Title: Random bipartite entanglement from W and W-like states

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Abstract: We describe a protocol for distilling maximally entangled bipartite states between random pairs of parties (``random entanglement") from those sharing a tripartite W state, and show that this may be done at a higher rate than distillation of bipartite entanglement between specified pairs of parties (``specified entanglement"). Specifically, the optimal distillation rate for specified entanglement for the W has been previously shown to be the asymptotic entanglement of assistance of 0.92 EPR pairs per W, while our protocol can distill 1 EPR pair per W between random pairs of parties, which we conjecture to be optimal. We further extend this to a more general class of W-like states and show by increasing the number of parties in the protocol that there exist states with fixed lower-bounded distillable random entanglement for arbitrarily small specified entanglement. [Work done in collaboration with Benjamin Fortescue. Preprint available at http://arxiv.org/abs/quant-ph/0607126]

Random distillation of bipartite entanglement from tripartite W states

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Outline

- Motivation/Background

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- Summary of entanglement distillation

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- *Future Work*

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Question: What about W states?

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$$\max_{N\to\infty} \frac{M}{N} = S(\rho_A) = -\mathrm{tr}\rho_A \log_2 \rho_A$$

where $\rho_A = \mathrm{tr}_B \rho_{AB}$ and *S()* is the Von Neumann entropy.

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- Consider instead distillation to 2-party MES (EPR pairs).

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$$E_{AB}^{\infty}(\rho) = \max_{N \to \infty} \frac{N'}{N} = \min\{S(\rho_A), S(\rho_B)\} \quad [1]$$

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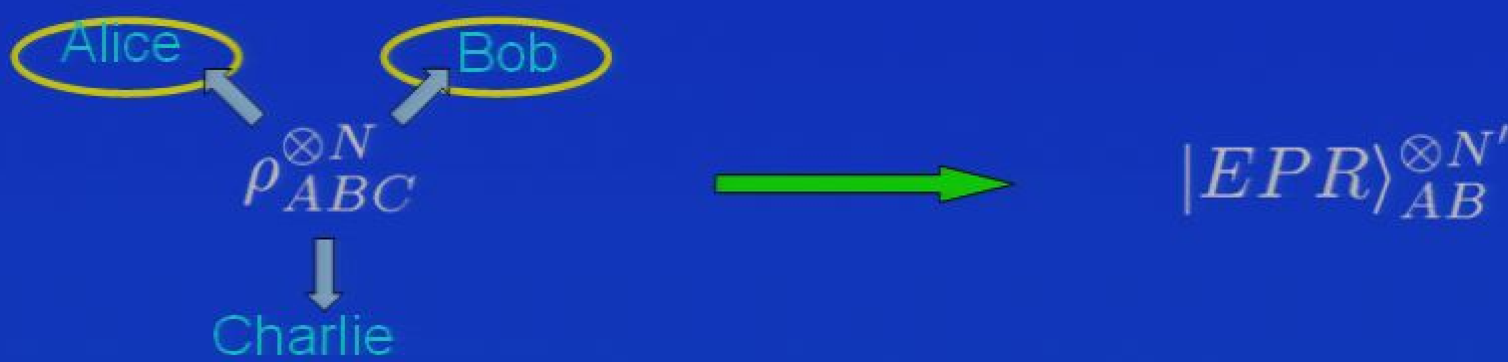

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Can *always* get an EPR from a W state

Classical analog: key distillation

Consider first distillation of random **classical key**:

Suppose Alice, Bob and Charlie share a W state:

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In summary, given a W-state, Alice, Bob and Charlie can get a one-bit key shared between a random pair of two parties.

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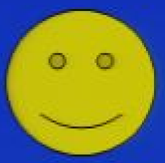

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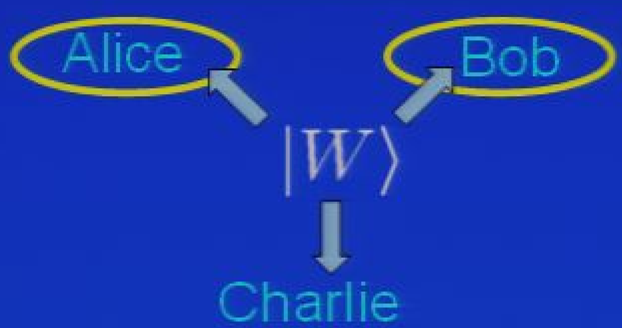
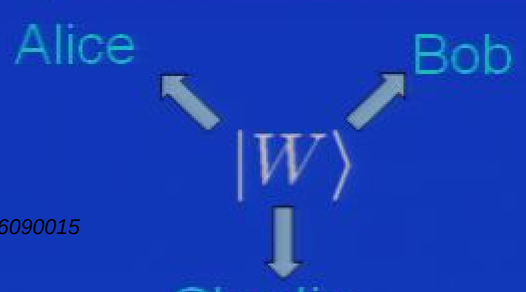


Asymptotic Rate=0.92 → $|EPR\rangle_{AB}$



Rate=1 →

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we can combine filtering with random distillation to achieve a rate

$$E_{rnd}(W') \geq 1 - (1-(a/c)^2)(b^2+c^2)\left(1 - H_2\left(\frac{b^2}{b^2+c^2}\right)\right)$$

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Outcome "B": Bob and Charlie share $\dfrac{b|10\rangle + c|01\rangle}{\sqrt{b^2+c^2}}$

