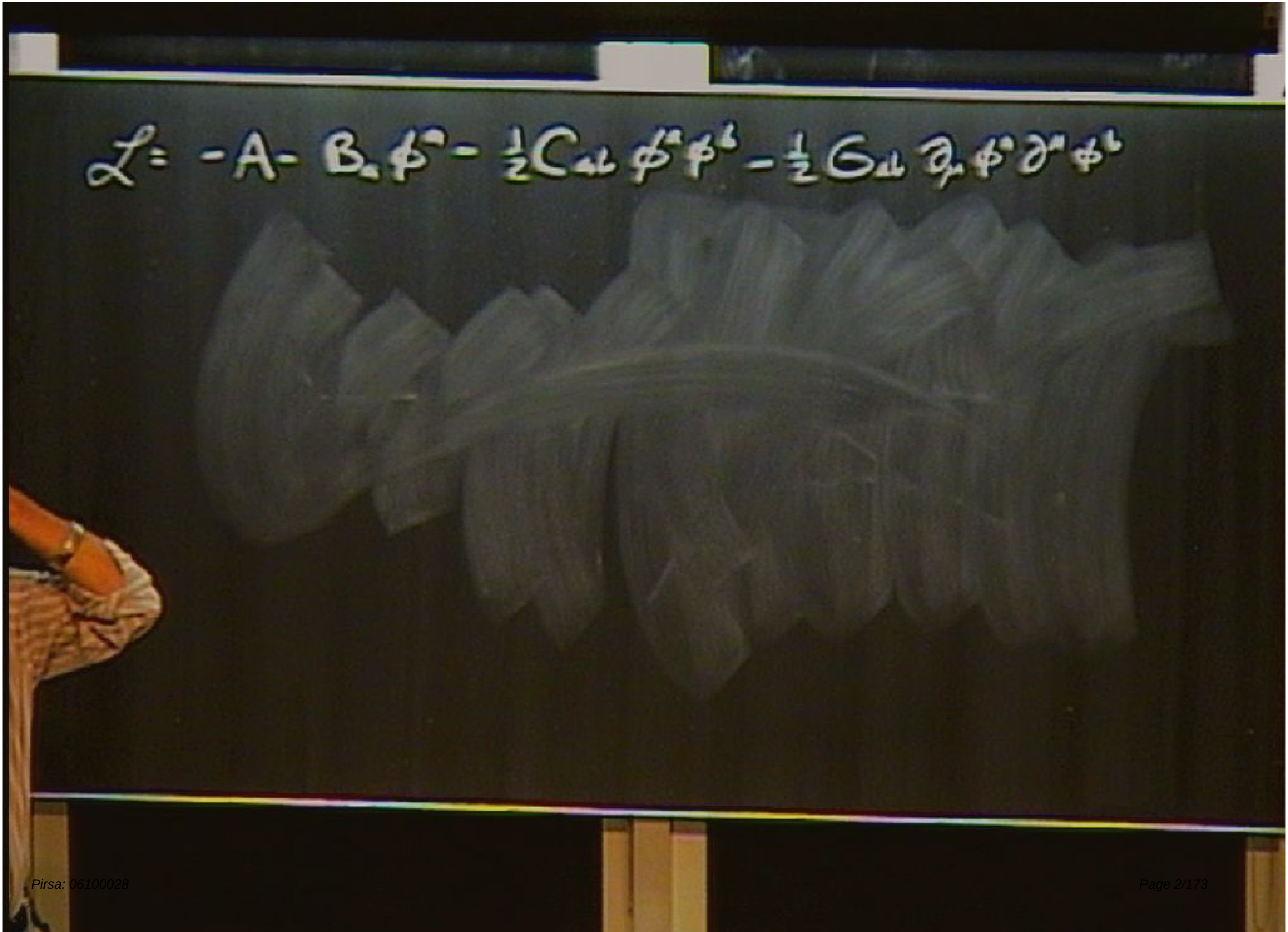
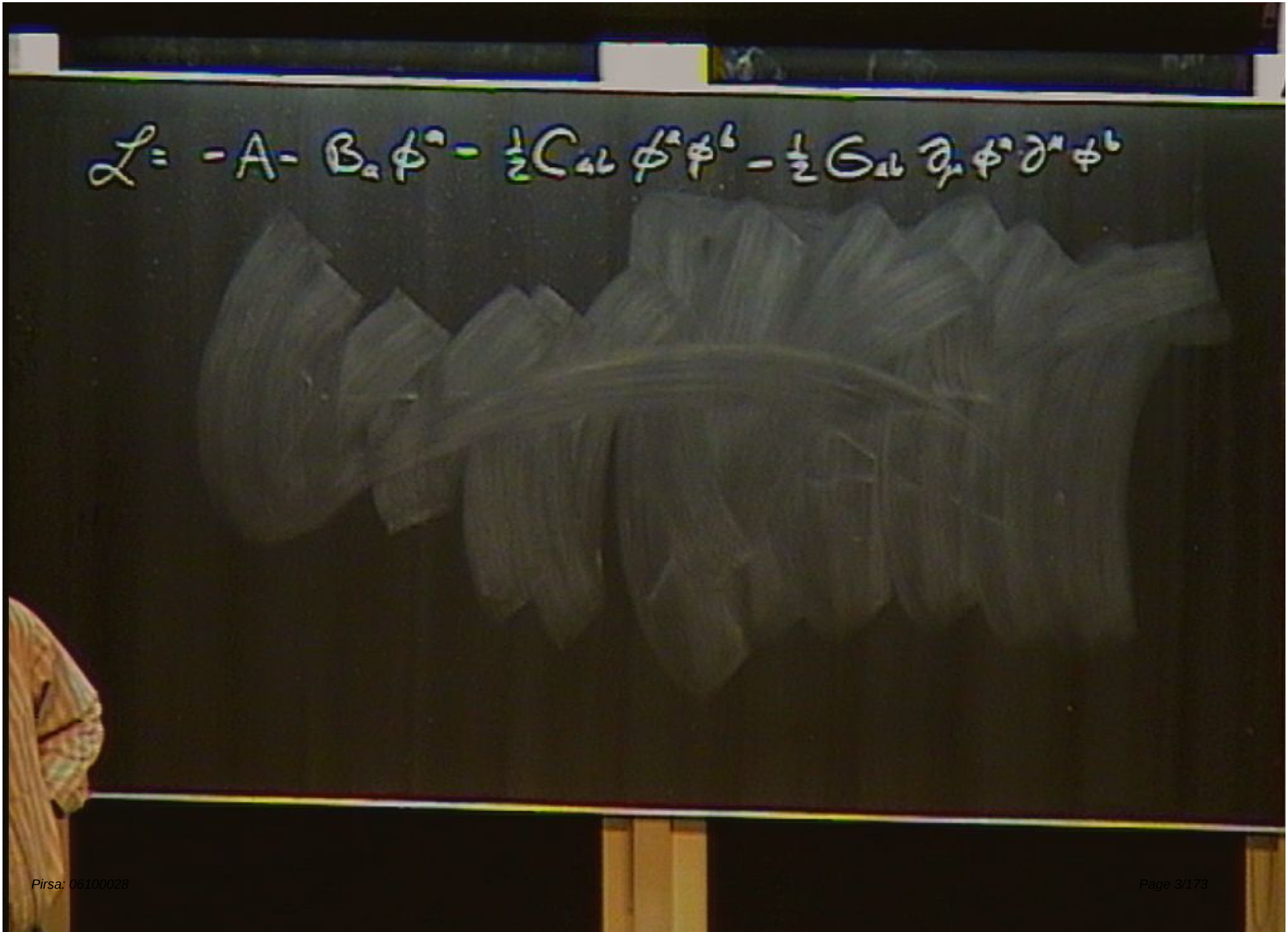
Title: Graduate Course on Standard Model & Quantum Field Theory - 2B

Date: Oct 11, 2006 03:30 AM

URL: http://pirsa.org/06100028

Abstract: Graduate Course on Standard Model & Quantum Field Theory


$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$



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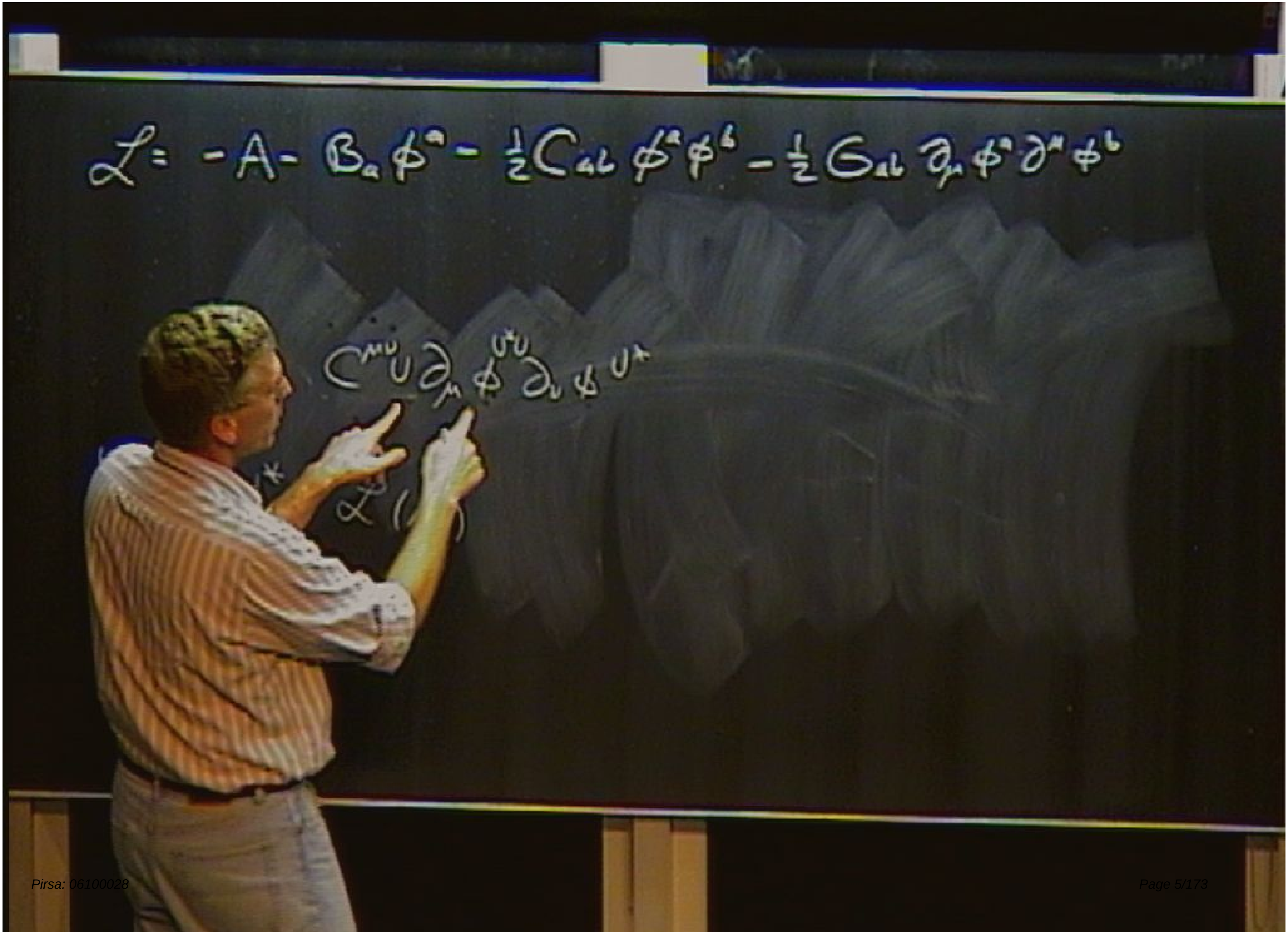
$$C^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi$$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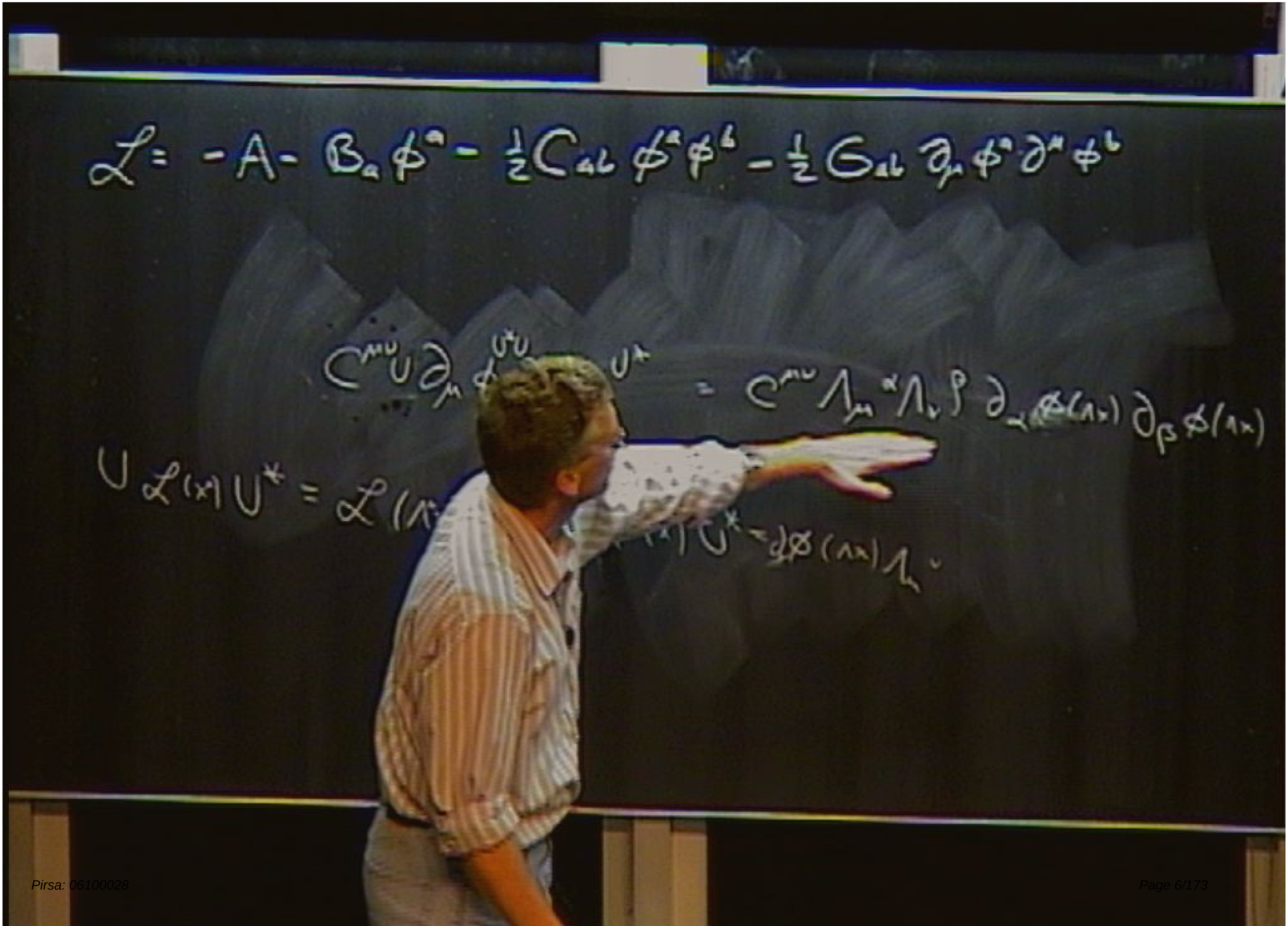
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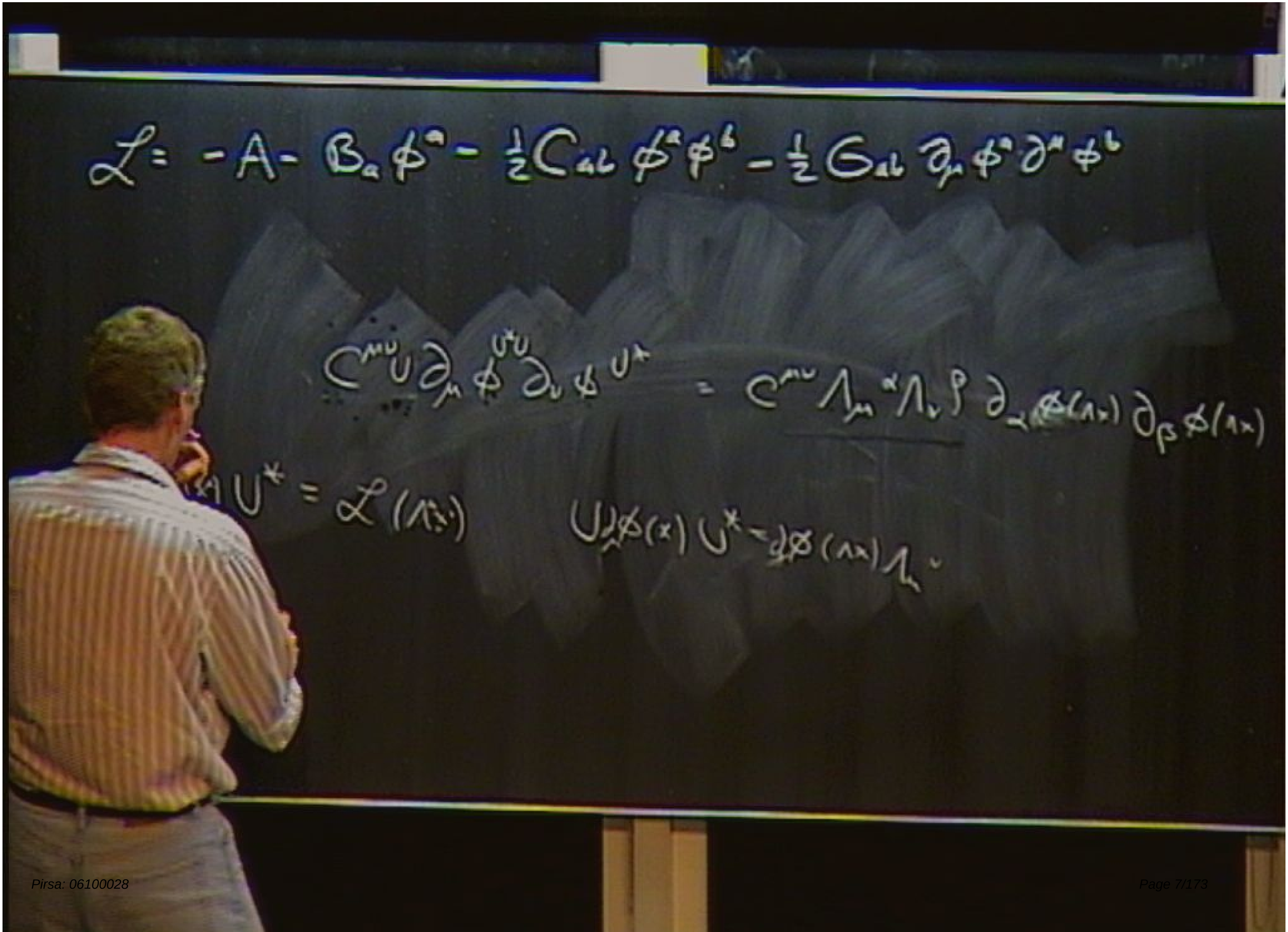
$= C^{\mu\nu} \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta \partial_\alpha \phi(\Lambda x) \partial_\beta \phi(\Lambda x)$
$U \mathcal{L}(x) U^* = \mathcal{L}(\Lambda$



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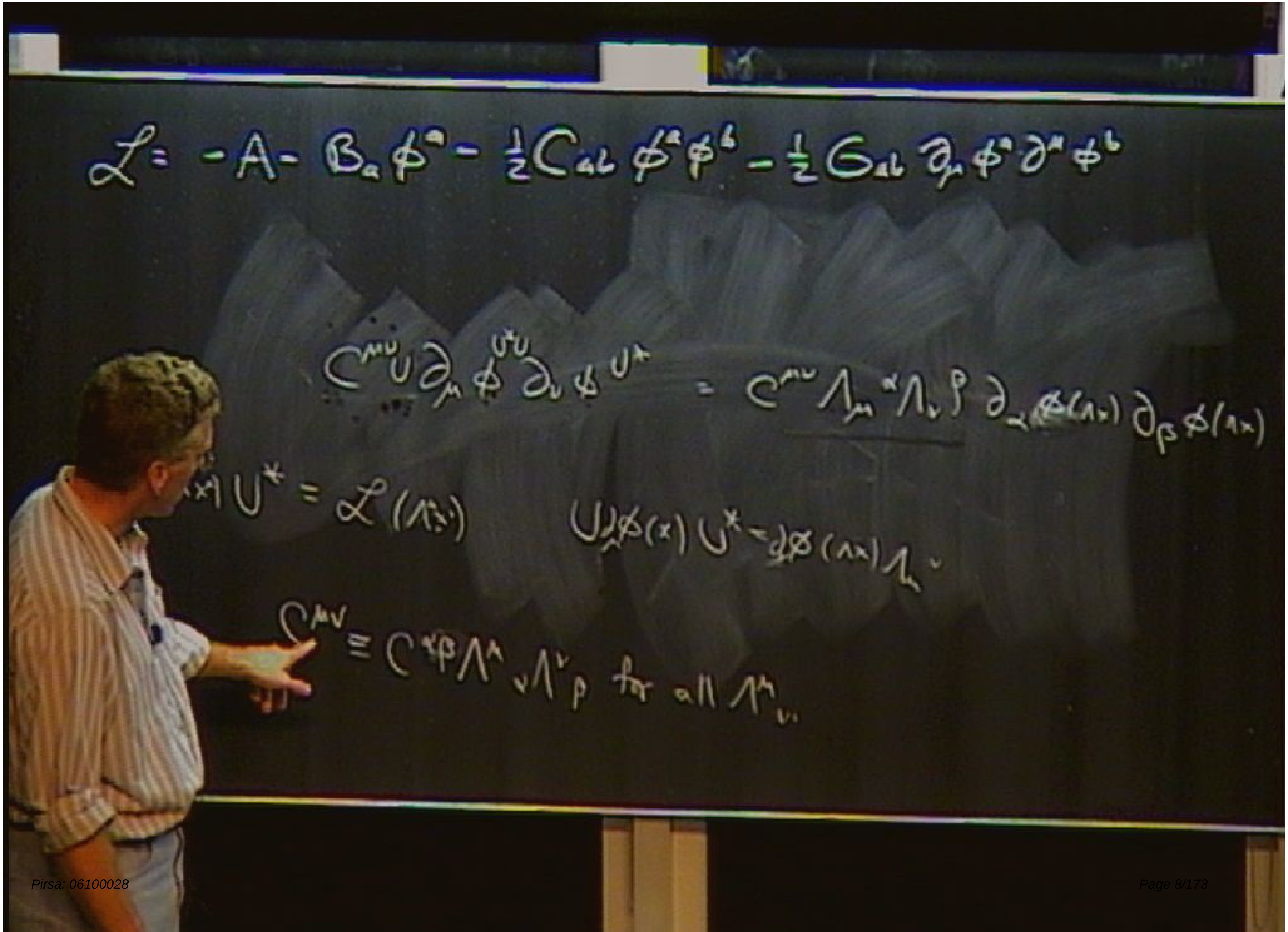
$C^{\mu\nu} U \partial_\mu \phi U^* U \partial_\nu \phi U^* = C^{\mu\nu} \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta \partial_\alpha \phi(\Lambda x) \partial_\beta \phi(\Lambda x)$
$(x) U^* = \mathcal{L}(\Lambda x)$
$U \partial_\mu \phi(x) U^* = \partial_\nu \phi(\Lambda x) \Lambda_\mu{}^\nu$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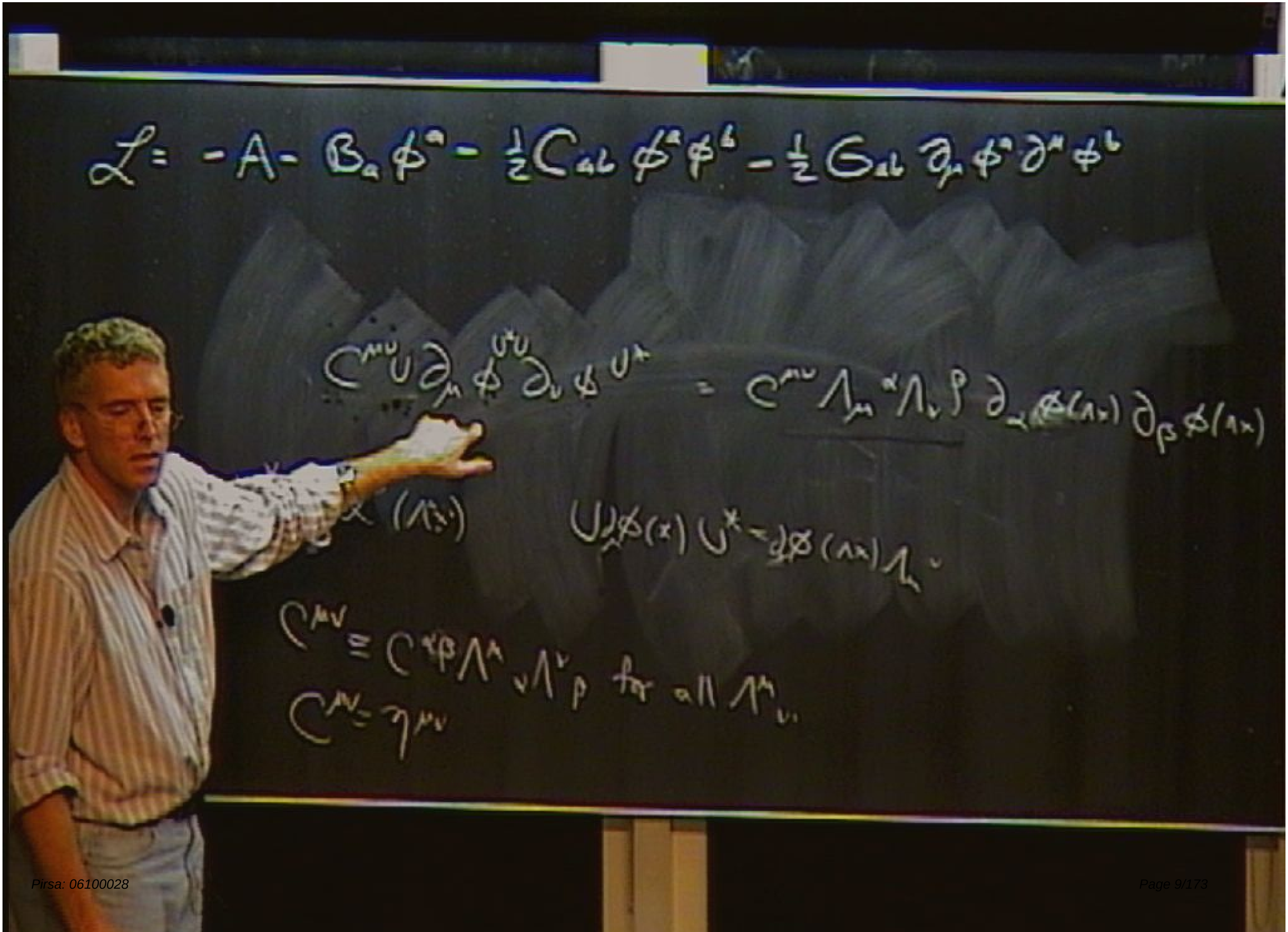
$C^{\mu\nu} = C^{\alpha\beta} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta$ for all $\Lambda^\mu{}_\nu$.



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$\mathcal{L}(\Lambda x)$
$U \partial_\mu \phi(x) U^\dagger = \partial \phi(\Lambda x) \Lambda_\mu{}^\nu$
$C^{\mu\nu} = C^{\alpha\beta} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta$ for all $\Lambda^\mu{}_\nu$.
$C^{\mu\nu} = \eta^{\mu\nu}$

$$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b$$

$$C^{\mu\nu}\,U\partial_\mu\phi U^*\,U\partial_\nu\phi U^* = C^{\mu\nu}\Lambda_\mu{}^\alpha\Lambda_\nu{}^\beta\,\partial_\alpha\phi(\Lambda x)\,\partial_\beta\phi(\Lambda x)$$

$$U\mathcal{L}(x)U^* = \mathcal{L}(\Lambda x) \qquad U\partial_\mu\phi(x)U^* = \partial_\nu\phi(\Lambda x)\Lambda_\mu{}^\nu$$

$$\epsilon^{\mu\nu\lambda\rho} \equiv \epsilon^{\alpha\beta\gamma\delta}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta\Lambda^\lambda{}_\gamma\Lambda^\rho{}_\delta\,(\det\Lambda)$$

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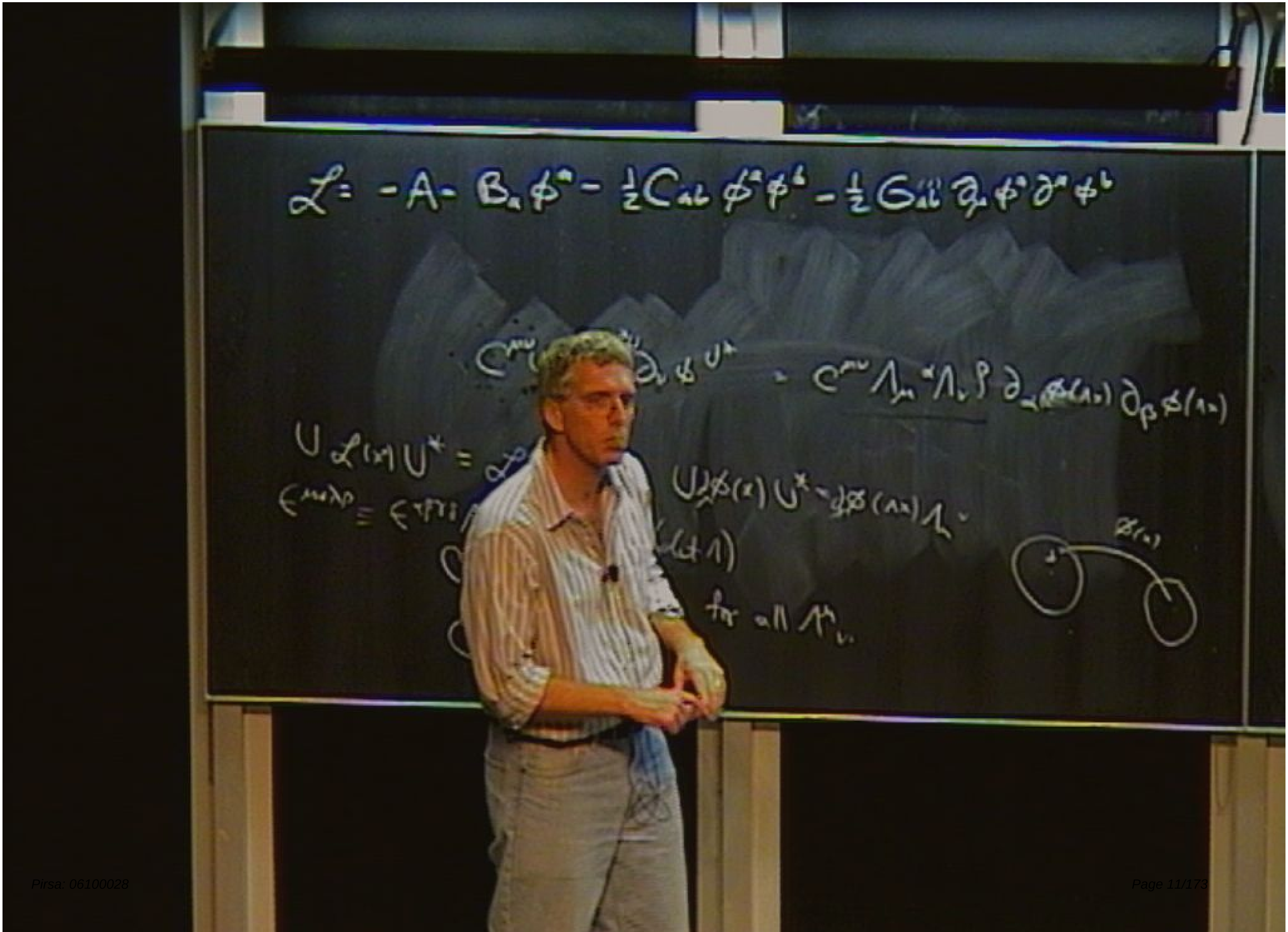
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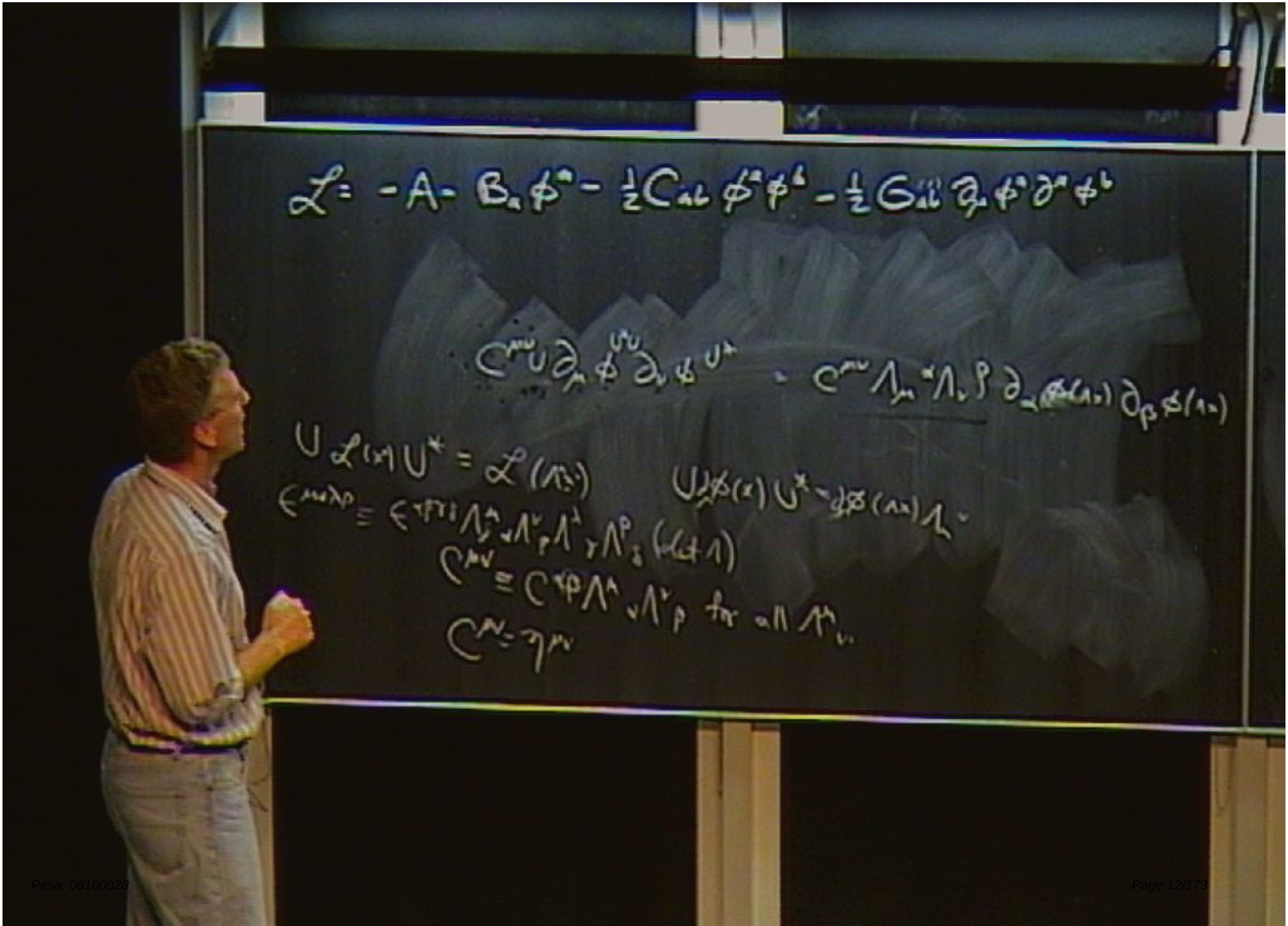

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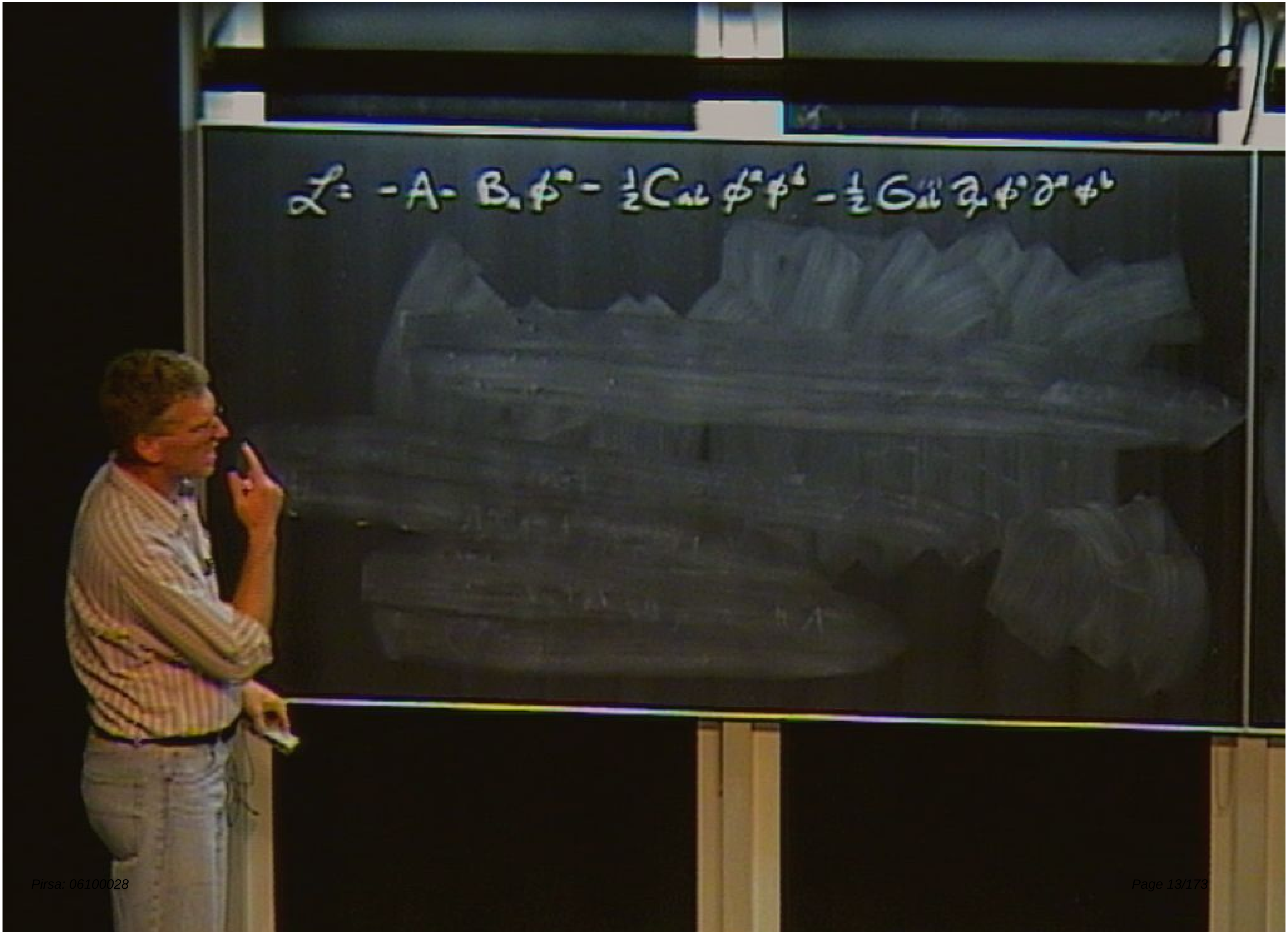
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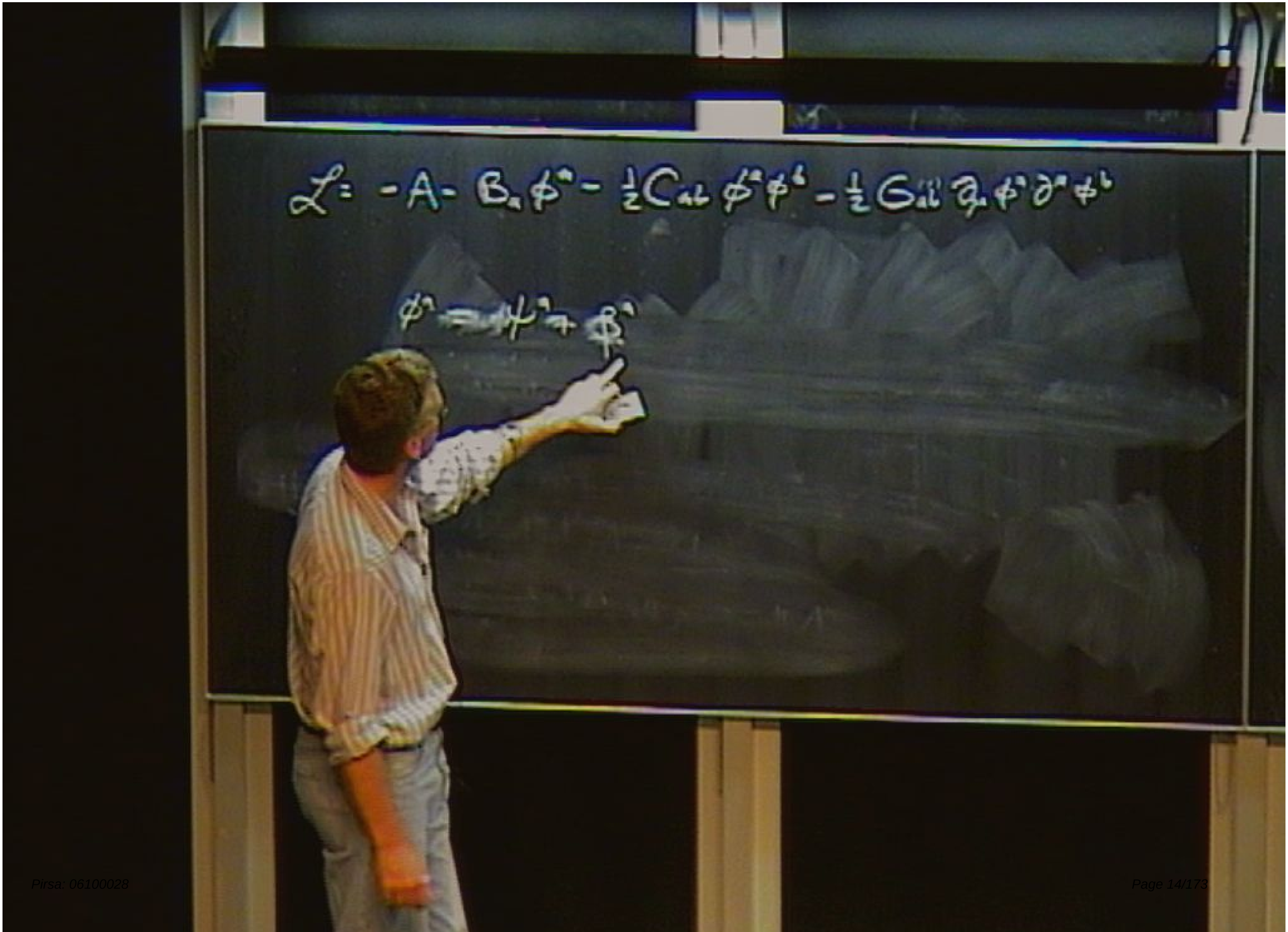
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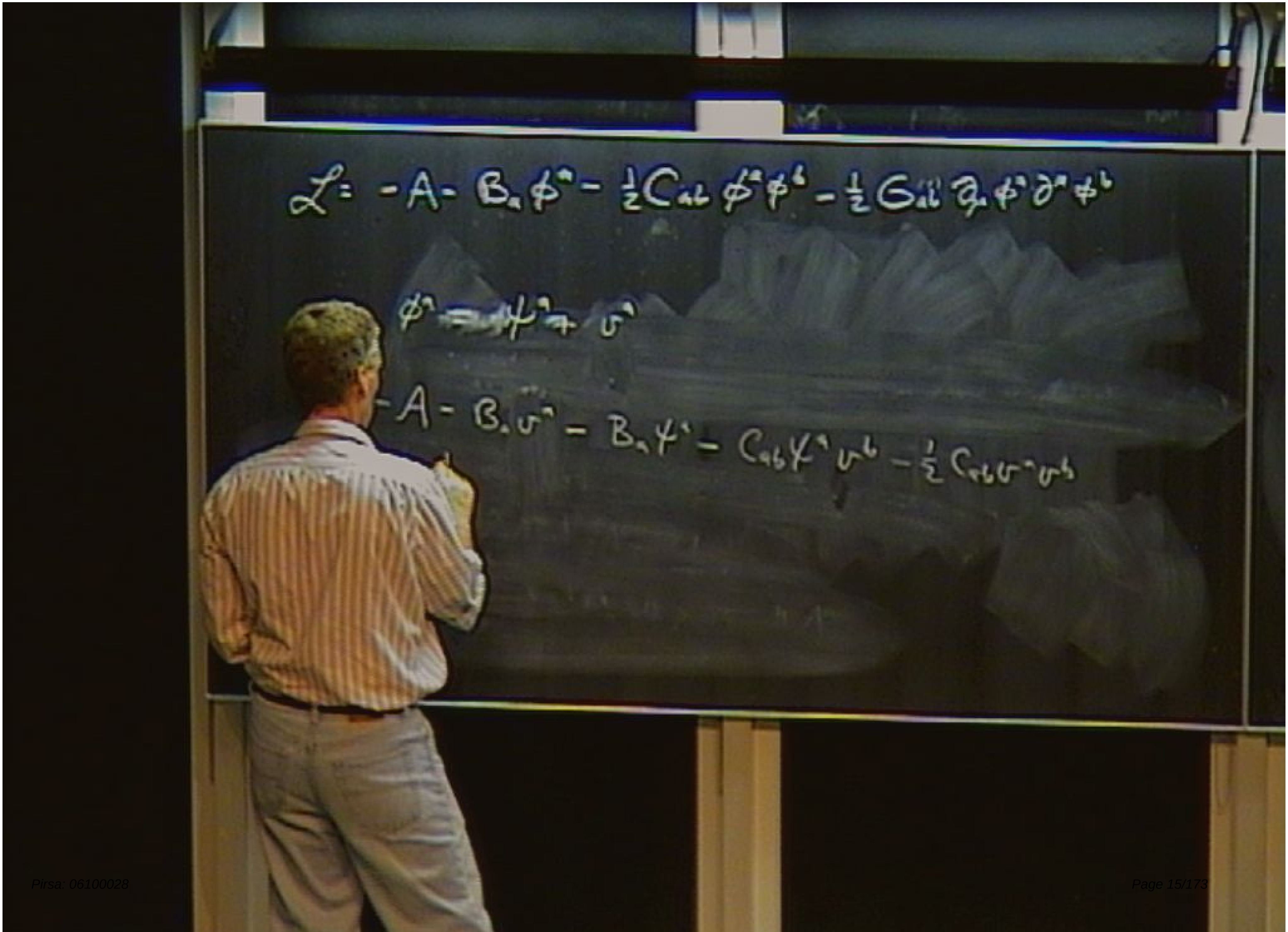
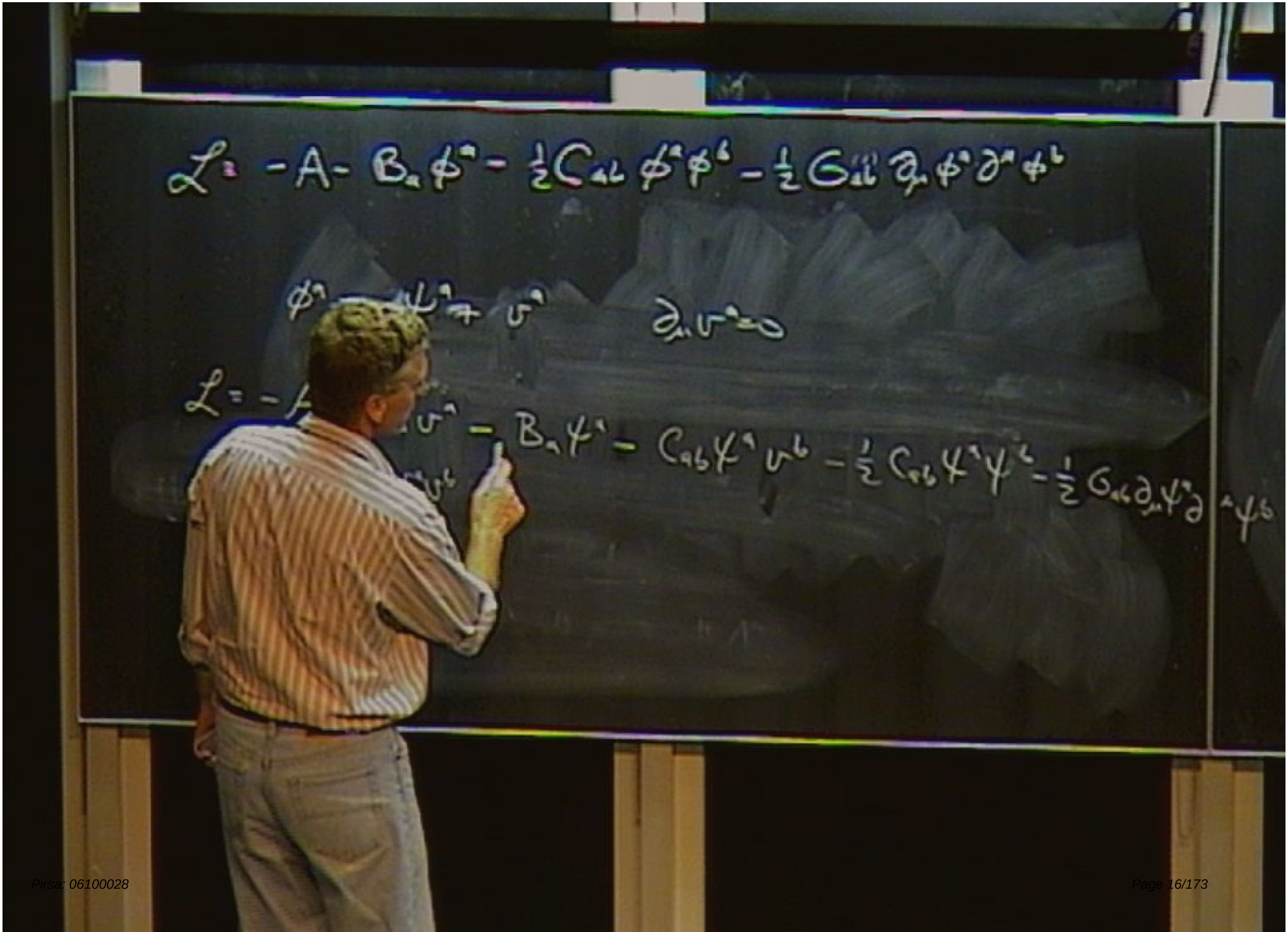


$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$
$\phi^a = \psi^a + v^a$
$-A - B_a v^a - B_a \psi^a - C_{ab} \psi^a v^b - \frac{1}{2} C_{ab} v^a v^b$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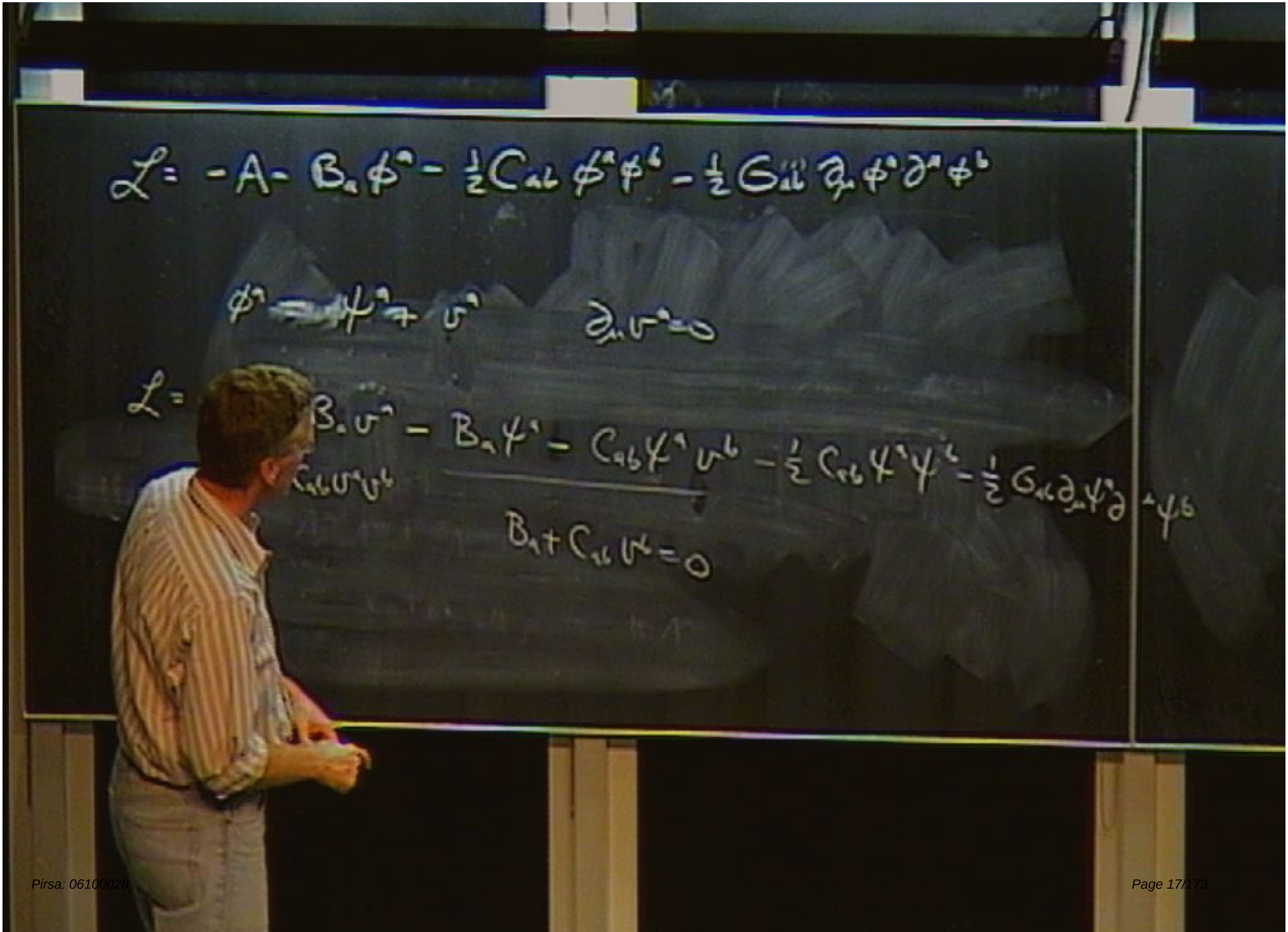
$\phi^a = \psi^a + v^a$
$\partial_\mu v^a = 0$
$- B_a \psi^a - C_{ab} \psi^a v^b - \frac{1}{2} C_{ab} \psi^a \psi^b - \frac{1}{2} G_{ab} \partial_\mu \psi^a \partial^\mu \psi^b$



