

Title: Cosmological Landscape From Nothing: Some Like It Hot

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Abstract:

# Cosmological Landscape from Nothing: Some Like It Hot

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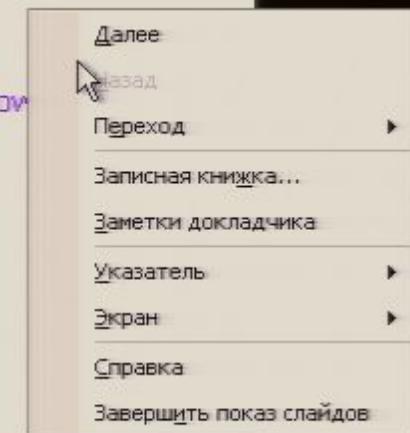
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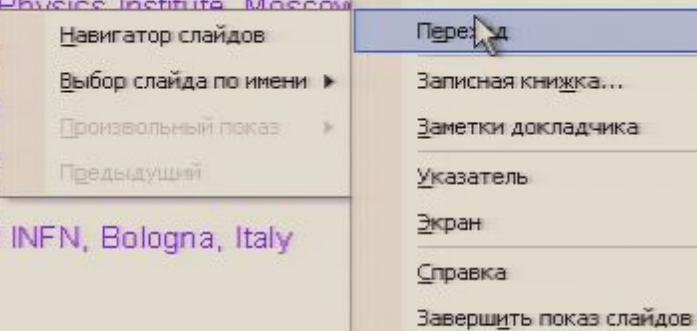
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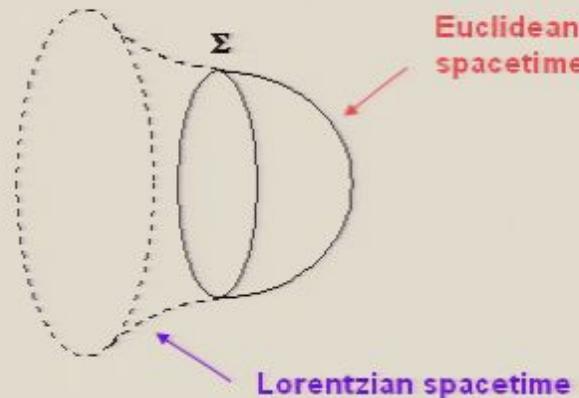
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Dipartimento di Fisica and INFN, Bologna, Italy

# Introduction

Motivation: search for selection rule restricting landscape of stringy vacua  $\rightarrow$  quantum cosmology and Euclidean quantum gravity

$$\Psi[\varphi] \sim$$



Infrared catastrophe:

$$\Psi_{\text{HH}} \sim \exp(-S_E) = \exp\left(\frac{3\pi}{2G\Lambda}\right) \rightarrow \infty, \quad \Lambda \rightarrow 0$$

Cosmology debate: no-boundary vs tunneling

$$\Psi_{\text{HH,T}} \sim \exp(\mp S_E)$$

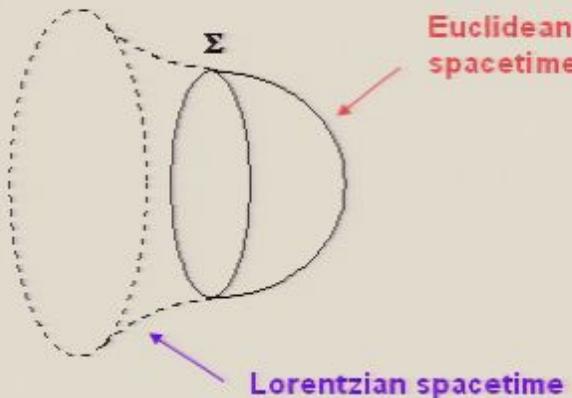


Wheeler-DeWitt equation

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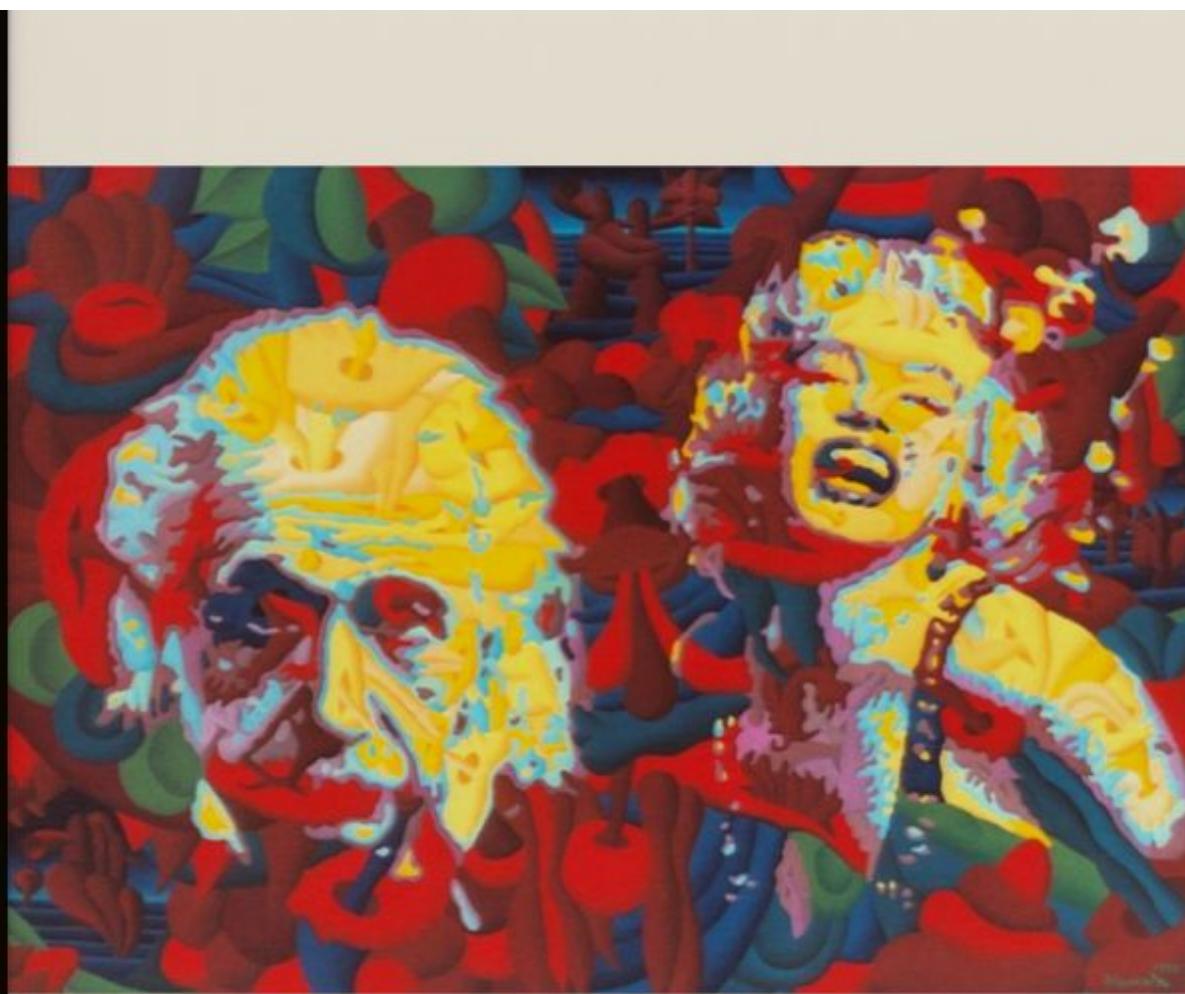
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Wheeler-DeWitt equation



