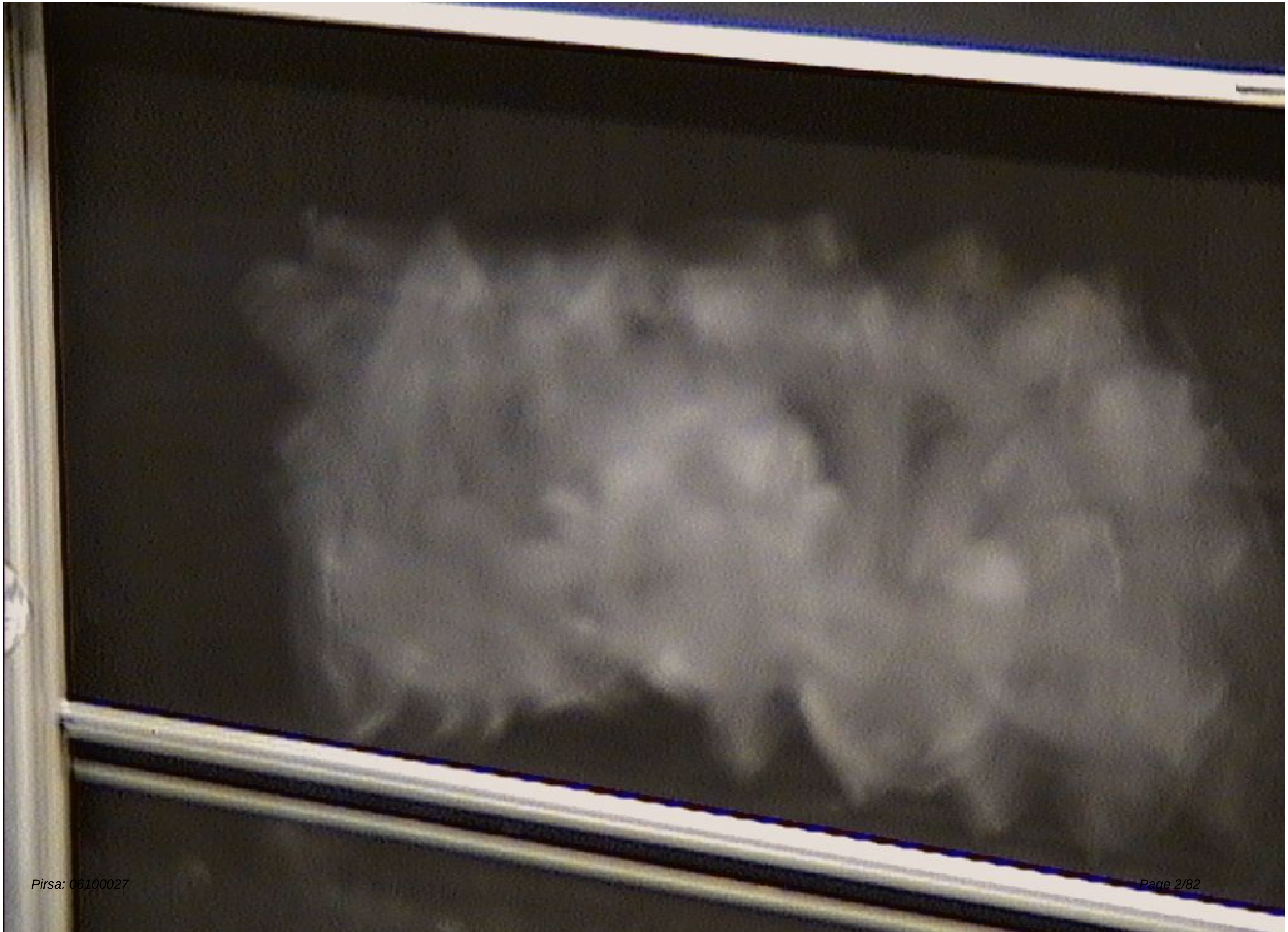
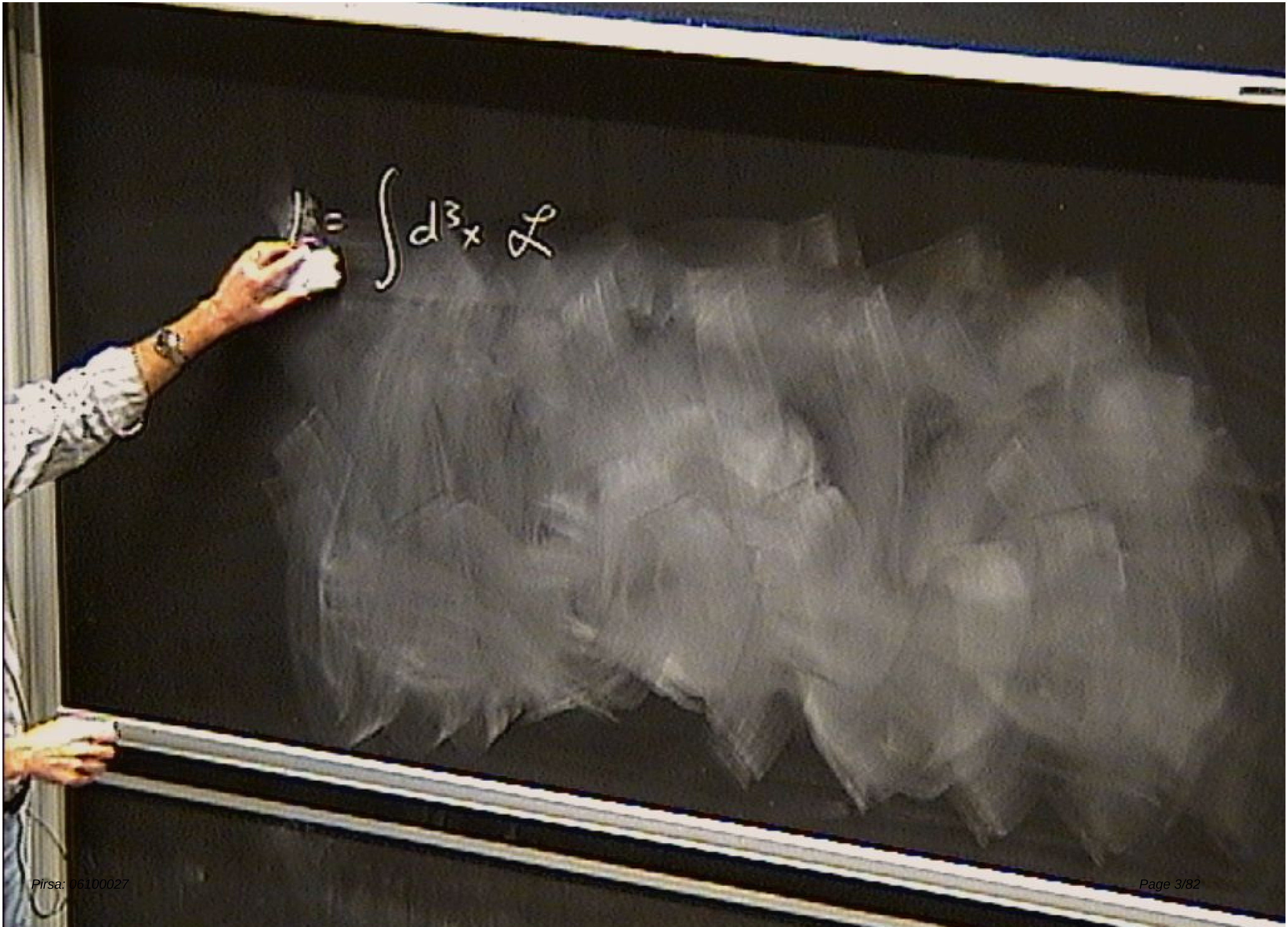
Title: Graduate Course on Standard Model & Quantum Field Theory - 2A (Part 1)

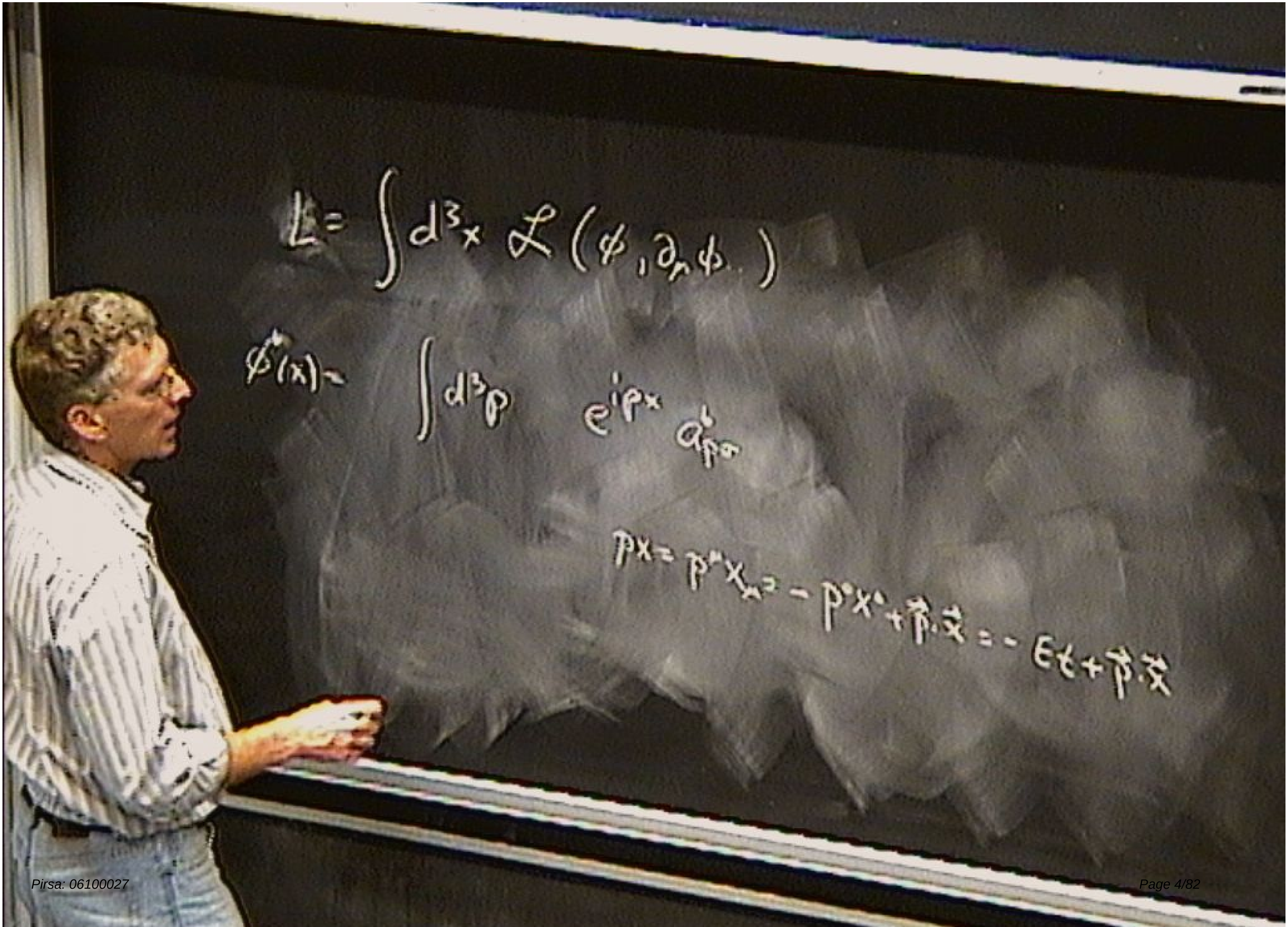
Date: Oct 11, 2006 11:00 AM

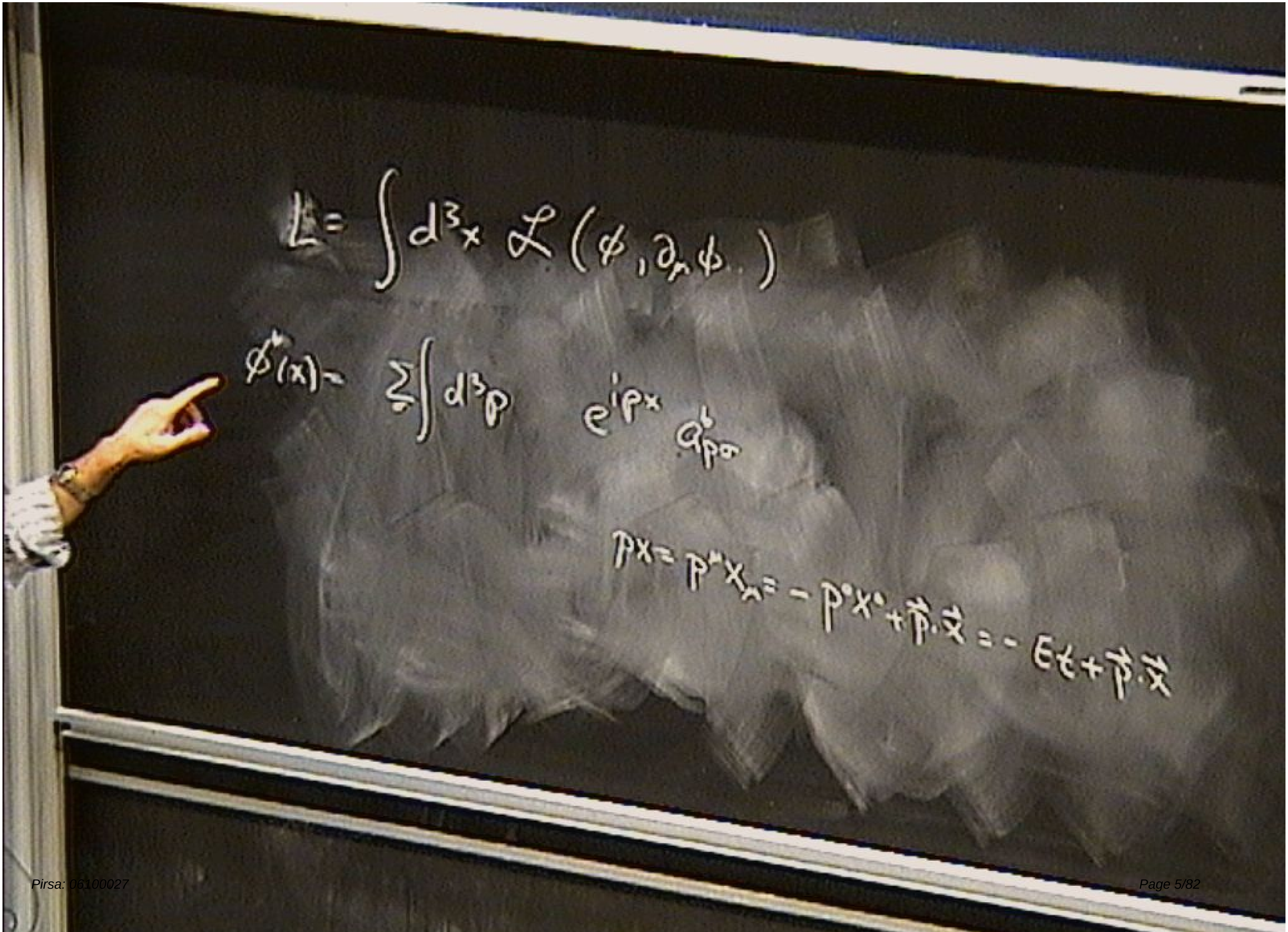
URL: http://pirsa.org/06100027

Abstract: Graduate Course on Standard Model & Quantum Field Theory




$= \int d^3x \, \mathcal{L}$


$L = \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$
$\phi(x) \sim \int d^3p \; e^{ipx} \; a^\dagger_{p\sigma}$
$px = p^\mu x_\mu = -p^0 x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$