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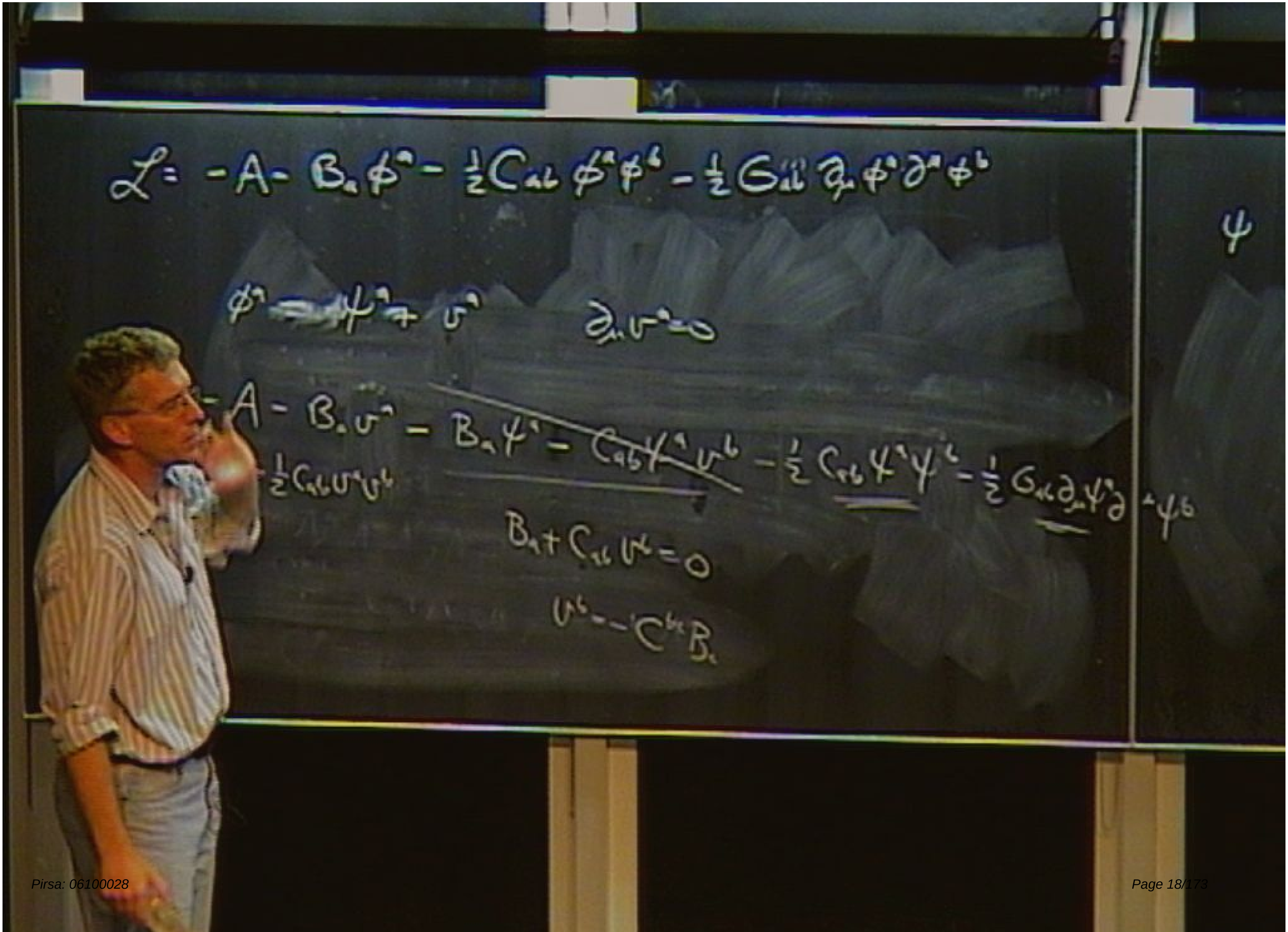
$\phi^a = \psi^a + v^a$
$\partial_\mu v^a = 0$
$\mathcal{L} =$
$B_a v^a - B_a \psi^a - C_{ab} \psi^a v^b - \frac{1}{2} C_{ab} \psi^a \psi^b - \frac{1}{2} G_{ab} \partial_\mu \psi^a \partial^\mu \psi^b$
$C_{ab} v^a v^b$
$B_a + C_{ab} v^b = 0$



$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$
ψ
$\phi^a = \psi^a + v^a$
$\partial_\mu v^a = 0$
$-A - B_a v^a - B_a\psi^a - \cancel{C_{ab}\psi^a v^b} - \frac{1}{2}C_{ab}\psi^a\psi^b - \frac{1}{2}G_{ab}\partial_\mu\psi^a\partial^\mu\psi^b$
$-\frac{1}{2}C_{ab}v^a v^b$
$B_a + C_{ab}v^b = 0$
$v^b = -C^{-1\,bc}B_c$

$$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$$

ψ

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$$-\frac{1}{2} C_{ab} v^a v^b$$

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ψ

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$$-\tfrac{1}{2}C_{ab}v^a v^b$$

$$B_a + C_{ab}v^b = 0$$

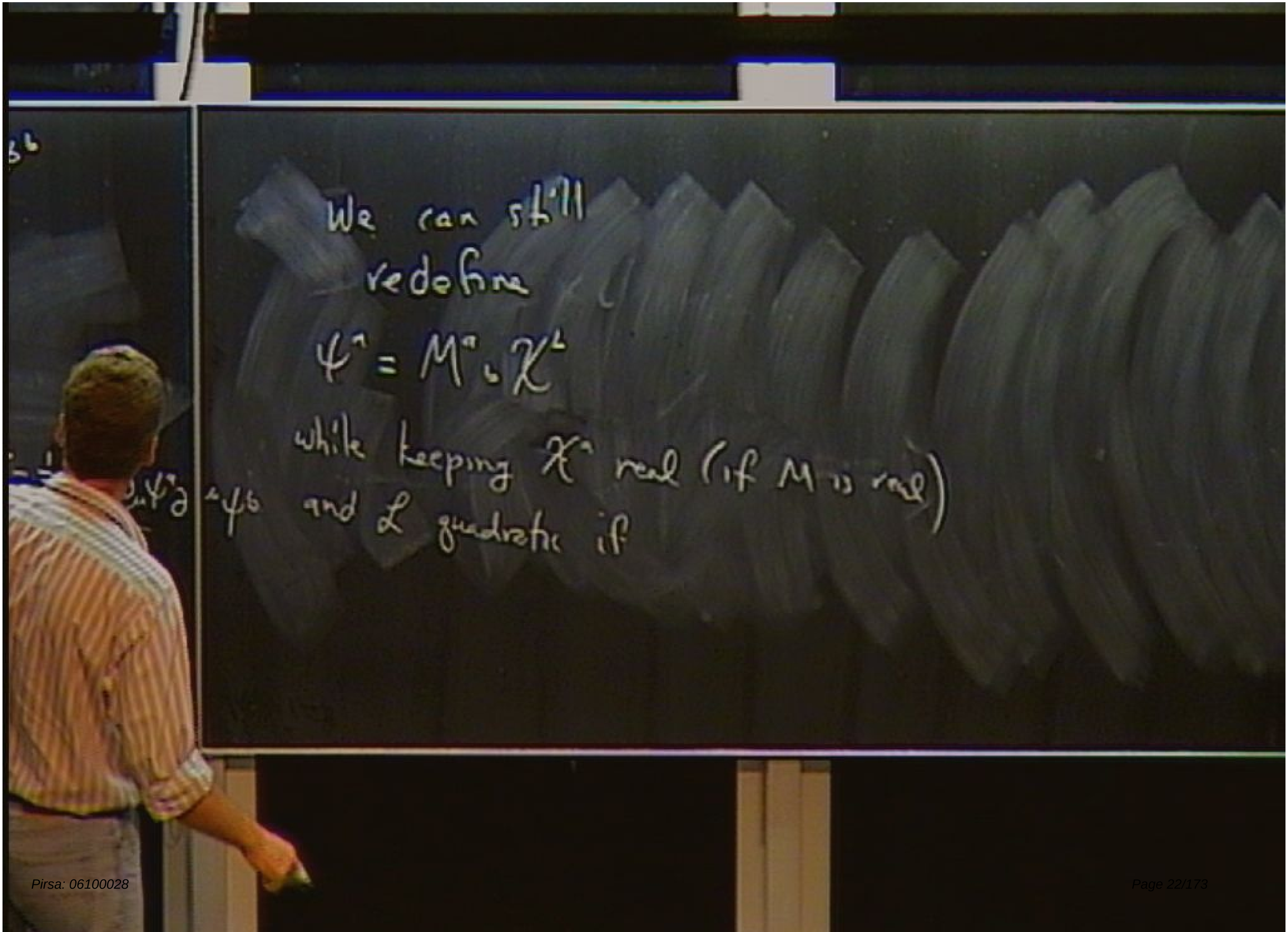
$$v^b = -C^{bc}B_c$$

We can still redefine

$$\psi^a = M^a{}_b \chi^b$$

while keeping

$-\frac{1}{2} G_{ab} \partial_\mu \psi^a \partial^\mu \psi^b$


We can still
redefine
$\psi^a = M^a{}_b \chi^b$
while keeping χ^a real (if M is real)
$\partial_\mu \psi^a \partial^\mu \psi^b$ and $\mathcal{L}$ quadratic if

^{b}

$-\frac{1}{2} G_{ab} \partial_\mu \psi^a \partial^\mu \psi^b$

We can still redefine

$$\psi^a = M^a{}_b \chi^b$$

while keeping χ^a real (if M is real) and $\mathcal{L}$ quadratic,

$$\mathcal{L} = -A'$$

$$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b$$

$$\phi^a = \psi^a + v^a \qquad \partial_\mu v^a = 0$$

$$\mathcal{L} = \underbrace{-A - B_a v^a - \frac{1}{2}C_{ab}v^a v^b}_{-A'} - B_a\psi^a - \cancel{C_{ab}\psi^a v^b} - \frac{1}{2}C_{ab}\psi^a\psi^b - \frac{1}{2}$$

$$B_a + C_{ab}v^b = 0$$

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$$\varphi^a = M^a{}_b \chi^b$$

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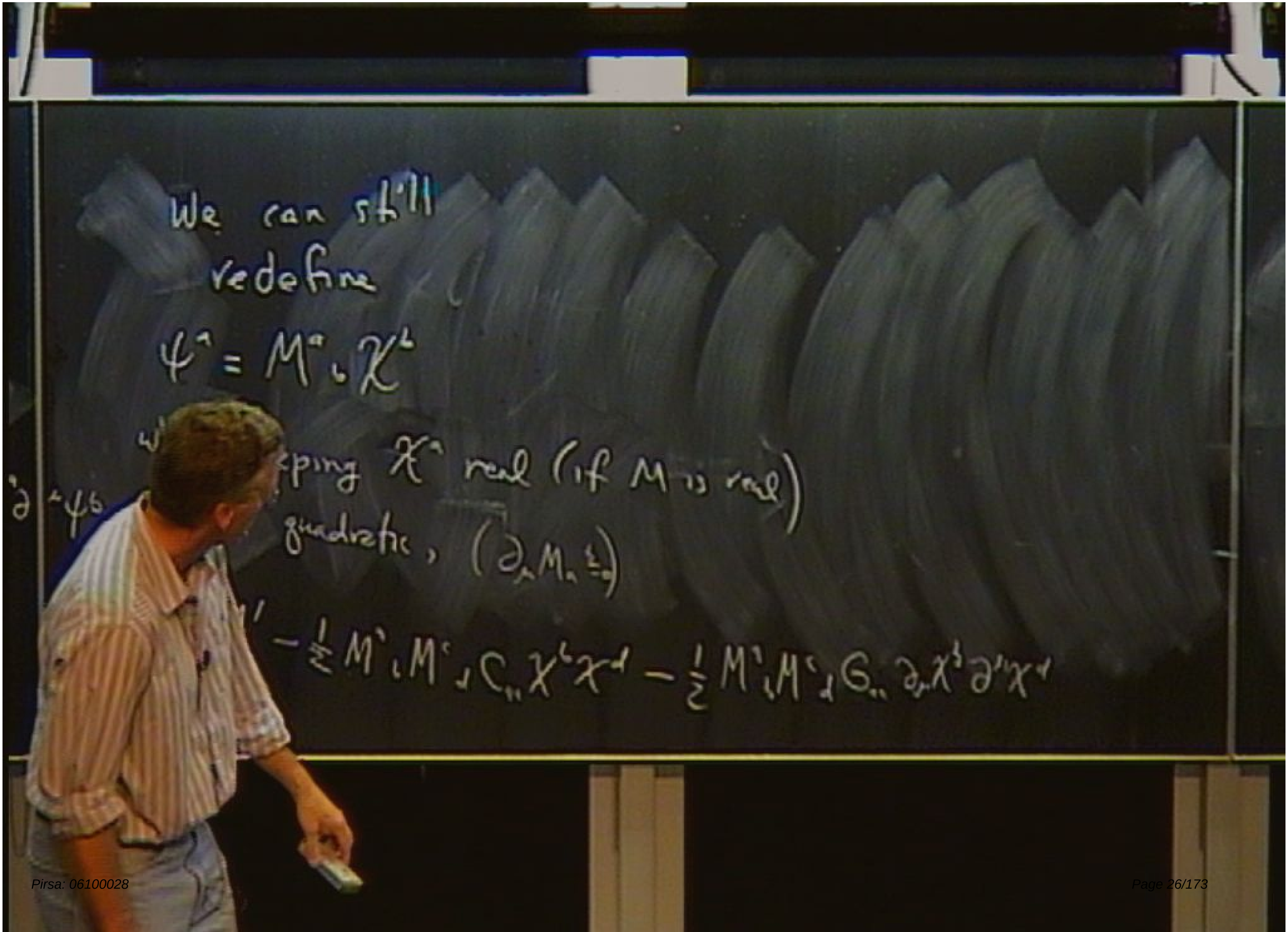
$\partial^\mu \varphi^b$ and $\mathcal{L}$ quadratic,

$$\mathcal{L} = -A' - \frac{1}{2} M^a{}_b M^c{}_d C_{ac} \chi^b \chi$$


We can still
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$\psi^a = M^a{}_b \chi^b$
χ^a real (if M is real)
quadratic, $(\partial_\mu M_a{}^b = 0)$
$- \frac{1}{2} M^a{}_b M^c{}_d C_{ac} \chi^b \chi^d - \frac{1}{2} M^a{}_b M^c{}_d G_{ac} \partial_\mu \chi^b \partial^\mu \chi^d$

We can still redefine

$$\varphi^a = M^a{}_b \chi^b$$

while keeping χ^a real (if M is real) and $\mathcal{L}$ quadratic, $\left(\partial_\mu M^a{}_b = 0\right)$

$$\mathcal{L} = -A' - \frac{1}{2} \underbrace{M^a{}_b M^c{}_d C_{ac}}_{(M^T C M)_{bd}} \chi^b \chi^d - \frac{1}{2} \underset{(M^T G M)_{bd}}{M^a{}_b M^c{}_d G_{ac}} \partial_\mu \chi^b \partial^\mu \chi^d$$

We can still redefine

$$\psi^a = M^a{}_b \chi^b$$

while keeping χ^a real (if M is real)

$\partial^\mu \psi^b$ and $\mathcal{L}$ quadratic, $(\partial_\mu M_{ab} = 0)$

$$\mathcal{L} = -A' - \frac{1}{2} \underbrace{M^a{}_b M^c{}_d C_{ac}}_{(M^T C M)_{bd}} \chi^b \chi^d - \frac{1}{2} \underbrace{M^a{}_b M^c{}_d G_{ac}}_{(M^T G M)_{bd}} \partial_\mu \chi^b \partial^\mu \chi^d$$

Since G is pos. definite, $G^{-1/2}$ exists.

We can still redefine

$$\psi^a = M^a{}_b \chi^b$$

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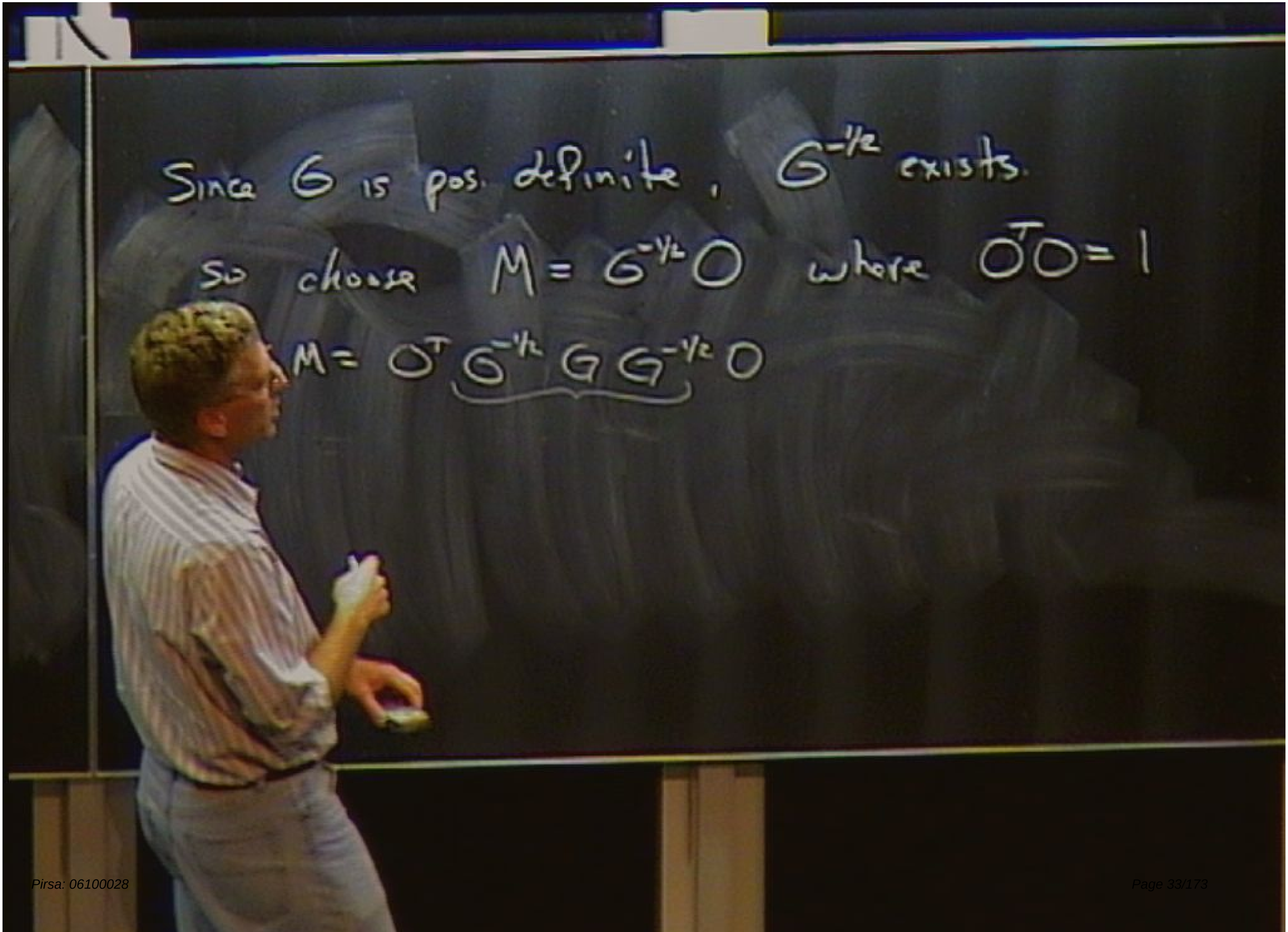
so choose $M = G^{-1/2} O$ where $O^T O = 1$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$$\psi^a = M^a{}_b \chi^b$$

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and $\mathcal{L}$ quadratic, $(\partial_\mu M_{ab} = 0)$

$$\mathcal{L} = -A' - \frac{1}{2} \underbrace{M^a{}_b M^c{}_d C_{ac}}_{(M^T C M)_{bd}} \chi^b \chi^d - \frac{1}{2} \underbrace{\overset{\delta_{bd}}{\overset{\shortparallel}{M^a{}_b M^c{}_d G_{ac}}}}_{(M^T G M)_{bd}} \partial_\mu \chi^b \partial^\mu \chi^d$$

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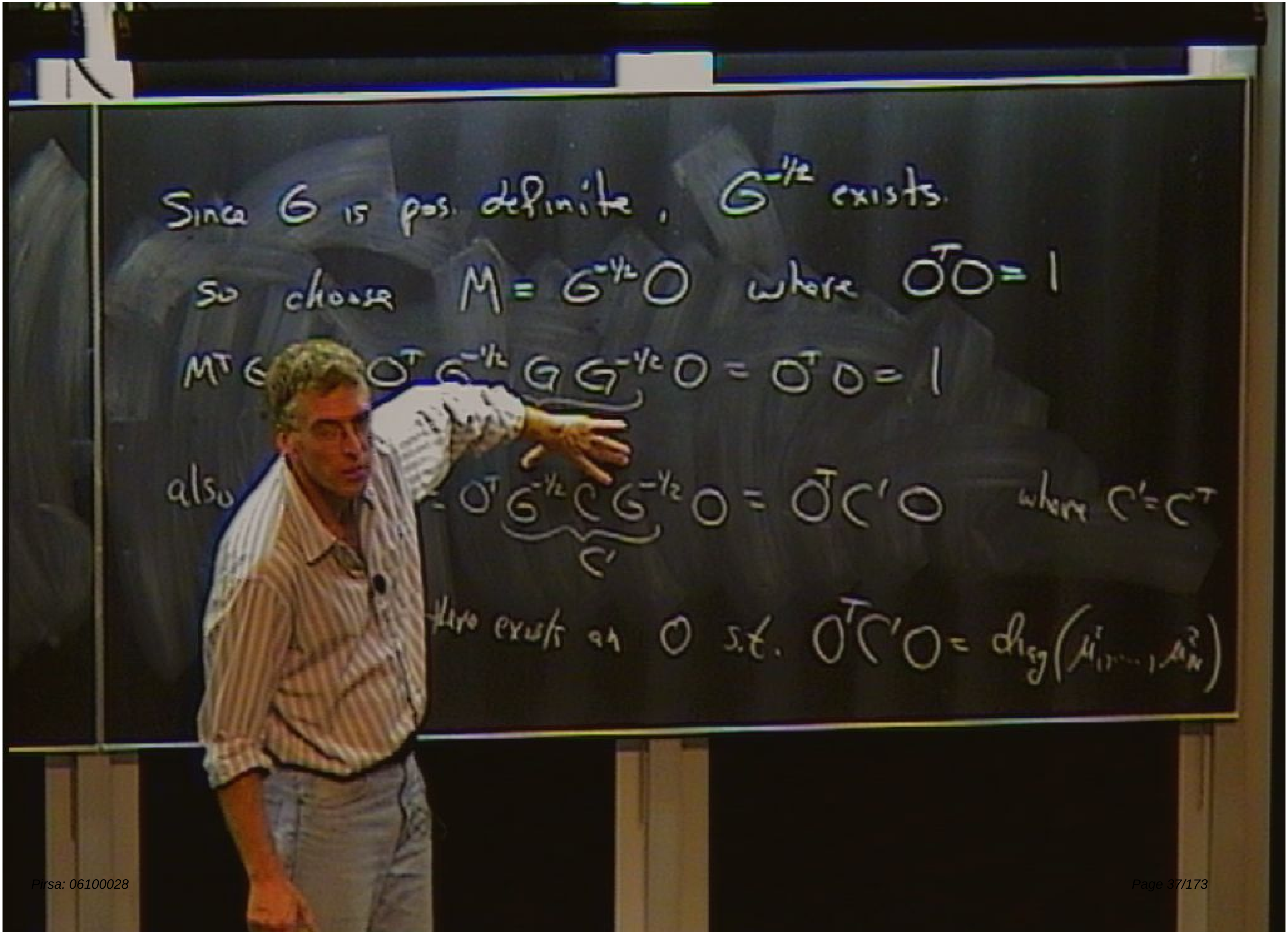
also $$M^T C M = O^T \underbrace{G^{-1/2} C G^{-1/2}}_{C'} O = O^T C' O$$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$M^T G$ … $O^T G^{-1/2} \underbrace{G G^{-1/2}}\, O = O^T O = 1$
also … $= O^T \underbrace{G^{-1/2} C G^{-1/2}}_{C'} O = O^T C' O$ where $C' = C'^T$
…there exists an O s.t. $O^T C' O = \mathrm{diag}(\mu_1^2, \ldots, \mu_N^2)$

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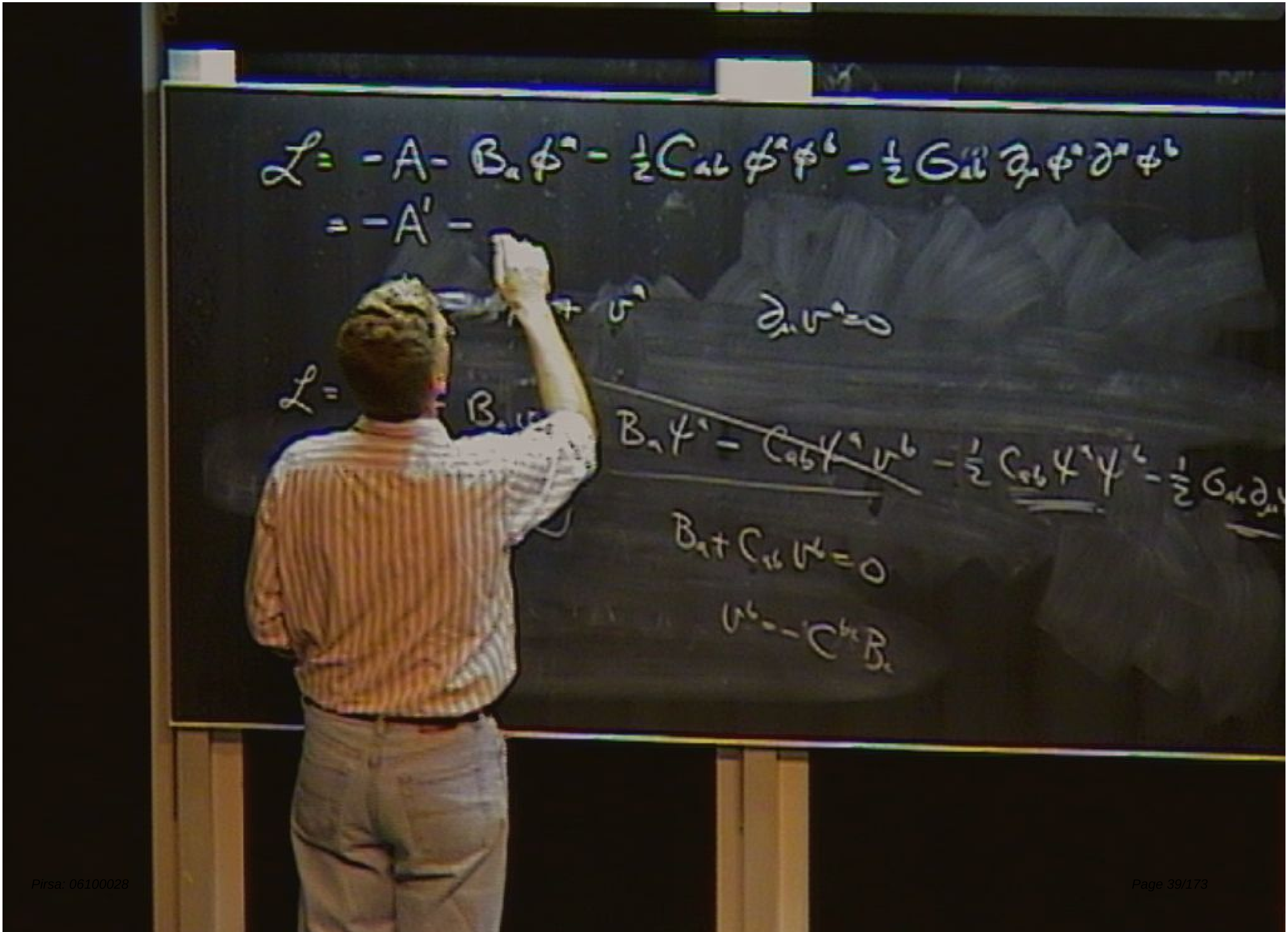
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$\partial_\mu v^a = 0$
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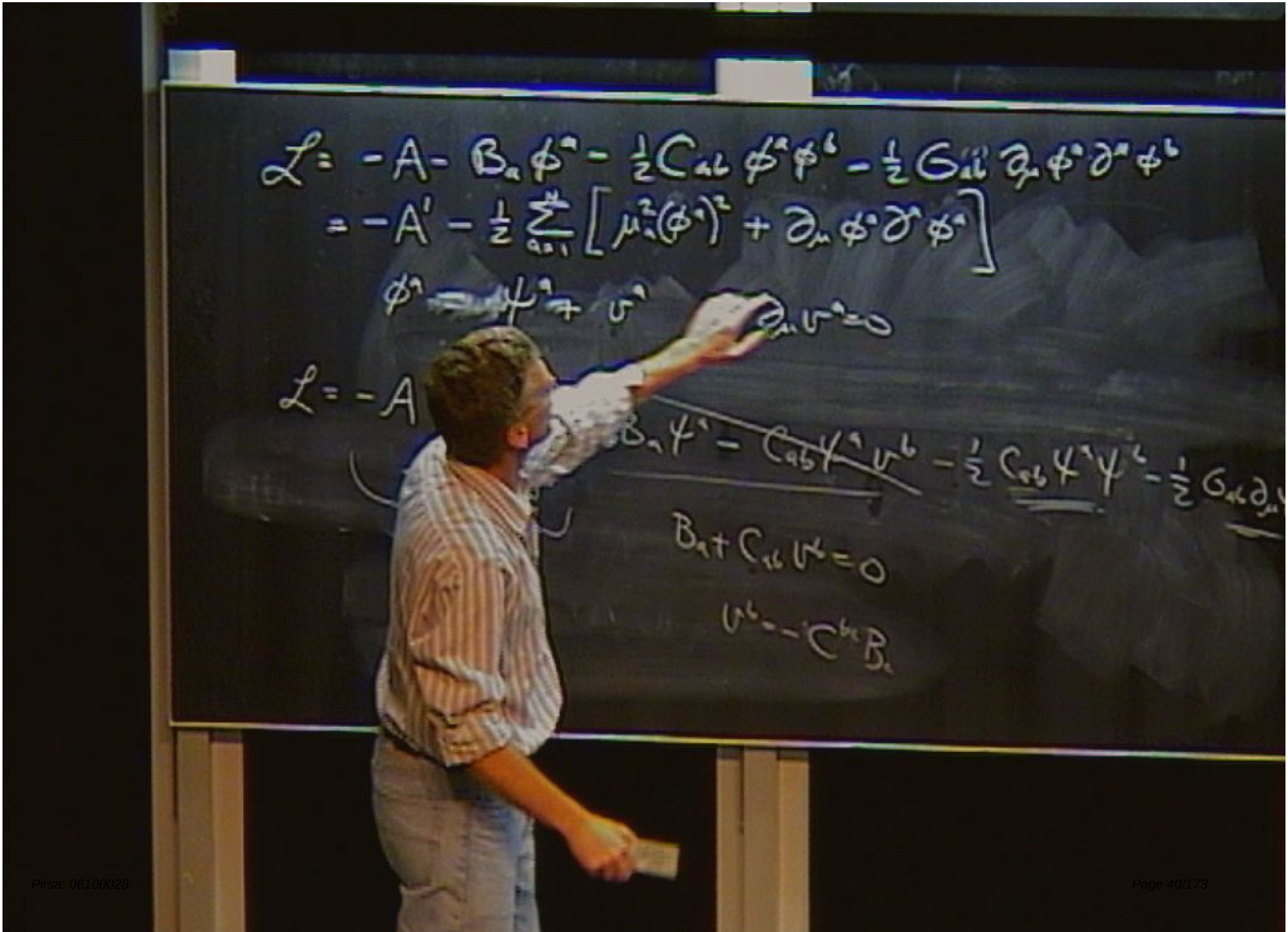
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$= -A' - \frac{1}{2} \sum_{a=1}^{N} \left[\mu_a^2 (\phi^a)^2 + \partial_\mu \phi^a \partial^\mu \phi^a \right]$
$\phi^a = \psi^a + v^a$
$\partial_\mu v^a = 0$
$\mathcal{L} = -A$
$B_a \psi^a - C_{ab} \psi^a v^b - \frac{1}{2} C_{ab} \psi^a \psi^b - \frac{1}{2} G_{ab} \partial_\mu \psi$
$B_a + C_{ab} v^b = 0$
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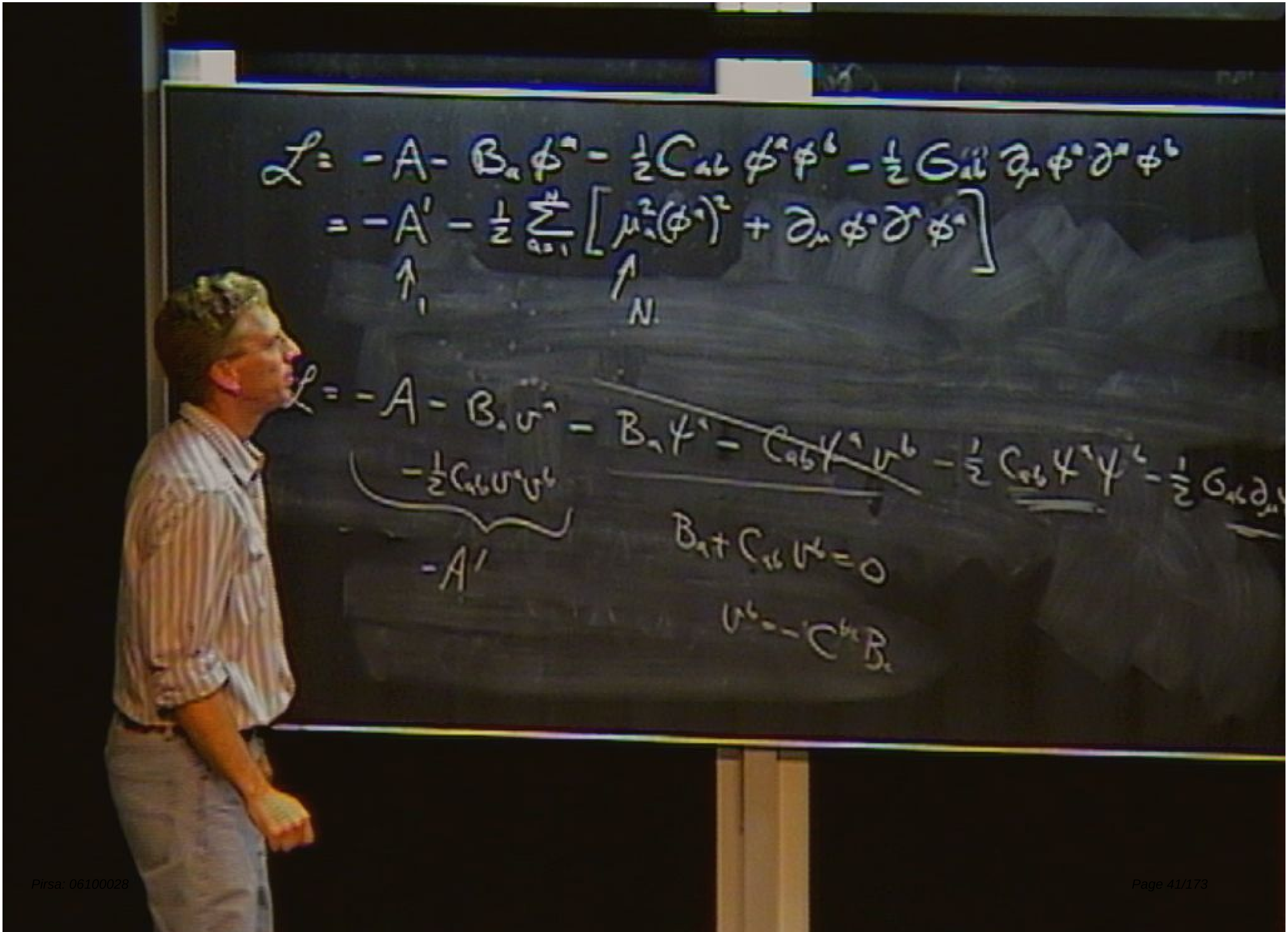
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1
N
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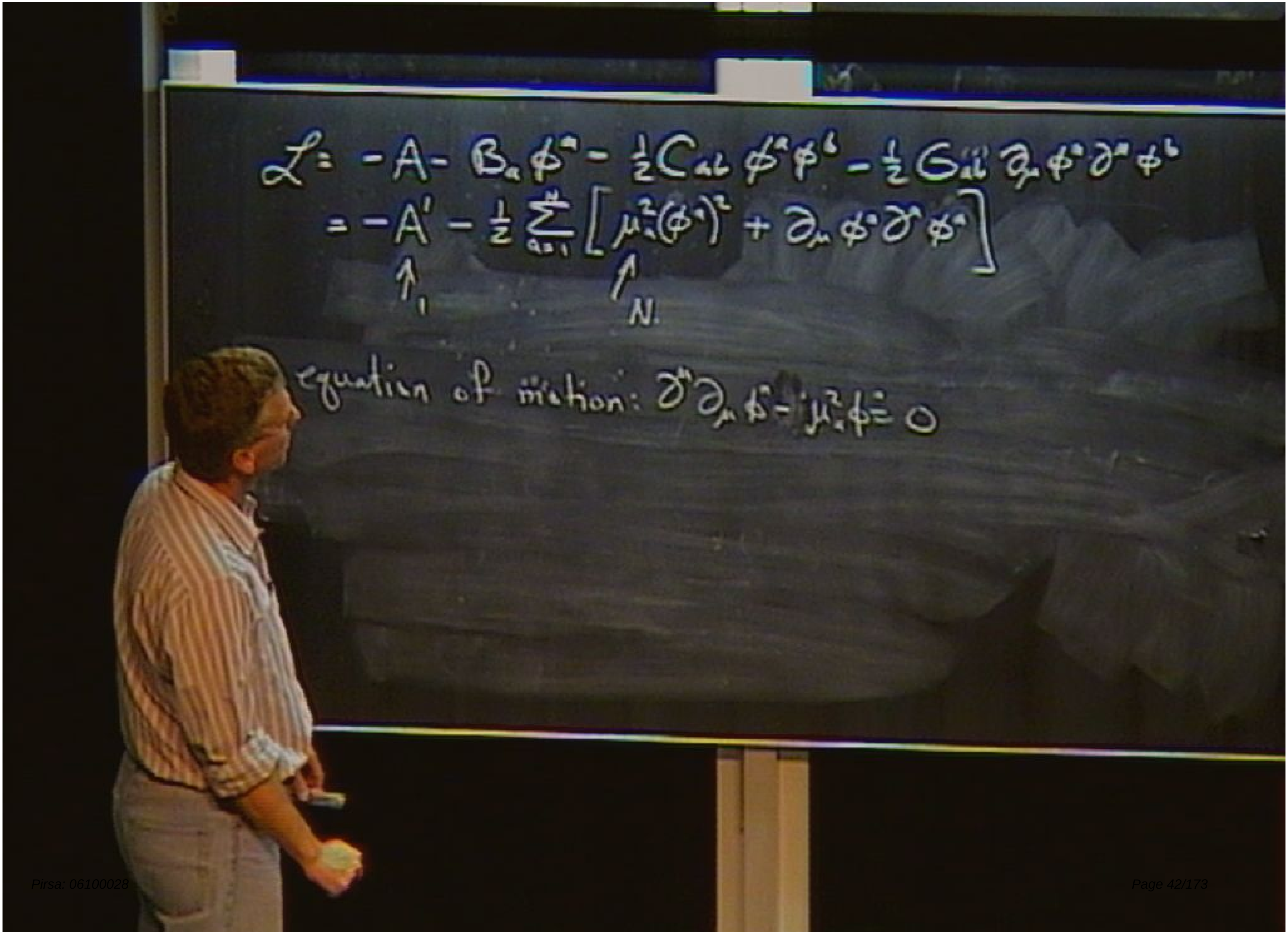
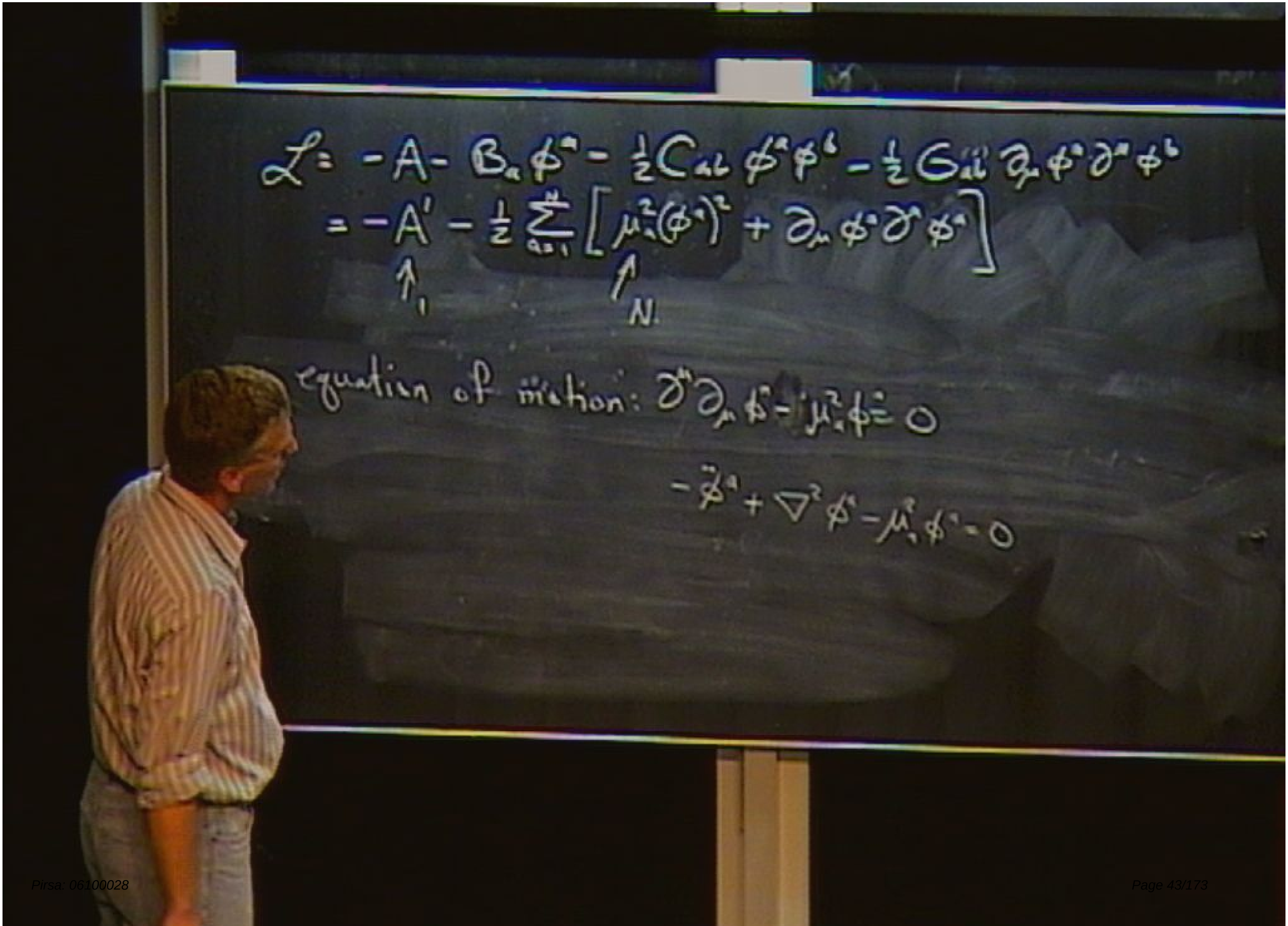
$\underbrace{-\frac{1}{2} C_{ab} v^a v^b}_{-A'}$
$B_a + C_{ab} v^b = 0$
$v^b = -C^{bc} B_c$


$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$
$= -A' - \frac{1}{2}\sum_{a=1}^{N}\left[\mu_a^2(\phi^a)^2 + \partial_\mu\phi^a\partial^\mu\phi^a\right]$
1
N
equation of motion: $\partial^\mu\partial_\mu\phi^a - \mu_a^2\phi^a = 0$


$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$
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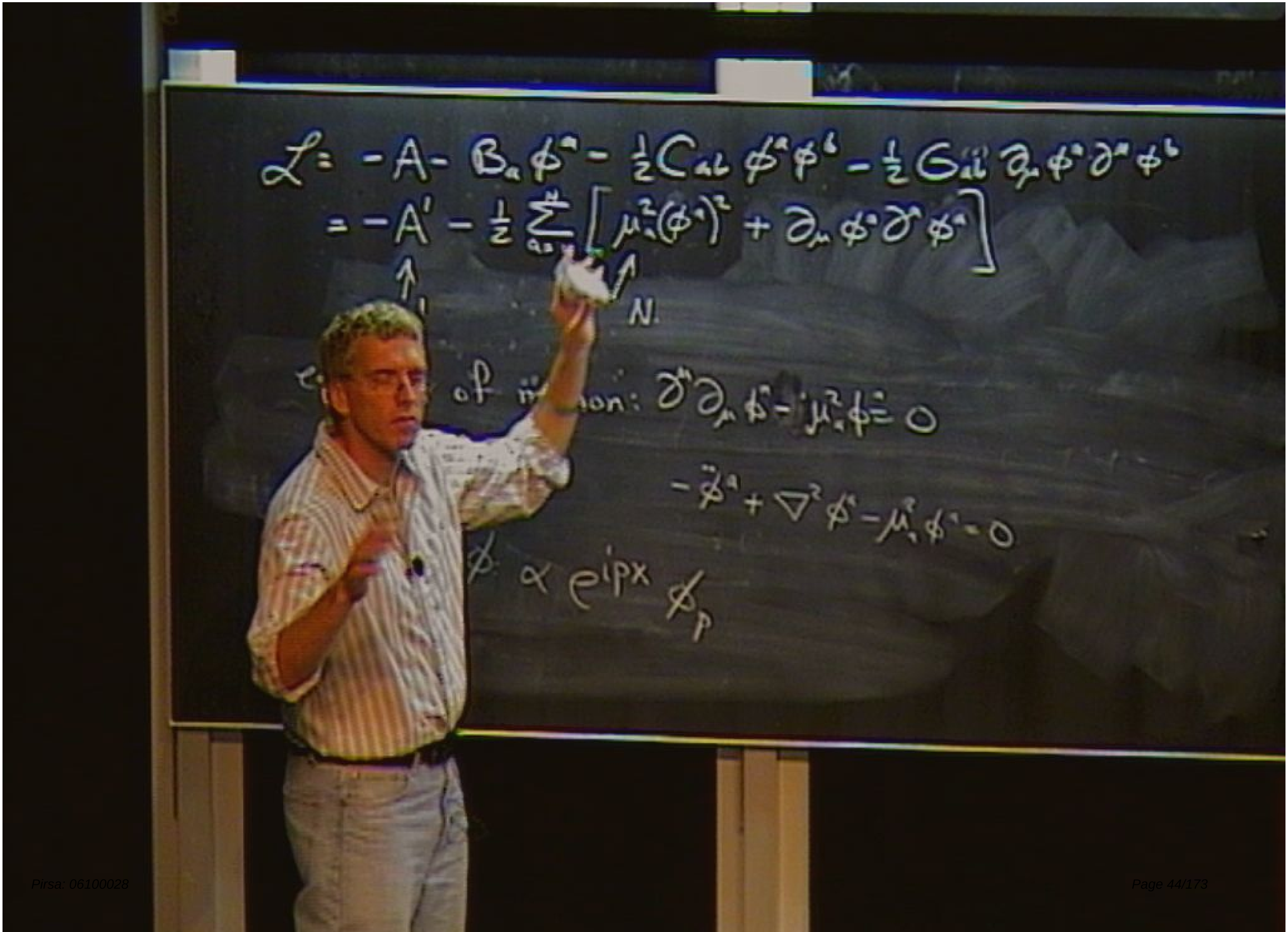
1
N
equation of motion: $\partial^\mu \partial_\mu \phi^a - \mu_a^2 \phi^a = 0$
$-\ddot{\phi}^a + \nabla^2 \phi^a - \mu_a^2 \phi^a = 0$


$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$
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N
of
$\partial^\mu \partial_\mu \phi^a - \mu_a^2 \phi^a = 0$
$-\ddot{\phi}^a + \nabla^2 \phi^a - \mu_a^2 \phi^a = 0$
$\phi \propto e^{ipx} \phi_p$

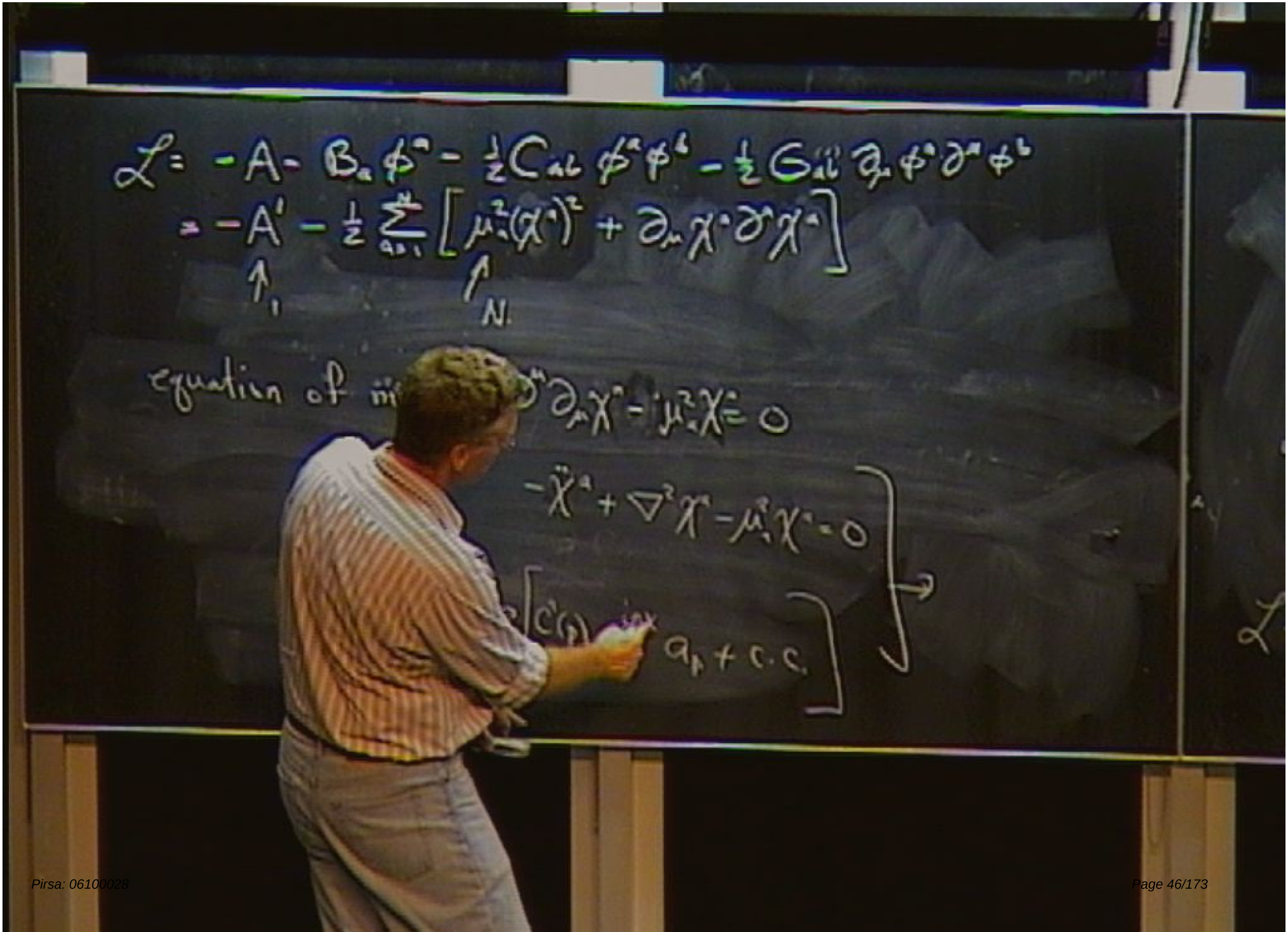
$$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\,\partial_\mu\phi^a\partial^\mu\phi^b$$

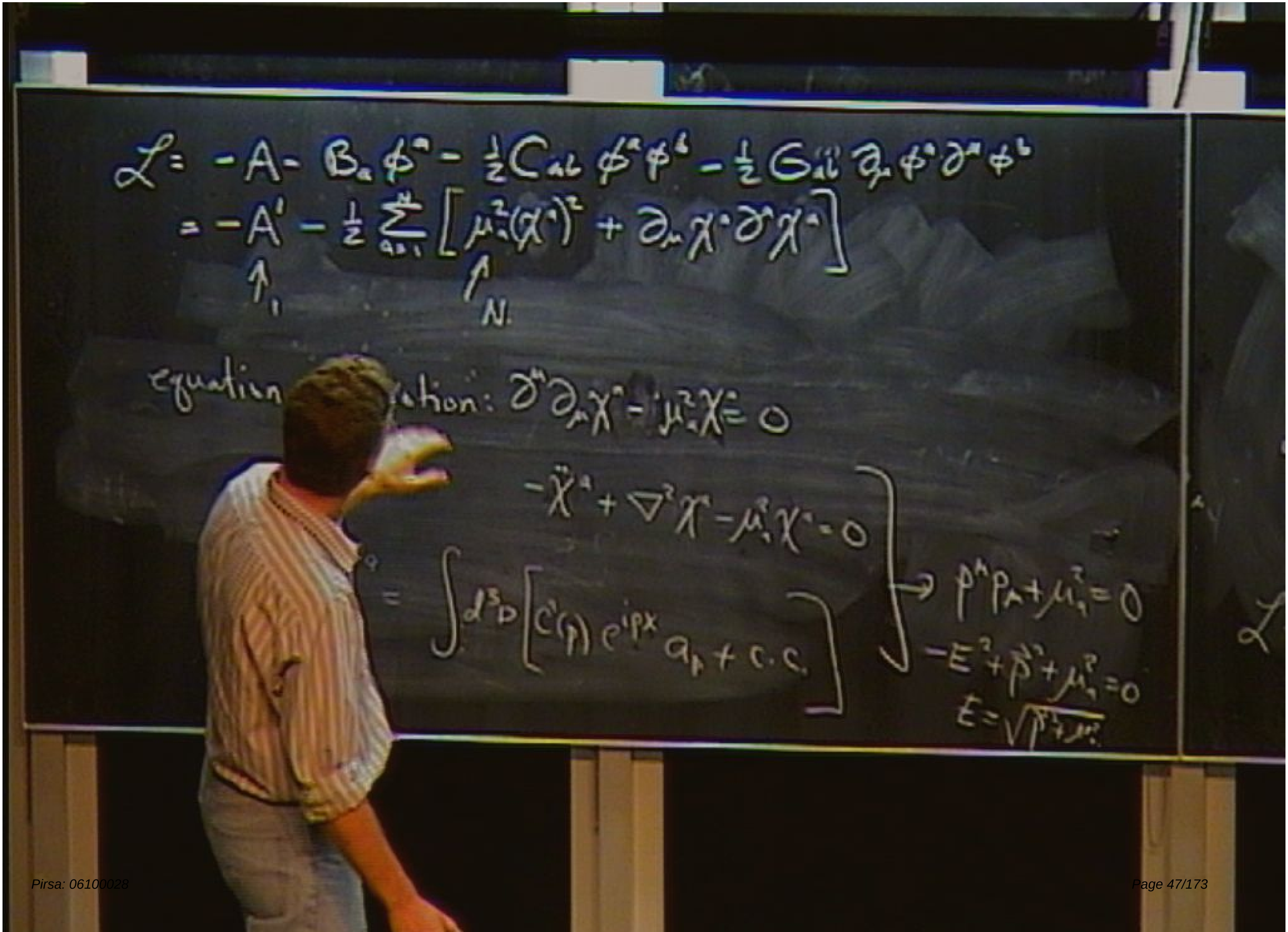
$$= -\underset{1}{\underset{\uparrow}{A'}} - \frac{1}{2}\sum_{a=1}^{N}\left[\underset{N}{\underset{\uparrow}{\mu_a^2}}(\phi^a)^2 + \partial_\mu\phi^a\partial^\mu\phi^a\right]$$

equation of motion: $\partial^\mu\partial_\mu\phi^a - \mu_a^2\phi^a = 0$

$$-\ddot{\phi}^a + \nabla^2\phi^a - \mu_a^2\phi^a = 0$$

$$\phi \propto e^{ipx}\,\phi_p$$


$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$
$= -A' - \frac{1}{2} \sum_{a=1}^{N} \left[\mu_a^2 (\chi^a)^2 + \partial_\mu \chi^a \partial^\mu \chi^a \right]$
1
N
equation of m
$\partial_\mu \chi^a - \mu_a^2 \chi^a = 0$
$-\ddot{\chi}^a + \nabla^2 \chi^a - \mu_a^2 \chi^a = 0$
$a_p + c.c.$


