

Title: Could quantum mechanics be an approximation to another, cosmological, theory?

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Abstract: We consider the hypothesis that quantum mechanics is an approximation to another, cosmological theory, accurate only for the description of subsystems of the universe. Quantum theory is then to be derived from the cosmological theory by averaging over variables which are not internal to the subsystem, which may be considered non-local hidden variables. I will explain the motivation for this view, give some examples of theories of this kind and investigate general conditions for such an approach to succeed.

Could quantum mechanics be an approximation to another theory?

Lee Smolin

- 1 Motivations
- 2 Nelson's formulation of quantum theory
- 3 Deriving Nelson from a hidden variables theory
- 4 Walstrom's objection
- 5 Particle model
- 6 Lattice model
- 7 Conclusions

gr-qc/0311059 with F. Markopoulou

Yidun Wan hep-th/0512210

Hal Finkel hep-th/0601163

quant-ph/0609109

The foundational problems of quantum theory remain unresolved.

- The measurement problem is still unsolved after 80 years.
- It gets worse when the observer is inside the system

Attempts to make sense of quantum cosmology are not convincing, *at least to me.*

There are some attempts in progress:

Fotini's quantum causal histories, connection with intuitionistic logic.

Lucien's generalized quantum theory incorporating dynamical causal structure

Isham's topos theoretical reformulation of quantum theory

What if none of these work?

What if quantum theory is in the end not the fundamental theory?

QIT theory suggests that QM is a theory of the information we have of quantum systems.

But then where are the limits to the validity of quantum theory?

One suggestion, given that the issues get qualitatively harder when we apply QM to a closed system, is that the limits are cosmological, i.e. the accuracy of QM is to order

$$r = N_{\text{subsystem}}/N_{\text{universe}}$$

What do we know about a theory that would replace QM?

- It must be manifestly non-local
(only a tiny loophole left in Bell issue)
- At the level of the additional degrees of freedom
(hidden variables) it must violate relativistic causality
- The vast web of entanglement in the quantum wavefunction
must be explicitly represented.
- It must also account for Fermi and Bose statistics

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Hypothesis: ***The hidden variables are relational.
This means that they represent a more detailed description,
not of a particle or a subsystem, but of the relations
that tie a subsystem to the rest of the universe.***

But if a theory is non-local and contradicts relativity, it must also imply a revision in our understanding of space and time.

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Leibniz, Mach, Einstein, Barbour, Stachel, Rovelli....

What does spacetime theory have to say about the possibility of a non-local, relational, hidden variables theory?

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2. *There is an expectation that a background independent quantum theory of gravity is not formulated in space. It is rather formulated purely in terms of algebra and combinatorics. Space must be shown to emerge from this.*

LQG, spin foams, matrix models for M theory, causal sets, causal dynamical triangulations....

Indeed, proposals for a fundamental formulation of string or M theory are described in terms of matrix models.

Fundamental degrees of freedom: Matrices

Classical local dof	Eigenvalues
---------------------	-------------

Non-local variables:	Matrix elements
----------------------	-----------------

So all these models have non-locality hidden in them.

Proposal: Matrix models of M theory are non-local hidden variables theories [hep-th/0201031](#), [hep-th/0206097](#)

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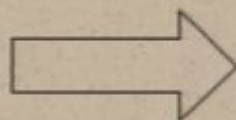
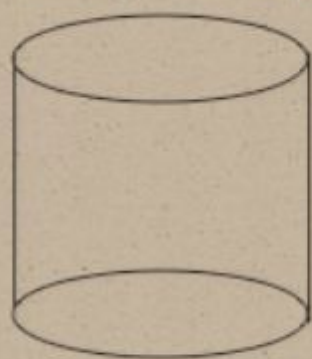
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3. *There are chronic difficulties with that emergence problem, which suggests that there may be macroscopic disorderings of locality (Fotini's mismatch of micro and macro locality).*

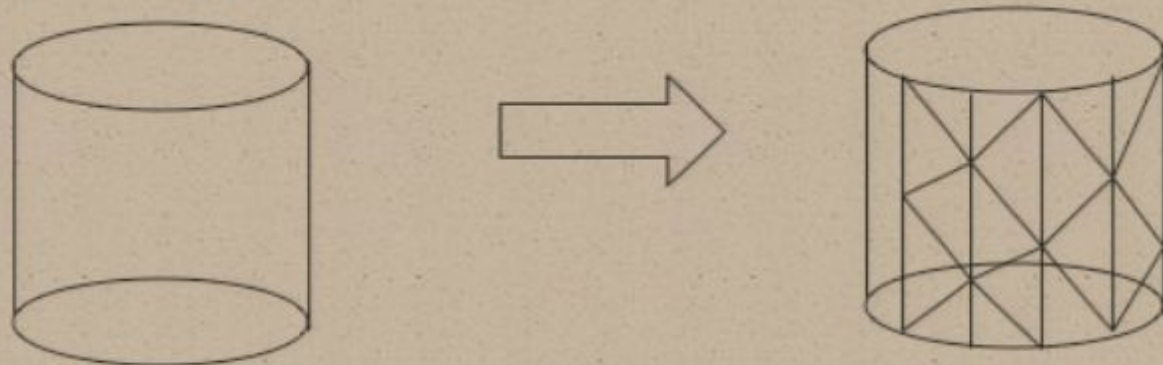
The inverse problem for discrete spacetimes:

Its easy to approximate smooth fields with discrete structures.

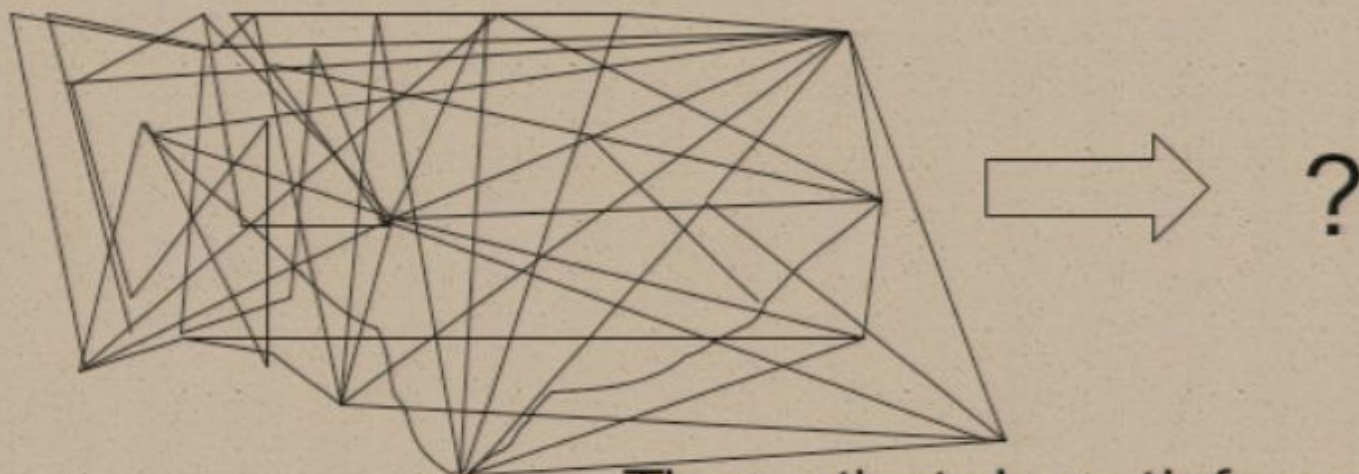


The inverse problem for discrete spacetimes:

Its easy to approximate smooth fields with combinatoric structures



But generic graphs do not embed in manifolds of low dimension, preserving even approximate distances.



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Γ : a graph with N nodes that has only links local in an embedding (or whose dual is a good manifold triangulation) in d dimensions.



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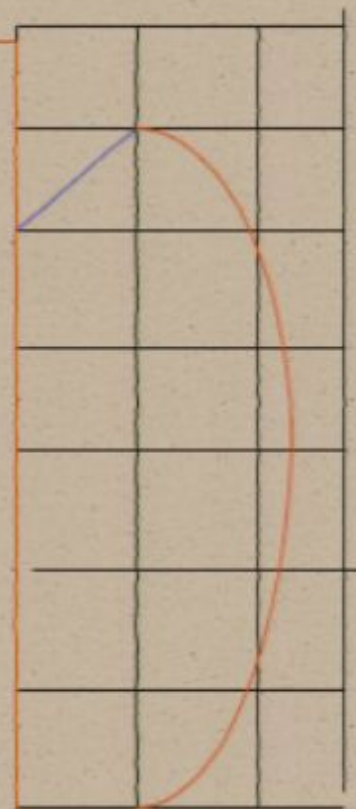
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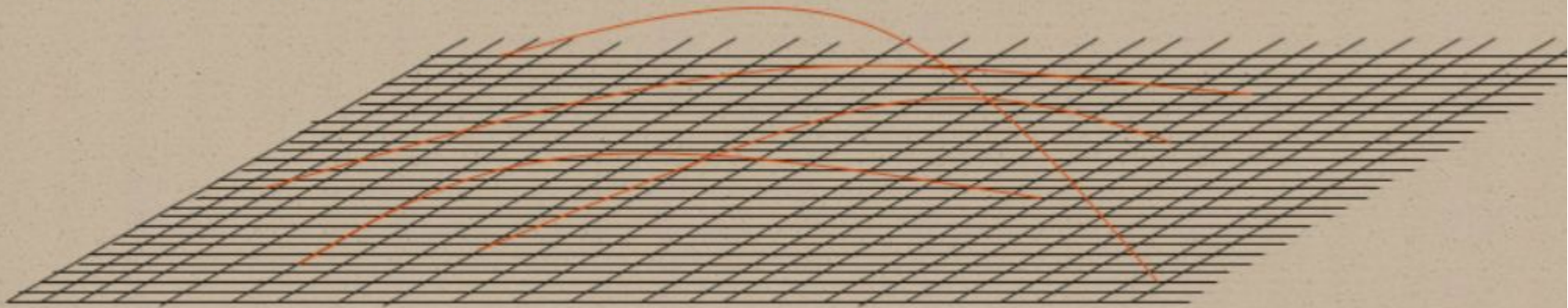
d N ways that don't.

N^2 ways that do.

Thus, if the low energy definition of locality comes from a coarse graining of a combinatorial graph, it will be easily violated in fluctuations.