$L = \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$
$\phi^{\ell}(x) = \sum_\sigma \int d^3p \; e^{ipx} \; a^{\ell}_{p\sigma}$
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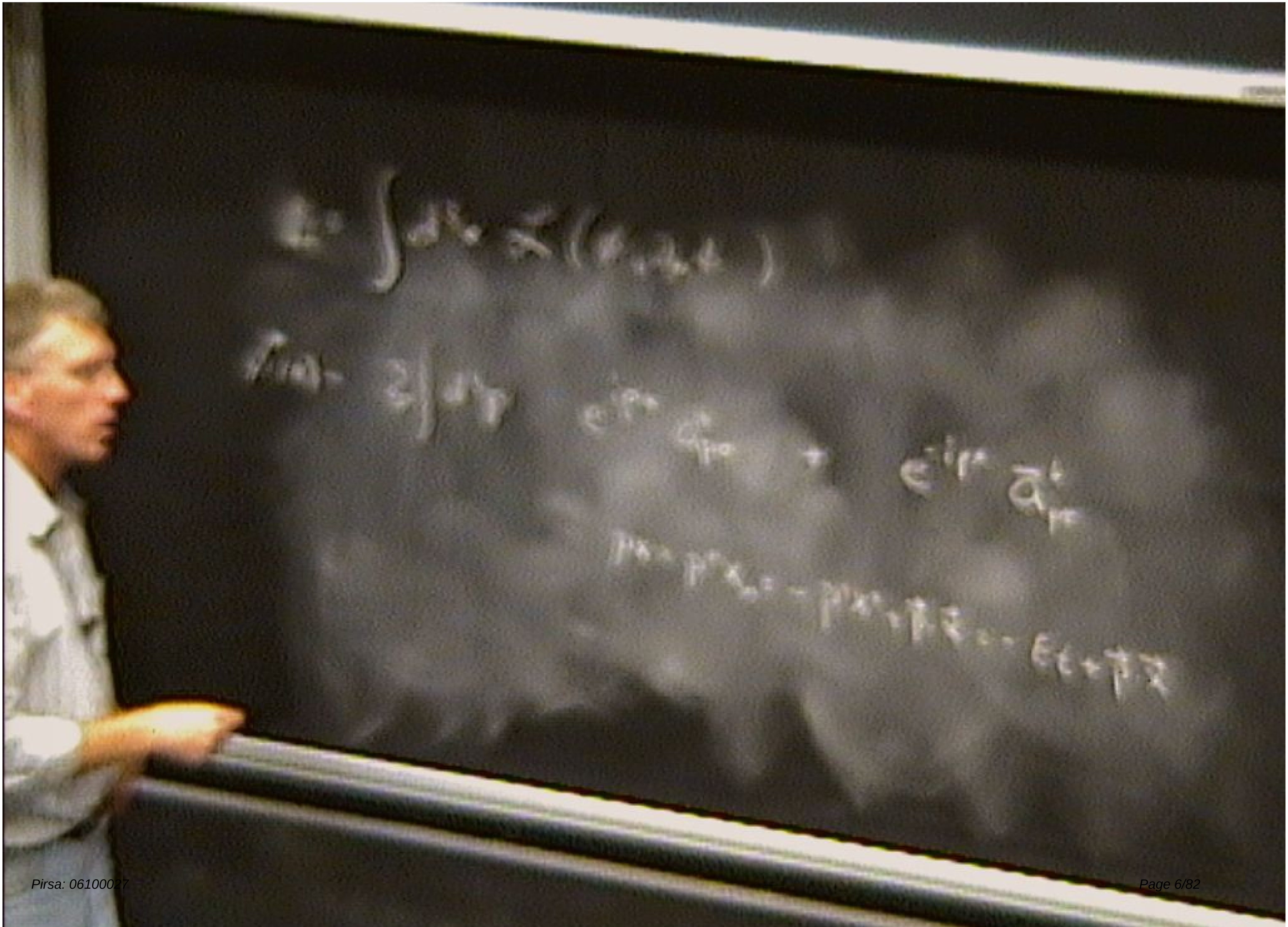

$$L = \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\phi^a(x) = \sum_\sigma \int d^3p \left[e^{ipx} a^b_{p\sigma} + e^{-ipx} \bar{a}^b_{p\sigma} \right]$$

$$px = p^\mu x_\mu = -p^0 x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$$

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Q: what are the u's and v's?

$$L = \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

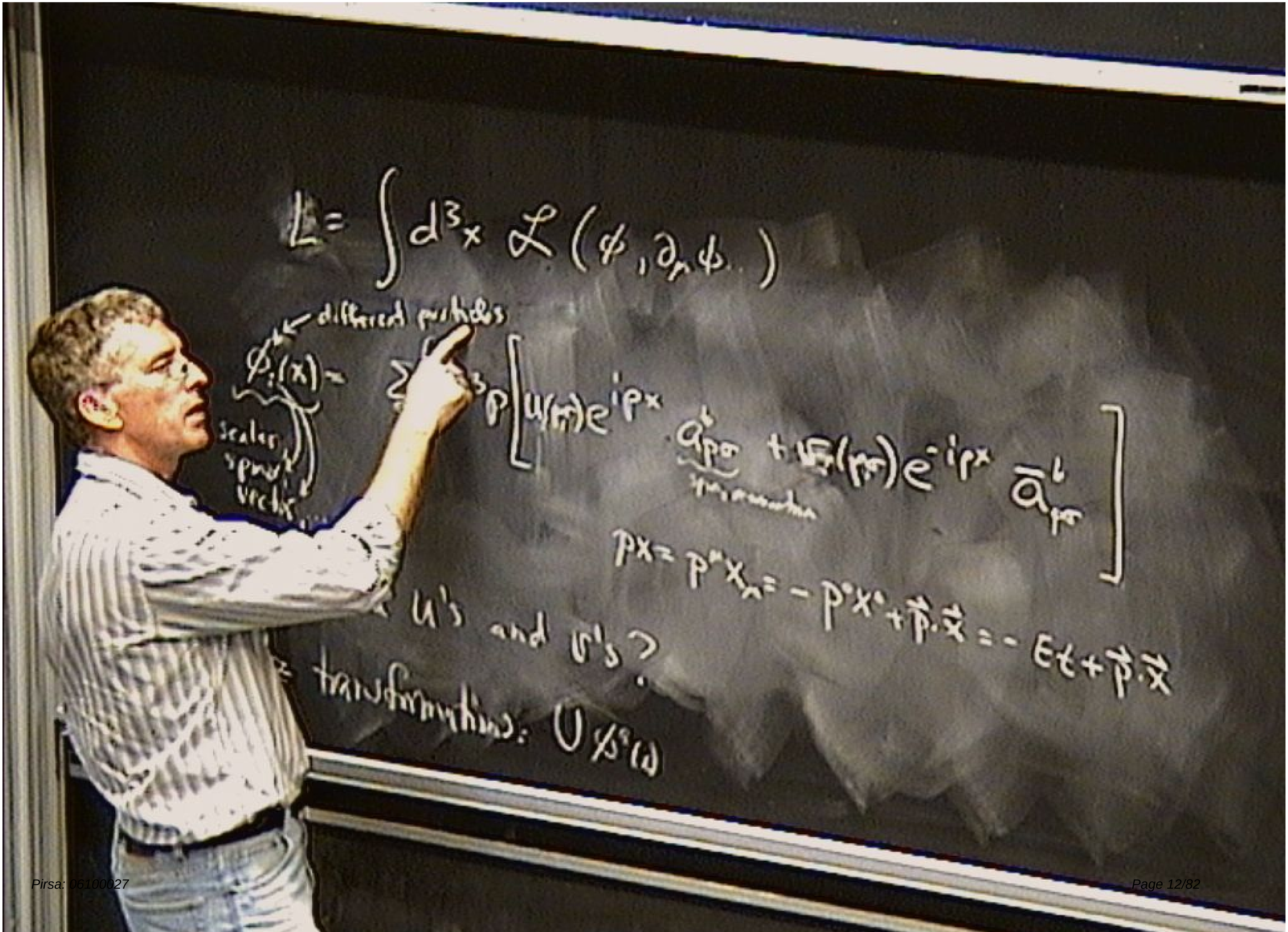
$$\phi^\ell(x) = \sum_\sigma \int d^3p \left[u^\ell(p,\sigma) e^{ipx} a_{p\sigma} + v^\ell(p,\sigma) e^{-ipx} \bar{a}^\dagger_{p\sigma} \right]$$

scalar, spinor, vector, ...

$a_{p\sigma}$: spin, momentum

$$px = p^\mu x_\mu = -p^0 x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$$

Q: What are the u's and v's?


L = ∫ d³x 𝓛(φ, ∂μφ)
different particles
scalar
spinor
vector
px = p^μ x_μ = − p⁰x⁰ + p⃗·x⃗ = − Et + p⃗·x⃗
u's and v's?
transformations:

$$L = \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

different particles

$$\underbrace{\phi_i(x)}_{\text{scalars, spinors, vectors, ...}} = \sum_\sigma \int d^3p \left[u_i(p\sigma) e^{ipx} \, a^\dagger_{p\sigma} + v_i(p\sigma) e^{-ipx} \, \bar{a}^\dagger_{p\sigma} \right]$$

$$px = p^\mu x_\mu = -p^0 x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$$

Q: What are the u's and v's?

Under Lorentz transformations: $U \phi^i(x)$

$\phi, \partial_\mu \phi\ )$

← different particles

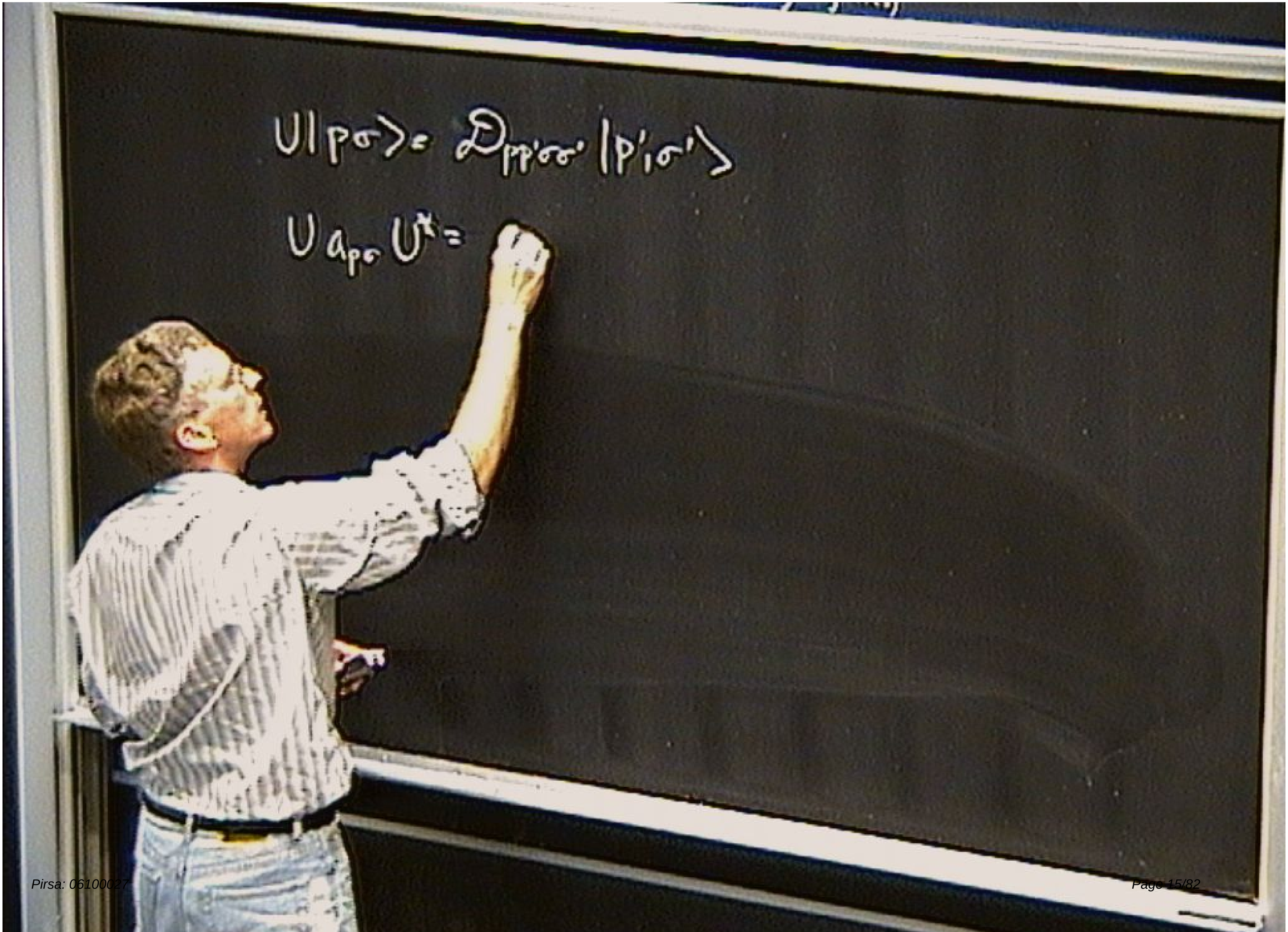
$$\underbrace{\phi_i(x)}_{\text{scalar, spinor, vector, ...}} = \sum_\sigma \int d^3p \left[u_i(p\sigma)\, e^{ipx}\, \underbrace{a^b_{p\sigma}}_{\text{spin, momentum}} + v_i(p\sigma)\, e^{-ipx}\, \bar{a}^b_{p\sigma} \right]$$

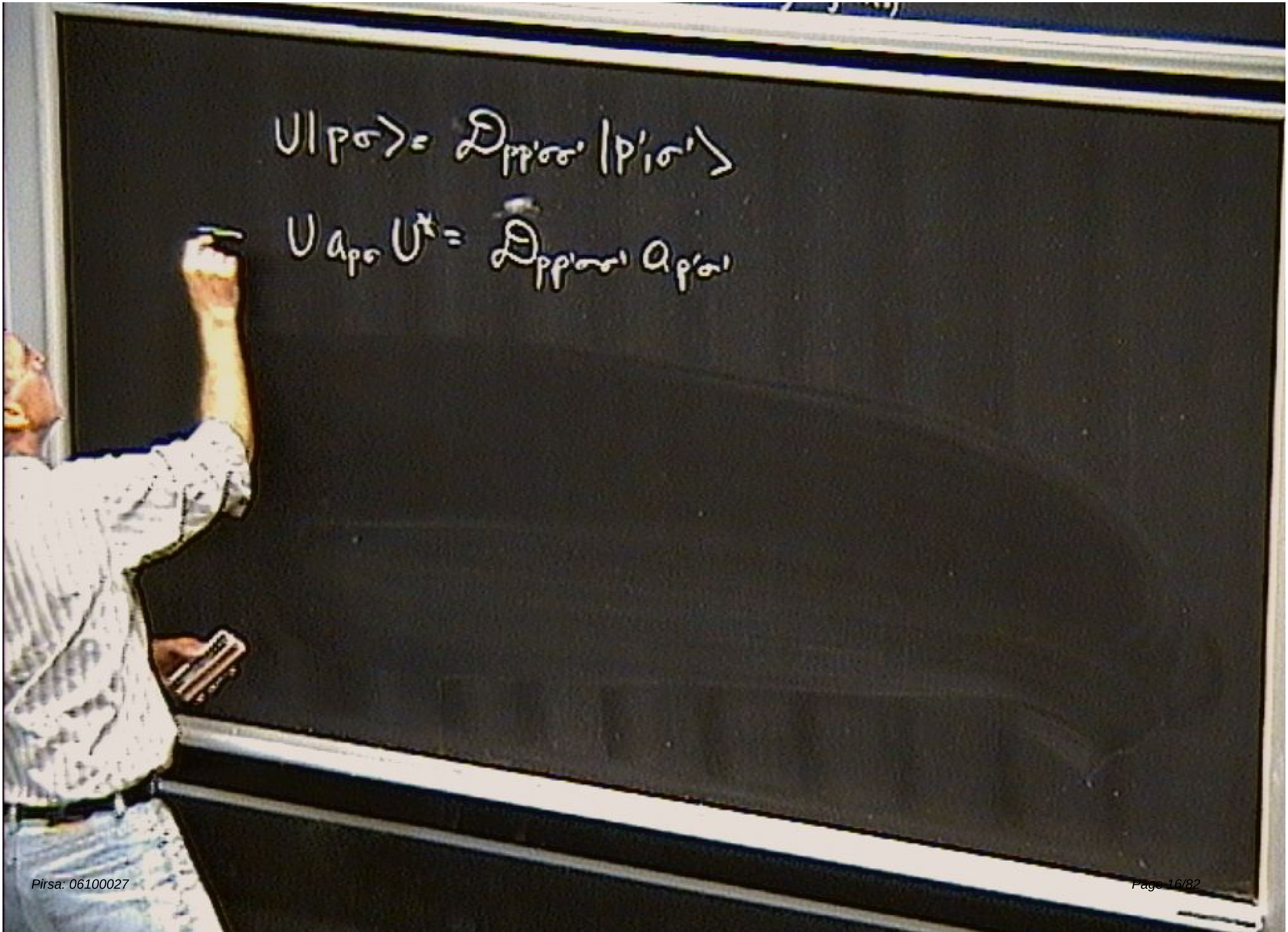
$$px = p^\mu x_\mu = -p^0x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$$

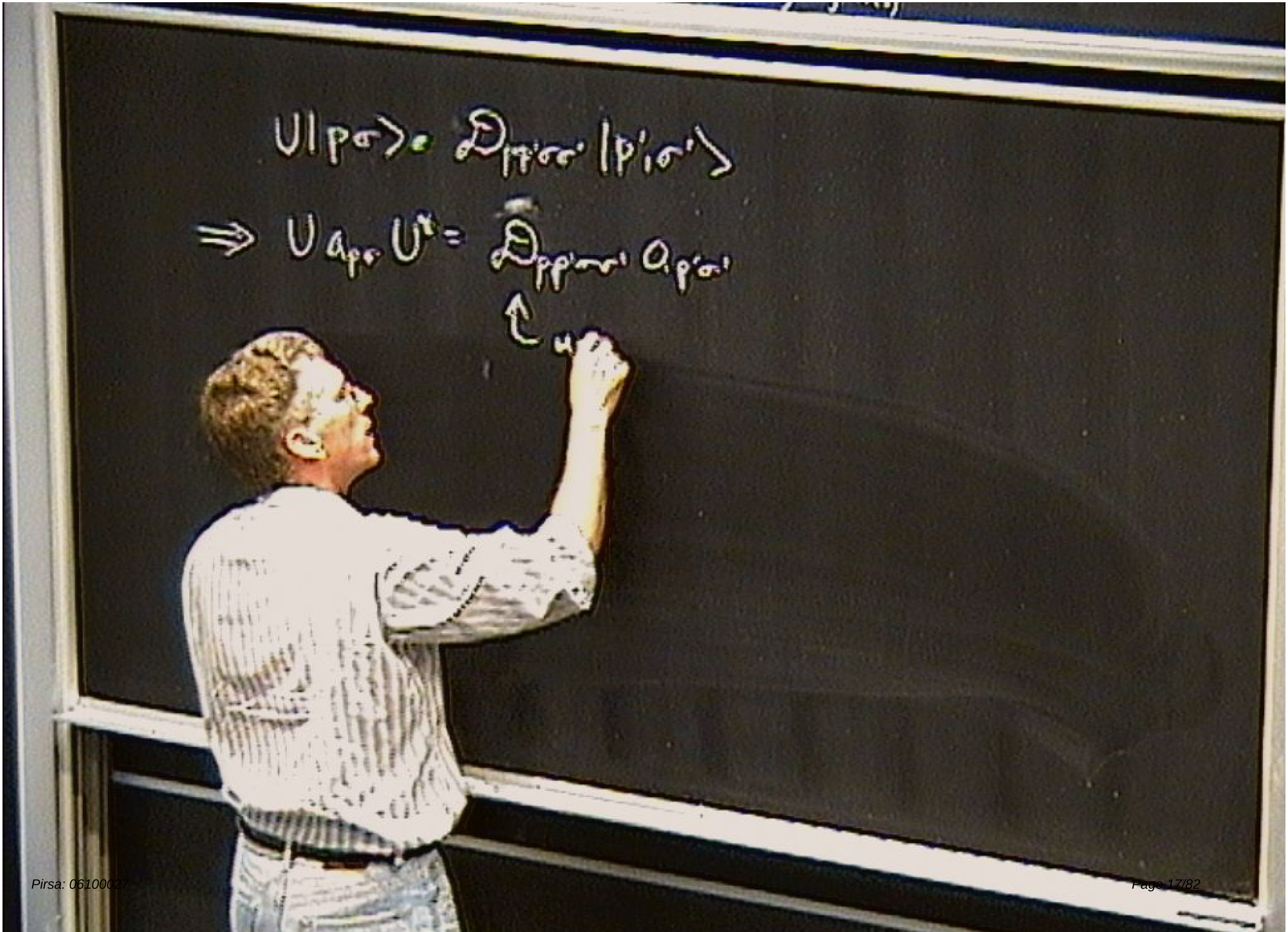
Q: What are the u's and v's?

