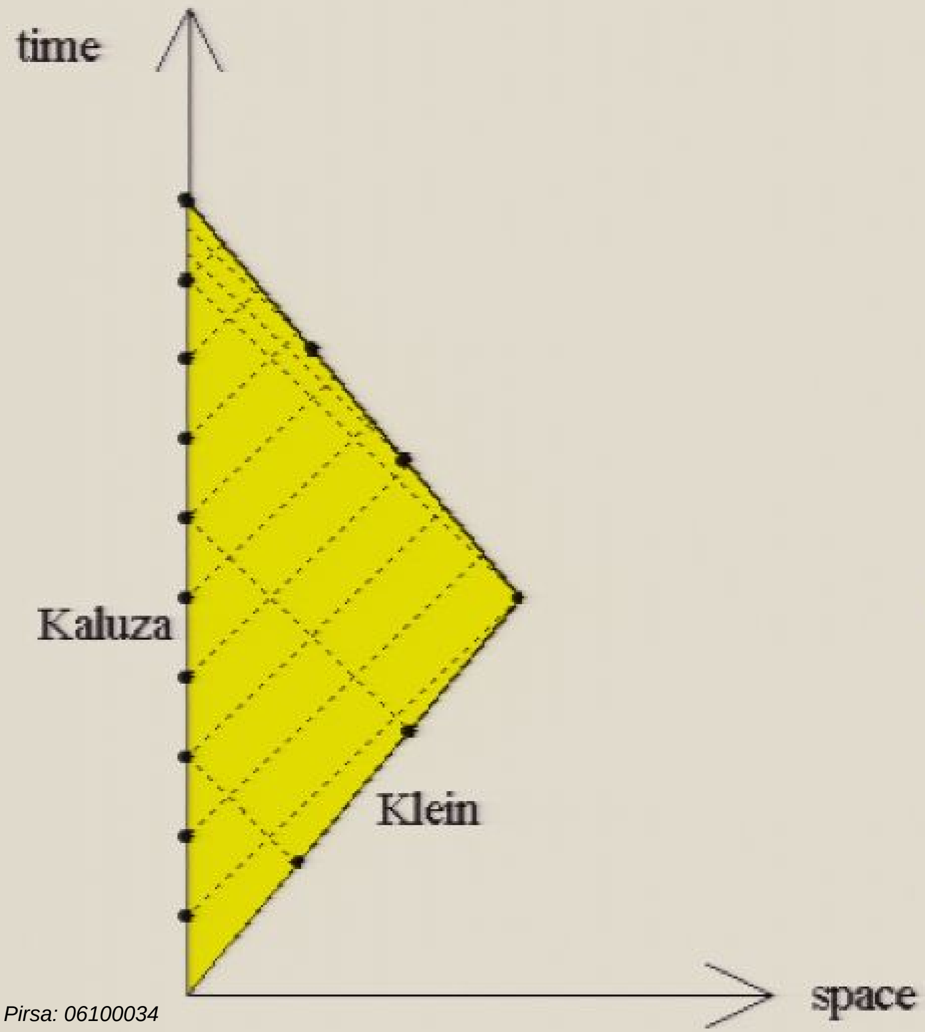


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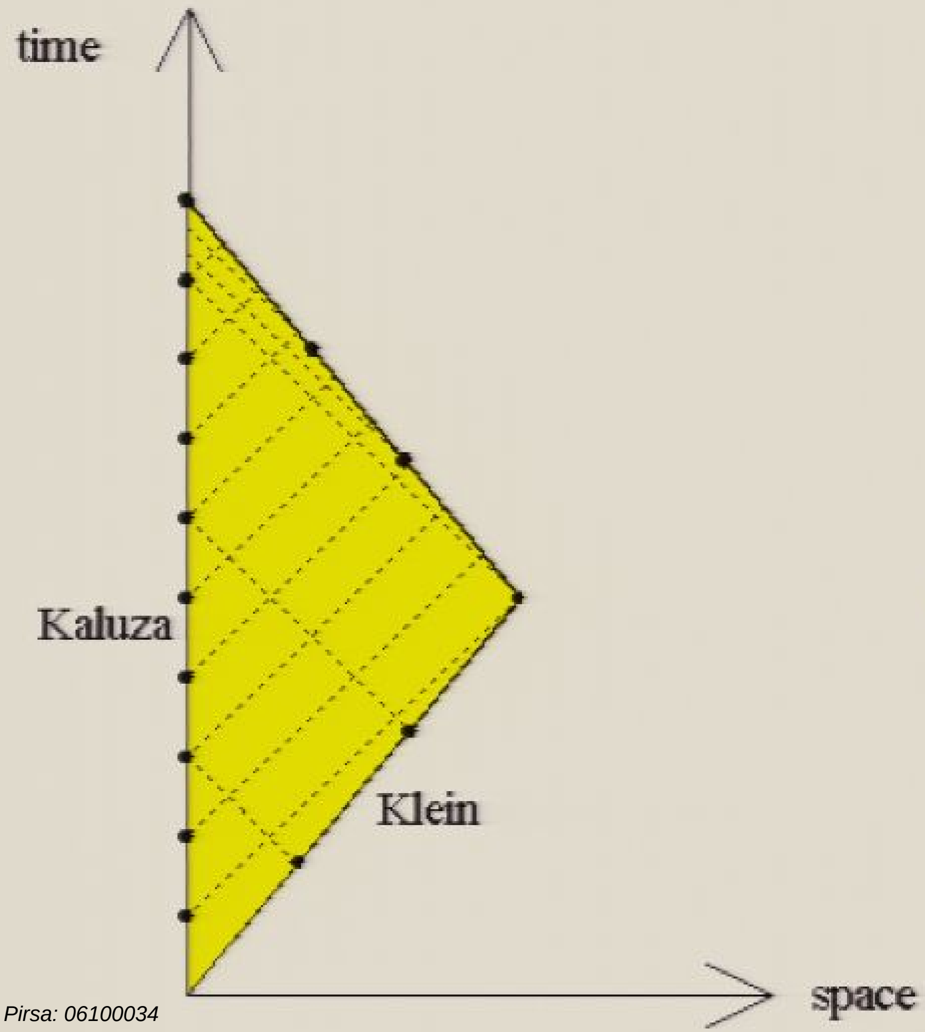


waywiser

# Did Einstein explain time dilation?

- Yes. It is a consequence of his 1905 principles.
- No. It took Minkowski's 1908 geometrization of the theory.
- No. Appeal must be made to the quantum theory of matter.

# Dynamical underpinning of kinematics

H. Weyl 1918, W. Pauli 1921, A.S. Eddington 1928,
W.F.G. Swann 1941, L. Jánossy 1971, J.S. Bell 1976, 1992,
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**Bell 1992** "If you are, for example, quite convinced of the second law of thermodynamics, of the increase of entropy, there are many things that you can get directly from the second law which are very difficult to get directly from a detailed study of the kinetic theory of gases, but you have no excuse for not looking at the kinetic theory of gases to see how the increase of entropy actually comes about. In the same way, although Einstein's theory of special relativity would lead you to expect the FitzGerald contraction, you are not excused from seeing how the detailed dynamics of the system also leads to the FitzGerald contraction."

# Last words

"The property that all non-gravitational fields should couple in the same manner to a single gravitational field is sometimes called ``universal coupling''. Because of it, one can discuss the metric as a property of spacetime itself rather than as a field over spacetime. This is because its properties may be measured and studied using a variety of different experimental devices, composed of different non-gravitational fields and particles, and, because of universal coupling, the results will be independent of the device. Thus, for instance, the proper time between two events is characteristic of spacetime and of the location of the events, not of the clocks used to measure it."

*Clifford Will 2001.*