Title: Questions about high - k phsyics in expanding spacetimes

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Abstract:

Questions about the high-k vacuum in expanding spacetimes

Jens Niemeyer
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Motivation and setting

Setting:

Semiclassical QFT, no QG

Motivation:

What constrains the vacuum at high (i.e., trans-Planckian = TP) wavenumbers? (consistency? principles? dynamics?)

Relevance:

1. Signatures in the CMBR ($\rightarrow$ Jerome's talk)
2. Cosmological particle creation and UHECRs (Starobinsky & Tkachev, Goldstein & Lowe)

$\rightarrow$ small deviations from the standard (Bunch-Davies) vacuum might be observable

1. Do TP modes exist or are they created?

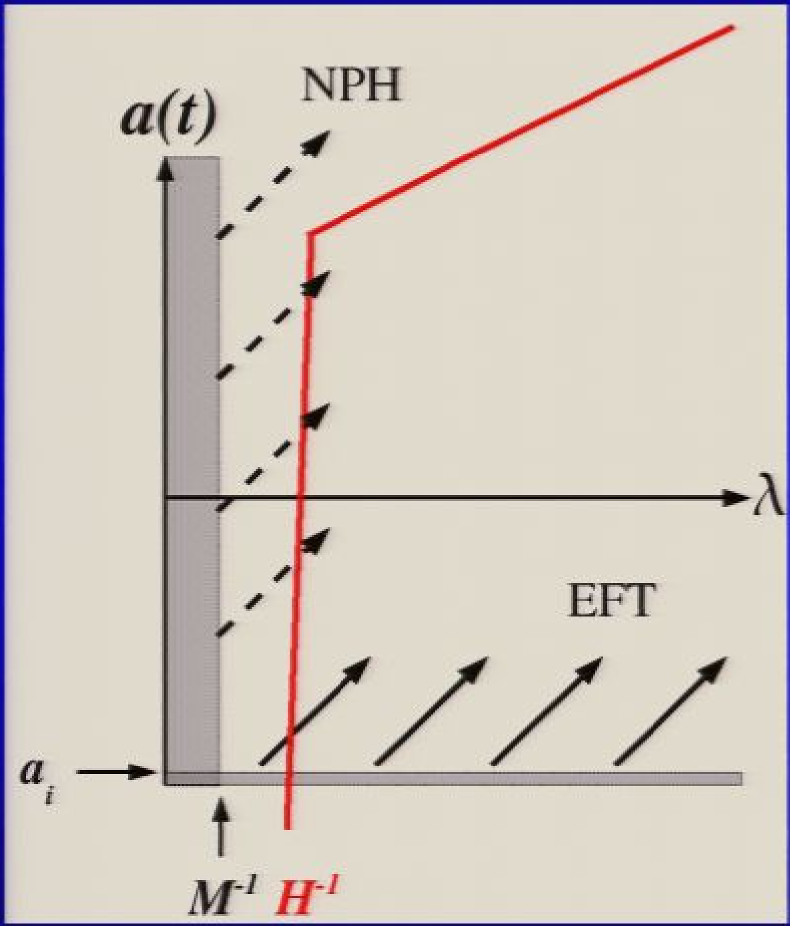
- "existence" $\leftrightarrow$ initial state formulation of EFT (e.g., Schalm, Shiu, van der Schaar, Greene)

  alternative: creation at Planck-scale crossing (Jacobson; Danielsson; JN, Parentani, Campo; Easther, Kinney, Shiu, Greene; ...)

- analogy: super-horizon modes in inflationless cosmology

  "existence": impose perturbations on homogeneous slice at initial time, including super-horizon modes

  "creation": impose growing mode at horizon entry



from Easther et al., astro-ph/0505426

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- Is this a useful question at all?

- Does "existence" imply observability?
  - –direct probes of $l < l_p \rightarrow$ bh formation $\rightarrow$ "locality bound"
  - –IR effects in expanding/curved spacetimes do not constrain existence vs. creation (?)