*This is how cosmological landscape comes out of  
Nothing, if some really like it hot*

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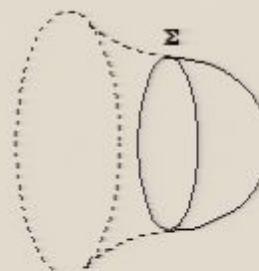
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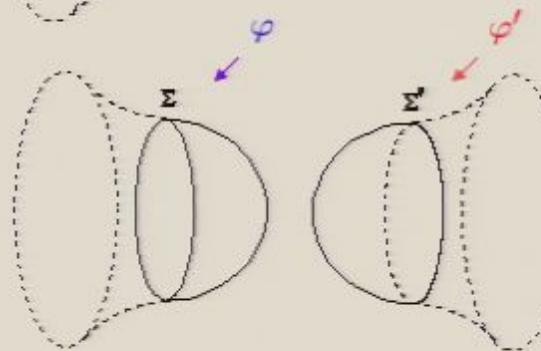
Dipartimento di Fisica and INFN, Bologna, Italy

**From the pure Hartle-Hawking state to the statistical ensemble – density matrix:**

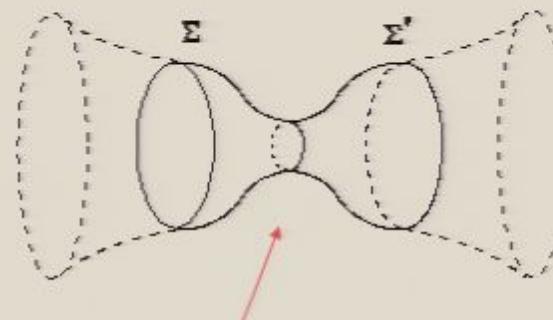
$$|\Psi_{HH}\rangle = \Psi_{HH}[\varphi]$$



$$\begin{aligned} |\Psi_{HH}\rangle \langle \Psi_{HH}| \\ = \rho_{HH}[\varphi, \varphi'] \end{aligned}$$



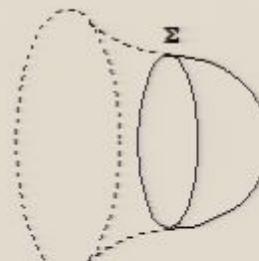
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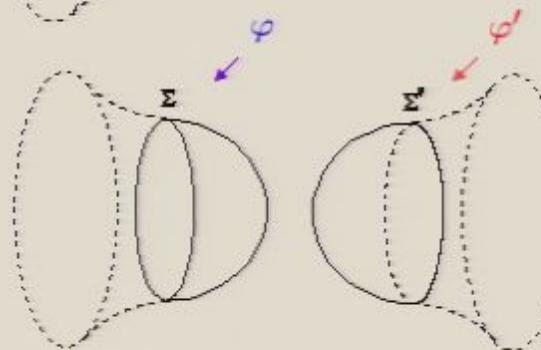
instanton bridge mediates  
density matrix correlations

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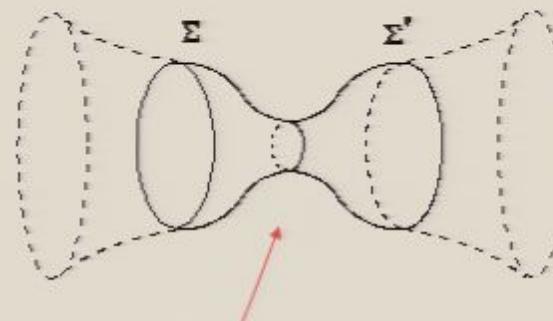
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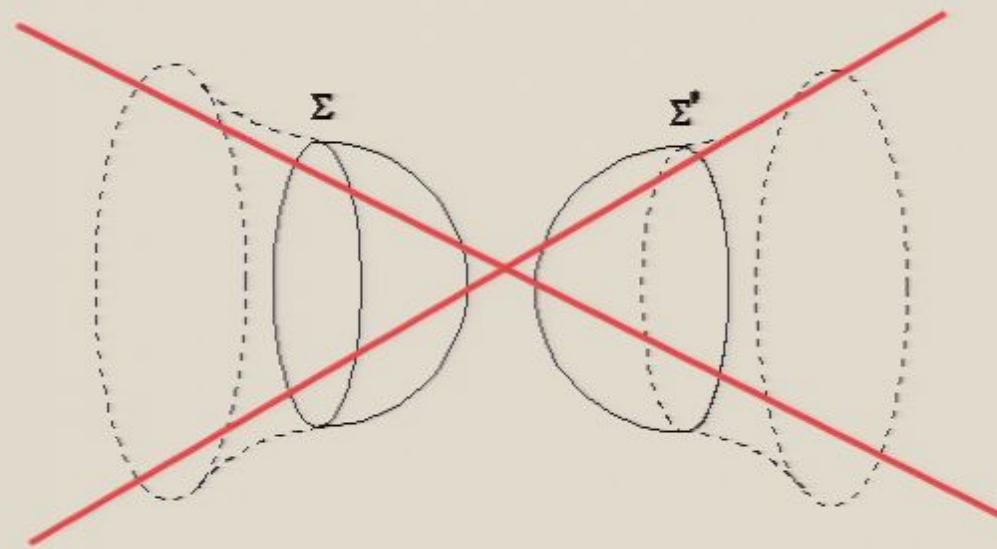


$$\begin{aligned} \hat{\rho}_{\text{mixed}} \\ = \rho_{\text{mixed}}[\varphi, \varphi'] \end{aligned}$$



instanton bridge mediates  
density matrix correlations

## Main effect



Back reaction of quantum matter destroys HH instanton by

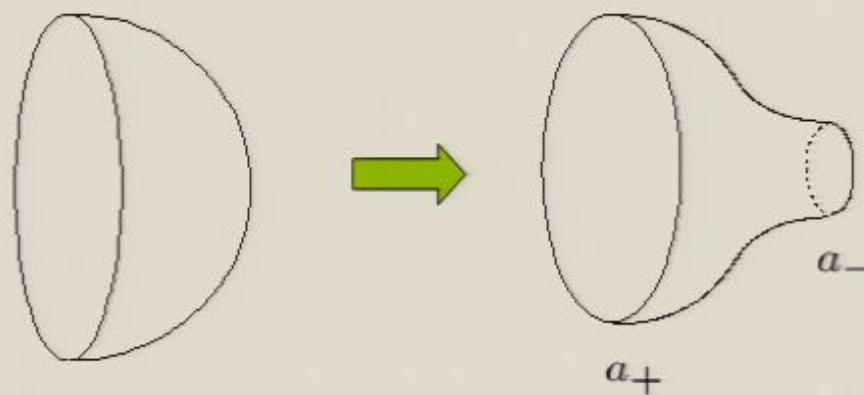
- i) radiation (classical effect) and
- ii) conformal anomaly contribution (quantum effect).

