Title: Incorporating Gravity into Bohmian Mechanics: A New Approach

Date: Sep 19, 2006 04:00 AM

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Abstract: My field is the foundations of quantum mechanics, in particular Bohmian mechanics, a non-relativistic theory that is empirically equivalent to standard quantum mechanics while solving all of its paradoxes in an elegant and simple way, essentially by assuming that particles have trajectories. Bohmian mechanics possesses a straightforward generalization to relativistic space-time, be it flat or curved, if one assumption against the spirit of relativity is granted: the existence of a "time foliation", i.e., a physical object mathematically represented by a slicing of space-time into spacelike 3-surfaces, which evolves according to a Lorentz-invariant law. On the basis of this kind of theory, describing particles in a background 4-geometry, I propose an extension in which the space-time geometry is dynamically generated, as in general relativity. Whether my model is empirically equivalent to any known type of quantum gravity I don't know. In this model, there is a Lorentzian metric on configuration-space-time, evolving according to the higher-dimensional analog of the Einstein field equation. The 4-metric is obtained from the configuration-space-time metric and the actual particle configuration. Thus, this Bohm-like model generates (up to diffeomorphisms) a 4-metric and particle world lines from a given wave function.

Bohmian mechanics

[Slater 1927, de Broglie 1927, Bohm 1952, Bell 1966]

- a nonrel. theory of N particles moving in R^3 along trajectories $\vec{Q}_i(t)$, $i = 1 \dots N$, defined by Bohm's eq of motion

$$\frac{d\vec{Q}_i}{dt} = \frac{\hbar}{m_i} \operatorname{Im} \frac{\psi^* \nabla_i \psi}{\psi^* \psi} \Big(\underbrace{\vec{Q}_1(t) \dots \vec{Q}_N(t)}_{Q(t) \in \mathcal{Q}} \Big)$$

$\psi_t : \mathcal{Q} \to \mathbb{C}^k$ obeys Schrödinger eq

$\mathcal{Q}$ = configuration space = R^{3N}

- $\Rightarrow$ equivariance: distribution$(Q(t)) = |\psi_t|^2$ $\forall t$ if initially

$$\boxed{\text{BM is empirically equivalent to QM}}$$

(for typical solutions)

- a quantum theory without observers: axioms talk of objective events (motion of particles), not of observations. The "measurement" formalism is a thm of BM.
- explicit and clear ontology
- limits of knowledge

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Bohmian mechanics

[Bohm, de Broglie 1927, Bohm 1952, Bell 1966]

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along trajectories $Q_i(t)$, $i = 1 \dots N$,

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$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\,\mathrm{Im}\,\frac{\psi^*\nabla_i\psi}{\psi^*\psi}\,(Q_1(t) \dots Q_N(t)) \qquad Q(t) \in \mathcal{Q}$$

$\psi_t : \mathcal{Q} \to \mathbb{C}^k$ obeys Schrödinger eq

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Extension of Bohmian Mechanics

Particle Creation and Annihilation

= Bell-Type Quantum Field Theory

[Bell 1986, Dürr–Goldstein–Tumulka–Zanghì 2003–5, Colin 2003–5]

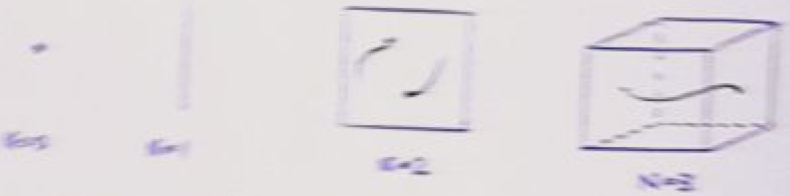
$Q(t) \in \mathcal{Q}$ = config. space of variable number of particles

$\equiv \Gamma(\mathbb{R}^3) = \{ q \subset \mathbb{R}^3 : \# q < \infty \}$

$= \bigcup_{n=0}^{\infty} {}^{n}\mathbb{R}^3 = \bigcup_{n=0}^{\infty} (\mathbb{R}^{3n} / S_n)$

config. space of n identical particles

Ex. $\Gamma(\mathbb{R})$



Q(t) jumps at creation/annihilation events, is continuous in between

Stochastic Process:

Jumps at random times with random destinations

$\Longrightarrow$ Q(t) is Markovian jump process, piecewise deterministic

- jump rate (density) $\sigma(q' \to q) = \frac{2}{\hbar} \frac{\mathrm{Im}^{+}\, \psi^*(q) \langle q | H_I | q' \rangle \psi(q')}{\psi^*(q') \psi(q')}$
- in between Bohm's eq of motion

