Title: M-theory Signatures in the CMB

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Abstract: I will demonstrate how one can realize Cascade inflation in M-theory. Cascade inflation is a realization of assisted inflation which is driven by non-perturbative interactions of N M5-branes. Its power spectrum possesses three distinctive signatures: a decisive power suppression at small scales, oscillations around the scales that cross the horizon when the inflaton potential jumps and stepwise decrease in the scalar spectral index. All three properties result from features in the inflaton potential. The features in the inflaton potential are generated whenever two M5-branes collide with the boundaries. The derived small-scale power suppression serves as a possible explanation for the dearth of observed dwarf galaxies in the Milky Way halo. The oscillations, furthermore, allow to directly probe M-theory by measurements of the spectral index and to distinguish cascade inflation observationally from other string inflation models.

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Amjad Ashoorioon
University of Waterloo
in collaboration with
Axel Krause

Based on
hep-th/0607001 & a work in progress

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  1. To achieve this goal a considerable amount of fine-tuning has to be applied.

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  Most of the approaches aim to derive a sufficiently flat potential.

- **Disadvantages:**

  1. To achieve this goal a considerable amount of fine-tuning has to be applied.

  2. To have the universe rolling along the flat direction, the potential of all the remaining moduli should be stabilized before the inflationary phase. This usually distorts the inflaton potential itself. For example in KKLMMT scenario, the stabilization of volume moduli leads to η problem.

- Construction of Inflation in String Theory:

- Realization of Assisted Inflation in String Theory

Although, individual moduli has a too steep potential to sustain inflation, their cumulative dynamics increases the Hubble friction and leads to inflation.

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- **Advantage:**

  This alleviates considerably the task to have all other moduli, except the inflaton, stabilized before the inflationary phase.

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- Conclusion & future directions

- **Assisted Inflationary setup:**

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For m scalar fields, $\phi_i, i = 1..m$, with potential

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and so the expansion rate is $a \propto t^{\tilde{p}}$. Even though $p < 1$, one can obtain inflation, if $\tilde{p} > 1$.

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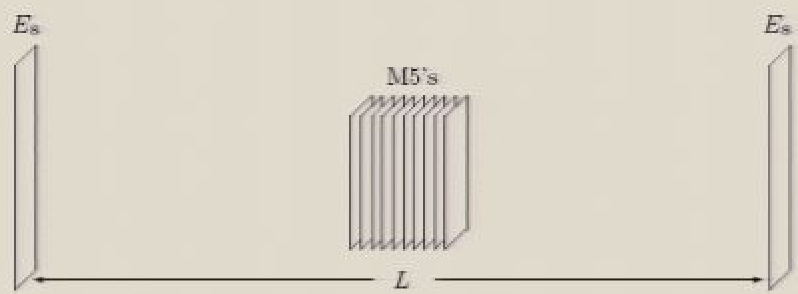
Starting point: M-theory in the presence of N parallel M5-branes distributed along the orbifold and compactified on a CY_3 preserving N=1 supersymmetry in 4D. Each M5-brane has wrapped the same 2-cycle Σ_2 on the CY_3 only once and fill the 4D space-time.



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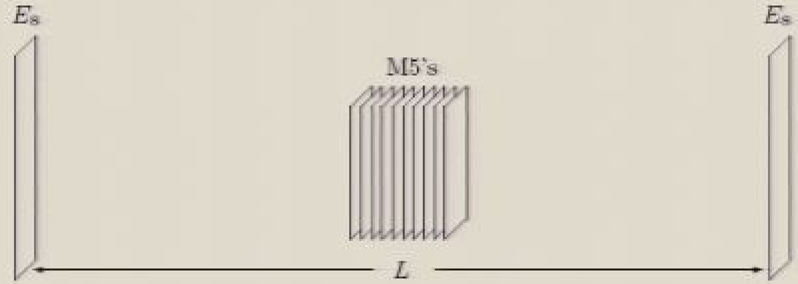
Setting to zero all charged matter fields, the superpotetial is

$$W=W_{99}+W_{55}+W_{59}+W_{95}+W_{GC}$$

W is open membrane isntantons wrapping the same 2-cycle and stretched between either the boundaries and M5-branes (W_{59}, W_{95}) or two boundaries (W_{99}) or two of the M5-branes (W_{55}). W_{GC} is the gaugino condensation on the hidden boundary.



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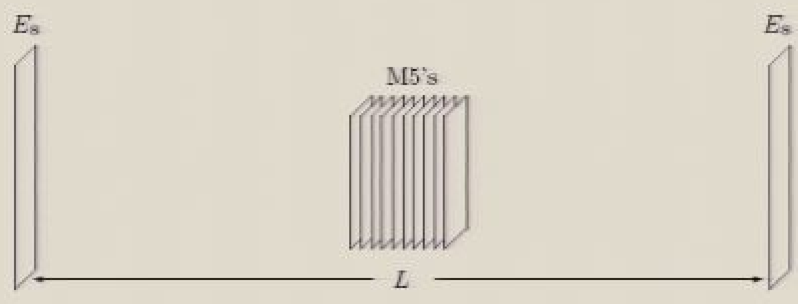

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$$S = v + v_{OM} \sum_{i=1}^{N} \left(\frac{x_i^{11}}{L} \right) + i\sigma_S;$$

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$$Q = s - \frac{y^2}{t} \qquad R = 3Q^2 - 2\frac{y^4}{t^2}$$

R is positive too

• Realization of assisted inflation in M-theory

N=1 SUGRA expression for F-term, yields the potential

$$U = M_{Pl}{}^4 e^K \left(\sum K^{\bar{I}J} D_{\bar{I}} \overline{W}_{55} D_J W_{55} - 3\left|W_{55}\right|^2\right)$$

Where

$$D_i W_{55} \equiv \partial W_{55} / \partial Y_i + W_{55} \partial K / \partial Y_i$$

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equidistant distribution of M5 branes

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Defining $\Delta Y + \overline{\Delta Y} \equiv \Delta y$ and applying the above constraints:

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To map the potential to assisted inflation potential

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During inflation $$\Delta x \equiv x_{i+1} - x_i = \frac{L}{2v_{\mathrm{OM}}} \Delta y$$

grows because of repulsive M_2-interactions. This continues until the two outermost M5 branes hit the boundaries. The ensuing non-perturbative small Instanton transition transforms the outmost M5 branes into small instantons on the boundaries [E. Witten, NPB 460 (1996)].

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$$N \to N-2$$

The instanton transitions which change the topological data on the boundaries can be either chirality or gauge group changing. We assume that only the first case happens here. As gauge group changing transitions do not happen, we do not have unwanted relics. Since N drops, the value of p_N and U_N drops too. After m transitions, we have:

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Exit from inflation happens when at the K -th phase where $p_{N_K} \leq 1.$

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At the end, we have a cascade of power-law inflations during which the scale factor evolves as:

$$a_m(t) = a_m t^{p_m}, \qquad t_{m-1} \le t \le t_m, \qquad m = 1\Lambda\, K$$

Where the continuity of the scale factor at transition times determine a_m s.

$$a_m = a_1 t_1^{p_{N_1}} \left(\frac{t_2}{t_1}\right)^{p_{N_2}} \left(\frac{t_3}{t_2}\right)^{p_{N3}} \Lambda \left(\frac{t_{m-1}}{t_{m-1}}\right)^{p_{N_{m-1}}} \frac{1}{(t_{m-1})^{p_{N_m}}}$$

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By inverting the exact power-law inflation solutions for $\varphi(t)$

$$\varphi(t) = \sqrt{2} M_{Pl} \ln\left(\sqrt{\frac{U_0}{p_N(p_N - 1)}} \frac{(N-1)}{M_{Pl}}\right)$$

and noting that $\Delta x(t_0)/L << 1$, one obtains

$$t_0 \approx \frac{2N^2}{3M_{Pl}} \sqrt{\frac{2td}{s}};$$

$$t_m - t_0 = \frac{1}{M_{Pl}} \sqrt{\frac{st^3 d}{6}} \left(\sum_{k=2}^{m} \frac{p_{N_k}(3p_{N_k} - 1)}{N_K - 1} e^{t\left(\frac{1}{N_K - 1} - \frac{1}{N_{K-1} - 1}\right)} + \frac{p_{N_1}(3p_{N_1} - 1)}{N_1 - 1} e^{\frac{1}{N_1}} \right)$$

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WMAP 3-year results indicate that

$$n_s = 0.951^{+0.015}_{-0.019}$$

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These values of N satisfy the constraint $Qt >> y^2$ which delivers the bound

$$N << 195$$

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$$N_e \equiv \ln\left(\frac{a(t_f)}{a(t_0)}\right) = \sum_{m=1}^{K} p_{N_m} \ln\left(\frac{t_m}{t_{m-1}}\right) \approx 238$$

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Assuming instant reheating, $(N_e)_{\rm min}\approx 60 \ll N_e$

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Requiring that the mode reduces to Bunch-Davies vacuum in the beginning of inflation ⇒

$$C_1(k) = \frac{\pi}{2} e^{i(\lambda_1 + 1/2)\pi/2} \qquad D_1(k) = 0$$

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• Through the transitions

$H\downarrow \Rightarrow l_H\uparrow \Rightarrow$ Some modes that have just gone outside the horizon are recaptured by the Hubble radius and starts oscillating again.

