Title: Inflationary cosmological pertubations of quantum - mechanical origin

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Abstract:

Inflationary Cosmological Perturbations of Quantum-Mechanical Origin

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Outline

- The inflationary Universe: cosmological perturbations of quantum-mechanical origin and the observational status of inflation

- Importance of the initial conditions: are inflationary predictions sensitive to very high energy physics?

- Criticisms and problems

- Planck/String effects in the sky?

- Conclusions

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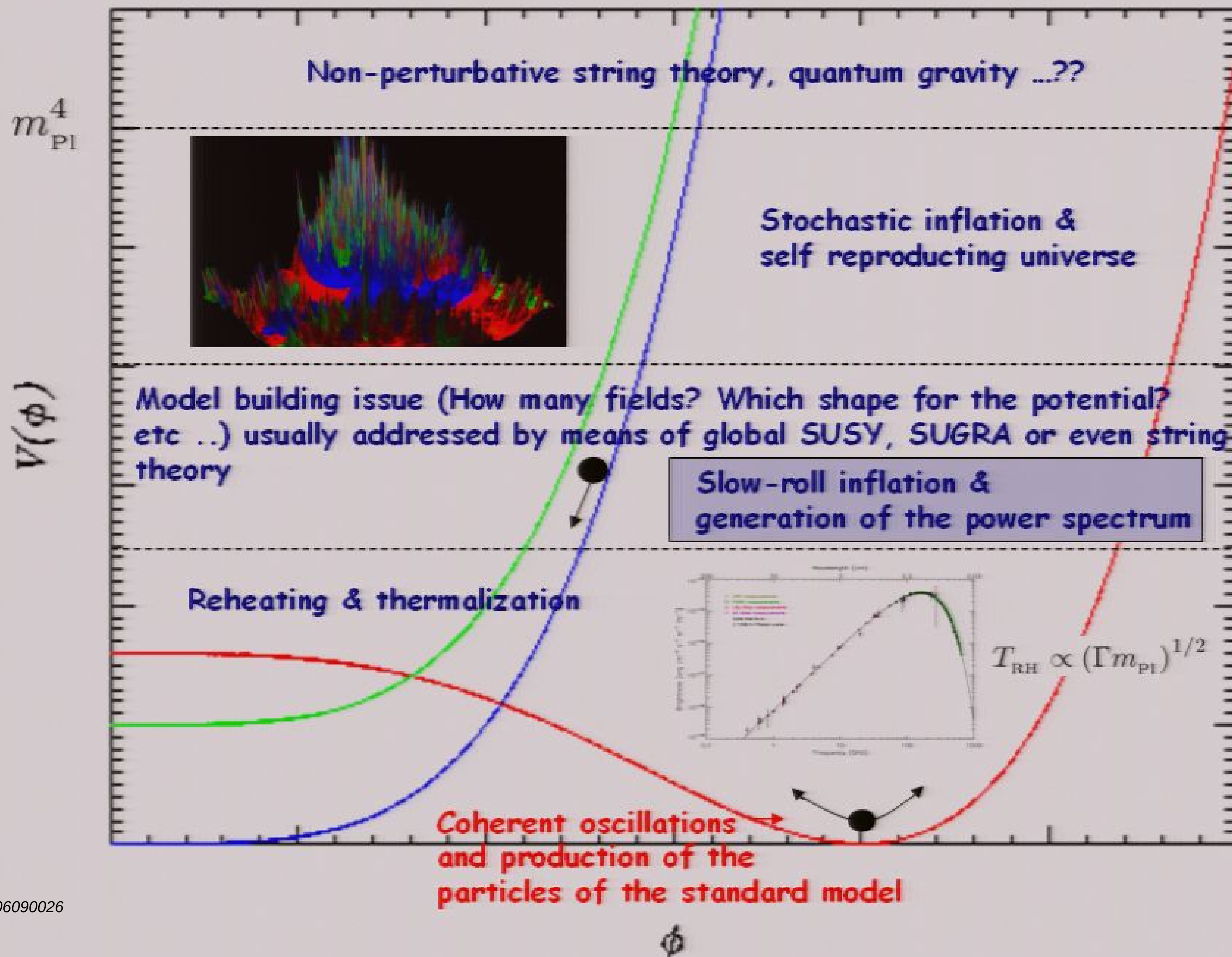
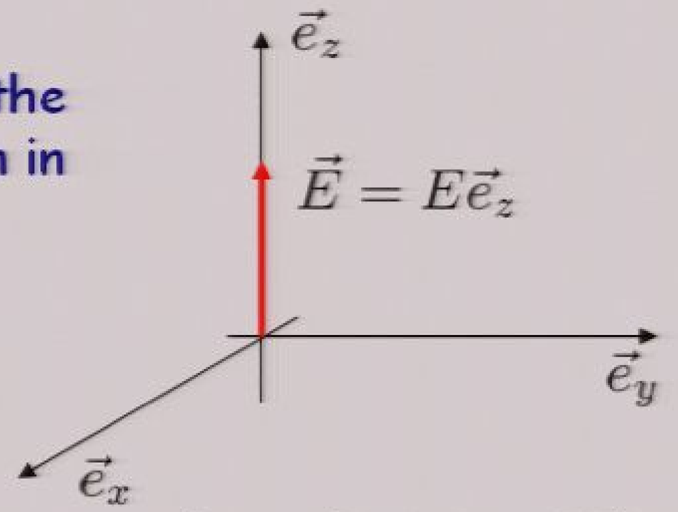
Inflation in General



The Schwinger Effect

- Production of cosmological perturbations in the Early universe is very similar to pair creation in a static electric field E



$$S = -\int \mathrm{d}^4x \left(\frac{1}{2}\eta^{\alpha\beta}\mathcal{D}_\alpha\phi\mathcal{D}_\beta\phi^* + \frac{1}{2}m^2\phi\phi^*\right)$$

$$\mathcal{D}_\alpha\phi = \partial_\alpha\phi + iq\phi A_\alpha$$

$$A_\alpha = (0, 0, 0, -Et)$$

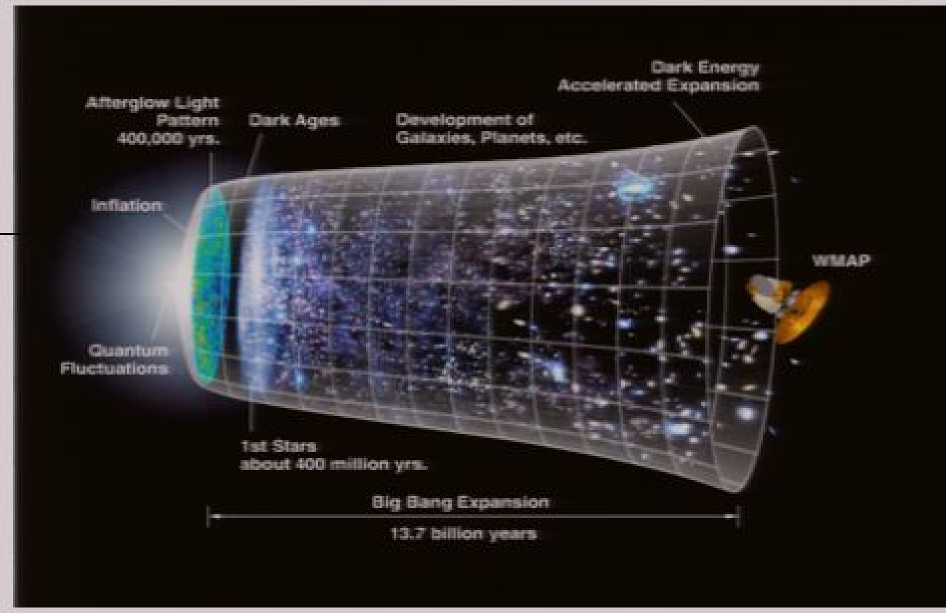
$$\frac{\mathrm{d}^2\phi_{\vec{k}}}{\mathrm{d}t^2} + \left(m^2 + k_\perp^2 + k_z^2 - 2qk_zEt + q^2E^2t^2\right)\phi_{\vec{k}} = 0$$

Because the frequency is time-dependent, the "in" vacuum is not the "out" vacuum and there is particles creation

- The role of the quantum scalar field is played by the perturbed metric (which is therefore a quantum operator) and the role of the external classical field is played by the classical background the "strength" of which is now characterized by H (instead of E before)

Cosmological Perturbations

- On top of the classical FLRW background, we have small quantum perturbations of the matter fields, and through Einstein equations, of the metric tensor. We have "gravitational phonons".



- In the early Universe, the inhomogeneities are small and, therefore, a linear treatment is possible

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1-2\phi)\,\mathrm{d}\eta^2 + 2(\partial_i B)\,\mathrm{d}x^i\mathrm{d}\eta + \left[(1-2\psi)\,\delta_{ij} + 2\partial_i\partial_j E + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

- The two types of perturbations (density perturbations and gravitational waves) are characterized by two time-dependent amplitudes in the Fourier space

$$h_{ij}(\eta,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\frac{1}{a(\eta)}\int \mathrm{d}\vec{k} \sum_{s=+,\times} p^s_{ij}\left(\vec{k}\right)\mu^s_{\vec{k}}(\eta)\,\mathrm{e}^{i\vec{k}\cdot\vec{x}}$$

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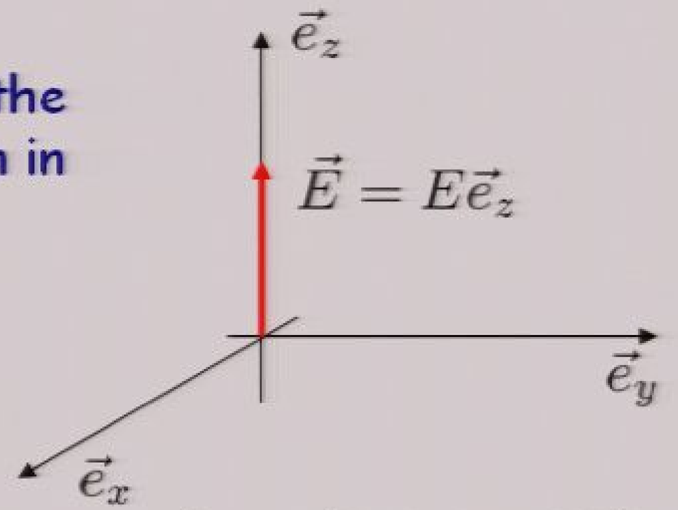

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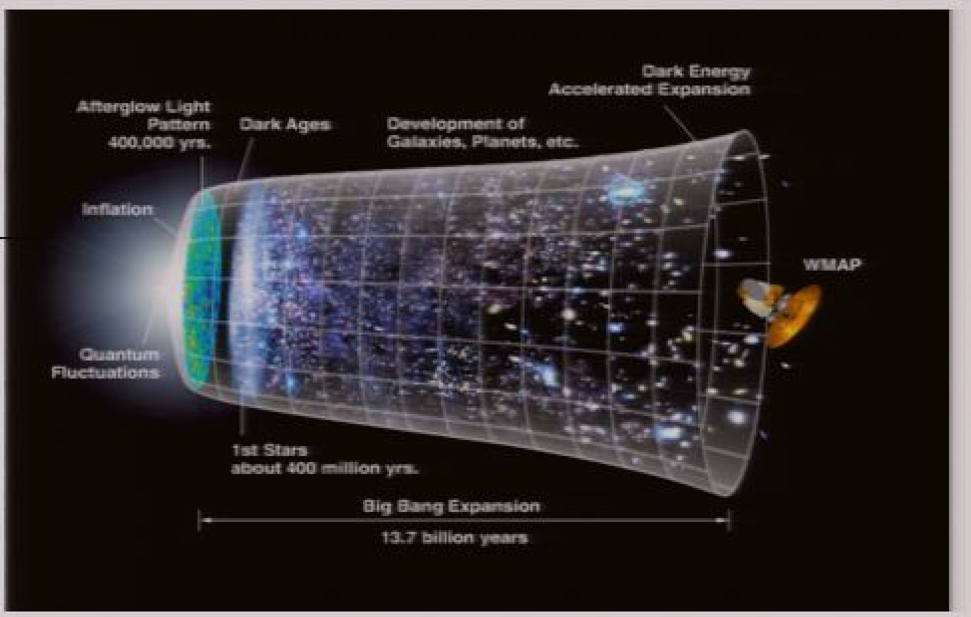

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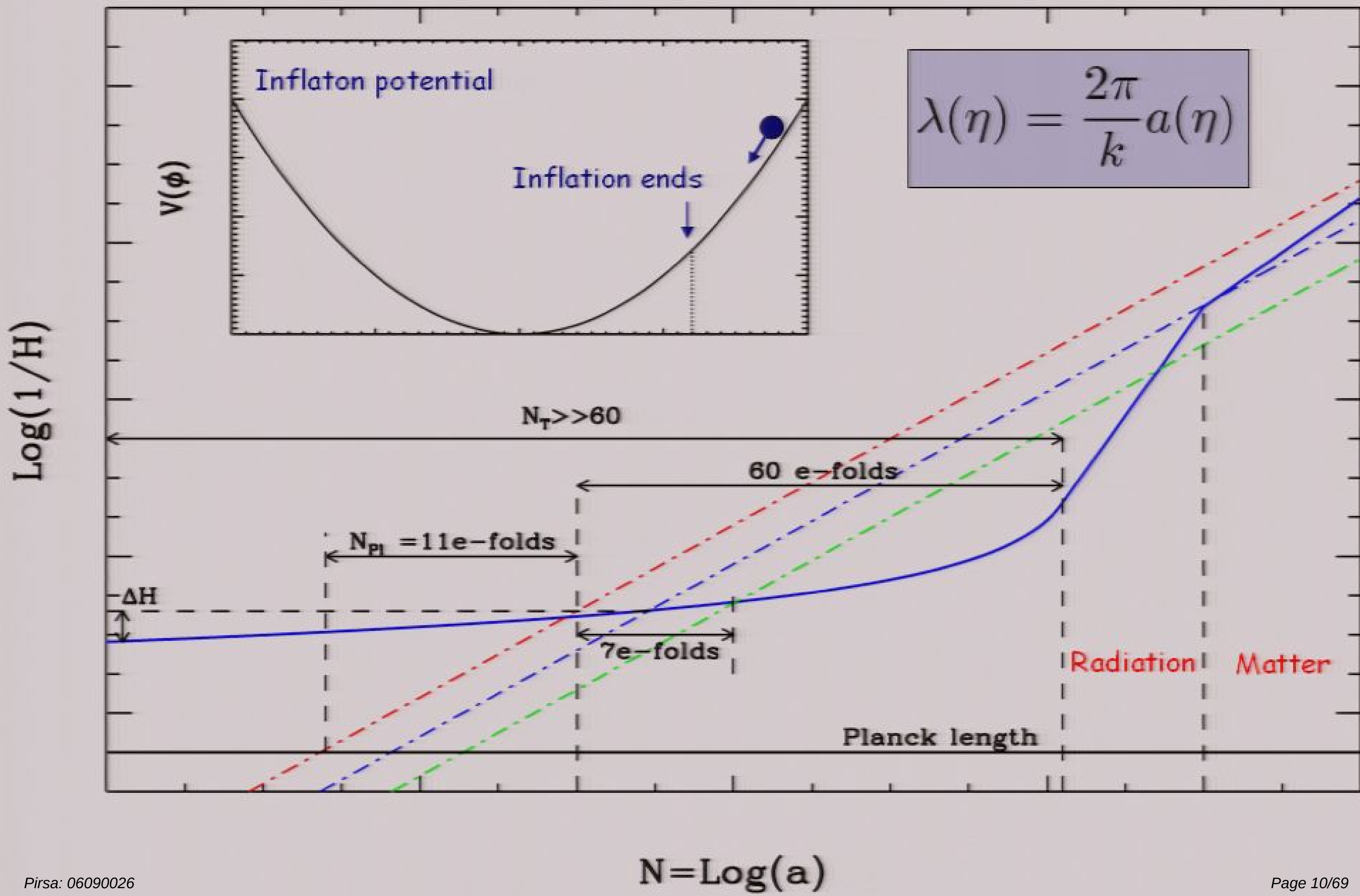
Evolution of the Fourier scales



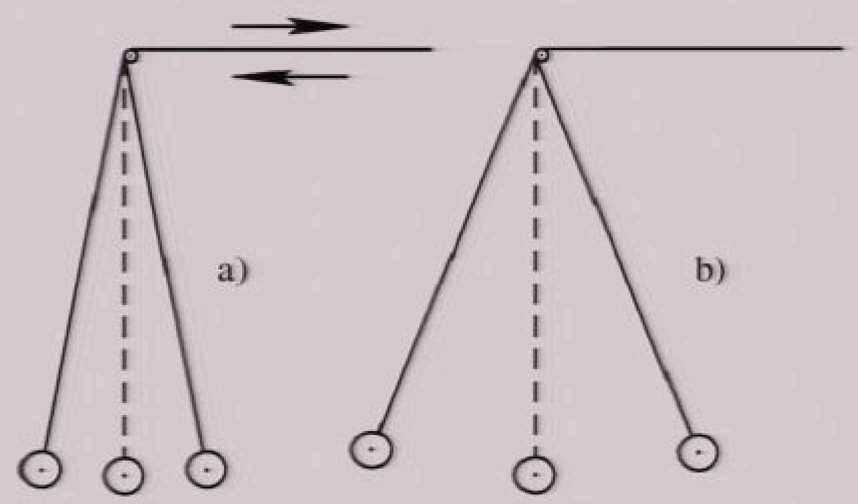
Parametric oscillators

Effective potentials

Power-law example

- The amplitudes of gravitational waves and density perturbations obey the equation of a parametric oscillator

$$\frac{\mathrm{d}^2\mu^s_{\vec{k}}}{\mathrm{d}\eta^2} + \omega^2(k,\eta)\,\mu^s_{\vec{k}} = 0$$



Parametric amplification. a) variation of the length of the pendulum, b) increased amplitude of oscillations.

Equations of Motion

Parametric oscillators

Effective potentials

Power-law example

$$\frac{\mathrm{d}^2\mu^s_{\vec{k}}}{\mathrm{d}\eta^2} + \omega^2(k,\eta)\,\mu^s_{\vec{k}} = 0$$

- Density perturbations

$$\omega^2(k,\eta) = k^2 - \frac{(a\sqrt{\gamma})''}{a\sqrt{\gamma}}$$

$$\gamma = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2}, \mathcal{H} \equiv \frac{a'}{a}$$

- Gravitational waves

$$\omega^2(k,\eta) = k^2 - \frac{a''}{a}$$

It can also be viewed as an effective Schroendinger equation with a effective ("time independent") potential U:

$$\frac{\mathrm{d}^2\mu^s_{\vec{k}}}{\mathrm{d}\eta^2} + \left[k^2 - U(\eta)\right]\mu^s_{\vec{k}} = 0$$

Equations of Motion

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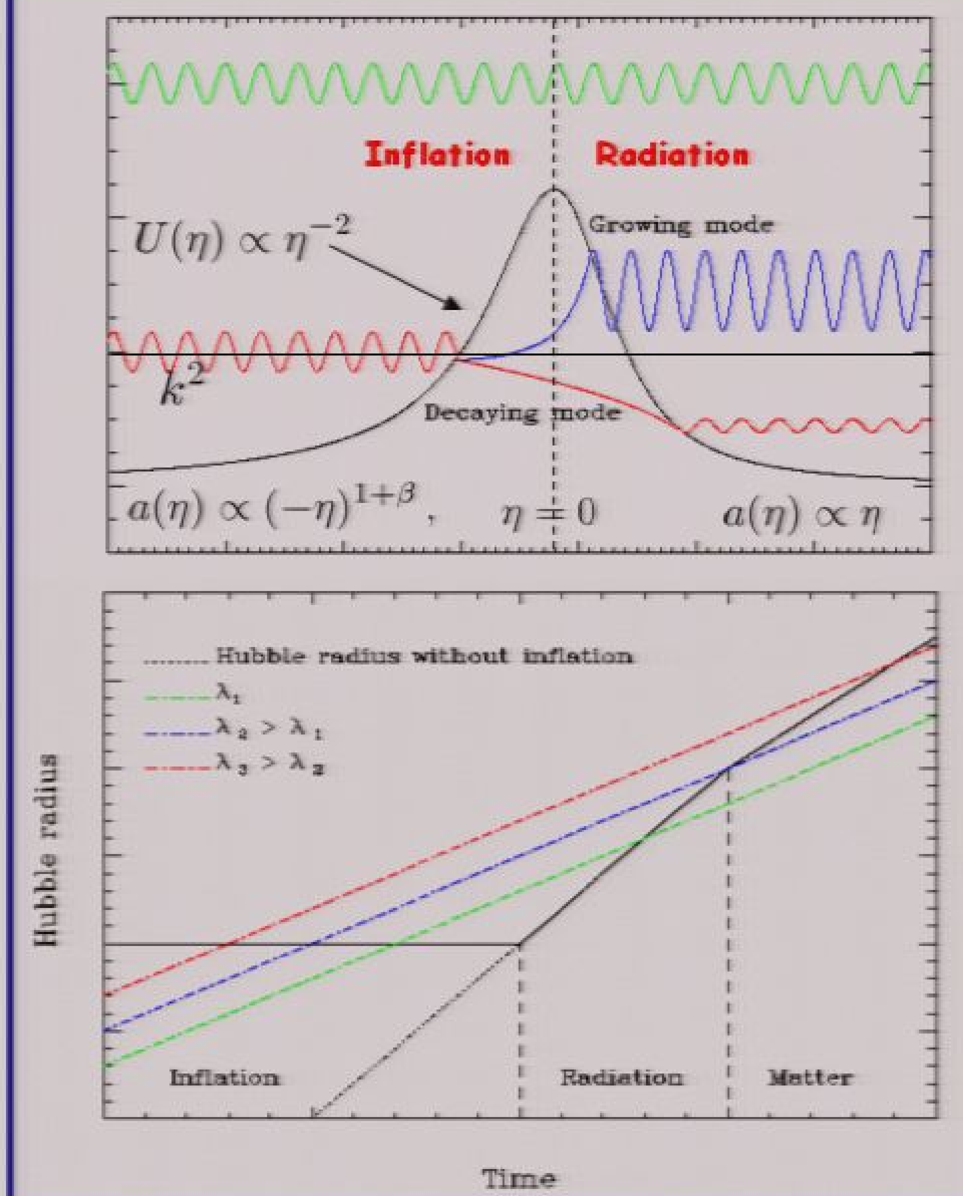
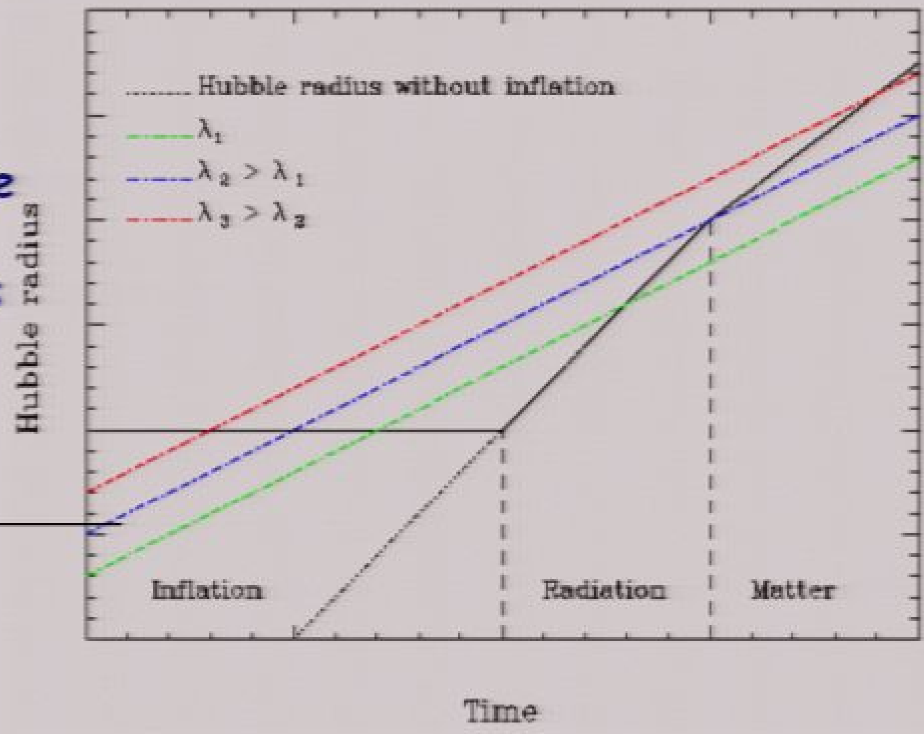
Valid also for slow-roll inflation



