Title: How a black hole emerges from a pure state

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Abstract:

How A Black Hole Emerges from A Pure State

Perimeter Institute
October 2006

Masaki Shigemori (Caltech)

V. Balasubramanian, P. Kraus, M.S. [hep-th/0508110]
N. Iizuka, P. Kraus, G. Mandal, S. Shenker, S. Trivedi, M.S., in progress

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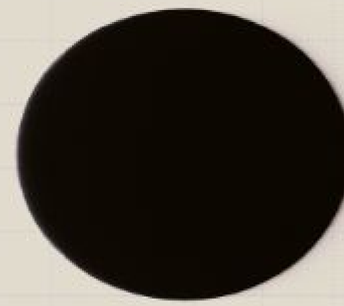
Introduction

Black hole

- BH = solution in classical gravity



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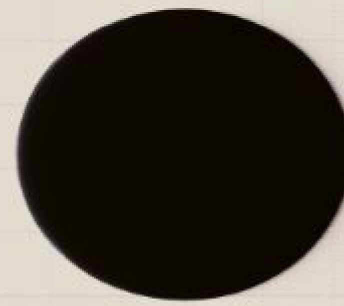

- Entropy

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}}$$

→ Underlying stat. mechanical nature?

→ Where are microstates (hair)?

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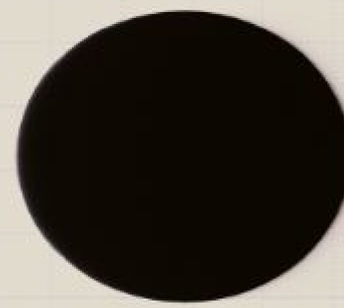

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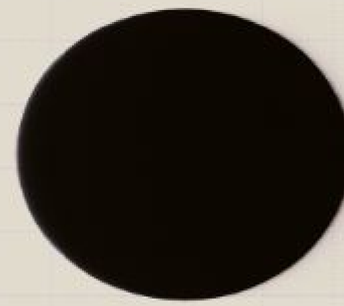

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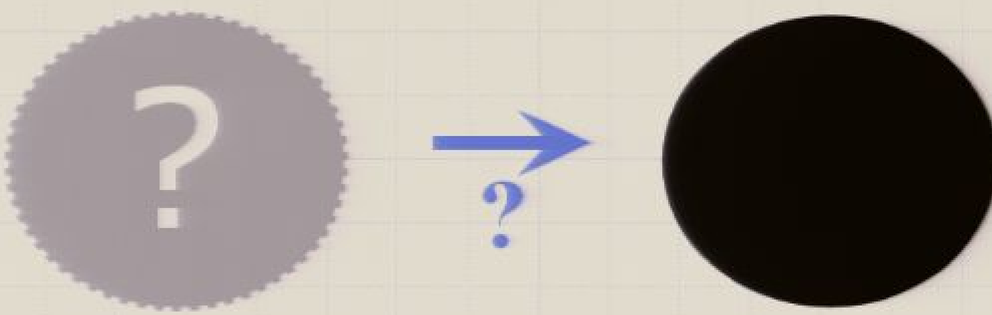
- Classical BH =
  "effective description" after "coarse graining"



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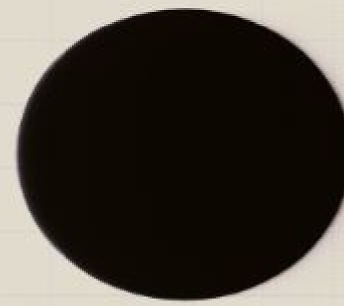

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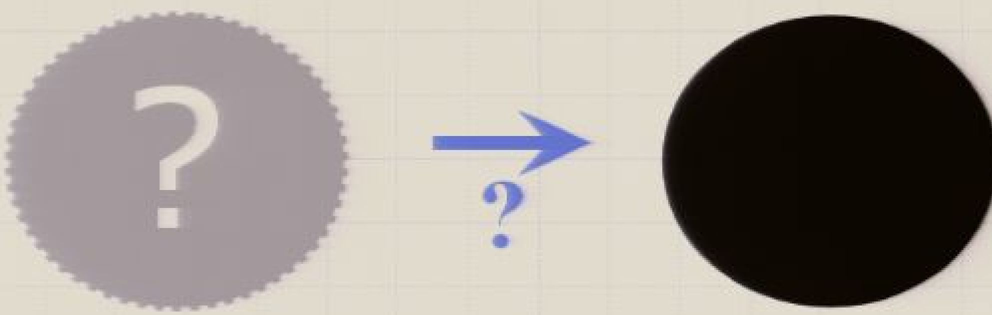

Classical BH

Such a statement doesn't mean much...

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unless one can really do something!

Explicit example: D1-D5 system

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- D1-D5 sys ($S \sim N^{1/2}$): small BH

small BH
[Dabholkar, Sen, ...]

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[Dabholkar, Sen, ...]

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sugra microstates
[Lunin-Mathur]

- Try to "coarse-grain" to get BH

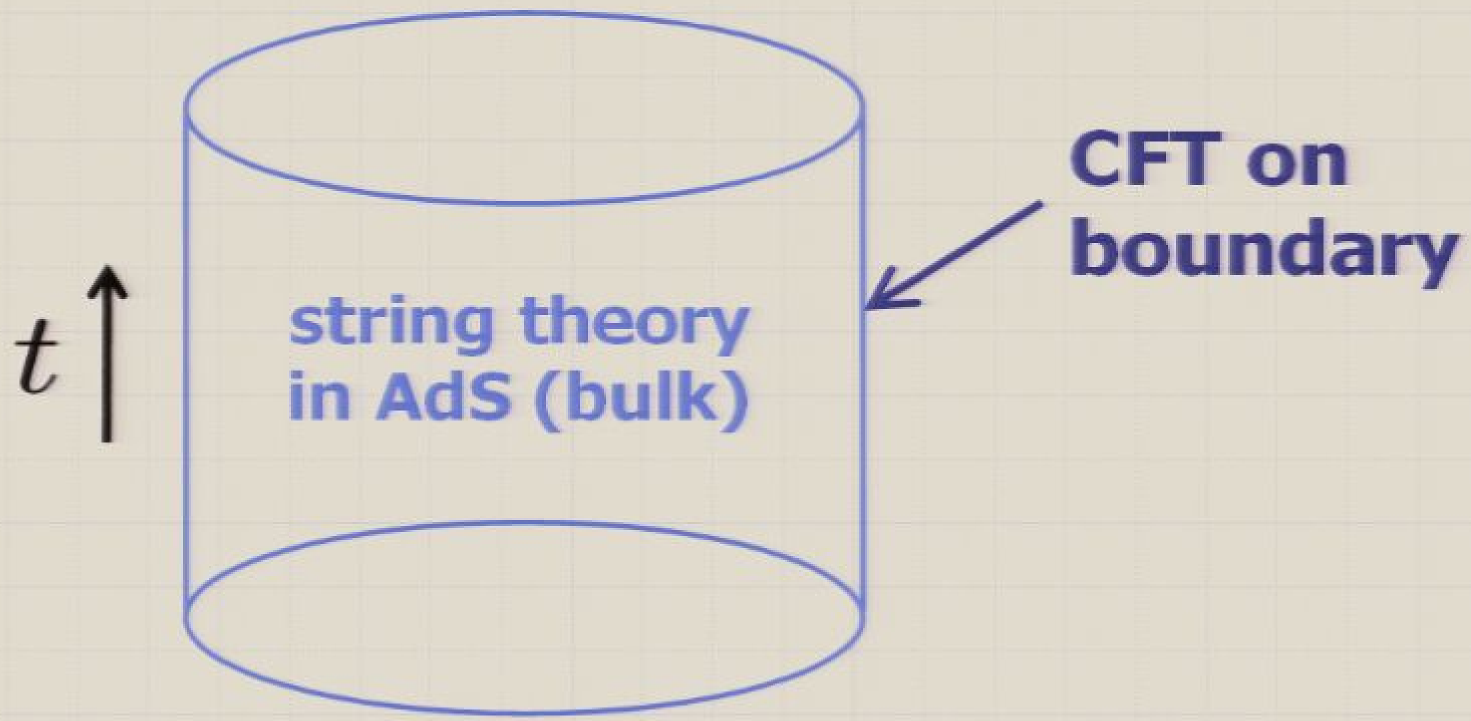
Plan

- Introduction ✓
- AdS/CFT
- D1-D5 system
- Typical States
- The effective geometry
- Conclusion

AdS/CFT

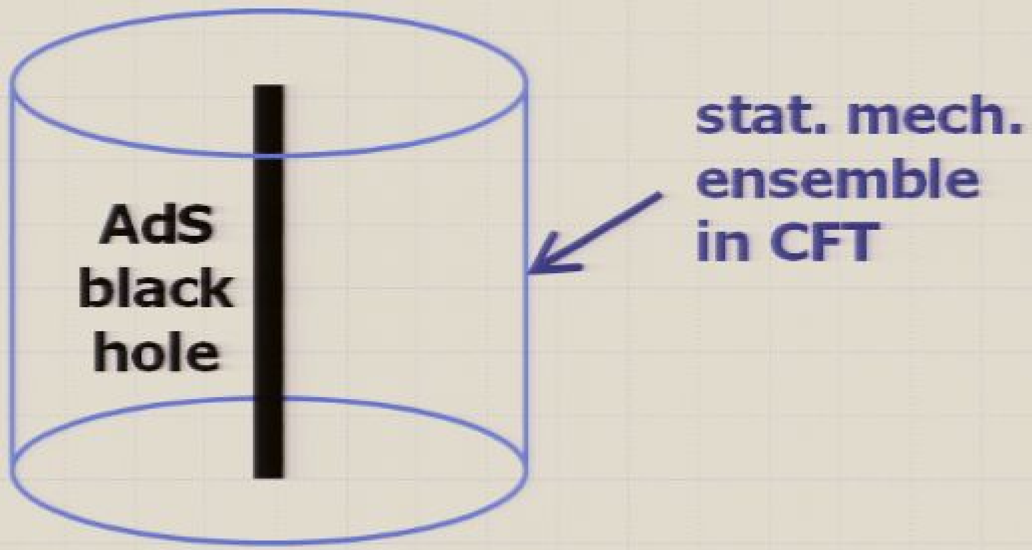
AdS/CFT

- Gravity/string theory in AdS (bulk)
  = CFT on boundary



AdS/CFT and BH

- Black hole ⟷ statistical mechanical ensemble in CFT
  - Can study BH from CFT: Bek.-Hawking entropy, correlation function, …
  - Even valid for "small BH" [Dabholkar 0409148], [DKM], …