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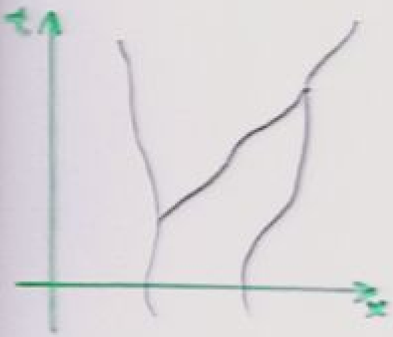

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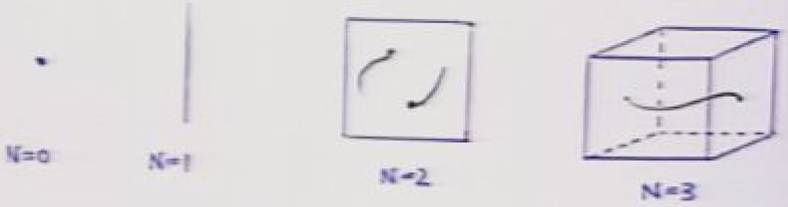
<u>Ex</u> $\Gamma(\mathbb{R}^1)$



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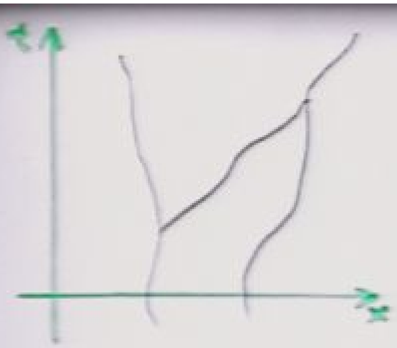
$\Longrightarrow$ equivariance: distribution



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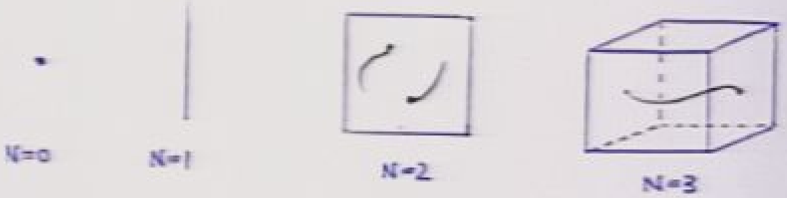

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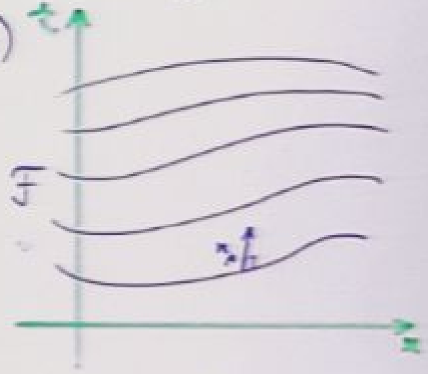
Extensions of Bohmian mechanics:

Relativity and the time foliation

[Bohm–Hiley 1993, Dürr et al. 1999, T 2001, 06]

- Relativistic space-time $(\mathcal{M}, g)$
- Assumption against the spirit of relativity:

> $\exists$ time foliation $\mathcal{F}$ = a physical object mathematically represented by a slicing of space-time into spacelike 3-surfaces.



n_μ = unit normal vector field on $\mathcal{F}$

- $\mathcal{F}$ could be governed by a Lorentz-invariant law

<u>Ex</u> $\nabla_\mu n_\nu - \nabla_\nu n_\mu = 0$

- Eq of motion BM$(\mathcal{F})$

$\mathcal{F} \ni \Sigma \ni Q_k(s)$

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arbitrary curve parameter

$$j^{\mu_1 \cdots \mu_N} = \overline{\psi}(\gamma^{\mu_1} \otimes \cdots \otimes \gamma^{\mu_N})\psi$$

quantum prob. current

for (say) Dirac wf, $i\hbar \gamma_k^\mu \nabla_{k\mu} \psi = m_k \psi$

- equivariance: $|\psi|^2$-distributed on every $\Sigma \in \mathcal{F}$

$\Rightarrow$ BM$(\mathcal{F})$ is empirically equivalent to QM

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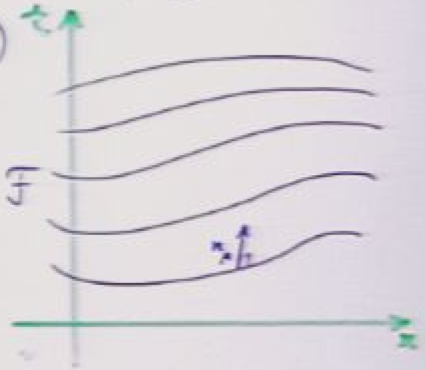

