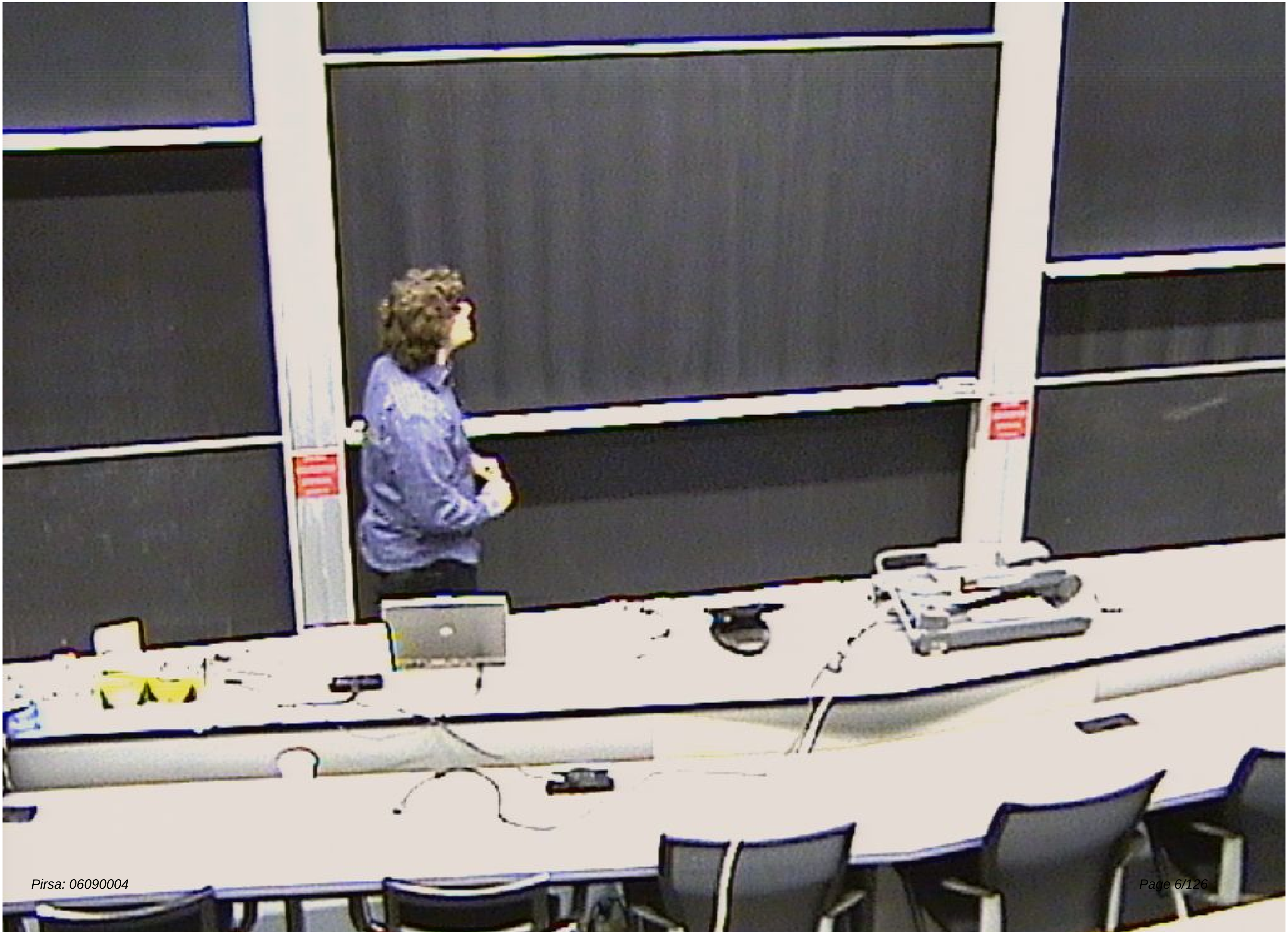
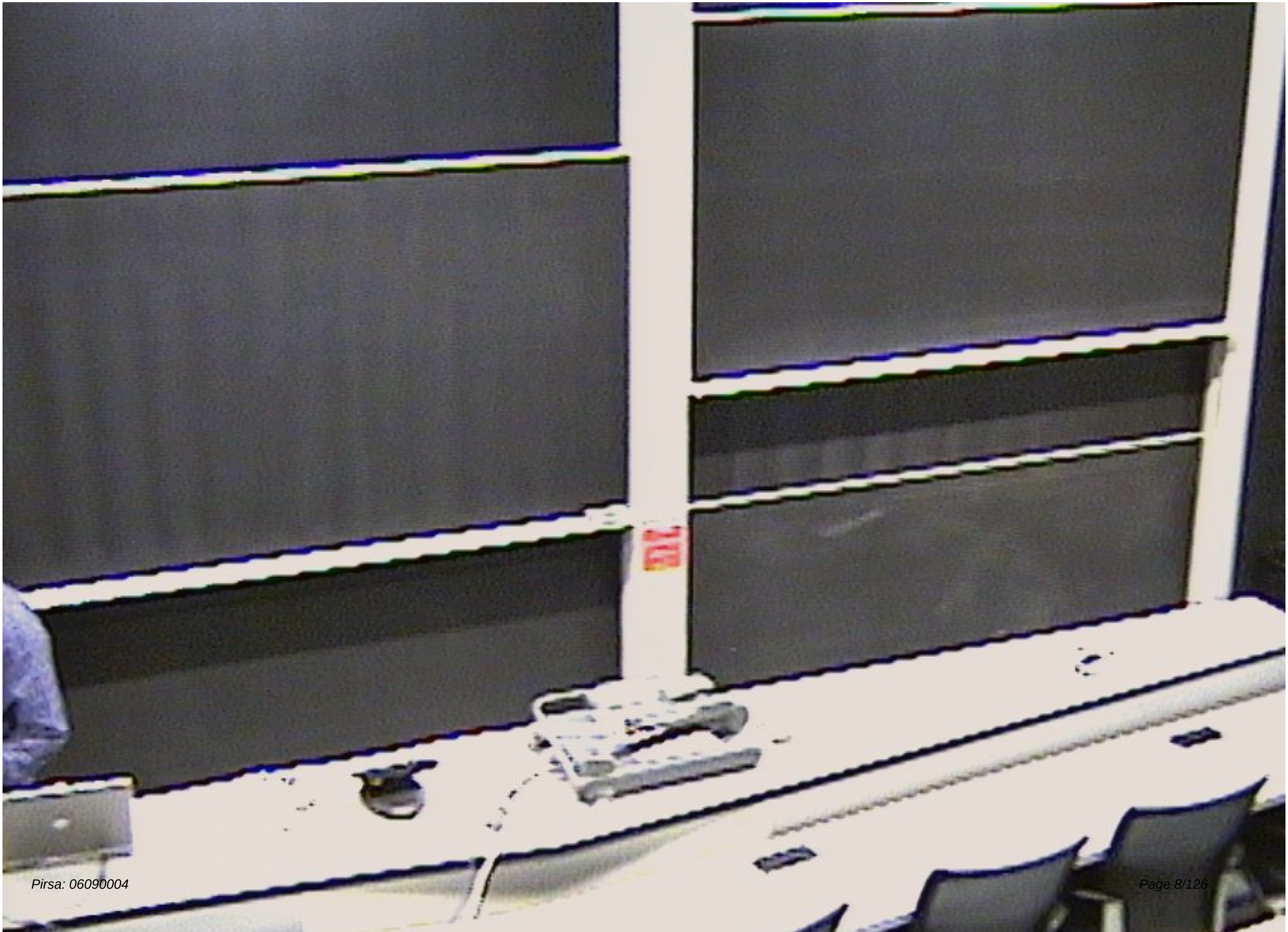
Title: Giant Magnons

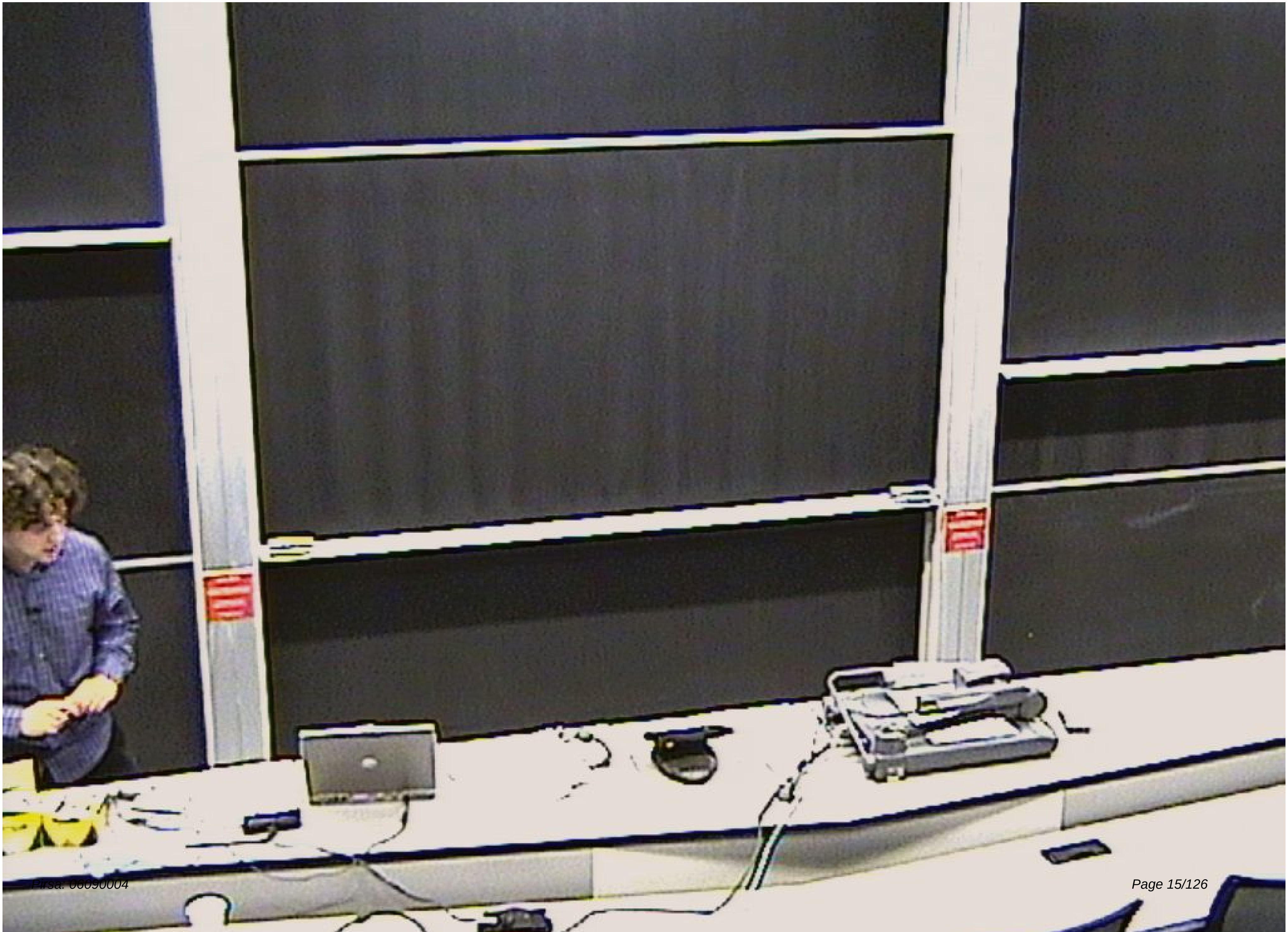
Date: Sep 12, 2006 11:00 AM

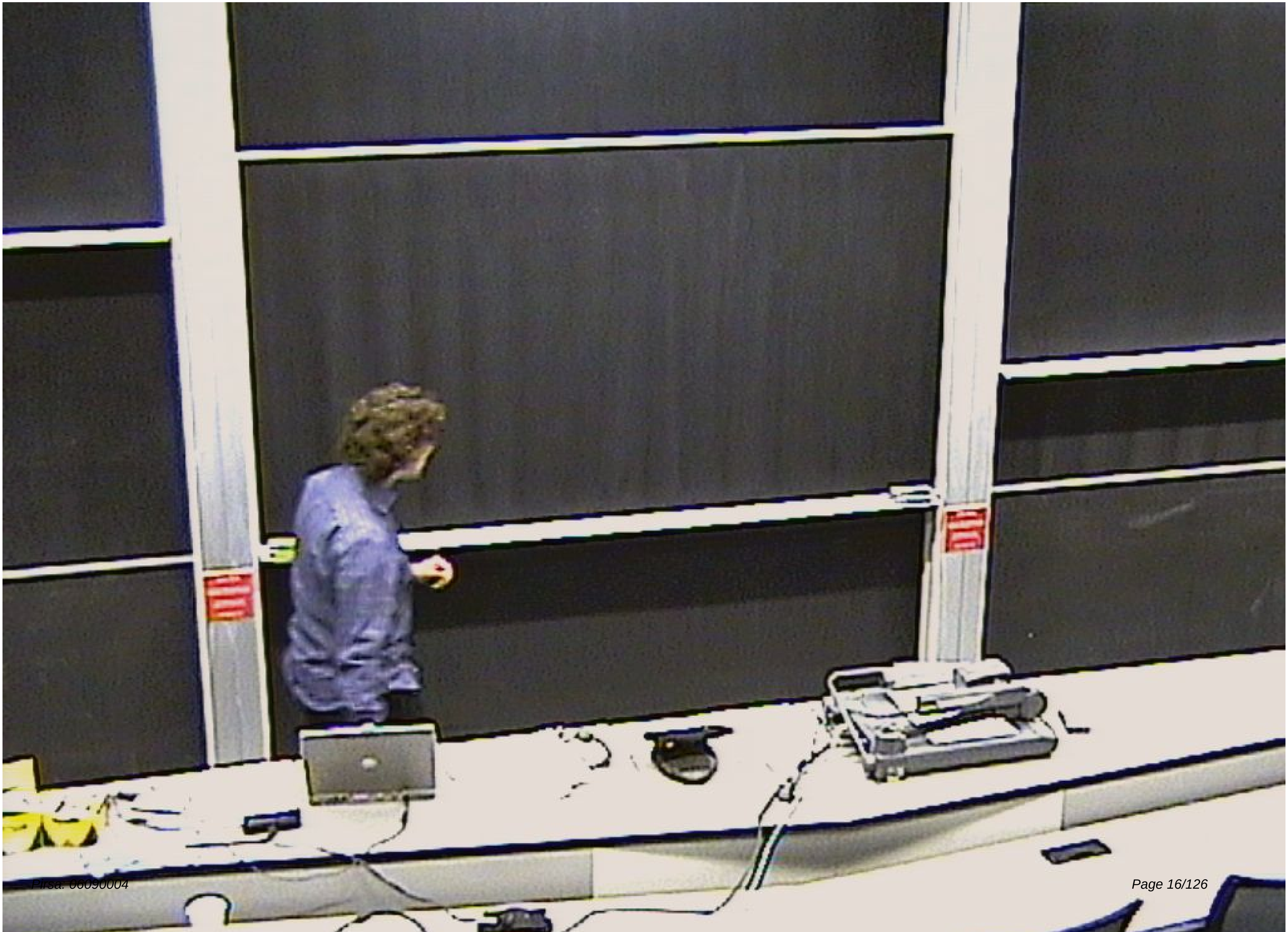
URL: http://pirsa.org/06090004

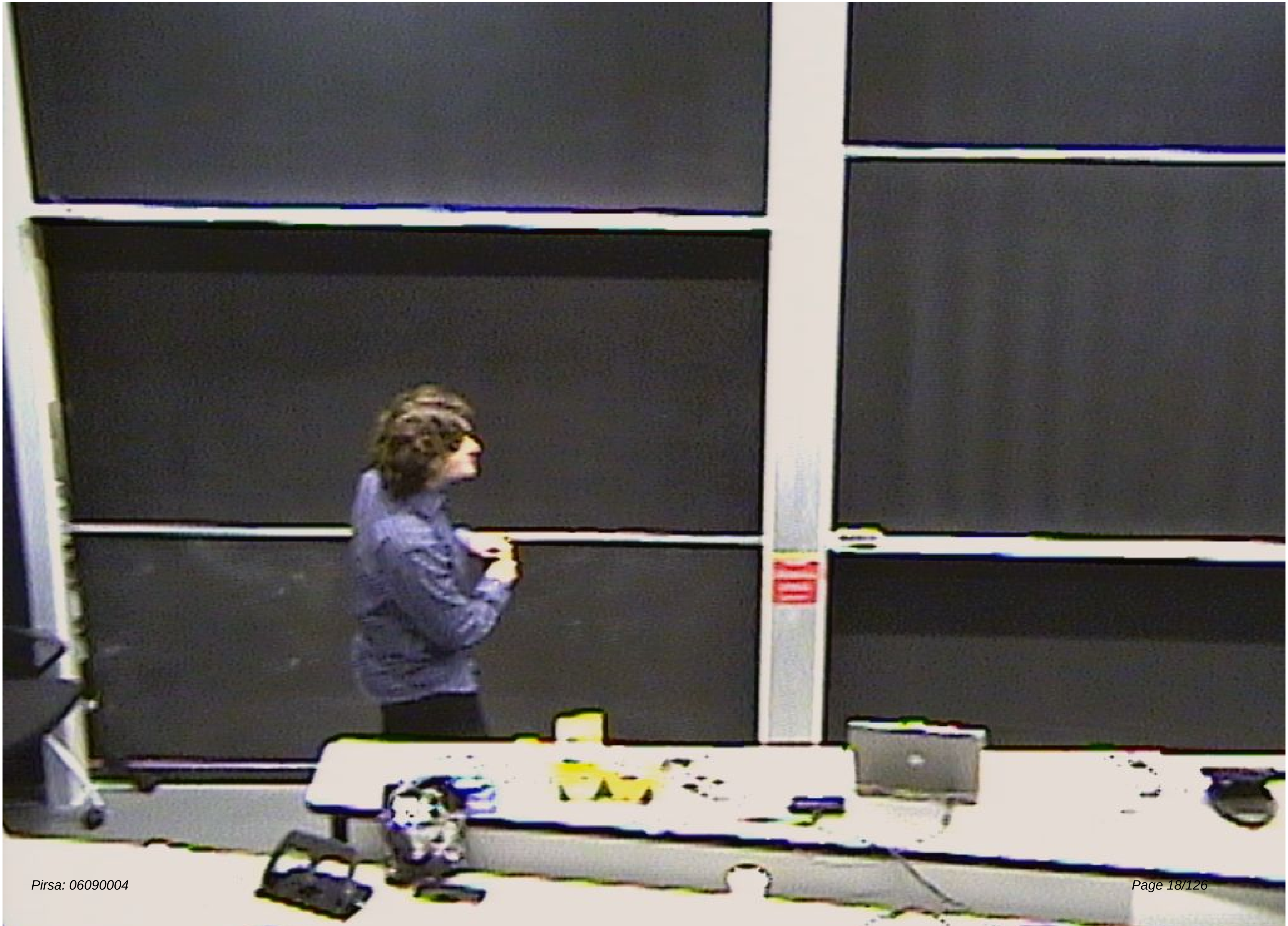
Abstract: Studies of ${cal N}=4$ super Yang Mills operators with large R-charge have shown that, in the planar limit, the problem of computing their dimensions can be viewed as a certain spin chain. These spin chains have fundamental ``magnon\'\' excitations which obey a dispersion relation that is periodic in the momentum of the magnons. This result for the dispersion relation was also shown to hold at arbitrary \'t Hooft coupling. Here we identify these magnons on the string theory side and we show how to reconcile a periodic dispersion relation with the continuum worldsheet description. The crucial idea is that the momentum is interpreted in the string theory side as a certain geometrical angle. We use these results to compute the energy of a spinning string. We also show that the symmetries that determine the dispersion relation and that constrain the S-matrix are the same in the gauge theory and the string theory. We compute the overall S-matrix at large \'t Hooft coupling using the string description and we find that it agrees with an earlier conjecture. We also find an infinite number of two magnon bound states at strong coupling, while at weak coupling this number is finite.