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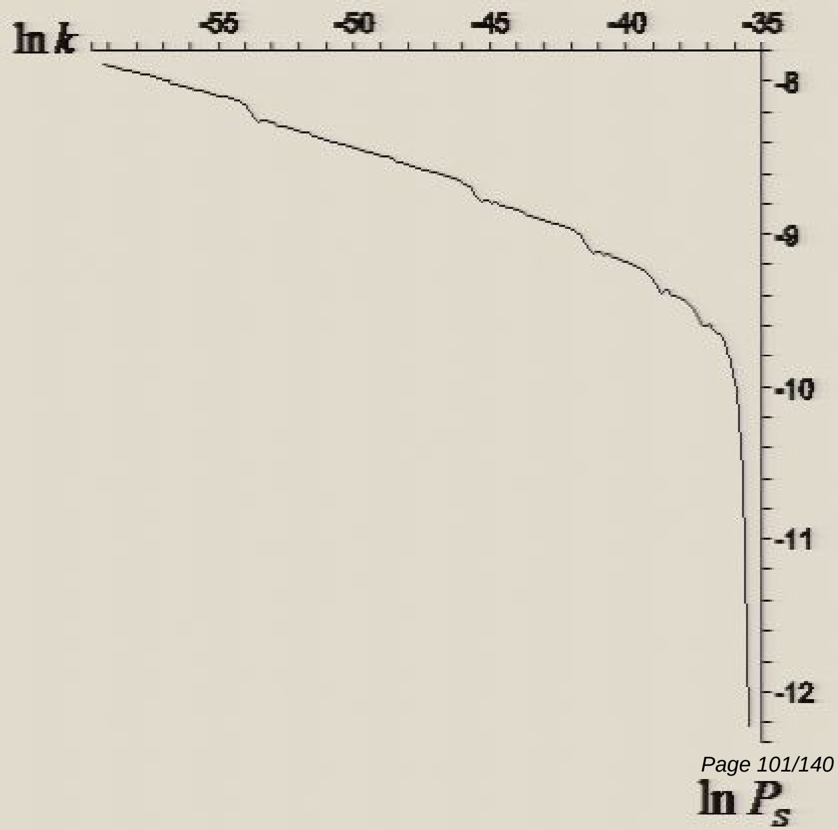

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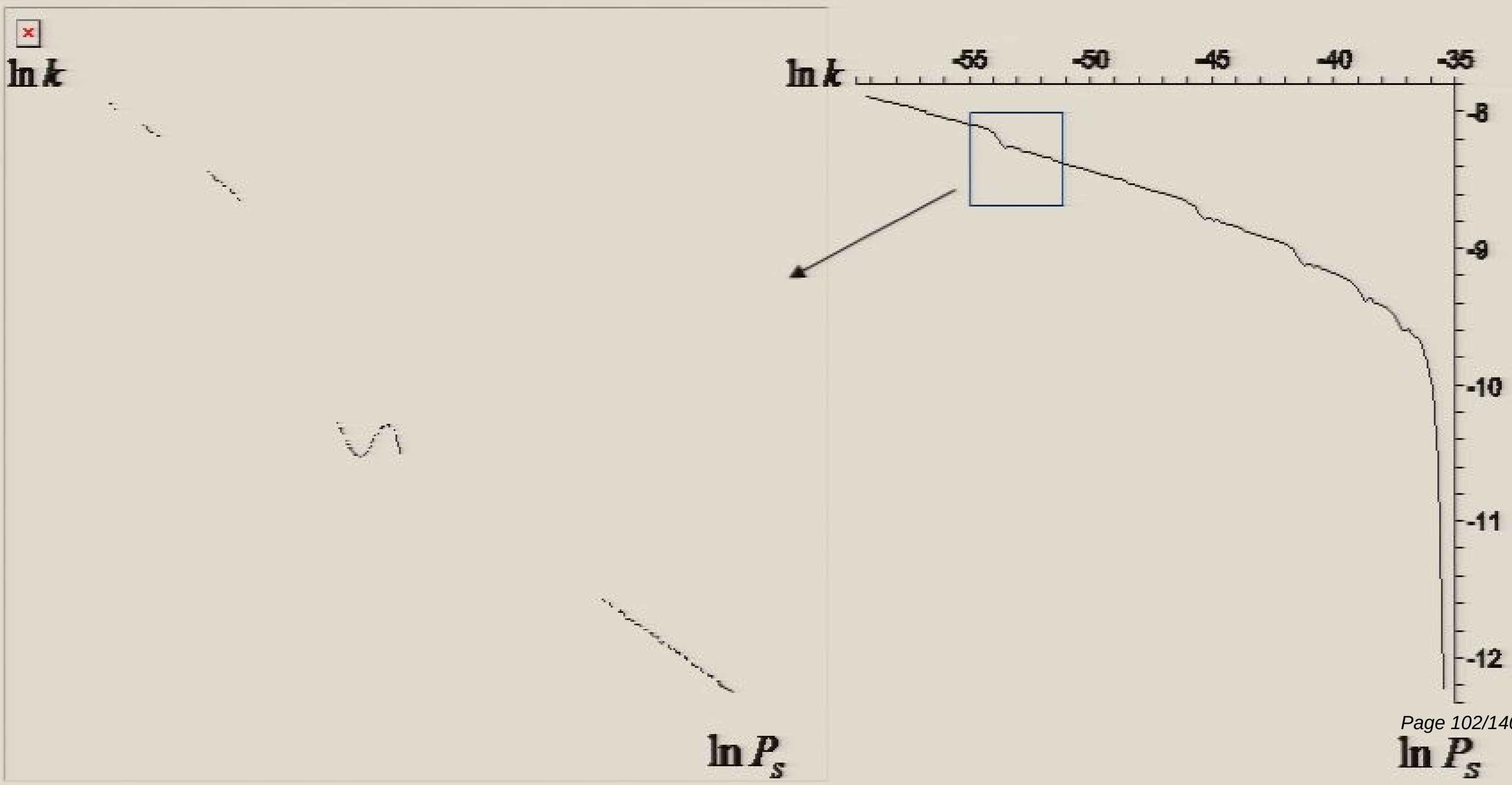

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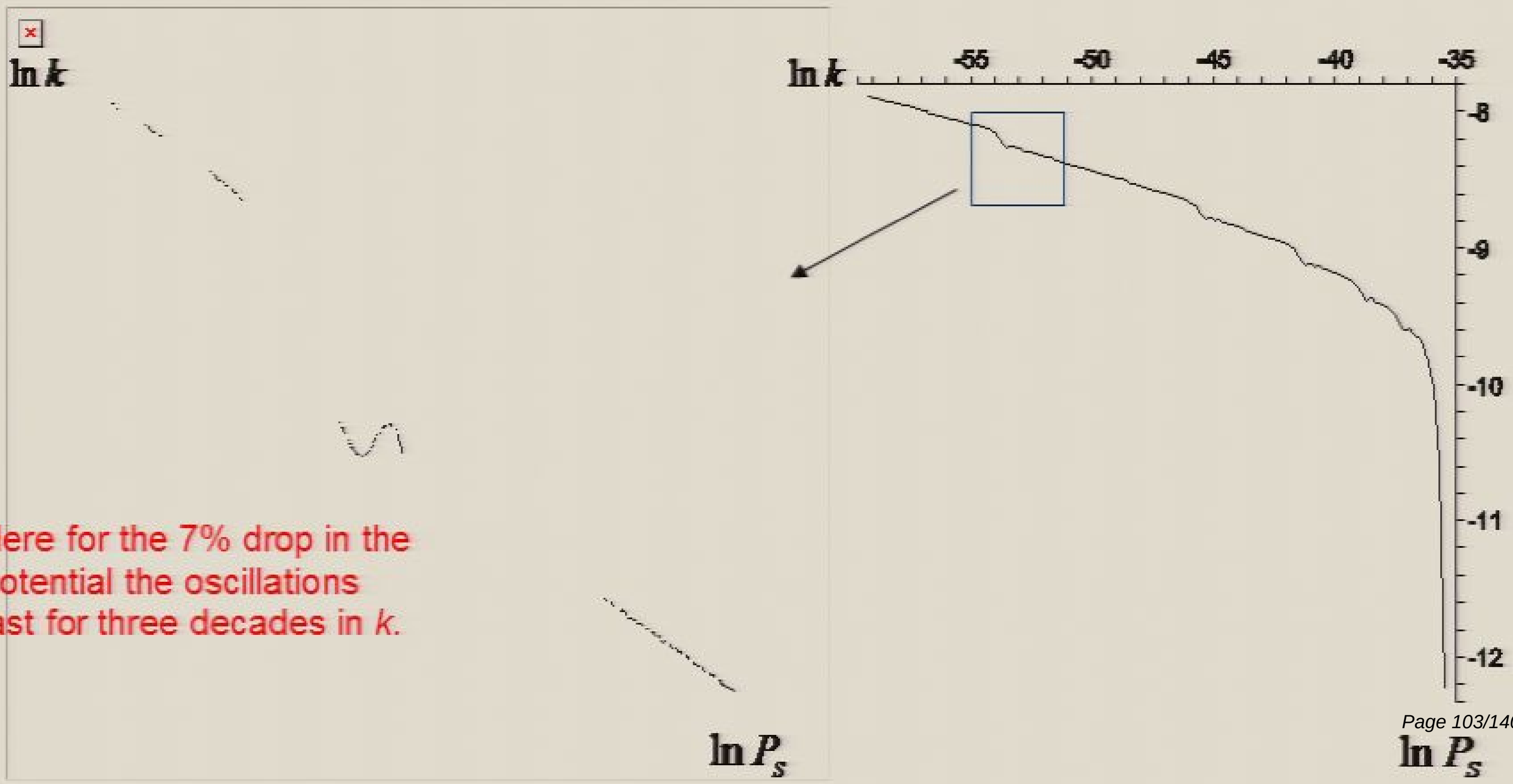