$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$
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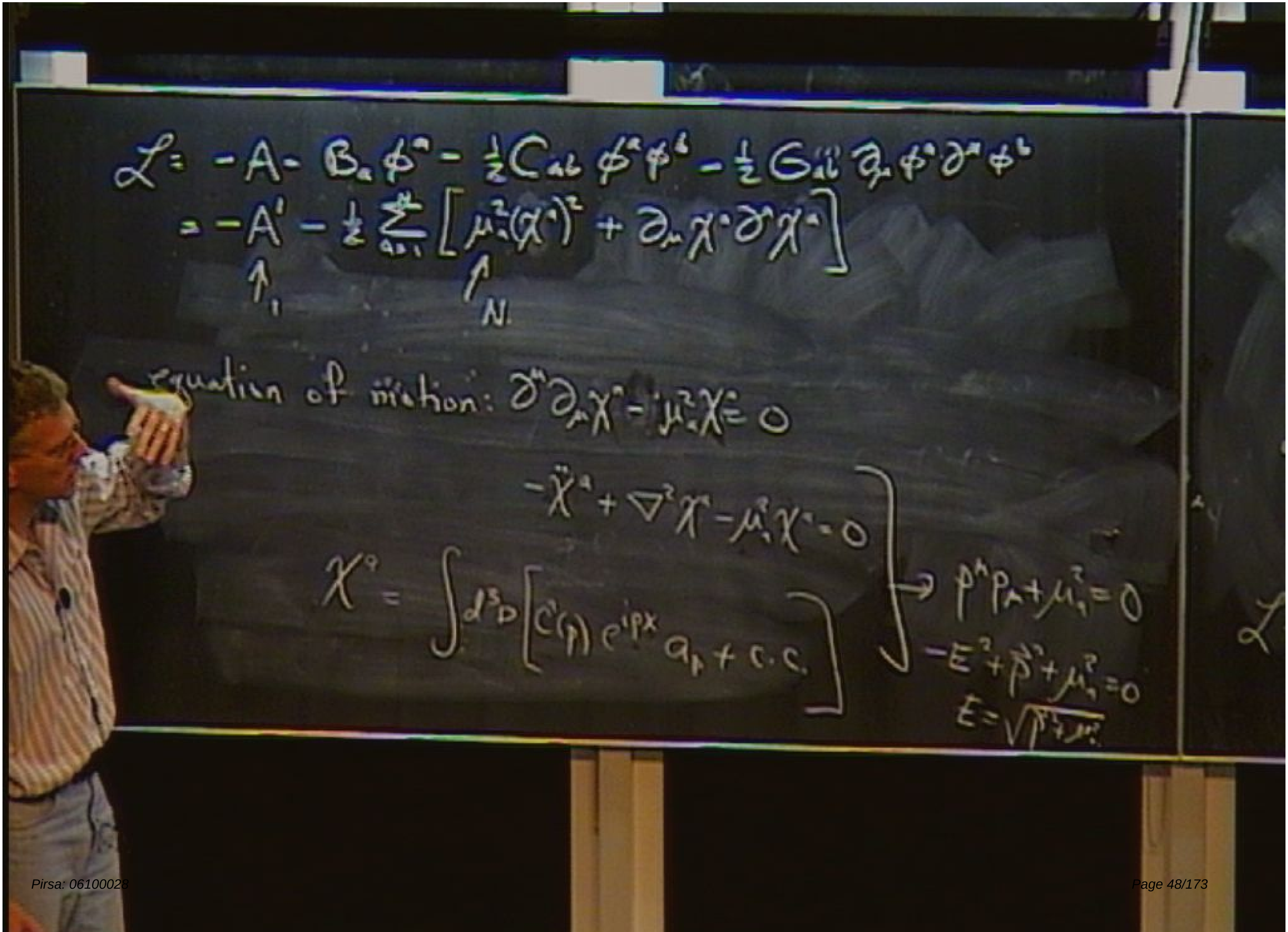
N.
equation ... ation: $\partial^\mu\partial_\mu\chi^a - \mu_a^2\chi^a = 0$
$-\ddot{\chi}^a + \nabla^2\chi^a - \mu_a^2\chi^a = 0$
$= \int d^3p\left[c^a(p)\, e^{ipx} a_p + c.c.\right]$
$\rightarrow p^\mu p_\mu + \mu_a^2 = 0$
$-E^2 + \vec{p}^2 + \mu_a^2 = 0$
$E = \sqrt{\vec{p}^2 + \mu_a^2}$



$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$
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equation of motion: $\partial^\mu \partial_\mu \chi^a - \mu_a^2 \chi^a = 0$
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$\chi^a = \int d^3p \left[c(p) e^{ipx} a_p + c.c. \right]$
$\rightarrow p^\mu p_\mu + \mu_a^2 = 0$
$-E^2 + \vec{p}^2 + \mu_a^2 = 0$
$E = \sqrt{\vec{p}^2 + \mu_a^2}$

$$\mathcal{L} = -A - B_a \phi^a - \tfrac{1}{2} C_{ab} \phi^a \phi^b - \tfrac{1}{2} G_{ab}^{ij} \partial_\mu \phi^a \partial^\mu \phi^b$$

$$= \underbrace{-A'}_{1} - \tfrac{1}{2} \sum_{a=1}^{N} \Big[\underbrace{\mu_a^2 (\chi^a)^2}_{N} + \partial_\mu \chi^a \partial^\mu \chi^a \Big]$$

equation of motion: $\partial^\mu \partial_\mu \chi^a - \mu_a^2 \chi^a = 0$

$$-\ddot{\chi}^a + \nabla^2 \chi^a - \mu_a^2 \chi^a = 0$$

$$\chi^a = \int d^3p \left[c(p)\, e^{ipx}\, a_p + \text{c.c.} \right]$$

$$\rightarrow p^\mu p_\mu + \mu_a^2 = 0$$

$$-E^2 + \vec{p}^{\,2} + \mu_a^2 = 0$$

$$E = \sqrt{\vec{p}^{\,2} + \mu_a^2}$$

$\mu_a \longleftrightarrow$ physical mass $\quad E = \sqrt{\vec{p}^{\,2} + m_a^2}$

$$\mathcal{L} = -A' - \frac{1}{2} \underbrace{M^a{}_b M^c{}_d C_{ac}}_{(M^T C M)_{bd}} \chi^b \chi^d - \frac{1}{2} \underbrace{M^a{}_b M^c{}_d G_{ac}}_{\overset{\delta_{bd}}{=}\,(M^T G M)_{bd}} \partial_\mu \chi^b \partial^\mu \chi^d$$

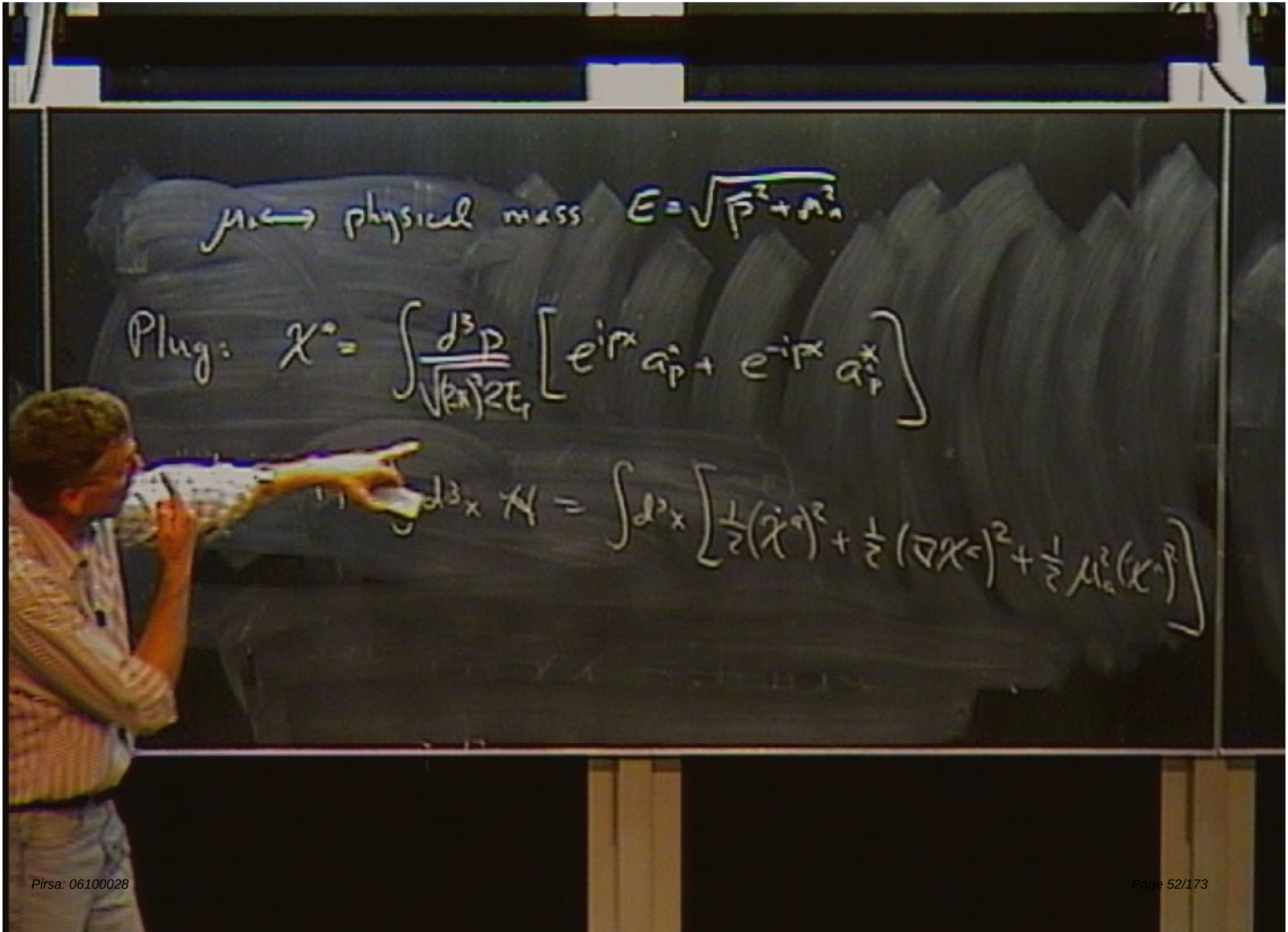
0

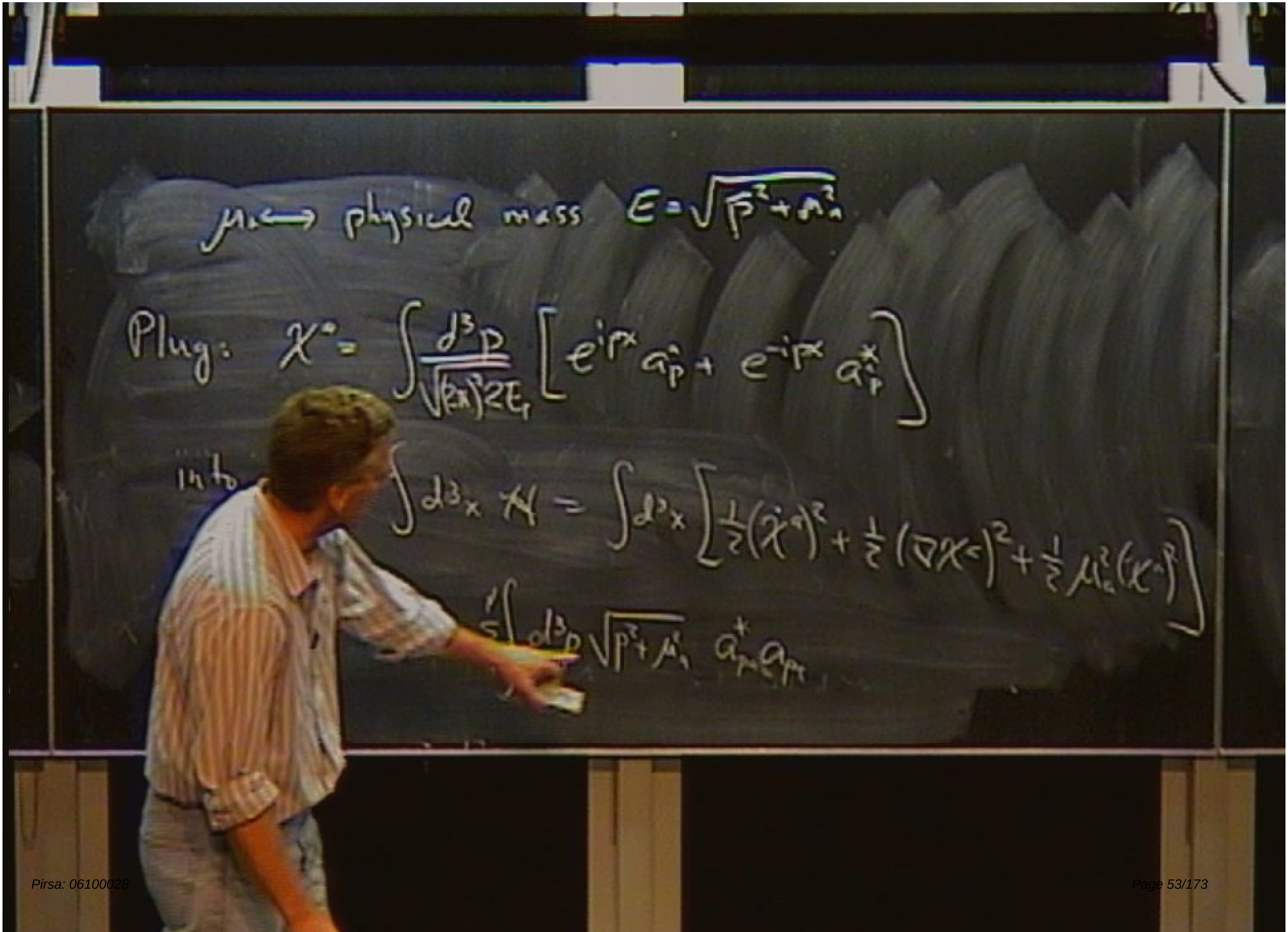
$=0$

$\mu_a \longleftrightarrow$ physical mass $\quad E = \sqrt{\vec{p}^{\,2} + m_a^2}$

Plug: $\quad \chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{a*} \right]$

into $\quad H = \int d^3x \, \mathcal{H}$


$\mu_a \longleftrightarrow$ physical mass $E = \sqrt{\vec{p}^2 + \mu_a^2}$
Plug: $\chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{a*} \right]$
$\int d^3x\, \mathcal{H} = \int d^3x \left[\frac{1}{2}(\dot{\chi}^a)^2 + \frac{1}{2}(\nabla \chi^a)^2 + \frac{1}{2}\mu_a^2 (\chi^a)^2 \right]$



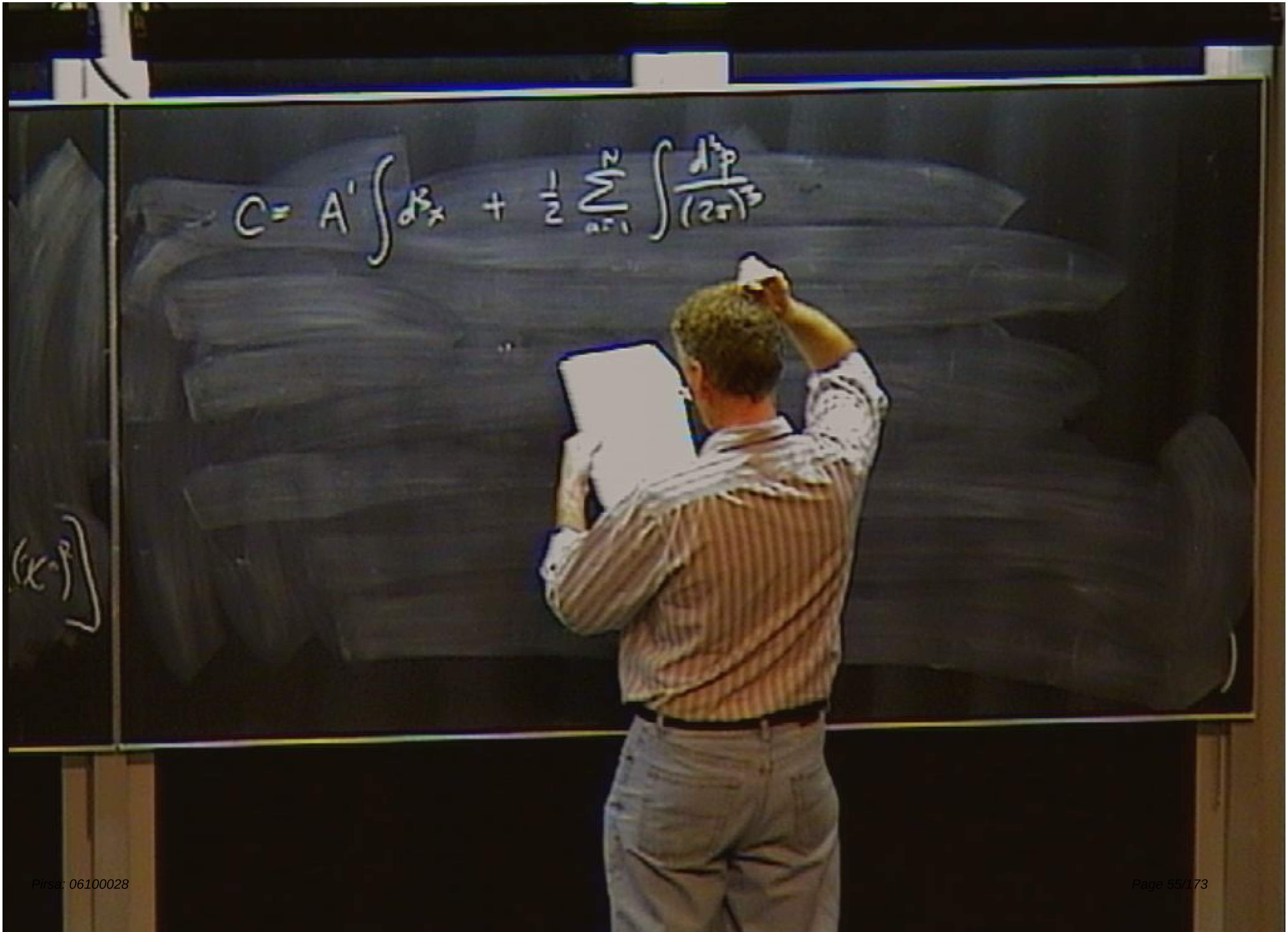
$\mu_n \longleftrightarrow$ physical mass $E = \sqrt{\vec{p}^2 + \mu_n^2}$
Plug: $\chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{a*} \right]$
into $\int d^3x \, \mathcal{H} = \int d^3x \left[\frac{1}{2}(\dot{\chi}^a)^2 + \frac{1}{2}(\nabla \chi^a)^2 + \frac{1}{2}\mu_n^2 (\chi^a)^2 \right]$
$\int d^3p \sqrt{p^2 + \mu_n^2} \; a_{p,a}^* a_{p,a}$

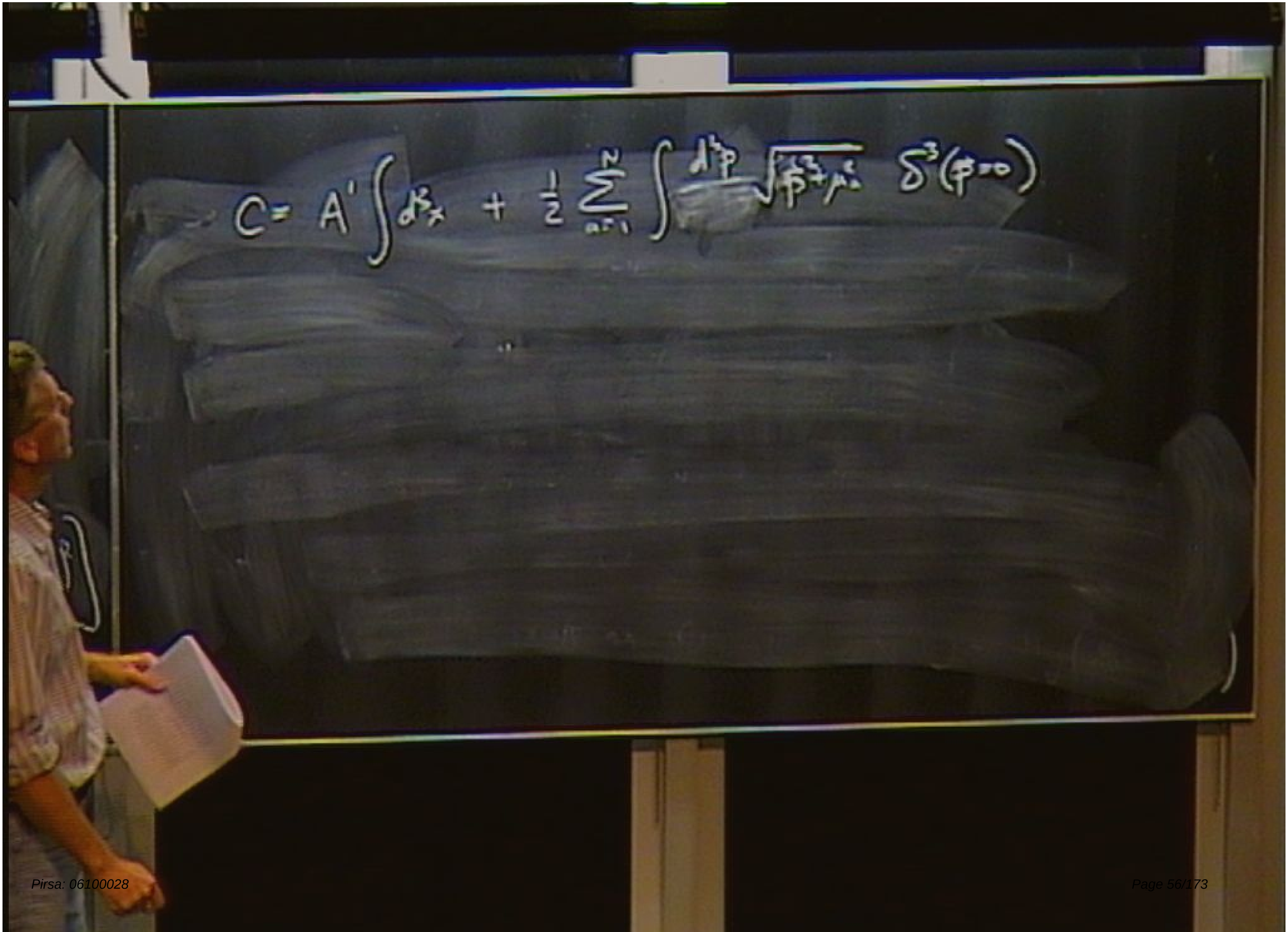
$\mu_a \longleftrightarrow$ physical mass $\quad E = \sqrt{\vec{p}^{\,2} + \mu_a^2}$

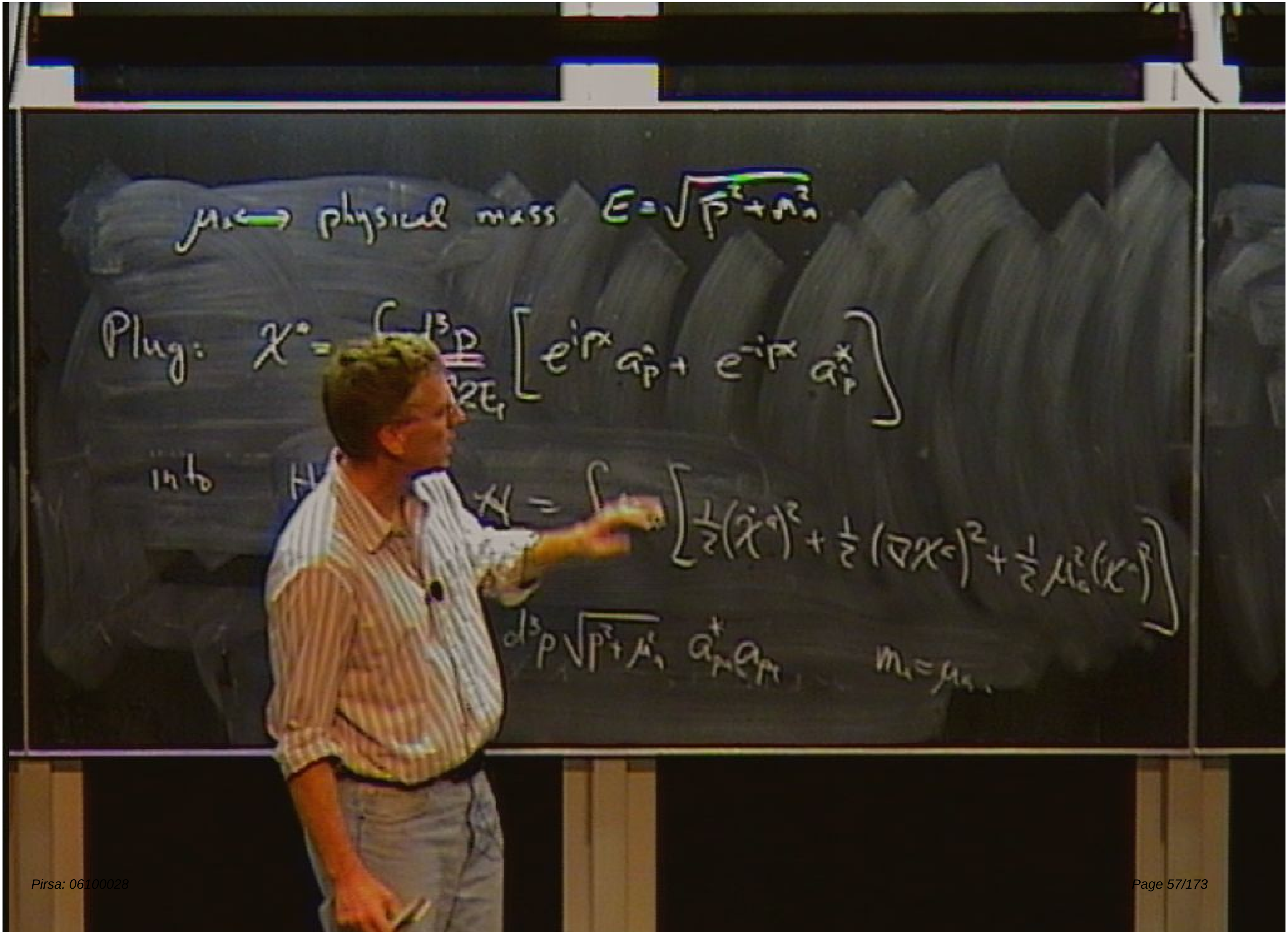
Plug: $\chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{a*} \right]$

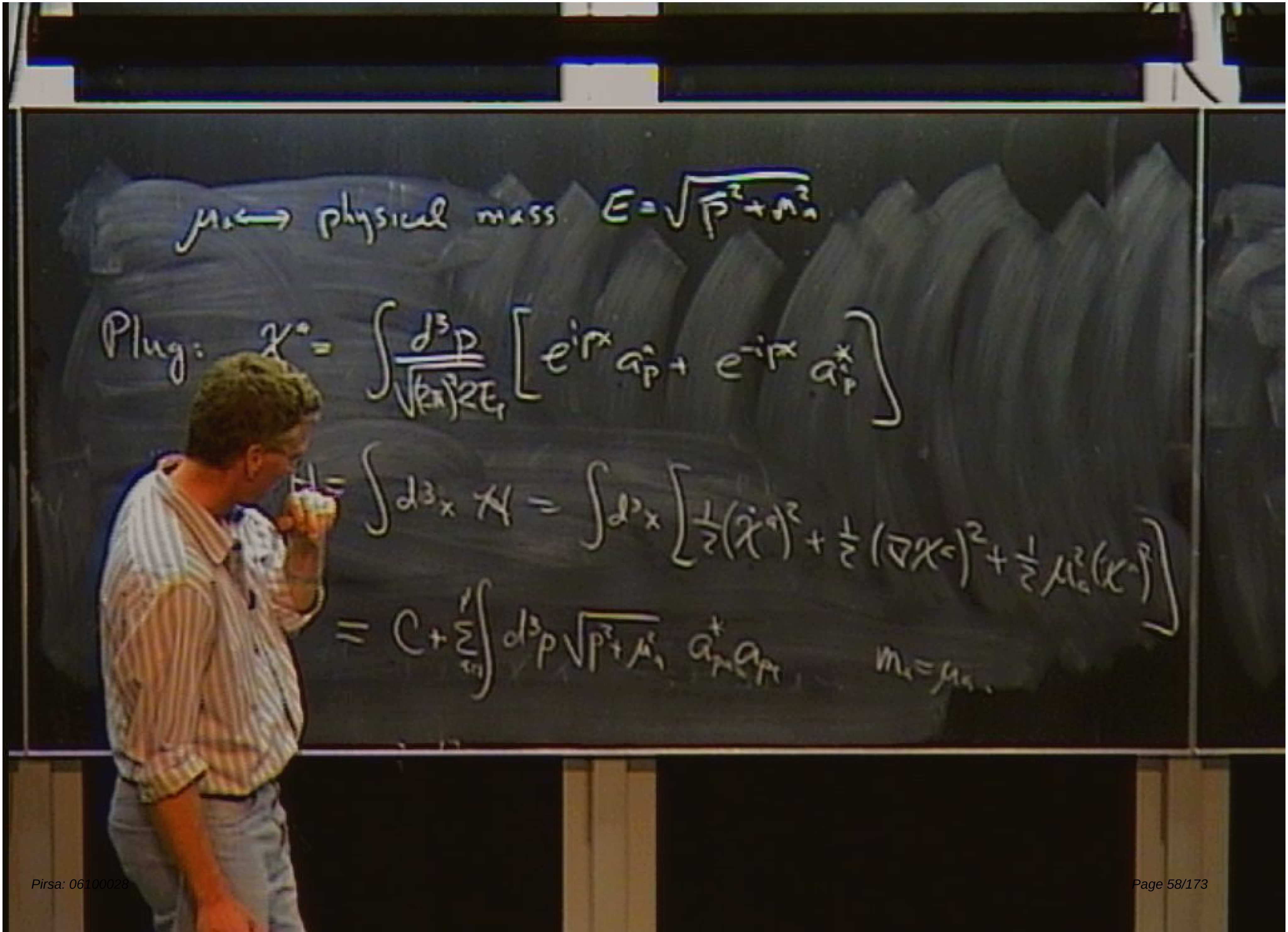
into $\quad H = \int d^3x \, \mathcal{H} = \int d^3x \left[\frac{1}{2}(\dot{\chi}^a)^2 + \frac{1}{2}(\nabla \chi^a)^2 + \frac{1}{2}\mu_a^2 (\chi^a)^2 \right]$

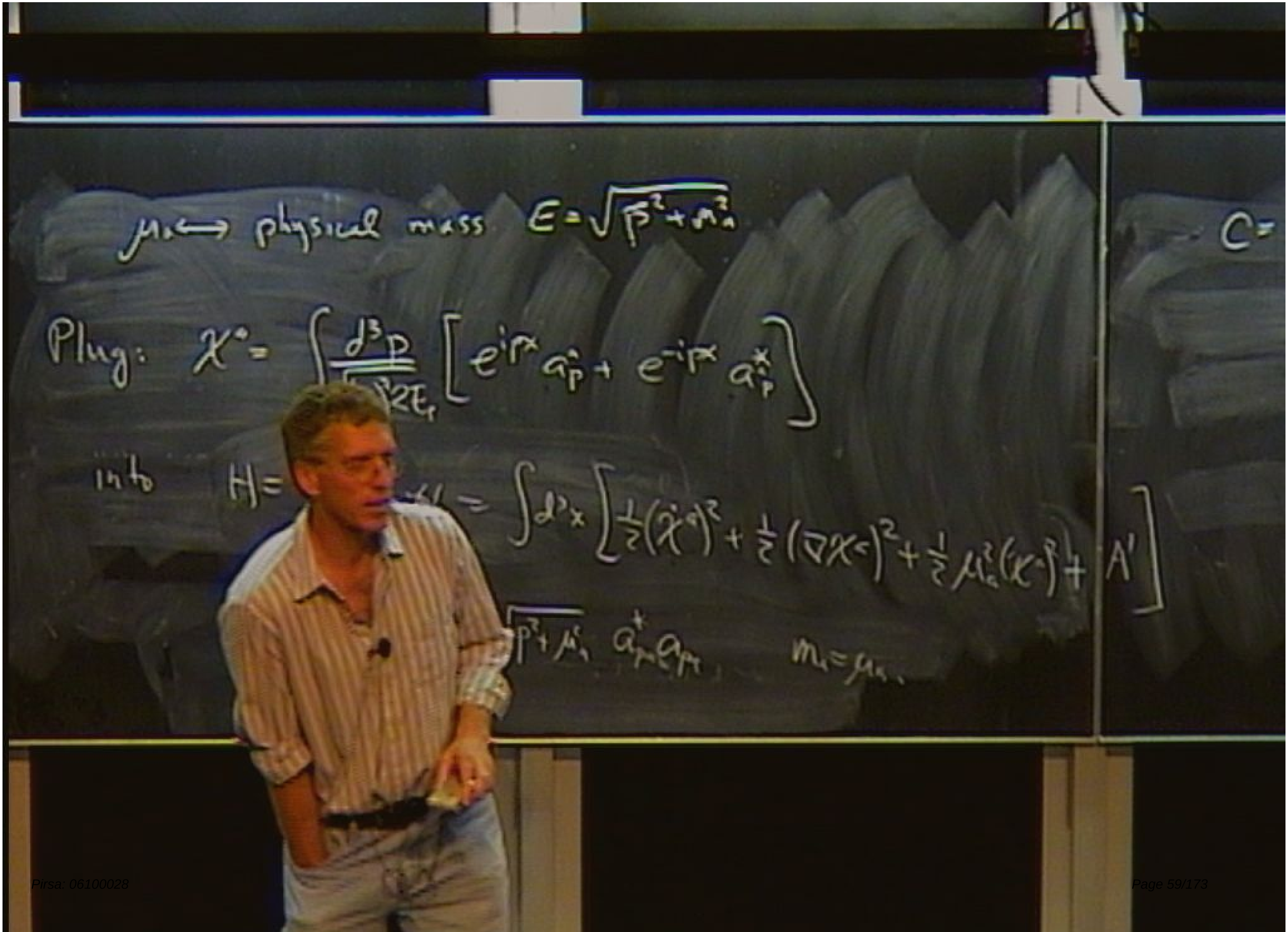
$$= C + \sum_{a=1}' \int d^3p \sqrt{p^2 + \mu_a^2} \; a^{*}_{pa} a_{pa} \qquad m_a = \mu_a$$


$C = A'\int d^3x + \frac{1}{2}\sum_{n=1}^{N}\int \frac{d^3p}{(2\pi)^3}$

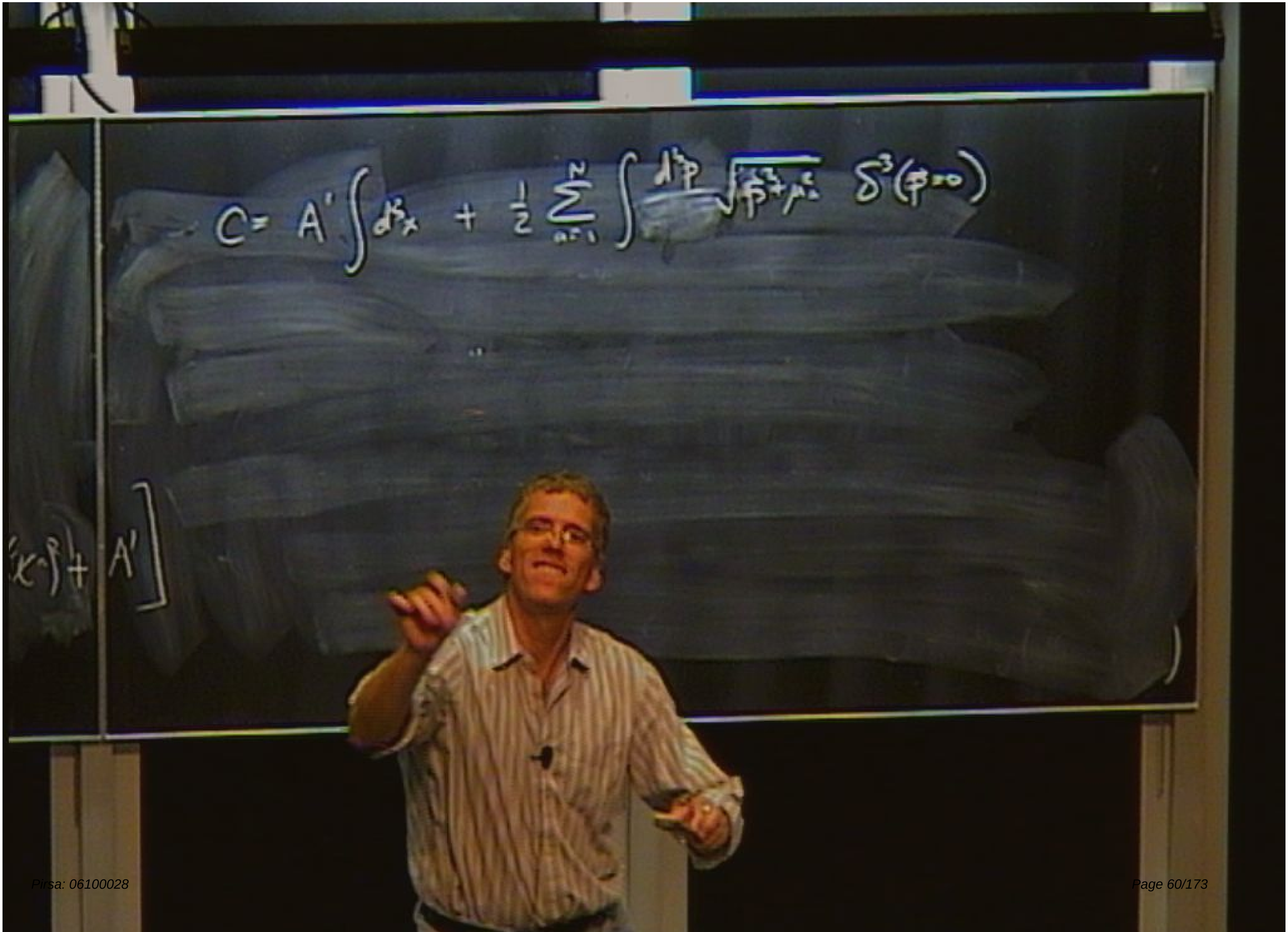

$C = A' \int d^3x + \frac{1}{2} \sum_{a=1}^{N} \int d^3p \sqrt{p^2 + \mu_a^2} \; \delta^3(\phi = 0)$


$\mu_1 \longleftrightarrow$ physical mass $E = \sqrt{\vec{p}^2 + m_1^2}$
Plug: $\chi^1 = \int \frac{d^3p}{\sqrt{2E_1}} \left[e^{ipx} a_p^1 + e^{-ipx} a_p^{1*} \right]$
into
$\mathcal{H} = \int \left[\frac{1}{2}(\dot{\chi}^1)^2 + \frac{1}{2}(\nabla\chi^1)^2 + \frac{1}{2}\mu_1^2(\chi^1)^2 \right]$
$d^3p \sqrt{p^2 + \mu_1^2}\; a_{p1}^* a_{p1}$
$m_1 = \mu_1$.


$\mu_n \longleftrightarrow$ physical mass $E = \sqrt{\vec{p}^2 + m_n^2}$
Plug: $\chi^n = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p^n + e^{-ipx} a_p^{n*} \right]$
$H = \int d^3x\, \mathcal{H} = \int d^3x \left[\frac{1}{2}(\dot{\chi}^n)^2 + \frac{1}{2}(\nabla \chi^n)^2 + \frac{1}{2}\mu_n^2 (\chi^n)^2 \right]$
$= C + \sum_{n}' \int d^3p \sqrt{p^2 + \mu_n^2}\, a_{pn}^* a_{pn}$
$m_n = \mu_n$.

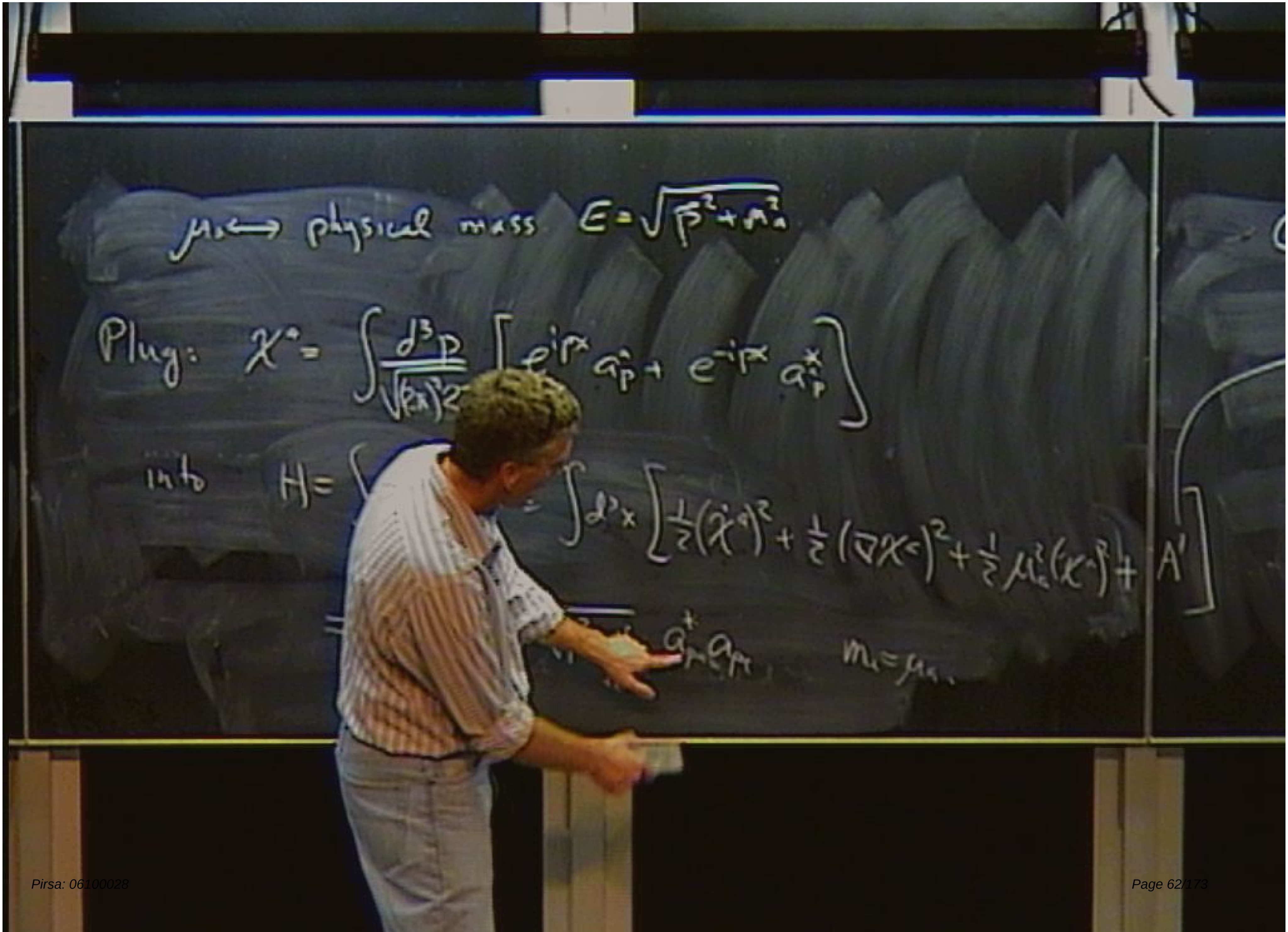


μn ⟷ physical mass E = √(p² + mn²)
Plug: χ^a = ∫ d³p / √(2E_p) [e^{ipx} a_p^a + e^{-ipx} a_p^{a*}]
into H =
= ∫ d³x [½(χ̇^a)² + ½(∇χ^a)² + ½ μn²(χ^a)² + A']
√(p² + μn²) a†pn apn
mn = μn
C=

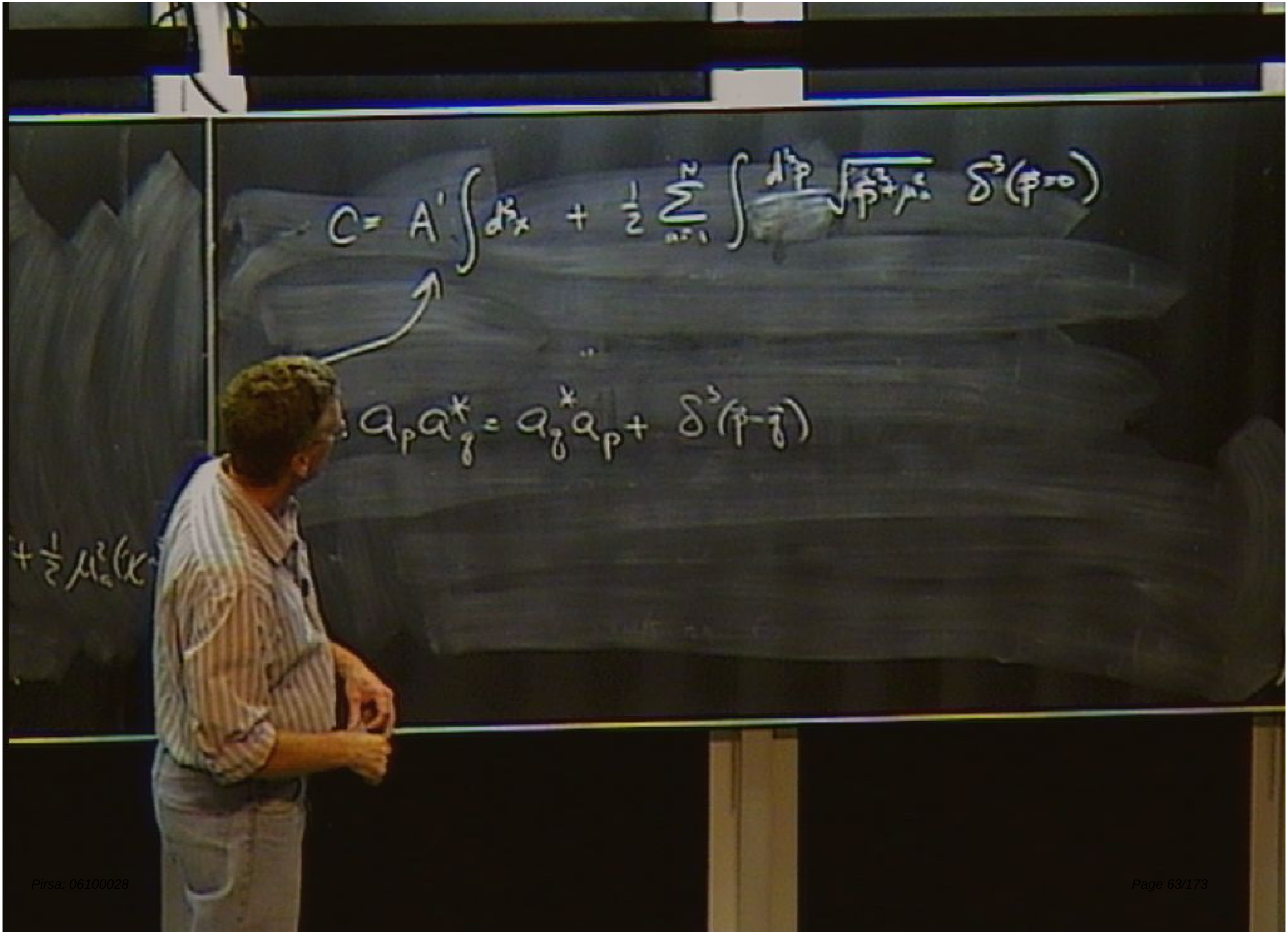

$C = A' \int d^3x + \frac{1}{2} \sum_{n=1}^{N} \int d^3p \sqrt{p^2 + \mu^2}\ \delta^3(p=0)$

$$C = A' \int d^3x + \frac{1}{2} \sum_{n=1}^{N} \int d^3p \, \sqrt{\vec{p}^{\,2} + \mu_n^2} \;\; \delta^3(\vec{p} = 0)$$

$$\ldots (x-)^2 + A' \,]$$



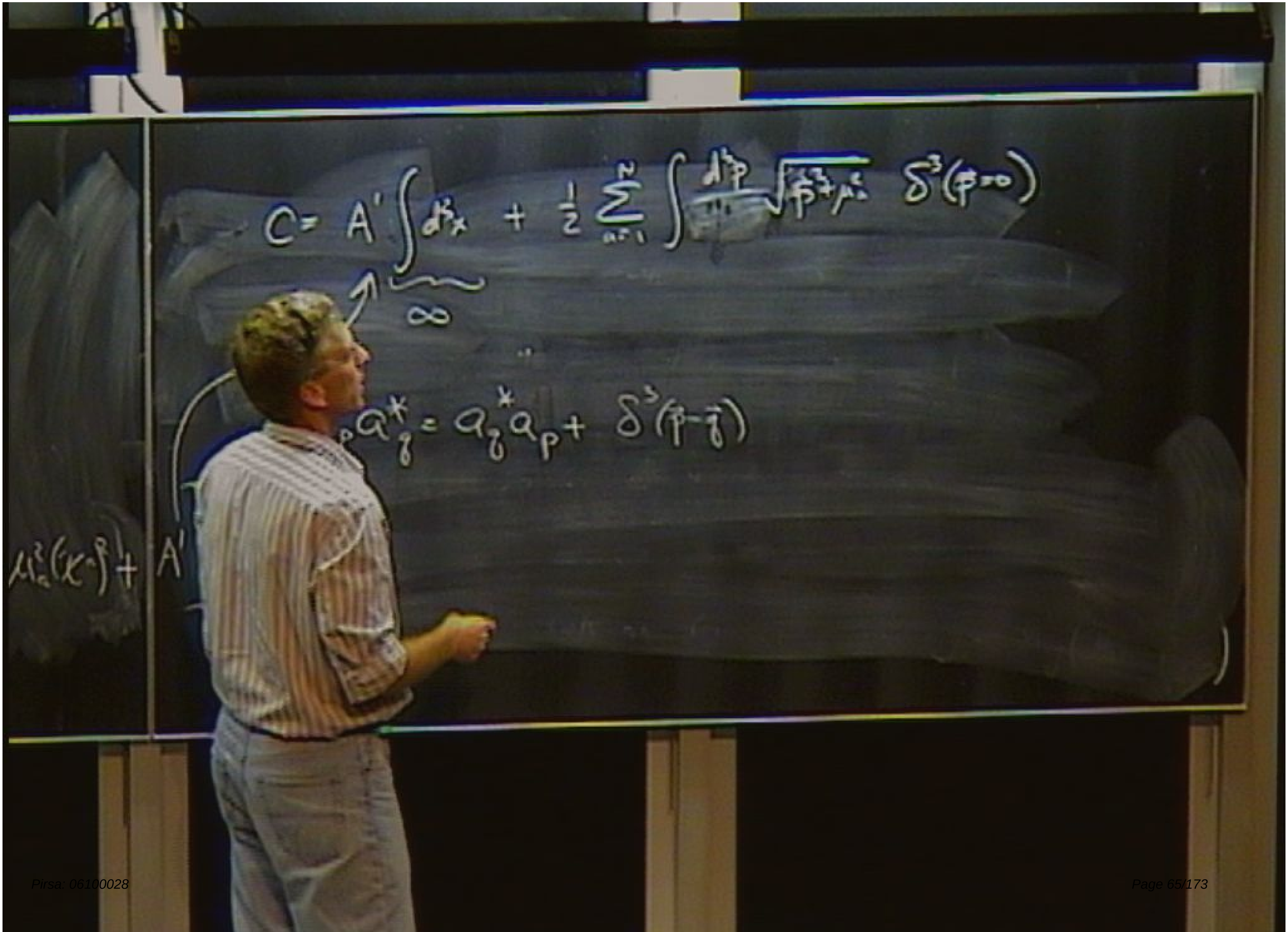
$\mu_a \longleftrightarrow$ physical mass $E = \sqrt{\vec{p}^2 + m_a^2}$
Plug: $\chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{a*} \right]$
into $H = \int \;\; = \int d^3x \left[\frac{1}{2}(\dot{\chi}^a)^2 + \frac{1}{2}(\nabla\chi^a)^2 + \frac{1}{2}\mu_a^2(\chi^a)^2 + A' \right]$
$= \; a_{p_a}^{*} a_{p_a}$ $\quad m_a = \mu_a$


$C = A' \int d^3x + \frac{1}{2} \sum_{n=1}^{N} \int d^3p \sqrt{\vec{p}^2 + \mu_n^2} \; \delta^3(\vec{p}=0)$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
$+ \frac{1}{2} \mu_n^2 (x$

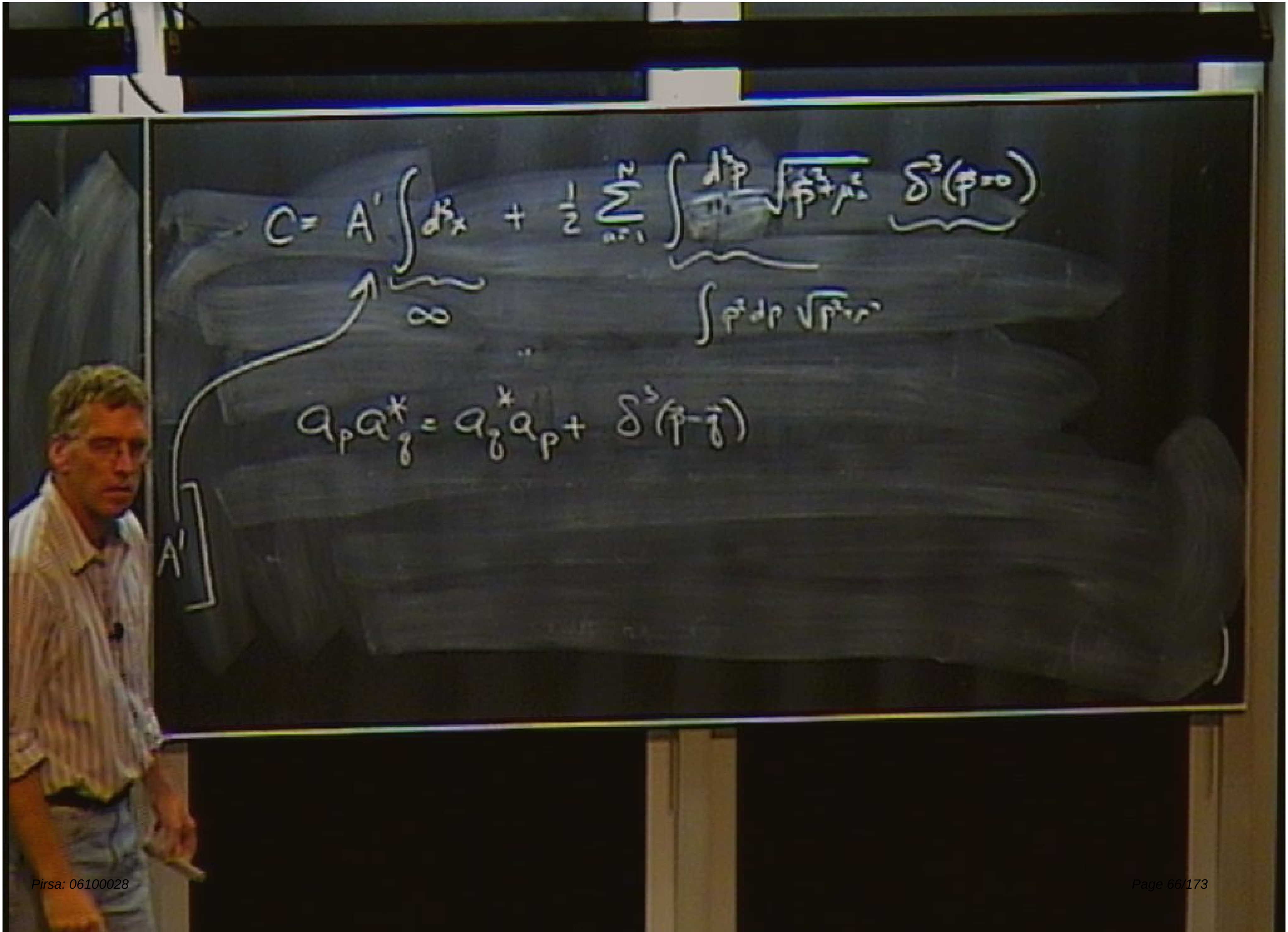
$$C = A' \int d^3x + \frac{1}{2} \sum_{n=1}^{N} \int d^3p \sqrt{\vec{p}^2 + \mu_n^2} \; \delta^3(\vec{p}=0)$$