2. If they exist, which criteria (or "principles") determine their state?

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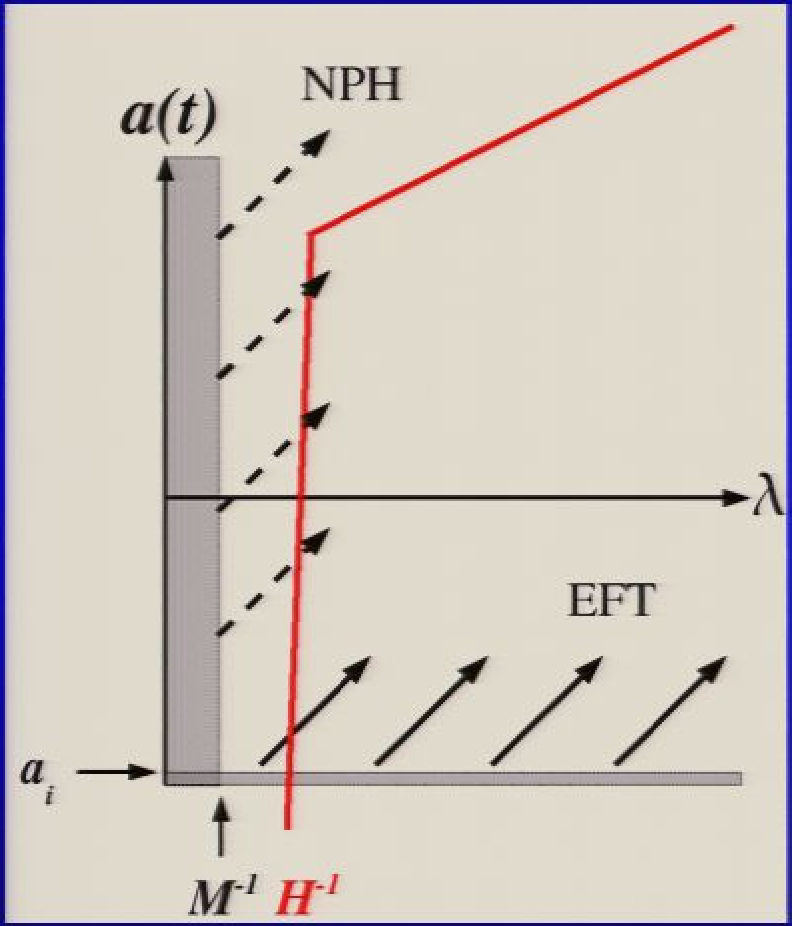

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   - e.g., dS invariance $\rightarrow$ "α-vacua" including Bunch-Davies (BD) vacuum
   - imprint of initial symmetry (cf. quantum cosmology, landscape,...)
   - why is large-scale symmetry inherited by infinitesimally small scales?

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2. decoupling / equivalence principle
   - conserved, renormalizable $\langle T_{\mu\nu} \rangle \leftrightarrow$ O(4)-adiabatic vacuum ("locally flat physics") (e.g., Anderson et al., hep-th/0504134)
   - doesn't require exact symmetry on TP scales
   - dS: 1) + 2) → BD vacuum

2. If they exist, which criteria (or "principles") determine their state?

3. regularity
   - quantum cosmology: prediction of BD vacuum for broad range of boundary conditions (Halliwell & Hawking; Wada; Vachaspati & Vilenkin; Vachaspati, Laflamme (?)
   - equivalent to 2) ?

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4. "naturalness"
   - "the deviation from BD must be small, so without fine-tuning the state must be exactly BD"
   - NB: ...and $\Lambda = 0$...

3. Does quantum cosmology really predict a BD vacuum on TP scales?

The argument in a nutshell (cf. Vachaspati ´89, PLB 217, 228):

–Wheeler-deWitt eq. for the wave function of the universe:

$$H\Psi = 0$$

–consider minimally coupled scalar field in closed dS background+perturbations

–expand field in spherical harmonics: $\phi \sim \sum_{n,l,m} f_{nlm}(t) Q^n_{lm}(x)$

