Title: Initial conditions in the presence of a UV cutoff: modified mode equations in inflation

Date: Sep 06, 2006 04:00 PM

URL: http://pirsa.org/06090027

Abstract:

Initial conditions in the presence of a UV cutoff:

Modified mode equations in inflation

Larissa Lorenz

Institut d'Astrophysique de Paris

PERIMETER INSTITUTE

Waterloo, September 5-8, 2006

A model for short distance physics

At the Planck scale, our notion of "distance" itself loses meaning: A photon with wavelength $\lambda = l_{Pl}$ inflicts too much curvature on spacetime to resolve structure of that size.

How can this cutoff be incorporated into quantum gravity?

- Allow QG corrections to the uncertainty relation:

$$\Delta x \, \Delta p \geq \frac{1}{2} \left(1 + \beta \, (\Delta p)^2 + \ldots \right)$$

where $\sqrt{\beta} = \Delta x_{min} = l_{Pl}$

- Corrections caused by QG modified commutation relations:

$$[x, \, p] = i \, (1 + \beta \, p^2 + \ldots)$$

To observe these effects, we need a Planck scale microscope.

Inflation as a Planck scale microscope

According to inflation, structures $\sim l_{Pl}$ in the early Universe are blown up to cosmological size today. Inflation also provides us with tools for calculating the CMB spectrum. How?

$$S = S_0 + \delta_2 S_{gr} + \delta_2 S_m$$

	Metric	Matter
background + perturbations	$ds^2 = (g^0_{\mu\nu} + \delta g_{\mu\nu})\, dx^\mu\, dx^\nu$	$\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$

- Describe **scalar** perturbations by φ
- Use perturbed Einstein equations: $\delta G^{\mu\,(gi)}_{\nu} = 8\pi G \delta T^{\mu\,(gi)}_{\nu}$
- Construct g.-i. variable **intrinsic curvature:** $\mathscr{R} = -\frac{a'}{a}\frac{\delta\phi}{\phi_0'} - \varphi$

Inflation as a Planck scale microscope

Re-write the perturbed action in terms of **intrinsic curvature**:

$$\delta_2 S = \frac{1}{2}\int d^4x\, z^2\, \left(\mathscr{R}'^2 - \delta^{ij}\partial_i\mathscr{R}\partial_j\mathscr{R}\right)$$

with $z = a\phi_0'/H$

Standard quantization procedure:

- Define canonically conjugate momentum
- Impose canonical commutation relations
- Use plane wave expansion
- Resulting mode equation:

$$\boxed{\mathscr{R}_{\boldsymbol{k}}'' + 2\,\frac{z'}{z}\,\mathscr{R}_{\boldsymbol{k}}' + k^2\,\mathscr{R}_{\boldsymbol{k}} = 0}$$

Inflation as a Planck scale microscope

According to inflation, structures $\sim l_{Pl}$ in the early Universe are blown up to cosmological size today. Inflation also provides us with tools for calculating the CMB spectrum. How?

$$S = S_0 + \delta_2 S_{gr} + \delta_2 S_m$$

	Metric	Matter
background + perturbations	$ds^2 = (g^0_{\mu\nu} + \delta g_{\mu\nu})\, dx^\mu\, dx^\nu$	$\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$

- Describe **scalar** perturbations by φ
- Use perturbed Einstein equations: $\delta G^{\mu}{}_{\nu}^{(gi)} = 8\pi G \delta T^{\mu}{}_{\nu}^{(gi)}$
- Construct g.-i. variable **intrinsic curvature:** $\mathcal{R} = -\frac{a'}{a}\frac{\delta\phi}{\phi_0'} - \varphi$

Inflation as a Planck scale microscope

Re-write the perturbed action in terms of **intrinsic curvature**:

$$\delta_2 S = \frac{1}{2}\int d^4x\, z^2\, \left(\mathscr{R}'^2 - \delta^{ij}\partial_i\mathscr{R}\partial_j\mathscr{R}\right)$$

with $z = a\phi_0'/H$

Standard quantization procedure:

- Define canonically conjugate momentum
- Impose canonical commutation relations
- Use plane wave expansion
- Resulting mode equation:

$$\boxed{\mathscr{R}_{\boldsymbol{k}}'' + 2\,\frac{z'}{z}\,\mathscr{R}_{\boldsymbol{k}}' + k^2\,\mathscr{R}_{\boldsymbol{k}} = 0}$$