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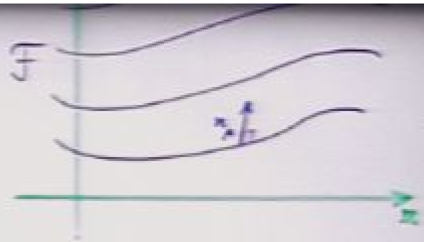

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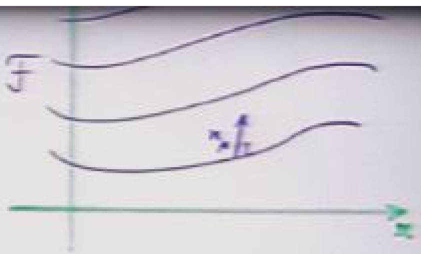

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A New Theory: $^{\sharp}$3M [T 2006]

Status of Lorentzian $g_{\mu\nu}$:

	flat $g_{\mu\nu}$	or	given background $g_{\mu\nu}$	or	dynamically generated $g_{\mu\nu}$
Ex	SR		geodesics as test-particle world lines		GR
Here	3M		3M($\mathcal{F}$)		$^{\sharp}$3M

$^{\sharp}$3M:

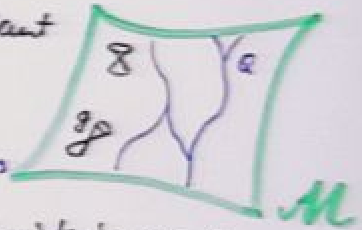
- is diffeomorphism-invariant
- solution:

$$\left(\mathcal{M},\ g_{\mu\nu},\ Q\right) \text{ mod diffeos}$$

manifold → $\mathcal{M}$

Lorentzian on $\mathcal{M}$ → $g_{\mu\nu}$

trajectories $Q \subset \mathcal{M}$ → Q



- uses a time foliation $\mathcal{F}$
- variables for generating solution:

$$^{\sharp}g,\ ^{\sharp}A,\ \psi,\ \mathcal{F},\ ^{\sharp}\mathcal{F}$$

- photon and graviton trajectories

$\Rightarrow$ dual nature of gravity:
graviton particles & geometry $g_{\mu\nu}$, $^{\sharp}g$
dual nature of e.m.:
photons & connection $^{\sharp}A$

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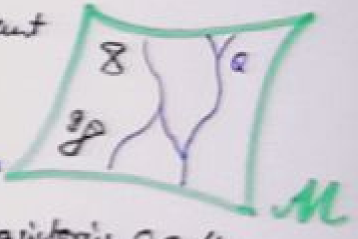

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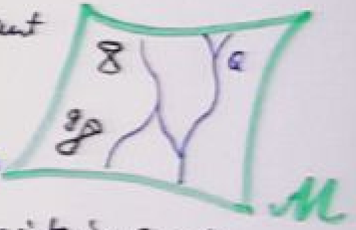

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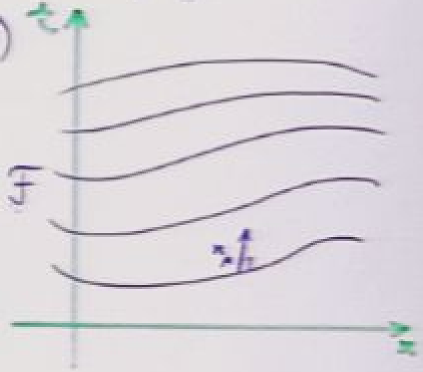

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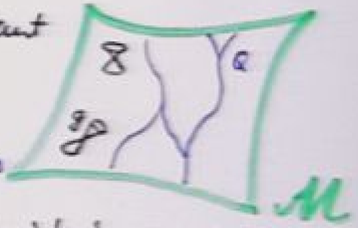


- uses a time foliation $\mathcal{F}$
- variables for generating solution:

$$^\#g,\ ^\#A,\ \psi,\ \mathcal{F},\ ^\#\mathcal{F}$$

- photon and graviton trajectories

$\Rightarrow$ dual nature of gravity:
graviton particles & geometry $g_{\mu\nu}$, $^\#g$
dual nature of e.m.:
photons & connection $^\#A$

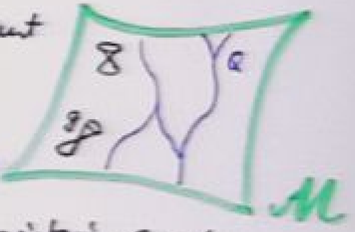
A New Theory: ${}^\#BM$ [T 2006]

Status of Lorentzian $g_{\mu\nu}$:

	flat $g_{\mu\nu}$	or given background $g_{\mu\nu}$	or	dynamically generated $g_{\mu\nu}$
Ex	SR	geodesics as test-particle world lines		GR
Here	BM	BM($\mathcal{F}$)		${}^\#BM$

${}^\#BM$:

