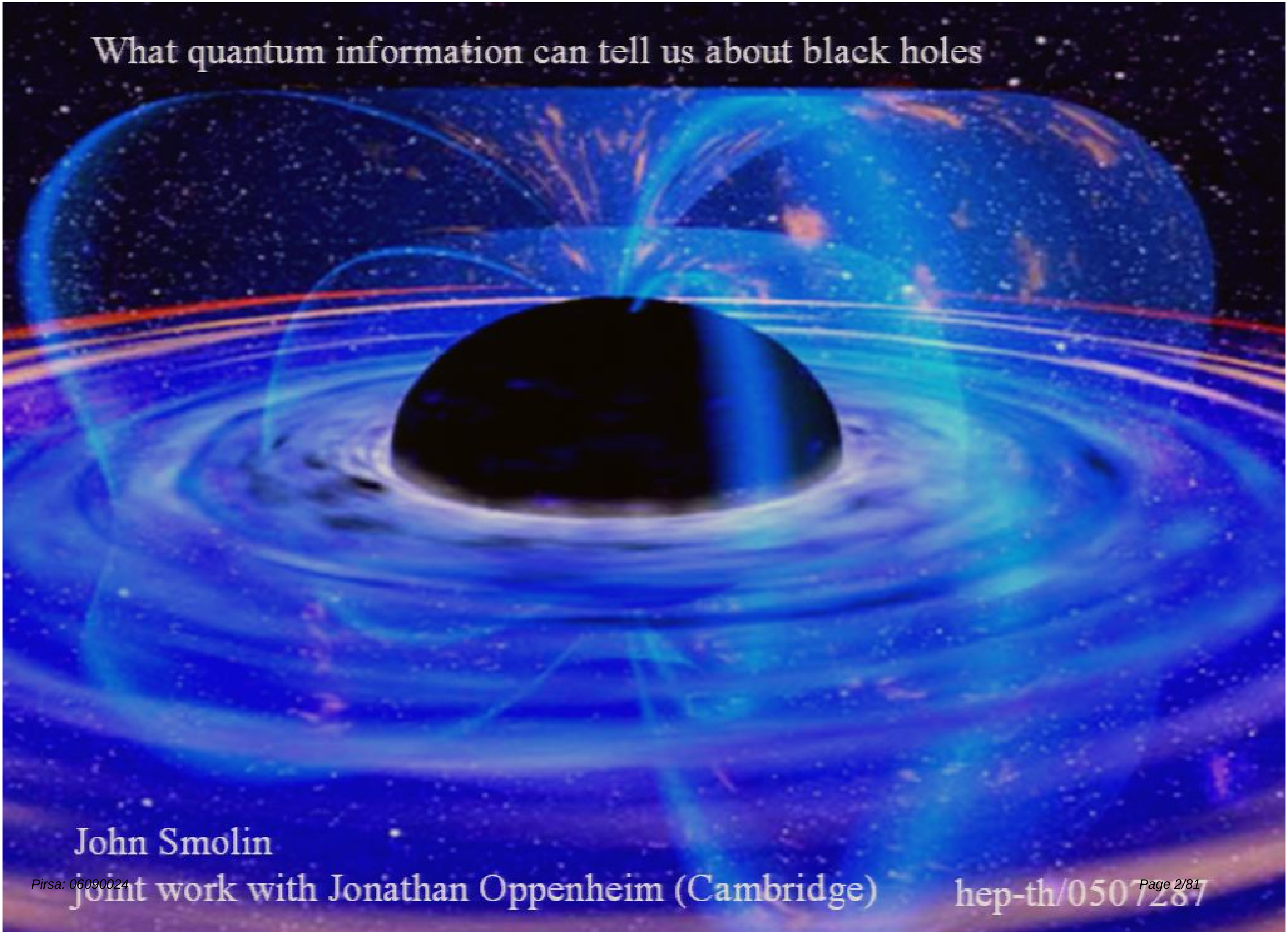
Title: Black hole evaporation and information

Date: Sep 05, 2006 06:00 PM

URL: http://pirsa.org/06090024

Abstract:

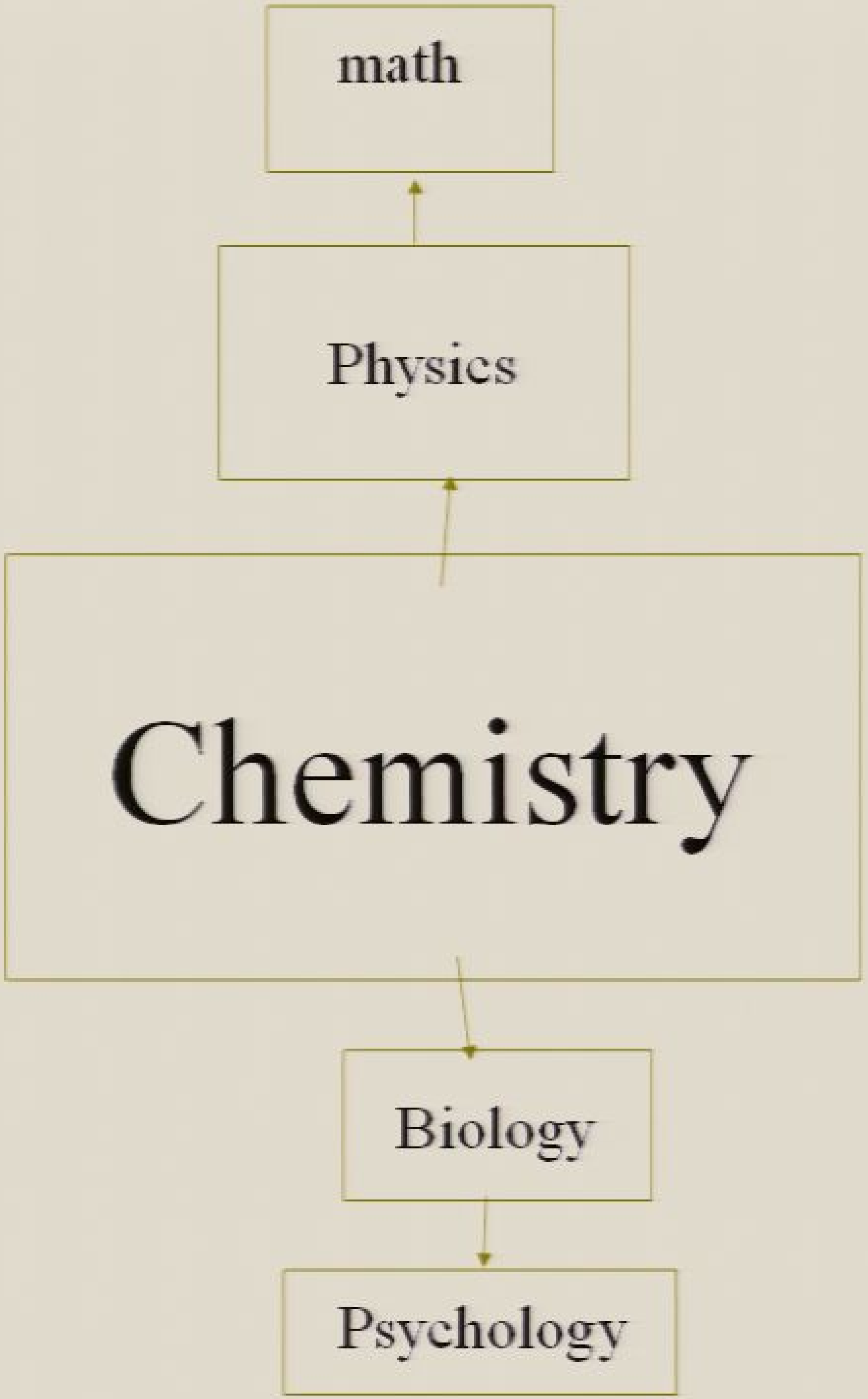


What quantum information can tell us about black holes

John Smolin

joint work with Jonathan Oppenheim (Cambridge)

hep-th/0507287



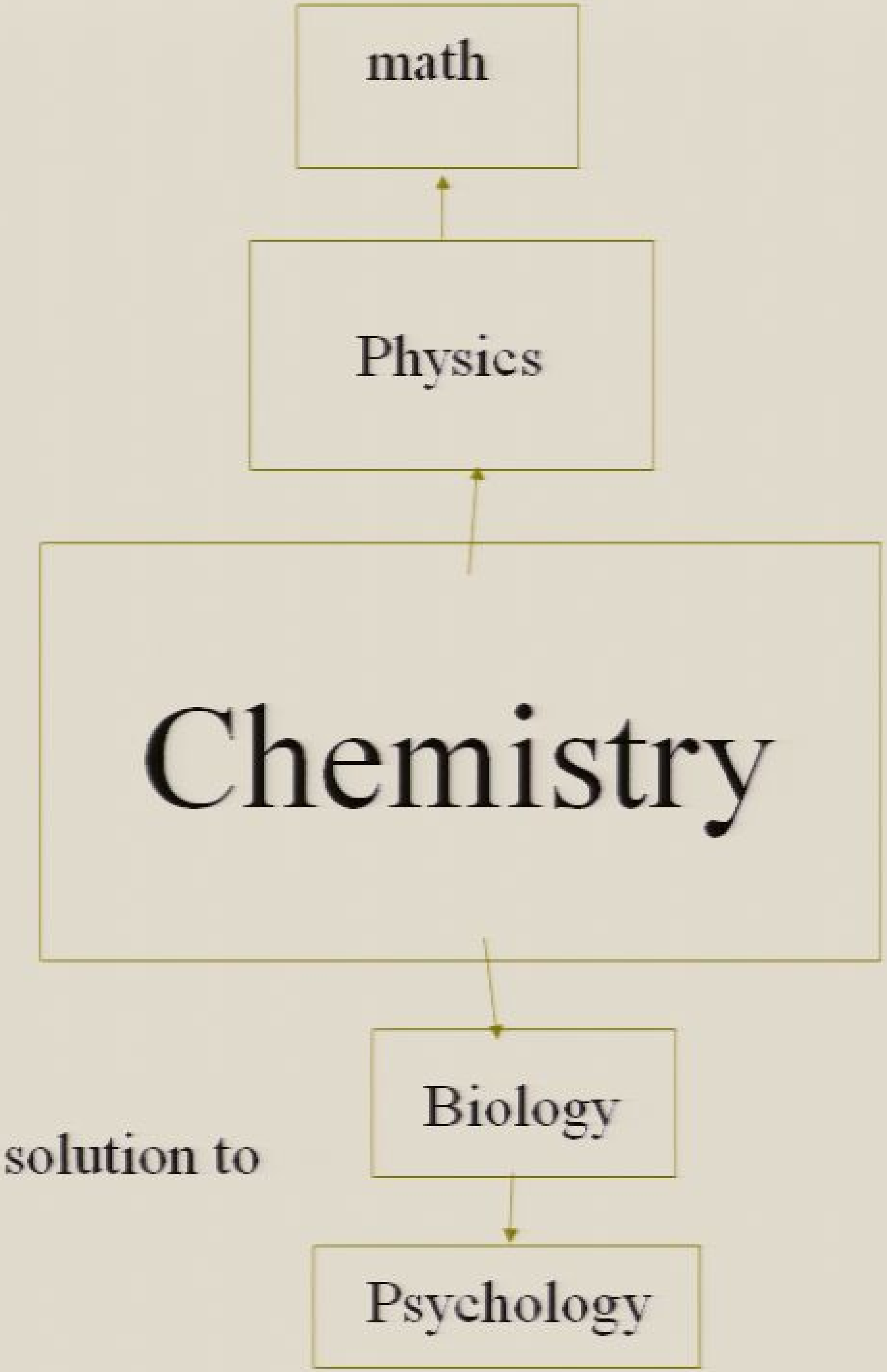
math

Physics

Chemistry

Biology

Psychology



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Bennett/Landauer
information-theoretic solution to
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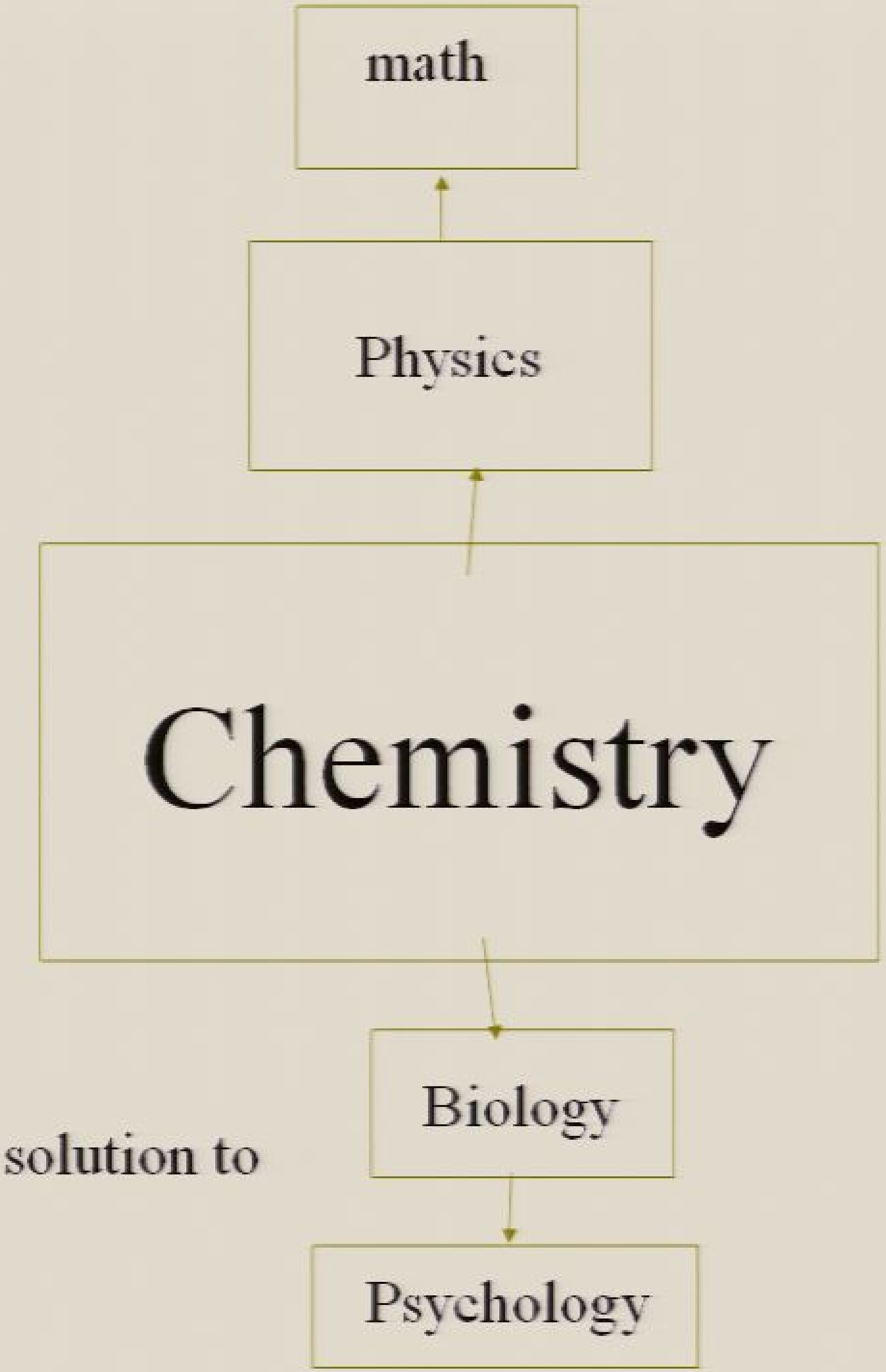

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Black hole information loss paradox is **the** reason quantum mechanics and general relativity are not unified.

1: New result in quantum information theory: Locking

2: What is the information loss paradox?

3: How locking helps solve the paradox

4: Some problems with the approach

Black hole information loss paradox is **the** reason quantum mechanics and general relativity are not unified.

1: New result in quantum information theory: Locking

Eating our vegetables

2: What is the information loss paradox?