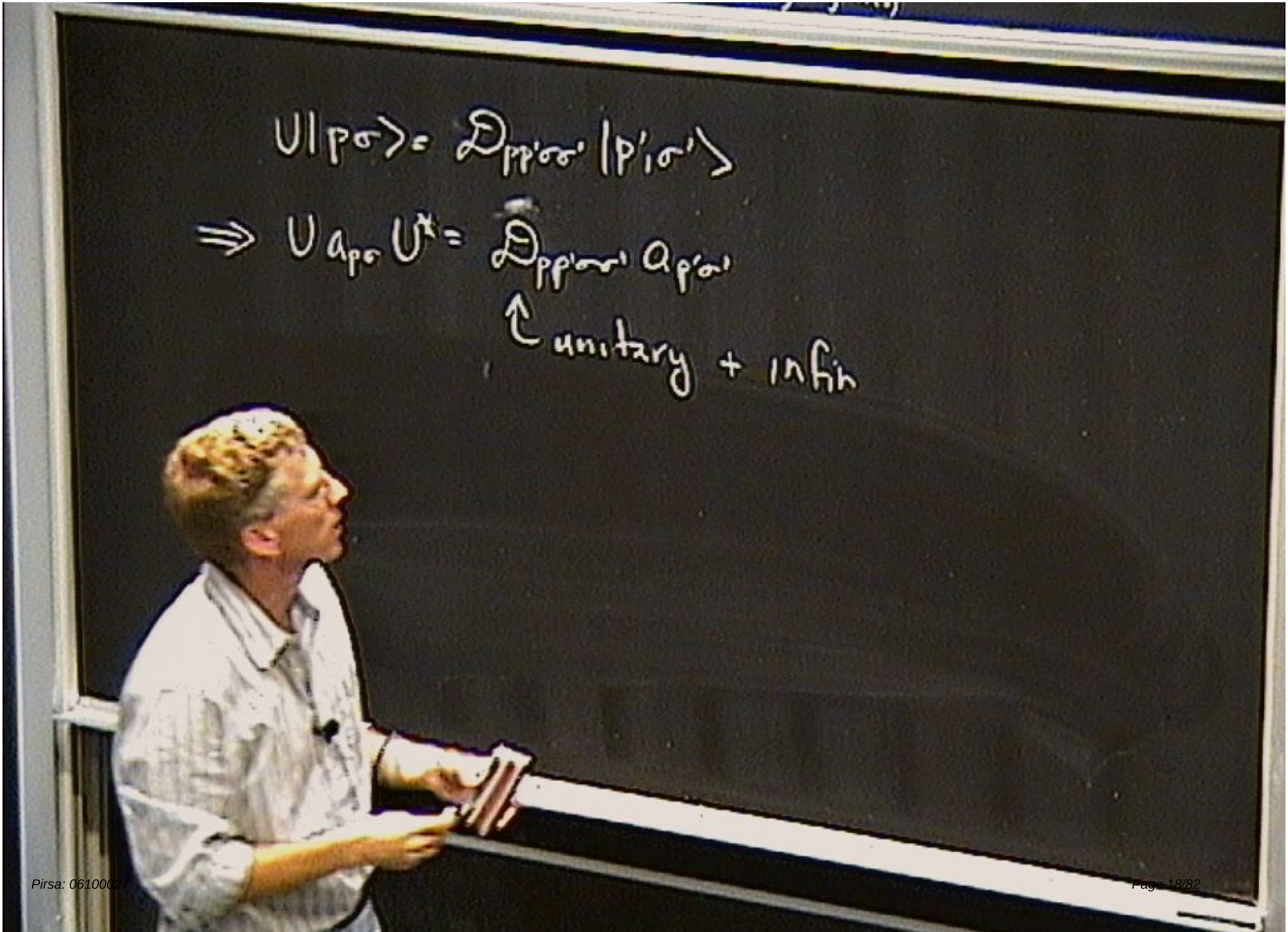
Under Lorentz transformations: $U\phi_i^a(x)U^* = \underbrace{D_{ij}}_{\text{finite-dim. matrix}}\phi_j^a(\Lambda x)$

$U|p\sigma\rangle =$


$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$
$U a_{p\sigma} U^{\dagger} =$


$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$
$U a_{p\sigma} U^{\dagger} = \mathcal{D}_{pp'\sigma\sigma'} a_{p'\sigma'}$


U|pσ> = D_{pp'σσ'} |p'σ'>
⇒ U a_{pσ} U† = D_{pp'σσ'} a_{p'σ'}


$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$
$\Rightarrow U a_{p\sigma} U^\dagger = \mathcal{D}_{pp'\sigma\sigma'} a_{p'\sigma'}$
unitary + infin

$$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$$

$$\Rightarrow U a_{p\sigma} U^{\dagger} = \mathcal{D}_{pp'\sigma\sigma'}\, a_{p'\sigma'}$$

$\uparrow$ unitary + infinite dimensional.

$$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$$

$$\Rightarrow U a_{p\sigma} U^{*} = \mathcal{D}_{pp'\sigma\sigma'} a_{p'\sigma'}$$

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Claim: u's v's are fixed up to normalization by group theory

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unitary + infinite dimensional.

Claim: u's v's are fixed up to normalization by group theory

(Weinberg: Vol I)

$$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p',\sigma'\rangle$$

$$\Rightarrow U a_{p\sigma} U^{\dagger} = \mathcal{D}_{pp'\sigma\sigma'} a_{p'\sigma'}$$

unitary + infinite dimensional.

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unitary + infinite dimensional.

<u>Claim</u>: u's v's are fixed up to normalization by group theory

(Weinberg: Vol I)

$A_{1/4}$

$B_{1/3}$ $\sigma = -A, -A+1, \ldots, A-1, A$

$$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}\,|p',\sigma'\rangle$$

$$\Rightarrow\ U a_{p\sigma} U^{\dagger} = \mathcal{D}_{pp'\sigma\sigma'}\, a_{p'\sigma'}$$

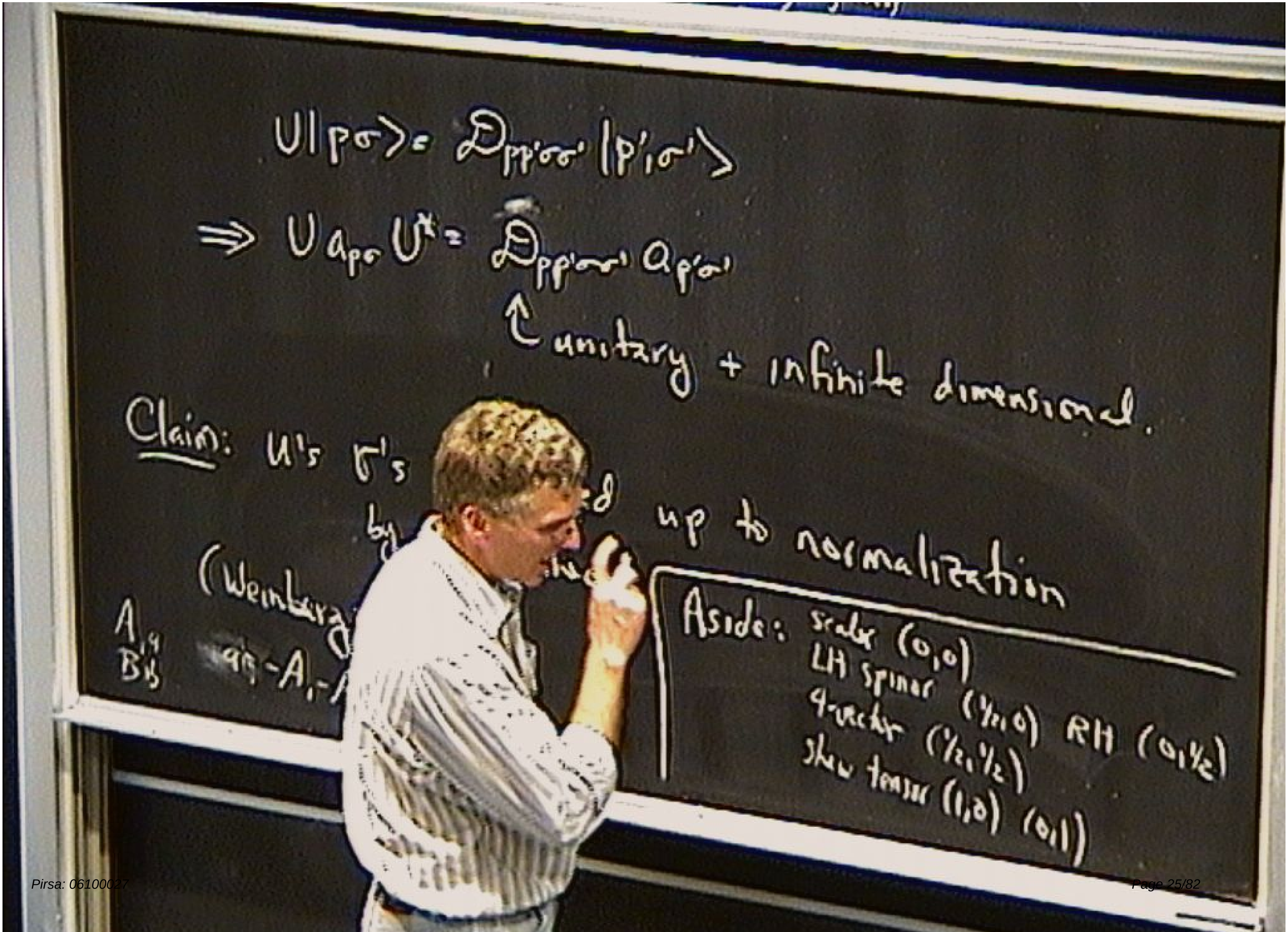
$\uparrow$ unitary + infinite dimensional.

Claim: u's v's are fixed up to normalization by group theory

(Weinberg: Vol I)

A, η

B, ξ $\quad \sigma = -A, -A+1, \ldots, A-1, A$


$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$
$\Rightarrow U a_{p\sigma} U^\dagger = \mathcal{D}_{pp'\sigma\sigma'} a_{p'\sigma'}$
unitary + infinite dimensional.
Claim: U's
up to normalization
Aside: Scalar (0,0)
LH spinor (½,0) RH (0,½)
4-vector (½,½)
Skew tensor (1,0) (0,1)

$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'}|p'\sigma'\rangle$

$\Rightarrow U a_{p\sigma} U^{\dagger} = \mathcal{D}_{pp'\sigma\sigma'} a_{p'\sigma'}$

unitary + infinite dimensional.

Claim: u's v's are fixed up to normalization by group theory

(Weinberg: Vol I)

$\to A_{ij}$

$\to B_{ij}$

$-A, -A+1, \ldots, A-1, A$

Aside: scalar $(0,0)$

LH spinor $(\frac{1}{2},0)$ RH $(0,\frac{1}{2})$

4-vector $(\frac{1}{2},\frac{1}{2})$

skew tensor $(1,0)$ $(0,1)$

Massive particles, of spin j:

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Massless " " helicity s: $A-B=s$.

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eg massless spin 1: $s=\pm 1 \to (1,0)\oplus(0,1) \to F_{\mu\nu}$

<u>not</u> A_μ.

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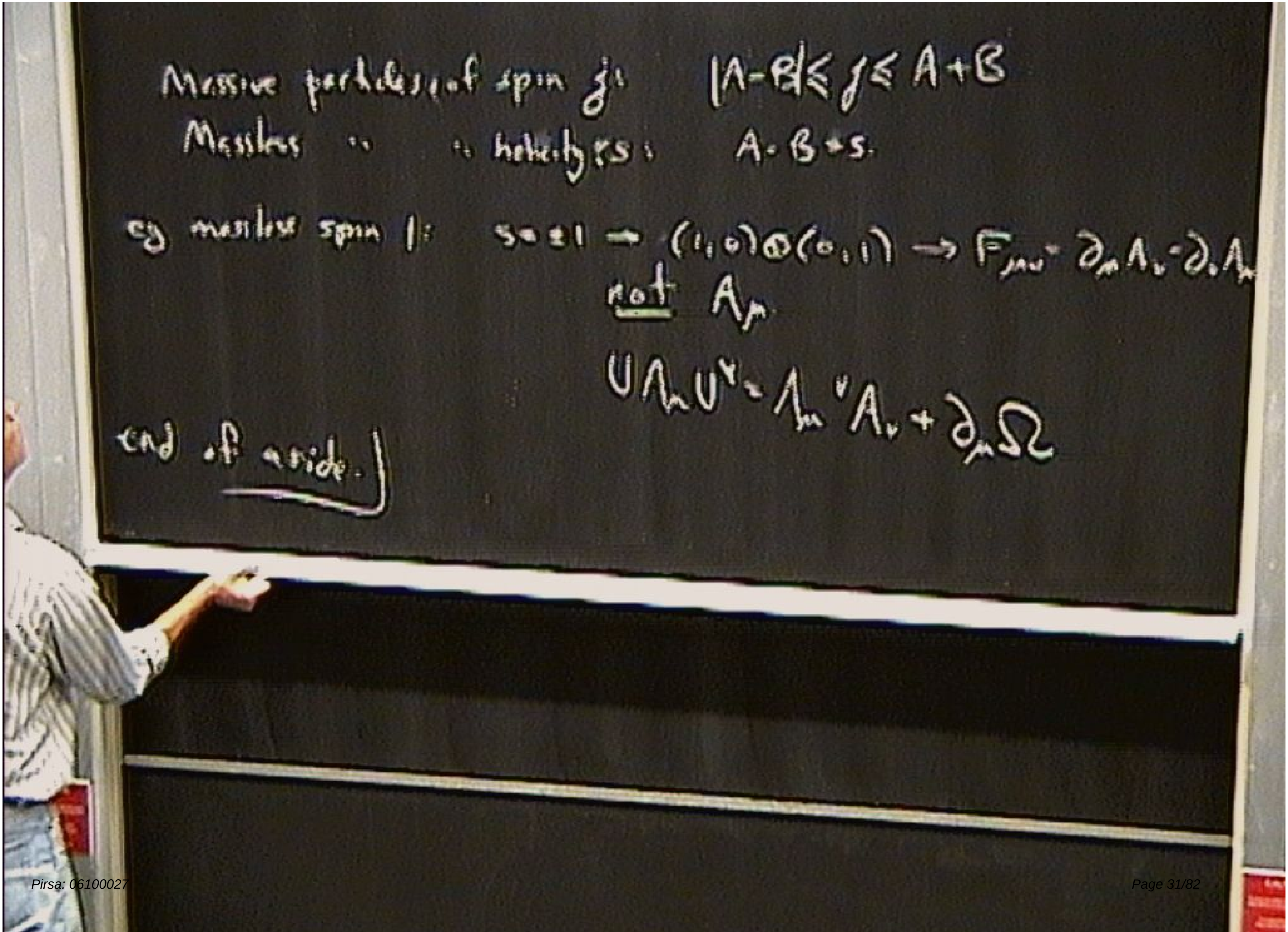
eg massless spin 1: $s = \pm 1 \rightarrow (1,0)\oplus(0,1) \rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

<u>not</u> A_μ

$$U A_\mu U^\dagger = \Lambda_\mu{}^\nu A_\nu + \partial_\mu \Omega$$


Massive particles of spin j: |A-B| ≤ j ≤ A+B
Massless " " helicity s: A - B = s.
eg massless spin 1: s = ±1 → (1,0)⊕(0,1) → F_μν = ∂_μ A_ν - ∂_ν A_μ
not A_μ
U A_μ U† = Λ_μ^ν A_ν + ∂_μ Ω
end of aside.