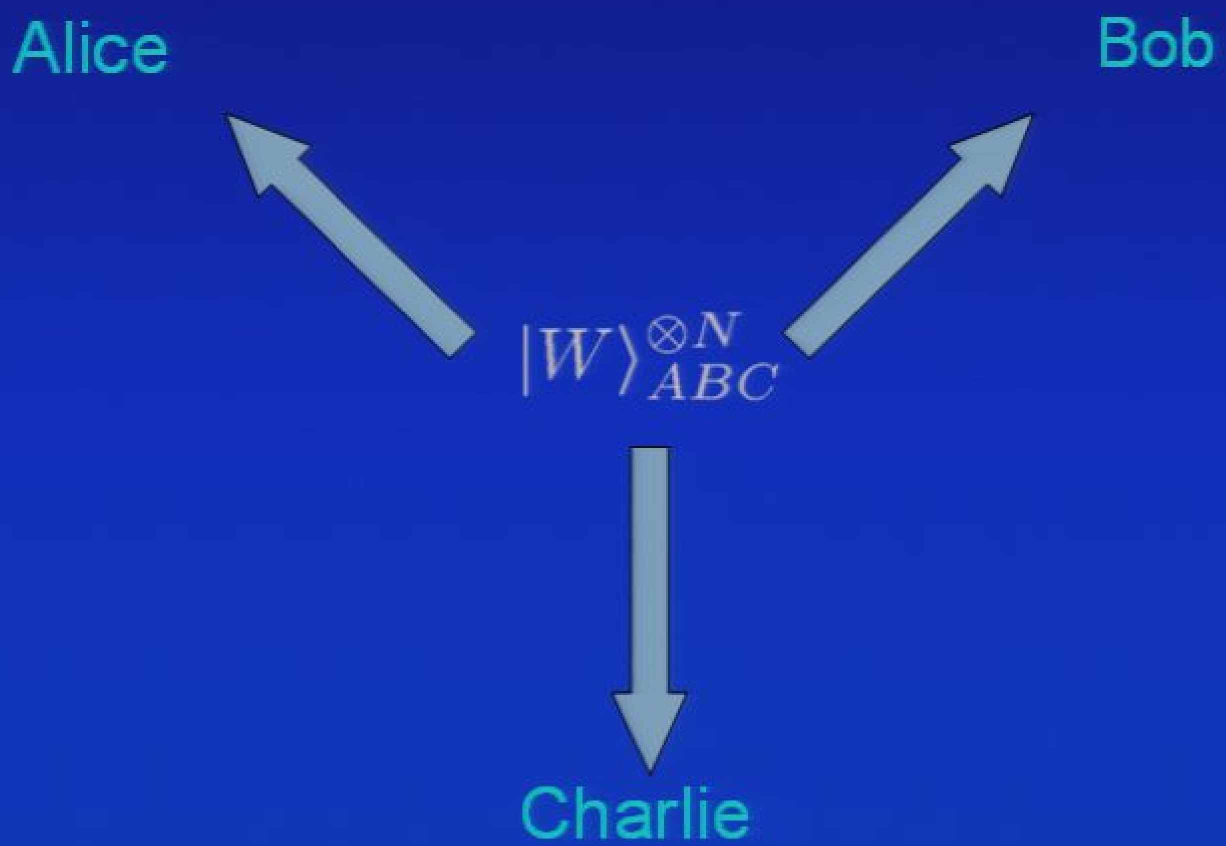
Outcome "A": State is

$$\frac{1}{\sqrt{2+(b/c)^2}}\left(\frac{\sqrt{3}b}{c}|W\rangle_{ABC} + \sqrt{2}|2\rangle_B\sqrt{1-(b/c)^2}|EPR\rangle_{AC}\right)$$

After a projection by Bob either an Alice-Charlie EPR or a *W* (from which an EPR can be distilled) is obtained.

Distillation of tripartite states from *W*

Can distill any 2⊗2⊗2 state ψ from *W* at rate ≥ 0.5



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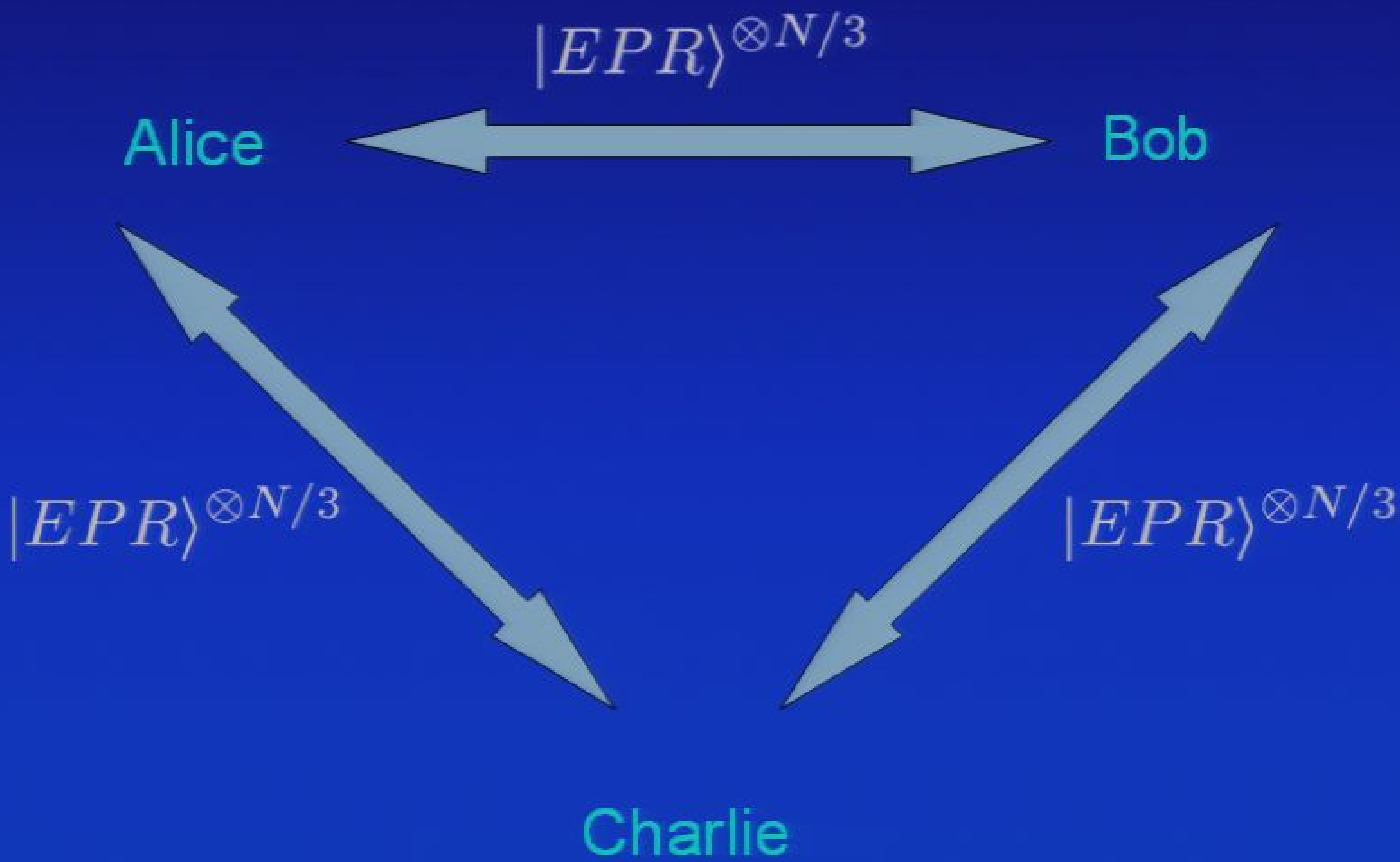

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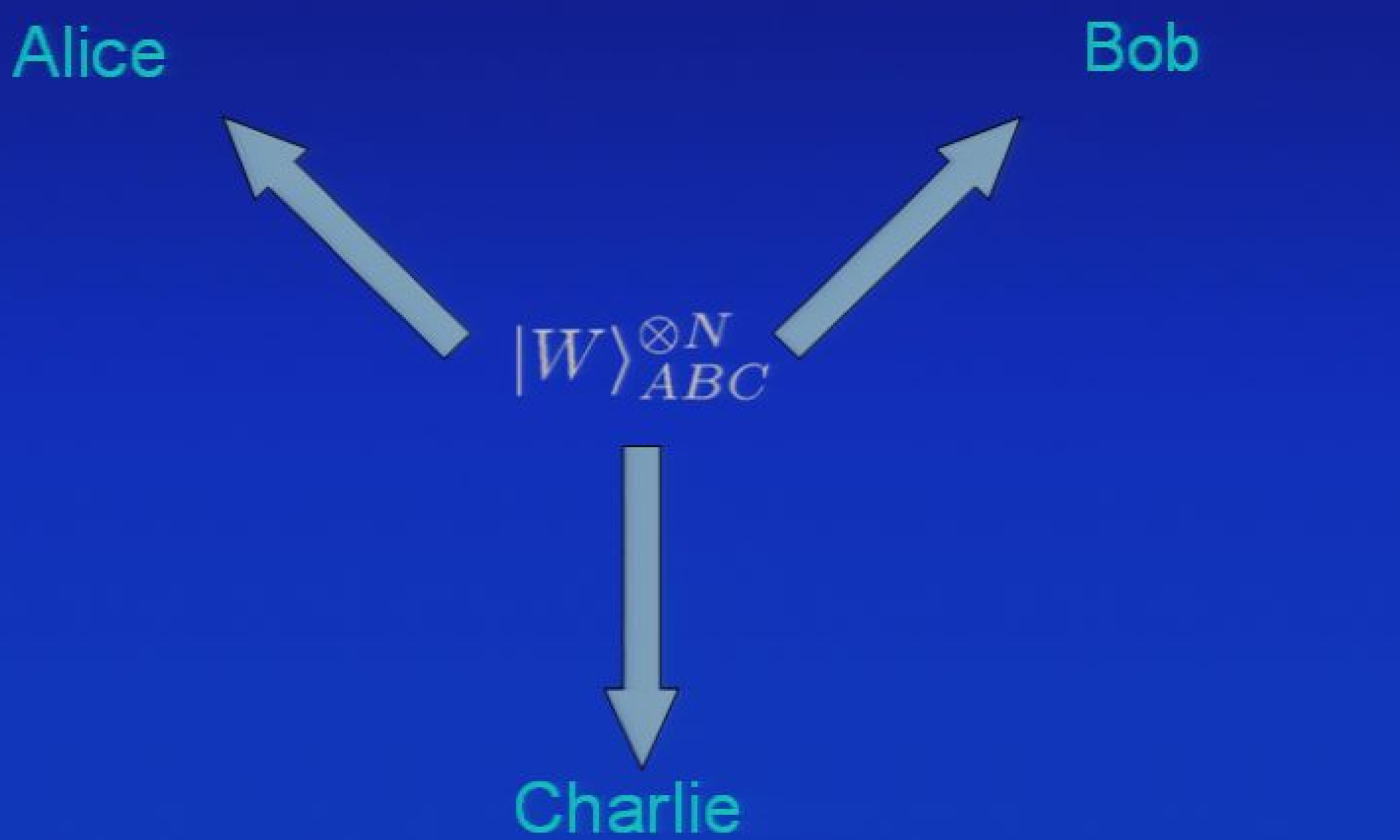
Can distill any 2⊗2⊗2 state ψ from *W* at rate ≥ 0.5 How?