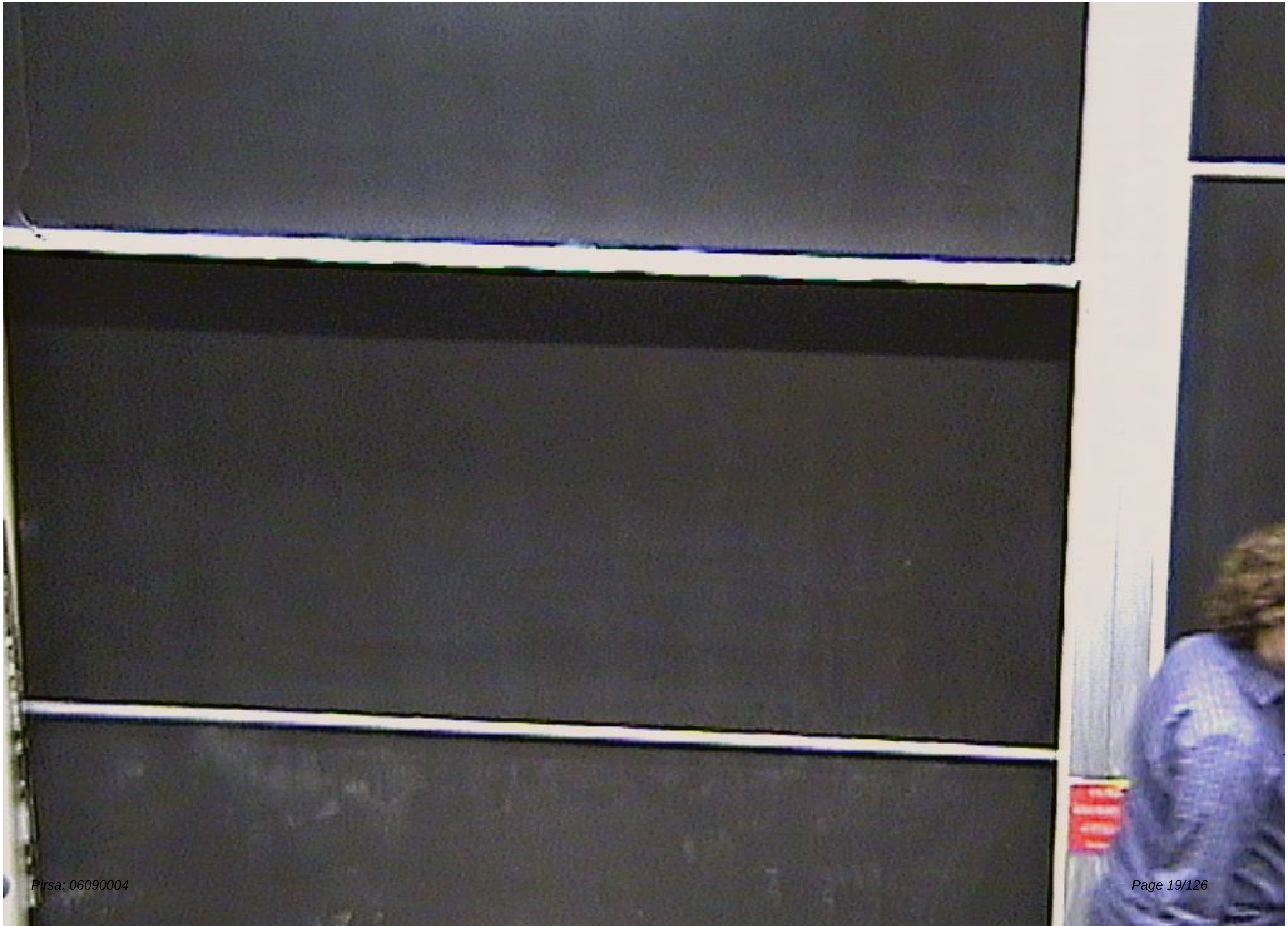


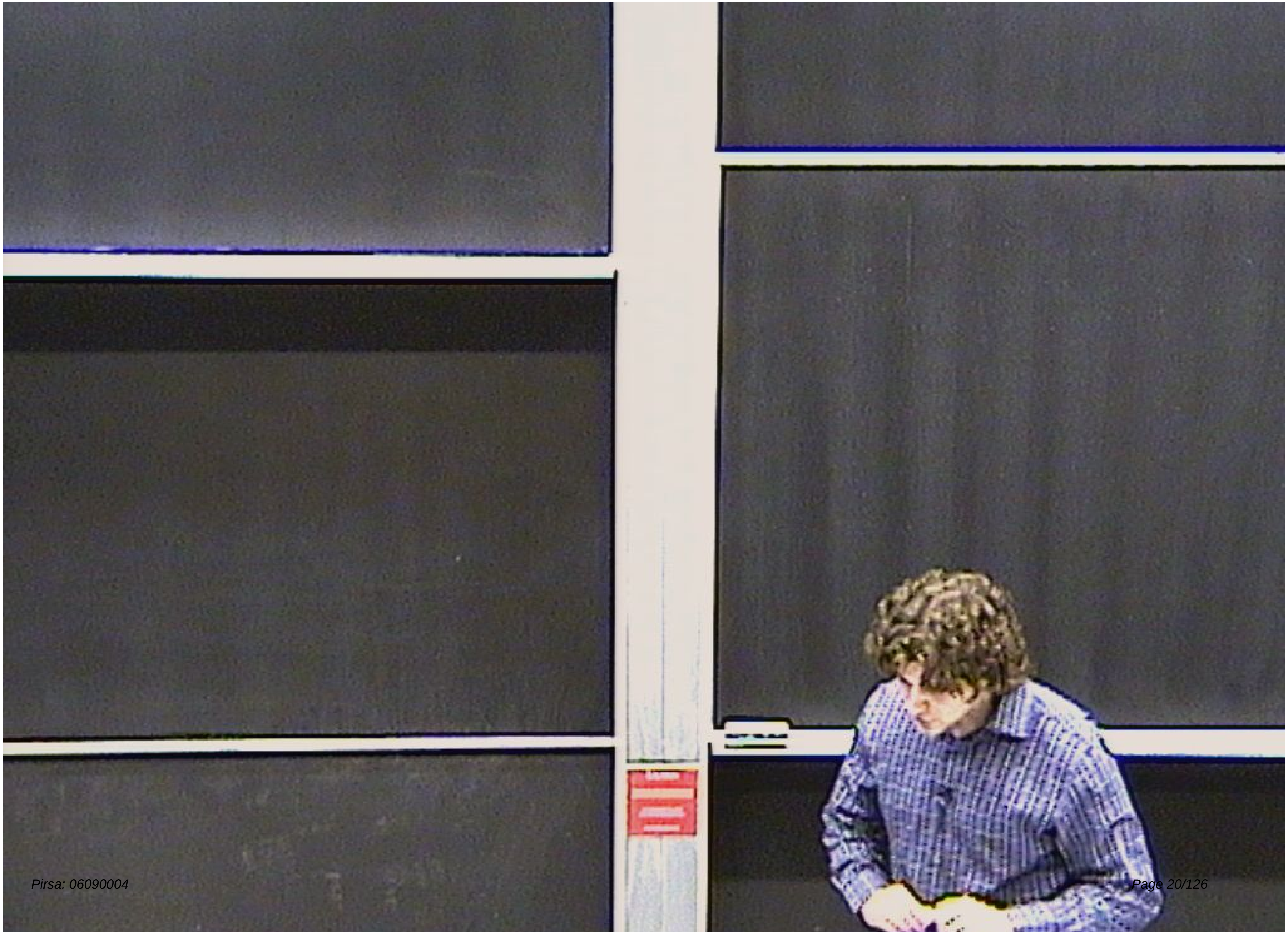


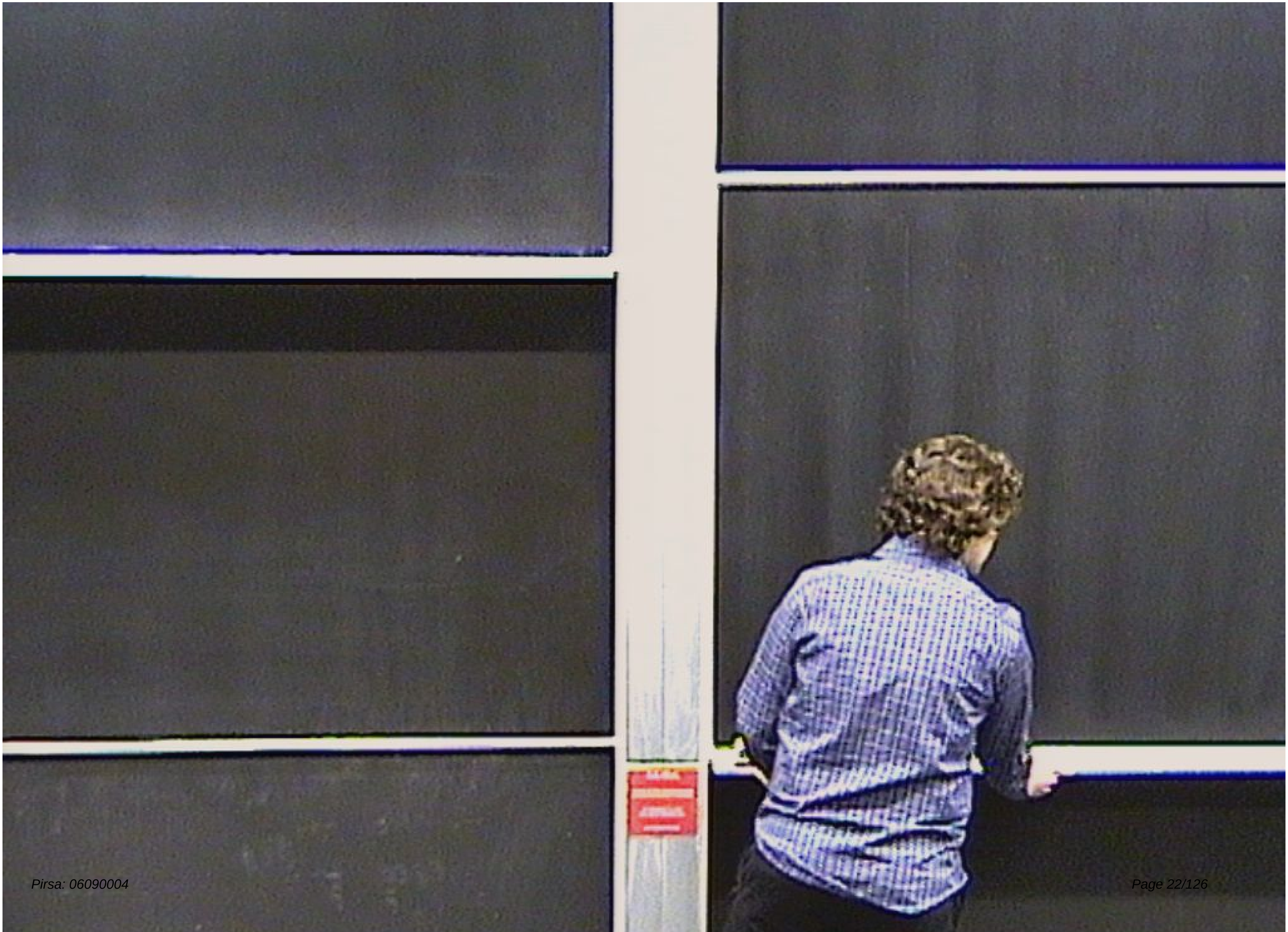


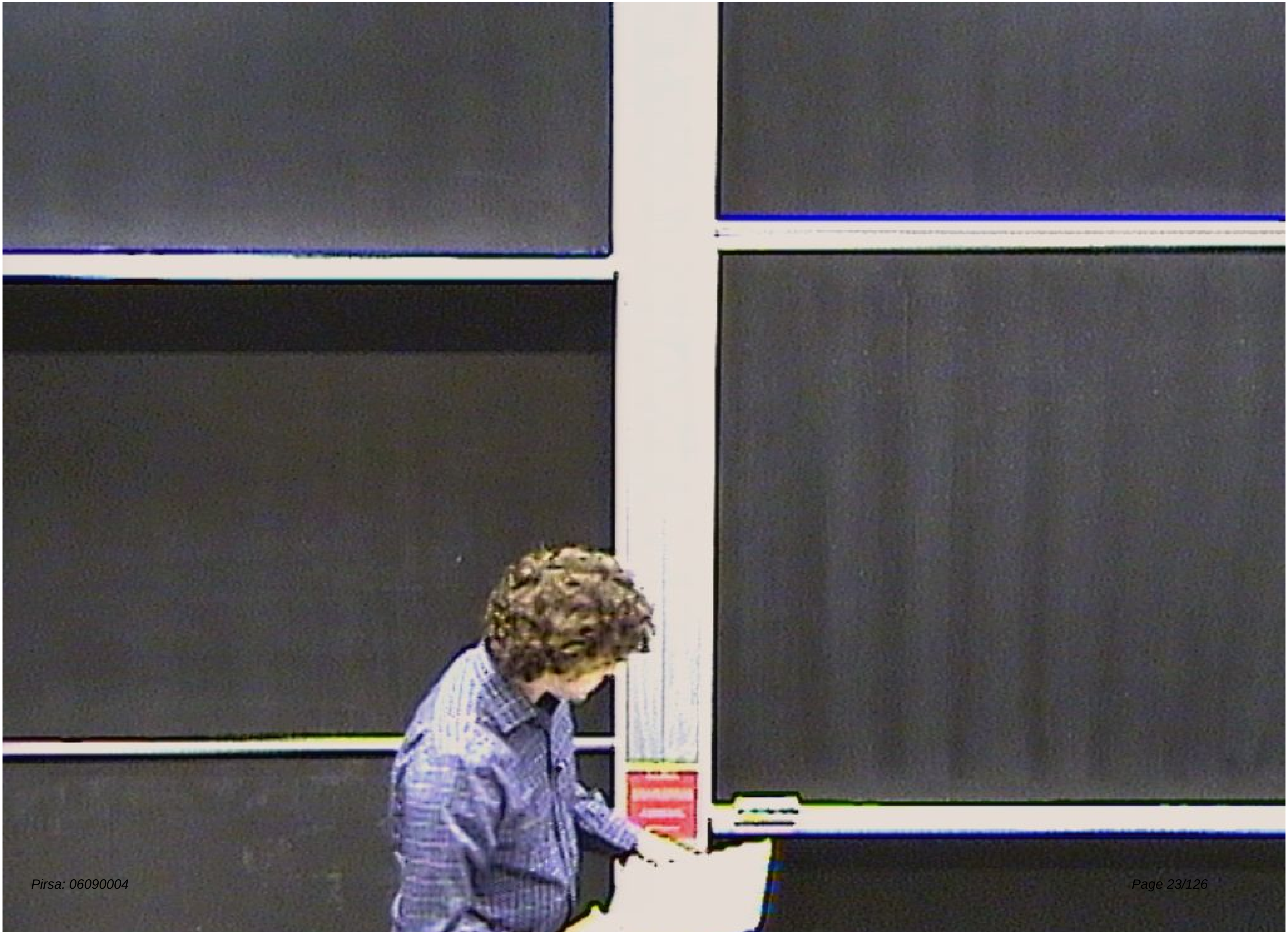


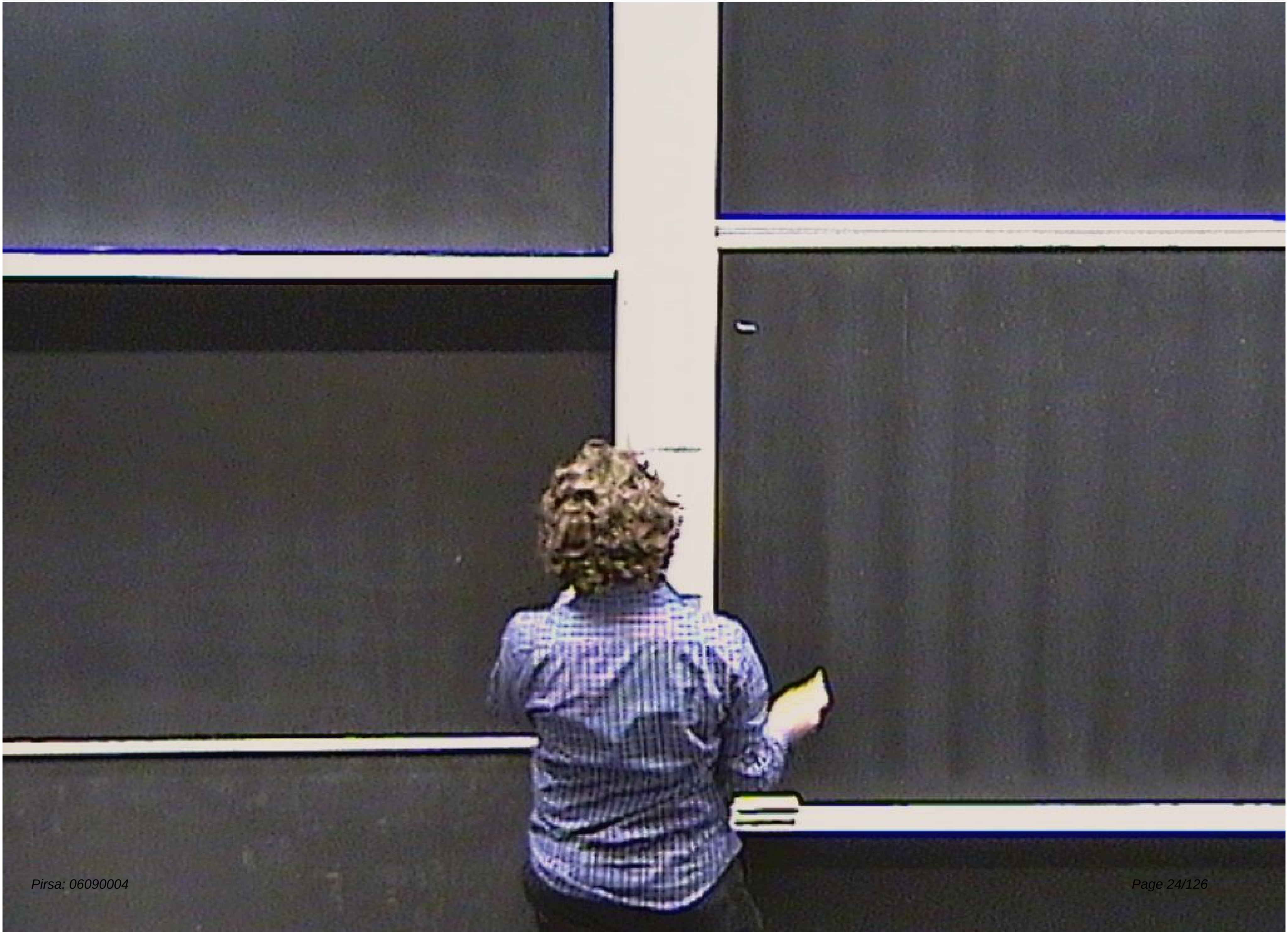


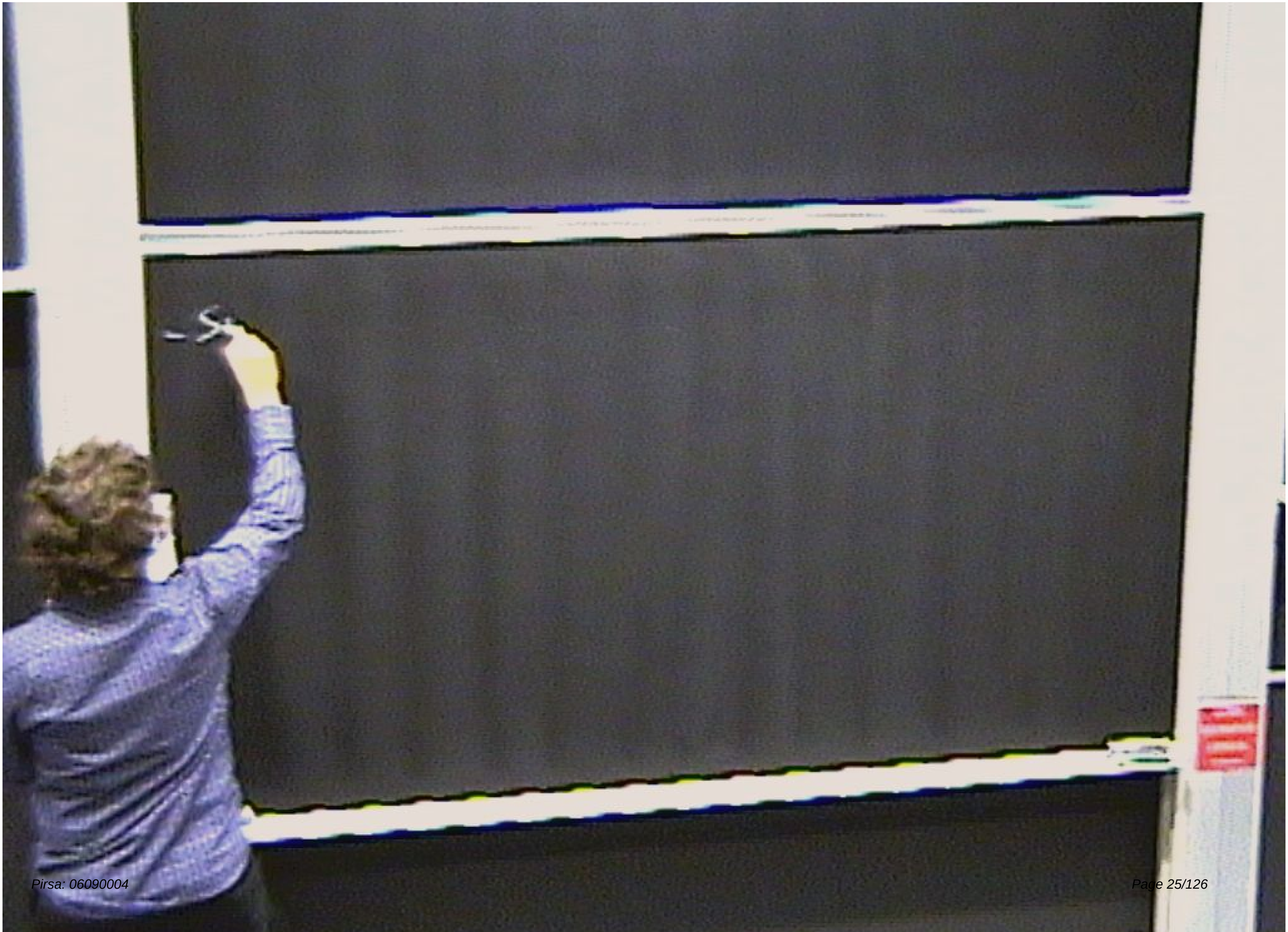


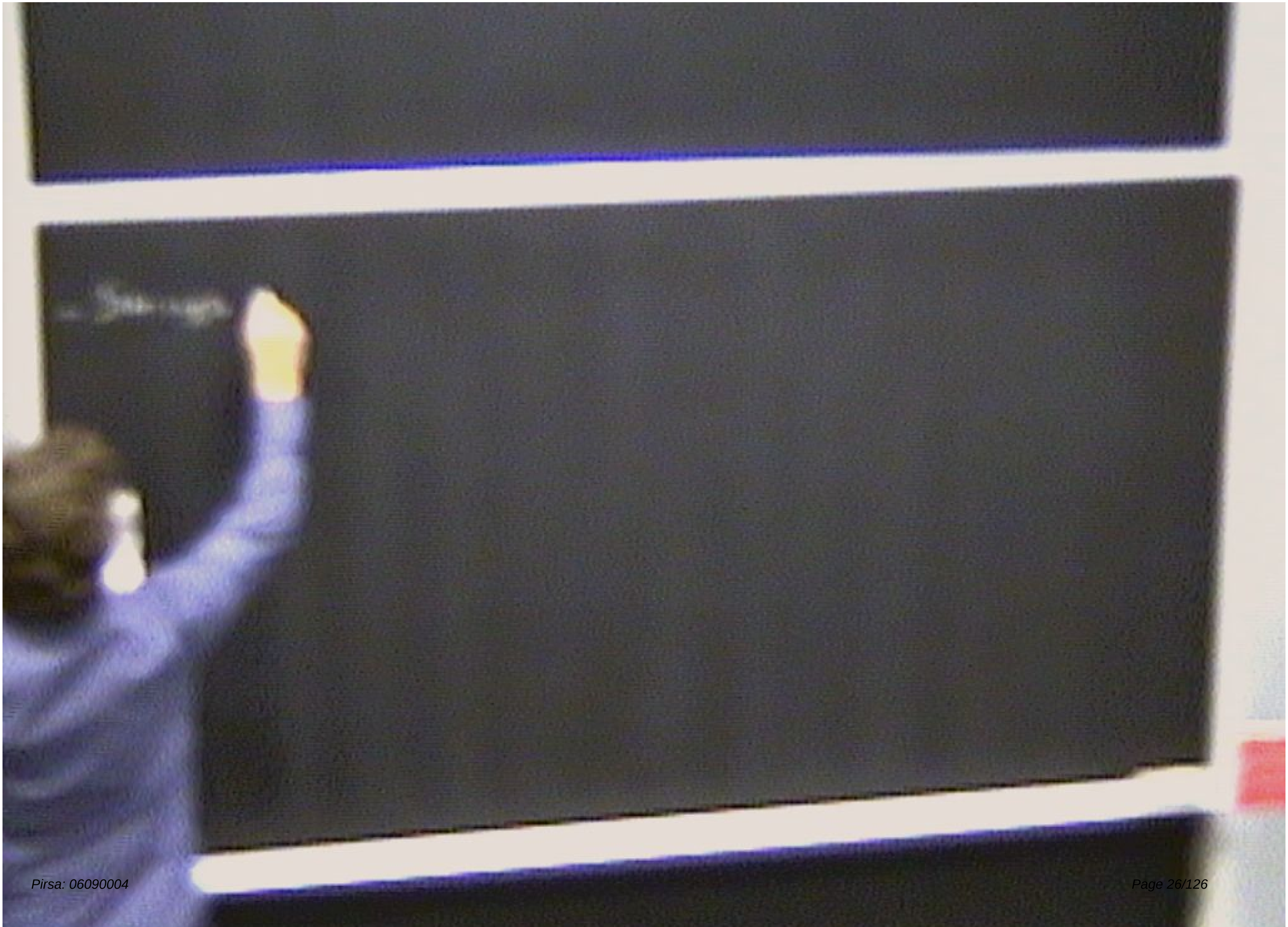


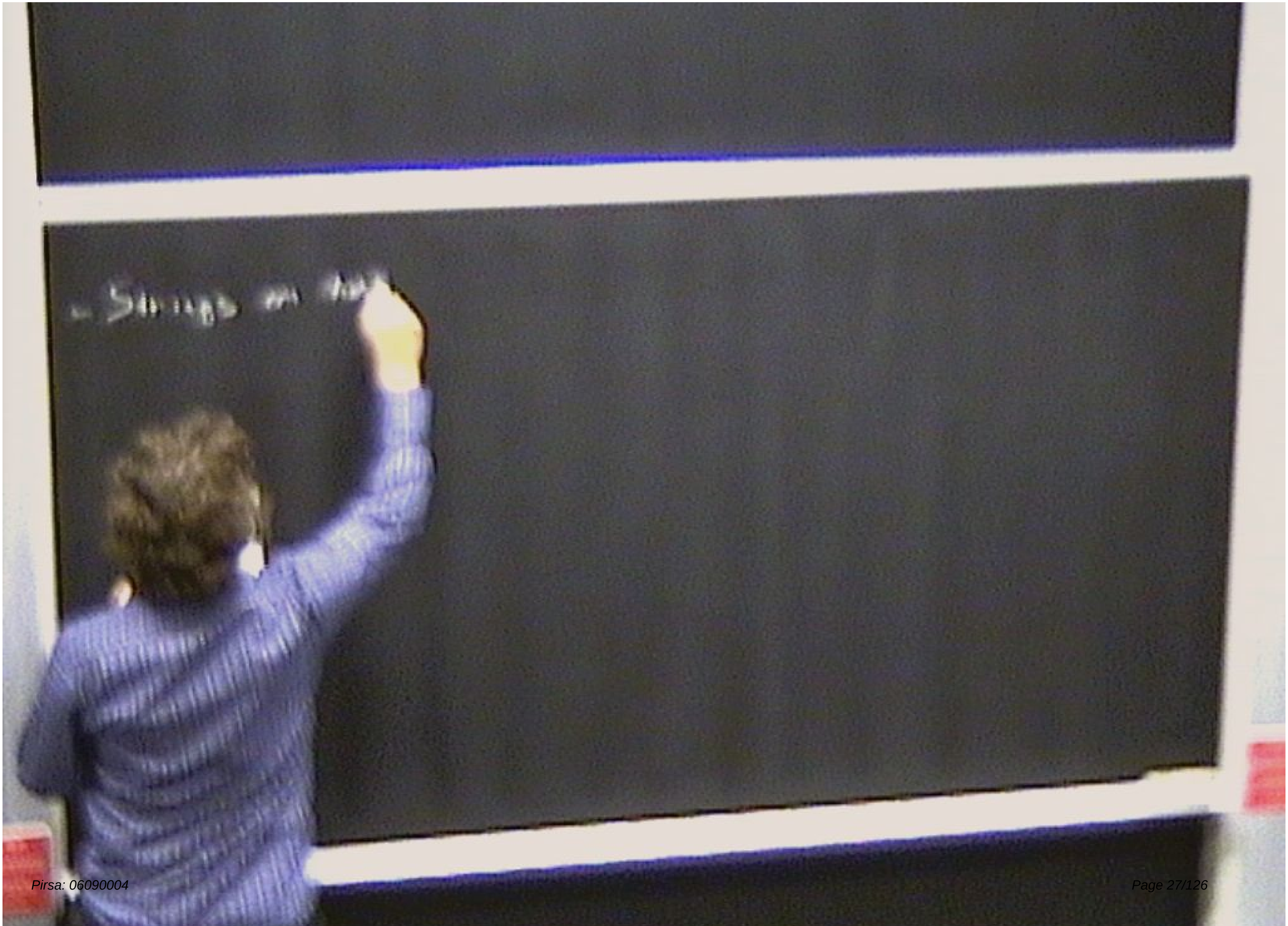


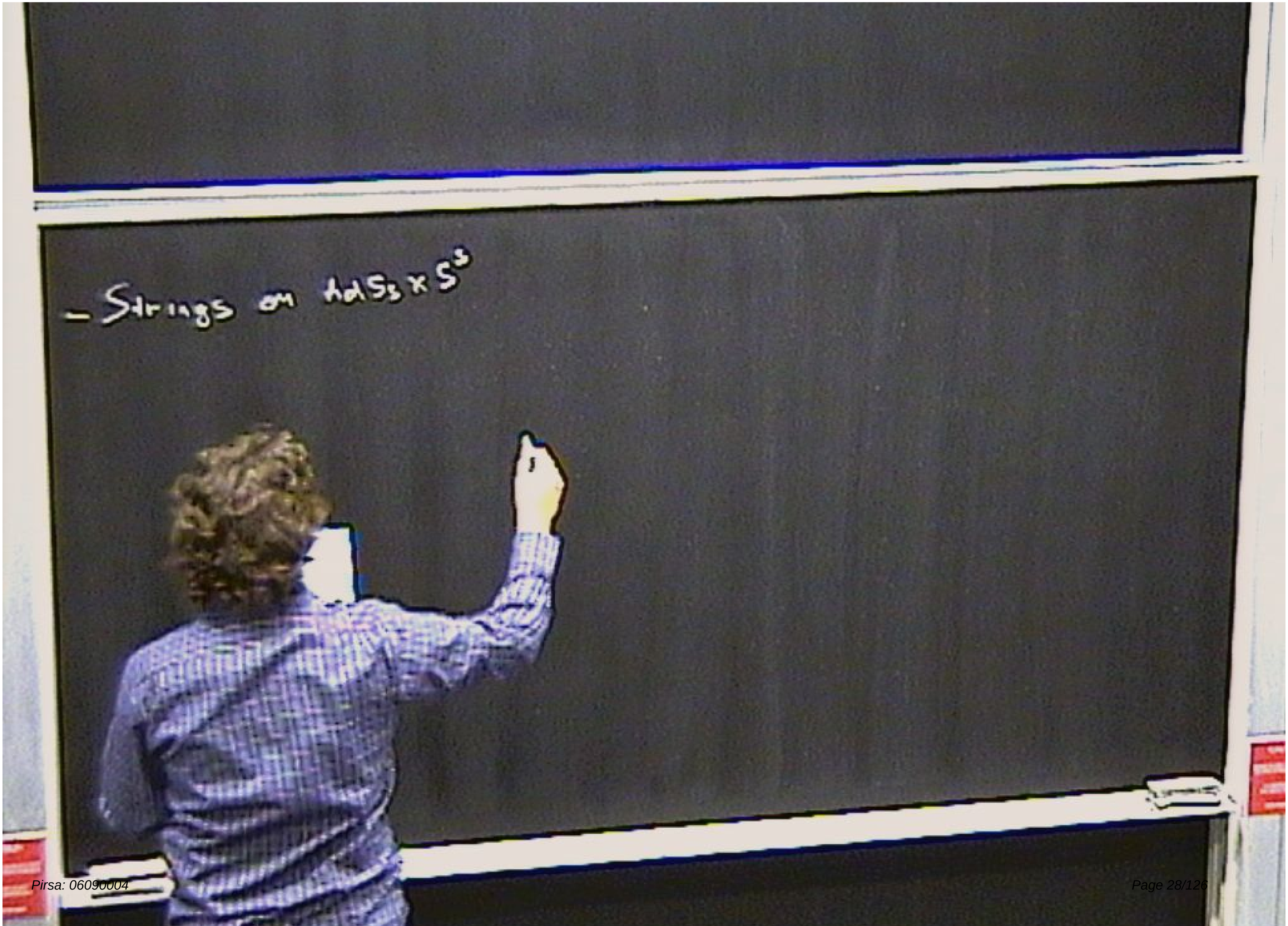


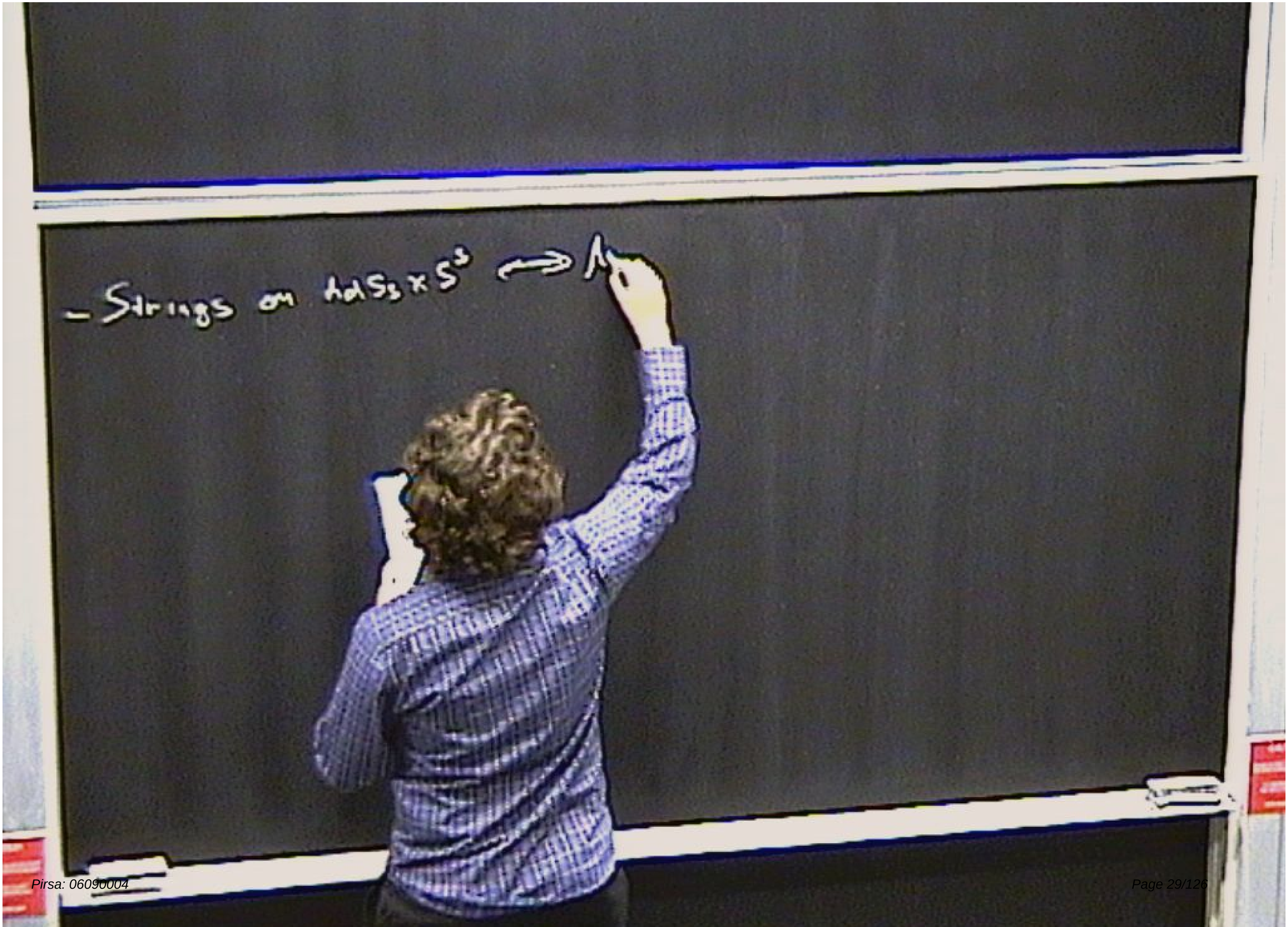


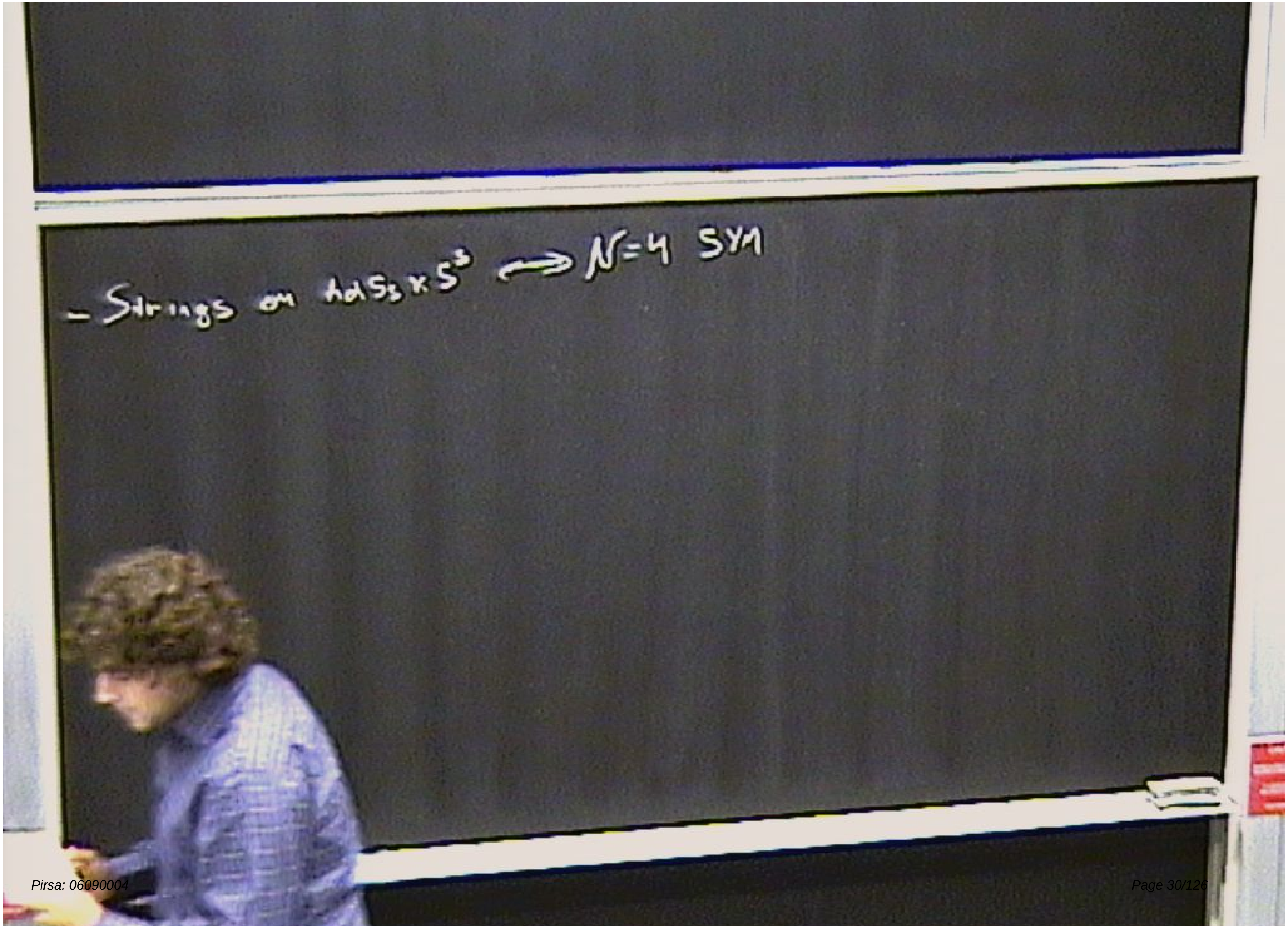





Strings on


– Strings on $AdS_3 \times S^3$

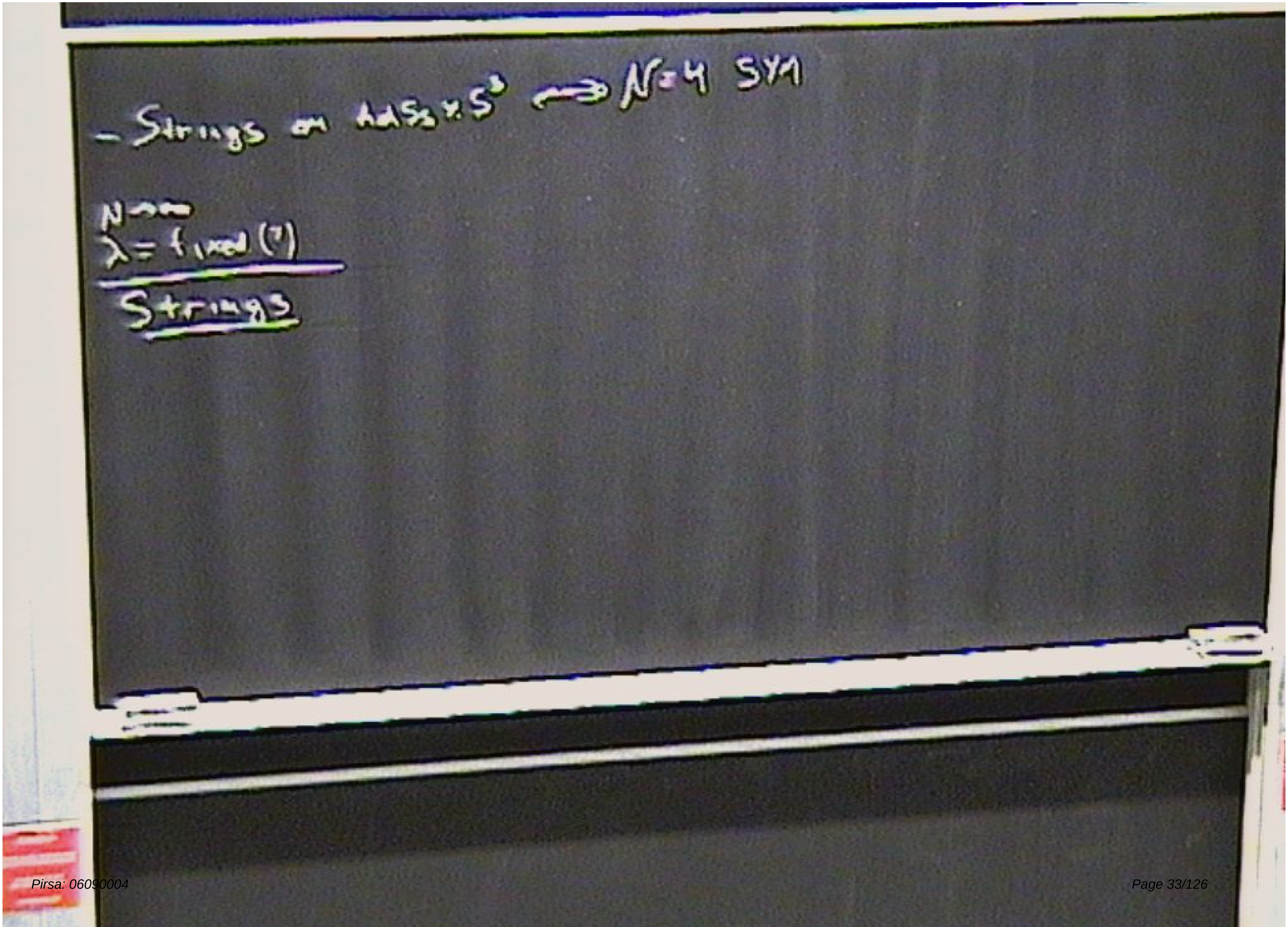

– Strings on $AdS_3 \times S^3 \longleftrightarrow$

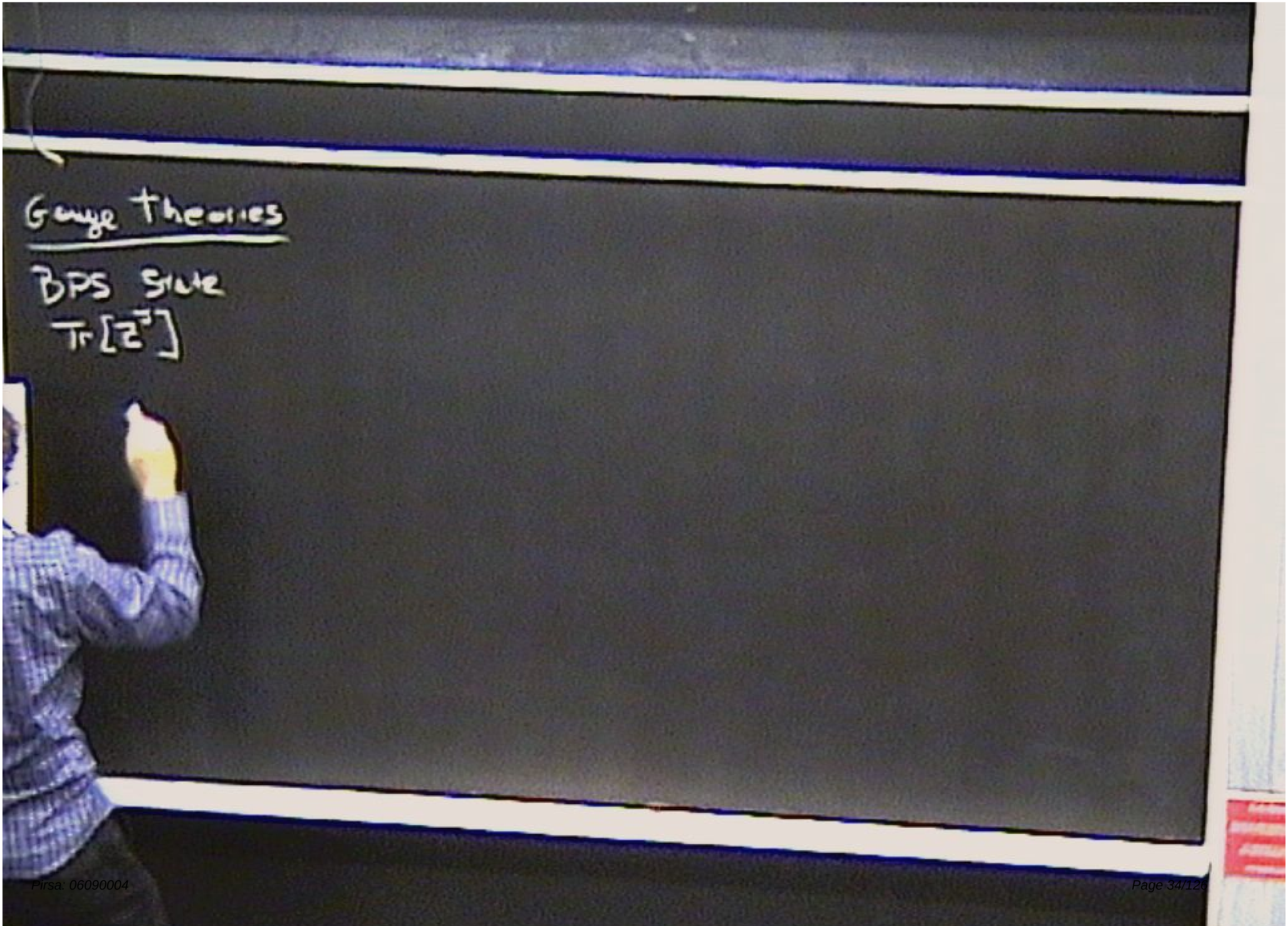

– Strings on AdS5 x S5 ⟷ N=4 SYM

– Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

– Strings on $AdS_5 \times S^5 \Longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$
$\lambda =$ fixed (?)


– Strings on $AdS_5 \times S^5$ $\Longleftrightarrow$ $\mathcal{N}=4$ SYM
$N \to \infty$
$\lambda =$ fixed (?)
Strings


Gauge Theories
BPS State
Tr[Z^J]