- is diffeomorphism-invariant
- solution:



$$(\mathcal{M},\ g_{\mu\nu},\ Q) \text{ mod diffeos}$$

manifold → $\mathcal{M}$; Lorentzian on $\mathcal{M}$ → $g_{\mu\nu}$; trajectories $Q \subset \mathcal{M}$ → Q

- uses a time foliation $\mathcal{F}$
- variables for generating solution:

$${}^\#g,\ {}^\#A,\ \psi,\ \mathcal{F},\ {}^\#\mathcal{F}$$

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$\Longrightarrow$ dual nature of gravity:

graviton particles & geometry $g_{\mu\nu}, {}^\#g$

dual nature of e.m.:

photons & connection

A New Theory: $^\sharp 3M$ [T 2006]

Status of Lorentzian $g_{\mu\nu}$:

	flat $g_{\mu\nu}$	or	given background $g_{\mu\nu}$	or	dynamically generated $g_{\mu\nu}$
Ex	SR		geodesics as test-particle world lines		GR
Here	3M		3M($\mathcal{F}$)		$^\sharp 3M$

$^\sharp 3M$:

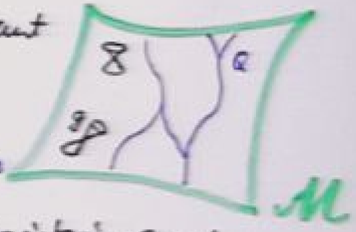
- is diffeomorphism-invariant
- solution:

$$(\mathcal{M},\ g_{\mu\nu},\ Q) \text{ mod diffeos}$$

manifold → $\mathcal{M}$

Lorentzian on $\mathcal{M}$ → $g_{\mu\nu}$

trajectories $Q \subset \mathcal{M}$ → Q



- uses a time foliation $\mathcal{F}$
- variables for generating solution:

$$^\sharp g,\ ^\sharp A,\ \psi,\ \mathcal{F},\ ^\sharp\mathcal{F}$$

- photon and graviton trajectories

$\Longrightarrow$ dual nature of gravity:

graviton particles & geometry $g_{\mu\nu}, {}^\sharp g$

dual nature of e.m.:

photons & connection $^\sharp A$

Status of Lorentzian $g_{\mu\nu}$:

	$\boxed{\text{flat } g_{\mu\nu}}$	or	$\boxed{\text{given background } g_{\mu\nu}}$	or	$\boxed{\text{dynamically generated } g_{\mu\nu}}$
Ex	SR		geodesics as test-particle world lines		GR
Here	3M		3M($\mathcal{F}$)		$^{\#}$3M

$^{\#}$3M:

- is diffeomorphism-invariant
- solution:

$$(\mathcal{M},\ g_{\mu\nu},\ Q) \text{ mod diffeos}$$

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Lorentzian on $\mathcal{M}$ → $g_{\mu\nu}$

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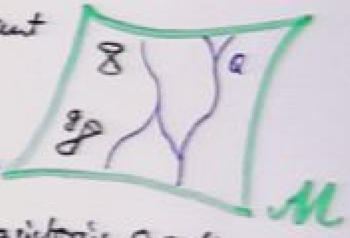

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$\Rightarrow$ dual nature of gravity:
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dual nature of e.m.:
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figuration – Space – Time $^*\mathcal{M}$

is $^*\mathcal{M}$? The space on which ψ is defined.

could be: $(\mathbb{R}^3)^N \times \mathbb{R}$ (usual, N dist.ble particles)

$\mathcal{M}^N$ (multitime ψ, N dist.ble particles)

$\mathcal{M}^N / S_N$ (identical particles, multitime)

$\bigcup_{\Sigma\in\mathcal{F}} \Sigma^N$ (using time foliation)

$\bigcup_{\Sigma\in\mathcal{F}} \Sigma^N / S_N$ (identical particles, using $\mathcal{F}$)

BM: $$^*\mathcal{M} = \bigcup_{\Sigma\in\mathcal{F}} \Gamma(\Sigma) = \bigcup_{\Sigma\in\mathcal{F}} \bigcup_{N=0}^{\infty} \Sigma^N/S_N = \bigcup_{N=0}^{\infty} \overset{\text{dim} = 3N+1}{{}^*\mathcal{M}_N}$$

$\Rightarrow$ $^*\mathcal{M} \ni q = \{x_1, \dots, x_N\}$ simultaneous configuration

$^*\mathcal{F}$ = lift of $\mathcal{F} = \{\Gamma(\Sigma) : \Sigma\in\mathcal{F}\}$ foliation of $^*\mathcal{M}$

Objects on $^*\mathcal{M}$: $\boxed{{}^*g,\ {}^*A,\ \psi}$

$^*g$ = Lorentzian metric on $^*\mathcal{M}$
of signature $(+\underbrace{-\,-\,-\,-\,-\,-\,-\,-\cdots-}_{3N \text{ on } {}^*\mathcal{M}_N})$

$^*A$ = connection (vector potential, 1-form) on $^*\mathcal{M}$

Configuration-Space-Time ${}^{\neq}\mathcal{M}$

What's ${}^{\neq}\mathcal{M}$? The space on which ψ is defined.

