Title: Graduate Course on Standard Model & Quantum Field Theory - 2A (Part 2)

Date: Oct 11, 2006 11:00 AM

URL: http://pirsa.org/06100029

Abstract: Graduate Course on Standard Model & Quantum Field Theory

we want $\mathcal{L}$ to be ① quadratic ② a Lorentz scalar ③ hermitian,

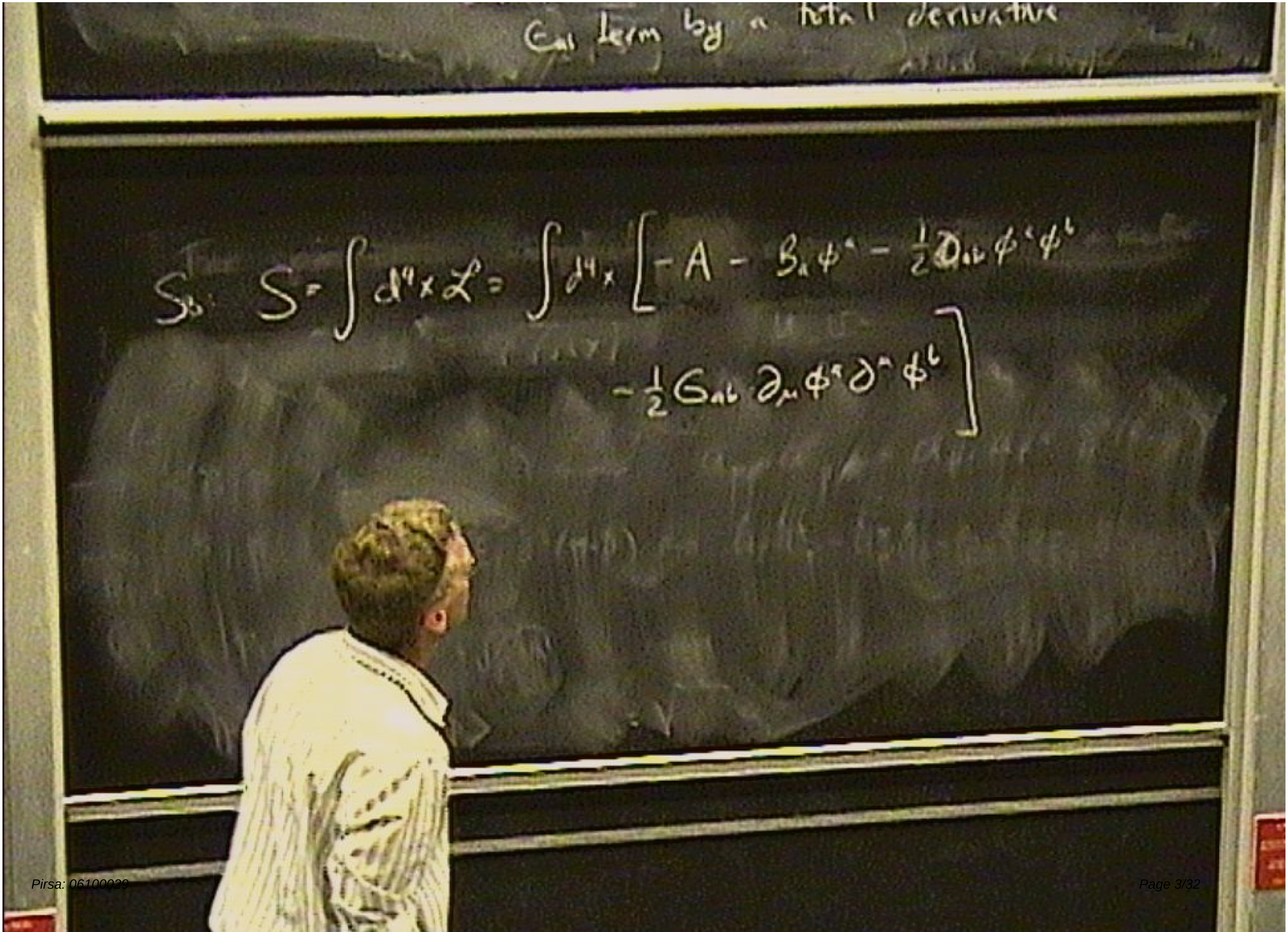
and we ask ④ H_0 to be bounded from below

$$\mathcal{L} = A + B_a\phi^a + \cancel{C^\mu_a \partial_\mu \phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\phi^a \partial_\mu \phi^b}$$

$$+ \cancel{F_{ab}\phi^a \partial_\mu \partial^\mu \phi^b} + G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b$$

F differs from G term by a total derivative

So: $$S = \int d^4x\, \mathcal{L} = \int d^4x \Big[-A - B_a\phi^a - \tfrac{1}{2} C_{ab}\phi^a\phi^b - \tfrac{1}{2} G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b \Big]$$


term by a total derivative
So: $S = \int d^4x\, \mathcal{L} = \int d^4x \left[-A - B_a\phi^a - \frac{1}{2} D_{ab}\phi^a\phi^b - \frac{1}{2} G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b \right]$

... term by a total derivative

$$S: \; S = \int d^4x\, \mathcal{L} = \int d^4x \left[-A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b \right]$$

$$\mathcal{L} = A + B_a\phi^a + \cancel{C^\mu_a\,\partial_\mu\phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\,\phi^a\partial_\mu\phi^b}$$

$$+ \cancel{F_{ab}\,\phi^a\partial_\mu\partial^\mu\phi^b} + G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b$$

differs from G_{ab} term by a total derivative

$$S_E:\quad S = \int d^4x\,\mathcal{L} = \int d^4x\left[-A - B_a\phi^a - \tfrac{1}{2}D_{ab}\phi^a\phi^b - \tfrac{1}{2}G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b\right]$$