Gauge Theories

BPS State

$\mathrm{Tr}[Z^J]$

$\epsilon = \Delta - J = 0$

- Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

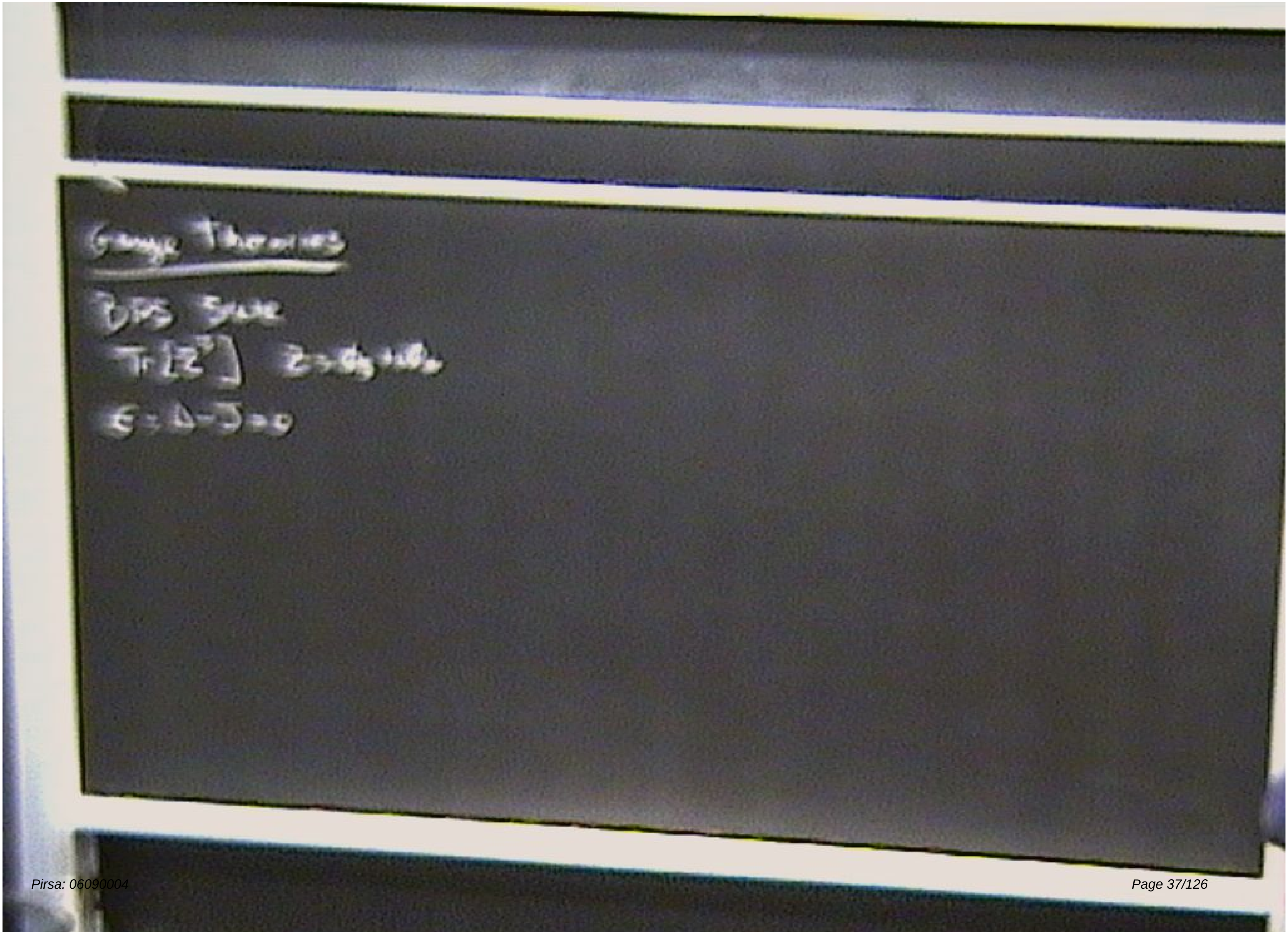
$N \to \infty$

$\lambda =$ fixed (?)

Strings

massless mode of 10d SUGRA

$\epsilon = E - J = 0$


Gauge Theories
BPS State
$Tr[Z^J]$ $Z = \phi_5 + i\phi_6$
$\epsilon = \Delta - J = 0$

Gauge Theories

BPS State

$\mathrm{Tr}[Z^J]$ $\quad Z = \Phi_5 + i\Phi_6$

$\epsilon = \Delta - J = 0$

– Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

$\lambda = \text{fixed}$ (?)

Strings

massless mode of AdS SUGRA

$\epsilon = E - J = 0$

- Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

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Strings

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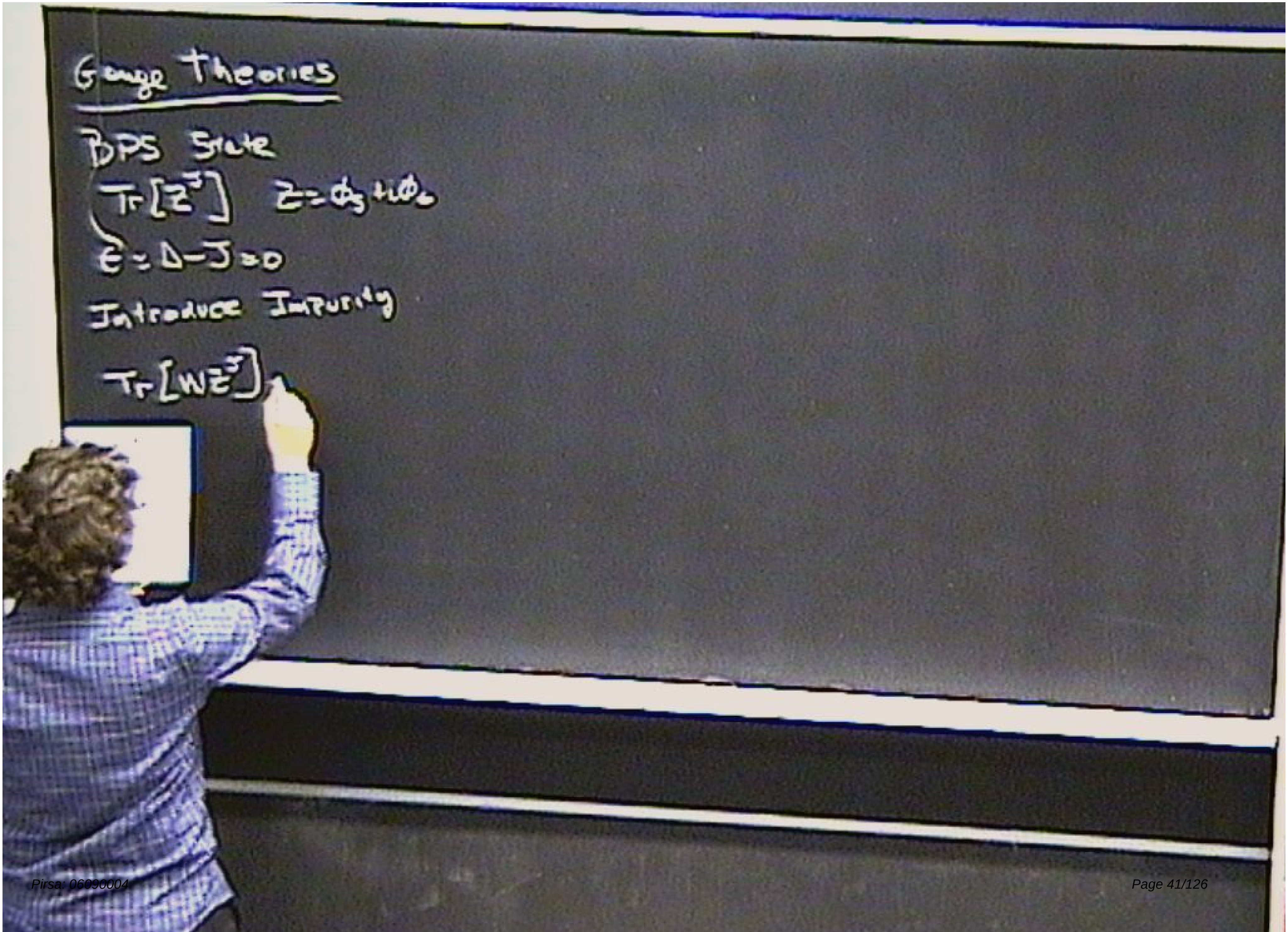
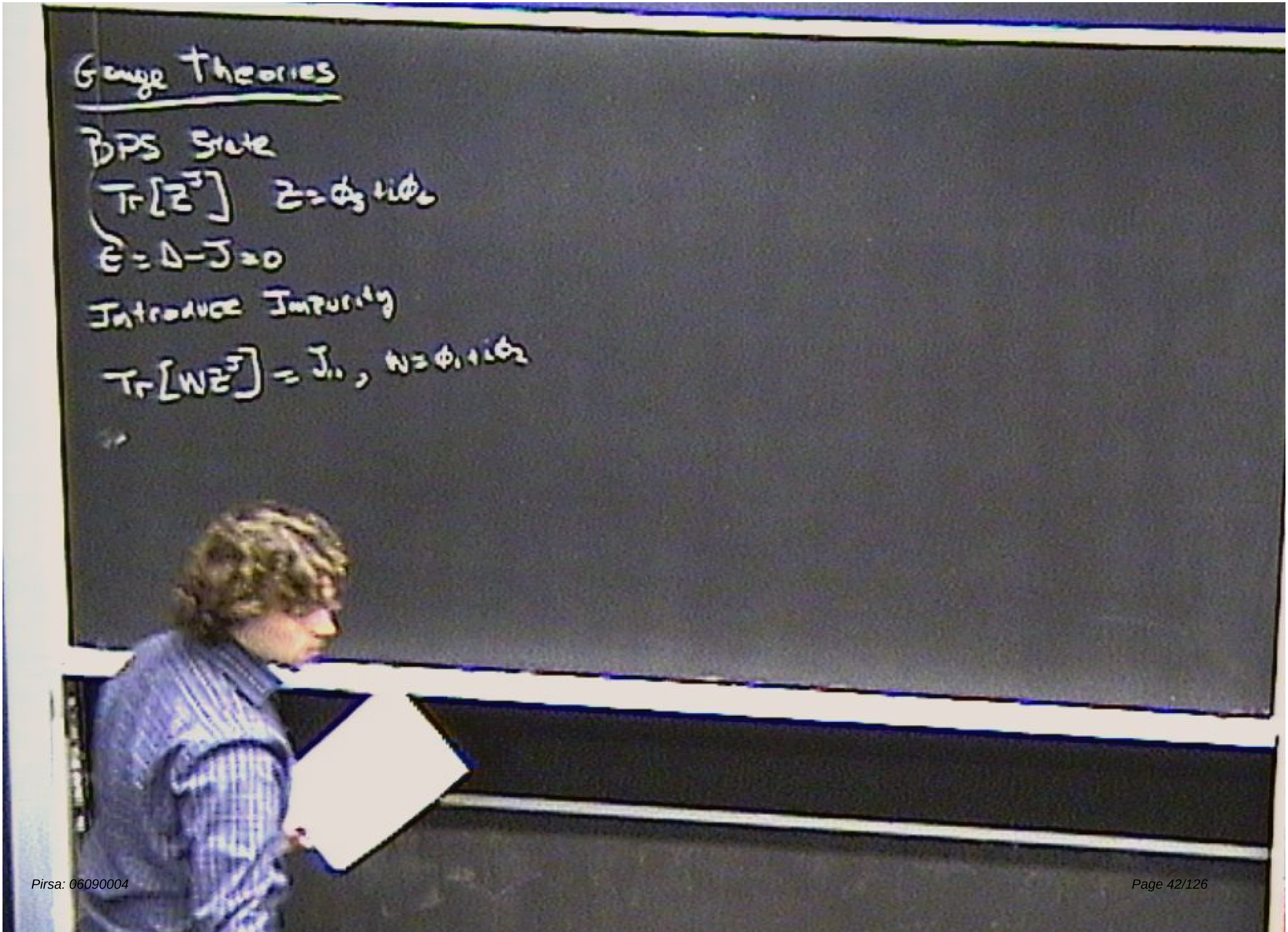
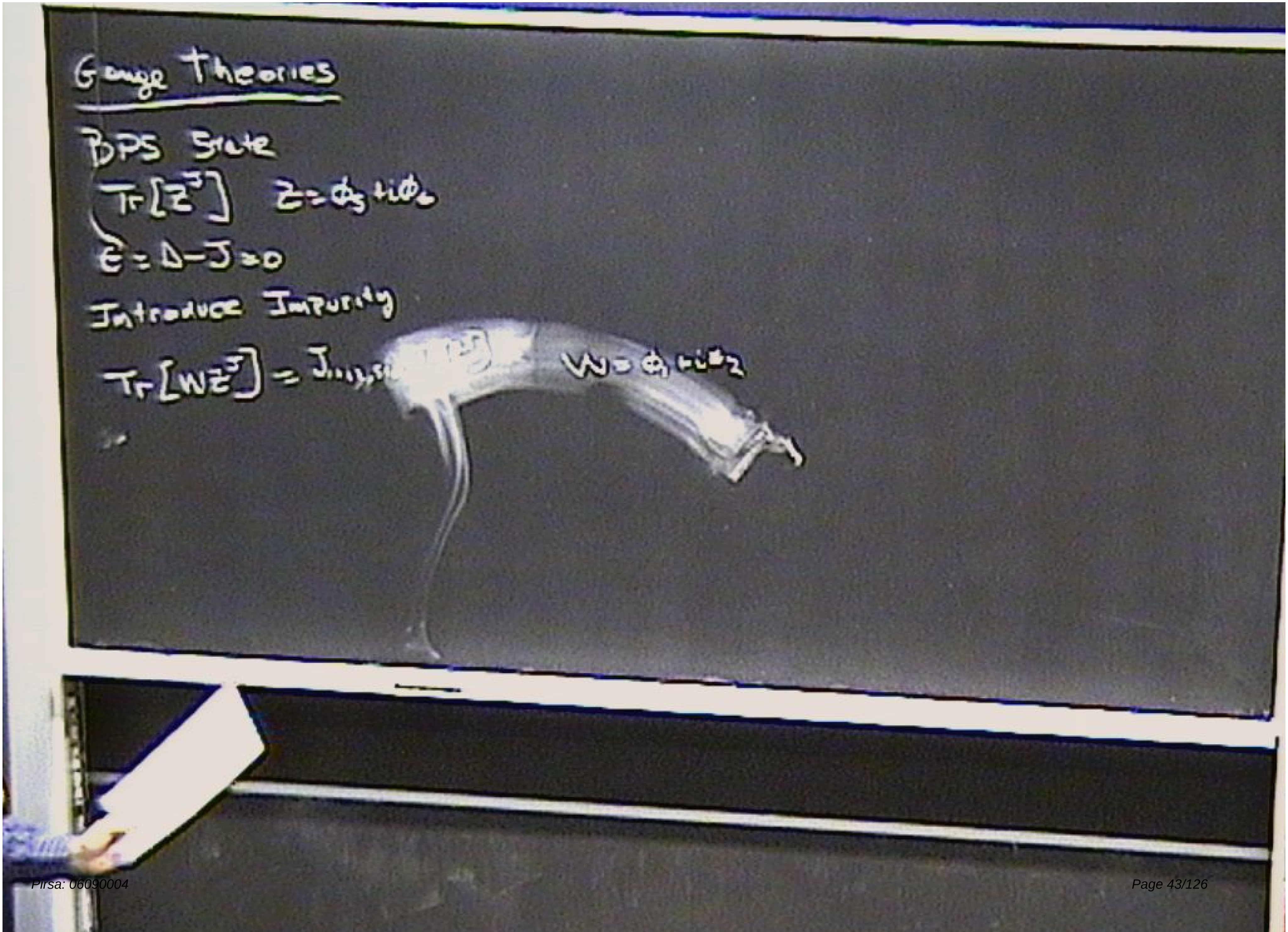
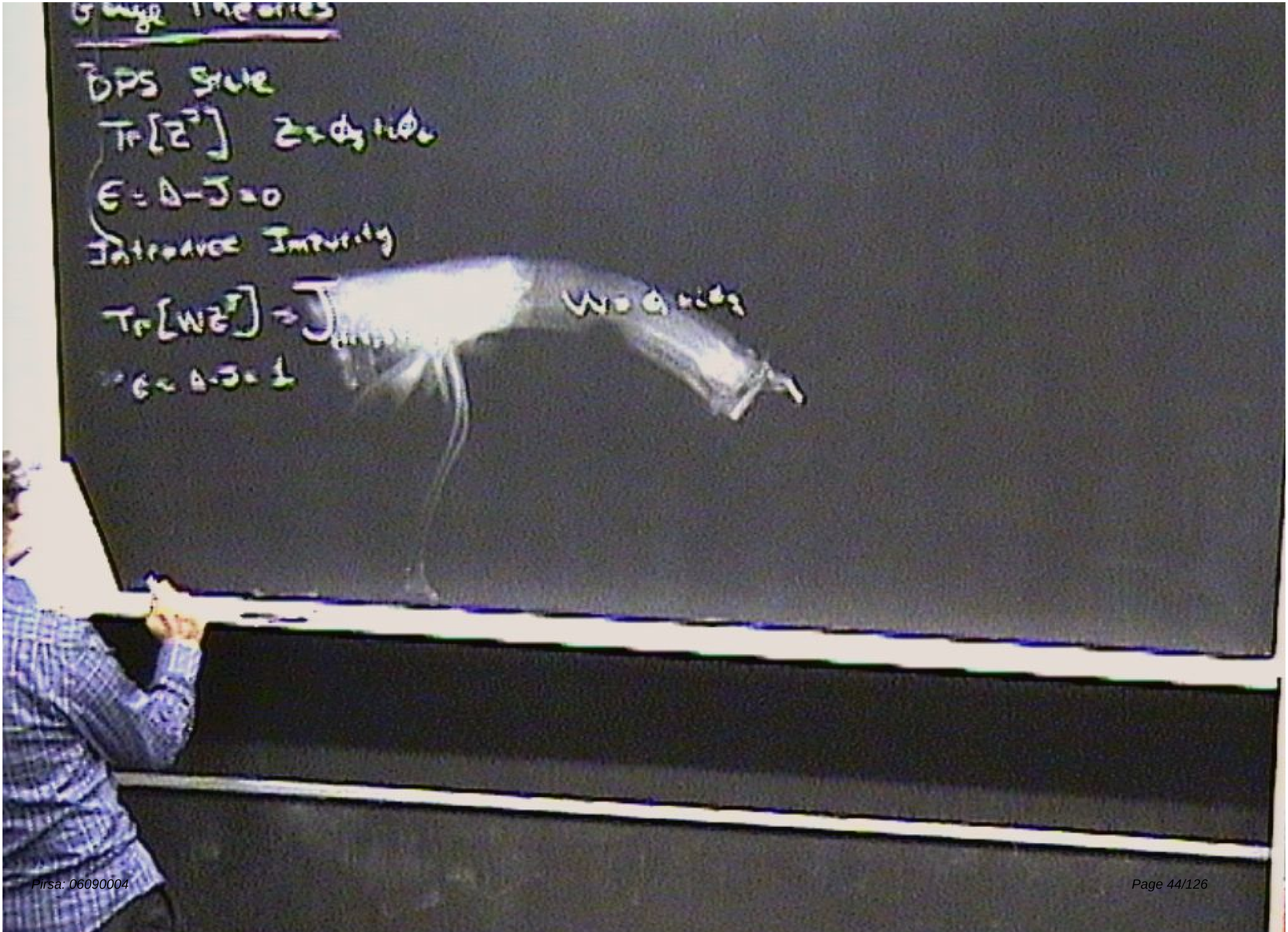
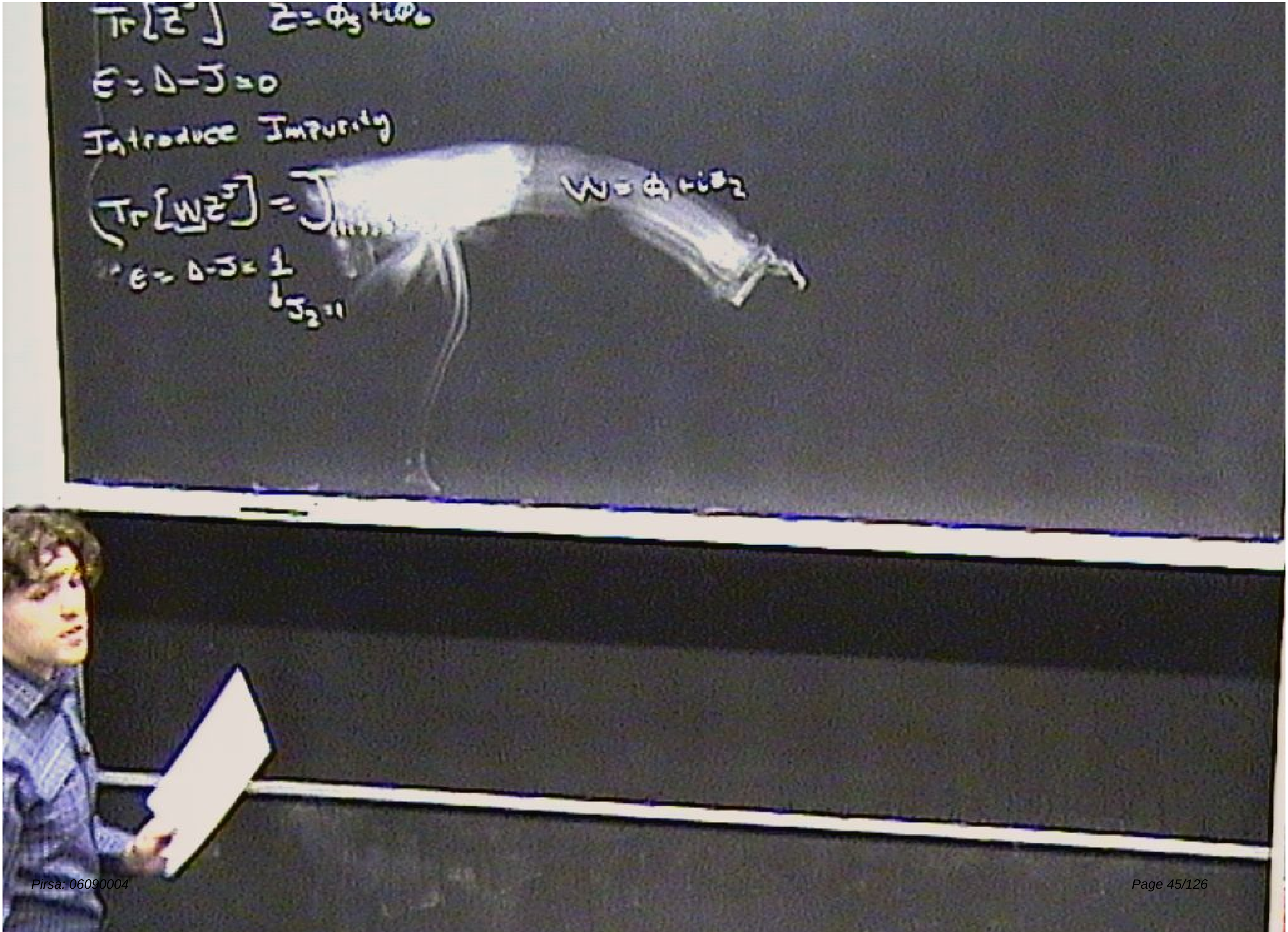
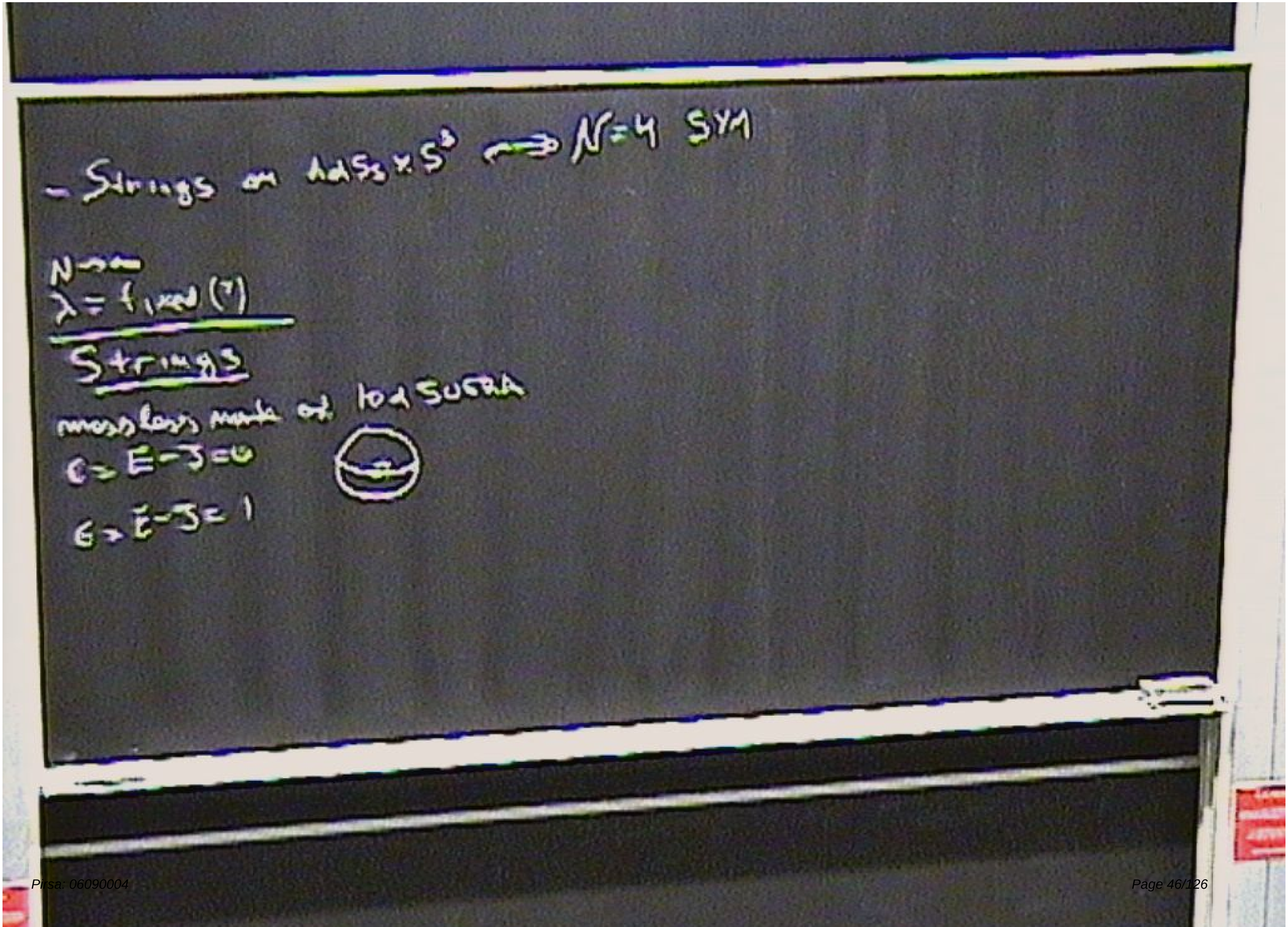