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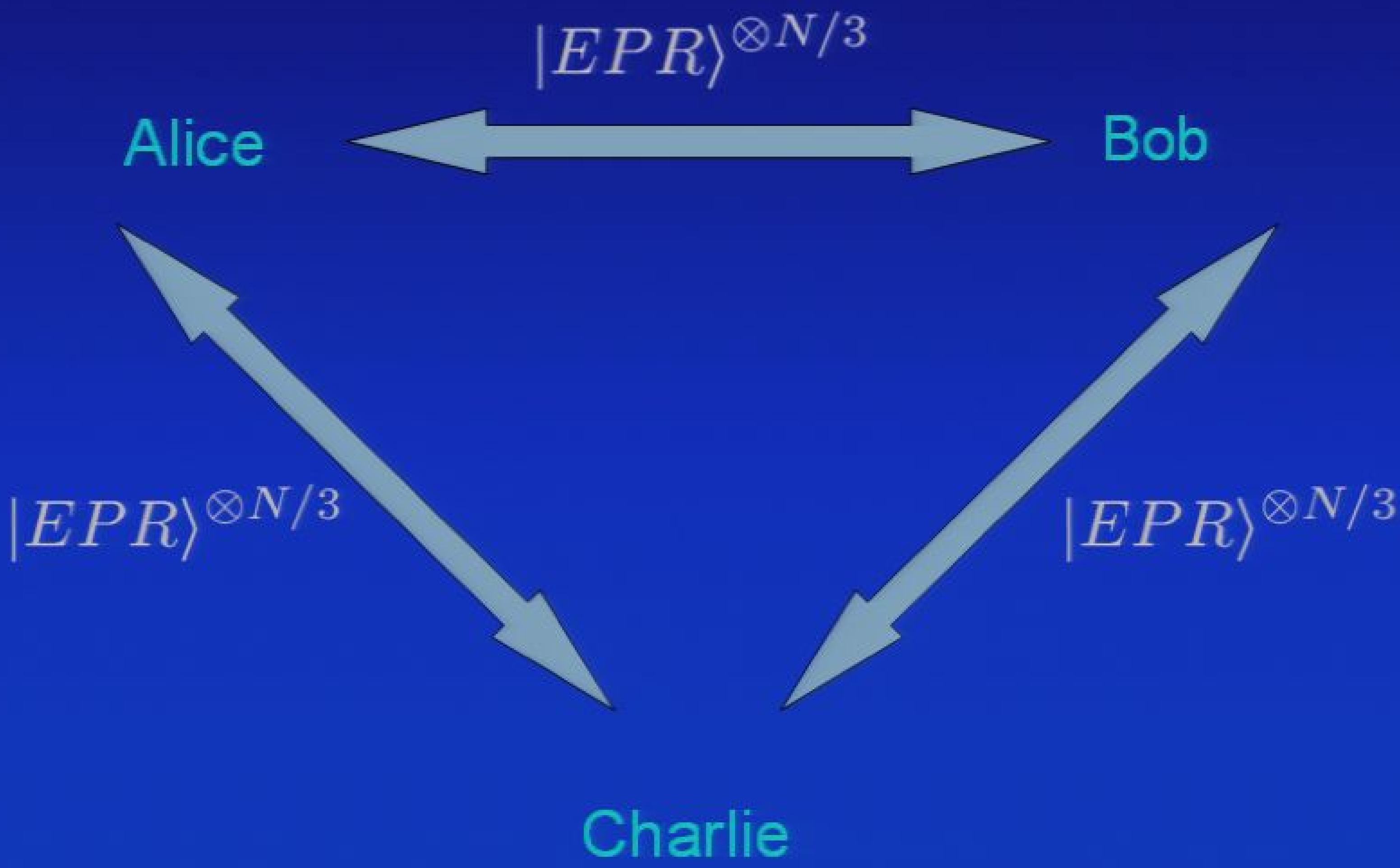

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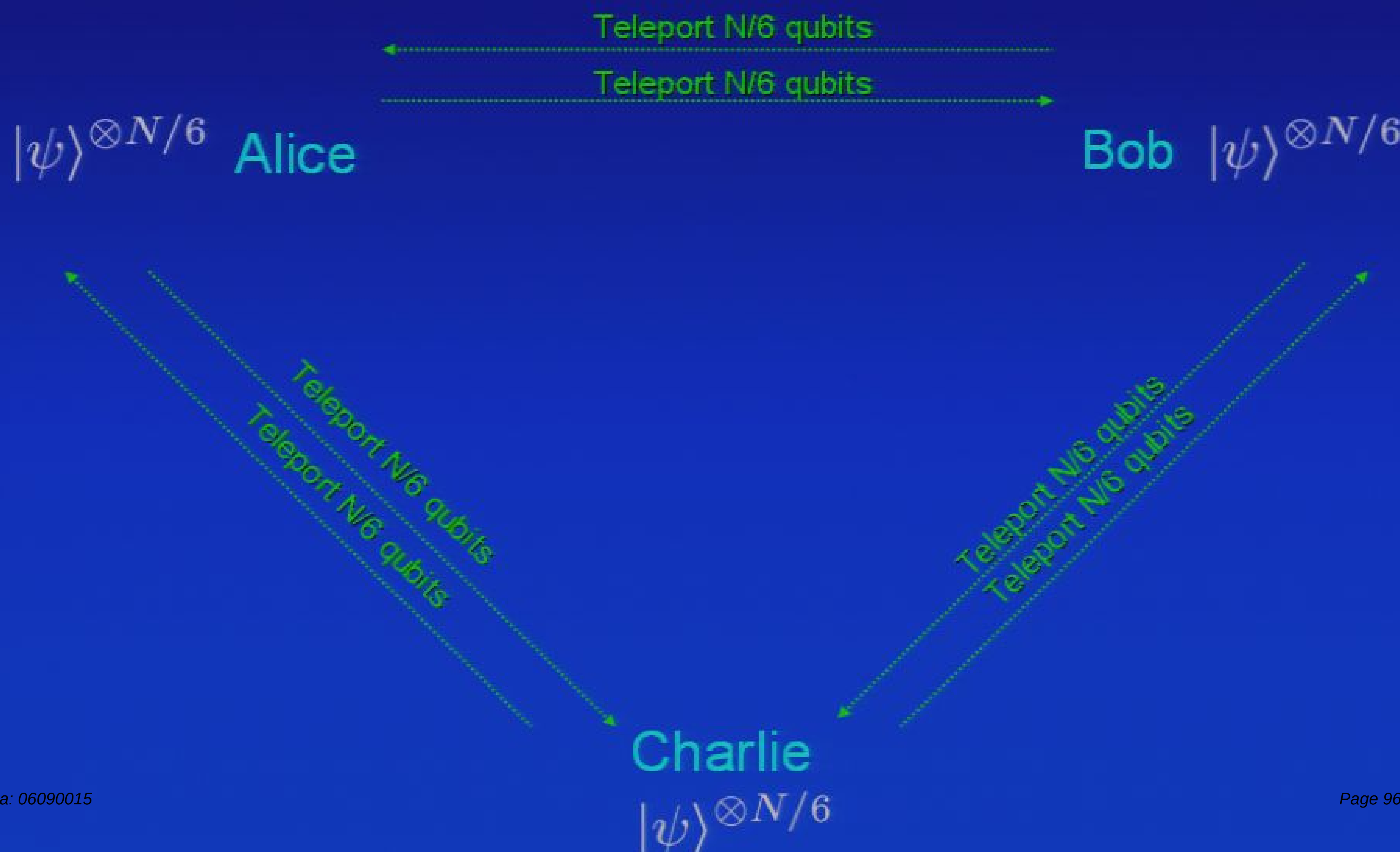