S … $*$

and we ask ① H to be bounded from below

$$\mathcal{L} = A + B_a \phi^a + \cancel{C^\mu_a \partial_\mu \phi^a} + D_{ab}\phi^a \phi^b + \cancel{E^\mu_{ab} \phi^a \partial_\mu \phi^b}$$

$$+ \cancel{F_{ab}\phi^a \partial_\mu \partial^\mu \phi^b} + G_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b$$

differs from G_{ab} term by a total derivative

So $$S = \int d^4x\, \mathcal{L} = \int d^4x \Big[-A - B_a \phi^a - \tfrac{1}{2} C_{ab} \phi^a \phi^b - \tfrac{1}{2} G_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b \Big]$$

$S = S^* \iff A, B_a, C_{ab}, G_{ab}$ are real matrices (given the choice $\phi^a = \phi^{a*}$)

and we ask ① H_L to be bounded from below

$$\mathcal{L} = A + B_a\phi^a + \cancel{C^\mu_a \partial_\mu \phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\phi^a\partial_\mu\phi^b}$$

$$+ \cancel{F_{ab}\phi^a\partial_\mu\partial^\mu\phi^b} + G_{ab}\,\partial_\mu\phi^a\,\partial^\mu\phi^b$$

differs from G term by a total derivative

$$S_0 \qquad S = \int d^4x\,\mathcal{L} = \int d^4x \Big[-A - B_a\phi^a - \tfrac{1}{2}D_{ab}\phi^a\phi^b$$

$$- \tfrac{1}{2}G_{ab}\,\partial_\mu\phi^a\,\partial^\mu\phi^b\Big]$$

$S = S^* \Longleftrightarrow A, B_a, C_{ab}, G$ ... real matrices

(given th... $\phi^a = \phi^{a*}$)

$$\mathcal{L} = A + B_a\phi^a + \cancel{C^\mu_a\partial_\mu\phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\phi^a\partial_\mu}\phi^b$$

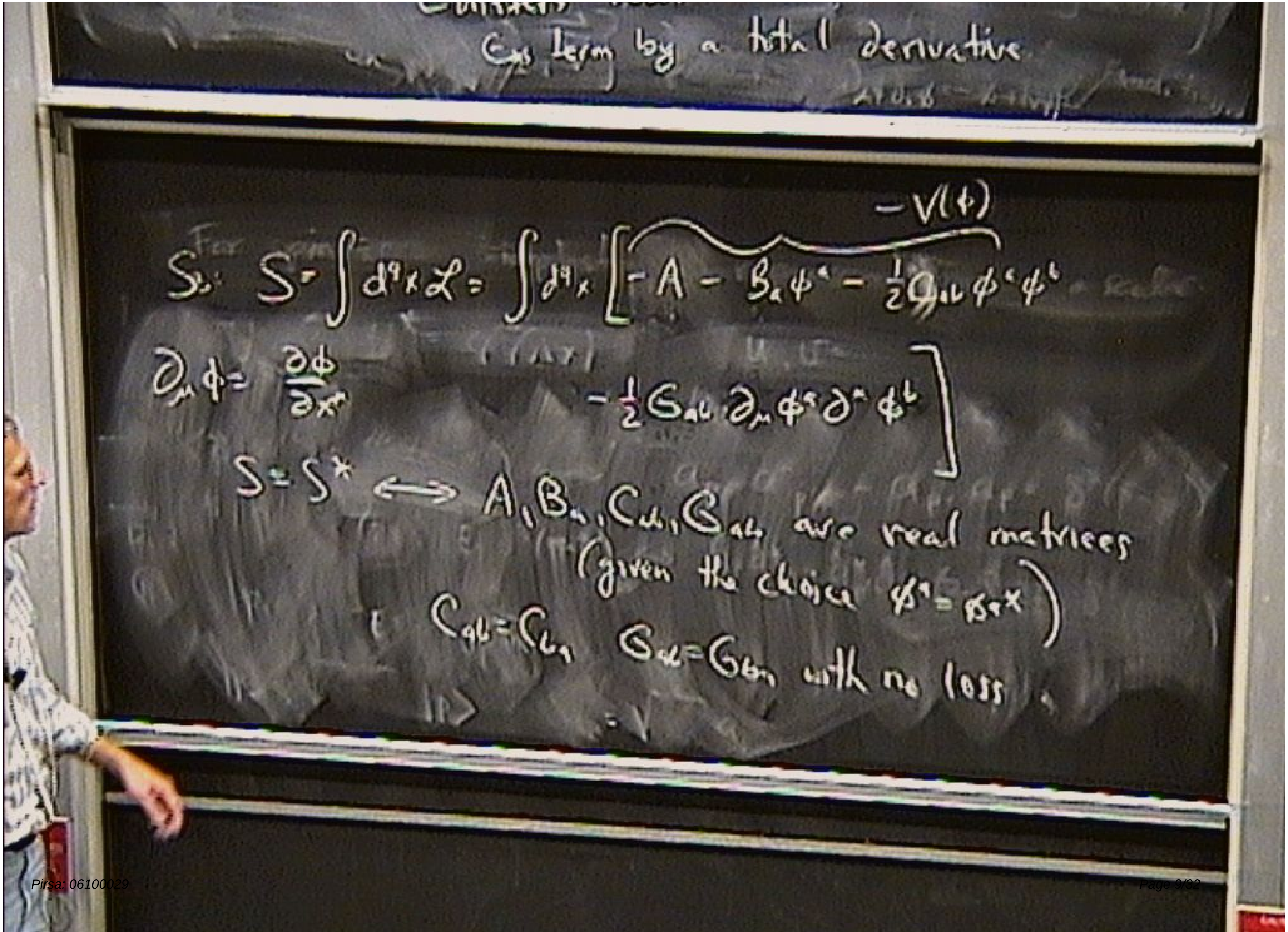
$$+ F_{ab}\cancel{\phi^a\partial_\mu}\partial^\mu\phi^b + G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$$

differs from G_{ab} term by a total derivative

So $$S = \int d^4x\,\mathcal{L} = \int d^4x\left[-A - B_a\phi^a - \tfrac{1}{2}C_{ab}\phi^a\phi^b - \tfrac{1}{2}G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b\right]$$