This prevents half-instantons from closure and maintains the thermal nature of the physical state

# Effect of radiation

$$ds^2 = N^2(\tau) d\tau^2 + a^2(\tau) d^2\Omega^{(3)},$$

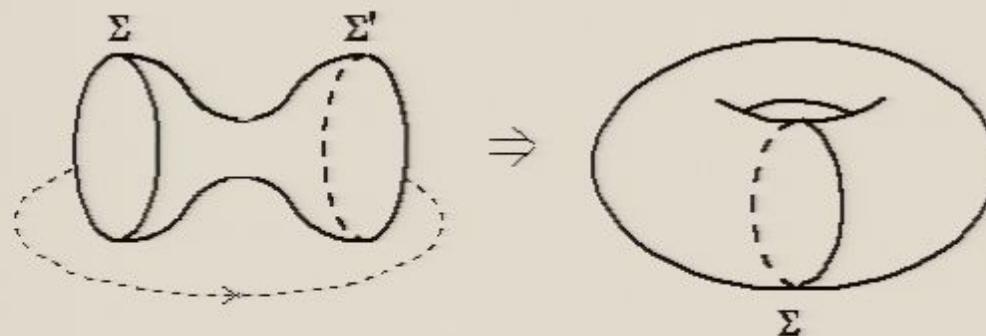
$$\frac{\dot{a}^2}{a^2} = \frac{1}{a^2} - H^2 - \frac{C}{a^4}$$



$$a_{\pm} = \frac{1}{\sqrt{2}H} \sqrt{1 \pm (1 - 4CH^2)^{1/2}}$$

$$4H^2C \leq 1$$

Calculation of partition function – toroidal instanton  
with periodically identified Euclidean time



Bootstrap: back reaction of radiation supports instanton background on top of which this radiation exists



Bounded cosmological landscape

$$\Lambda_{\min} < \Lambda < \Lambda_{\max}$$

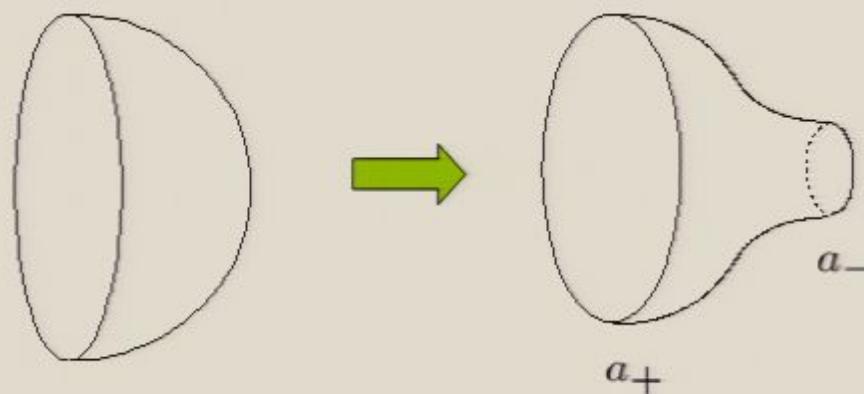
Boundedness of on-shell gravitational action in Euclidean QG

Infinite sequence of 1-parameter families of garland-type instantons saturating  $\Lambda_{\max}$  – new quantum gravity scale

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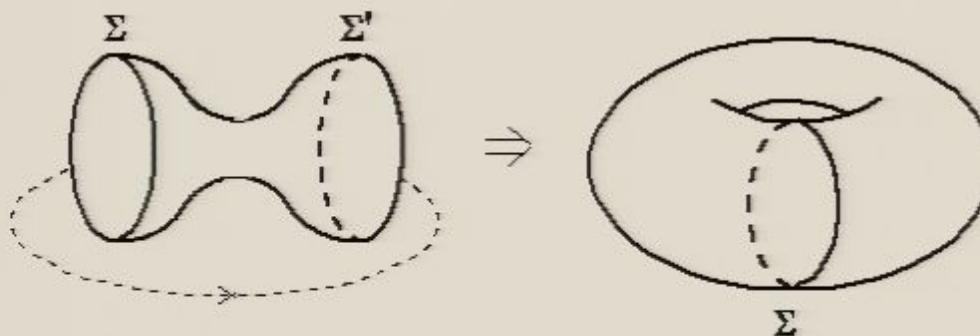
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# Outline

Density matrix vs pure Hartle-Hawking state

Back reaction of hot (radiation-like) quantum matter

Conformal anomaly and ghost-avoidance  
renormalization

Effective (modified Friedmann) and bootstrap  
equations

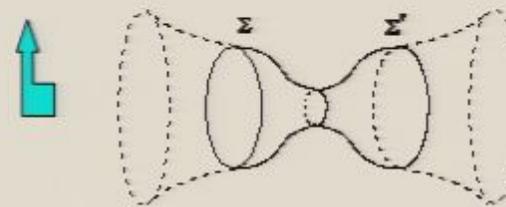
Elimination of infrared catastrophe of small  $\Lambda$

Bounded cosmological landscape of instanton  
garlands --- new quantum scale

Limiting the landscape of stringy vacua?

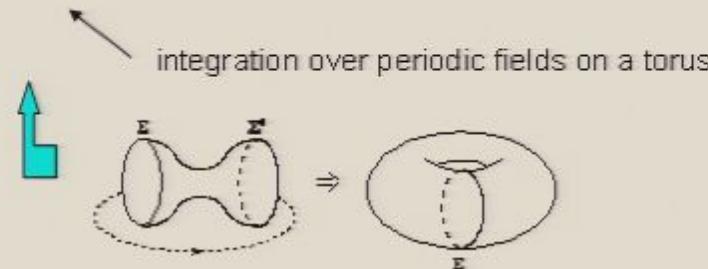
# Density matrix

$$\rho[\varphi, \varphi'] = e^{\Gamma} \int_{g, \phi|_{\Sigma, \Sigma'} = (\varphi, \varphi')} D[g, \phi] \exp(-S_E[g, \phi])$$



$\text{tr } \hat{\rho} = 1 \longrightarrow$  effective action:

$$e^{-\Gamma} = \int_{g, \phi|_{\Sigma} = g, \phi|_{\Sigma'}} D[g, \phi] \exp(-S_E[g, \phi])$$



## Back reaction: nature of approximation

$$[g, \phi] = [g_0(\tau), \Phi(x)]$$

$$g_0(\tau) = (a(\tau), N(\tau))$$

minisuperspace  
background

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \dots)$$

quantum "matter" –  
cosmological perturbations

$$e^{-\Gamma} = \int Dg_0(\tau) \exp(-\Gamma[g_0(\tau)])$$

$$e^{-\Gamma[g_0]} = \int D\Phi(x) \exp(-S_E[g_0, \Phi(x)])$$

$$\Gamma[g_0] = S_E[g_0] + \Gamma_{\text{1-loop}}[g_0]$$

$$\Gamma = \Gamma_{\text{tree-level}}$$

nature of approximation



$$\frac{\delta \Gamma[g_0]}{\delta g_0} = 0$$

**Effective equation of motion**

# Conformal anomaly and ghosts

Conformally invariant quantum matter

Conformal transform to static Einstein universe

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \quad \xrightarrow{\text{conformal time}} \quad d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$$
$$g_{\mu\nu} \frac{\delta\Gamma_{\text{1-loop}}}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2} (\alpha \square R + \beta E + \gamma C_{\mu\nu\alpha\beta}^2)$$

Euler density

$$\Gamma_{\text{1-loop}}[g] = \Gamma_{\text{1-loop}}[\bar{g}] + \Delta\Gamma[g, \bar{g}], \quad g_{\mu\nu}(x) = e^{\sigma(x)} \bar{g}_{\mu\nu}(x)$$

$$\begin{aligned} \Delta\Gamma[g, \bar{g}] &= \frac{1}{2(4\pi)^2} \int d^4x \bar{g}^{1/2} \left\{ \frac{1}{2} \left[ \gamma \bar{C}_{\mu\nu\alpha\beta}^2 + \beta \left( \bar{E} - \frac{2}{3} \bar{\square} \bar{R} \right) \right] \sigma \right. \\ &\quad \left. + \frac{\beta}{2} \left[ (\bar{\square}\sigma)^2 + \frac{2}{3} \bar{R} (\bar{\nabla}_\mu \sigma)^2 \right] \right\} \\ &\quad - \frac{1}{2(4\pi)^2} \left( \frac{\alpha}{12} + \frac{\beta}{18} \right) \int d^4x \left( g^{1/2} R^2(g) - \bar{g}^{1/2} R^2(\bar{g}) \right) \end{aligned}$$



$\alpha \times \ddot{a} \longrightarrow \text{ghosts (BAD!)}$

Coefficient in the conformal anomaly

### Finite ghost-avoidance renormalization:

$$\alpha \rightarrow \alpha_R = 0$$

$$\Gamma_{\text{1-loop}}[g] \rightarrow \Gamma_R[g] = \Gamma_{\text{1-loop}}[g] + \frac{1}{2(4\pi)^2} \frac{\alpha}{12} \int d^4x g^{1/2} R^2(g)$$

positive!

