

Title: Mode creation in expanding universes, through dissipative effects

Date: Sep 05, 2006 09:00 AM

URL: <http://pirsa.org/06090020>

Abstract:

PI '66

Mode creation
in
expanding universes
(by dissipative effects)

Parentani
LPT-Orsay

1st Motivation.

unknown high energy processes
might break Lorentz invariance
by **dissipative effects.**

⇒ introducing

- a UV scale Λ_{LV}
- a preferred frame

1st Aim:

The phenomenology of this possibility
in Minkowski space, in vacuo.

2^d Motivation.

address the trans-Planckian question
(of inflationary cosmology)
when (strong) dissipative effects
exist above $\omega = \Lambda_{\text{LV}}$

- So far only dispersion was studied

2^d Aim:

provide a class of ^{dissipative} models
wherein $P(k)$ can be computed

3rd Motivation.

Confront the question of
mode creation in expanding universe

- assuming that the mode density

$$\rho_{\Omega} < \infty,$$

- • Either the number of modes = c^{3t}

⇒ $\rho_{\Omega} \propto 1/a^3$ (absurd)

• Or $\rho_{\Omega} = c^{3t}$

⇒ Mode creation.

⇒ 2 questions:

1. how to describe mode creation in QFT?
2. What fixes the **state** of the newly born modes?

3rd Aim:

provide hamiltonian (closed) models wherein (underdamped) modes are "created" so that their density stays fixed as the universe expands.

4th Motivation / Aim

- identify the relevant properties of the environment Ψ governing dissipative effects on ϕ so as to

1. identify equivalent classes
2. cover the generic case (anti-ad-hoc)

in order to 3. be able to meet additional requirements.
(e.g. general covariance)

Two basic requirements

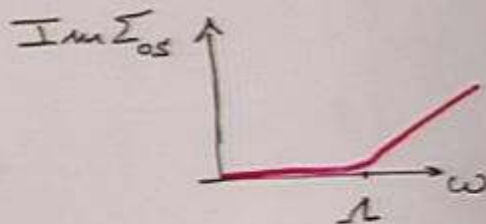
QFT

1) Models with dissipation

- on shell
- in Mink. space time
- in vacuum (tree interacting)
- above $\omega = \Lambda_{UV}$
- preserving {homogeneity, isotropy, stat:onsarity.

$$\Rightarrow G_F(\omega, k) = \left(-\omega^2 + k^2 + \underbrace{\Sigma(\omega, k)}_{\text{separately}} \right)^{-1}$$

$$\text{ex: } \Sigma_m = \frac{i\omega}{\Lambda} k^2 \left(\frac{k}{\Lambda} \right)^2$$



TWO BASIC requirements

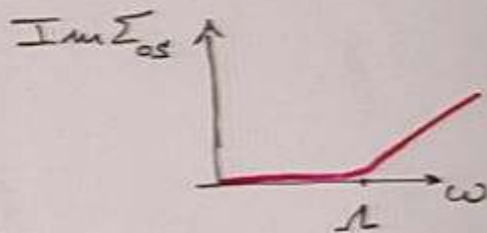
QFT

1] Models with dissipation

- on shell
- in Mink. space time
- in vacuum (true interacting)
- above $\omega = \Lambda_{UV}$
- preserving {homogeneity, isotropy, stat:anarity.

$$\Rightarrow G_F(\omega, k) = (-\omega^2 + k^2 + \underbrace{\Sigma(\omega, k)}_{\text{separately}})^{-1}$$

$$\text{ex: } \Sigma_m = \frac{i\omega}{\Lambda} k^2 \left(\frac{k}{\Lambda}\right)^n$$



Note: the dispersion relation is defined from the poles of $G(\omega, k)$

2^d requirement.

"hamiltonian (closed)" : unitary

$$G(x, x') = \text{Tr} \left[\hat{J}_{\text{in}} \hat{\phi}_H(x) \hat{\phi}_H(x') \right]$$

where $\cdot \hat{J}_{\text{in}} = \hat{J}_{\phi} \otimes \hat{J}_{\text{env. } \psi} >$

$\cdot \hat{\phi}_H(x)$: Heisenberg f. op.
evolving with \hat{H}_T

Note: Forster-Jacobson's model is not H and U.
nor Kempf's and Ted's ones

Link with Quantum Gravity.