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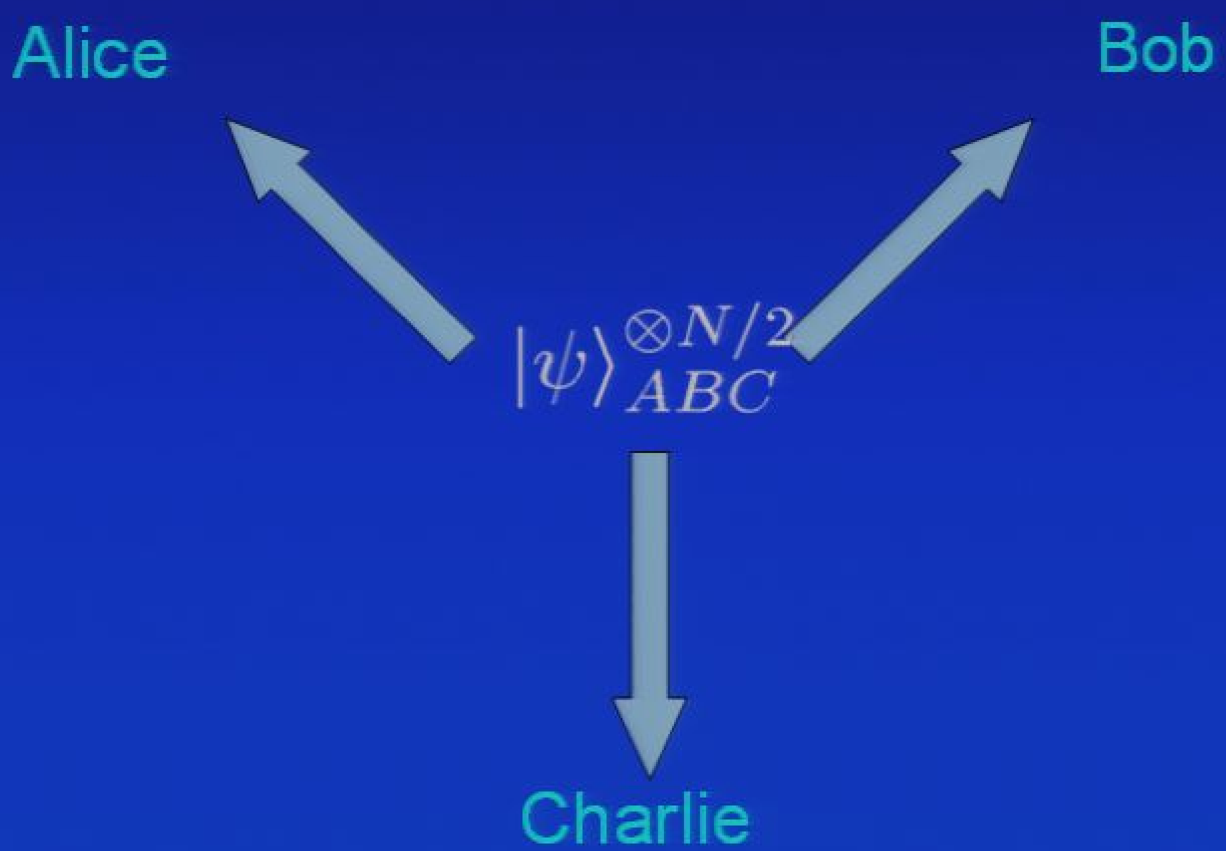

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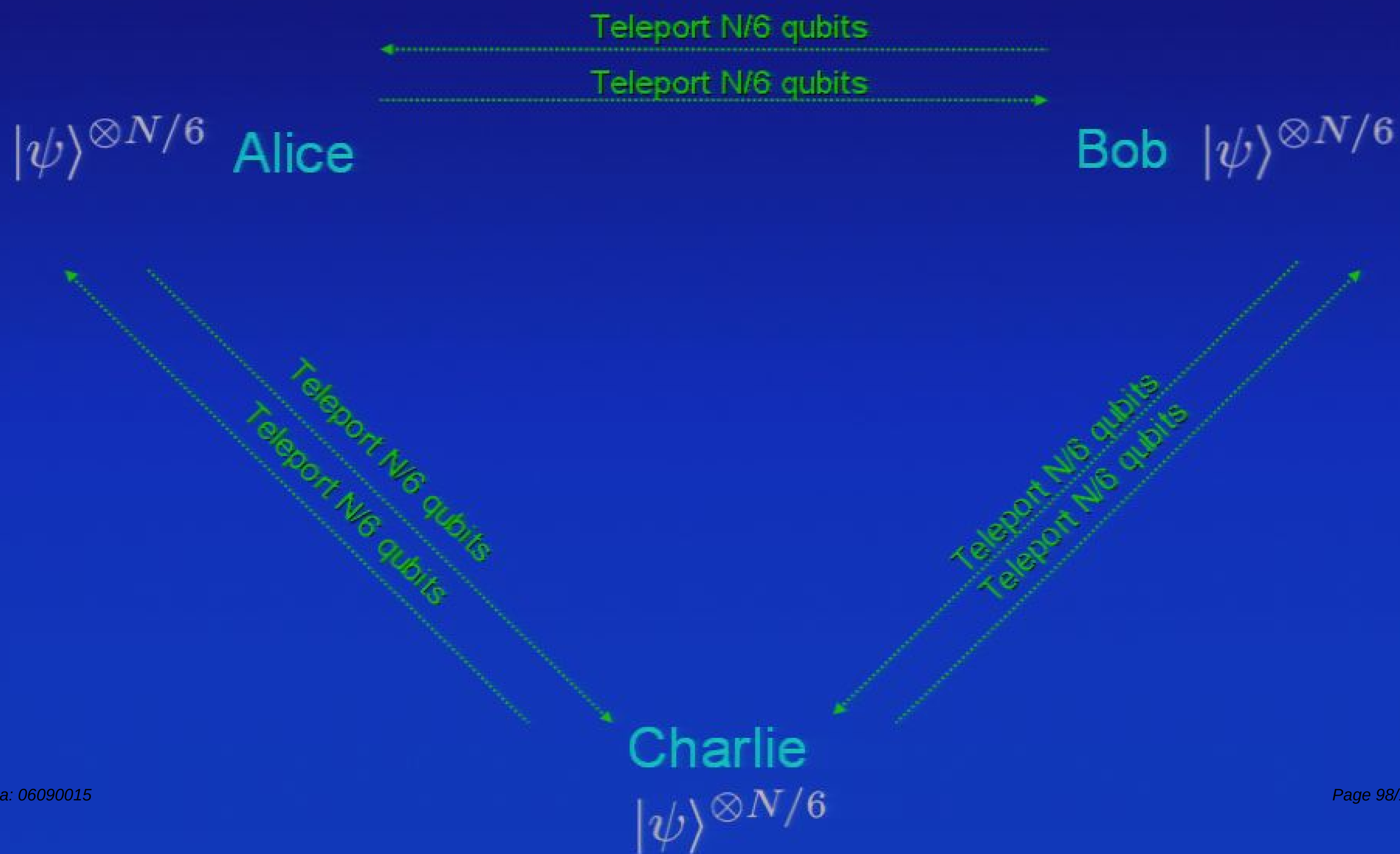