Hypothesis: the low energy limit of QG is characterized by a small worlds network



Dislocations in locality are scale invariant up to the Hubble scale

QUERY: How much of such non-locality can we tolerate before contradicting experiment?

Suppose the ground state is contaminated by a small proportion of non-local links (locality defects)??

What is the effect of a small proportion of non-local edges in a regular lattice field theory?

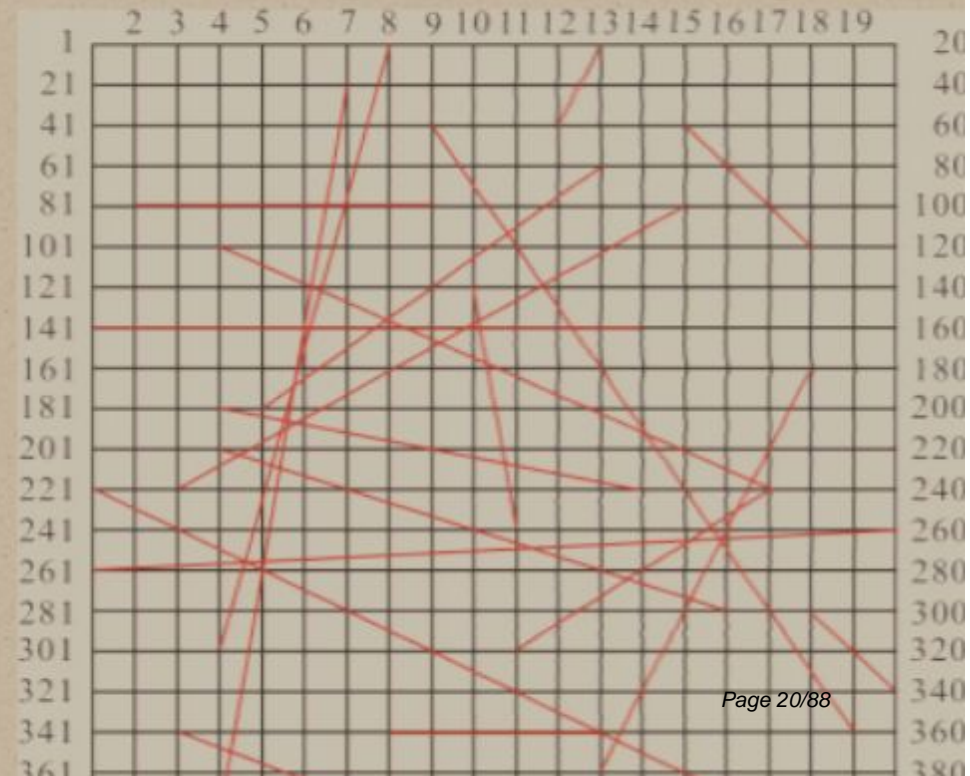
If this room had a small proportion of non-local link, with no two nodes in the room connected, but instead connecting to nodes at cosmological distances, could we tell?

Yidun Wan studied the Ising model on a lattice contaminated by random non-local links.

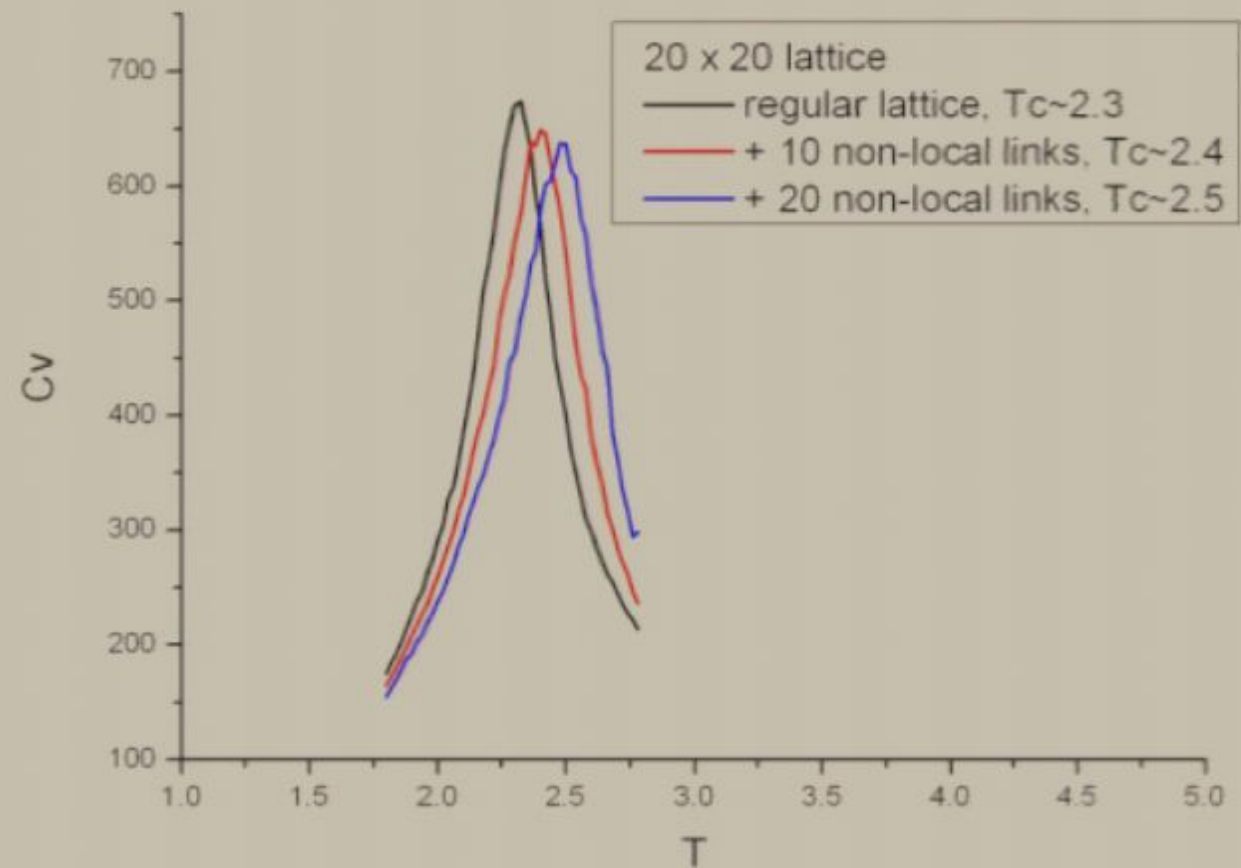
hep-th/0512210

$r = \text{non-local links} / \text{local links}$

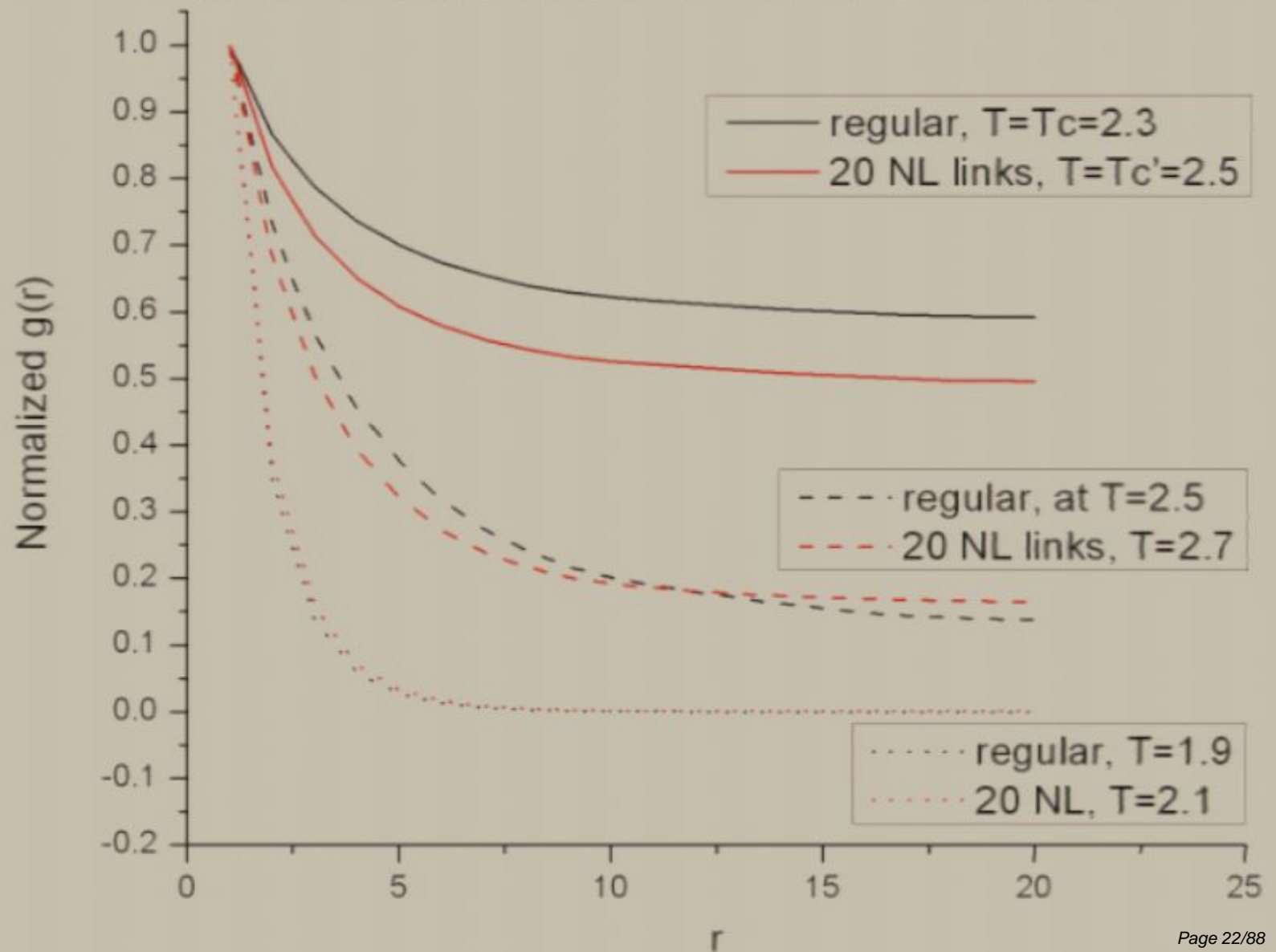
$$= 20/800 = 1/40$$



The critical phenomena is the same, but the Curie temperature increases slightly.