$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p} - \vec{q})$$

$$+ \frac{1}{2} \mu_n^2 (x_n)^2 + A' \Big]$$



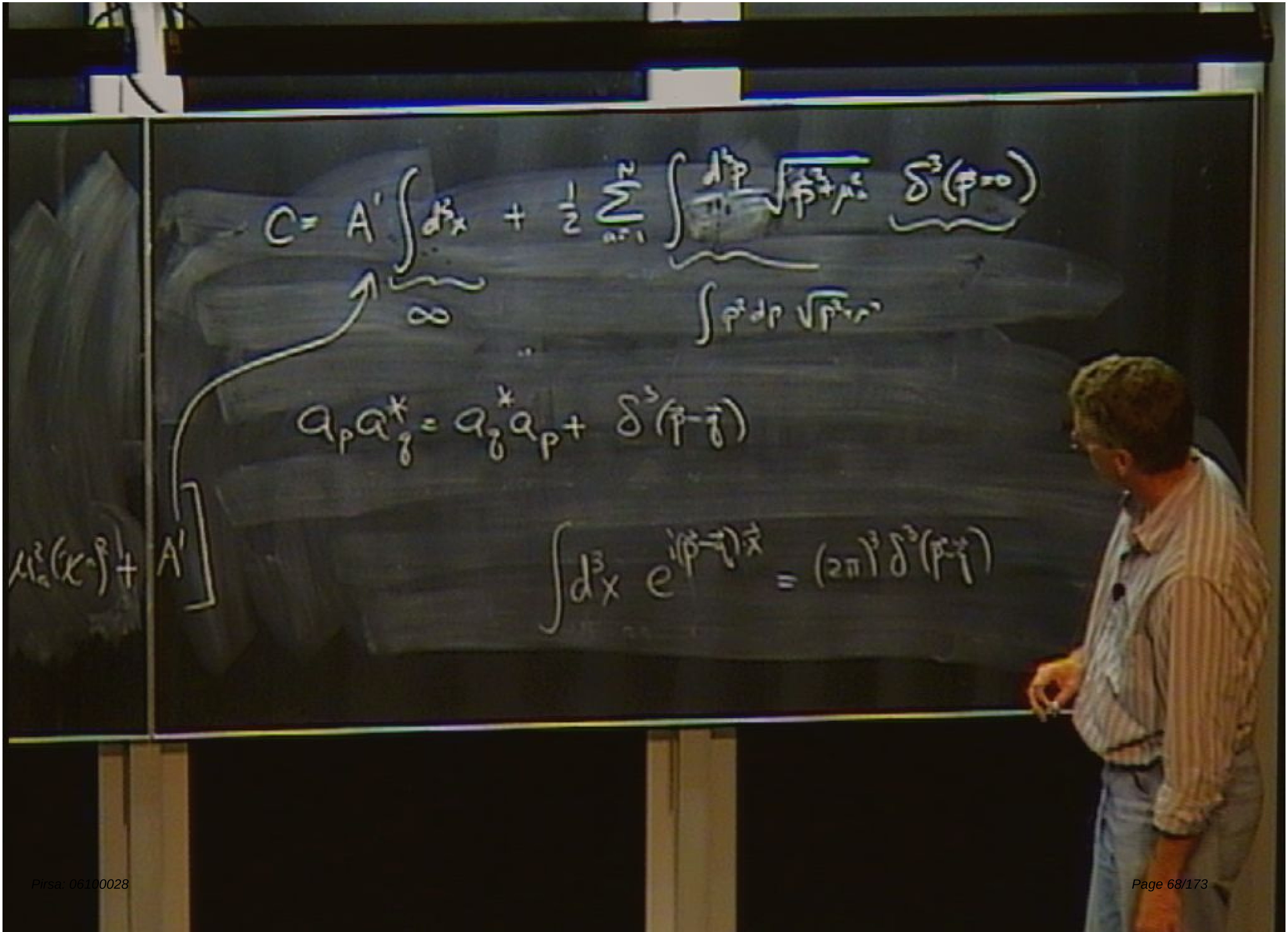
C= A' ∫d³x + ½ Σ ∫ d³p √(p²+μ²) δ³(p=0)
∞
a*_q = a_q a_p + δ³(p−q)
A'


$C = A' \int d^3x + \frac{1}{2} \sum_{n=1}^{N} \int d^3p \sqrt{\vec{p}^2 + \mu_n^2}\ \delta^3(\vec{p}=0)$
∞
$\int p^2 dp \sqrt{p^2+\mu^2}$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
A'

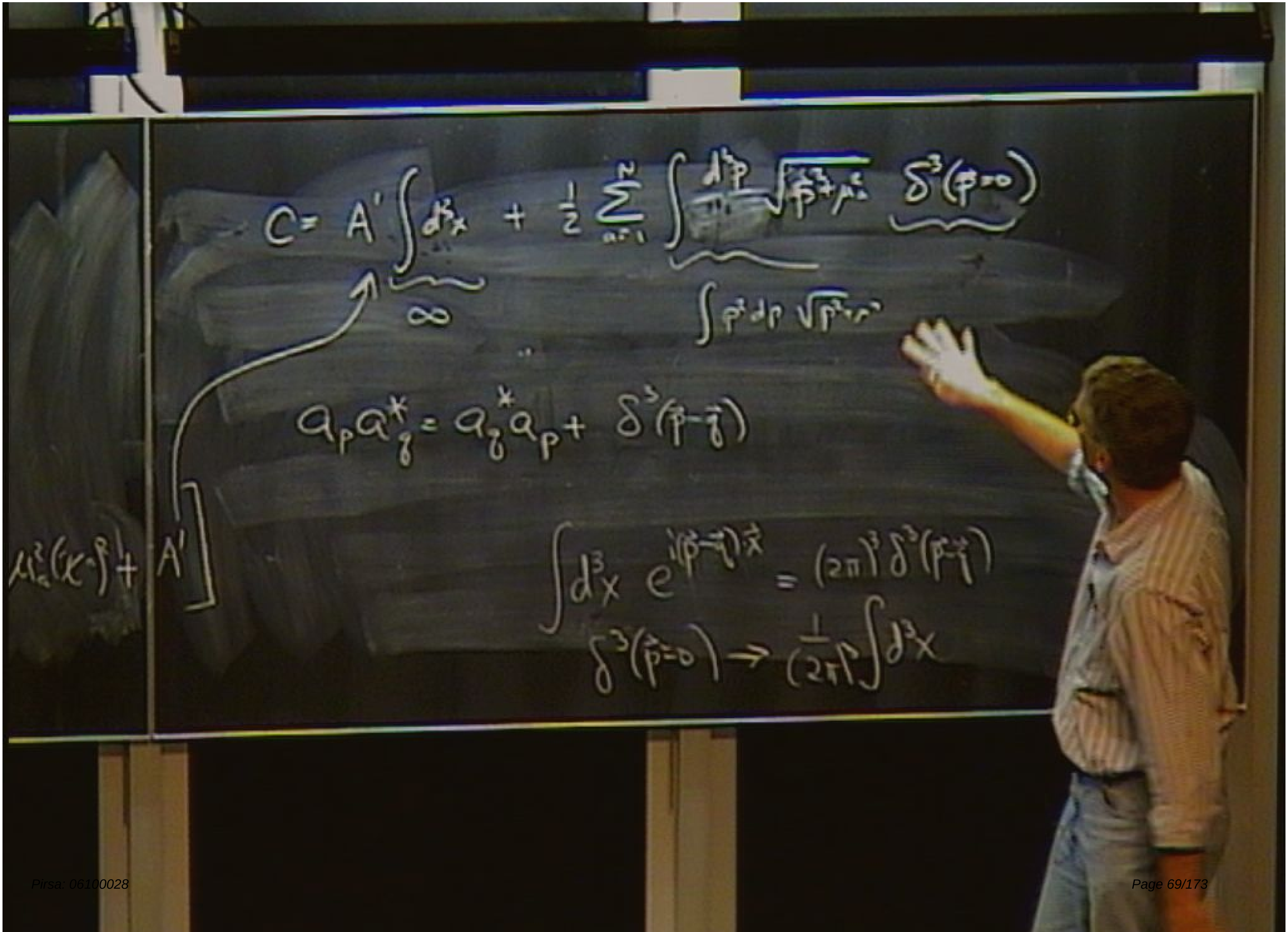
$$C = A' \underbrace{\int d^3x}_{\infty} + \frac{1}{2}\sum_{a=1}^{N} \underbrace{\int \frac{d^3p}{\ldots}\sqrt{\vec{p}^{\,2}+\mu_a^2}}_{\int p^2\,dp\,\sqrt{p^2+\mu^2}}\ \underbrace{\delta^3(\vec{p}=0)}$$

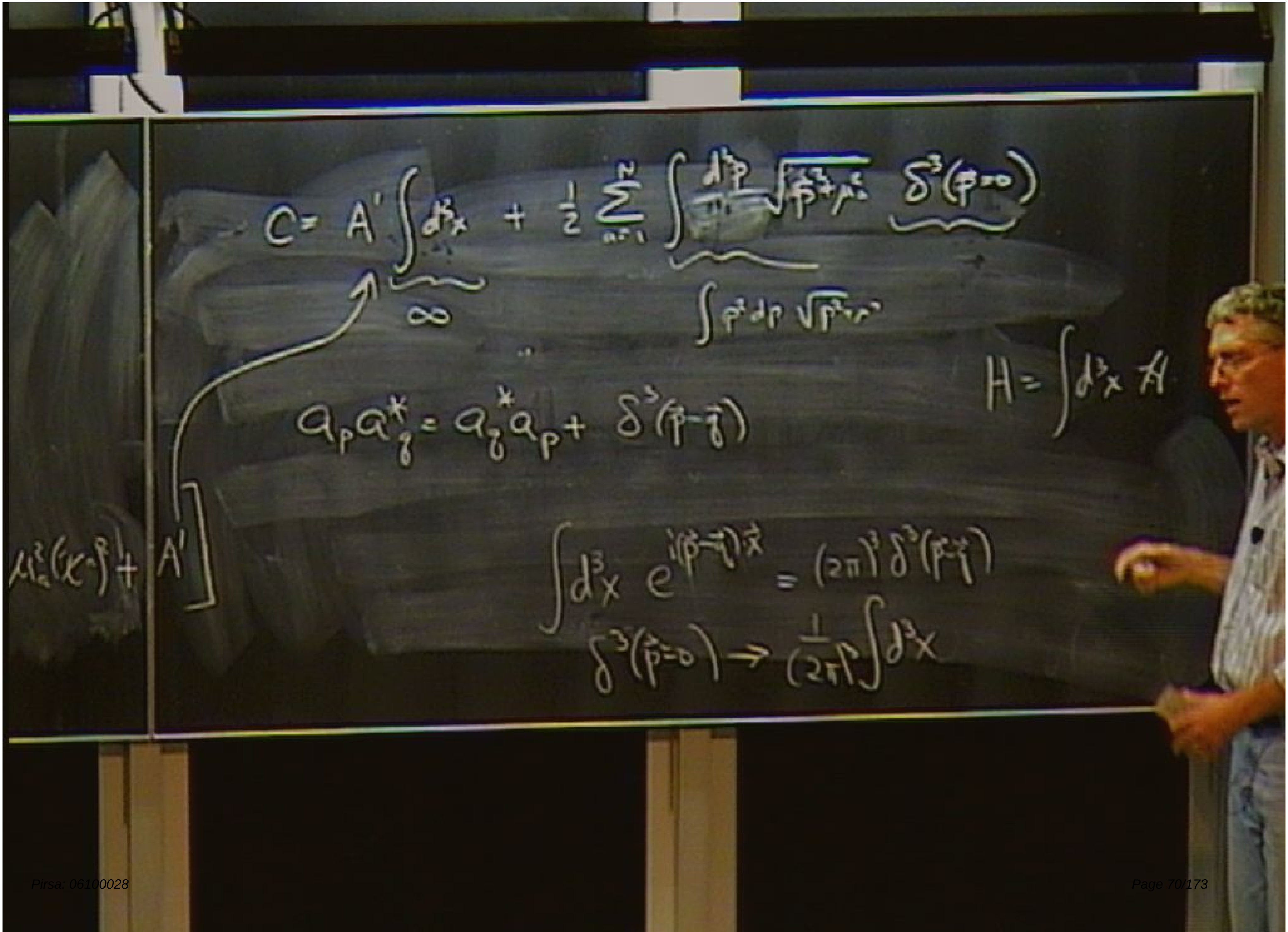
$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$$

$$\mu_a^2 (x^a)^2 + \left. A' \right]$$



$C = A' \int d^3x + \frac{1}{2} \sum_{n=1}^{N} \int d^3p \sqrt{\vec{p}^2+\mu_n^2}\ \delta^3(\vec{p}=0)$
∞
$\int p^2 dp \sqrt{p^2+\mu^2}$
$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$
A'


$C = A' \underbrace{\int d^3x}_{\infty} + \frac{1}{2} \sum_{n=1}^{N} \underbrace{\int \frac{d^3p}{} \sqrt{\vec{p}^2+\mu_n^2}}_{\int p^2 dp \sqrt{p^2+\mu^2}} \underbrace{\delta^3(\vec{p}=0)}$
$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$
$\mu_n^2 (x-\beta)^{...} + A']$
$\int d^3x \, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$
$\delta^3(\vec{p}=0) \to \frac{1}{(2\pi)^3} \int d^3x$


$C = A' \underbrace{\int d^3x}_{\infty} + \frac{1}{2} \sum_{a=1}^{N} \underbrace{\int \frac{d^3p}{\ } \sqrt{\vec{p}^2 + \mu_a^2}}_{\int p^2 dp \sqrt{p^2+\mu^2}} \ \underbrace{\delta^3(\vec{p}=0)}$
$H = \int d^3x \, \mathcal{H}$
$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$
$\mu_a^2 (\chi^a)^2 + A'$
$\int d^3x \, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$
$\delta^3(\vec{p}=0) \rightarrow \frac{1}{(2\pi)^3} \int d^3x$

$$\frac{C}{V} = A' + \frac{1}{2}\sum_{a=1}^{N} \underbrace{\int \frac{d^3p}{\ldots} \sqrt{\vec{p}^2+\mu_a^2}}_{\int p^2\,dp\,\sqrt{p^2+\mu^2}} \frac{1}{(2\pi)^3}$$

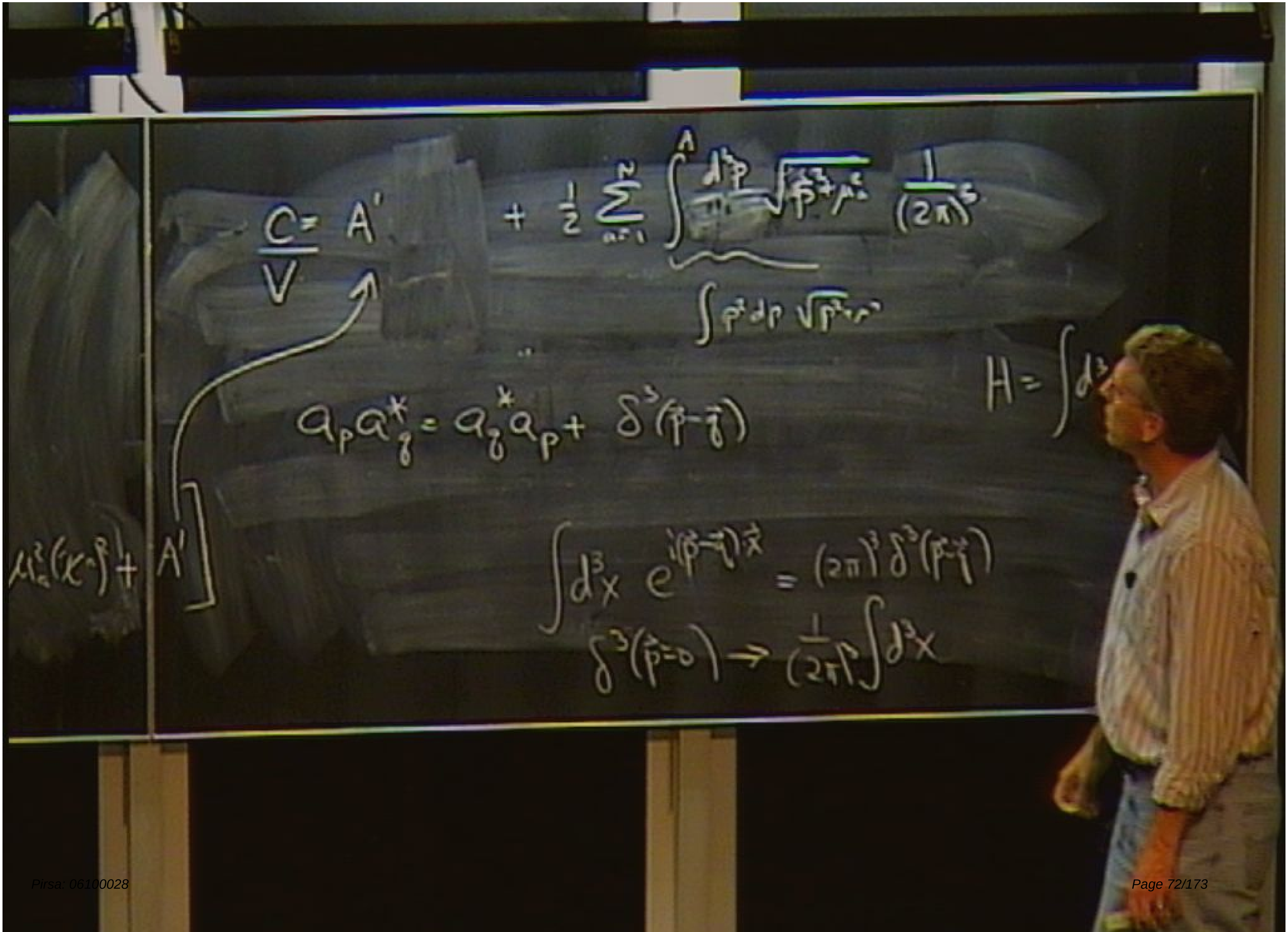
$$H = \int d^3x\, \mathcal{H}$$

$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$$

$$\mu_a^2(\ldots)^2 + A' \Big]$$

$$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$$

$$\delta^3(\vec{p}=0) \to \frac{1}{(2\pi)^3}\int d^3x$$


$\frac{C}{V} = A' + \frac{1}{2}\sum_{a=1}^{N}\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+\mu_a^2}$
$\int p^2 dp \sqrt{p^2+\mu^2}$
$H = \int d^3$
$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$
$\delta^3(\vec{p}=0) \to \frac{1}{(2\pi)^3}\int d^3x$

$$\frac{C}{V} = A' + \frac{1}{2}\sum_{a=1}^{N} \underbrace{\int^{\Lambda} d^3p \sqrt{\vec{p}^2+\mu_a^2}}_{\int p^2\,dp\,\sqrt{p^2+\mu^2}} \frac{1}{(2\pi)^3}$$

$$H = \int d^3x\, \mathcal{H}$$

$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$$

$$\mu_a^2(x_a \ldots)^2 + A' \,]$$

$$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$$

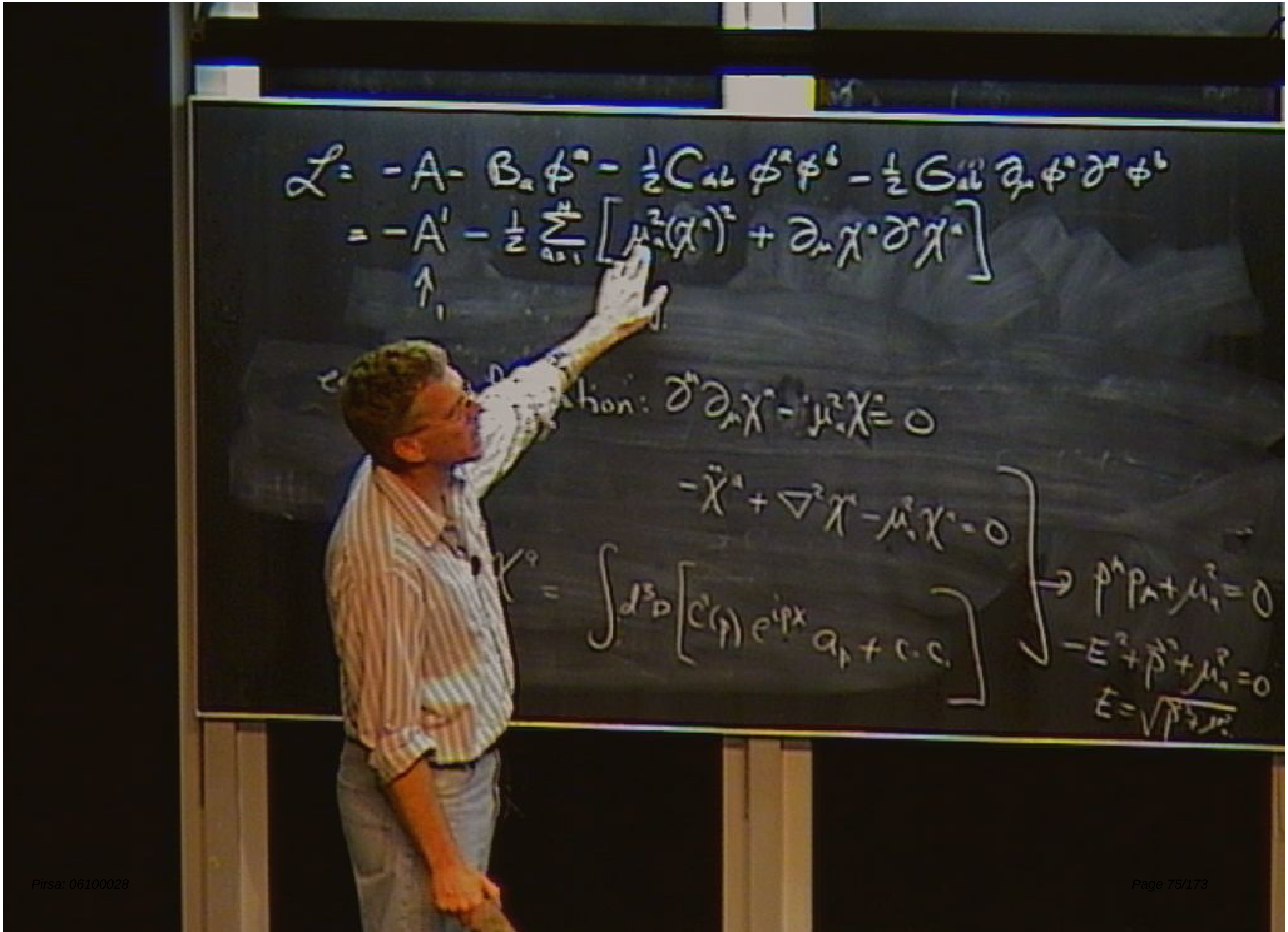
$$\delta^3(\vec{p}=0) \rightarrow \frac{1}{(2\pi)^3}\int d^3x$$

$\mu_a \longleftrightarrow$ physical mass $\quad E = \sqrt{\vec{p}^2 + m_a^2}$

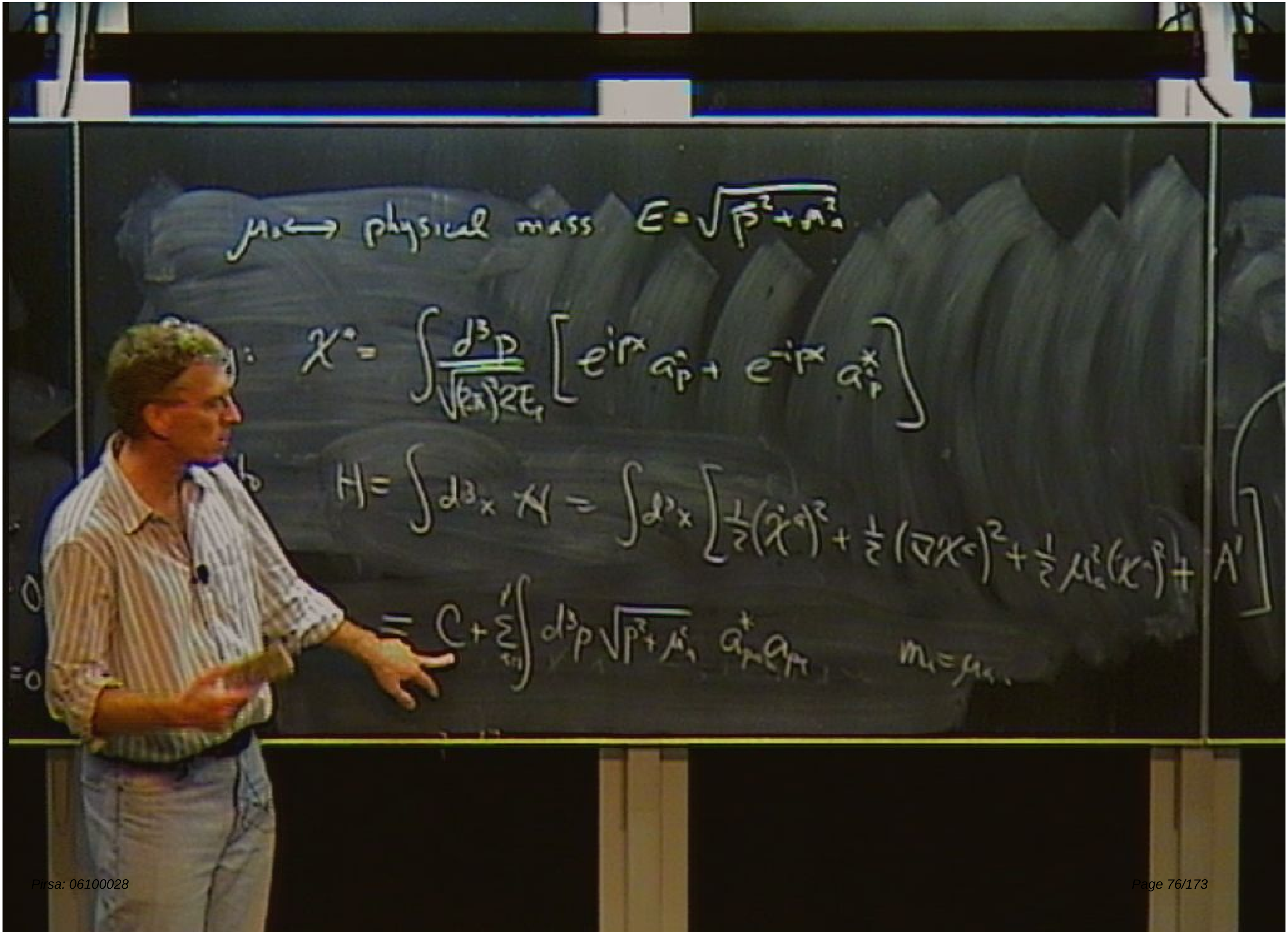
Plug: $\quad \chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{*a} \right]$

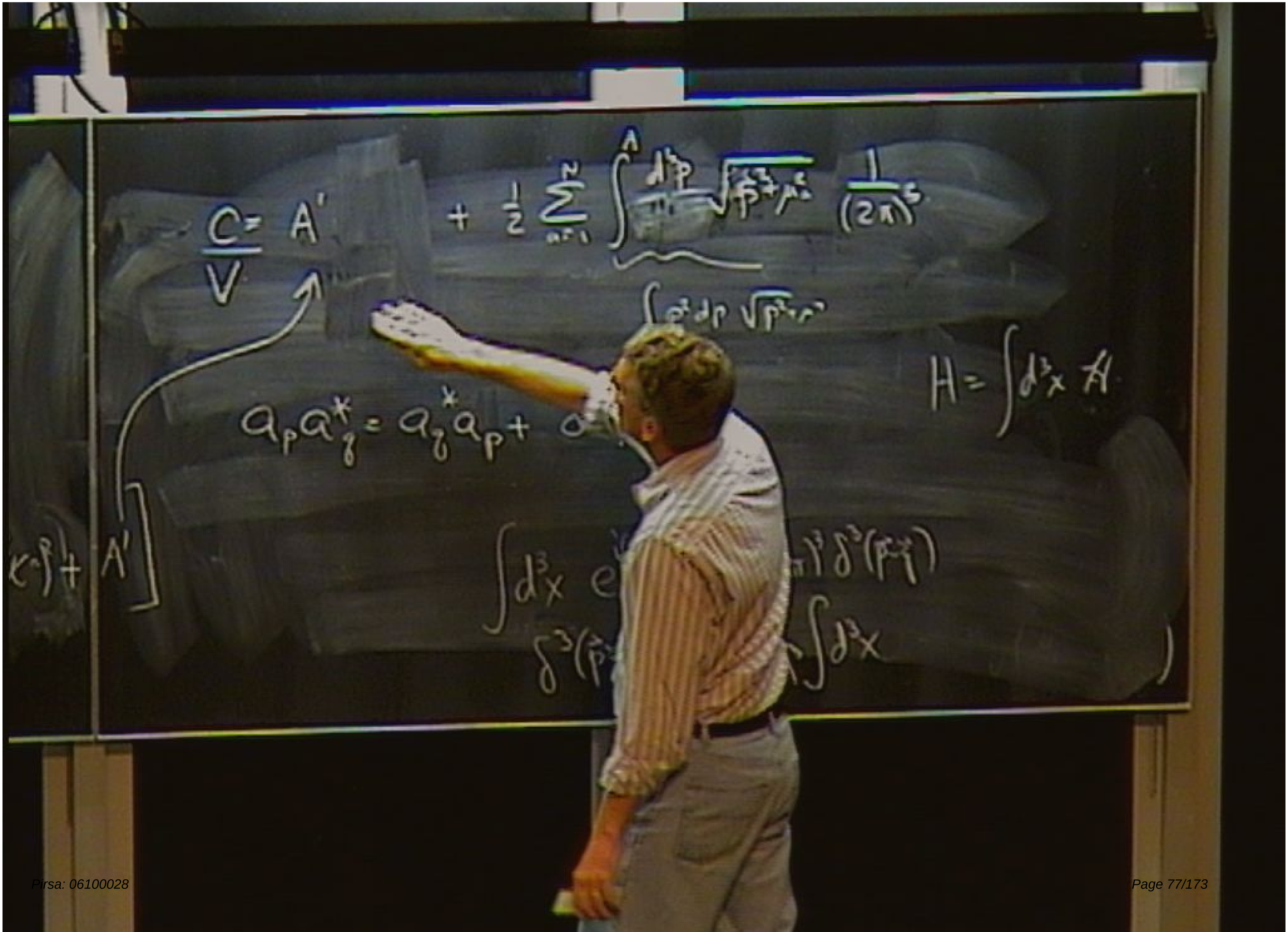
into $\quad H = \int d^3x \, \mathcal{H} = \int d^3x \left[\frac{1}{2} (\dot{\chi}^a)^2 + \frac{1}{2} (\nabla \chi^a)^2 + \frac{1}{2} \mu_a^2 (\chi^a)^2 + A' \right]$

$= C + \sum_{a=1}^{N} \int d^3p \sqrt{\vec{p}^2 + \mu_a^2} \; a_{pa}^* a_{pa} \qquad m_a = \mu_a$

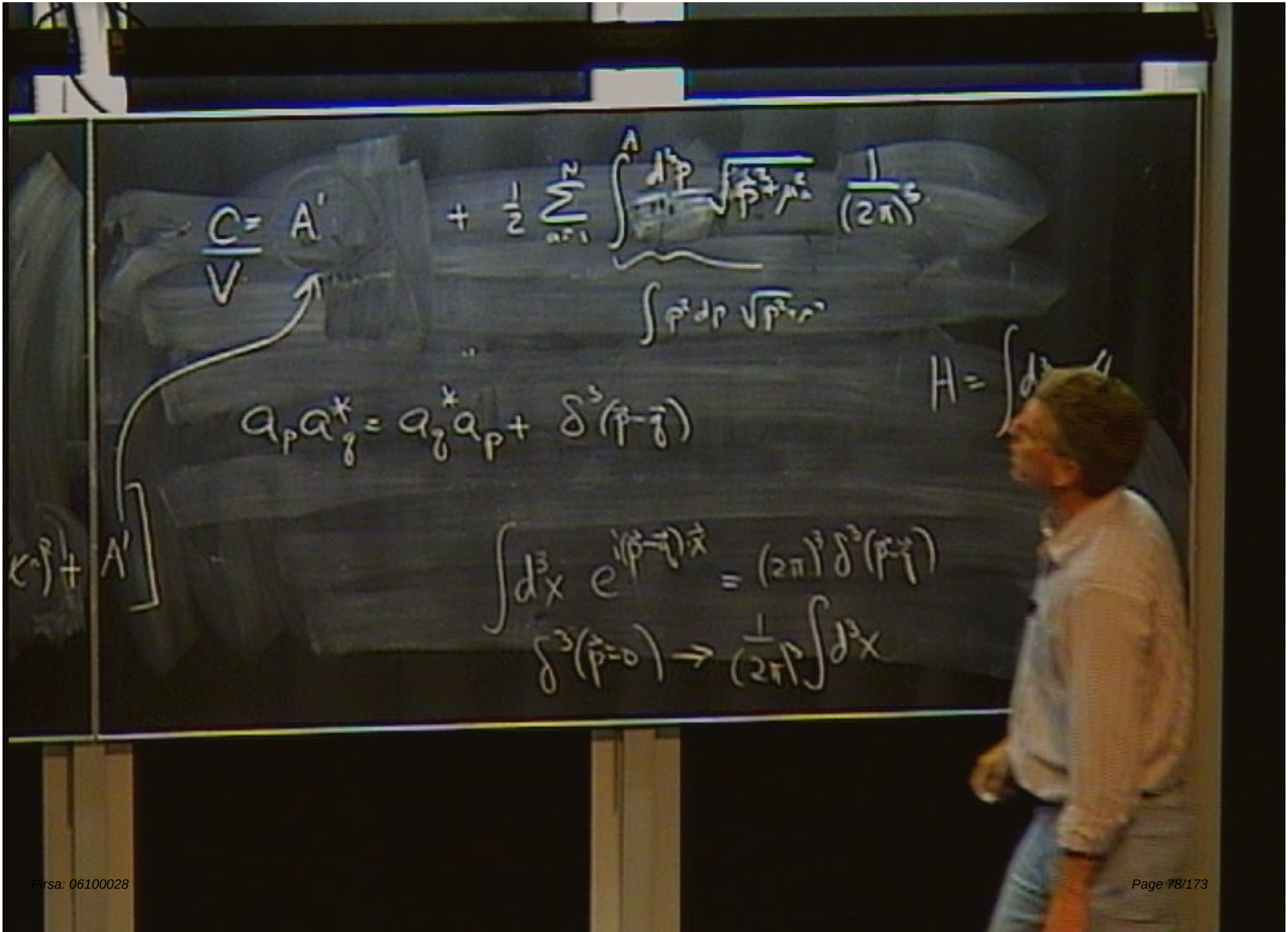


$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$
$= -A' - \frac{1}{2} \sum_{a=1}^{n} \left[\mu_a^2 (\chi^a)^2 + \partial_\mu \chi^a \partial^\mu \chi^a \right]$
ation: $\partial^\mu \partial_\mu \chi^a - \mu_a^2 \chi^a = 0$
$-\ddot{\chi}^a + \nabla^2 \chi^a - \mu_a^2 \chi^a = 0$
$\chi^a = \int d^3p \left[c^a(p)\, e^{ipx} a_p + c.c. \right]$
$\rightarrow p^\mu p_\mu + \mu_a^2 = 0$
$-E^2 + \vec{p}^2 + \mu_a^2 = 0$
$E = \sqrt{\vec{p}^2 + \mu_a^2}$

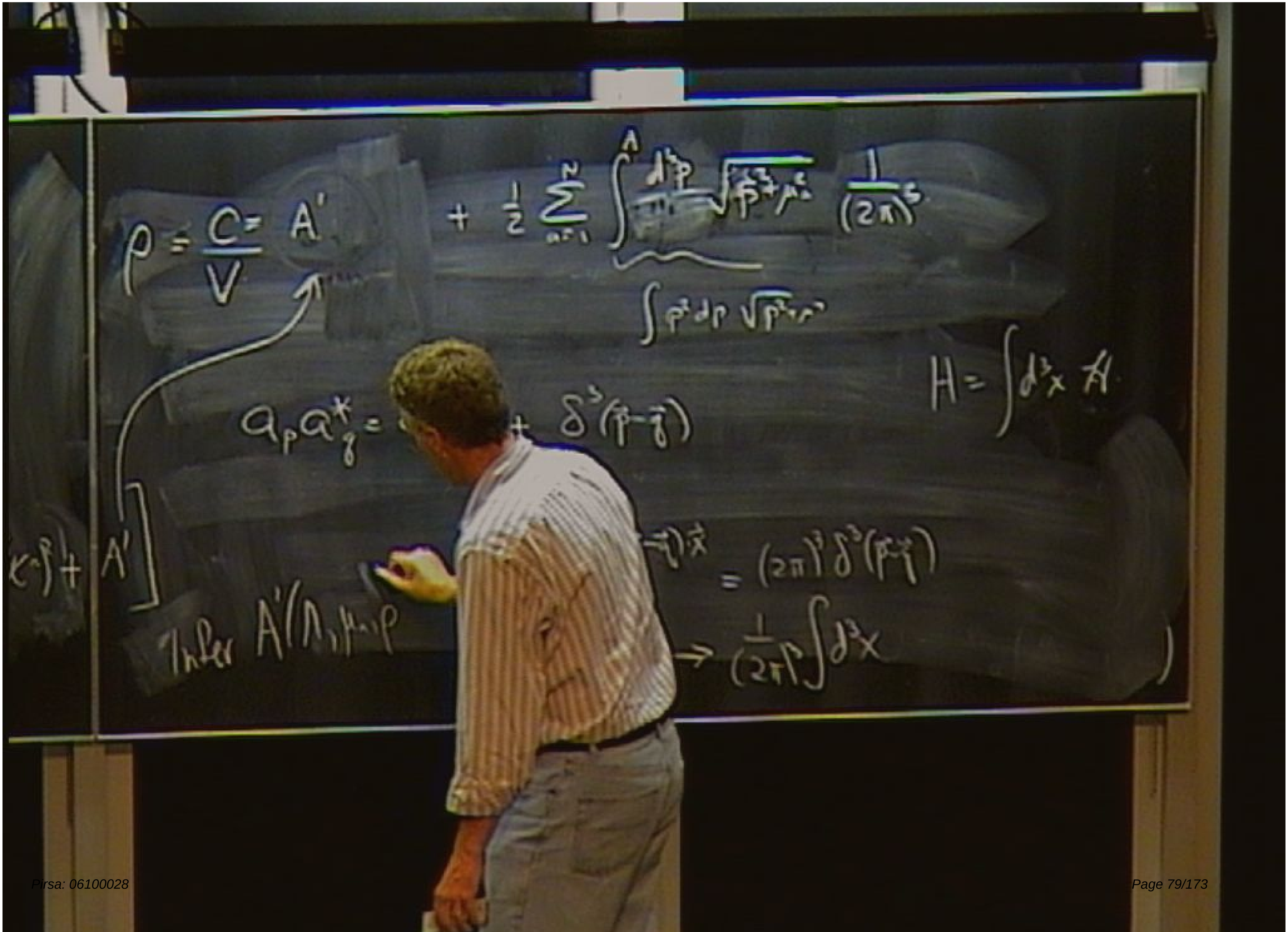

$\mu_a \longleftrightarrow$ physical mass $E = \sqrt{\vec{p}^2 + m_a^2}$
$\chi^a = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_1}} \left[e^{ipx} a_p^a + e^{-ipx} a_p^{a*} \right]$
$H = \int d^3x \, \mathcal{H} = \int d^3x \left[\frac{1}{2}(\dot{\chi}^a)^2 + \frac{1}{2}(\nabla\chi^a)^2 + \frac{1}{2}\mu_a^2(\chi^a)^2 + A' \right]$
$= C + \sum_{a=1}^{N} \int d^3p \sqrt{p^2 + \mu_a^2} \; a_{pa}^{+} a_{pa}$ $m_a = \mu_a$

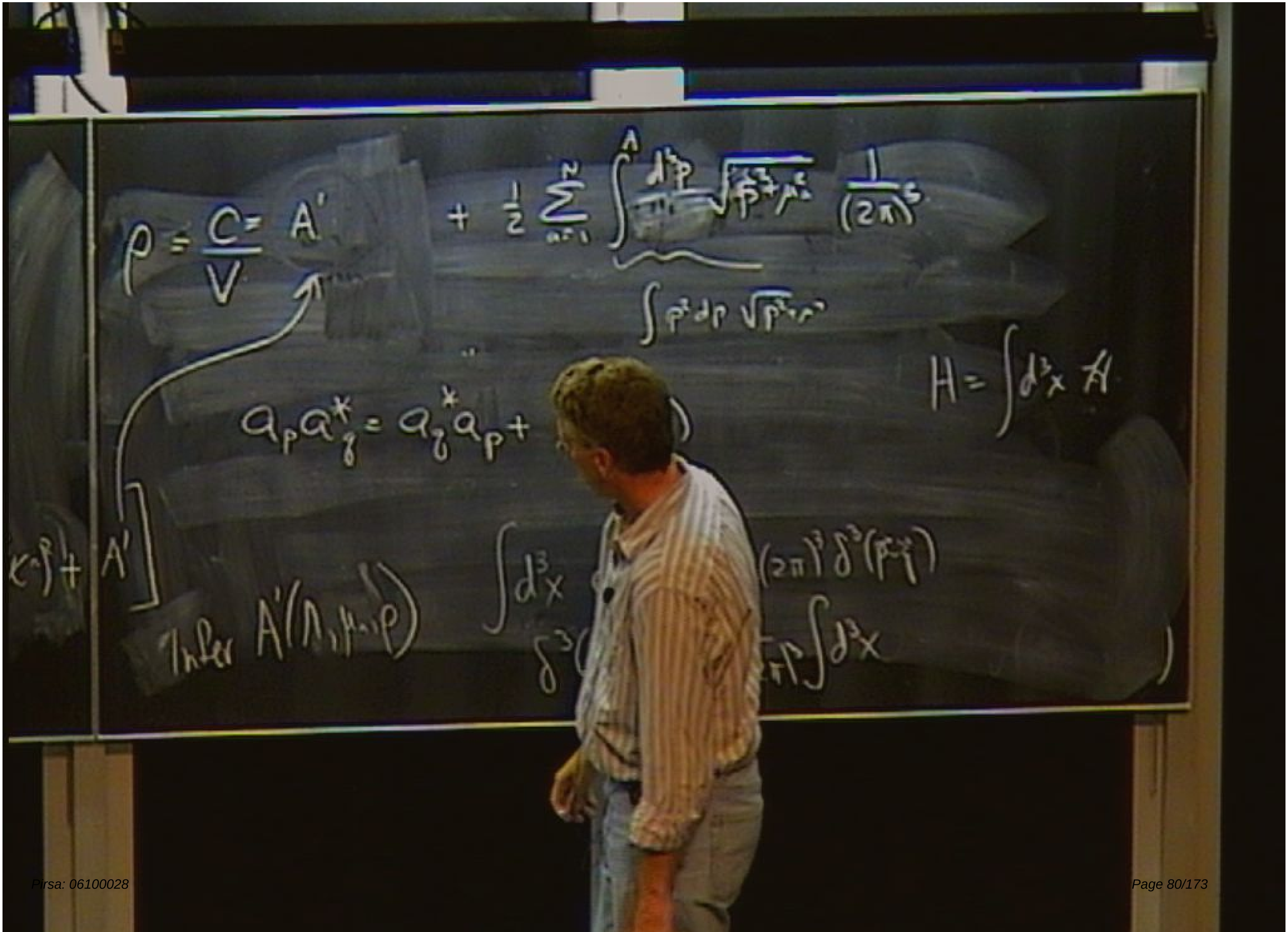


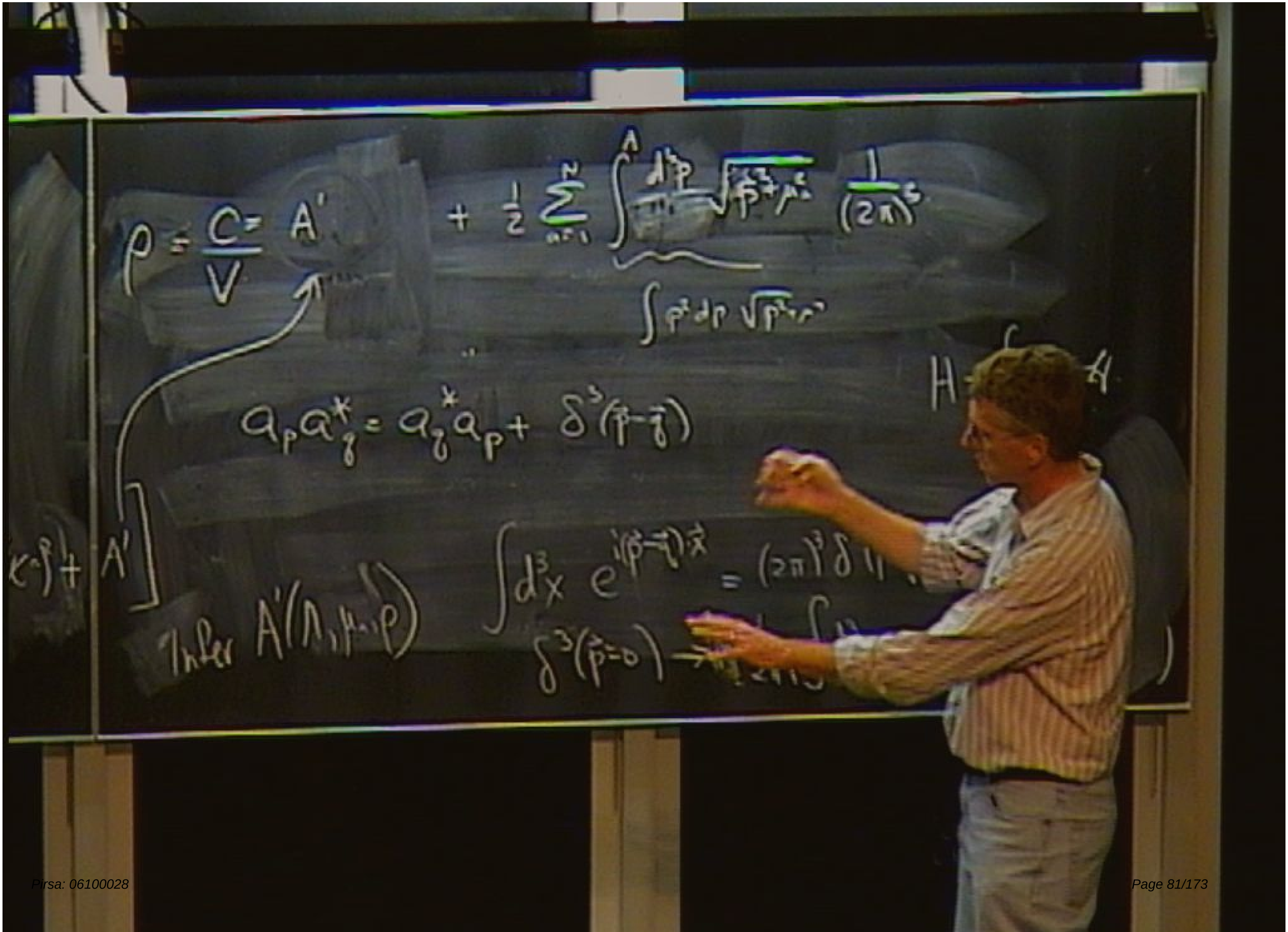
C/V = A' + 1/2 Σ ∫ ... 1/(2π)^3
H = ∫d^3x ℋ
a_p a*_q = a*_q a_p +
∫d^3x



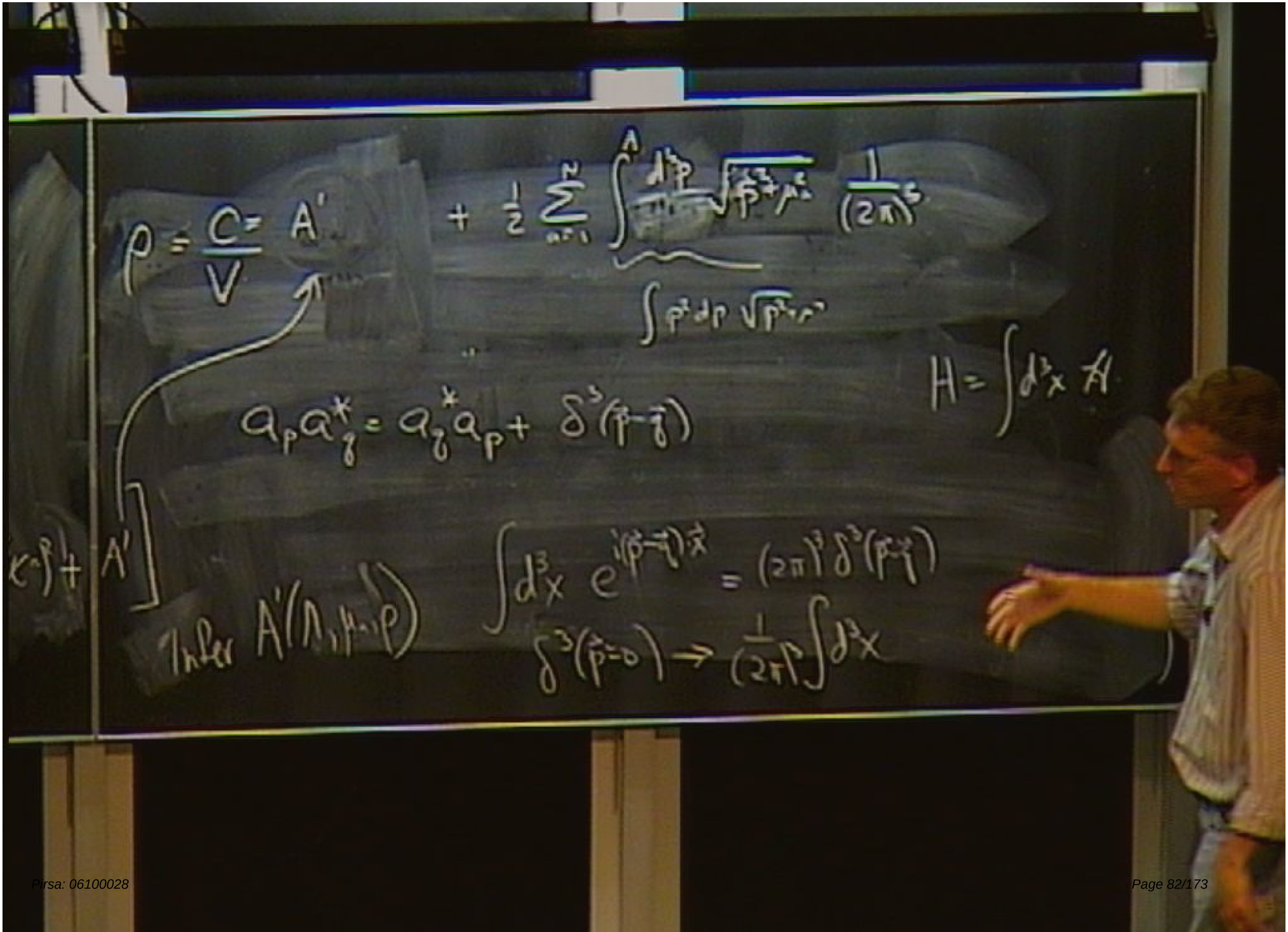
$\frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda} d^3p \sqrt{p^2+\mu_n^2}\ \frac{1}{(2\pi)^3}$
$\int p^2\, dp \sqrt{p^2+\mu^2}$
$H = \int d$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
A'
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$
$\delta^3(\vec{p}=0) \to \frac{1}{(2\pi)^3}\int d^3x$


ρ = C/V = A'
+ 1/2 Σ_{n=1}^{N} ∫^Λ d³p √(p²+μ_n²) 1/(2π)³
∫ p² dp √(p²+μ²)
H = ∫ d³x ℋ
a_p a*_q = + δ³(p−q)
= (2π)³ δ³(p−q)
Infer A'(Λ, μ_n, ρ
→ 1/(2π)³ ∫ d³x


ρ = C/V = A' + 1/2 Σ ∫ d³p √(p²+μ²) 1/(2π)³
∫ p² dp √(p²+μ²)
H = ∫ d³x ℋ
a_p a*_q = a*_q a_p +
Infer A'(Λ, μ_n, ρ)
∫ d³x
(2π)³ δ³(p−q)
∫ d³x



$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda} d^3p \sqrt{\vec{p}^2+\mu^2}\,\frac{1}{(2\pi)^3}$
$\int p^2\,dp\,\sqrt{p^2+\mu^2}$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta$
$\delta^3(\vec{p}=0)$
Infer $A'(\Lambda, \mu, \rho)$



$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda} d^3p \sqrt{\vec{p}^2+\mu_n^2}\,\frac{1}{(2\pi)^3}$
$\int p^2\,dp\,\sqrt{p^2+\mu^2}$
$H = \int d^3x\,\mathcal{H}$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
$+ A']$
Infer $A'(\Lambda, \mu_n, \rho)$
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$
$\delta^3(\vec{p}=0) \rightarrow \frac{1}{(2\pi)^3}\int d^3x$

$$\mathcal{L} = -A - B_a \phi^a - \frac{1}{2} C_{ab} \phi^a \phi^b - \frac{1}{2} G_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$$

$$= -A' - \frac{1}{2} \sum_{a=1}^{N} \left[\mu_a^2 (\chi^a)^2 + \partial_\mu \chi^a \partial^\mu \chi^a \right]$$

$$\uparrow_1 \qquad \uparrow_N$$

equation of motion: $\partial^\mu \partial_\mu \chi^a - \mu_a^2 \chi^a = 0$

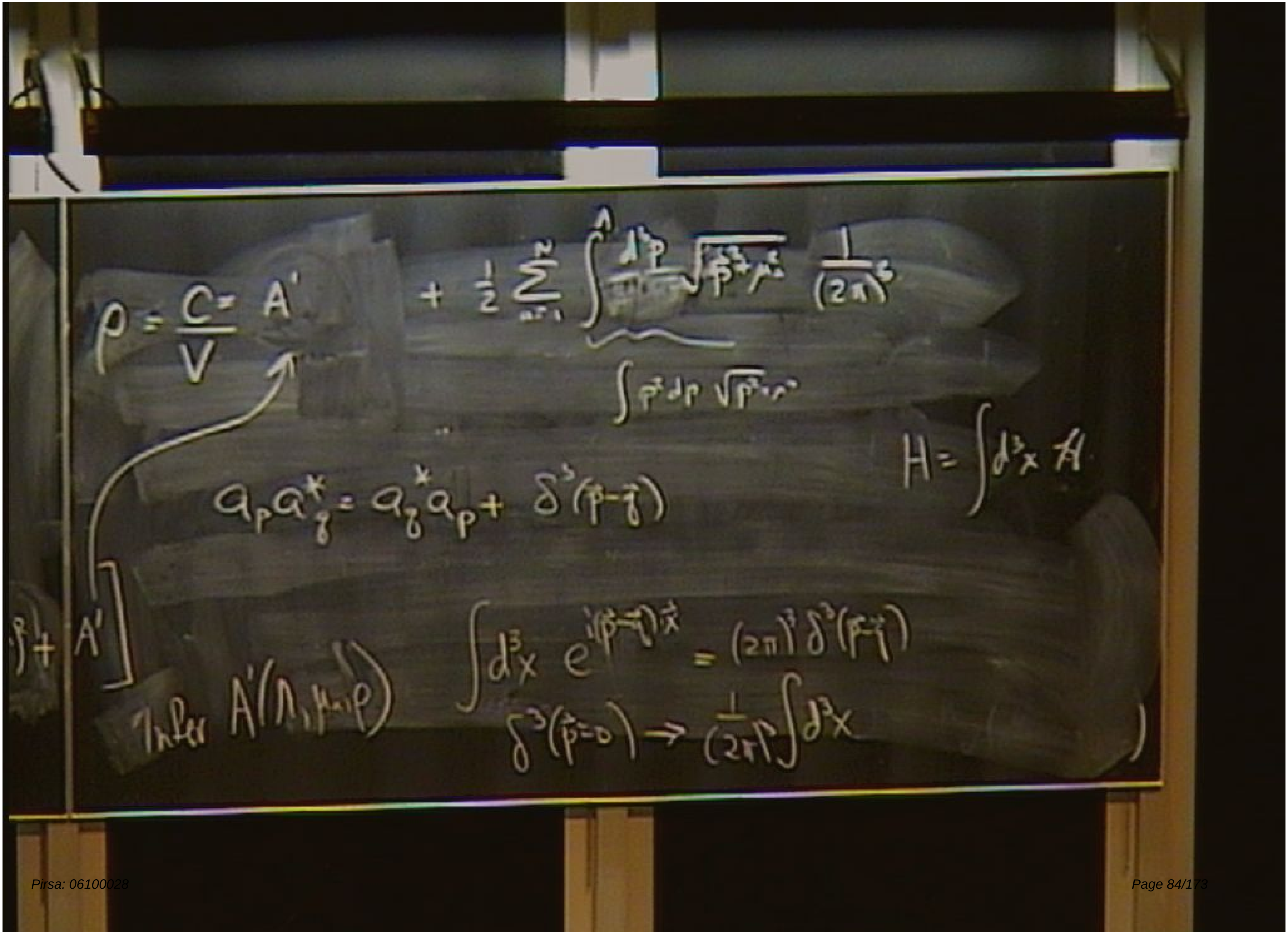
$$-\ddot{\chi}^a + \nabla^2 \chi^a - \mu_a^2 \chi^a = 0$$

$$\chi^a = \int d^3p \left[C(p)\, e^{ipx} a_p + \text{c.c.} \right]$$

$$\rightarrow p^\mu p_\mu + \mu_a^2 = 0$$

$$-E^2 + \vec{p}^{\,2} + \mu_a^2 = 0$$

$$E = \sqrt{\vec{p}^{\,2} + \mu_a^2}$$


ρ = C/V = A' + 1/2 Σ_{n=1}^{N} ∫^Λ d³p √(p²+μ²) 1/(2π)³
∫ p² dp √(p²+μ²)
H = ∫ d³x ℋ
a_p a*_q = a*_q a_p + δ³(p−q)
A'
∫ d³x e^{i(p−q)·x} = (2π)³ δ³(p−q)
Infer A'(Λ, μ_n, ρ)
δ³(p=0) → 1/(2π)³ ∫ d³x

$$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N} \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^2+\mu_n^2}$$

$$\int p^2\, dp\, \sqrt{p^2+\mu^2}$$

$$H = \int d^3x\, \mathcal{H}$$

$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$$

$$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$$

$$\delta^3(\vec{p}=0) \to \frac{1}{(2\pi)^3}\int d^3x$$

Infer $A'(\Lambda, \mu_n, \rho)$

$$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+\mu_n^2} \quad \frac{1}{(2\pi)^3}$$

$$\int p^2\,dp\,\sqrt{p^2+\mu^2}$$

$$H = \int d^3x\,\mathcal{H}$$

$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$$

$$+ A' \Big]$$

Infer $A'(\Lambda, \mu_n, \rho)$

$$\int d^3x\; e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\,\delta^3(\vec{p}-\vec{q})$$

$$\delta^3(\vec{p}=0) \rightarrow \frac{1}{(2\pi)^3}\int d^3x$$

$$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{a=1}^{N} \underbrace{\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+\mu_a^2}}_{\int p^2\,dp\,\sqrt{p^2+\mu^2}}$$

$$H = \int d^3x\, \mathcal{H}$$

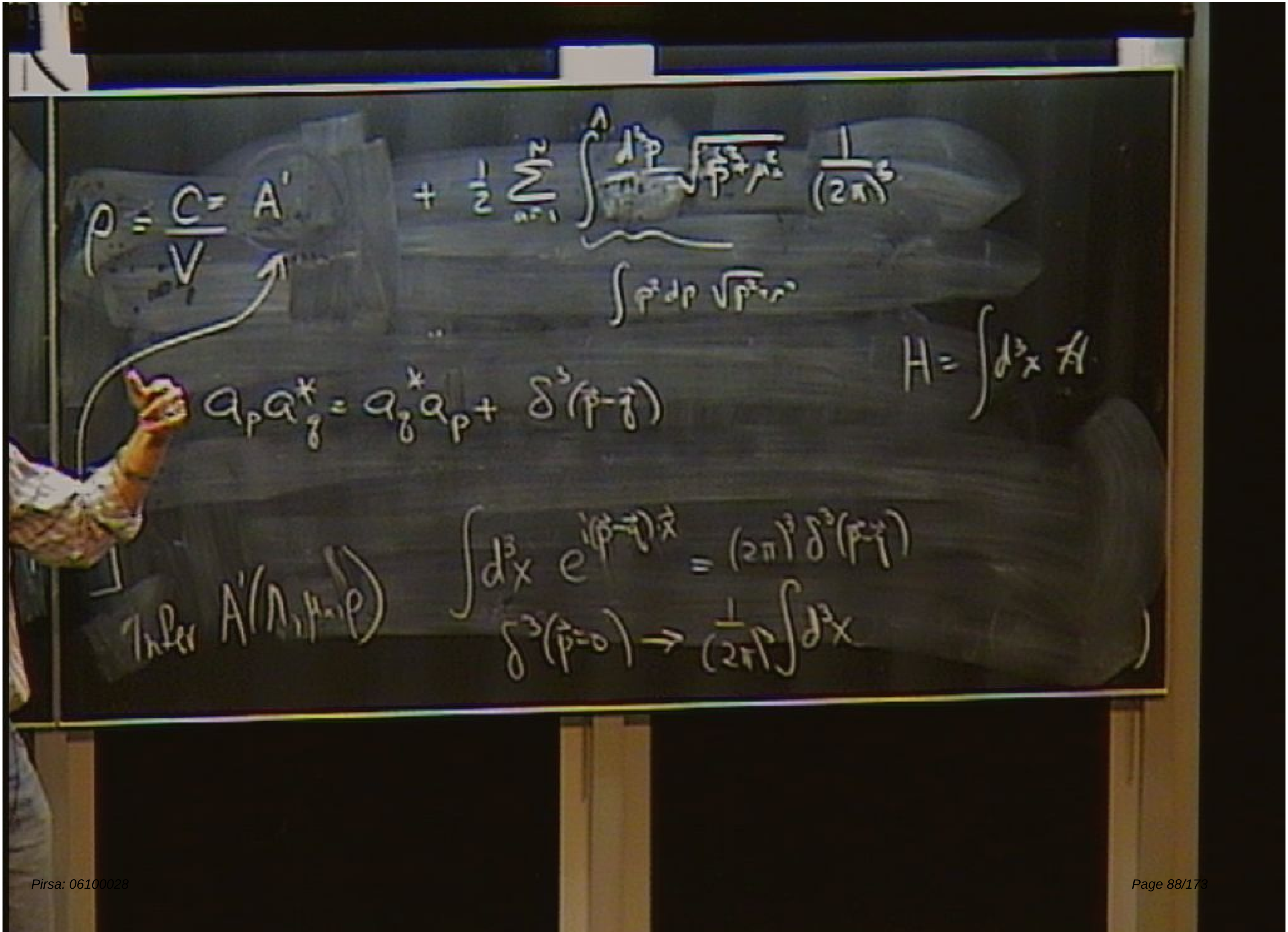
$$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$$