Ensemble vs. microstates — CFT side

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Stat. mech. ensemble
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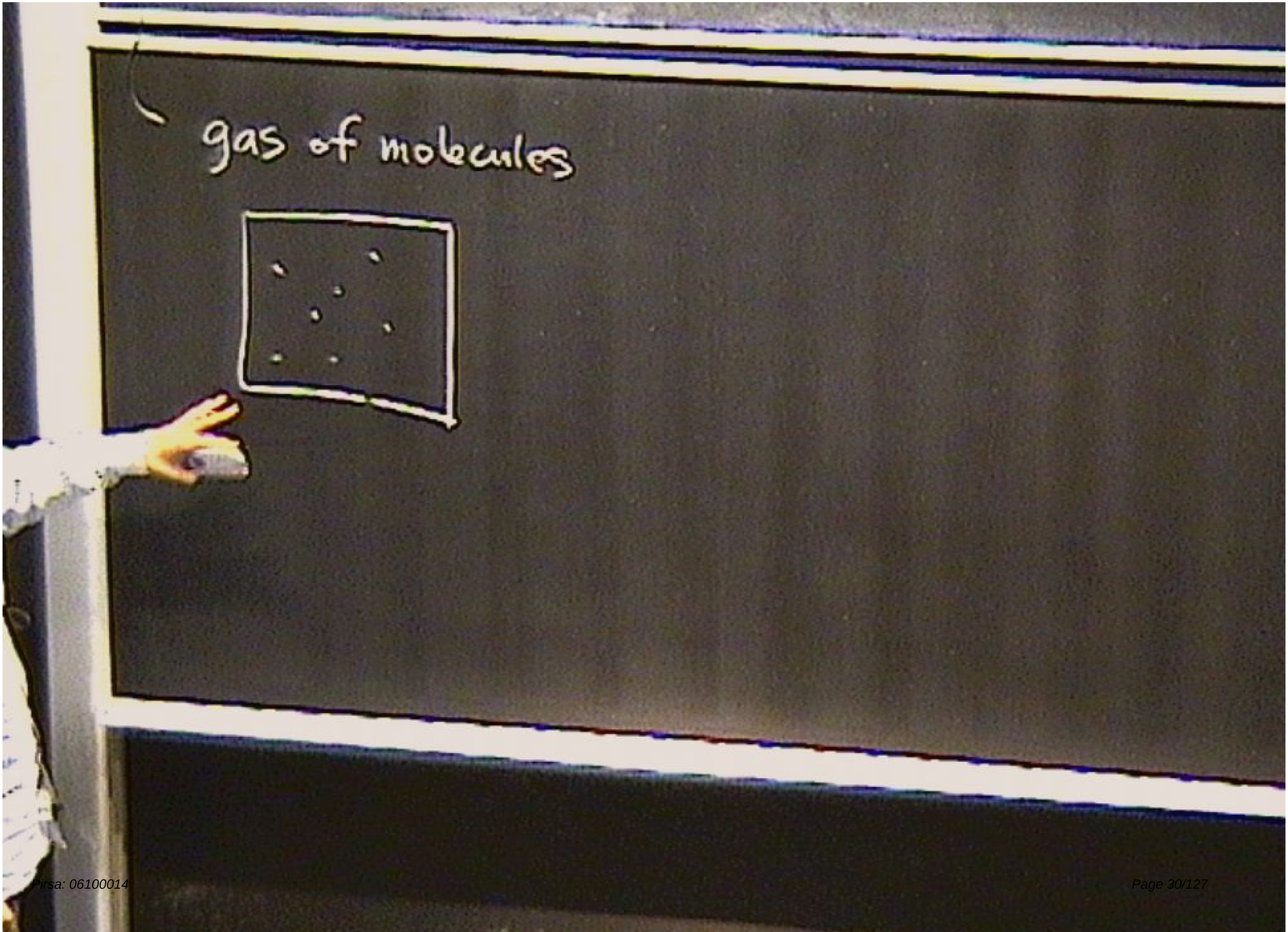
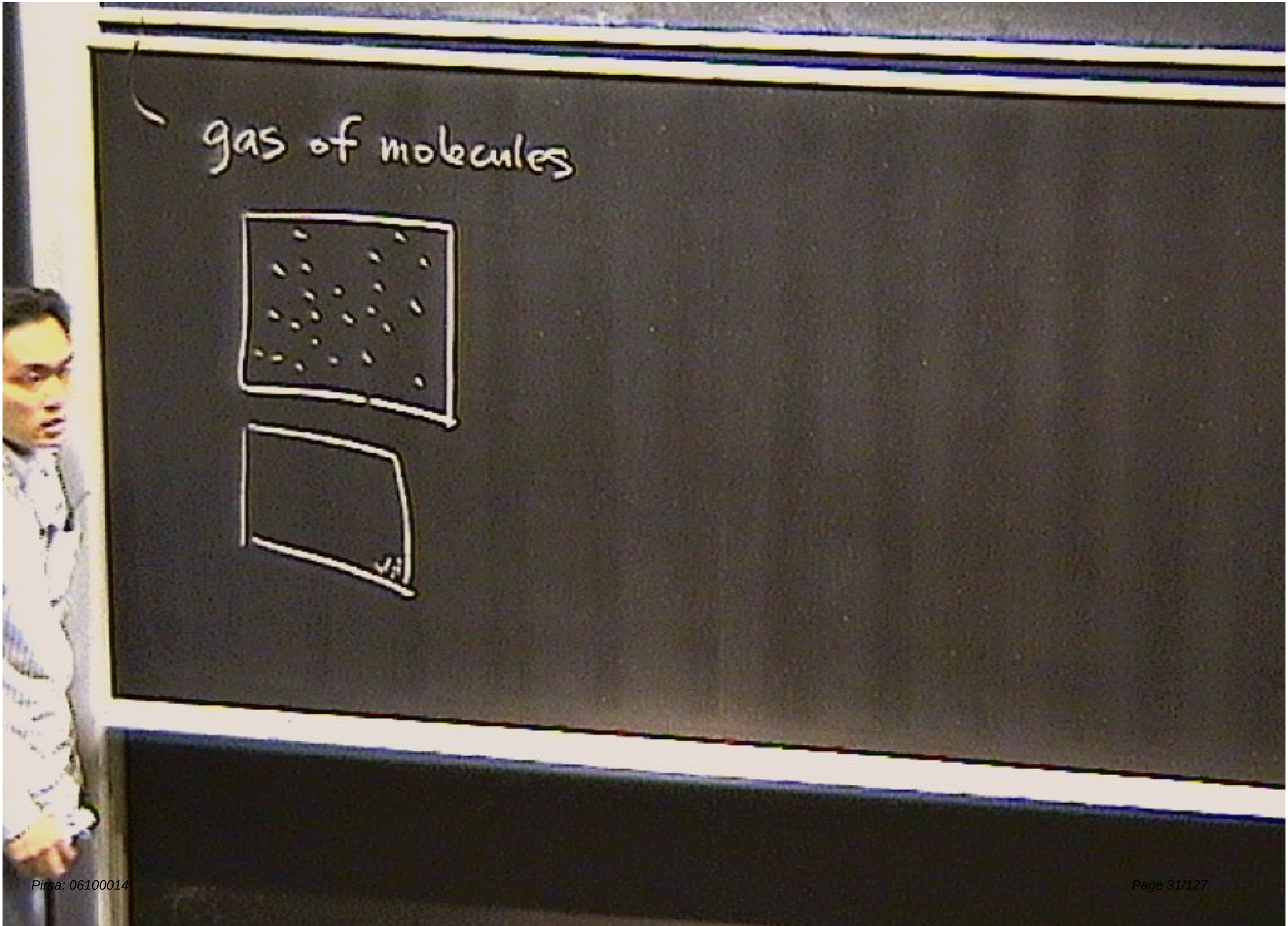
  - For large N, most states are very similar to each other —— "typical state"


gas of molecules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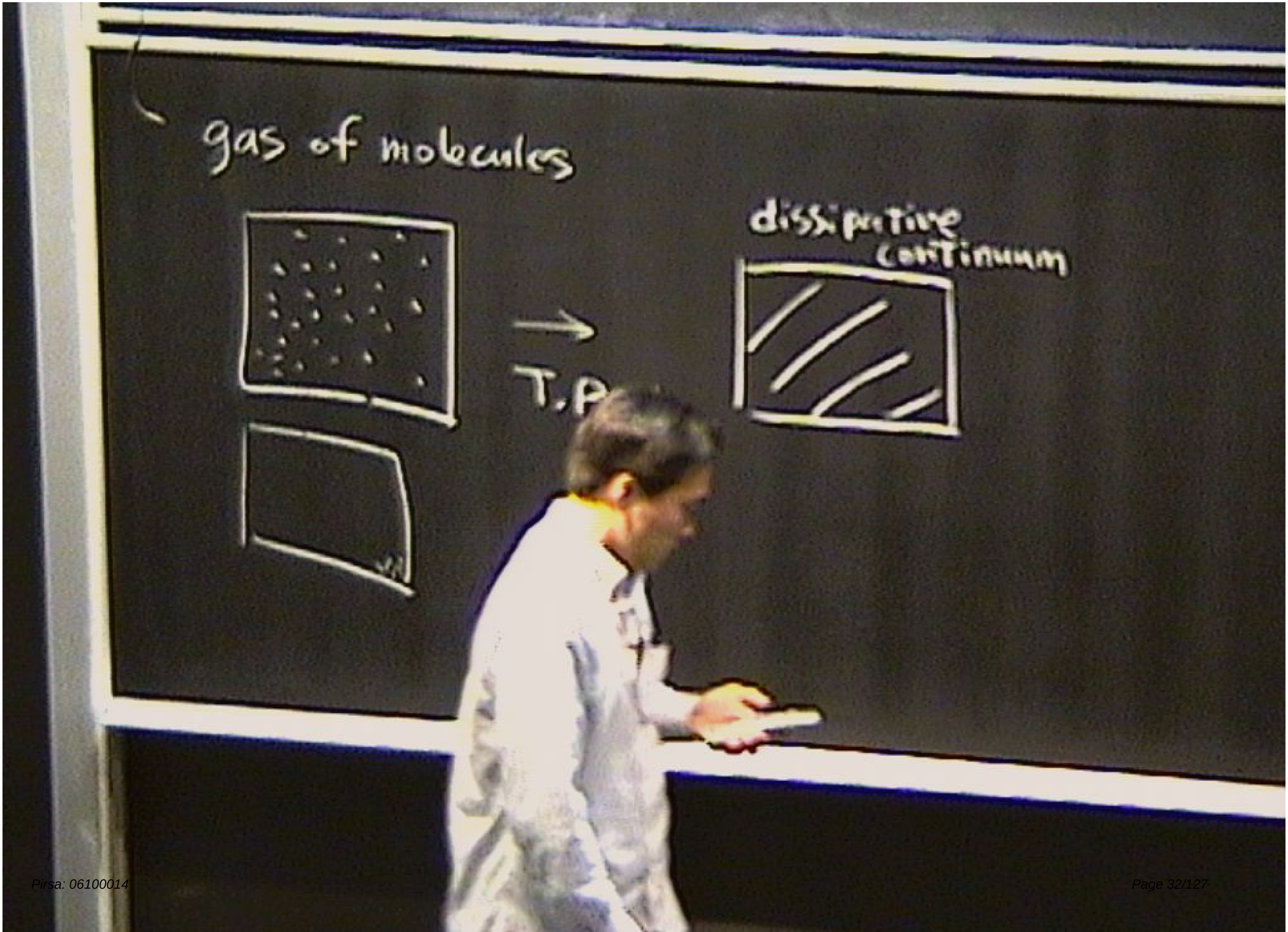
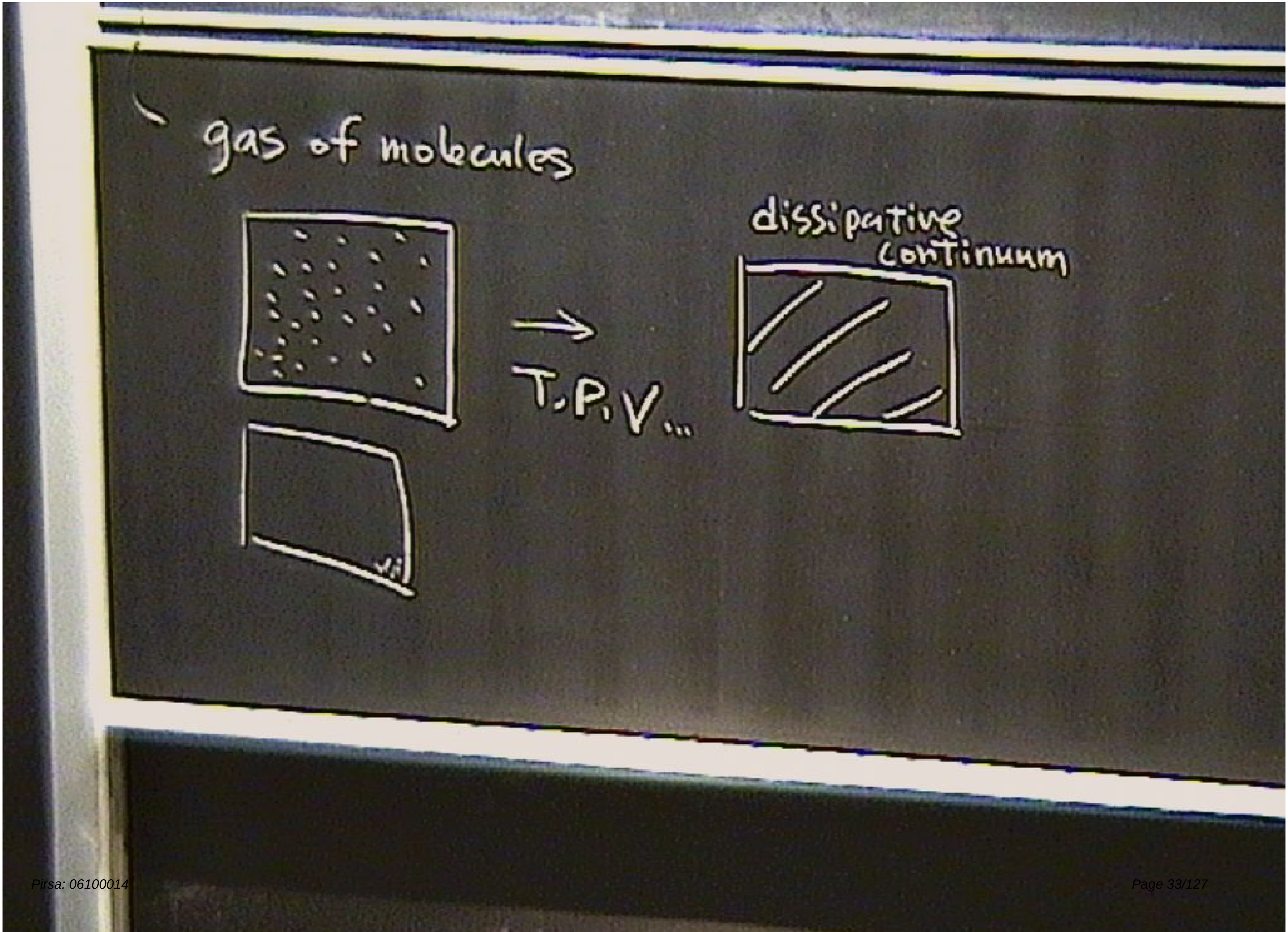

gas of molecules
dissipative
continuum
T, P


gas of molecules
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T, P, V ...