${}^{\neq}\mathcal{M}$ could be: $(\mathbb{R}^3)^N \times \mathbb{R}$ (nonrel., N dist.ble particles)

$\mathcal{M}^N$ (multitime ψ, N dist.ble particles)

$\mathcal{M}^N / S_N$ (identical particles, multitime)

$\bigcup_{\Sigma \in \mathcal{F}} \Sigma^N$ (using time foliation)

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Here in ${}^{\neq}$BM:

$$ {}^{\neq}\mathcal{M} = \bigcup_{\Sigma \in \mathcal{F}} \Gamma(\Sigma) = \bigcup_{\Sigma \in \mathcal{F}} \bigcup_{N=0}^{\infty} \Sigma^N / S_N = \bigcup_{N=0}^{\infty} \overset{\text{dim} = 3N+1}{{}^{\neq}\mathcal{M}_N} $$

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Objects on ${}^{\neq}\mathcal{M}$:

$$\boxed{{}^{\neq}g, \quad {}^{\neq}A, \quad \psi}$$

${}^{\neq}g$ = Lorentzian metric on ${}^{\neq}\mathcal{M}$

of signature $(+ \underbrace{- - - - - - - - \cdots}_{3N \text{ on } {}^{\neq}\mathcal{M}_N})$

${}^{\neq}A$ = connection (vector potential, 1-form) on ${}^{\neq}\mathcal{M}$

Configuration – Space – Time ${}^{\sharp}\mathcal{M}$

What is ${}^{\sharp}\mathcal{M}$? The space on which ψ is defined.

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Objects on $^\#\mathcal{M}$: $\boxed{{}^\#g,\ {}^\#A,\ \mathbb{C}}$

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${}^{\neq}A =$ connection (vector potential, 1-form) on ${}^{\neq}\mathcal{M}$

Dynamical Laws

Hierarchical Structure: ${}^{\#}g, {}^{\#}A \longrightarrow \psi \longrightarrow Q, g$

law for g

$$g(x) = {}^{\#}g(Q_\Sigma \cup x) \quad \text{for } \Sigma \ni x$$

"the metric that the next ($N+1$st) particle would see if it were at x"