Gauge Theories
BPS State
Tr[Z^J]   Z = Φ5 + iΦ6
ε = Δ − J = 0
Introduce Impurity
Tr[WZ^J]

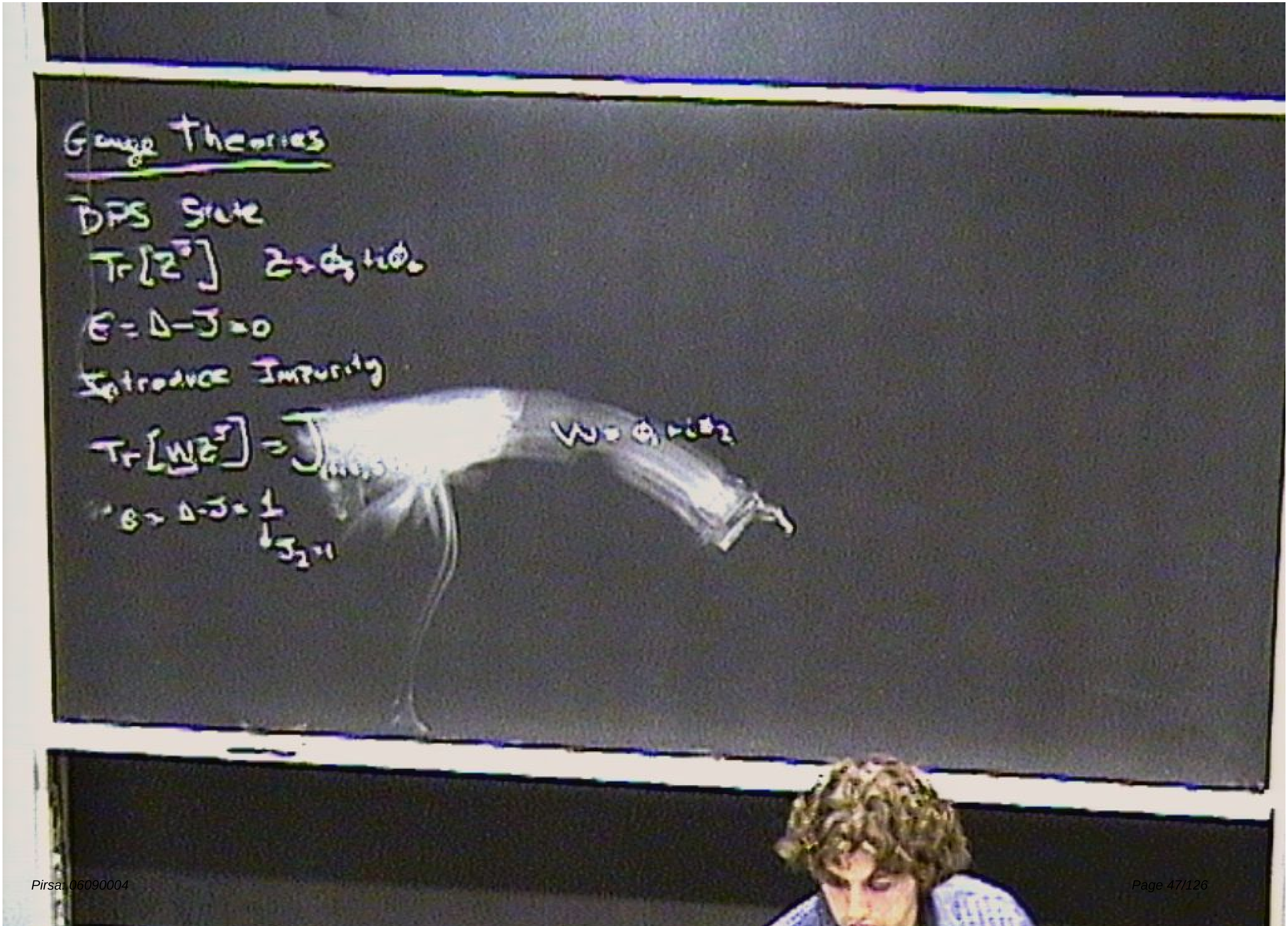

Gauge Theories
BPS State
Tr[Z^J] Z = Φ3 + iΦ6
E = Δ − J = 0
Introduce Impurity
Tr[WZ^J] = J,1 , W = Φ1 + iΦ2

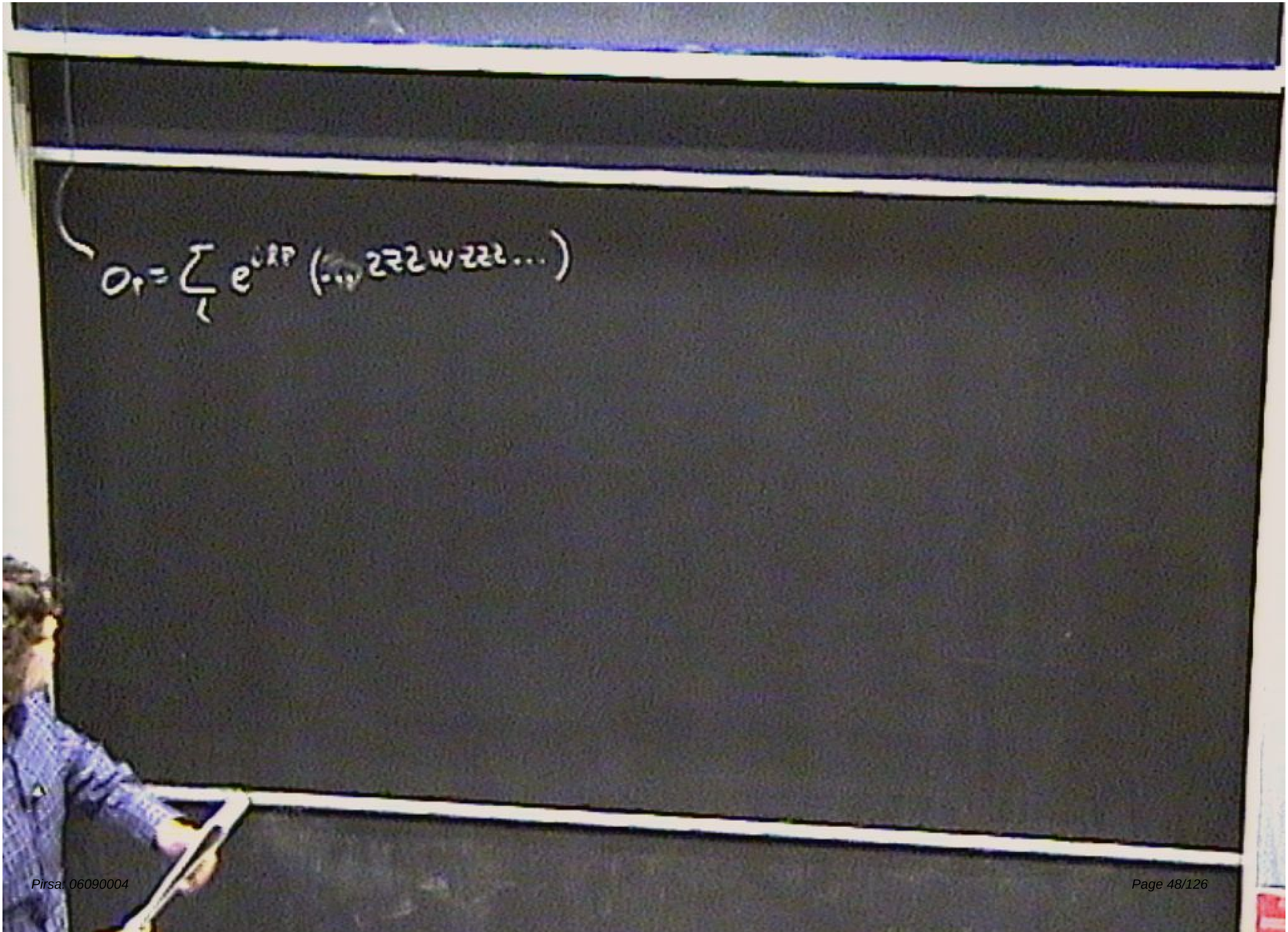

Gauge Theories
BPS State
Tr[Z^J] Z = Φ_5 + iΦ_6
E = Δ − J = 0
Introduce Impurity
Tr[WZ^J] = J…
W = Φ_1 + iΦ_2

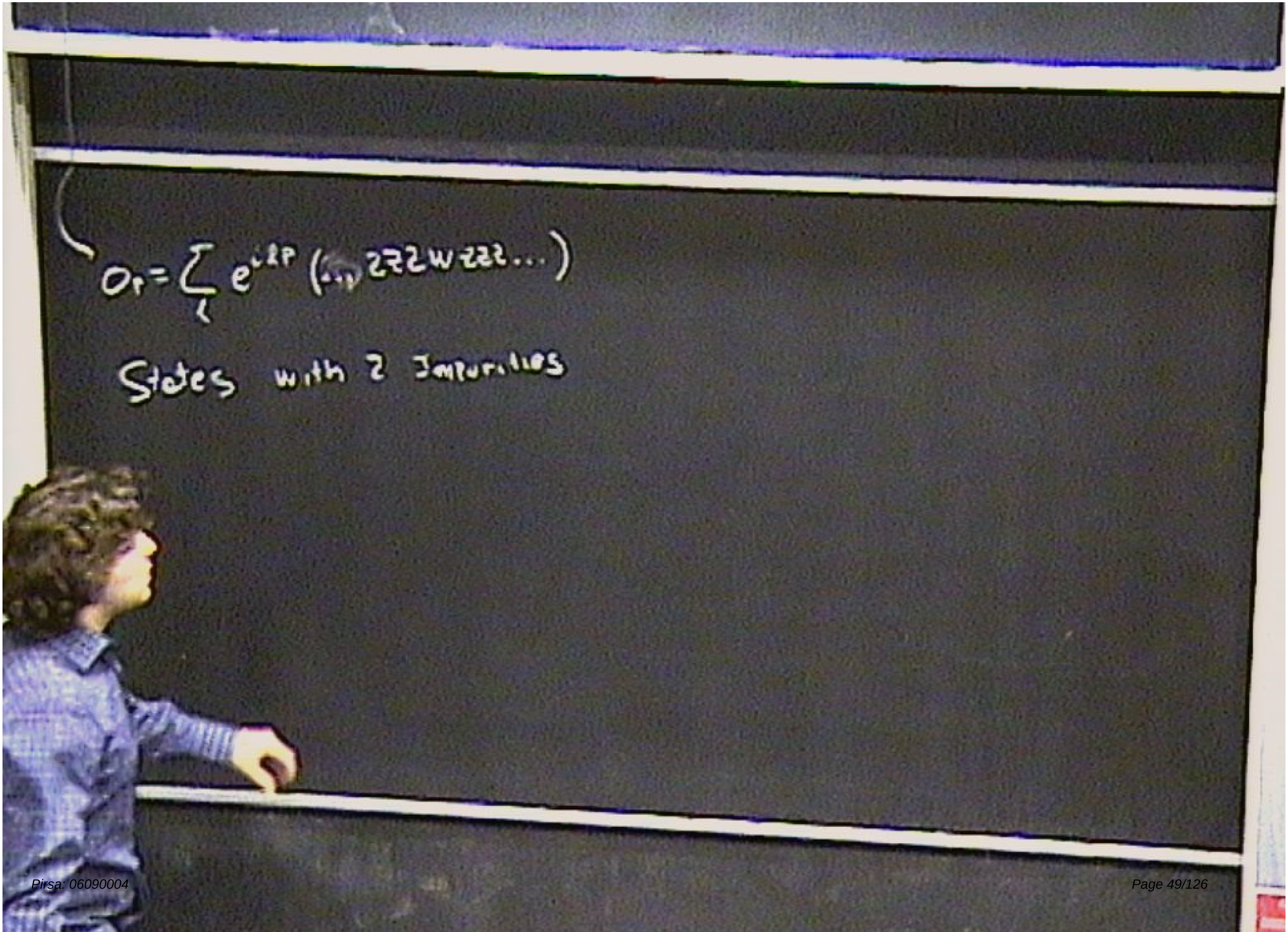

Gauge Theories
BPS State
Tr[Z^J] Z=Φ_3+iΦ_4
ε = Δ−J = 0
Introduce Impurity
Tr[WZ^J]
W = Φ_1+iΦ_2
ε = Δ−J = 1

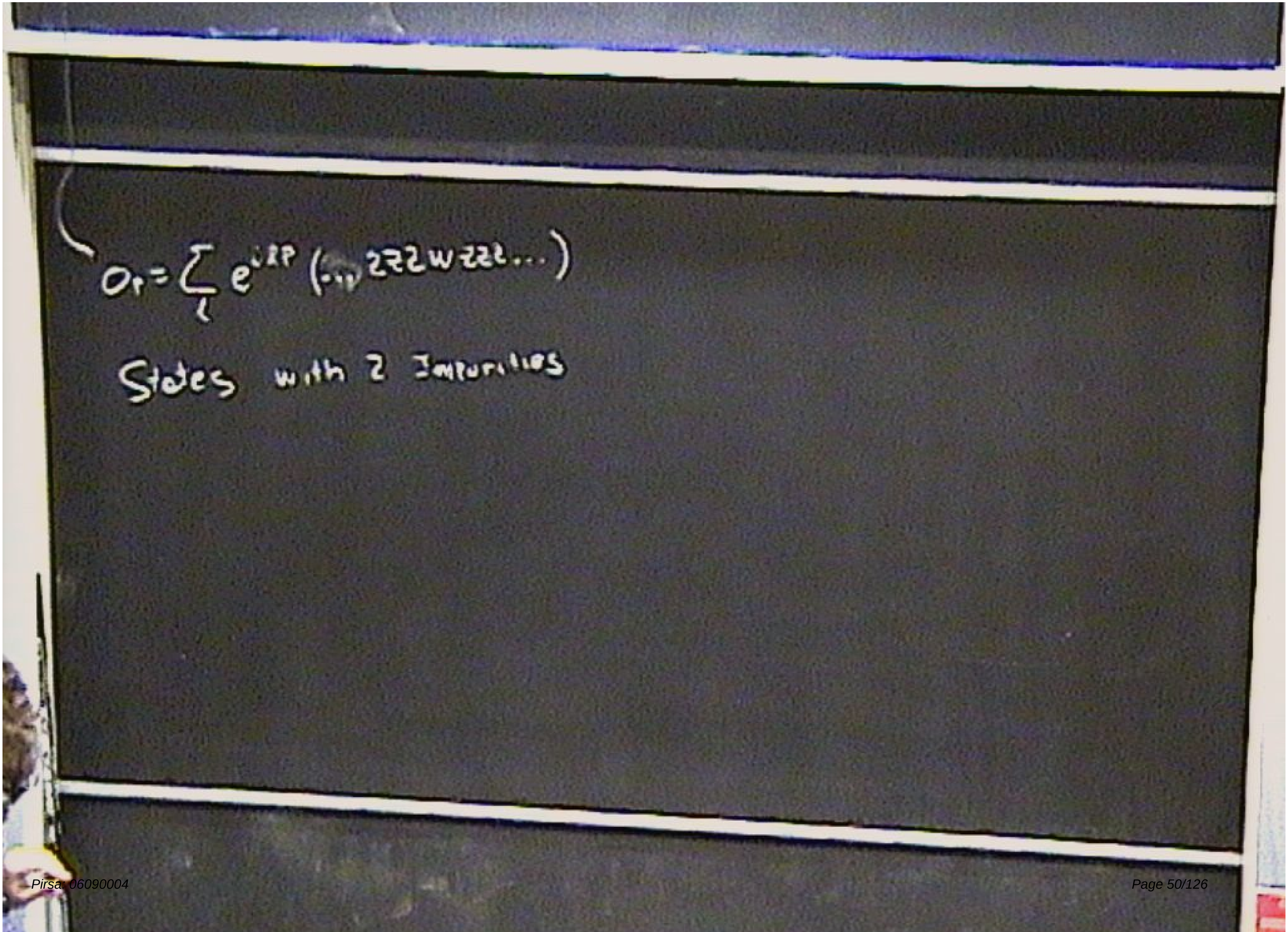

Tr[Z^J]   Z = Φ5 + iΦ6
ε = Δ − J = 0
Introduce Impurity
Tr[W Z^J] =   W = Φ1 + iΦ2
ε = Δ − J = 1
J2 = 1

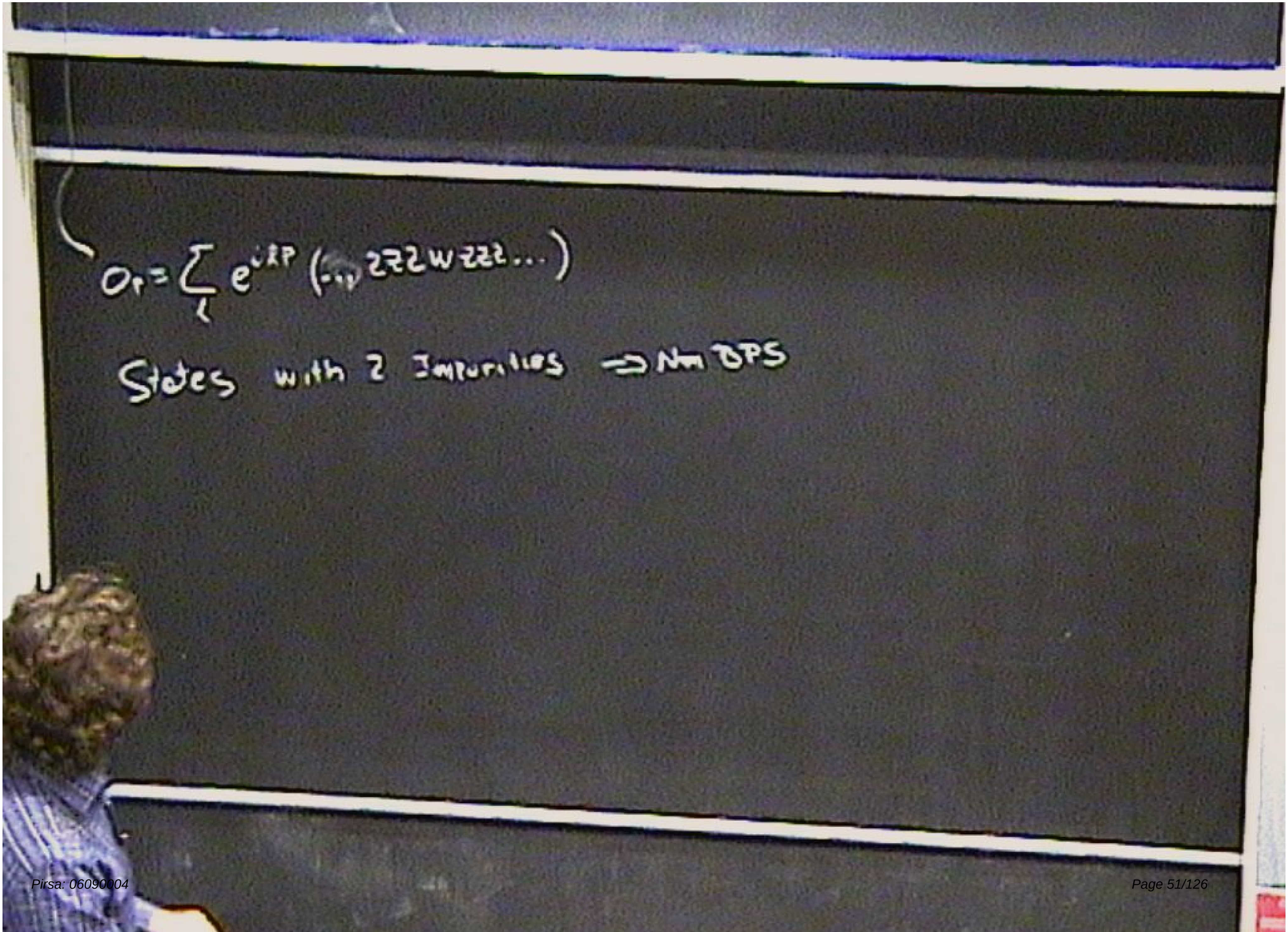

- Strings on AdS5 x S5 ⟶ N=4 SYM
N→∞
λ = fixed (?)
Strings
massless mode of 10d SUGRA
ε = E−J = 0
ε = E−J = 1