Massive particles, of spin j: $|A-B| \leq j \leq A+B$

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end of aside.

of aside.

For spin zero:

of aside.

For spin zero: simplest field representation is a scalar.

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$$\langle \vec{p} | \vec{p}' \rangle = \delta^3(\vec{p} - \vec{p}') \longleftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = \delta^3(\vec{p} - \vec{p}')$$

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$$(\vec{p}|\vec{p}\,') = (2\pi)^3 2E_p \delta^3(p-p') \leftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = (2\pi)^3 2E_p \delta^3(\vec{p}-\vec{p}\,')$$

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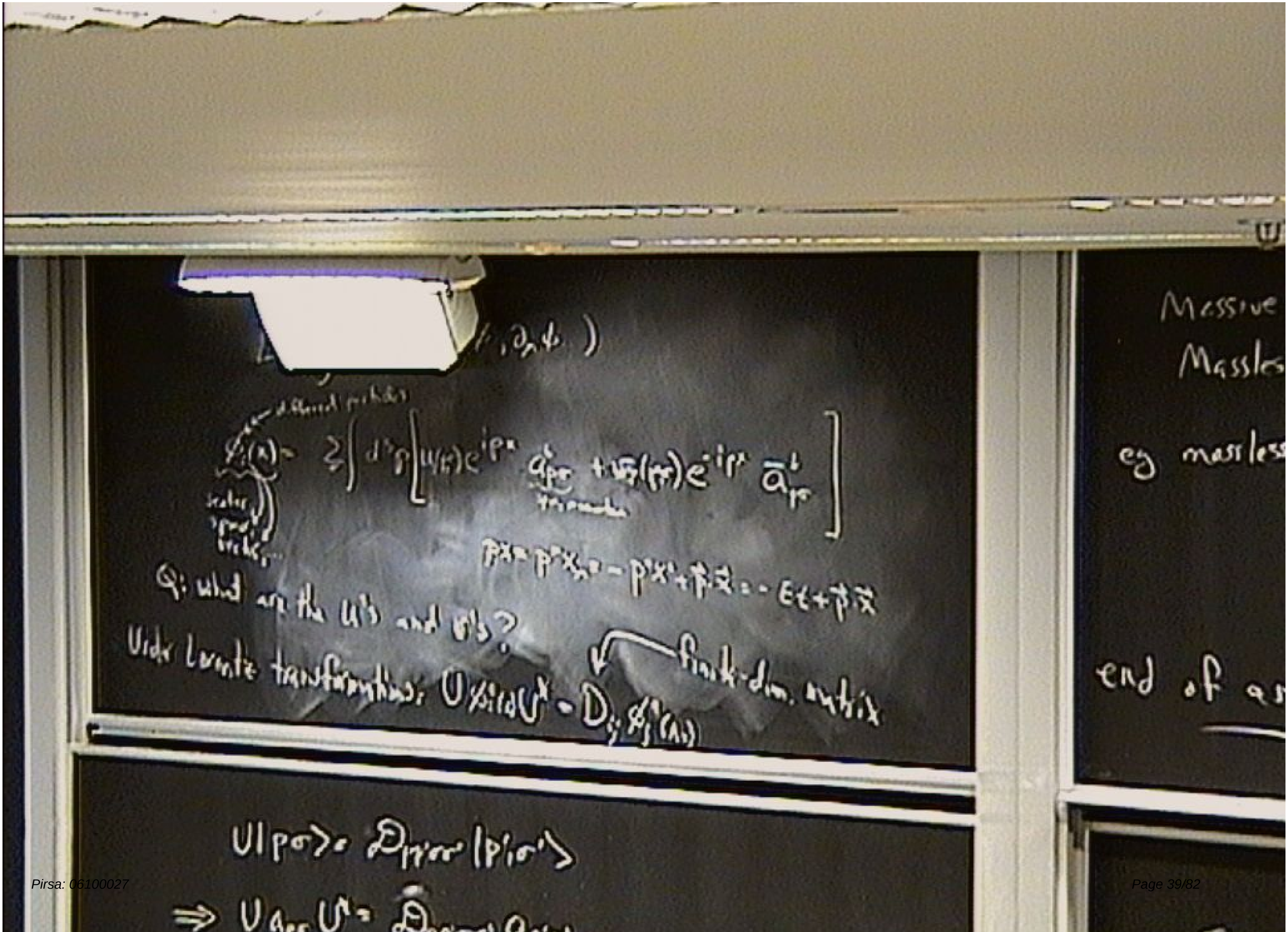
$$|\vec{p}, \sigma\rangle \qquad E = p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$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$$(\vec{p} | \vec{p}') = (2\pi)^3 2E_p \, \delta^3(\vec{p} - \vec{p}') \leftrightarrow a_p a^{*}_{p'} - a^{*}_{p'} a_p = (2\pi)^3 2E_p \, \delta^3(\vec{p} - \vec{p}')$$

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For spin zero: simplest field representation is a scalar.

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$$|\vec{p}, \sigma\rangle \qquad E = p^0 = \sqrt{\vec{p}^2 + m^2}$$

For spin zero: simplest field representation is a scalar:

$$U\phi^0(x)U^* = \phi^0(\Lambda x)$$

$\xrightarrow{\text{Class}}$ $\langle\vec{p}|\vec{p}'\rangle = \delta^3(\vec{p}-\vec{p}') \longleftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = \delta^3(\vec{p}-\vec{p}')$

$\xrightarrow[\text{Back}]{}$ $(\vec{p}|\vec{p}') = (2\pi)^3 2E_p \delta^3(\vec{p}-\vec{p}') \leftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = (2\pi)^3 2E_p \delta^3(\vec{p}-\vec{p}')$

$|\vec{p},\sigma\rangle \quad E = p^0 = \sqrt{\vec{p}^2+m^2}$

For spin zero: simplest field representation is a scalar.

$$U\,\phi(x)\,U^* = \phi(\Lambda x)$$

(A)

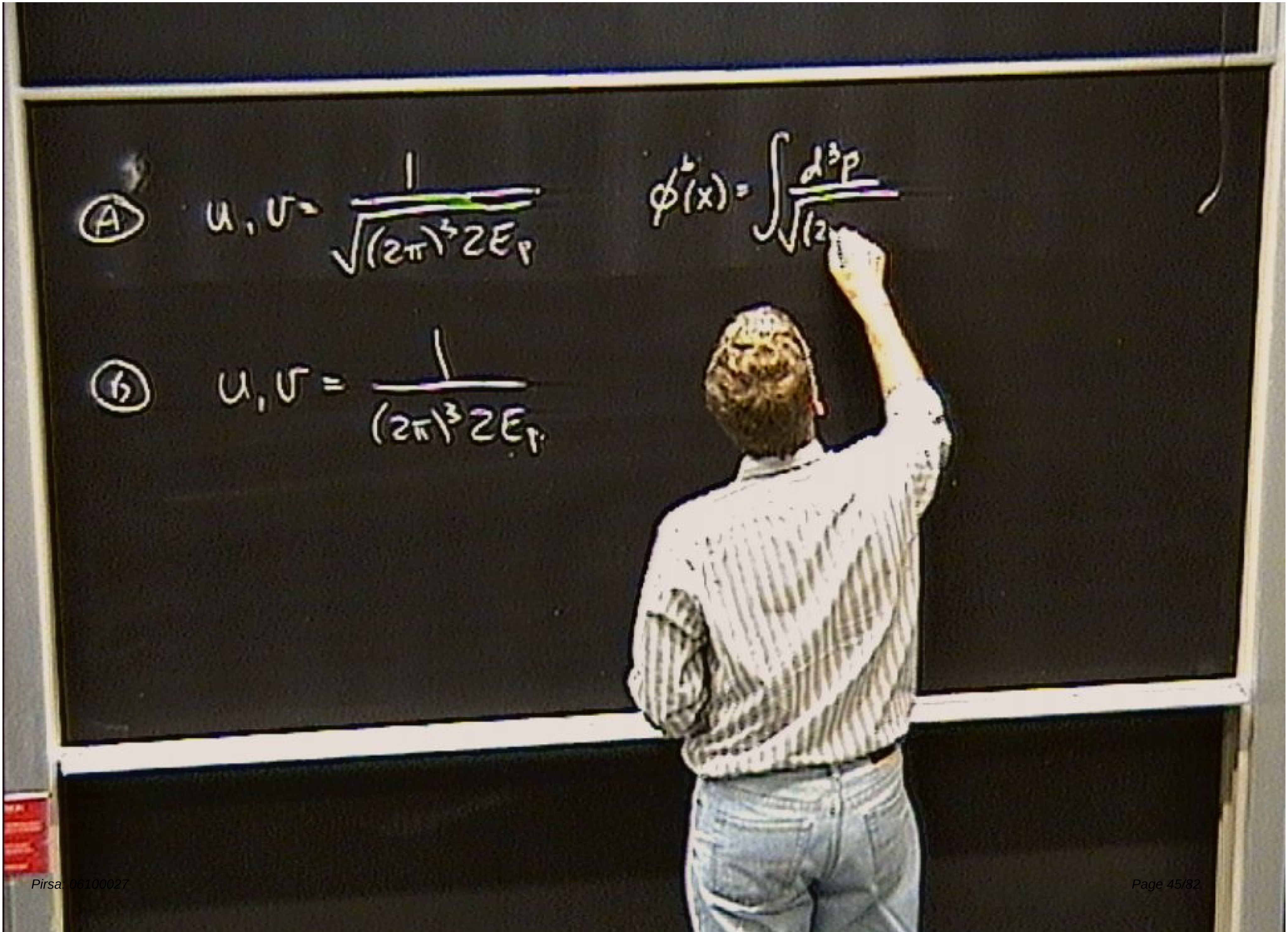
Class $\rightarrow$ $\langle \vec{p}\,|\,\vec{p}'\rangle = \delta^3(\vec{p}-\vec{p}') \longleftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = \delta^3(\vec{p}-\vec{p}')$

Book $\rightarrow$ $(\vec{p}\,|\,\vec{p}') = (2\pi)^3\, 2E_p\, \delta^3(\vec{p}-\vec{p}') \leftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = (2\pi)^3\, 2E_p\, \delta^3(\vec{p}-\vec{p}')$

(B) $|\vec{p},\sigma\rangle$ $\quad E = p^0 = \sqrt{\vec{p}^2+m^2}$

(A) $$u, v = \frac{1}{\sqrt{(2\pi)^3 2E_p}}$$

(B) $$u, v = \frac{1}{(2\pi)^3 2E_p}$$

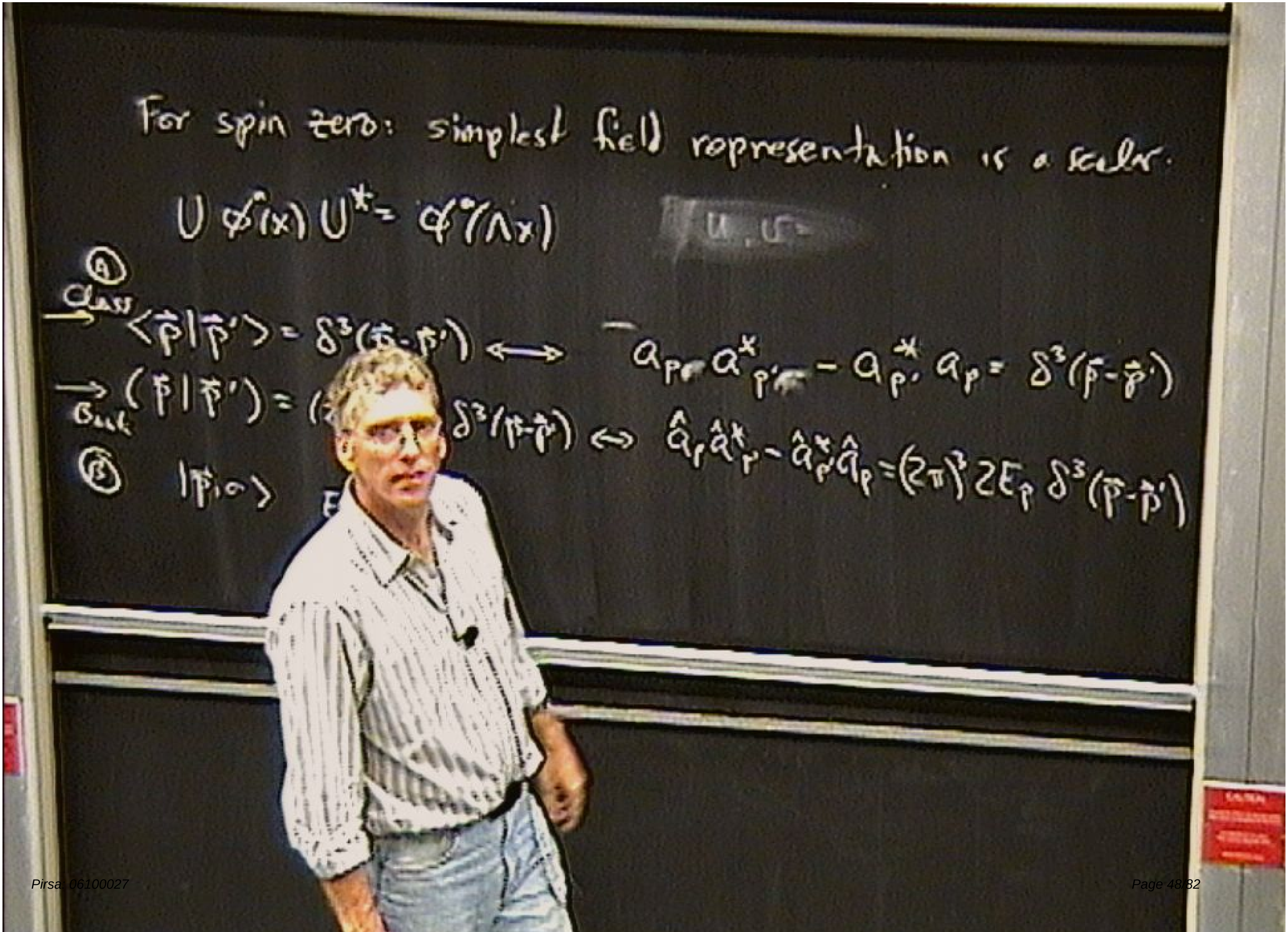

A
$u, v = \frac{1}{\sqrt{(2\pi)^3 2E_p}}$
$\phi^{\dagger}(x) = \int \frac{d^3p}{\sqrt{(2}}$
b
$u, v = \frac{1}{(2\pi)^3 2E_p}$

(A) $u, v = \frac{1}{\sqrt{(2\pi)^3 2E_p}}$ $\phi^b(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p + e^{-ipx} a_p^\dagger \right]$

(B) $u, v = \frac{1}{(2\pi)^3 2E_p}$ $\phi^b(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[e^{ipx} a_p \right.$

(A) $u, v = \dfrac{1}{\sqrt{(2\pi)^3 2E_p}}$ $\qquad \phi^b(x) = \displaystyle\int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} \hat{a}_p + e^{-ipx} \hat{a}_p^\dagger \right]$

(B) $u, v = \dfrac{1}{(2\pi)^3 2E_p}$ $\qquad \phi^b(x) = \displaystyle\int \frac{d^3p}{(2\pi)^3 2E_p} \left[e^{ipx} \hat{a}_p + e^{-ipx} \hat{a}_p^\dagger \right]$


For spin zero: simplest field representation is a scalar.
U φ(x) U* = φ'(Λx)
Ⓐ
Class
⟨p⃗|p⃗'⟩ = δ³(p⃗-p⃗') ⟷ a_p a*_p' - a*_p' a_p = δ³(p⃗-p⃗')
Bank
(p⃗|p⃗') = δ³(p⃗-p⃗') ⟷ â_p â*_p' - â*_p' â_p = (2π)³ 2E_p δ³(p⃗-p⃗')
Ⓑ
|p⃗, σ⟩

For spin zero: simplest field representation is a scalar.