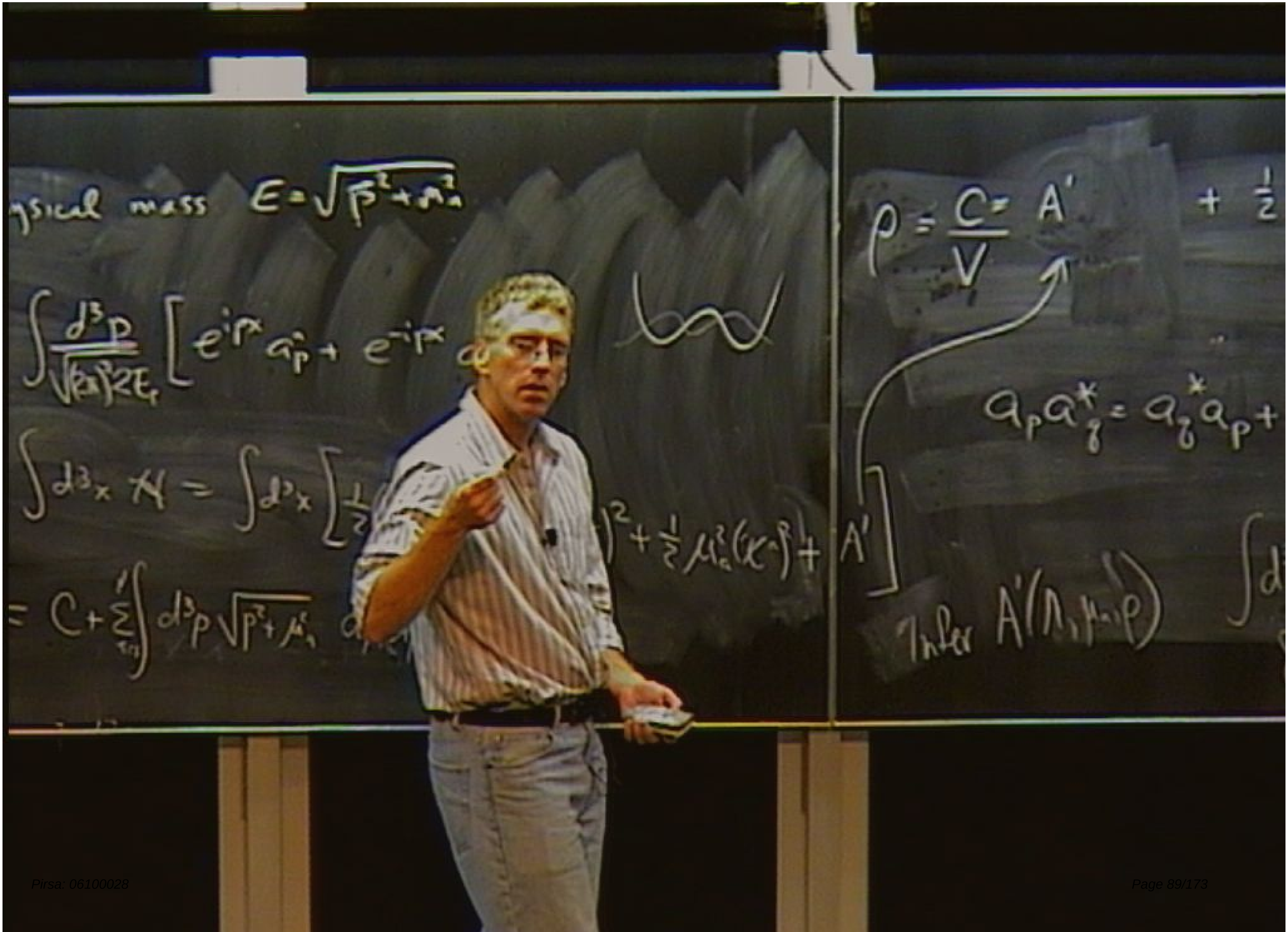
$$+ A'$$

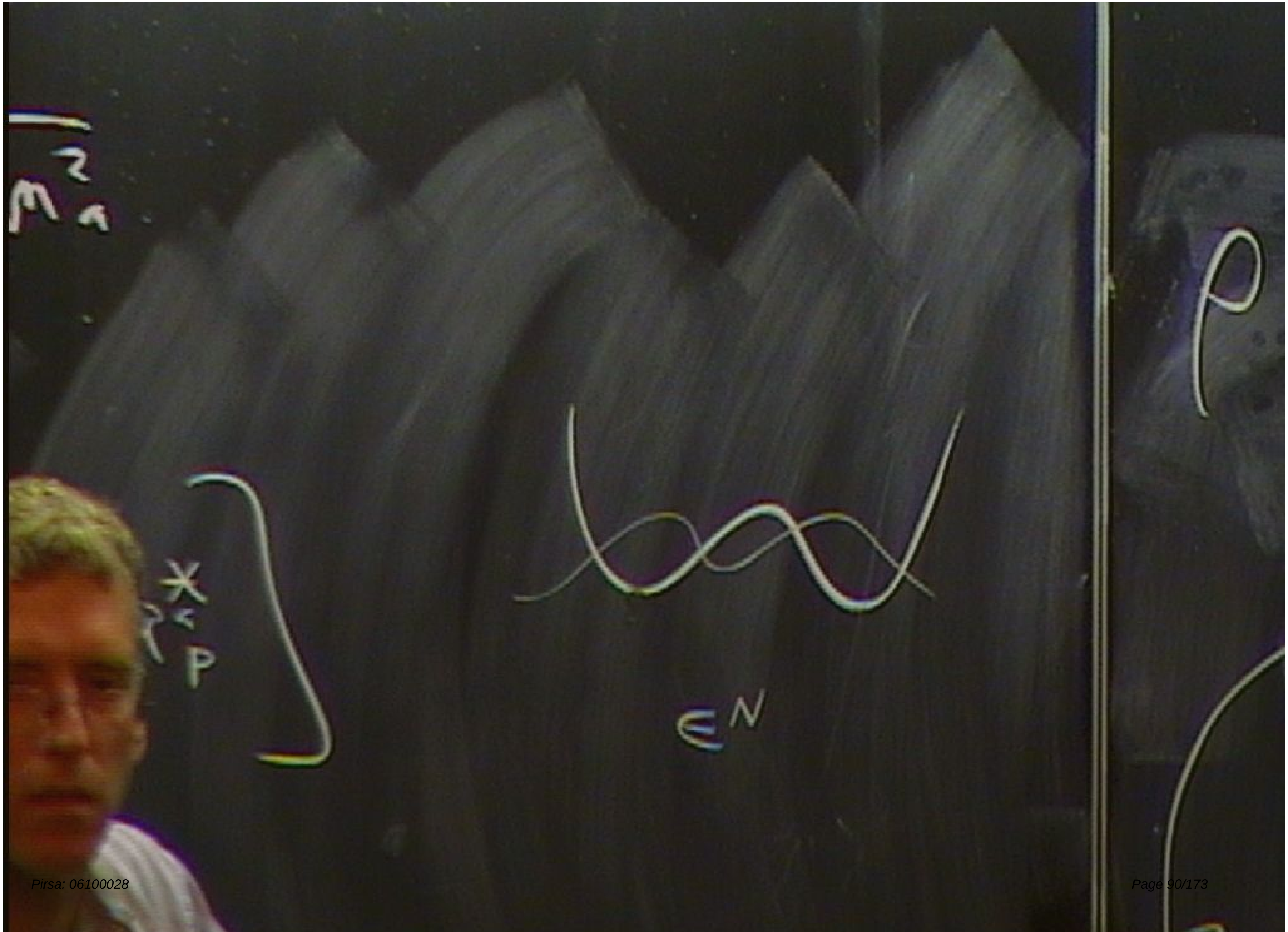
$$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$$

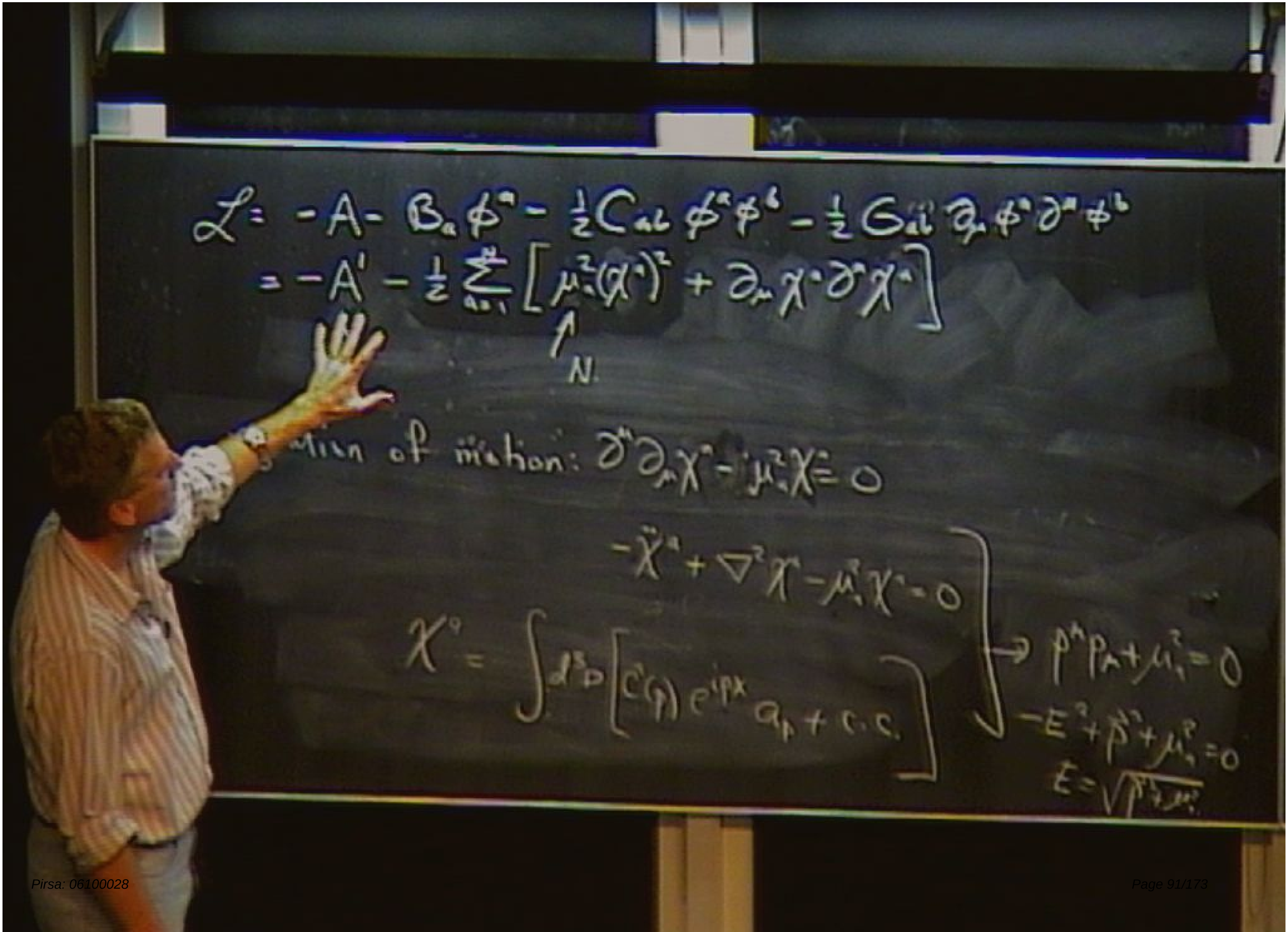
Infer $A'(\Lambda, \mu_a, \rho)$

$$\delta^3(\vec{p}=0) \Rightarrow \frac{1}{(2\pi)^3}\int d^3x$$


$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+\mu_n^2}$
$\int p^2\, dp\, \sqrt{p^2+\mu^2}$
$H = \int d^3x\, \mathcal{H}$
$a_p a_q^* = a_q^* a_p + \delta^3(\vec{p}-\vec{q})$
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$
$\delta^3(\vec{p}=0) \to \frac{1}{(2\pi)^3}\int d^3x$
Infer $A'(\Lambda, \mu_n, \rho)$


ysical mass
ρ = C/V = A'
Infer A'(Λ, μ, ρ)




$\mathcal{L} = -A - B_a\phi^a - \frac{1}{2}C_{ab}\phi^a\phi^b - \frac{1}{2}G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$
$= -A' - \frac{1}{2}\sum_{a=1}^{N}\left[\mu_a^2(\chi^a)^2 + \partial_\mu\chi^a\partial^\mu\chi^a\right]$
N
of motion: $\partial^\mu\partial_\mu\chi^a - \mu_a^2\chi^a = 0$
$-\ddot{\chi}^a + \nabla^2\chi^a - \mu_a^2\chi^a = 0$
$\chi^a = \int d^3p\left[C(p)\, e^{ipx} a_p + c.c.\right]$
$p^\mu p_\mu + \mu_a^2 = 0$
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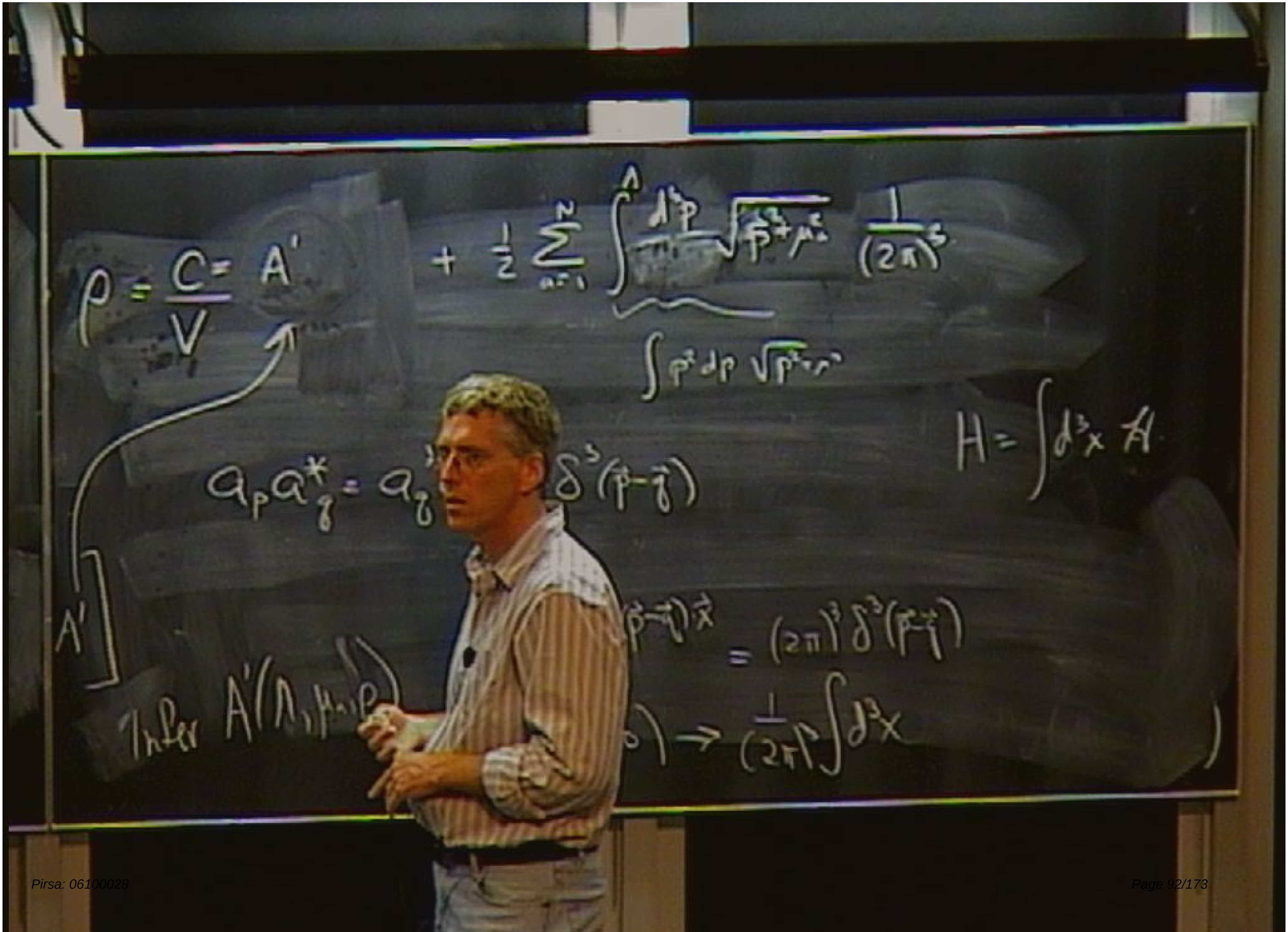
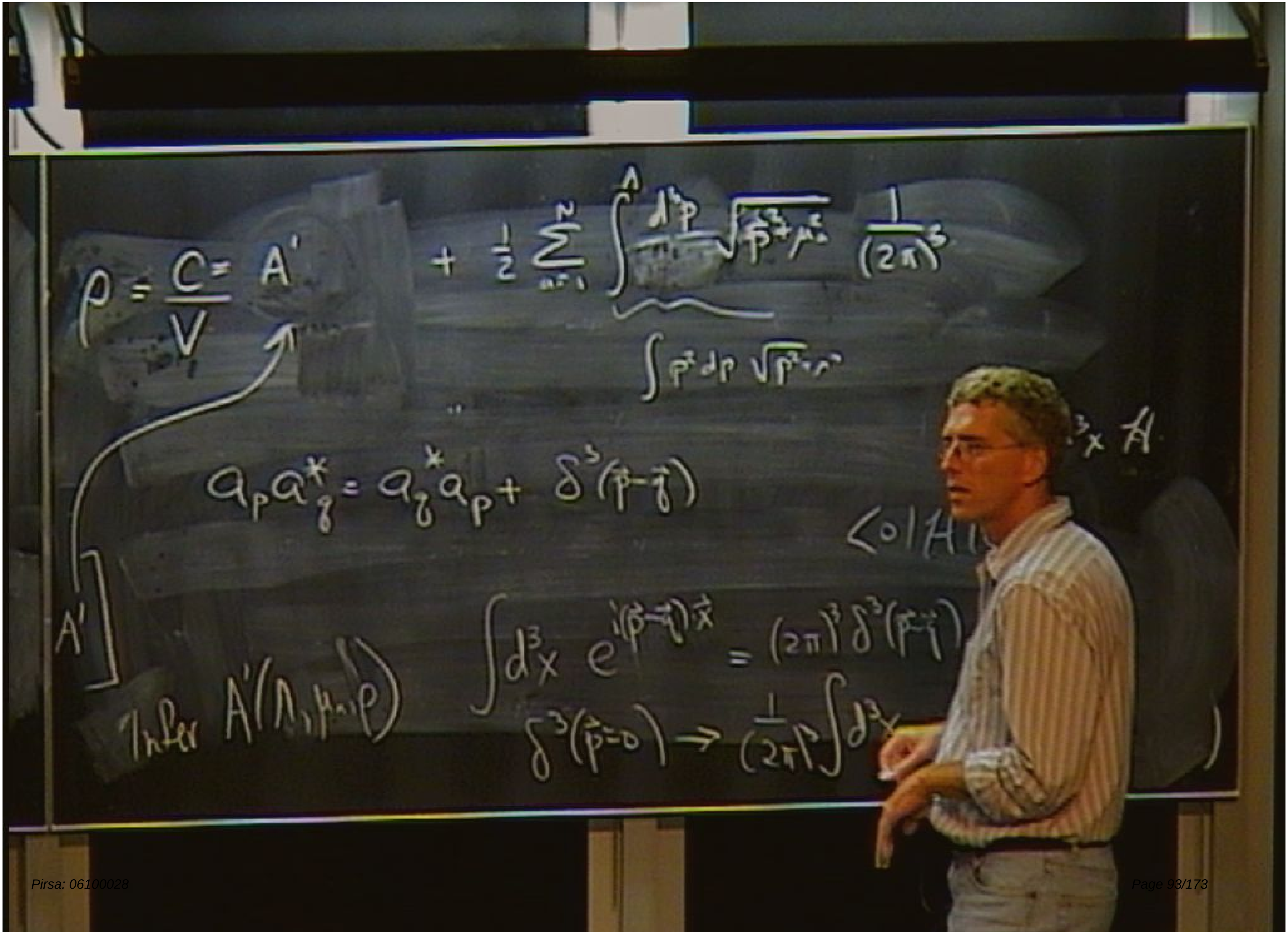

ρ = C/V = A'
+ 1/2 Σ_{a=1}^{N} ∫^Λ d³p √(p²+μ_a²) 1/(2π)³
∫ p² dp √(p²+μ²)
H = ∫d³x ℋ
a_p a*_q = a_q
δ³(p−q)
A'
= (2π)³ δ³(p−q)
→ 1/(2π)³ ∫d³x
Infer A'(Λ, μ_a, ρ)



$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+\mu_n^2}$
$\int p^2\,dp\,\sqrt{p^2+\mu^2}$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
$\langle 0|A|$
A'
Infer $A'(\Lambda, \mu_n, \rho)$
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3\delta^3(\vec{p}-\vec{q})$
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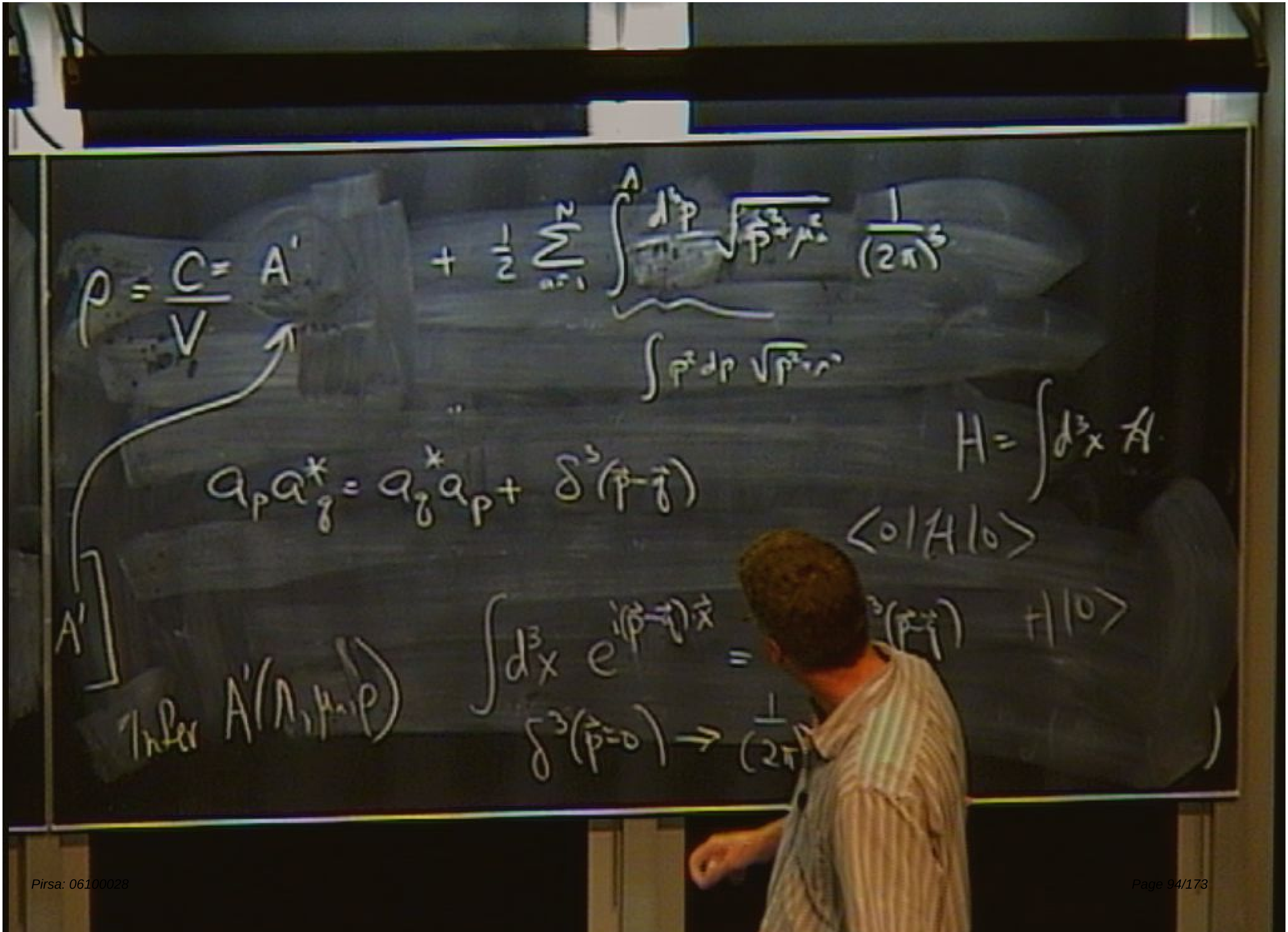
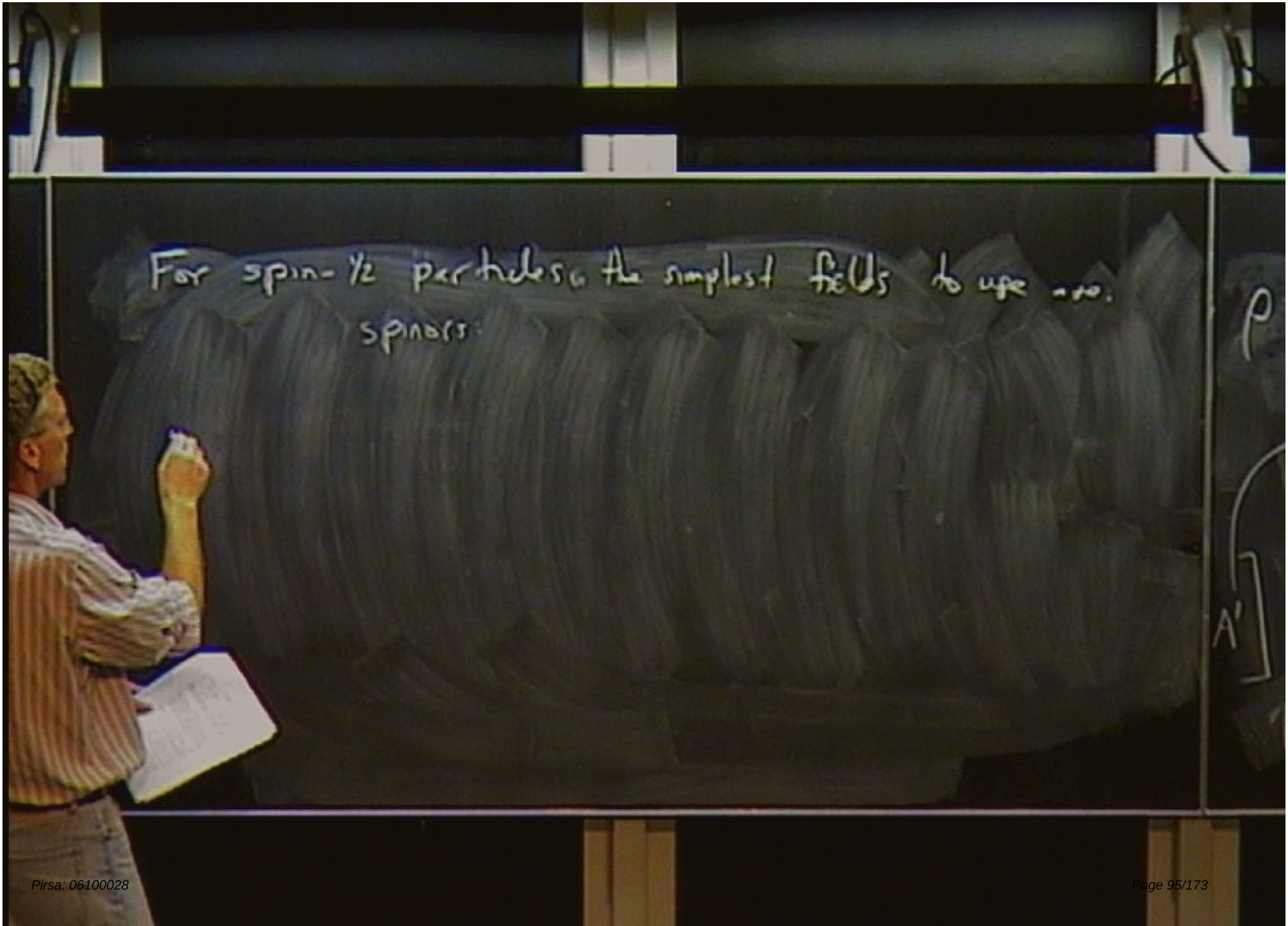


$\rho = \frac{C}{V} = A' + \frac{1}{2}\sum_{n=1}^{N}\int^{\Lambda} d^3p \sqrt{\vec{p}^2+\mu_n^2}\ \frac{1}{(2\pi)^3}$
$\int p^2 dp \sqrt{p^2+\mu^2}$
$H = \int d^3x\, \mathcal{H}$
$a_p a^*_q = a^*_q a_p + \delta^3(\vec{p}-\vec{q})$
$\langle 0|\mathcal{H}|0\rangle$
A'
$\int d^3x\, e^{i(\vec{p}-\vec{q})\cdot\vec{x}} =$
$\mathcal{H}|0\rangle$
Infer $A'(\Lambda, \mu_n, \rho)$
$\delta^3(\vec{p}=0) \rightarrow \frac{1}{(2\pi)^3}$


For spin-1/2 particles, the simplest fields to use are
spinors:

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spinors:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \quad (\underline{\text{Dirac}}\ \text{spinor})$$

complex

For spin-1/2 particles, the simplest fields to use are:

spinors:

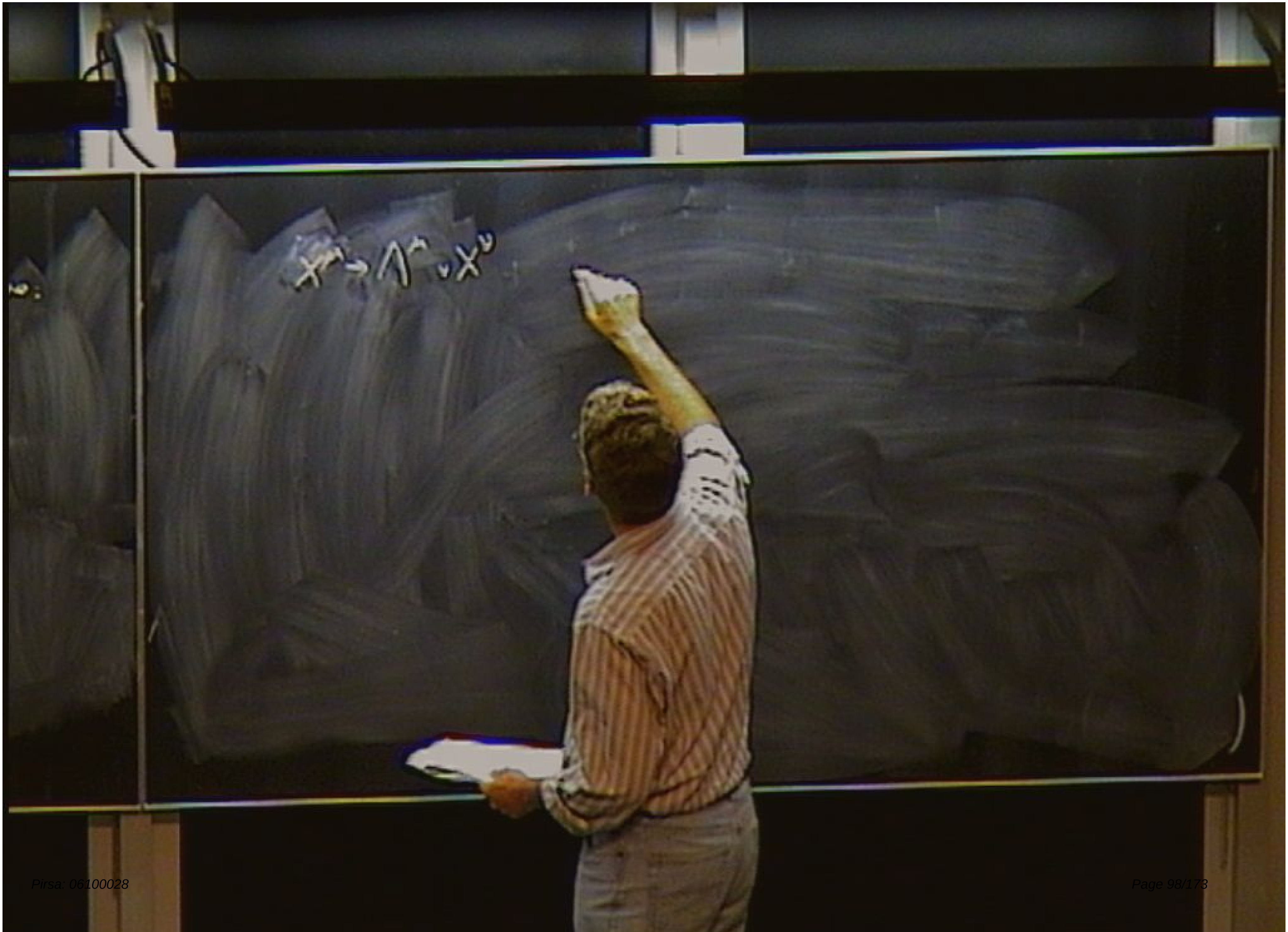
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \text{ (Dirac spinor)}$$

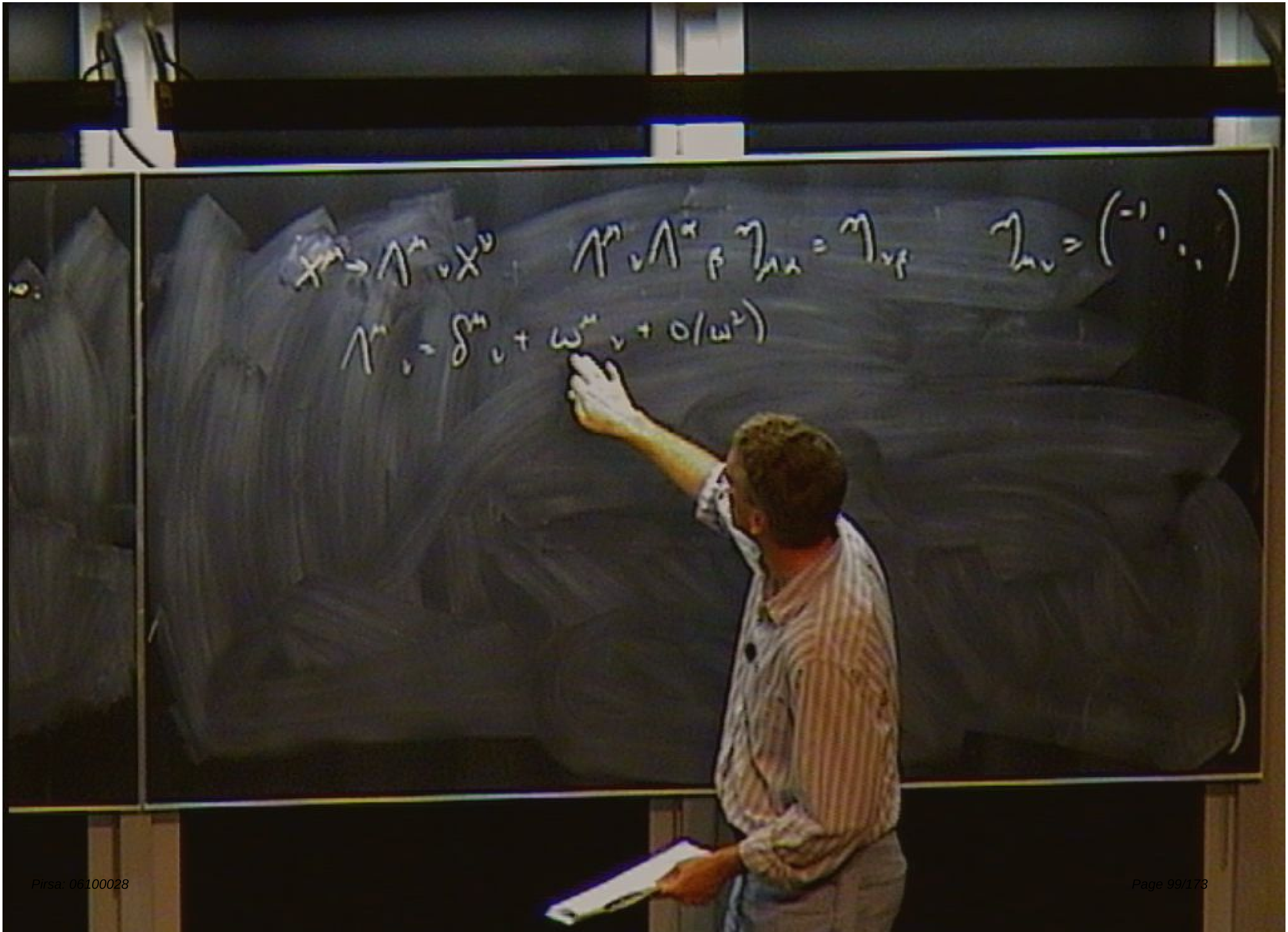
complex

Under Lorentz transformations: $U \psi_i(x) U^\dagger = \boxed{D_{ij}(\Lambda)} \psi_j(\Lambda x)$

ρ

A'

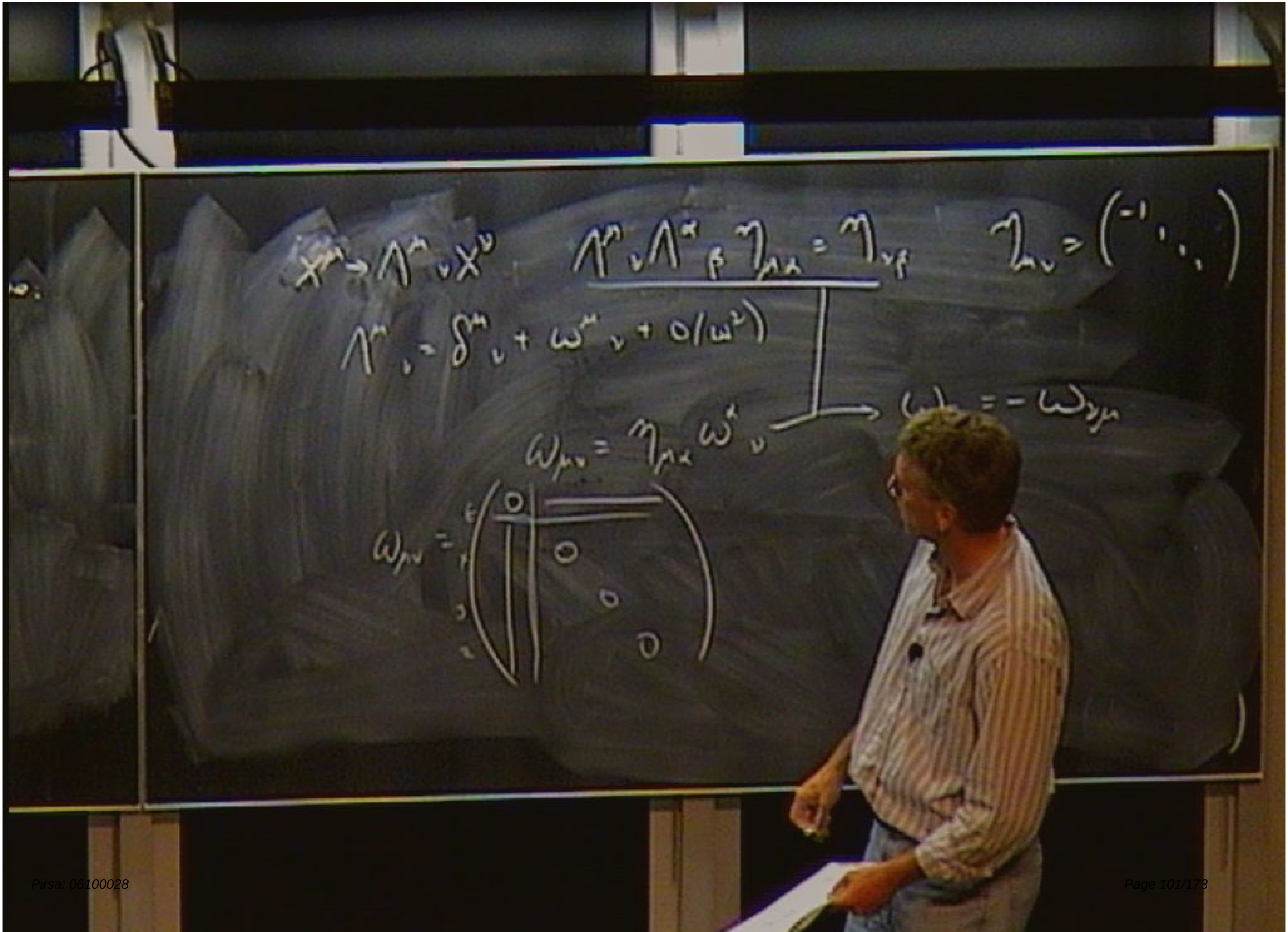

$x^\mu \to \Lambda^\mu{}_\nu x^\nu$

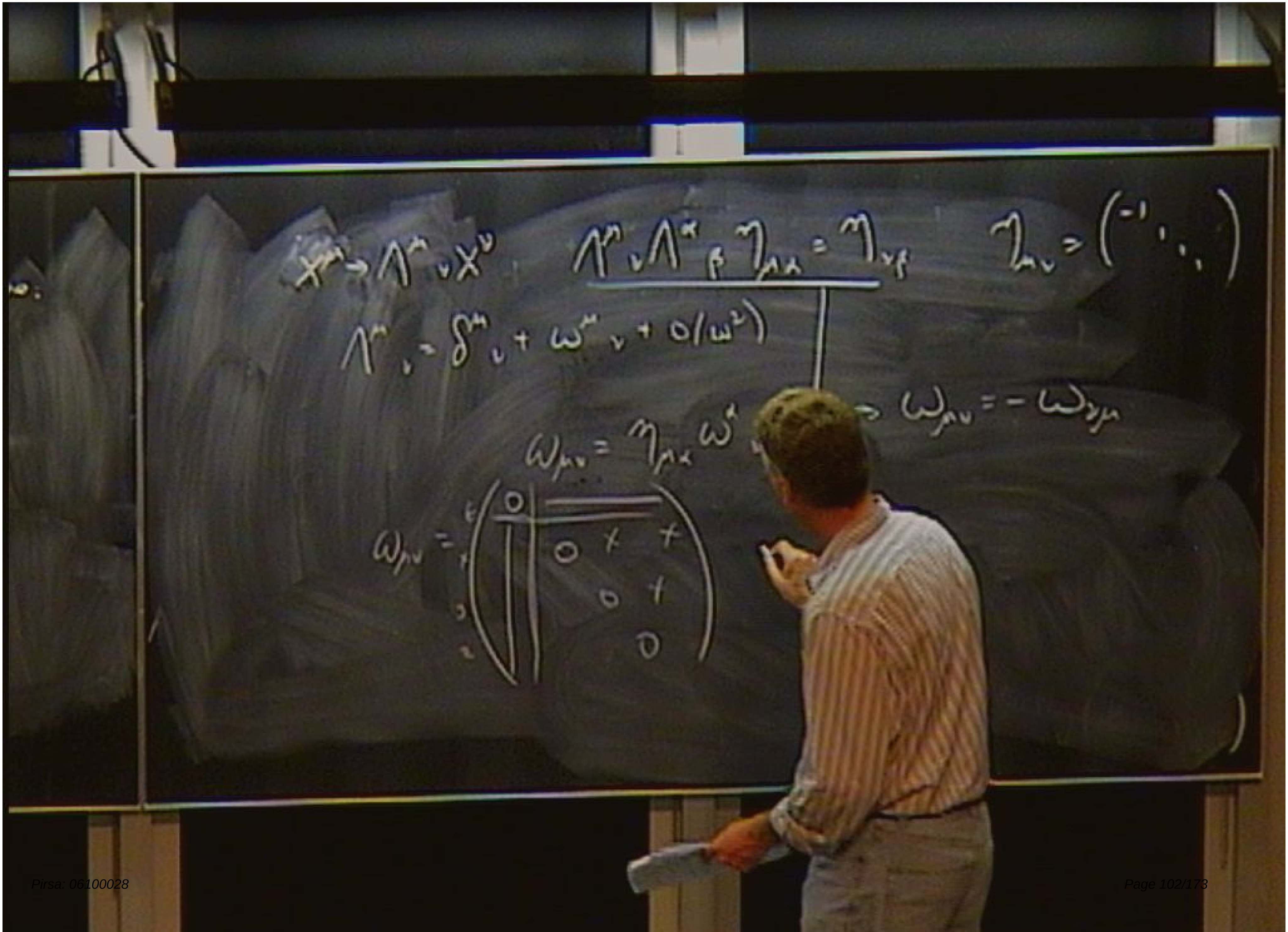

$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\Lambda^\mu{}_\nu \Lambda^\alpha{}_\beta \eta_{\mu\alpha} = \eta_{\nu\beta}$
$\eta_{\mu\nu} = (-1, 1, 1)$
$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu + O(\omega^2)$

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu \qquad \Lambda^\mu{}_\nu \Lambda^\alpha{}_\beta \eta_{\mu\alpha} = \eta_{\nu\beta} \qquad \eta_{\mu\nu} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$$

$$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu + O(\omega^2)$$

$$\omega_{\mu\nu} = \eta_{\mu\alpha}\omega^\alpha{}_\nu \qquad \omega_{\mu\nu} = -\omega_{\nu\mu}$$


$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\Lambda^\mu{}_\nu \Lambda^\alpha{}_\beta \eta_{\mu\alpha} = \eta_{\nu\beta}$
$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & \ddots & \end{pmatrix}$
$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu + O(\omega^2)$
$\omega_{\nu\beta} = -\omega_{\beta\nu}$
$\omega_{\mu\nu} = \eta_{\mu\alpha}\omega^\alpha{}_\nu$
$\omega_{\mu\nu} =$



$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\Lambda^\mu{}_\nu \Lambda^\alpha{}_\beta \eta_{\mu\alpha} = \eta_{\nu\beta}$
$\eta_{\mu\nu} = (-1, 1, \ldots)$
$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu + O(\omega^2)$
$\Rightarrow \omega_{\mu\nu} = -\omega_{\nu\mu}$
$\omega_{\mu\nu} = \eta_{\mu\alpha} \omega^\alpha$
$\omega_{\mu\nu} =$

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu \qquad \underline{\Lambda^\mu{}_\nu \Lambda^\alpha{}_\beta \eta_{\mu\alpha} = \eta_{\nu\beta}} \qquad \eta_{\mu\nu} = \left(^{-1}{}_{1}{}_{\cdot}{}_{\cdot}\right)$$

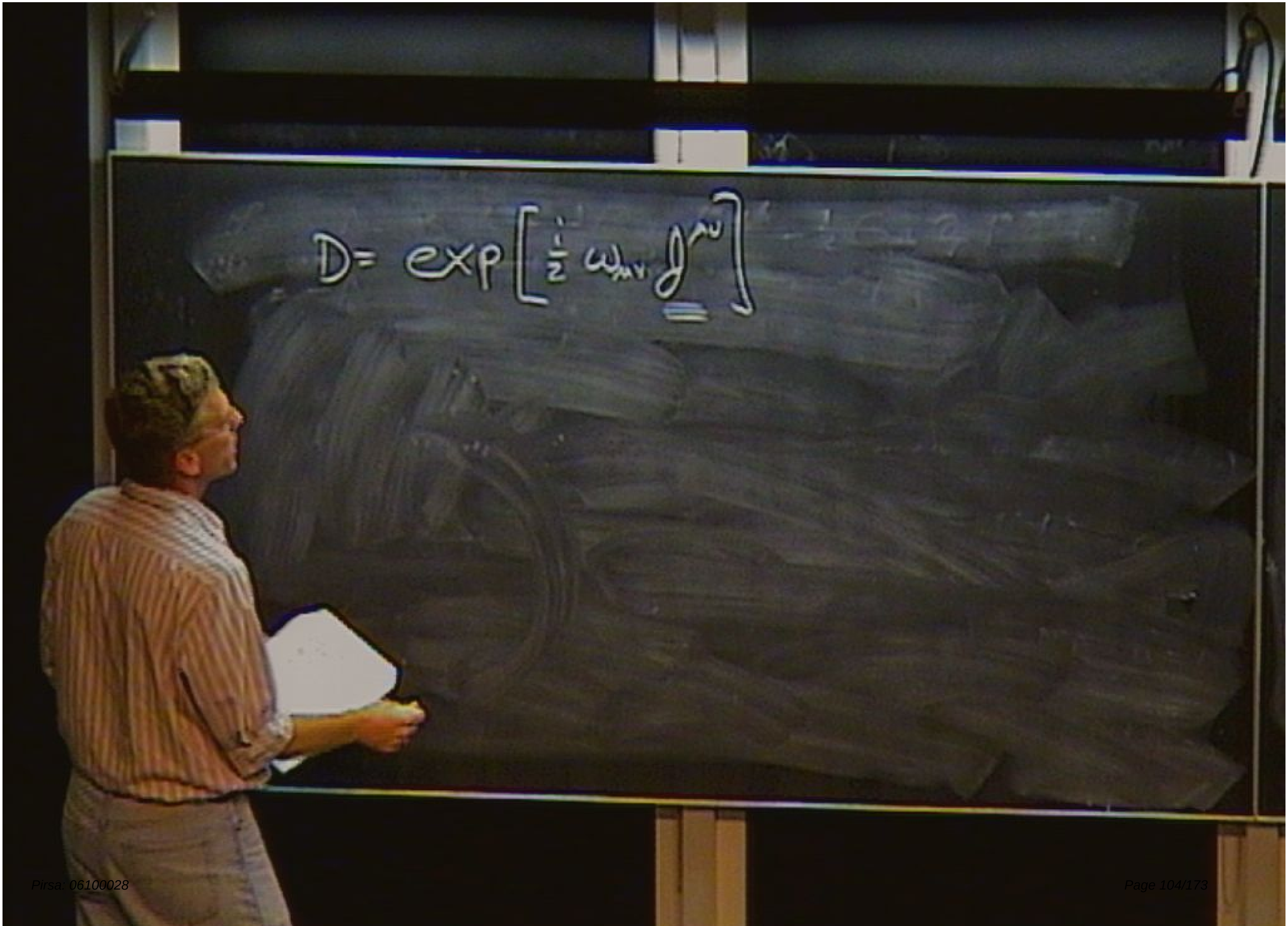
$$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu + O(\omega^2)$$