Inflation as a Planck scale microscope

The same action also describes **tensor perturbations**:

$$\delta_2 S = \frac{1}{2} \int d^4x \, a^2 \, \left(\Phi'^2 - \delta^{ij} \partial_i \Phi \partial_j \Phi \right)$$

with $z \rightarrow a$

Standard quantization procedure:

- Define canonically conjugate momentum
- Impose canonical commutation relations
- Use plane wave expansion
- Resulting mode equation:

$$\boxed{\Phi_k'' + 2 \frac{a'}{a} \Phi_k' + k^2 \Phi_k = 0}$$

Inflation as a Planck scale microscope

The same action also describes **tensor perturbations**:

$$\delta_2 S = \frac{1}{2} \int d^4x \, a^2 \, \left(\Phi'^2 - \delta^{ij} \partial_i \Phi \partial_j \Phi\right)$$

with $z \rightarrow a$

Standard quantization procedure:

- Define canonically conjugate momentum
- Impose canonical commutation relations
- Use plane wave expansion
- Resulting mode equation:

$$\boxed{\Phi_k'' + 2 \frac{a'}{a} \Phi_k' + k^2 \Phi_k = 0}$$

Inflation as a Planck scale microscope

The same action also describes **tensor perturbations**:

$$\delta_2 S = \frac{1}{2}\int d^4x\, a^2\, \left(\Phi'^2 - \delta^{ij}\partial_i\Phi\partial_j\Phi\right)$$

with $z \rightarrow a$

Modified quantization procedure to introduce the cutoff Δx_{min}:

- Go to proper coordinates
- Define canonically conjugate momentum
- Impose modified commutation relations
- Use plane wave expansion
- Resulting mode equation ?

Mode creation at the cutoff

The effect of the cutoff is twofold:

- Each mode is "created" with a minimum wavelength $\lambda = \Delta x_{min}$.

without cutoff	with cutoff
$\delta_2 S = \frac{1}{2}\int d^4x\, a^2 \left(\Phi'^2 - \delta^{ij}\partial_i\Phi\partial_j\Phi\right)$	$\boldsymbol{\delta_2 S = \frac{1}{2}\int d\eta \int_{\tilde{k}^2 < \frac{a^2}{e\beta}} d^3\tilde{k}\, \mathscr{L}}$

with $\mathscr{L}$ a complicated Lagrangian

Mode creation at the cutoff

The effect of the cutoff is twofold:

- Each mode is "created" with a minimum wavelength $\lambda = \Delta x_{min}$.

without cutoff	with cutoff
$\delta_2 S = \frac{1}{2}\int d^4x\, a^2 \left(\Phi'^2 - \delta^{ij}\partial_i\Phi\partial_j\Phi\right)$	$\boldsymbol{\delta_2 S = \frac{1}{2}\int d\eta \int_{\tilde{k}^2 < \frac{a^2}{e\beta}} d^3\tilde{k}\, \mathscr{L}}$

with $\mathscr{L}$ a complicated Lagrangian

In other words:

$$\boldsymbol{H = \int_{\tilde{k}^2 < \frac{a^2}{e\beta}} d^3\tilde{k}\, \left(\pi_{\tilde{k}}(\eta)\phi_{\tilde{k}}(\eta) - \mathscr{L}\right)}$$

Mode creation at the cutoff

The effect of the cutoff is twofold:

- Each mode is "created" with a minimum wavelength $\lambda = \Delta x_{min}$.

without cutoff	with cutoff
$\delta_2 S = \frac{1}{2}\int d^4x\, a^2\, \left(\Phi'^2 - \delta^{ij}\partial_i\Phi\partial_j\Phi\right)$	$\delta_2 S = \frac{1}{2}\int d\eta \int_{\tilde{k}^2 < \frac{a^2}{e\beta}} d^3\tilde{k}\, \mathscr{L}$

with $\mathscr{L}$ a complicated Lagrangian

- The modes obey a modified equation of motion.

without cutoff	with cutoff
$\Phi_k'' + 2\,\frac{a'}{a}\,\Phi_k' + k^2\,\Phi_k = 0$	$\phi_{\tilde{k}}'' + \frac{\nu'}{\nu}\phi_{\tilde{k}}' + \left[\mu - 3\,\left(\frac{a'}{a}\right)' - 9\,\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu}\right]\phi_{\tilde{k}} = 0$

