Title: The Missing Link Between Dark Matter And Structure Formation

Date: Oct 18, 2006 02:00 PM

URL: http://pirsa.org/06100015

Abstract: Weakly interacting massive particles (WIMPs) are excellent candidates for cold dark matter. After the first millisecond, WIMPs have decoupled from standard model matter, both chemically and kinetically, they enter the free streaming regime and the formation of cosmic structure begins. Another 40 million years pass before the typical first structures enter the nonlinear regime and collapse to the first WIMPy halos. Therefore, it has been assumed that structure formation is insensitive to the WIMP field theory and can be neglected. However,
this leads to a monotonically increasing power of structure formation on small scales and some kind of regularization procedure would be required to make the hierarchical picture of structure formation well defined. It will be shown that nonequilibrium processes give rise to a physical regularization of hierarchical structure formation. This has important consequences for indirect and direct dark matter searches which are sensitive to sub-galactic and sub-milli-parsec scales. Furthermore, due the existence of a physical regulator, the problem of structure formation can consistently be solved using N-body simulations.

THE 1st WIMPy HALOs

with :

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MOTIVATION

Folklore from large scale structure formation :

$$\frac{\delta\epsilon}{\epsilon} \equiv \Delta(k,z) = T_{\Delta}^{1/2}(k,z)\, \Delta(k,z_i)$$

Continuation to small scales ?

Problem : 'small scale structure crisis'

$$\Delta \propto \ln(k/k_{eq}) \sim \ln(M_{eq}/M(k))$$

- monotonically increasing power of density fluctuations on small scales
- impossible to consistently solve structure formation

What is missing ?

'small scale structure crisis' is a serious problem because

- resolution of numerical experiments (until recently)

  $$M_{res} \approx 10^6 M_\odot$$

  (Stoehr et al. 2003, astro-ph/0307026)

- # substructures is growing with resolution

  (Moore et al. 1998, ApJ 499, L5-L8)

- real experiments, e.g.

  $$\Phi_\gamma = \text{diffuse flux} + \underbrace{\text{line contribution}}_{\propto \Delta^2}$$

  (Bergström et al. 2001, PRL 87, 251301;
  Ullio et al. 2002, PRD 66, 123501;
  Bergström et al. 2005, JCAP 0504:004)

Need for a consistent theory of small scale structure formation !

WIMPs

Weakly Interacting Massive Particles are generic ($\cong$ natural in extensions of the Standard Model) CDM candidates.

Assumptions:

(1) $\nexists$ WIMP anti WIMP asymmetry

(2) WIMPs have been in chemical and thermal equilibrium for $T \gg m$

(3) $\sigma_0^{el} \approx (G_F\, m_W^2)^2\, m^2/m_Z^4$

■ Chemical decoupling

$$\frac{m}{T_{cd}} \equiv x_{cd}(m,\omega) \approx 23 + \ln\left(\frac{m}{100\,\text{GeV}}\,\frac{5}{\omega}\,\frac{g_*|_0}{g_*|_{cd}}\right)$$

■ Kinetic decoupling $\neq$ last scattering

$$\frac{m}{T_{kd}} \equiv x_{kd}(m,\tau_{relax}) \approx \left[7\cdot 10^{13}\,(g_e|_{kd})^{1/2}\left(\frac{m}{100\,\text{GeV}}\right)^3\right]^{\frac{1}{3+\ell}}$$

For $\ell = 0$ (Dirac) $T_{kd} \approx 2{,}4\,(g_e|_{kd})^{-1/6}$ MeV

For $\ell = 1$ (Majorana) $T_{kd} \approx 34{,}2\,(g_e|_{kd})^{-1/8}$ MeV $\left(\frac{m}{100\,\text{GeV}}\right)^{1/4}$

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HYDRODYNAMICS

- $T \gg T_{cd}$: single radiation fluid

$$J_{rad}^{(0)} = n_{rad}\, U \quad , \quad T_{rad}^{(0)} = \epsilon_{rad}\, U \otimes U - P_{rad}\, h$$