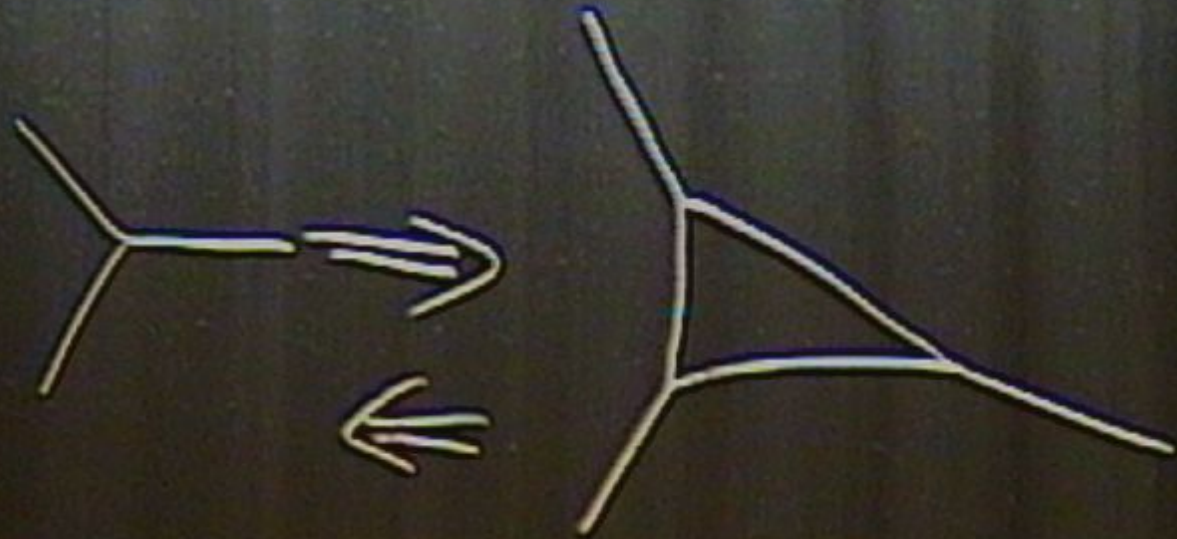


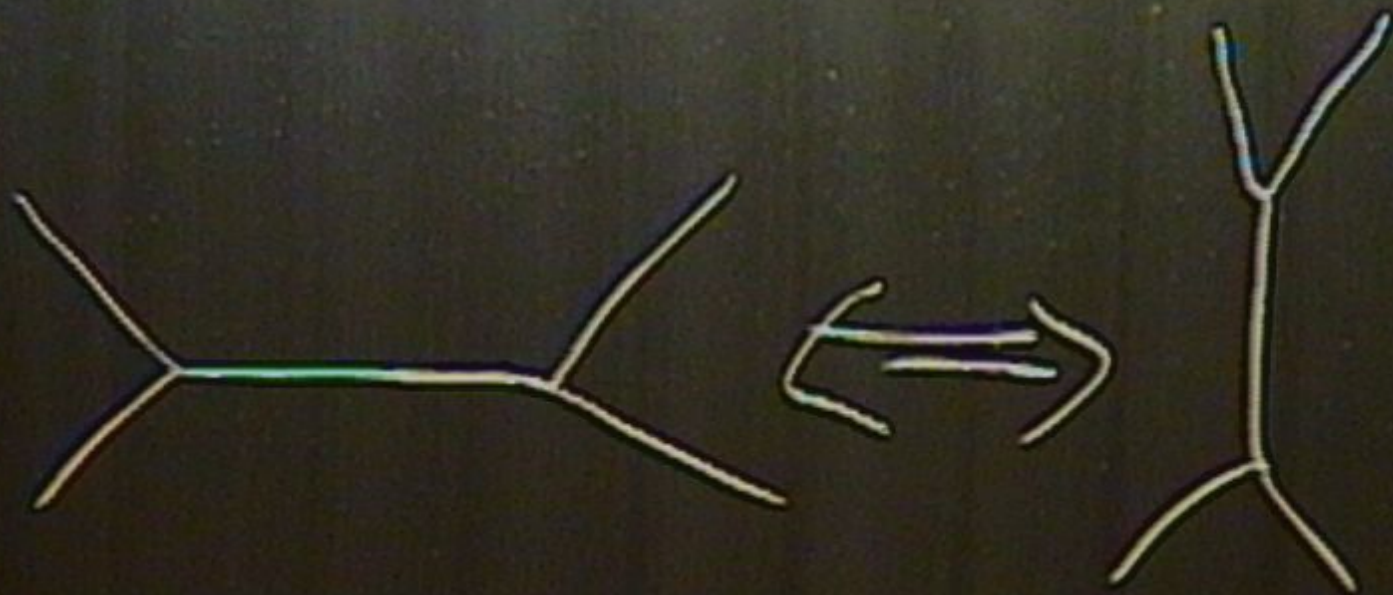
Comparison between the correlation function of a 20x20 lattice and that of the same lattice with 20 non-local (NL) links



LQG cosmological scenario

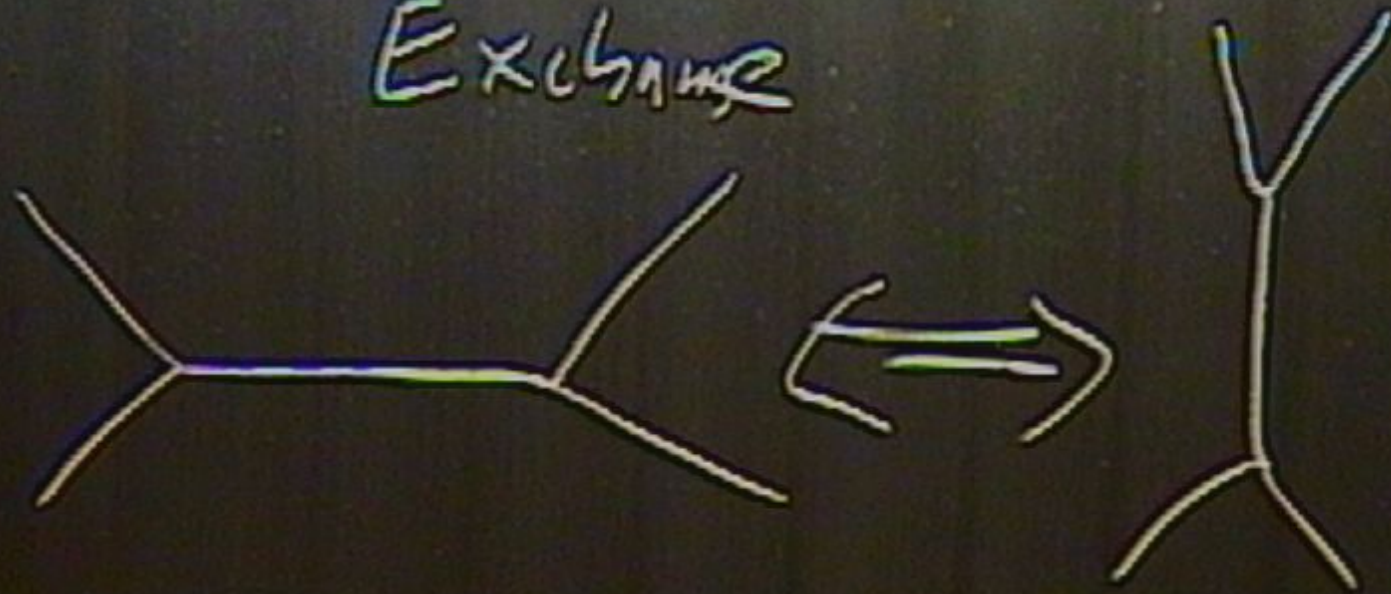
- Universe starts with a random spinnet
- Expands by a combination of expansion and exchange moves
- What really happens? Do the networks become local?







Exchange



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Hal Finkel did a series of numerical experiments using stochastic evolution with various mixes of evolution moves

- No quantum mechanics
- No labels, only graphs
- Random start ~ 200 nodes
- Grow to $\sim 5,000$ nodes
- Vary $R = \text{exchange moves} / \text{expansion moves}$

Finkel hep-th/0601163

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$$R = \frac{\text{prob of exchange}}{\text{prob of expansion}}$$

2.12.05

Start:

Random graph

200 nodes

400 edges

Red local

Magenta non-local

$L=5$



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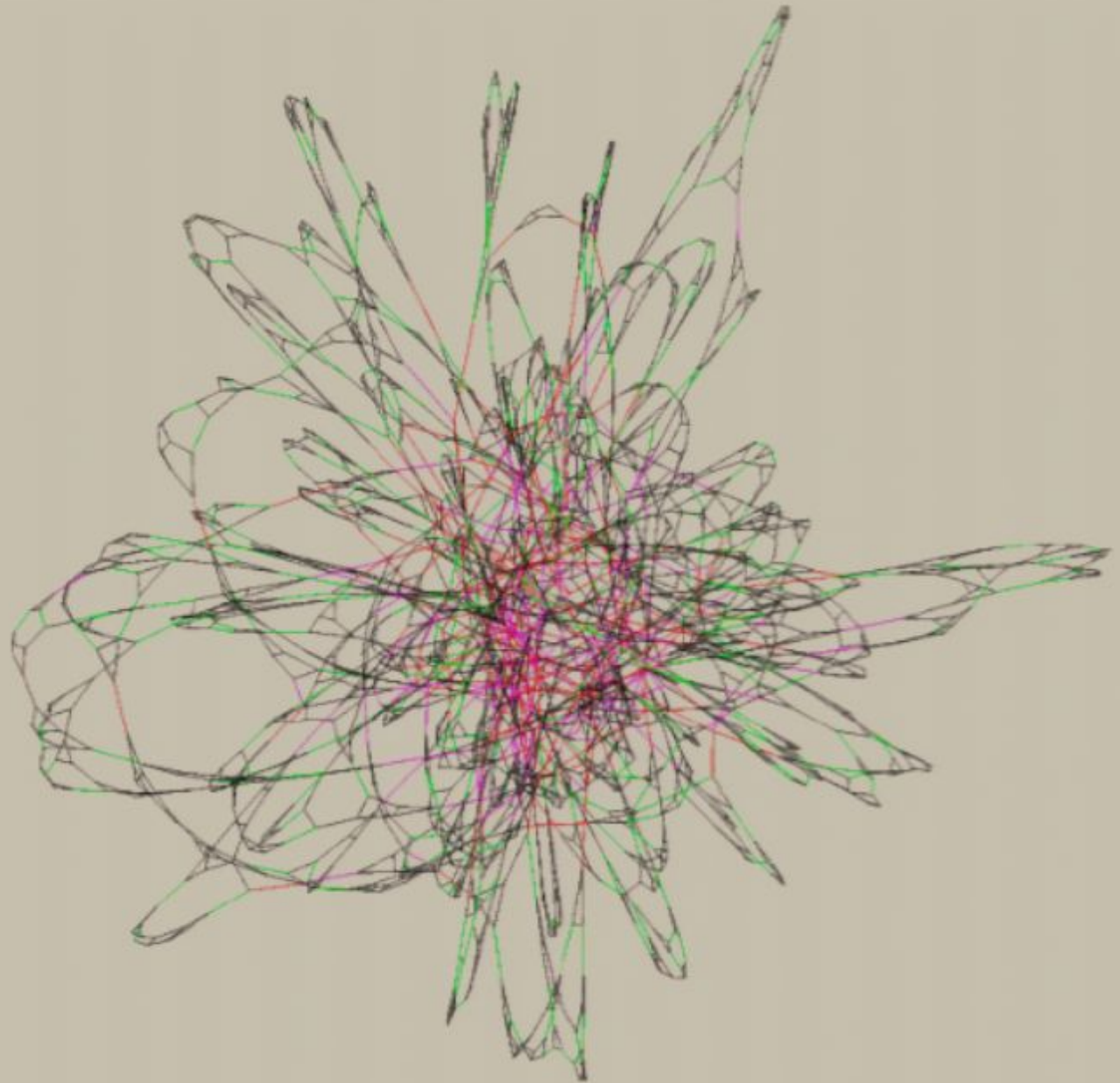
Evolved to
2200 nodes,
3400 edges

$R=0$

All expansion
Moves

Red and black
are local

Green, magenta,
Non-local



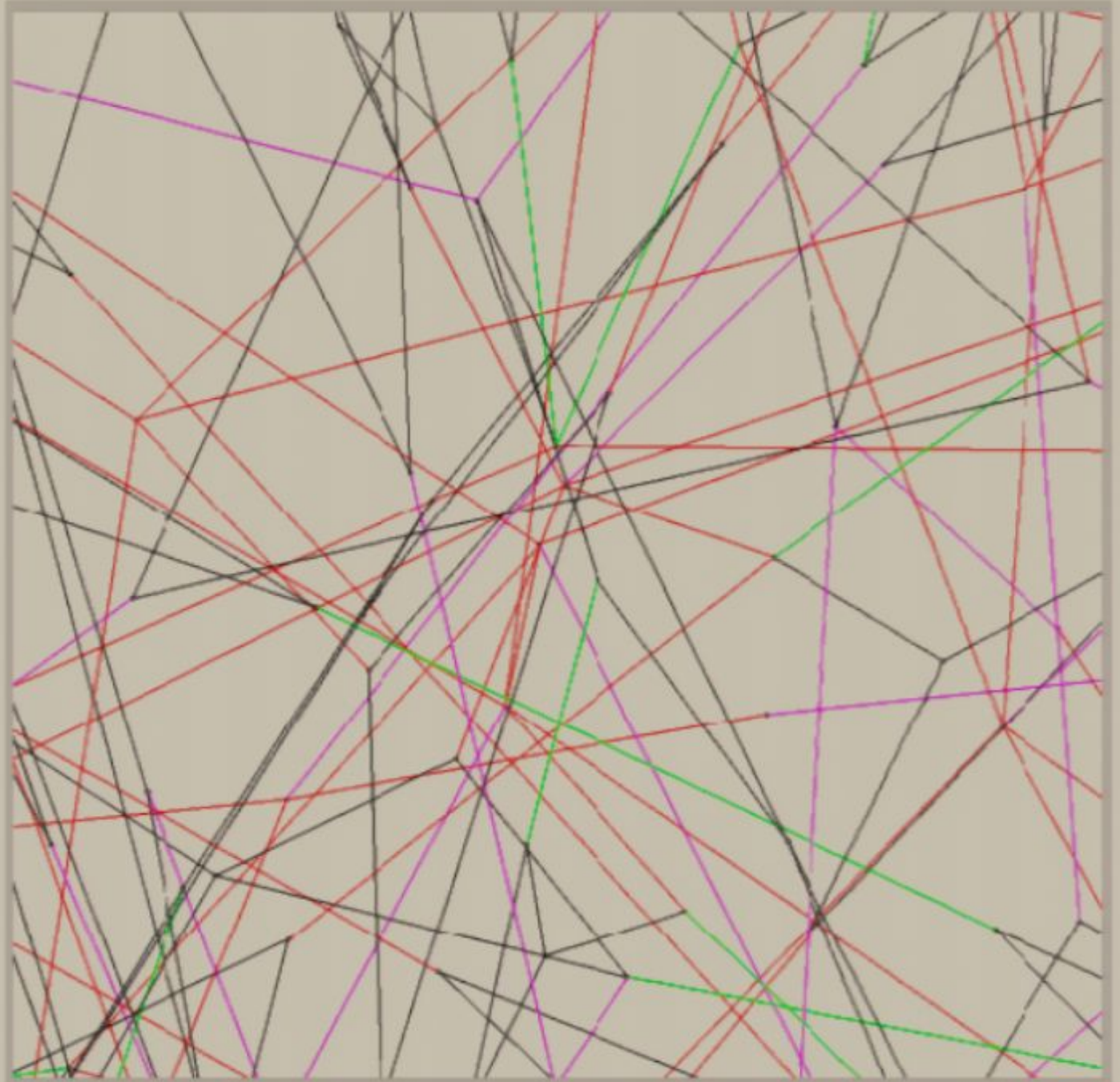
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BLOW UP
OF CENTER

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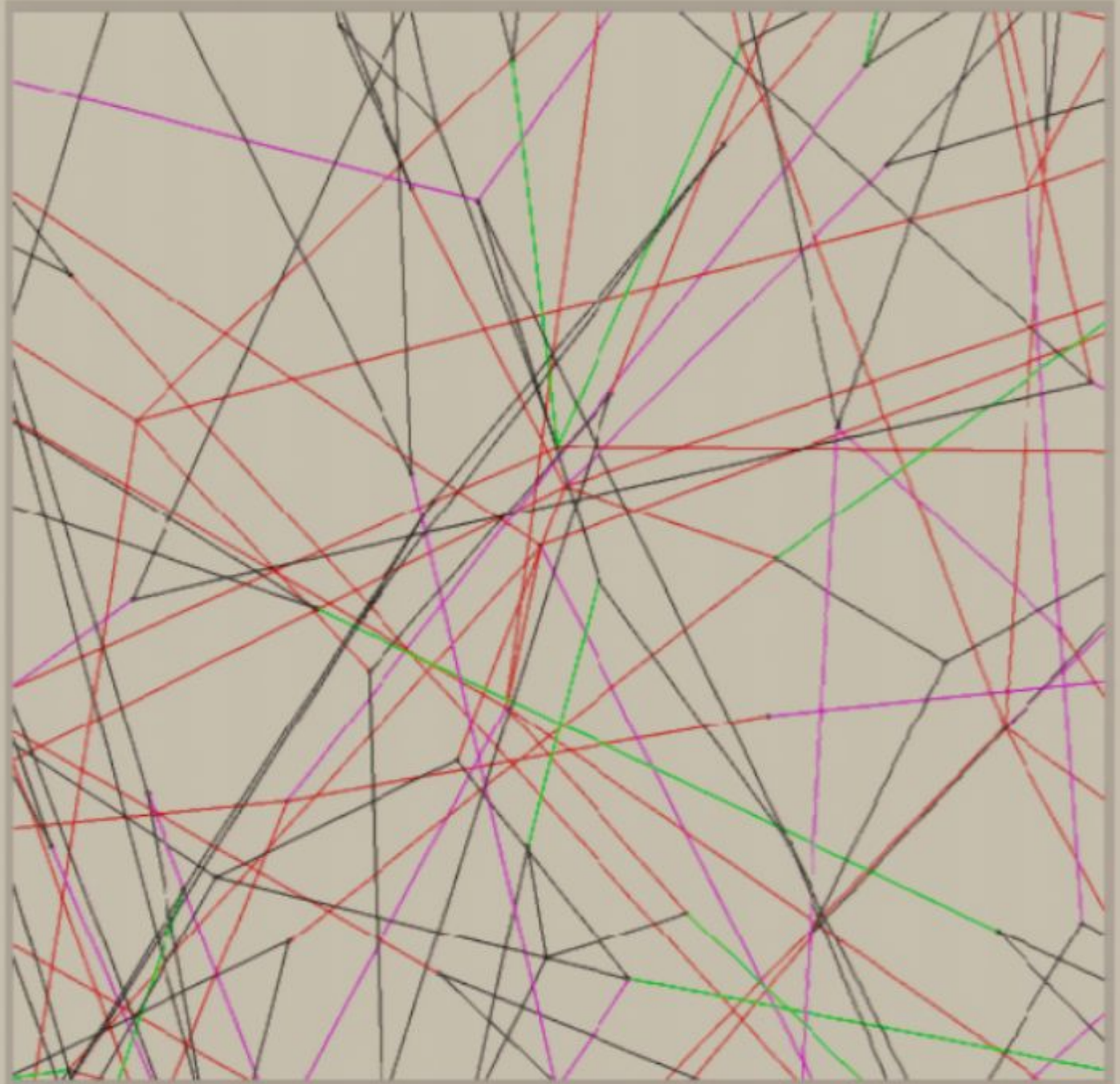
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Initial: local red
nonlocal magenta

R=1 blowup

Added: local black
nonlocal green

Expansion dominated phase:

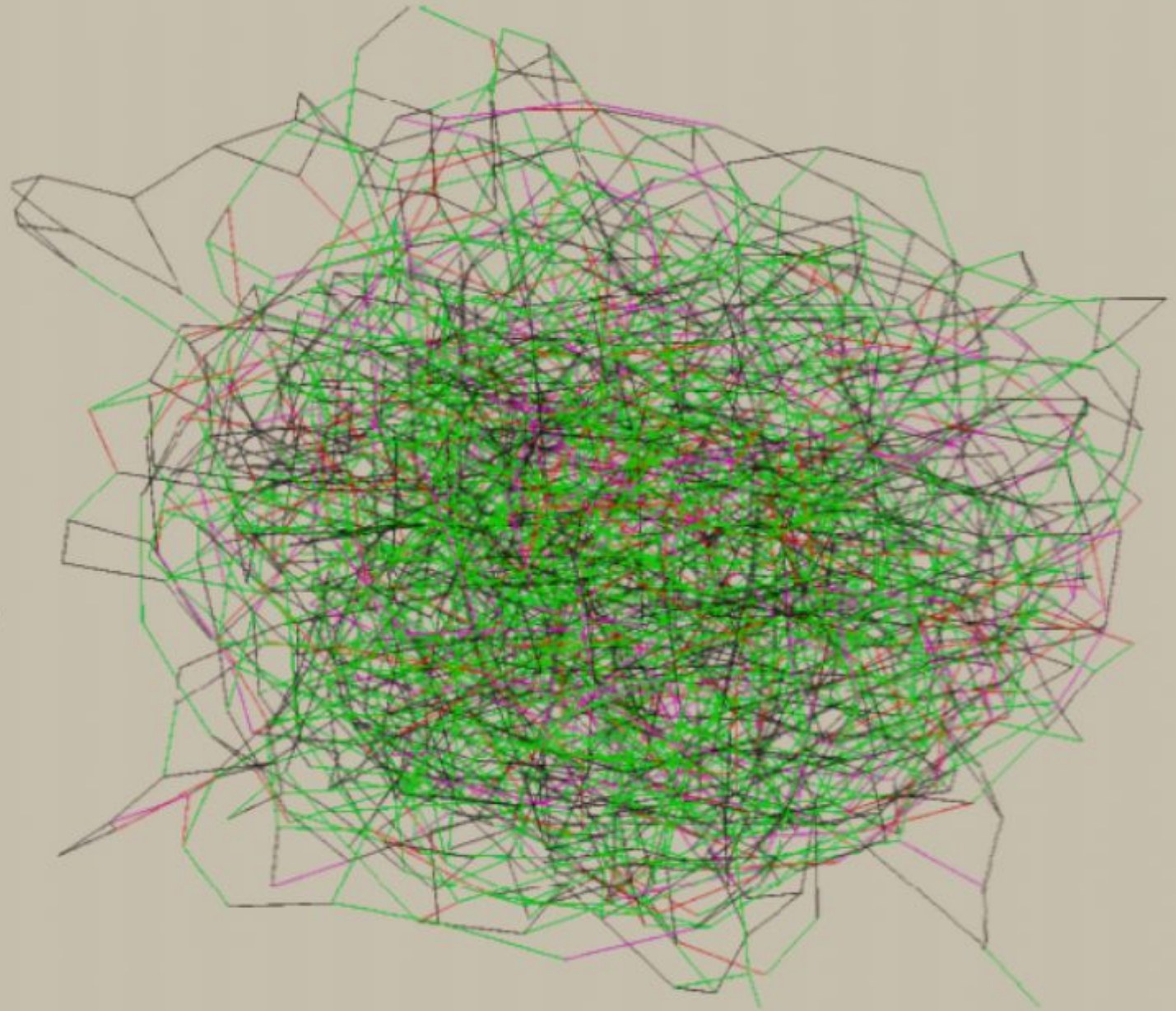
spiky, not a random sampling of any manifold

Evolved to
2200 nodes

$R=100$

Red, black local

Green, magenta,
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Evolved to
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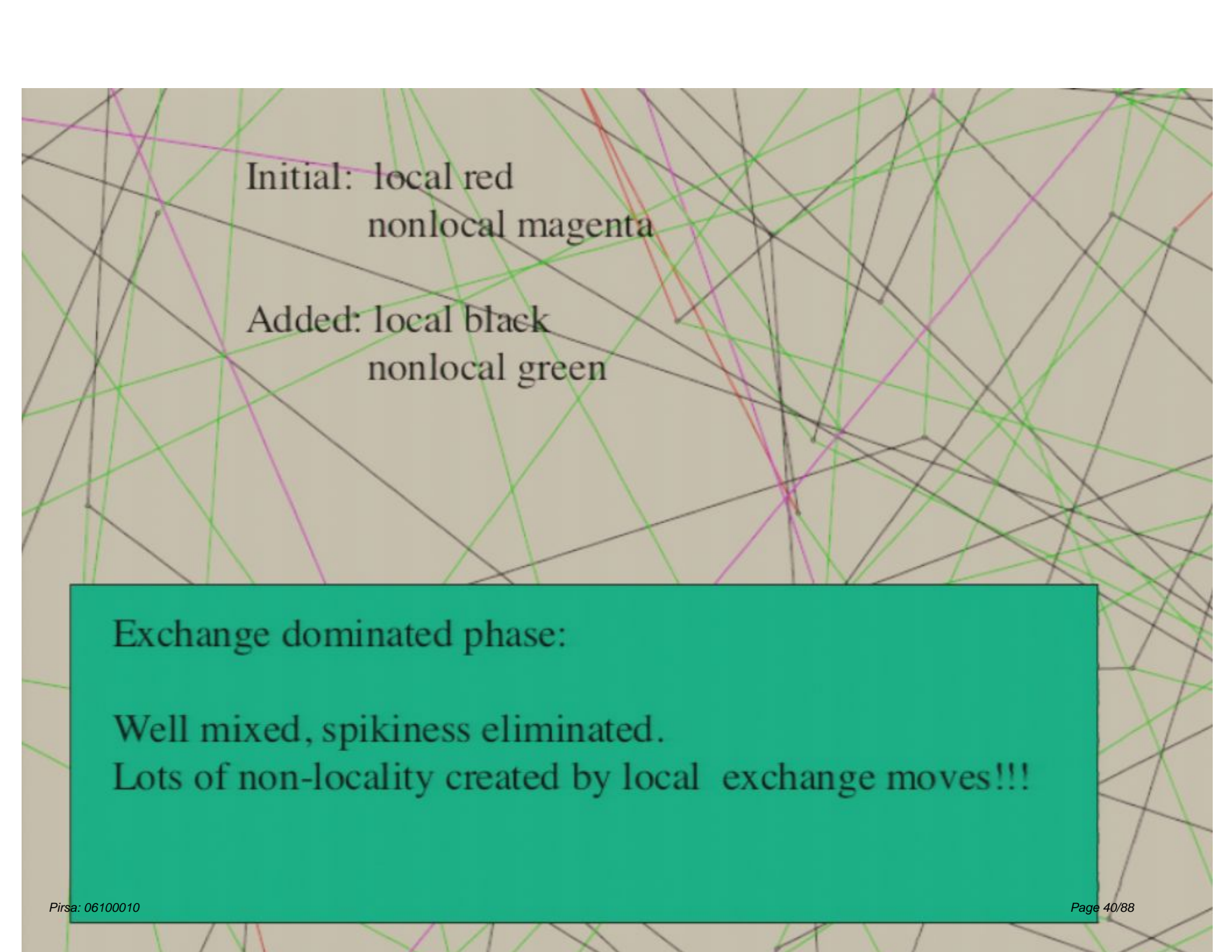
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Red, black local

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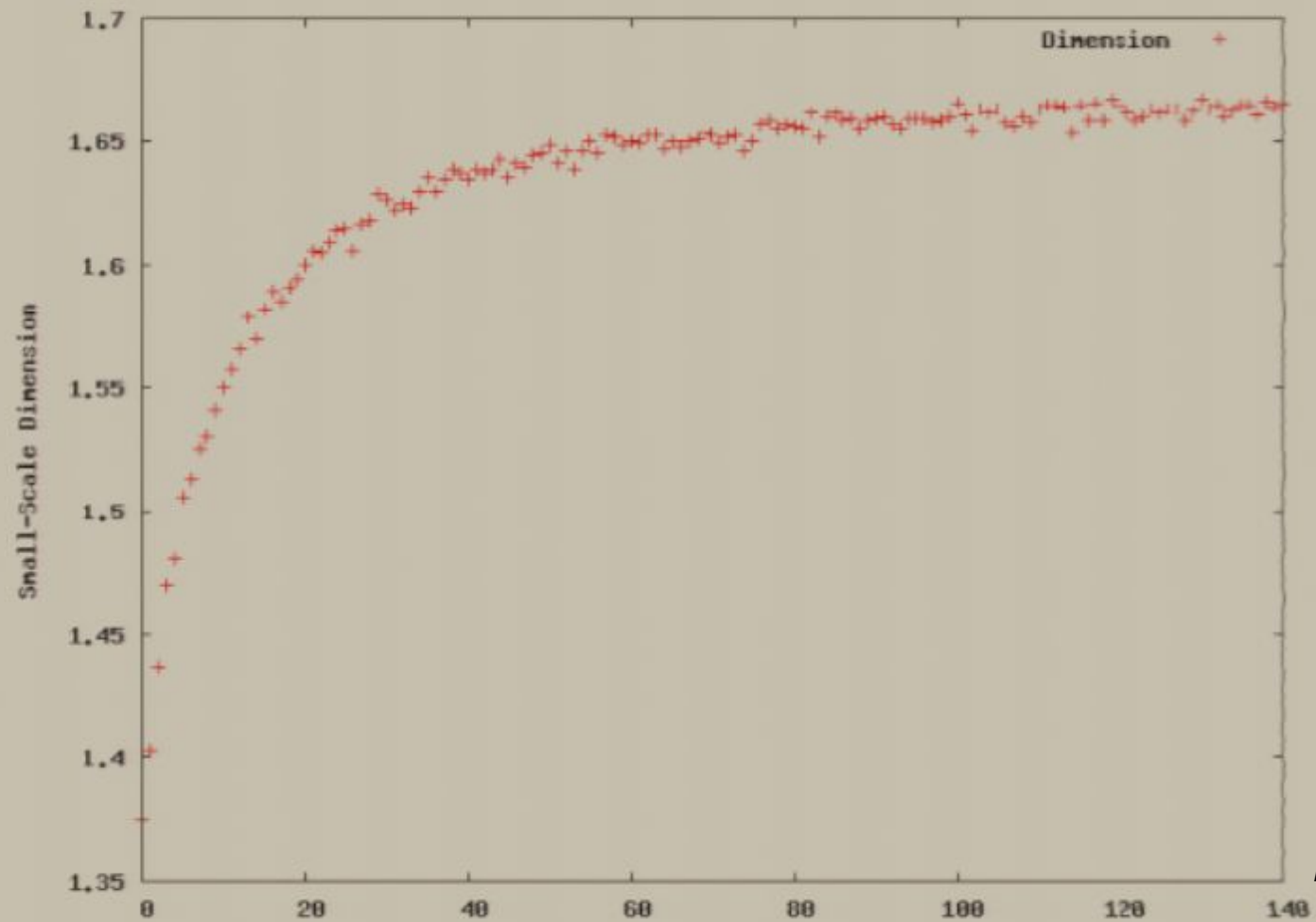
Exchange dominated phase:

Well mixed, spikiness eliminated.