$$\Longrightarrow \quad {}^{\#}g, Q \longrightarrow g$$

laws for Q

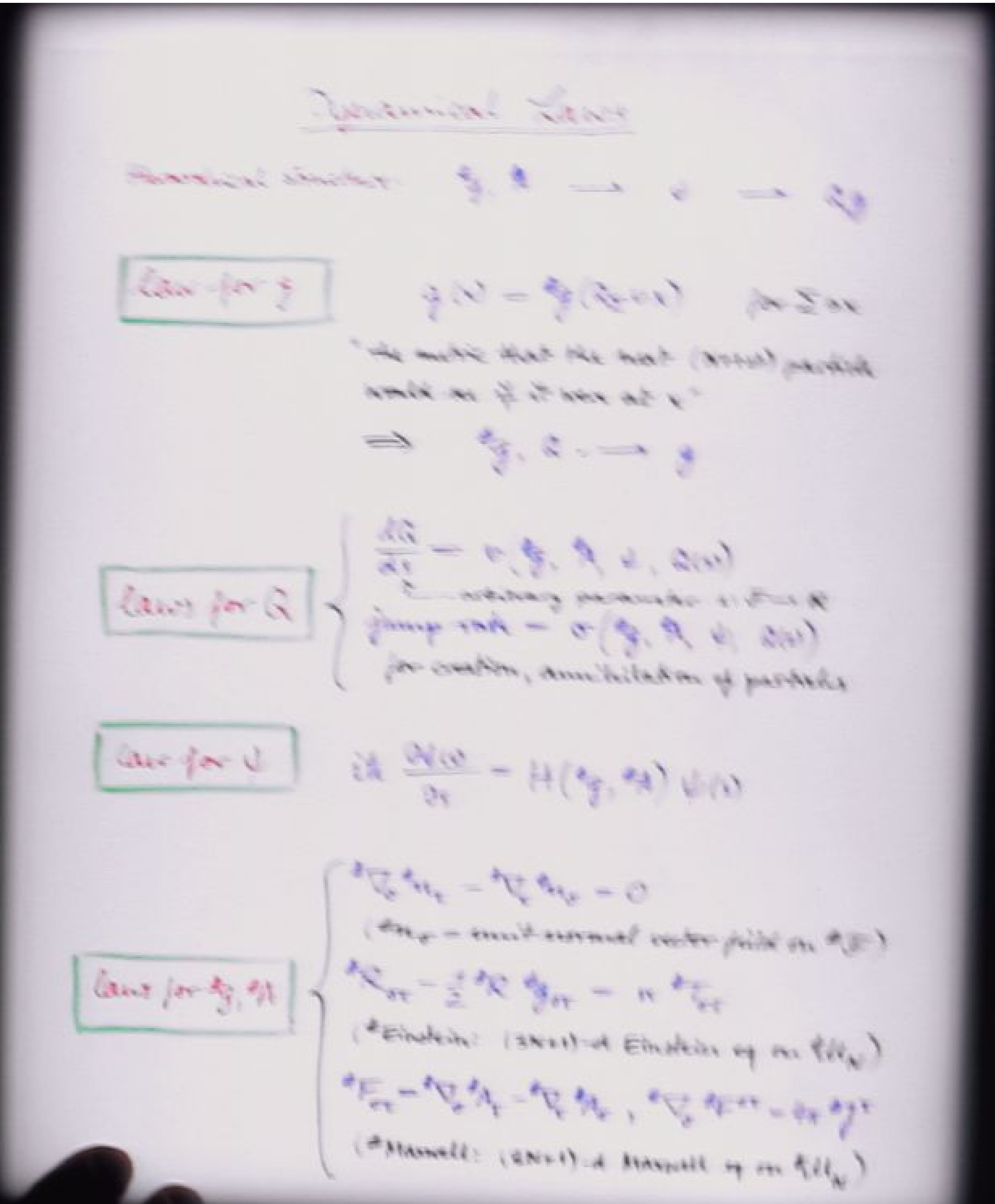
$$\left\{ \begin{array}{l} \dfrac{dQ}{ds} = v({}^{\#}g, {}^{\#}A, \psi, Q(s)) \\ \uparrow \text{arbitrary parameter } s: {}^{\#}\mathcal{F} \to \mathbb{R} \\ \text{jump rate} = \sigma({}^{\#}g, {}^{\#}A, \psi, Q(s)) \\ \text{for creation, annihilation of particles} \end{array} \right.$$

law for ψ

$$i\hbar \frac{\partial \psi(s)}{\partial s} = H({}^{\#}g, {}^{\#}A)\, \psi(s)$$

laws for ${}^{\#}g, {}^{\#}A$

$$\left\{ \begin{array}{l} {}^{\#}\nabla_\sigma {}^{\#}n_\tau - {}^{\#}\nabla_\tau {}^{\#}n_\sigma = 0 \\ ({}^{\#}n_\sigma = \text{unit normal vector field on } {}^{\#}\mathcal{F}) \\ {}^{\#}R_{\sigma\tau} - \frac{1}{2} {}^{\#}R\, {}^{\#}g_{\sigma\tau} = \kappa\, {}^{\#}T_{\sigma\tau} \\ ({}^{\#}\text{Einstein: } (3N+1)\text{-d Einstein eq on } {}^{\#}\mathcal{M}_N) \\ {}^{\#}F_{\sigma\tau} = {}^{\#}\nabla_\sigma {}^{\#}A_\tau - {}^{\#}\nabla_\tau {}^{\#}A_\sigma, \quad {}^{\#}\nabla_\sigma {}^{\#}F^{\sigma\tau} = 4\pi\, {}^{\#}j^\tau \\ ({}^{\#}\text{Maxwell: } (3N+1)\text{-d Maxwell eq on } {}^{\#}\mathcal{M}_N) \end{array} \right.$$



Dynamical Laws

Dynamical structure: $^*g, {}^*A \longrightarrow \psi \longrightarrow Q$

Law for g $g(x) = {}^*g(Q_\Sigma, x)$ for Σ on

"the metric that the next (actual) particle would see if it were at x"

$\Longrightarrow \quad {}^*g, Q \longrightarrow g$

Laws for Q

$$\frac{dQ}{ds} = v({}^*g, {}^*A, \psi, Q(s))$$

arbitrary parameter $s: \Sigma \to \mathbb{R}$

jump rate $= \sigma({}^*g, {}^*A, \psi, Q(s))$

for creation, annihilation of particles

Law for ψ

$$i\hbar \frac{\partial \psi(t)}{\partial t} = H({}^*g, {}^*A)\,\psi(t)$$

Laws for *g, *A

$$^*\nabla_\mu {}^*n_\nu - {}^*\nabla_\nu {}^*n_\mu = 0$$

($^*n_\mu$ = unit normal vector field on $^*\Sigma$)

$$^*R_{\mu\nu} - \frac{1}{2}{}^*R\,{}^*g_{\mu\nu} = \kappa\,{}^*T_{\mu\nu}$$

(*Einstein: (3N+1)-d Einstein eq on $^*\mathcal{U}_N$)

$$^*F_{\mu\nu} = {}^*\nabla_\mu {}^*A_\nu - {}^*\nabla_\nu {}^*A_\mu\,, \quad {}^*\nabla_\mu {}^*F^{\mu\nu} = 4\pi\,{}^*j^\nu$$