- $T_{cd} > T > T_{kd}$: radiation $\xleftrightarrow[\text{coupled}]{\text{tightly}}$ CDM

$$J_a^{(0)} \;,\; T_a^{(0)} \quad , \quad a \in \{ rad,\, cdm \}$$

- $T \sim T_{kd}$: radiation $\xleftrightarrow[\text{decoupled}]{\not\;}$ CDM

$$J_{cdm} = J_{cdm}^{(0)} + \underbrace{\cancel{J_{cdm}^{(1)}}}_{\text{choice}} \quad , \quad T_{cdm} = T_{cdm}^{(0)} + T_{cdm}^{(1)}$$

$T_{cdm}^{(1)} = \zeta \Big($  $\Big) +$

bulk viscosity

$\eta \Big($  $\Big)$

shear viscosity

KINETIC THEORY

- $T \sim T_{kd}$: WIMP phase space distribution

$$F = F^{(0)} + F^{(1)} \quad \text{with} \quad |F^{(1)}| \ll F^{(0)}$$

non-equilibrium

$$(p \cdot \nabla) F^{(0)} = \mathcal{L}\left[F^{(1)}\right] \quad , \quad p = \omega\, \mathcal{U} + |\vec{p}|\, n$$

Ansatz :

$$F^{(1)} = A(\omega, x) + B(\omega, x) \cdot n + C(\omega, x) \cdot \left(n \otimes n + \frac{1}{3} h\right) + \cdots$$

The 1st and 2nd moments of $F^{(1)}$ are

$$J^{(1)} = 0$$

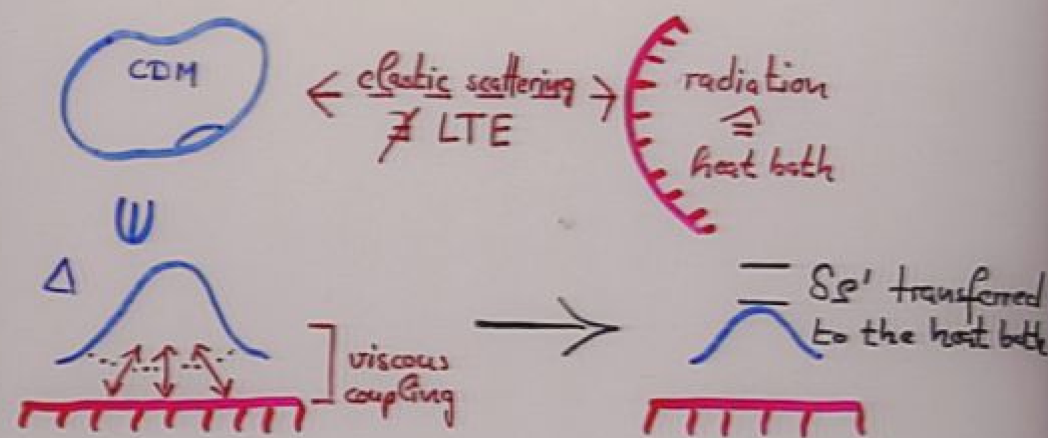
$$T^{(1)} = \epsilon\, c_s^2\, \tau_{relax} \cdot$$

$\Big[\frac{5}{3}$  +  $\Big]$

KINETIC DECOUPLING

Effect of ζ, η on Δ:

collisional damping at $T \sim T_{kd}$



$$\Rightarrow \Delta'' + \frac{\zeta + \frac{4}{3}\eta}{\epsilon}\frac{k^2}{a}\Delta' + c^2 k^2 \Delta = 0$$

$$\rightsquigarrow D_d(k) \equiv \left|\frac{\Delta(k, z_{kd})}{\Delta(k, z_i)}\right| = \exp\left[-\left(\frac{k}{k_d}\right)^2\right]$$

characteristic damping wavenumber:

$$k_d = \frac{3{,}76 \cdot 10^7}{\mathrm{Mpc}} \left(\frac{m}{100\,\mathrm{GeV}}\right)^{1/2} \left(\frac{T_{kd}}{30\,\mathrm{MeV}}\right)^{1/2}$$