Lots of non-locality created by local exchange moves!!!

Evidence for two phases

Figure 14: Small-scale dimension vs. R



Compare $R=100$ to $R=1$

$R=100$



$R=1$



Tentative conclusion: dominance by exchange moves is needed to recover macro-geometry

But the non-locality never goes away.

The evolution of N_{NL} , the number of non-local connections.

- There are microscopic processes by which non-local links split into two and processes in which pairs annihilate.
- We assume these come to equilibrium. This gives us a rough Estimate of the dependence of N_{NL} with volume, V

So we have to confront the problem on non-locality in background independent formulations of quantum gravity

Three options:

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This kills theories

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Non-locality within quantum theory

Tests for observable effects:

emergent matter, dark energy, inflation, etc

(in progress with Fotini, Sundance, Chanda, Joao...)

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Non-locality is the origin of quantum theory

The rest of this talk...

The basic idea:

- The fundamental theory is combinatorial and deterministic or stochastic. It is based on an evolving graph like a spin network.
- The low energy theory recovers spacetime as an averaged description, with the graph embedded in it.
- Because the coarse grained notion of locality incompletely represents the fundamental notion, there are stray edges, connecting nodes that are far away in the coarse grained notion of locality.
- Statistical mechanics for the whole system, plus reasonable conditions, implies quantum mechanics for subsystems.
- The stray non-local links are the missing hidden variables.

There are several claims to derive QM from a statistical or stochastic non-local theory:

- Edward Nelson
- Steve Adler
- Markopoulou and LS
- Artem Starodubstev [hep-th/0206097](#)

How do these work?

Are there general conditions that must be satisfied for such a derivation to work?

Principles:

P1 Averaging over the hidden variables induces a random noise in the evolution of the subsystem variables. This allows the subsystem to be described in terms of stochastic differential equations.

P2 There is a notion of averaged energy in the subsystem, which is conserved in time.

P3 The averaging over the effects of the hidden variables does not impose a direction of time. After the hidden variables are averaged over the stochastic dynamics of the subsystem is invariant under time reversal.

P4 We can approximate the averaged dynamics by keeping terms only of lowest order in spacial and time derivatives. In particular, the probability current for the subsystem variables is assumed to be irrotational.

The basics of Brownian motion on configuration space.

probability density: $\rho(x^a, t)$ probability current $v^a(x, t)$

Conservation: $\frac{d\rho}{dt} = -\frac{\partial \rho v^a}{\partial x^a}$ $\int d^n x \rho(x) = 1$

Evolution

Forward: $dx^a = b^a dt + dw^a$ $\langle dw^a dw^b \rangle = 2\nu dt q^{ab}$ $dt > 0$

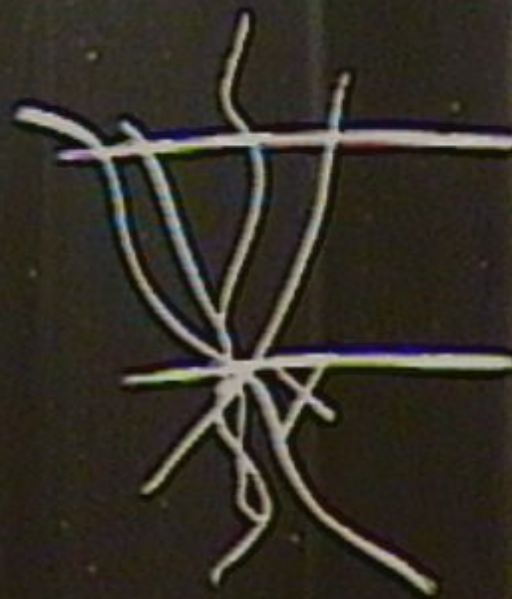
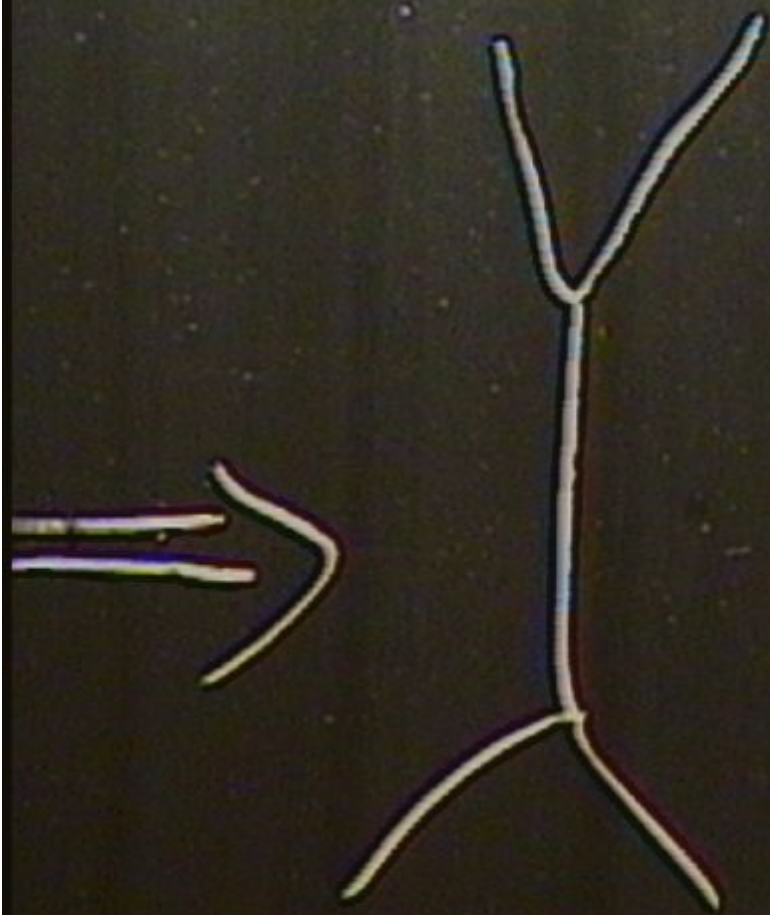
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Current velocity: $v^a = \frac{1}{2}(b^a + b^{*a})$

Osmotic velocity: $u^a = \frac{1}{2}(b^a - b^{*a}) = \nu q^{ab} \nabla_b \ln \rho$

$$Dx(t) = \lim_{\Delta t \rightarrow 0+} \left\langle \frac{x(t + \Delta t) - x(t)}{\Delta t} \right\rangle$$

$$D_*x(t) = \lim_{\Delta t \rightarrow 0+} \left\langle \frac{x(t) - x(t - \Delta t)}{\Delta t} \right\rangle$$