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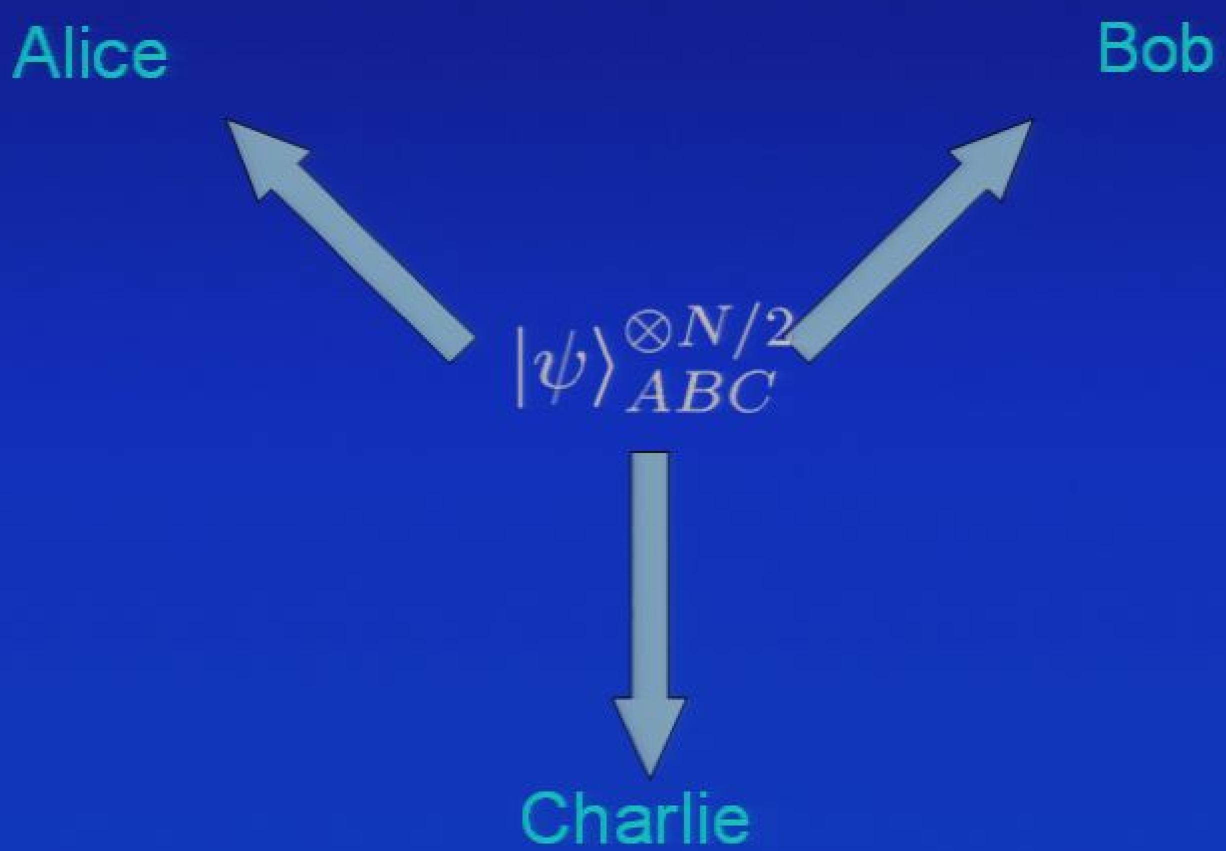

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(E_r = relative entropy of entanglement, E^{∞}_r=asymptotic E_r)

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Corollary:

$$E_{rnd}(\sigma_{ABC}) \leq E_r^{\infty}(\sigma_{ABC})$$

[4]

[3] N. Linden *et al.* Quant Inf Proc. 4, 241 (2005)

[4] Martin Plenio (private communication)

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Result:

$$\boldsymbol{E_{EPR}(W'(b=c)) > E_{rnd}(W'(b=c)),\ (0 < a < 1)}$$

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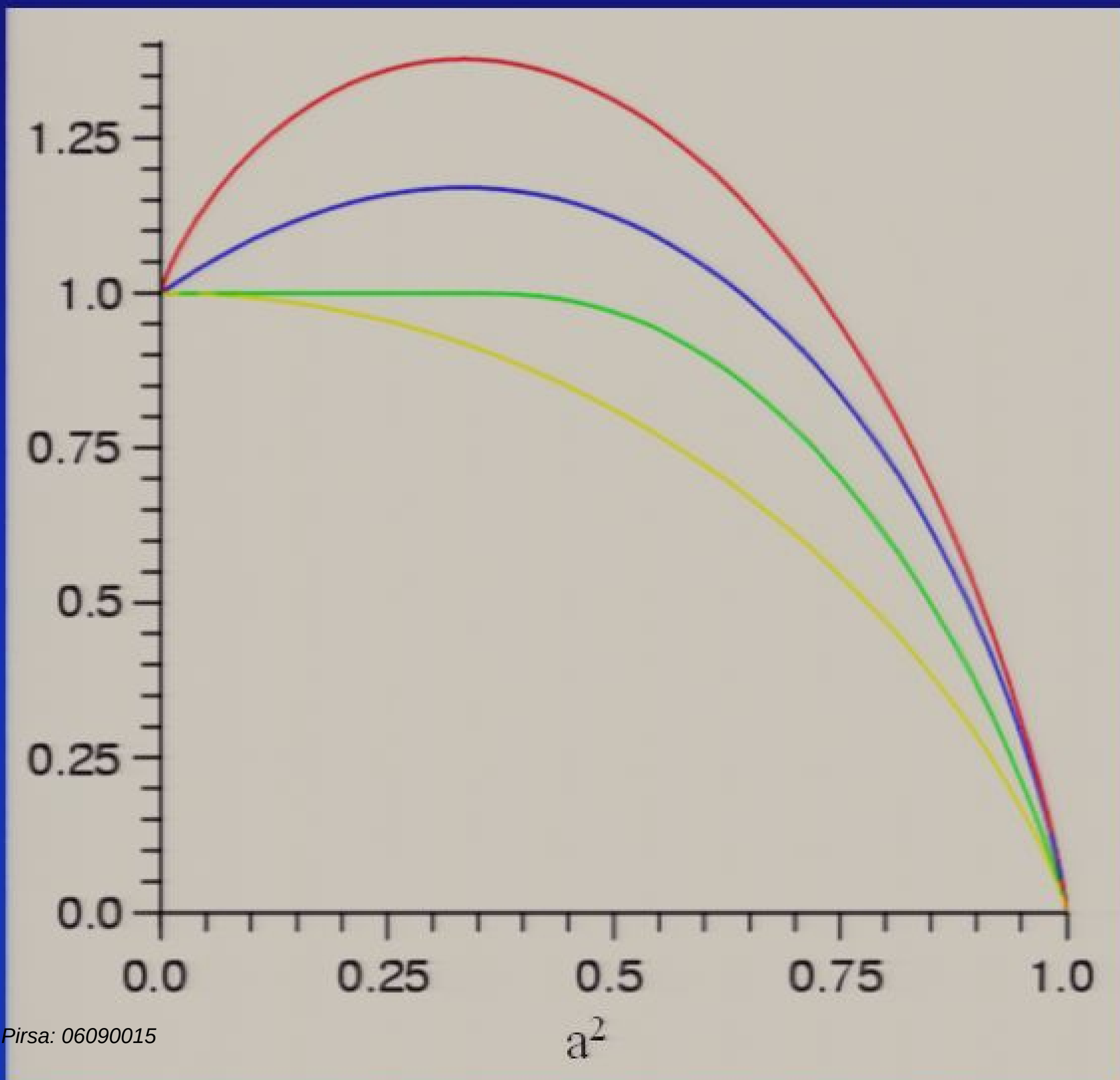