FREE STREAMING

Effect of geodesic motion on Δ:

collisionless damping at $T < T_{kd}$

$$\hat{=}$$

$$(p\cdot\nabla)\,F = 0 \quad \wedge \quad F\big|_{kd} \propto D_d(k)$$

$$\rightarrow \quad D_{fs}(k,z) \equiv \frac{\Delta(k,z)}{\Delta(k,z_{kd})}$$

$$= \left[1-\frac{2}{3}\left(\frac{k}{k_{fs}}\right)^2\right]\exp\left[-\left(\frac{k}{k_{fs}}\right)^2\right]$$

Characteristic damping wavenumber:

$$k_{fs} = \frac{1{,}7\cdot 10^6}{Mpc}\;\frac{(m_\chi/100\,GeV)^{1/2}\,(T_{kd}/30\,MeV)^{1/2}}{1+\ln(T_{kd}/30\,MeV)\,/\,19{,}2}$$

BENCHMARK WIMP MODELS

So far,

$$\Delta(k,z) = T_\Delta^{1/2}(k,z)\, D(k)\, \Delta(k,z_i)$$

$$\propto \ln(k/k_{eq}) \exp[-(k/k_d)^2]$$

Contribution from CDM microphysics:

$$D(k) = \left[1 - \frac{2}{3}\left(\frac{k}{k_{fs}}\right)^2\right] \exp\left[-\left(\frac{k}{k_{fs}}\right)^2 - \left(\frac{k}{k_d}\right)^2\right]$$

Scales:

$$k_d \sim 10^7 \, Mpc^{-1} \hat{\approx} 10^{-2} H_{kd}^{-1} \hat{\approx} 10^{-10} M_\odot$$

$$k_{fs} \sim 10^6 \, Mpc^{-1} \hat{\approx} 10^{-8} H_{eq}^{-1} \hat{\approx} 10^{-6} M_\odot$$

TRANSFERFUNCTION

- **Baryons:**

  $z > 10^6$: baryons are tightly coupled to photons
  $\Rightarrow$ photon diffusion damping erases baryon density perturbations

  $z \sim 10^4$: tight coupling breaks down, but
  $$\Delta_b \ll \Delta_{cdm}$$

  $10^3 > z > z_b \sim 150$: post decoupling residual electrons prevent Δ_b from growing on scales $k > k_b \sim 10^3\, Mpc^{-1}$

$\Rightarrow$ neglect perturbations in the baryon fluid

- **CDM:**

$$\Delta_{cdm} = 6\, \zeta_0\, c \left[\ln(k/k_{eq}) + b\right] \left(\frac{1+z_{eq}}{1+z}\right)^{\nu/2}$$

Note: $\Delta_{cdm}(z=300;\, \omega_{cdm} = 0{,}116;\, \omega_b = 0{,}024)$

$\approx 0{,}4\ \Delta_{cdm}(z=300;\, \omega_{cdm} = 0{,}14;\, \omega_b = 0{,}00)$