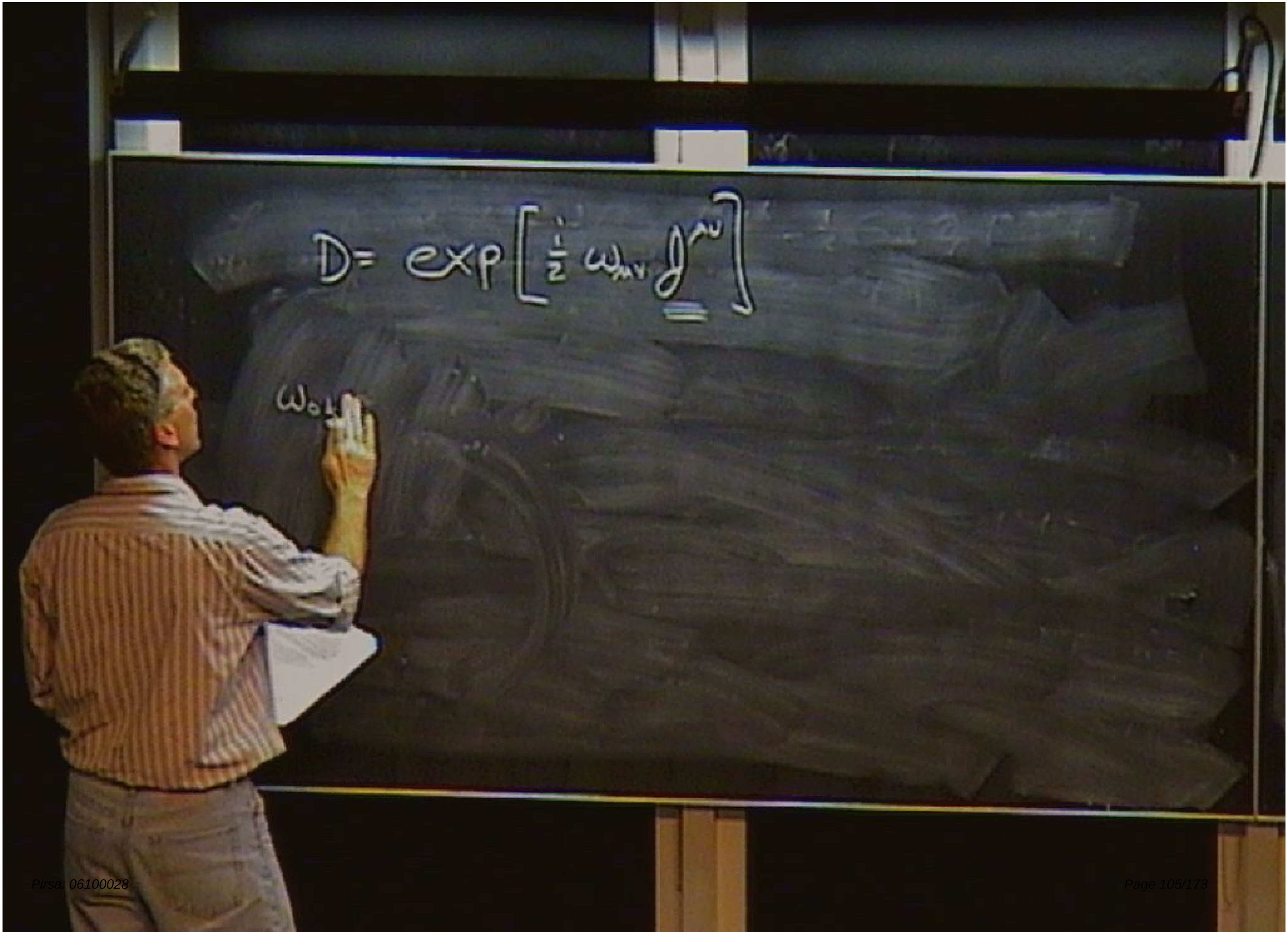
$$\omega_{\mu\nu} = -\omega_{\nu\mu}$$

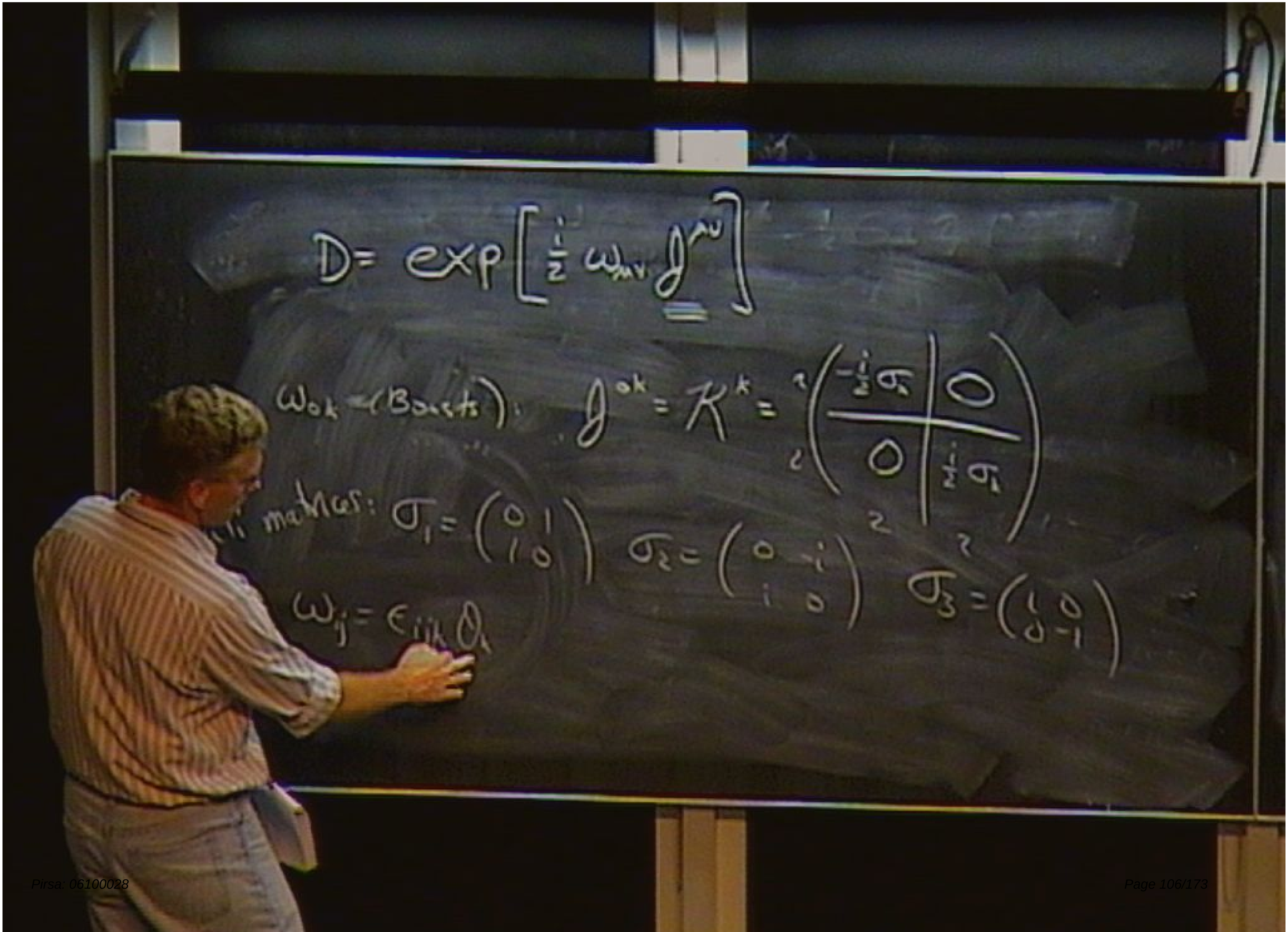
$$\omega_{\mu\nu} = \eta_{\mu\alpha}\omega^\alpha{}_\nu$$

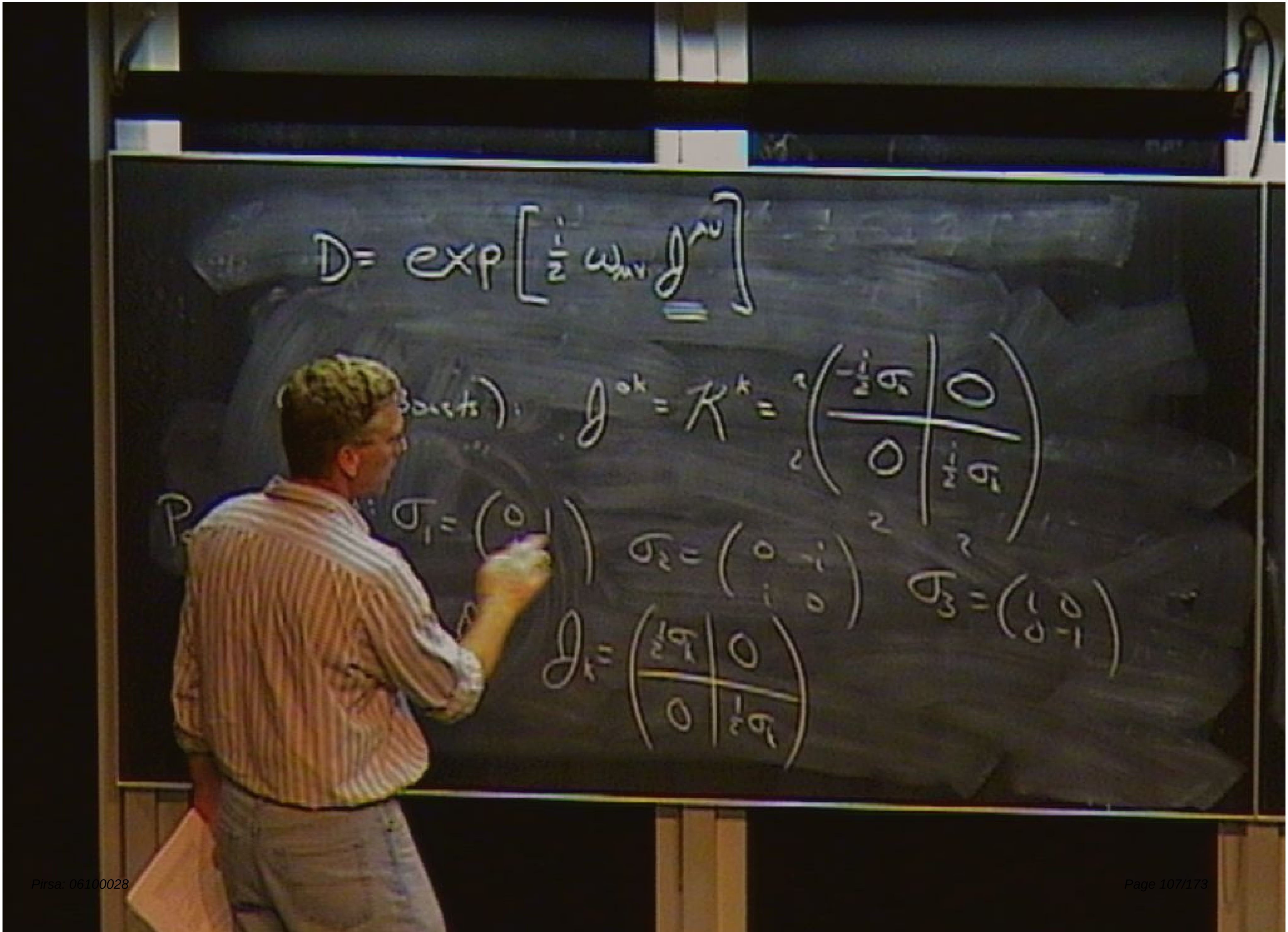
$$\omega_{\mu\nu} = \begin{array}{c} t \\ x \\ y \\ z \end{array} \begin{pmatrix} 0 & & & \\ & 0 & \times & \times \\ & & 0 & \times \\ & & & 0 \end{pmatrix} \text{— rotations}$$

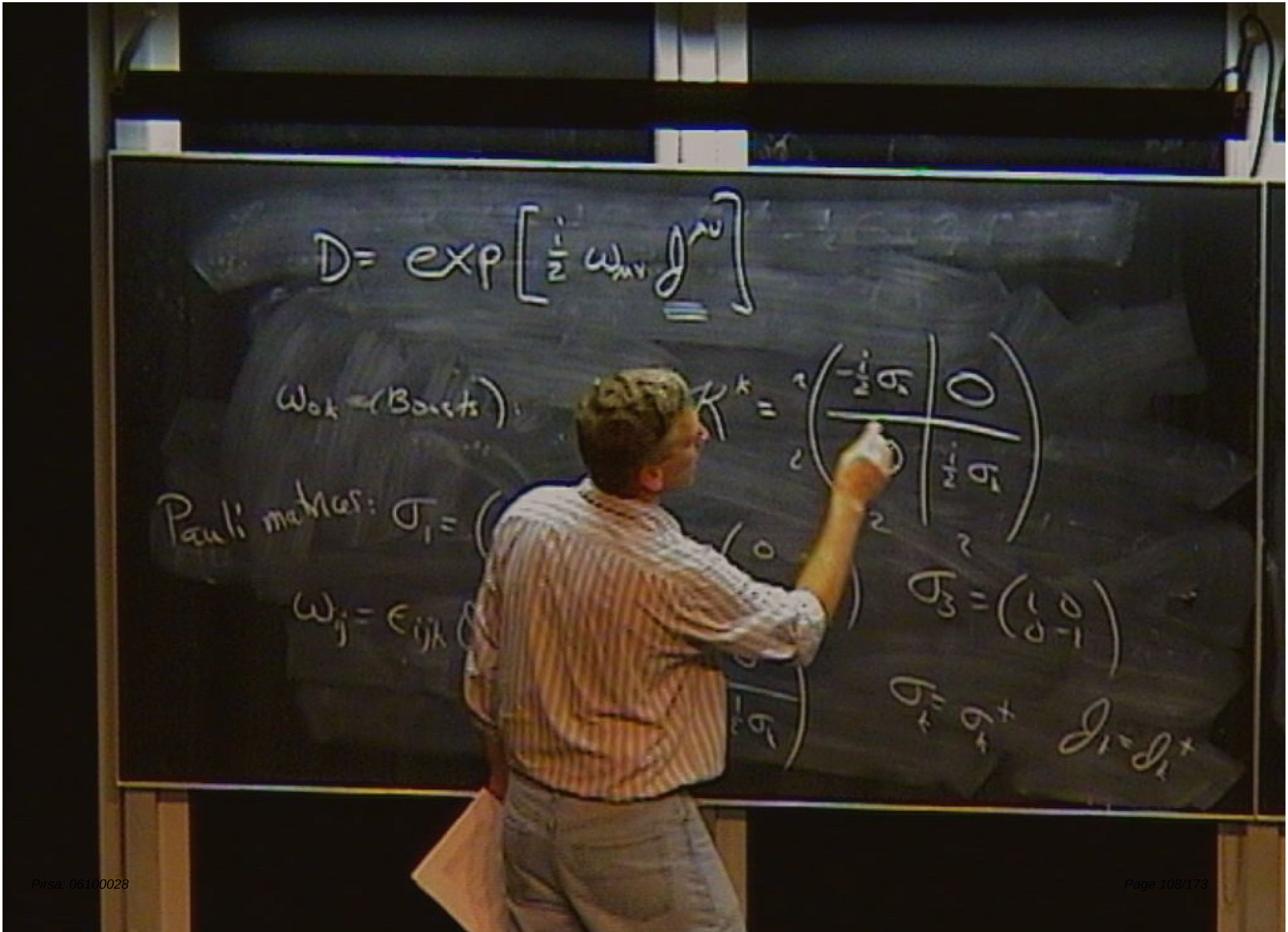
boosts

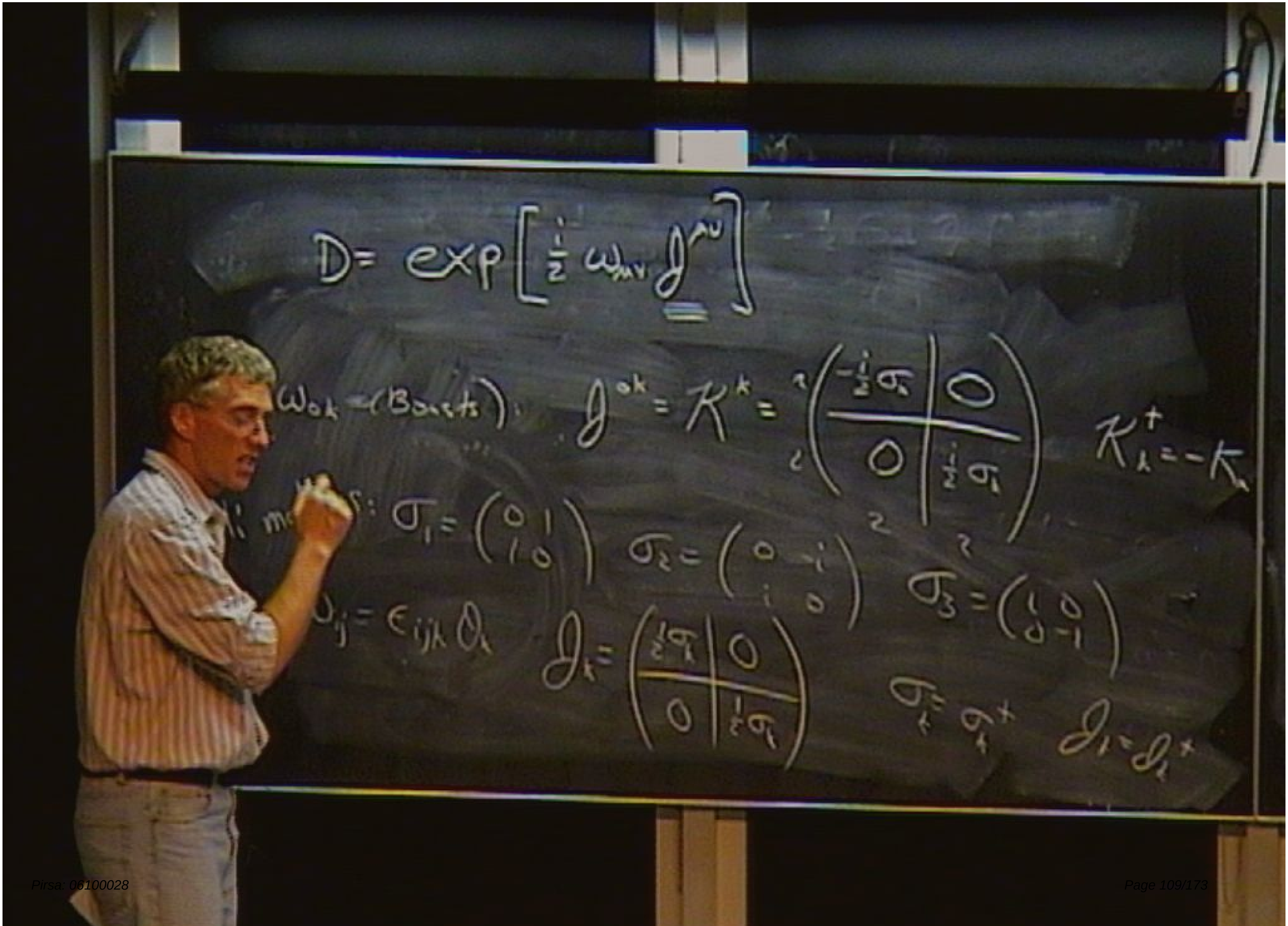

D= exp[i/2 ω_μν J^μν]

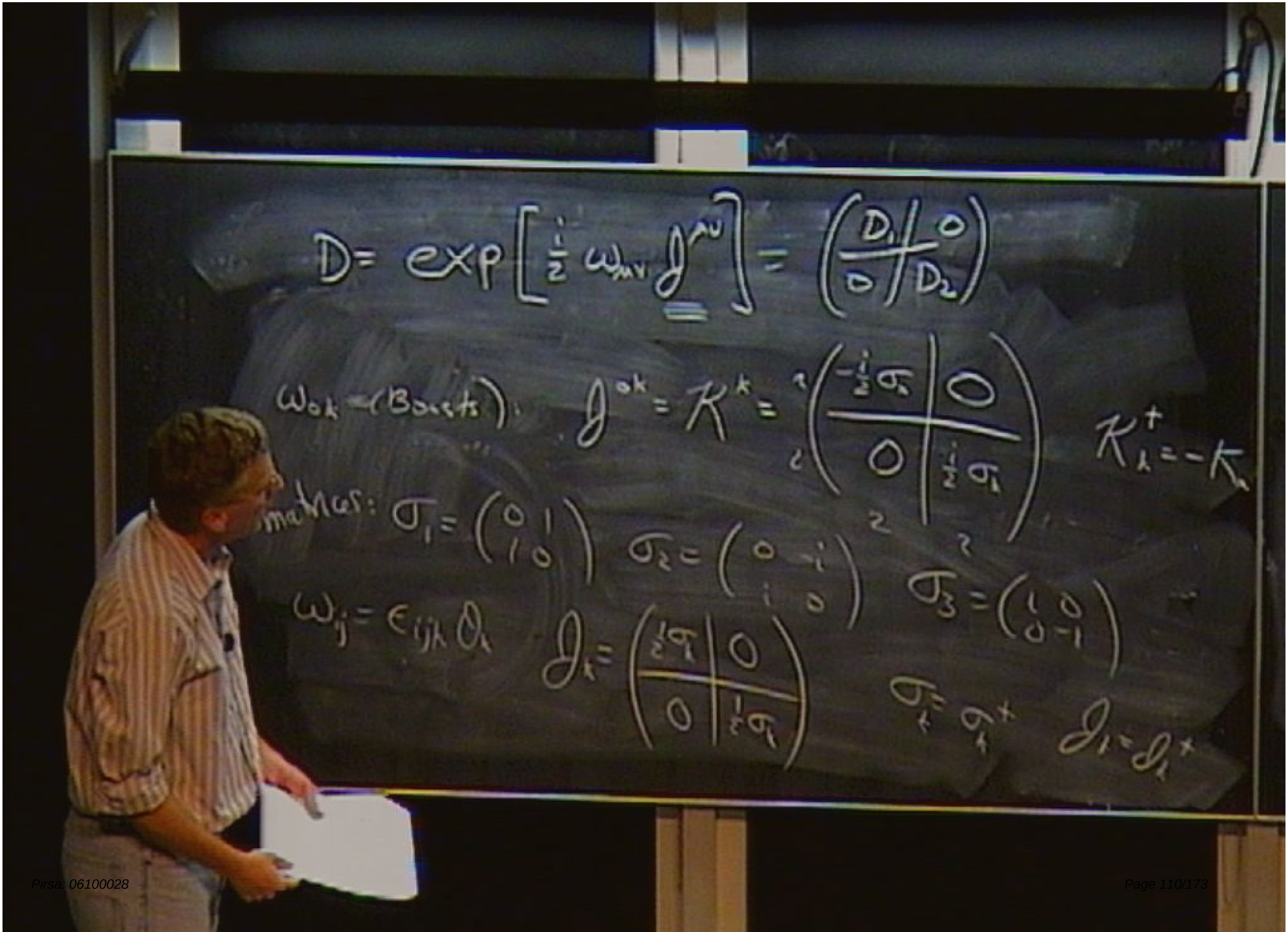

$D = \exp\left[\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu}\right]$

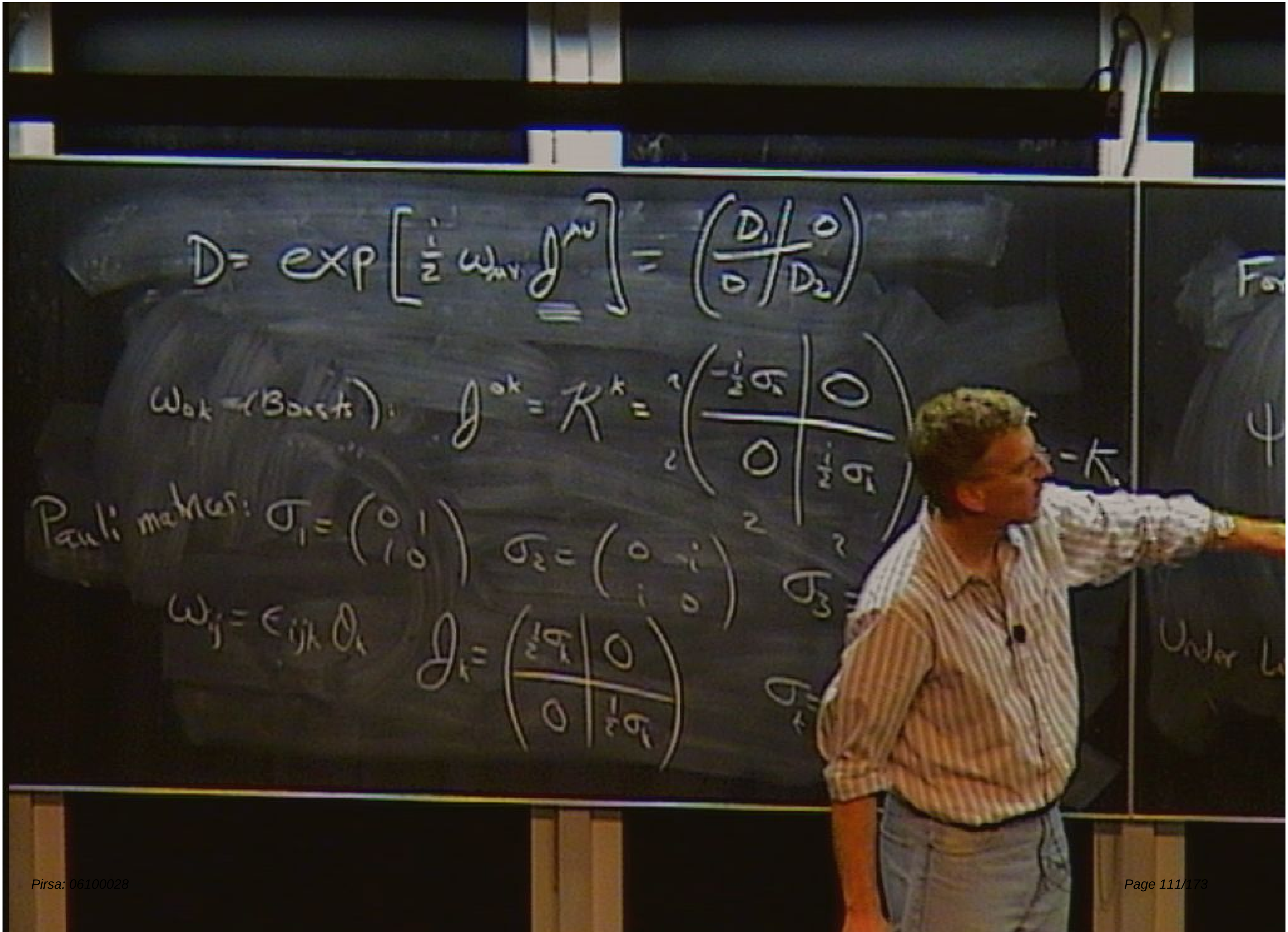

$D = \exp\left[\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu}\right]$
ω_{0k} (Boosts):
$J^{0k} = K^k = \begin{pmatrix} -\frac{i}{2}\sigma_k & 0 \\ 0 & \frac{i}{2}\sigma_k \end{pmatrix}$
matrices: $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\omega_{ij} = \epsilon_{ijk}\theta_k$


D= exp[i/2 ω_μν J^μν]
(Boosts)
J^0k = K^k =
σ_1 =
σ_2 = (0 -i ; i 0)
σ_3 = (1 0 ; 0 -1)
J_k =


D= exp[i/2 ω_μν J^μν]
ω_0k = (Boosts)
K^k =
Pauli matrices: σ_1 =
ω_ij = ε_ijk
σ_3 = (1 0 / 0 -1)
σ_k = σ_k^+
J_k = J_k^+


$D = \exp\left[\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu}\right]$
ω_{0k} (Boosts)
$J^{0k} = K^k = \begin{pmatrix} -\frac{i}{2}\sigma_k & 0 \\ 0 & \frac{i}{2}\sigma_k \end{pmatrix}$
$K_k^+ = -K_k$
$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\omega_{ij} = \epsilon_{ijk}\theta_k$
$J_k = \begin{pmatrix} \frac{1}{2}\sigma_k & 0 \\ 0 & \frac{1}{2}\sigma_k \end{pmatrix}$
$\sigma_k = \sigma_k^+$ $J_k = J_k^+$


D= exp[i/2 ω_μν J^μν] = (D1 0 / 0 D2)
ω_0k = (Boosts). J^0k = K^k = (−i/2 σ_k 0 / 0 i/2 σ_k)
K_k^+ = −K_k
maths: σ_1 = (0 1 / 1 0)
σ_2 = (0 −i / i 0)
σ_3 = (1 0 / 0 −1)
ω_ij = ε_ijk θ_k
J_k = (1/2 σ_k 0 / 0 1/2 σ_k)
σ_k = σ_k^+
J_k = J_k^+


D= exp[i/2 ω_μν J^μν] = (D_1 0 / 0 D_2)
ω_0k (Boosts):
J^0k = K^k =
Pauli matrices: σ_1 = (0 1 / 1 0)
σ_2 = (0 -i / i 0)
σ_3 =
ω_ij = ε_ijk θ_k
J_k =
Under L

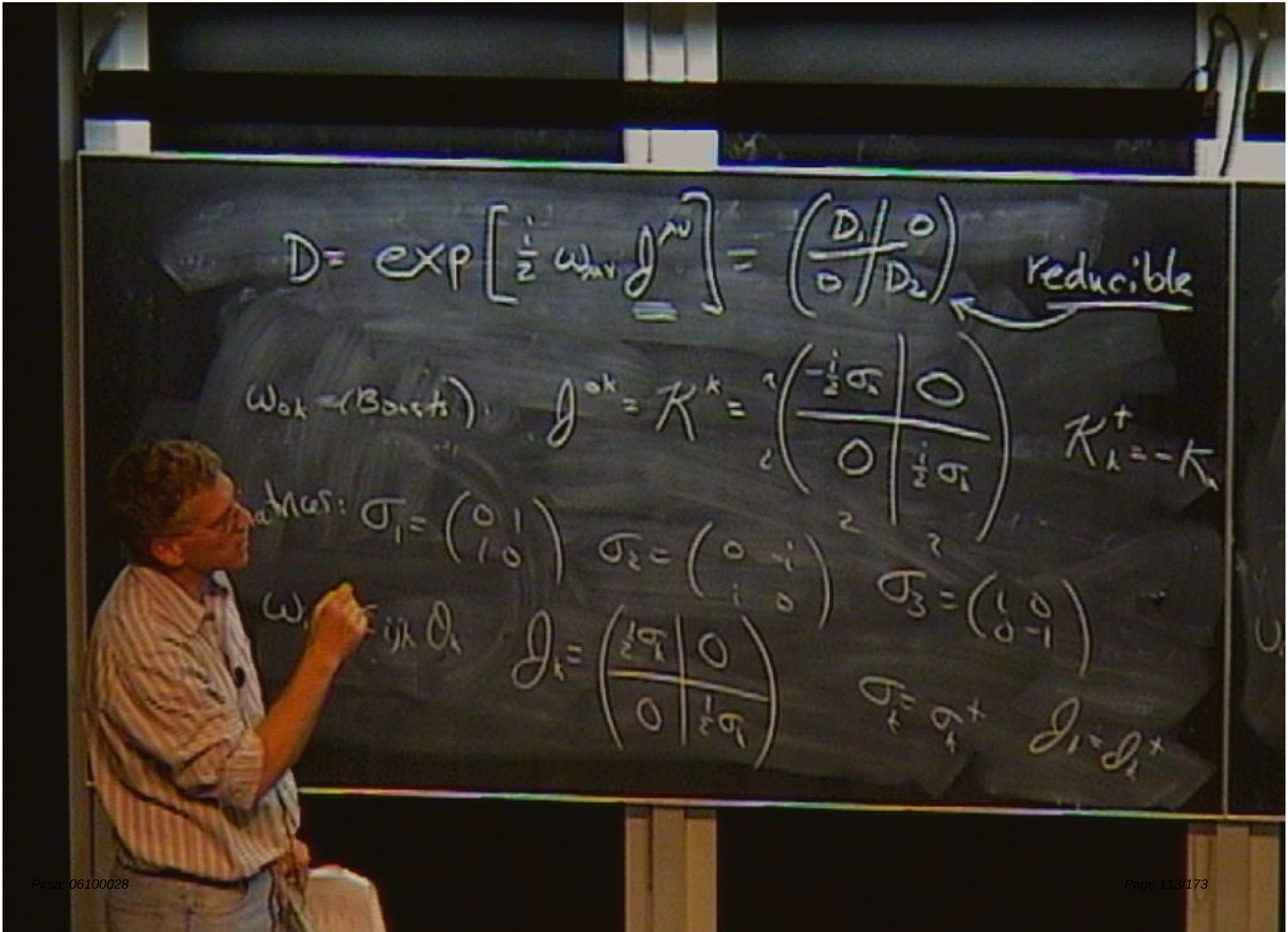
For spin-1/2 particles, the simplest fields to use are:

spinors:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \quad \text{(\underline{Dirac} spinor)}$$

complex

Under Lorentz transformations: $U \psi_i(x) U^* = \boxed{D_{ij}(\Lambda)} \psi_j(\Lambda x)$


$D = \exp\left[\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu}\right] = \left(\begin{array}{c|c} D_1 & 0 \\ \hline 0 & D_2 \end{array}\right)$
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ω_{0k} (Boosts). $J^{0k} = K^k = \left(\begin{array}{c|c} -\frac{i}{2}\sigma_k & 0 \\ \hline 0 & \frac{i}{2}\sigma_k \end{array}\right)$
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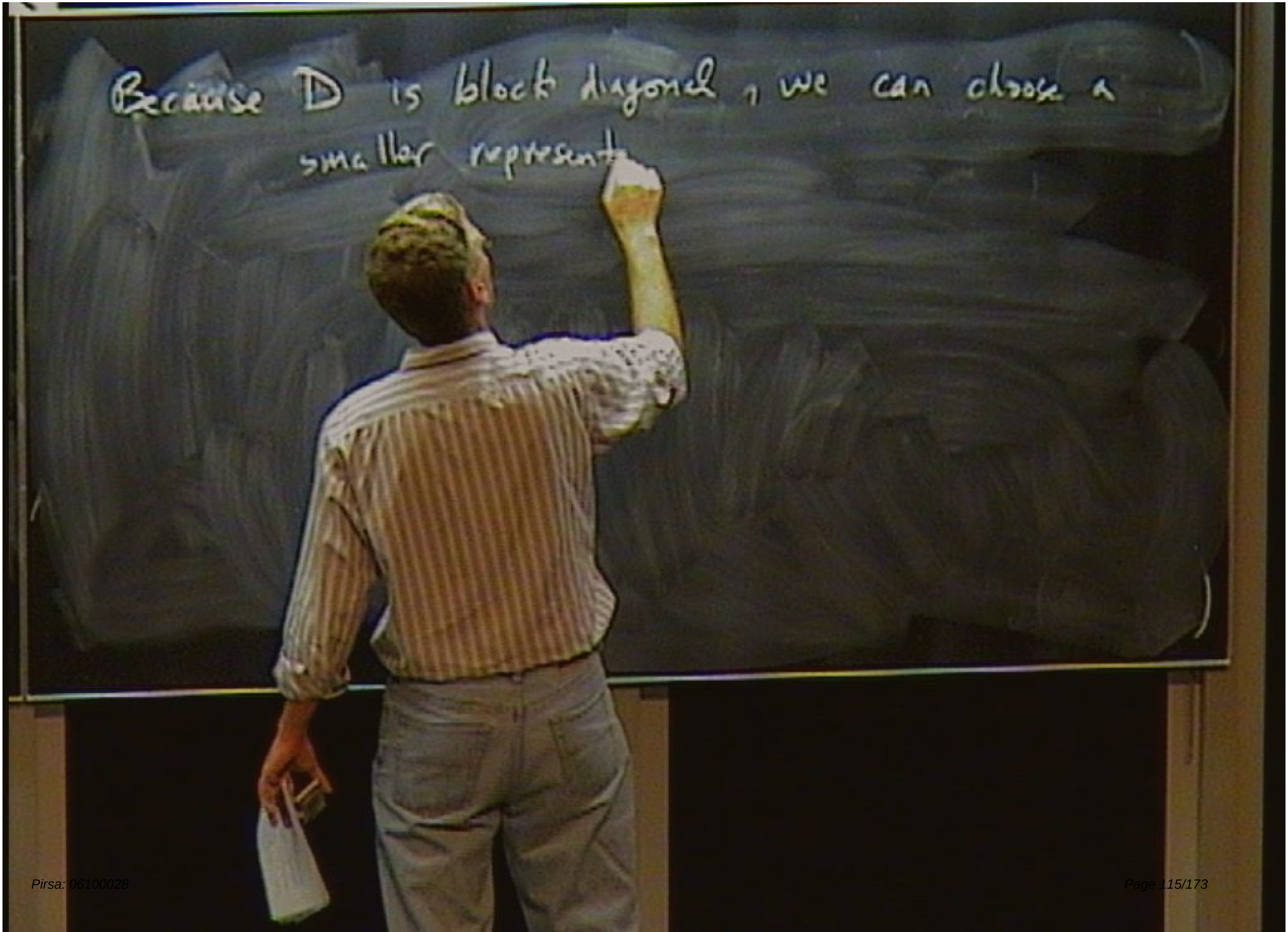
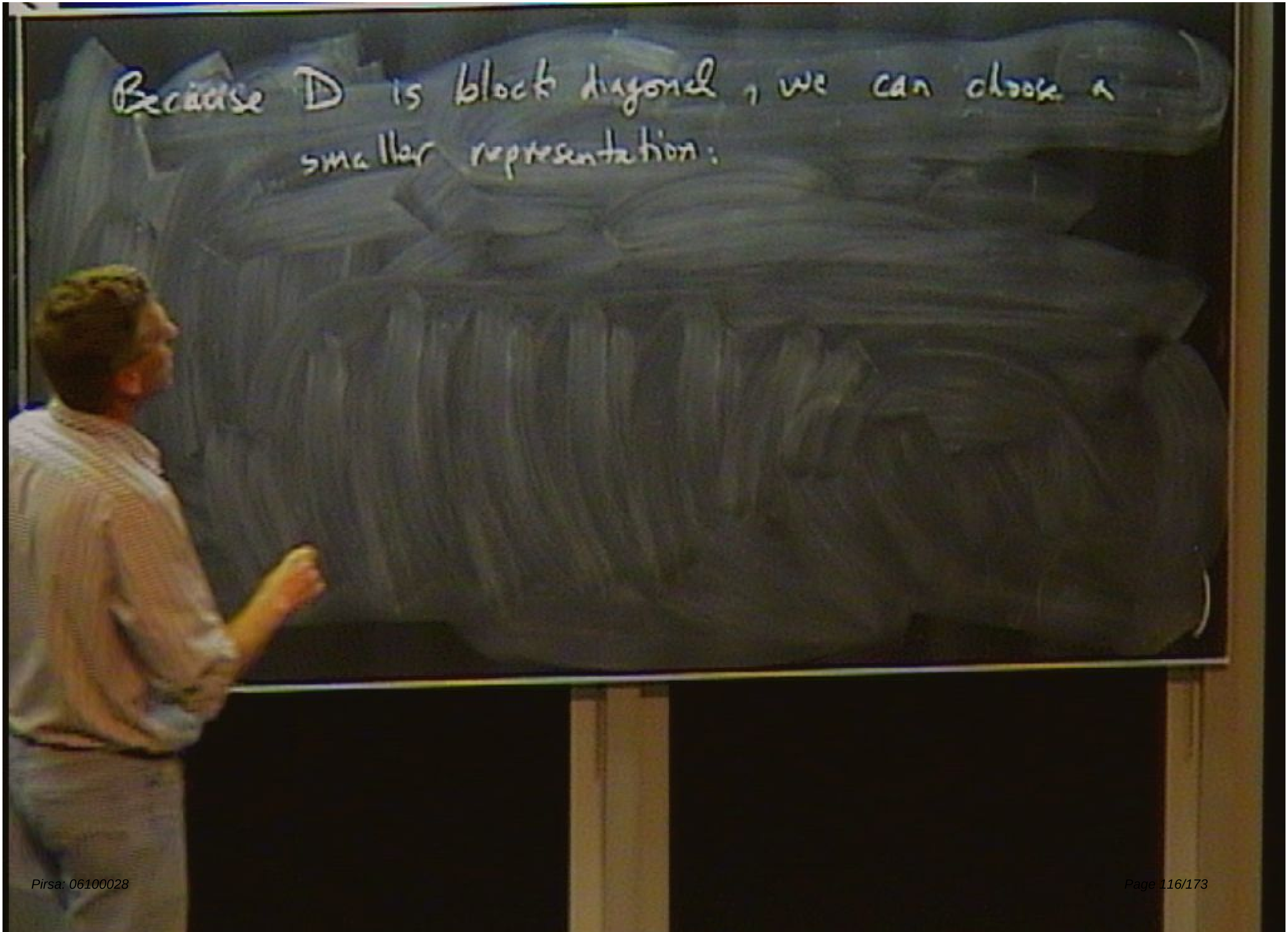
$\sigma_k = \sigma_k^\dagger$ $J_k = J_k^\dagger$

Because D is block diagonal.


Because D is block diagonal, we can choose a
smaller represent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2 conventional choices:

① Weyl spinors:

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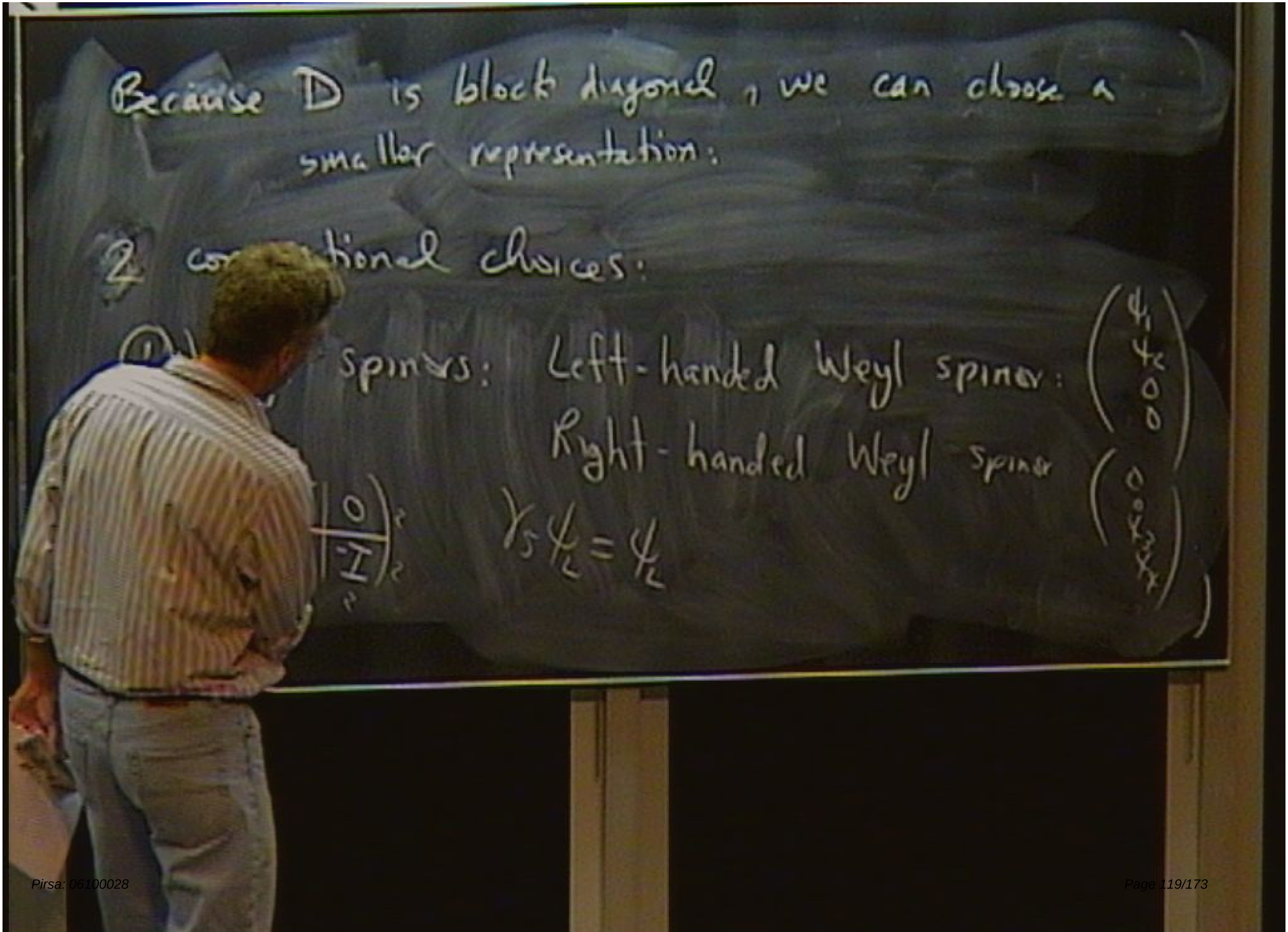
Right-handed Weyl spinor $\begin{pmatrix} 0 \\ 0 \\ \chi_3 \\ \chi_4 \end{pmatrix}$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$$\gamma_5 = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & -I \end{array}\right) \qquad \gamma_5 \psi_L = \psi_L \qquad \gamma_5 \psi_R = -\psi_R$$

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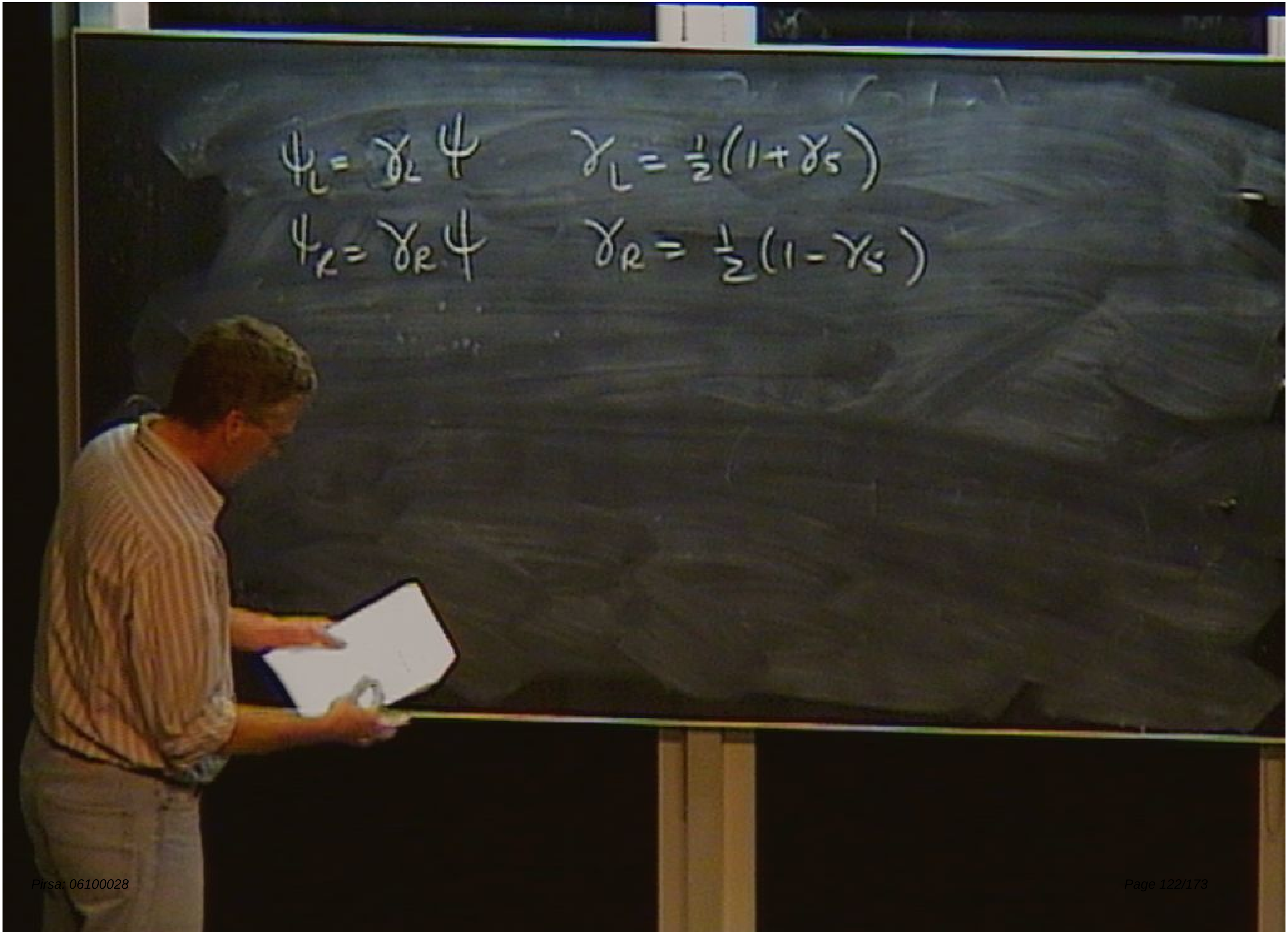
"chirality"


$\psi_L = \gamma_L \psi$ $\gamma_L = \frac{1}{2}(1+\gamma_5)$
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(2) Majarana spinor:

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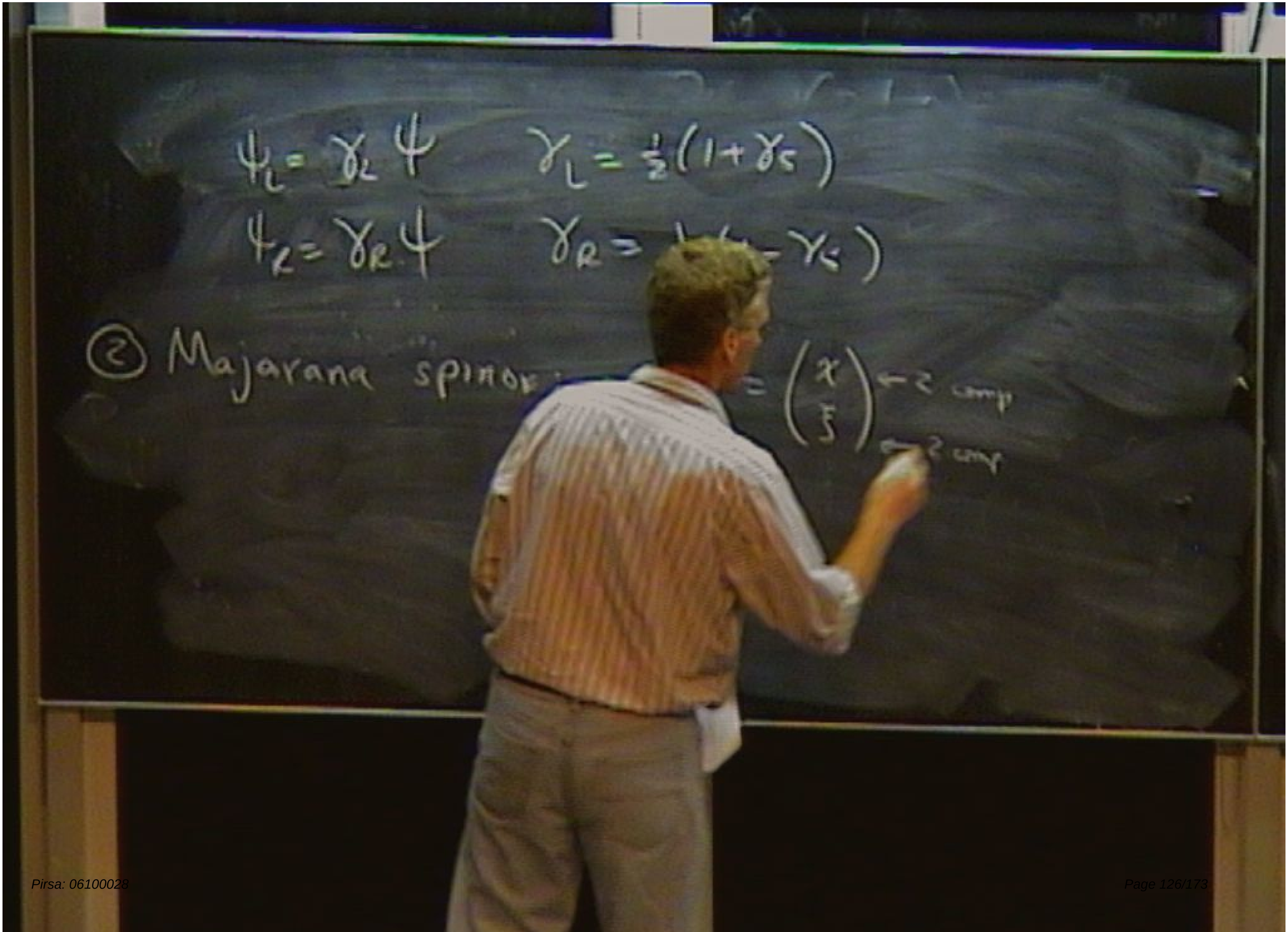
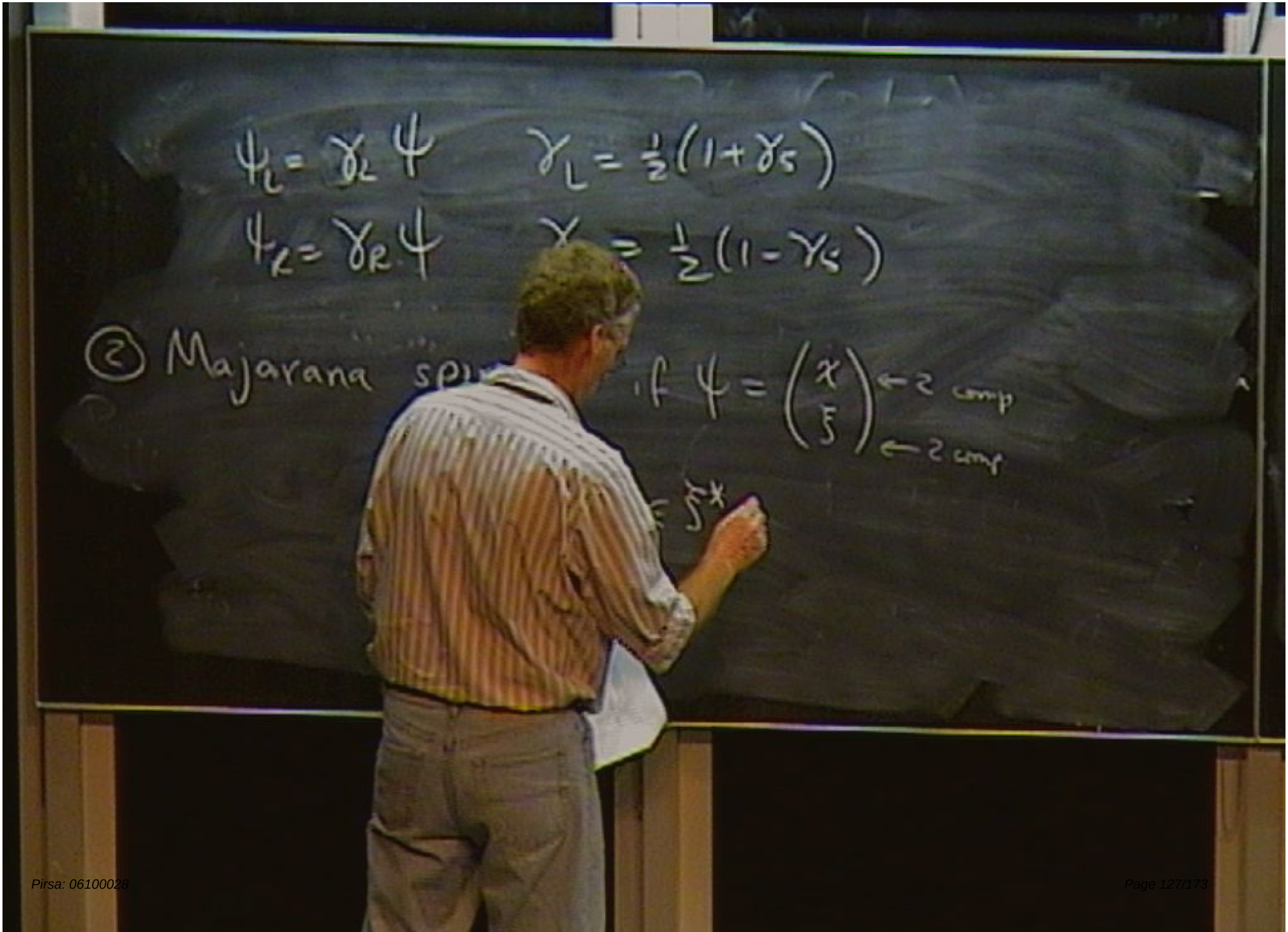

ψL = γL ψ
γL = ½ (1 + γ5)
ψR = γR ψ
γR =
② Majarana spinor
χ
ξ
← 2 comp
2 comp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if $\psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$
← 2 comp
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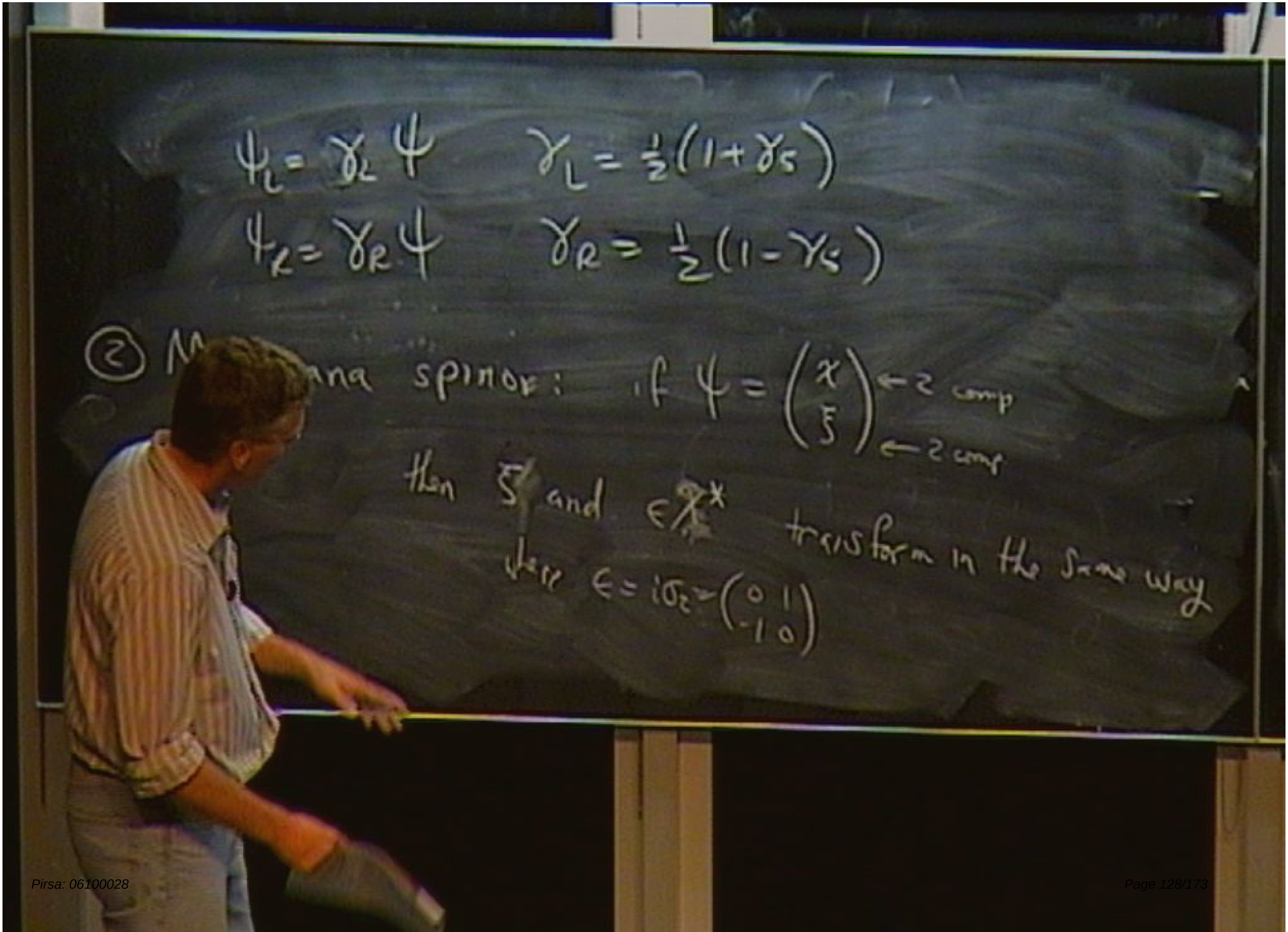
ξ^*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② M
ana spinor:
if $\psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$
← 2 comp
← 2 comp
then ξ and $\epsilon\chi^*$ transform in the same way
where $\epsilon = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