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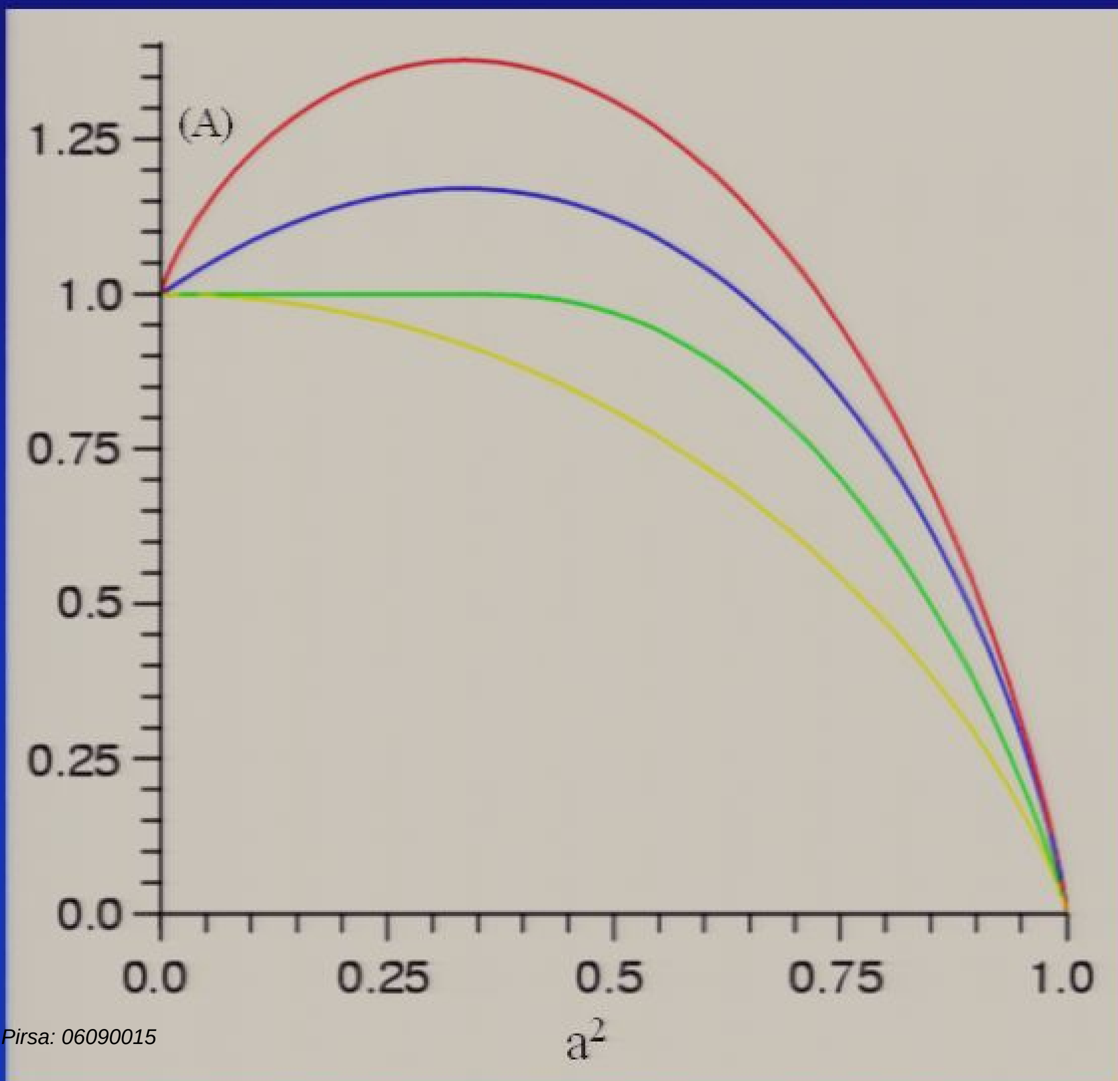

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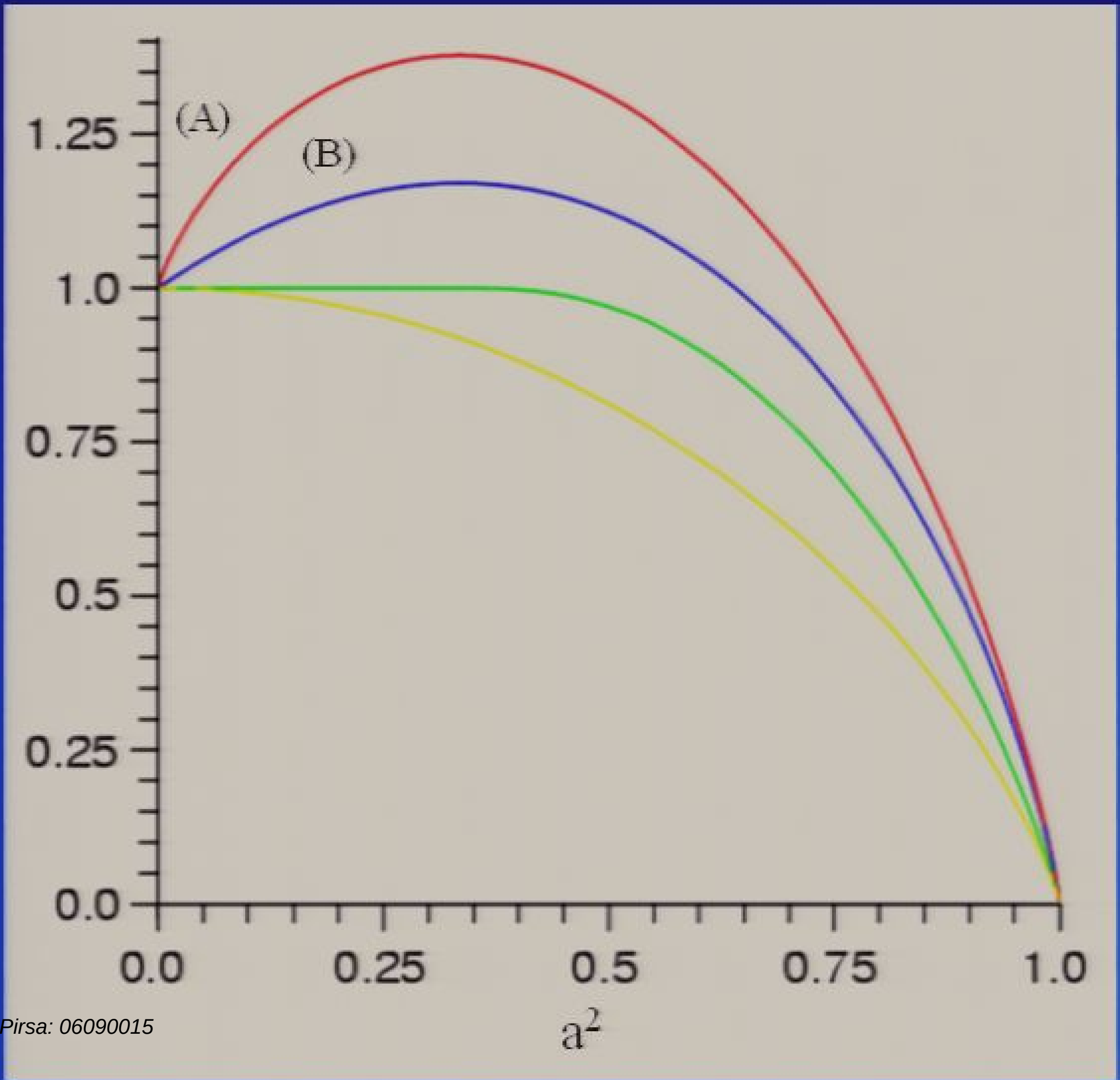