3: How locking helps solve the paradox

Hopefully it's almost obvious by now

4: Some problems with the approach

Classical mutual information:

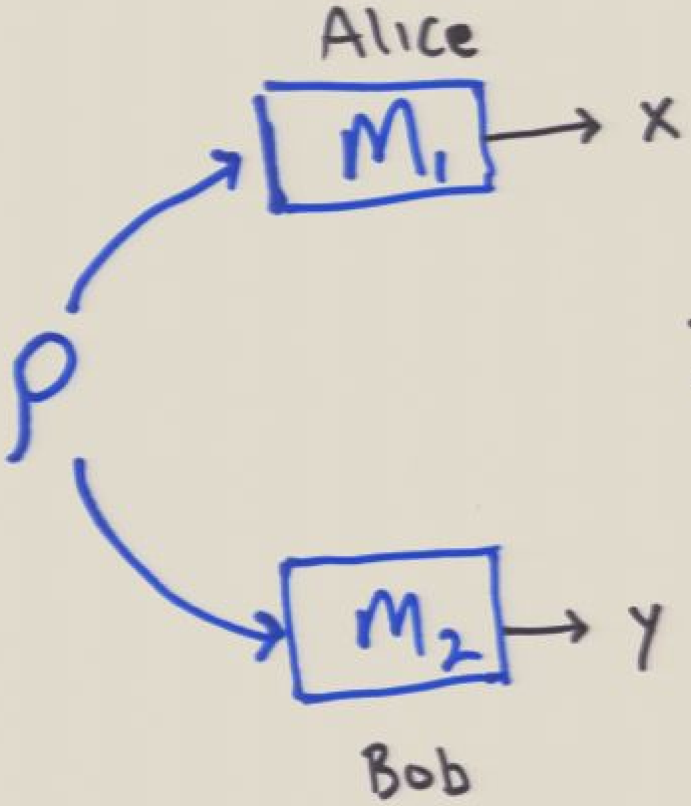
$$I(X:Y) = H(X) + H(Y) - H(XY)$$

$$I(XZ:Y) - I(X:Y) \leq I(Z:Y) \leq H(Z)$$

That is to say, the additional information about Y one gets by getting Z can be no more that the information in Z about Y.

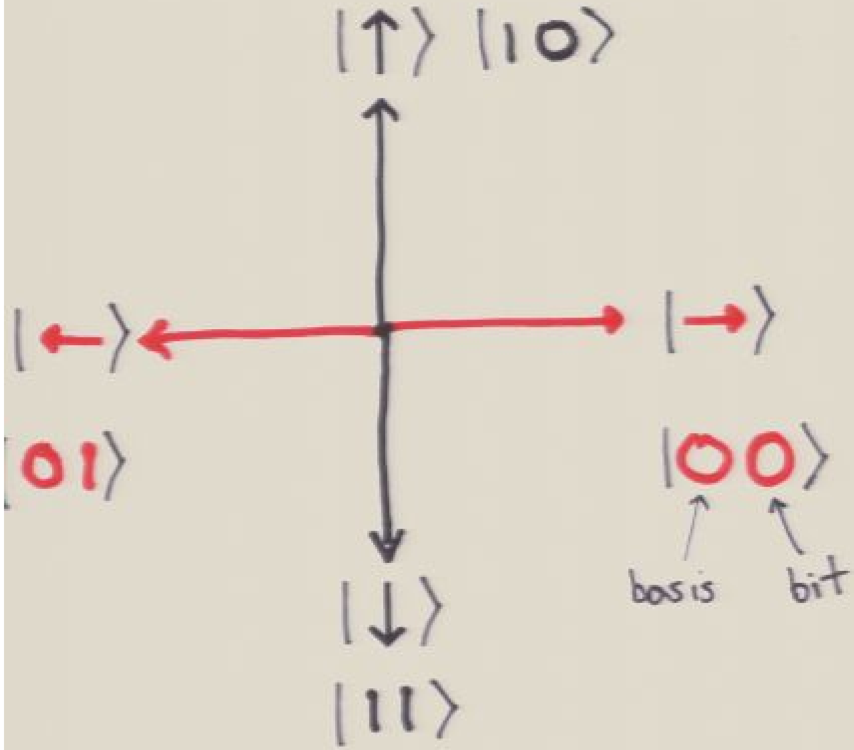
So if someone sends you a bit, your information increases by at most 1 bit---can be less if you already knew the information.

This is obvious, indeed it is why mutual information is designed this way



$$I_{acc}(\rho) \equiv \max_{M_1, M_2} I(x:y)$$

Conjugate bases from BB84 Quantum Cryptography

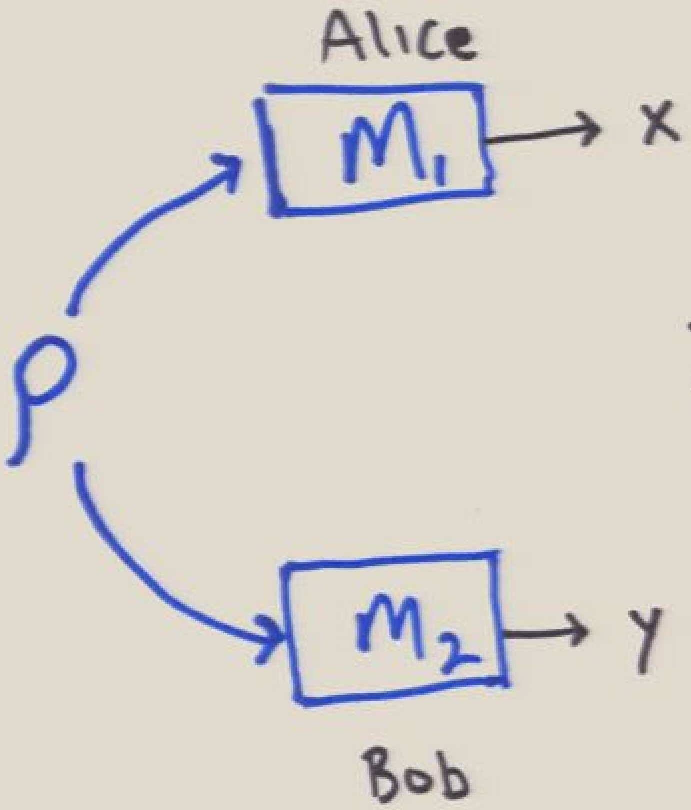


Not an orthonormal basis!

$$\rho = \frac{1}{4}\sum_{ij} |i\rangle\langle i|_A \otimes |j\rangle\langle j|_A \otimes |ij\rangle\langle ij|_B$$

Alice has 2 bits, Bob learns nothing about the basis, gets the "bit" half the time

$$I_{acc}(\rho) = 1/2 \text{ bit}$$



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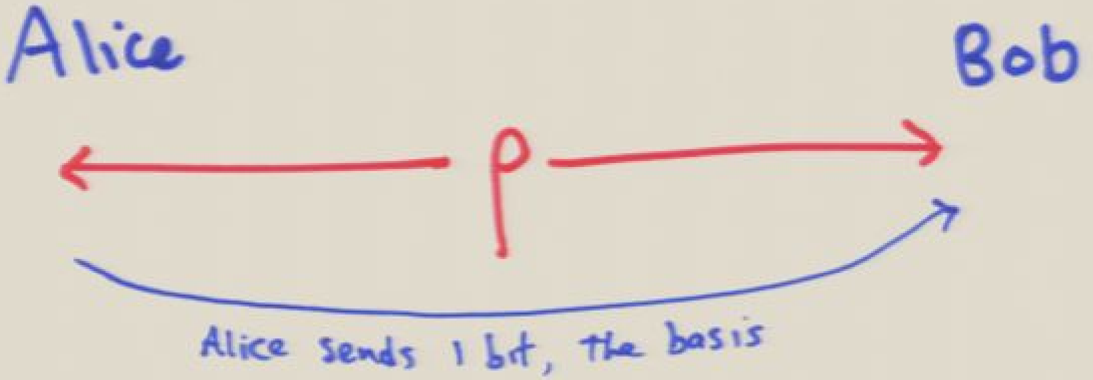

$$\rho' = \frac{1}{4}\sum_{ij} |i\rangle\langle i|_A \otimes |j\rangle\langle j|_A \otimes |ij\rangle\langle ij|_B \otimes |i\rangle\langle i|_B$$

↙ Bob knows the basis

Bob finds out the bit by measuring in appropriate basis. Now he has 2 bits

$$I_{acc}(\rho') = 2$$

$$\boxed{I_{acc}(\rho') - I_{acc}(\rho) = \frac{3}{2} > 1}$$

For large dimension D :

$$\rho = \frac{1}{2D} \sum_{i=1}^{2} \sum_{j=1}^{D} |i\rangle\langle i|_A \otimes |j\rangle\langle j|_A \otimes |ij\rangle\langle ij|_B$$

$\nearrow$ Still not a basis

$$I_{acc}(\rho) = \frac{\log D}{2}$$

$$I_{acc}(\rho') = \log D + 1$$

$$I_{acc}(\rho') - I_{acc}(\rho) = \frac{\log D}{2} + 1 \gg 1$$

Arbitrarily large violation!

DiVincenzo, M. Horodecki, Leung, Smolin and Terhal, *Phys. Rev. Lett.* **92** 067902 (2004)

$$\rho = \frac{1}{dn}\sum_{i=1}^{d}\sum_{j=1}^{n} |i\rangle\langle i|_A \otimes |j\rangle\langle j|_A \otimes U_j|i\rangle\langle i|_B U_j^\dagger$$

If the U_j are chosen randomly then, following Hayden, Leung, Shor and Winter, Commun. Math. Phys. 240 (2):371-391 (2004):

$$n = (\log d)^3 + k$$

$$I_{\text{acc}}(\rho) < \delta = C^{-k}$$

$$I_{\text{acc}}(\rho') \approx \log d$$

$$\rho = \frac{1}{dn}\sum_{i=1}^{d}\sum_{j=1}^{n} \underset{\underset{\text{message}}{\uparrow}}{|i\rangle\langle i|_A} \otimes \underset{\underset{\text{key}}{\uparrow}}{|j\rangle\langle j|_A} \otimes \underset{\underset{\text{encrypted message}}{\uparrow}}{U_j|i\rangle\langle i|_B U_j^\dagger}$$

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message key encrypted message

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$$n = (\log d)^3 + k \qquad S(n) \approx 3 \log\log d$$

$$I_{\mathrm{acc}}(\rho) < \delta = C^{-k}$$

$$I_{\mathrm{acc}}(\rho') \approx \log d$$

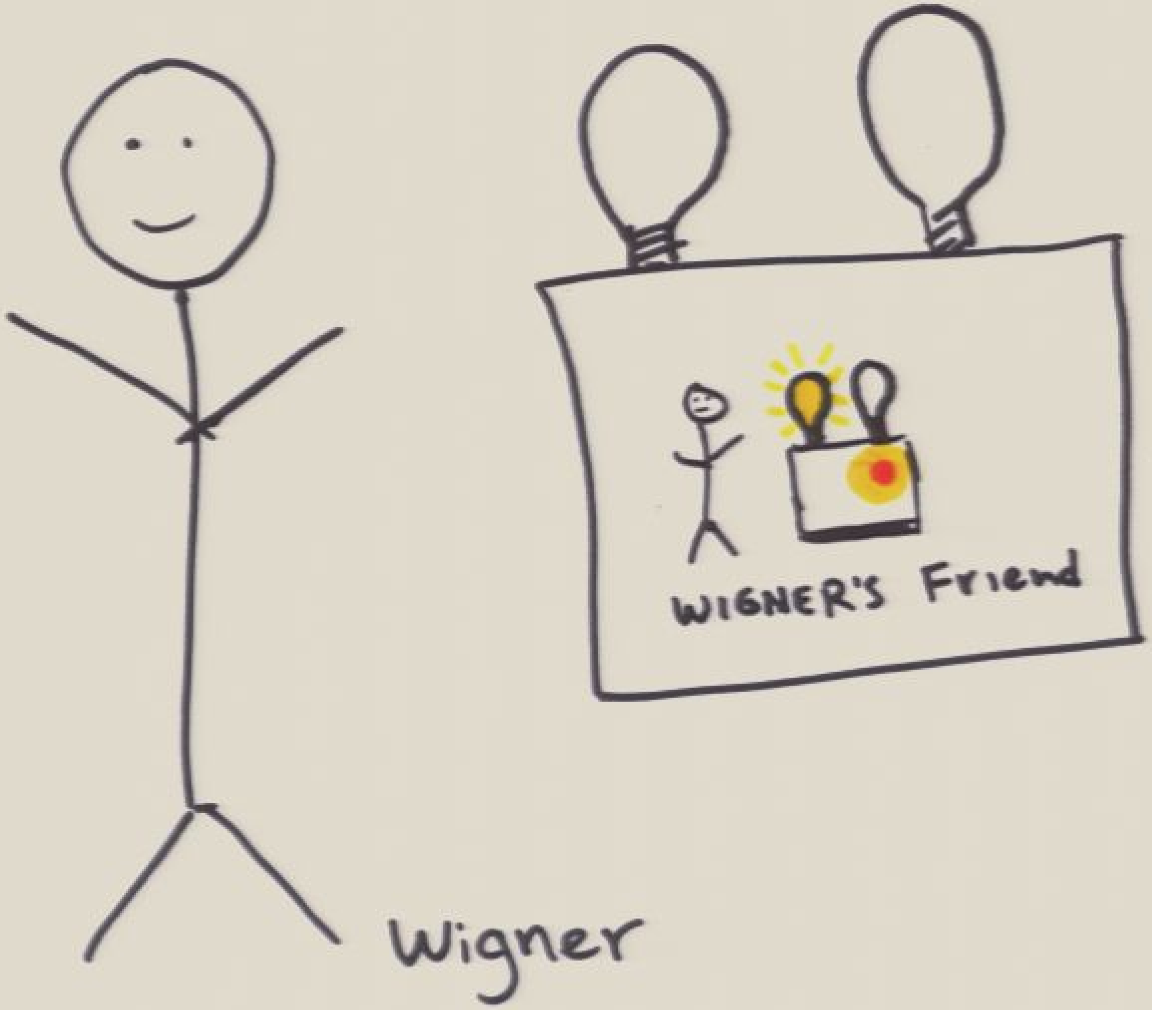
Quantum Mechanics is Unitary

An initial pure state ψ will remain forever pure.

Except for measurement?

Wigner's friend

WIGNER'S Friend


WIGNER'S Friend
Wigner

Black holes are not unitary

Irreversible—Things that fall in don't come out

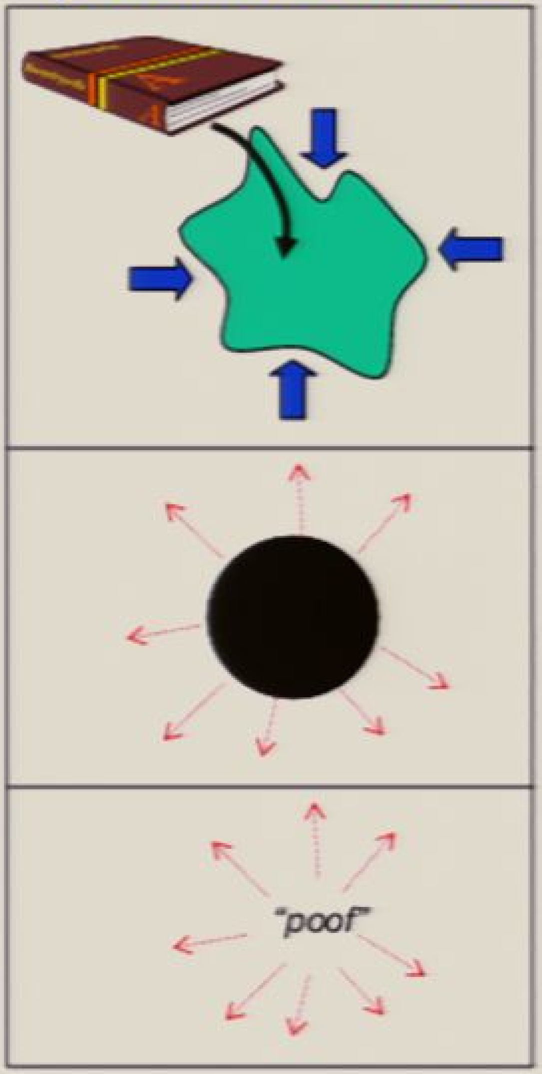
Not so bad. This is a restriction on the evolution of states, like superselection, rather than a total breakdown.

But, black holes *evaporate* (Hawking 1975)
(and the way they do is nonunitary)

$$S(\psi) = \rho_{\text{themal}}$$

Black hole evaporation

Suppose we prepare a quantum state, encoding some information, as pressureless dust on the brink of gravitational collapse.



It collapses, and begins to emit Hawking radiation. This radiation is featureless, not dependent on the information encoded in original collapsing body.

Eventually, all the mass is radiated away, and the black hole disappears. What happened to the information?

If it is gone, unitarity fails and quantum mechanics is wrong. If it comes out, GR must be wrong.