### Conformal anomaly contribution:

$$\Delta\Gamma[g, \bar{g}] \equiv \Gamma_R[g] - \Gamma_R[\bar{g}] = m_P^2 B \int d\tau \left( \frac{\dot{a}^2}{a} - \frac{1}{6} \frac{\dot{a}^4}{a} \right)$$

$$m_P^2 B = \frac{3}{4} \beta$$

Euler density coefficient in the  
conformal anomaly

### For low spins:

$$\alpha = \frac{1}{90} \times \begin{cases} -1 \\ -3 \\ 18 \end{cases}, \quad \beta = \frac{1}{360} \times \begin{cases} 2 \\ 11 \\ 124 \end{cases}$$

scalar  
Weyl spinor  
vector (e-m field)



zeta-functional and  
point-separation regularizations

## Effective action on a static Einstein instanton

$$\Gamma_{\text{1-loop}}[\bar{g}] = m_P^2 E_0 \eta_0 + F(\eta_0)$$

$$m_P^2 E_0 = \pm \sum_n \frac{\omega_n}{2} \quad \text{vacuum energy } (\pm \text{ for bosons and fermions})$$

$$F(\eta_0) = \pm \sum_n \ln(1 \mp e^{-\omega_n \eta_0}) \quad \text{free energy}$$



ghost avoidance renormalization

$$\Gamma_R[\bar{g}] = m_P^2 C_0 \eta_0 + F(\eta_0)$$

$$m_P^2 C_0 = m_P^2 E_0 + \frac{3}{16} \alpha \leftarrow \begin{matrix} \text{coefficient of } \square R \text{ in anomaly} \\ \text{modified vacuum energy} \end{matrix}$$

$$m_P^2 E_0 = \frac{1}{960} \times \begin{cases} 4 \\ 17 \\ 88 \end{cases} \quad \text{UV renormalized vacuum (Casimir) energy for scalars, spinors and vectors (positive for all statistics)}$$

Important universality

$$m_P^2 C_0 = \frac{1}{2} m_P^2 B = \frac{3}{8} \beta$$

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Euler density coefficient in anomaly

## Effective Friedmann and bootstrap equations

conformal time  $\eta_0 = 2 \int_{\tau_-}^{\tau_+} \frac{d\tau N(\tau)}{a(\tau)}$  ← lapse function

$\tau_{\pm}$  – turning points of solutions:  $\dot{a}(\tau_{\pm}) = 0$

### Total effective action

$$\Gamma[a(\tau), N(\tau)] =$$

$$2m_P^2 \int_{\tau_-}^{\tau_+} d\tau \left( -\frac{a\dot{a}^2}{N} - Na + NH^2 a^3 \right) \quad \text{classical}$$

$$+ 2Bm_P^2 \int_{\tau_-}^{\tau_+} d\tau \left( \frac{\dot{a}^2}{Na} - \frac{1}{6} \frac{\dot{a}^4}{N^3 a} \right) \quad \text{anomaly}$$

$$+ Bm_P^2 \int_{\tau_-}^{\tau_+} \frac{d\tau N}{a} \quad \text{vacuum energy}$$

$$+ F \left( 2 \int_{\tau_-}^{\tau_+} \frac{d\tau N}{a} \right) \quad \text{thermal}$$

nonlocality (polylocality)  
breezing of instanton in  $\tau$ -direction

## Effective Friedmann equation (via variation of $N(\tau)$ )

$$\frac{\dot{a}^2}{a^2} + B \left( \frac{1}{2} \frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^4} \right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}$$

B anomaly contribution      C radiation and vacuum energy term

$$m_P^2 C = m_P^2 \frac{B}{2} + \frac{dF(\eta_0)}{d\eta_0} \quad \text{Bootstrap equation}$$

$$\eta_0 = 2 \int_{\tau_-}^{\tau_+} \frac{d\tau}{a(\tau)}$$

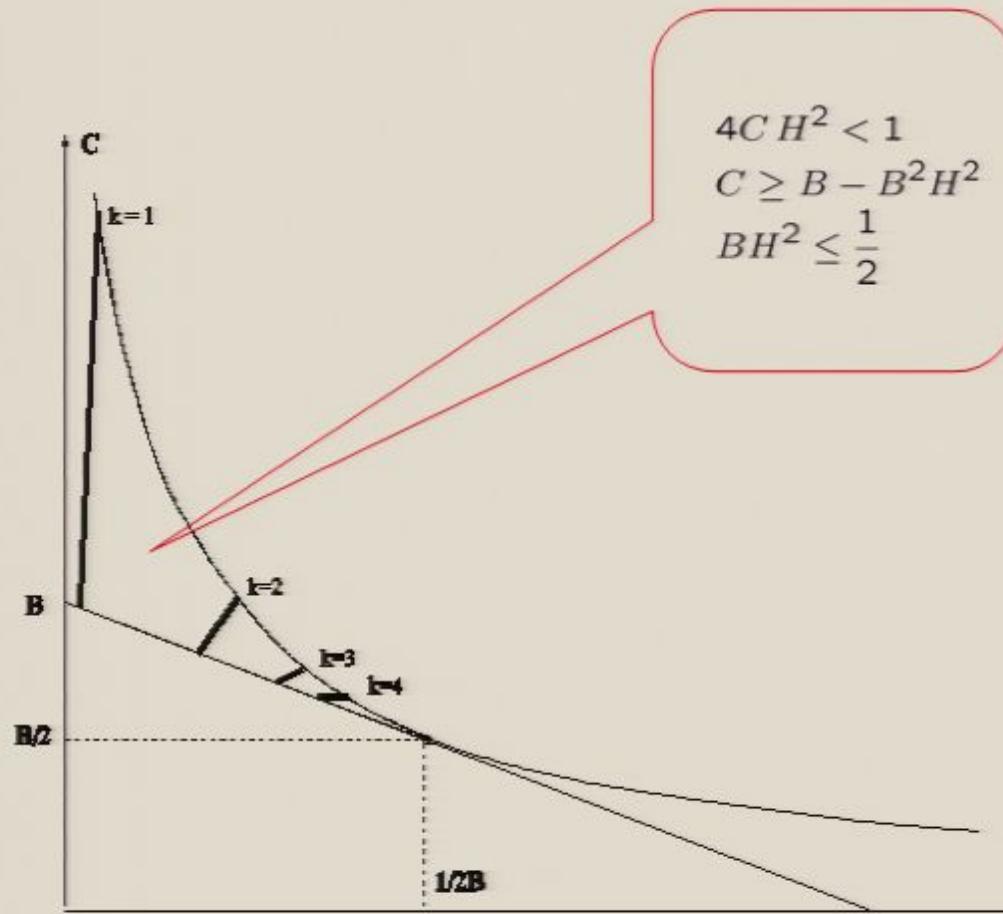
**On-shell action:**

$$\begin{aligned} \Gamma_0 &= F(\eta_0) - \eta_0 \frac{dF(\eta_0)}{d\eta_0} && \text{thermal part} \\ &+ 4m_P^2 \int_{a_-}^{a_+} \frac{da \dot{a}}{a} \left( B - a^2 - \frac{B \dot{a}^2}{3} \right) && \text{geometric part} \end{aligned}$$

positive!