- Suppose that when approaching Λ , the smooth manifold is replaced by a new structure.

• Question:

how will **this** manifest itself in Green functions of matter fields?

$$G(x, x') = \langle T_2 [\phi(x) \phi(x')] \rangle$$

$$G(t, t'; k) = \frac{1}{2\omega_k} e^{-i\omega_k(t-t')}$$

$$= ? \text{ in QG}$$

in the case of vacuum in Minkowski sp.

Link with Quantum Gravity.

- Suppose that when approaching Λ , the smooth manifold is replaced by a new structure.
- Question:
how will **this** manifest itself in Green functions of matter fields?

$$G(x, x') = \text{Tr} [\rho \phi(x) \phi(x')] "$$

$$G(t, t'; k) = \frac{1}{2\omega_k} e^{-i\omega_k(t-t')}$$

$$= ? \text{ in QG}$$

in the time QFT vacuum in Thirring's sp.

Link with Quantum Gravity.

- Suppose that when approaching Λ , the smooth manifold is replaced by a new structure.
- Question:
how will **this** manifest itself in Green functions of matter fields?

$$G(x, x') = \text{"} T_2 [\rho \phi(x) \phi(x')] \text{"}$$

$$G(t, t'; k) = \frac{1}{2\omega_k} e^{-i\omega_k(t-t')} \quad \text{in the true QFT vacuum in Minkowski sp.}$$
$$= ? \quad \text{in QG}$$

Note: $P_h(t) = G(t, t; k)$ in inflation

- Suppose that when approaching Λ , the smooth manifold is replaced by a new structure.

- Question:

how will **this** manifest itself in Green functions of matter fields?

$$G(x, x') = \text{"} T_2 [\rho \phi(x) \phi(x')] \text{"}$$

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in the true QFT vacuum in Minkowski sp.

$$= ? \text{ in QG}$$

Note: $\bar{P}_k(t) = G(t, t; k)$ in inflation

Finite mode density

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- Free field, solution of $\square \phi = 0$
in Minkowski sp. time.

- $$\phi(t, \underline{x}) = \underbrace{\int d^3k}_{\text{SOM}} e^{i\underbrace{\underline{k} \cdot \underline{x}}_{\text{mode op.}}} \phi_{\underline{k}}(t)$$

Sum over modes

- each mode $(\underbrace{\partial_t^2}_{\underline{L} = |\underline{k}|^2} + \omega_{\underline{k}}^2) \phi_{\underline{k}} = 0$. (HO)

- hamiltonian:

$$\hat{H} = \underbrace{\int d^3k}_{\text{SOM}} \hat{H}_{\underline{k}}, \quad \hat{H}_{\underline{k}} = \omega_{\underline{k}} \hat{a}_{\underline{k}}^{\dagger} \hat{a}_{\underline{k}}$$

- state of the field

$$|\Psi\rangle = \underbrace{\prod_{\underline{k}}}_{\text{POM}} \underbrace{|\Psi_{\underline{k}}\rangle}_{\text{state of the mode } \underline{k}}$$

• density of particles / quanta: \bar{n}_Q ⁶

$$\bullet \frac{\bar{N}_Q}{V_s} = \bar{n}_Q = \int d^3k \underbrace{\bar{n}_k}_{\text{mean occ. number}} \\ (= (e^{\beta\omega} - 1)^{-1} \text{ in th. bath})$$

• density of modes : ρ_M

$$\frac{N_{\text{Modes}}}{V_s} = \rho_M = \int d^3k \quad 1$$

• IF Lorentz invar. $\Rightarrow \rho_M = \infty$.