Whereas Stephen Hawking and Kip Thorne firmly believe that information swallowed by a black hole is forever hidden from the outside universe, and can never be revealed even as the black hole evaporates and completely disappears,

And whereas John Preskill firmly believes that a mechanism for the information to be released by the evaporating black hole must and will be found in the correct theory of quantum gravity,

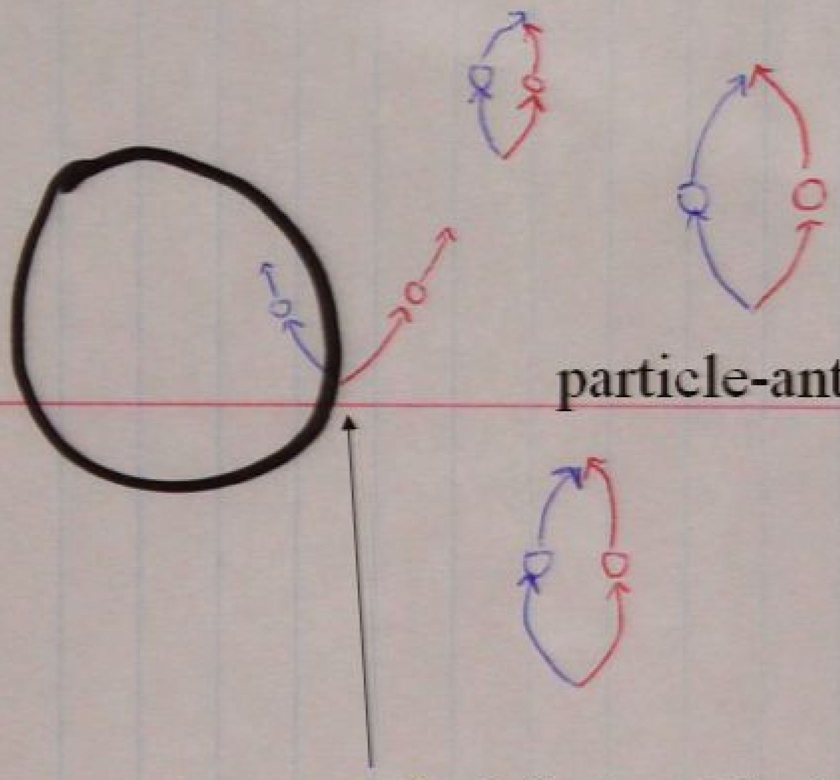
Therefore Preskill offers, and Hawking/Thorne accept, a wager that:

When an initial pure quantum state undergoes gravitational collapse to form a black hole, the final state at the end of black hole evaporation will always be a pure quantum state.

The loser(s) will reward the winner(s) with an encyclopedia of the winner's choice, from which information can be recovered at will.

Stephen W. Hawking, Kip S. Thorne, John P. Preskill
Pasadena, California, 6 February 1997

Other hot objects look thermal, but there is no information-loss paradox. Why are black holes counted as a special problem?



particle-antiparticle pairs

negative particle falls into black hole reducing its mass
other escapes

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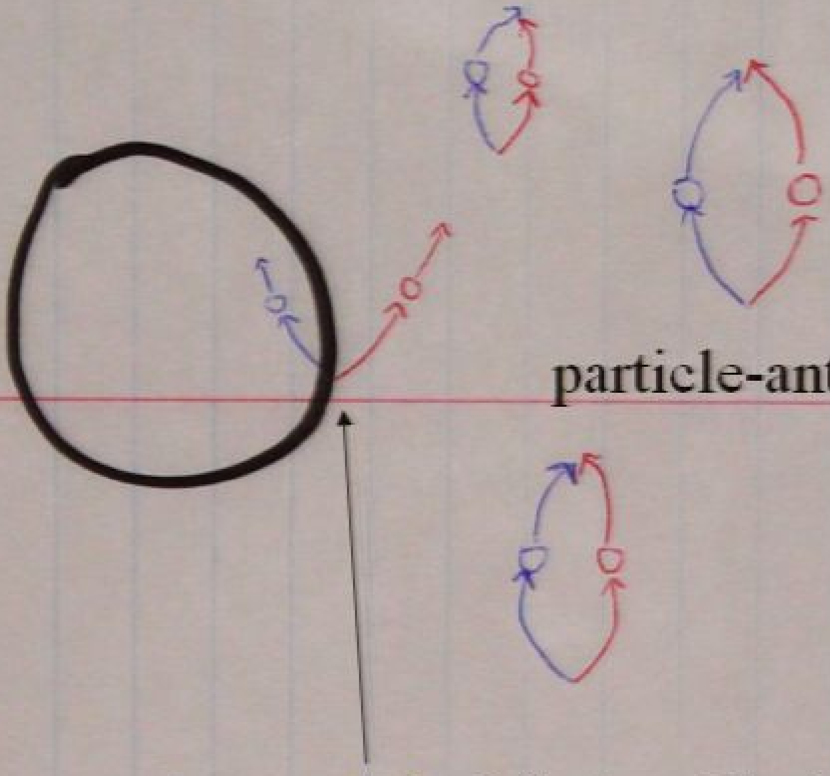

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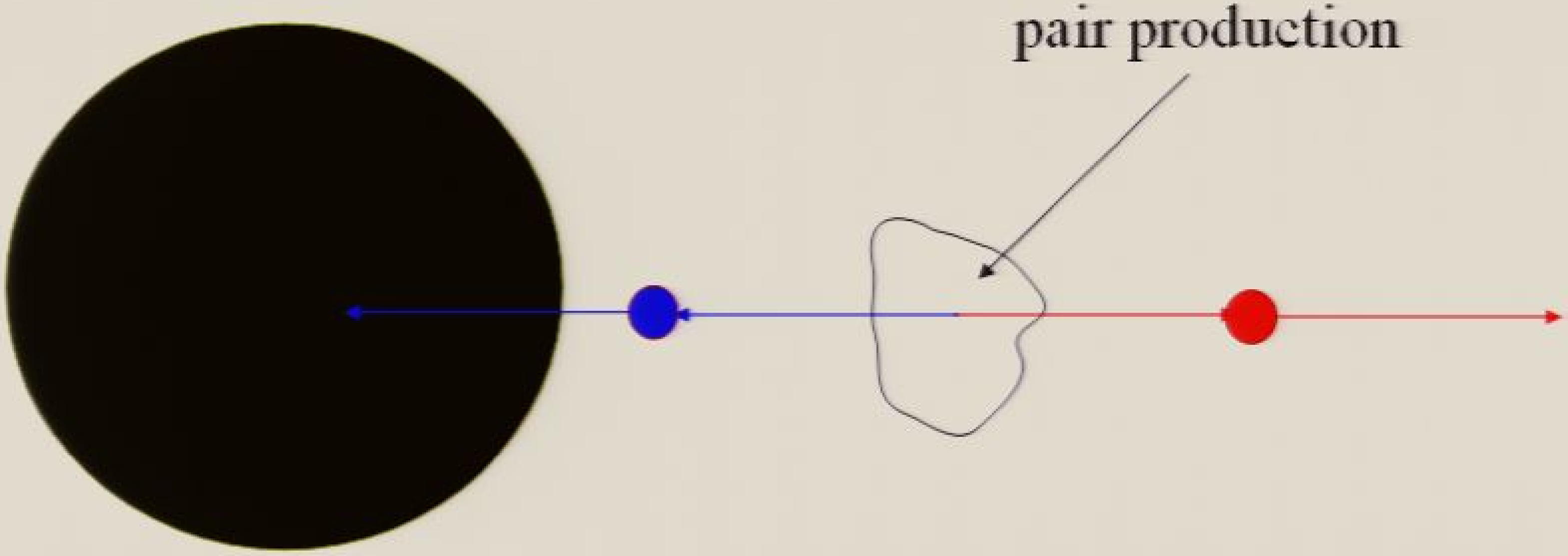
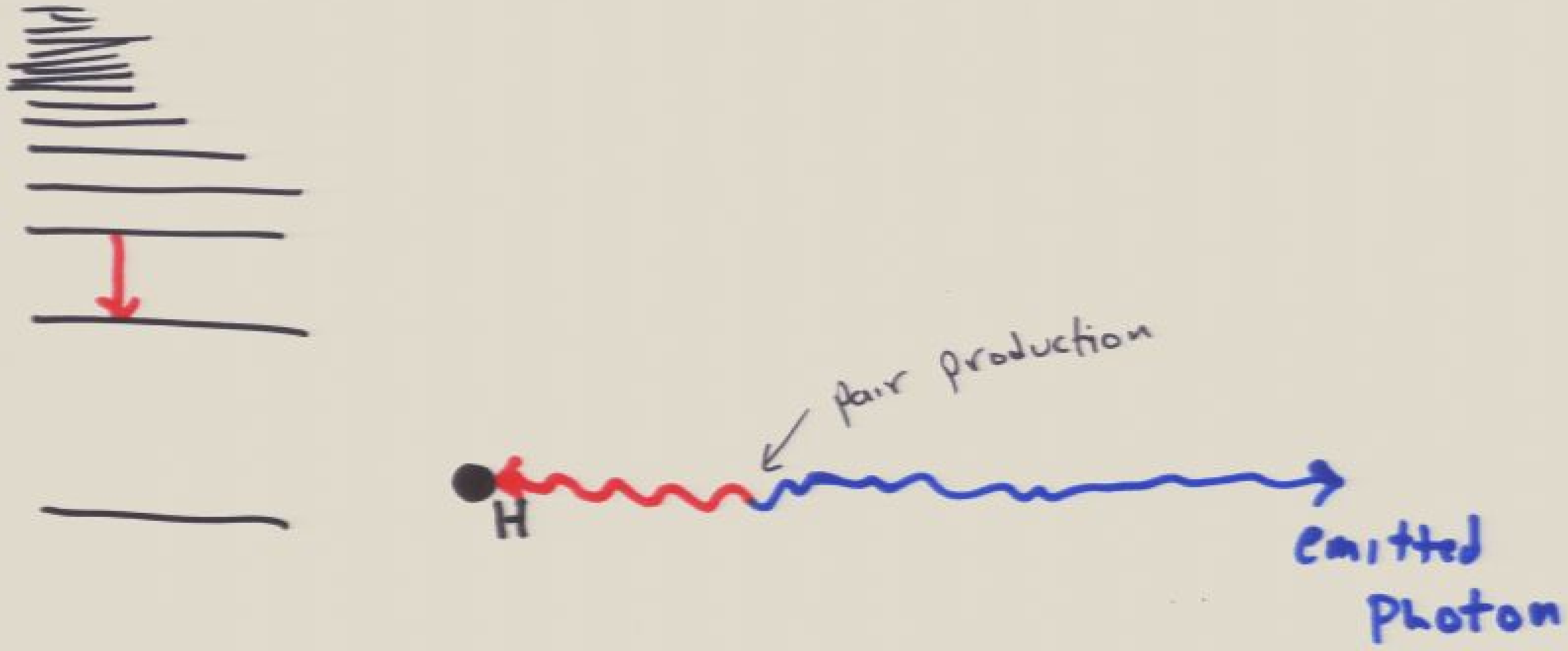
Since the creation happens outside the event horizon, the outgoing particle has no information about what's inside the black hole.



Thermal spectrum comes from a cross-section calculation involving wavelength vs. black hole size in full GR.



QED view of all radiation

System only "eats" certain wavelengths

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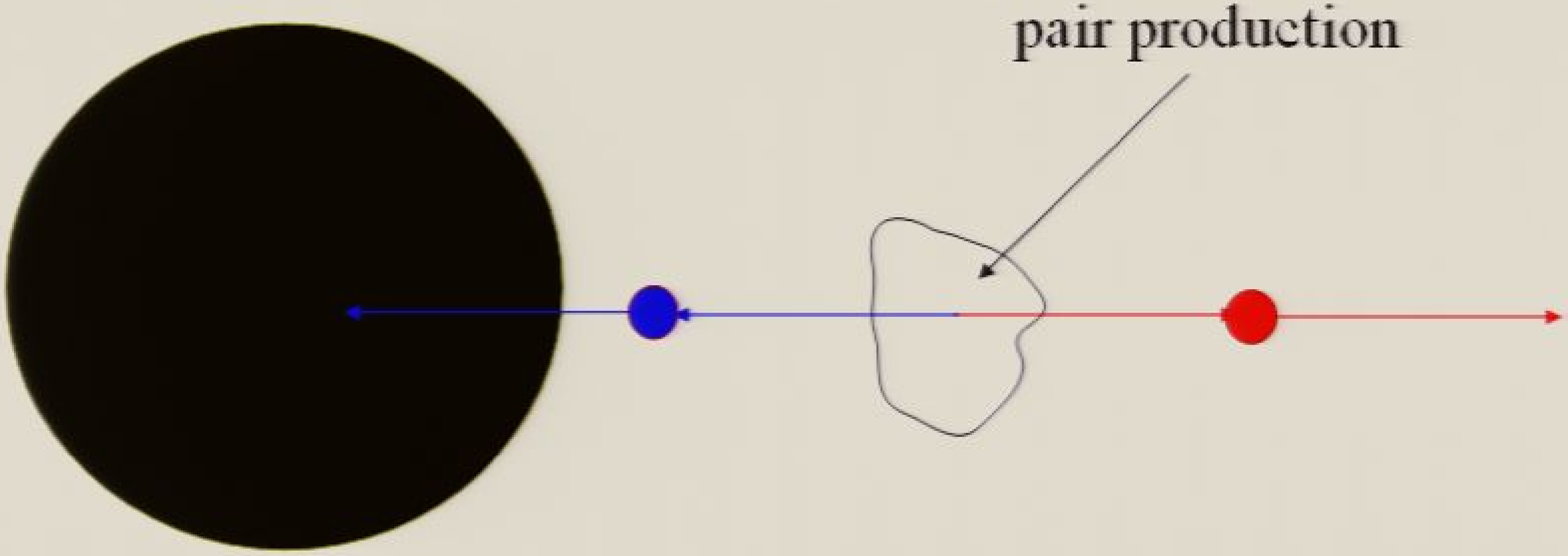

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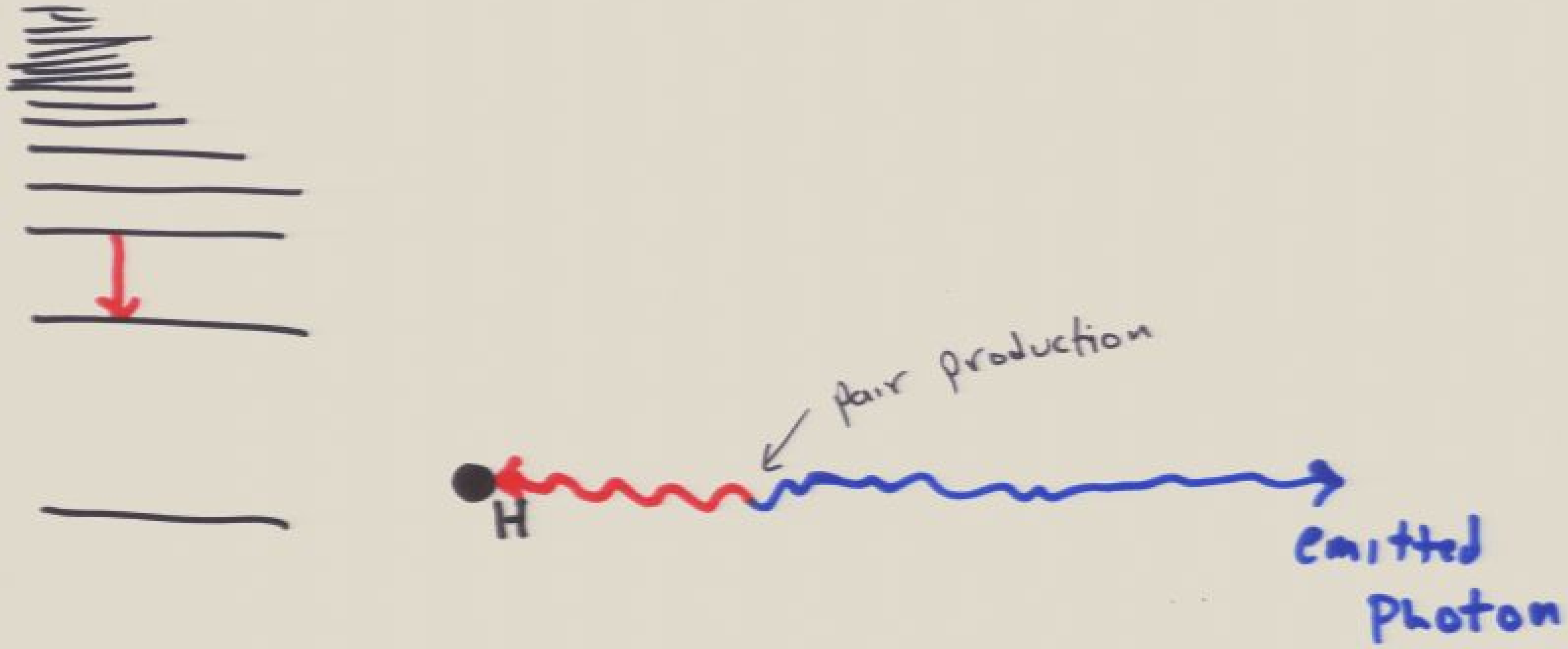

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Evolution to a thermal state is bad for **two** reasons:

1) Initial pure state evolves into highly mixed state
(anomalous entropy production)

2) Final state is uncorrelated with the initial state
(information loss paradox)

Any unitary theory will disagree with Hawking on these so we will assume his calculation is only true approximately, and for large black holes.