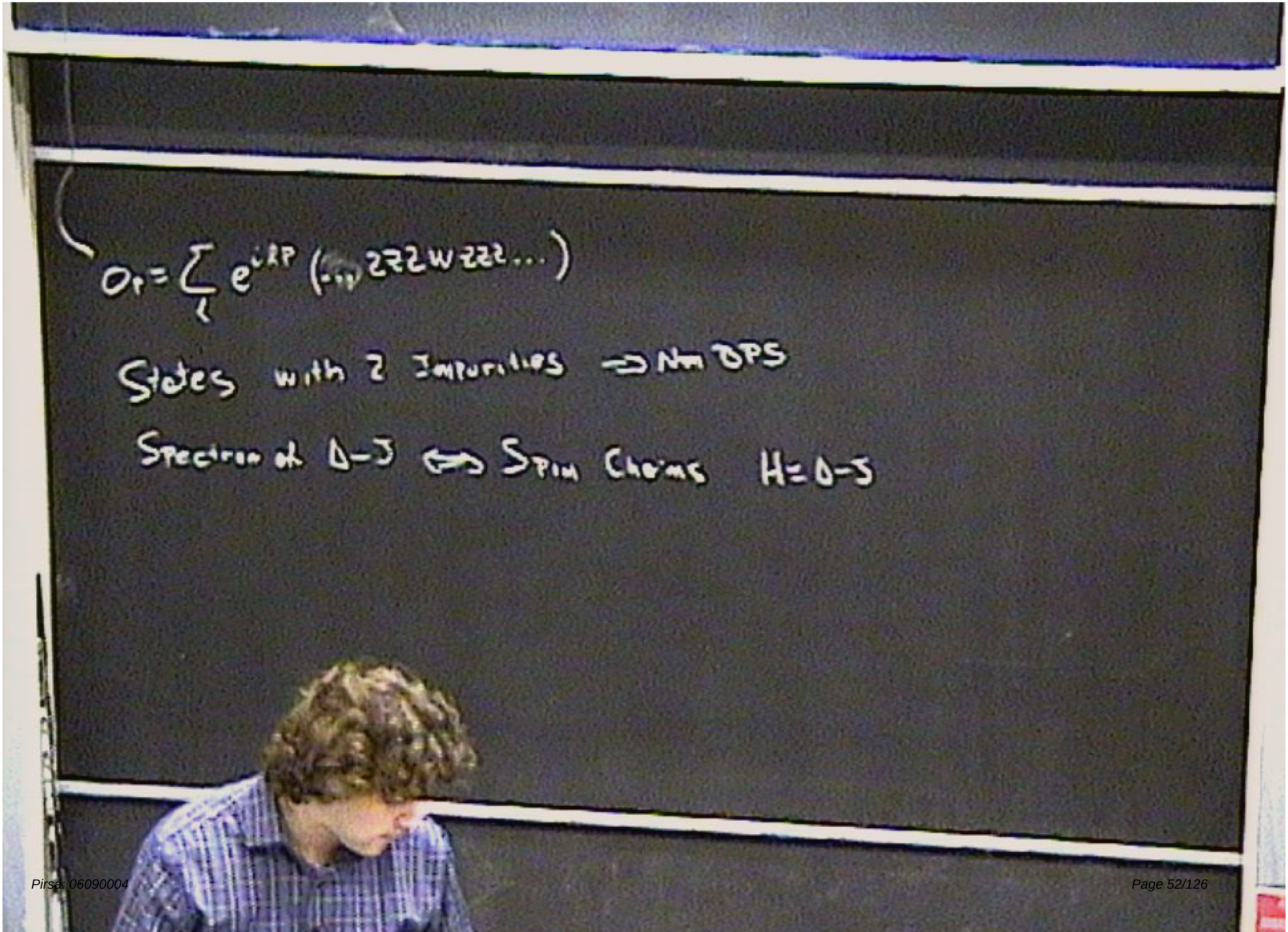

Gauge Theories
BPS State
Tr[Z^J]   Z = Φ5 + iΦ6
E = Δ − J = 0
Introduce Impurity
Tr[W Z^J] =
W = Φ1 + iΦ2
E = Δ − J = 1


$O_p = \sum_l e^{ilp} (\ldots zzzwzzz\ldots)$


$O_P = \sum_l e^{ilP} (\ldots zzzWzzz\ldots)$
States with 2 Impurities


$O_p = \sum_l e^{ilp} (\ldots zzzwzzz\ldots)$
States with 2 Impurities


$O_P = \sum_l e^{ilP} (\ldots ZZZWZZZ\ldots)$
States with 2 Impurities ⇒ Non BPS

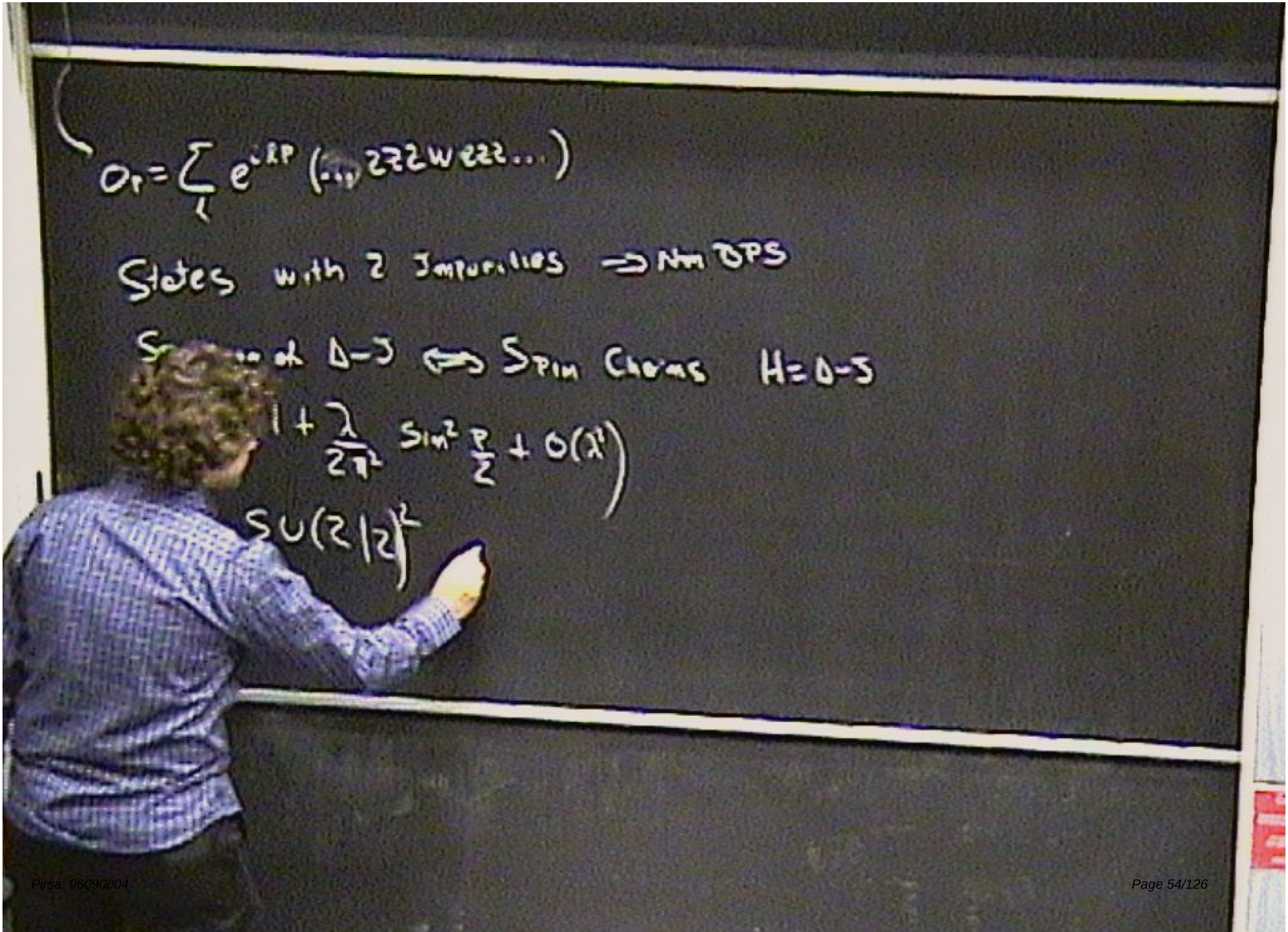

O_r = Σ_l e^{ilP} (... ZZZWZZZ...)
States with 2 Impurities ⇒ Non BPS
Spectrum of Δ−J ⟺ Spin Chains H=Δ−J

$$O_P = \sum_{l} e^{ilP} (\ldots ZZZWZZZ\ldots)$$

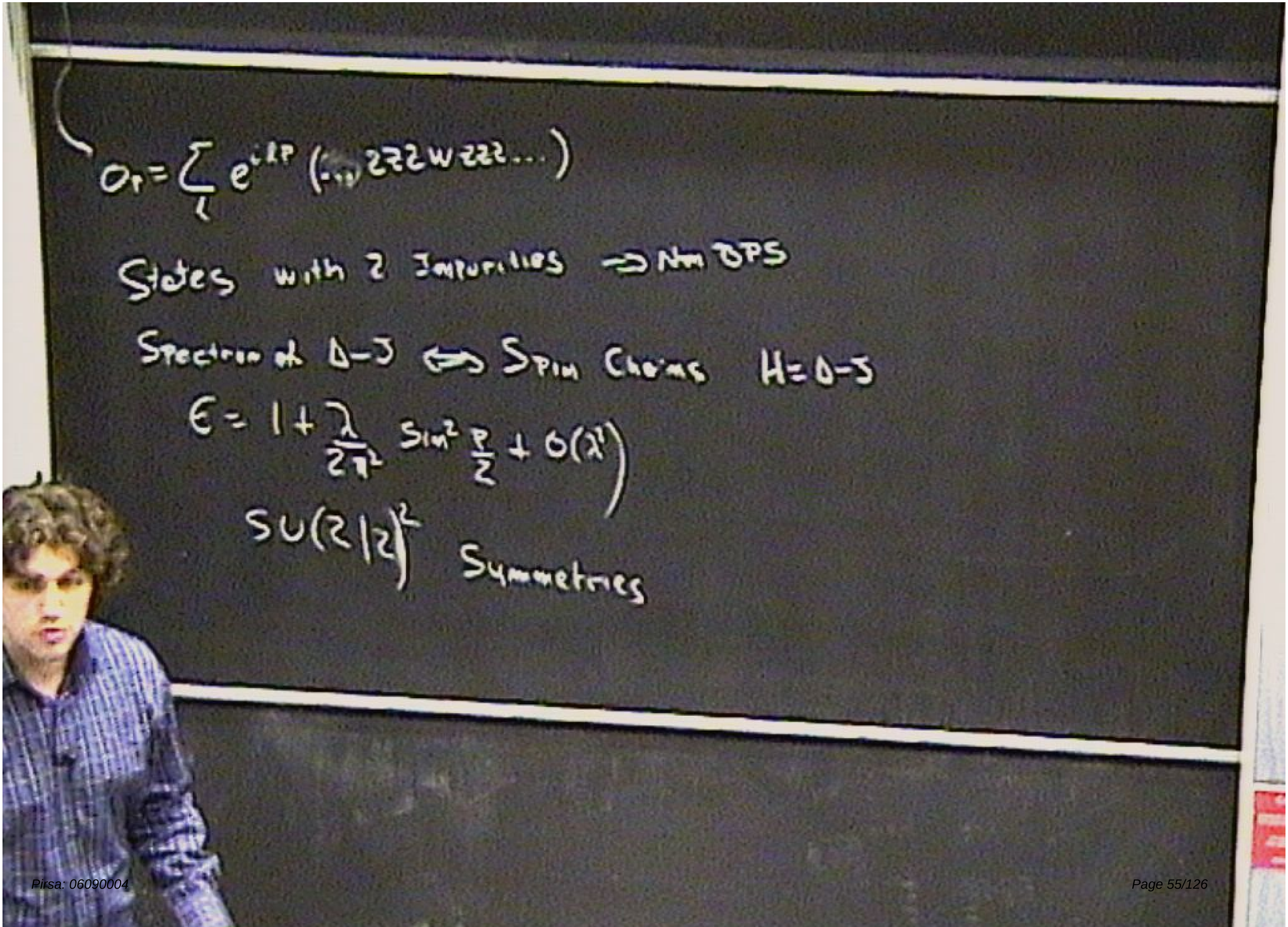
States with 2 Impurities $\Rightarrow$ Non BPS

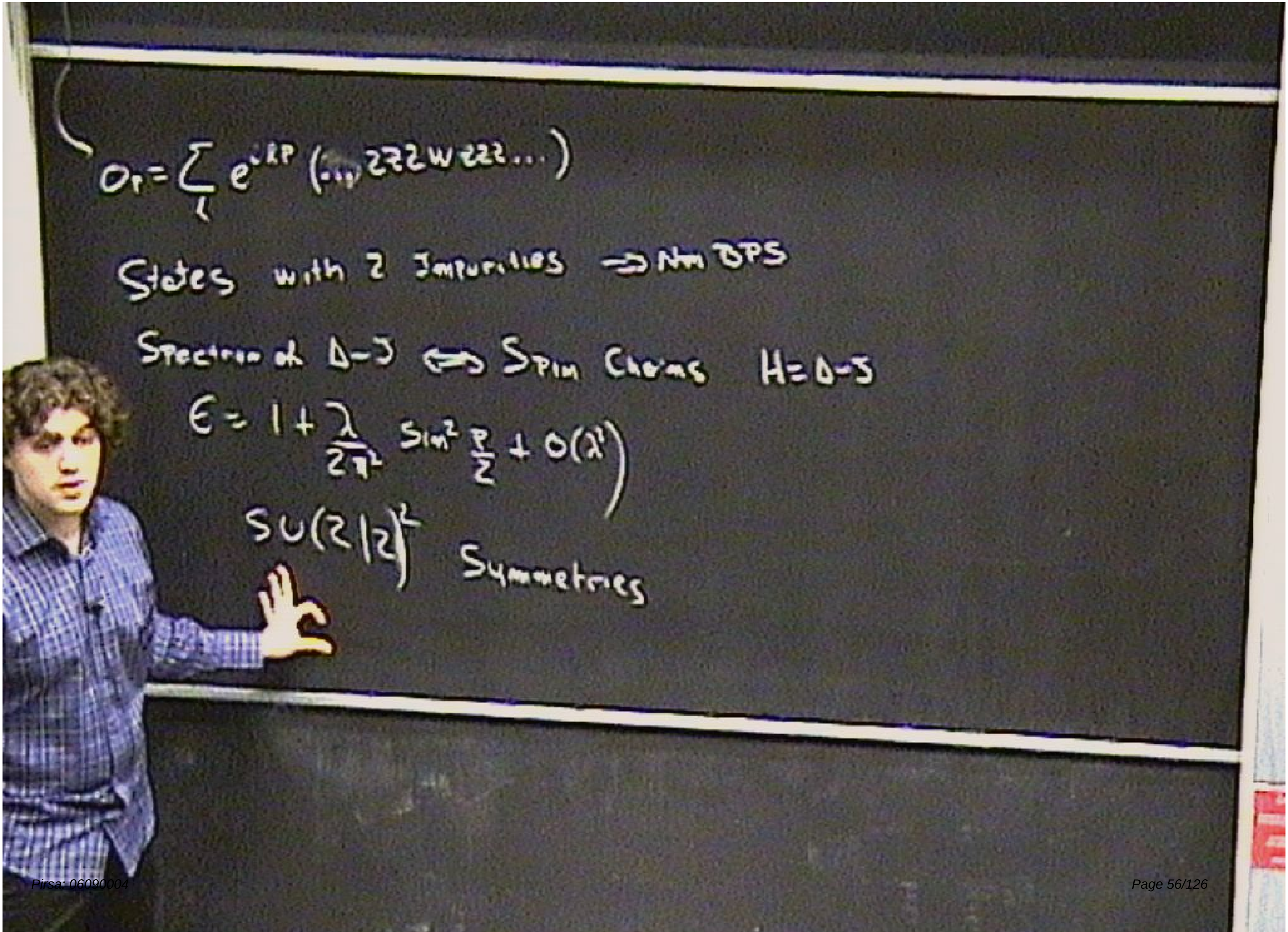
Spectrum of $\Delta - J$ $\Leftrightarrow$ Spin Chains $H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$$



O_r = Σ_l e^{ilp} (... ZZZWZZZ...)
States with 2 Impurities → Non BPS
Δ−J ⟺ Spin Chains H=Δ−J
1 + λ/2π² sin² p/2 + O(λ²)
SU(2|2)²


O_P = Σ_l e^{ilP} (... ZZZWZZZ...)
States with 2 Impurities ⟹ Non BPS
Spectrum of Δ−J ⟺ Spin Chains H=Δ−J
E = 1 + λ/2π² sin² p/2 + O(λ²)
SU(2|2)² Symmetries


O_P = Σ_l e^{ilp} (... ZZZWZZZ...)
States with 2 Impurities ⟹ Non BPS
Spectrum of Δ−J ⟺ Spin Chains H=Δ−J
ε = 1 + λ/2π² sin² p/2 + O(λ²)
SU(2|2)² Symmetries

$$O_p = \sum_l e^{ilp} (\ldots ZZZWZZZ\ldots)$$

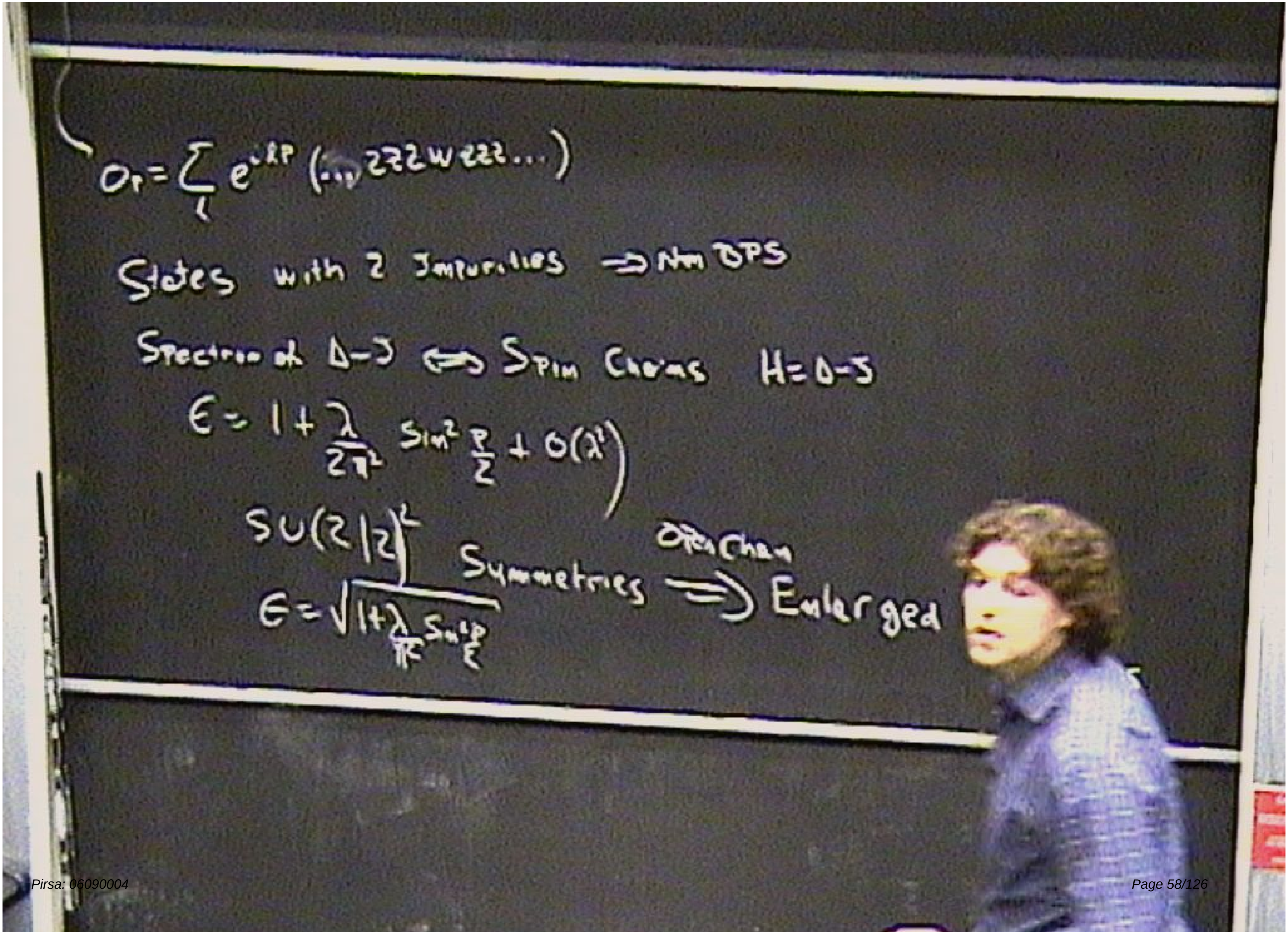
States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J$ $\Leftrightarrow$ Spin Chains $H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$$

$SU(2|2)^2$ Symmetries $\Rightarrow$ Enlarged by Central charges (Often Chain)

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$$


O_p = Σ_l e^{ilp} (... ZZZWZZZ...)
States with 2 Impurities ⇒ Non BPS
Spectrum of Δ−J ⟺ Spin Chains H=Δ−J
ε = 1 + λ/2π² sin² p/2 + O(λ²)
SU(2|2)² Symmetries ⇒ Enlarged
Open Chain
ε = √(1+λ/π² sin² p/2)

$O_P = \sum_l e^{ilP} (\ldots ZZZWZZZ\ldots)$

States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J \Leftrightarrow$ Spin Chains $H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{P}{2} + O(\lambda^2)$$

$SU(2|2)^2$ Symmetries $\Rightarrow$ Enlarged by Central charges

Open Chain

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{P}{2}}$$

$O_p = \sum_l e^{ilp} (\ldots ZZZWZZZ\ldots)$

States with 2 Impurities $\Rightarrow$ Non BPS

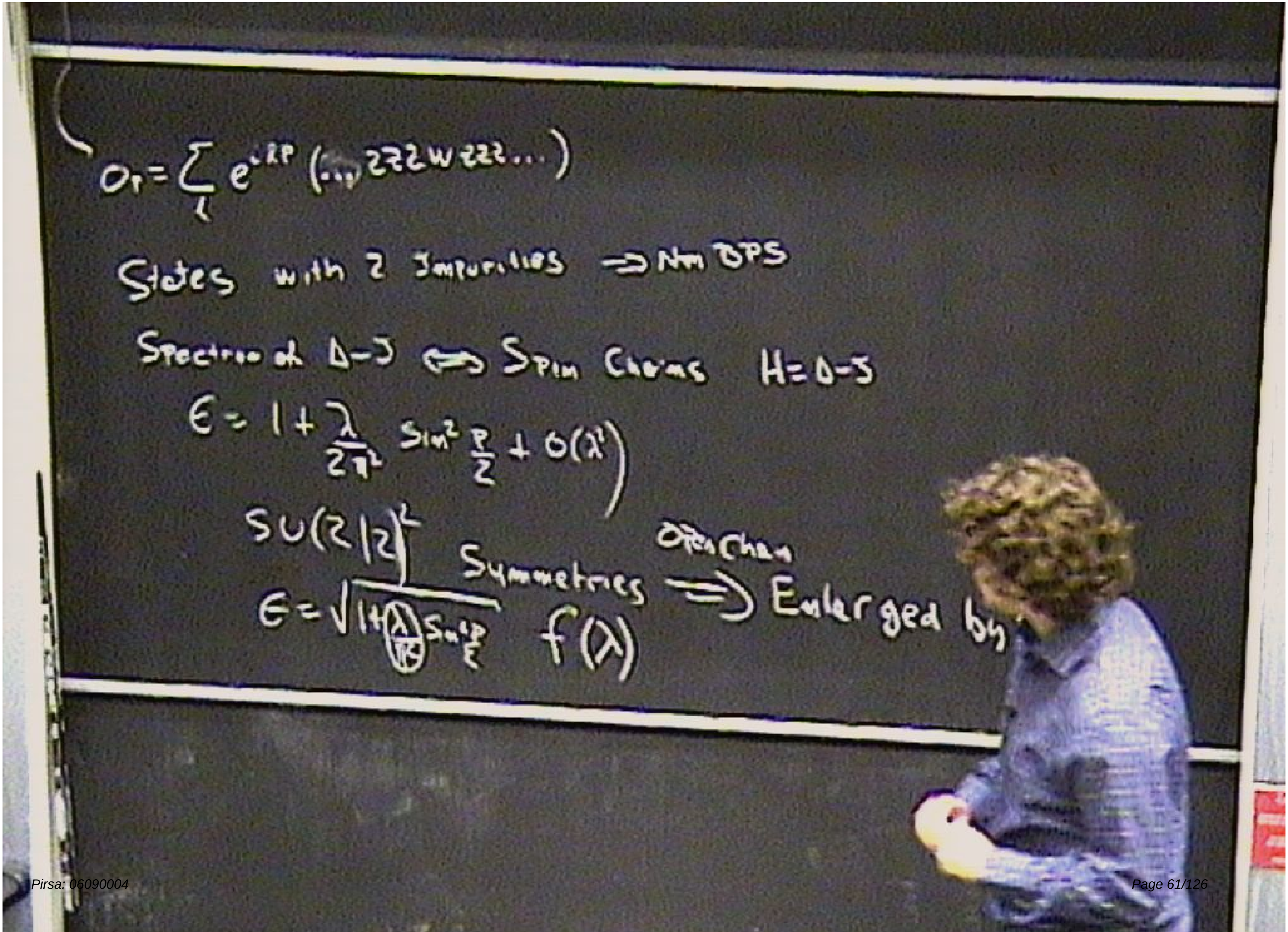
Spectrum of $\Delta - J$ $\Leftrightarrow$ Spin Chains $H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$$

$SU(2|2)^2$ Symmetries $\Rightarrow$ Enlarged by Central charges

Open Chain

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \quad f(\lambda)$$


O_p = Σ_l e^{ilp} (...ZZZWZZZ...)
States with 2 Impurities ⇒ Non BPS
Spectrum of Δ−J ⇔ Spin Chains H=Δ−J
ε = 1 + λ/2π² sin² p/2 + O(λ²)
SU(2|2)²
Symmetries ⇒ Enlarged by
Open Chain
ε = √(1+ λ/π² sin² p/2) f(λ)

$O_p = \sum_l e^{ilp} (\ldots ZZZWZZZ\ldots)$

States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J$ $\Longleftrightarrow$ Spin Chains $H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$$

$SU(2|2)^2$ Symmetries $\Longrightarrow$ (Open Chain) Enlarged by Central charges

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \quad f(\lambda)$$

– Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

λ = fixed (?)

Strings

massless mode of 10d SUGRA

$\epsilon = E - J = 0$

$\epsilon = E - J = 1$

pp-wave limit of

– Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

$\lambda =$ fixed (?)

Strings

massless mode of AdS SUGRA

$\epsilon = E - J = 0$

$\epsilon = E - J = 1$

pp-wave limit of

$\lambda \to \infty \qquad \lambda \sim J^2$

- Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

$\lambda =$ fixed (?)

Strings

massless mode of AdS SUGRA

$\epsilon = E - J = 0$

$\epsilon = E - J = 1$

pp-wave limit of

$\lambda \to \infty \qquad \lambda \sim J^2$

$J \to \infty \qquad \frac{J}{\sqrt{\lambda}} =$ fixed

– Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

$\lambda =$ fixed (?)

Strings

massless mode of AdS SUGRA

$\epsilon = E - J = 0$

$\epsilon = E - J = 1$

pp-wave limit of

$\lambda \to \infty \qquad \lambda \sim J^2$

$J \to \infty \qquad \frac{J}{\sqrt{\lambda}} =$ fixed

$$O_p = \sum_l e^{ilp} (\ldots ZZZWZZZ\ldots)$$

States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J$ $\Leftrightarrow$ Spin Chains $H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$$

$\hookrightarrow$ Integrability.

$SU(2|2)^2$ Symmetries $\Rightarrow$ Enlarged by Central charges

Open Chain

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \quad f(\lambda)$$

- Strings on $AdS_5 \times S^5$ $\longleftrightarrow$ $\mathcal{N}=4$ SYM

$$\epsilon = \sqrt{1+\underset{\text{fixed.}}{\lambda p^2}}$$

$N \to \infty$

λ = fixed (?)