• IF UV cutoff: $|\vec{k}_{\text{max}}|^2 = \Lambda^2$

$$\Rightarrow \boxed{\rho_\Lambda = \Lambda^3}$$

In the absence of cosmic expansion,^{L7}
 Λ does not modify observables in QFT.


ex: • in a thermal bath:

$$\begin{aligned}\bar{m}_Q^\Lambda &= \int d^3k \bar{m}_k \\ &= \bar{m}_Q^{\Lambda=0} + \underbrace{O(e^{-\beta\Lambda})}_{\text{negligible}} \\ &\quad \text{when } T \ll \Lambda.\end{aligned}$$

• Casimir. (vacuum effect)

$$F_\Lambda = F_{\Lambda=0} + O\left(\frac{1}{L\Lambda}\right)^\alpha ; \alpha \geq 1$$

L : size of the cavity.

• However  to large boosts!
(Unruh effect).

- In expanding RW universes, L^3

$$ds^2 = -dt^2 + a^2(t) \underbrace{d\underline{x}^2}_{\text{comoving coord.}}$$

- Modes still exist:

$$\hat{\phi} = \underbrace{\int d^3k}_{\text{SOM}} e^{i\underline{k} \cdot \underline{x}} \underline{\phi}_{\underline{k}}(t)$$

$$\hat{H} = \underbrace{\int d^3k}_{\text{POM}} \hat{H}_{\underline{k}} ; |\Psi\rangle = \underbrace{\prod_{\underline{k}}}_{\text{POM}} |\Psi_{\underline{k}}\rangle$$

- $\omega_{\underline{k}}^2 = \underline{k}^2 - \frac{a''}{a}$, conformal frequency
($dz = \frac{dt}{a}$)

- proper frequency $\Omega(z) = \frac{\omega_{\underline{k}}}{a(z)}$, $\vec{p} = \frac{\vec{k}}{a(z)}$

\Rightarrow Modes are labeled by conf. momentum \vec{k} .
and not by physical mom. \vec{p} .

• proper volume: $V_p(t) = a^3(t) V_{\text{conf.}}$ [3]

⇒ particle density

$$\bar{n}_Q(t) = n_Q(t_0) \left(\frac{a_0}{a(t)} \right)^3 \propto \frac{1}{a^3}$$

⇒ What about mode density?

• if today $\rho_0 = (\pi_{pl})^3$; and no mode creation

⇒ the density of these modes was:

$$\rho_{\pi}(t) = \rho_0 \left(\frac{a_0}{a(t)} \right)^3$$

$$= \rho_0 (10^{10})^3 \text{ at PNs}$$

$$= \rho_0 (10^{50})^3 \text{ at the beginning of}$$

The Forster/Jacobson proposal (2004). [11]

- growing lattice:

"a pair of points
is added at t_{i+1} ."



- \Rightarrow a pair of modes is "created" at t_{i+1} .
- choose some state for these "birth modes"
- compute the "growth-induced particle creation".

Remarks

- mode $\left\{ \begin{array}{l} \text{addition} \\ \text{creation} \end{array} \right.$ is specified from the outset.
- the state is also chosen " " ".
- virtue of their analysis:
reveal the problems raised by
formalizing mode creation.

The dissipative model

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- Why dissipation?
- Dissipation is GENERIC in the absence of Lorentz invariance:
- When increasing the energy, **NEW** channels open. \Rightarrow **dissipation**.

Ex.: GZK, above a critical energy, protons scattered by γ_{CMB} "decay" into pions.

$$\text{Im } \Sigma_k \neq 0 \quad \text{for } E_k > \bar{E}_c.$$

So:

- If there exist Planckian d^3 of freedom, when the mode energy $\omega_p = \frac{k}{a} \sim \pi_{\text{Planck}}$ NEW channels open causing $\text{Im } \Sigma_k \neq 0$.

How to handle **dissipation** in QFT.

I Free modes \Rightarrow Wronskian conserved.

II Interactions with new d^0 of freedom Ψ

III Simple models $\rightarrow \text{Im} \Sigma_h \neq 0$.

- unitarity
- the **relevant** kernels (N and D)
- the limit $g^2 t_{in} \rightarrow \infty$, $g^2 \rightarrow 0$.

IV Application to QFT and cosmology.

- Stationarity of Λ_{UV}
dynamically realized by
that of D and N .