Here for the 7% drop in the potential the oscillations last for three decades in *k*.

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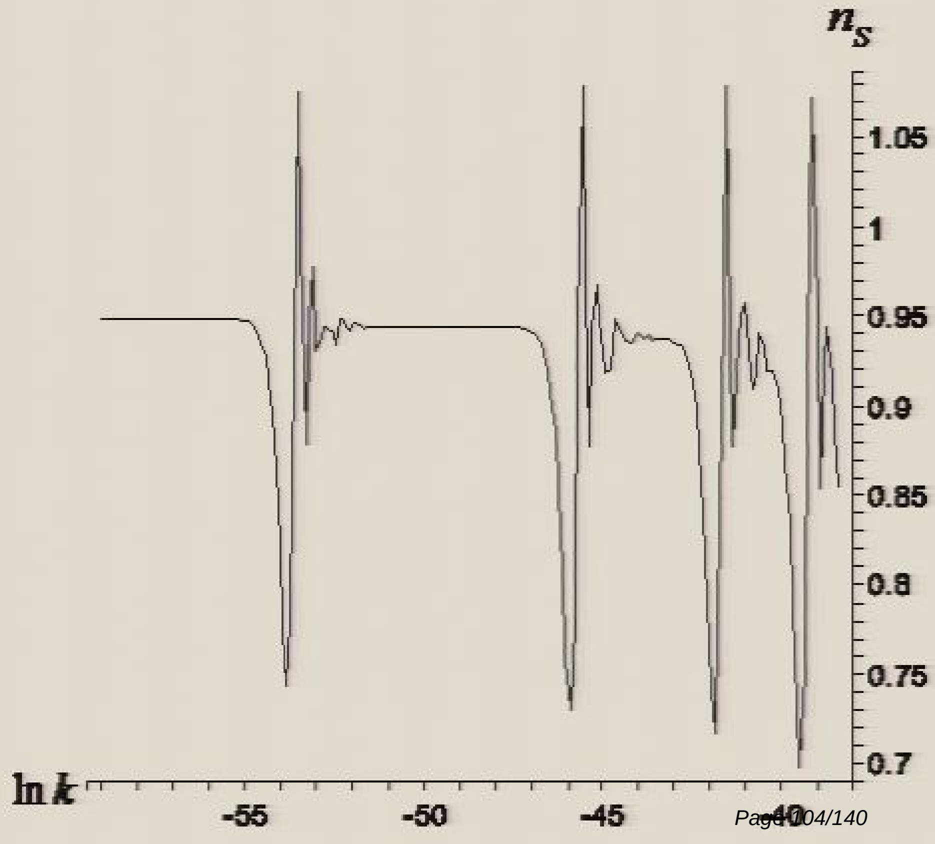

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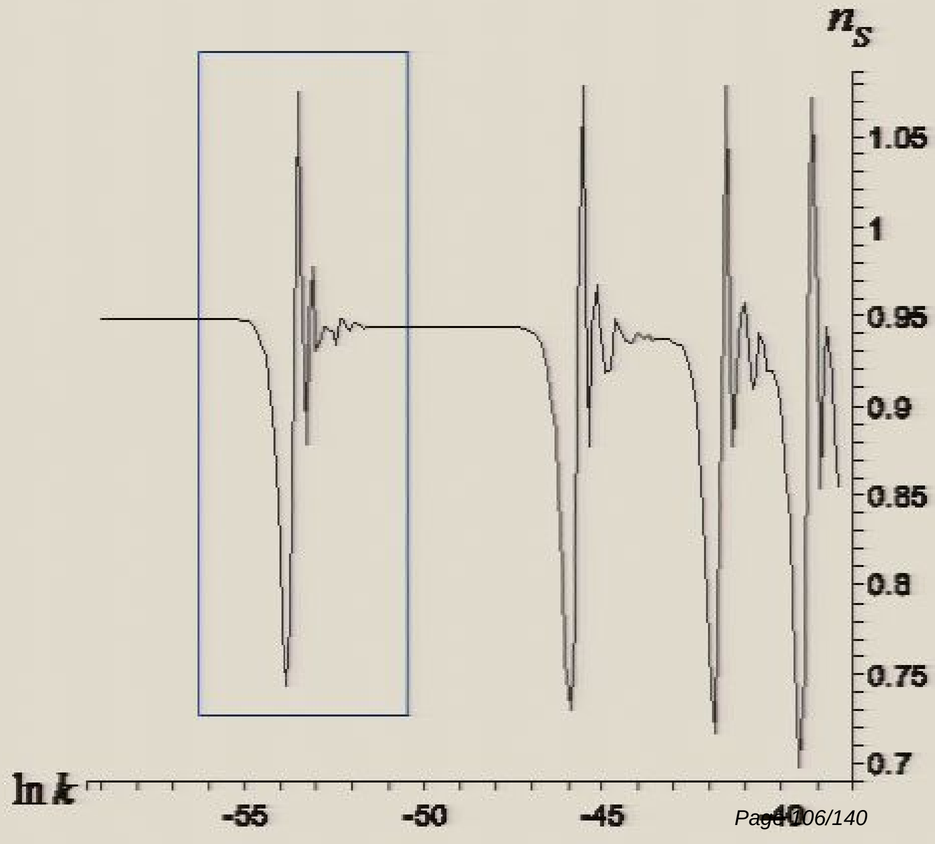

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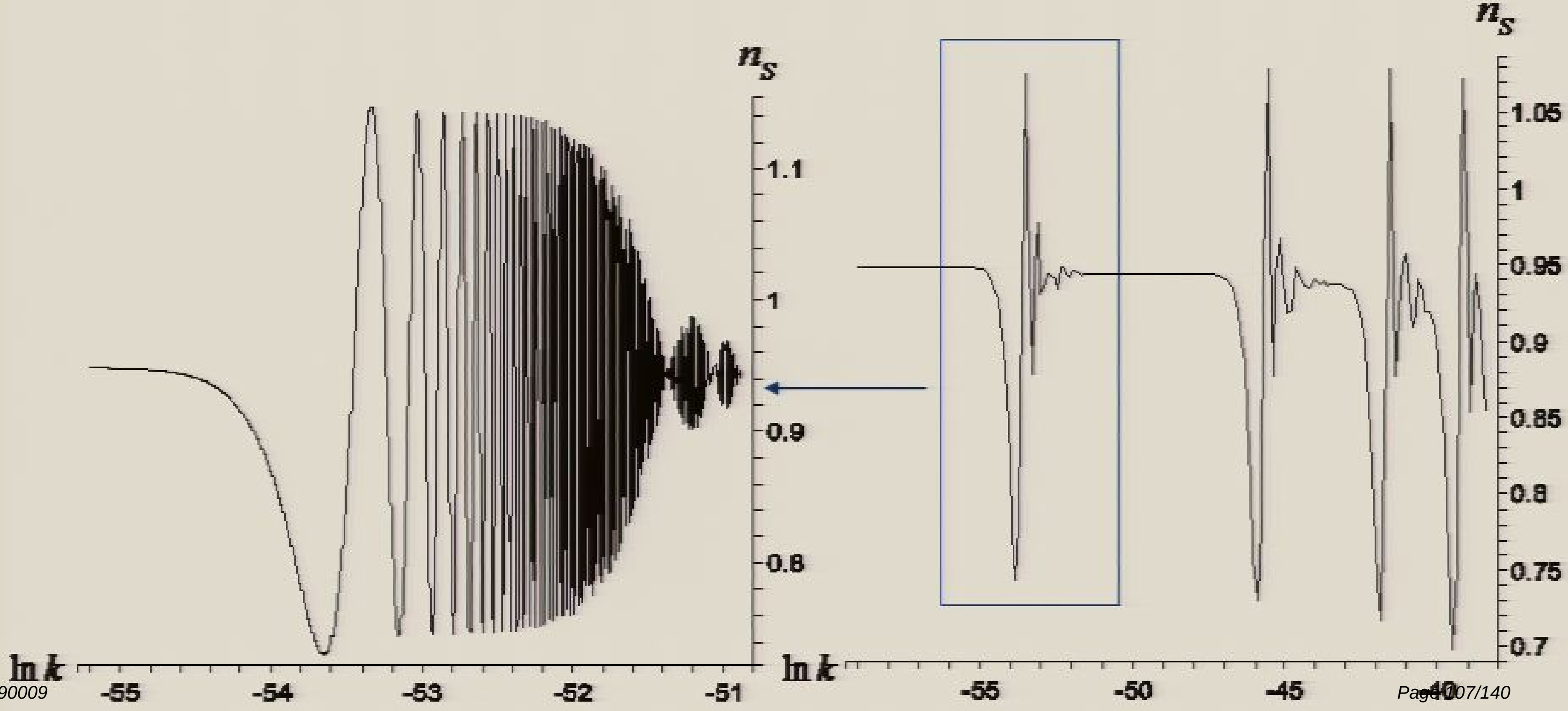