## Effect of anomaly and bootstrap: beginning of landscape

The same two turning points  $a_{\pm}$  provided  $a_-^2 > B$



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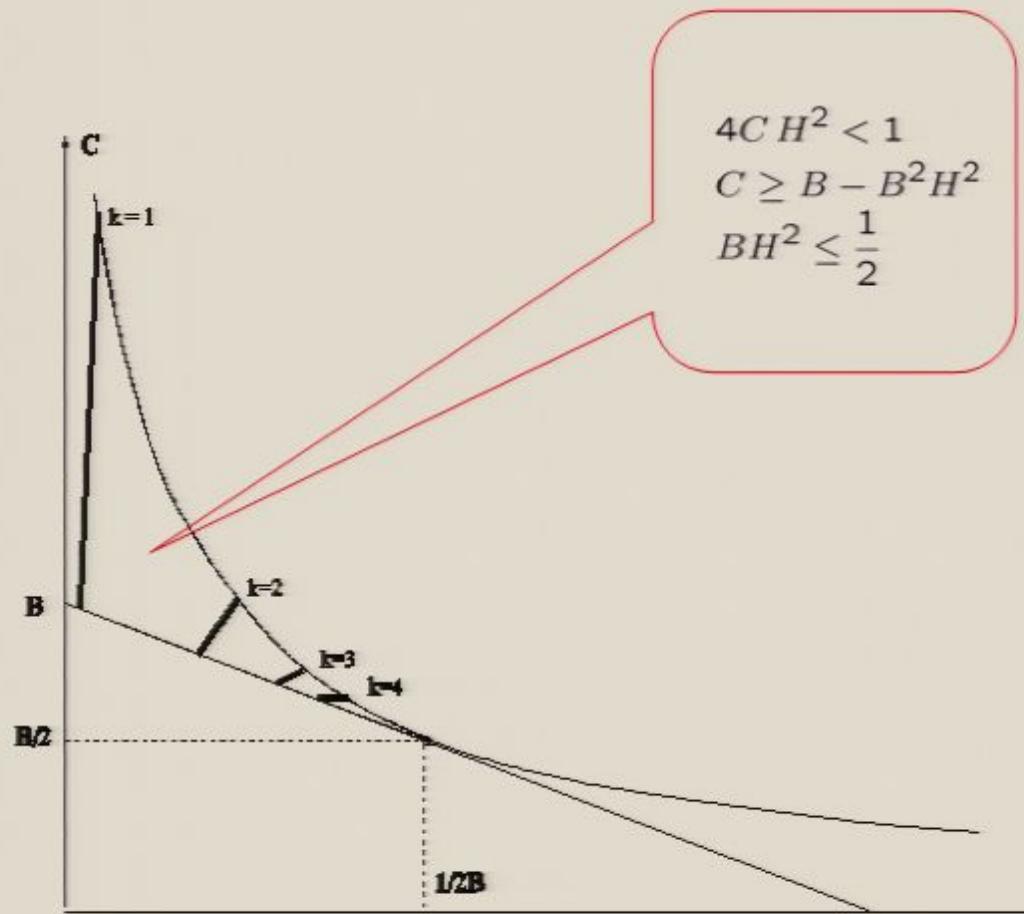
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## Elimination of infrared catastrophe

Below the straight line boundary the turning point  $a_-$  disappears and the instanton **closes** at  $a = 0$ :

$$\eta_0 = \int_0^{a_+} \frac{da}{a\dot{a}} \rightarrow \infty$$

$$F(\eta_0) \sim F'(\eta_0) \rightarrow 0$$

bootstrap  $\implies C \rightarrow \frac{B}{2}, \dot{a} \rightarrow 1, a \rightarrow 0$

This is **exact deSitter HH instanton** with the effective Hubble constant  $H_{\pm} = (1 \pm (1 - 2BH^2)^{1/2})/2B$  (Euclidean version of the Starobinsky solution in  $R^2$ -inflation)

**Effective action of the HH instanton**  $\Gamma_0 \equiv \Gamma_{HH}$

$$\Gamma_{HH} = 4m_P^2 \int_0^{a_+} \frac{da\dot{a}}{a} \left( B - a^2 - \frac{B\dot{a}^2}{3} \right) \rightarrow +\infty$$

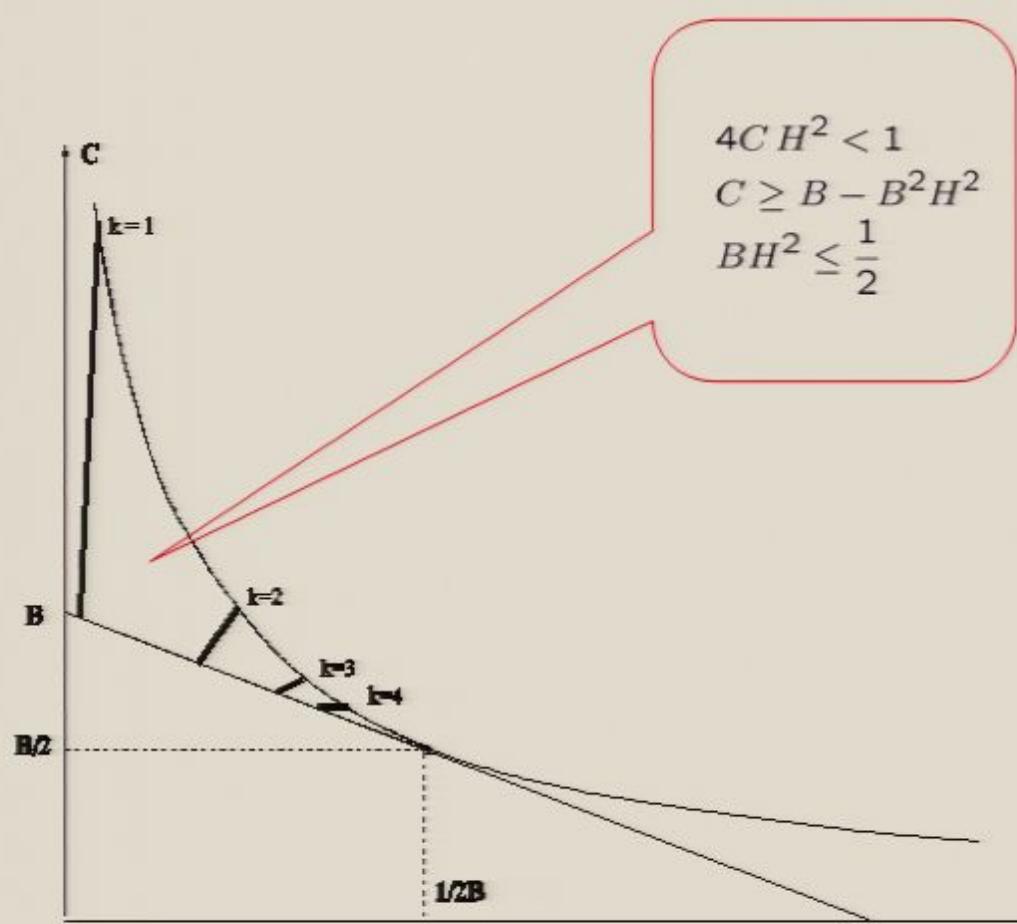


$$\exp(-\Gamma_{HH}) = 0$$

**HH vacuum instanton is dynamically ruled out by the conformal anomaly**

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**HH vacuum instanton is dynamically ruled out by the conformal anomaly**