Ensemble vs. microstates — CFT side

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  - For **large N**, most states are very similar to each other —— "typical state"

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Cf. gas of molecules is well described by thermodynamics, although it's in pure microstate

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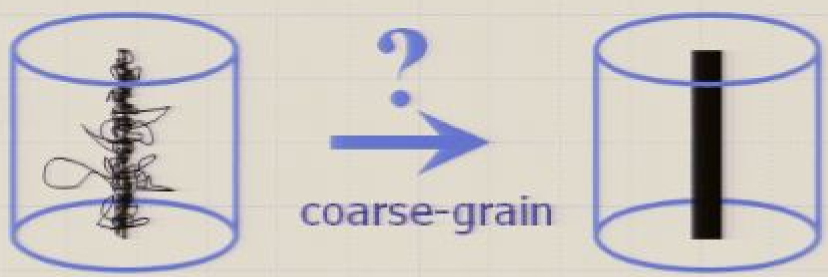
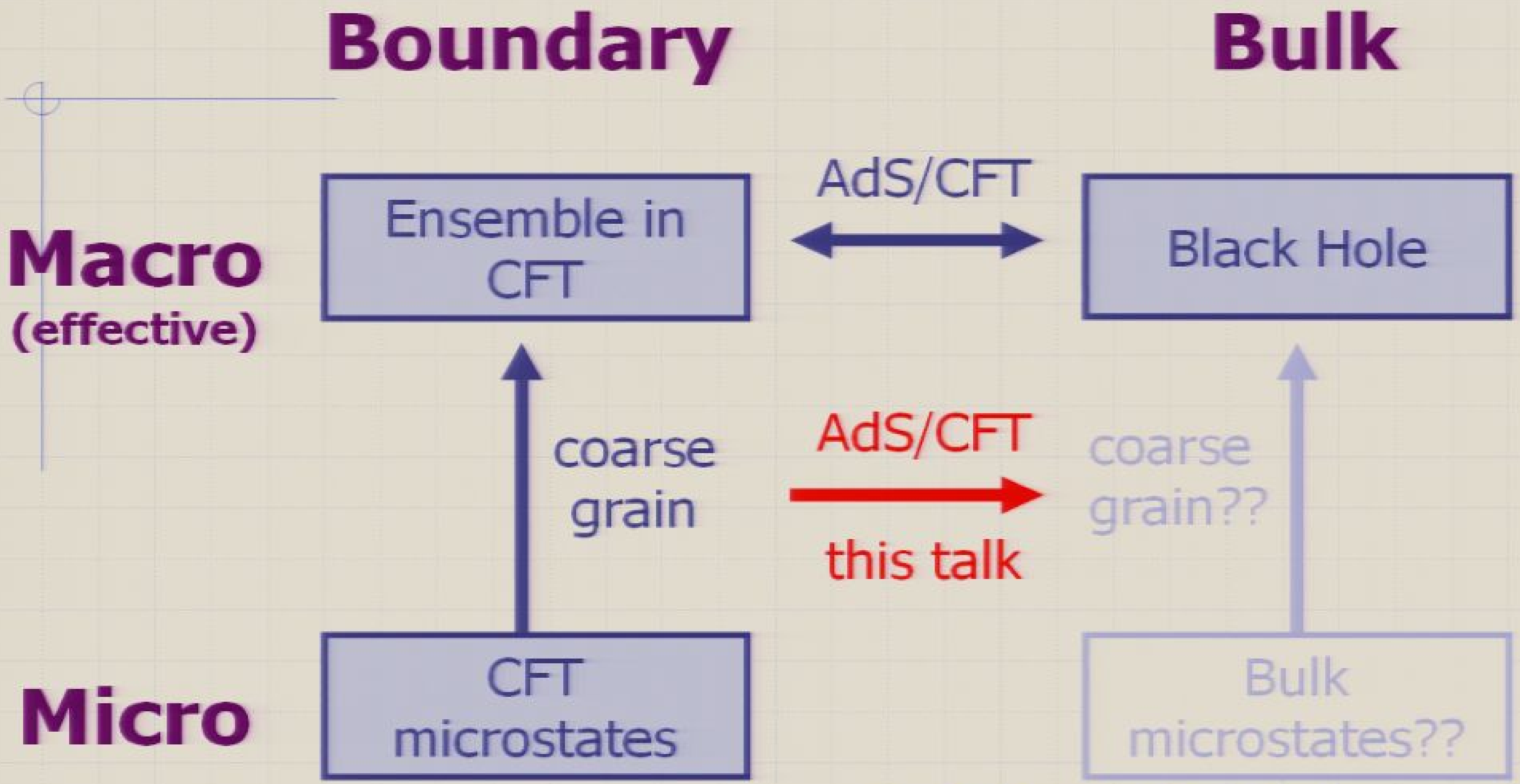
- Expectation for bulk microstates:
  - Result of "typical" measurements for "typical state" is very well approximated by that for stat. mech. ensemble, i.e. classical BH!

bulk microstate
(whatever they may be…)



effective geometry
= classical BH



cf. Mathur's conjecture

AdS5: Balasubramanian et al. "Library of Babel"

D1-D5 system

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  - Classically, horizon size vanishes
  - Still, must have some properties of BH
  - Stringy corrections makes it a small BH [Sen, Dabholkar, DKM]

D1-D5 system

Most importantly,

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- Large class of bulk microstate geometries are known [Lunin-Mathur]

Setup: D1-D5 System

D1-D5 system

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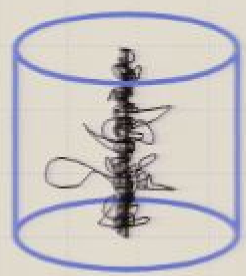
$$|\sigma\rangle = \sigma(0)|0\rangle$$

CFT state
(R ground state)

↔



bulk microstate geometry

$$ds^2 = \ldots$$

explicitly written down

D1-D5 system

- Simplest link between BH & CFT
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  - $SO(4)_E \times SO(4)_I$ symmetry

	$\mathbb{R}^4$	S^1	T^4
D1	$\cdot$	$\bigcirc$	$\sim$
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  - N=(4,4) supersymmetric sigma model
  - Target space: $(T^4)^N/S_N$, $N = N_1 N_5$

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  - Target space: $(T^4)^N/S_N$, $N = N_1 N_5$
- We use orbifold point (free) approximation

D1-D5 CFT

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  - Building blocks: 8+8 single-trace twist ops.

$$\sigma_n^{s\tilde{s}},\ \sigma_n^{\tilde{\alpha}\tilde{\beta}},\ \tau_n^{s\tilde{\alpha}},\ \tau_n^{\tilde{\alpha}\tilde{s}} \quad \equiv \quad \sigma_n^{\mu}, \tau_n^{\mu}$$

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with s labelling $SU(2)_R$.

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$$\sum_{n,\mu} n(N_{n\mu} + N'_{n\mu}) = N, \quad N_{n\mu} = 0, 1, 2, \ldots, \quad N'_{n\mu} = 0, 1 .$$

Map to FP system

Map to FP system

- D1-D5 sys is U-dual to FP sys:
  - F1 winds N_5 times around S^1
  - N_1 units of momentum along S^1

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D1-D5 FP D1-D5 FP

D1-D5 microstate geometries

R ground state of D1-D5 sys → (U-dual) → BPS state of FP sys → Classical profile of F1

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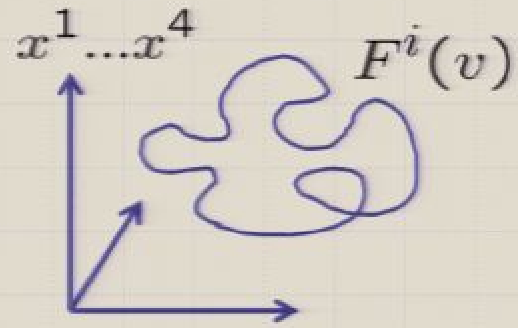
→ FP sys geometry → (U-dual) → D1-D5 geometry

Dabholkar-Gauntlett -Harvey-Waldram

Lunin-Mathur hep-th/0109154



$$ds^2_{\text{string}} = \frac{1}{\sqrt{f_1 f_5}}[-(dt-A)^2 + (dy+B)^2] + \sqrt{f_1 f_5}\, dx^i dx^i + \sqrt{\frac{f_1}{f_5}} dz^a dz^a,$$

$$e^{2\Phi} = \frac{f_1}{f_5}, \qquad f_5 = \frac{Q_5}{L}\int_0^L \frac{dv}{|\mathbf{x}-\mathbf{F}(\mathbf{v})|^2}, \qquad f_1 = \frac{Q_5}{L}\int_0^L \frac{|\dot{\mathbf{F}}(v)|^2 dv}{|\mathbf{x}-\mathbf{F}(\mathbf{v})|^2},$$

$$A_i = -\frac{Q_5}{L}\int_0^L \frac{\dot{F}_i(v) dv}{|\mathbf{x}-\mathbf{F}(\mathbf{v})|^2}, \qquad dB = -*_4 dA.$$

Typical states

Statistics & typical states

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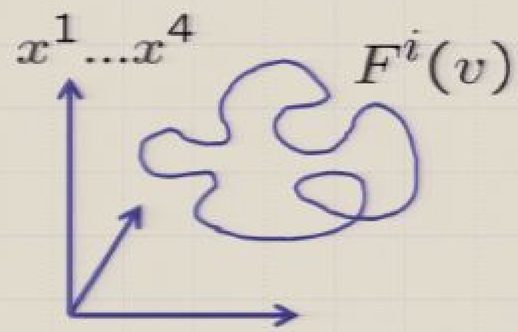
$$F^i(v), \ \ v = t - y$$
$$i = 1, \ldots, 4$$

→ FP sys geometry → (U-dual) → D1-D5 geometry



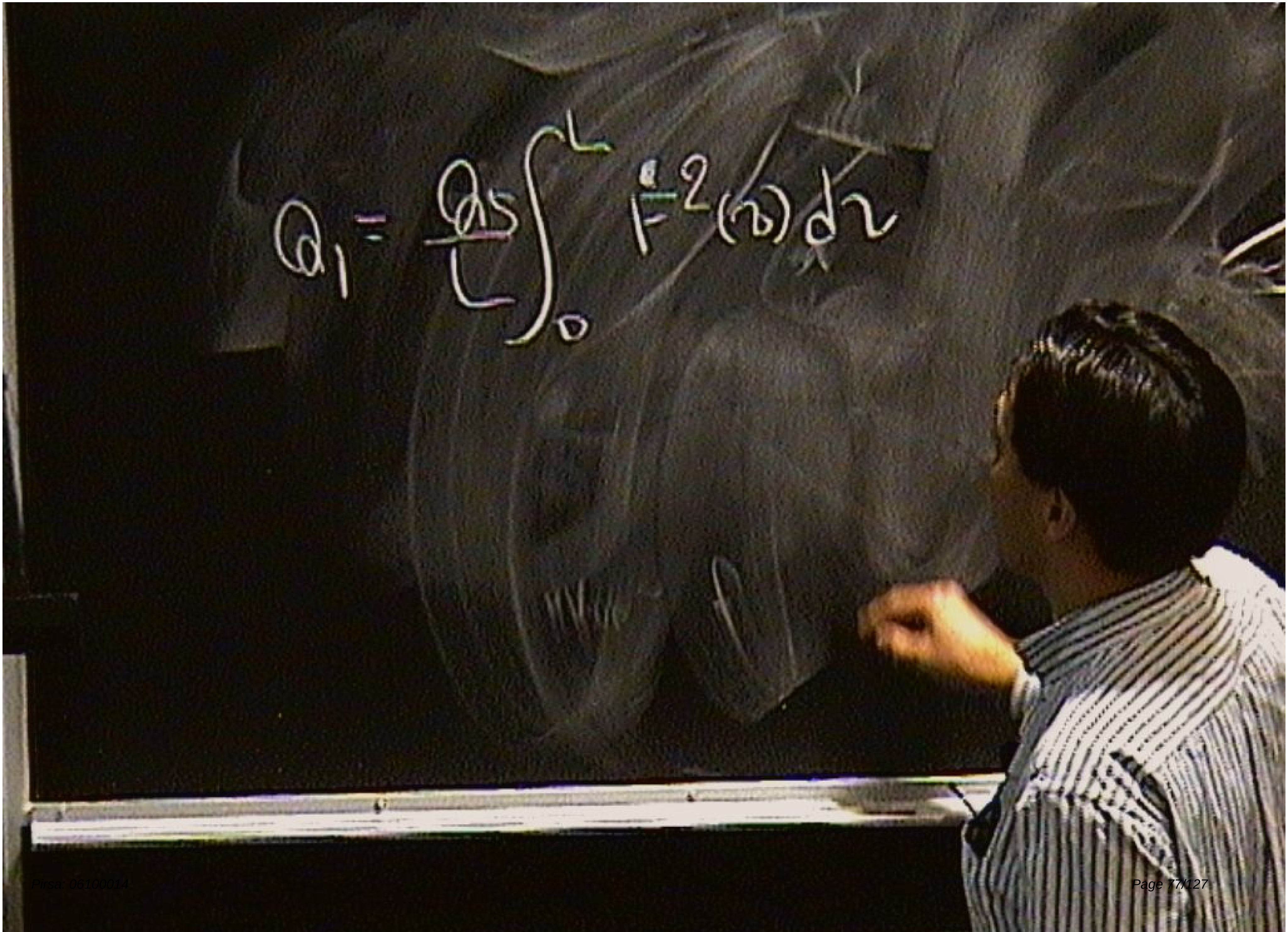
Dabholkar-Gauntlett -Harvey-Waldram

Lunin-Mathur hep-th/0109154

$$ds^2_{\text{string}} = \frac{1}{\sqrt{f_1 f_5}}[-(dt - A)^2 + (dy + B)^2] + \sqrt{f_1 f_5}\, dx^i dx^i + \sqrt{\frac{f_1}{f_5}} dz^a dz^a,$$