Mode creation at the cutoff

The effect of the cutoff is twofold:

- Each mode is "created" with a minimum wavelength $\lambda = \Delta x_{min}$.

without cutoff	with cutoff
$\delta_2 S = \frac{1}{2}\int d^4x\, a^2\, \left(\Phi'^2 - \delta^{ij}\partial_i\Phi\partial_j\Phi\right)$	$\boldsymbol{\delta_2 S = \frac{1}{2}\int d\eta \int_{\tilde{k}^2 < \frac{a^2}{e\beta}} d^3\tilde{k}\, \mathscr{L}}$

with $\mathscr{L}$ a complicated Lagrangian — creation time $\eta_c(\tilde{k})$

- The modes obey a modified equation of motion.

without cutoff	with cutoff
$\Phi_k'' + 2\,\frac{a'}{a}\,\Phi_k' + k^2\,\Phi_k = 0$	$\boldsymbol{\phi_{\tilde{k}}'' + \frac{\nu'}{\nu}\phi_{\tilde{k}}' + \left[\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu}\right]\phi_{\tilde{k}} = 0}$

modified e of m

Modified mode equation

After its creation at $\eta_c(\tilde{k})$, a mode obeys the equation

$$\phi''_{\tilde{k}} + \frac{\nu'}{\nu}\phi'_{\tilde{k}} + \left[\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu}\right]\phi_{\tilde{k}} = 0$$

with the functions μ and ν given by:

$$\mu(\eta,\tilde{k}) = -\frac{a^2\,\mathrm{plog}(-\beta\tilde{k}^2/a^2)}{\beta\left(1+\mathrm{plog}(-\beta\tilde{k}^2/a^2)\right)^2}$$

$$\nu(\eta,\tilde{k}) = -\frac{\exp\left(-\frac{3}{2}\mathrm{plog}(-\beta\tilde{k}^2/a^2)\right)}{a^4\left(1+\mathrm{plog}(-\beta\tilde{k}^2/a^2)\right)}$$

The plog-function is the inverse of the function $x \rightarrow x \cdot e^{x}$.
Therefore $\mathrm{plog}\left(-\frac{1}{e}\right) = -1$.

Modified mode equation

. . . and why it is difficult to solve

After its creation at $\eta_c(\tilde{k})$, a mode obeys the equation

$$\phi''_{\tilde{k}} + \frac{\nu'}{\nu}\phi'_{\tilde{k}} + \left[\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu}\right]\phi_{\tilde{k}} = 0.$$

For each mode, we have: $\text{plog}\left(-\frac{\beta\tilde{k}^2}{a^2(\eta_c)}\right) = -1$

Recall:

$$\mu(\eta,\tilde{k}) \propto -\frac{1}{\left(1+\text{plog}(-\beta\tilde{k}^2/a^2)\right)^2}$$

$$\nu(\eta,\tilde{k}) \propto -\frac{1}{\left(1+\text{plog}(-\beta\tilde{k}^2/a^2)\right)}$$

The coefficients in the e of m have an irregular singular point at creation time!

Modified mode equation

… and approaches to solving it

After its creation at $\eta_c(\tilde{k})$, a mode obeys the equation

$$\phi''_{\tilde{k}} + \frac{\nu'}{\nu}\phi'_{\tilde{k}} + \left[\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu}\right]\phi_{\tilde{k}} = 0.$$

The coefficients in the e of m have an irregular singular point at creation time. How can we deal with it?

- **Extract dominant behaviour close to $\eta_c(\tilde{k})$**
  using series expansions of the coefficients
  Approach pursued by Easther et al. (2001) – see later
- **Turn $\eta_c(\tilde{k})$ into a regular singular point**
  using a variable transformation

Modified mode equation

... and how to solve it **exactly**

After its creation at $\eta_c(\tilde{k})$, a mode obeys the equation

$$\phi''_{\tilde{k}} + \frac{\nu'}{\nu}\phi'_{\tilde{k}} + \left[\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu}\right]\phi_{\tilde{k}} = 0.$$

Recall:

$$\mu(\eta,\tilde{k}) \propto -\frac{1}{\left(1+\text{plog}(-\beta\tilde{k}^2/a^2)\right)^2}$$

$$\nu(\eta,\tilde{k}) \propto -\frac{1}{\left(1+\text{plog}(-\beta\tilde{k}^2/a^2)\right)}$$

We need a variable transformation to turn $\eta_c(\tilde{k})$ into a regular singular point.

$$\boxed{\tau = \text{plog}(-\beta\tilde{k}^2/a^2)}$$

Creation time then corresponds to $\tau_c = -1$.