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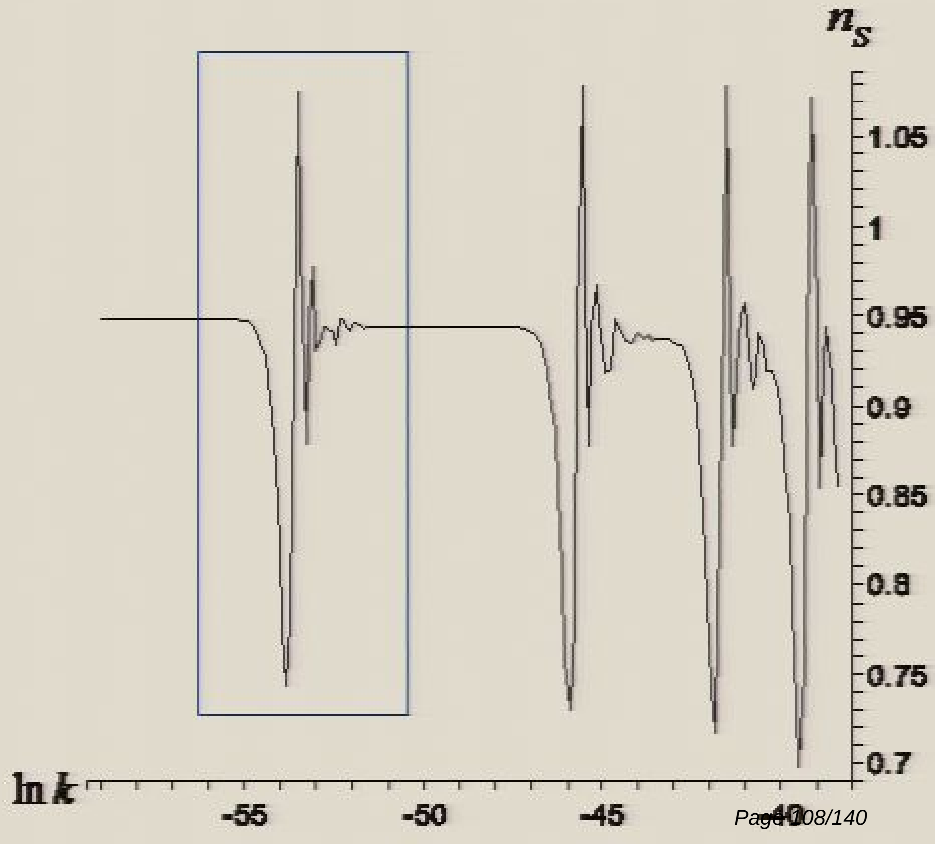

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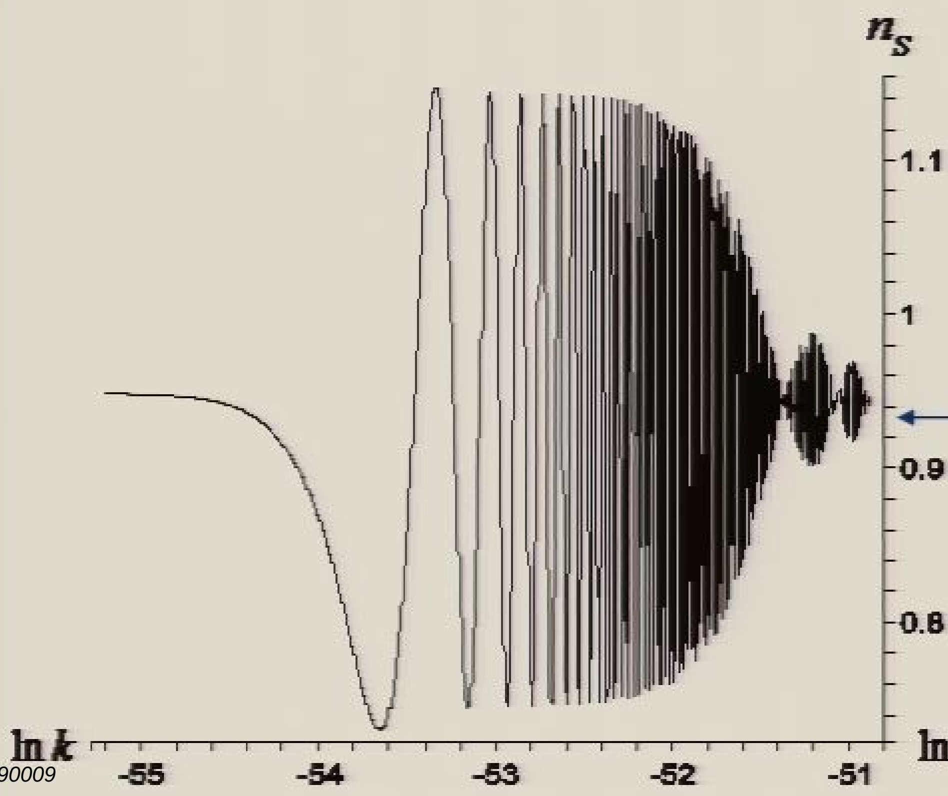