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σ

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where $\epsilon = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

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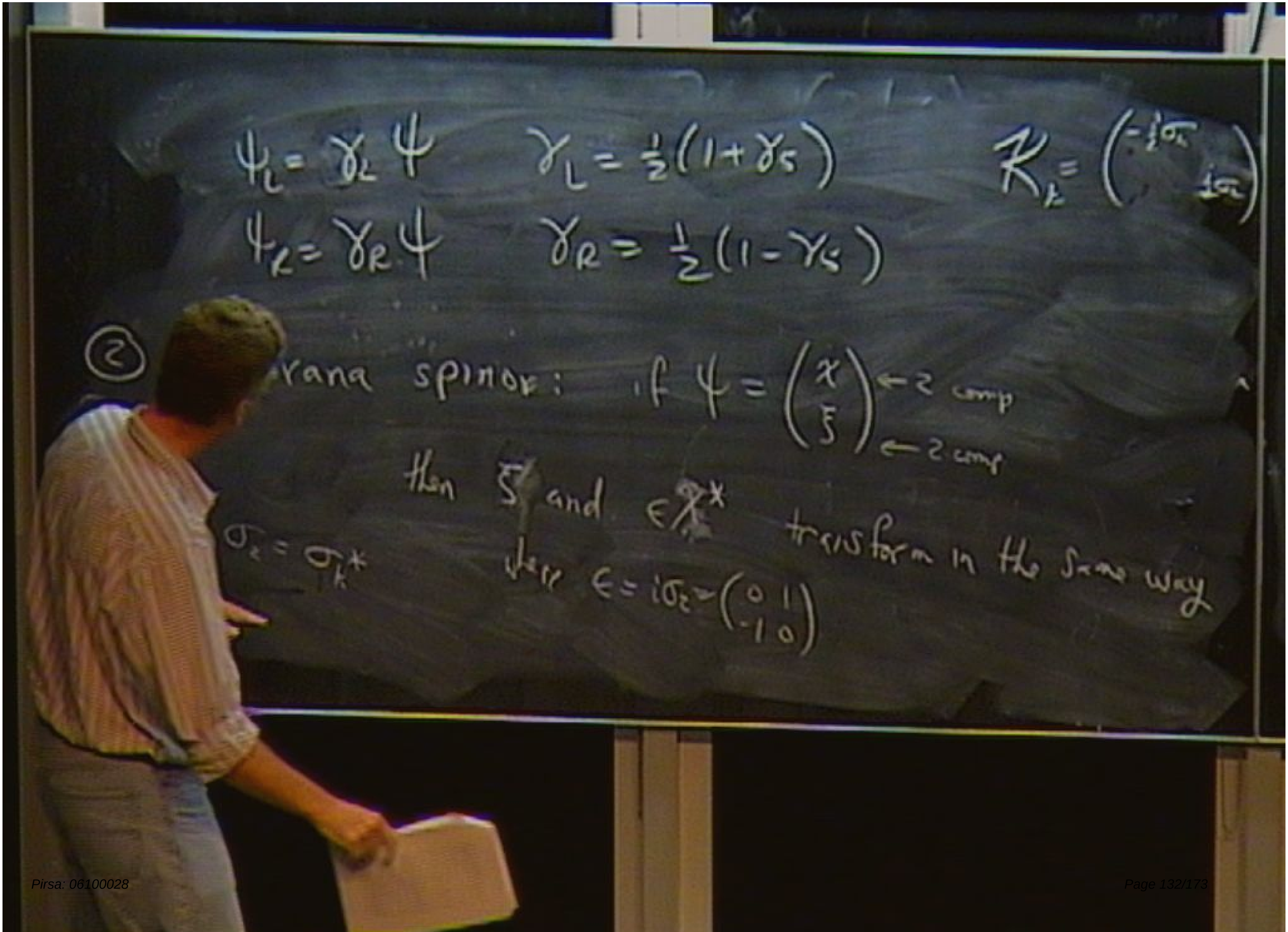
$$-\sigma_2 \sigma_k \sigma_2 = \sigma_k^*$$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2
rana spinor: if $\psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$ ← 2 comp ← 2 comp
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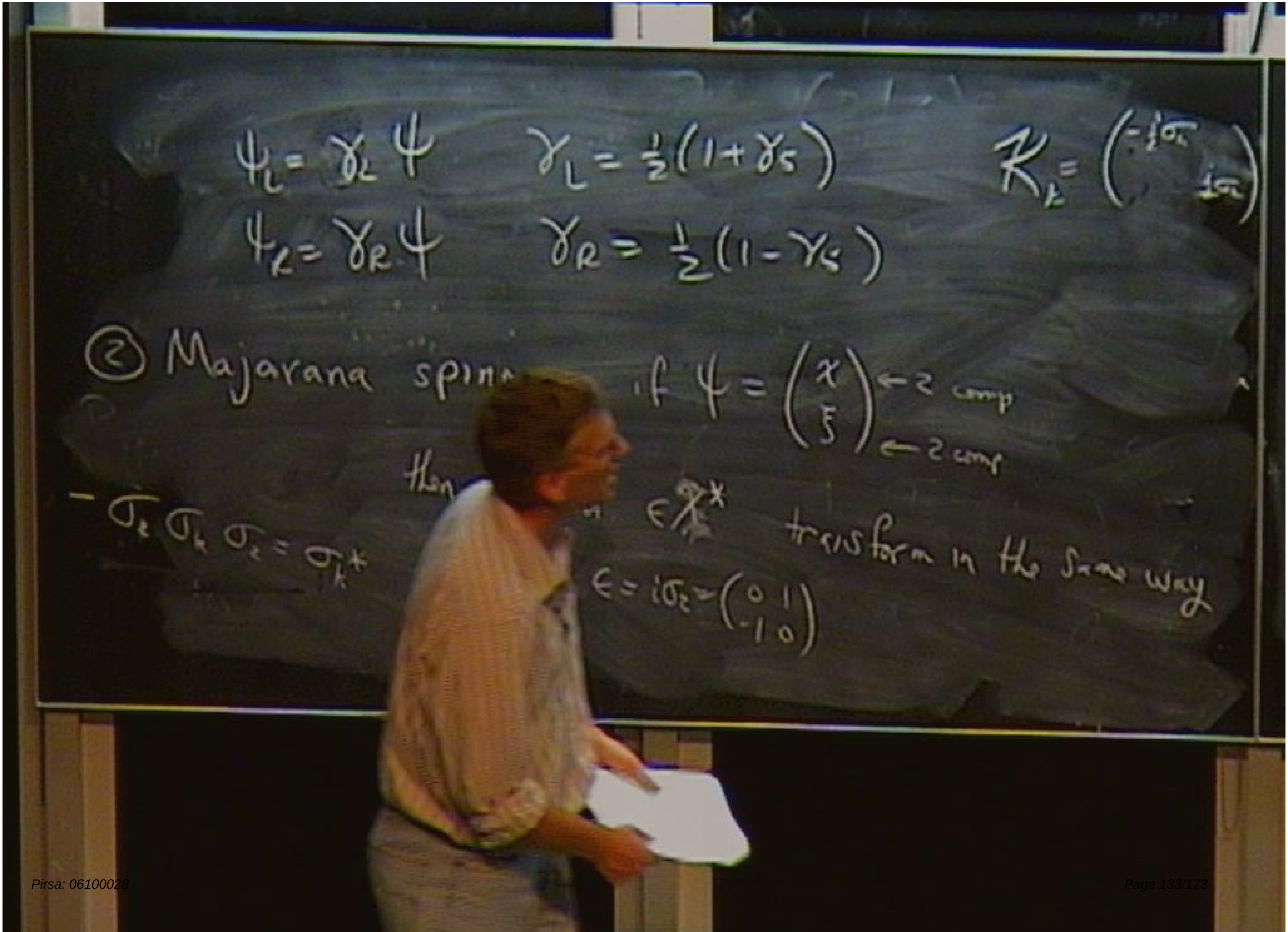
ψ_L = γ_L ψ   γ_L = ½(1 + γ_5)
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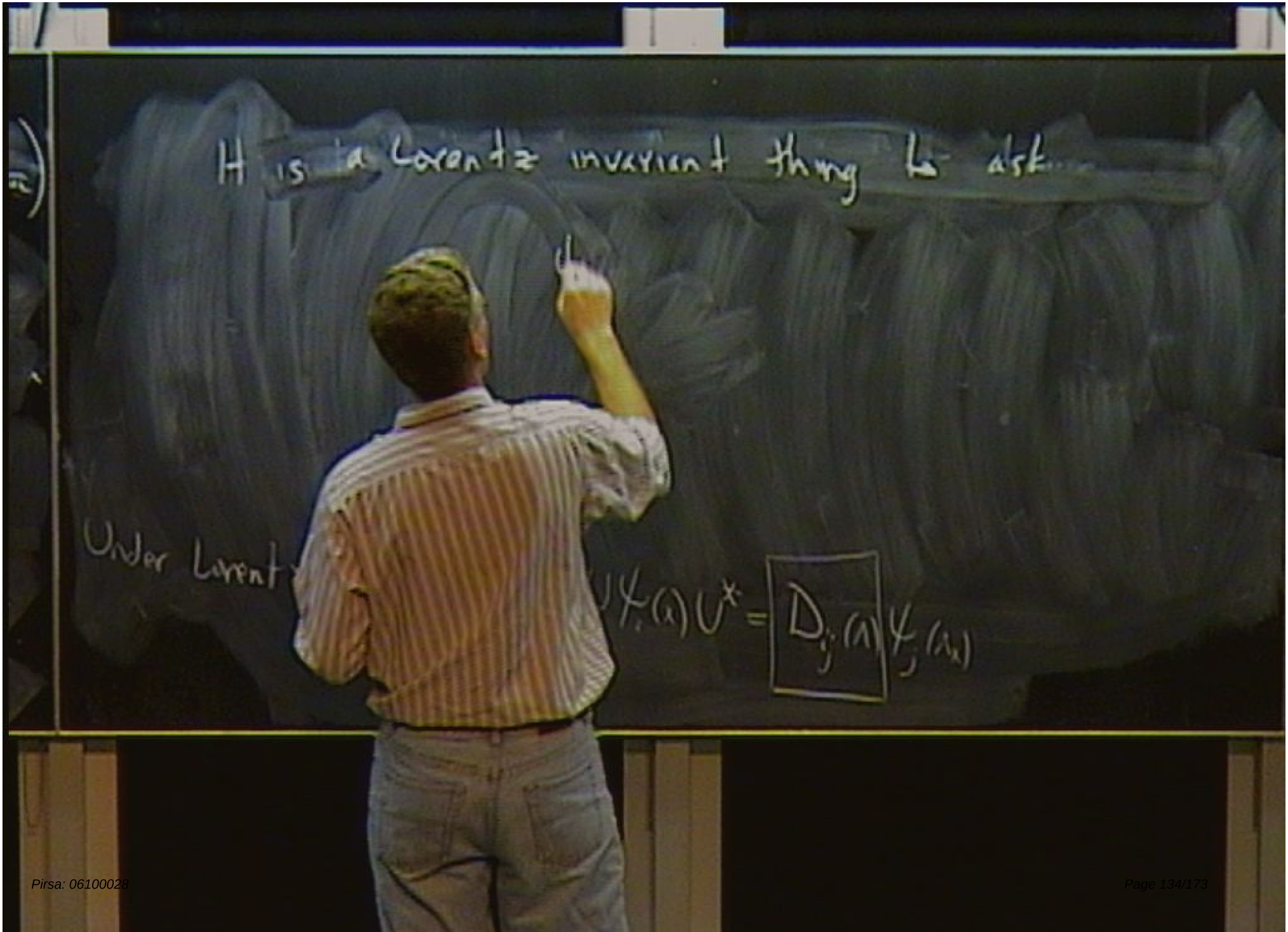
② Majarana spin
if ψ = (χ ξ) ← 2 comp ← 2 comp
then
− σ_2 σ_k σ_2 = σ_k*
ε χ* transform in the same way
ε = iσ_2 = (0 1 −1 0)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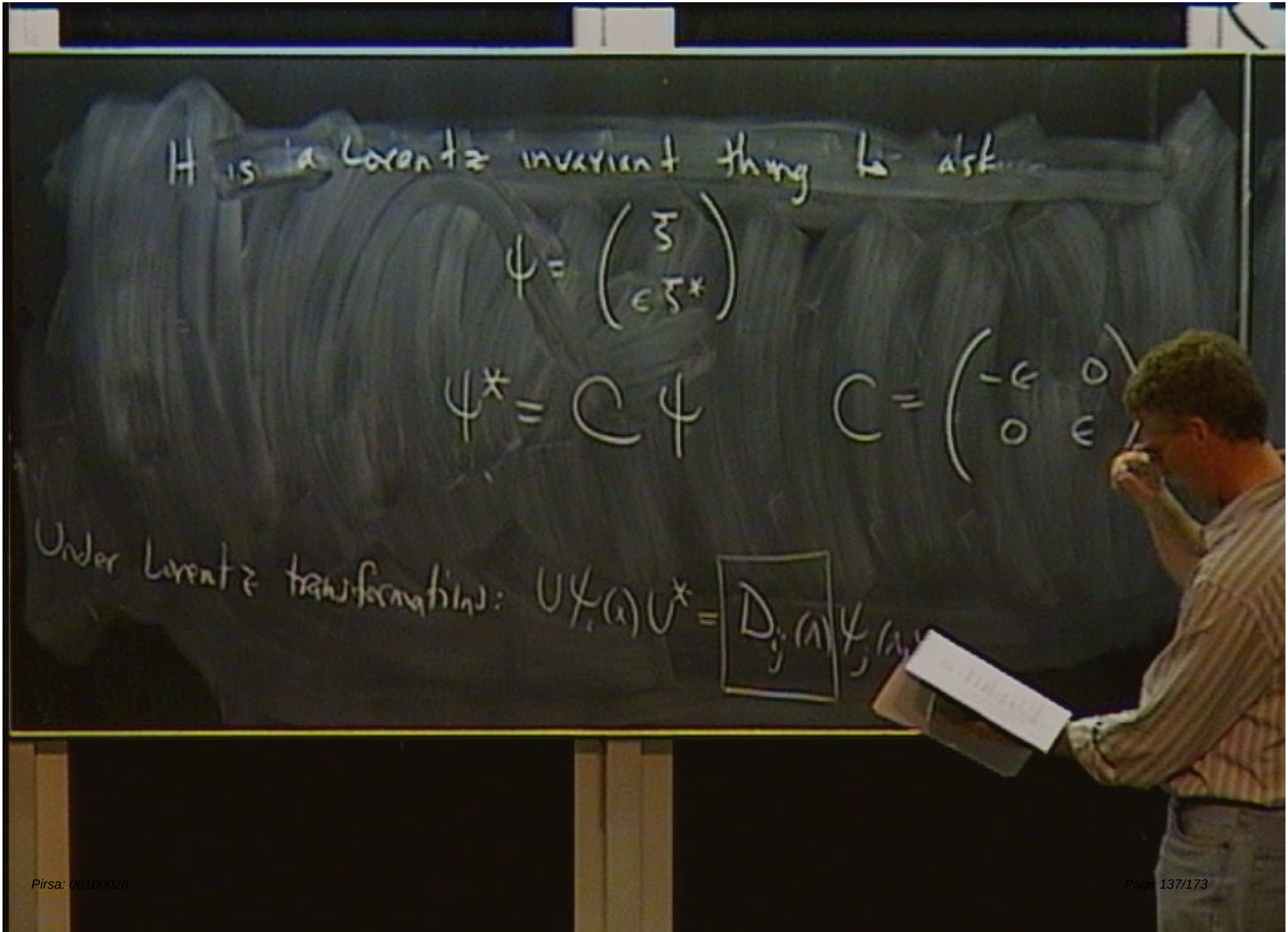
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$\psi = \begin{pmatrix} \xi \\ \epsilon \xi^* \end{pmatrix}$
$\psi^* = C \psi$
$C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}$
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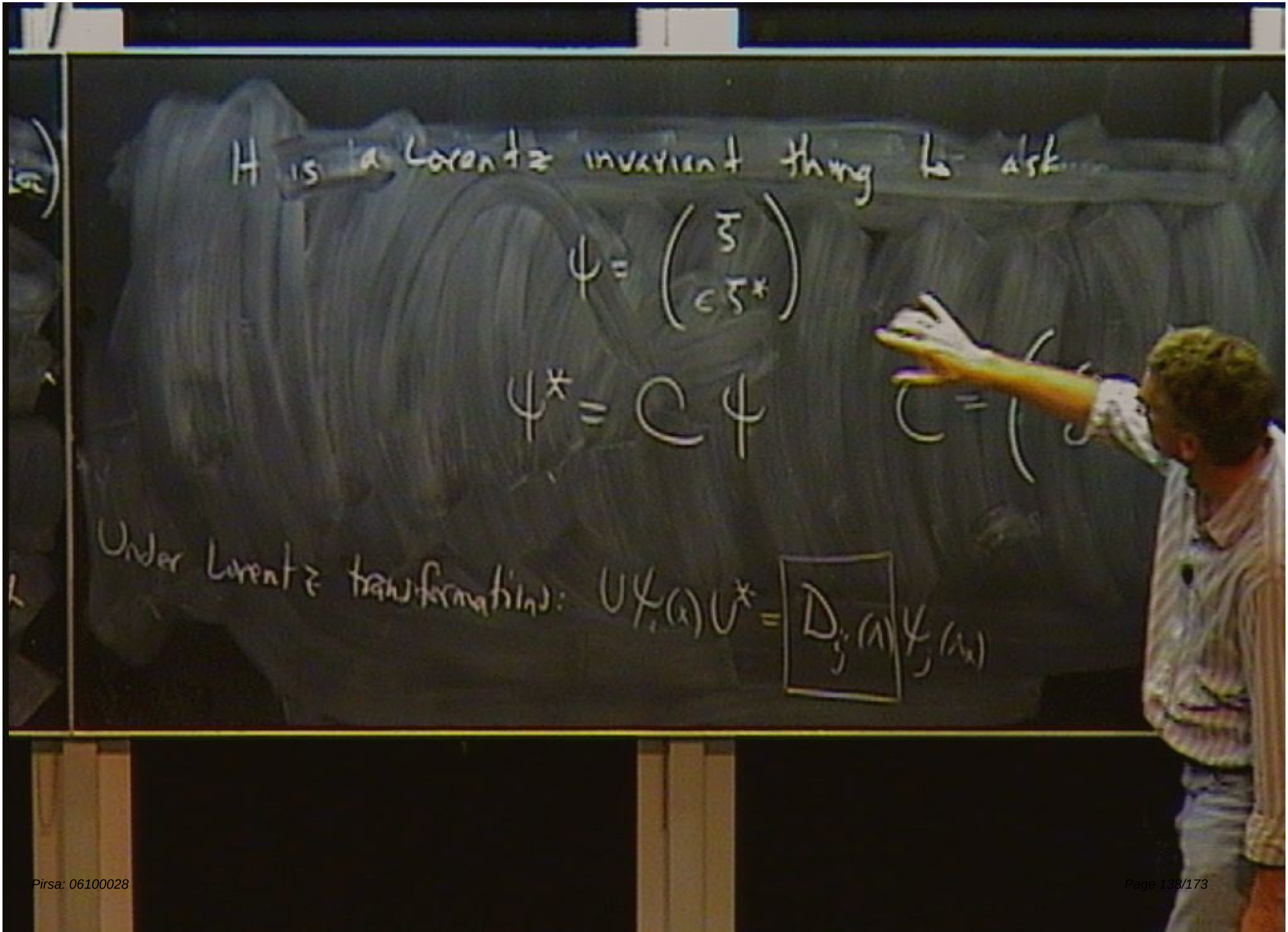
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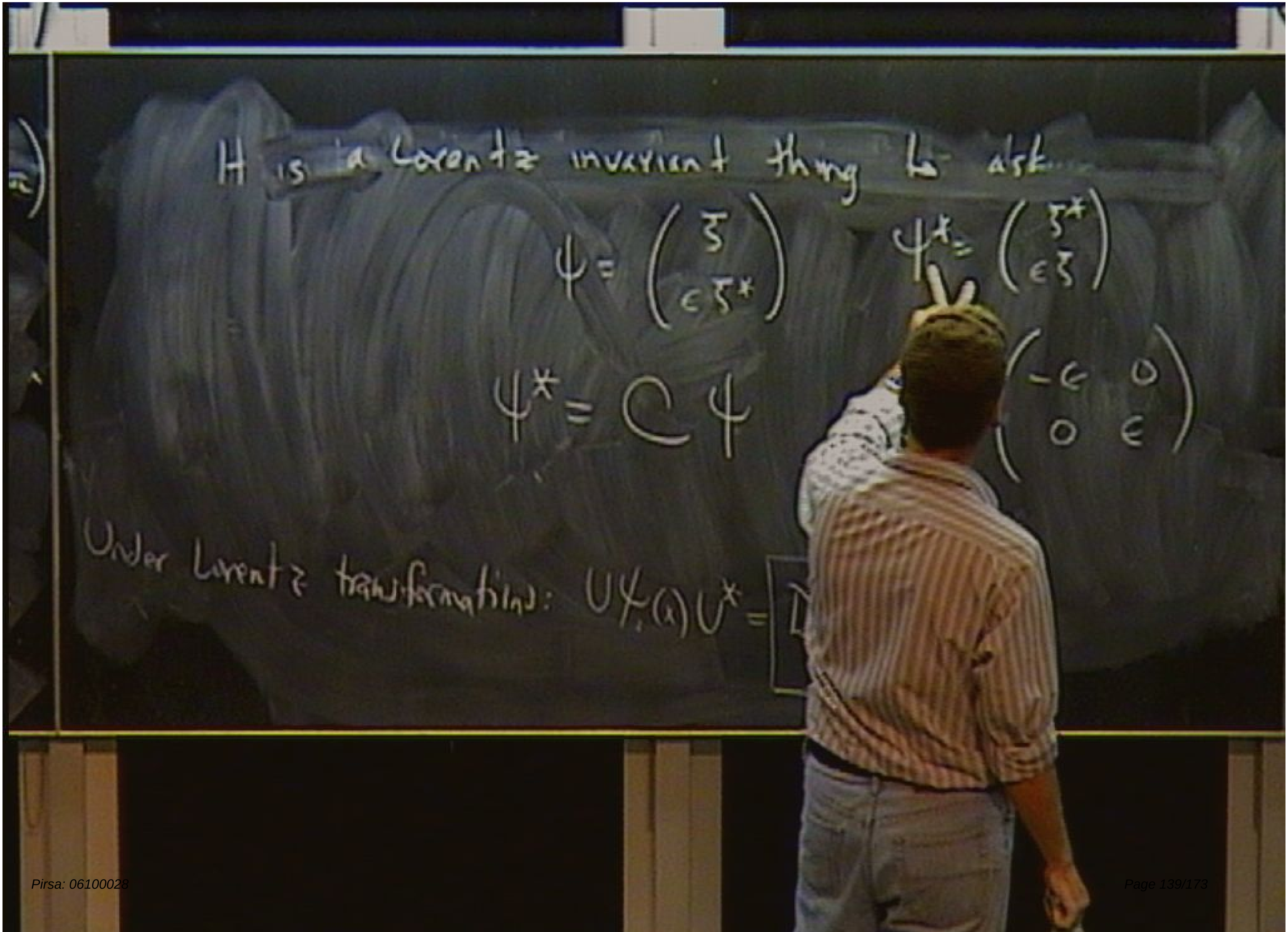
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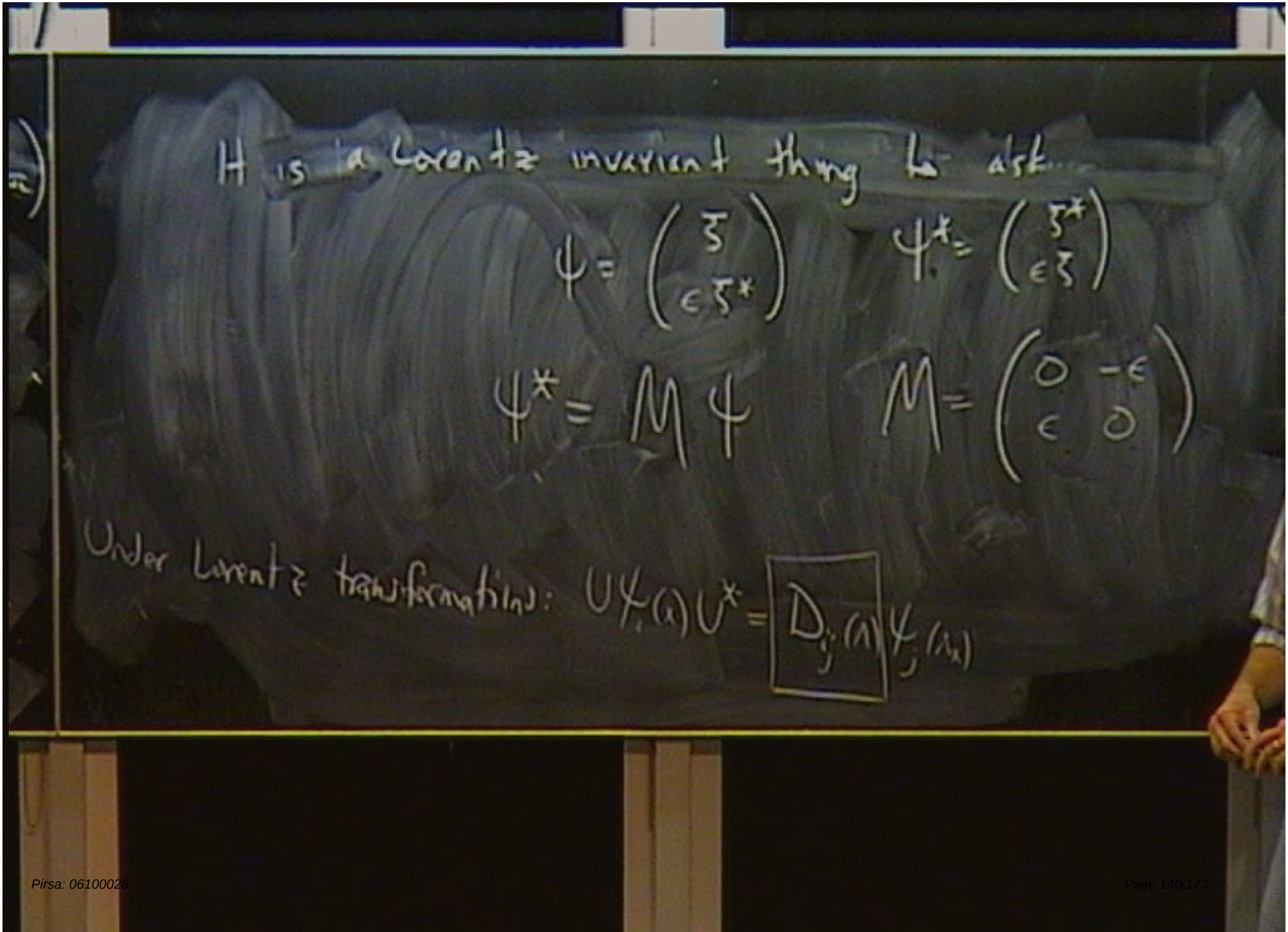
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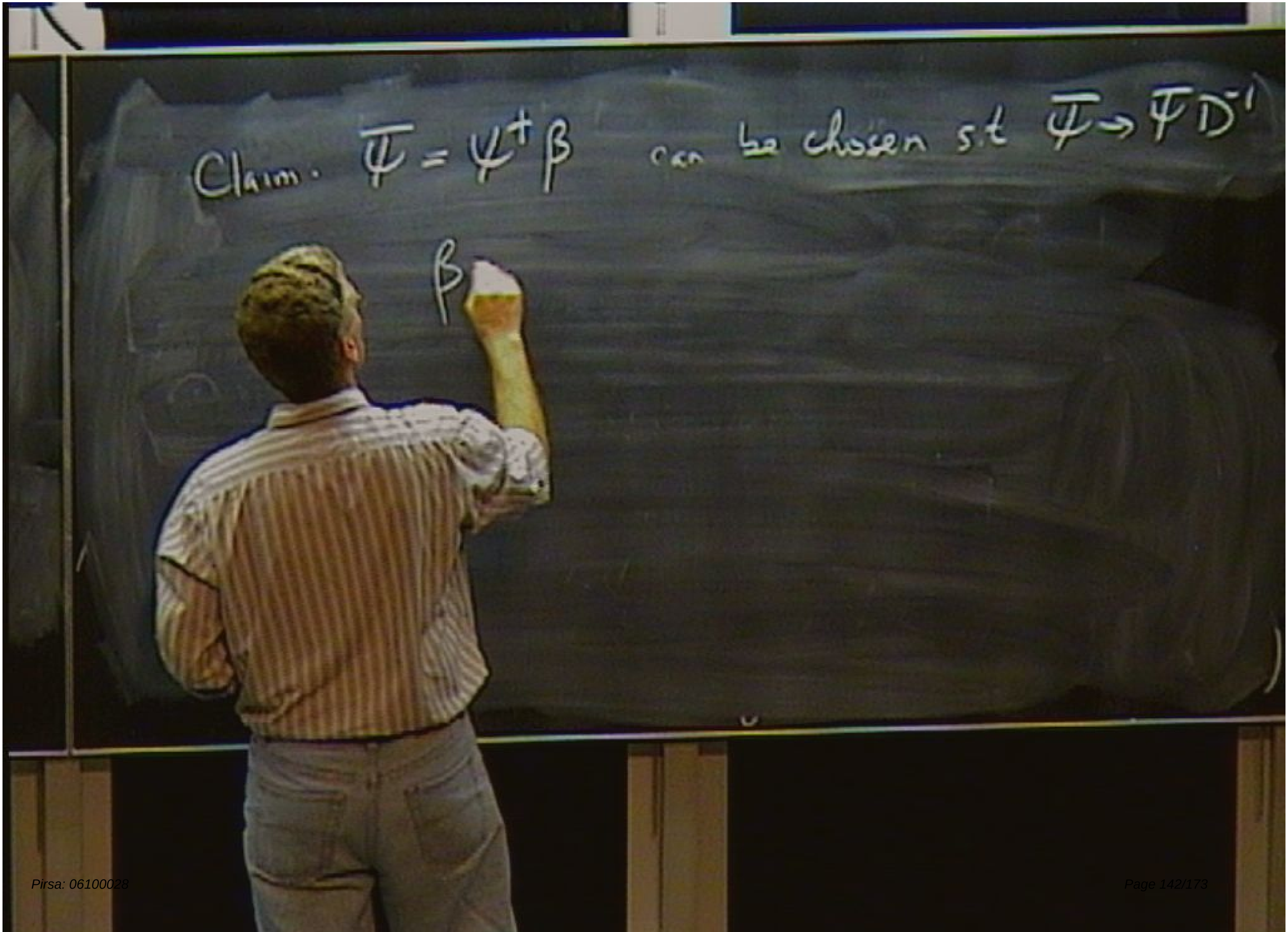
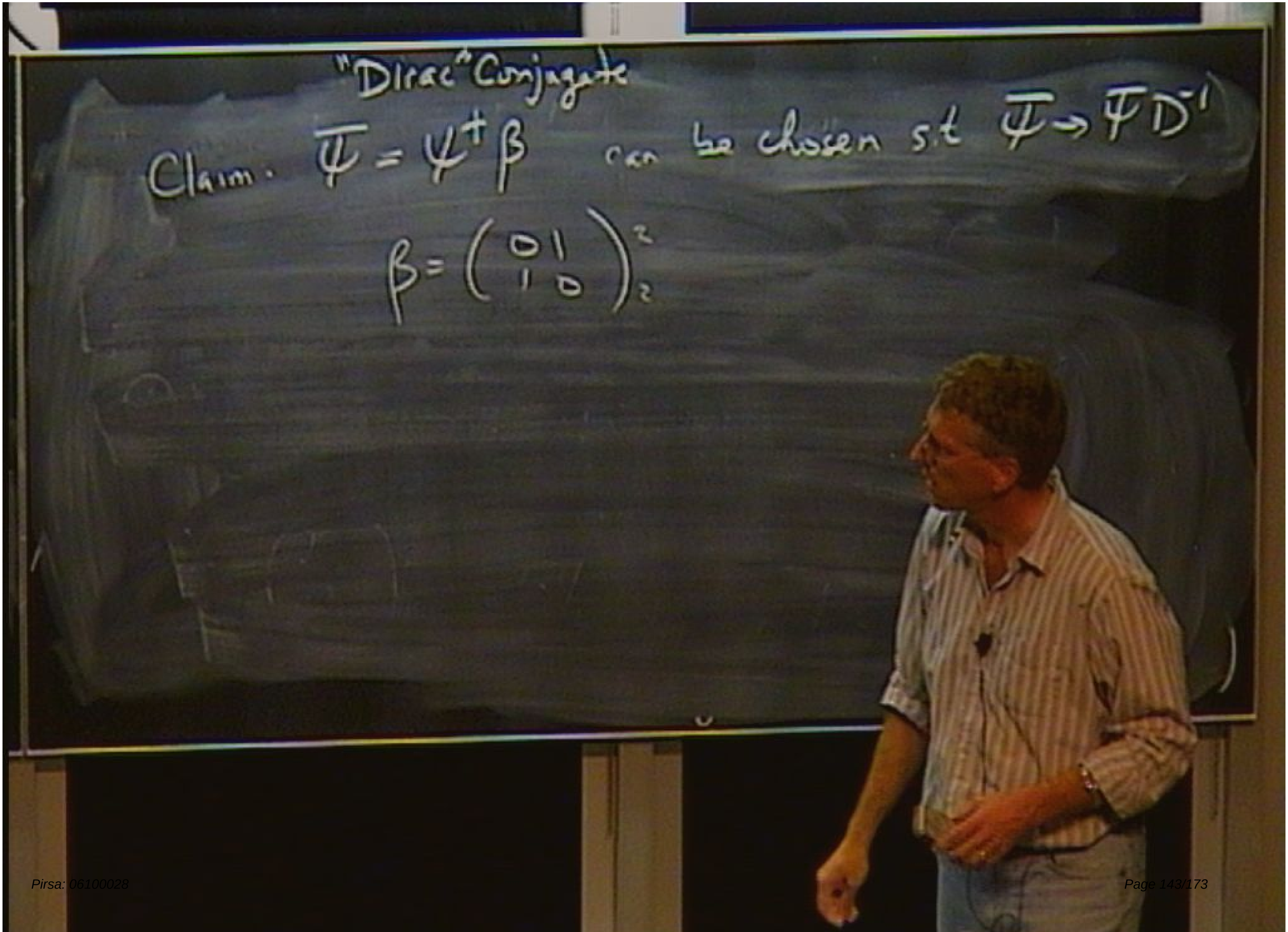
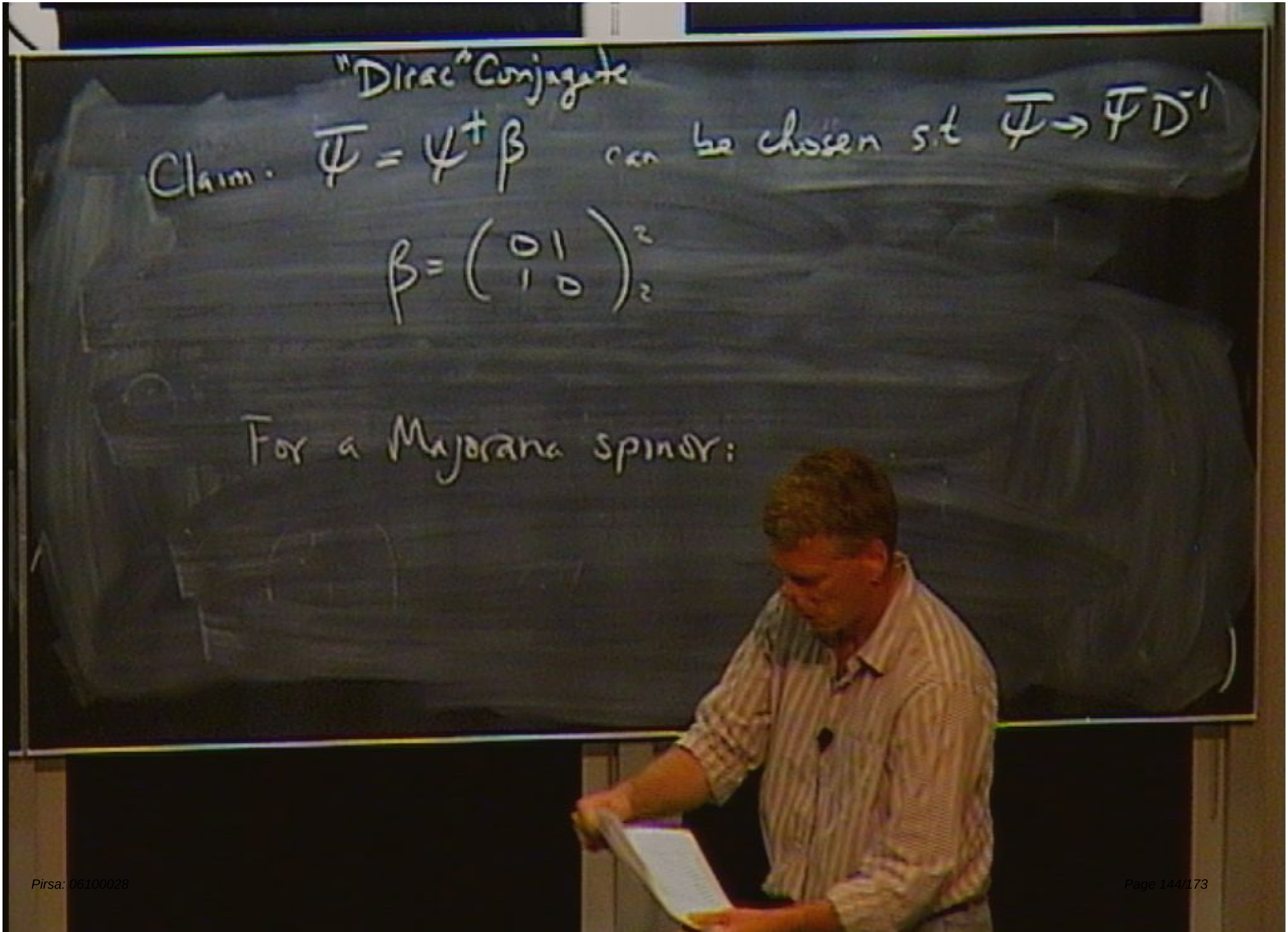
$$\psi \rightarrow D\psi$$

$$\psi^\dagger \rightarrow \psi^\dagger D^\dagger \qquad D^\dagger D \neq 1$$


Claim. $\bar{\Psi} = \psi^{\dagger}\beta$ can be chosen s.t. $\bar{\Psi} \to \bar{\Psi} D^{-1}$
β


"Dirac" Conjugate
Claim. $\bar{\Psi} = \psi^\dagger \beta$ can be chosen s.t $\bar{\Psi} \to \bar{\Psi} D^{-1}$
$\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 2 2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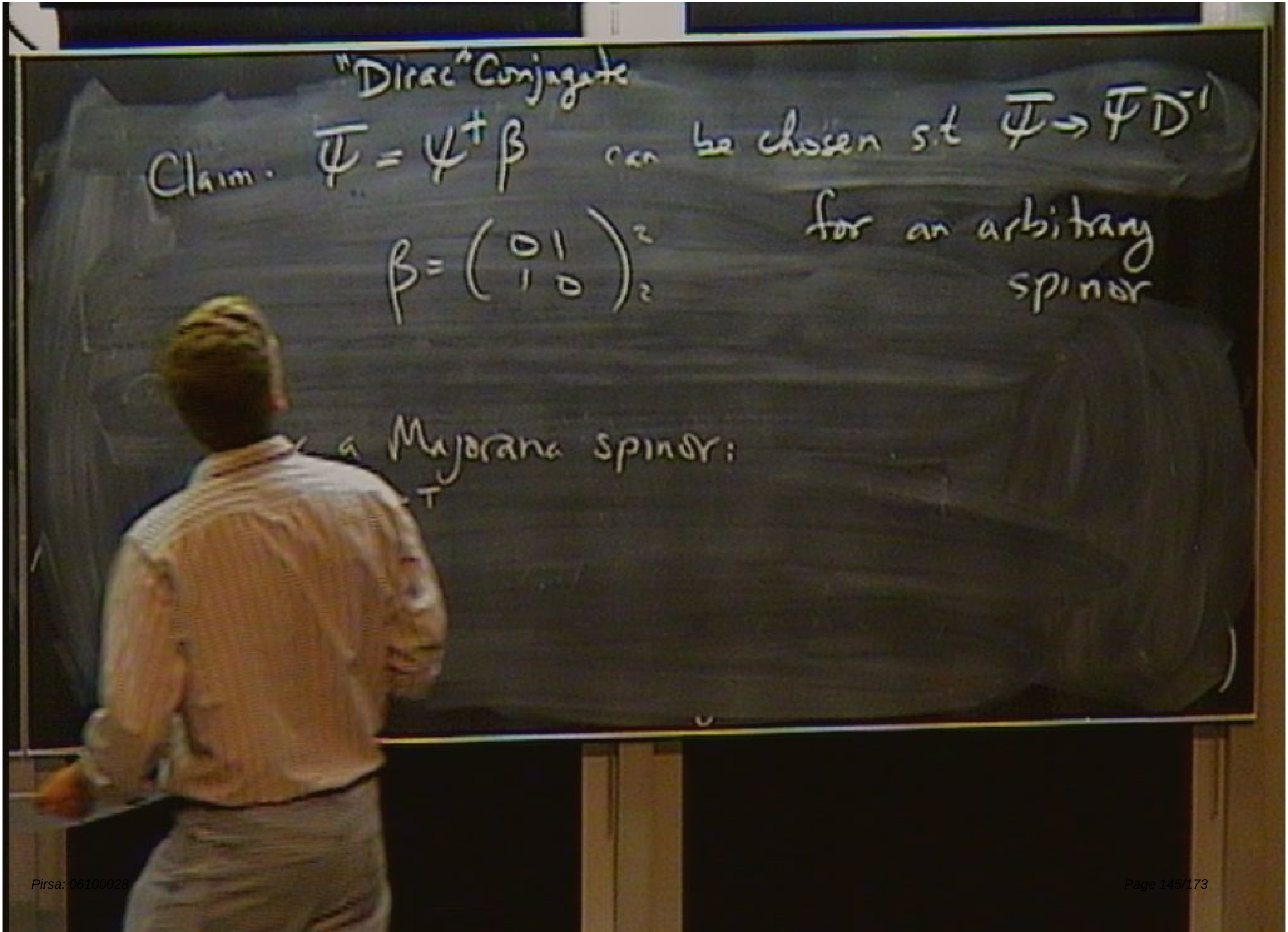
For a Majorana spinor: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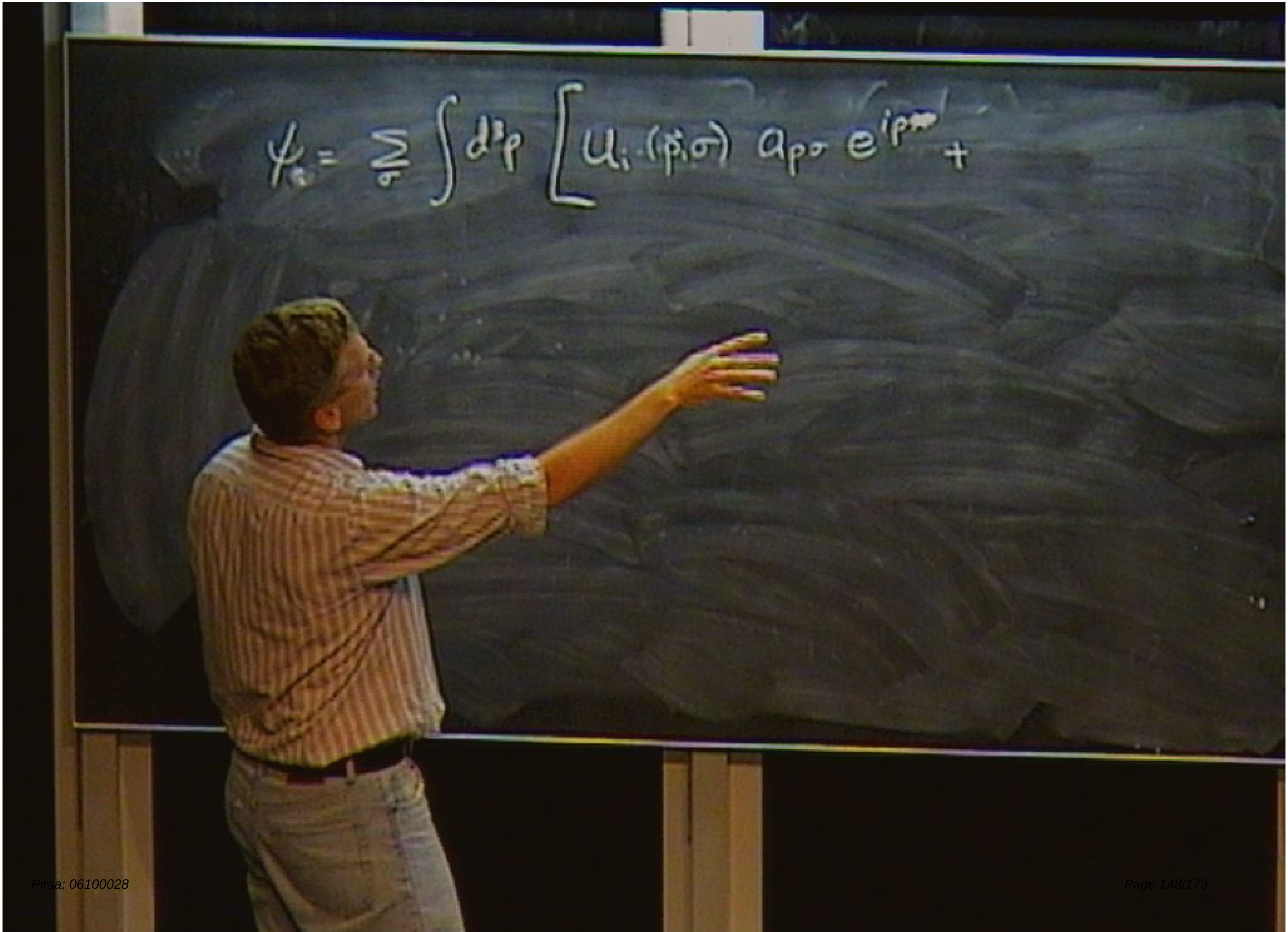
For a Majorana spinor:

$$\psi = C\overline{\Psi}^{T} \qquad C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}$$

"C

$$\psi_{i} = \sum_{\sigma} \int d^3p \Big[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ip}$$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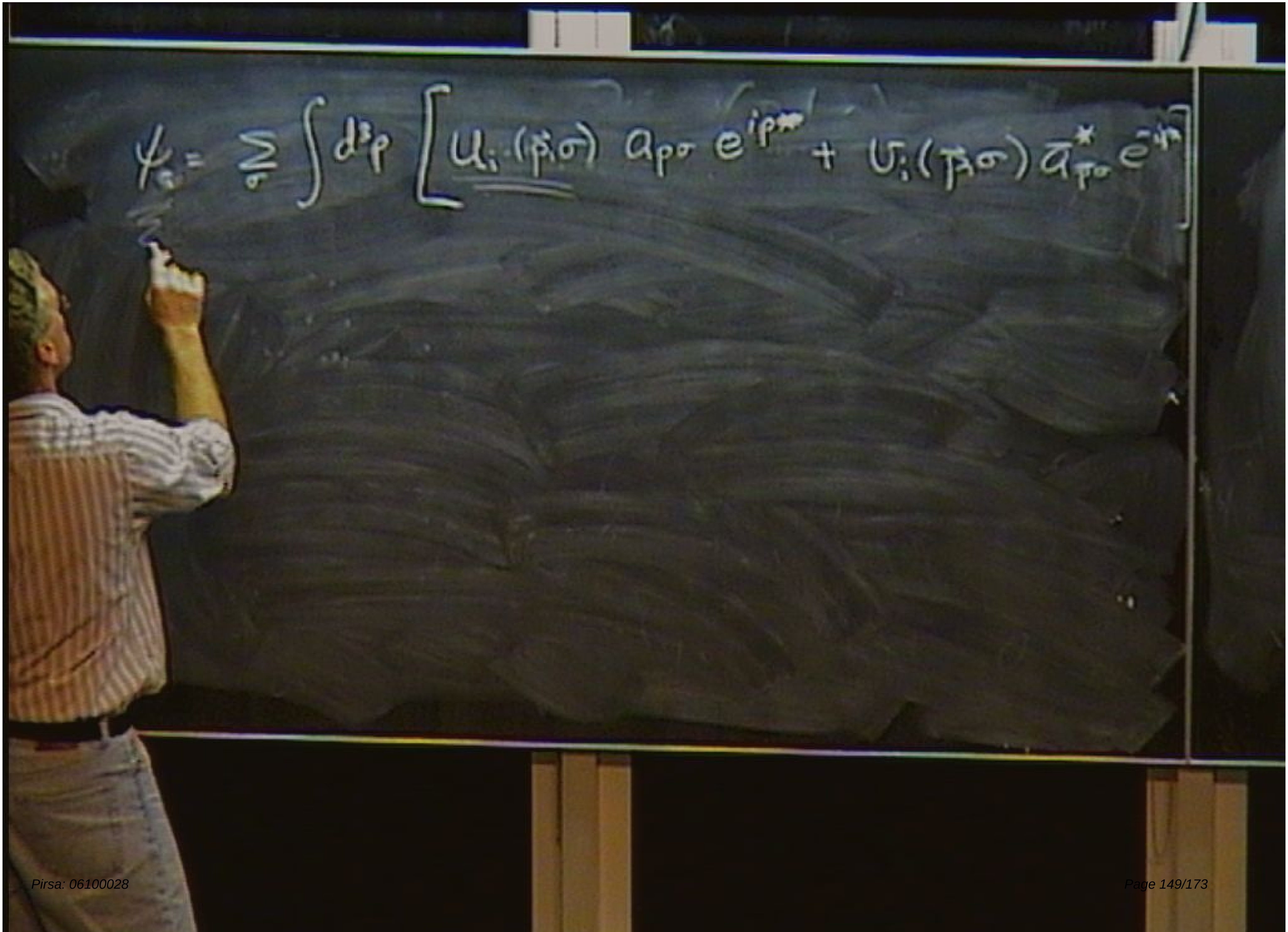
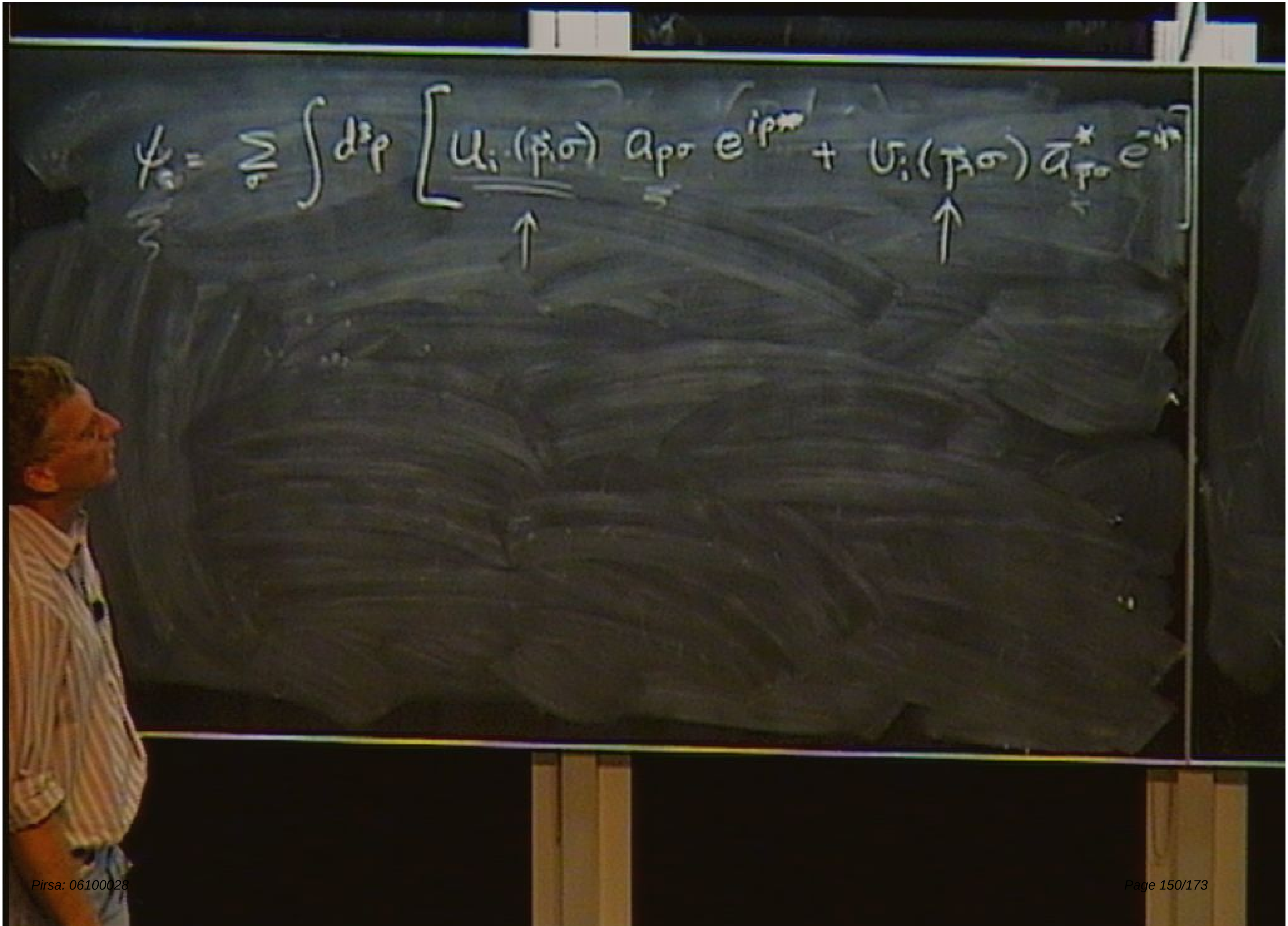