$S = S^* \iff A, B_a, C_{ab}, G_{ab}$ are real matrices (given the choice $\phi^a = \phi^{a*}$)

$C_{ab} = C_{ba}$ $G_{ab} = G_{ba}$ with no loss



$C_{\mu\nu}$ term by a total derivative
$S = \int d^dx\, \mathcal{L} = \int d^dx \Big[\overbrace{-A - B_a\phi^a - \tfrac{1}{2} C_{ab}\phi^a\phi^b}^{-V(\phi)}$
$\partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu}$
$- \tfrac{1}{2} G_{ab}\, \partial_\mu\phi^a \partial^\mu \phi^b \Big]$
$S = S^* \Longleftrightarrow A, B_a, C_{ab}, G_{ab}$ are real matrices
(given the choice $\phi^a = \phi^{a*}$)
$C_{ab} = C_{ba}$ $G_{ab} = G_{ba}$ with no loss.

C_{ab} term by a total derivative

$$S_0: \quad S = \int d^4x\, \mathcal{L} = \int d^4x \Big[\overbrace{-A - B_a\phi^a - \tfrac{1}{2}C_{ab}\phi^a\phi^b}^{-V(\phi)}$$

$$\partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu} \qquad\qquad -\tfrac{1}{2} G_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b \Big]$$

$S = S^* \Longleftrightarrow A, B_a, C_{ab}, G_{ab}$ are real matrices (given the choice $\phi^a = \phi^{a*}$)

$\partial_0 \phi = \dfrac{\partial \phi}{\partial x^0} = \dot\phi \qquad x^0 = t$

$C_{ab} = C_{ba} \quad G_{ab} = G_{ba}$ with no loss.

C_{ab} term by a total derivative

$$S_{3}: \; S = \int d^4x\, \mathcal{L} = \int d^4x \Big[\overbrace{-A - B_a \phi^a - \tfrac{1}{2} C_{ab} \phi^a \phi^b}^{-V(\phi)}$$

$$\partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu} \qquad - \tfrac{1}{2} G_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b \Big]$$

$S = S^* \Longleftrightarrow A, B_a, C_{ab}, G_{ab}$ are real matrices

$\eta_{\mu\nu} = (-1, \ldots)$ (given the choice $\phi^a = \phi^{a*}$)

$\partial_0 \phi = \frac{\partial \phi}{\partial x^0}$, $x^0 = t$ $\quad C_{ab} = C_{ba} \quad G_{ab} = G_{ba}$ with no loss.

$= \dot{\phi} \qquad \partial_i \phi = (\nabla\phi)_i$

C_{ab} term by a total derivative

$$S = \int d^4x\,\mathcal{L} = \int d^4x \Big[\overbrace{-A - B_a \phi^a - \tfrac{1}{2} C_{ab} \phi^a \phi^b}^{-V(\phi)}$$

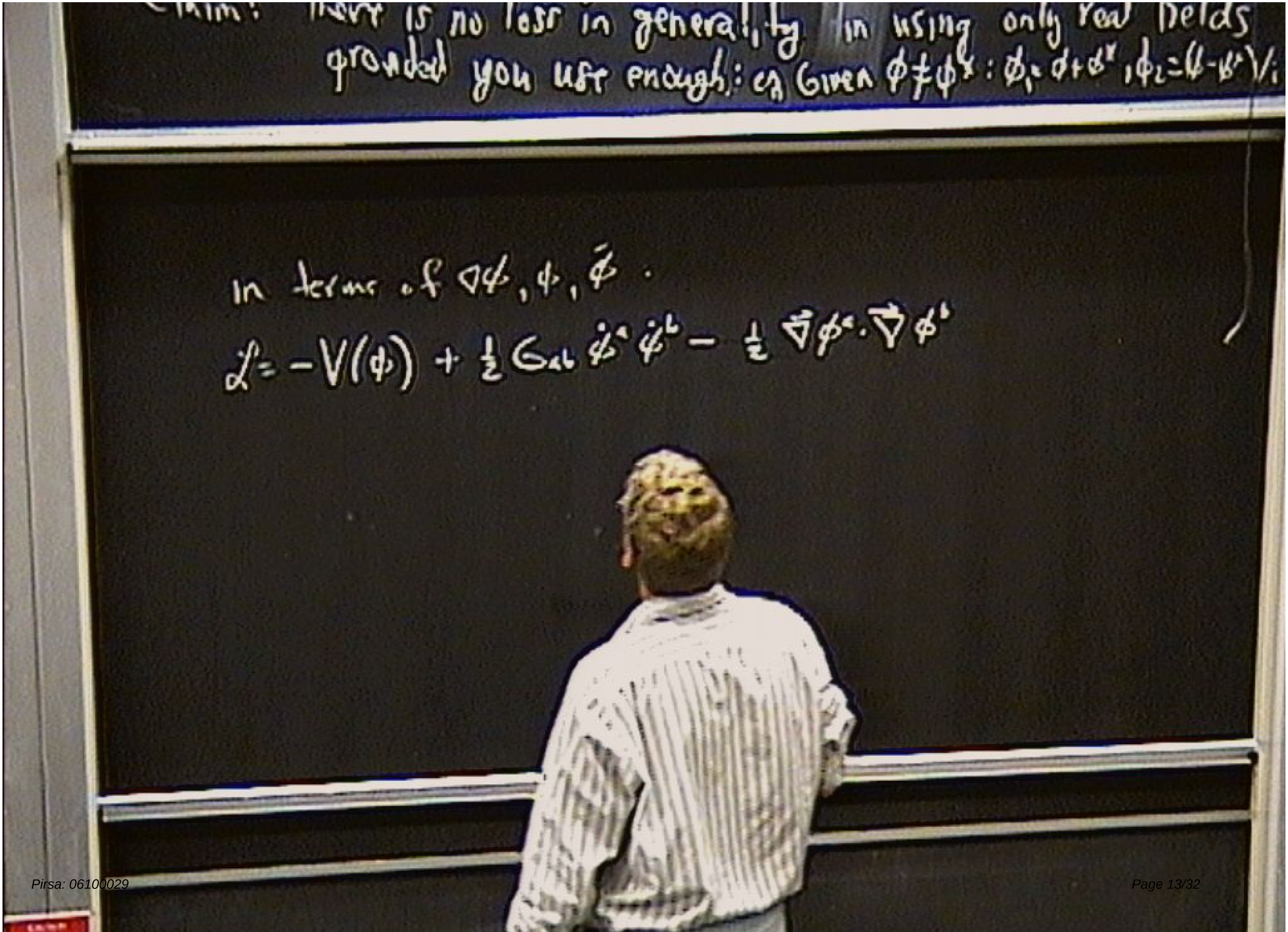
$$\partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu} \qquad - \tfrac{1}{2} G_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b \Big] \qquad \partial_\mu \phi \partial^\mu \phi = -\dot{\phi}^2 + (\nabla\phi)^2$$