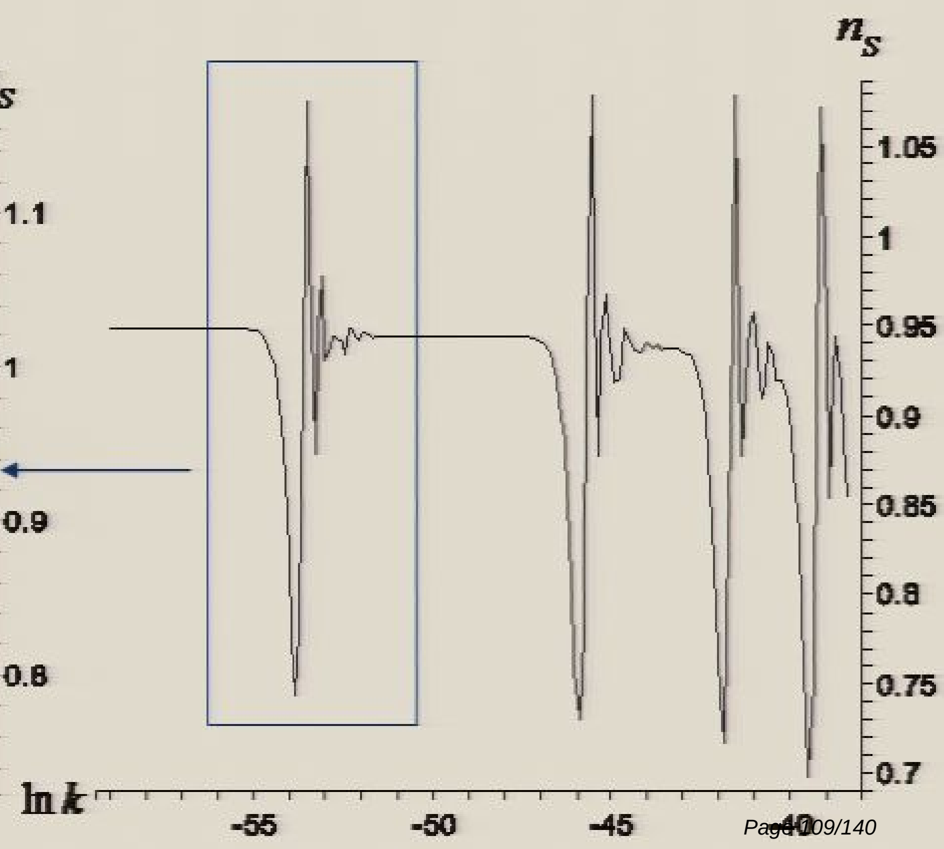

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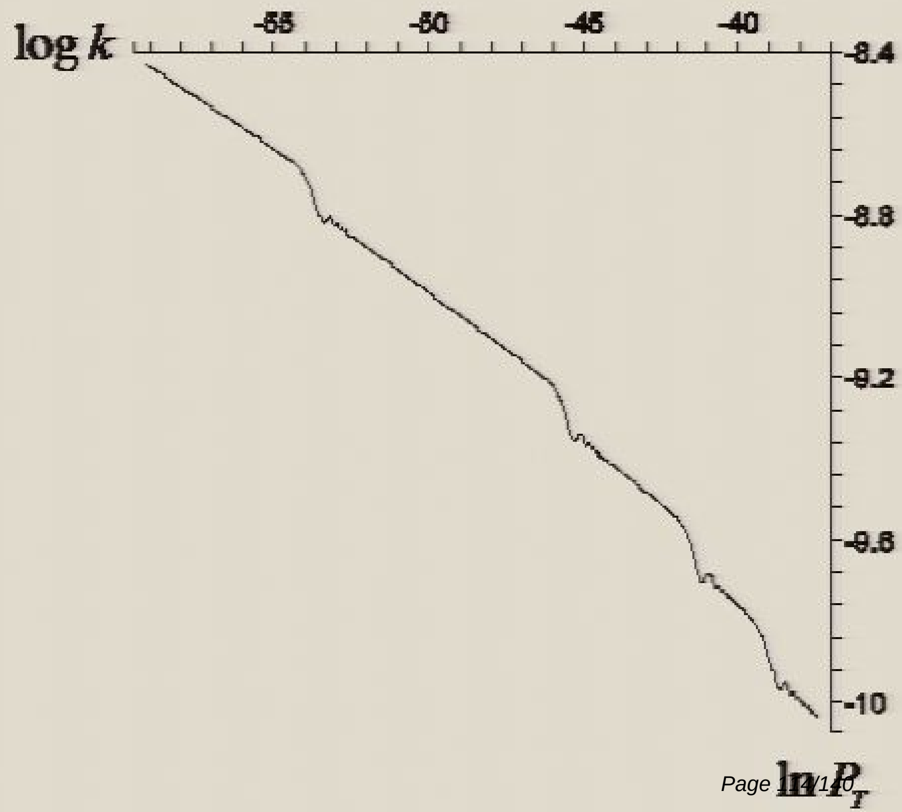

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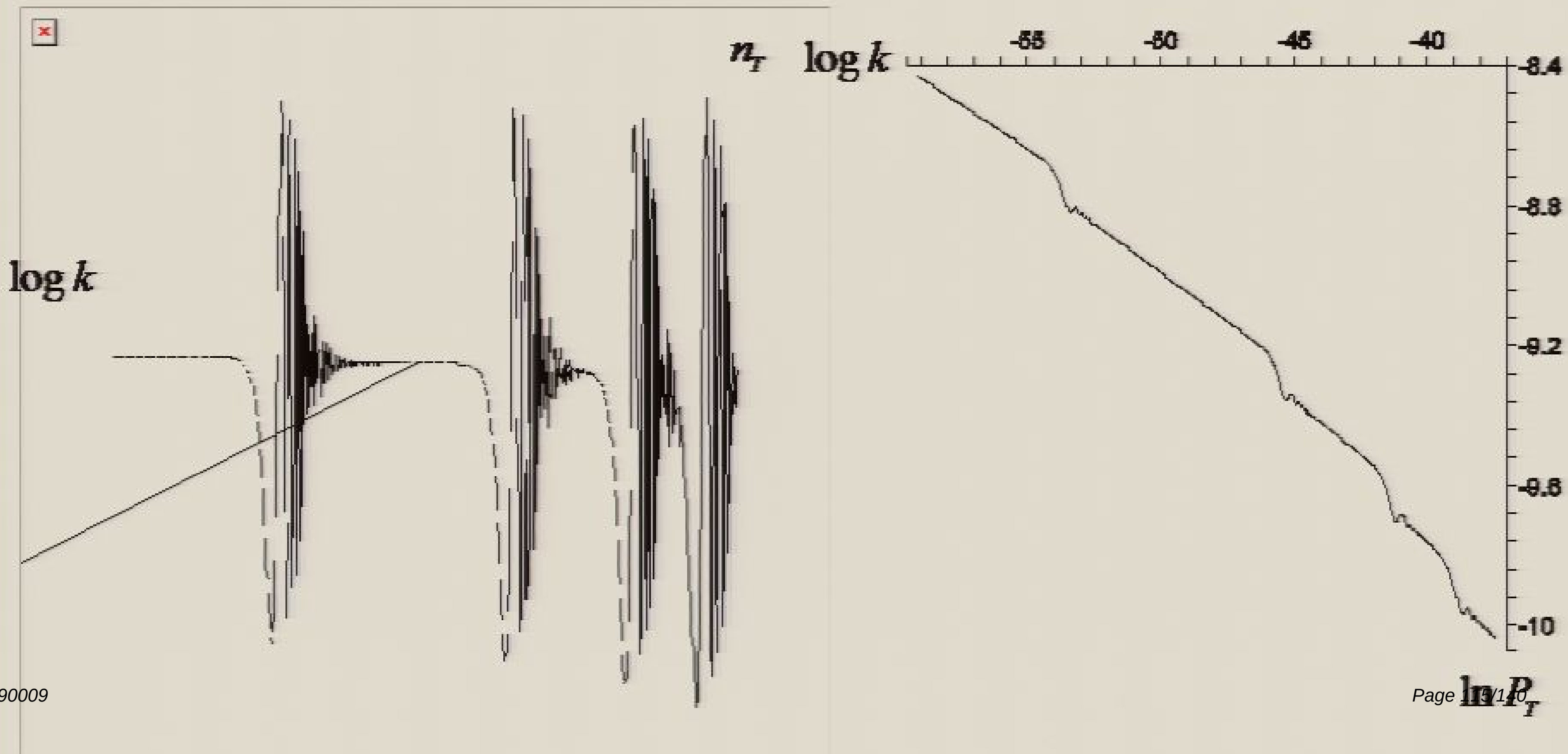

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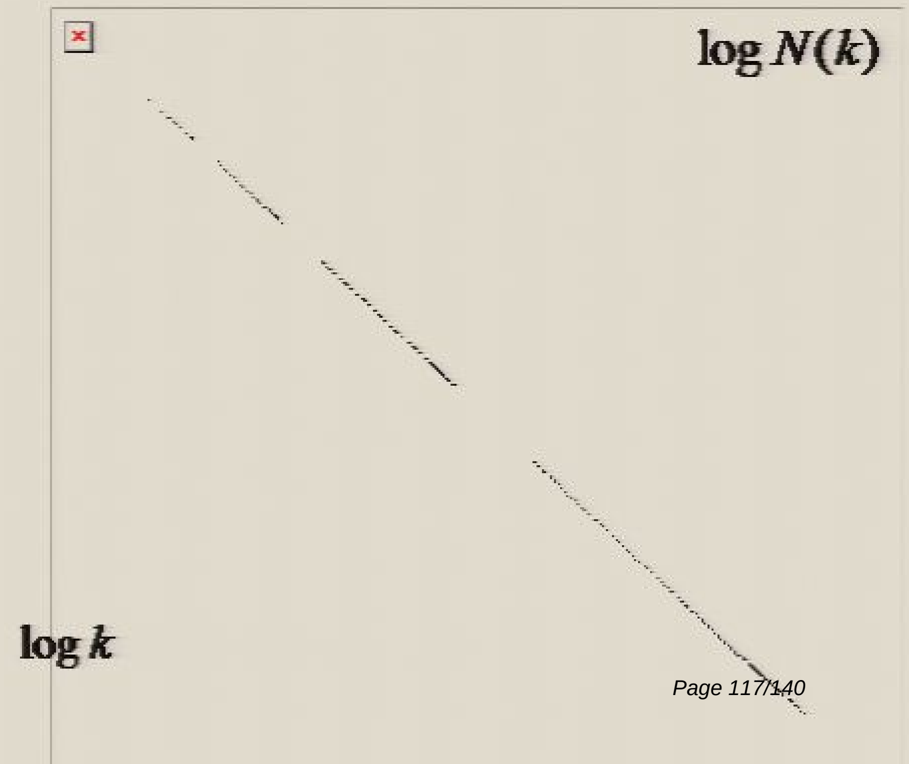