$$e^{2\Phi} = \frac{f_1}{f_5}, \qquad f_5 = \frac{Q_5}{L}\int_0^L \frac{dv}{|\mathbf{x} - \mathbf{F}(\mathbf{v})|^2}, \qquad f_1 = \frac{Q_5}{L}\int_0^L \frac{|\dot{\mathbf{F}}(v)|^2 dv}{|\mathbf{x} - \mathbf{F}(\mathbf{v})|^2},$$

$$A_i = -\frac{Q_5}{L}\int_0^L \frac{\dot{F}_i(v) dv}{|\mathbf{x} - \mathbf{F}(\mathbf{v})|^2}, \qquad dB = -*_4 dA.$$



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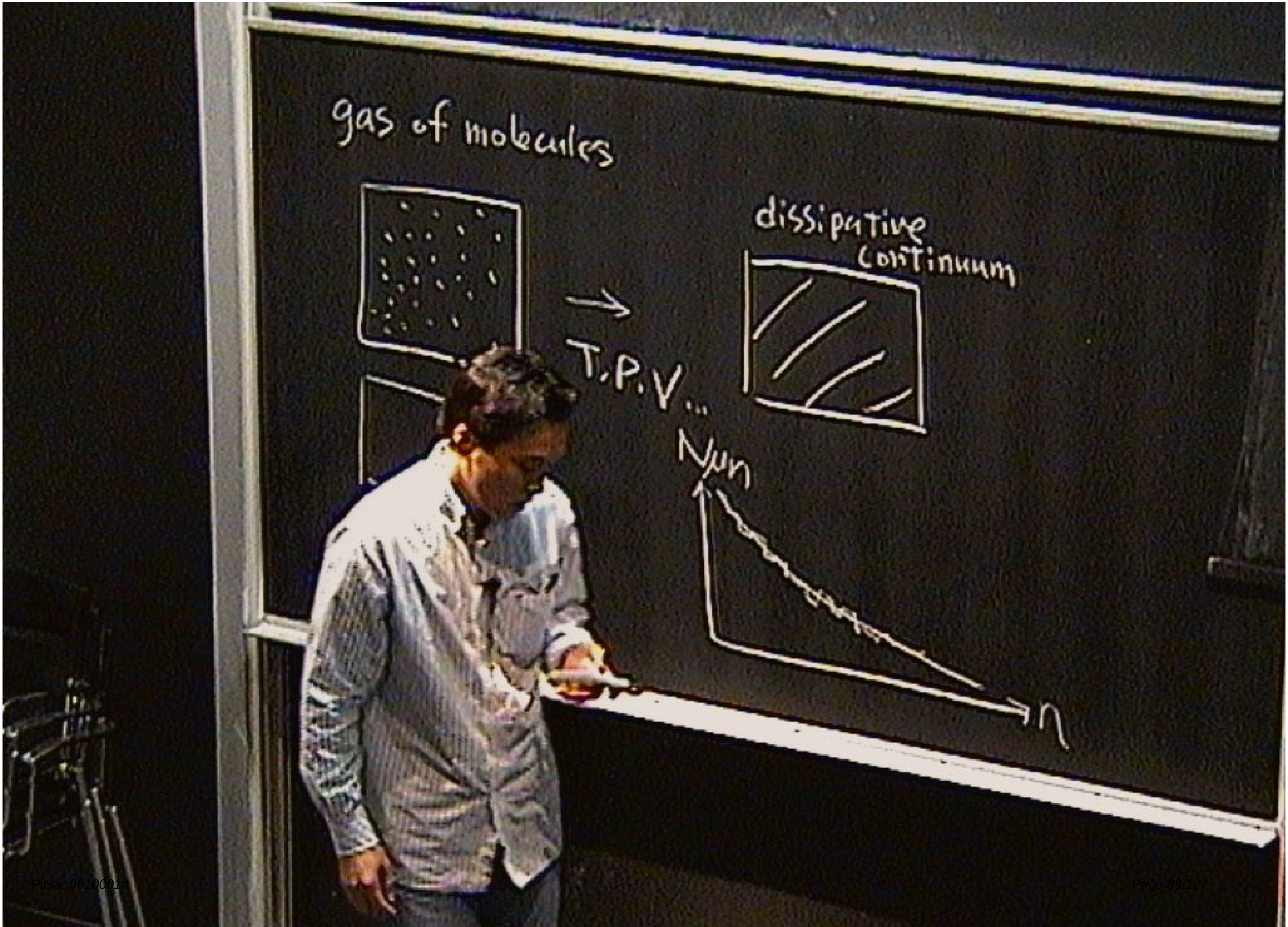
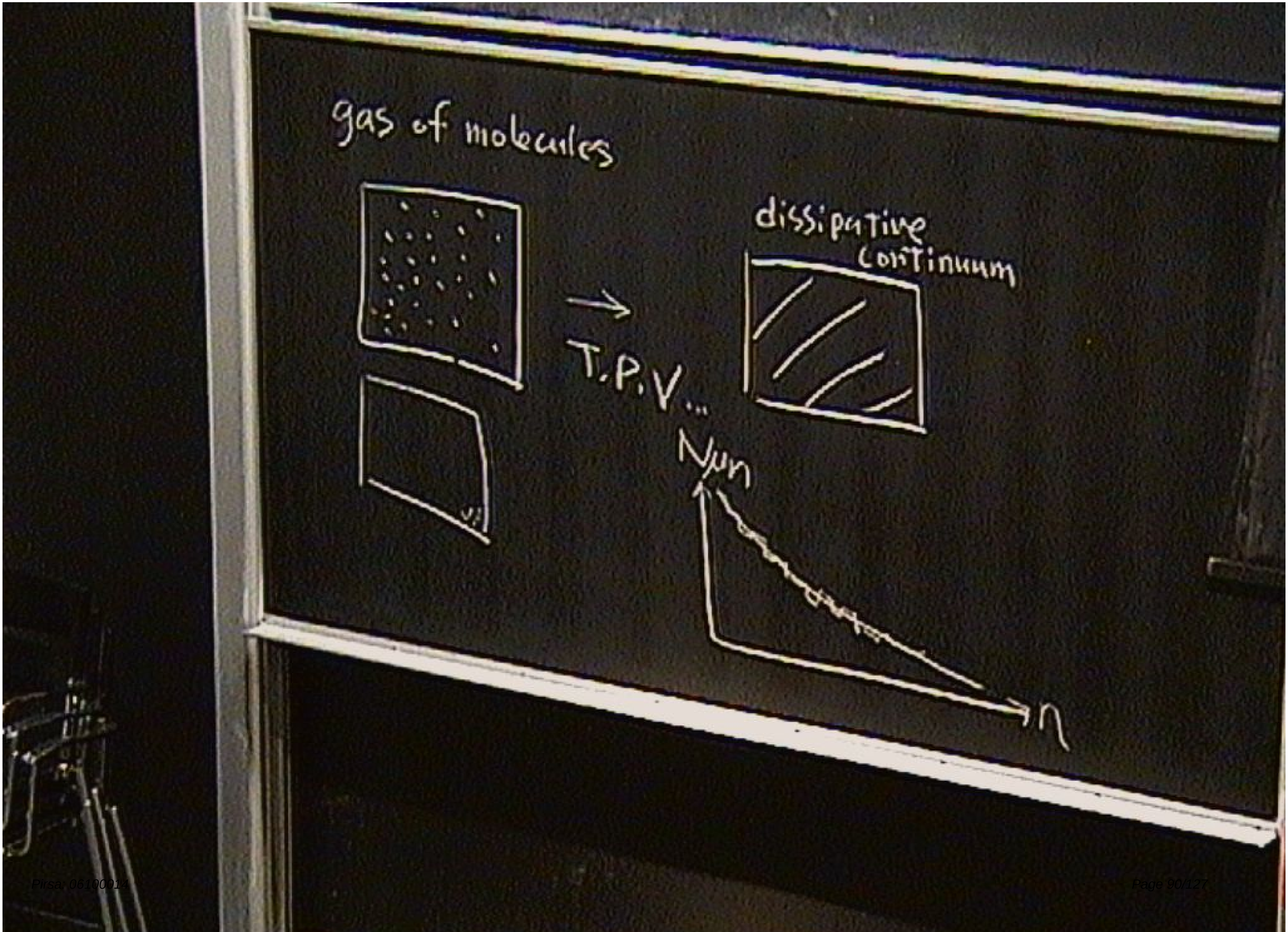

gas of molecules
T, P, V ...
dissipative continuum
n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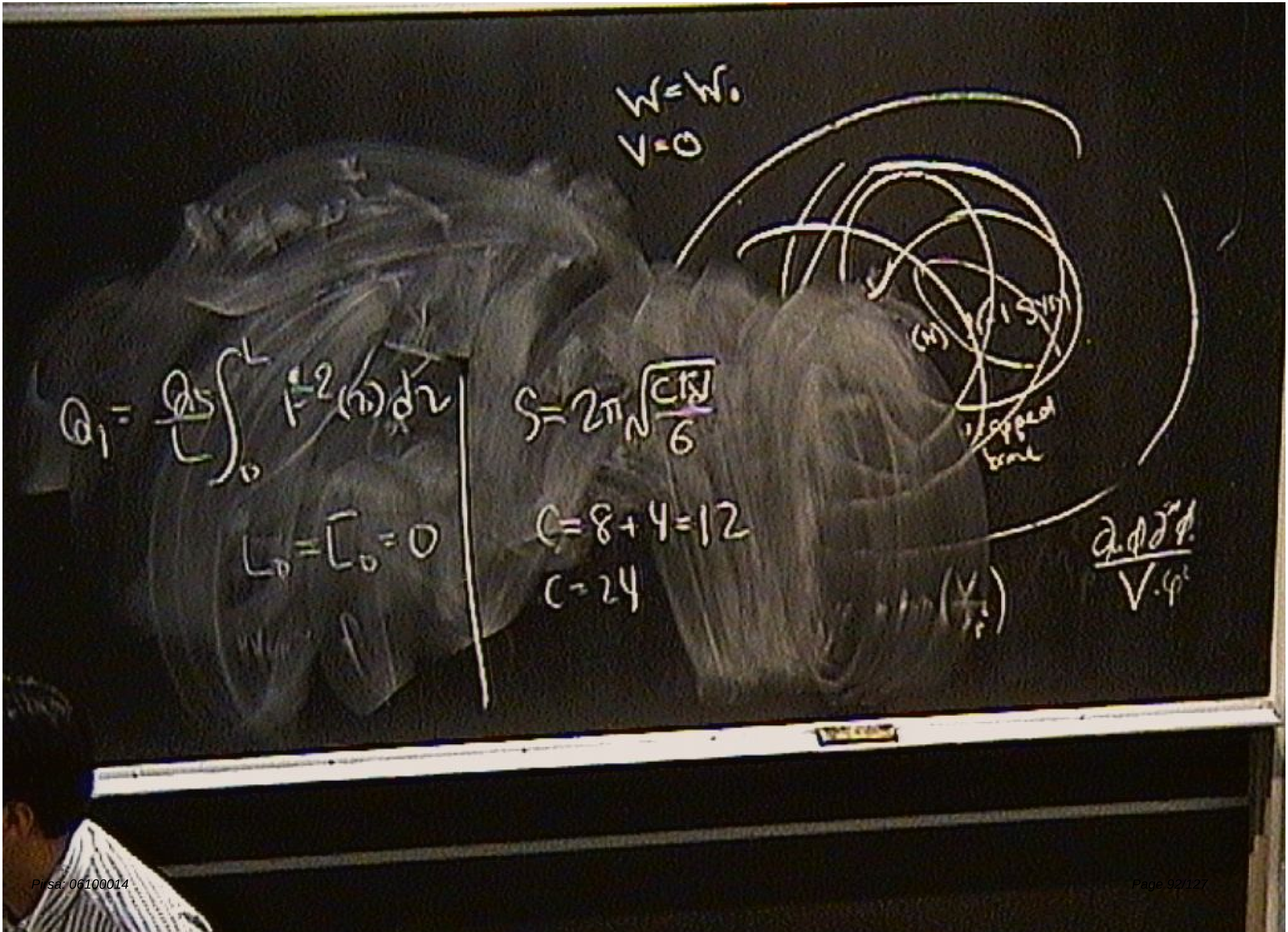
W=W0
V=0
C=8+4=12
C=24