$S = S^* \Longleftrightarrow A, B_a, C_{ab}, G_{ab}$ are real matrices

$\eta_{\mu\nu} = (^{-1}{}_{1}{}_{1}{}_{1})$ (given the choice $\phi^a = \phi^{a*}$)

$\partial_0 \phi = \frac{\partial \phi}{\partial x^0}$ $\quad x^0 = t \quad C_{ab} = C_{ba} \quad G_{ab} = G_{ba}$ with no loss.

$= \dot{\phi} \qquad \partial_i x \cdot (\nabla \phi)$


Claim: There is no loss in generality in using only real fields
provided you use enough: eg Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$
in terms of $\nabla\phi, \phi, \dot{\phi}$.
$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab} \dot{\phi}^a \dot{\phi}^b - \frac{1}{2} \vec{\nabla}\phi^a \cdot \vec{\nabla}\phi^b$

Claim: There is no loss in generality in using only real fields provided you use enough: e.g. Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

In terms of $\nabla\phi, \phi, \dot{\phi}$:

$$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab}\, \dot{\phi}^a \dot{\phi}^b - \frac{1}{2} \vec{\nabla}\phi^a \cdot \vec{\nabla}\phi^b$$

$$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot{\phi}^a} = G_{ab}\, \dot{\phi}^b \quad ; \quad \dot{\phi}^b = G^{bc}\, \pi_c$$

inverse m

Claim: There is no loss in generality in using only real fields provided you use enough: eg Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

in terms of $\nabla\phi, \phi, \dot{\phi}$.

$$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab} \dot{\phi}^a \dot{\phi}^b - \frac{1}{2} \vec{\nabla}\phi^a \cdot \vec{\nabla}\phi^b$$

inverse matrix

$$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot{\phi}^a} = G_{ab} \dot{\phi}^b \quad ; \quad \dot{\phi}^b = G^{bc} \pi_c$$

Claim: There is no loss in generality in using only real fields provided you use enough: eg Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

In terms of $\nabla\phi, \phi, \dot\phi$:

$$\mathcal{L} = -V(\phi) + \tfrac{1}{2} G_{ab} \dot\phi^a \dot\phi^b - \tfrac{1}{2} \vec\nabla\phi^a \cdot \vec\nabla\phi^b$$

inverse metric

$$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot\phi^a} = G_{ab}\dot\phi^b \quad ; \quad \dot\phi^b = G^{bc}\pi_c \quad : \quad G^{ab}G_{bc} = \delta^a_c$$

$$\mathcal{H} = \pi_a \dot\phi^a - \mathcal{L}$$

Claim: There is no loss in generality in using only real fields provided you use enough: eg Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

In terms of $\nabla\phi, \phi, \dot{\phi}$:

$$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab} \dot{\phi}^a \dot{\phi}^b - \frac{1}{2} \vec{\nabla}\phi^a \cdot \vec{\nabla}\phi^b$$

inverse metric

$$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot{\phi}^a} = G_{ab} \dot{\phi}^b \quad ; \quad \dot{\phi}^b = G \quad : \quad G^{ab} G_{bc} = \delta^a_c$$

$$\mathcal{H} = \pi_a \dot{\phi}^a - \mathcal{L} = \frac{1}{2} G_{ab} \dot{\phi}^a \dot{\phi}^b + \frac{1}{2} \nabla\phi \quad V(\phi)$$

Claim: There is no loss in generality in using only real fields provided you use enough: eg Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

In terms of $\nabla\phi$, ϕ, $\dot\phi$:

$$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab} \dot\phi^a \dot\phi^b - \frac{1}{2} \vec\nabla \phi^a \cdot \vec\nabla \phi^b$$

$$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot\phi^a} = G_{ab} \dot\phi^b \quad ; \quad \dot\phi^b = \underbrace{G^{bc}}_{\text{inverse metric}} \pi_c \quad : \quad G^{ab} G_{bc} = \delta^a_c$$

$$\mathcal{H} = \pi_a \dot\phi^a - \mathcal{L} = \underbrace{\frac{1}{2} G_{ab} \dot\phi^a \dot\phi^b}_{\frac{1}{2} G^{ab} \pi_a \pi_b} + \frac{1}{2} \nabla\phi^a \cdot \nabla\phi^b + V(\phi)$$

In terms of $\nabla\phi, \phi, \dot\phi$:

$$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab}\dot\phi^a\dot\phi^b - \frac{1}{2} G_{ab}\vec\nabla\phi^a\cdot\vec\nabla\phi^b$$

inverse metric

$$\pi_a = \frac{\partial\mathcal{L}}{\partial\dot\phi^a} = G_{ab}\dot\phi^b \quad ; \quad \dot\phi^b = G^{bc}\pi_c \quad : \quad G^{ab}G_{bc} = \delta^a_c$$

$$\mathcal{H} = \pi_a\dot\phi^a - \mathcal{L} = \underbrace{\frac{1}{2} G_{ab}\dot\phi^a\dot\phi^b}_{\frac{1}{2} G^{ab}\pi_a\pi_b} + \frac{1}{2} G_{ab}\nabla\phi^a\cdot\nabla\phi^b + V(\phi)$$