## Lower bound on the cosmological constant range

Above the straight line boundary:

$$H^2 \rightarrow 0 \implies \eta_0 \rightarrow \infty \implies C \rightarrow B/2$$

bootstrap

contradicts  
 $C \geq B - B^2 H^2$

↙

$$H^2 \not\rightarrow 0$$

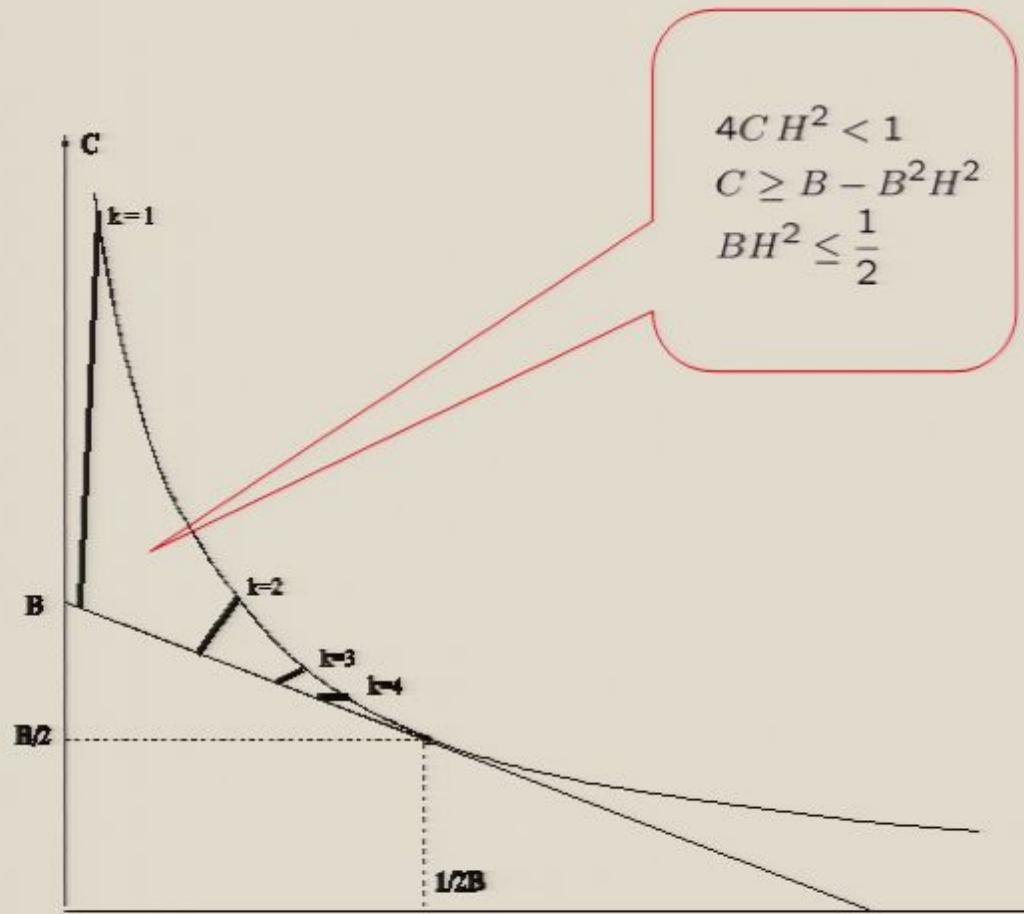
One-parameter family in  $(H^2, C)$ -plane can only interpolate between the upper (hyperbolic) and lower (straight) boundaries at  $H^2 > 0$

Cosmological constant has a positive lower bound

$$\Lambda = 3H^2 > \Lambda_{\min} > 0$$

## Effect of anomaly and bootstrap: beginning of landscape

The same two turning points  $a_{\pm}$  provided  $a_-^2 > B$



## Elimination of infrared catastrophe

Below the straight line boundary the turning point  $a_-$  disappears and the instanton **closes** at  $a = 0$ :

$$\eta_0 = \int_0^{a_+} \frac{da}{a\dot{a}} \rightarrow \infty$$

$$F(\eta_0) \sim F'(\eta_0) \rightarrow 0$$

bootstrap  $\implies C \rightarrow \frac{B}{2}, \dot{a} \rightarrow 1, a \rightarrow 0$

This is **exact deSitter HH instanton** with the effective Hubble constant  $H_{\pm} = (1 \pm (1 - 2BH^2)^{1/2})/2B$  (Euclidean version of the Starobinsky solution in  $R^2$ -inflation)

**Effective action of the HH instanton**  $\Gamma_0 \equiv \Gamma_{HH}$

$$\Gamma_{HH} = 4m_P^2 \int_0^{a_+} \frac{da\dot{a}}{a} \left( B - a^2 - \frac{B\dot{a}^2}{3} \right) \rightarrow +\infty$$



$$\exp(-\Gamma_{HH}) = 0$$

**HH vacuum instanton is dynamically ruled out by the conformal anomaly**

## Lower bound on the cosmological constant range

Above the straight line boundary:

$$H^2 \rightarrow 0 \implies \eta_0 \rightarrow \infty \implies C \rightarrow B/2$$

bootstrap

contradicts  
 $C \geq B - B^2 H^2$

↙

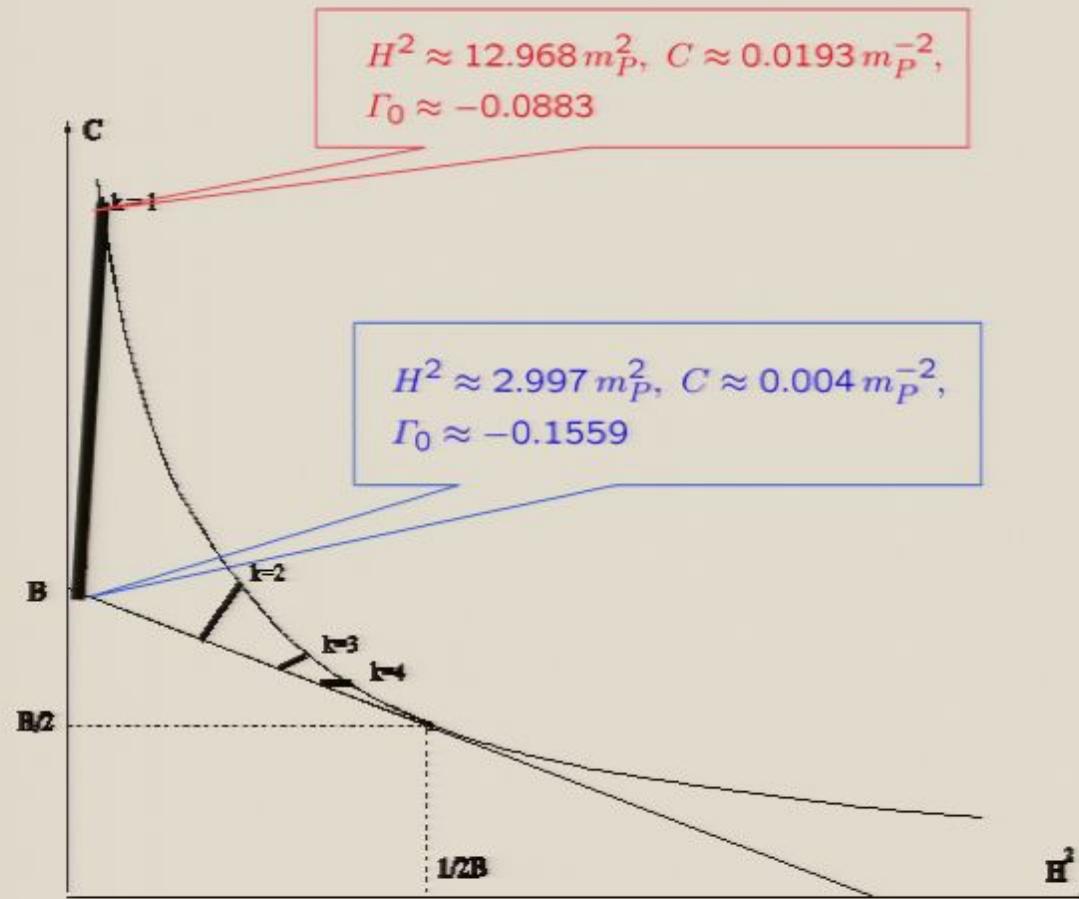
$$H^2 \not\rightarrow 0$$

One-parameter family in  $(H^2, C)$ -plane can only interpolate between the upper (hyperbolic) and lower (straight) boundaries at  $H^2 > 0$