I Free evolution

(1)

modes obeys:

$$\left[\frac{\partial^2}{\partial t^2} + \omega_k^2 \right] \phi_k(t) = 0 \quad (1)$$

- second order
- homogeneous: no source
- time reversible: no odd term in $\frac{\partial}{\partial t}$

$$\Rightarrow \text{ETC: } \frac{1}{i} [\dot{\phi}_k(t), \phi_k(t)] = 1 \quad (\text{all time})$$

$$\Leftrightarrow [\hat{a}_k, \hat{a}_k^\dagger] = 1$$

because Wronskian $\phi_k^* \overset{\leftarrow}{\partial_t} \phi_k = 1$ all time

But: IF $\gamma \frac{\partial}{\partial t}$ present in (1)

\Rightarrow no Wronskian conservation

I Free evolution

(16)

modes obeys:

$$\left[\frac{\partial^2}{\partial t^2} + \omega_k^2 \right] \phi_k(t) = 0 \quad (1)$$

- second order
- homogeneous: no source
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$$\Rightarrow \text{ETC: } \frac{1}{i} [\phi_k(t), \dot{\phi}_k(t)] = 1 \quad (\text{all time})$$

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because Wronskian $\phi_k^* \overleftrightarrow{\partial}_t \phi_k = 1$ all time

But: IF $\gamma \frac{\partial}{\partial t}$ present in (1)

\Rightarrow no Wronskian conservation

\Rightarrow unitarity? ETC?

II Interactions with Ψ

(15)

$$\bullet \mathcal{L}_T = \mathcal{L}_\phi + \mathcal{L}_\Psi + \mathcal{L}_{\phi, \Psi}$$

When: Lorentz + isotropy preserved,

$$\begin{aligned} \Rightarrow G(x, x') &\doteq T_2 [\rho_T \phi_k \phi_k] \\ &= \int d^3k e^{-ik(x-x')} G_k(t, t') \end{aligned}$$

$G_k(t, t')$

- c-number
- always exists (All $\hat{\Psi}$)
(UNLIKE modes, Bogliubov coef, ...)

• in the true interacting vacuum, in T -state,

$$G_F(\omega, k) = [-\omega^2 + k^2 + \Sigma_F(\omega, k)]^{-1}$$

dissipation: $\text{Im} \Sigma_{0S} \neq 0$.

II Interactions with ψ

(15)

$$\bullet \mathcal{L}_T = \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_{\phi,\psi}$$

When: hom + isotropy preserved,

$$\Rightarrow G(x, x') \doteq T_2 \left[\rho \phi \phi_H \right]$$

$$= \left(\frac{d^3k}{(2\pi)^3} e^{-ik(x-x')} G_H(k, \omega) \right)$$

$G_H(k, \omega)$

- c-number

always exists (All $\hat{\psi}$)

(UNLIKE modes,

Boglinbov coef.)

• in the free interacting vacuum, in Thir, ω ,

$$G_F(\omega, k) = [-\omega^2 + k^2 + \Sigma_F(\omega, k)]^{-1}$$

dissipation: $\text{Im} \Sigma_{0s} \neq 0$.

III Simple hamiltonian models ≤ 5
 with $\text{Im} \Sigma_k \neq 0$.

- QIT, (Cold. legget)
- decoherence

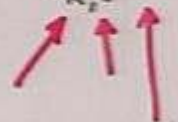
Assume:

$$S_T = \int d^3k \underbrace{S_T(k)}_{\text{indep. from each other.}} ;$$

$$S_T(k) = \frac{1}{2} \int dt [|\partial_t \phi_k|^2 + k^2 |\phi_k|^2]$$

$$+ \sum_i \frac{1}{2} \int dt [|\partial_t \psi_{k,i}|^2 + \Omega_{k,i}^2 |\psi_{k,i}|^2]$$

$$- \sum_i \int dt g_{k,i}(t) \phi_k \psi_{k,i}^*$$



ANY dep. is allowed

EoM:

16'

$$\begin{cases} [-\partial_t^2 + \omega_k^2] \phi_k = \sum_i g_{i,k} \psi_{k,i} \\ \underbrace{[-\partial_t^2 + \Omega_{k,i}^2]}_{\equiv \square_{i,k}} \psi_{k,i} = g_{i,k} \phi_k \end{cases}$$