$\ldots -A, -A+1, \ldots, A-1, A$

skew tensor $(1,0)$ $(0,1)$

1) For constant fields, $\partial_\mu \phi = 0$,

$$\mathcal{H}(\phi) = V(\phi)$$

$\rightarrow B_3$ $-A, -A+1, \ldots, A-1, A$

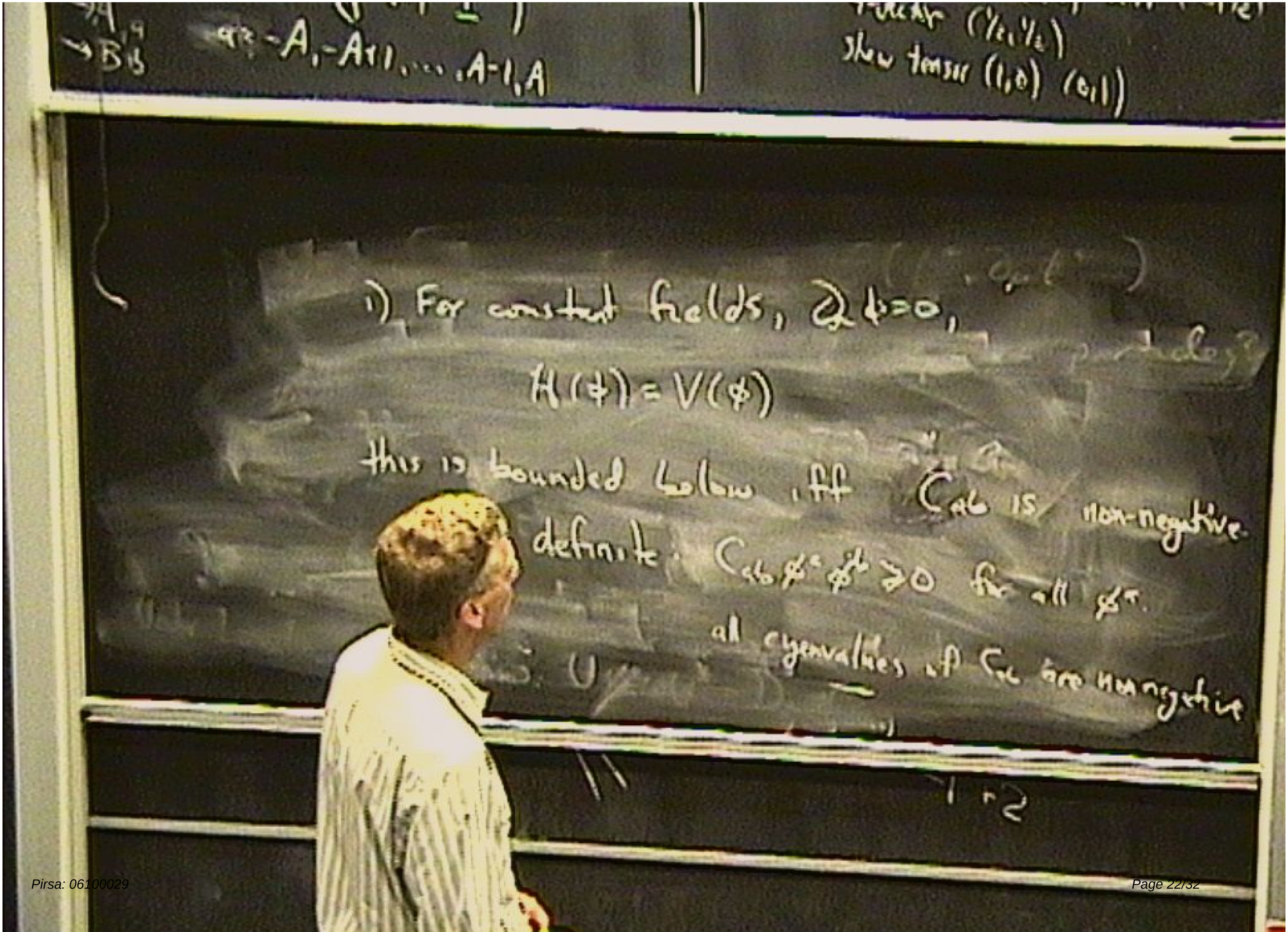
then tensor $(1,0)$ $(0,1)$