$$U \phi^0(x) U^* = \phi^0(\Lambda x)$$

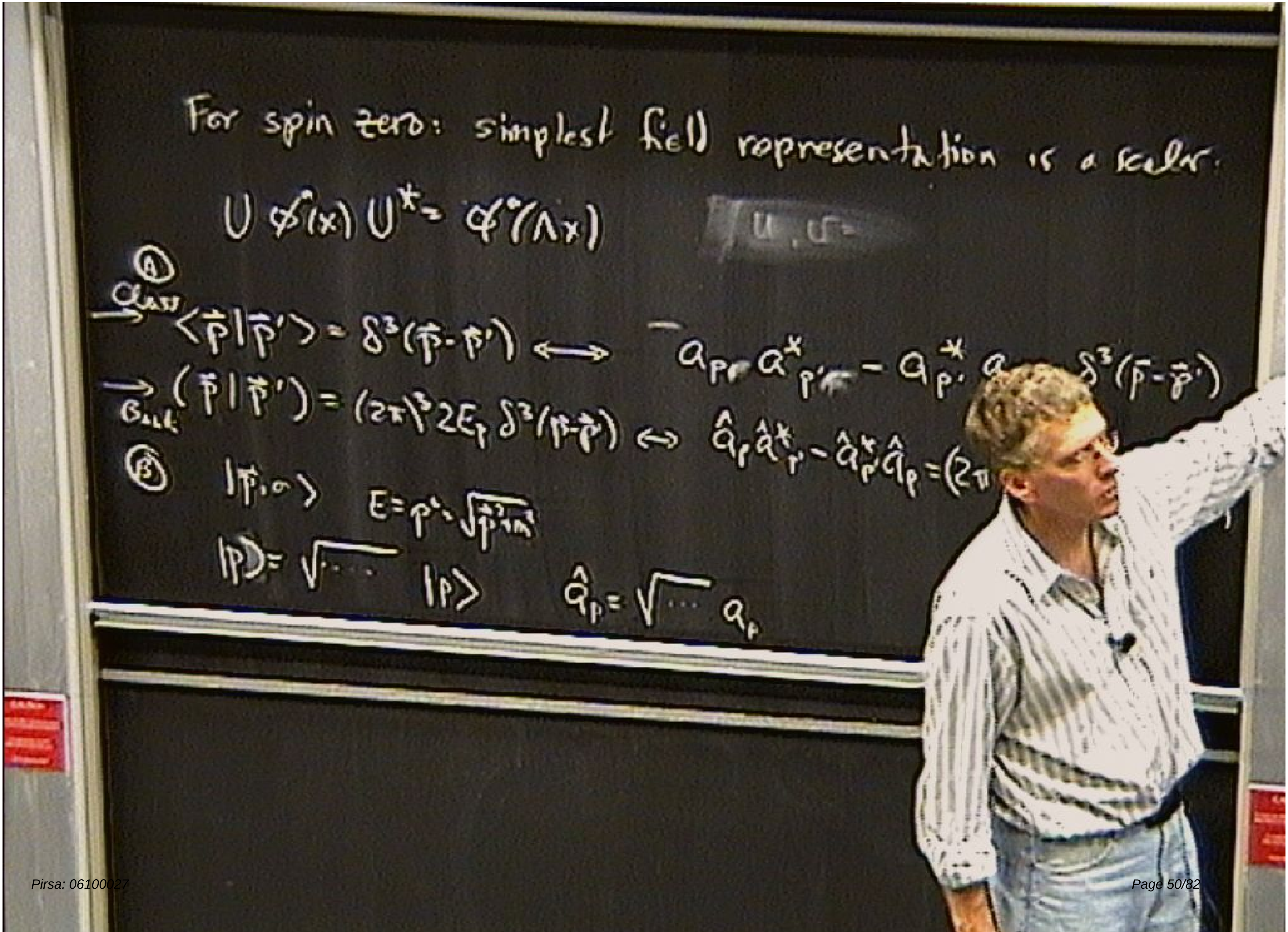
(A)

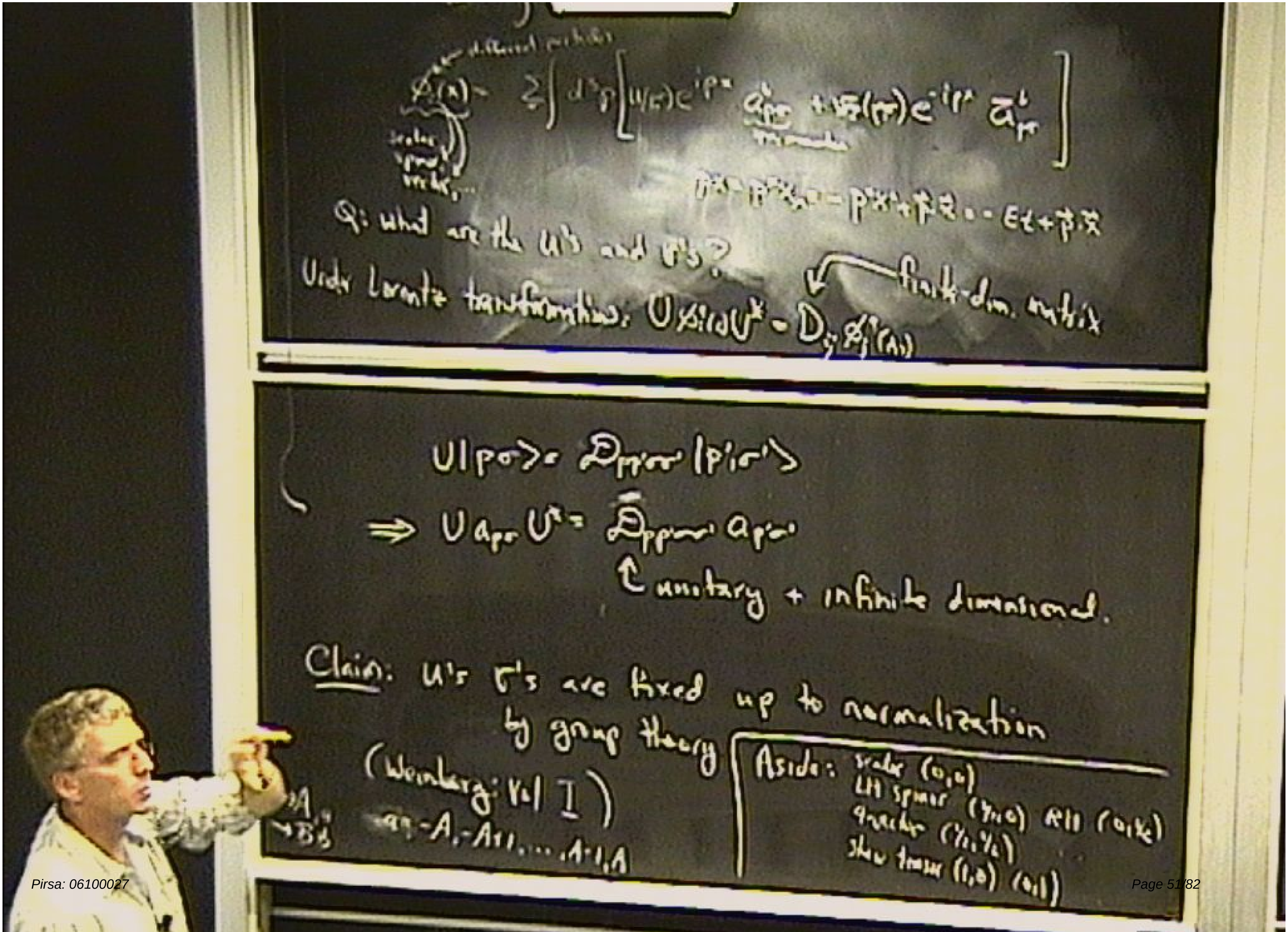
Class. $\langle \vec{p} | \vec{p}' \rangle = \delta^3(\vec{p} - \vec{p}') \longleftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = \delta^3(\vec{p} - \vec{p}')$

Bult. $(\vec{p} | \vec{p}') = (2\pi)^3 2E_p \delta^3(\vec{p} - \vec{p}') \longleftrightarrow \hat{a}_p \hat{a}^*_{p'} - \hat{a}^*_{p'} \hat{a}_p = (2\pi)^3 2E_p \delta^3(\vec{p} - \vec{p}')$

(B) $|\vec{p}, \sigma\rangle$ $\quad E = p^0 = \sqrt{\vec{p}^2 + m^2}$

$|p) = \sqrt{\cdots}\ |p\rangle$


For spin zero: simplest field representation is a scalar:
$U \phi(x) U^* = \phi(\Lambda x)$
A
Class
$\langle \vec p | \vec p' \rangle = \delta^3(\vec p - \vec p') \longleftrightarrow a_p a^*_{p'} - a^*_{p'} a_p = \delta^3(\vec p - \vec p')$
$(\vec p | \vec p') = (2\pi)^3 2E_p \delta^3(\vec p - \vec p') \leftrightarrow \hat a_p \hat a^*_p - \hat a^*_p \hat a_p = (2\pi$
B
$|\vec p, \sigma\rangle$
$E = p^0 = \sqrt{\vec p^2 + m^2}$
$|p) = \sqrt{\cdots}\ |p\rangle$
$\hat a_p = \sqrt{\cdots}\ a_p$


different particles
$\phi_i(x) \sim \sum \int d^3p \left[u(p) e^{ip\cdot x} a_{p\sigma} + v(p) e^{-ip\cdot x} a^\dagger_{p\sigma} \right]$
scalar
spinor
vector, ...
$p\cdot x = p^\mu x_\mu = -p^0x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$
Q: what are the u's and v's?
finite-dim. matrix
Under Lorentz transformations: $U \phi_i(x) U^\dagger = D_{ij} \phi_j(\Lambda x)$
$U|p\sigma\rangle = \mathcal{D}_{pp'\sigma\sigma'} |p'\sigma'\rangle$
$\Rightarrow U a_{p\sigma} U^\dagger = \bar{\mathcal{D}}_{pp'\sigma\sigma'} a_{p'\sigma'}$
unitary + infinite dimensional.
Claim: u's v's are fixed up to normalization
by group theory
(Weinberg: Vol I)
Aside: scalar (0,0)
LH spinor (1/2,0) RH (0,1/2)
4-vector (1/2,1/2)
Skew tensor (1,0) (0,1)
$-A, -A+1, \dots, A-1, A$

unitary + infinite dimensional.

Claim: u's v's are fixed up to normalization by group theory

(Weinberg: Vol I)

$\to A, B$

$-A, -A+1, \ldots, A-1, A$

Aside: scalar $(0,0)$
LH spinor $(\tfrac{1}{2},0)$ RH $(0,\tfrac{1}{2})$
4vector $(\tfrac{1}{2},\tfrac{1}{2})$
skew tensor $(1,0)$ $(0,1)$

$$\psi(x) = \sum_\sigma \int d^3p \left[u(p,\sigma)\, e^{ip\cdot x}\, a^\dagger_{p\sigma} + v(p,\sigma)\, e^{-ip\cdot x}\, a^{*}_{p\sigma} \right]$$

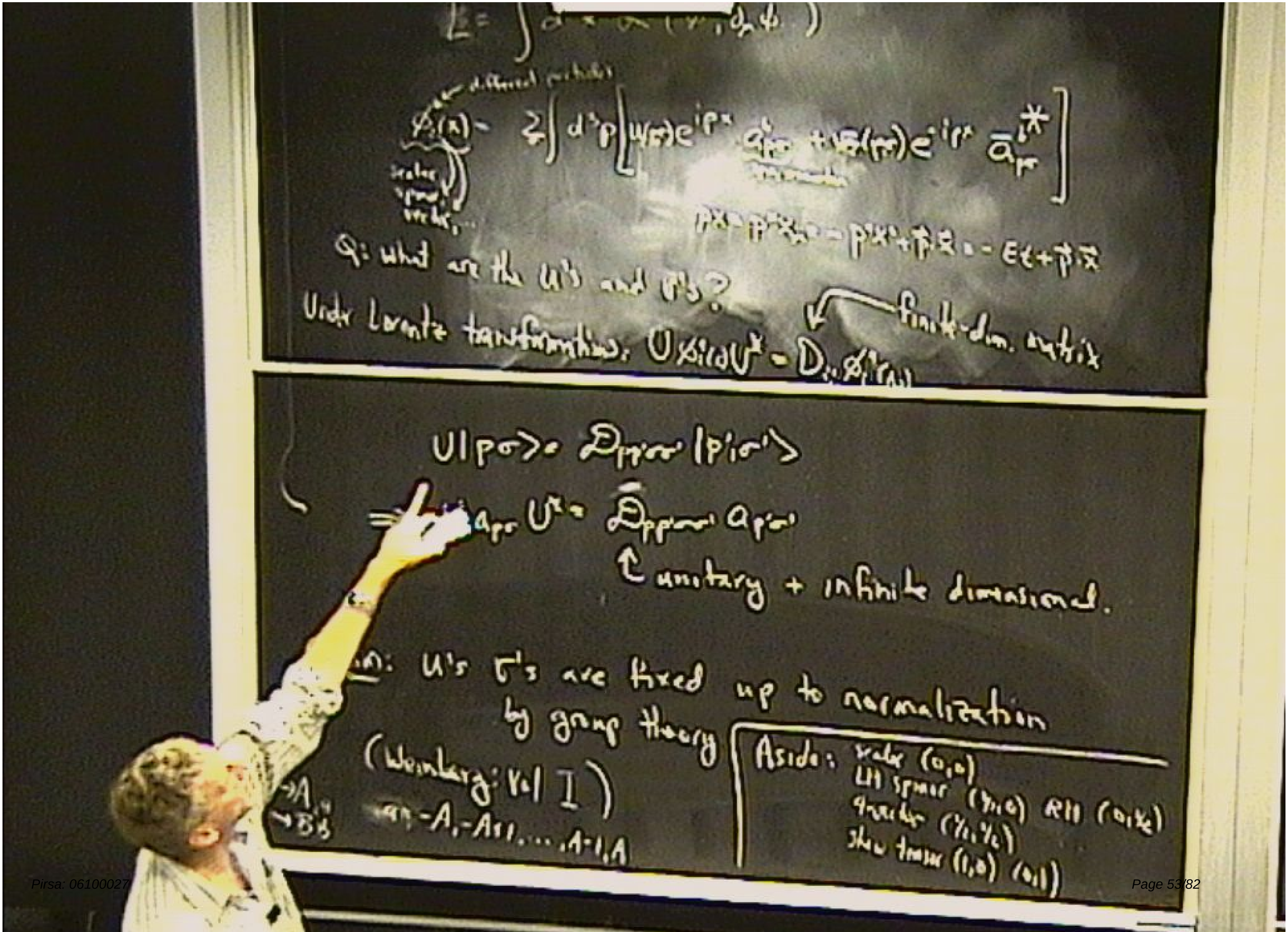
$p\cdot x = p^\mu x_\mu = -p^0 x^0 + \vec p\cdot\vec x = -Et + \vec p\cdot\vec x$

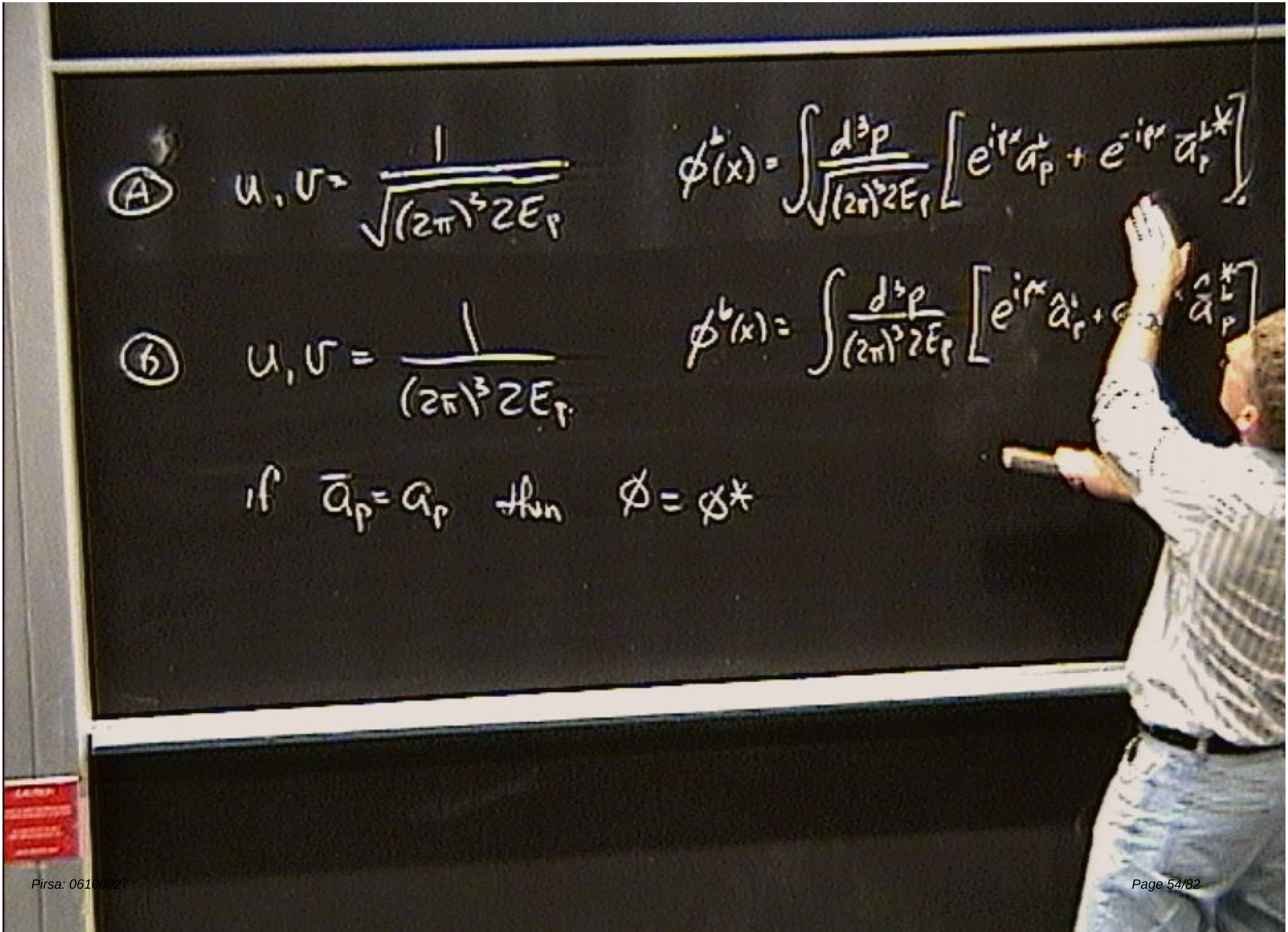
Q: what are the u's and v's?