- Initially, the wave-lengths of the Fourier modes of astrophysical interest today were much smaller than the Hubble scale. They do not feel the curvature of spacetime and, therefore, the Minkowski vacuum quantum state is well-defined and chosen as the initial state $c_{\vec{k}}(\eta_{\rm ini})|0\rangle = 0$

$$\lim_{k\eta\rightarrow -\infty} \mu = \frac{e^{-ik\eta}}{\sqrt{2k}}$$



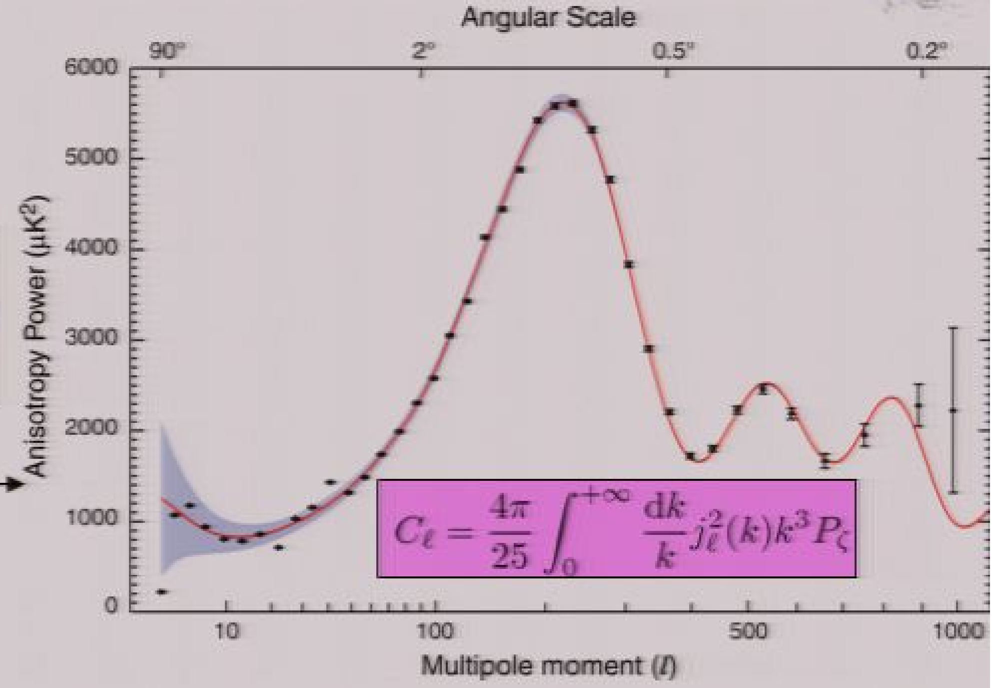
- Initially, the state of the quantum graviton operator is the vacuum and, then, it evolves into a strongly squeezed states with a non-vanishing number of particles

$$\hat{h}_{ij}(\eta,\vec{x}) = \frac{4\sqrt{\pi}}{m_{\rm Pl}a(\eta)}\frac{1}{(2\pi)^{3/2}}\int d\vec{k}\sum_{s=+,\times} p_{ij}^s\left(\vec{k}\right)\left[\mu_{\vec{k}} c_{\vec{k}}^s(\eta_{\rm ini})e^{i\vec{k}\cdot\vec{x}} + \mu_{\vec{k}}^{s*} c_{\vec{k}}^{s\dagger}(\eta_{\rm ini})e^{-i\vec{k}\cdot\vec{x}}\right]$$

$$|0\rangle \longrightarrow |\Psi\rangle \quad \text{such that} \quad \langle\Psi|N_{\vec{k}}|\Psi\rangle \neq 0$$

- What we measure are the two-point correlation functions

$$\left\langle 0\left|\frac{\delta\hat{T}}{T}(\vec{e}_1)\frac{\delta\hat{T}}{T}(\vec{e}_2)\right|0\right\rangle=\sum_{\ell=2}^{+\infty}\frac{2\ell+1}{4\pi}C_\ell P_\ell(\cos\theta)$$



$$C_\ell=\frac{4\pi}{25}\int_0^{+\infty}\frac{\mathrm{d}k}{k}j_\ell^2(k)k^3P_\zeta$$

- The power spectra that are used to compute the multipole moments are

$$\left\langle 0\left|\hat{h}_{ij}(\eta,\vec{x})\,\hat{h}^{ij}(\eta,\vec{x}+\vec{r})\right|0\right\rangle=\int_0^{+\infty}\frac{\mathrm{d}k}{k}\frac{\sin kr}{kr}k^3P_h(k)$$

$$\left\langle 0\left|\hat{\zeta}(\eta,\vec{x})\,\hat{\zeta}(\eta,\vec{x}+\vec{r})\right|0\right\rangle=\int_0^{+\infty}\frac{\mathrm{d}k}{k}\frac{\sin kr}{kr}k^3P_\zeta(k)$$

with

$$k^3P_h(k)=\frac{16k^3}{\pi m_{\rm Pl}^2}\left|\frac{\mu_{\vec{k}}(\eta)}{a(\eta)}\right|^2,\quad k^3P_\zeta(k)=\frac{k^3}{8\pi m_{\rm Pl}^2}\left|\frac{\mu_{\vec{k}}(\eta)}{a(\eta)\sqrt{\gamma}}\right|^2$$

A Simple Calculation of the Spectrum: importance of the IC

- For inflation, the effective frequency (and/or effective potential) is given by

$$\omega^2(k,\eta) = k^2 - \frac{\beta(\beta+1)}{\eta^2} = k^2 - \frac{a''}{a}$$

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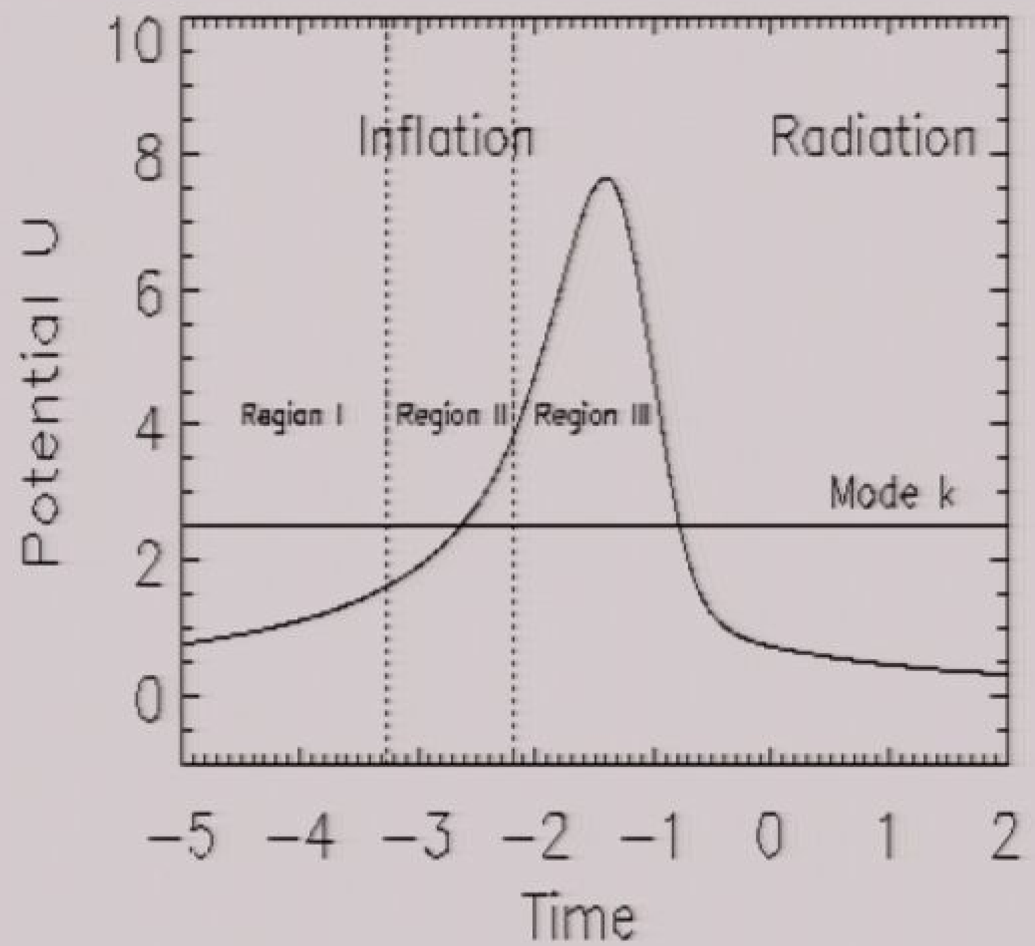

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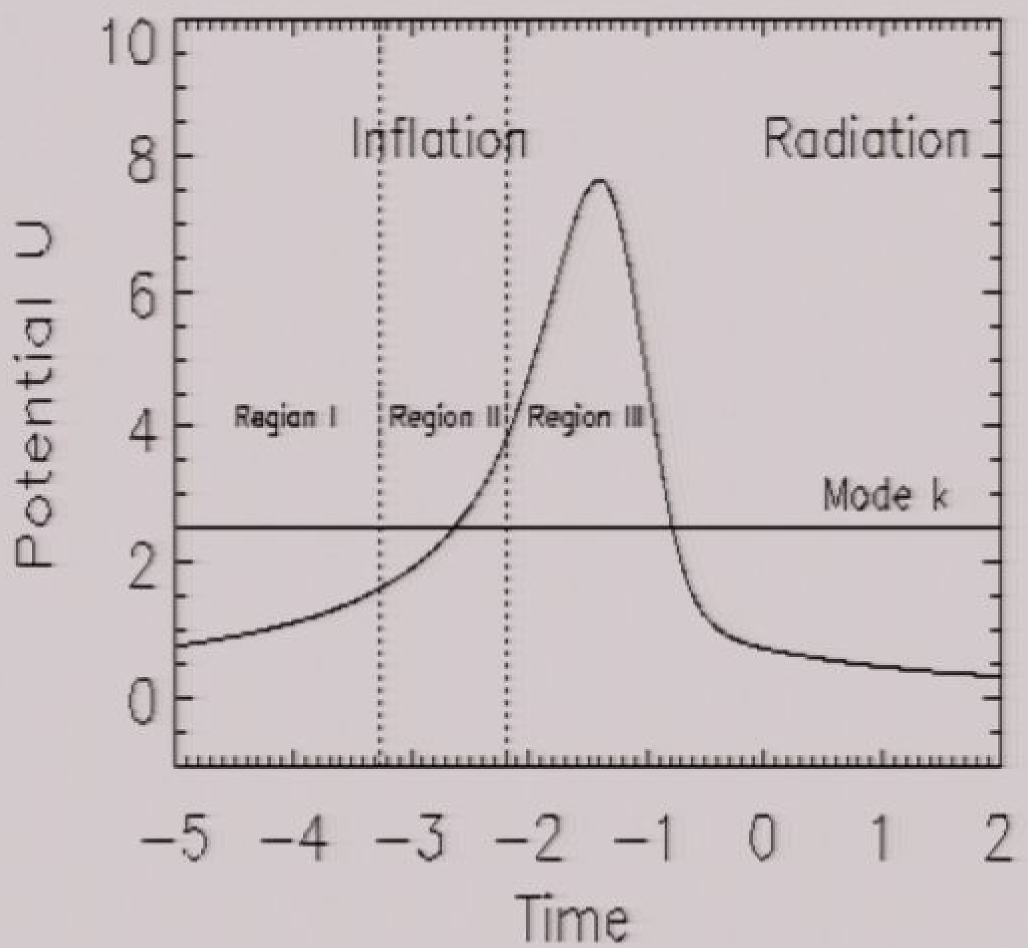

$$\mu^s_{\vec{k}} + \left(k^2 - \frac{a''}{a}\right)\mu^s_{\vec{k}} = 0$$

$$\mu^s_{\vec{k}} = \frac{e^{-ik(\eta-\eta_{\rm ini})}}{\sqrt{2k}}$$

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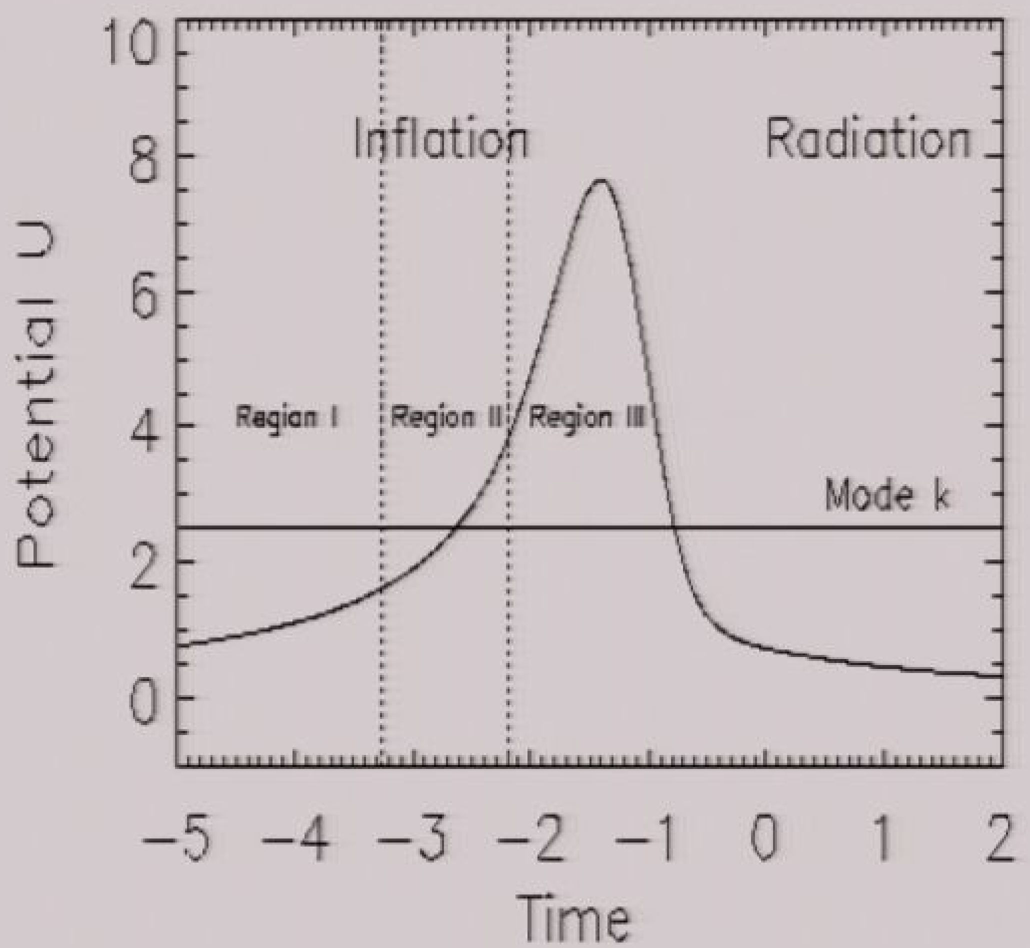

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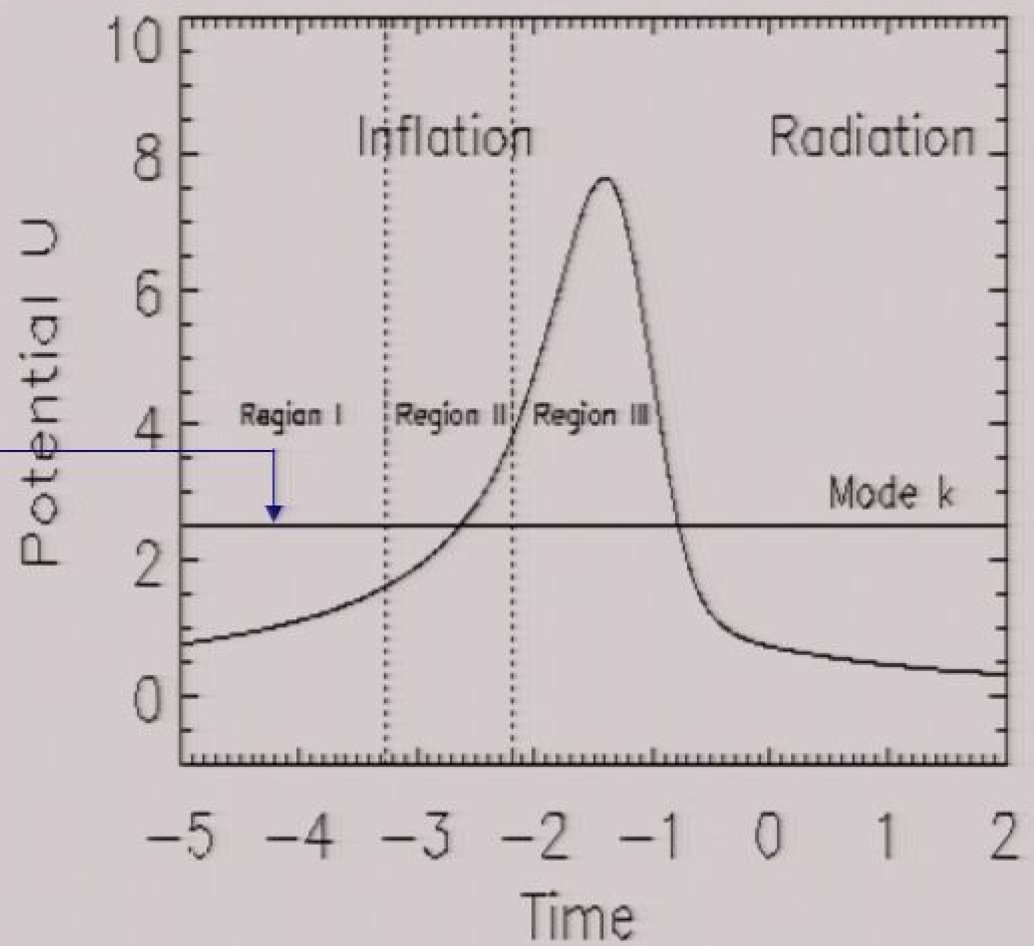

$$\mu_{\vec{k}}^{s} + \left(k^2 - \frac{a''}{a}\right)\mu_{\vec{k}}^{s} = 0$$

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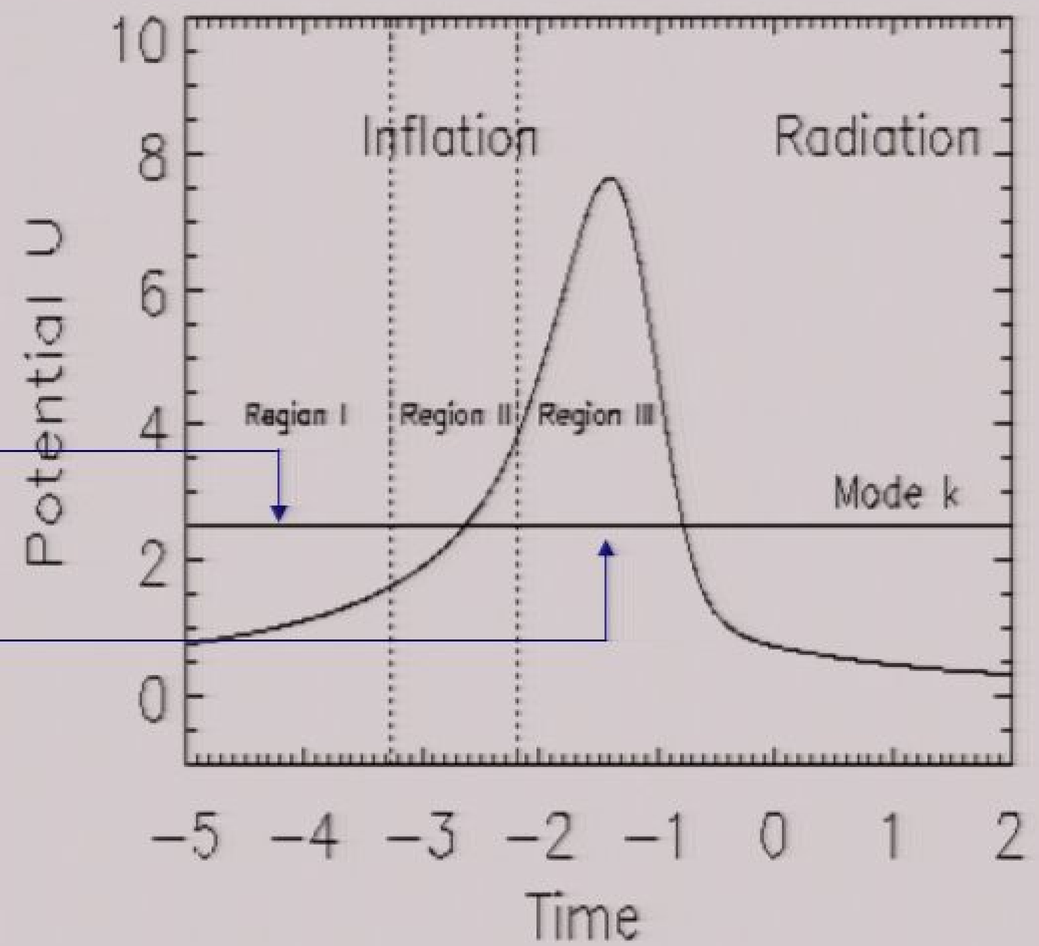

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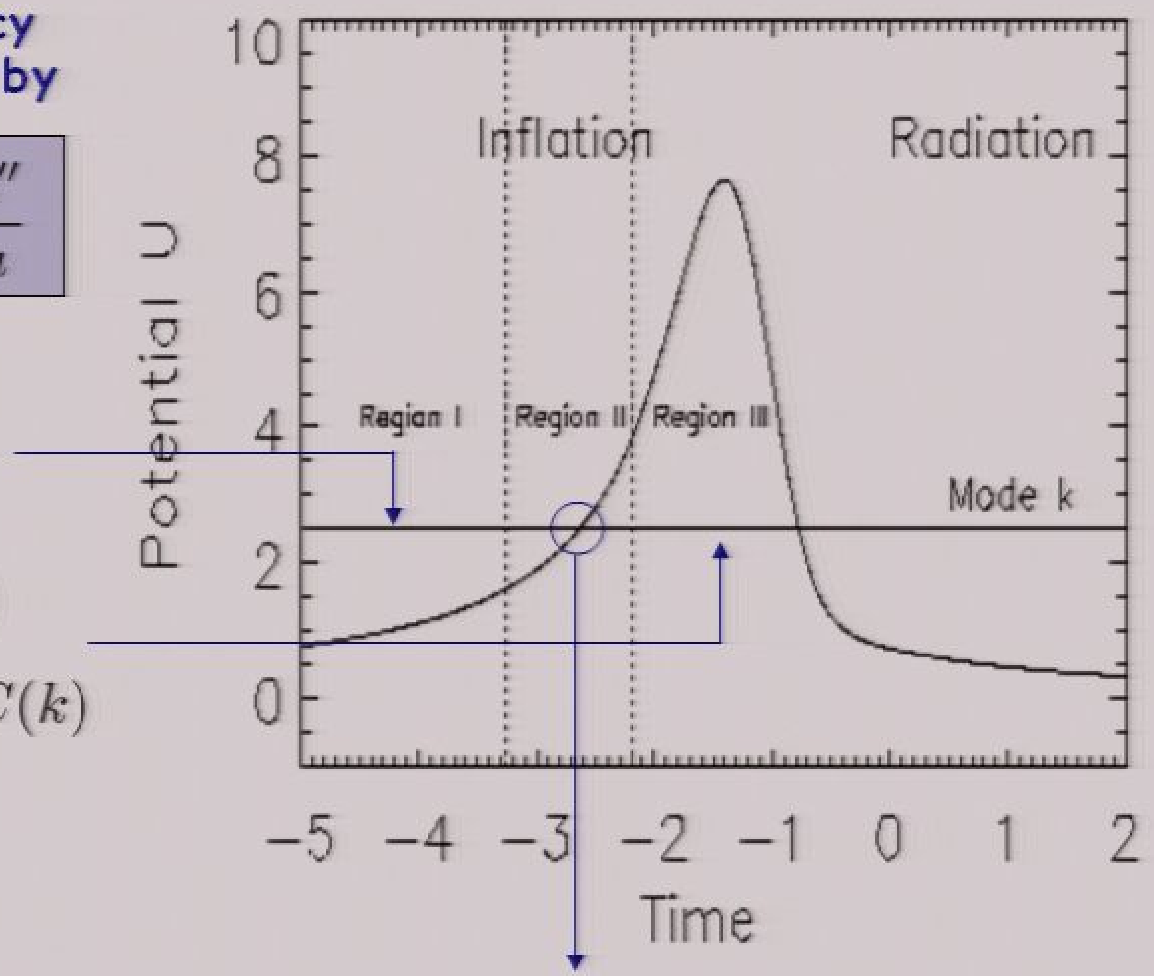