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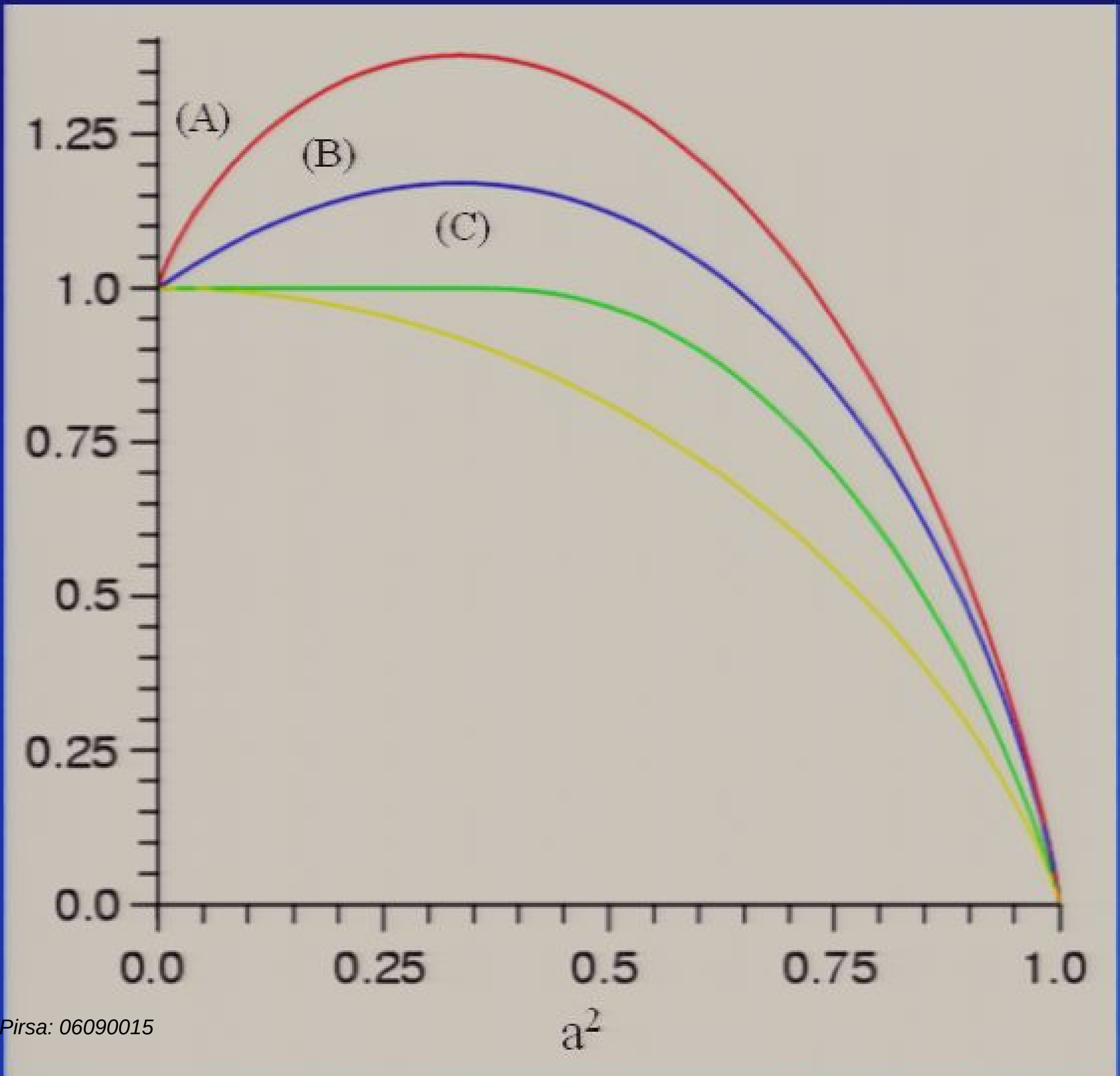

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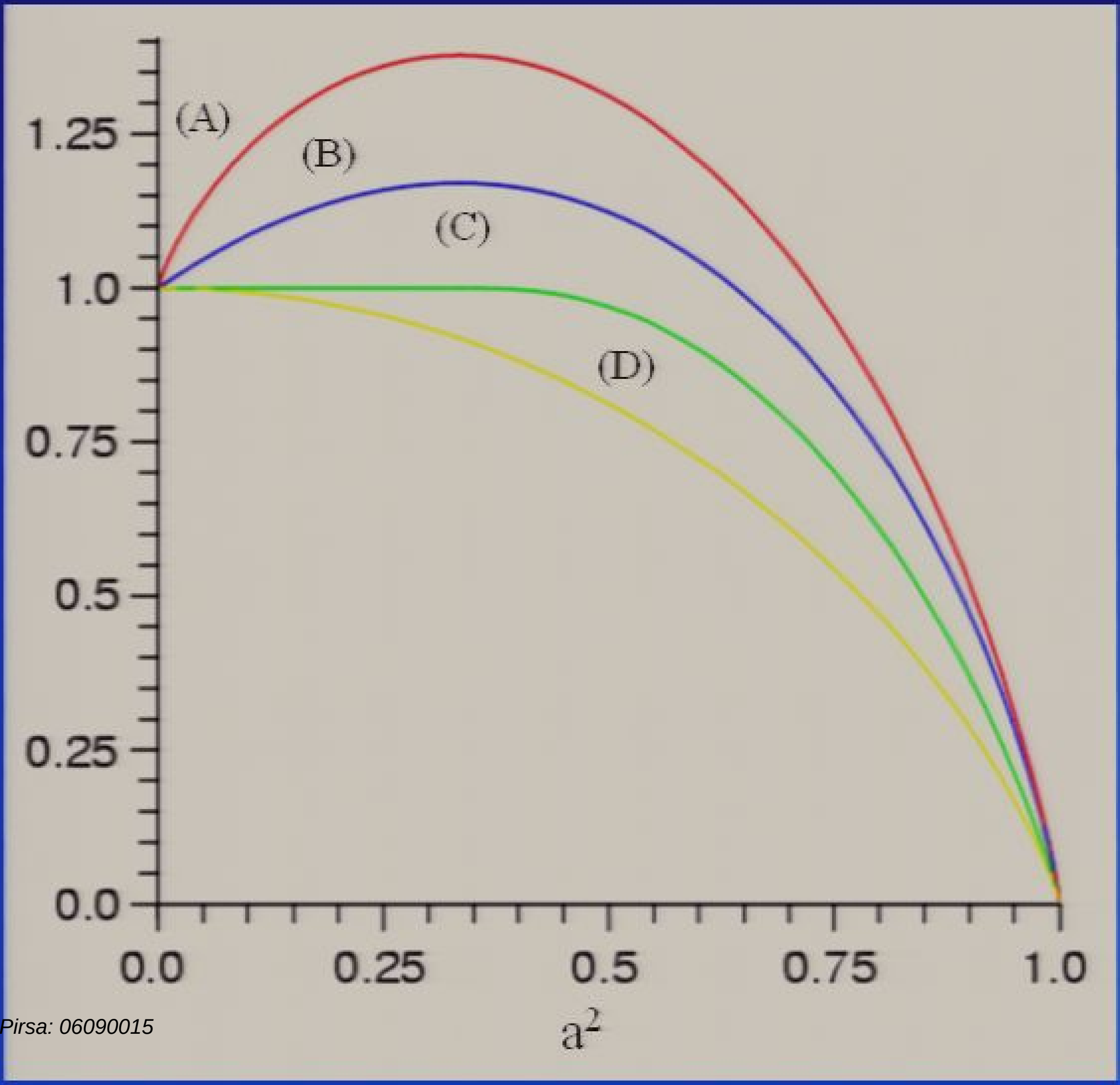