Solution:

retarded Green f. : $\square R = \delta$

$$\psi_{k,i}(t) = \underbrace{\psi_{k,i}^{(0)}(t)}_{\text{homogeneous solution}} + \int dt' \overline{R_{ki}^{(0)}(t', t)} (g_{i,k} \phi_k(t'))$$

$$[-\partial_t^2 + \omega_k^2] \phi_k = \sum_i g_{i,k} \psi_{k,i}^{(0)} + \underbrace{\sum_i g_{i,k} \int dt' R_{ki}^{(0)} g_{i,k} \phi_k(t')}_{\text{non local in } t}$$

\Rightarrow expand:

$$\left(= \underline{\lambda_1(t) \phi_k(t)} + \underline{\lambda_1 \partial_t^2 \phi_k} + \underline{\lambda_2 \partial_t^4 \phi_k} + \underline{\lambda_3 \dots} \right)$$

Even terms: mass^2 , wave function z , dispersion

Odd terms: dissipation (generically present).

$$\begin{cases} [-\partial_t^2 + \omega_k^2] \phi_k = \sum_i g_{i,k} \psi_{k,i} \\ \underbrace{[-\partial_t^2 + \Omega_{k,i}^2]}_{\equiv \square_{i,k}} \psi_{k,i} = g_{i,k} \phi_k \end{cases}$$

Solution:

retarded Green f. : $\square R = \delta$

$$\psi_{k,i}^{(t)} = \underbrace{\psi_{k,i}^{(co)}(t)}_{\text{homogeneous solution}} + \int dt' \overline{R_{ki}^{(co)}(t', t)} (g_{i,k} \phi_k(t'))$$

$$[-\partial_t^2 + \omega_k^2] \phi_k = \sum_i g_{i,k} \psi_{k,i}^{(co)} + \sum_i g_{i,k} \underbrace{\int dt' R_{ki}^{(co)} g_{i,k} \phi_k(t')}_{\text{non local in } t}$$

\Rightarrow expand:

$$\left(= \underline{\lambda_0(t) \phi_k(t)} + \underline{\lambda_1 \partial_t^2 \phi_k} + \underline{\lambda_2 \partial_t^4 \phi_k} + \underline{\lambda_3 \dots} \right)$$

Even terms: $mass^2$, wave function ω_k , dispersion

Odd terms: dissipation (generically present).

The general solution:

16''

$$\phi_k(t) = \underbrace{\phi^d(t)}_{\text{DECAYING}} + \underbrace{\int dt' G_{\tilde{z}}(t, t') \Psi_0(t')}_{\text{DRIVEN}} \quad (2)$$

where

- * $\phi^d(t)$:
 - exponentially decays $e^{-\gamma(t-t_0)}$
 - depends on **IC** on ϕ

- * $G_{\tilde{z}}(t, t')$: the dressed retarded Green f.

$$\Delta_g G_{\tilde{z}} = \delta(t-t')$$

$$(\Delta_g \phi^d = 0)$$

- * the **IC** of Ψ_0 are in the driven solution

$$\Psi_0 = \sum_i g_i \Psi_i^{(0)}$$

- * (1) : the exact exp. of the Heisenberg op. $\hat{\phi}_k(t)$

Rem: if $\int \phi_{,in}$, $\int \psi_{,in}$ Gaussian. (19)

$\Rightarrow G_c$ and G_a are the only
 ϕ -observable blocks.

\Rightarrow only $\begin{cases} N(t_1, t_2) \\ D(t_1, t_2) \end{cases}$ matter.

\Rightarrow equivalent class.

\Rightarrow How much should be
known (imposed)
on ψ ?

Fourier analysis

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• Suppose 1. g_{ik} , ω_k^2 , Ω_{ik}^2 constant.