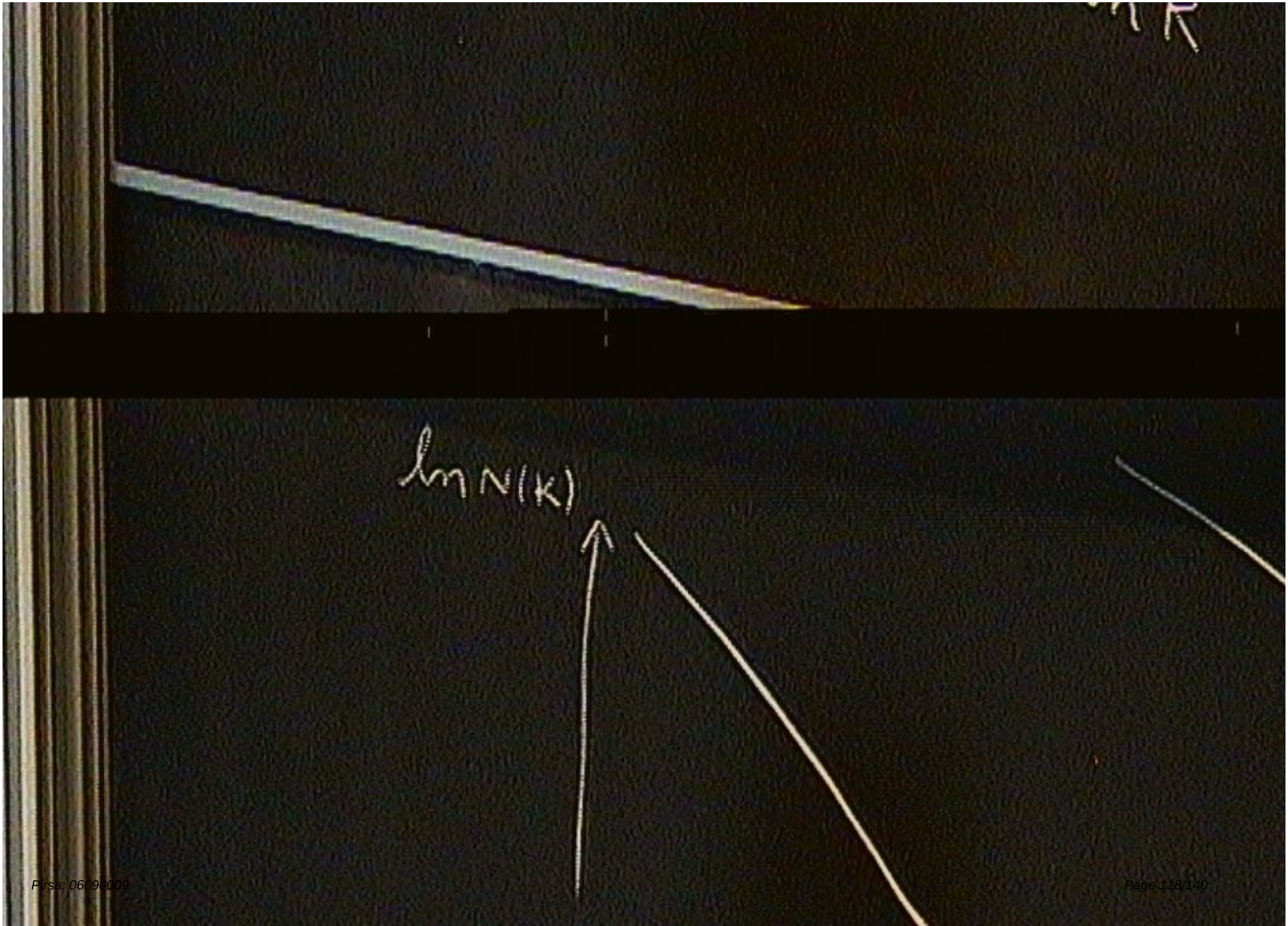
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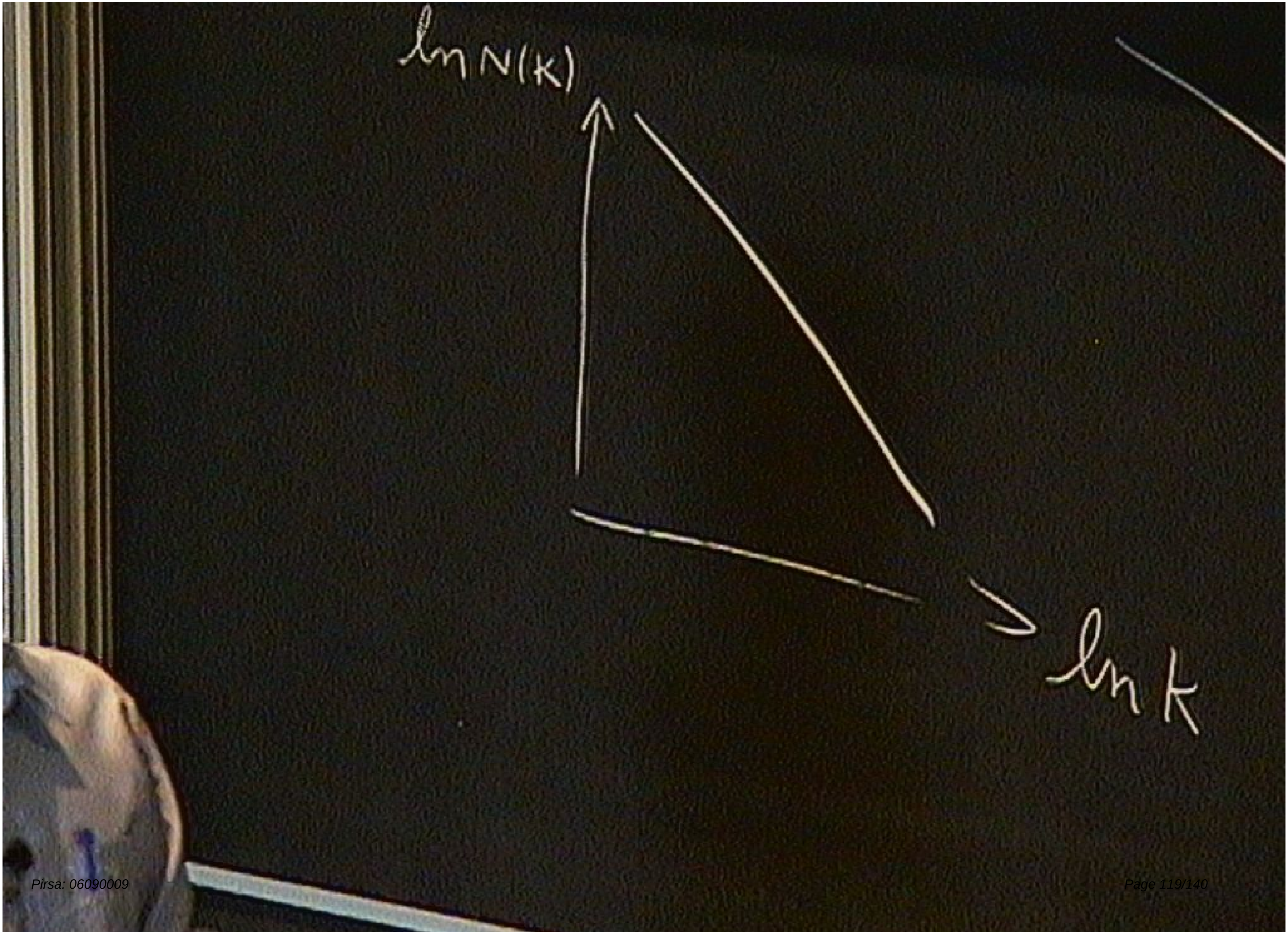
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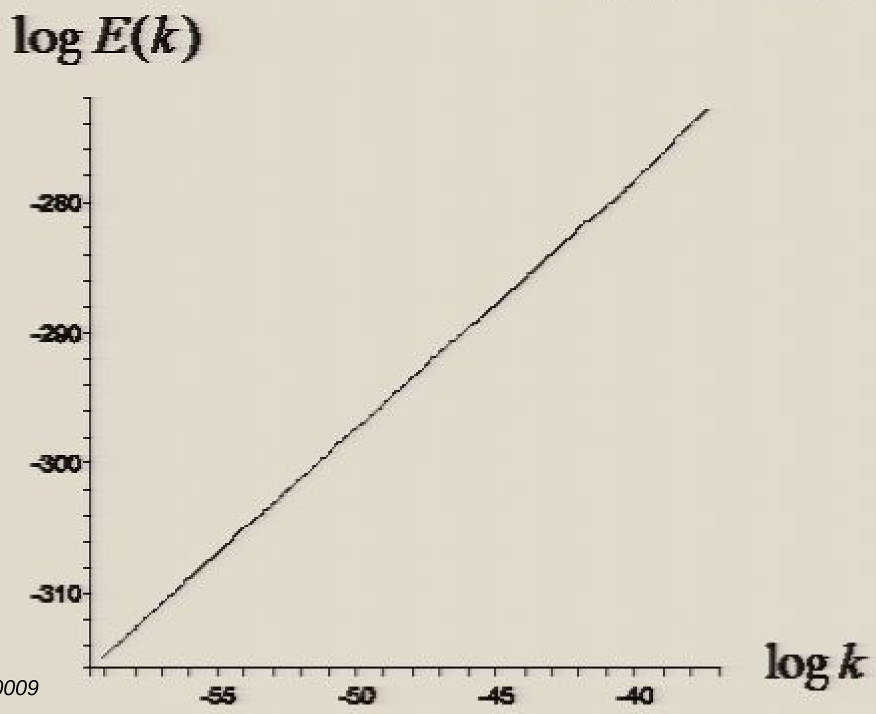