(*Maxwell: (3N+1)-d Maxwell eq on $^*\mathcal{U}_N$)

Dynamical Laws

Hierarchical Structure: ${}^\# g, {}^\# A \longrightarrow \psi \longrightarrow Q, g$

law for g

$$g(x) = {}^\# g(Q_\Sigma \cup x) \quad \text{for } \Sigma \ni x$$

"the metric that the next (N+1st) particle would see if it were at x"

$$\Longrightarrow \quad {}^\# g, Q \longrightarrow g$$

laws for Q

$$\frac{dQ}{ds} = v({}^\# g, {}^\# A, \psi, Q(s))$$

↑ arbitrary parameter $s : \mathcal{F} \to R$

jump rate $= \sigma({}^\# g, {}^\# A, \psi, Q(s))$

for creation, annihilation of particles

law for ψ

$$i\hbar \frac{\partial \psi(s)}{\partial s} = H({}^\# g, {}^\# A)\, \psi(s)$$

laws for ${}^\# g, {}^\# A$

$${}^\# \nabla_\sigma {}^\# n_\tau - {}^\# \nabla_\tau {}^\# n_\sigma = 0$$

(${}^\# n_\sigma$ = unit normal vector field on ${}^\# \mathcal{F}$)

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(${}^\#$Einstein: $(3N+1)$-d Einstein eq on ${}^\# \mathcal{M}_N$)

$${}^\# F_{\sigma\tau} = {}^\# \nabla_\sigma {}^\# A_\tau - {}^\# \nabla_\tau {}^\# A_\sigma \, , \quad {}^\# \nabla_\sigma {}^\# F^{\sigma\tau} - 4\pi \, {}^\# j^\tau$$

(${}^\#$Maxwell: $(3N+1)$-d Maxwell eq on ${}^\# \mathcal{M}_N$)

$\boxed{\text{law for } g}$

$$g(x) = {}^{\#}g(Q_\Sigma \cup x) \quad \text{for } \Sigma \ni x$$

"the metric that the next (N+1st) particle would see if it were at x"

$$\Longrightarrow \quad {}^{\#}g,\, Q \longrightarrow g$$

$\boxed{\text{laws for } Q}$

$$\left\{ \begin{array}{l} \dfrac{dQ}{ds} = v({}^{\#}g,\, {}^{\#}A,\, \psi,\, Q(s)) \\ \quad \uparrow\!\text{— arbitrary parameter } s: \mathcal{F} \to R \\ \text{jump rate} = \sigma({}^{\#}g,\, {}^{\#}A,\, \psi,\, Q(s)) \\ \quad \text{for creation, annihilation of particles} \end{array} \right.$$

$\boxed{\text{law for } \psi}$

$$i\hbar \frac{\partial \psi(s)}{\partial s} = H({}^{\#}g, {}^{\#}A)\, \psi(s)$$

$\boxed{\text{laws for } {}^{\#}g, {}^{\#}A}$

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for creation, annihilation of particles

law for ψ

$$i\hbar \frac{\partial \psi(s)}{\partial s} = H\left({}^{\#}g, {}^{\#}A\right) \psi(s)$$

laws for #g, #A

$${}^{\#}\nabla_\sigma {}^{\#}n_\tau - {}^{\#}\nabla_\tau {}^{\#}n_\sigma = 0$$

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The Source Terms

- $^\sharp$Einstein: $\;{}^\sharp R_{\sigma\tau} - \frac{1}{2}\,{}^\sharp R\,{}^\sharp g_{\sigma\tau} = \kappa\,{}^\sharp T_{\sigma\tau}$

$$^\sharp T = {}^\sharp T_{e.m.} + {}^\sharp T_{\text{particles}}$$

$$({}^\sharp T_{e.m.})_{\sigma\tau} = -\,{}^\sharp F_{\sigma\rho}\,{}^\sharp F_{\tau}{}^{\rho} - *{}^\sharp F_{\sigma\rho}\,{}^\sharp F_{\tau}{}^{\rho}$$

($*$ – Hodge dual)

$({}^\sharp T_{\text{particles}})_{\sigma\tau}$ = something like ${}^\sharp u_\sigma\,{}^\sharp u_\tau \sum_{i<j} m_i m_j\, \delta(x_i - x_j)$

- $^\sharp$Maxwell: $\;{}^\sharp\nabla_\sigma\,{}^\sharp F^{\sigma\tau} = 4\pi\,{}^\sharp J^\tau$

${}^\sharp J_\sigma$ = something like ${}^\sharp u_\sigma \sum_{i<j} q_i q_j\, \delta(x_i - x_j)$

↑ charge of particle i

- source terms $^\sharp T$, $^\sharp J$ are independent of ψ, Q, g.