$\Delta = T^{th} \mathcal{D}$

$$\Delta = T^{1h} \, \mathcal{D} \, \Delta_i$$

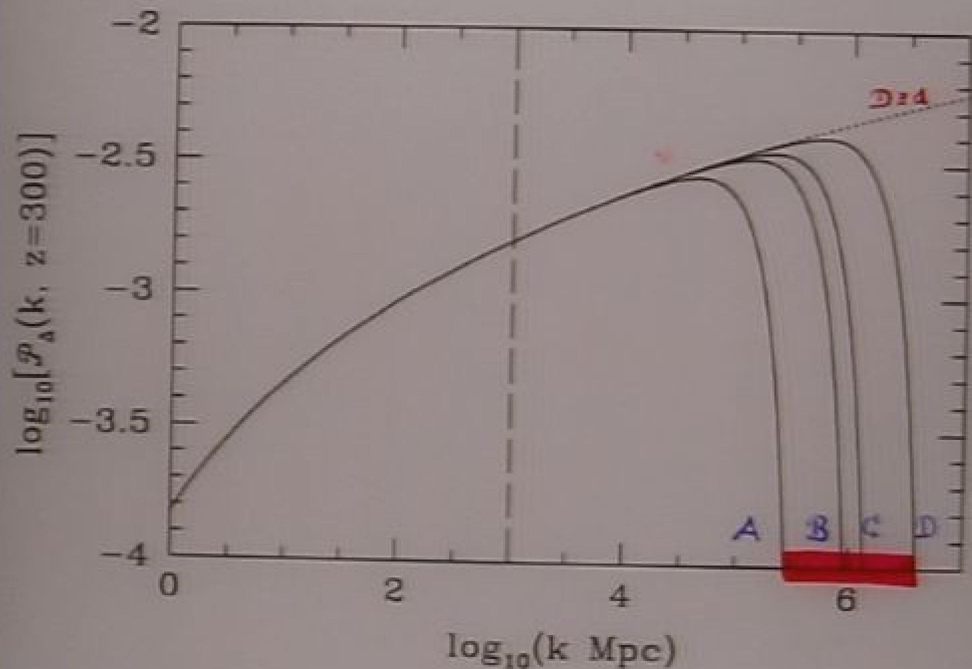
$$\Delta = T^{1/2} D \, \Delta_i$$

$$P(k, z) = \frac{k^3}{2\pi^2} |\Delta|^2$$

$\mathcal{P}_\Delta$OWER SPECTRUM

For $k > k_b$ and $z_{eq} \gg z \gg z_b$:

$$\frac{\mathcal{P}_\Delta(k,z)}{10^{-7}A} = 1{,}06\, c^2 \left[\ln\left(\frac{k}{k_{eq}}\right) + b \right] D^2(k) \left(\frac{1+z_{eq}}{1+z}\right)^{\nu}$$



Parameters :

$k_{eq} = \frac{10^{-2}}{\text{Mpc}} \frac{\omega_m}{0{,}14}$, $1+z_{eq} = 3371 \frac{\omega_m}{0{,}14}$

$\omega_{cdm} = 0{,}116$, $f_b \equiv \frac{\omega_b}{\omega_m} = 0{,}17$ (WMAP best fit)

$n = 1$

TYPICAL 1st HALOS

Mass variance on comoving scale R :

$$\sigma^2(R,z) = \int_0^\infty \frac{dk}{k} W^2(Rk)\, P_\Delta(k,z)$$

Criterium for nonlinear regime :

$$\sigma^2(R, z_{ne}) = 1$$

For WMAP best fit + $n=1$:

$$z_{ne} = 60 \pm 10$$

Mass of the 1st generation of subhalos :