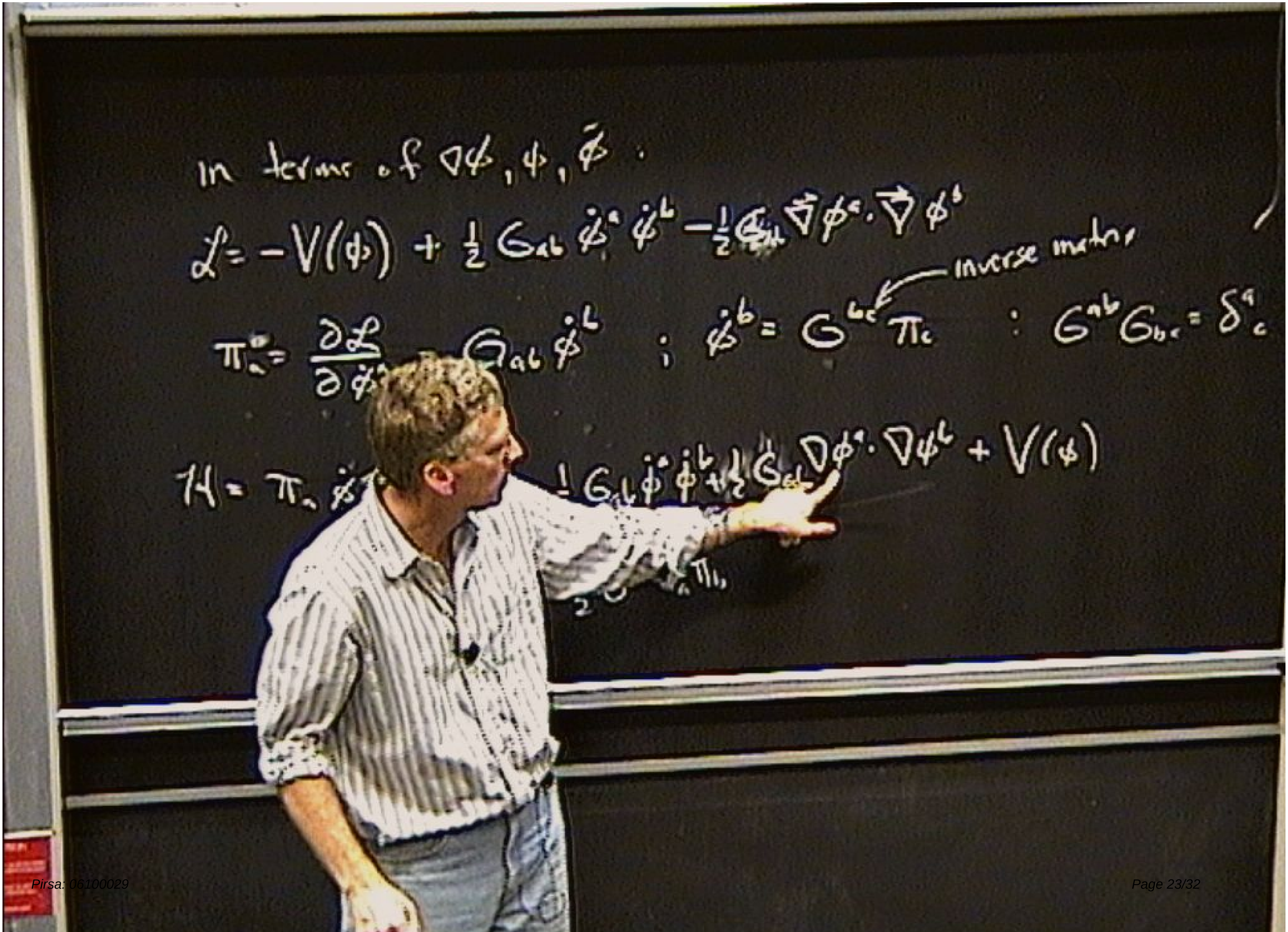
i) For constant fields, $\partial_\mu \phi = 0$,

$$H(\phi) = V(\phi)$$

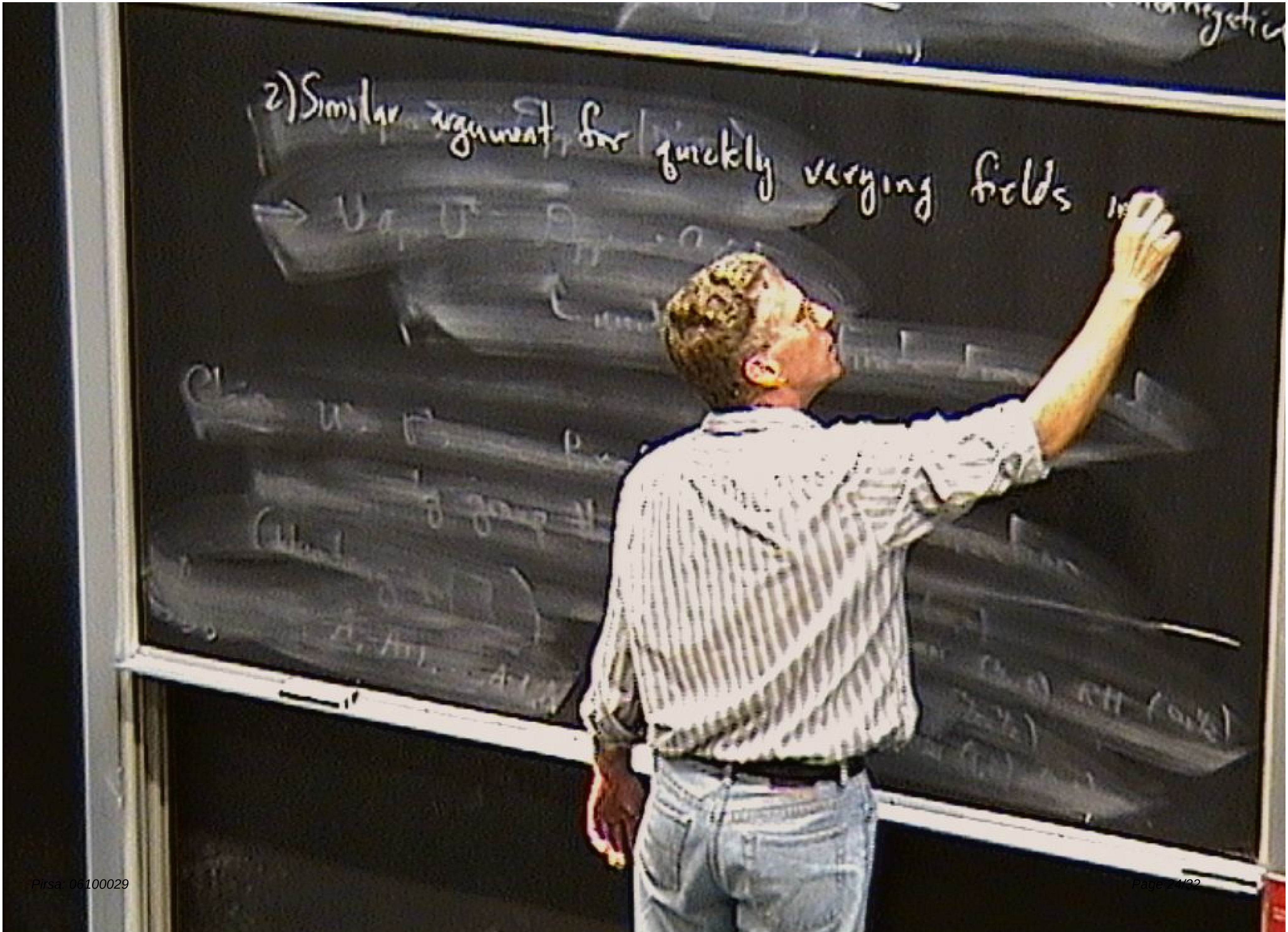
this is bounded below iff C_{ab} is non-negative definite: $C_{ab}\phi^a\phi^b \geq 0$ for all ϕ^a