Under Lorentz transformations, $U(\Lambda)$ ... finite-dim matrix

$U(k)$

$\dfrac{-l^2+l}{-l+2}$


different particles
$\phi_i(x) = \sum_\sigma \int d^3p \left[u_i(p\sigma) e^{ipx} a_{p\sigma} + v_i(p\sigma) e^{-ipx} \bar{a}^*_{p\sigma} \right]$
scalar, spinor, vector, ...
$p\cdot x = p^\mu x_\mu = -p^0 x^0 + \vec{p}\cdot\vec{x} = -Et + \vec{p}\cdot\vec{x}$
Q: what are the u's and v's?
Under Lorentz transformations: $U \phi_i(x) U^* = D_{ij} \phi_j(\Lambda x)$
finite-dim. matrix
$U|p\sigma\rangle = \mathcal{D}_{p\sigma p'\sigma'} |p'\sigma'\rangle$
$\Rightarrow U a_{p\sigma} U^* = \mathcal{D}_{p\sigma p'\sigma'} a_{p'\sigma'}$
unitary + infinite dimensional.
A: u's v's are fixed up to normalization by group theory
(Weinberg: Vol I)
$-A, -A+1, \ldots, A-1, A$
Aside: scalar (0,0)
LH spinor (1/2,0) RH (0,1/2)
4-vector (1/2,1/2)
skew tensor (1,0) (0,1)



A $u, v = \frac{1}{\sqrt{(2\pi)^3 2E_p}}$
$\phi^b(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a^b_p + e^{-ipx} a^{b*}_p \right]$
B $u, v = \frac{1}{(2\pi)^3 2E_p}$
$\phi^b(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[e^{ipx} a^b_p + \quad a^{b*}_p \right]$
if $\bar{a}_p = a_p$ then $\phi = \phi^*$

(A) $u, v = \frac{1}{\sqrt{(2\pi)^3 2E_p}}$ $\qquad \phi^L(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a^L_p + e^{-ipx} a^{L*}_p \right]$

(B) $u, v = \frac{1}{(2\pi)^3 2E_p}$ $\qquad \phi^L(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[e^{ipx} \hat{a}^L_p + e^{-ipx} \hat{a}^{L*}_p \right]$

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if $\bar{a}_p = a_p$ then $\phi = \phi^*$.

Claim: There is no loss in generality in using only real fields provided you use enough: eg Given $\phi = \phi^*$: $\phi_1 = \phi + \phi^*$, $\phi_2 = (\phi - \phi^*)/i$

What is the most general $\mathcal{L}(\phi^a, \partial_\mu \phi^a, \ldots)$
which corresponds to N spinless particles?

$$\longrightarrow \quad H_0 = C + \sum_{a=1}^{N} \int d^3\vec{p} \; E_a(\vec{p})\, a^\dagger_{pa}\, a_{pa}$$

$$E_a = \sqrt{\vec{p}^2 + m_a^2}$$

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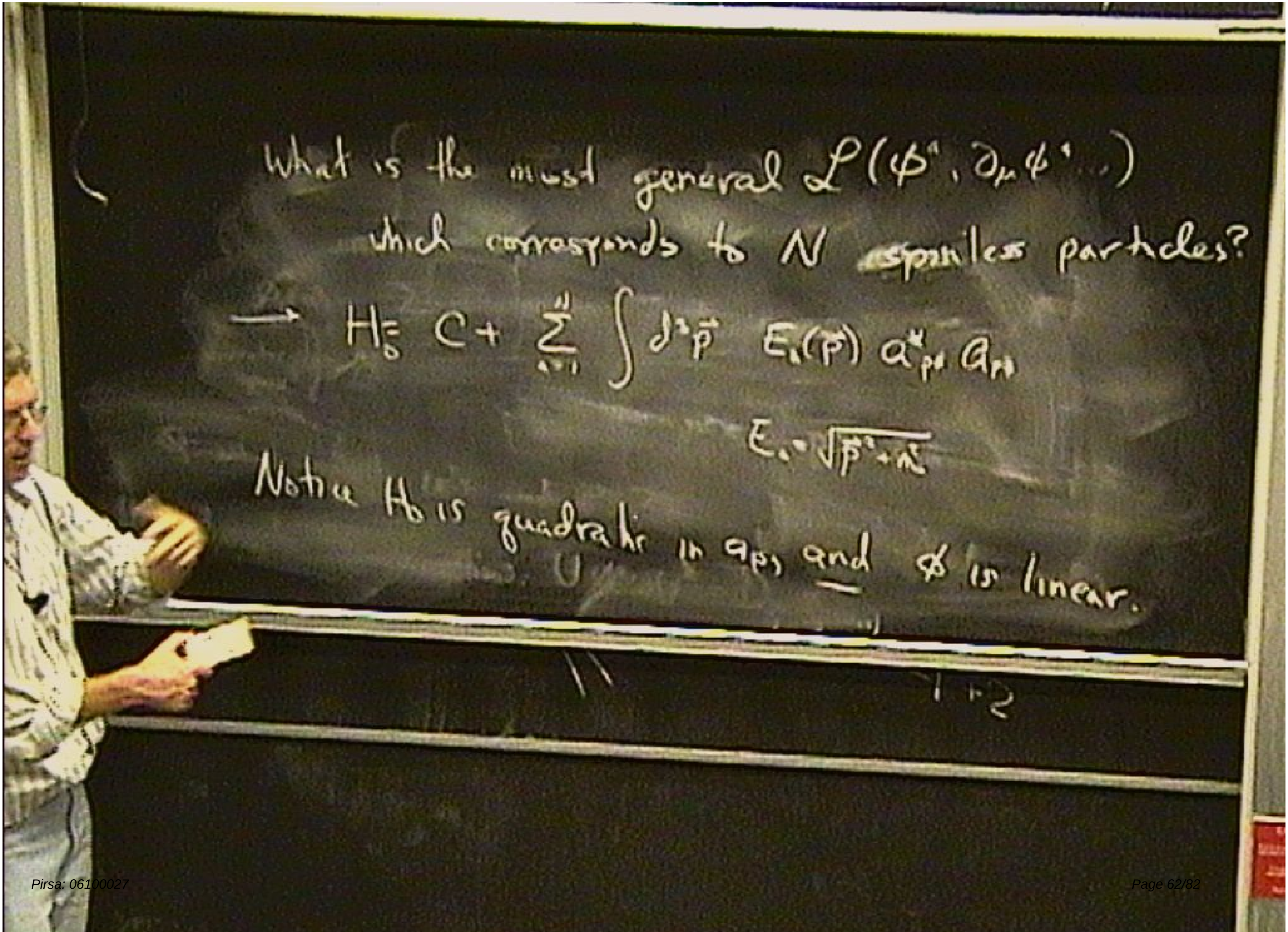
Notice H_0 is quadratic in a_p.


What is the most general $\mathcal{L}(\phi^a, \partial_\mu \phi^a \ldots)$
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$\longrightarrow H_0 = C + \sum_{n=1}^{N} \int d^3\vec{p}\; E_n(\vec{p})\, a^\dagger_{pn} a_{pn}$
$E_n = \sqrt{\vec{p}^2 + m_n^2}$
Notice H_0 is quadratic in a_p's and ϕ is linear.

Ⓐ $u, v = \frac{1}{\sqrt{(2\pi)^3 2E_p}}$ $\qquad \phi^a(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[e^{ipx} a_p + e^{-ipx} a_p^{b\dagger} \right]$

$\phi^b(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[e^{ipx} \hat{a}_p + e^{-ipx} \hat{a}_p^{\dagger} \right]$

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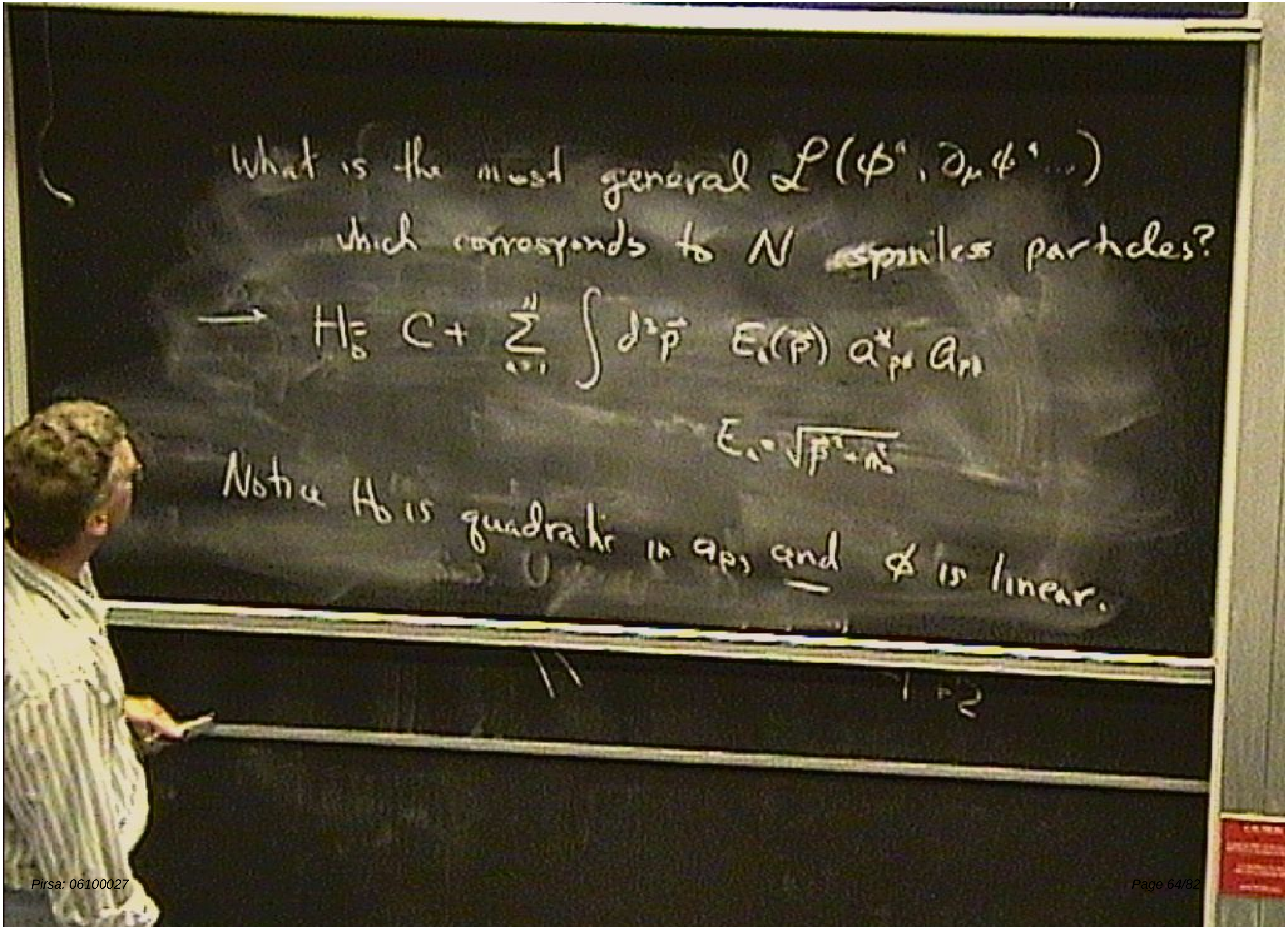
if $\bar{a}_p = a_p$ then $\phi = \phi^*$

Claim: There is no loss in generality in using only real fields provided you use enough: eg Given $\phi \neq \phi^\dagger$: $\phi_1 = \phi + \phi^\dagger$, $i\phi_2 = \phi - \phi^\dagger$.


What is the most general $\mathcal{L}(\phi^a, \partial_\mu \phi^a \ldots)$
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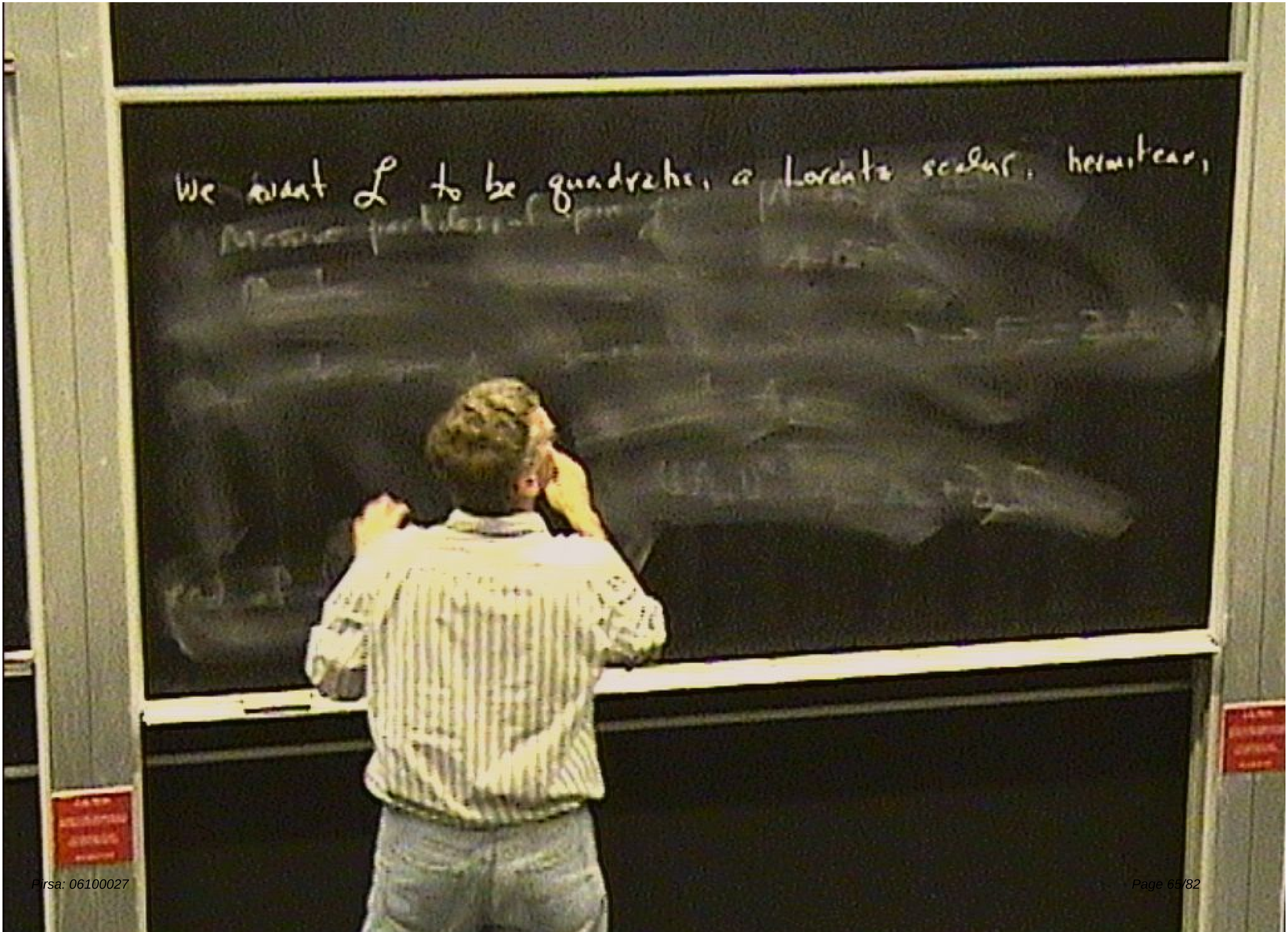
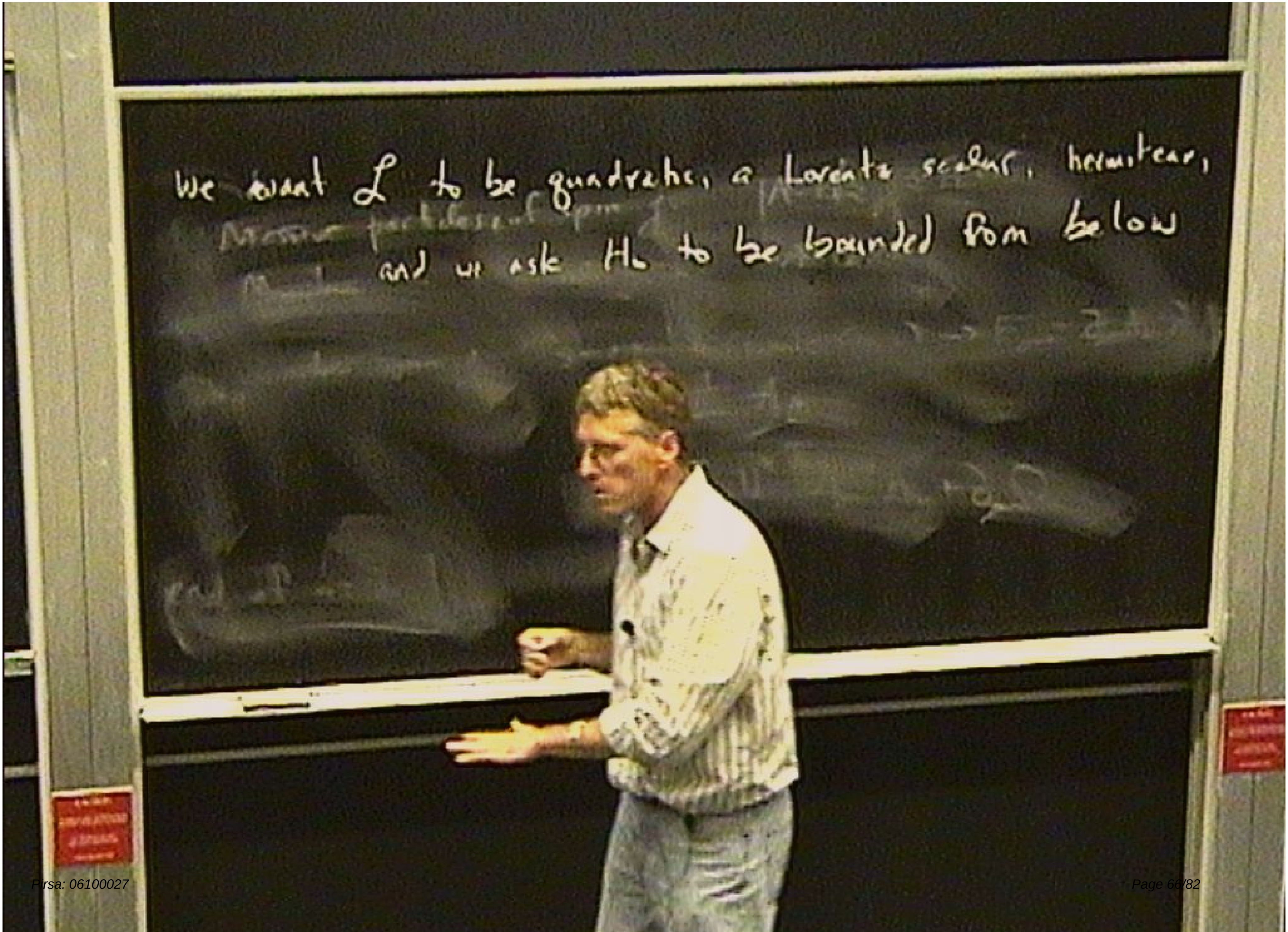
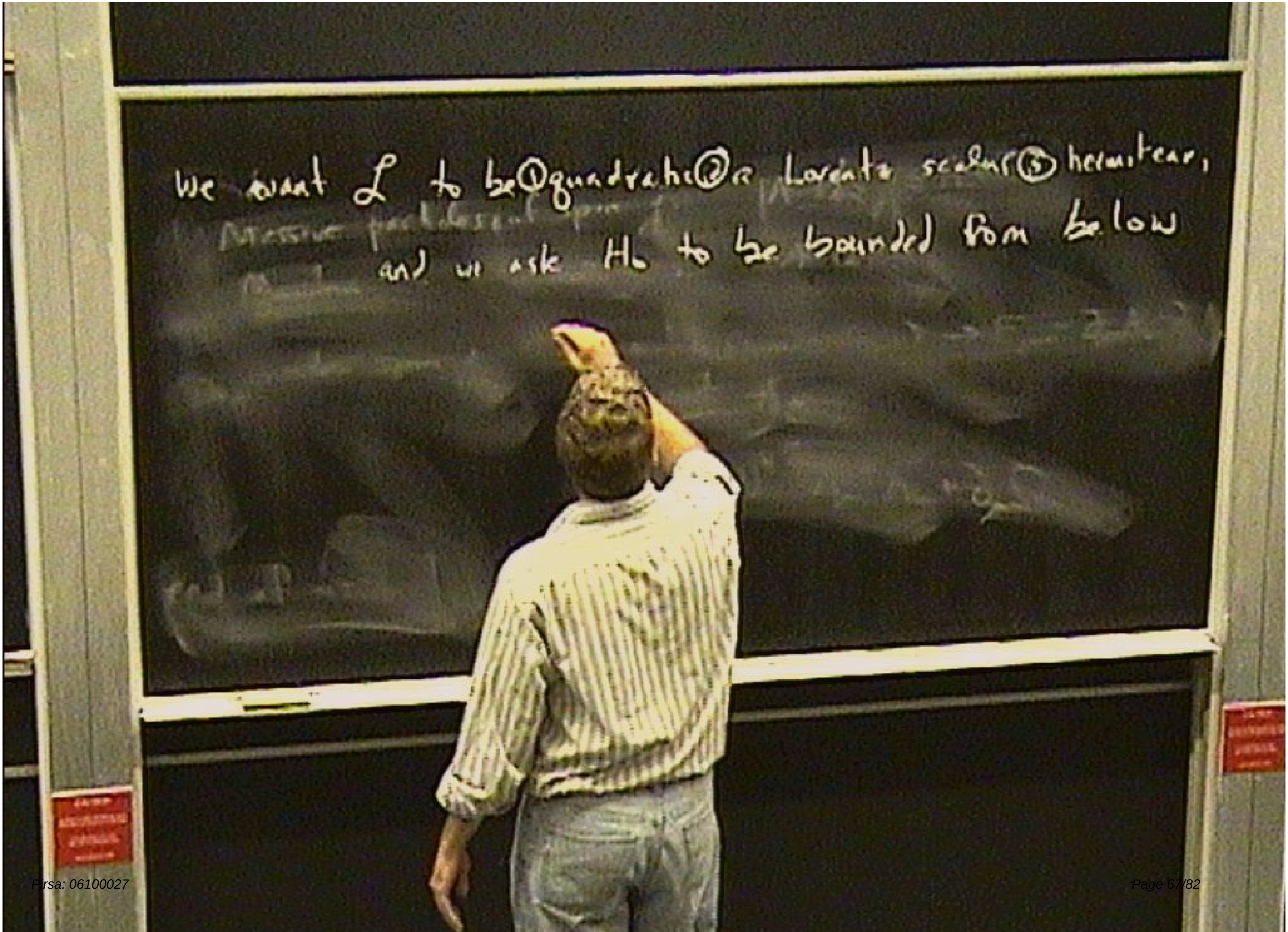
Notice H_0 is quadratic in a_p, and ϕ is linear.