The Effective Geometry

What we've learned so far:

- R ground states of D1-D5 system is specified by $\{N_{n\mu}, N'_{n\mu}\}$
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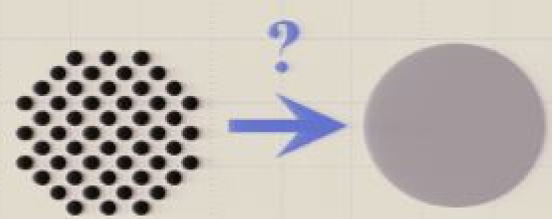
What we'll see:

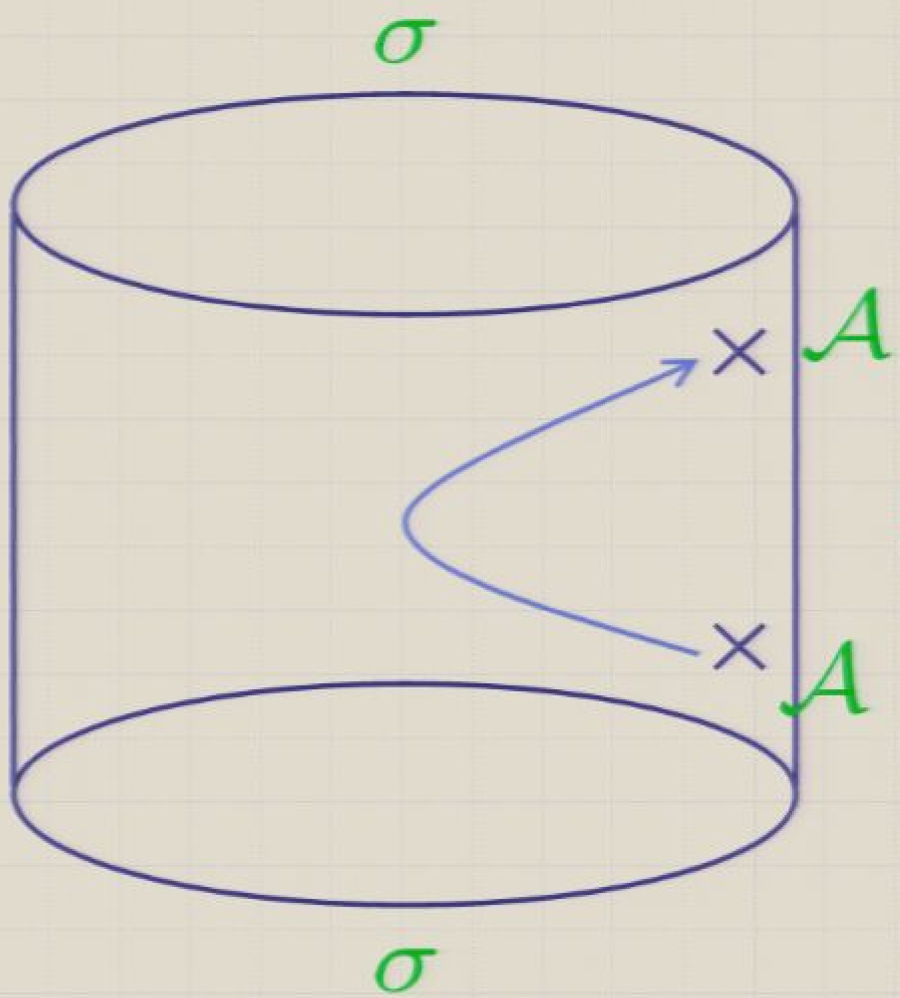
- We will compute CFT correlator $\langle\sigma|\mathcal{A}^\dagger\mathcal{A}|\sigma\rangle$
- For generic probes, almost all states give universal responses
  → effective geometry: M=0 BTZ
- For non-generic probes (e.g. late time correlator), different microstates behave differently

* How about bulk side? Why not "coarse-grain" bulk metric?

- Technically hard
- LLM/Lunin-Mathur is at sugra level



2-point func of D1-D5 CFT



Probe the bulk geometry corresponding to R ground state σ

$$= \langle \sigma^\dagger \mathcal{A}^\dagger \mathcal{A} \sigma \rangle$$

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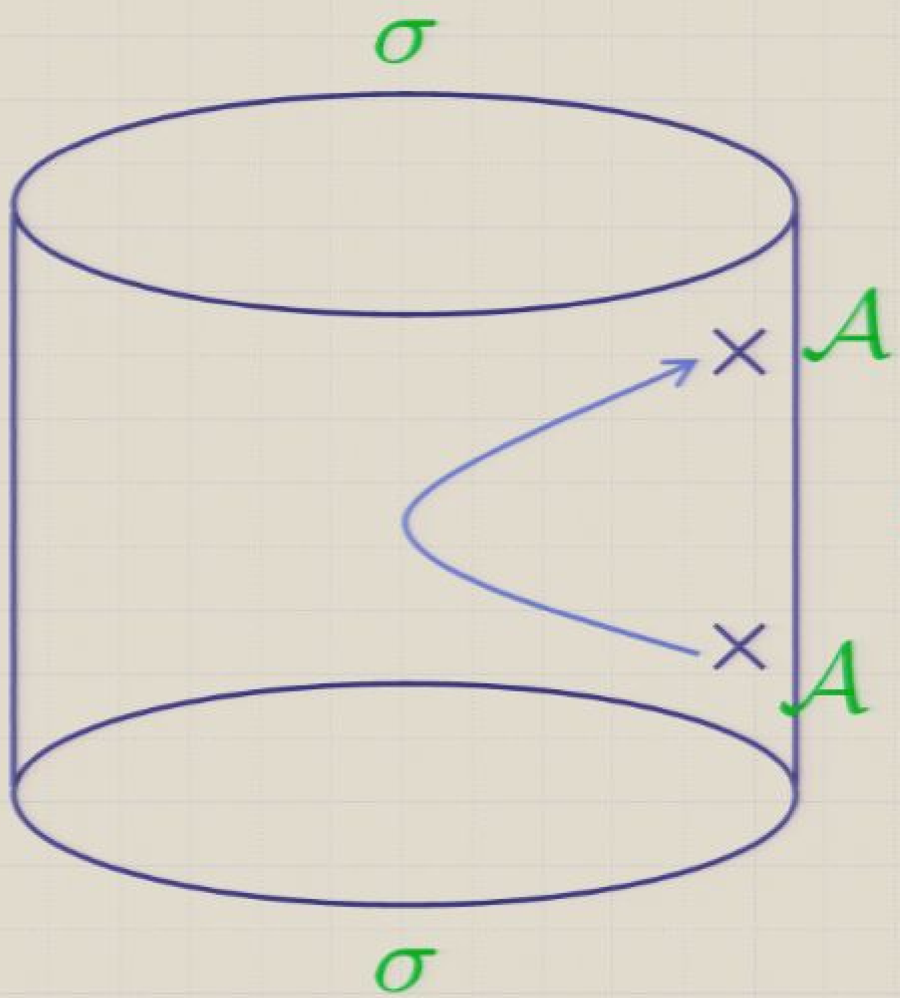

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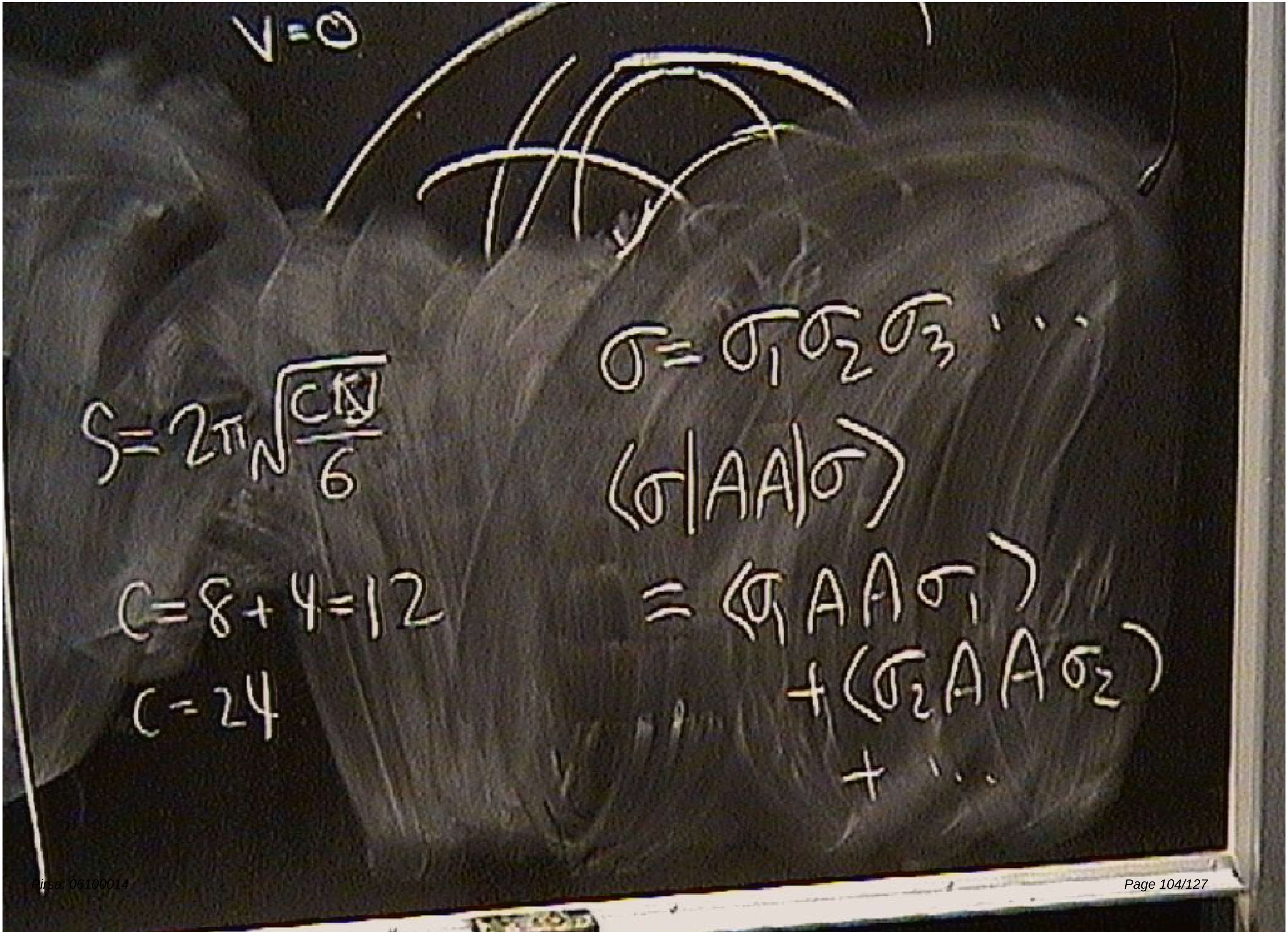
V=0
S=2π√(cN/6)
c=8+4=12
c=24
σ=σ1σ2σ3 ...
⟨σ|AA|σ⟩
=⟨σ1AAσ1⟩
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+ ...