2. $D = D(t_1 - t_2)$

$N = N(t_1 - t_2)$

⇒ Stationary situation.

$$\Rightarrow \tilde{N}(\omega) = \tilde{D}(\omega) \underbrace{(2n(\omega) + 1)}_{\substack{\uparrow \\ \text{FD}}} \text{sign}(\omega)$$

↳ mean occupation number.
(thermal state)

$$\Rightarrow \tilde{G}_a(\omega) = \tilde{G}_c(\omega) (2n + 1) \text{sign}(\omega)$$

meaning: the ϕ field has reached equilibrium with its environment

Proof:

(20)

$$\tilde{G}_D = |\tilde{G}_{\text{ref}}(\omega)|^2 \tilde{D}$$

$$\tilde{G}_A = |\tilde{G}_{\text{ref}}(\omega)|^2 \tilde{N}$$

(when the decaying part can be neglected)

⇒ 2 consequences.

1. in the (true) vacuum: $M=0$,

$$\Rightarrow \tilde{G}_C(\omega) = \tilde{G}_A(\omega) \operatorname{sign}(\omega)$$

$$\begin{aligned}\Rightarrow \tilde{G}_W(\omega) &= \tilde{G}_C + \tilde{G}_A \\ &= \tilde{G}_C \Theta(\omega)\end{aligned}$$

only positive frequency.

(• non perturbative in g^2)

$$S_T = \int d^3k \underbrace{S_T(k)}_{\text{arrow}}$$

2. this obtains for ALL
but Ψ obeying the vacuum FID

cannot be
stochastic variables
(not even a subset of
Quantum and class. ones)

But. $\hat{\Psi} = \sum_i g_i \hat{\Psi}_i^{(0)}$ can be
any composite operator
made of canon. $\{\hat{\chi}_i\}$

linearity not necessary

but Ψ obeying the vacuum F

Ψ_i cannot be
stochastic variables

(not even a cocktail of
Quantum and classic ones.)

But. $\hat{\Psi} = \sum_i g_i \hat{\Psi}_i^{(0)}$ can be
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linearity not necessary!

but obeying the vacuum FJ

$\Rightarrow \Psi_i$ cannot be
stochastic variables

(nor even a cocktail of
Quantum and classic ones.)

But. $\hat{\Psi} = \sum_i g_i \hat{\Psi}_i^{(0)}$ can be
any composite operator
made of canon. $\{\hat{\chi}_i\}$

linearity not necessary!

Rem: when $\hat{\Psi}$ is composite,

- $\Sigma(\omega, k) = \text{series in } g^2$
- higher order correlation functions $\neq 0$.
- $\tilde{G}_a = \tilde{G}_c (2m+1) \text{ sign }(\omega)$
still exact.

The limit: $g^2 t_{in} \rightarrow \infty$ (1) (22)

followed by: $g^2 \rightarrow 0$ (2)

(1) implies that *only* the driven part remains.

$\Rightarrow \hat{\sigma}_n^1$ is a composite op. in \mathcal{Y}_0 Hilbert space.

(2) the free limit applied to this composite op.

• as $\lim_{g^2 \rightarrow 0} \hat{A} = g^2 \rightarrow 0$
 $\hat{B} = g^2 \rightarrow 0$

• But $\left\{ \begin{array}{l} \hat{G}_0 = i \hat{G}_0^* \hat{J} \rightarrow \frac{1}{2\omega_n} \\ \hat{G}_{20} = i \hat{G}_{20}^* \hat{A} \rightarrow \frac{2\omega_n + 1}{2\omega_n} \end{array} \right.$

i.e. the USUAL values of \hat{G} for free modes

The limit: $g^2 \frac{1}{\omega_n} \rightarrow \infty$ (1) (22)

followed by: $g^2 \rightarrow 0$ (2)

(1) implies that *only* the driven part remains.

$\Rightarrow \hat{\phi}_H$ is a composite op. in Ψ_0 Hilbert Space.

(2) the free limit applied to this compos. op.