Cosmological constant has a positive lower bound

$$\Lambda = 3H^2 > \Lambda_{\min} > 0$$

## One-parameter family of instantons (conformal scalar field)

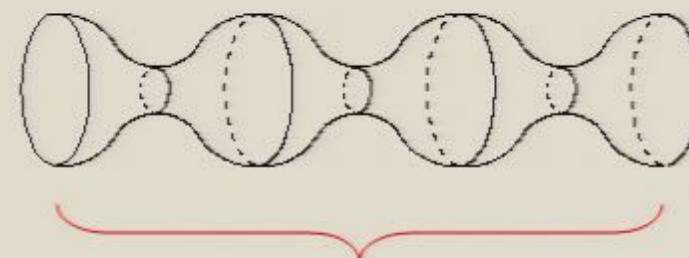
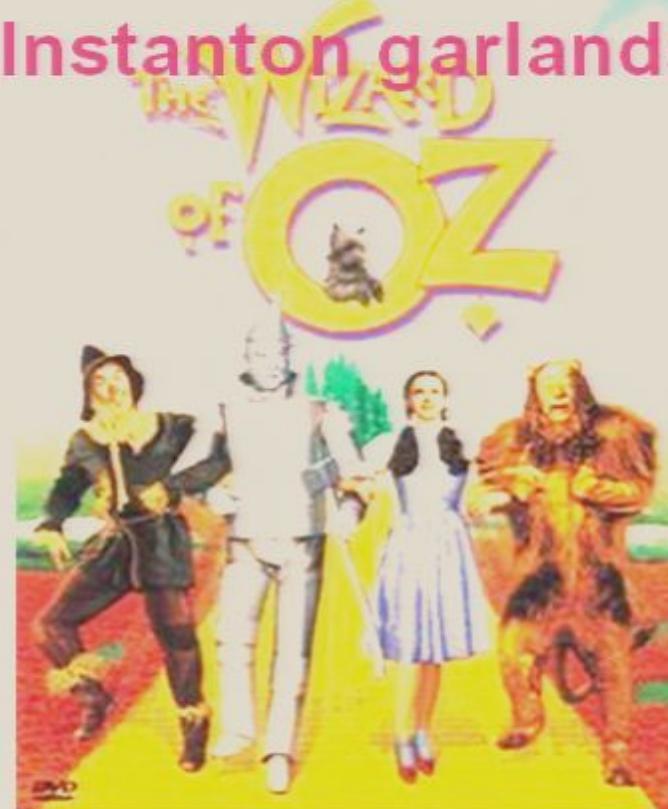


Upper (hyperbolic) boundary: static Einstein Universe  
with hot equilibrium gas of temperature

$$T = \frac{H}{\pi \sqrt{1 - 2BH^2}}$$

WB Warner Bros. FAMILY ENTERTAINMENT

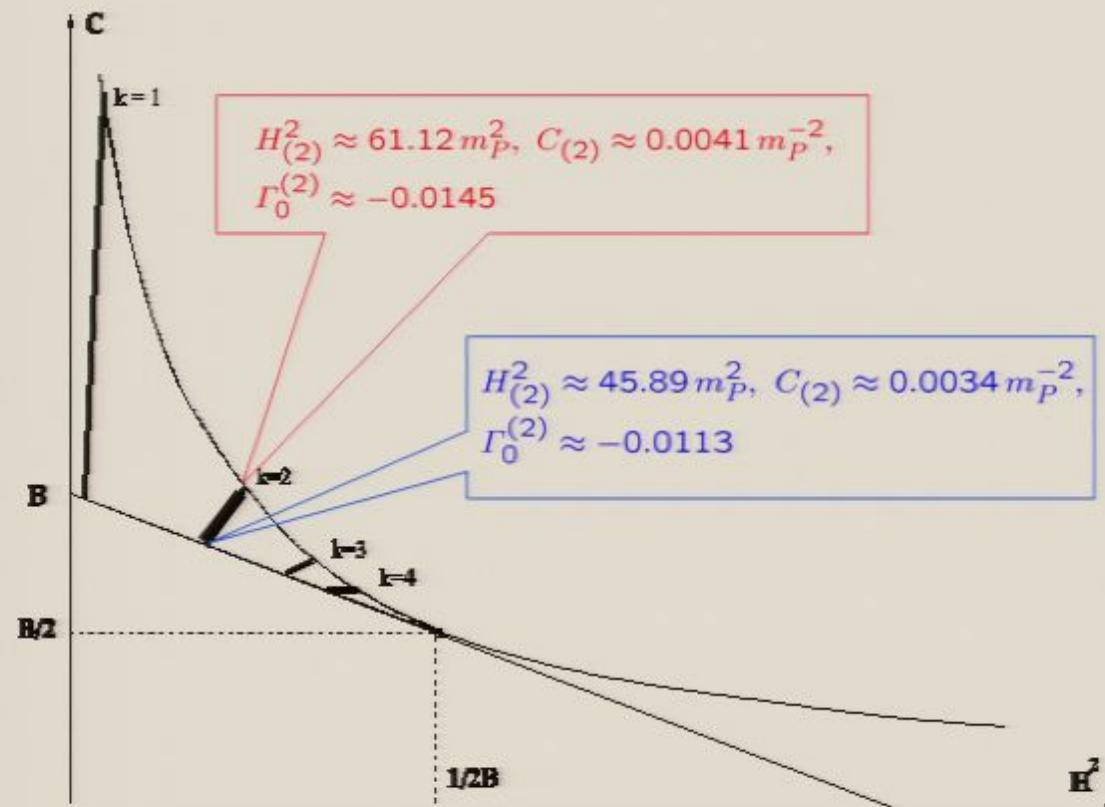
# Instanton garlands



k-folded garland,  $k=2,3,\dots$

**k-folded conformal time**       $\eta_0^{(k)} = 2k \int_{\tau_-}^{\tau_+} \frac{d\tau}{a}$

## k=2 garland



$k \rightarrow \infty$  sequence of garlands

$$H_{(k)}^2 \simeq \frac{1}{2B} \left( 1 - \frac{\ln^2 k^2}{2k^2\pi^2} \right) \rightarrow H_{\text{crit}}^2 = \frac{1}{2B}$$

$$C_{(k)} \simeq \frac{B}{2} \left( 1 + \frac{\ln^2 k^2}{2k^2\pi^2} \right)$$

$$\Gamma_0^{(k)} \simeq -m_P^2 B \frac{\ln^3 k^2}{4k^2\pi^2} \rightarrow 0 \quad \text{no additivity in } k !$$

$$T_{(k)} = \frac{1}{\sqrt{B} \ln k^2} \rightarrow 0$$

Increasingly long  $\eta_0 \sim \log k^2$ , increasingly static and more and more cold garlands saturating the upper bound of landscape -- a new scale in quantum gravity:

$$\Lambda_{\max} = \frac{3}{2B} = \frac{2}{\beta} m_P^2 \quad \text{Planck mass} \quad m_P^2 \equiv \frac{3\pi}{4G}$$

coefficient of the Gauss-Bonnet term in the conformal anomaly

Final bounds of landscape for scalar and vector matter:

$$\Lambda_{\min}^{\text{scalar}} \approx 8.99 m_P^2, \quad \Lambda_{\max}^{\text{scalar}} = 360 m_P^2,$$

$$\Lambda_{\min}^{\text{vector}} \approx 3.19 m_P^2, \quad \Lambda_{\max}^{\text{vector}} = \frac{180}{31} m_P^2$$