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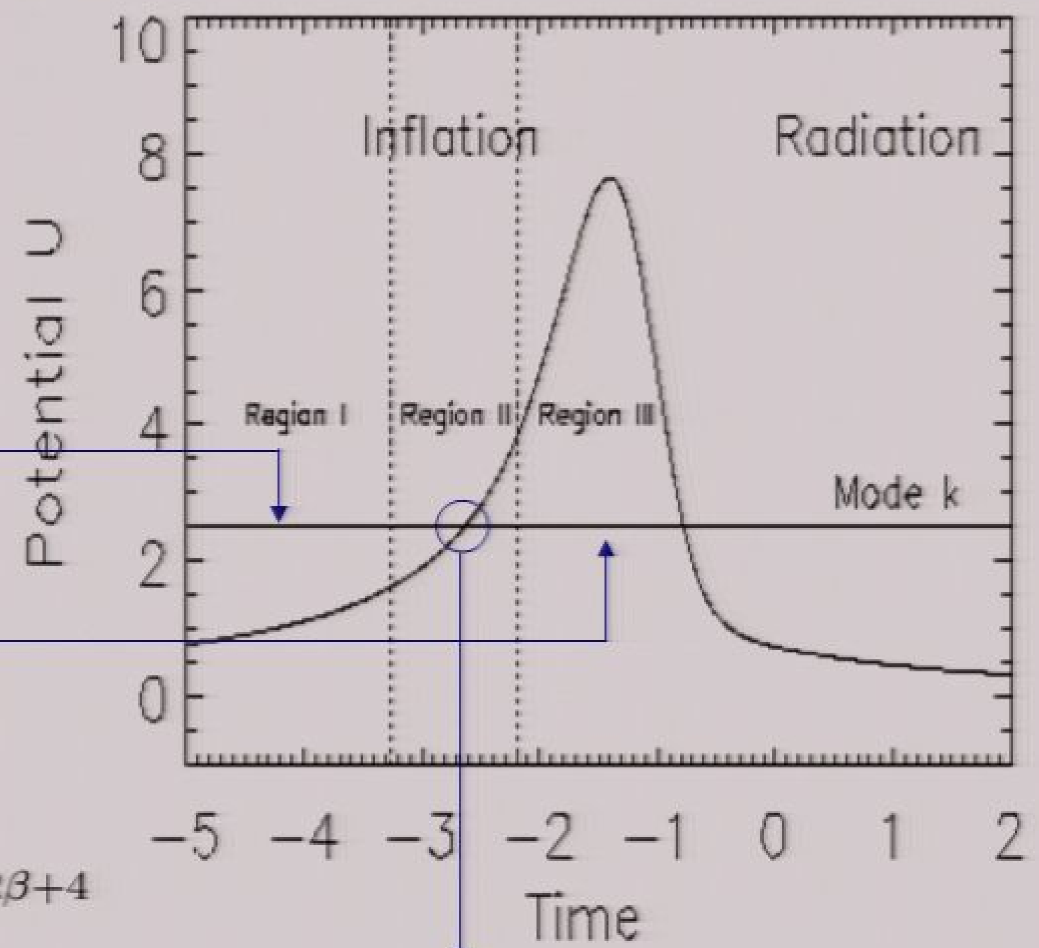

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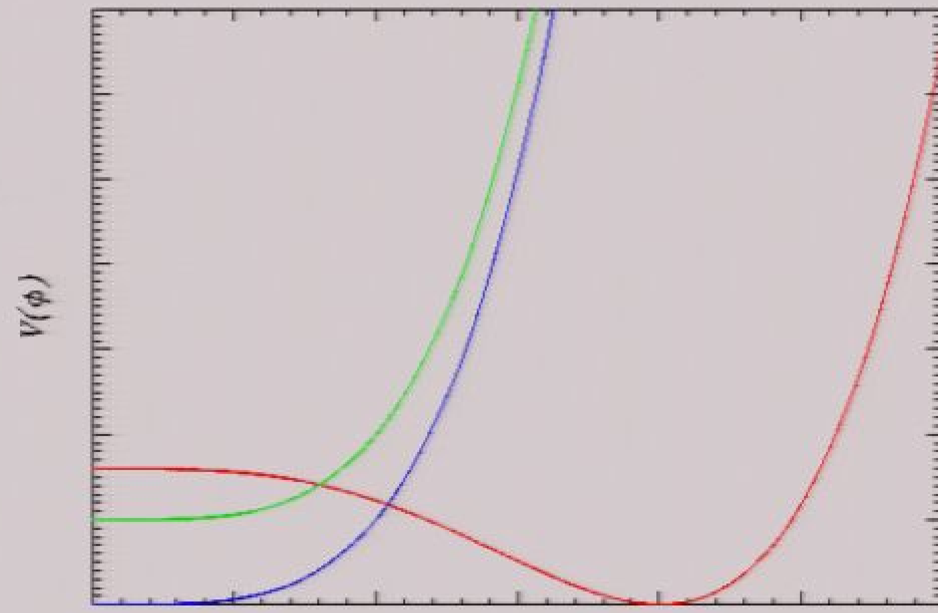
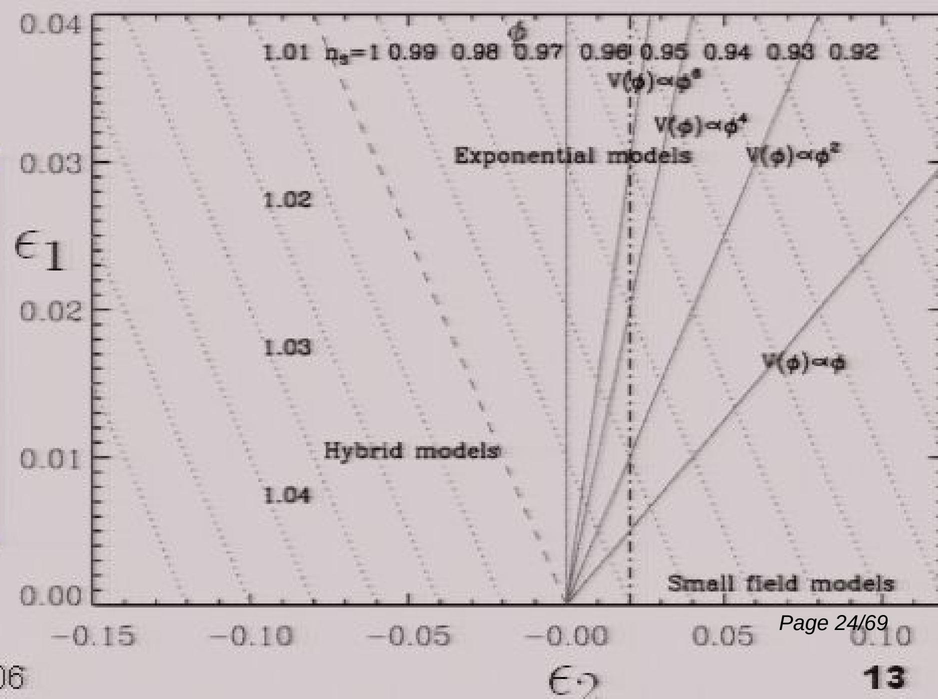
- $k^3 P \propto k^3 \left|\frac{\mu}{a}\right|^2 \propto k^3 |C(k)|^2 \propto \mathcal{O}(1) k^{2\beta+4}$

$$k^3 P(k) = k^{n-1}, \quad n = 2\beta + 5$$

In practice we have different models characterized by the slow-roll parameters

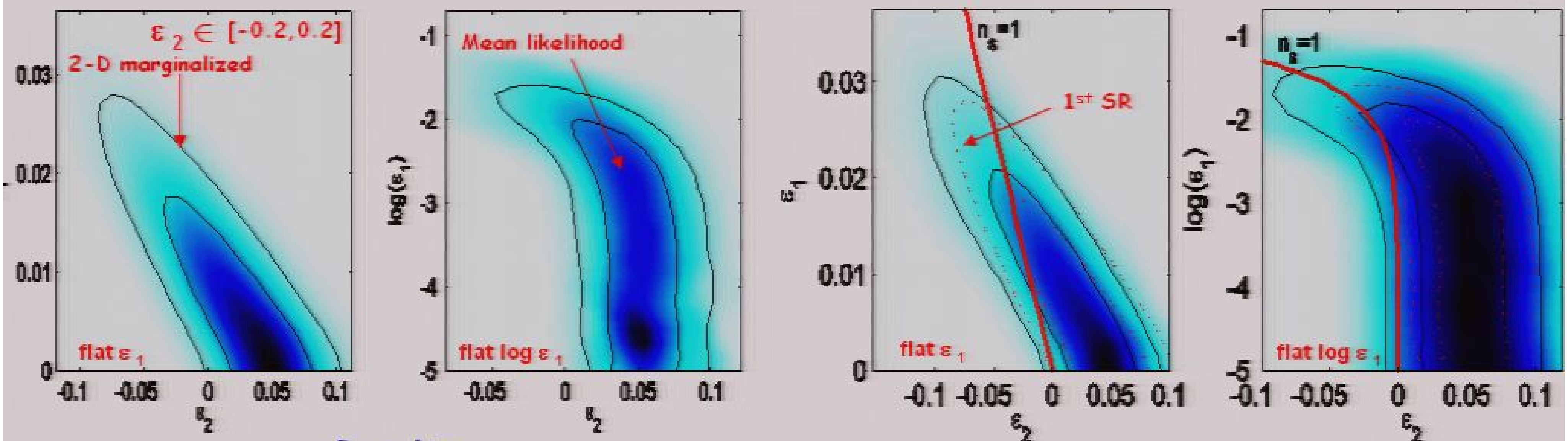
- Large field models
- Small field models
- Hybrid models etc ...





$$\epsilon_1 = \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{1}{V(\phi)}\left(\frac{{\rm d}V}{{\rm d}\phi}\right)\right]^2$$

$$\epsilon_2 = \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{1}{V^2(\phi)}\left(\frac{{\rm d}V}{{\rm d}\phi}\right)^2 - \frac{1}{V(\phi)}\left(\frac{{\rm d}^2V}{{\rm d}\phi^2}\right)\right]$$



Results

1- $\epsilon_1 < 0.022\,,\ -0.07 < \epsilon_2 < 0.07\ \ (95\%\text{CL})$

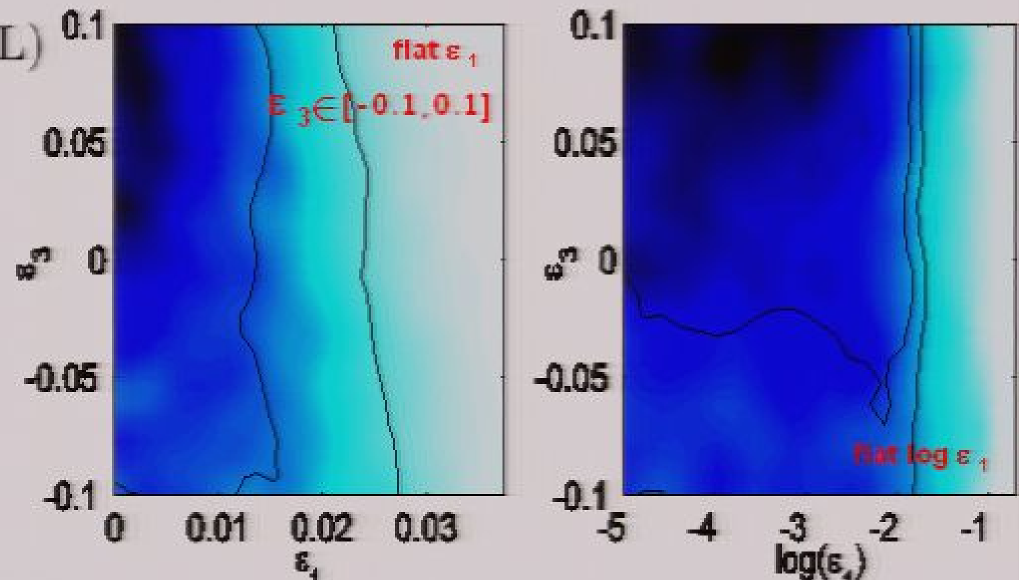
2- $n_{\rm S} = 1$ still ok ...

3- $\dfrac{H}{m_{\rm Pl}} < 1.3\times 10^{-5}$

4- $r_{10} < 0.21$

5- No running (prior independent way)

6- Very mild constraints on the reheating temperature for small fields models



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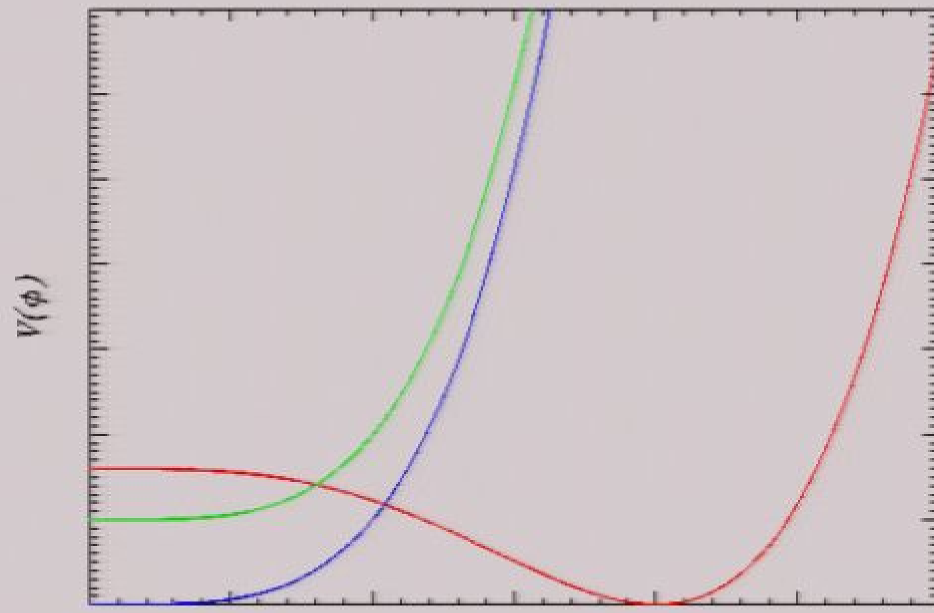

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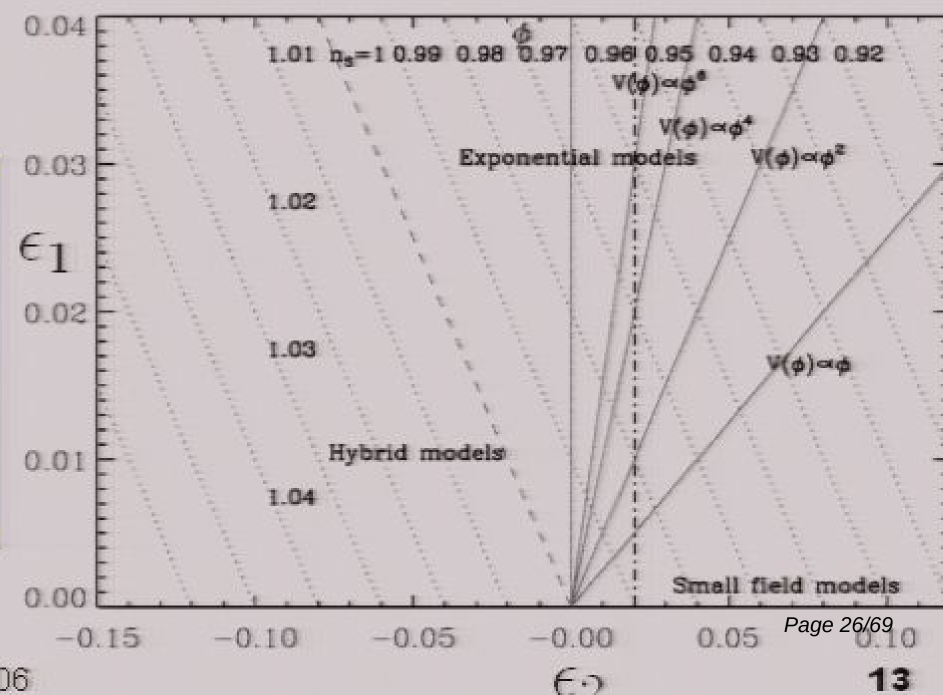

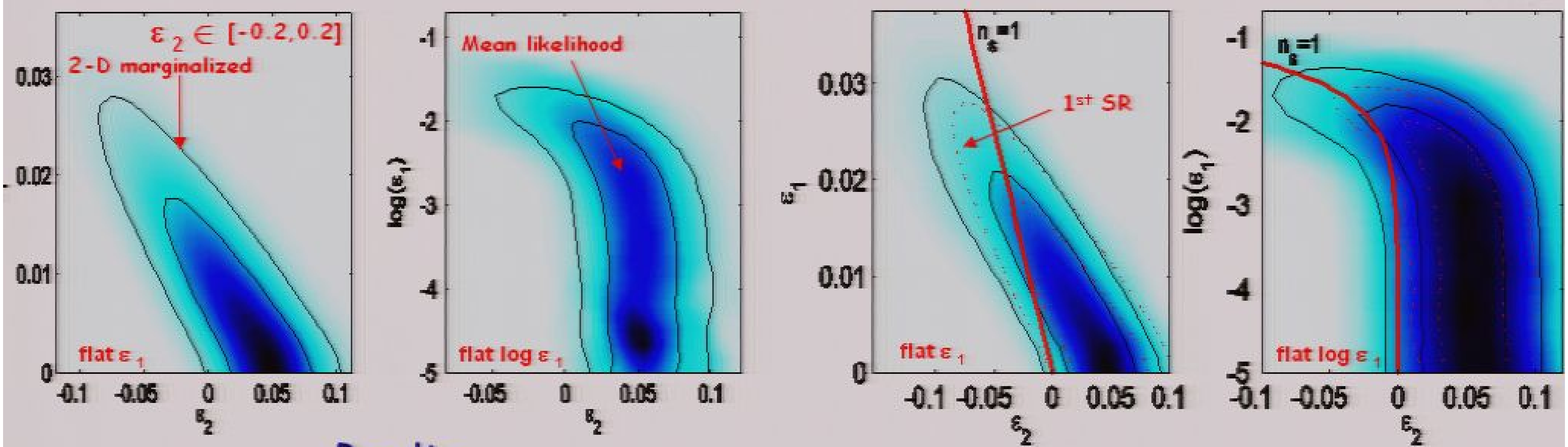

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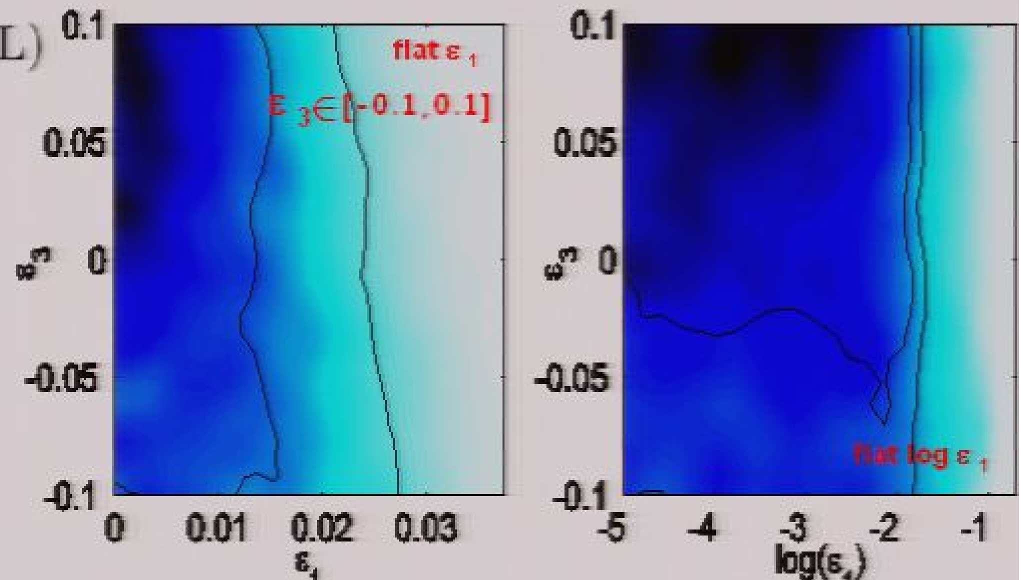

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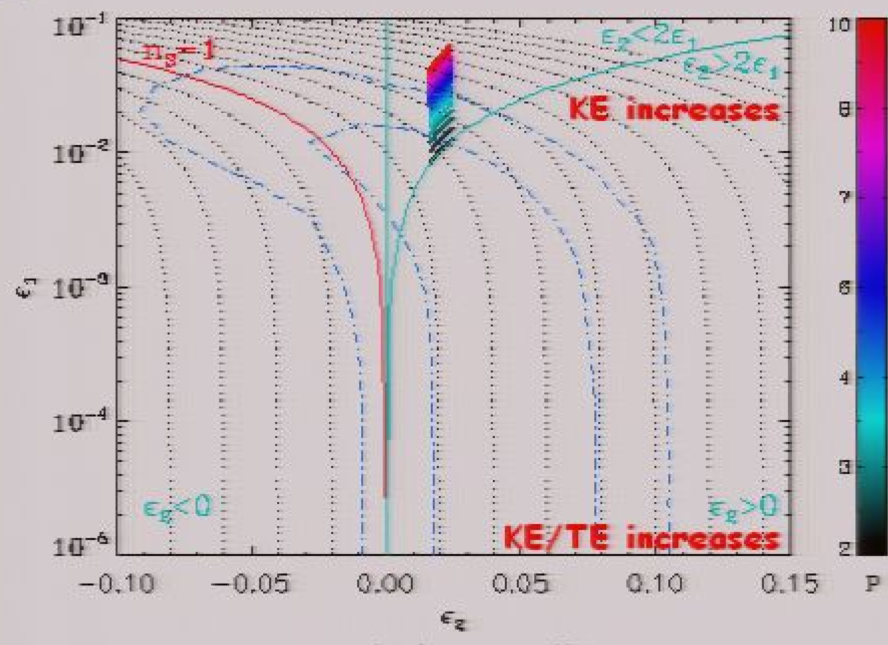
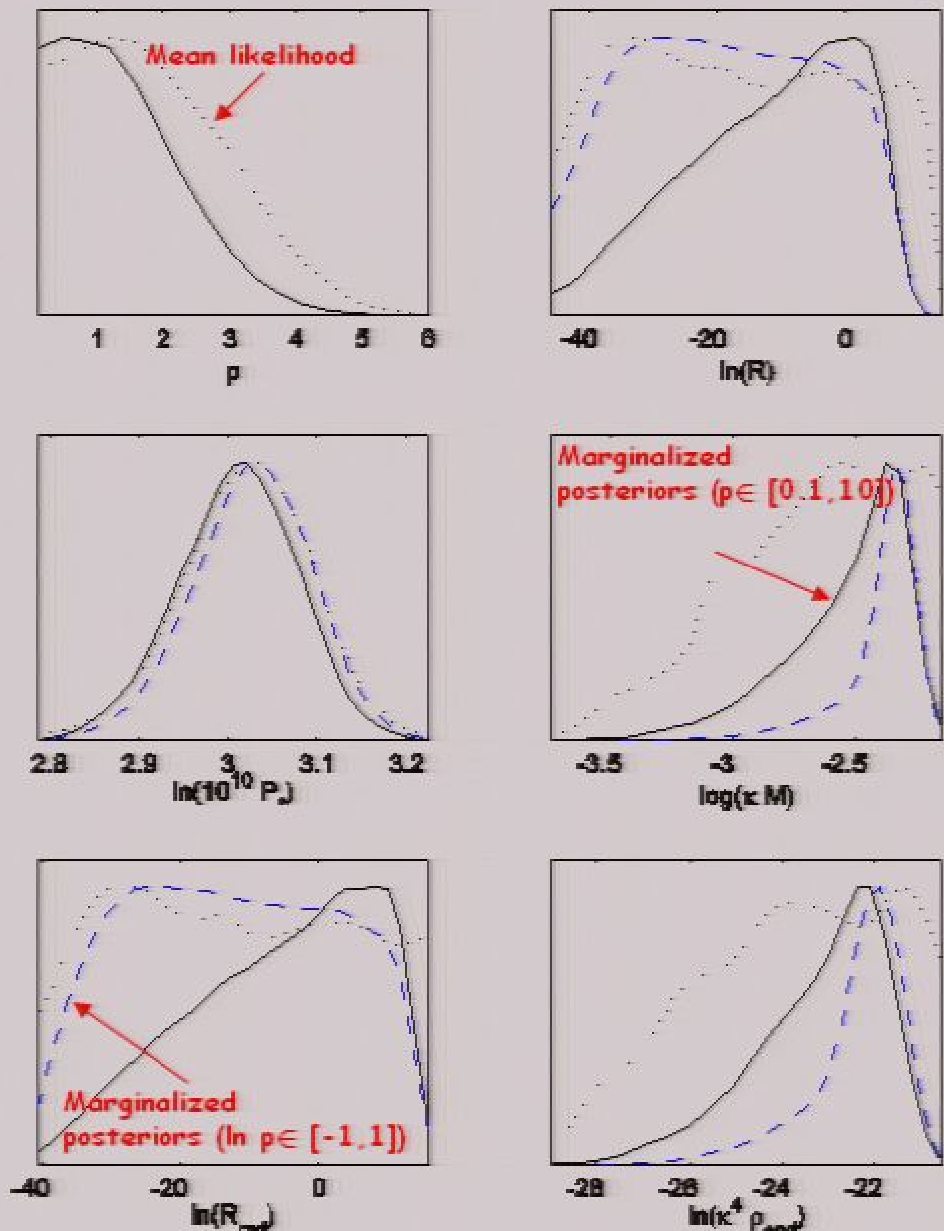
WMAP3 and inflation: large field



$$V(\phi) \propto \phi^p$$



Results

1- $p < 3.1 (95\% \mathrm{CL})$

2- No constraint on T_{reh}

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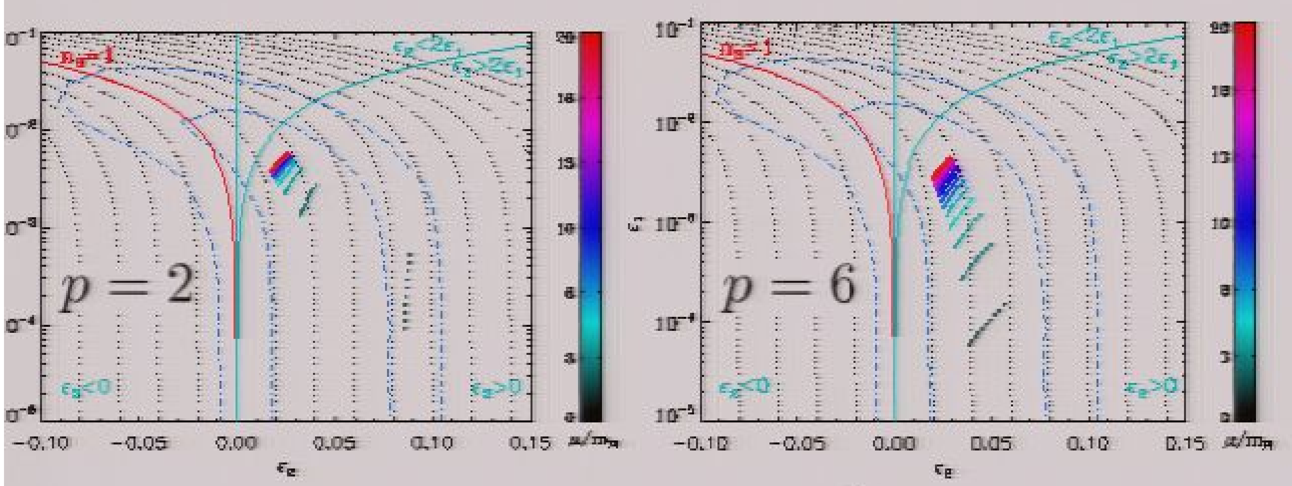