all eigenvalues of C_{ab} are nonnegative


q: -A, -A+1, ..., A-1, A
Skew tensor (1,0) (0,1)
1) For constant fields, ∂φ=0,
H(φ) = V(φ)
this is bounded below iff C_ab is non-negative
definite: C_ab φ^a φ^b ≥ 0 for all φ^a.
all eigenvalues of C_ab are nonnegative



in terms of $\nabla\phi, \phi, \dot\phi$:
$\mathcal{L} = -V(\phi) + \frac{1}{2} G_{ab} \dot\phi^a \dot\phi^b - \frac{1}{2} G_{ab} \vec\nabla\phi^a \cdot \vec\nabla\phi^b$
inverse matrix
$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot\phi^a} = G_{ab} \dot\phi^b \; ; \; \dot\phi^b = G^{bc} \pi_c \; : \; G^{ab} G_{bc} = \delta^a_c$
$\mathcal{H} = \pi_a \dot\phi^a$
$\frac{1}{2} G_{ab} \dot\phi^a \dot\phi^b + \frac{1}{2} G_{ab} \nabla\phi^a \cdot \nabla\phi^b + V(\phi)$


2) Similar argument for quickly varying fields

2) Similar argument for quickly varying fields implies G_{ab} is non negative definite

2) Similar argument for quickly varying fields implies

G_{ab} is non negative definite

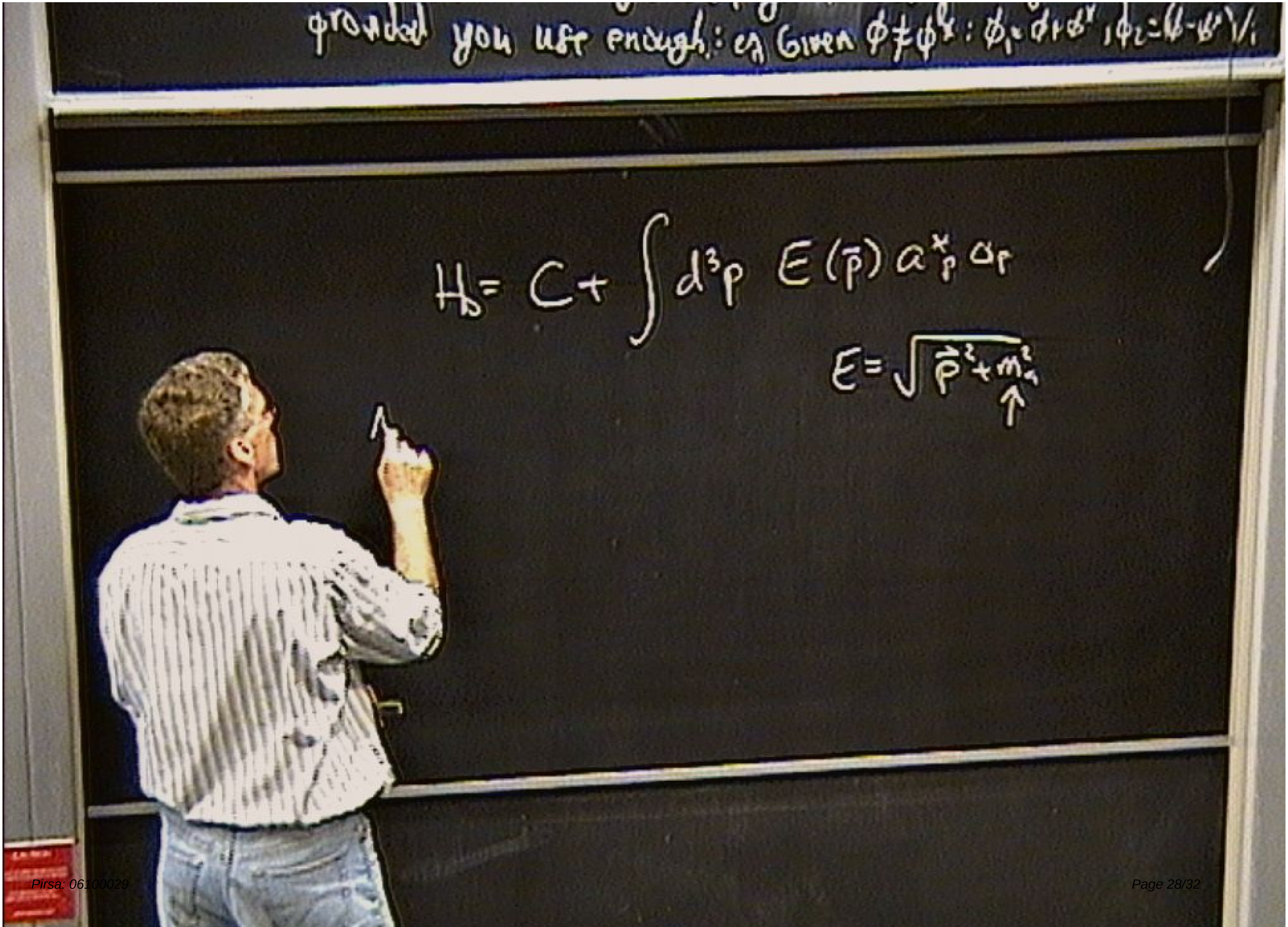
(exclude the case where G_{ab} has a zero eigenvector)

2) Similar argument for quickly varying fields implies

G_{ab} is non negative definite

(exclude the case where G_{ab} has a zero eigenvector)

Any choice of real A, B_a, and symmetric, pos. (or nonneg) C_{ab}, G_{ab} satisfies all our criteria.