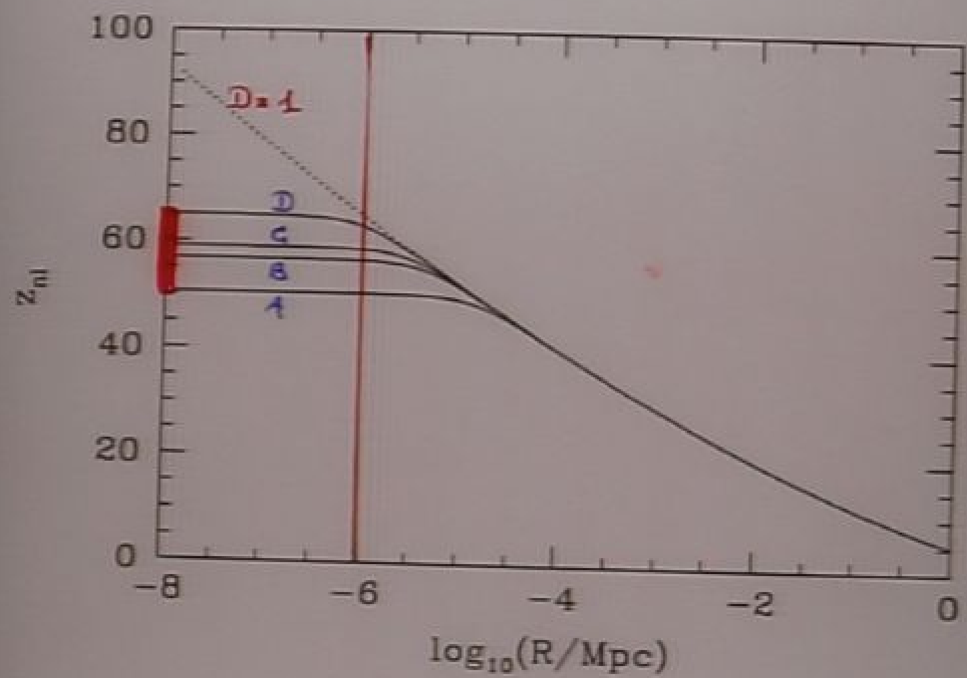
$$M(R) = 21{,}6 \cdot 10^{-7} M_\odot \frac{\omega_m}{0{,}14}\left(\frac{R}{pc}\right)^3 \sim M_♂$$

Physical size at turn-around :

$$r = 1{,}05 \frac{R}{1+z_{ne}^{max}} \sim 0{,}02\ pc \text{ for } R_{min} = 1\,pc$$

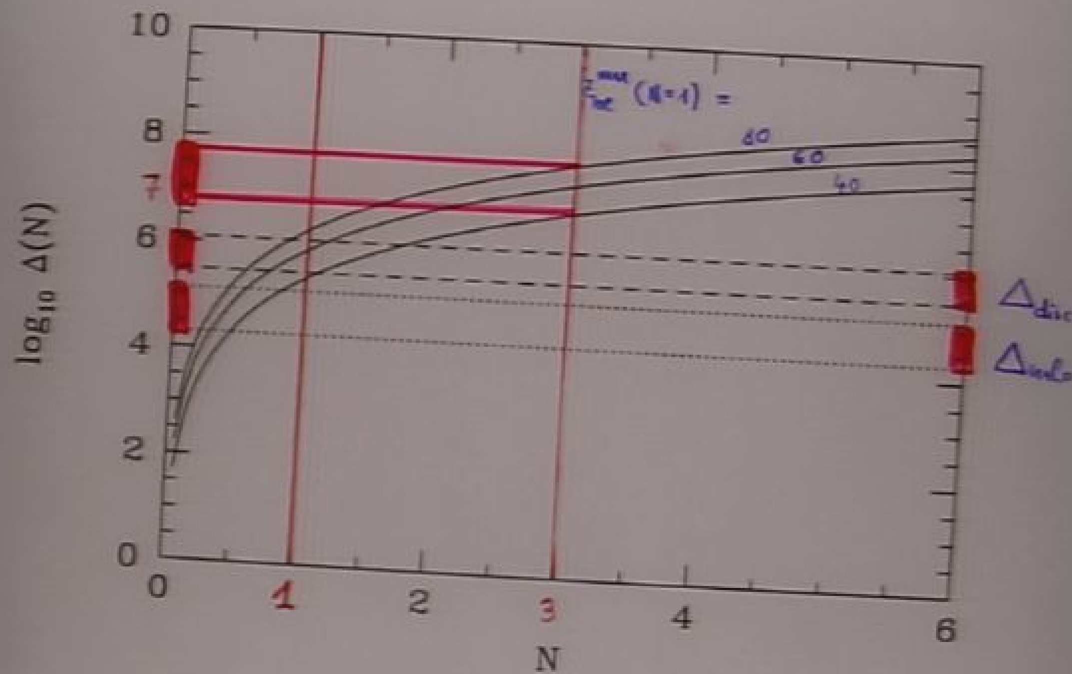
Present day density contrast :