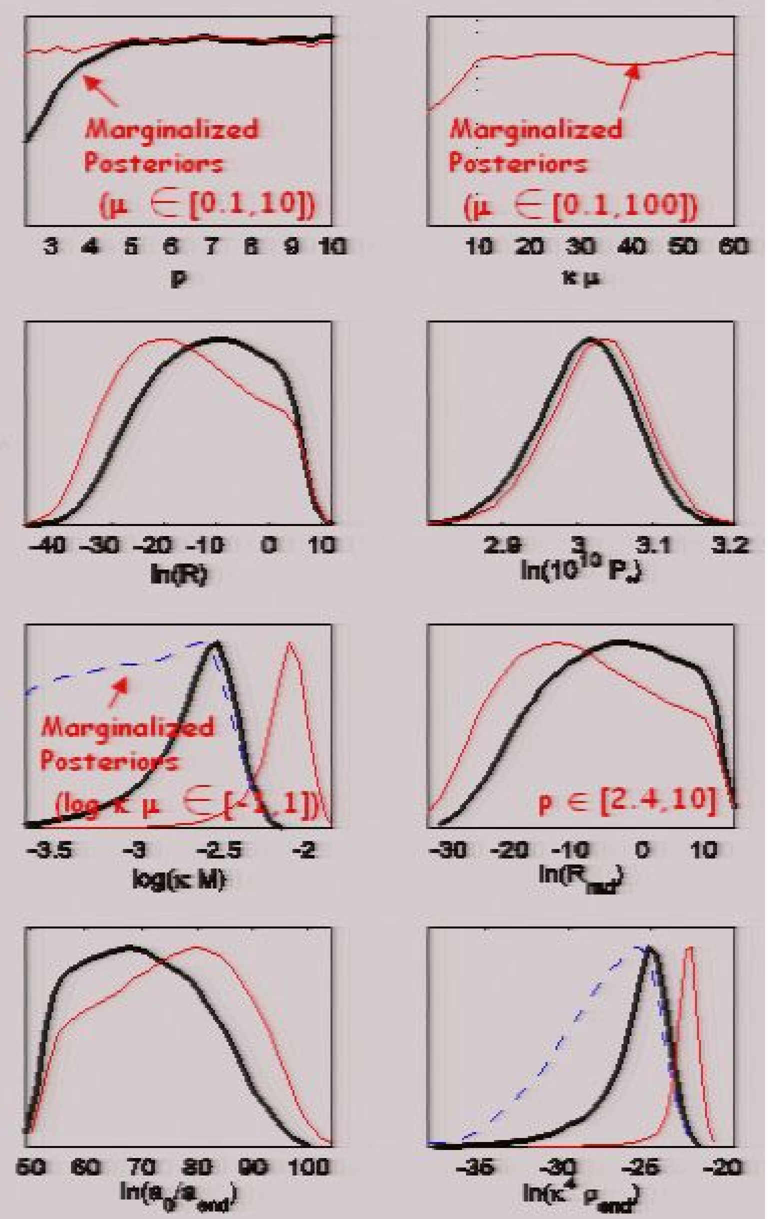

$$V(\phi) \propto 1 - \left(\frac{\phi}{\mu}\right)^p$$

Results

1- No (prior independent) constraint on p

2- $T_{\rm reh} > 2\,\mathrm{TeV}\,(95\%\mathrm{CL})\,,\,(w \sim -1/3)$

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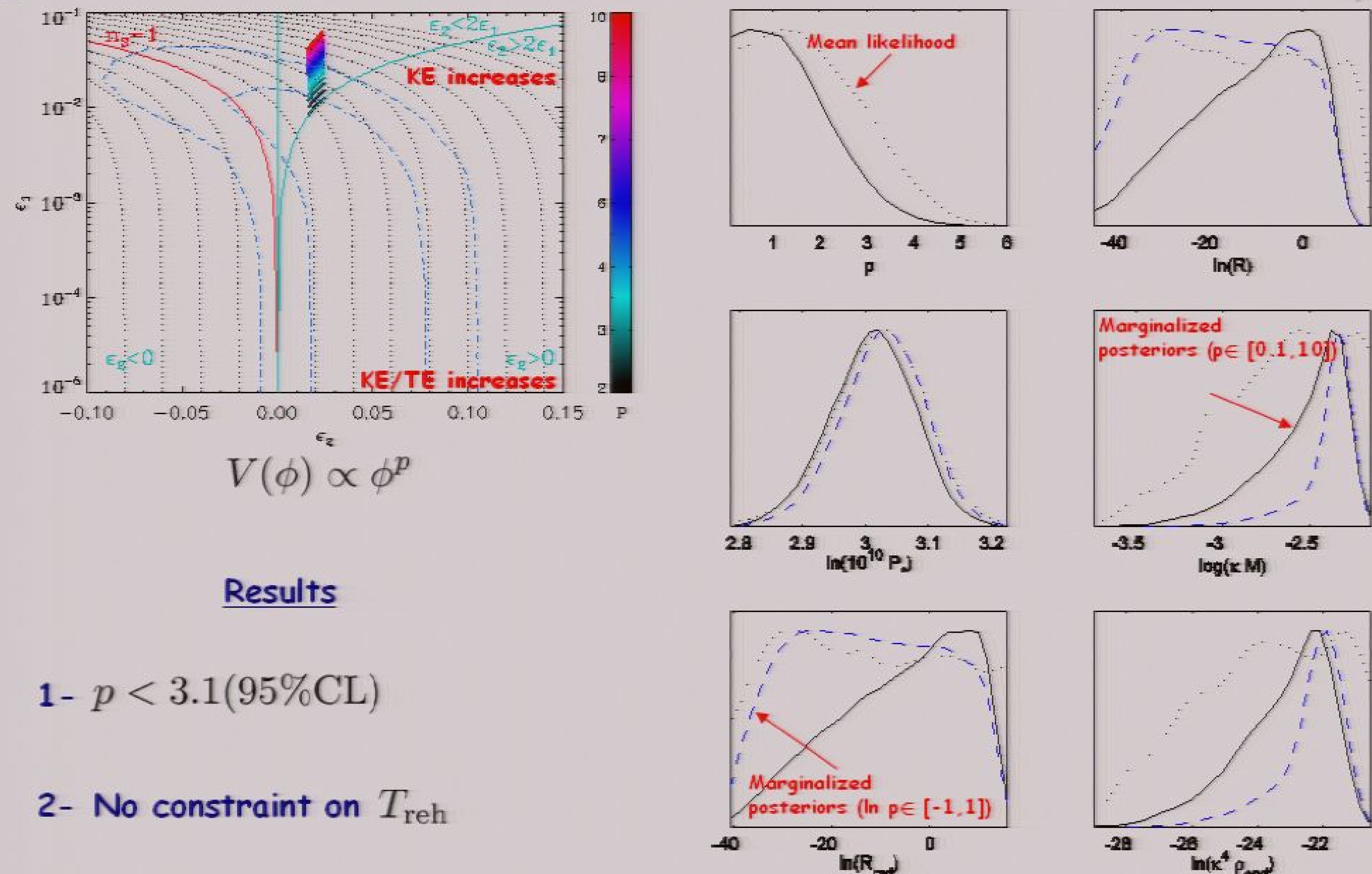

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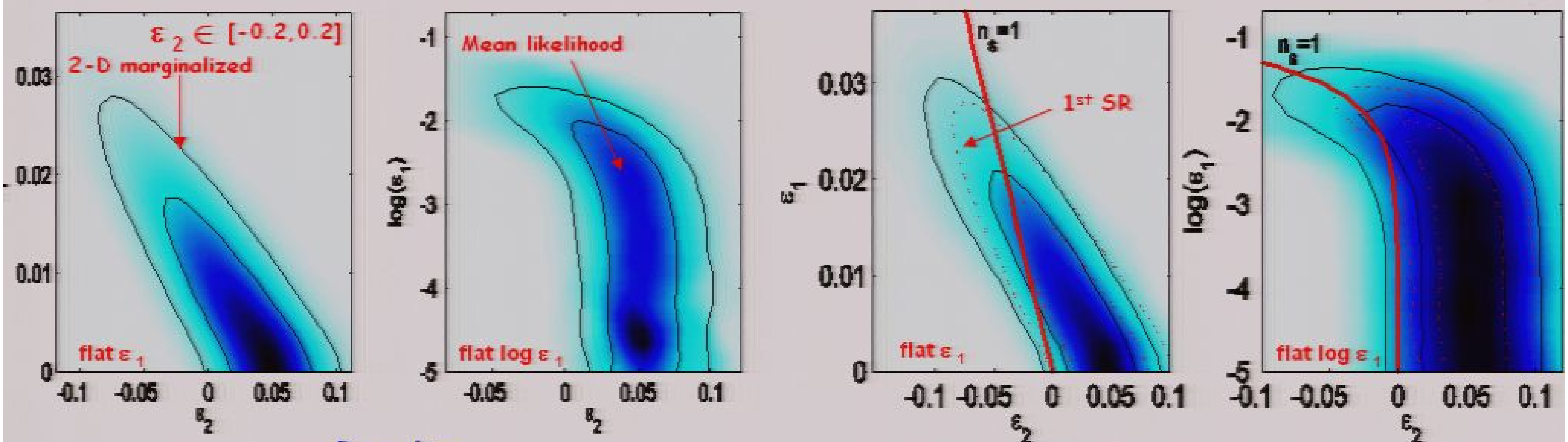

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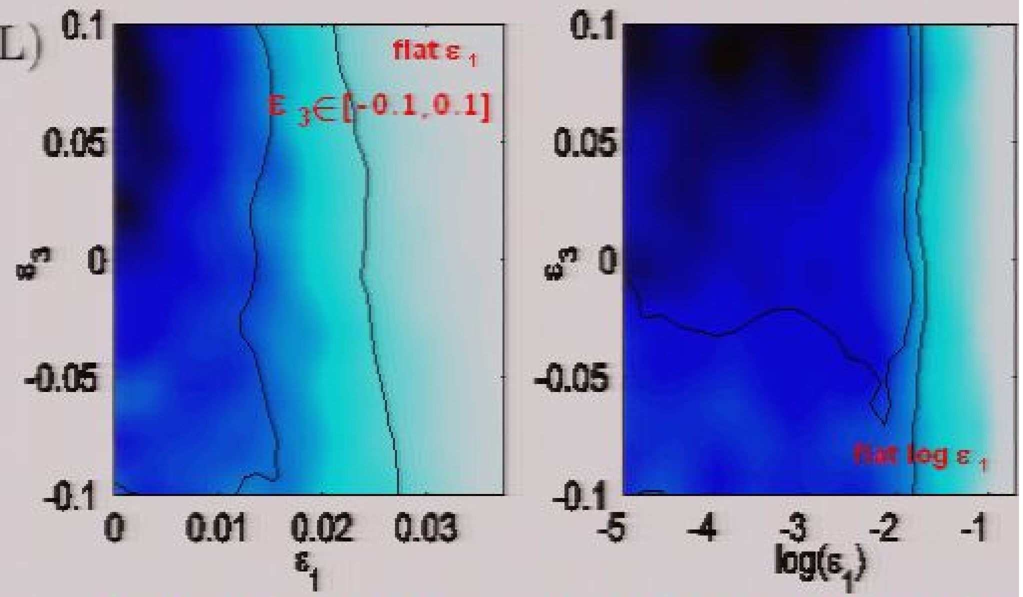

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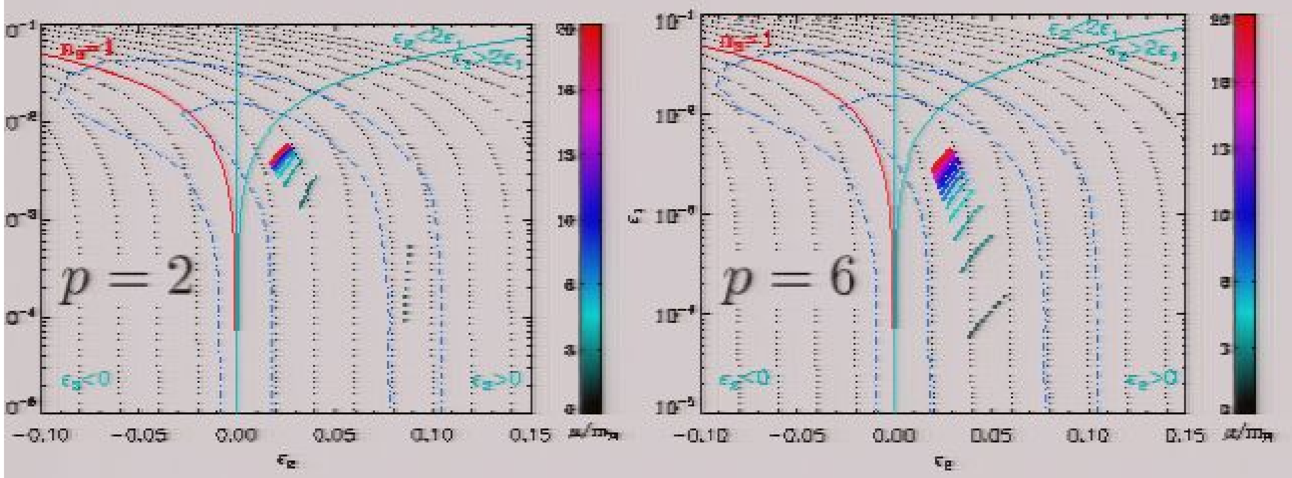
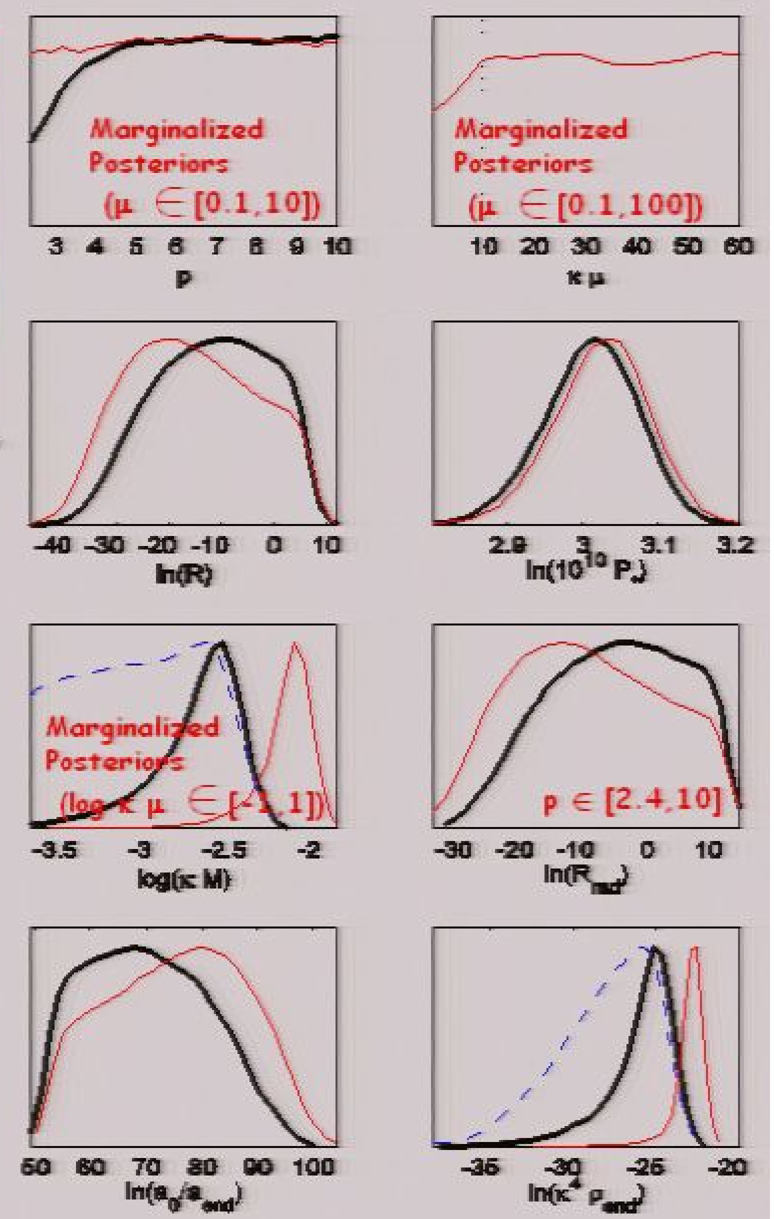
WMAP3 and inflation: small field





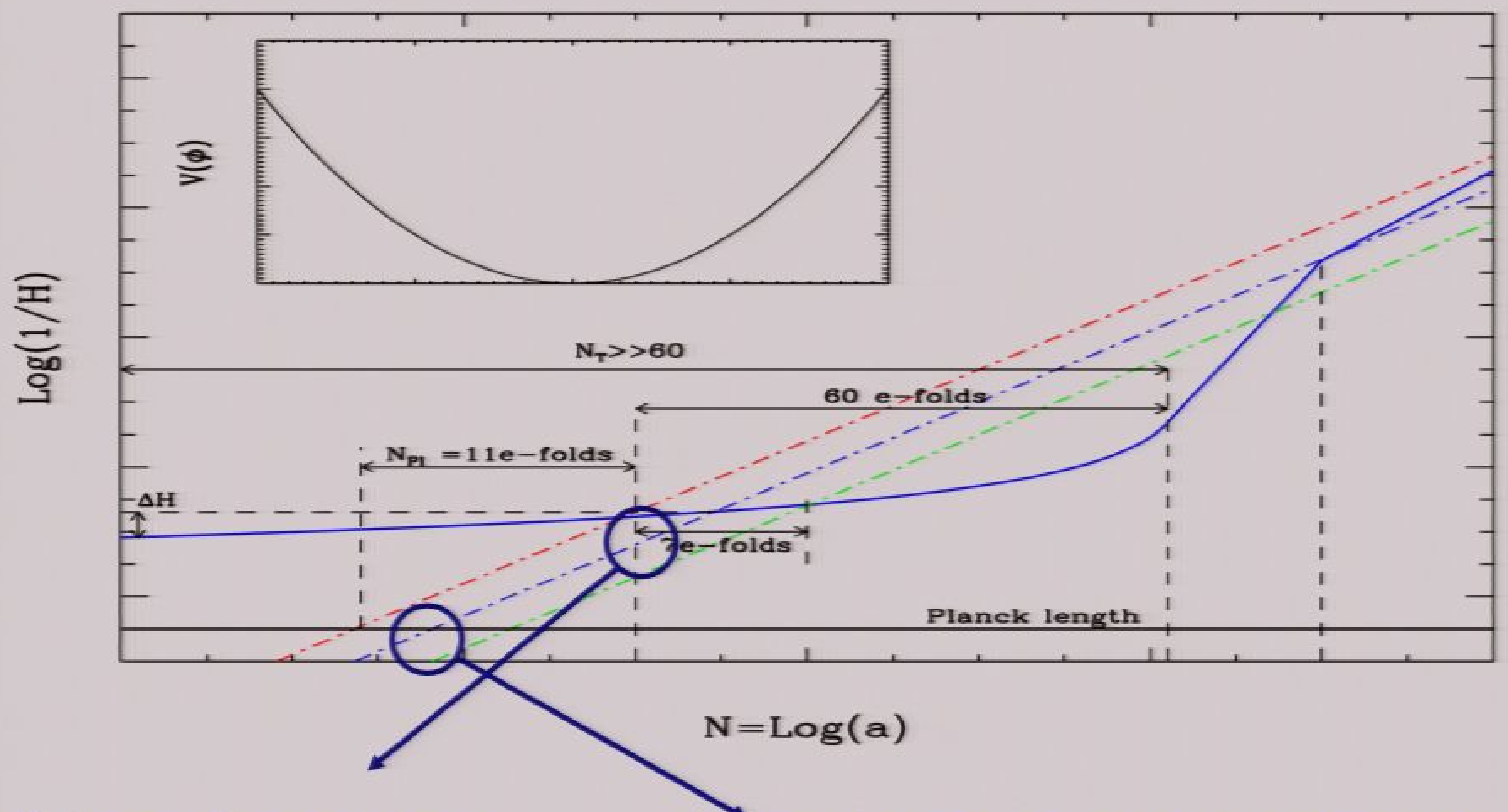
Results

1- No (prior independent) constraint on

2-

J. Martin & C. Ringeval, JCAP 08, 009 (2006)
astro-ph/0605367

Importance of the IC



If the IC do not scale as $k^{-1/2}$, it is clear that we no longer obtain a scale-invariant power-spectrum

But, the IC are fixed when the modes are <u>well-below the Planck length</u>, where the framework used (QFT in curved space-time) certainly breaks down

J. Martin & R. H. Brandenberger, PRD 63 123501 (2001), hep-th/0005209

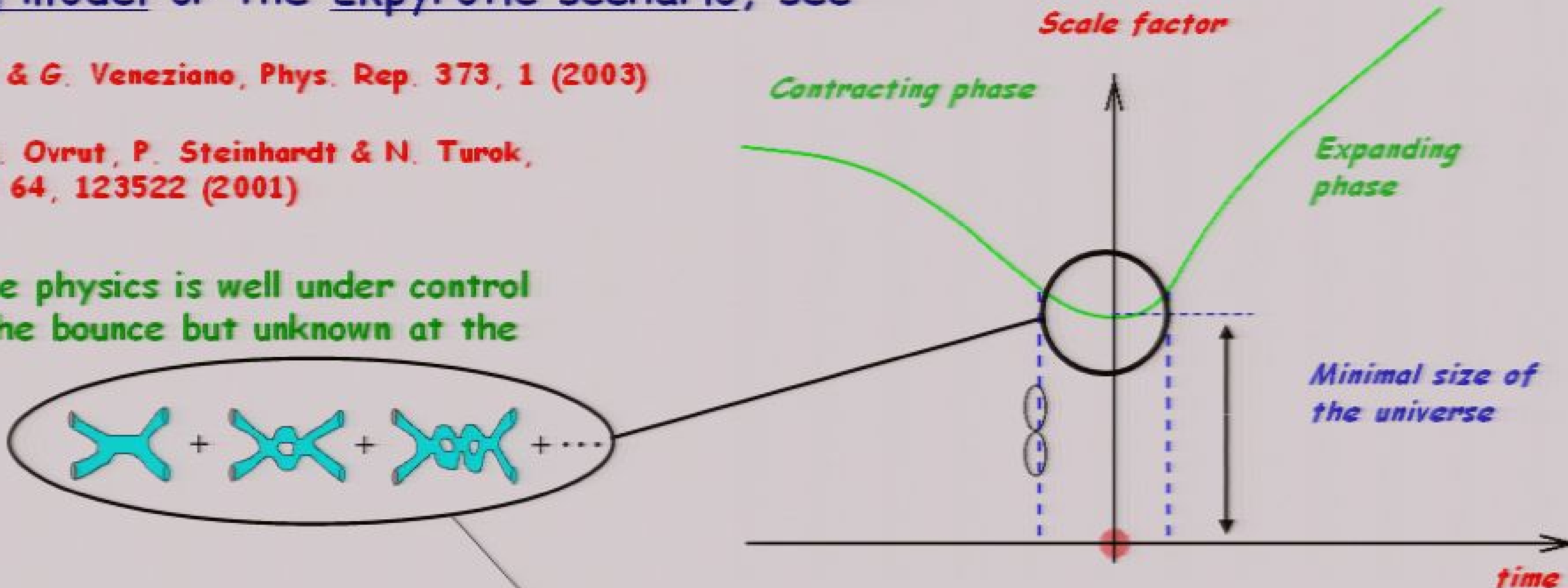
Importance of the IC

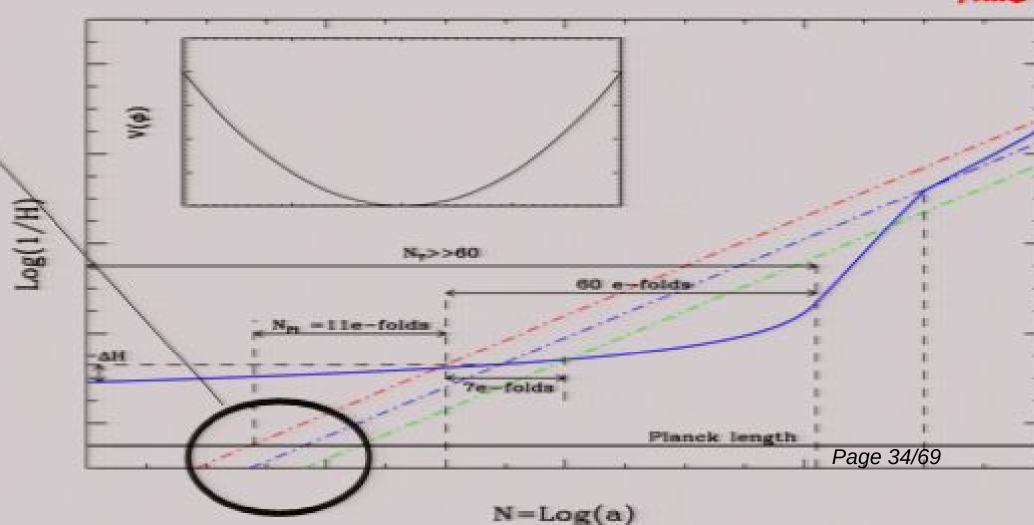
The attempts to apply string theory to time-dependent backgrounds, i.e. to cosmology, have revived the interest for bouncing models as e.g. the Pre Big Bang model or the Ekpyrotic scenario, see