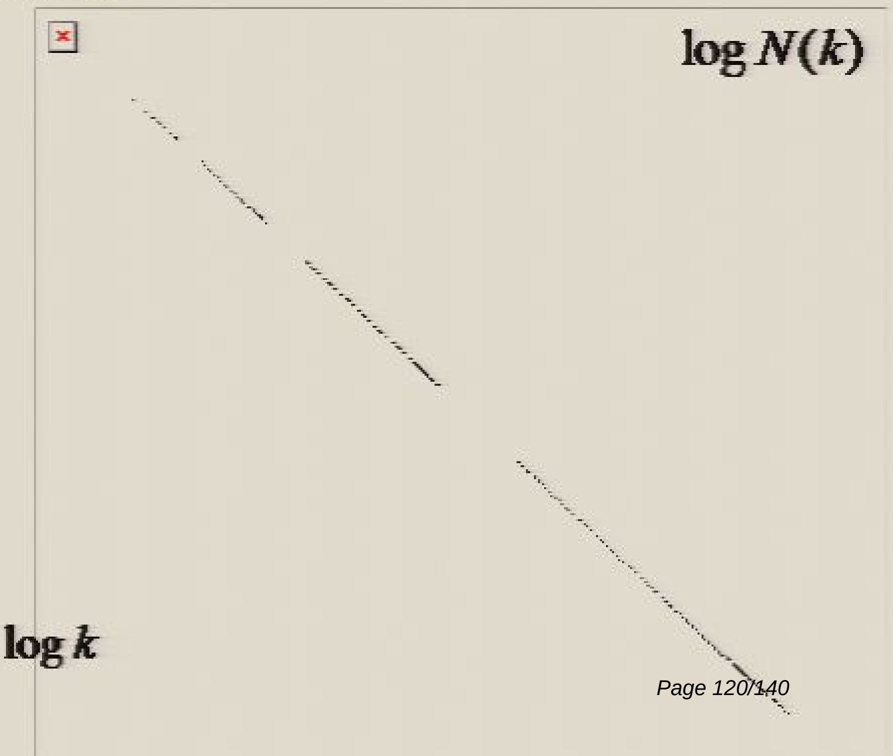

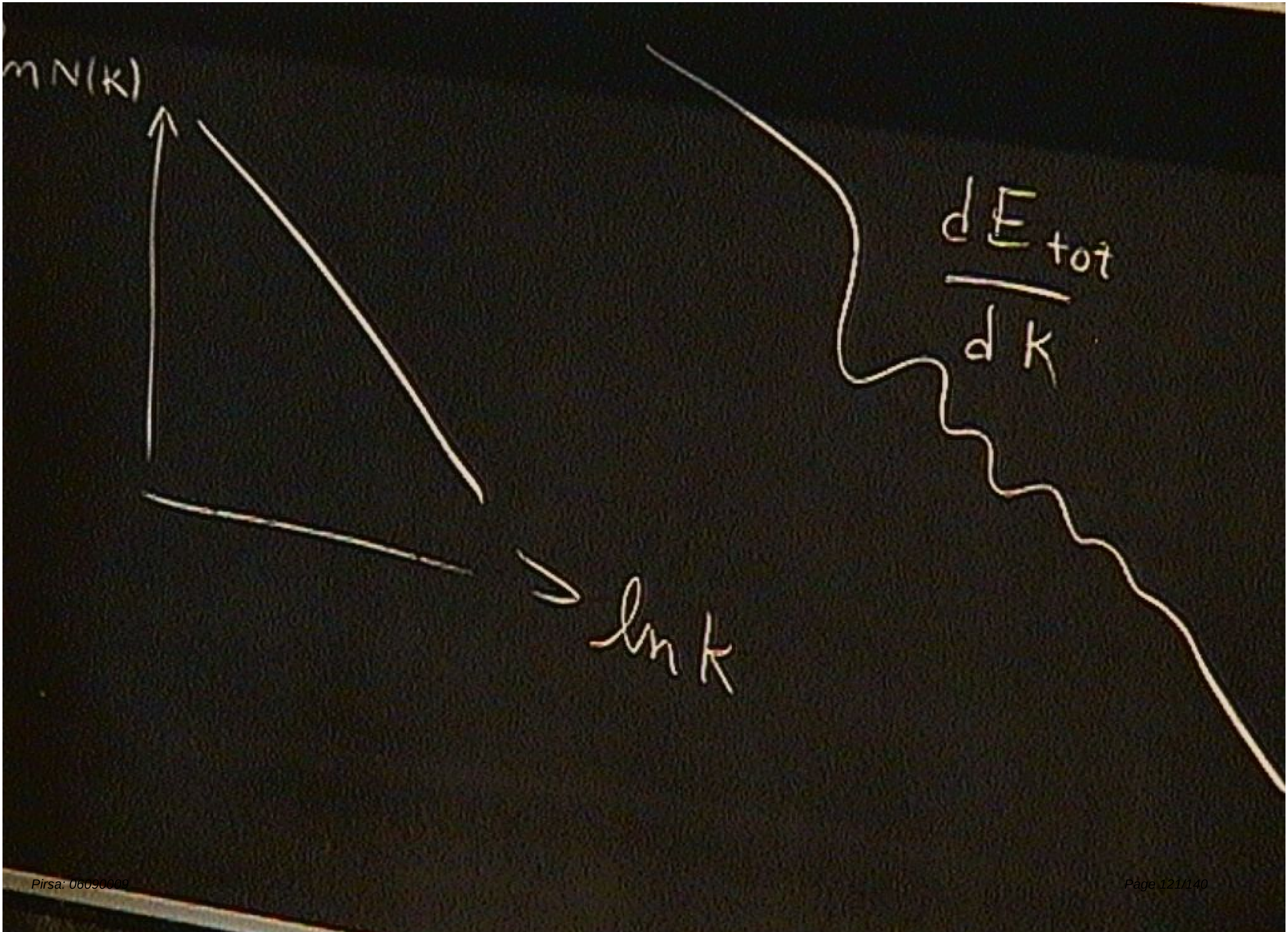

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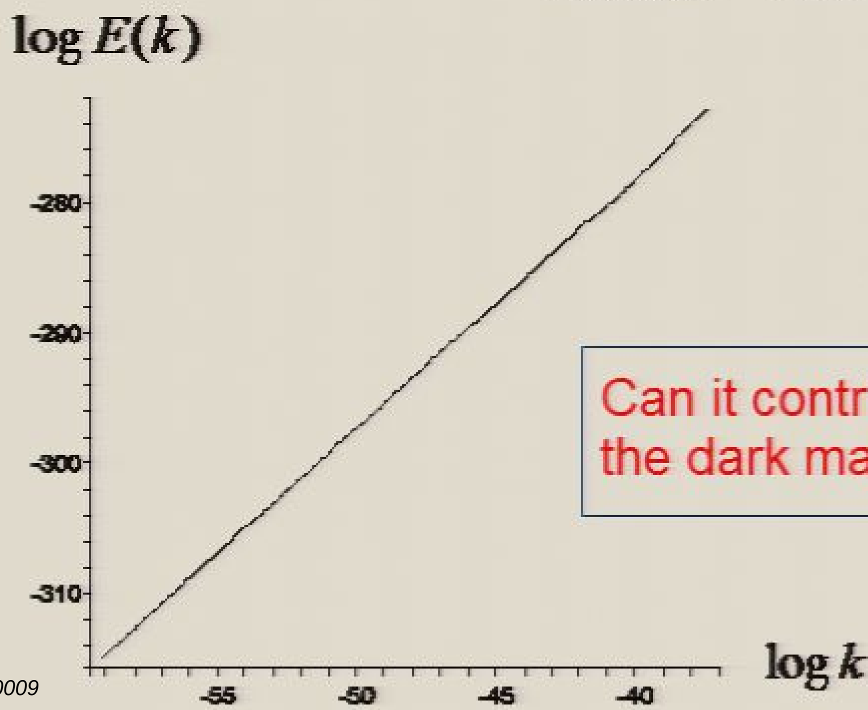