We want $\mathcal{L}$ to be quadratic, a Lorentz scalar, hermitean,


We want $\mathcal{L}$ to be quadratic, a Lorentz scalar, hermitean,
and we ask H_0 to be bounded from below


We want $\mathcal{L}$ to be (1) quadratic (2) a Lorentz scalar (3) hermitean,
and we ask H_0 to be bounded from below

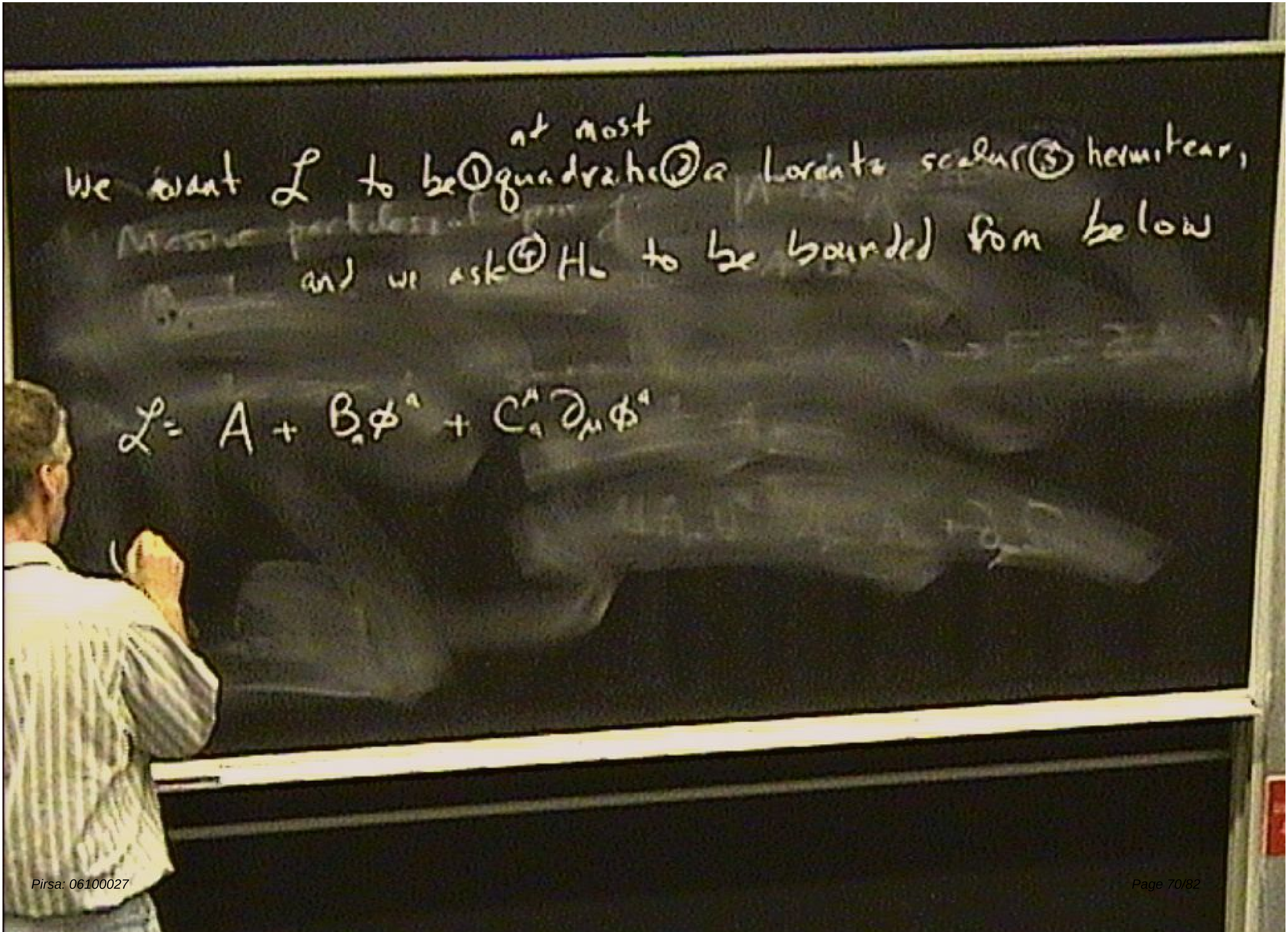
We want $\mathcal{L}$ to be (1) quadratic (2) a Lorentz scalar (3) hermitean,

and we ask (4) H_0 to be bounded from below

We want $\mathcal{L}$ to be (1) quadratic (2) a Lorentz scalar (3) hermitean,

and we ask (4) $H_{\mathcal{L}}$ not to be bounded from below

$$\mathcal{L} = A +$$

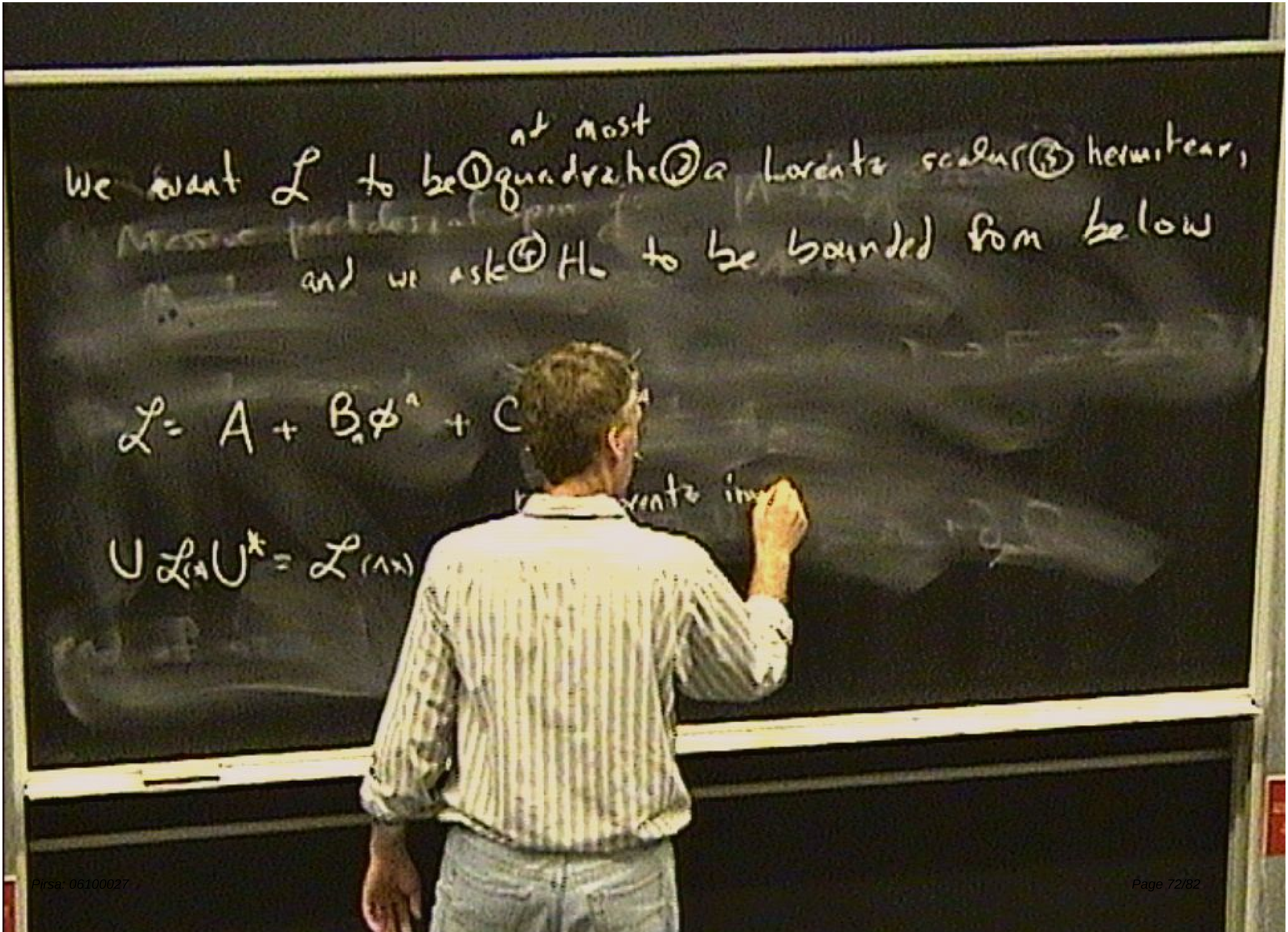

We want $\mathcal{L}$ to be ① at most quadratic ② a Lorentz scalar ③ hermitean,
and we ask ④ H to be bounded from below
$\mathcal{L} = A + B_a \phi^a + C^\mu_a \partial_\mu \phi^a$

We want $\mathcal{L}$ to be (1) at most quadratic (2) a Lorentz scalar (3) hermitean,

and we ask (4) H_0 to be bounded from below

$$\mathcal{L} = A + B_a \phi^a + C^\mu_a \partial_\mu \phi^a$$

$$U \mathcal{L}(x) U^\dagger = \mathcal{L}(\Lambda x)$$


at most
We want $\mathcal{L}$ to be (1) quadratic (2) a Lorentz scalar (3) hermitean,
and we ask (4) H_0 to be bounded from below
$\mathcal{L} = A + B_a \phi^a + C$
Lorentz inv
$U \mathcal{L}(x) U^* = \mathcal{L}(\Lambda x)$

We want $\mathcal{L}$ to be (1) at most quadratic (2) a Lorentz scalar (3) hermitean,

and we ask (4) H to be bounded from below

$$\mathcal{L} = A + B_a \phi^a + \cancel{C^\mu_a \partial_\mu \phi^a}$$

$\rightarrow$ not Lorentz invariant

$$U \mathcal{L}(x) U^* = \mathcal{L}(\Lambda x) \rightarrow$$

We want $\mathcal{L}$ to be ① at most quadratic ② a Lorentz scalar ③ hermitean,

and we ask ④ H_0 to be bounded from below

$$\mathcal{L} = A + B_n \phi^n + \cancel{C^m_n \partial_m \phi^n}$$

$\rightarrow$ not Lorentz invariant

$$U \mathcal{L}(x) U^* = \mathcal{L}(\Lambda x) \quad \rightarrow$$

We want $\mathcal{L}$ to be (1) at most quadratic (2) a Lorentz scalar (3) hermitean,