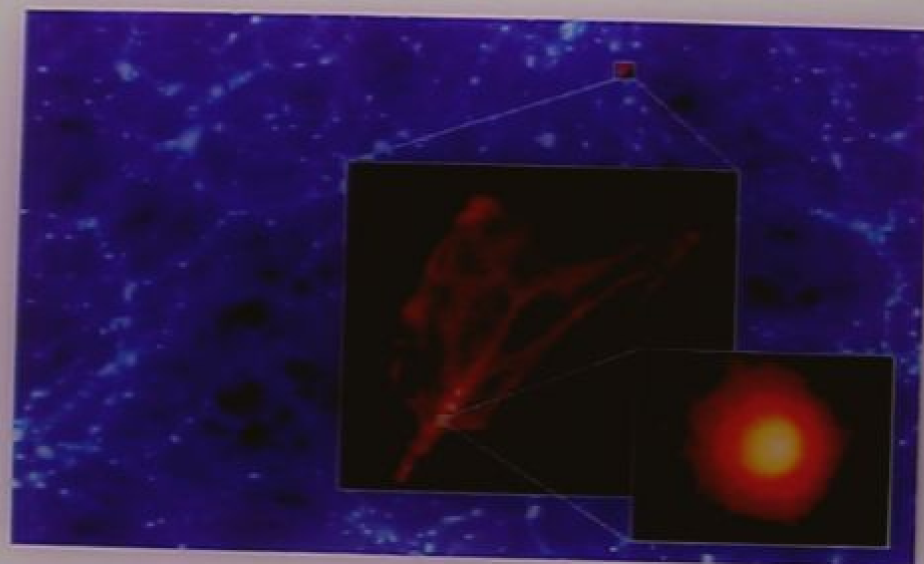
$$\Delta = 3{,}7\,(60 \pm 10)^3 = (0{,}2 - 1{,}8)\, 10^6$$



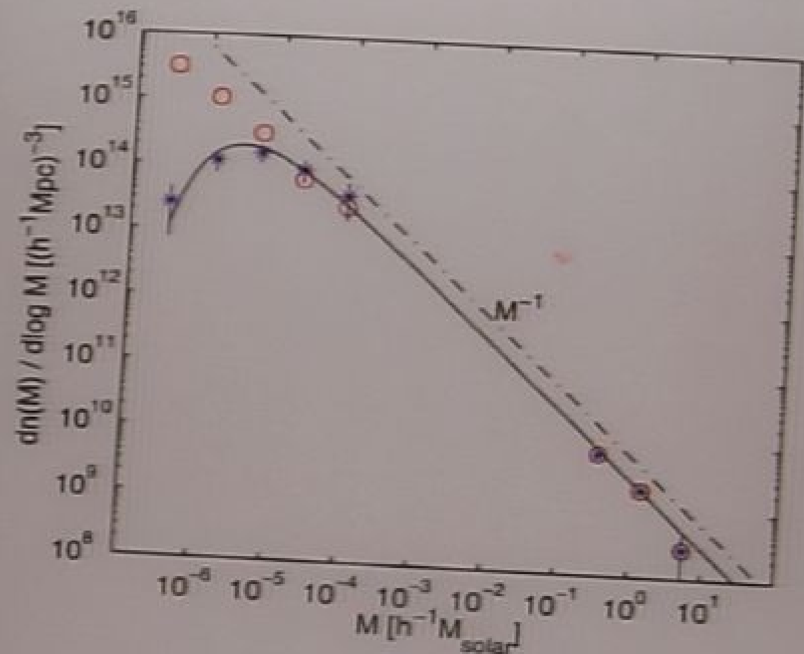
100
80
60
40
20
0
−8
−6
−4
−2
0
z_{nl}
$\log_{10}(R/\mathrm{Mpc})$
D = 1
D
C
B
A

RARE FLUCTUATIONS





J. Diemand, B. Moore and J. Stachel,
Nature 433, 389, 2005



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