Re-write μ, ν as functions of τ and translate derivatives $\frac{\partial}{\partial\eta} \to \frac{\partial}{\partial\tau}\ldots$

Modified mode equation

... and how to solve it **exactly**

After the variable transformation $\eta \to \tau$, the mode equation reads

$$\phi''_{\tilde{k}} + \left(\frac{\nu'}{\nu} + \frac{\left(\frac{\partial}{\partial\tau}\frac{\partial\tau}{\partial\eta}\right)}{\frac{\partial\tau}{\partial\eta}} \right) \phi'_{\tilde{k}} + \left[\frac{\mu}{\left(\frac{\partial\tau}{\partial\eta}\right)^2} - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\frac{a'\nu'}{a\nu} - 3\left(\frac{a'}{a}\right)\frac{\left(\frac{\partial}{\partial\tau}\frac{\partial\tau}{\partial\eta}\right)}{\frac{\partial\tau}{\partial\eta}} \right] \phi_{\tilde{k}} = 0.$$

- The coefficients of the transformed e of m are regular singular.
- At a regular singular point, the **Frobenius method** can be applied.
- It allows to find two solutions in the form of **power series in** $(\tau - \tau_c)$.

In the following, we will focus on the de Sitter case $a = -\frac{1}{H\eta}$.
The same procedure is possible for a general power-law background.

Exact solution

After the variable transformation $\eta \to \tau$, the mode equation reads

$$\phi_{\tilde{k}}'' + \left(\frac{\nu'}{\nu} + \frac{\left(\frac{\partial}{\partial\tau}\frac{\partial\tau}{\partial\eta}\right)}{\frac{\partial\tau}{\partial\eta}} \right) \phi_{\tilde{k}}' + \left[\frac{\mu}{\left(\frac{\partial\tau}{\partial\eta}\right)^2} - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - 3\,\frac{a'\nu'}{a\nu} - 3\left(\frac{a'}{a}\right)\frac{\left(\frac{\partial}{\partial\tau}\frac{\partial\tau}{\partial\eta}\right)}{\frac{\partial\tau}{\partial\eta}} \right] \phi_{\tilde{k}} = 0.$$

Here, the Frobenius method gives:

$$\phi_{\tilde{k},1}(\tau) = \sum_{n=0}^{\infty} a_n (\tau - \tau_c)^{n+3}$$

$$\phi_{\tilde{k},2}(\tau) = \mathscr{A} \cdot \ln(\tau - \tau_c) \cdot \phi_{\tilde{k},1}(\tau) + \sum_{n=0}^{\infty} c_n (\tau - \tau_c)^n$$

General solution:

$$\boxed{\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)}$$

The choice of $\mathscr{C}_1$ and $\mathscr{C}_2$ selects a vacuum state.

Exact solution

. . . and why it is important

After the variable transformation $\eta \to \tau$, the mode equation reads

$$\phi''_{\tilde{k}} + \left(\frac{\nu'}{\nu} + \frac{\left(\frac{\partial}{\partial \tau} \frac{\partial \tau}{\partial \eta} \right)}{\frac{\partial \tau}{\partial \eta}} \right) \phi'_{\tilde{k}} + \left[\frac{\mu}{\left(\frac{\partial \tau}{\partial \eta} \right)^2} - 3 \left(\frac{a'}{a} \right)' - 9 \left(\frac{a'}{a} \right)^2 - 3 \frac{a' \nu'}{a \nu} - 3 \left(\frac{a'}{a} \right) \frac{\left(\frac{\partial}{\partial \tau} \frac{\partial \tau}{\partial \eta} \right)}{\frac{\partial \tau}{\partial \eta}} \right] \phi_{\tilde{k}} = 0.$$

General solution:

$$\boxed{\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)}$$

Why bother with a complicated exact solution?

- Effectively, any impact of trans-Planckian physics corresponds to an unsual choice of initial conditions for the usual mode equation (Starobinsky).
- But only the exact solution to the modified mode equation allows to identify these initial conditions on physical grounds.