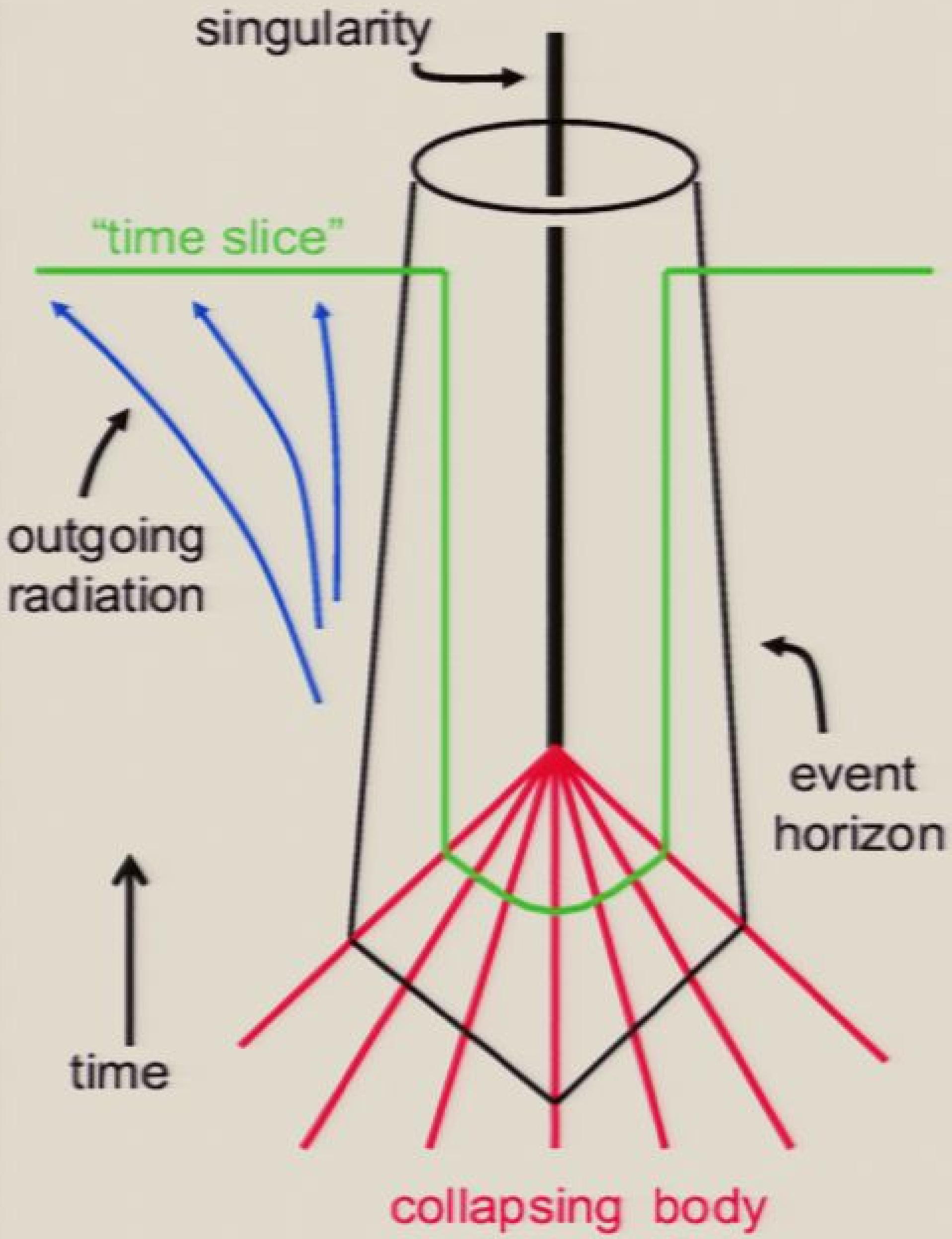
Hydrogen atom analogy explains away both points, but is satisfactory only for point 1 as we shall see

Black hole: a quantum cloning machine?

Suppose that the information about the collapsing body is subtly encoded in correlations among the quanta in the Hawking radiation; the information is *thermalized*, not destroyed.

The green time slice crosses both the collapsing body behind the horizon and the radiation outside the horizon. *Thus the same information is in two places at the same time.*

A quantum cloning machine has operated, which is not allowed by the linearity of quantum mechanics.



Quantum Information cannot be cloned.

Bah!

Unitarity implies no cloning

Quantum Information cannot be cloned.

Bah!

Unitarity implies no cloning

We insist on unitarity

Therefore information escapes

Therefore we can clone?

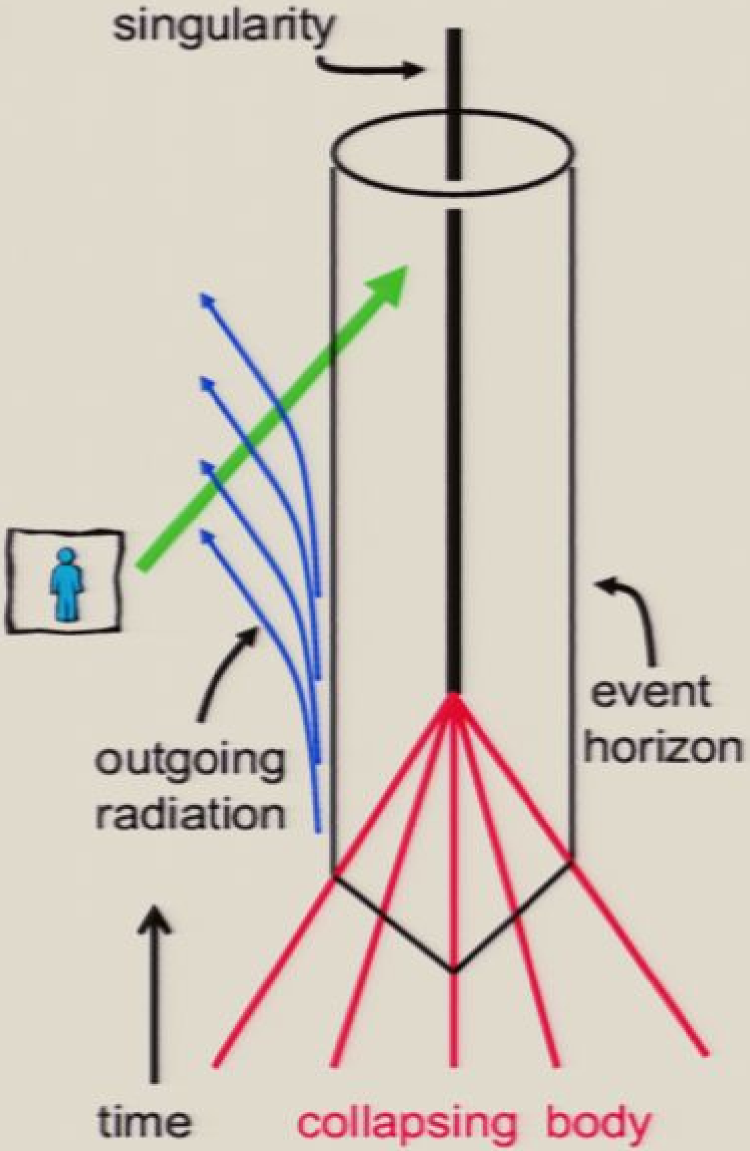
Black hole complementarity

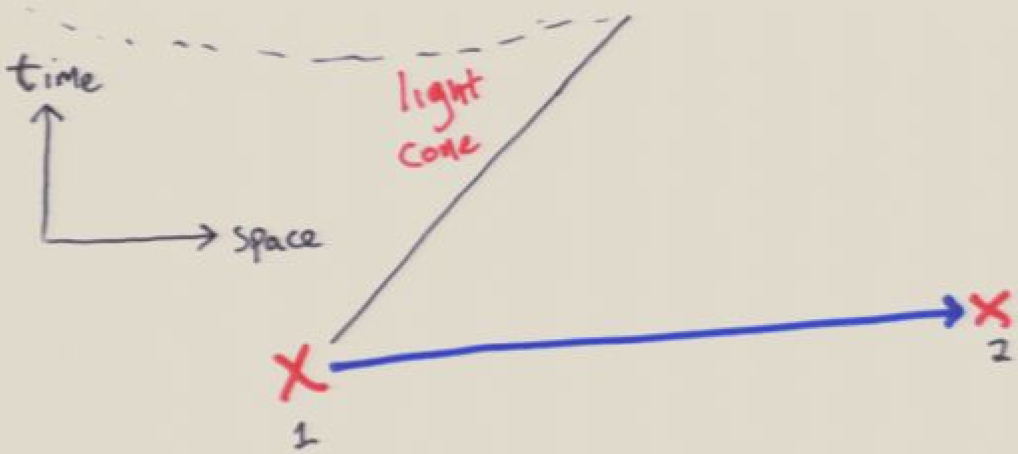
An observer who reads the outgoing radiation concludes that the information must be erased as the collapsing body crosses the event horizon.

A freely falling observer who follows the collapsing body across the horizon knows otherwise.

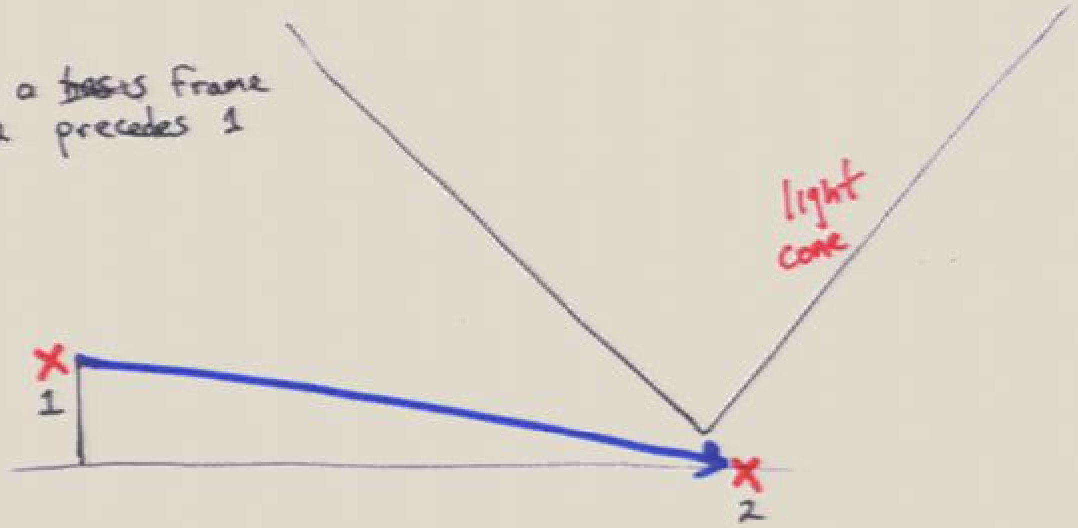
But they can never compare notes…

Perhaps it is okay for quantum information to be copied, if no one can ever find out!



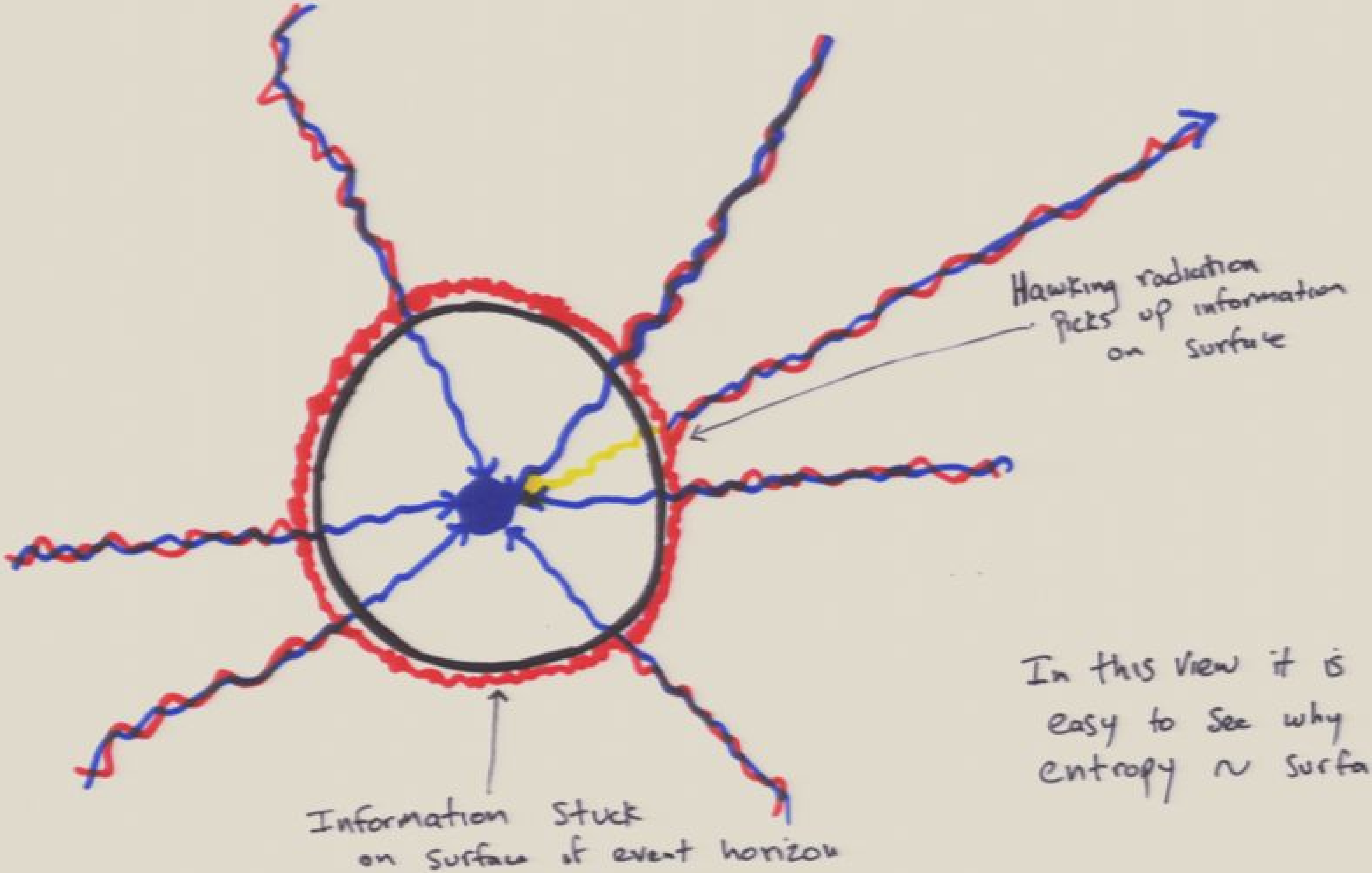


There is a ~~basis~~ frame where 2 precedes 1

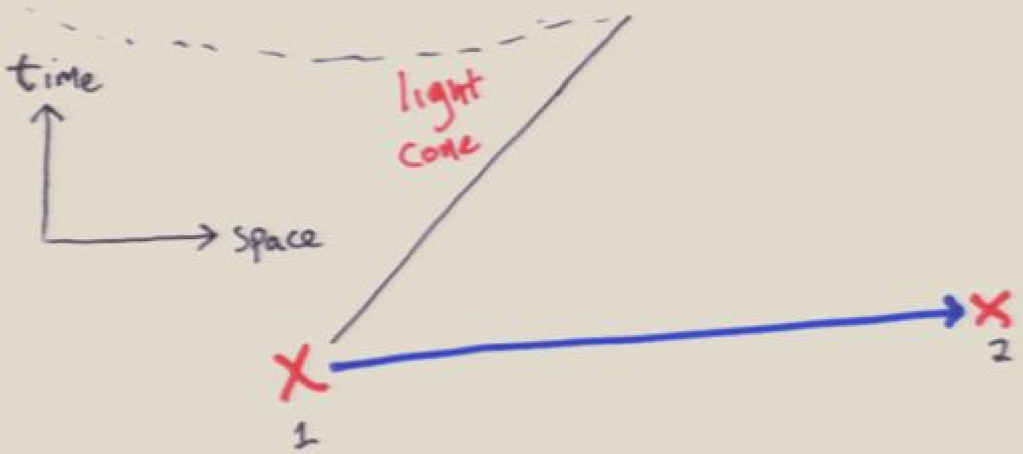


but $\Delta t \leq \frac{c}{d}$ in all frames

Cloning, but no grandfather paradox without 2-way FTL



In this view it is easy to see why entropy ~ surface area



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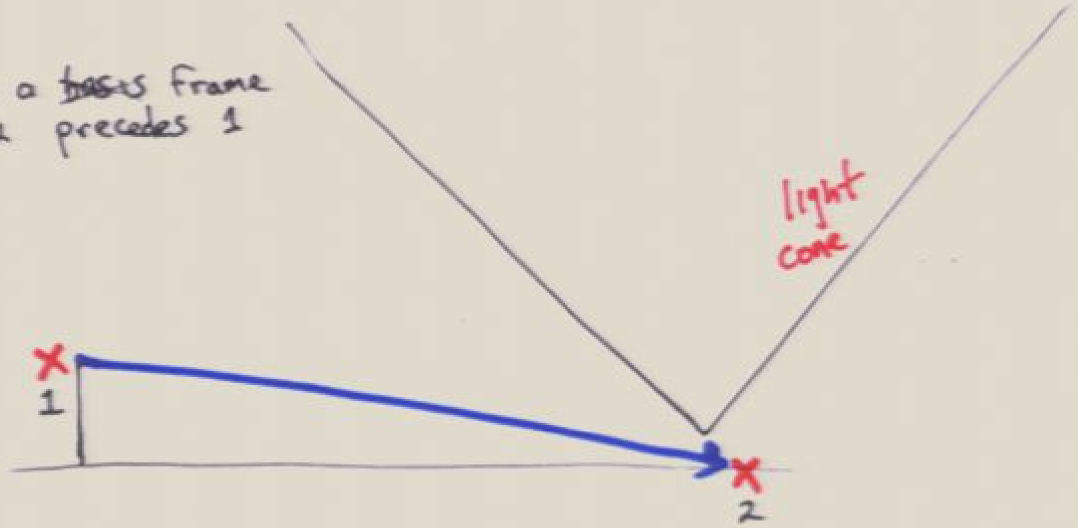