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Typical state correlator: example

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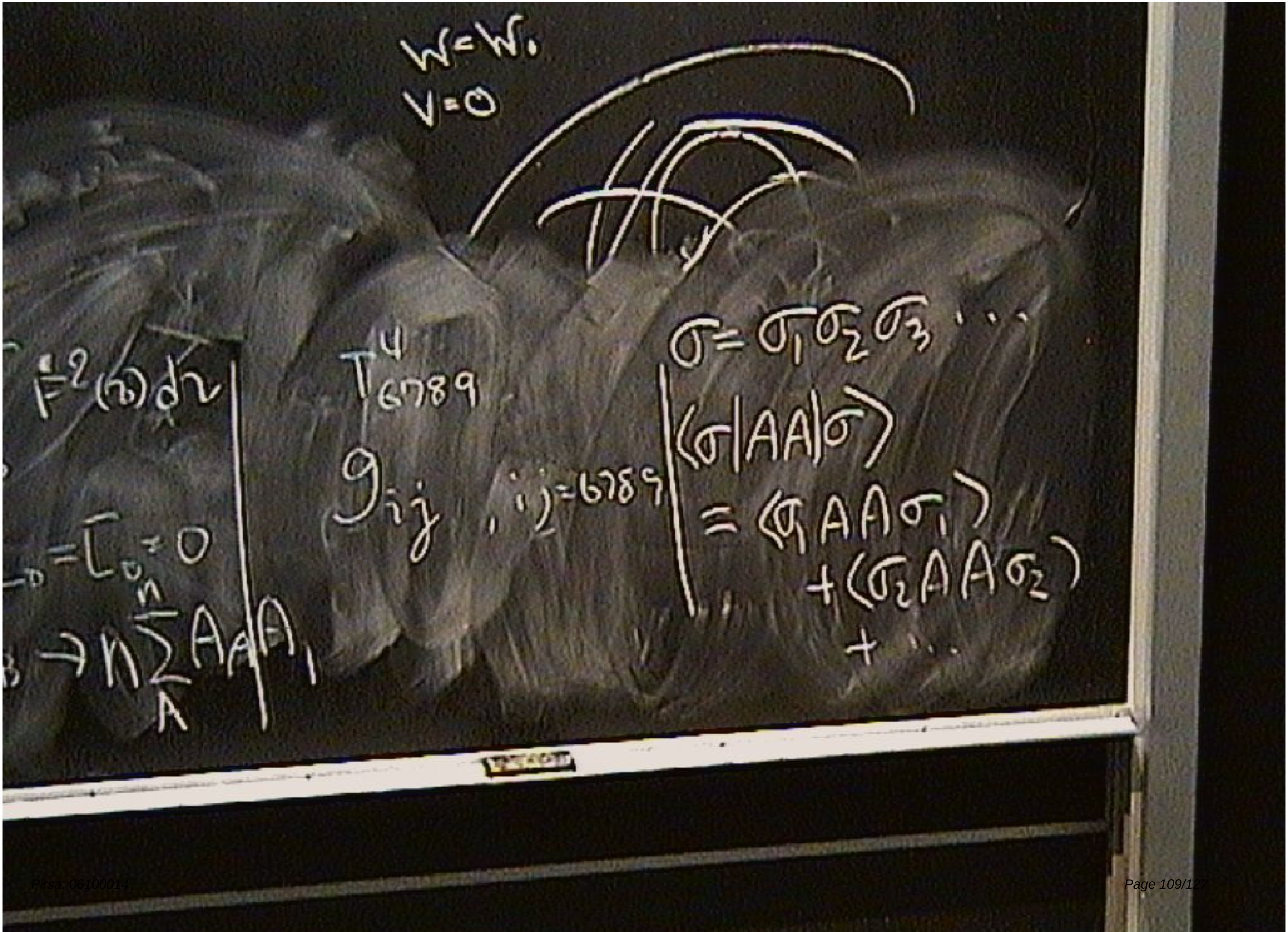
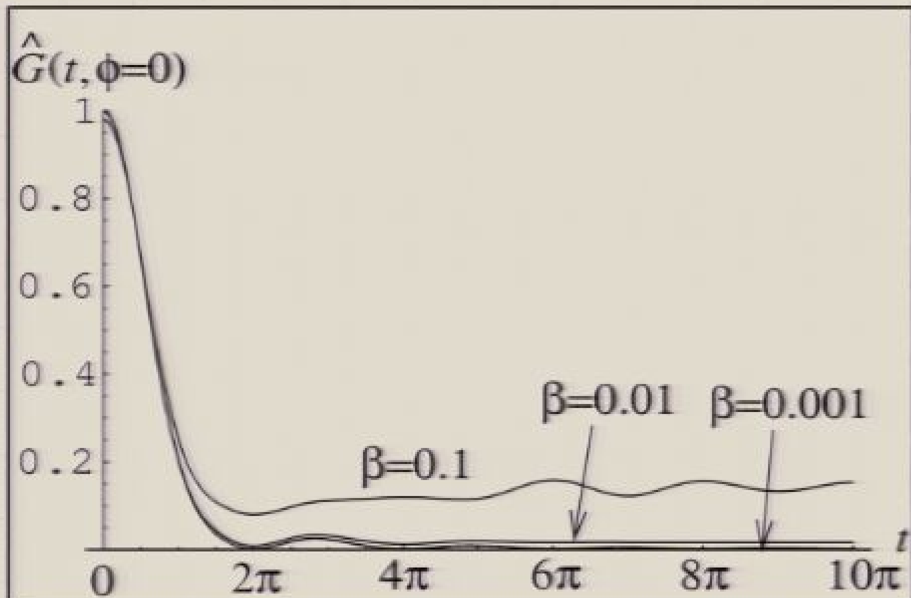
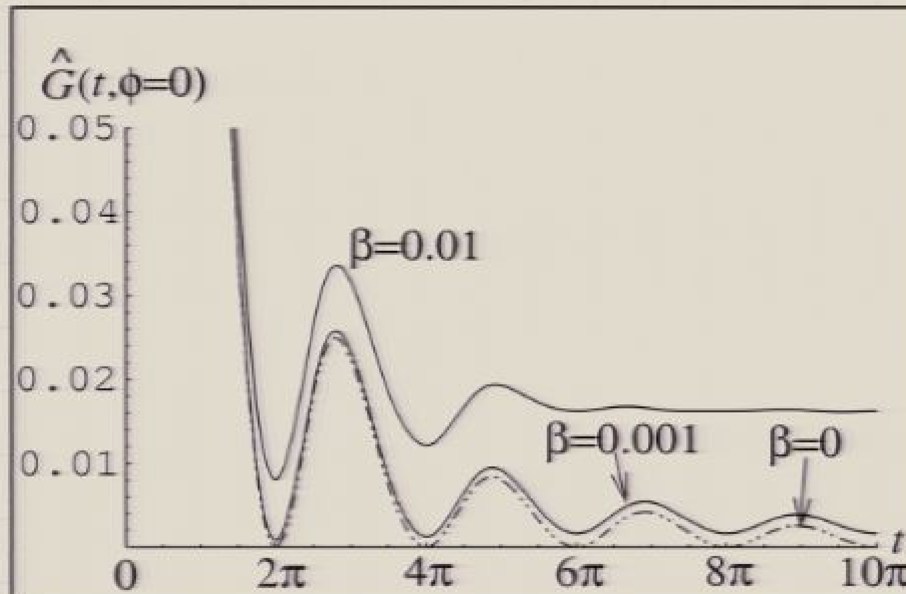

W=W0
V=0
T^4 6789
g ij
ij=6789
σ=σ1σ2σ3 . . .
⟨σ|AA|σ⟩
=⟨σ1AAσ1⟩
+⟨σ2AAσ2⟩
+ . . .

Typical state correlator: example

◆Short-time behavior:

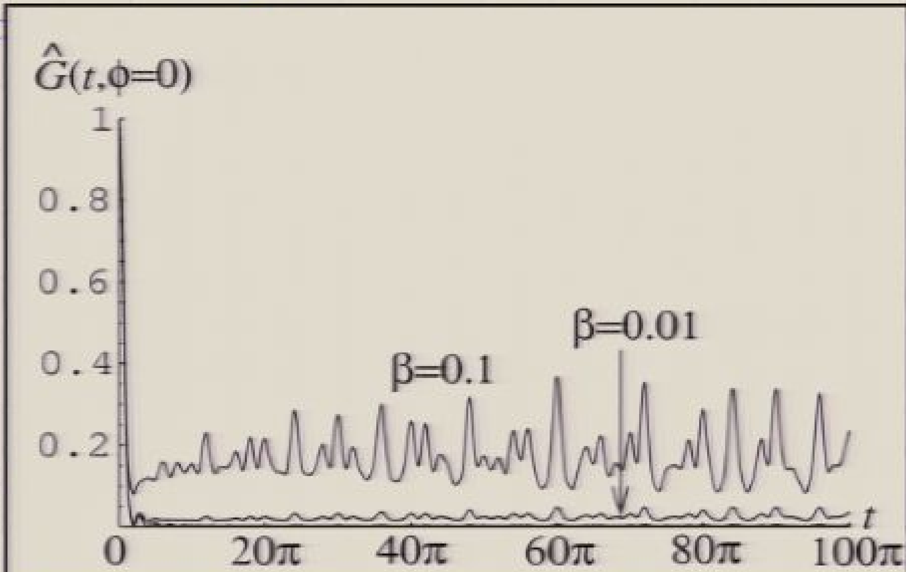


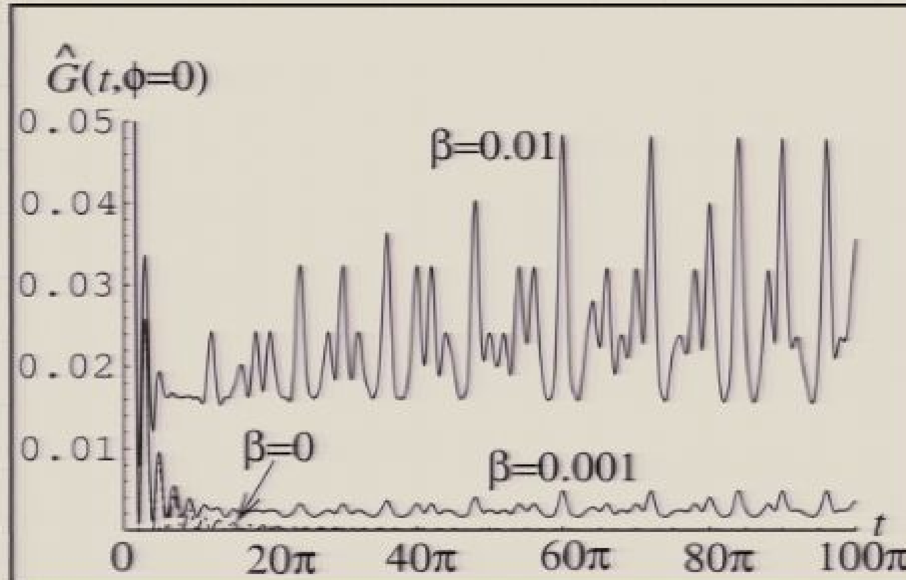


- Decays rapidly at initial times $(t \lesssim \pi\ell_{\mathrm{AdS}})$
- As $N\to\infty$ ($\beta\to 0$), approaches a certain limit shape (actually M=0 BTZ correlator!)

Typical state correlator: example

◆Long-time behavior:





- Becomes random-looking, quasi-periodic
- The larger N is, the longer it takes until the quasi-periodic regime
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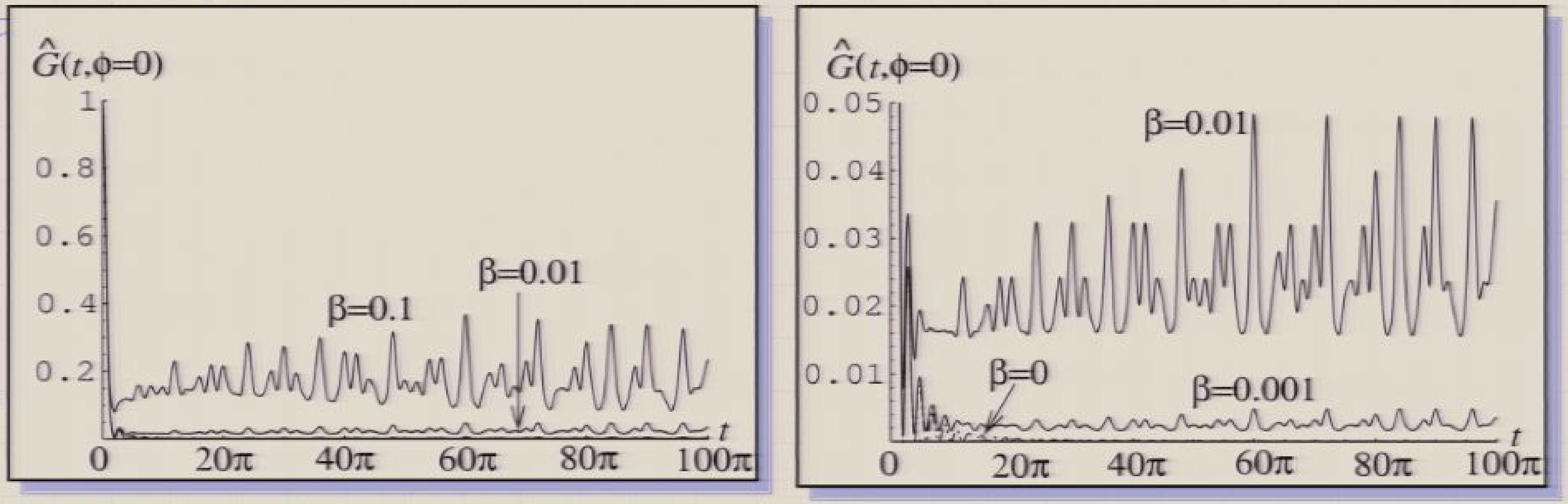

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Effective geometry of microstates