Exact solution

. . . and how to formulate initial conditions

We have found an exact solution to the modified mode equation:

$$\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)$$

where

$$\begin{aligned} \phi_{\tilde{k},1}(\tau) &= \sum_{n=0}^{\infty} a_n (\tau - \tau_c)^{n+3} \\ \phi_{\tilde{k},2}(\tau) &= \mathscr{A} \cdot \ln(\tau - \tau_c) \cdot \phi_{\tilde{k},1}(\tau) + \sum_{n=0}^{\infty} c_n (\tau - \tau_c)^{n} \end{aligned}$$

To fix $\mathscr{C}_1$ and $\mathscr{C}_2$, choose initial conditions at creation time τ_c.

Expectation:

$\phi_{\tilde{k}}(\tau = \tau_c)$

$\phi'_{\tilde{k}}(\tau = \tau_c)$ allow to determine $\mathscr{C}_1, \mathscr{C}_2$

Exact solution

... and how to formulate initial conditions

We have found an exact solution to the modified mode equation:

$$\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)$$

where

$$\phi_{\tilde{k},1}(\tau) = \sum_{n=0}^{\infty} a_n (\tau - \tau_c)^{n+3}$$

$$\phi_{\tilde{k},2}(\tau) = \mathscr{A} \cdot \ln(\tau - \tau_c) \cdot \phi_{\tilde{k},1}(\tau) + \sum_{n=0}^{\infty} c_n (\tau - \tau_c)^{n}$$

To fix $\mathscr{C}_1$ and $\mathscr{C}_2$, choose initial conditions at creation time τ_c.

But here:

$\phi_{\tilde{k}}(\tau = \tau_c) \propto \mathscr{C}_2$ because $\phi_{\tilde{k},1}(\tau = \tau_c) = 0$

$\phi'_{\tilde{k}}(\tau = \tau_c) \equiv 0$ because $\phi'_{\tilde{k},1}(\tau = \tau_c) = 0$ and $c_1 \equiv 0$

Exact solution

... and how to formulate initial conditions

We have found an exact solution to the modified mode equation:

$$\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)$$

where

$$\phi_{\tilde{k},1}(\tau) = \sum_{n=0}^{\infty} a_n (\tau - \tau_c)^{n+3}$$

$$\phi_{\tilde{k},2}(\tau) = \mathscr{A} \cdot \ln(\tau - \tau_c) \cdot \phi_{\tilde{k},1}(\tau) + \sum_{n=0}^{\infty} c_n (\tau - \tau_c)^{n}$$

To fix $\mathscr{C}_1$ and $\mathscr{C}_2$, choose initial conditions at creation time τ_c.

But here: **$\mathscr{C}_1$ still unknown!**

$\phi_{\tilde{k}}(\tau = \tau_c) \propto \mathscr{C}_2$ because $\phi_{\tilde{k},1}(\tau = \tau_c) = 0$

$\phi'_{\tilde{k}}(\tau = \tau_c) \equiv 0$ because $\phi'_{\tilde{k},1}(\tau = \tau_c) = 0$ and $c_1 \equiv 0$

Exact solution

. . . and how to formulate initial conditions

We have found an exact solution to the modified mode equation:

$$\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)$$

where

$$\begin{aligned} \phi_{\tilde{k},1}(\tau) &= \sum_{n=0}^{\infty} a_n (\tau - \tau_c)^{n+3} \\ \phi_{\tilde{k},2}(\tau) &= \mathscr{A} \cdot \ln(\tau - \tau_c) \cdot \phi_{\tilde{k},1}(\tau) + \sum_{n=0}^{\infty} c_n (\tau - \tau_c)^{n} \end{aligned}$$

To fix $\mathscr{C}_1$ and $\mathscr{C}_2$, choose initial conditions at creation time τ_c.

Try higher derivatives:

$\phi''_{\tilde{k}}(\tau = \tau_c) \propto \mathscr{C}_2$ because $\phi''_{\tilde{k},1}(\tau = \tau_c) = 0$

$\phi'''_{\tilde{k}}(\tau = \tau_c)$ diverges! because of $\ln(\tau - \tau_c)$

Encoding of initial conditions

We have found an exact solution to the modified mode equation:

$$\phi_{\tilde{k}}(\tau) = \mathscr{C}_1 \cdot \phi_{\tilde{k},1}(\tau) + \mathscr{C}_2 \cdot \phi_{\tilde{k},2}(\tau)$$

	no. parameters
$\mathscr{C}_1$, $\mathscr{C}_2$ are complex	4
$\phi_{\tilde{k}}(\tau = \tau_c) \propto \mathscr{C}_2$	–2
Wronskian condition	–1
no. parameters left to fix	1

We can encode this missing parameter into a phase condition, i.e.:

$$\Im\left(\phi_{\tilde{k}}(\tau = \tau_{aux})\right) \equiv 0$$

Which criteria can we use to identify a vacuum state?