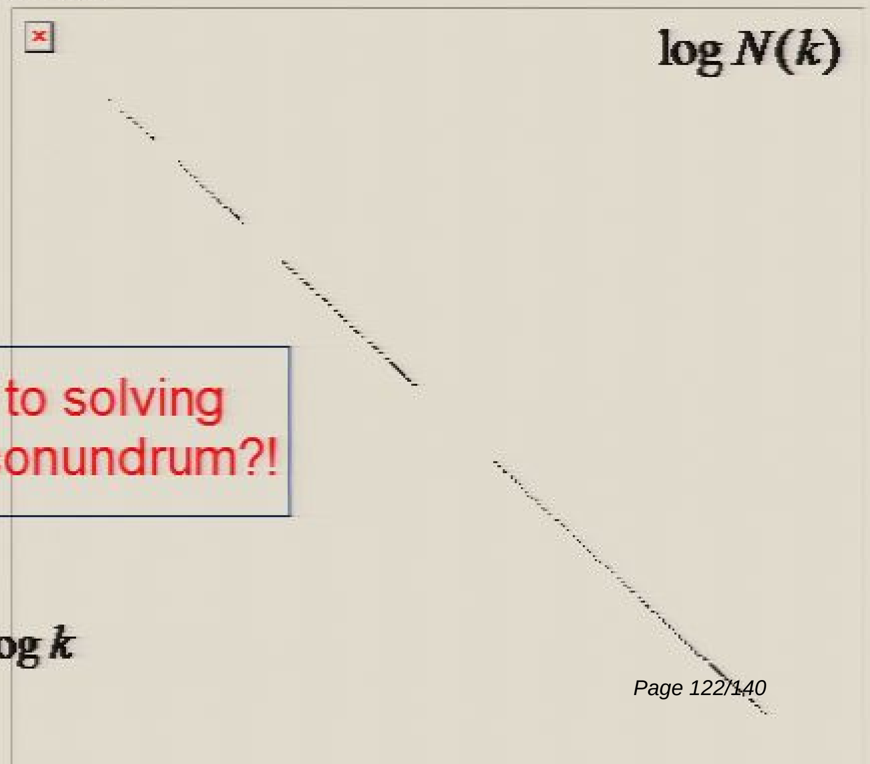

Can it contribute to solving the dark matter conundrum?!

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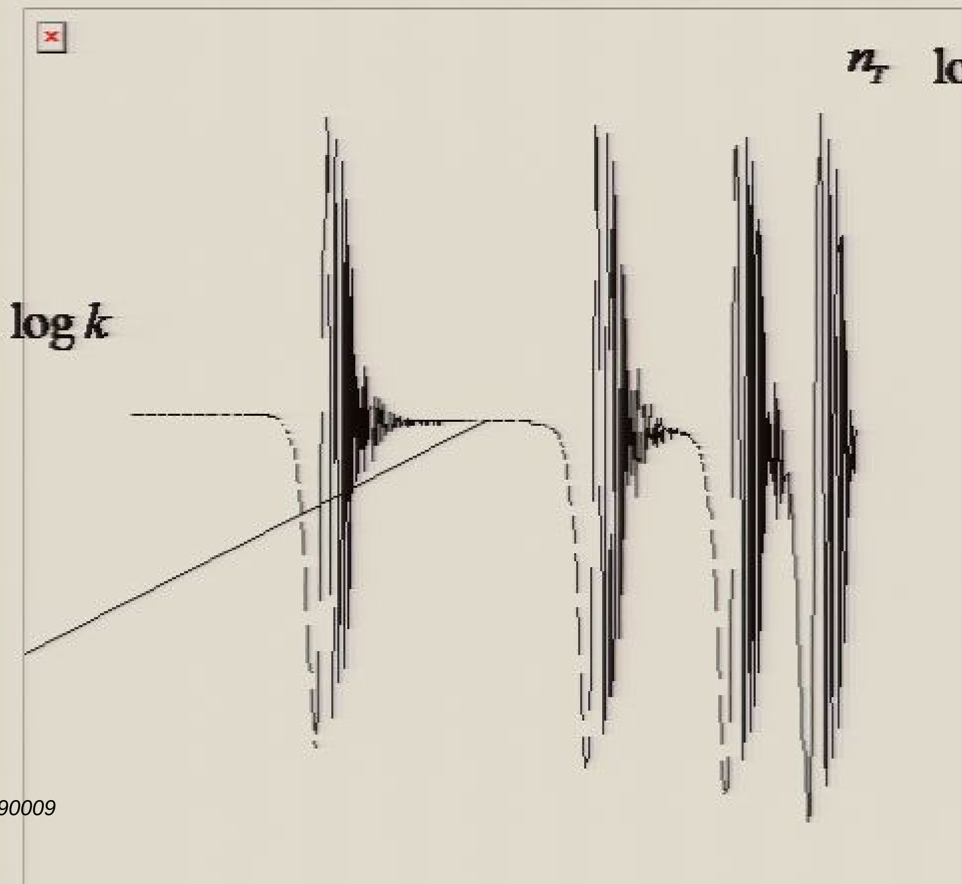

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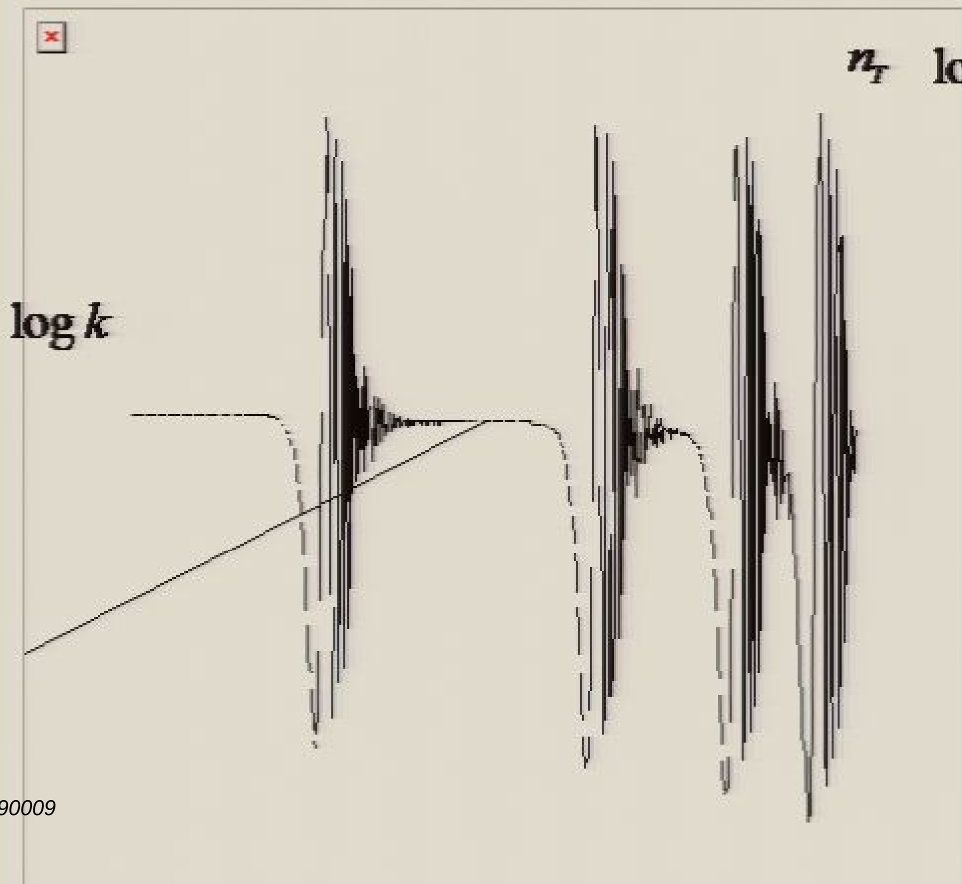

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• Recently Covi, et. al. [astro-ph/0606452], have tried to explain the measured deviation of the WMAP3 from featureless power spectrum, using potentials with step and found interesting constraints on the location and magnitude of possible steps. One may be able to use the results of that paper to derive M-theory parameters!

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• Since in cascade inflation, we have multiple fields and these fields reach their minima at different times, this possibility exists that successive collision of the M5-branes with the boundaries create a large (detectable!) non-Gaussianity.

Ashoorioon and Krause
Work in progress.