$t + \delta t$

t

$R =$

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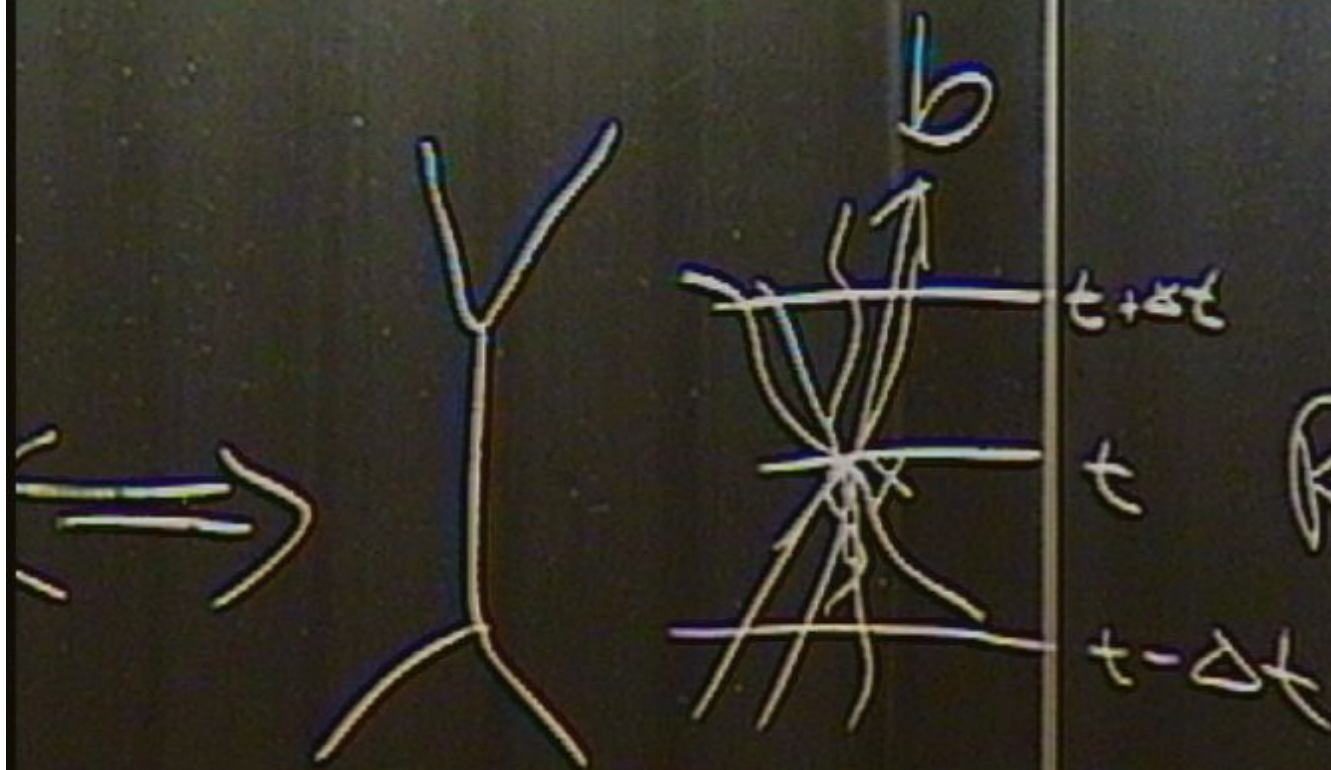
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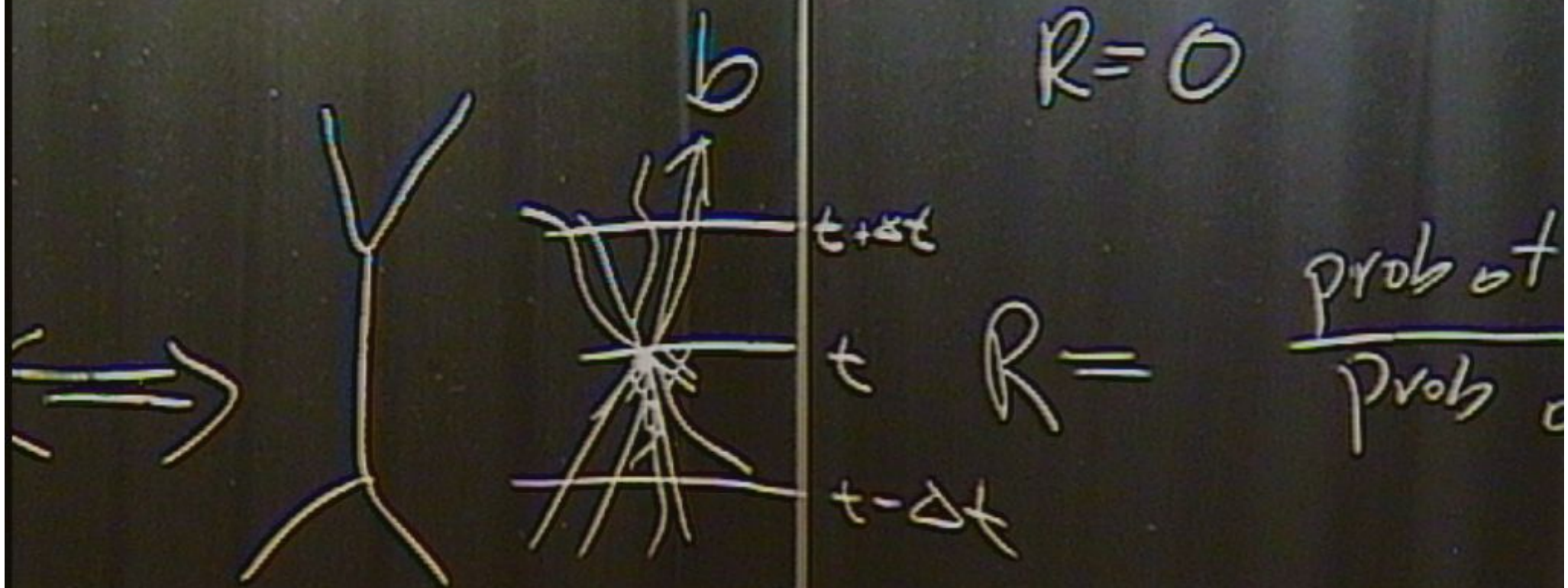
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The Fokker-Planck equations

$$\begin{aligned}\dot{\rho} &= -\nabla_a(\rho b^a) + \nu \nabla^2 \rho \\ \dot{\rho} &= \nabla_a(\rho b^{*a}) - \nu \nabla^2 \rho\end{aligned}$$

Imply:

$$\frac{d\rho}{dt} = -\frac{\partial \rho v^a}{\partial x^a}$$

$$u^a = \frac{1}{2}(b^a - b^{*a}) = \nu q^{ab} \nabla_b \ln \rho$$

Behavior under time reversal

$$b^a \rightarrow -b^{*a}, \quad v^a \rightarrow -v^a, \quad u^a \rightarrow u^a$$

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We posit an averaged Hamiltonian with the following conditions:

- H is a function of $\rho(x)$, $v(x)$ and position.
- H is conserved in time, (**P2**).
- H is invariant under time reversal invariance, (**P3**).
- H contains only those terms that dominate in the low velocity and long long wavelength limit (**P4**).
- The current velocity $v_a(x)$ is irrotational (**P4**).
- H is positive definite.
- H is invariant under rotations.

• H is local

$$H = \int d^N x \tilde{\mathcal{H}}$$

$$\tilde{\mathcal{H}} = \rho h$$

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- H is invariant under time reversal invariance, (**P3**).
- H is positive definite and invariant under rotations.

$$h(x) = F_1[v^2] + F_2[u^2] + F_3[(u \cdot v)^2] + \mathcal{U}(x) + \text{terms in } \partial v \text{ and } \partial u \quad F_i, \mathcal{U} \text{ pos, definite}$$

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In the low velocity and long long wavelength limit (**P4**):

$$h(x) = \frac{m}{2} v^2 + \frac{b}{2} u^2 + \mathcal{U}(x) \quad m, b \geq 0.$$

So anything new comes from the b term

$$H_{quantum} = \int d^N x \rho \frac{b}{2} u^2 = \frac{b\nu^2}{2} \int d^N x \rho (\nabla \ln \rho)^2$$

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We now impose: The current velocity $v^a(x)$ is irrotational (**P4**).

$$\frac{\partial v^a}{\partial x^b} - \frac{\partial v^b}{\partial x^a} = 0$$

This implies

$$v^a = \frac{1}{m} q^{ab} \frac{\partial S}{\partial x^b}$$

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Putting everything together we have finally:

$$H = \int \rho \left(\frac{1}{2m} (\nabla_a S)^2 + \frac{b\nu^2}{2} (\nabla_a \ln \rho)^2 + \mathcal{U} \right)$$

Derivation of the Schroedinger equation

$$H = \int \rho \left(\frac{1}{2m} (\nabla_a S)^2 + \frac{b\nu^2}{2} (\nabla_a \ln \rho)^2 + \mathcal{U} \right)$$

We now impose

$$0 = \dot{H} = \int \left\{ \dot{\rho} \left[\frac{1}{2m} (\nabla_a S)^2 + \mathcal{U} - \frac{b\nu^2}{2} \left(\frac{(\nabla_a \rho)^2}{\rho^2} + 2\nabla^a \frac{1}{\rho} \nabla_a \rho \right) \right] - \dot{S} \frac{1}{2m} \nabla^a (\rho \nabla_a S) \right\}$$

Derivation of the Schroedinger equation

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We use

$$\dot{\rho} = -\frac{1}{m} \nabla^a (\rho \nabla_a S),$$

Which gives:

$$0 = \int \dot{\rho} \left[\dot{S} + \frac{1}{2m} (\nabla_a S)^2 + \mathcal{U} - \frac{b\nu^2}{2} \left(\frac{(\nabla_a \rho)^2}{\rho^2} + 2\nabla^a \frac{1}{\rho} \nabla_a \rho \right) \right]$$

This implies for all ρ that

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We also have the conservation law

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These are the real and imaginary parts of the Schroed eq

$$i\hbar \frac{d\Psi}{dt} = \left[-\frac{\hbar^2}{2m} \nabla^2 + \mathcal{U} \right] \Psi$$

$$\Psi(x, t) = \sqrt{\rho} e^{iS/\hbar}$$

$$\hbar \equiv \nu \sqrt{mb}$$

The scene of the crime:

When we write:

$$H = \int d^N x \rho(x) h(x)$$

We expect that h is a property of an individual system.
So h cannot depend on ρ .

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Either:

We accept that the energy of a system can depend on the gradient of its probability distribution.

Or

We derive this from a more fundamental theory

Derivation of Nelson from a hidden variables theory

$$H = H^{subsystem}(x, p) + H^{hidden}(y, \pi) + H^{int}(x, y, p, \pi)$$

$$\langle O \rangle = \int_{\Gamma} \tilde{\rho} O$$

$$H^{subsystem}(x, p) = \frac{1}{2m} q^{ab} p_a p_b + \mathcal{U}(x)$$

We study special probability distributions defined by a Hamilton-Jacobi function, $S(x, y)$

$$\tilde{\rho}_S[x, y; p, \pi] = \tilde{\rho}(x, y) \prod_a \delta(p_a - \frac{\delta S}{\delta x^a}) \prod_{\alpha} \delta(p_{\alpha} - \frac{\delta S}{\delta y^{\alpha}})$$

On solutions:

$$p_a = \frac{\delta S}{\delta x^a} = m q_{ab} \dot{x}^b(x, y)$$

Hence

$$\langle H^{subsystem} \rangle = \int dx dy \tilde{\rho}(x, y) \left[\frac{m}{2} q_{ab} \dot{x}^a(x, y) \dot{x}^b(x, y) + V(x) \right]$$

The task is now to average over the y 's to find the stochastic dynamics for the x 's.

We study the averaged kinetic energy:

$$\langle k \rangle = \int dy \int dx \tilde{\rho}(x, y) \frac{m}{2} \dot{x}^2(x, y)$$

With the hidden variables specified everything is classical
So we can write:

$$k(x, y) = \frac{m}{2} \lim_{dt \rightarrow 0} \left(\frac{x(t + dt)(x, y) - x}{dt} \right)^2$$

We can make it explicitly time symmetric

$$k(x, y) = \frac{m}{4} \lim_{dt \rightarrow 0} \left[\left(\frac{x(t + dt)(x, y) - x}{dt} \right)^2 + \left(\frac{x - x(t - dt)(x, y)}{dt} \right)^2 \right]$$

Since everything is smooth this doesn't change anything

Now we take the average:

$$\langle k \rangle = \int dy \int d^N x \tilde{\rho}(x, y) \frac{m}{4} \lim_{dt \rightarrow 0} \left[\left(\frac{x(t + dt)(x, y) - x}{dt} \right)^2 + \left(\frac{x - x(t - dt)(x, y)}{dt} \right)^2 \right]$$

To get effective local equations we have to integrate out the hidden variables, y . This will make the evolution of the x 's stochastic. Hence, non-differentiable.