Vacuum identification

Recall:

	no. parameters
$\mathscr{C}_1, \mathscr{C}_2$ are complex	4
$\phi_{\tilde{k}}(\tau = \tau_c) \propto \mathscr{C}_2$	–2
Wronskian condition	–1
Phase condition	–1
Solution completely fixed!	**0**

How to choose a vacuum state?

Two approaches:

- Bogolyubov transformations to other solution sets which suggest a certain vacuum selection
- "Physical" criteria which minimize uncertainty, vacuum energy, . . .

Vacuum identification

. . . using Bogolyubov transformations

We have studied two other sets of solutions:

- solutions to the approximate mode equation
  Easther et al. (2001)
  These solutions have an oscillatory part similar to the Bunch-Davies vacuum, which suggests a certain vacuum choice.

- solutions of WKB type
  One can re-write the modified equation of motion without a friction term and find WKB solutions in its adiabatic regime.

The matching procedure is non-trivial.
These vacuum choices lead to almost adiabatic evolution.

Vacuum identification

... using physical criteria

We have studied three different possible criteria:

- minimum uncertainty criterion (Danielsson)
- diagonalisation of the Hamiltonian
- minimize vacuum energy

At creation time, none of this criteria allow to identify a vacuum because of divergences.

- Single out another time for their application?
- Understand divergence better?

Conclusions

- In inflationary perturbation theory, a UV cutoff Δx_{min}
  - introduces a creation time η_c for every mode and
  - changes the mode evolution equations.
- With an appropriate variable transformation $\eta \to \tau$, this equation can be solved **exactly**.
  - The solutions are polynomials in $(\tau - \tau_c)$.
  - Specifying their initial conditions is nontrivial.
- Two strategies for selecting a vacuum state can be pursued:
  - Bogolyubov transformations to other solution sets
  - applying "physical" criteria

Lessons learned:

Initial conditions here need to be "paraphrased".

Is there a underlying principle from which to derive initial conditions?

Vacuum identification

. . . using physical criteria

We have studied three different possible criteria:

- minimum uncertainty criterion (Danielsson)
- diagonalisation of the Hamiltonian
- minimize vacuum energy

At creation time, none of this criteria allow to identify a vacuum be-cause of divergences.

- Single out another time for their application?
- Understand divergence better?

Vacuum identification

... using physical criteria

We have studied three different possible criteria:

- minimum uncertainty criterion (Danielsson)
- diagonalisation of the Hamiltonian
- minimize vacuum energy

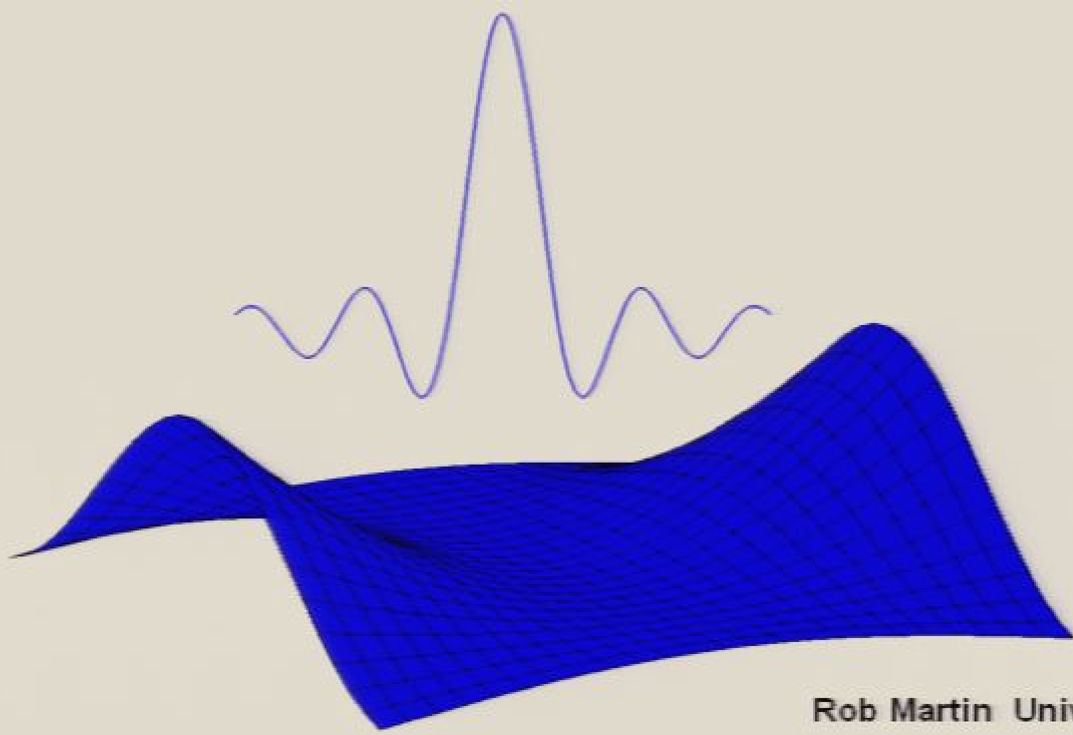
At creation time, none of this criteria allow to identify a vacuum because of divergences.