The Source Terms

- ${}^\sharp$Einstein: $${}^\sharp R_{\sigma\tau} - \frac{1}{2}\, {}^\sharp R \, {}^\sharp g_{\sigma\tau} = \kappa \, {}^\sharp T_{\sigma\tau}$$

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$(* - \text{Hodge dual})$

$$({}^\sharp T_{particles})_{\sigma\tau} = \text{something like } {}^\sharp u_\sigma \, {}^\sharp u_\tau \sum_{i<j} m_i m_j \, \delta(x_i - x_j)$$

- ${}^\sharp$Maxwell: $${}^\sharp\nabla_\sigma \, {}^\sharp F^{\sigma\tau} = 4\pi \, {}^\sharp J^\tau$$

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charge of particle i

- source terms ${}^\sharp T$, ${}^\sharp J$ are independent of ψ, Q, g.

Similar Theories

- M. Kiessling 2004:

  Classical and Bohmian mechanics with vector potential ${}^{\ddagger}A$ on (more or less) config. space

- QED in Coulomb gauge (flat space-time):

$$\mathcal{H} = \mathcal{H}_{\text{charges}} \otimes \mathcal{H}_{\text{e.m.}}$$

$$H = H_{\text{charges}} + H_{\text{free photons}} +$$

$$+ H_{\text{creation/ann.}} + \underbrace{V_{\text{Coulomb}}}_{= {}^{\ddagger}A_0,}$$

$${}^{\ddagger}A_\sigma = 0 \text{ for } \sigma \neq 0$$

a special solution of ‡Maxwell for

$${}^{\ddagger}J_r = {}^{\ddagger}n_\sigma \sum_{i<j} q_i q_j \, \delta(x_i - x_j)$$

$${}^{\ddagger}g = \text{flat}$$

$${}^{\ddagger}\mathcal{F} = \text{parallel hyperplanes}$$

Field operators $\hat{A}_\mu(x)$, $x \in \mathcal{M}$:

$\hat{A}_\mu$ = multiplication by ${}^{\ddagger}A$ +

+ photon creati

classical and Bohmian mechanics with vector potential $^\# A$ on (more or less) config. space

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$$^\# J_\sigma = {}^\# n_\sigma \sum_{i<j} q_i q_j \, \delta(x_i - x_j)$$

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Field operators $\hat{A}_\mu(x)$, $x \in \mathcal{M}$:

$\hat{A}_\mu =$ multiplication by $^\# A$ +

+ photon creation + photon annihilation

Link to Quantum Gravity

- operators $\hat{g}_{\mu\nu}(x)$, $x \in \mathcal{M}$
- from *BM:

$$\hat{g}_{\mu\nu} = \text{multiplication by } {}^{*}g \; +$$
$$+ \text{ graviton creation } + \text{ graviton ann.}$$

- conversely, $g_{\mu\nu}$ from $\hat{g}_{\mu\nu}$:

$$g_{\mu\nu}(x) = \frac{\langle\psi|\, \hat{P}_{\Sigma}(Q_{\Sigma})\; \hat{g}_{\mu\nu}(x)\; \hat{P}_{\Sigma}(Q_{\Sigma})\,|\psi\rangle}{\langle\psi|\, \hat{P}_{\Sigma}(Q_{\Sigma})\,|\psi\rangle}$$

where $\exists\, \Sigma \ni x$

Q_{Σ} = actual configuration on Σ

$\hat{P}_{\Sigma}$ = position POVM on Σ

Conclusions

- BM = QM − weirdness
- If BM is right, need to revise Relativity:
  introduce $\mathcal{F}$
- $^{\#}$BM is a vision of

  QG − weirdness

  based on

$$^{\#}\mathcal{M} = \left\{ \{x_1 \dots x_N\} : N \geq 0,\ x_i \in \mathcal{M},\ \text{simultaneous} \right\}$$

$$^{\#}g = \text{Lorentzian metric on } {}^{\#}\mathcal{M}.$$

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  introduce $\mathcal{F}$

- ${}^{\#}$BM is a vision of
  QG − weirdness

based on

$${}^{\#}\mathcal{M} = \left\{ \{x_1 \ldots x_N\} : N \geq 0,\ x_i \in \mathcal{M},\ \text{simultaneous} \right\}$$

${}^{\#}g$ = Lorentzian metric on ${}^{\#}\mathcal{M}$.