Strings

massless mode of 10d SUGRA

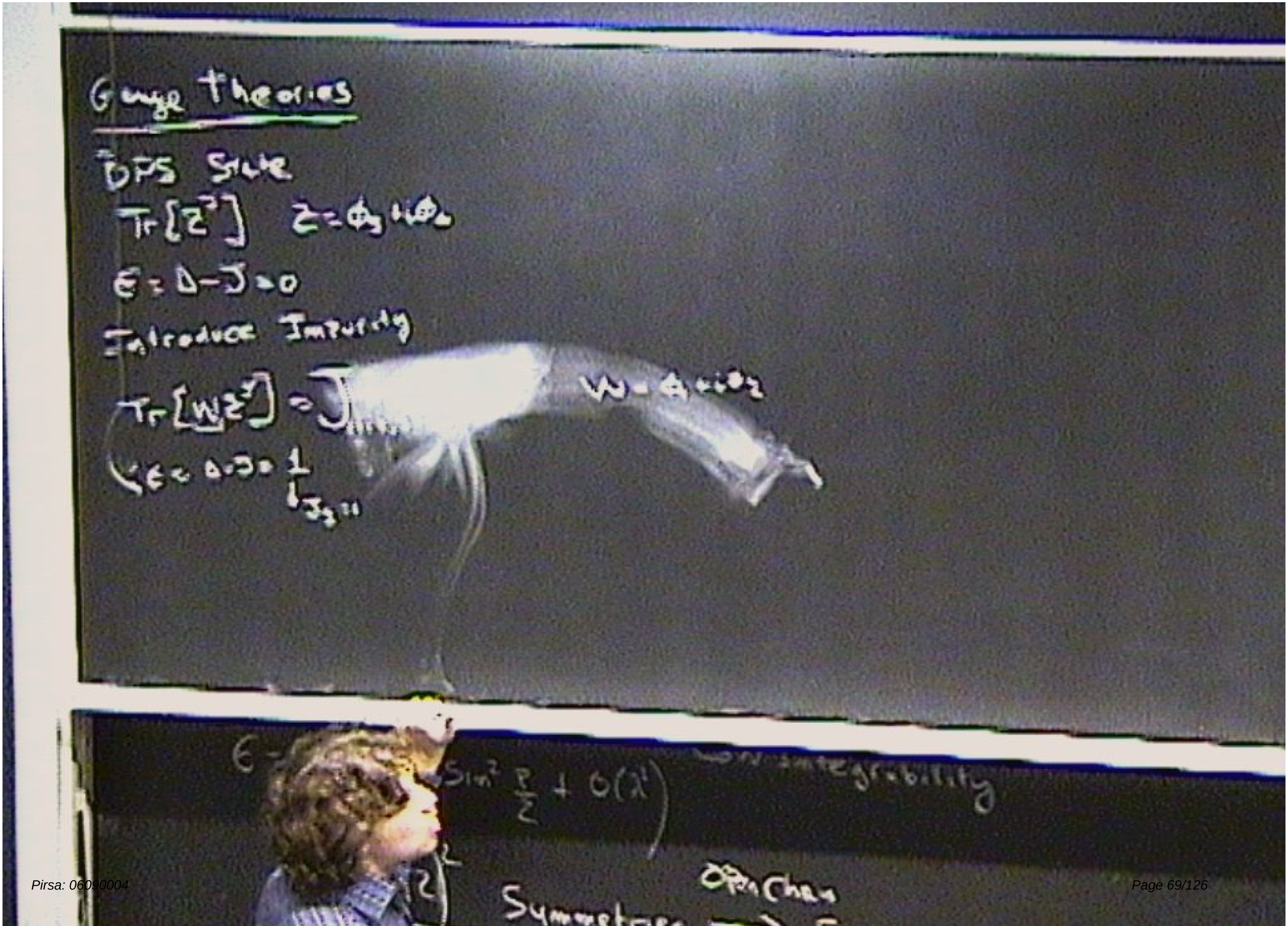
$\epsilon = E - J = 0$

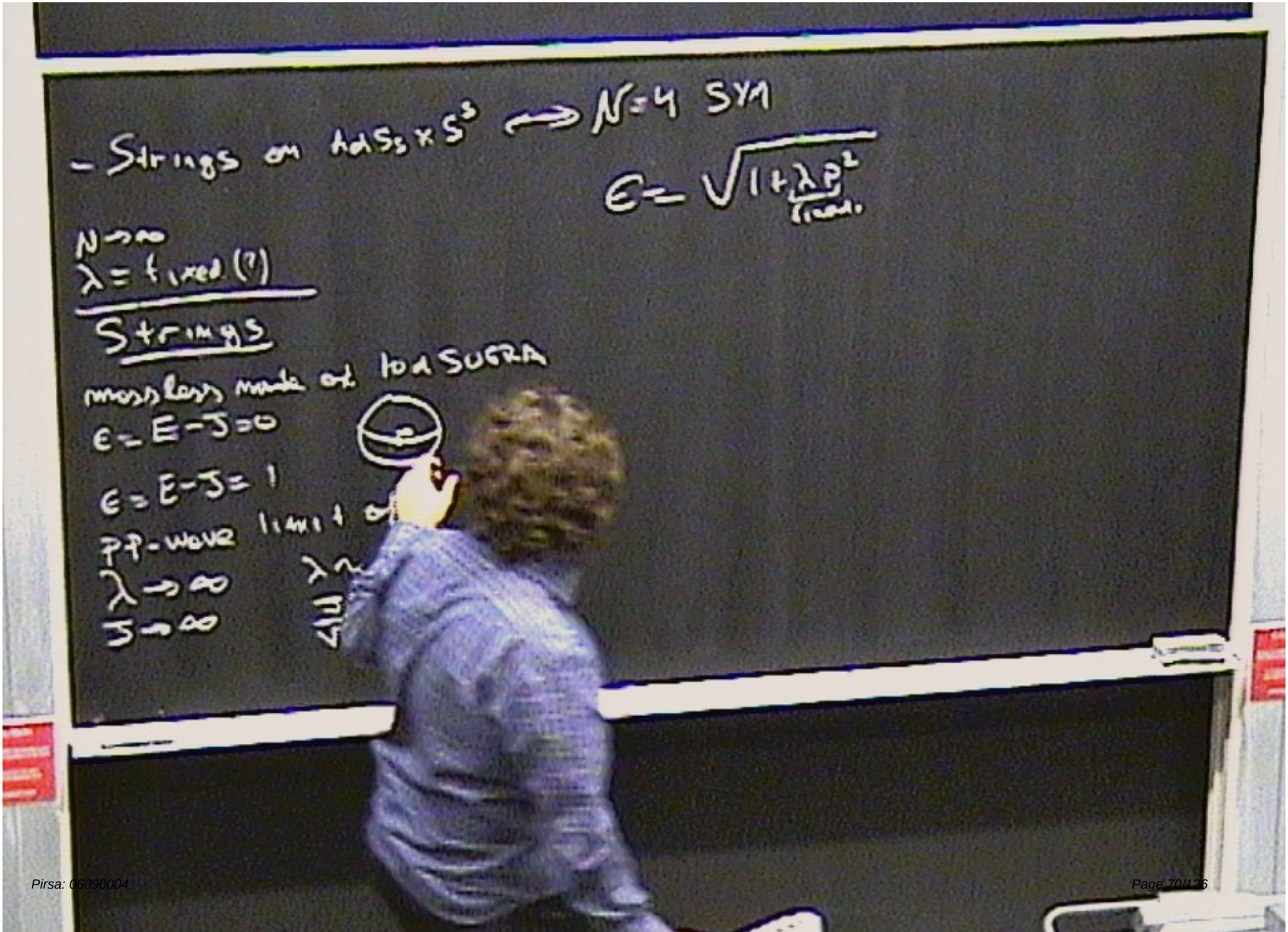
$\epsilon = E - J = 1$

pp-wave limit of

$\lambda \to \infty$ $\qquad \lambda \sim J^2$

$J \to \infty$ $\qquad \frac{\sqrt{\lambda}}{J}$ = fixed.


Gauge Theories
BPS State
Introduce Impurity
integrability
Symmetries


- Strings on AdS5 x S5 ⟷ N=4 SYM
ε = √(1 + λp²)
N → ∞
λ = fixed (?)
Strings
massless mode of 10d SUGRA
ε = E − J = 0
ε = E − J = 1
pp-wave limit of
λ → ∞
J → ∞

- Strings on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM

$$\epsilon = \sqrt{1+\frac{\lambda p^2}{(\text{fixed})}} \qquad \lambda p^2 = \text{fixed}$$

$$= \sqrt{1+\lambda' n^2} \qquad \lambda' = \frac{\lambda}{J^2}$$

$$p = \frac{n}{J}$$

$N \to \infty$

$\lambda =$ fixed (?)

Strings

massless mode of 10d SUGRA

$\epsilon = E - J = 0$

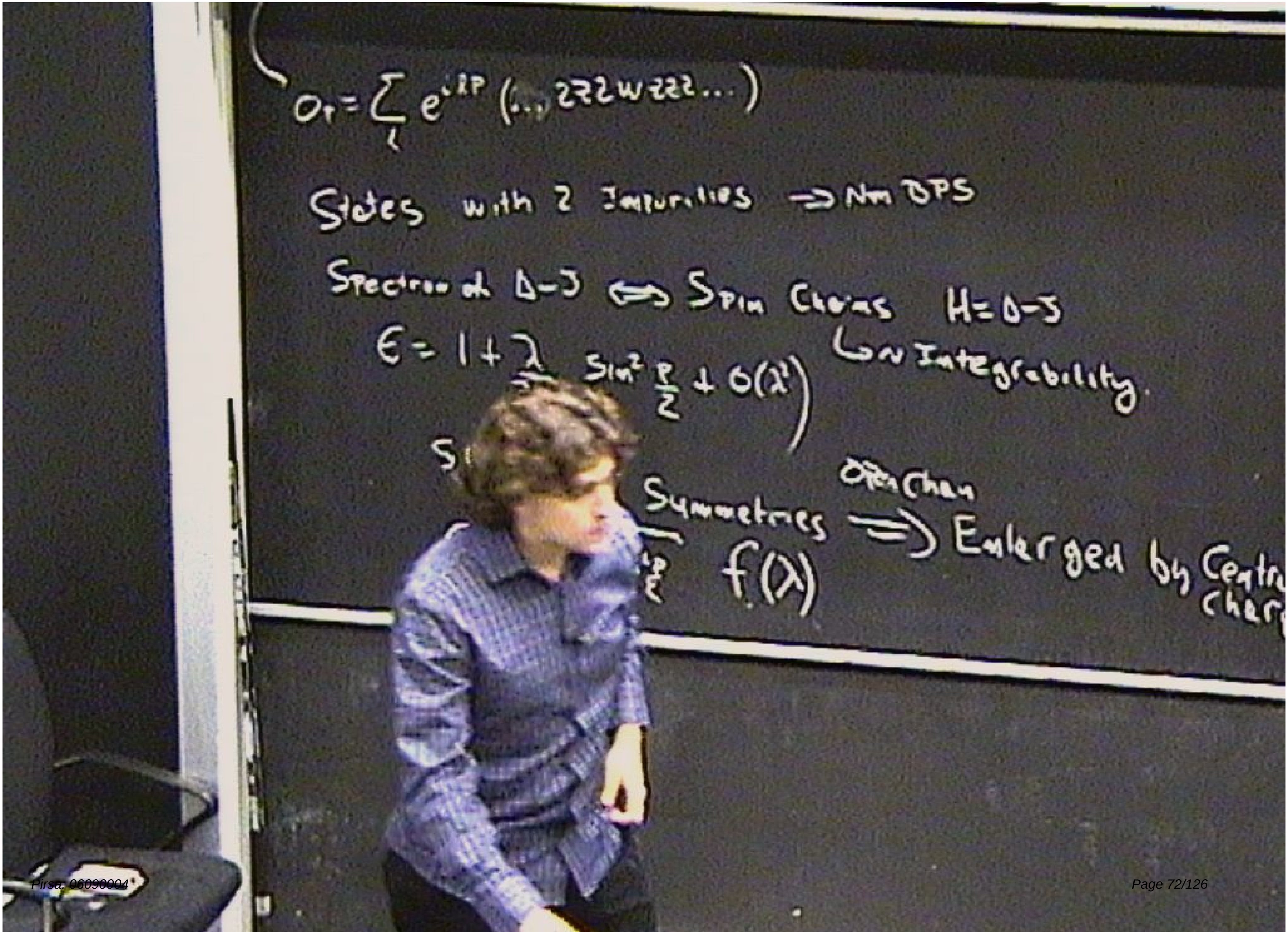
$\epsilon = E - J = 1$

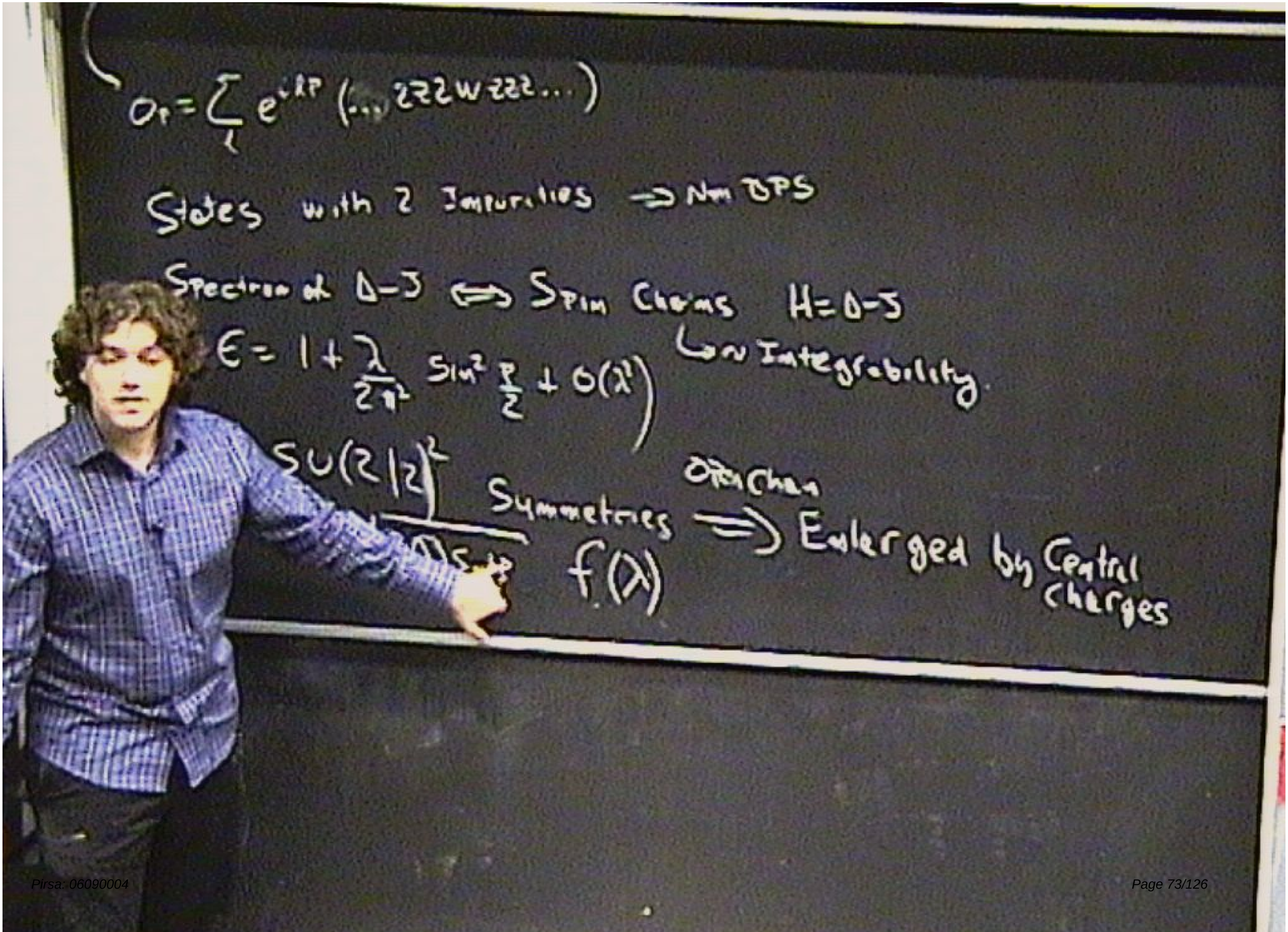
pp-wave limit of

$\lambda \sim J^2$

$\lambda \to \infty$

$J \to \infty$ $\qquad \frac{J}{\sqrt{\lambda}} =$ fixed.


States with 2 Impurities ⇒ Non BPS
Spectrum of Δ−J ⇔ Spin Chains H=Δ−J
Integrability
Symmetries ⇒ Enlarged by Central charge
f(λ)


O_P = Σ_l e^{ilP} (... ZZZWZZZ...)
States with 2 Impurities → Non BPS
Spectrum of Δ−J ⟺ Spin Chains H=Δ−J
ε = 1 + λ/2π² sin² P/2 + O(λ²)
Integrability
SU(2|2)²
Symmetries
On Chain
⟹ Enlarged by Central charges
f(λ)

$O_p = \sum_l e^{ilp} (\ldots zzzWzzz\ldots)$

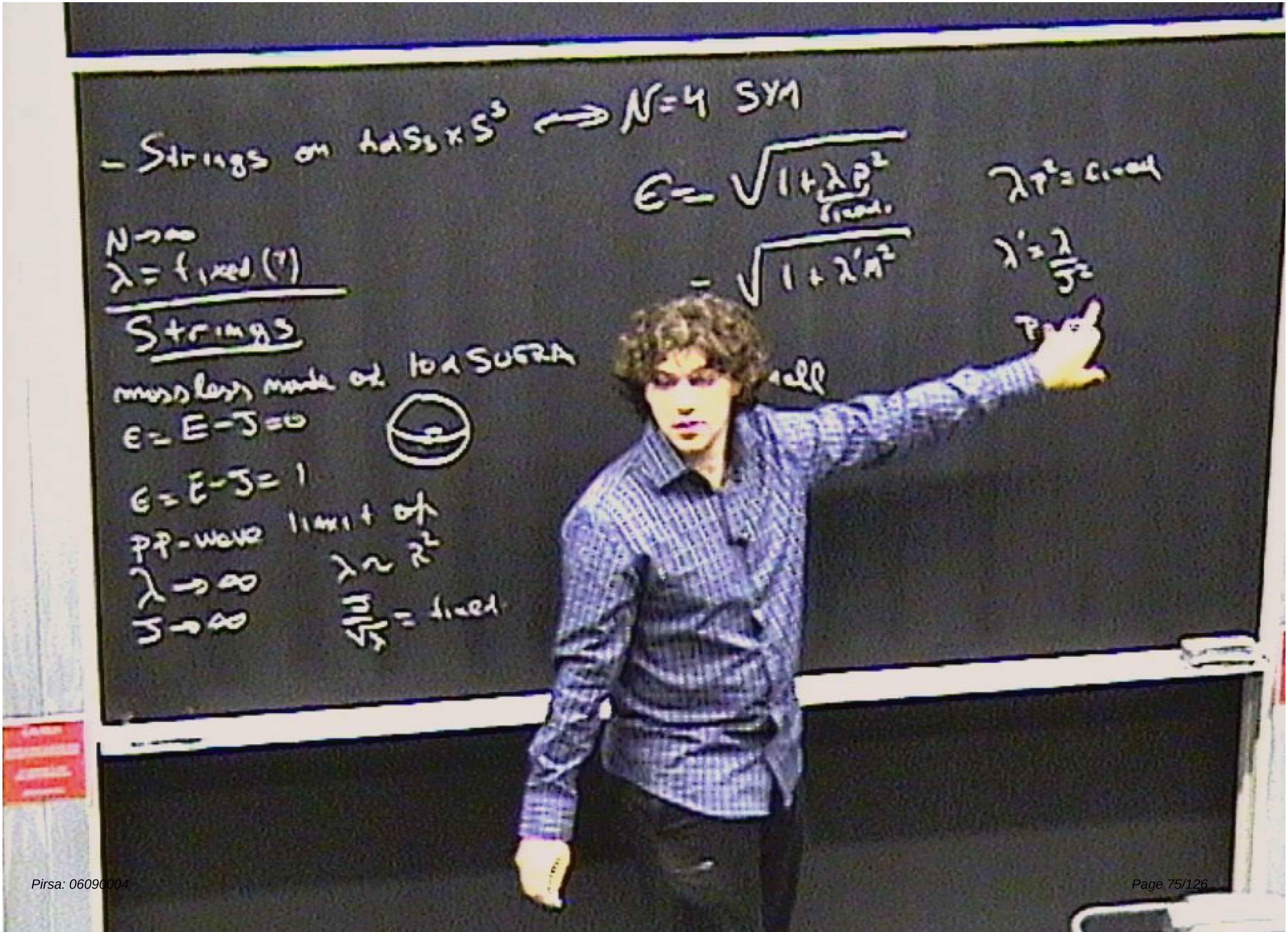
States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J \Leftrightarrow$ Spin Chains $H = \Delta - J$

$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$ — Integrability

$SU(2|2)^2$ Symmetries — Open Chain $\Rightarrow$ Enlarged by Central charges

$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$ $f(\lambda)$


- Strings on AdS5 x S5 ⟷ N=4 SYM
N → ∞
λ = fixed (?)
Strings
massless mode of 10d SUGRA
ε = E − J = 0
ε = E − J = 1
pp-wave limit of
λ → ∞
J → ∞
λ ~ J²
J/√λ = fixed
ε = √(1 + λp²)
λp² = fixed
λ' = λ/J²

– Strings on $AdS_5 \times S^5 \Longleftrightarrow \mathcal{N}=4$ SYM

$N \to \infty$

$\lambda = \text{fixed}\ (?)$

Strings

massless mode of AdS SUGRA

$\epsilon = E - J = 0$

$\epsilon = E - J = 1$

pp-wave limit of

$\lambda \to \infty \qquad \lambda \sim R^2$

$J \to \infty \qquad \frac{J}{\sqrt{\lambda}} = \text{fixed}$

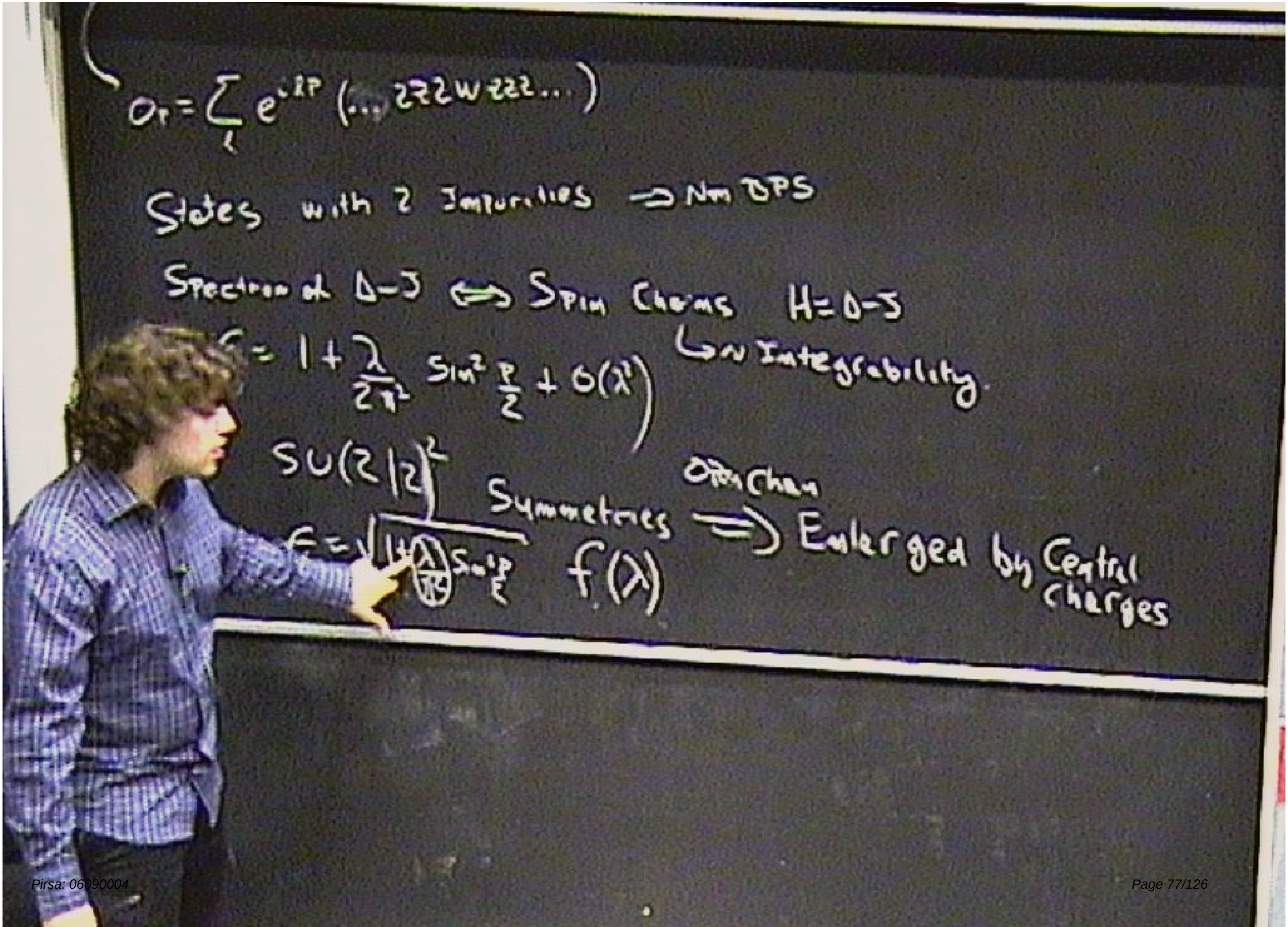
$$\epsilon = \sqrt{1 + \underset{\text{fixed}}{\lambda p^2}} \qquad \lambda p^2 = \text{fixed}$$

$$= \sqrt{1 + \lambda' n^2} \qquad \lambda' = \frac{\lambda}{J^2}$$

$$p = \frac{n}{J}$$

λ' small

λ large


States with 2 Impurities ⟹ Non BPS
Spectrum of Δ−J ⟺ Spin Chains H=Δ−J
Integrability
SU(2|2)²
Symmetries ⟹ Enlarged by Central charges
f(λ)

$O_P = \sum_l e^{ilP} (\ldots ZZZWZZZ\ldots)$

States with 2 Impurities $\Rightarrow$ Non BPS

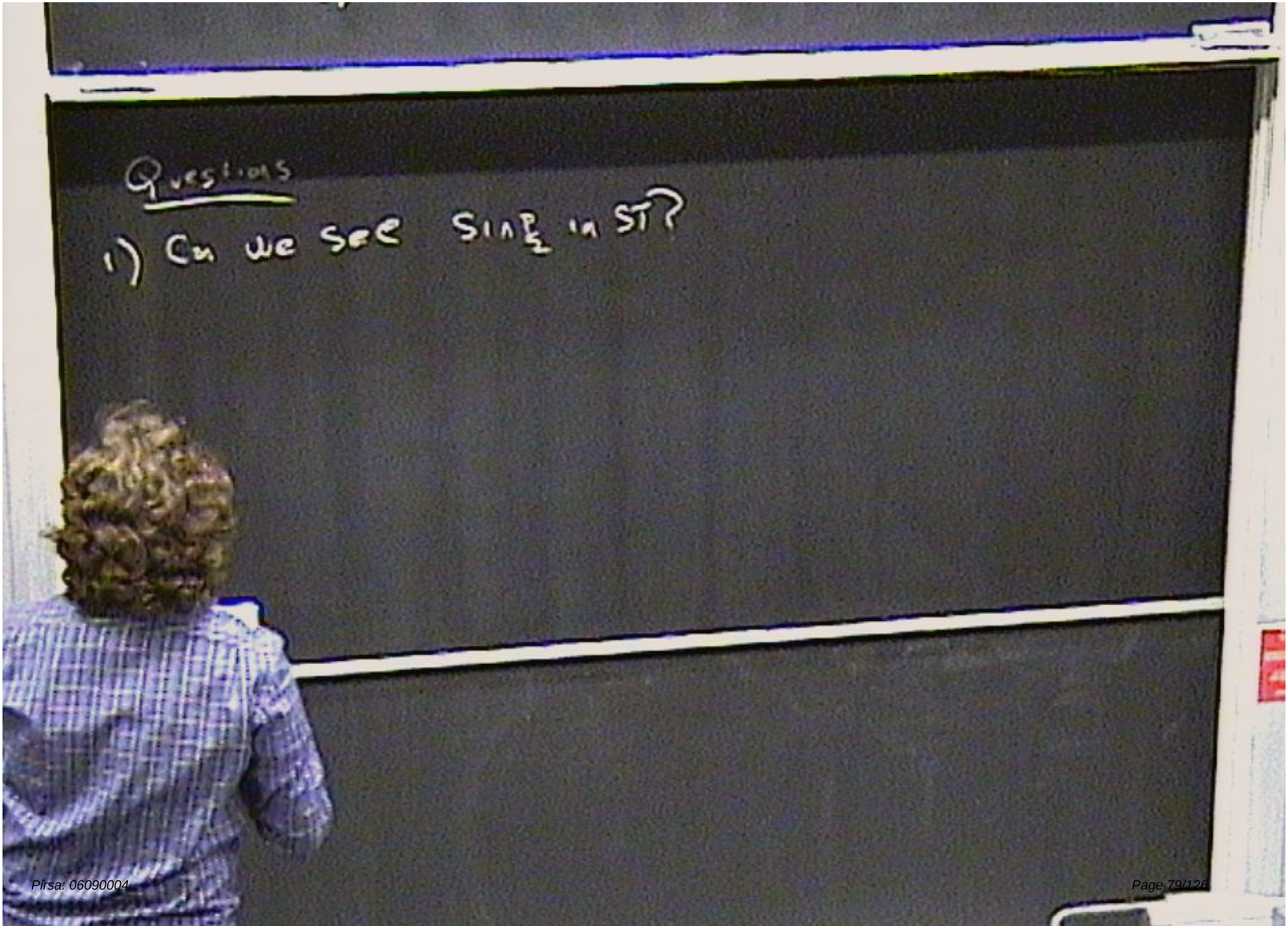
Spectrum of $\Delta - J \Longleftrightarrow$ Spin Chains $H = \Delta - J$

$\epsilon = 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} + O(\lambda^2)$ ↳ Integrability

$SU(2|2)^2$ Symmetries $\Longrightarrow$ Enlarged by Central charges

Open Chain

$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$ $f(\lambda)$


Questions
1) Can we see
in ST?