M. Gasperini & G. Veneziano, Phys. Rep. 373, 1 (2003)

J. Khoury, B. Ovrut, P. Steinhardt & N. Turok, Phys. Rev. D 64, 123522 (2001)

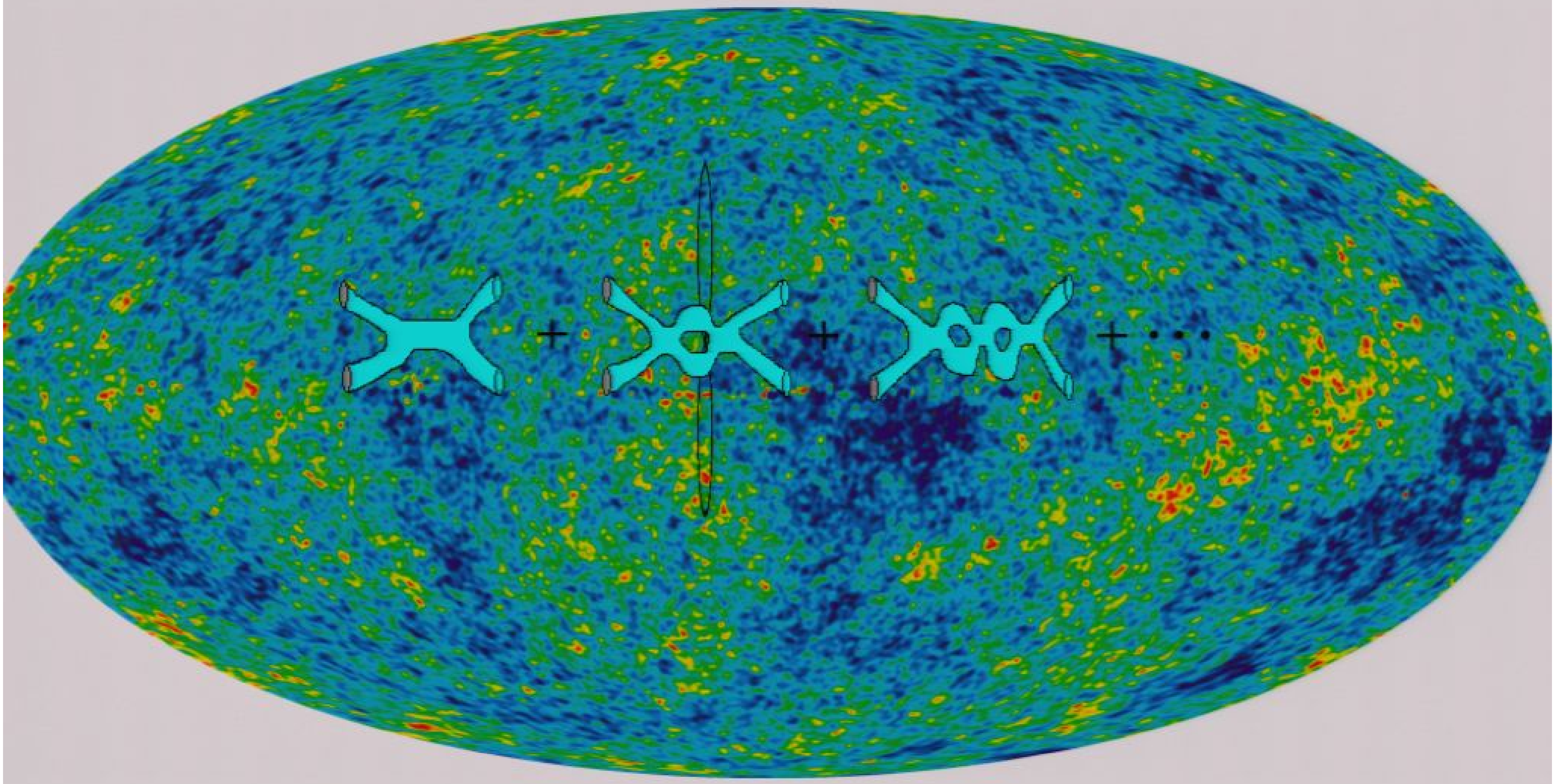
Usually, the physics is well under control far from the bounce but unknown at the bounce.



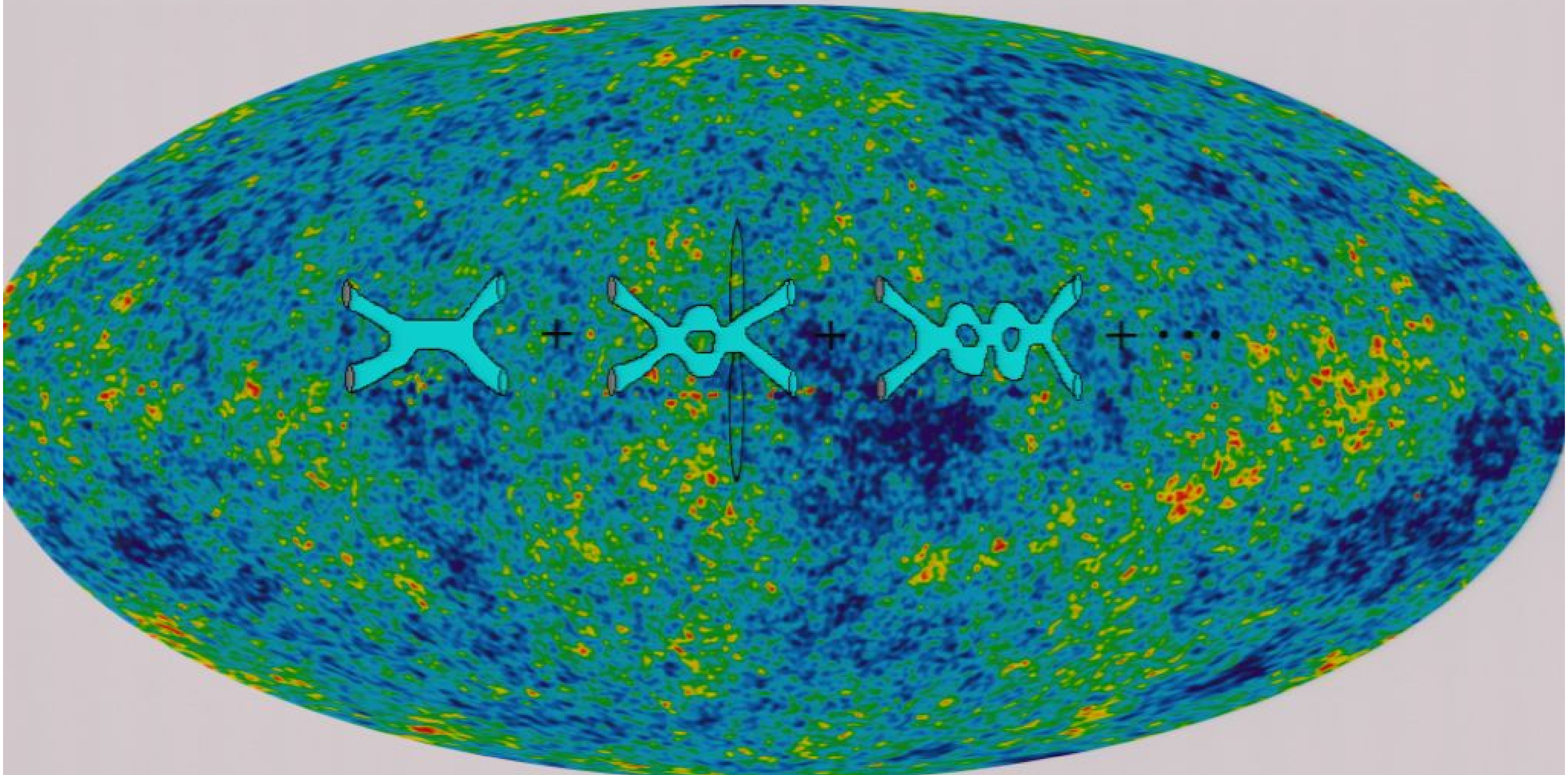


In some sense, in the case of inflation, we face the same problem again … (but not for the background!)

Stringy effects in the sky?



Stringy effects in the sky?



Remarks

1- We do not know physics beyond the Planck scale: it is therefore perfectly legitimate to contemplate the problem and stop at this stage ...

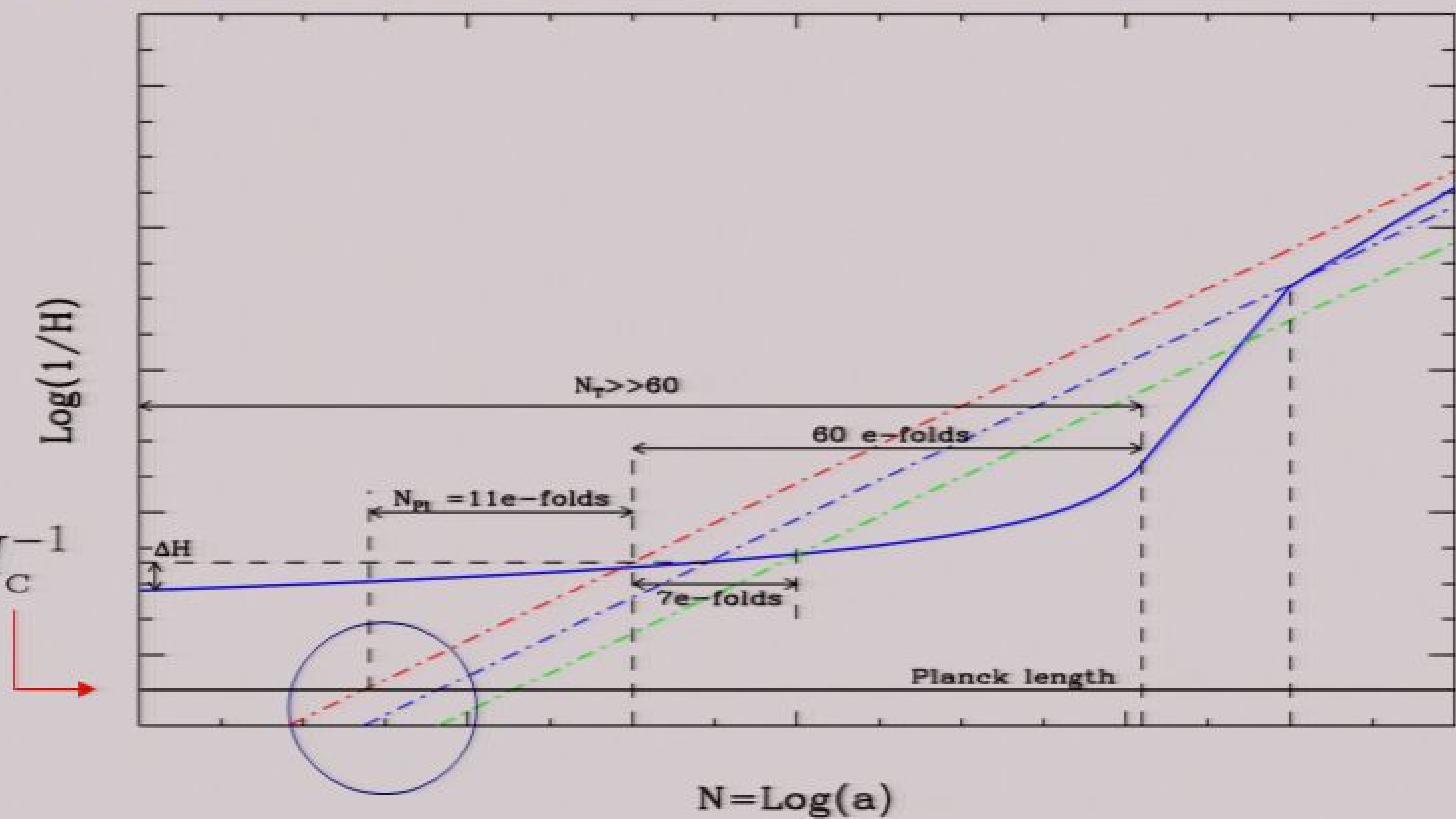
2- One can also try to test the robustness of the inflationary predictions to modifications of high energy physics.

3- These modifications are only phenomenological. Obviously, one has no guarantee that, in reality, physics beyond the Planck scale is as we imagine. Therefore, these considerations are necessarily very speculative and, hence, of "quite limited" impact.

4- The relevant scale is not necessarily the Planck scale, it could be the string scale or something else.

Planck/String Physics in the CMB?

$$\ell_c = M_c^{-1}$$

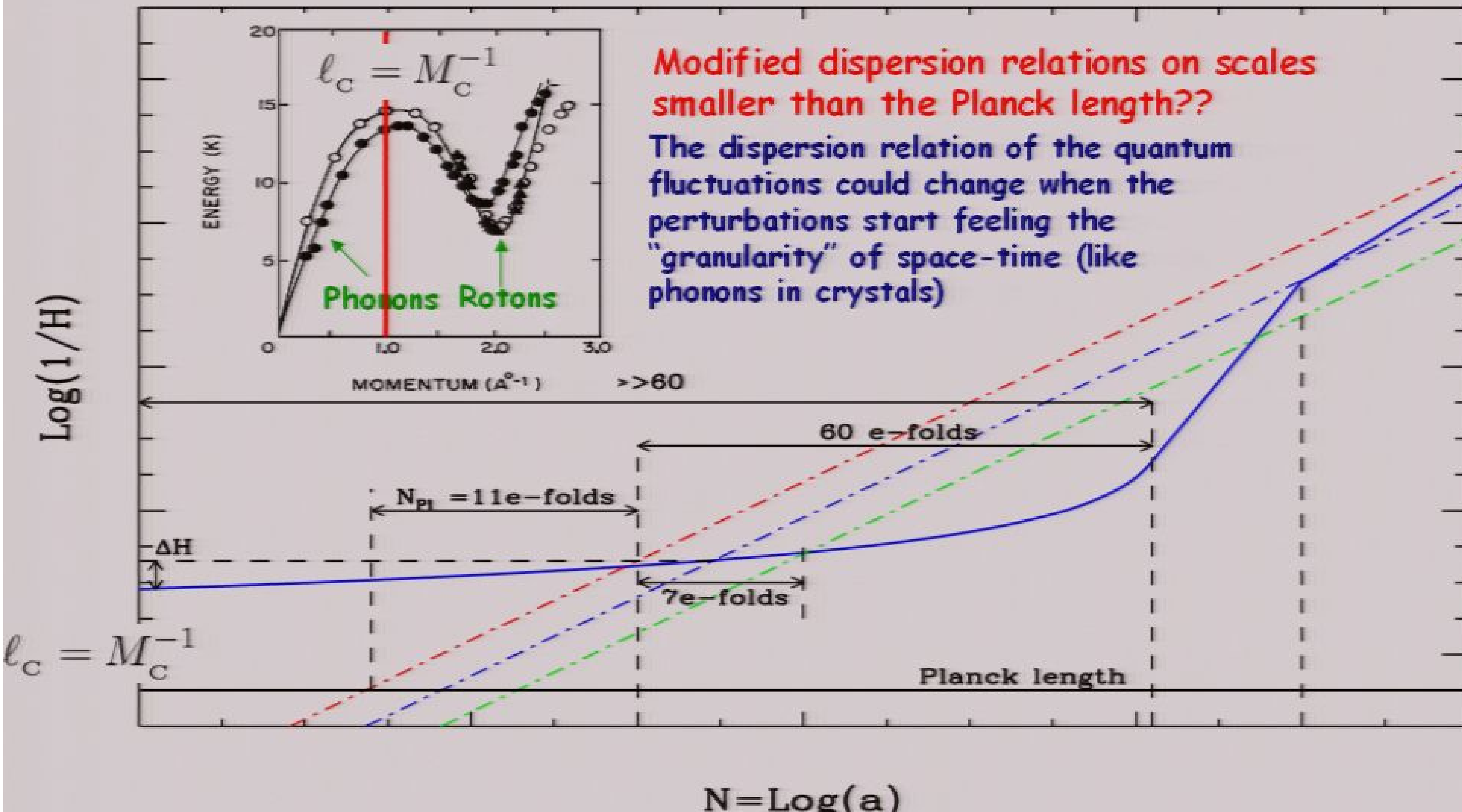


All approaches to Quantum Gravity (String theory, Non-commutative space-time, LQG, DSR etc …) predicts new physics at some new characteristic scale

$$\ell_c = M_c^{-1}$$

Which modifications??

Which modifications?



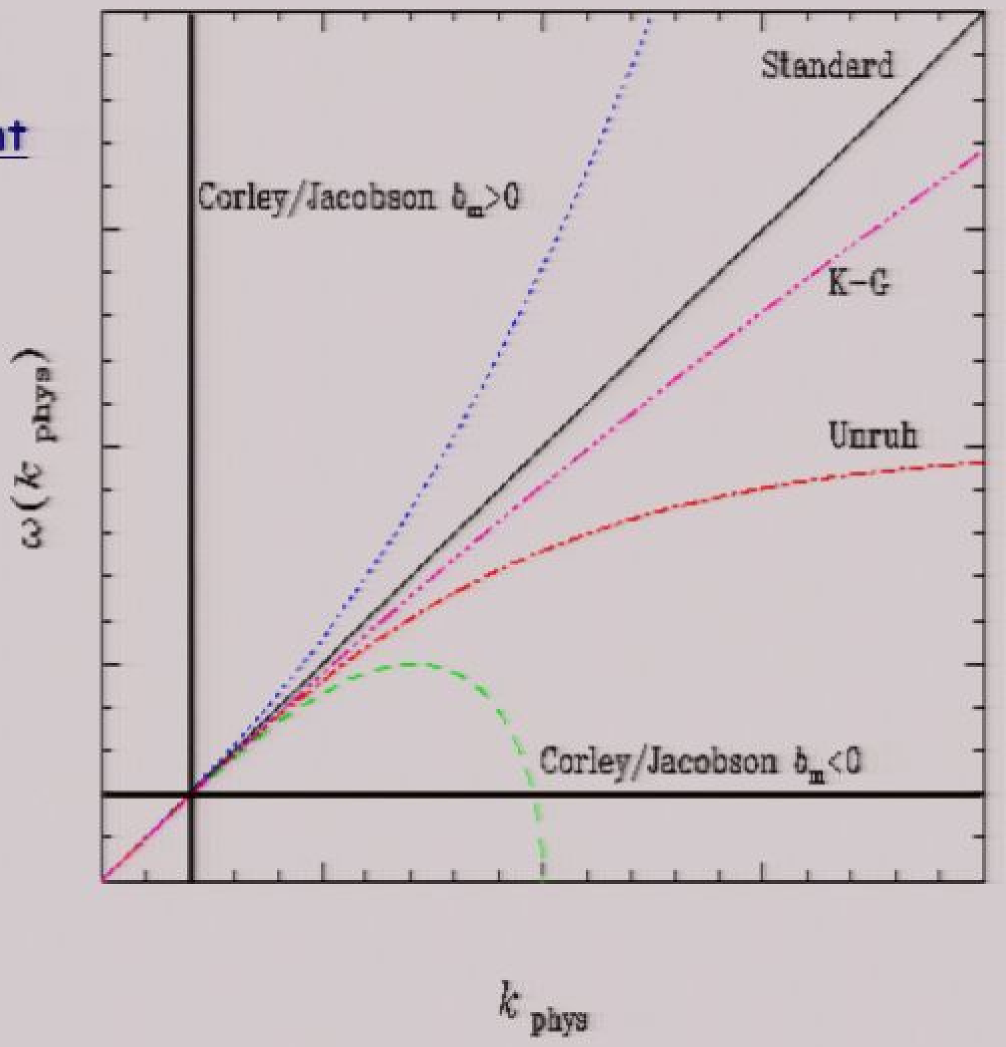
This allows to identify what is needed beyond the Planck scale in order to get a modification of the inflationary spectrum: is the adiabatic regime still satisfied below the Planck length?

The new dispersion relation does not change the type of equation describing the perturbations but the time-dependent frequency is modified

$$\frac{d^2\mu}{d\eta^2} + \omega^2(k,\eta)\,\mu = 0$$

with

$$\omega^2(k,\eta) = k_{\rm eff}^2(\eta) - \frac{\beta(\beta+1)}{\eta^2}$$
$$k_{\rm eff}^2(\eta) = a^2(\eta)\omega_{\rm phys}^2\left[\frac{k}{a(\eta)}\right]$$



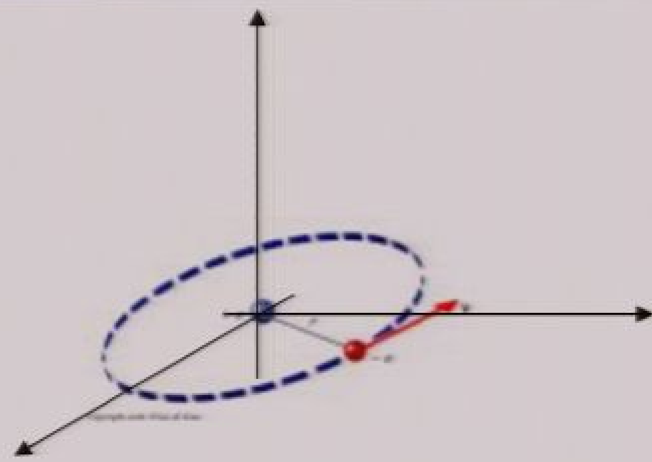
When do we get a modification of the spectrum?