# Conclusions

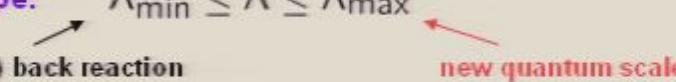
- 1) Lorentzian evolution of the cosmological landscape:  
expansion, radiation dilution, inflation, usual structure formation scenario;  
two branches of expansion at late time

$$a(t) \sim e^{H_{\pm}t}, \quad H_{\pm} = \frac{1}{B}(1 \pm \sqrt{1 - 2BH^2})$$

$H_-^2 \simeq H^2 \rightarrow 0$  decay of a composite  $H^2$

$H_+^2 \rightarrow \frac{2}{B}$  cosmological acceleration at a new scale!

Braneworld mechanism is needed to handle hierarchy problem  $B \sim m_P^{-2} \rightarrow 10^{120} m_P^{-2}$

- 2) Bounded landscape:  $\Lambda_{\min} \leq \Lambda \leq \Lambda_{\max}$   

  - vacuum HH instantons ruled out by  $\Gamma_0 = +\infty$ ;
  - boundedness of on-shell gravitational action in infrared domain of Euclidean gravity – nonlocal effect of anomaly and cosmological bootstrap

- 3) Beyond conformal quantum matter? Instantons are nearly static. Universality relation can be broken:  
 $C_0 < B/2$  – sequence of garlands becomes hot;  
 $B > C_0 > B/2$  -- sequence of garlands is truncated;  
 $C_0 > B$  – infrared catastrophe develops

- 4) No renormalization ambiguity – all UV divergences are absorbed by renormalized  $\Lambda = 3H^2$  and  $m_P^{-2}$ . Logarithmic divergences are vanishing – their finite remnant is the conformal anomaly

5) Normalizability of partition function – sum over instantons.

Measure and prefactors:

$$\sum_k (\Lambda_{\max}^{(k)} - \Lambda_{\min}^{(k)}) \sim \sum_k (1/k^4) < \infty$$

6) Semiclassical validity – scaling behavior with  $N$  – number of fields:  $C \rightarrow NC, B \rightarrow NB, F(\eta_0) \rightarrow NF(\eta_0)$



$$H^2 \rightarrow H^2/N \Rightarrow 1/N\text{-expansion}$$

7) Climbing up phenomenology energy scale and higher spins:  
growing  $B$  for growing spin and  $N$

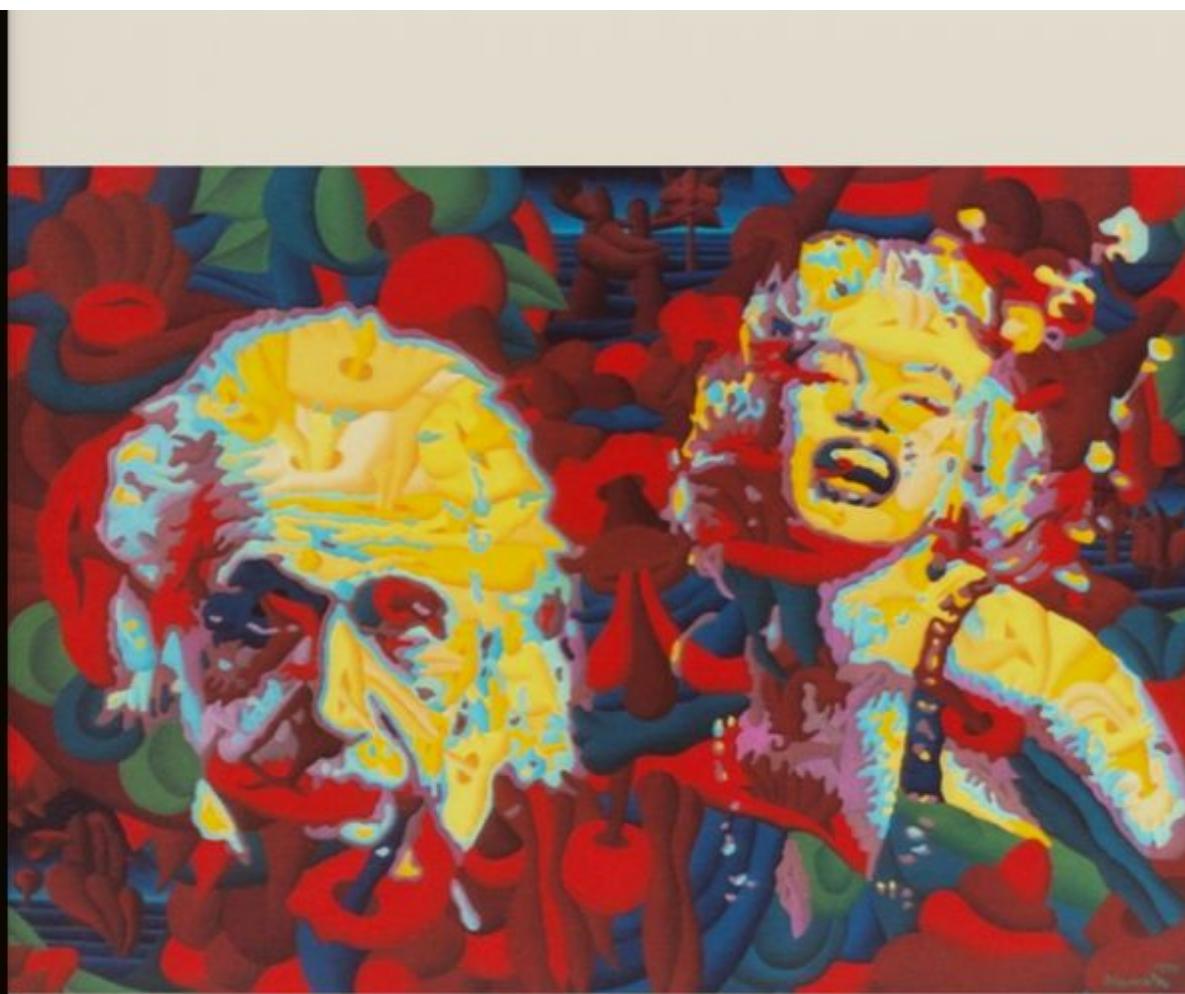
$$B \gg 1, H^2 \sim 1/B, -\Gamma_0 \sim m_P \sqrt{B} > 0$$



Selection mechanism for landscape of string vacua  
at the string scale  $m_s^2 \sim 2/B \ll m_P^2$ ?

8) Domination of (underbarrier) Euclidean quantum gravity configurations over possible Lorentzian stationary phase points for the Wheeler-DeWitt equation:  
in view of the negative action  $\Gamma_0$

$$e^{-\Gamma_0} \sim \exp(\#m_P \sqrt{B}) \sim \exp\left(\#\frac{m_P}{m_s}\right) \gg 1$$



*This is how cosmological landscape comes out of  
Nothing, if some really like it hot*