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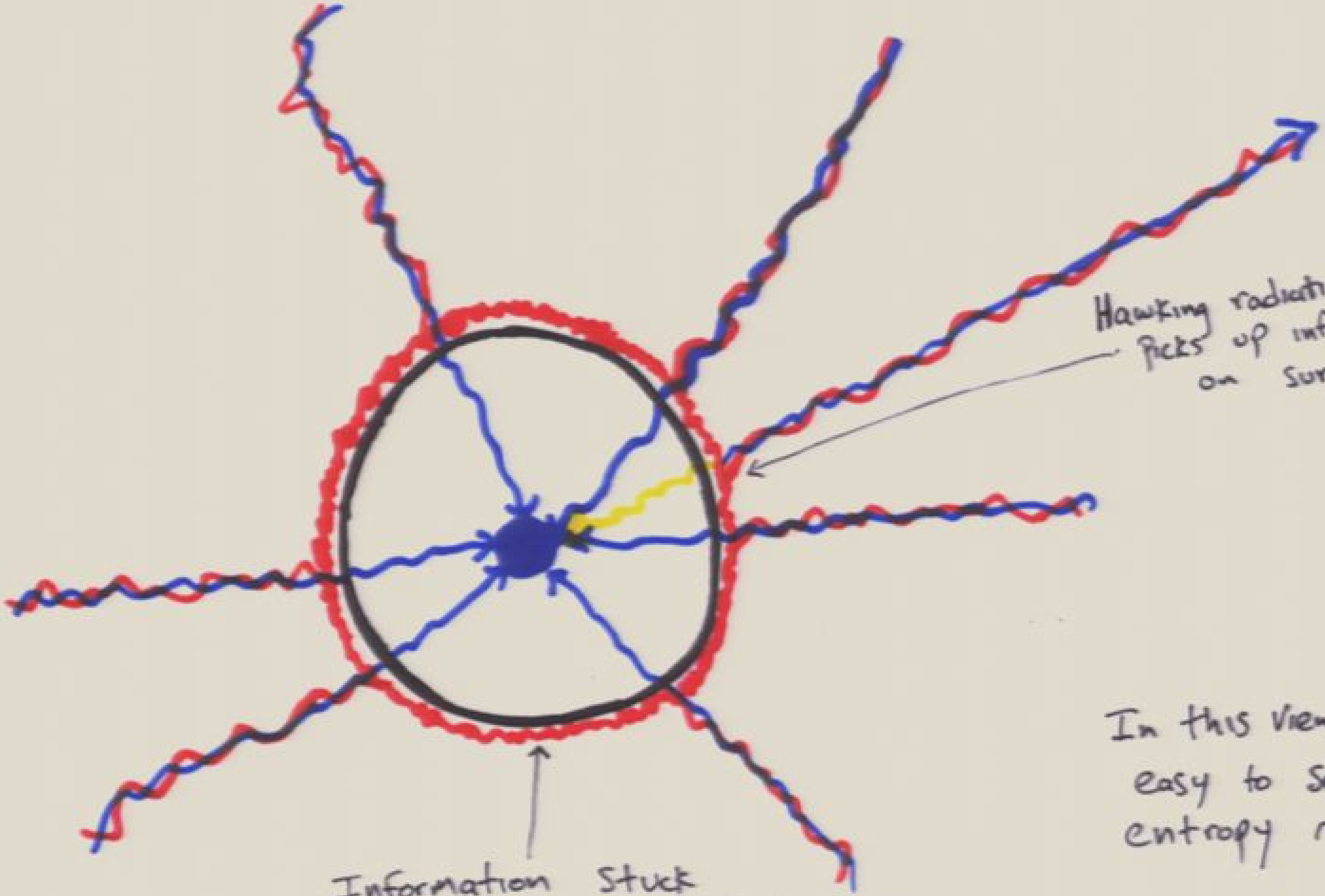

Hawking radiation picks up information on surface

Information stuck on surface of event horizon

In this view it is easy to see why entropy $\sim$ surface area

Complementarity unsatisfying because we want a theory that works for everyone.

To preserve causality for information that really falls into the hole, we must insist that no information leaks out.

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Loophole—Information all gets out at the end of the evaporation, when the black hole is Planck scale and semi-classical ideas surely fail

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But this means all the information has to live in a very small hole

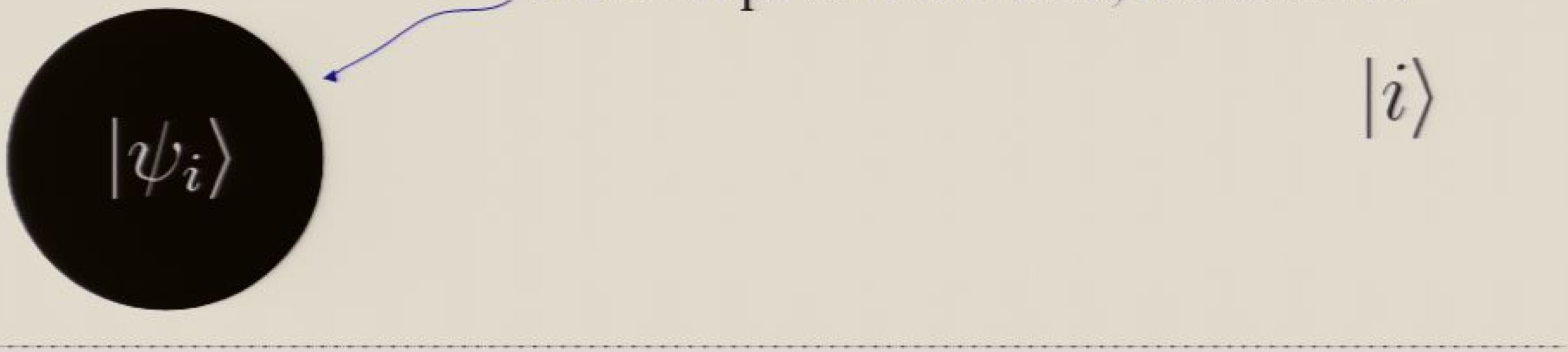
Tiny black holes shouldn't be able to hold large amount of information:

1: We have a black hole entropy forumula: $S=M^2$

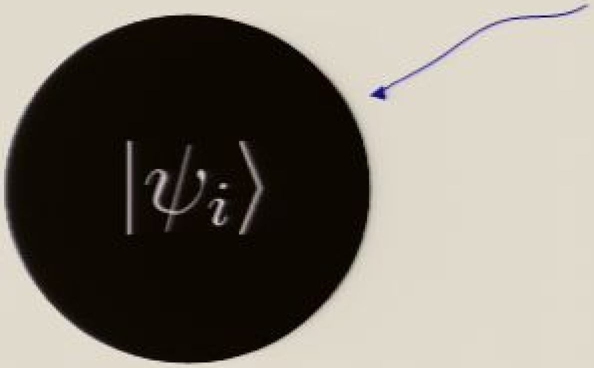
we're discarding semi-classical ideas anyway,
so not too convincing

2: It takes a long time for such an information-dense hole to evaporate, of order M^4 Aharonov, Casher and Nussinov/Carliz and Willey (1987)

This is so long this is effectively a stable remnant. There would be $(M/M_p)^2$ different species. This degeneracy would change low-energy physics.



Alice drops state into hole, remembers i



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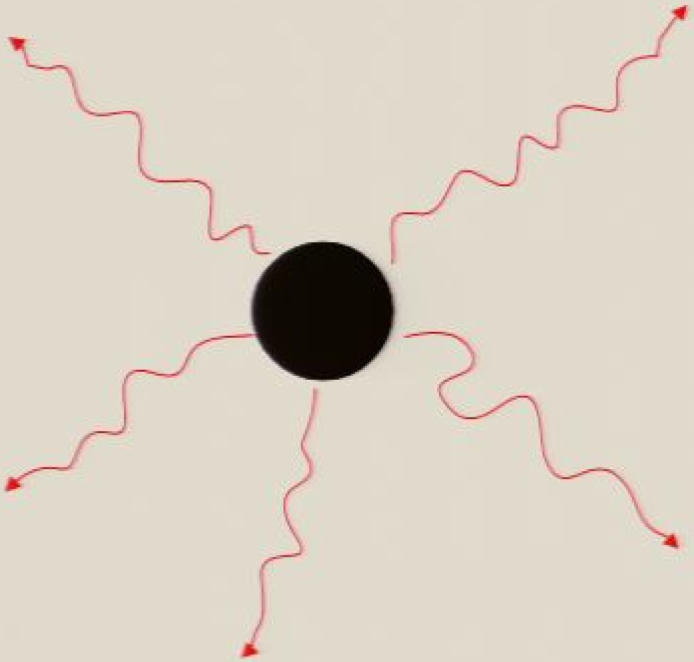

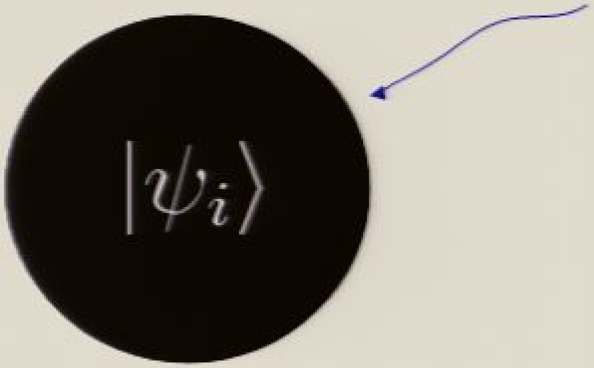

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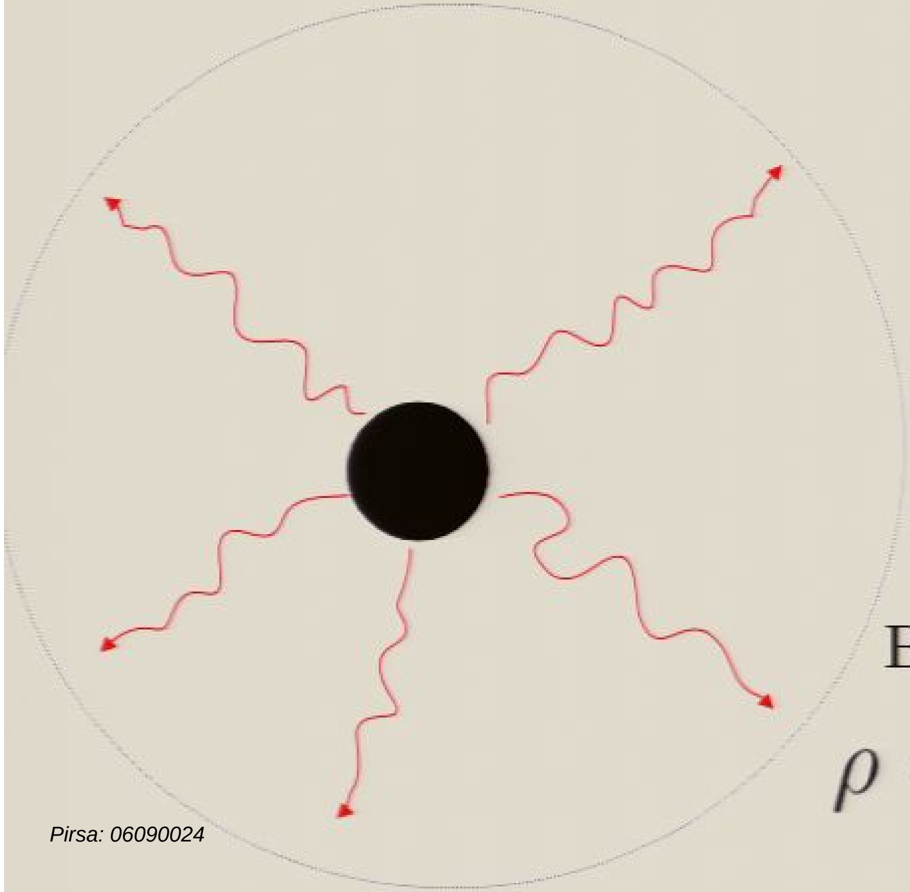
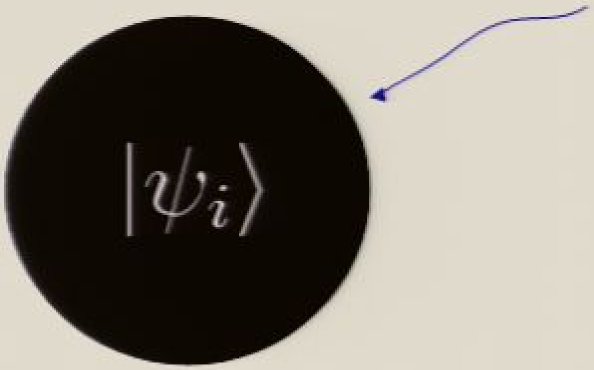


Alice $|i\rangle$

Bob

$$\rho = S_t(|\psi_i\rangle\langle\psi_i|)$$

Alice drops state into hole, remembers *i*



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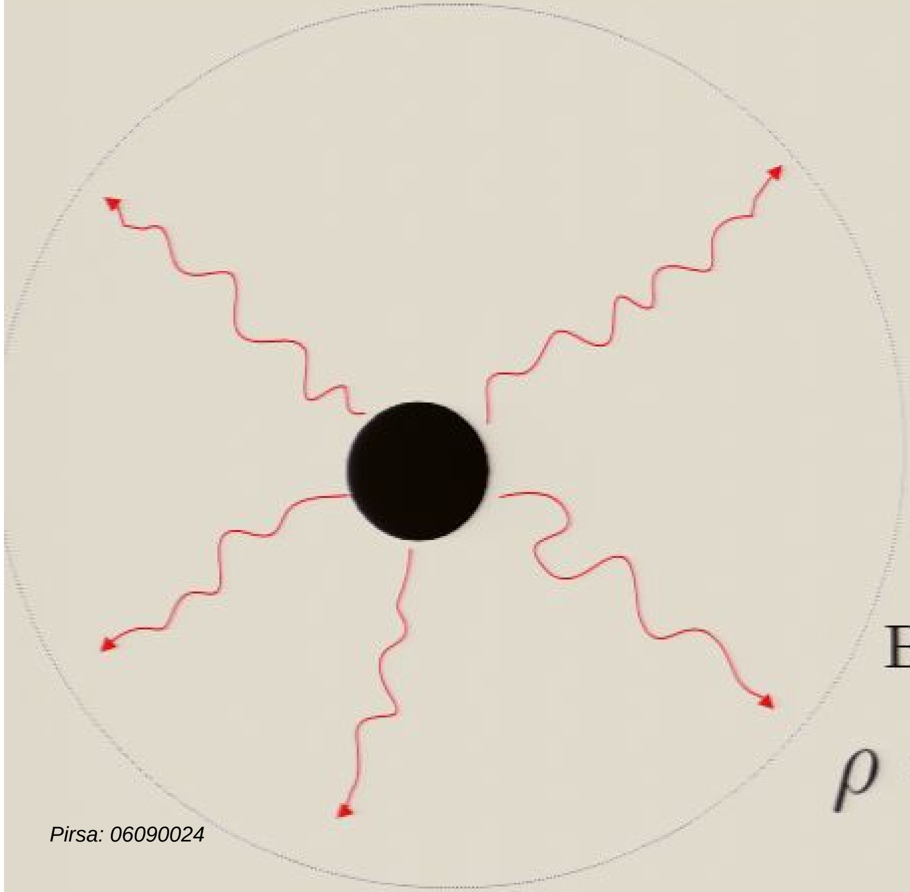