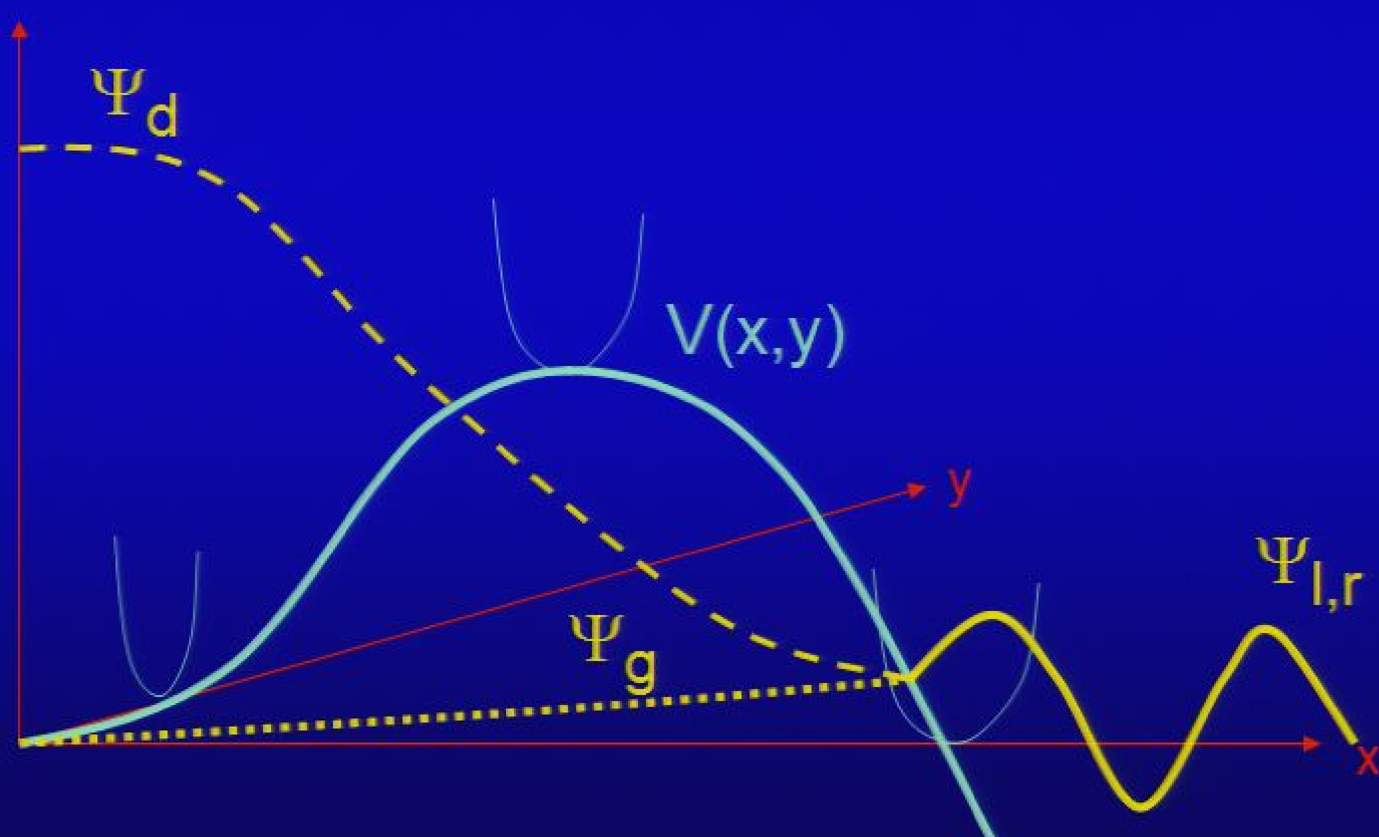
–WdW eq. (x = ln a): $\left[\frac{\partial^2}{\partial x^2} - \sum_n \frac{\partial^2}{\partial f_n^2} - V(x, f_n)\right]\Psi = 0$

where

$$V(x, f_n) = e^{4x}(1 - \Lambda e^{2x}) - e^{4x}\sum_n (n^2 - 1) f_n^2$$
$$\equiv V_0(x) + \sum_n V_n(x) f_n^2$$

–separate the wavefunction: $\Psi = \Pi\Psi(x, f_n)$

$\rightarrow$ problem equivalent to n independent 2-d QM problems for each mode $f_n = y$



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$$(\partial_x S_0)^2 = V_0 \qquad (1)$$

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–introduce (conformal) time via $\frac{dS_0}{da} = i\frac{da}{d\tau}$

(1) yields (with $\tau \to i\tau$ for $a < \Lambda^{-1/2}$): $a = \frac{1}{\Lambda^{1/2}\cos\tau}$