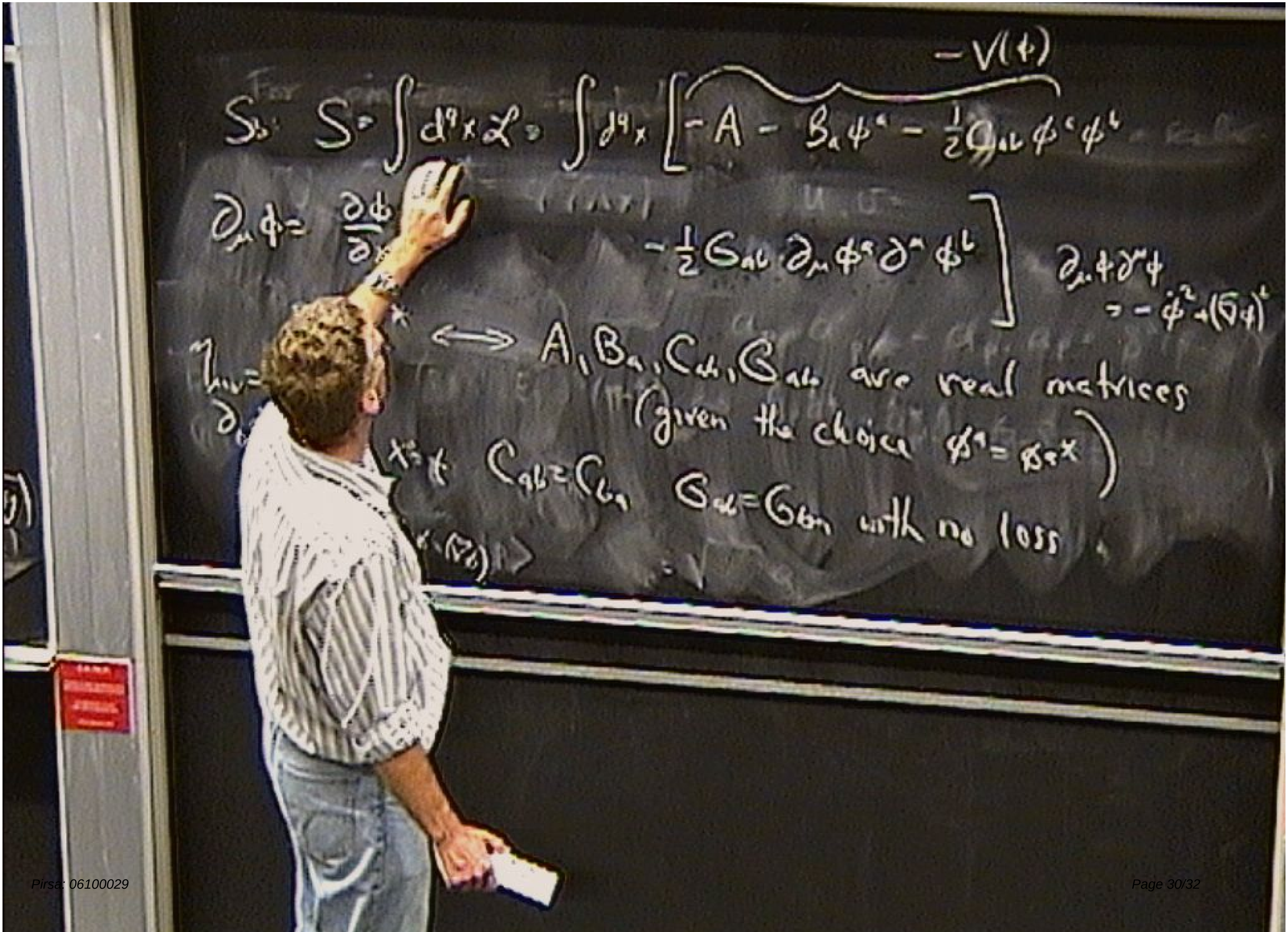
provided you use enough: eg Given $\phi \neq \phi^\dagger$: $\phi_1 = \phi + \phi^\dagger$, $\phi_2 = (\phi - \phi^\dagger)/i$
$H_0 = C + \int d^3p \; E(\bar{p}) \, a^*_p a_p$
$E = \sqrt{\vec{p}^2 + m^2}$

provided you use enough: eg Given $\phi \neq \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

$$H_0 = C + \int d^3p \; E(\vec{p}) \, a^*_p a_p$$

$$E = \sqrt{\vec{p}^2 + m_a^2}$$

N+1 parameters
for N particles.


$-V(\phi)$
$S = \int d^4x\, \mathcal{L} = \int d^4x \left[-A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b \right.$
$\left. - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b \right]$
$\partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu}$
$\partial_\mu \phi \partial^\mu \phi = -\dot{\phi}^2 + (\nabla \phi)^2$
A, B_a, C_{ab}, G_{ab} are real matrices
(given the choice $\phi^a = \phi_a^*$)
$C_{ab} = C_{ba}$ $G_{ab} = G_{ba}$ with no loss

$$H_0 = C + \int d^3p \; E(\vec{p})\, a_p^* a_p$$

$$E = \sqrt{\vec{p}^2 + m_n^2}$$

N+1 parameters
for N particles.

... are nonnegative

2) Similar argument for quickly varying fields implies
G_{ab} is non negative definite
(exclude the case where G_{ab} has a zero eigenvector)

Any choice of real A, B_a, and symmetric, pos. (or nonneg.) C_{ab}, G_{ab} satisfies all our criteria.