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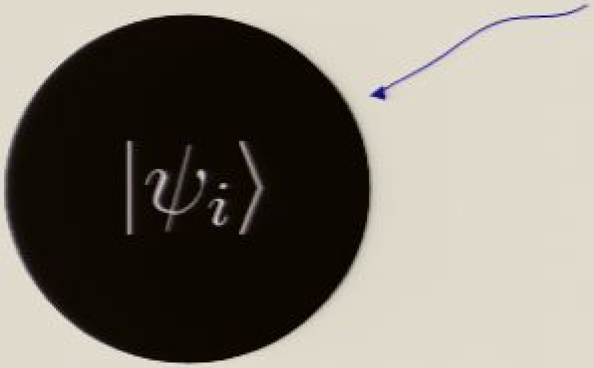

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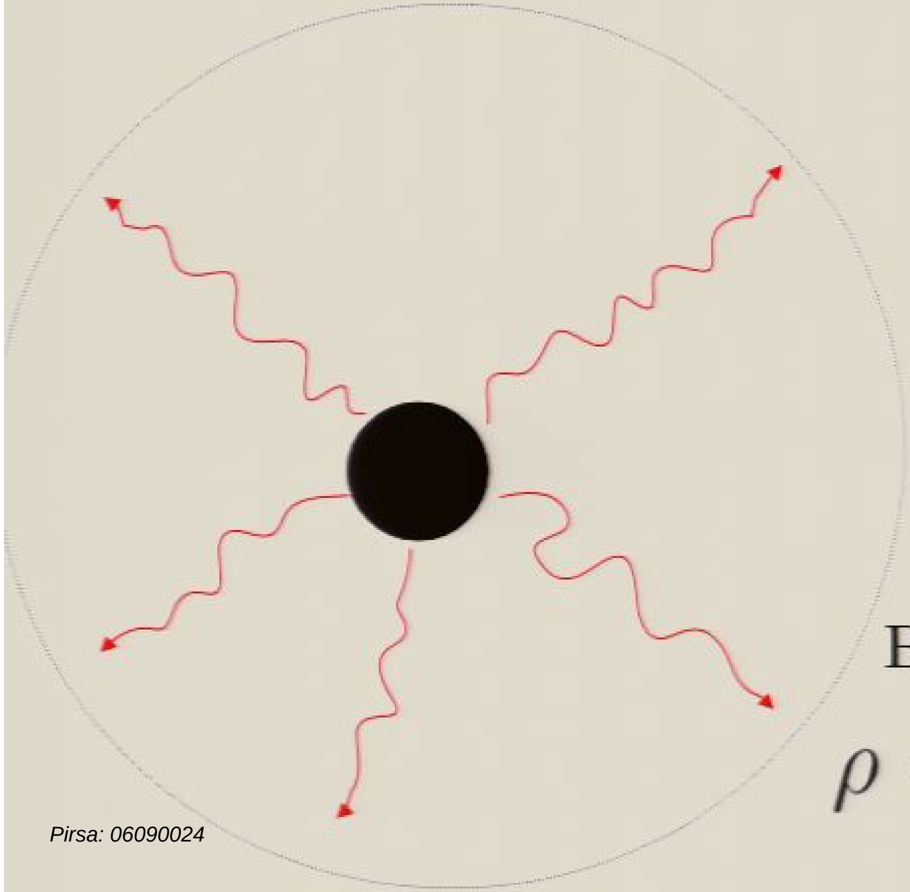

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$$I_{\text{acc}}(\rho_{AB}) \leq \epsilon$$

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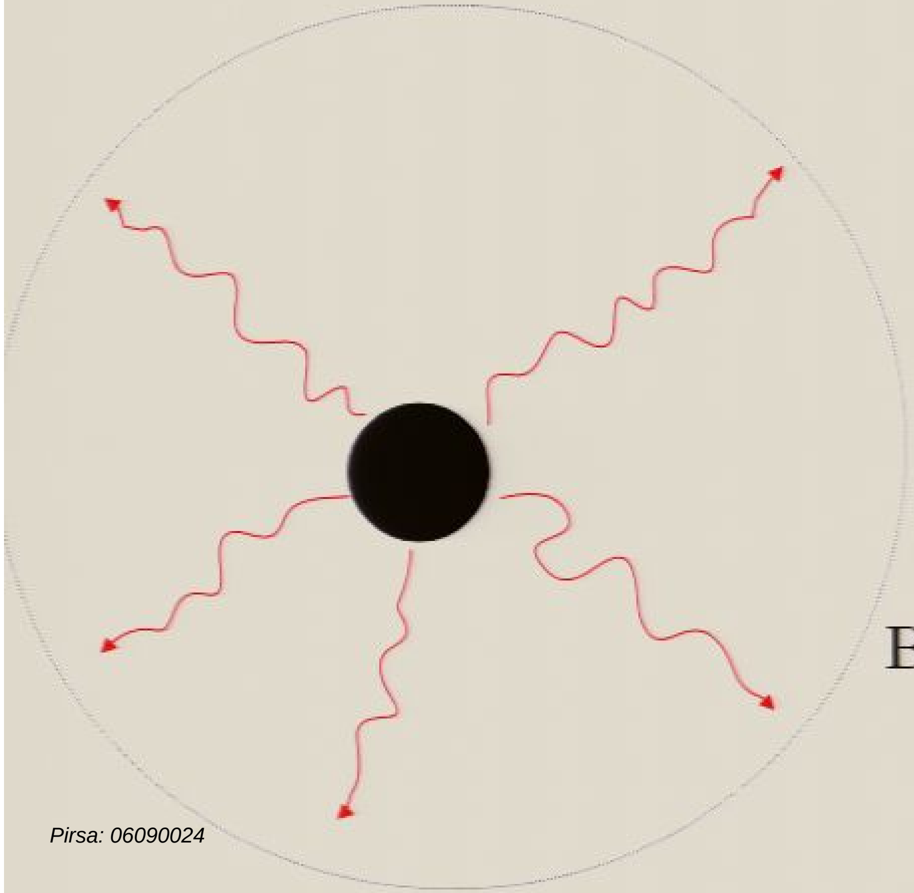

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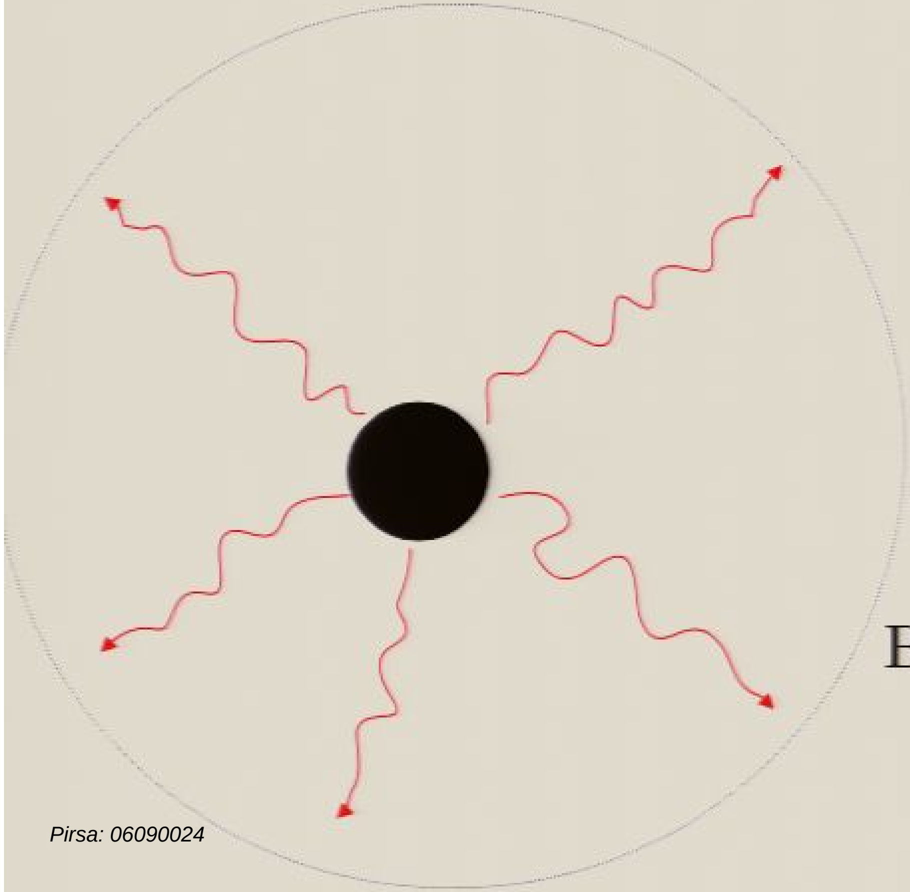

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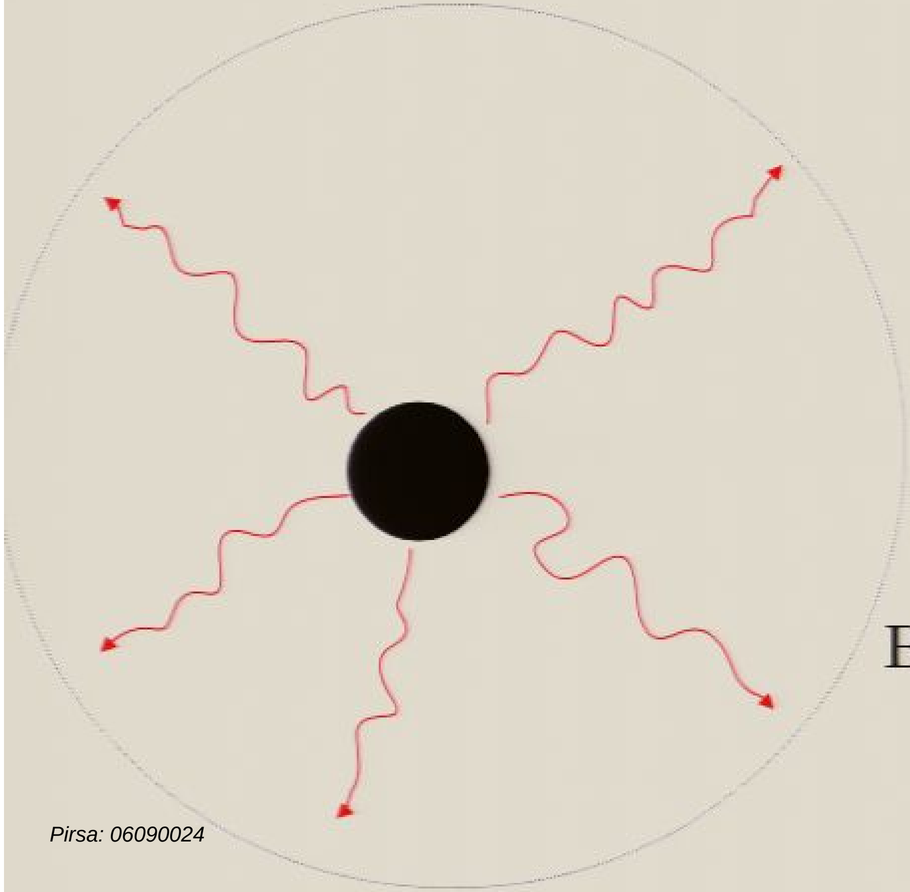

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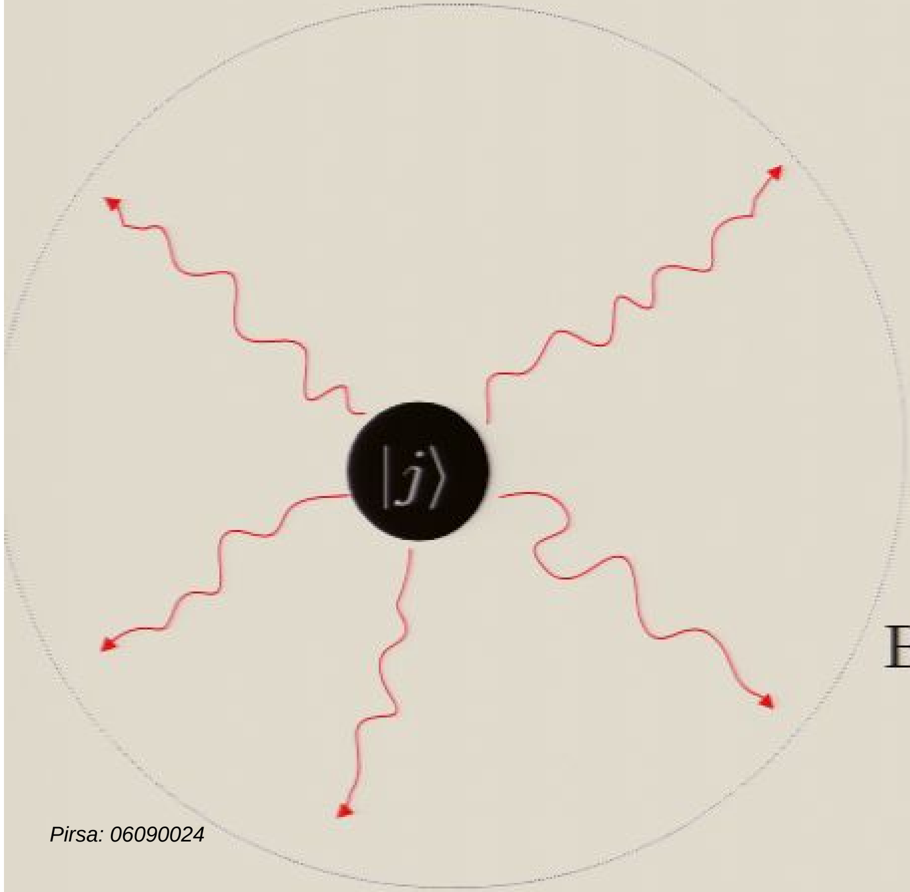
$$\rho_{ABH} = \frac{1}{dn}\sum_{i=1}^{d}\sum_{j=1}^{n} |i\rangle\langle i|_A \otimes U_j|i\rangle\langle i|_B U_j^\dagger \otimes |j\rangle\langle j|_H$$



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To preserve causality we want:

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$$n = (\log d)^3 + k$$

$$S(n) \approx 3\log\log d$$

Size of small black hole in information or Planck units

$$I_{\mathrm{acc}}(\rho') \approx \log d$$

All information that formed hole

$$I_{\mathrm{acc}}(\rho) < \delta = C^{-k}$$

Amount of information that leaks ahead of schedule

Objections (and solutions?)

No mechanism:

Some information still leaks: $I_{\mathrm{acc}}(\rho) < \delta = C^{-k}$

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True, but at least the locking effect is very generic. Almost any random partitioning will work (and therefore many not-so-random ones as well).

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Some information still leaks: $I_{\mathrm{acc}}(\rho) < \delta = C^{-k}$

Semi-classical Hawking calculation always allowed some leakage (actually much more than this) and this leakage is arbitrarily small

Objection: Size of final black hole still grows (though slowly):

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For some d this will be larger than Planck scale.