$\psi_i = \sum_\sigma \int d^3p \left[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ip\cdot x} + v_i(\vec{p},\sigma)\, a^*_{p\sigma}\, e^{-ip\cdot x} \right]$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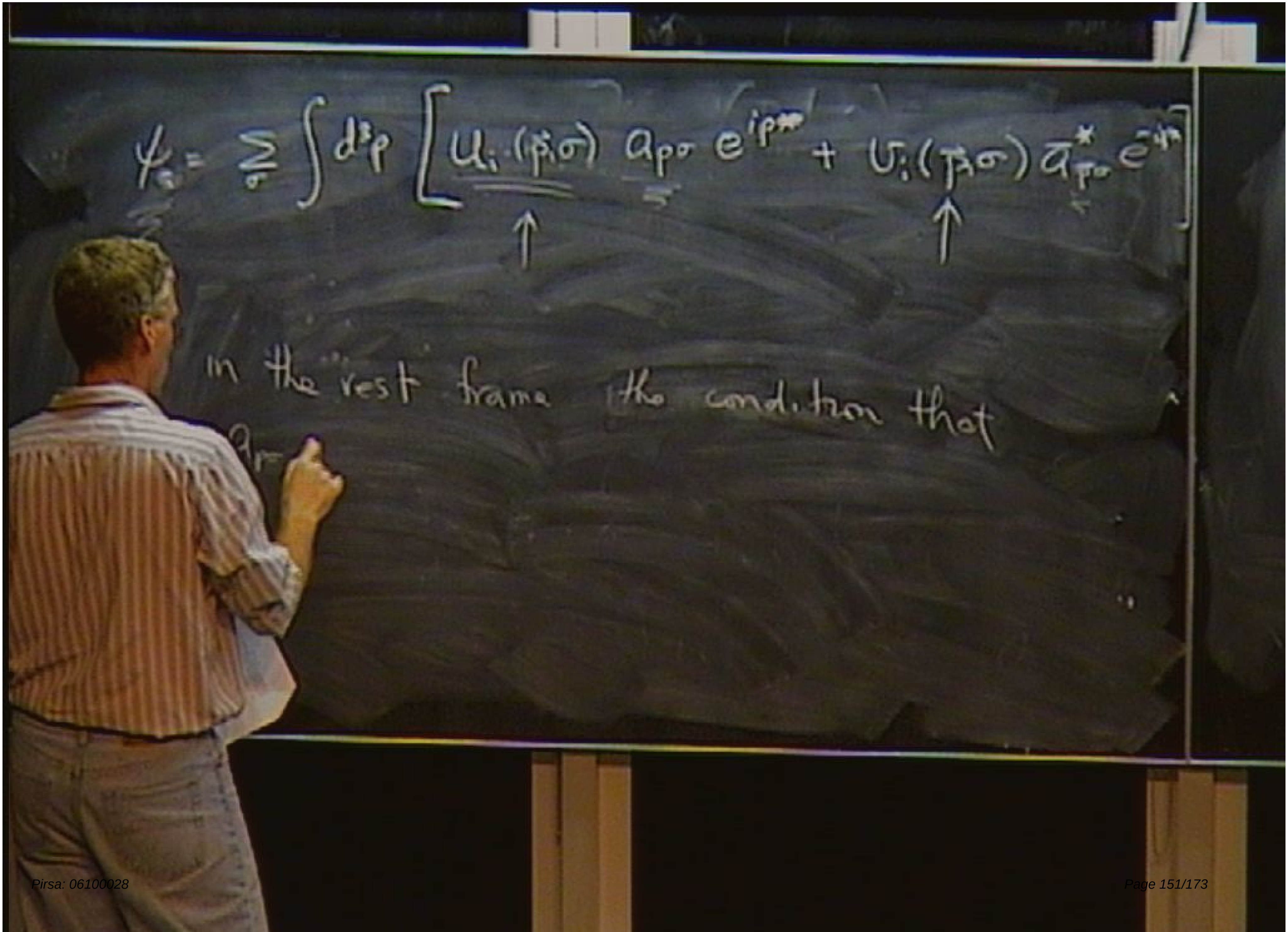
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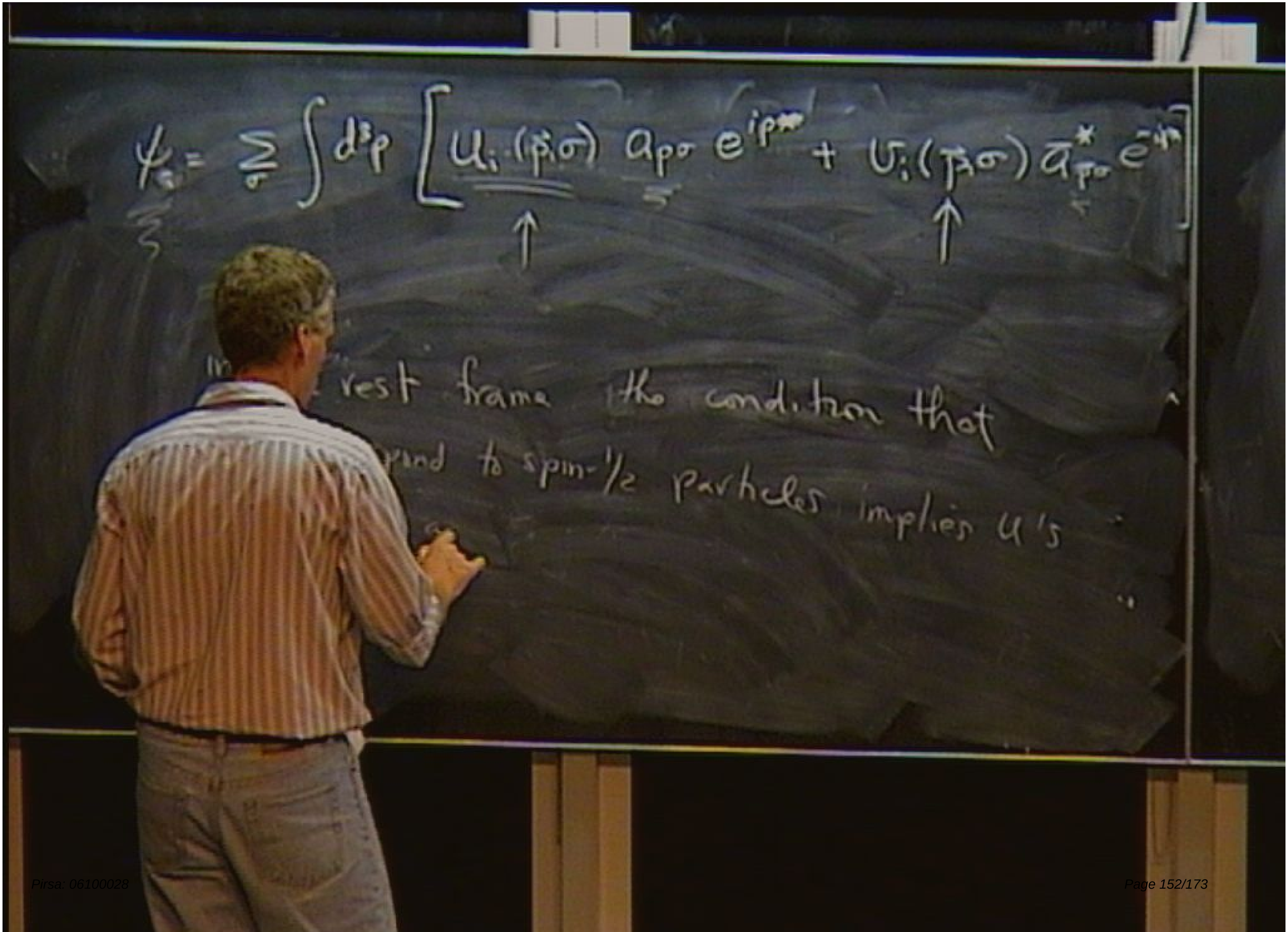
in the "rest" frame the condition that
$a_{p\sigma}$


$\psi_i = \sum_\sigma \int d^3p \, [u_i(\vec{p},\sigma) \, a_{p\sigma} \, e^{ip\cdot x} + v_i(\vec{p},\sigma) \, a^*_{p\sigma} \, e^{-ip\cdot x}]$
rest frame the condition that
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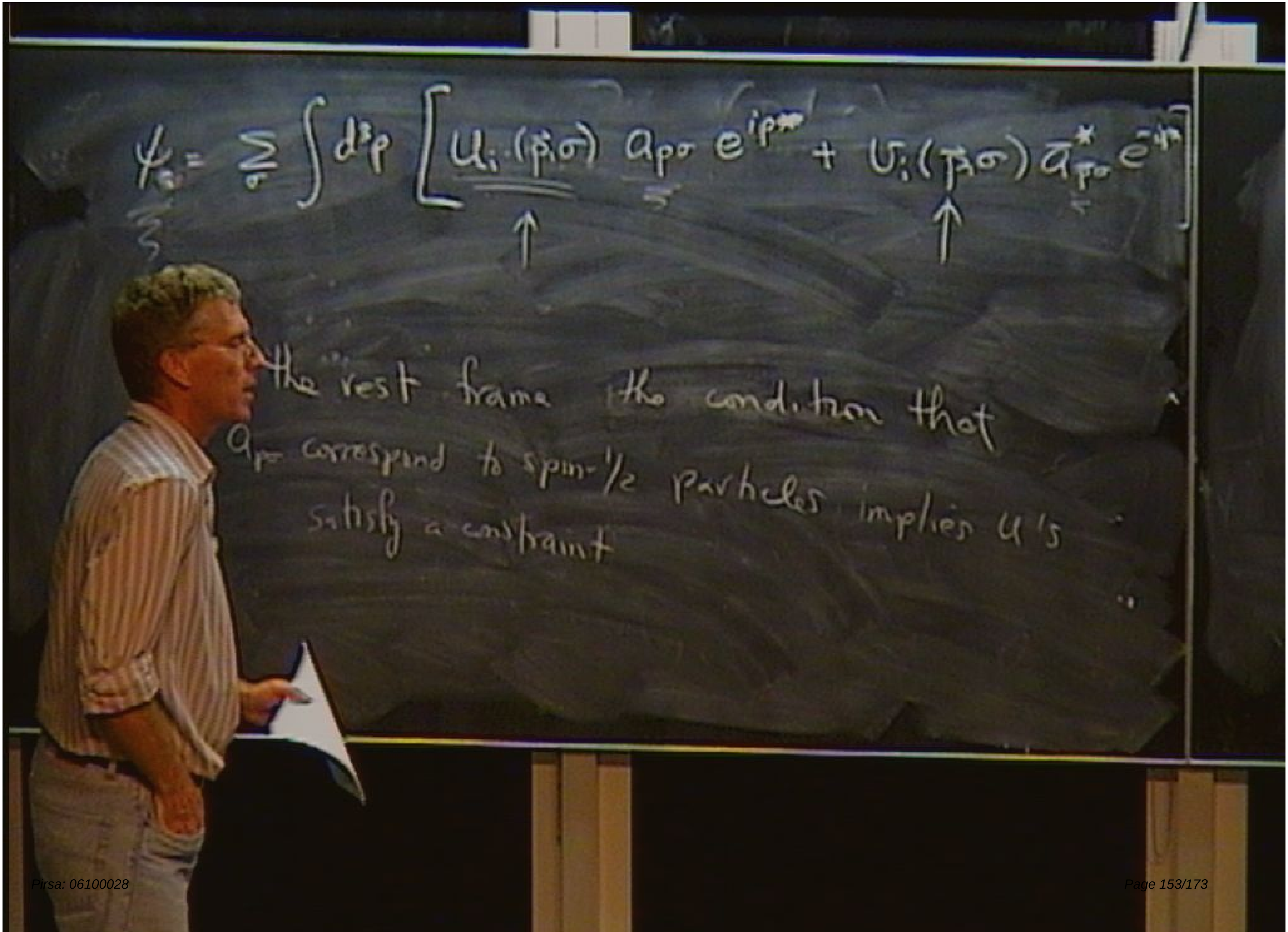
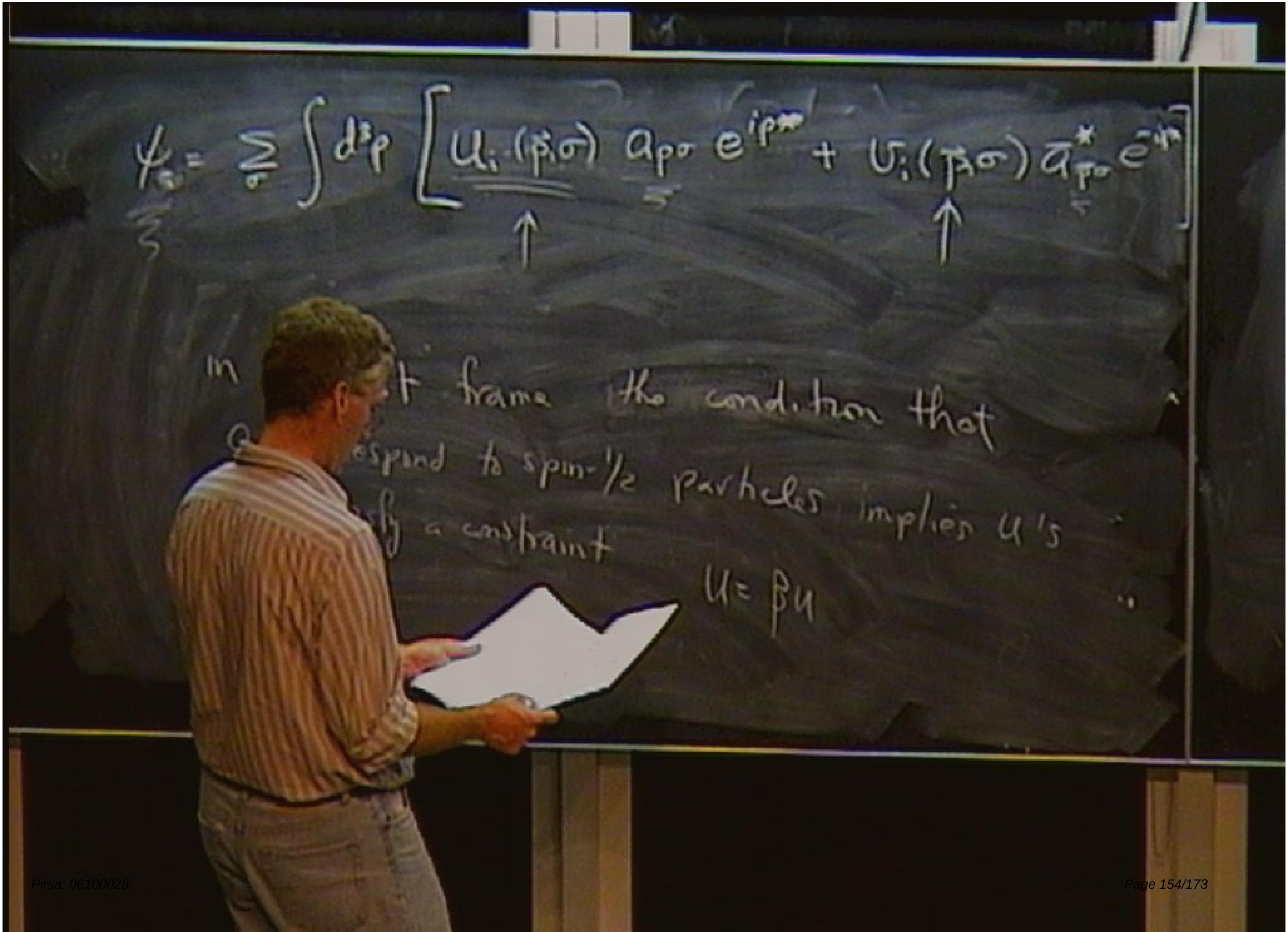

$\psi_i = \sum_\sigma \int d^3p \left[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ipx} + v_i(\vec{p},\sigma)\, a^*_{p\sigma}\, e^{-ipx} \right]$
the rest frame the condition that
$a_{p\sigma}$ correspond to spin-1/2 particles implies u's
satisfy a constraint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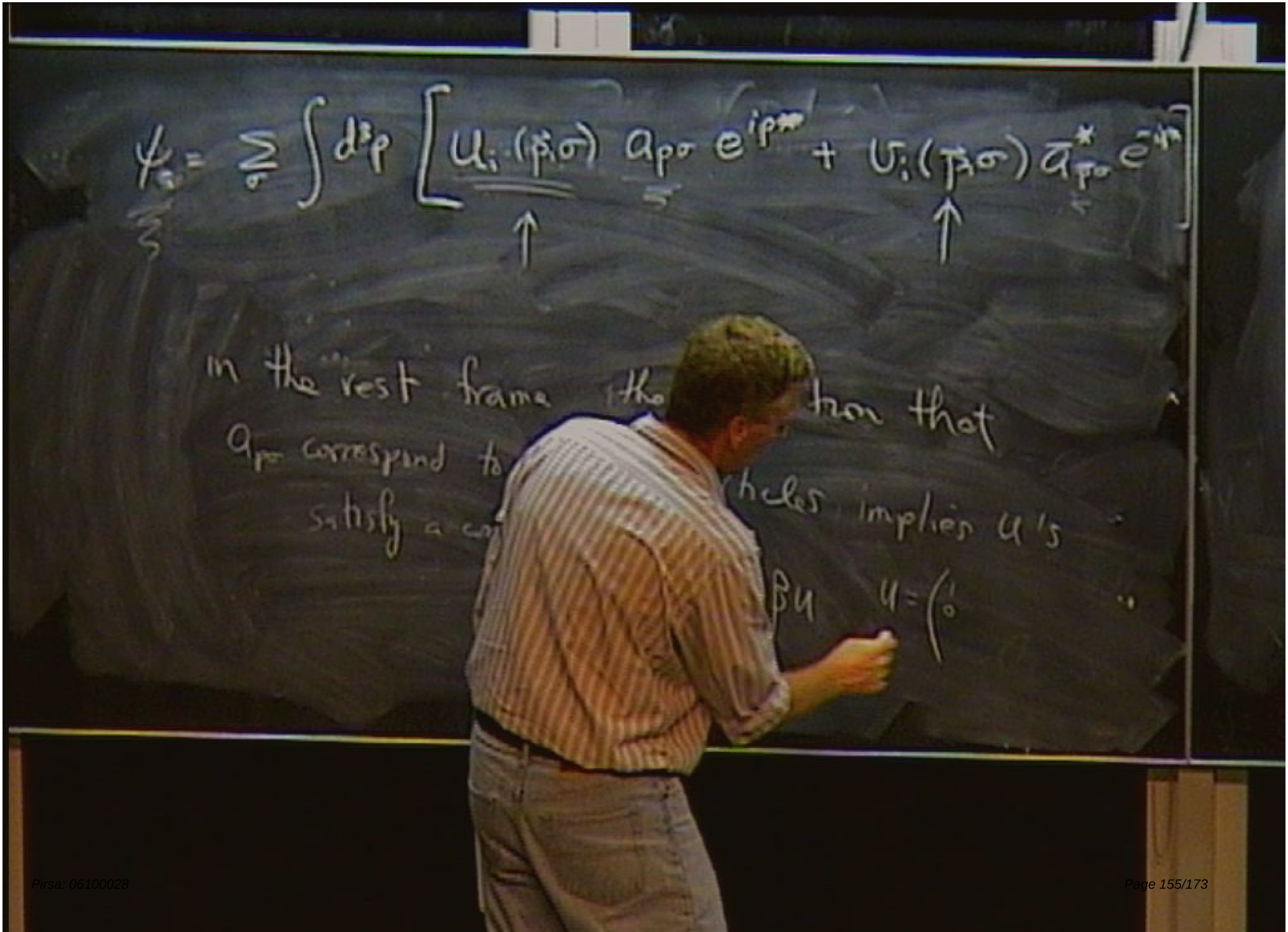
in
t frame the condition that
espond to spin-½ particles implies u's
a constraint
$u = \beta u$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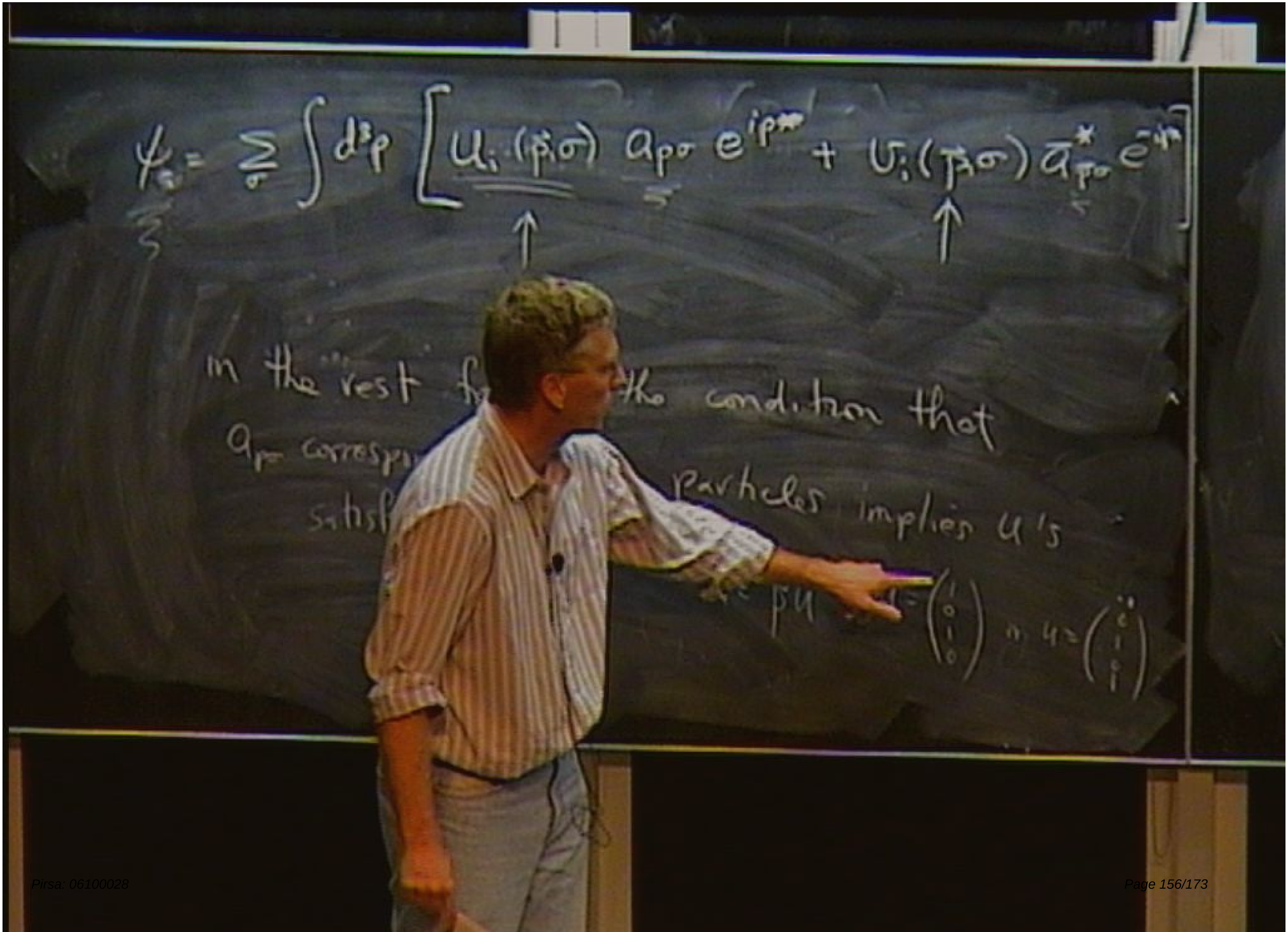
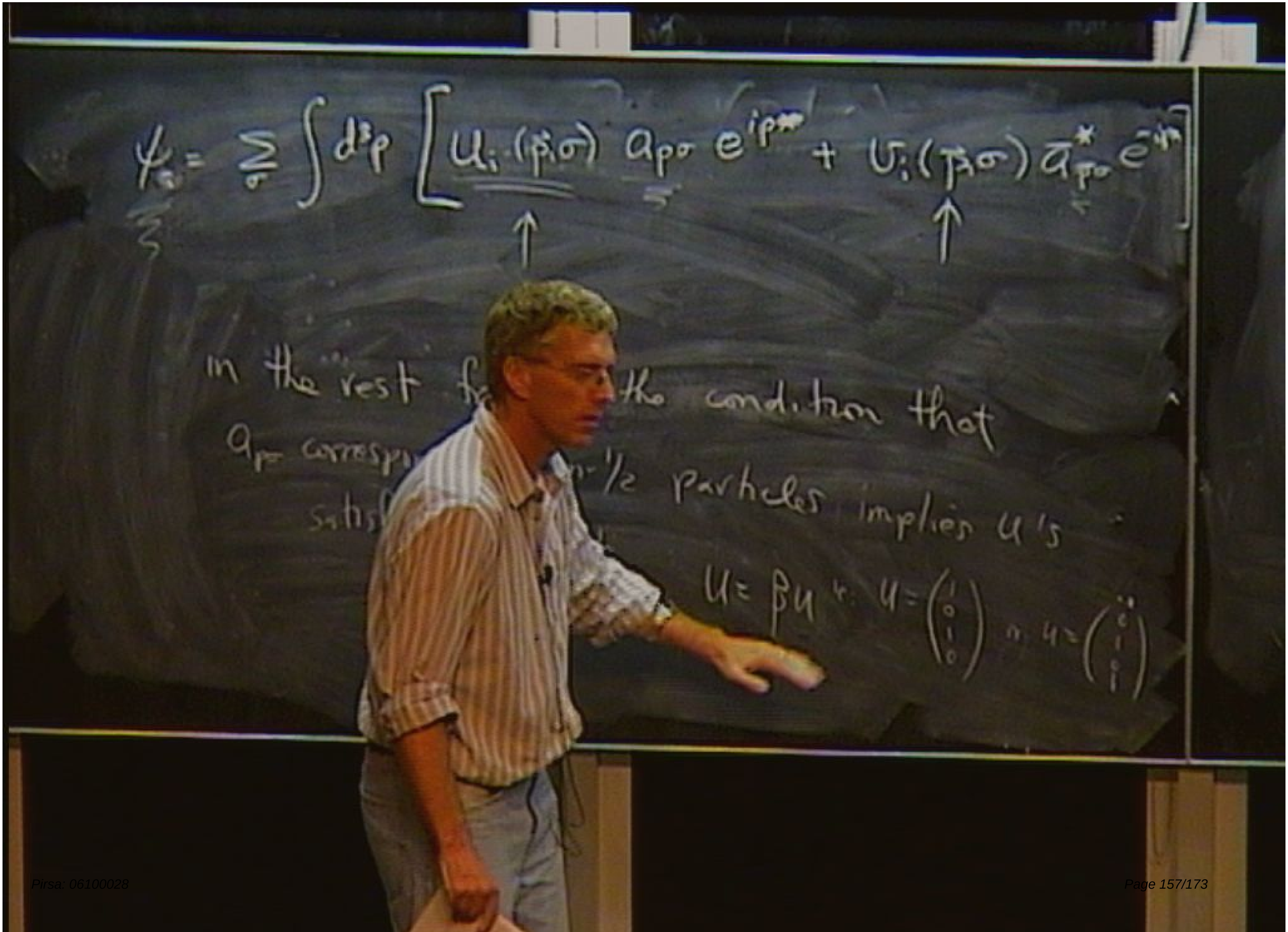
implies u's
satisfy a
βu
$u = ($


ψ = Σ ∫ d³p [u(p,σ) a e^{ip} + v(p,σ) a* e^{-i}]
in the rest f the condition that
a corresp particles implies u's
satisf
pu


in the rest
the condition that
particles implies u's
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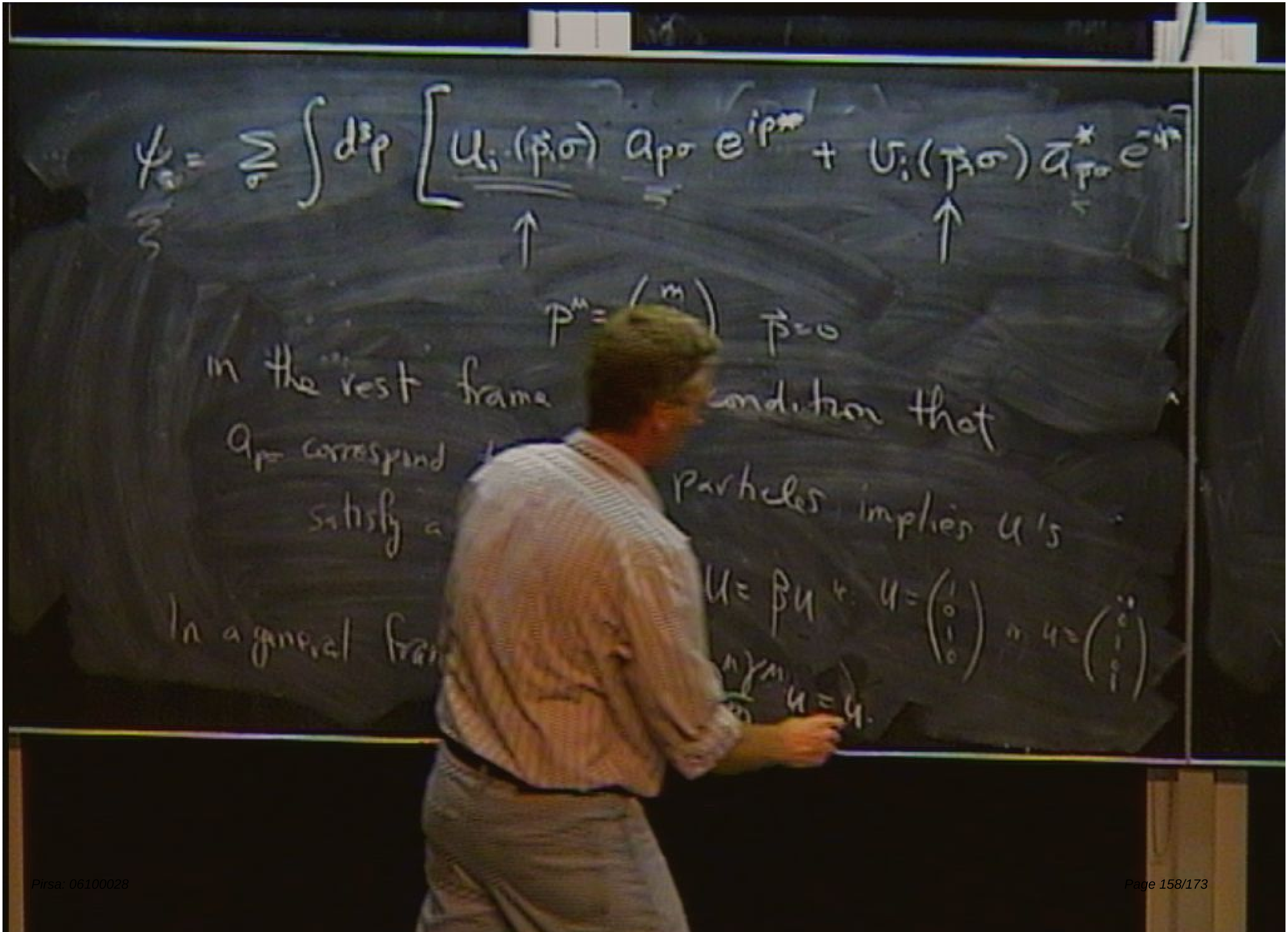

In the rest frame
condition that
correspond
particles implies u's
satisfy
In a general fra

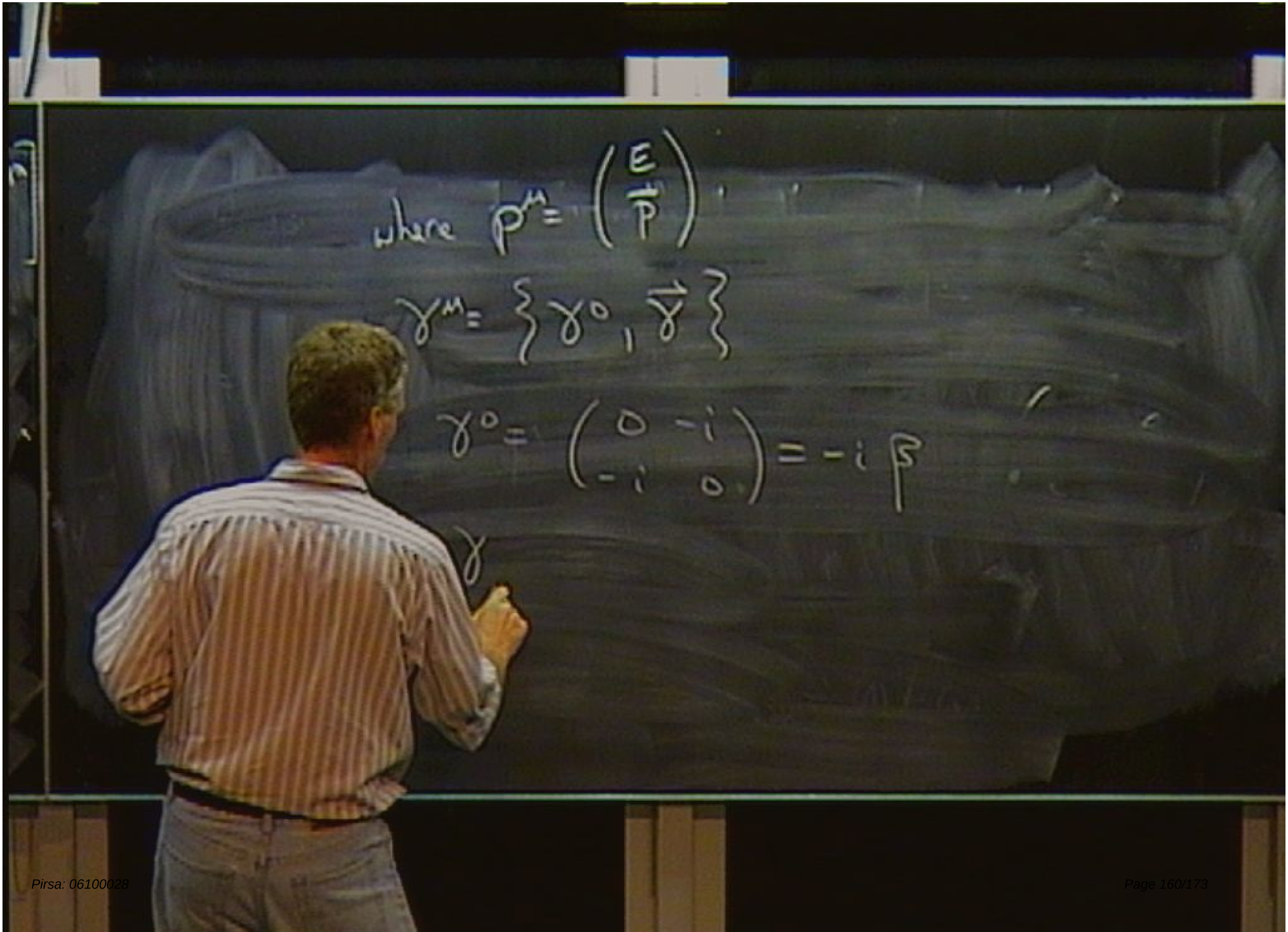
$$\psi_i = \sum_\sigma \int d^3p \left[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ipx} + v_i(\vec{p},\sigma)\, a^*_{p\sigma}\, e^{-ipx} \right]$$

$$p^\mu = \begin{pmatrix} m \\ 0 \\ \vdots \end{pmatrix} \quad \vec{p} = 0$$

In the rest frame the condition that $a_{p\sigma}$ correspond to spin-1/2 particles implies u's satisfy a constraint

$$u = \beta u \quad \text{e.g. } u = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ or } u = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

In a general frame: $u \to Du$, $\frac{p_\mu \gamma^\mu}{m} u = u$.



where $p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$
$\gamma^\mu = \{\gamma^0, \vec{\gamma}\}$
$\gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = -i\beta$
γ

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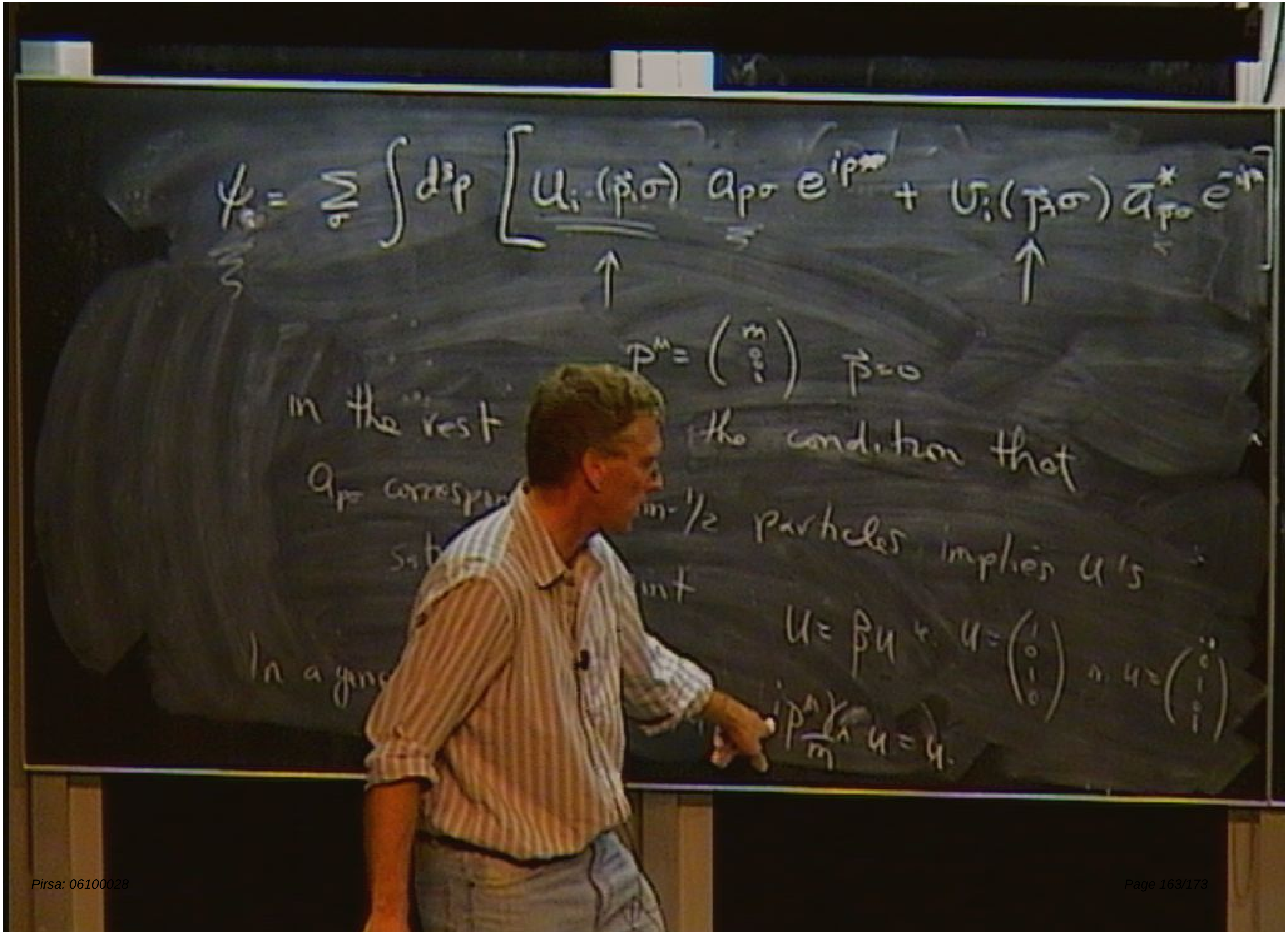
$$D\gamma^\mu D^{-1} = \Lambda^\mu{}_\nu \gamma^\nu$$


$\psi_i = \sum_\sigma \int d^3p \, [u_i(\vec{p},\sigma) \, a_{p\sigma} \, e^{ipx} + v_i(\vec{p},\sigma) \, a^*_{p\sigma} \, e^{-ipx}]$
$p^\mu = (m, 0, 0, 0)$ $\vec{p} = 0$
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$u = \beta u$
$u = (1, 0, 1, 0)$
$u = (0, 1, 0, 1)$
In a
$\frac{ip^\mu \gamma_\mu}{m} u = u.$

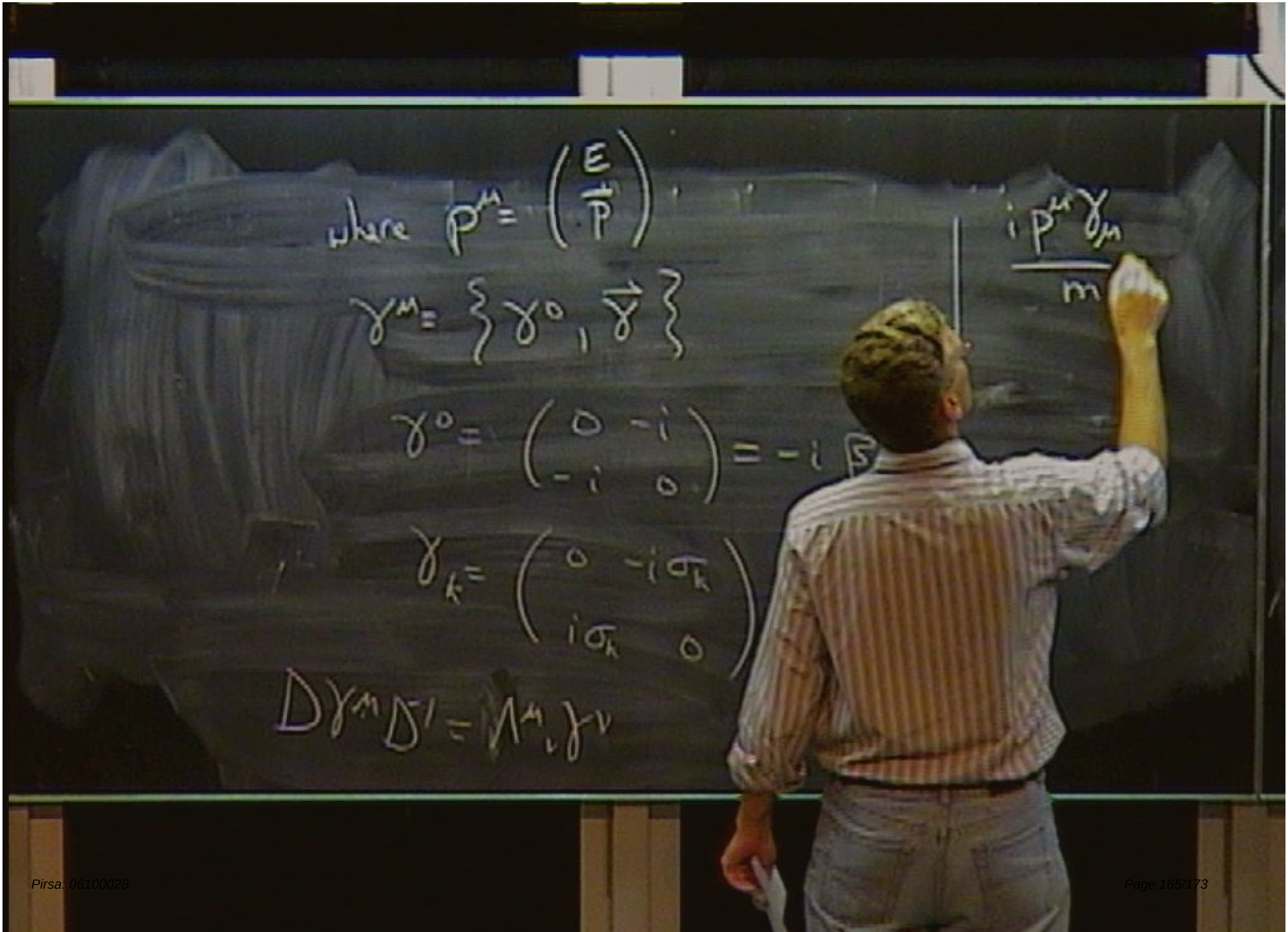
$$\psi_\alpha = \sum_\sigma \int d^3p \left[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ipx} + v_i(\vec{p},\sigma)\, a^*_{p\sigma}\, e^{-ipx} \right]$$

$$p^\mu = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \vec{p} = 0$$

In the rest frame the condition that $a_{p\sigma}$ correspond to spin-1/2 particles implies u's satisfy a constraint

$$u = \beta u \quad : \quad u = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ or } u = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

In a general frame: $u \to Du$: $\frac{i p_\mu \gamma^\mu}{m} u = u$.


where $p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$
$\gamma^\mu = \{\gamma^0, \vec{\gamma}\}$
$\gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = -i$
$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}$
$D\gamma^\mu D^{-1} = \Lambda^\mu{}_\nu \gamma^\nu$
$\frac{i p^\mu \gamma_\mu}{m}$

where $p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$

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$$D\gamma^\mu D^{-1} = \Lambda^\mu{}_\nu \gamma^\nu$$

$$\frac{i p^\mu \gamma_\mu}{m} u = u$$

$$(i\not{p} - m)u = 0$$

Dirac equation.

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Dirac equation.

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\psi_a = \sum_\sigma \int d^3p \left[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ipx} + v_i(\vec{p},\sigma)\, a^*_{p\sigma}\, e^{-ipx} \right.$$

$$(\not{\partial} + m)\psi = 0$$

$$\not{\partial} = \gamma^\mu \partial_\mu$$

$$p^\mu = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \vec{p} = 0$$

In the rest frame, the condition that $a_{p\sigma}$ correspond to spin-1/2 particles implies u's satisfy a constraint

$$u = \beta u \quad u = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

In a general frame: $u \to Du$, $\frac{ip^\mu \gamma_\mu}{m} u = u$.

$$\psi_i = \sum_\sigma \int d^3p \left[u_i(\vec{p},\sigma)\, a_{p\sigma}\, e^{ipx} + v_i(\vec{p},\sigma)\, \bar{a}^*_{p\sigma}\, e^{-ipx} \right.$$

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In a general frame: $u \to Du$ $\cdot$ $\frac{ip^\mu \gamma_\mu}{m} u = u$.

$$\psi_a = \sum_\sigma \int d^3p \left[\underline{u_i(\vec{p},\sigma)}\, a_{p\sigma}\, e^{ip\cdot x} + v_i(\vec{p},\sigma)\, \bar{a}^*_{p\sigma}\, e^{-ip\cdot x} \right.$$

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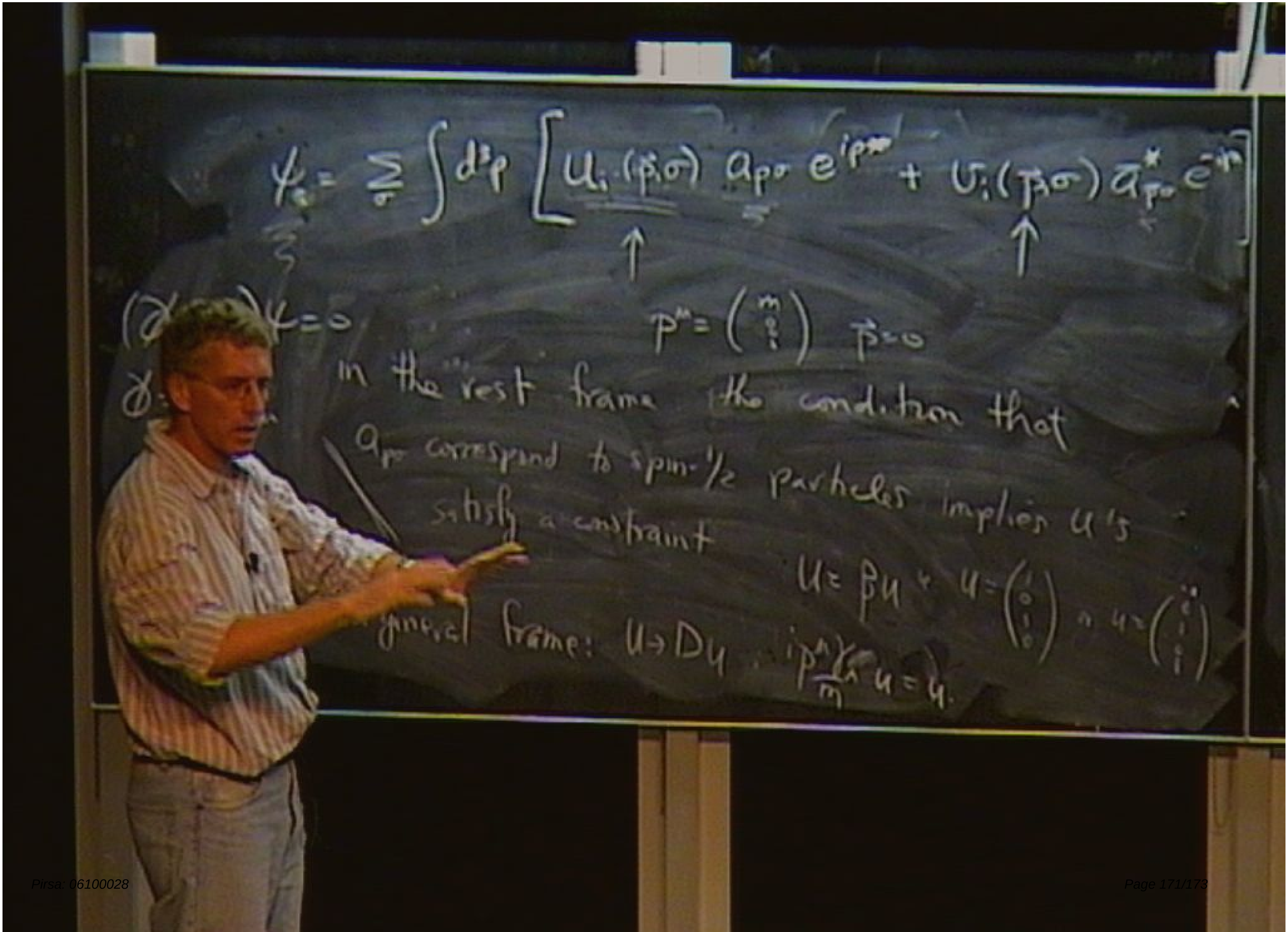

$p^\mu = \begin{pmatrix} m \\ 0 \\ 0 \end{pmatrix}$ $\vec{p} = 0$
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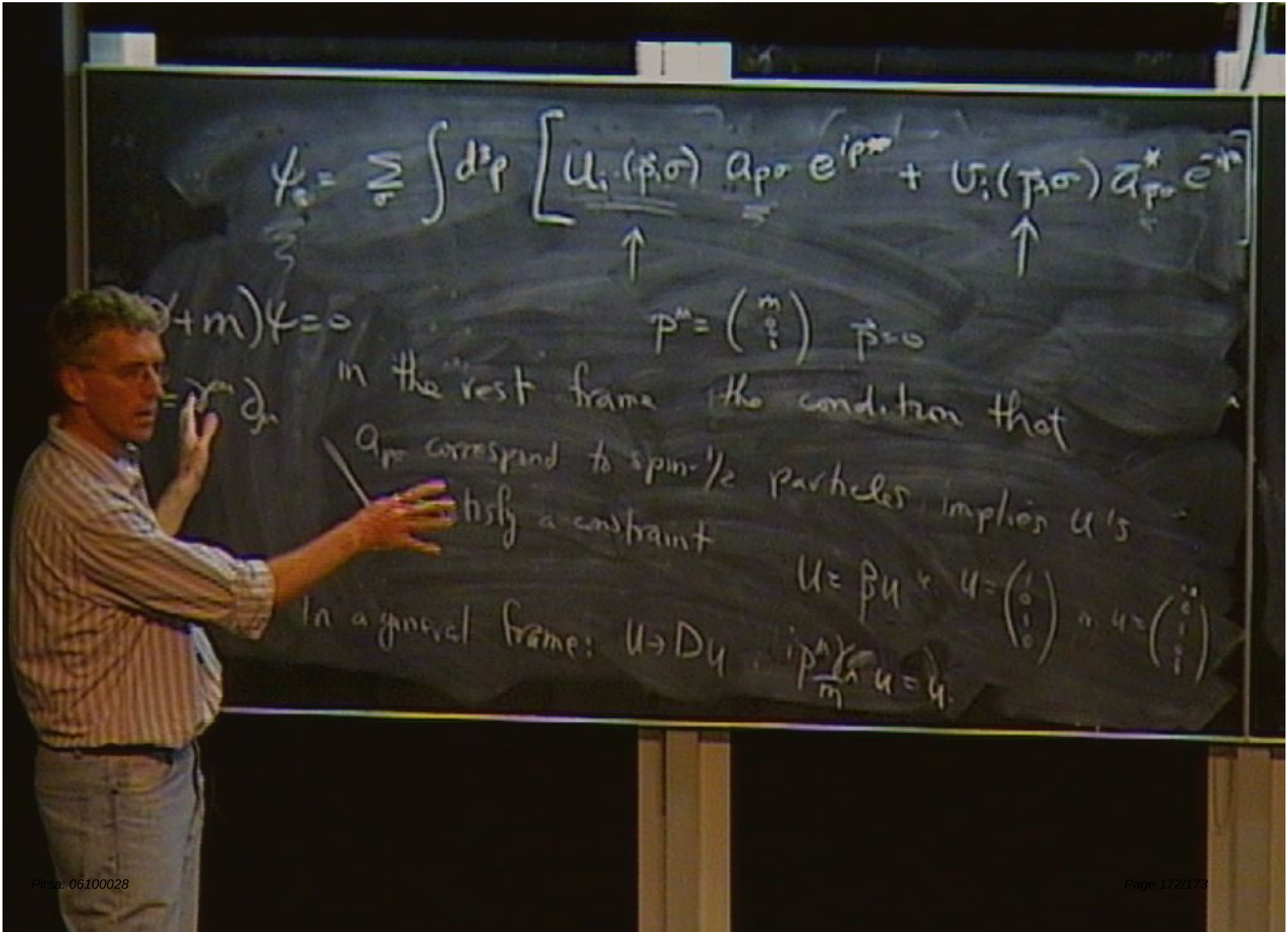
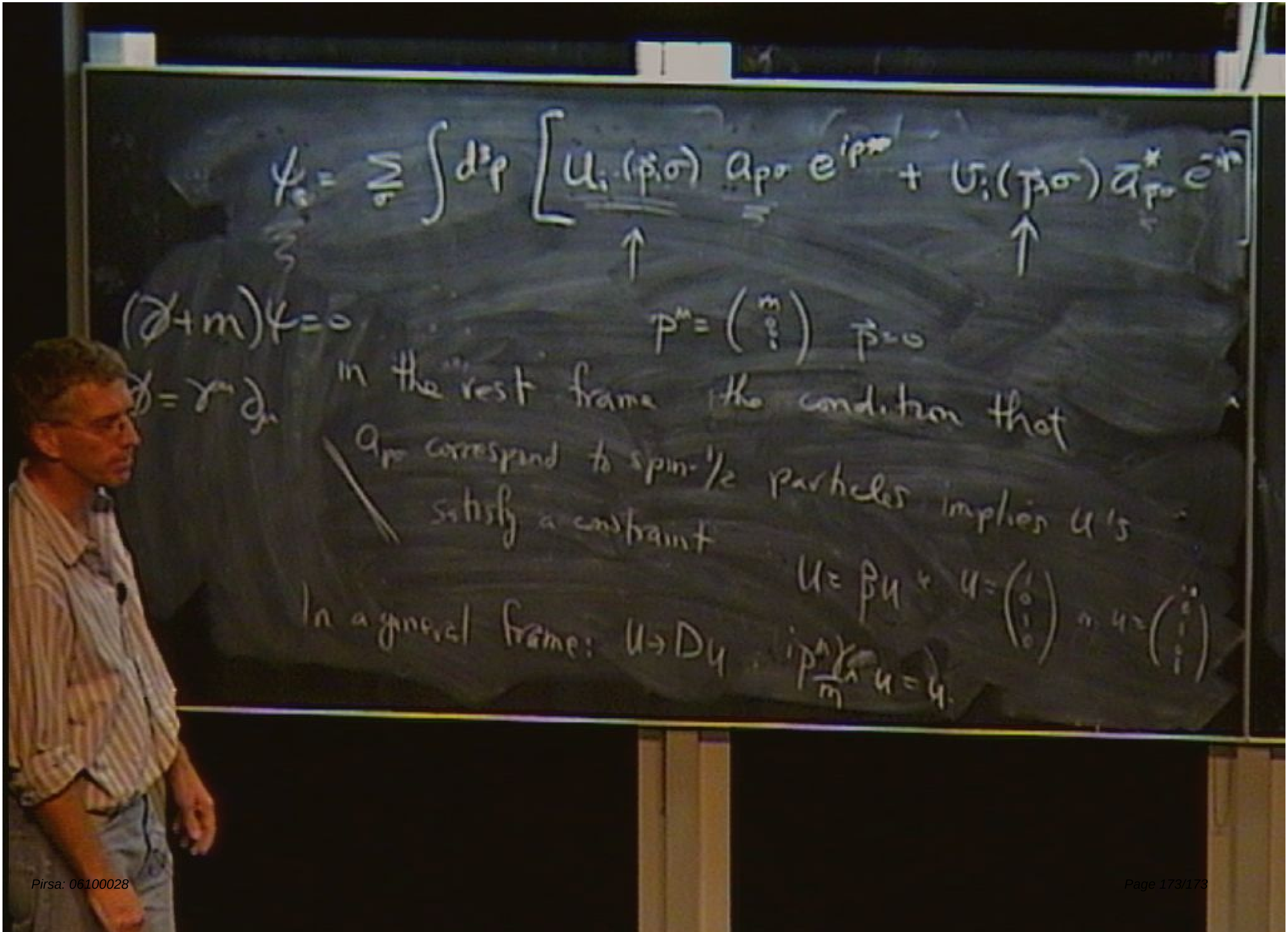
general frame: $u \to Du$



ψ = Σ ∫ d³p [u(p,σ) a e^{ipx} + v(p,σ) a* e^{-ipx}]
(γ∂+m)ψ = 0
p^μ = (m, 0)   p = 0
In the rest frame the condition that
a correspond to spin-1/2 particles implies u's
satisfy a constraint
u = βu
In a general frame: u → Du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