Hence, we exchange the order of the limit and averaging

ie we assume there is some micro time scale τ such that

$$\int dy \lim_{dt \rightarrow 0} \approx \lim_{dt \rightarrow \tau} \int dy.$$

The result must be expressed in the language of stoch. diff eq's

Thus, exchanging the order we have

$$\langle k \rangle = \lim_{dt \rightarrow 0} \frac{m}{4} \int dy \int d^N x \tilde{\rho}(x, y) \left[\left(\frac{x(t+dt)(x, y) - x}{dt} \right)^2 + \left(\frac{x - x(t-dt)(x, y)}{dt} \right)^2 \right]$$

We now use

$$x^a(t+dt)(x, y) - x^a = b^a(x, t)dt + dw^a(x, y)$$

To write:

$$\langle k \rangle = \lim_{dt \rightarrow \tau} \frac{m}{4} \int dy \int d^N x \tilde{\rho}(x, y) \left[\left(\frac{b^a(x, t)dt + dw^a(x, y)}{dt} \right)^2 + \left(\frac{b^{*a}(x, t)dt + \Delta^* w^a(x, y)}{dt} \right)^2 \right]$$

$$= \frac{m}{4} \int d^N x \rho(x) [b^2 + b^{*2}] + C + O(\tau)$$

$$C = \frac{\nu m}{2\tau}$$

$$= \frac{m}{2} \int d^N x \rho(x) [v^2 + u^{*2}] + C + O(\tau)$$

This leads to QM!

Hence, our procedure, reversing the order of limits and integration, reproduces the u^2 term.

Note: assuming time reversal invariance was essential.

This is no longer enforced once the limit and integral are exchanged.

Had we taken:

$$k(x, y) = \frac{m}{4} \lim_{dt \rightarrow 0} \left[\alpha \left(\frac{x(t + dt)(x, y) - x}{dt} \right)^2 + \beta \left(\frac{x - x(t - dt)(x, y)}{dt} \right)^2 \right]$$

The result would have been different:

$$\alpha + \beta = 1$$

$$\langle K \rangle = \frac{m}{2} \int d^N x \rho(x) [v^2 + u^{*2} + 2(\alpha - \beta)v \cdot u] + C$$

This does not lead to QM!

CONCLUSION:

Quantum mechanics can be a consequence of the more fundamental theory envisioned in these derivations only if the effect of the hidden variables on the observables does not disturb time reversal invariance.

In many circumstances in which a system is put into interaction with a reservoir, the effect of the reservoir is to make the system tend to equilibrium, which gives a preferred direction of time to the thermodynamics of the system. If hidden variables are responsible for quantum dynamics, their effect must be to disorder the dynamics of the observable quantities in a way that preserves both the conservation of an average conserved energy and the time reversal invariance of the evolution of the ensemble representing the system.

ANSWERING WALLSTROM'S OBJECTION

We consider quantum mechanics on a circle

The variables in Nelson's formulation are $\rho(\theta)$, $v(\theta)$

Conservation:

$$\dot{\rho} = \partial(\rho v)$$

Locally, but not globally:

$$v = \frac{1}{m} \partial S$$

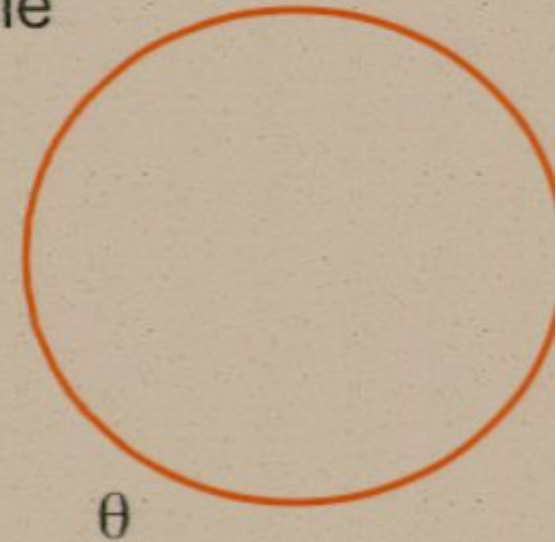
$$\dot{S} = \frac{1}{2m} (\partial S)^2 + \mathcal{U} + \frac{b\nu^2}{2} \left(\frac{(\partial \rho)^2}{\rho^2} + 2\partial \left(\frac{1}{\rho} \partial \rho \right) \right)$$

$$\mathcal{U}=0$$

Solution:

$$\rho = \frac{1}{2\pi} \quad v(\theta) = w$$

w arbitrary



We consider quantum mechanics on a circle

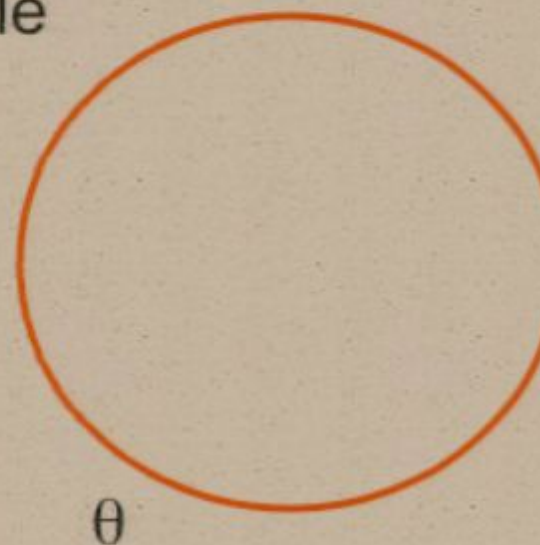
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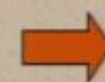
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$$S = w m \theta + \omega t$$

Solution:

$$\rho = \frac{1}{2\pi}$$

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w arbitrary

$$\omega = \frac{m w^2}{2}$$

$$\psi(\theta, t) = \sqrt{\rho} e^{i \frac{S}{\hbar}} = \frac{1}{\sqrt{2\pi}} e^{i \frac{1}{\hbar} (m w \theta - \omega t)}$$

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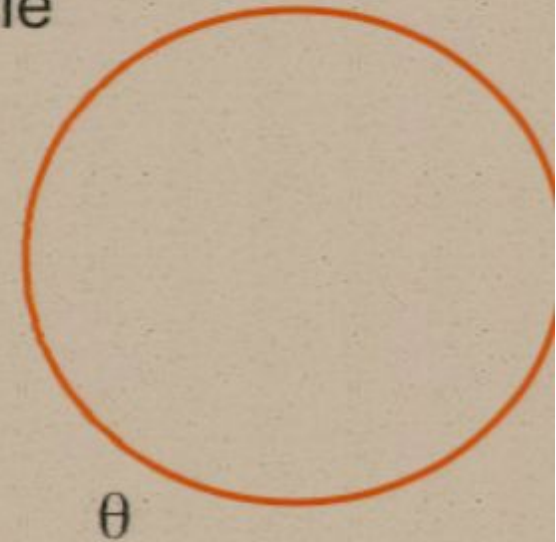
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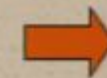
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Problem: this is not continuous at $\theta=0$

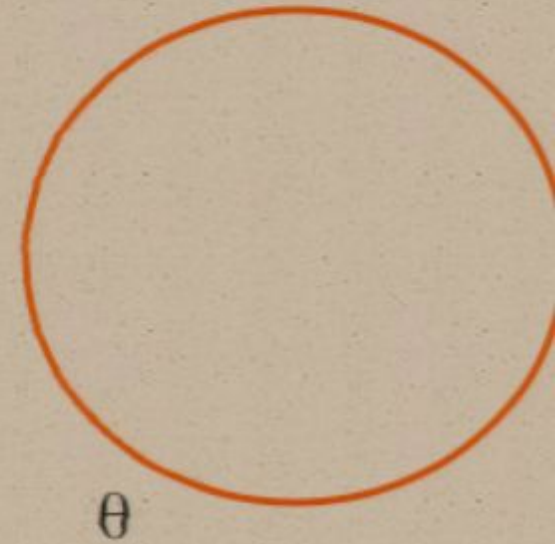
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Answer: This wavefunction is in $L^2(S^1)$. So it is a good state.

Hence it must be a solution to Nelson's equations, as it is.

It is not a momentum or energy eigenstate, but so what.

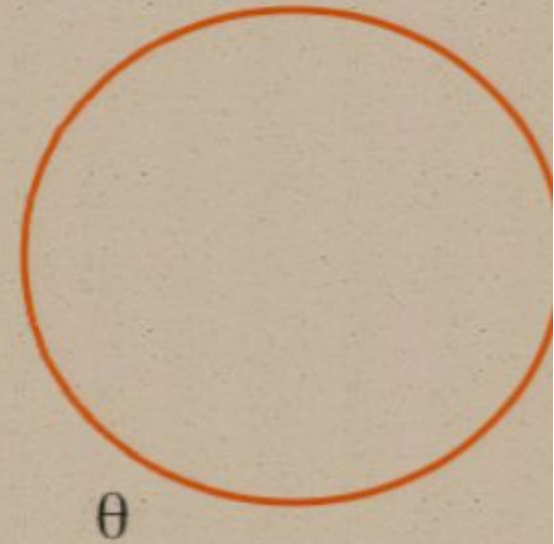
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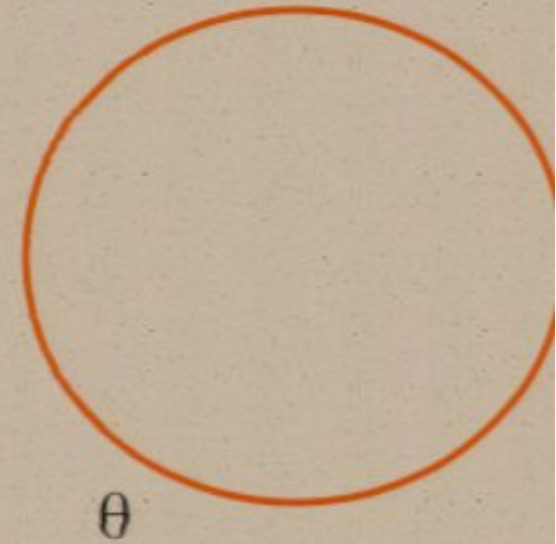
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Answer, yes it is, by the averaged energy of Nelson.