J. Martin & R. H. Brandenberger, PRD 63 123501 (2001), hep-th/0005209

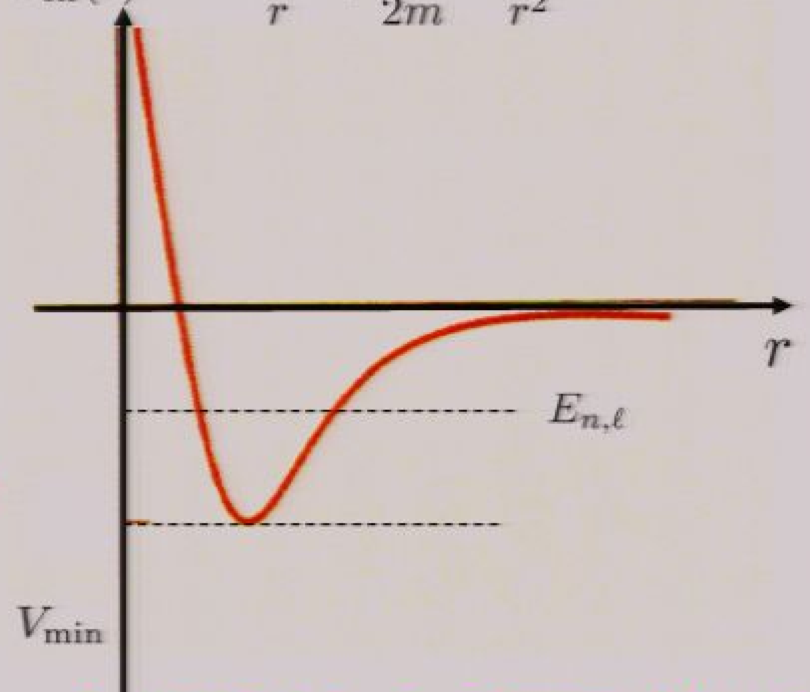
- The Schroedinger equation reads:

$$\frac{\mathrm{d}^2 u_{n,\ell}}{\mathrm{d}r^2} + \omega^2(E,r)\, u_{n,\ell} = 0$$

$$\omega^2(E,r) = \frac{2m}{\hbar^2}\left(E + \frac{Ze^2}{r}\right) - \frac{\ell(\ell+1)}{r^2}$$



$$V_{\rm eff}(r) = -\frac{Ze^2}{r} + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}$$



- This is similar to the cosmological case

$$\omega^2(k,\eta) = k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

- Close to the nucleus ~ super-Hubble scales

- Far from the nucleus ~ sub-Hubble scales

J. Martin & D. J. Schwarz, PRD 67 083512 (2003), astro-ph/0210090

Historical Digression

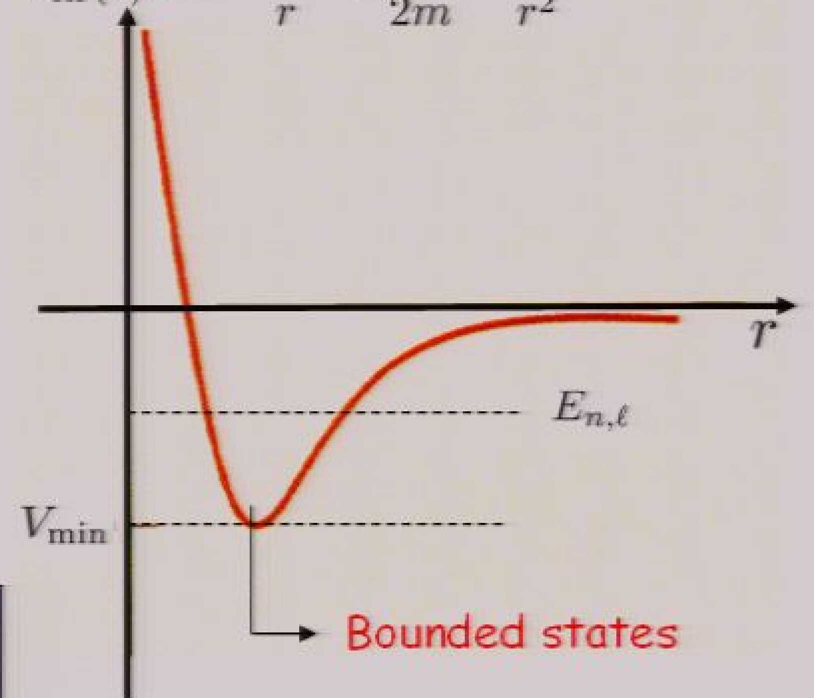
- The WKB approximation leads to the Bohr-Sommerfeld quantization rule

$$\oint \omega\,(E, r)\,\mathrm{d}r = 2\pi\left(n + \frac{1}{2}\right)$$

which gives

$$E_{n,\ell} = -\frac{mZ^2e^4}{2\hbar^2}\left\{\left(n+\frac{1}{2}\right) + \left[\ell\,(\ell+1)\right]^{1/2}\right\}^{-2}$$

$$V_{\text{eff}}(r) = -\frac{Ze^2}{r} + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}$$



The factor $\ell(\ell+1)$ is wrong and should be replaced by $\left(\ell+\frac{1}{2}\right)^2$:
" failure of the WKB method" ?

APRIL 15, 1937 PHYSICAL REVIEW VOLUME 51

On the Connection Formulas and the Solutions of the Wave Equation

RUDOLPH E. LANGER
Department of Mathematics, University of Wisconsin, Madison, Wisconsin

Part 1 gives a general discussion of asymptotic representations of the solutions of the one-dimensional wave equation. The forms ordinarily used in the so-called W. K. B. method are multiple valued and consequently necessitate a consideration of the Stokes' phenomenon, in any region about a turning point, i.e., a point in which the kinetic energy changes sign. Except under restrictive hypotheses they give no description of the solutions near the turning points. The author's method for representing the solutions of such differential equations by means of single valued functions is discussed, and the formulas applicable to the wave equation are given. These formulas are usable over the whole of an interval which includes a turning point. The Stokes' phenomenon is not involved. It need be considered only if expressions of the older type are desired, and then the connection formulas of the W. K. B. method are immediately evolved. An appropriate formal development of the solutions of the wave equation as power series in $\hbar$ is given.

Part 2 deals with the radial wave equation for motion in a central field of force. Both the attractive and repulsive Coulomb field are considered. It is shown that the application of the W. K. B. analysis to this equation as it has generally been made is uncritical and in error. The solution commonly identified thereby as the wave function is in fact not the wave function. The "failure" of the W. K. B. formulas, and the apparent necessity for modifying them by replacing the number $l(l+1)$ by $(l+\frac{1}{2})^2$, has been noted by many investigators. This is traced to the misapplication of the theory. When correctly applied the theory naturally yields the formulas which have been found to be called for on other grounds.

Finally the case is discussed in which a turning point lies too near the point $r=0$ for the W. K. B. method to be effectively applicable. It is shown how the solutions are describable in this case, the formulas given specializing, when the field is an attractive field and the energy is zero, to formulas which were given for that special case by Kramers.

- During many years, atomic physicists have used the trick to replace " by hand" the wrong factor $\ell(\ell+1)$ by the correct one $(\ell+1/2)^2$!
- The problem was solved by Langer in 1937.

The WKB approximation is violated close to the nucleus

A change of variable can make the approximation well-defined at small distances

Cosmological consequences

- The wkb wave-function exactly obeys the equation

$$u_{\rm wkb}(r) = \frac{1}{\sqrt{2\omega(r)}} e^{\pm i \int^r \omega(\rho) d\rho}$$

$$\frac{d^2 u_{\rm wkb}}{dr^2} + \left[\omega^2(E,r) - Q(E,r)\right] u_{\rm wkb} = 0$$

with $Q \equiv \frac{3}{4\omega^2}\left(\frac{d\omega}{dr}\right)^2 - \frac{1}{2\omega}\frac{d^2\omega}{dr^2}$

- Langer remarked that, for very small radii, one does not have

$$\left|\frac{Q}{\omega^2}\right| \ll 1$$

wkb is violated for $r \to 0$

The Langer's Solution

The WKB approximation
is violated close to the nucleus

A change of variable can
make the approximation
well-defined at small distances

Cosmological consequences

- With the following change of variables

$$r = \mathrm{e}^{x}\,, \quad \mu = \mathrm{e}^{x/2} u_{n,\ell}\,,$$

the effective frequency reads

$$\omega^2(x) = \frac{2m}{\hbar^2}\left(E\mathrm{e}^{2x} + Ze^2\mathrm{e}^{x}\right) - \left(\ell + \frac{1}{2}\right)^2$$

and wkb is valid for small radii

- Expressed back into the original variables, this method leads to the correct Balmer formula

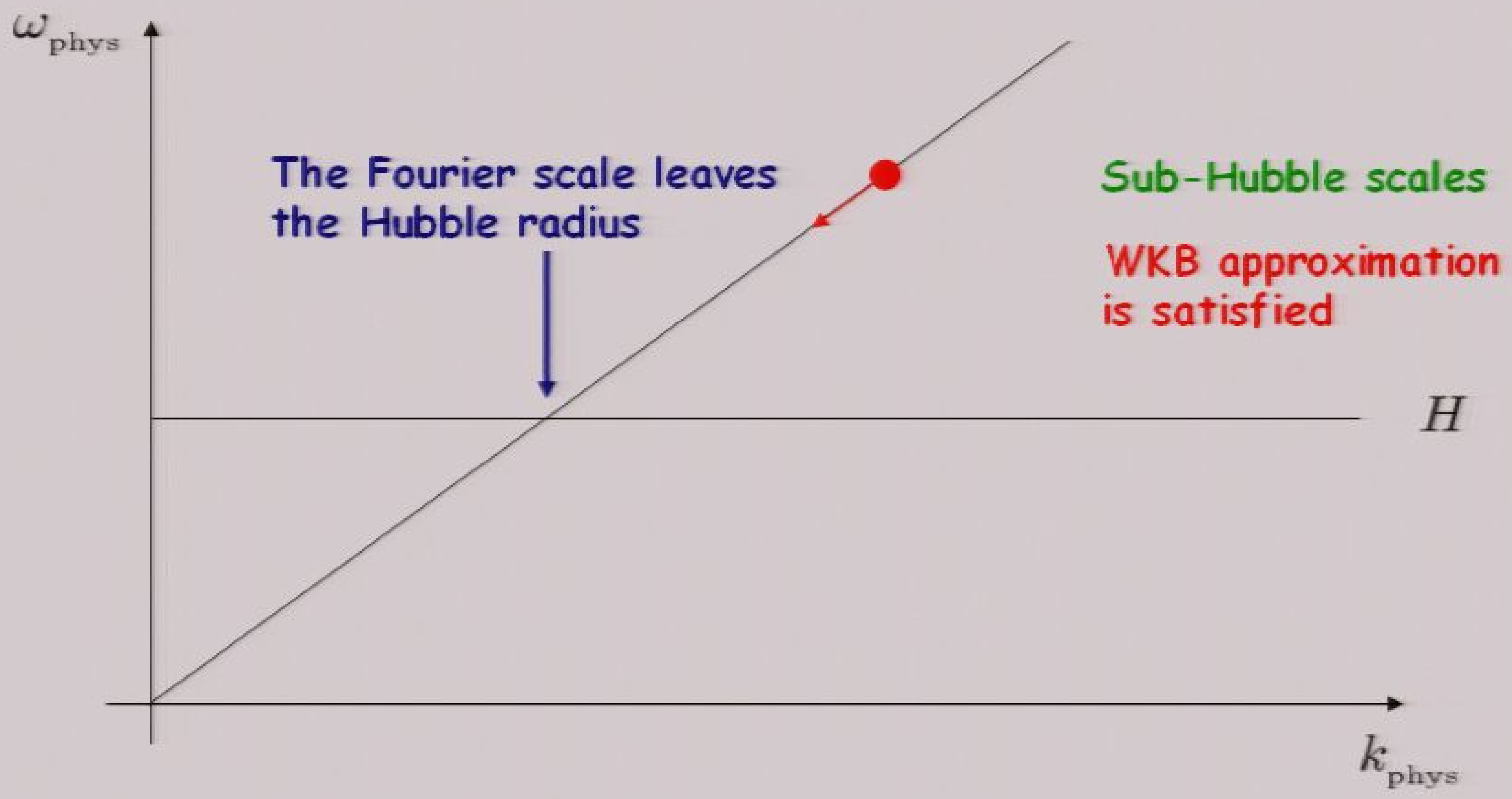
The Langer's Solution

The WKB approximation
is violated close to the nucleus

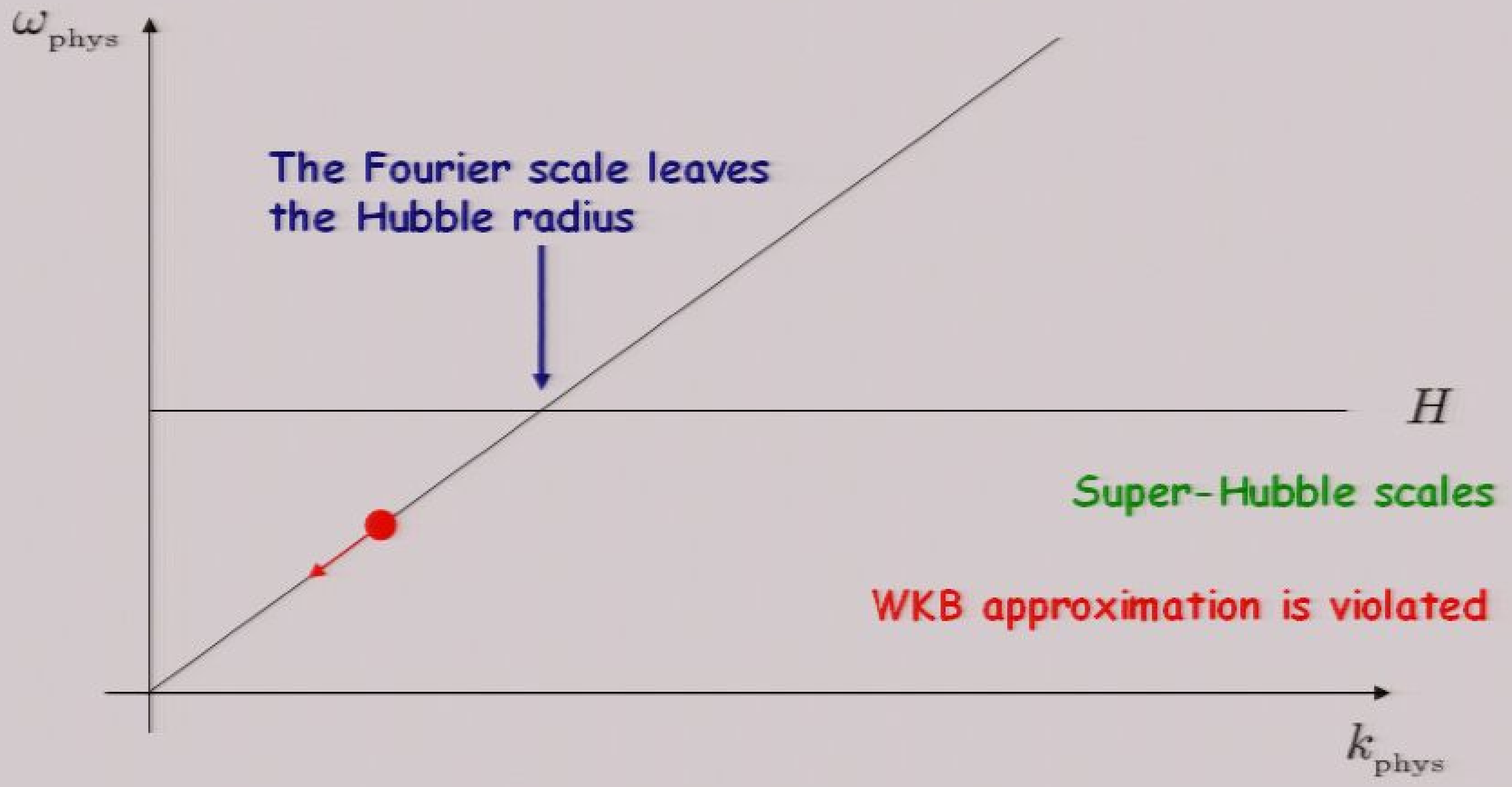
A change of variable can
make the approximation
well-defined at small distances

Cosmological consequences

- The wkb or adiabatic approximation is violated on super-Hubble scales during inflation
- This is because violation of the wkb approximation is associated to particles production, or, equivalently to a modification of the initial vacuum state and/or spectrum
- The cosmological version of the Langer's transformation can be used in order to derive the inflationary power spectra

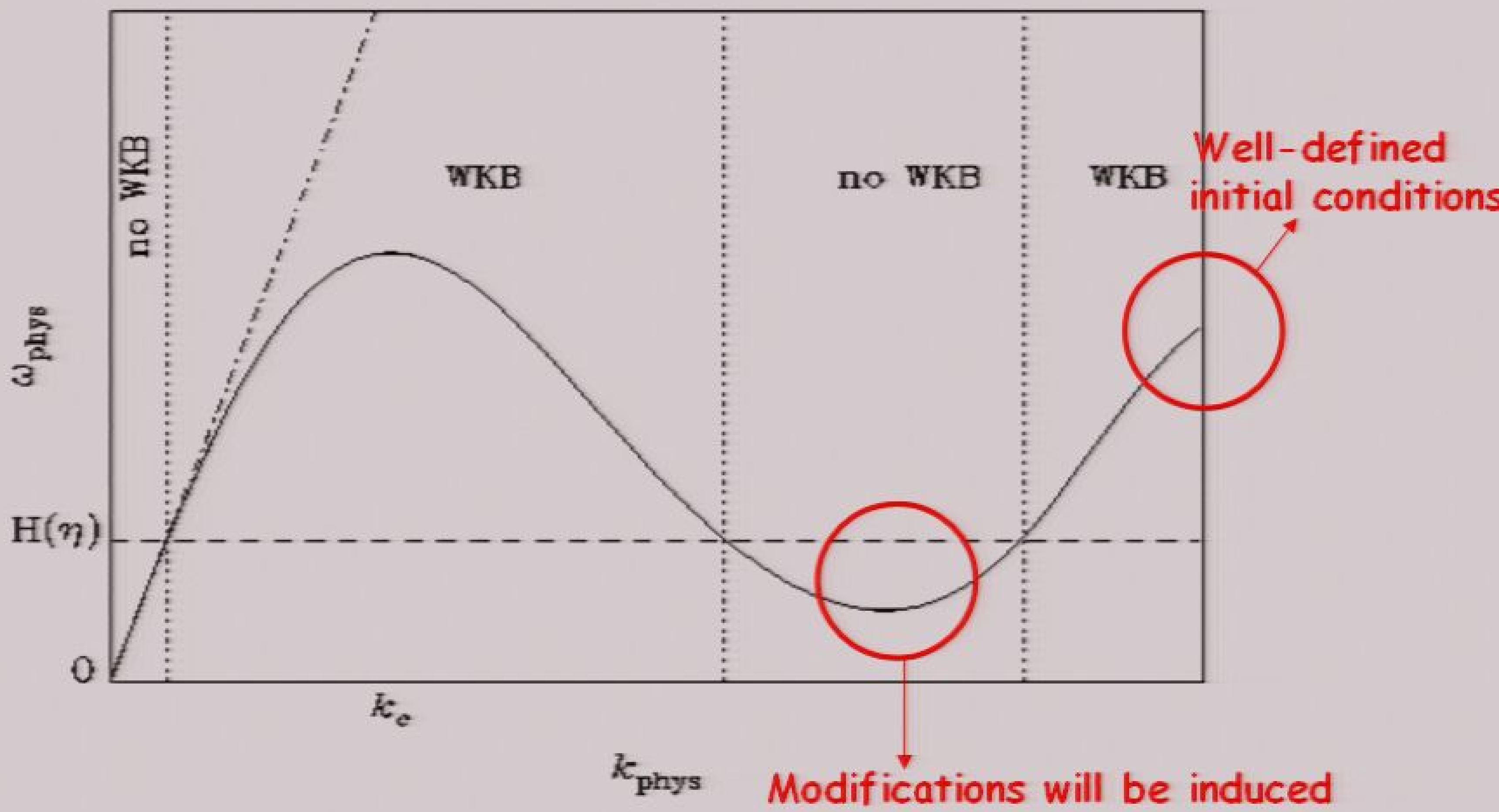


This allows us to set up well-defined initial conditions



Lesson: when the "frequency is below the Hubble scale", WKB is violated

Example: the Landau-Feynmann dispersion relation

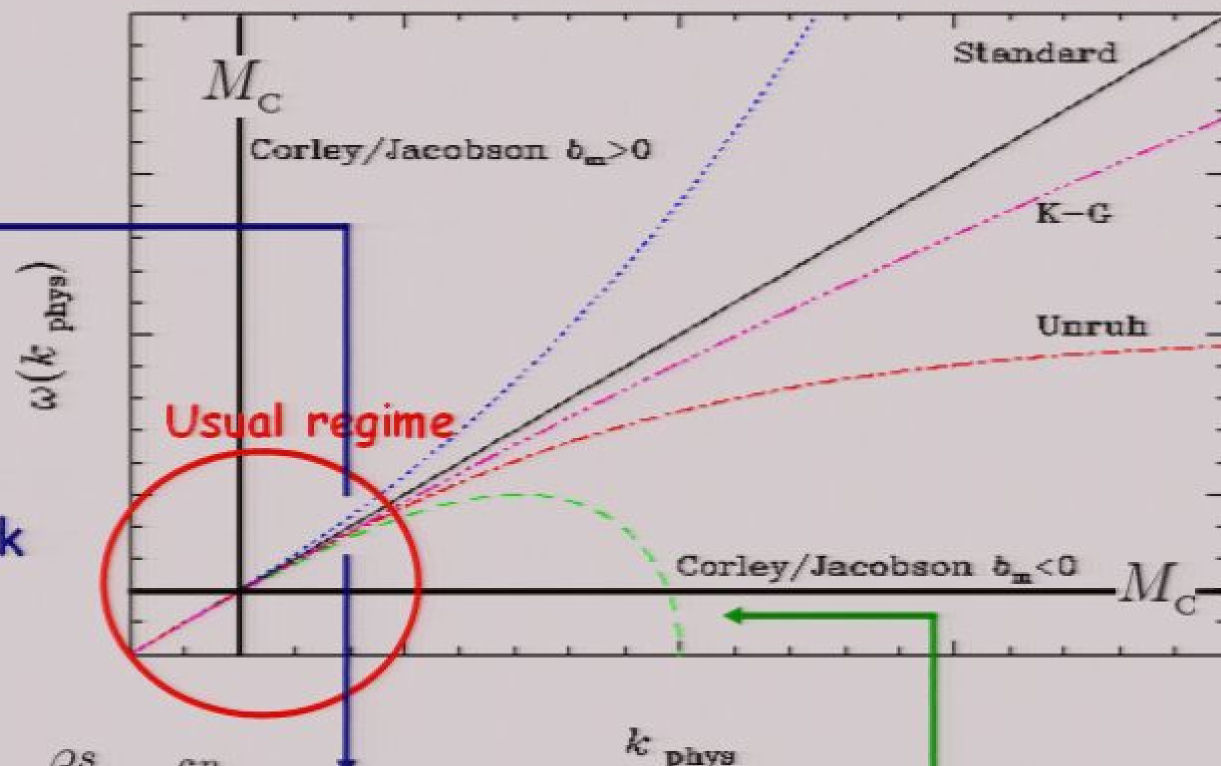


The WKB criterion

1- Standard scenario $\beta^s_{\vec{k}} = 0$

2- What happened before results (is described by) in the β branch

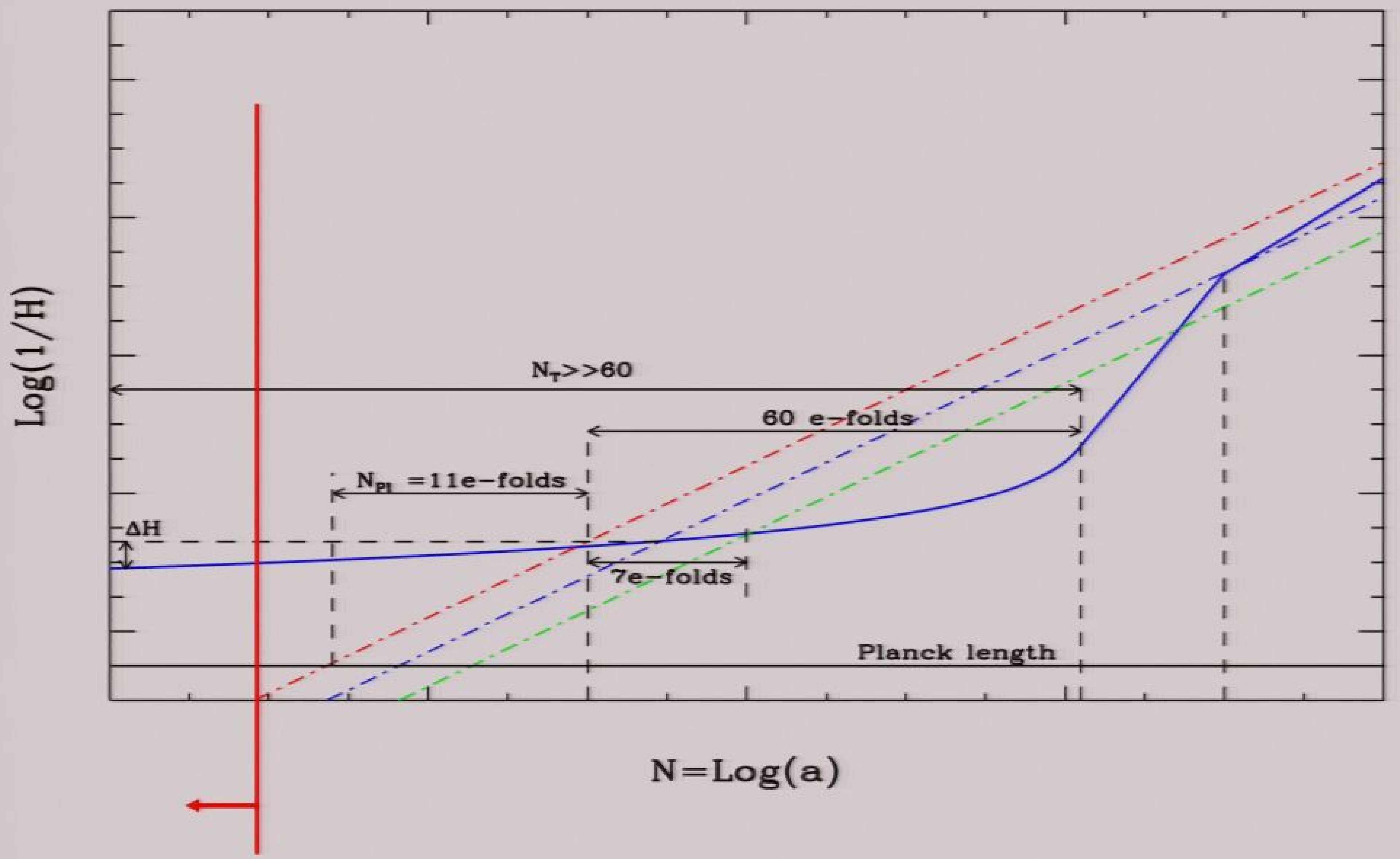
3- If the WKB approximation was satisfied beyond the Planck scale then $\beta^s_{\vec{k}} = 0$



$$\mu^s_{\vec{k}}(\eta) = \frac{\alpha^s_{\vec{k}}}{\sqrt{2k}} e^{-\int_{\eta_{\rm ini}}^{\eta} \omega(k,\tau)\mathrm{d}\tau} + \frac{\beta^s_{\vec{k}}}{\sqrt{2k}} e^{\int_{\eta_{\rm ini}}^{\eta} \omega(k,\tau)\mathrm{d}\tau}$$