General non-twist bosonic correlator for $(h, \tilde{h})$

$$\langle \mathcal{A}(w_1)\mathcal{A}(w_2)\rangle = \frac{1}{N}\sum_n nN_n \sum_{k=0}^{n-1} \frac{C}{\left[2n\sin\left(\frac{w-2\pi k}{2n}\right)\right]^{2h}\left[2n\sin\left(\frac{\overline{w}-2\pi k}{2n}\right)\right]^{2\tilde{h}}},$$

$$N_n = \frac{8}{\sinh\beta n}, \quad w = \phi - \frac{t}{\ell}, \quad \overline{w} = \phi + \frac{t}{\ell}$$

- Substantial contribution comes from terms with

$$n \sim 1/\beta \sim \sqrt{N}$$

- For $t \ll n$, can approximate the sum:

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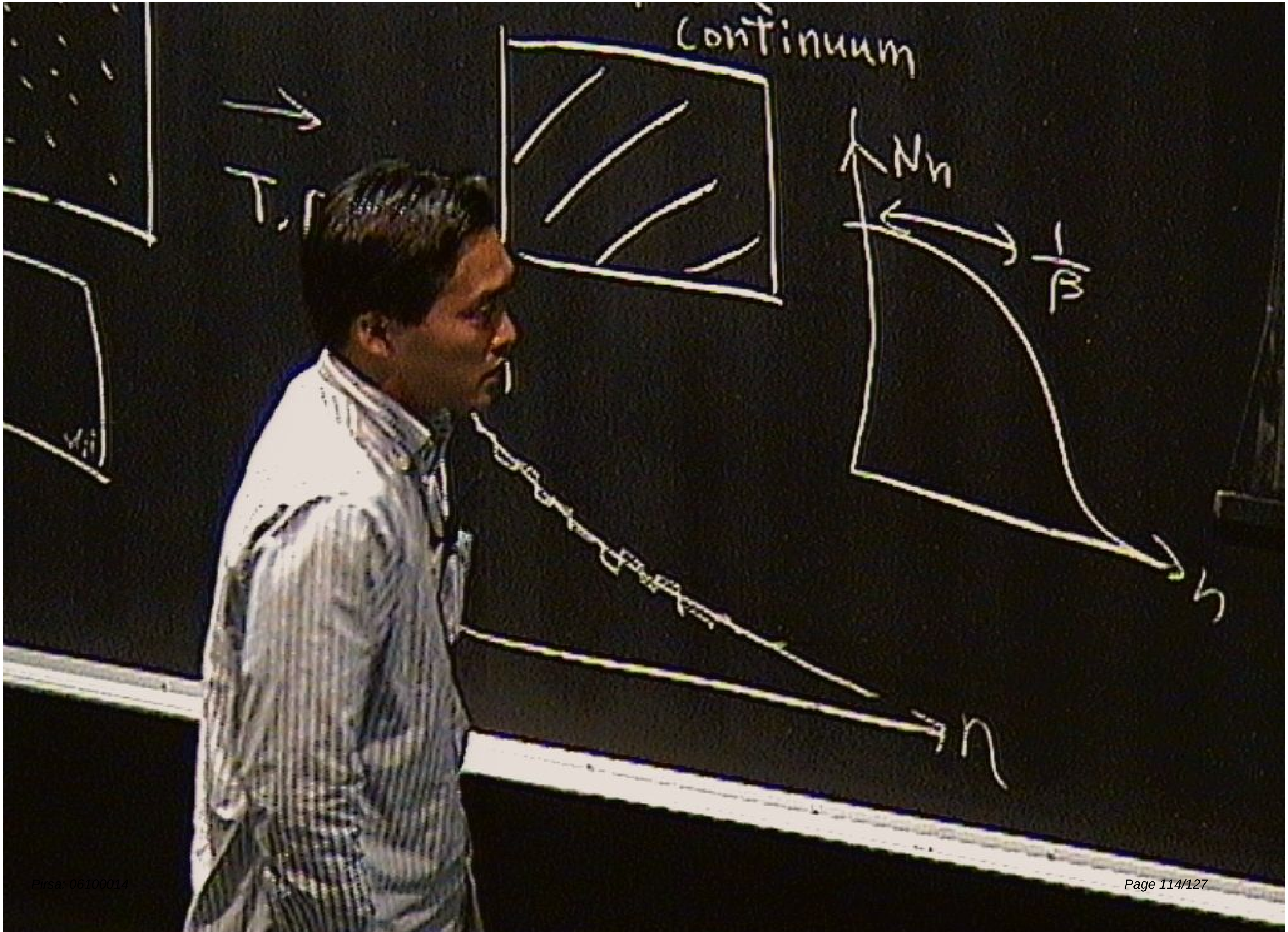
continuum
Nn
1/β
n
n

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Correlator for M=0 BTZ black hole

◆Crucial points:

- For N≫1, correlator for any state is very well approximated by that for the "typical state"
- Typical state is determined solely by statistics
- Correlator decomposed into constituents

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Comments:

- M=0 BTZ has no horizon
  - → we ignored interaction
- Still, M=0 BTZ has BH properties
  - Correlation function decays to zero at late times
  - Well-defined classical geometry

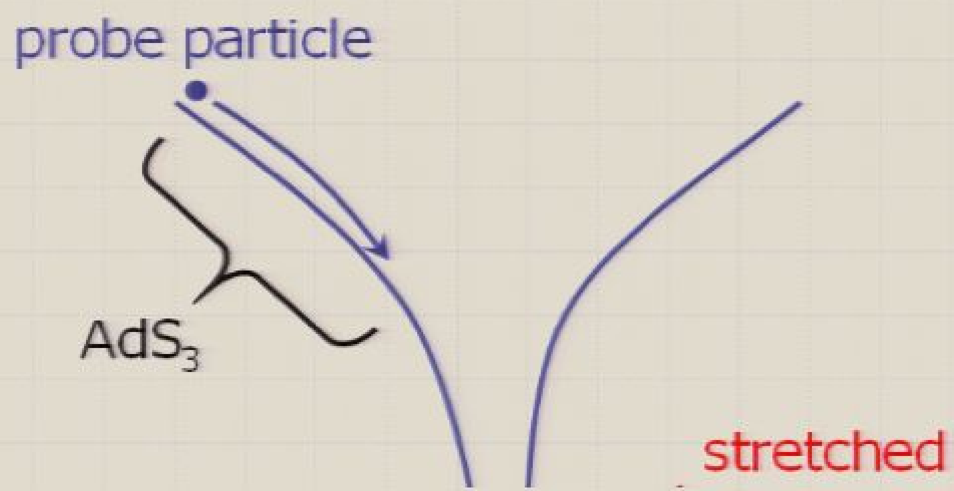
Notes on correlator

- M=0 BTZ correlator decays like $1/t^2$
- Microstate correlators have quasi-periodic fluctuations with mean $\sim 1/\sqrt{N}$
  Cf. for finite system: mean $\sim e^{-cS}$
  →need effect of interaction?
- Periodicity: $\Delta t \sim \mathrm{LCM}(1,\ldots,\sqrt{N}) \sim e^{c\sqrt{N}} = e^{c'S}$
  as expected of finite system

Fermion correlator also sees M=0 BTZ

Can do J≠0 case too (small black ring)

Consistency check: where are we probing?



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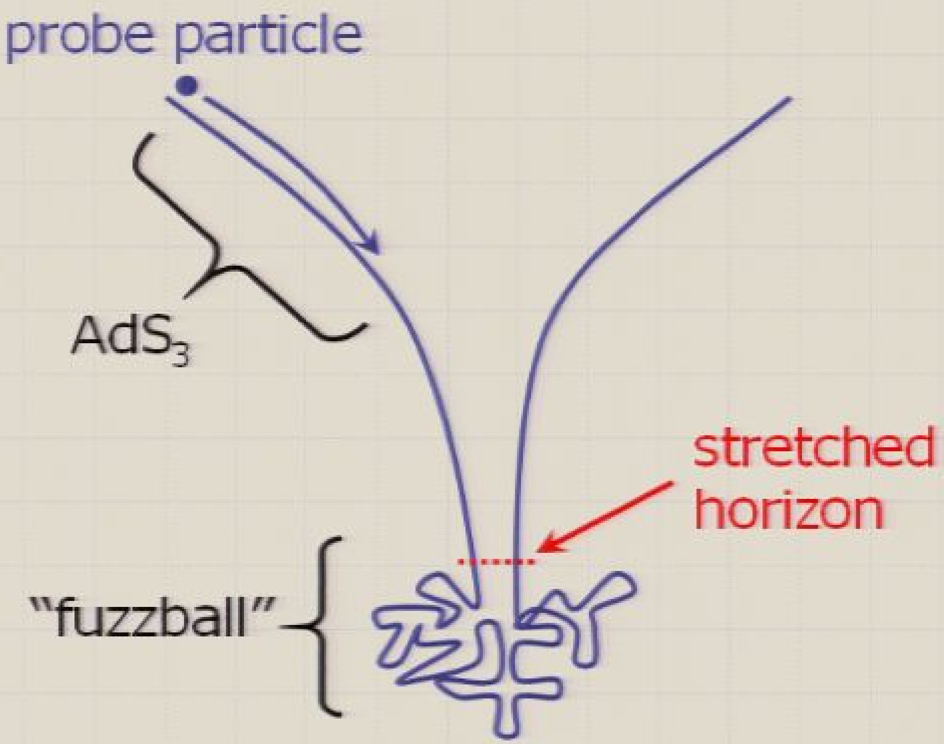


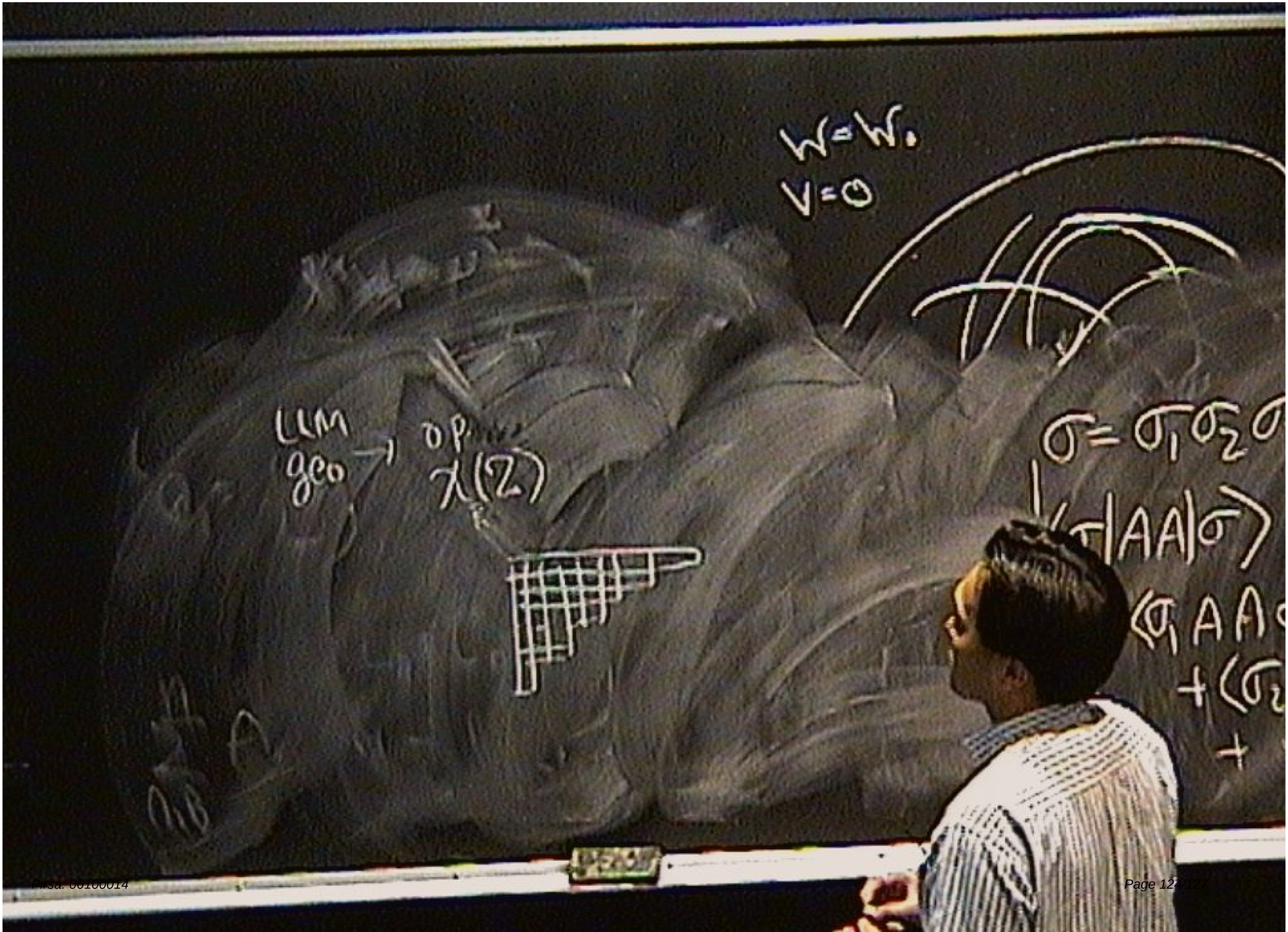
$$t_c \sim \begin{bmatrix} \text{time until probe} \\ \text{reaches stretched horizon} \end{bmatrix} \sim \sqrt{N}$$