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(D) E_s – rate of EPR distillation to a specified pair of parties

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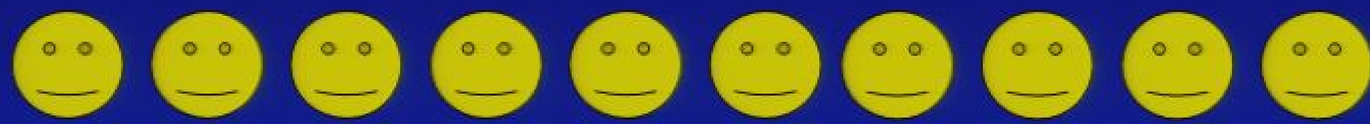

M parties

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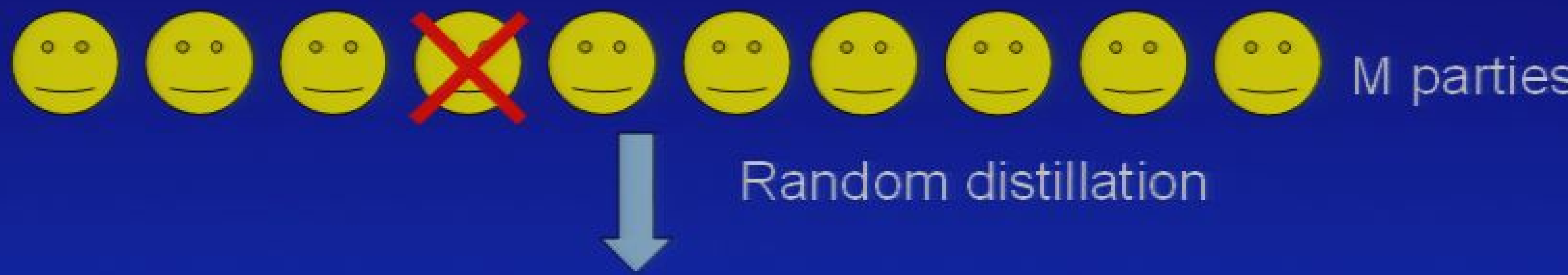

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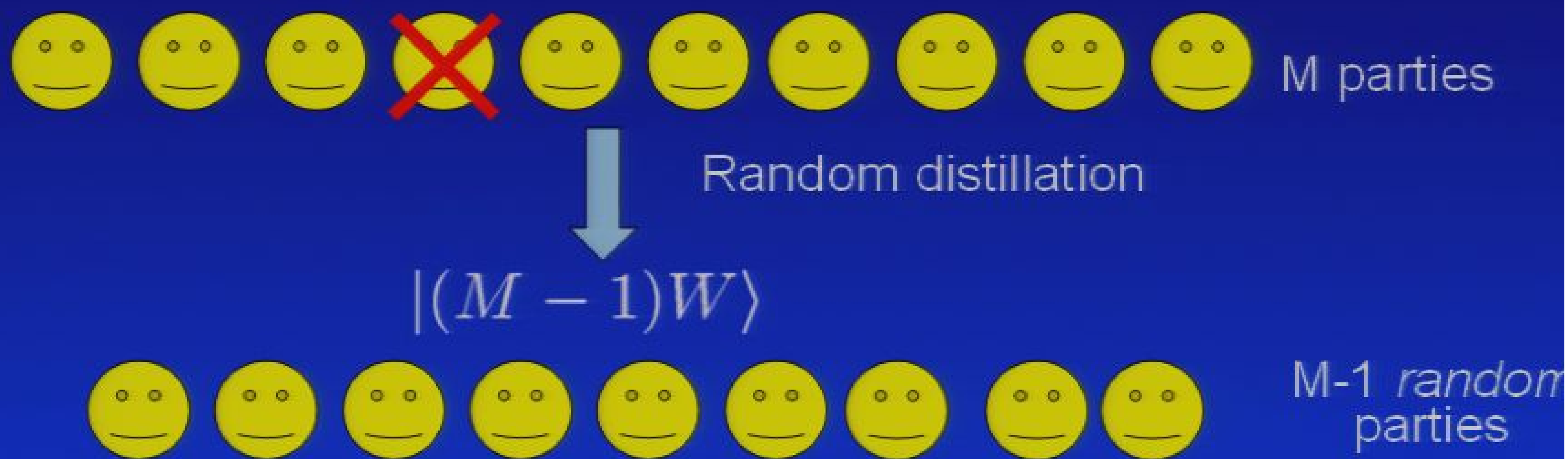

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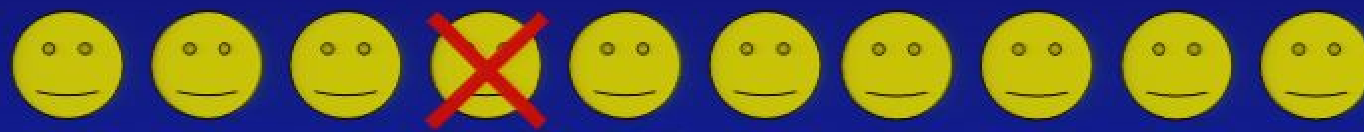

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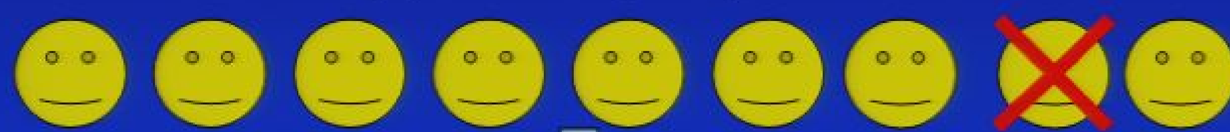

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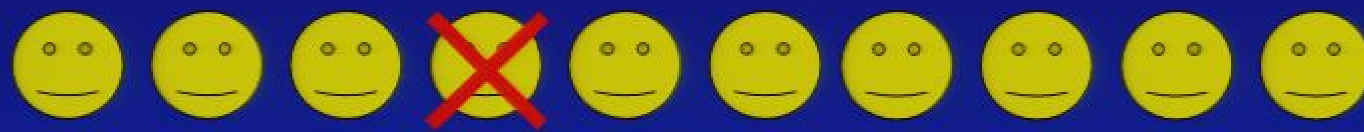

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- Distillation from N to M<N party states gives new degrees of freedom – which parties share the final state.
- *Not* specifying the parties is also an option and can have qualitatively different results.

Other conclusions

- General upper bound for E_{rnd} for tripartite states in terms of E_r and S for subsystems.

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- *W*-like states not reversibly obtainable from EPRs

- Randomly-shared EPR pairs can sometimes be reliably obtained even when the probability of obtaining an EPR shared between specified parties is arbitrarily small.

Future Work - general

- Tighten bounds above and below

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Generalisation.

- *Extend upper bounds on E_{rnd} to multiparty states.*
- *Consideration of more general pure states including stabiliser states.*

Future work - Distillation "vectors"

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Complete description of asymptotic random distillation comprises achievable N_{AB}, N_{BC}, N_{AC} for given ψ.

E_{rnd} alone does not specify e.g. tripartite distillation rate through teleportation.

Acknowledgements

Thanks to: Daniel Gottesman, Martin Plenio, Debbie Leung, Matthias Christandl and Andreas Winter. HKL thanks PI for financial support for his one-year visit. BF thanks University of Toronto EF Burton Fellowship.

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