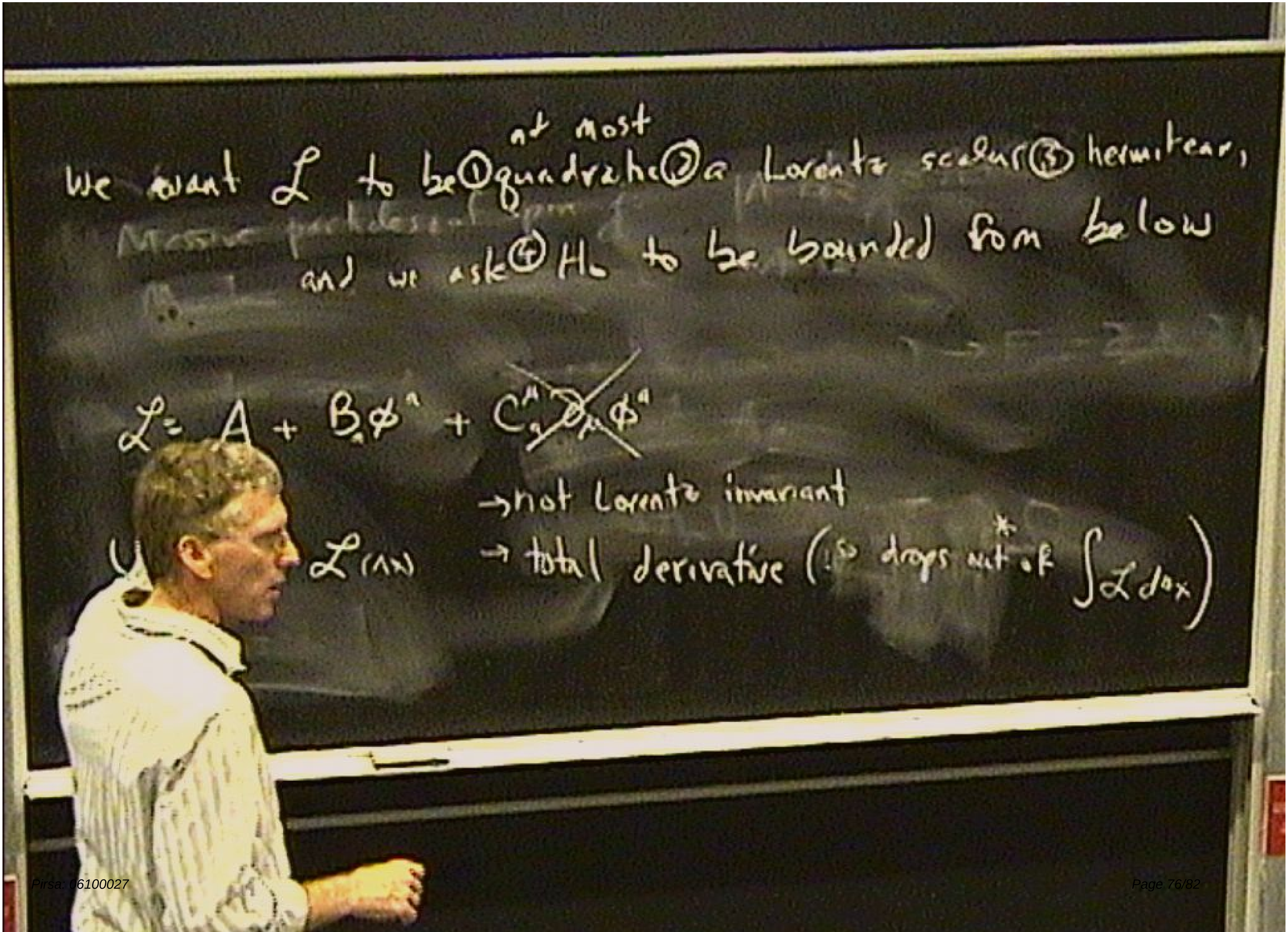
and we ask (4) H_0 to be bounded from below

$$\mathcal{L} = A + B_a \phi^a + \cancel{C^\mu_a \partial_\mu \phi^a}$$

→ not Lorentz invariant

$$U \mathcal{L}(x) U^* = \mathcal{L}(\Lambda x)$$

→ total derivative


We want L to be ① at most quadratic ② a Lorentz scalar ③ hermitean,
and we ask ④ H to be bounded from below
L = A + B_a φ^a + C^μ_a ∂_μ φ^a
→ not Lorentz invariant
L(Λx)
→ total derivative (so drops out of ∫ L d^4x)

We want $\mathcal{L}$ to be (1) at most quadratic (2) a Lorentz scalar (3) hermitean,

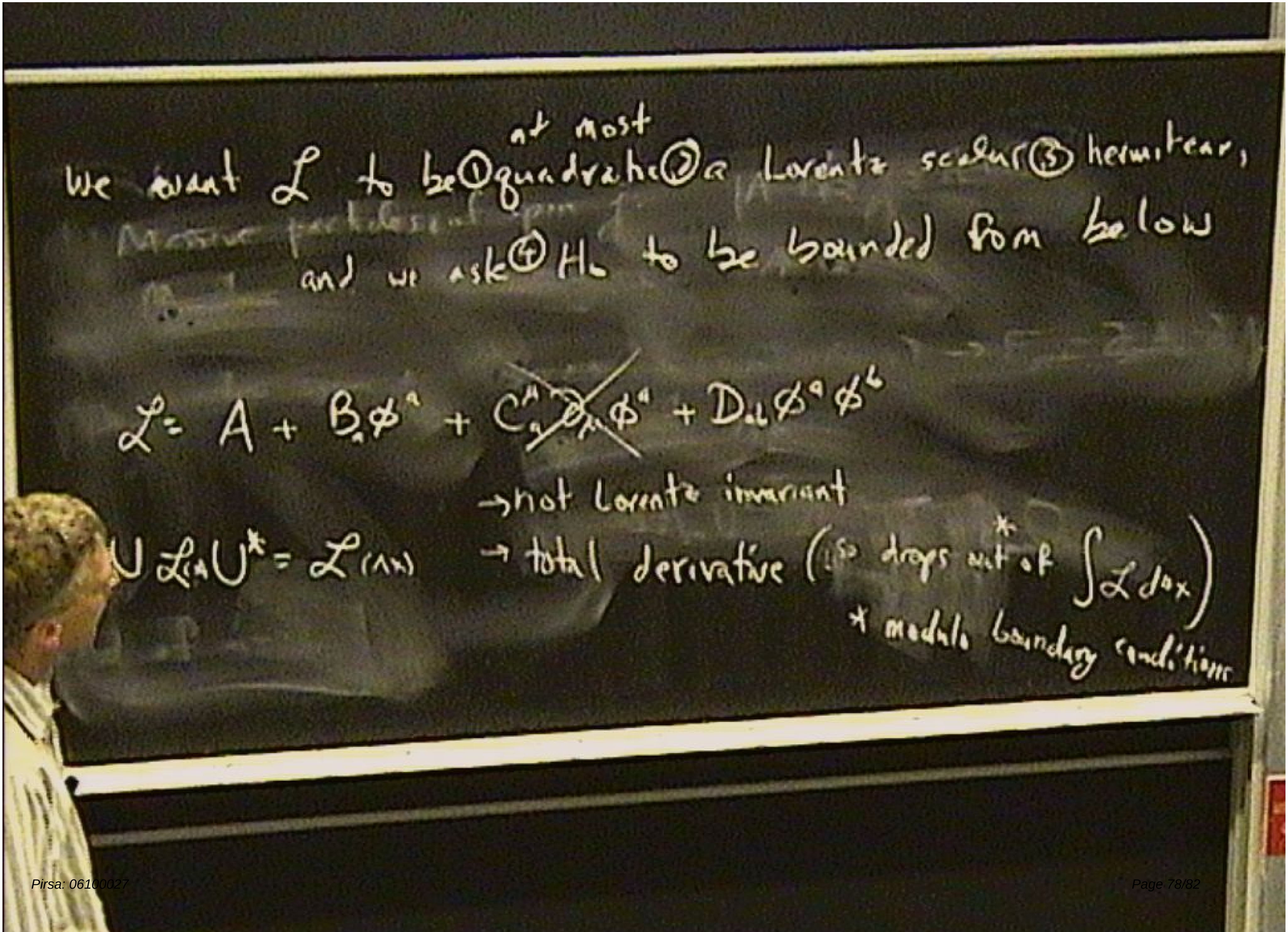
and we ask (4) H_L to be bounded from below

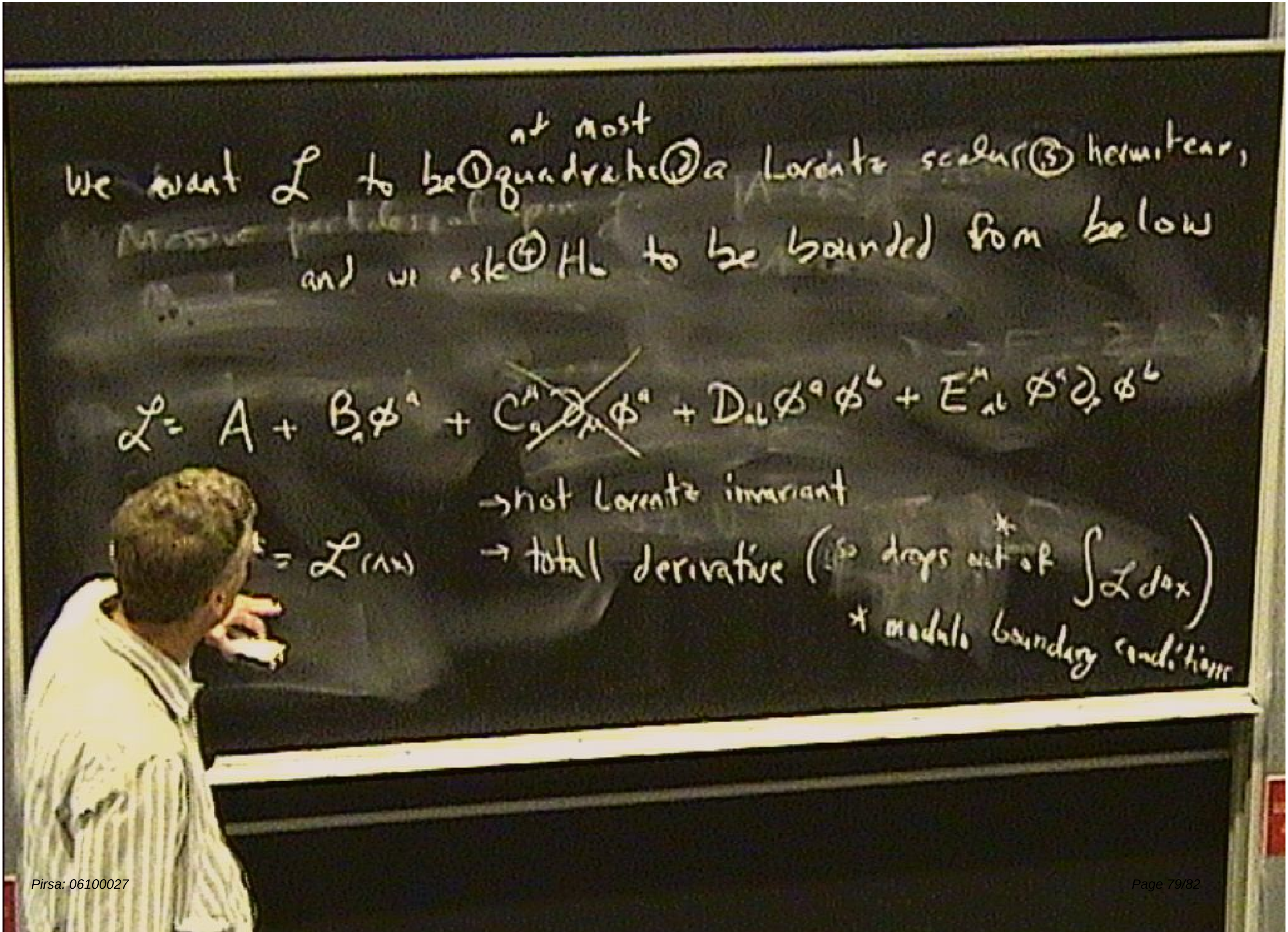
$$\mathcal{L} = A + B_a \phi^a + \cancel{C^\mu_a \partial_\mu \phi^a}$$

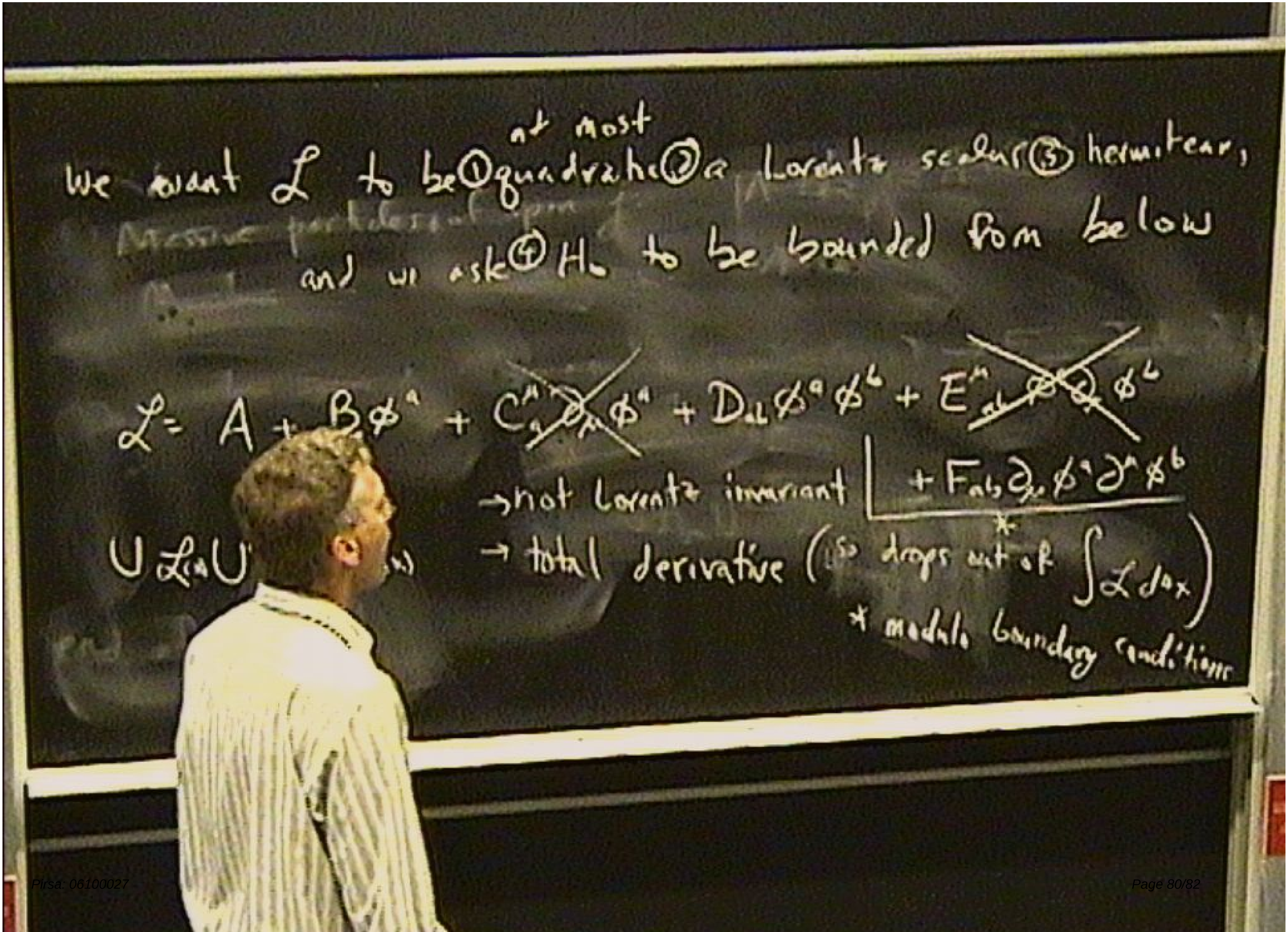
$\rightarrow$ not Lorentz invariant

$U \mathcal{L}(x) U^* = \mathcal{L}(\Lambda x)$ $\rightarrow$ total derivative (so drops out of* $\int \mathcal{L} \, d^4x$)

* modulo boundary conditions


We want L to be ① at most quadratic ② a Lorentz scalar ③ hermitean,
and we ask ④ H₀ to be bounded from below
L = A + B_a ϕ^a + C^μ_a ∂_μ ϕ^a + D_ab ϕ^a ϕ^b
→ not Lorentz invariant
U L(x) U* = L(Λx)
→ total derivative (so drops out* of ∫ L d⁴x)
* modulo boundary conditions


We want ℒ to be ① at most quadratic ② a Lorentz scalar ③ hermitean,
and we ask ④ H₀ to be bounded from below
ℒ = A + B_a φ^a + C^μ_a ∂_μ φ^a + D_ab φ^a φ^b + E^μ_ab φ^a ∂_μ φ^b
→ not Lorentz invariant
= ℒ(Λx) → total derivative (drops out* of ∫ℒ d⁴x)
* modulo boundary conditions


We want $\mathcal{L}$ to be ① at most quadratic ② a Lorentz scalar ③ hermitian,
and we ask ④ H_0 to be bounded from below
$\mathcal{L} = A + B\phi^a + \cancel{C^\mu_a \partial_\mu \phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\phi^a \partial_\mu \phi^b}$
$+ F_{ab}\partial_\mu \phi^a \partial^\mu \phi^b$
→ not Lorentz invariant
→ total derivative (so drops out of $\int \mathcal{L}\, d^4x$)
* modulo boundary conditions

We want $\mathcal{L}$ to be ① at most quadratic ② a Lorentz scalar ③ hermitean,

and we ask ④ H_0 to be bounded from below

$$\mathcal{L} = A + B_a\phi^a + \cancel{C^\mu_a \partial_\mu \phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\partial_\mu\phi^a}\,\phi^b + F_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$$

→ not Lorentz invariant

$U\mathcal{L}(x)U^* = \mathcal{L}(\Lambda x)$ → total derivative (also drops out of $\int \mathcal{L}\, d^4x$)*

* modulo boundary conditions

$$\partial_\mu = \{\partial/\partial x^\mu\} = \{\partial_t, \vec{\nabla}\}$$

we want $\mathcal{L}$ to be ① quadratic ② a Lorentz scalar ③ hermitian,

and we ask ④ H_0 to be bounded from below

$$\mathcal{L} = A + B_a\phi^a + \cancel{C^\mu_a \partial_\mu \phi^a} + D_{ab}\phi^a\phi^b + \cancel{E^\mu_{ab}\phi^a\partial_\mu\phi^b}$$

→ not Lorentz invariant

$$+ F_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$$

$$U\mathcal{L}(x)U^* = \mathcal{L}(\Lambda x)$$

→ total derivative (also drops out of $\int \mathcal{L}\, d^4x$)*

* modulo boundary conditions

$$\partial_\mu = \left\{\partial/\partial x^\mu\right\} = \left\{\partial_t, \vec{\nabla}\right\}$$

$$\partial_\mu\phi\,\partial^\mu\phi = \eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = -\dot{\phi}^2 + (\nabla\phi)^2$$

$$U\phi^a(x)U^* = \phi^a(\Lambda x)$$