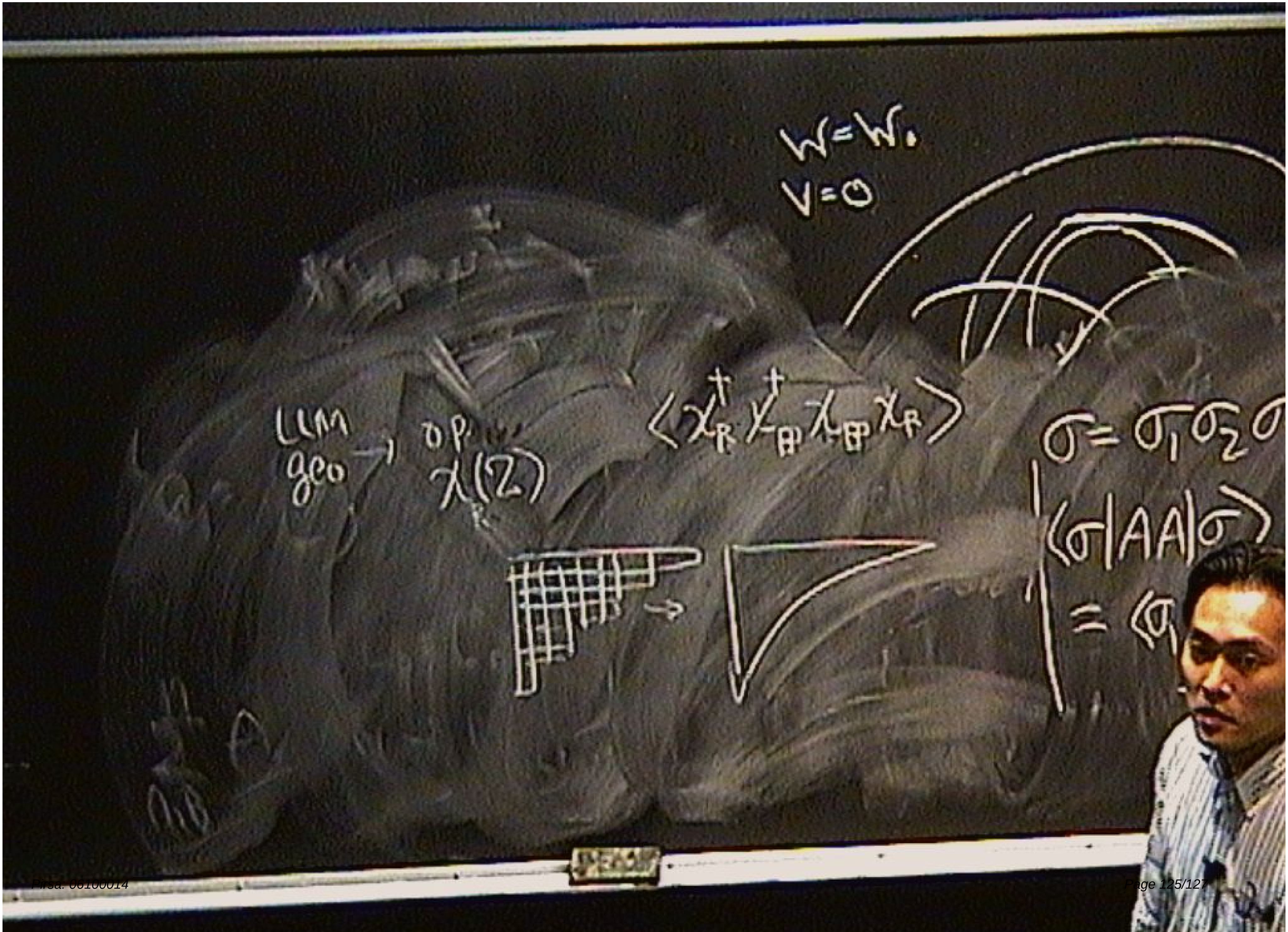
Conclusion

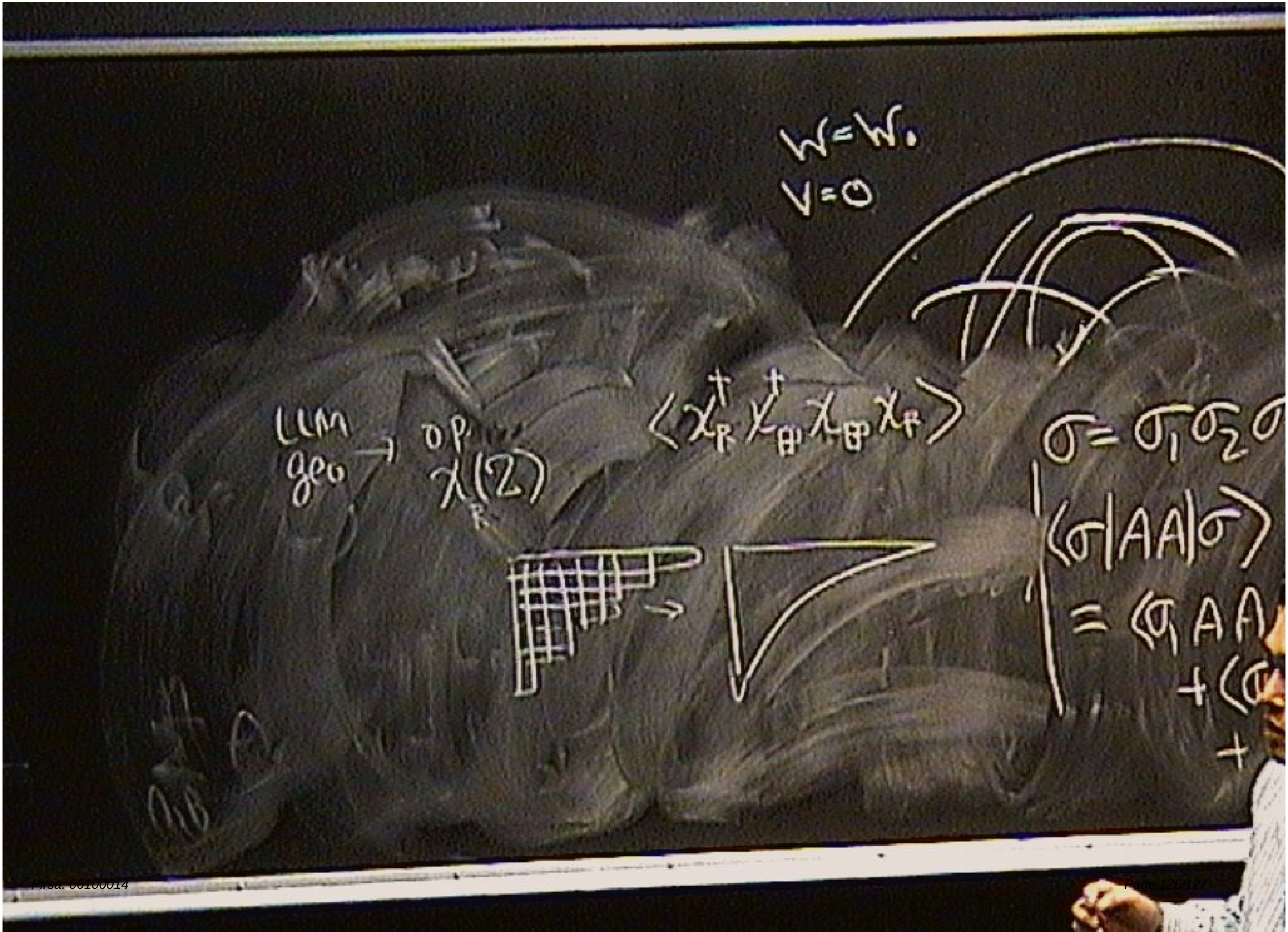
- For large N, almost all microstates in D1-D5 ensemble is well approximated by typical state
- Form of typical state is governed solely by statistics
- At sufficiently early times, bulk geometry is effectively described by M=0 BTZ BH
- At later times ($t \gtrsim t_c \sim N^{1/2}$), description by effective geometry breaks down

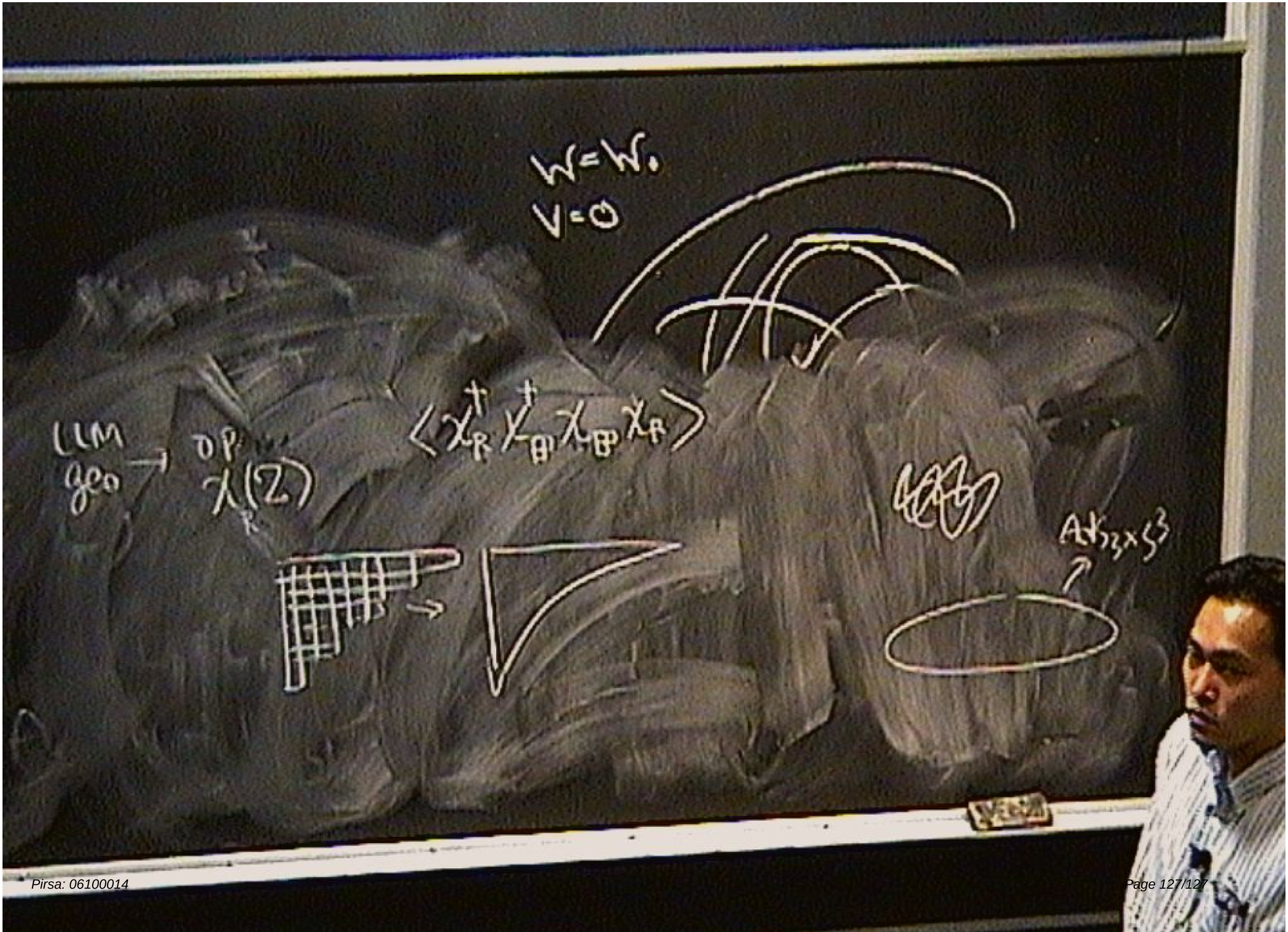
Message:

A black hole geometry should be understood as an effective coarse-grained description that accurately describes the results of "typical" measurements, but breaks down in general.


W=W0
V=0
LLM
geo
op.
λ(Z)
σ=σ1σ2σ


W=W0
V=0
LLM
geo
op.
σ=σ1σ2σ
⟨σ|AA|σ⟩


W=W0
V=0
lim
geo
op
χR(z)
⟨χ†R χ† χ χR⟩
σ=σ1σ2σ
⟨σ|AA|σ⟩
=⟨σ1 A A


W=W₀
V=0
LLM
geo
op.
$\chi_R(Z)$
$\langle \chi_R^\dagger \chi_{\boxplus}^\dagger \chi_{\boxplus} \chi_R \rangle$
$AdS_5 \times S^5$