- Single out another time for their application?
- Understand divergence better?


EPSON
otoCenter
QuickTime
Player
Maple 9
DivXInstaller
Aicrosoft
eyboard
LimeWire
4.12.6
minuvet
ogle Earth
Creative
WebC...
MSN
ssenger 7.5
NetMeeting
Mozilla
underbird
zilla Firefox
SightSpeed
ecycle Bin

A covariant UV cutoff:

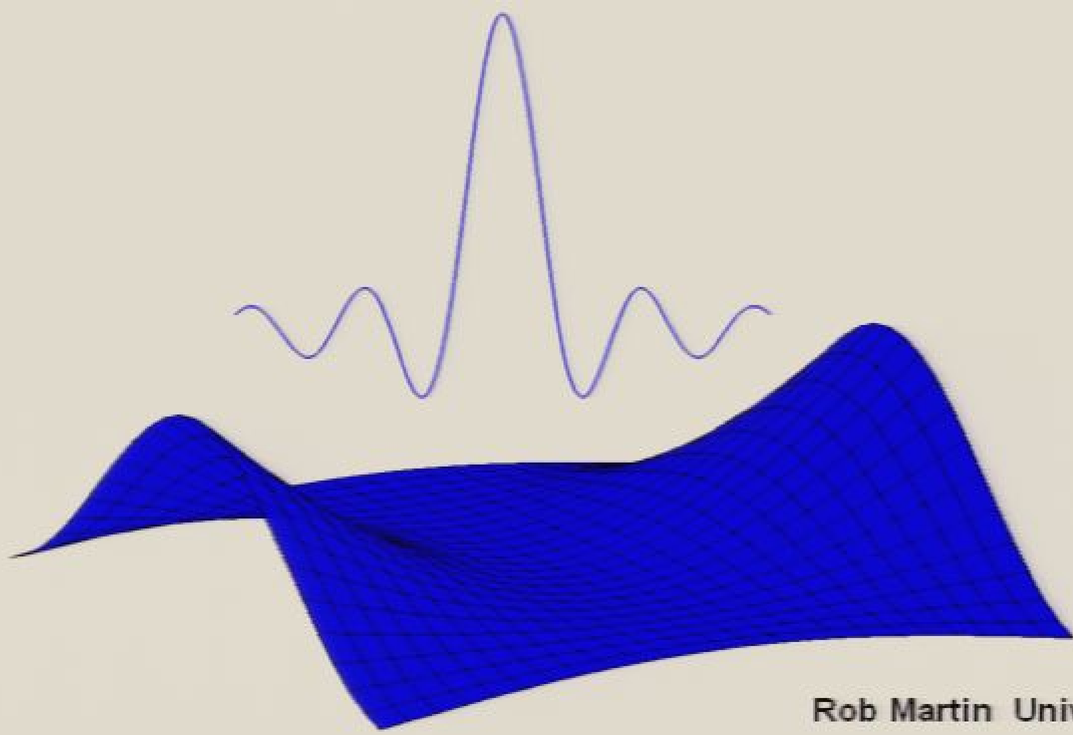
Sampling Theory on curved spacetime



Rob Martin University of Waterloo 06/09/06

A covariant UV cutoff:

Sampling Theory on curved spacetime



Rob Martin University of Waterloo 06/09/06