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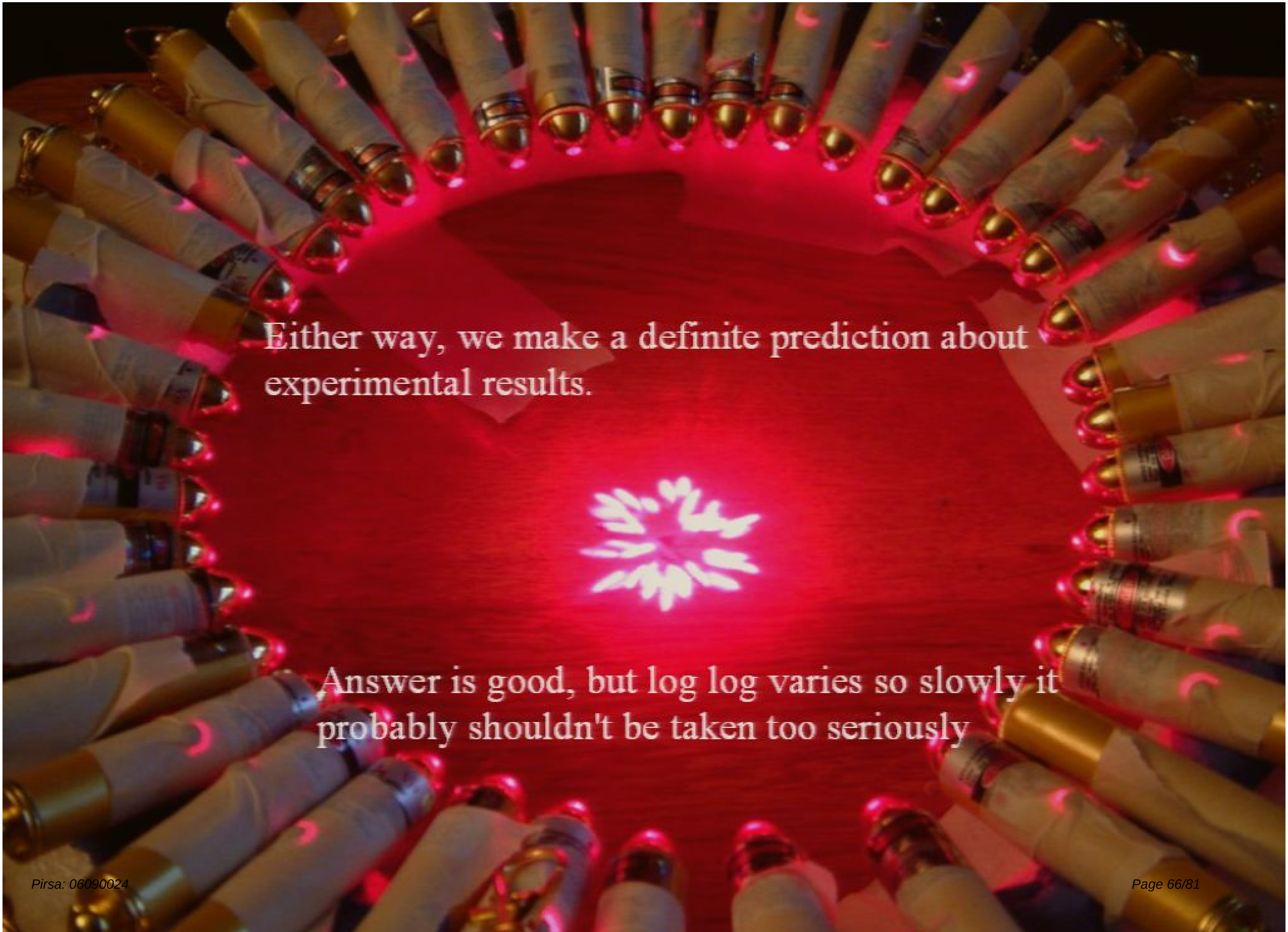
Even more fun (from (Karol Horodecki): Turn this on its head--size of universe determines Planck's constant.

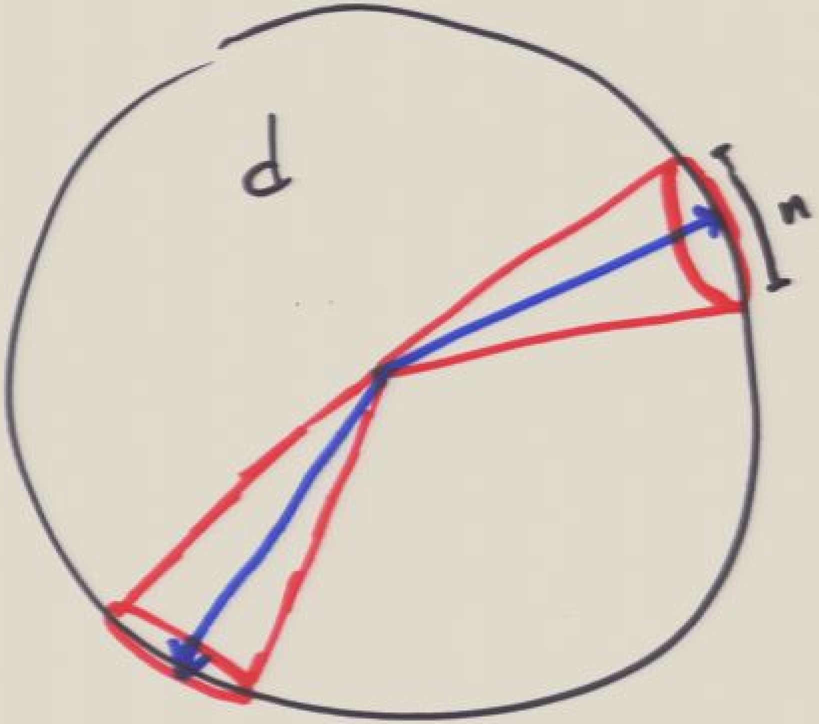
Either way, we make a definite prediction about experimental results.



Either way, we make a definite prediction about experimental results.

Answer is good, but log log varies so slowly it probably shouldn't be taken too seriously

A State is mapped to a cone of size n

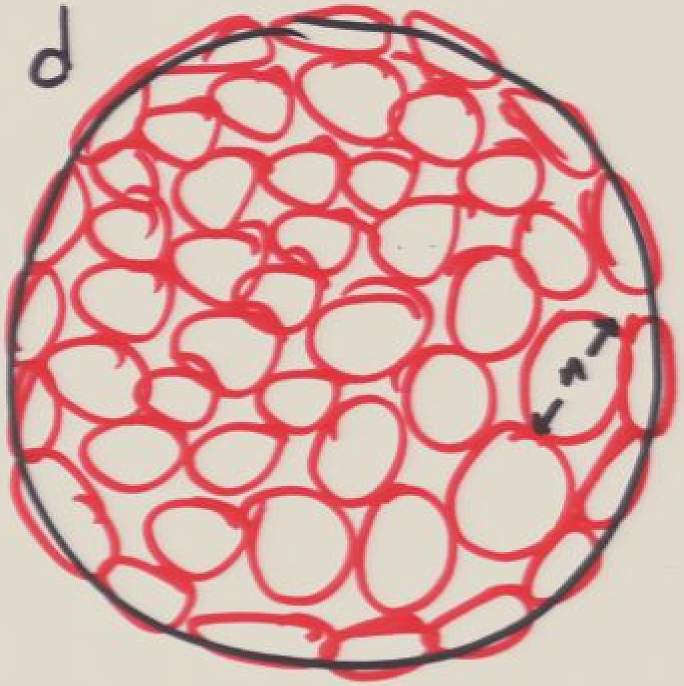


if $n \ll d$

chance that two random States overlap is Small

$$n \simeq (\log d)^3$$

Sphere is covered by roughly $\frac{d}{n} \equiv K$ Circles



$$\log d - \log n = \log K$$

total possible information

key size

encoded bits

Packing Lemma (Devetak): if we send only $K = \frac{d}{n}$ vectors, then $I_{acc}(P) = \log K$

All of it / No locking!

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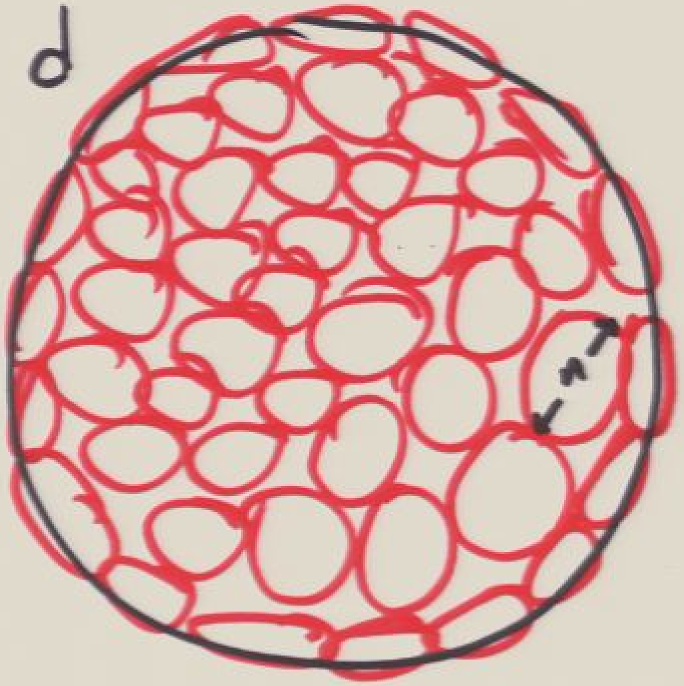

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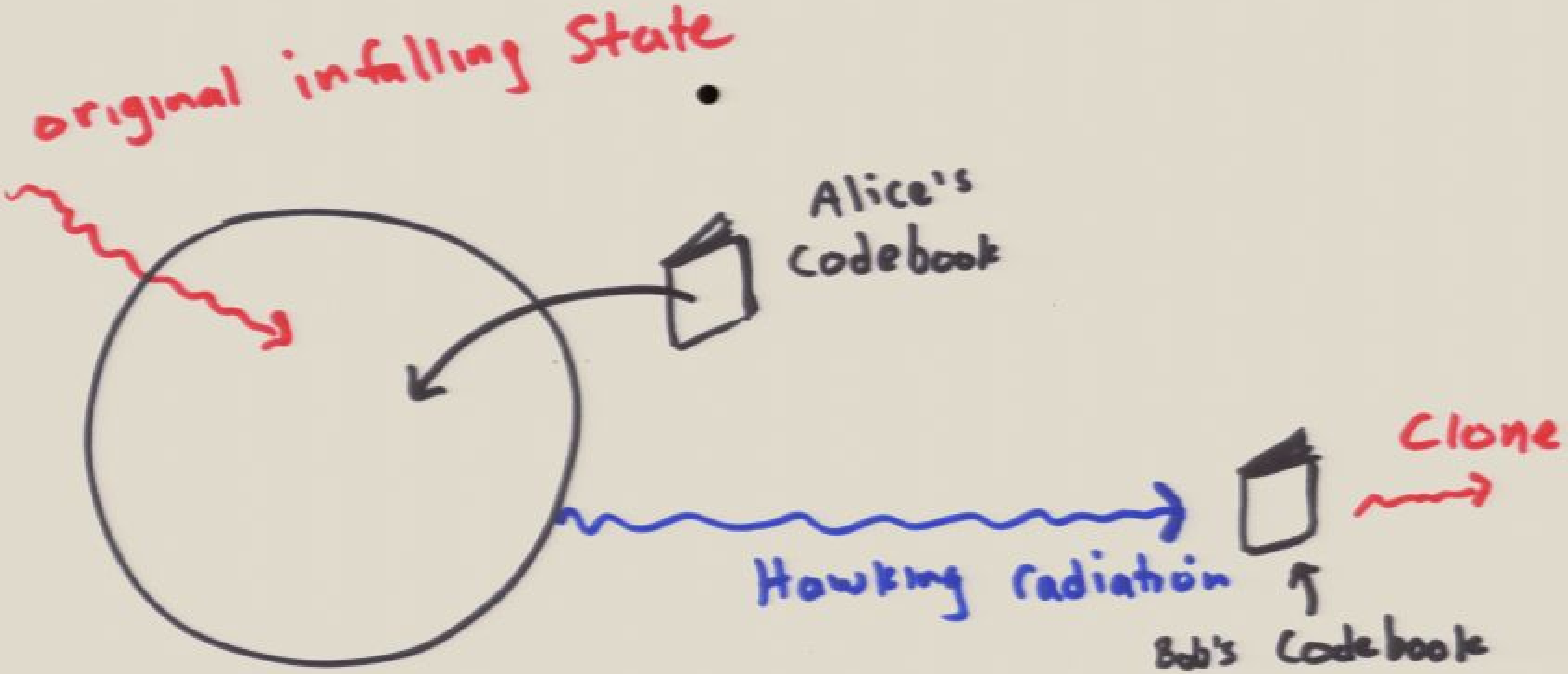
"Mule Rescue" : Information Complementarity



Alice needs the codebook to "interpret" the State

But the Codebook is bigger than the black hole!

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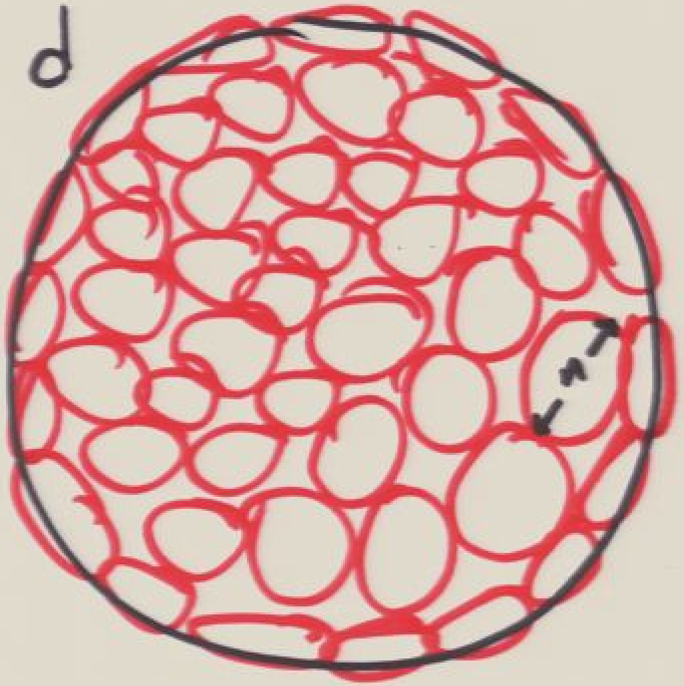

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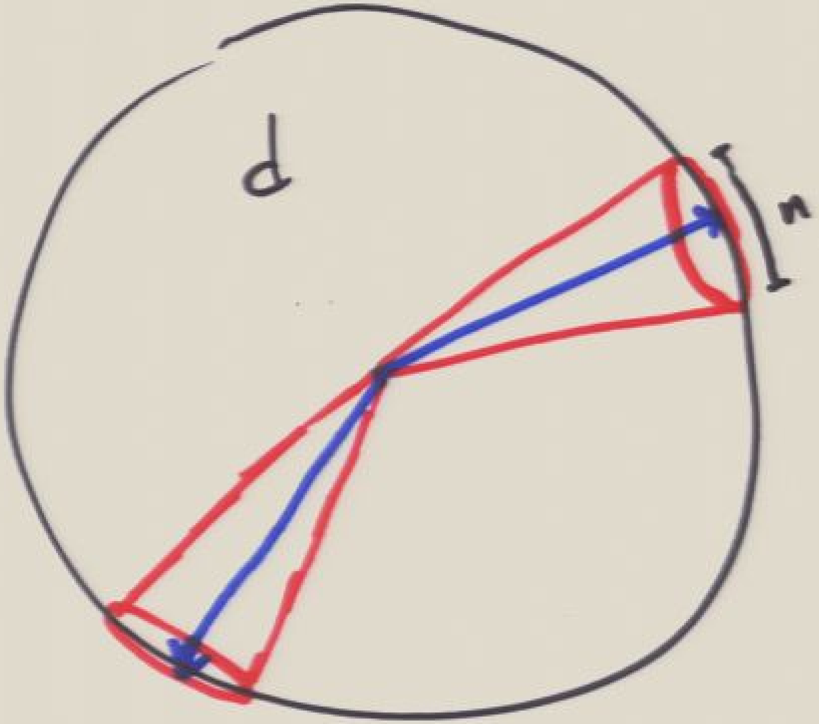

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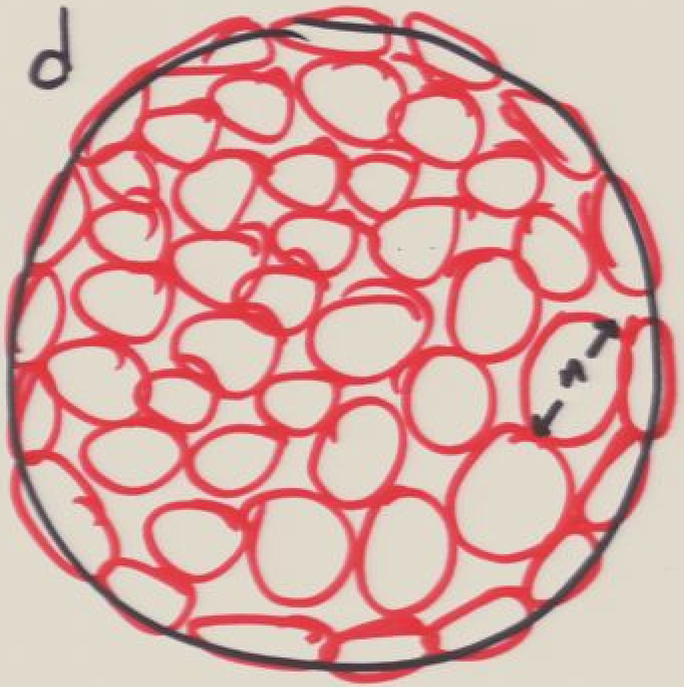