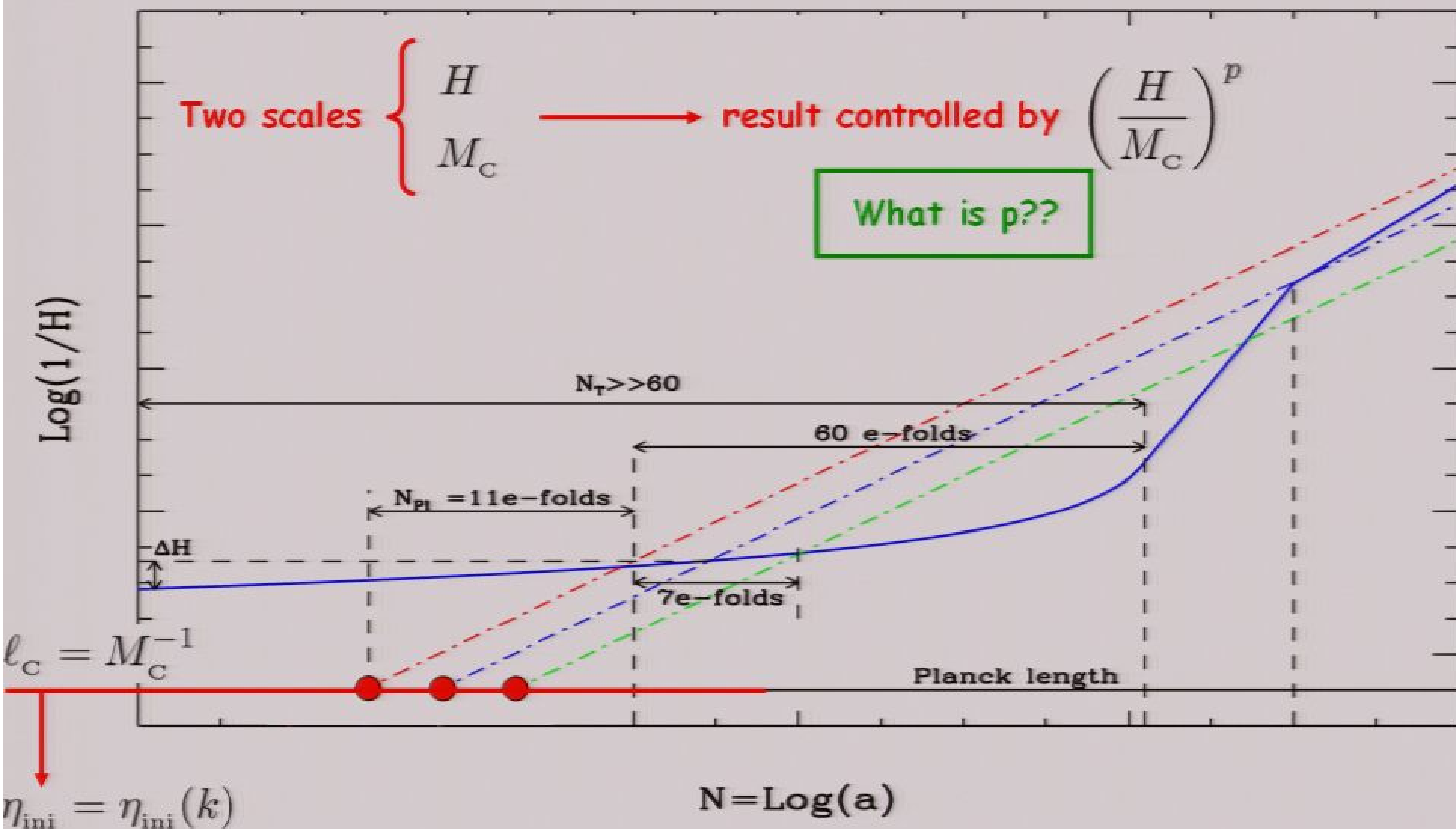
If the WKB approximation is violated in the trans-Planckian region, then one gets corrections to standard inflationary predictions. Typically super-imposed oscillations appear $\left|e^{-ik\eta} + e^{ik\eta}\right| \sim \cos(k\eta)$

Which modifications?



$\eta_{ini} = \mathrm{cte} = -\infty$

Which modifications?



The scales "pop up" at the preferred scale: "scale-dependent" initial conditions

Criticisms

- The approach is only phenomenological since physics beyond the new scale is not known. Th or at present time equation?
- The initial state
This is true for relation, it is true that but not the Lan the Hubble parameter at instance). ecause particle
- There is a back goes on now. Any tpl the energy dens severely limited. But this ed anyway ...

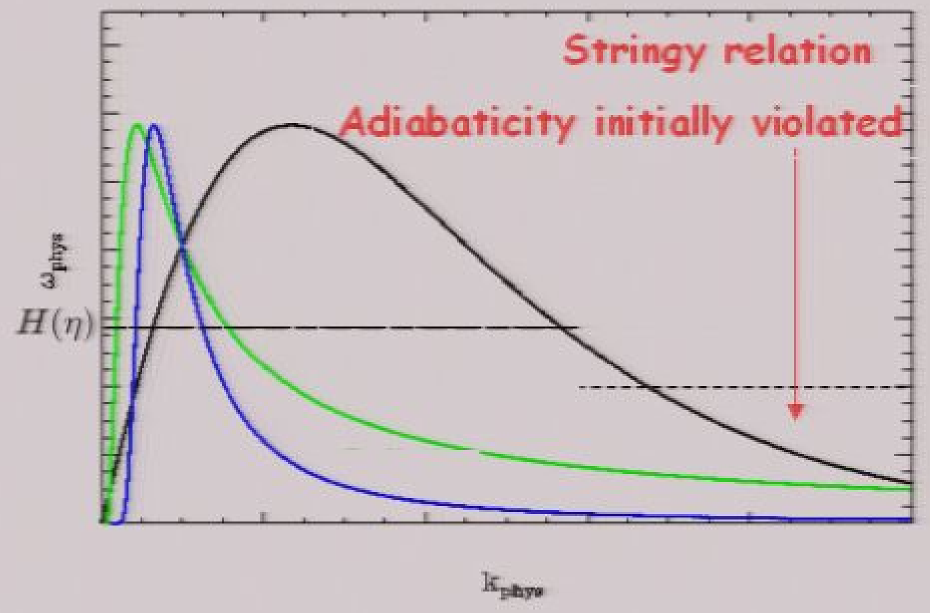
$$\langle \rho \rangle_{\rm UV} = \int_{k_{\rm phys}=H}^{k_{\rm phys}=M_{\rm C}}$$

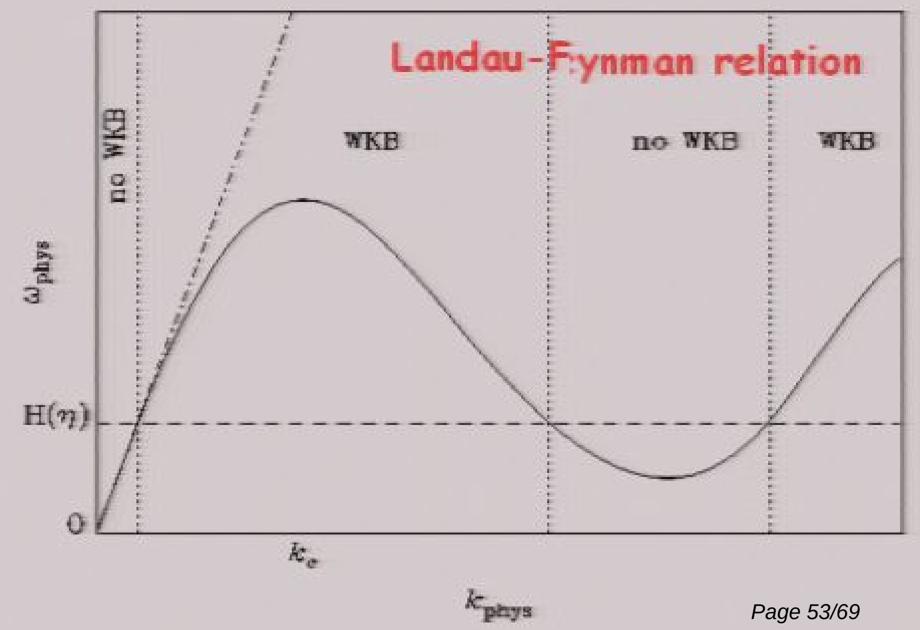
-Feynman case, hould use the Hubble ng inflation because

one gets the fol tion has now stopped

s: what is the

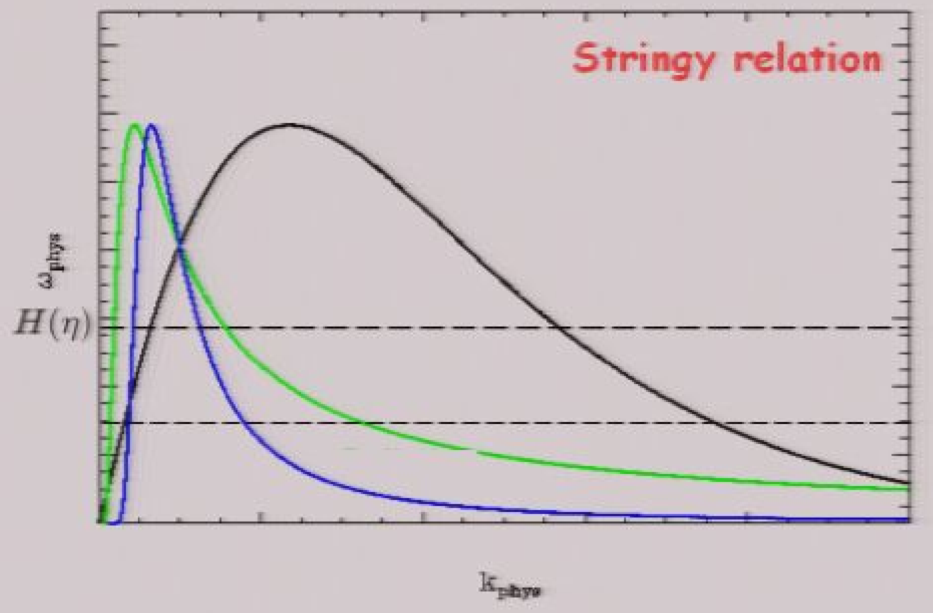
$$|\beta|^2 < M_{\rm C}^4$$

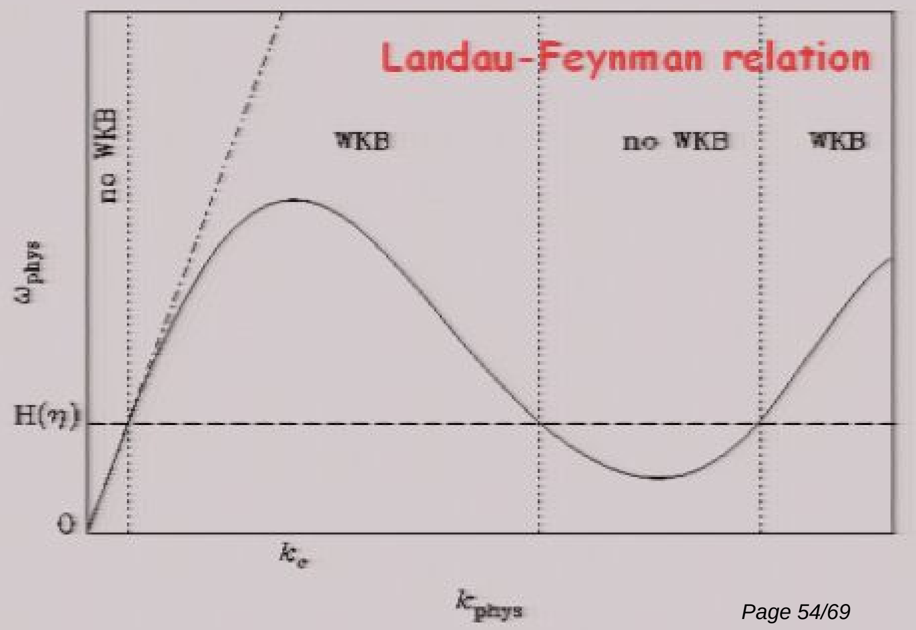




Criticisms

- A crucial question is whether we should put the Hubble parameter during inflation or at present time in the previous equation?
- For the stringy relation, it is true that we should use the Hubble parameter at present time because particle production still goes on now. Any tpl effect is thus severely limited. But this case is discarded anyway …
- For the Landau-Feynman case, crucially, one should use the Hubble parameter during inflation because particle production has now stopped
- Other loopholes: what is the equation of state of the fluctuations?? One should compute the back-reaction of cosmological perturbations with UV physics modified … very complicated!

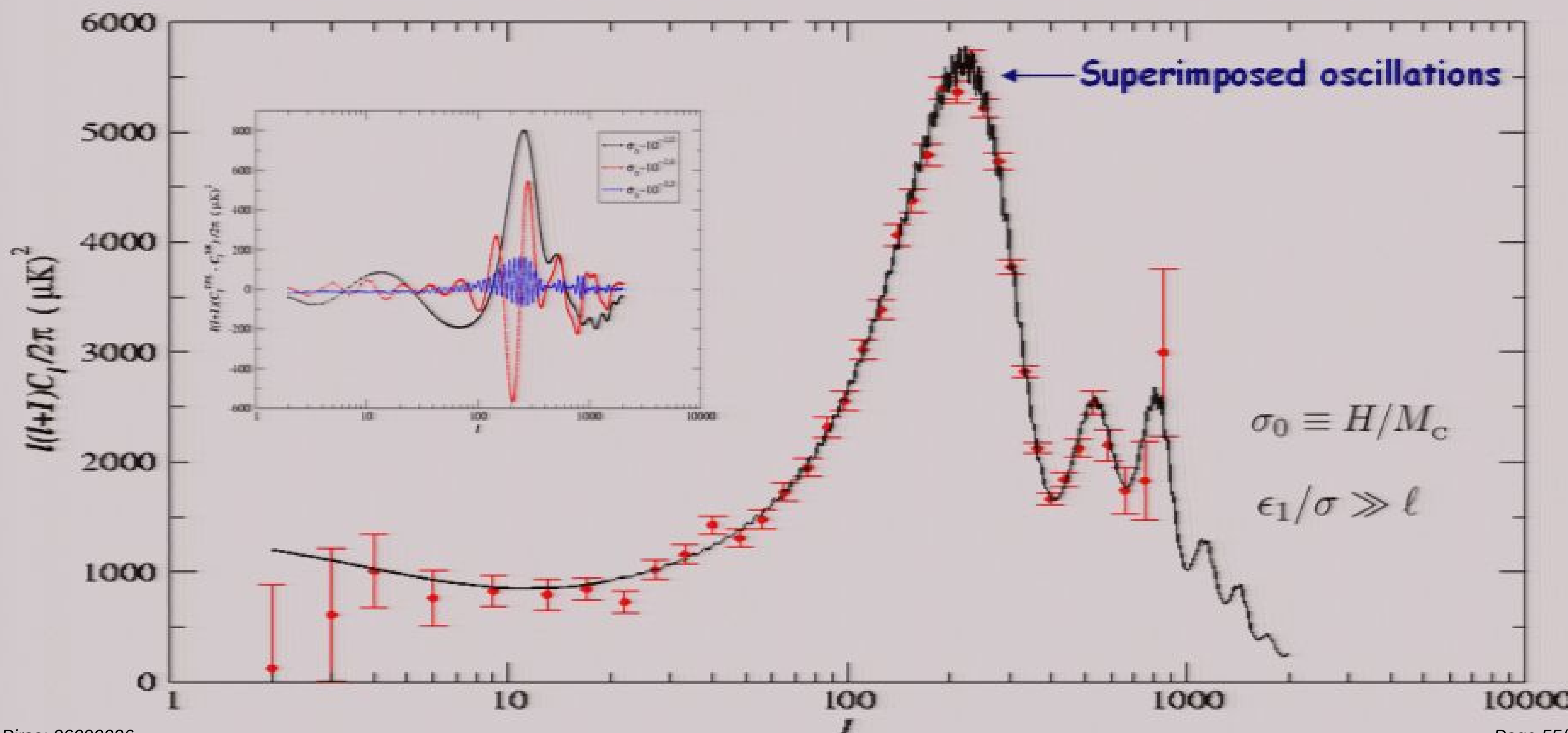




WMAP and super-imposed oscillations

The oscillations in the power spectrum are transferred to the multipole moments

$$\ell(\ell+1)C_\ell^{\rm S} \simeq \frac{2H^2}{25\epsilon_1 m_{\rm Pl}^2}(1-2\epsilon_1)\left\{1+\pi^{1/2}\frac{|x|\sigma_0\ell(\ell+1)}{(\epsilon_1/\sigma_0)^{5/2}}\cos\left[\pi\ell+\frac{2}{\sigma_0}\left(1+\epsilon_1\ln\frac{\epsilon_1/\sigma_0}{a_0 M_{\rm c} r_{\rm lss}}\right)+\varphi-\frac{\pi}{4}\right]\right\}$$



J. Martin & C. Ringeval, PRD 69 083515 (2041), astro-ph/0310382

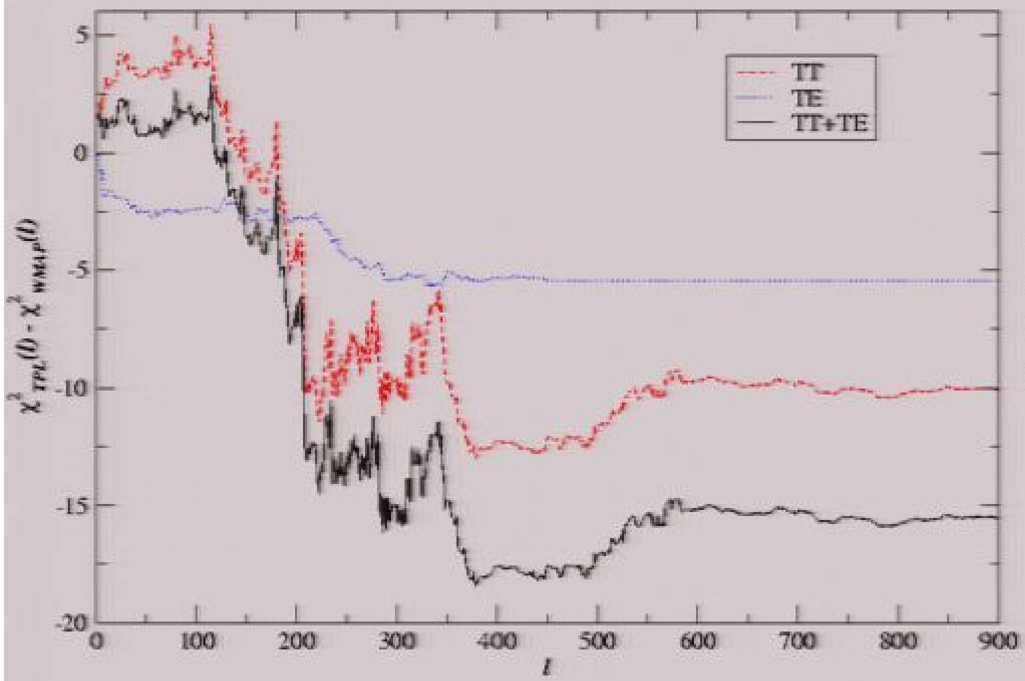
WMAP and super-imposed oscillations

The best fit model is obtained with the (superimposed) oscillations

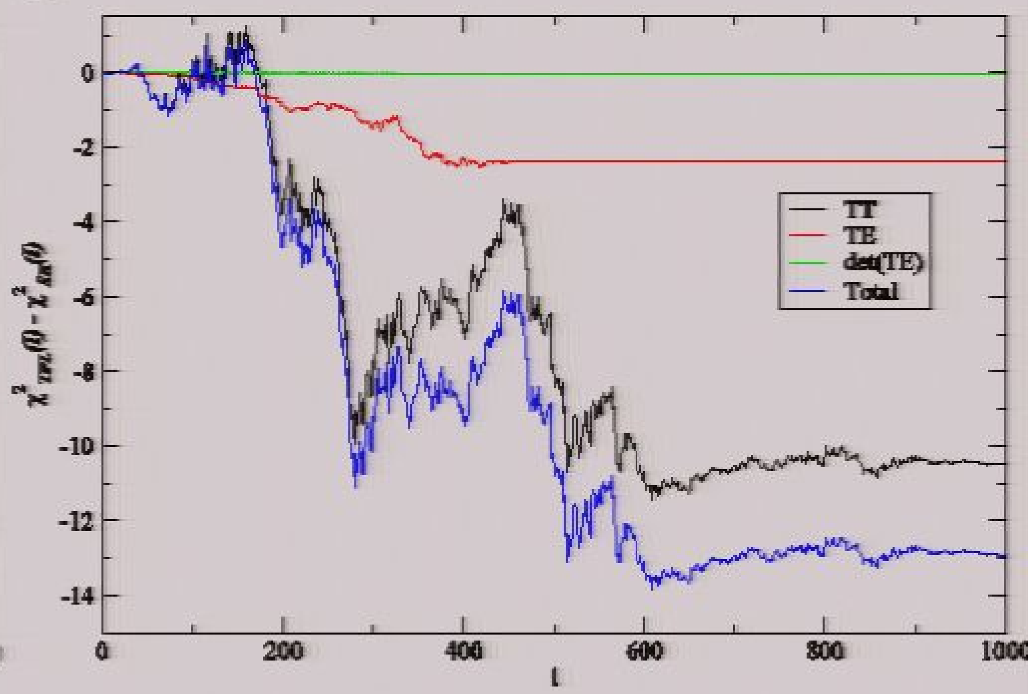
1- $\Delta\chi^2 \sim -12$ (3 extra parameters)

2- $2|x|\sigma_0 < 0.76$ (95% CL)

3- $\log(\epsilon_1/\sigma_0) \sim 2.23\,, \epsilon_1 \sim 2.1\times 10^{-3}\,, |x|\sigma_0 \sim 0.268 \Rightarrow$
$\sigma_0 \sim 1.2\times 10^{-5}\,, M_c \sim 0.3 m_{\rm Pl}$



WMAP 1.0



WMAP 3.0

J. Martin & C. Ringeval, JCAP 08, 009 (2006)
astro-ph/0605367

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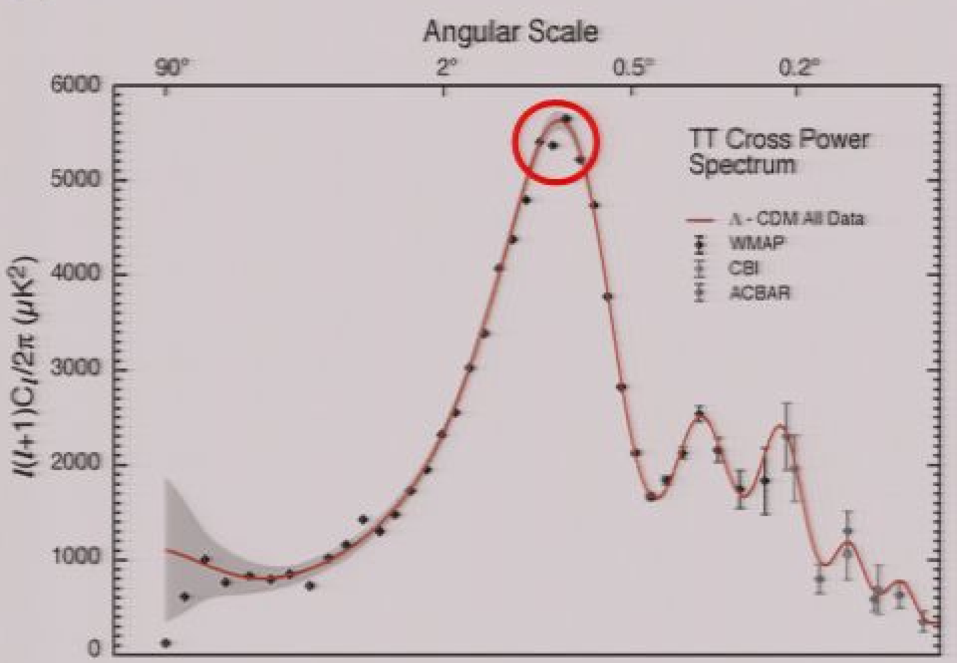

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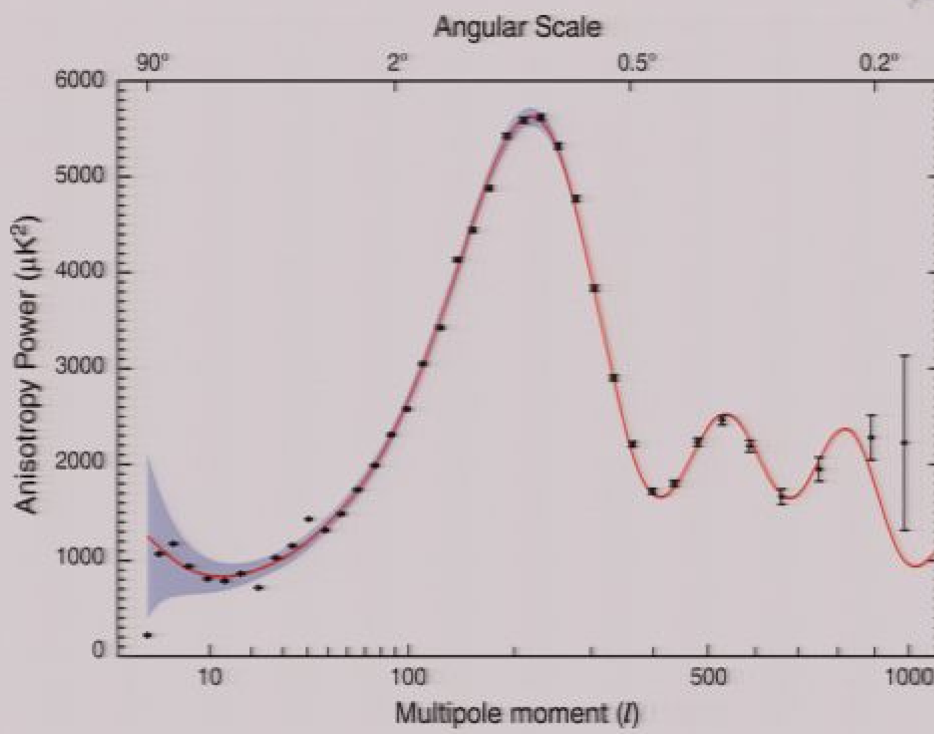


WMAP3.0

This shows that the presence of a good fit is (was) not linked to the presence of a particular outliers ...