$O_p = \sum_l e^{ilp} (\ldots ZZZWZZZ\ldots)$

States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J$ $\Longleftrightarrow$ Spin Chains $\quad H = \Delta - J$

$$\epsilon = 1 + \frac{\lambda}{2\pi^2}\sin^2\frac{p}{2} + O(\lambda^2)$$

↳ Integrability

$SU(2|2)^2$ Symmetries $\Longrightarrow$ Enlarged by Central charges

[illegible] Chen

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} \qquad f(\lambda)$$

$O_P = \sum_l e^{ilP} (\ldots ZZZWZZZ\ldots)$

States with 2 Impurities $\Rightarrow$ Non BPS

Spectrum of $\Delta - J$ $\Leftrightarrow$ Spin Chains $H = \Delta - J$

↳ Integrability

$$\epsilon = 1 + \frac{\lambda}{2\pi^2}\sin^2\frac{p}{2} + O(\lambda^2)$$

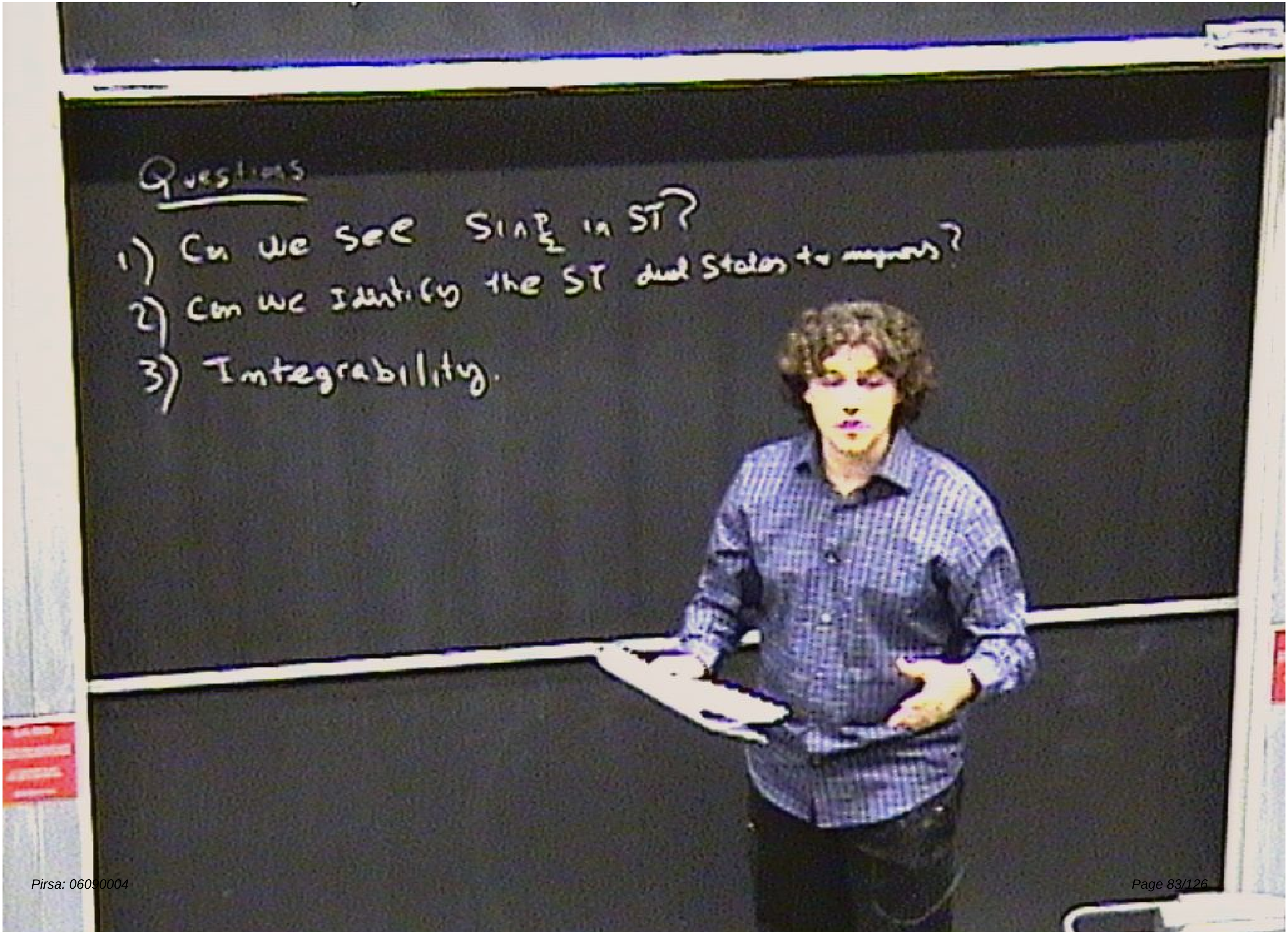
$SU(2|2)^2$ Symmetries $\Rightarrow$ Enlarged by Central charges

Oth. Chan

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} \quad f(\lambda)$$

Questions

1) Can we see $\sin\frac{p}{2}$ in ST?
2) Can we identify the ST dual states to magnons?


Questions
1) Can we see $\sin\frac{p}{2}$ in ST?
2) Can we Identify the ST dual States to magnons?
3) Integrability.

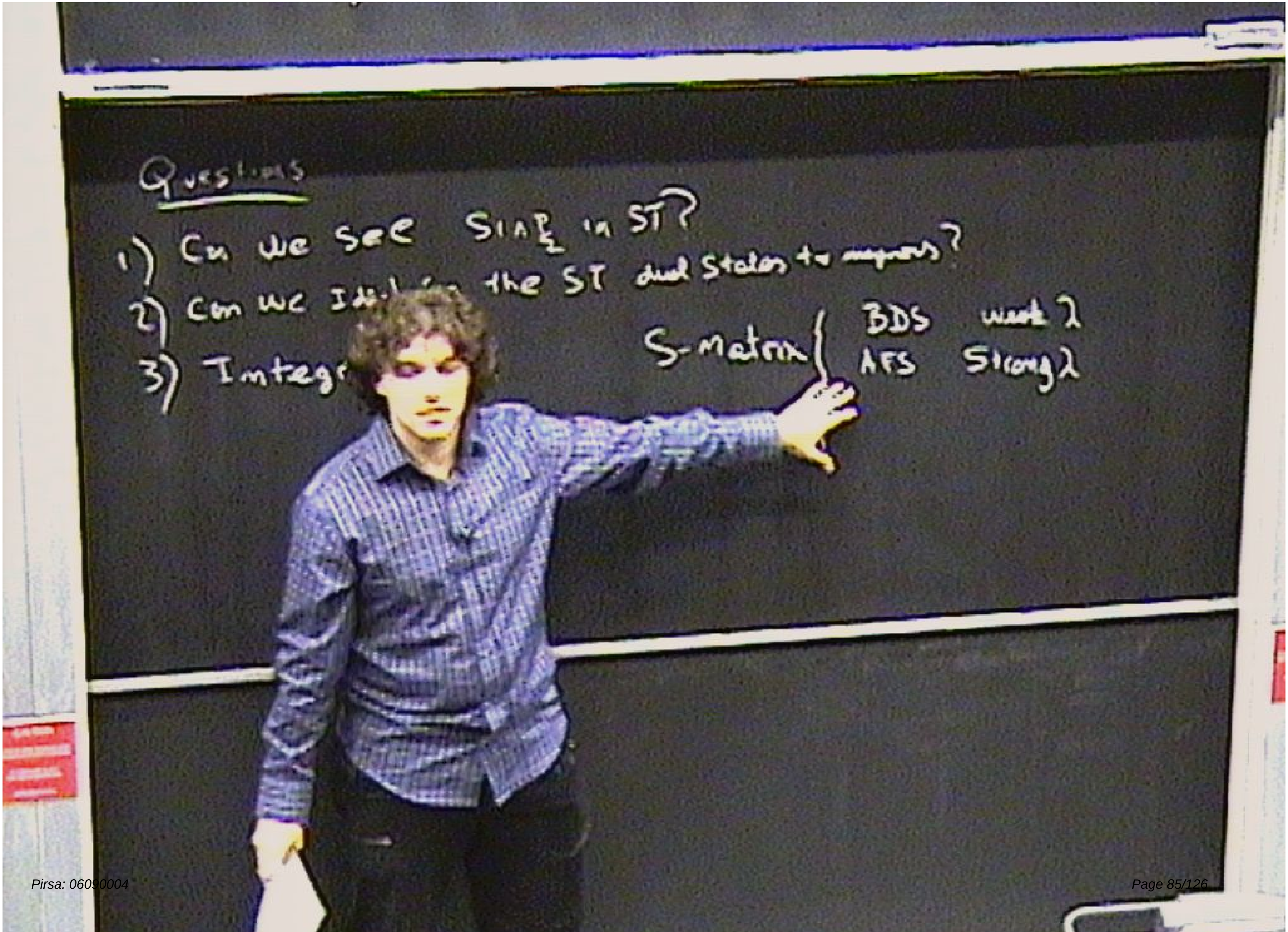
Questions

1) Can we see $\sin\frac{p}{2}$ in ST?

2) Can we Identify the ST dual States to magnons?

3) Integrability.

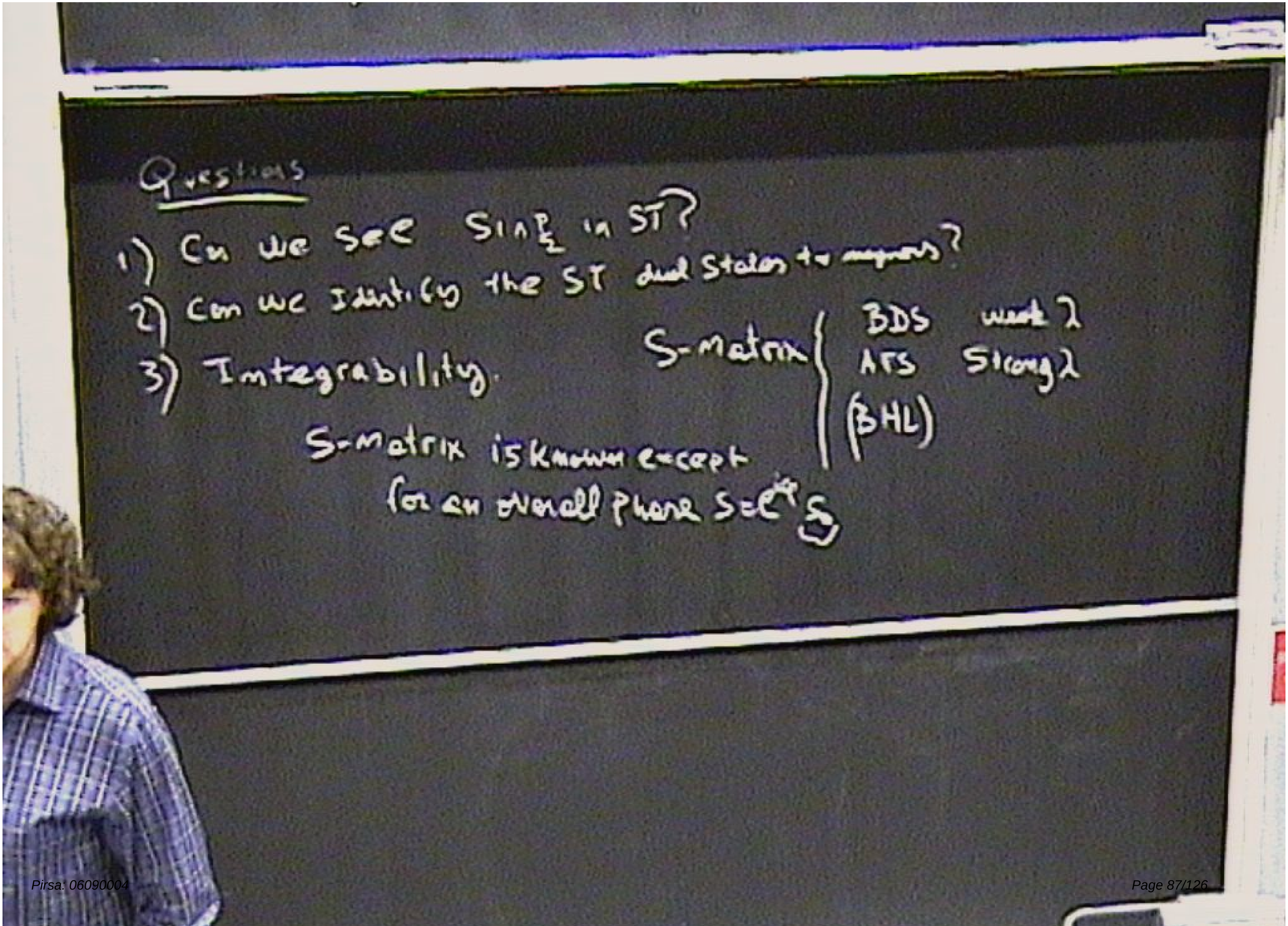
S-Matrix { BDS

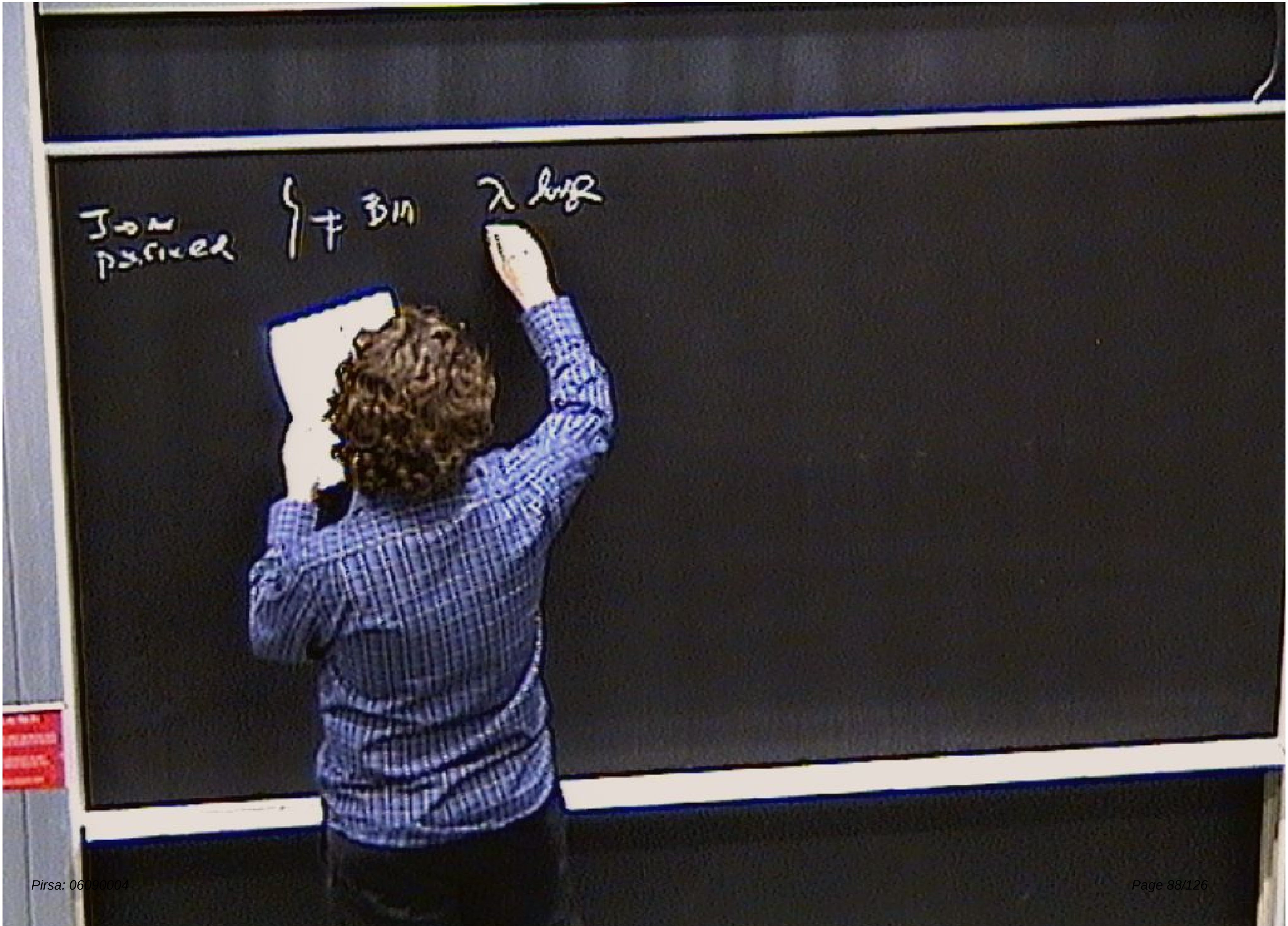

Questions
1) Can we see in ST?
2) Can we the ST dual States to magnons?
3)
S-Matrix
BDS weak λ
AFS Strong λ

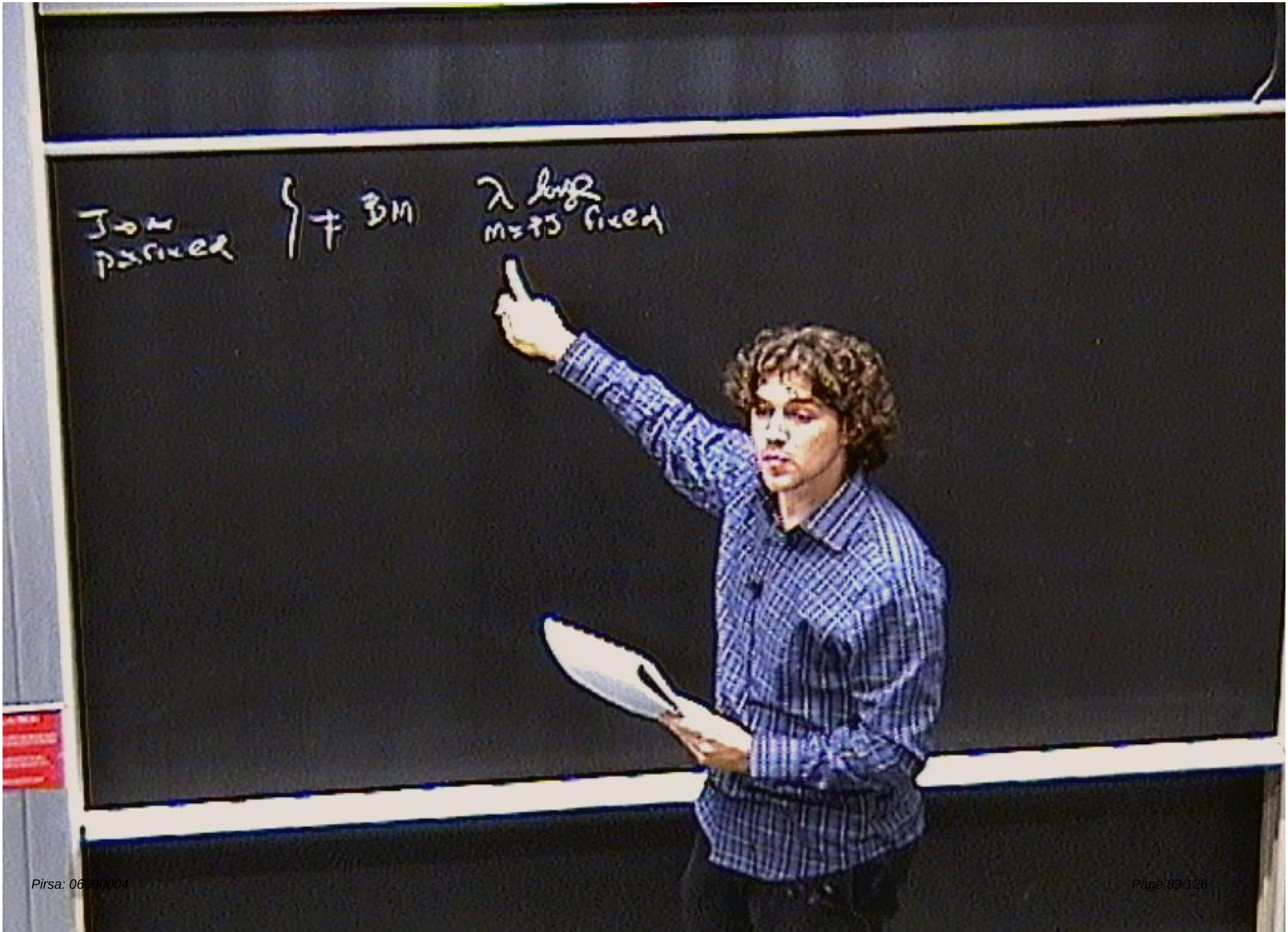
Questions

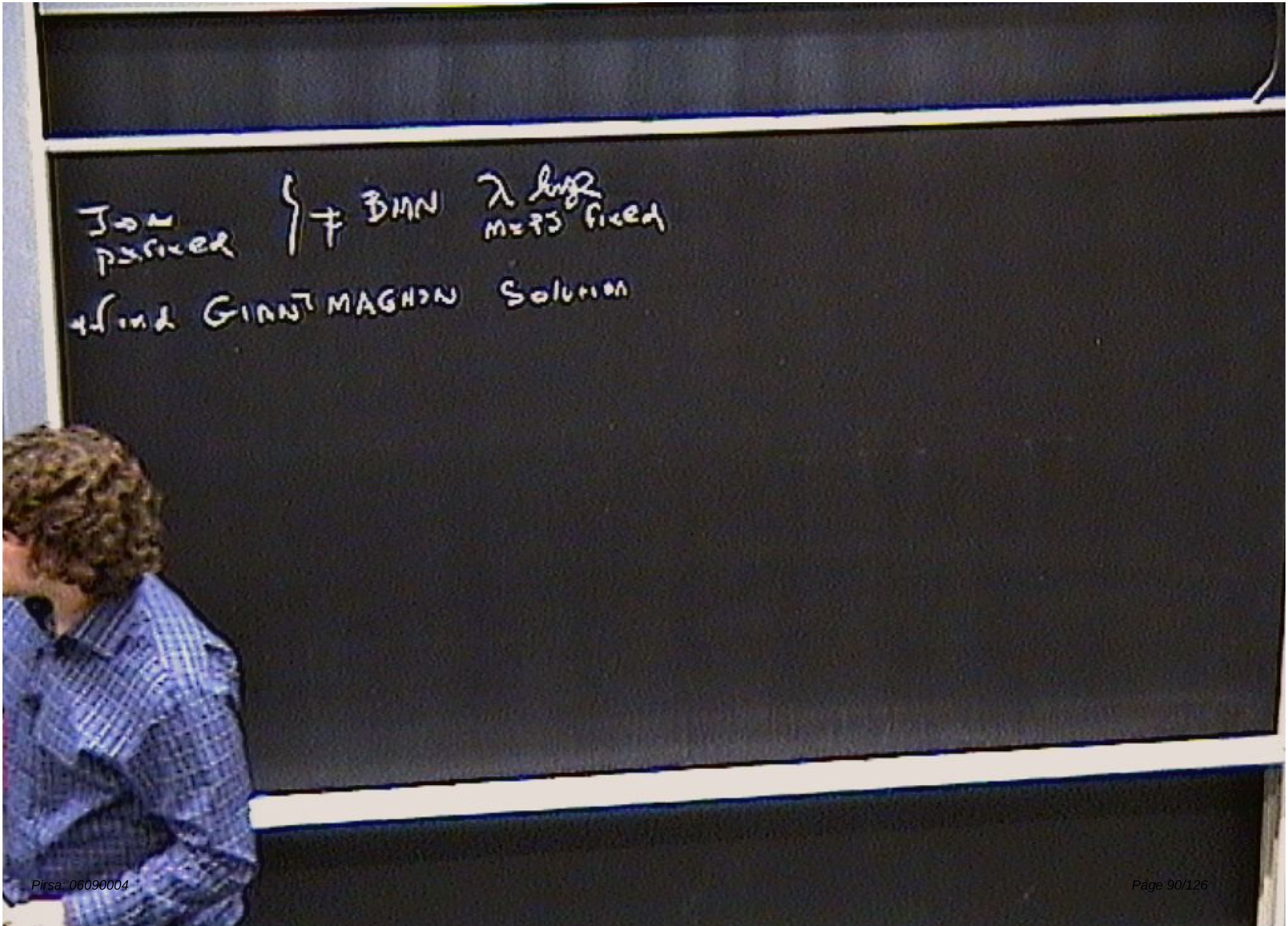
1) Can we see $\sin\frac{p}{2}$ in ST?
2) Can we identify the ST dual states to magnons?
3) Integrability.