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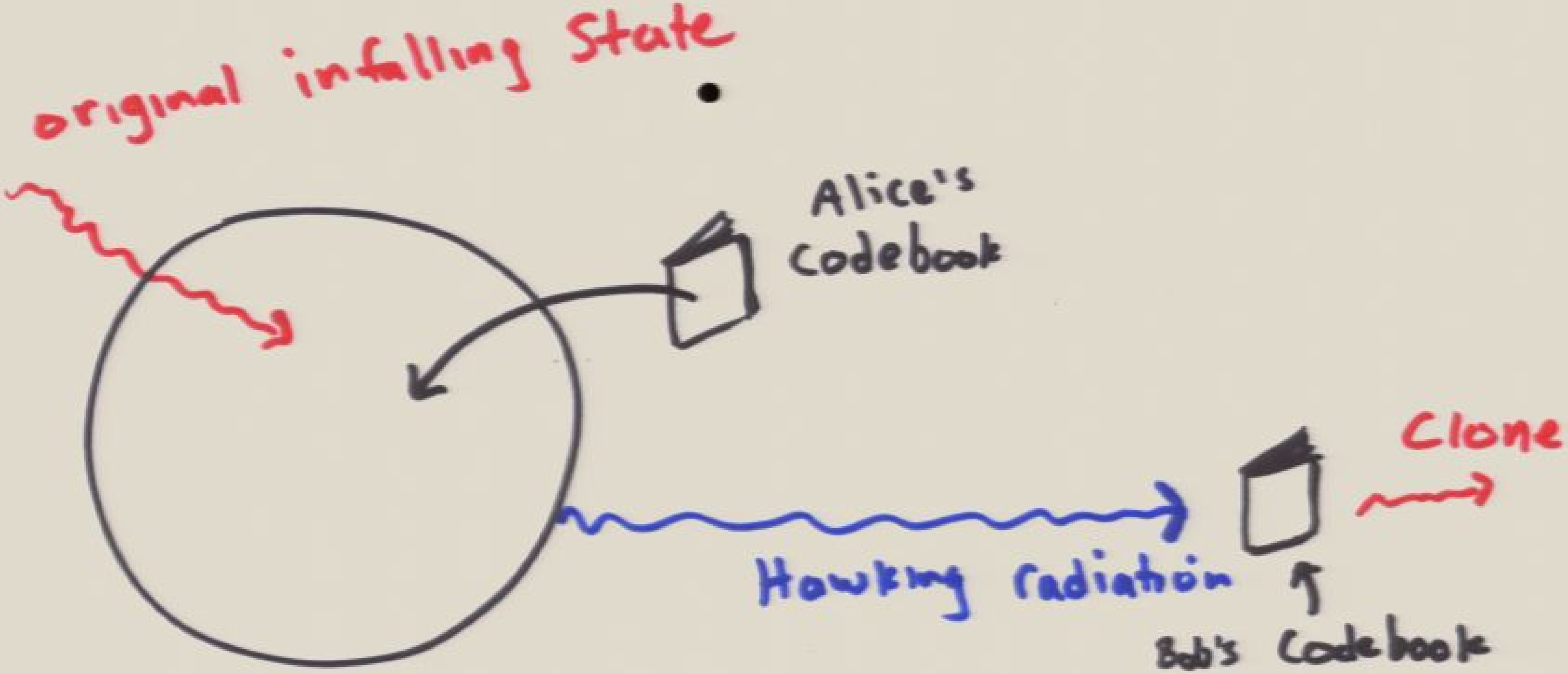

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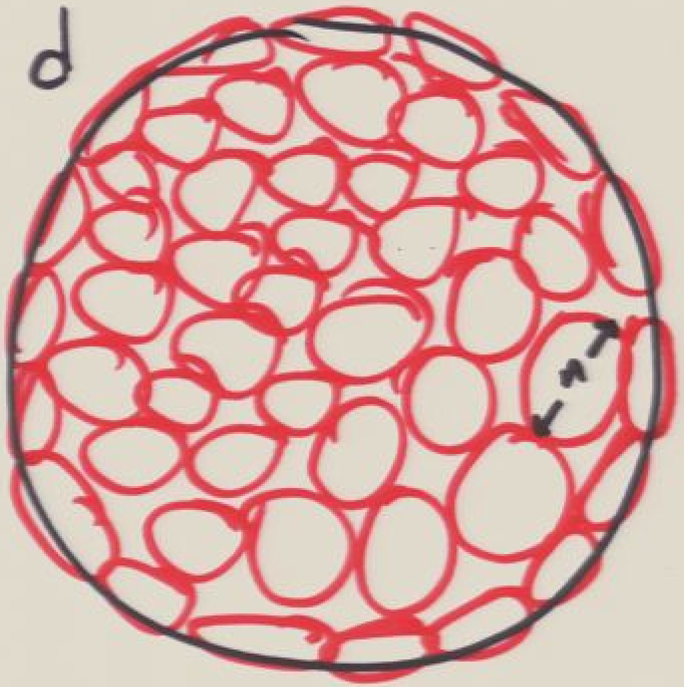

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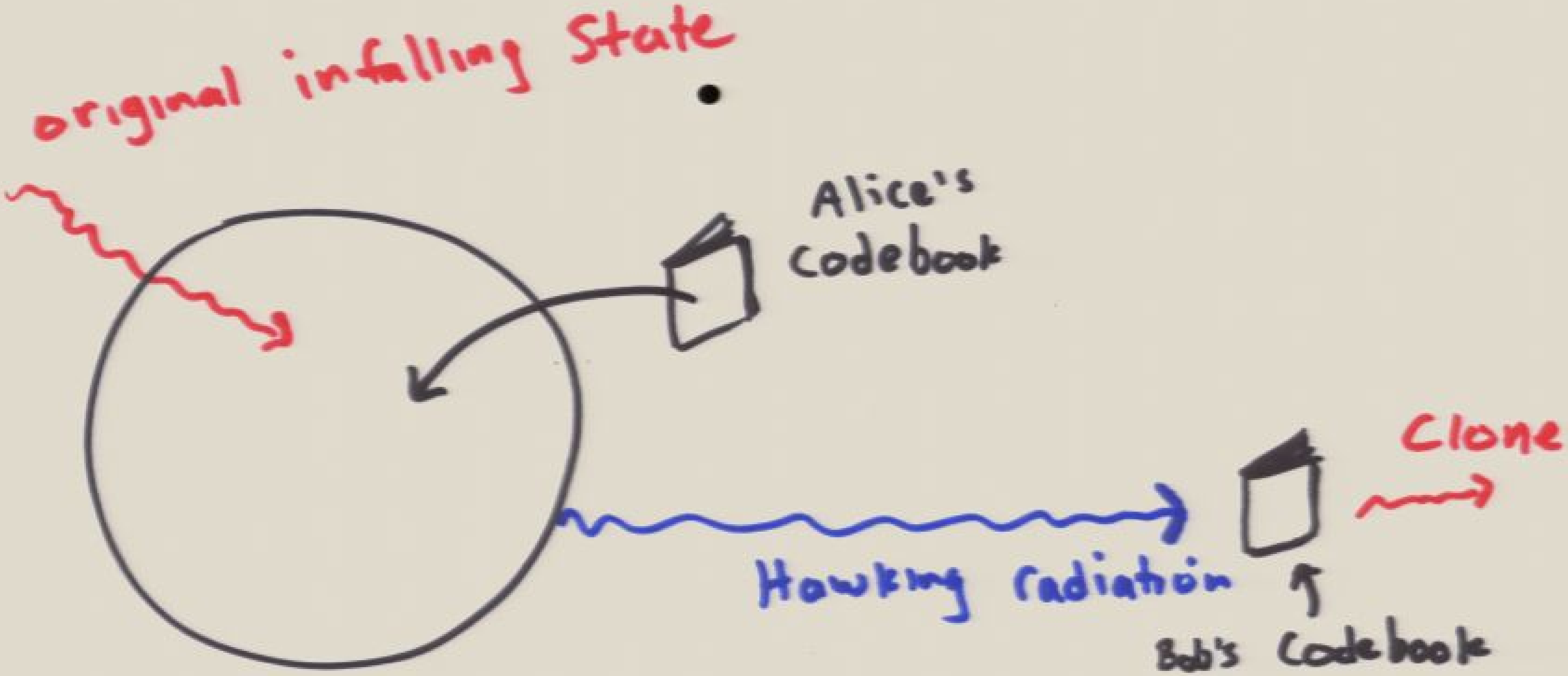

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Conclusions and future work

The quantum "locking" effect helps solve the black hole information loss paradox.

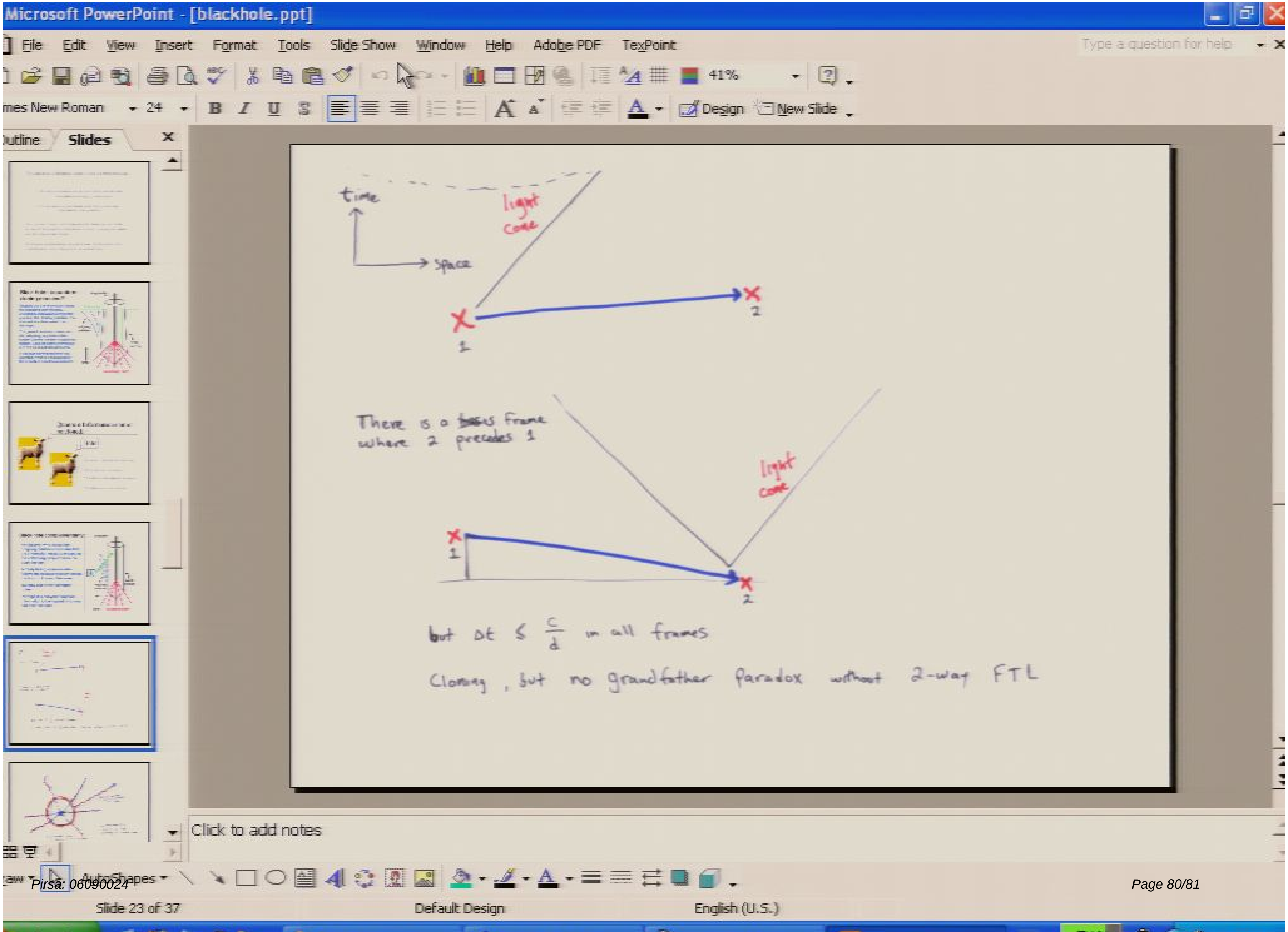
Information can escape only at the end of the evaporation, but the small black hole need not hold very much.

Generalize locking:

- Symmetric locking
- Locking other forms of information

Deal with coding problem—towards a full quantum gravity

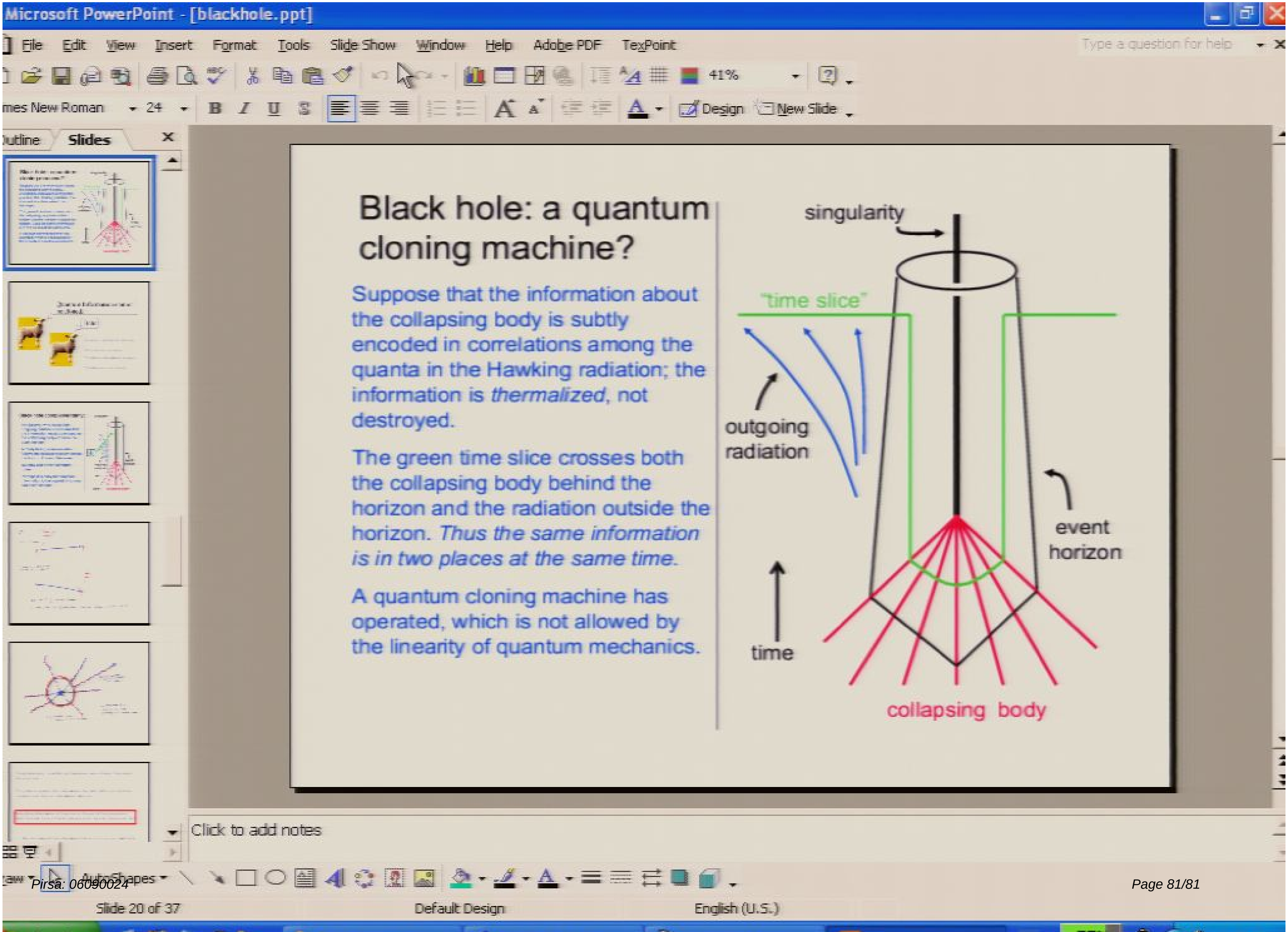
End of slide show, click to exit.



There is a ~~basis~~ frame where 2 precedes 1

but $\Delta t \leq \frac{c}{d}$ in all frames

Cloning, but no grandfather paradox without 2-way FTL



Black hole: a quantum cloning machine?

Suppose that the information about the collapsing body is subtly encoded in correlations among the quanta in the Hawking radiation; the information is *thermalized*, not destroyed.

The green time slice crosses both the collapsing body behind the horizon and the radiation outside the horizon. *Thus the same information is in two places at the same time.*

A quantum cloning machine has operated, which is not allowed by the linearity of quantum mechanics.