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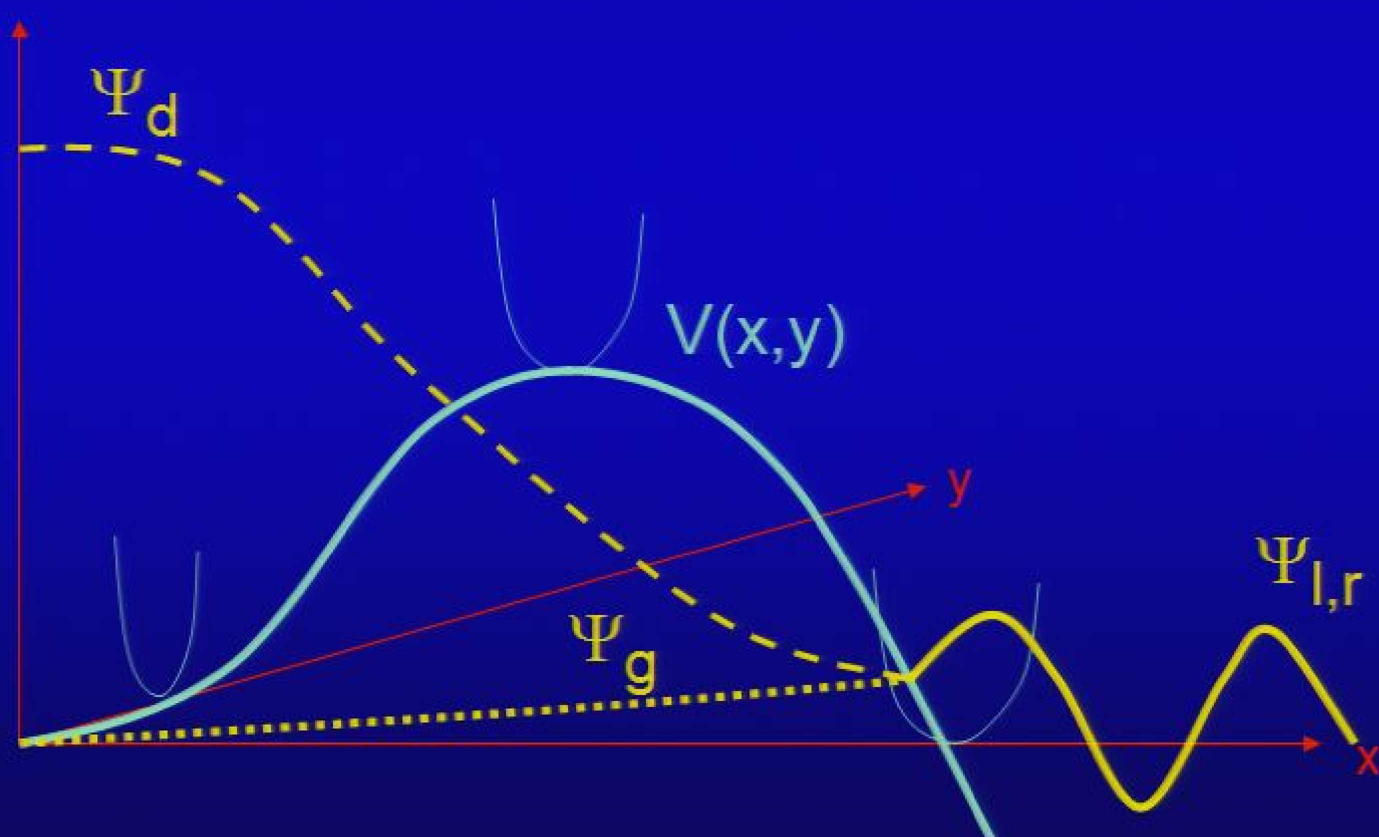

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- asymptotic solution of (3) has the form $A\,e^{-n\tau} + B\,e^{n\tau}$, so that:

$$S_n = \frac{1}{\Lambda \cosh^2\tau}\left(n - \frac{2An}{A + B\exp(-2n\tau)} - \tanh\tau\right)$$

- this is only positive definite for $\tau \to \infty$ ($a \to 0$) if A=0 $\to$ BD vacuum

Assumptions made:

1. minisuperspace reduction
2. f_n perturbatively small
3. semiclassical approximation
4. all modes (including TP) are oscillatory

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Mode creation by quantum tunneling

Idea (not even half-baked):

Shift the turning point to n- (or k-)dependent time (or scale factor), so that Ψ_n becomes oscillatory at the Planck-scale crossing time of n.

Realization:

"Manual" implementation by introducing n-dependent term into V_0,

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Example:

$$\mathcal{F}_n(a) = \frac{n^2 l_p^2}{a^2}$$

Questions:

1. Does any of this make any sense?
2. Can this effect be produced by "reverse engineering" the formalism, i.e. relaxing the assumptions of semiclassical minisuperspace quantum cosmology?
3. Does regularity of $|\Psi|$ still predict the state? If yes, what is it?