• we have $\begin{cases} \tilde{N} \rightarrow 0 \\ \tilde{D} \rightarrow 0 \end{cases}$

• But $\begin{cases} \tilde{D} = |\tilde{G}_{out}|^2 \tilde{D} \rightarrow \frac{1}{2\omega_H} \\ \tilde{N} = |\tilde{G}_{out}|^2 \tilde{N} \rightarrow \frac{2N\omega + 1}{2\omega_H} \end{cases}$

ie. the *values* of \tilde{G} for free modes

$\Rightarrow \hat{\phi}_H$, composite op., behaves

as if it belongs to the Hilbert Space!

$\Rightarrow \hat{\phi}_H^{\sim}$ is a composite op. in \mathcal{H}_0 Hilbert space.

(2) the free limit applied to this compos. op.

• one has
$$\begin{cases} \tilde{N} \propto g^2 \rightarrow 0 \\ \tilde{D} \propto g^2 \rightarrow 0 \end{cases}$$

• But
$$\begin{cases} \tilde{G}_C = |\tilde{G}_{\text{ret}}|^2 \tilde{D} \rightarrow \frac{1}{2\omega_H} \\ \tilde{G}_{AC} = |\tilde{G}_{\text{ret}}|^2 \tilde{N} \rightarrow \frac{2\omega + 1}{2\omega_H} \end{cases}$$

i.e. the USUAL values of \tilde{G} for free modes.

$\rightarrow \hat{\phi}_H^{\sim}$, composite op., behaves
as if it possesses its own Hilbert space!

(1) implies that only the driven part remains.
 $\Rightarrow \hat{\phi}_H^1$ is a composite op. in \mathcal{H}_0 Hilbert space.

(2) the free limit applied to this compos. op.

• one has
$$\begin{cases} \tilde{N} \propto g^2 \rightarrow 0 \\ \tilde{D} \propto g^2 \rightarrow 0 \end{cases}$$

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i.e. the USUAL values of \tilde{G} for free modes.

$\Rightarrow \hat{\phi}_H^1$, composite op., behaves as if it possesses its own Hilbert space!

- **Implication**

by analyzing $\hat{\phi}$ -observables
we have no access
to the $\hat{\phi}$ -constituents $\{\hat{X}_i\}$
in this limit.

• ————— •

- **Remark:**

to get $\text{Im } \Sigma \neq 0$, a dense
set of Ψ_i is needed.

by analyzing $\hat{\phi}$ -observables
we have no access
to the $\hat{\phi}$ -constituents $\{\hat{X}_i\}$
in this limit.

• ————— •

• Remark:

to get $\text{Im} \Sigma \neq 0$, a dense
set of ψ_i is needed.

$$\Rightarrow \sum_i \psi_i \rightarrow \int dy \psi(y)$$

"extra dimension" **Brane World
Scenario.**



$$S_T = \int d^3k \underbrace{S_T(k)}_{\psi_i}$$

ψ_i ψ_i ψ_i

$$S_T = \int d^3k \underbrace{S_T(k)}$$

$$\sum_i g_i \psi_i$$
$$\int dy \psi(y)$$

The basic $G_F = \tilde{G}_{ret}$.

(23)

- it obeys:

$$[-\omega^2 - \underbrace{2i\gamma_A \omega} + \omega_k^2] \tilde{G}_{ret}(\omega) = 1$$

$$\text{Im} \tilde{\Sigma}_{ret}(\omega)$$

- Analytic with two roots:

$$\omega_{\pm} = -i\gamma_k \pm \sqrt{\omega_k^2 - \gamma^2} \in \text{Im}^-$$

$$\gamma_k = g^2/4 > 0$$

- overdamped for $\omega_k < \gamma_k$
underdamped for $\omega_k > \gamma_k$



The basic $G_F = G_{ret}$.

- it obeys:

$$[-\omega^2 - \underbrace{2i\gamma_R\omega} + \omega_R^2] \tilde{G}_{ret}(\omega) = 1$$

$$\text{Im} \tilde{\Sigma}_{ret}(\omega)$$

- Analytic with two roots:

$$\omega_{\pm} = -i\gamma_R \pm \sqrt{\omega_R^2 - \gamma_R^2} \in \text{Im}^-$$

$\gamma_R = g^2/4 > 0$

- overdamped for $\omega_R < \gamma_R$
underdamped for $\omega_R > \gamma_R$

→
"as g increases"