WMAP and super-imposed oscillations

Power-spectrum of super-imposed oscillations

$$k^3 P_\zeta \simeq k^3 P_{\rm sr} \left\{ 1 - 2|x| \frac{H}{M_c} \cos\left[2\epsilon_1 \frac{M_c}{H} \ln\left(\frac{k}{k_*}\right) + \psi \right] \right\}$$

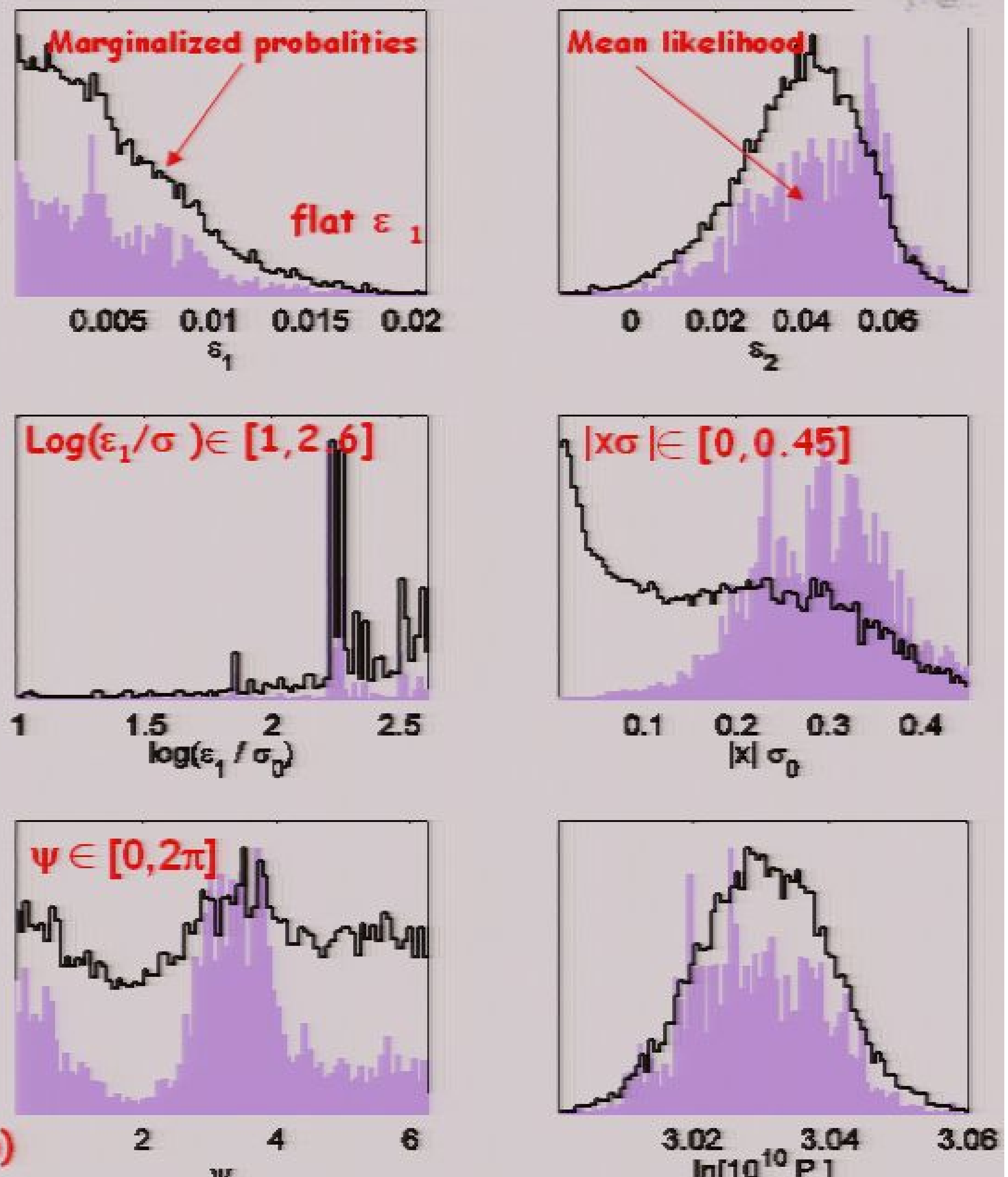
Usual SR power spectrum

$$\sigma_0 \equiv \frac{H}{M_C}$$

Logarithmic oscillations

Results

From the Baeysian point of view (ie taking into account volume effects in the parameter space), the no-oscillation solution remains favored



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WMAP and super-imposed oscillations

Testing the shape of the oscillations

$$k^3 P_\zeta = P_* \left(\frac{k}{k_*}\right)^{n_S - 1} \left(1 - A_\omega \cos\left\{\frac{\omega}{p}\left[\left(\frac{k}{k_*}\right)^p - 1\right] + \psi\right\}\right)$$

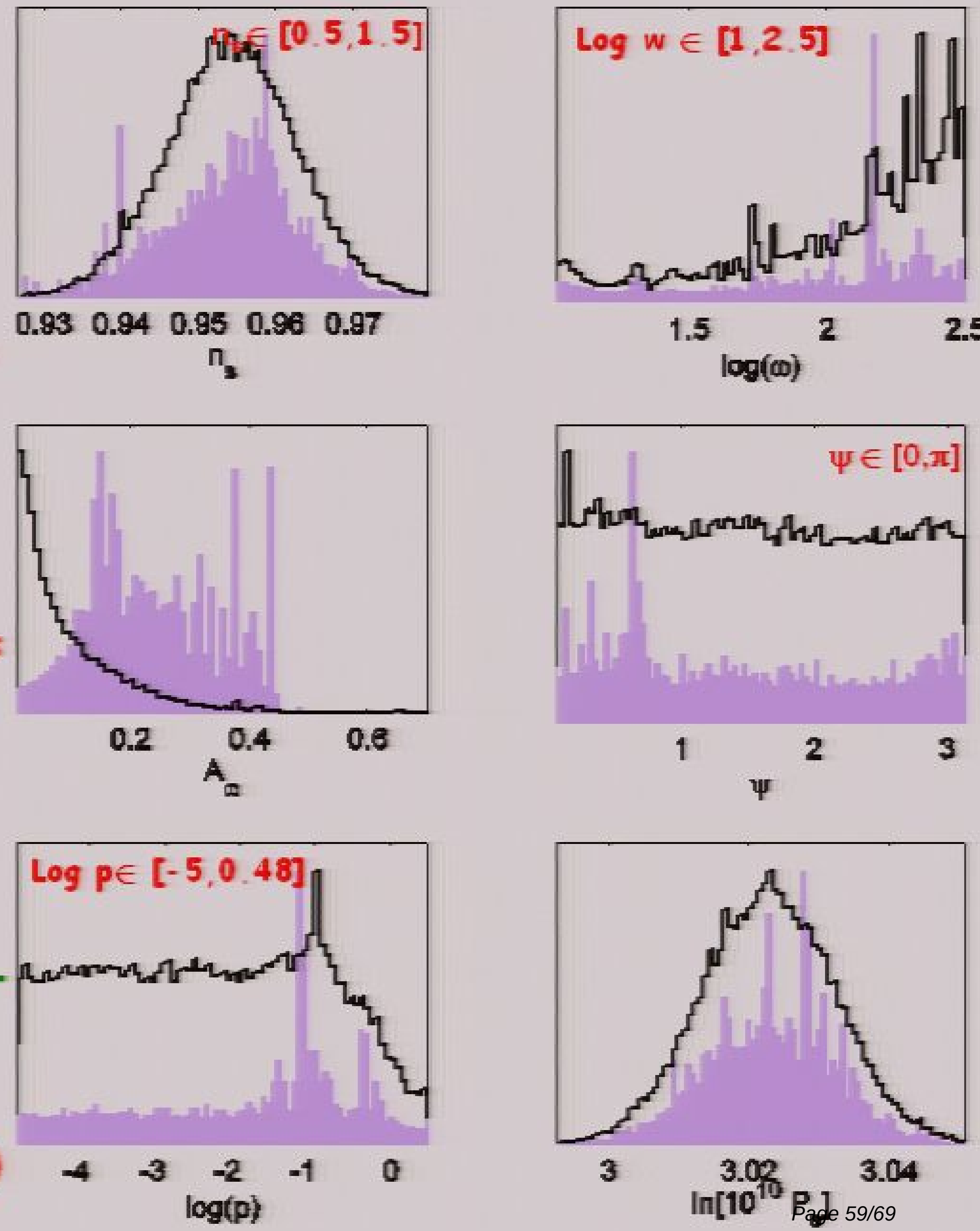
Spectral index

Amplitude of The oscillations

Phase

Amplitude of The corrections

Shape of the oscillations



Results

1- $\Delta\chi^2 \sim -10$ (4 extra parameters)

2- The logarithmic shape is favored $p < 0.68$ (95%CL)

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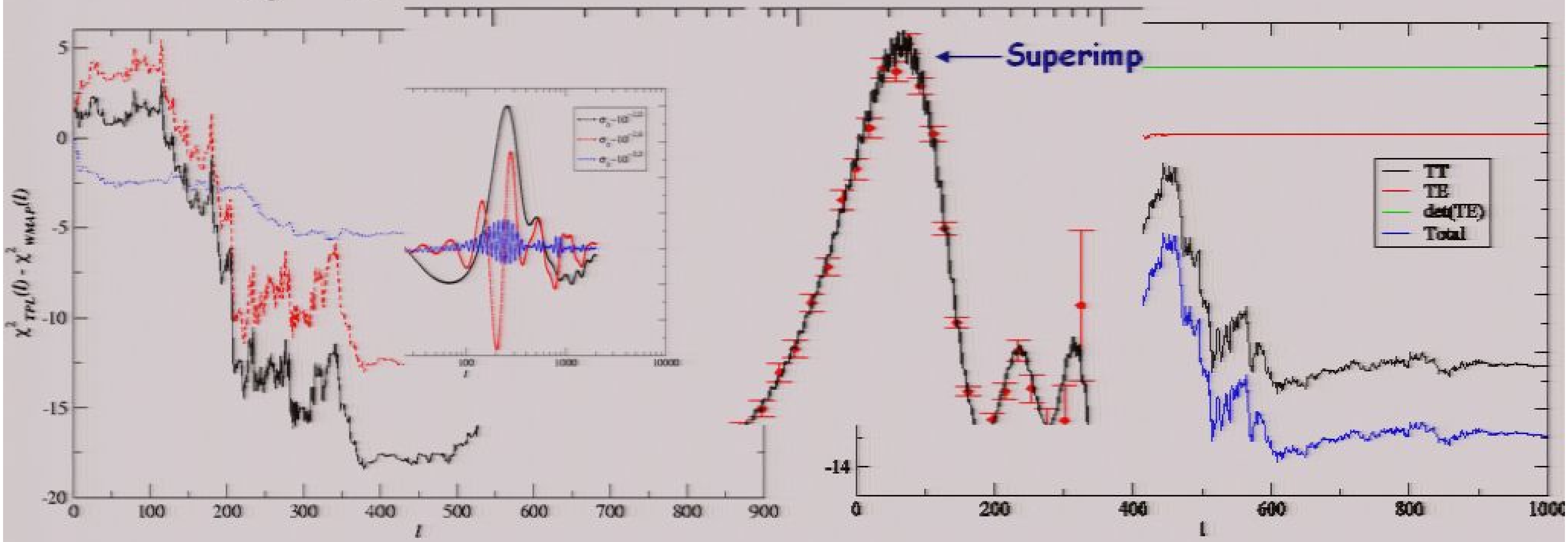
The best fit model is obtained with the (superimposed) oscillations

1- $\Delta\chi^2 \sim -12$ (3 extra parameters)

2- $2|x|\sigma_0 < 0.76$ (95% CL)

3- $\log(\epsilon_1/\sigma_0$ ... $68 \Rightarrow$

$\sigma_0 \sim 1.2$



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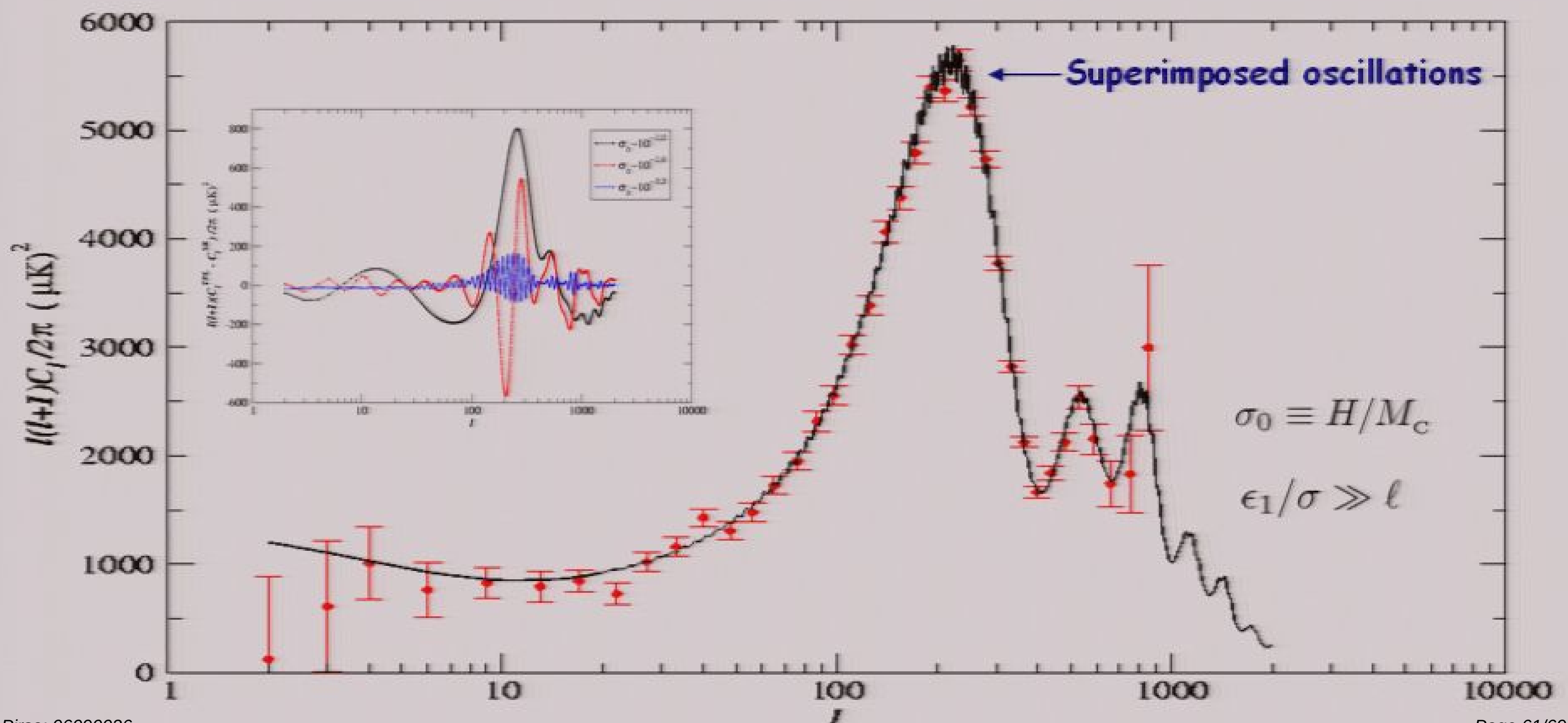

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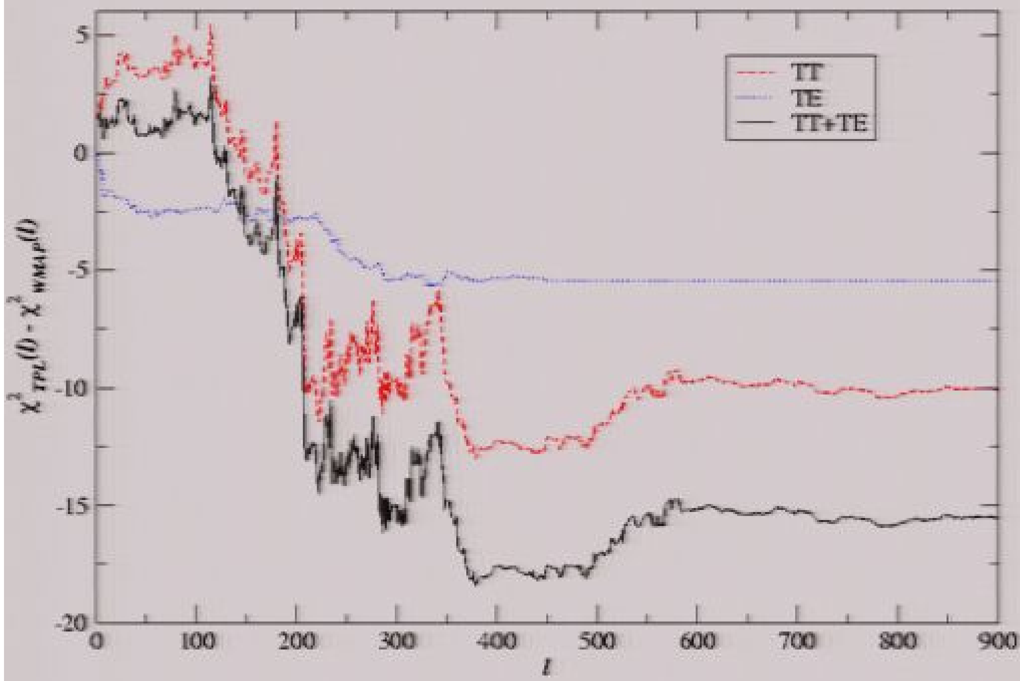
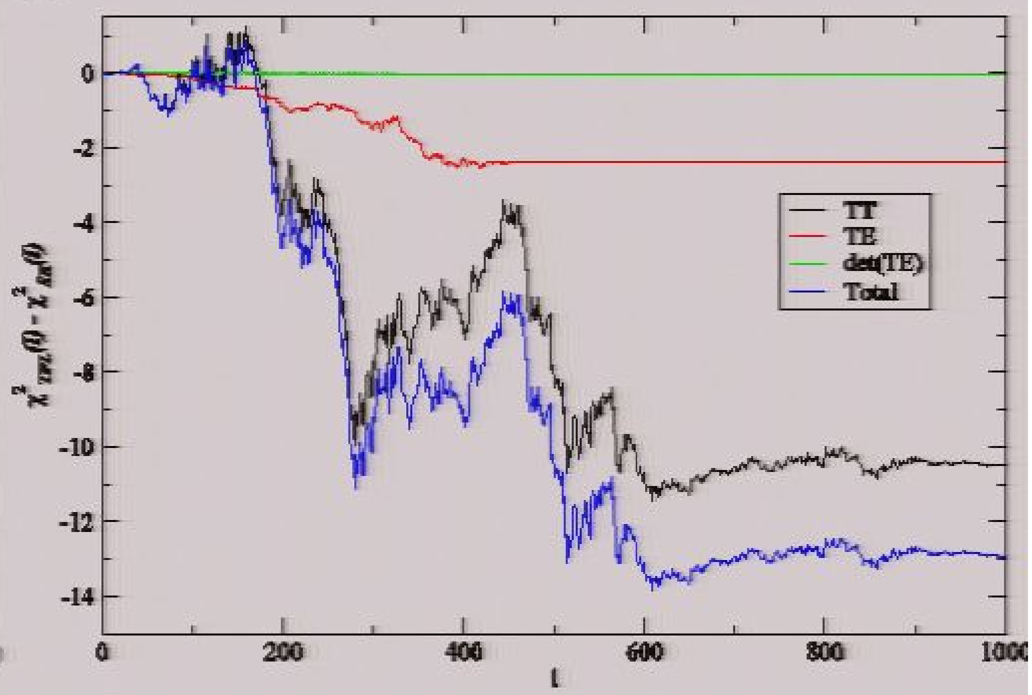


WMAP 1.0



WMAP 3.0

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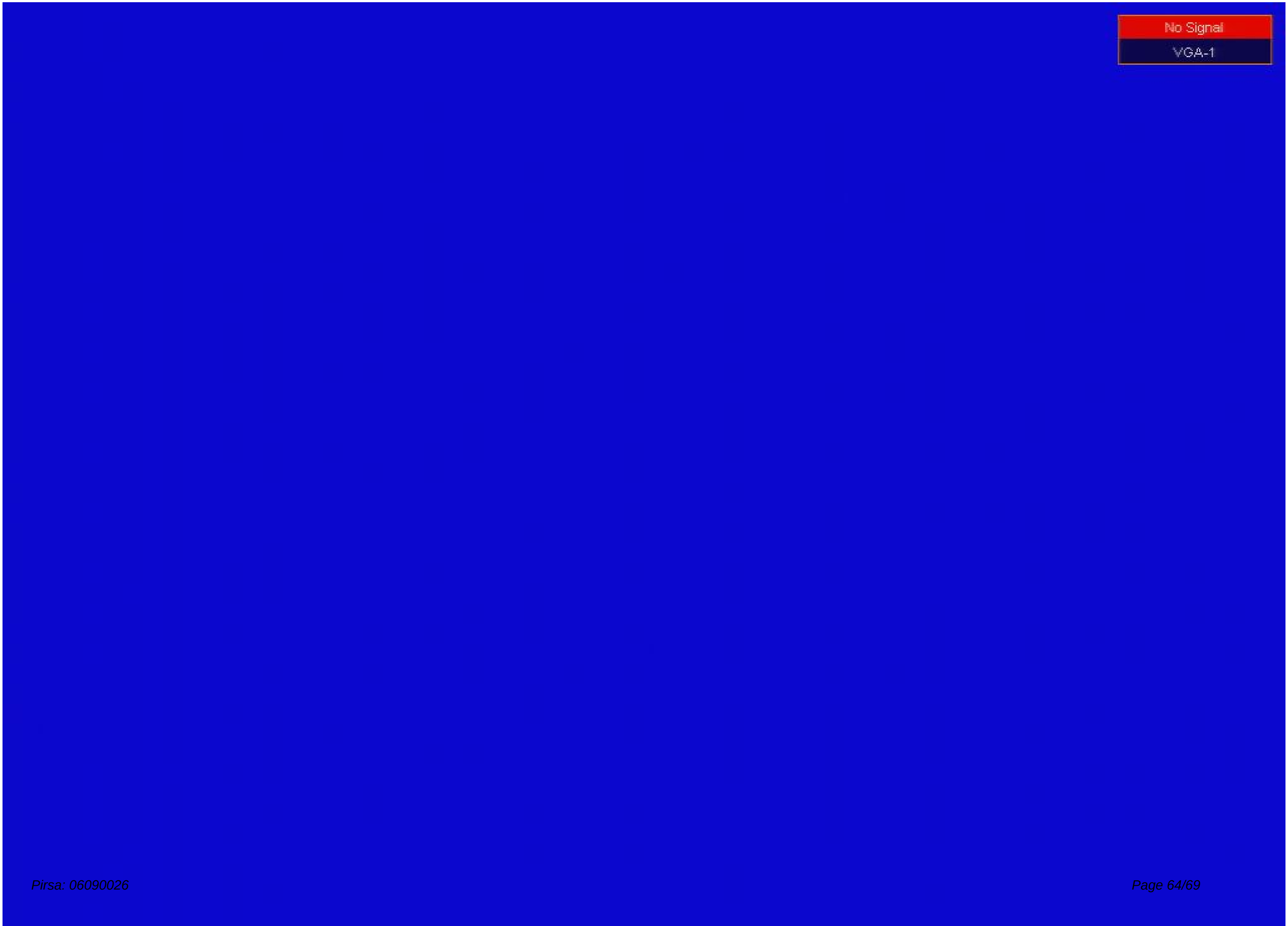
astro-ph/0605367

No Signal

VGA-1


No Signal
VGA-1

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VGA-1

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