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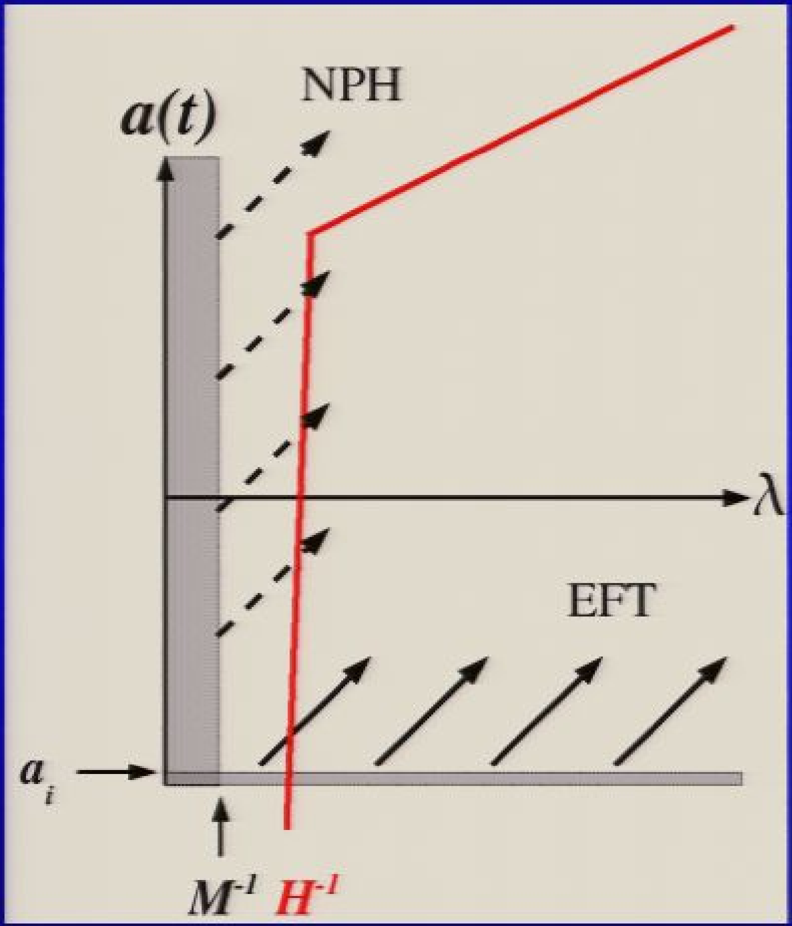

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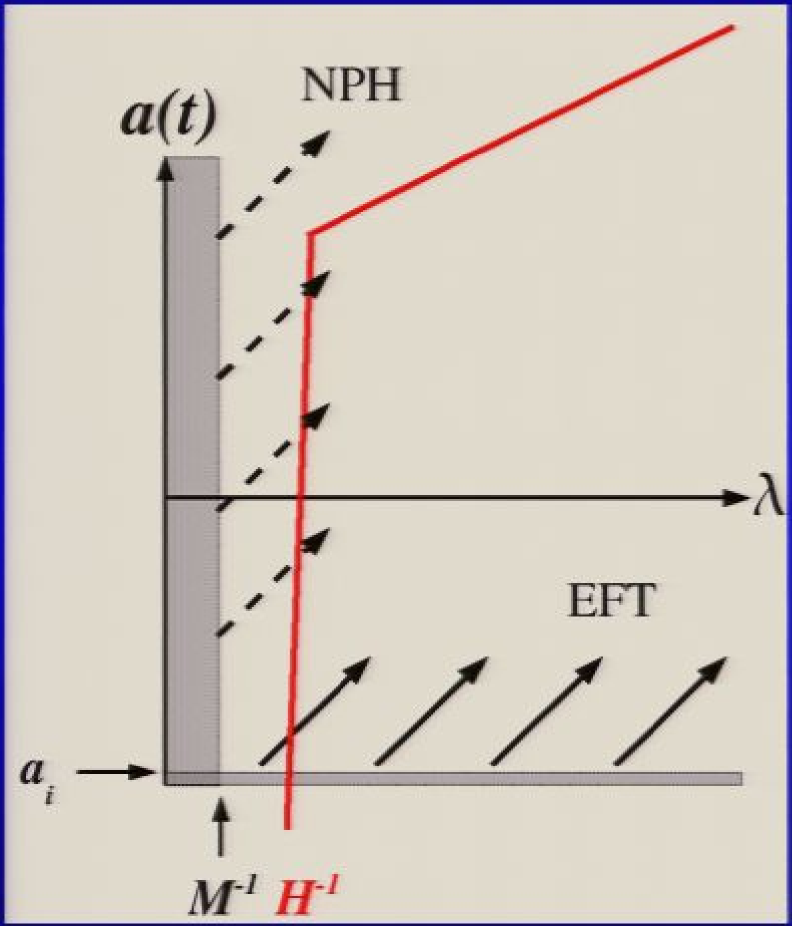

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