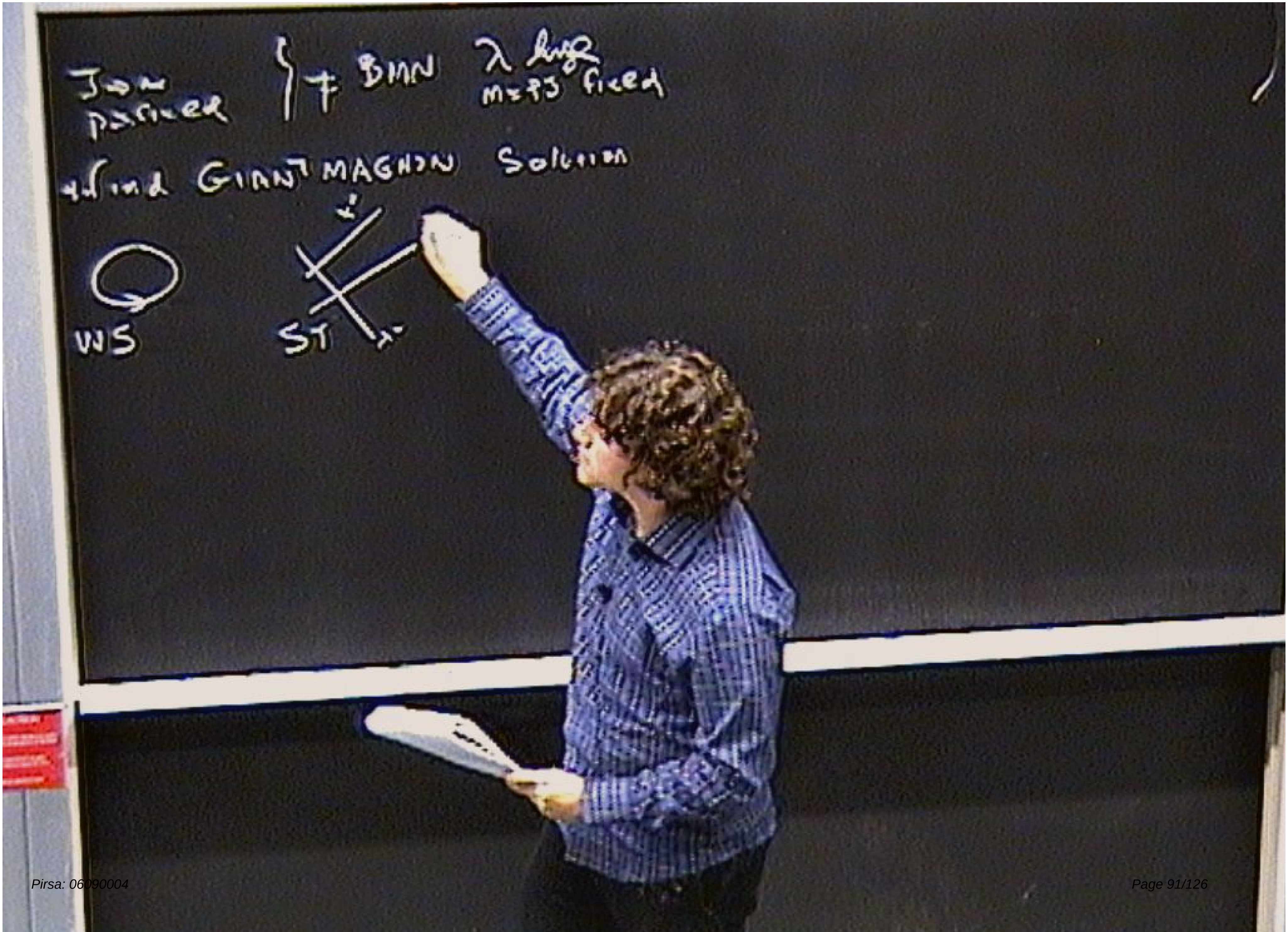
S-Matrix {
- BDS — weak λ
- AFS — strong λ
- (BHL)

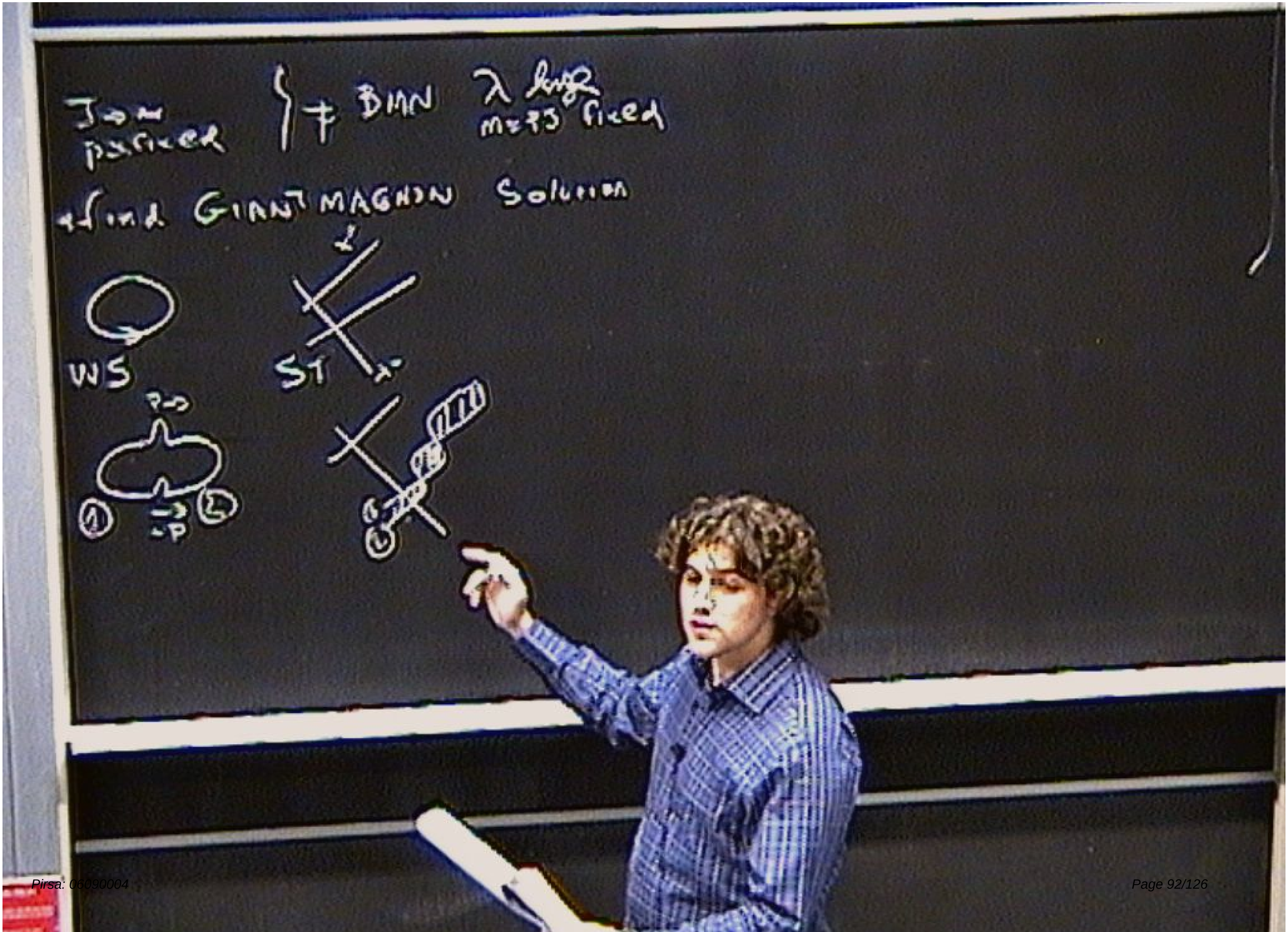

Questions
1) Can we see sin p/2 in ST?
2) Can we identify the ST dual states to magnons?
3) Integrability.
S-matrix
BDS weak λ
AFS strong λ
(BHL)
S-matrix is known except
for an overall phase S=e^{iθ} S_0

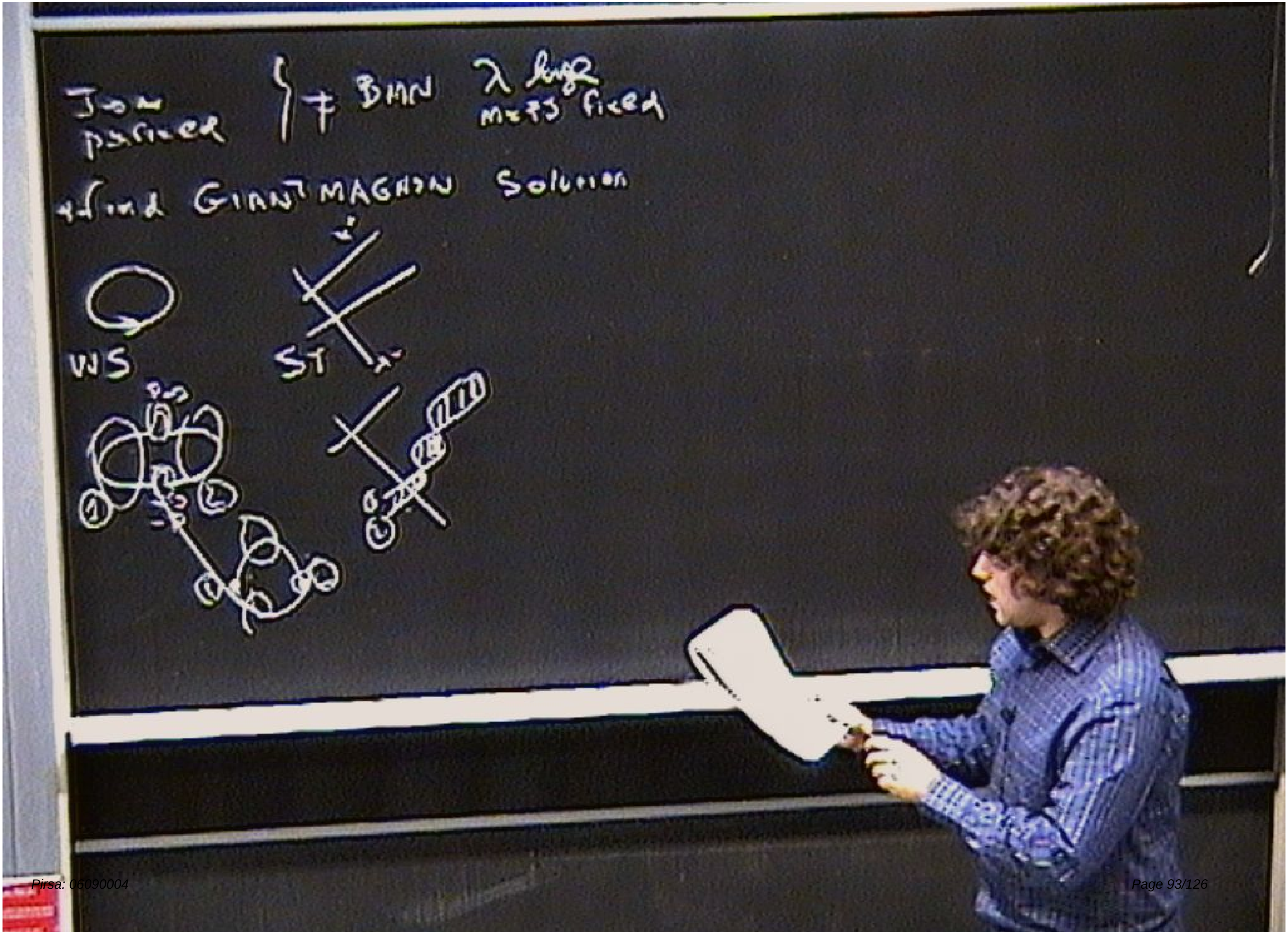


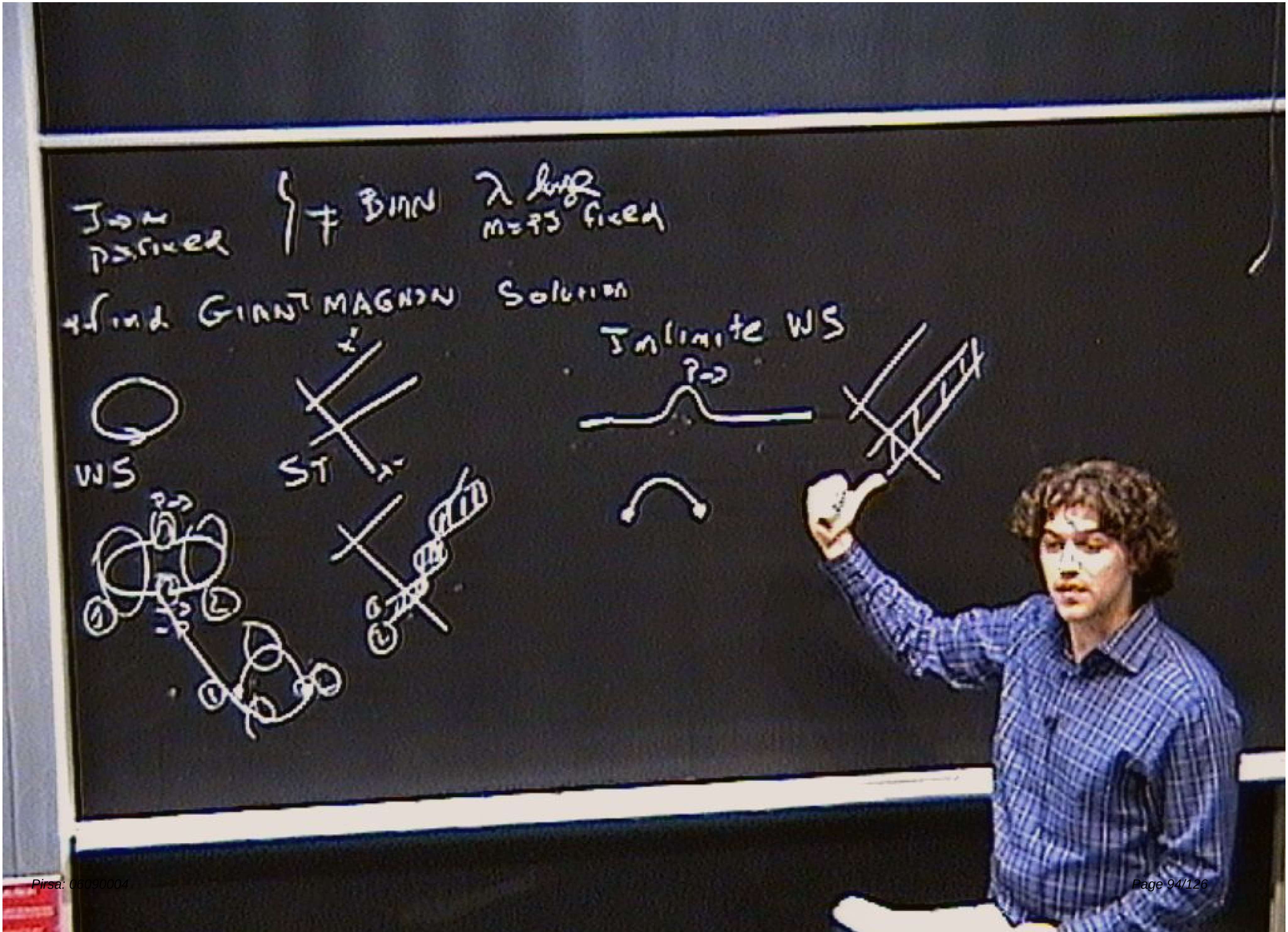

J→∞
p fixed
≠ BM
λ large
fixed


J → ∞
p fixed
≠ BMN
λ large
fixed
GIANT MAGNON
Solution


BMN
fixed
GIANT MAGNON
Solution
ST


BMN
fixed
find GIANT MAGNON Solution
WS
ST


BMN
WS
ST


BMN
fixed
GIANT MAGNON Solution
WS
ST

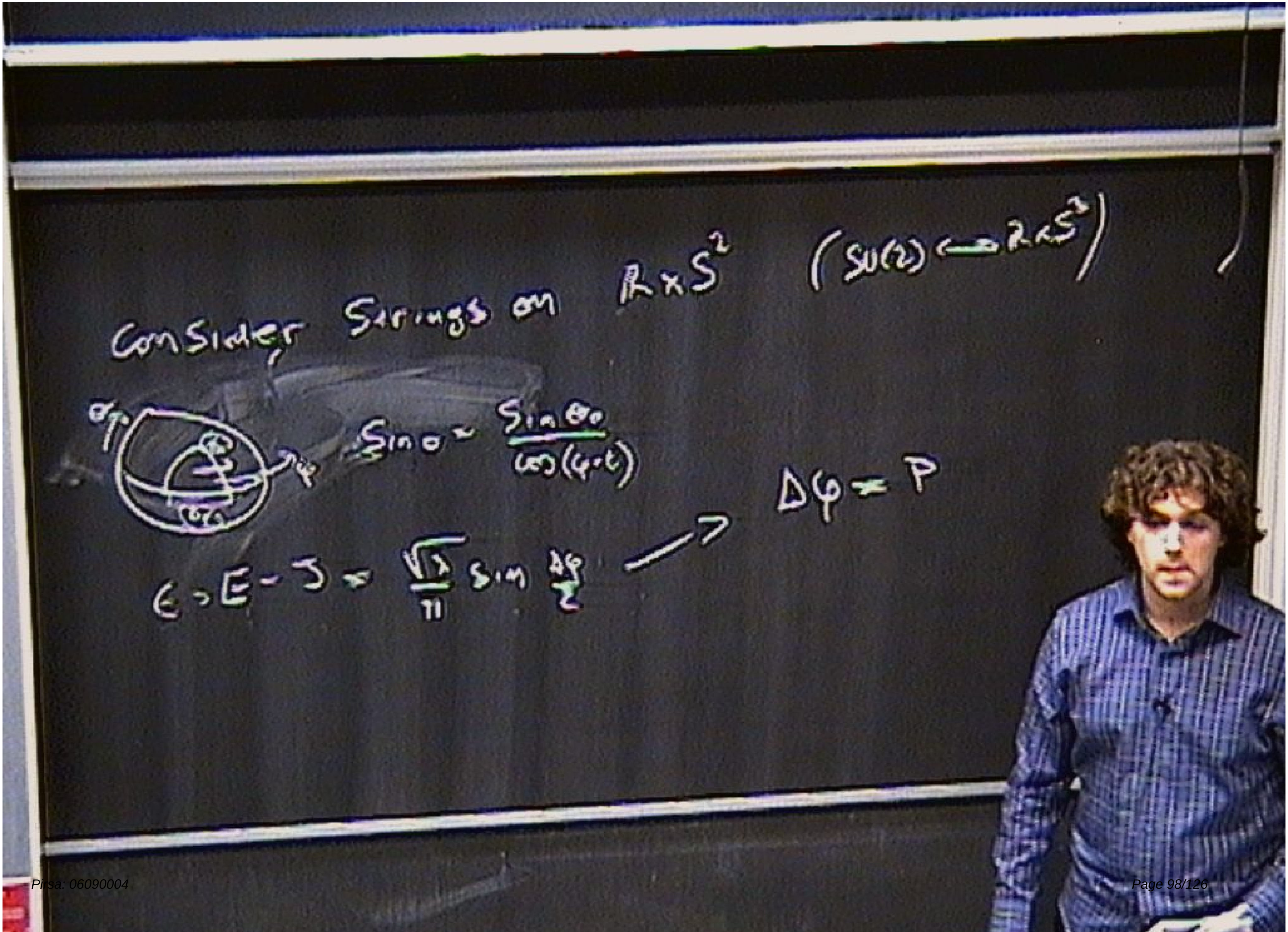
Consider Strings on $\mathbb{R} \times S^2$

Consider Strings on $\mathbb{R} \times S^2$ $\left(SU(2) \leftrightarrow \mathbb{R} \times S^3 \right)$

Consider Strings on $\mathbb{R}\times S^2$ $\left(SU(2) \longrightarrow \mathbb{R}\times S^2\right)$

$$\sin\theta = \frac{\sin\theta_0}{\cos(\varphi - t)}$$

$$E - J = \frac{\sqrt{\lambda}}{\pi}\sin\frac{\Delta\varphi}{2}$$



Consider Strings on R×S^2 (SU(2) ⟶ R×S^3)
Δφ = P
ε = E − J = √λ/π sin Δφ/2

Consider Strings on $\mathbb{R}\times S^2$ $\left(SU(2) \hookrightarrow \mathbb{R}\times S^3\right)$

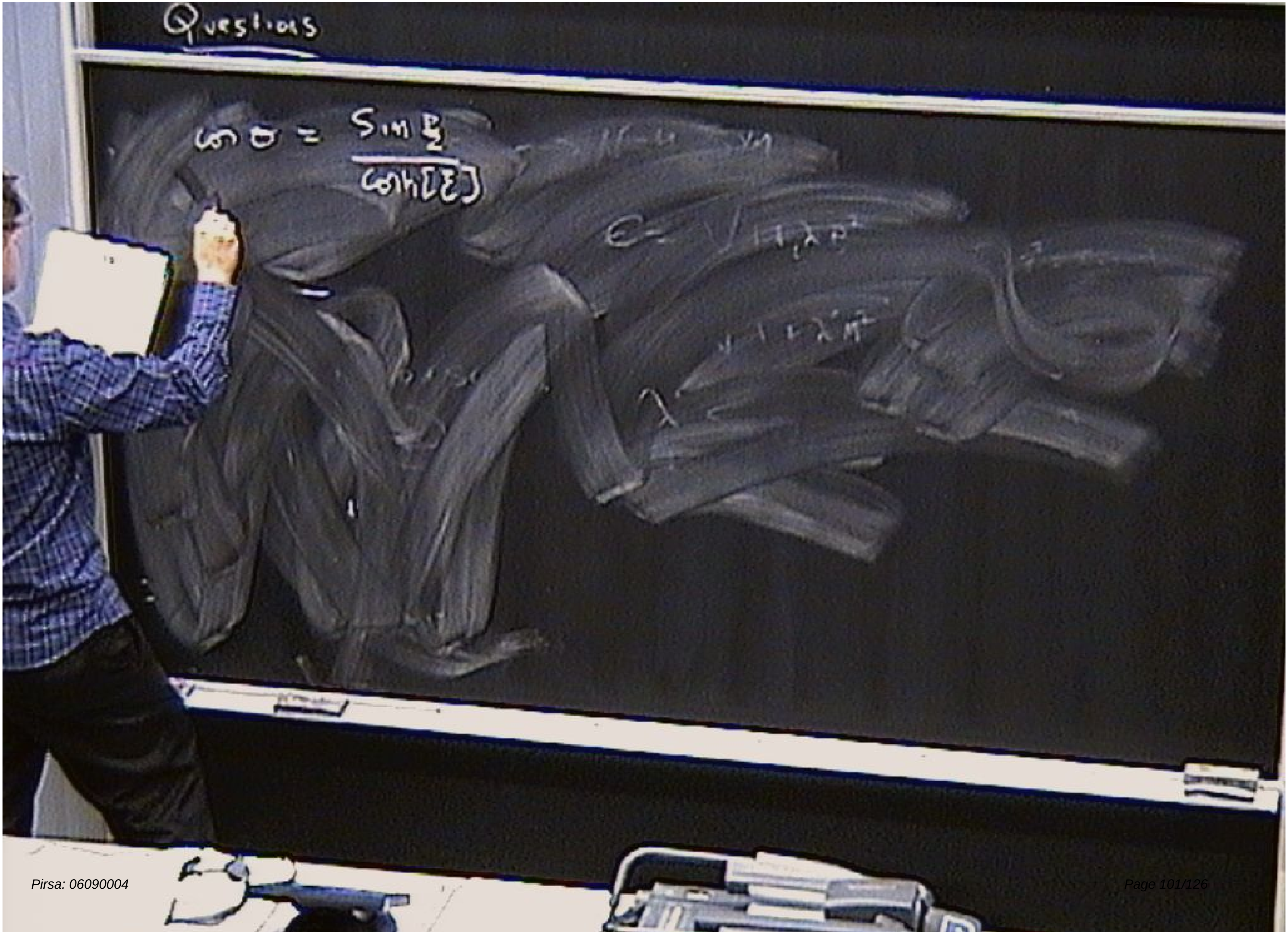
$$\sin\theta = \frac{\sin\theta_0}{\cos(\varphi - t)}$$

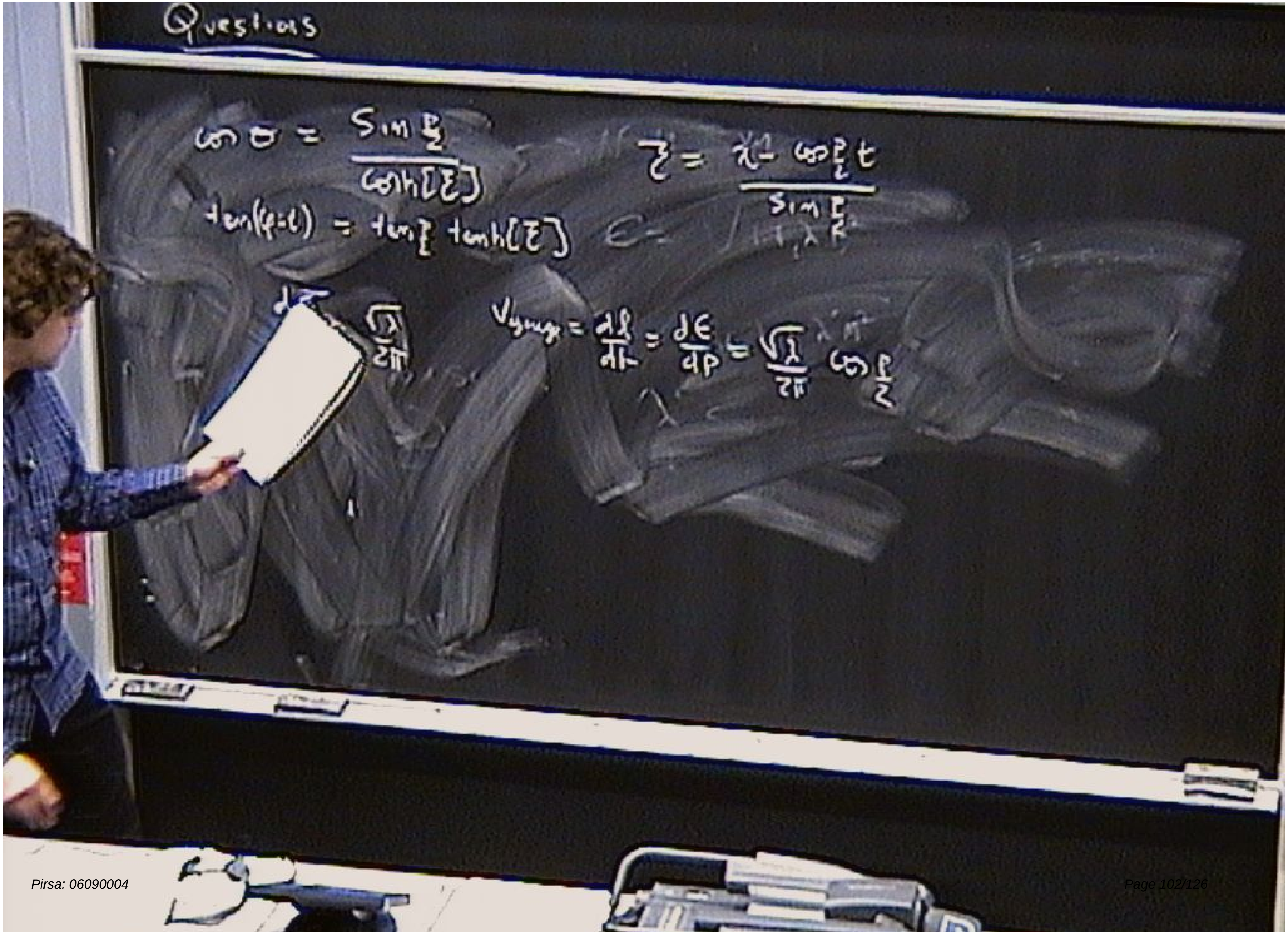
$$\epsilon = E - J = \frac{\sqrt{\lambda}}{\pi}\sin\frac{\Delta\varphi}{2} \longrightarrow \begin{cases} \Delta\varphi = p \\ \text{where's the } 1 \end{cases}$$

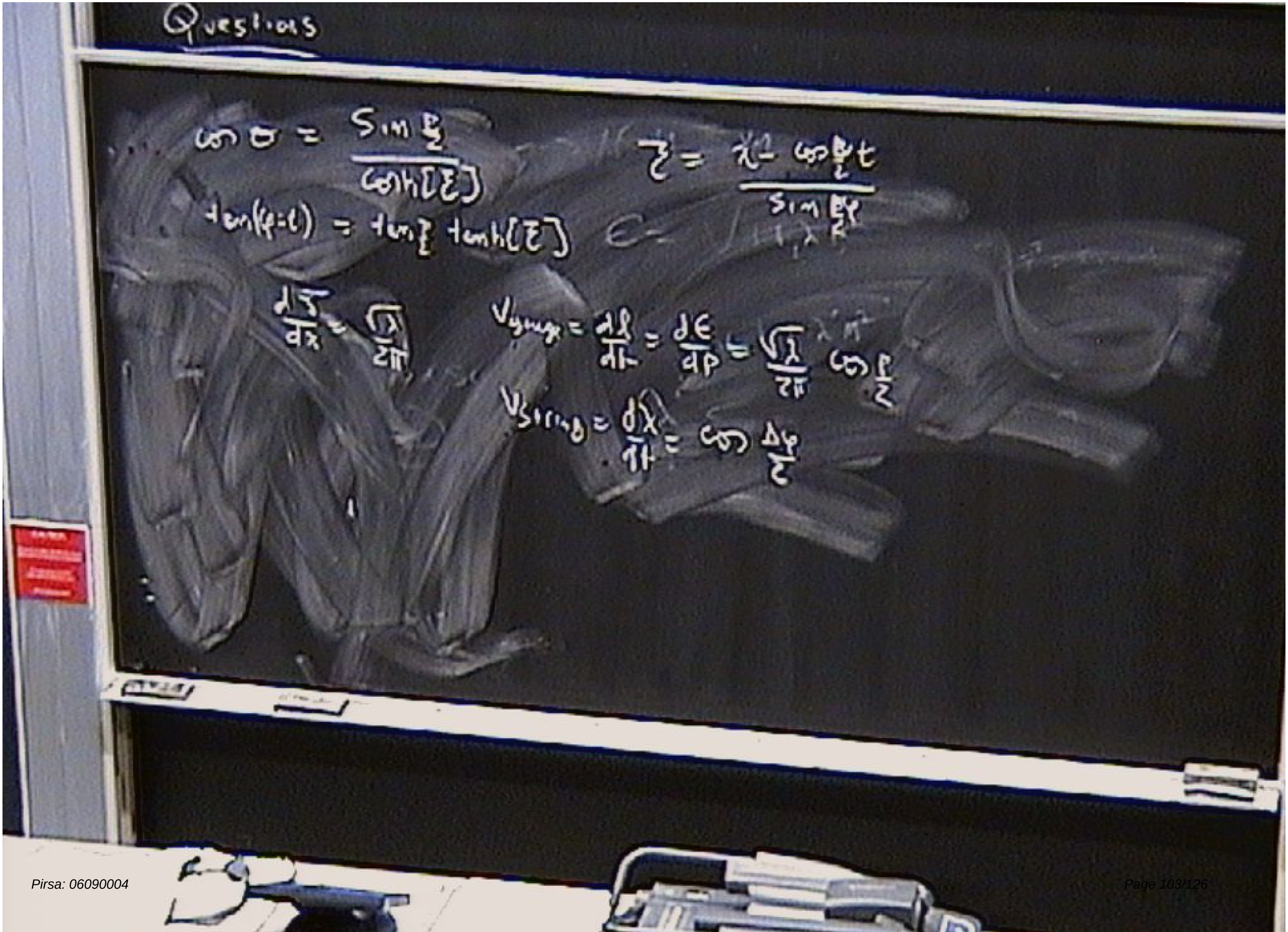
Consider Strings on $\mathbb{R}\times S^2$ $\left(SU(2) \hookrightarrow \mathbb{R}\times S^3 \right)$

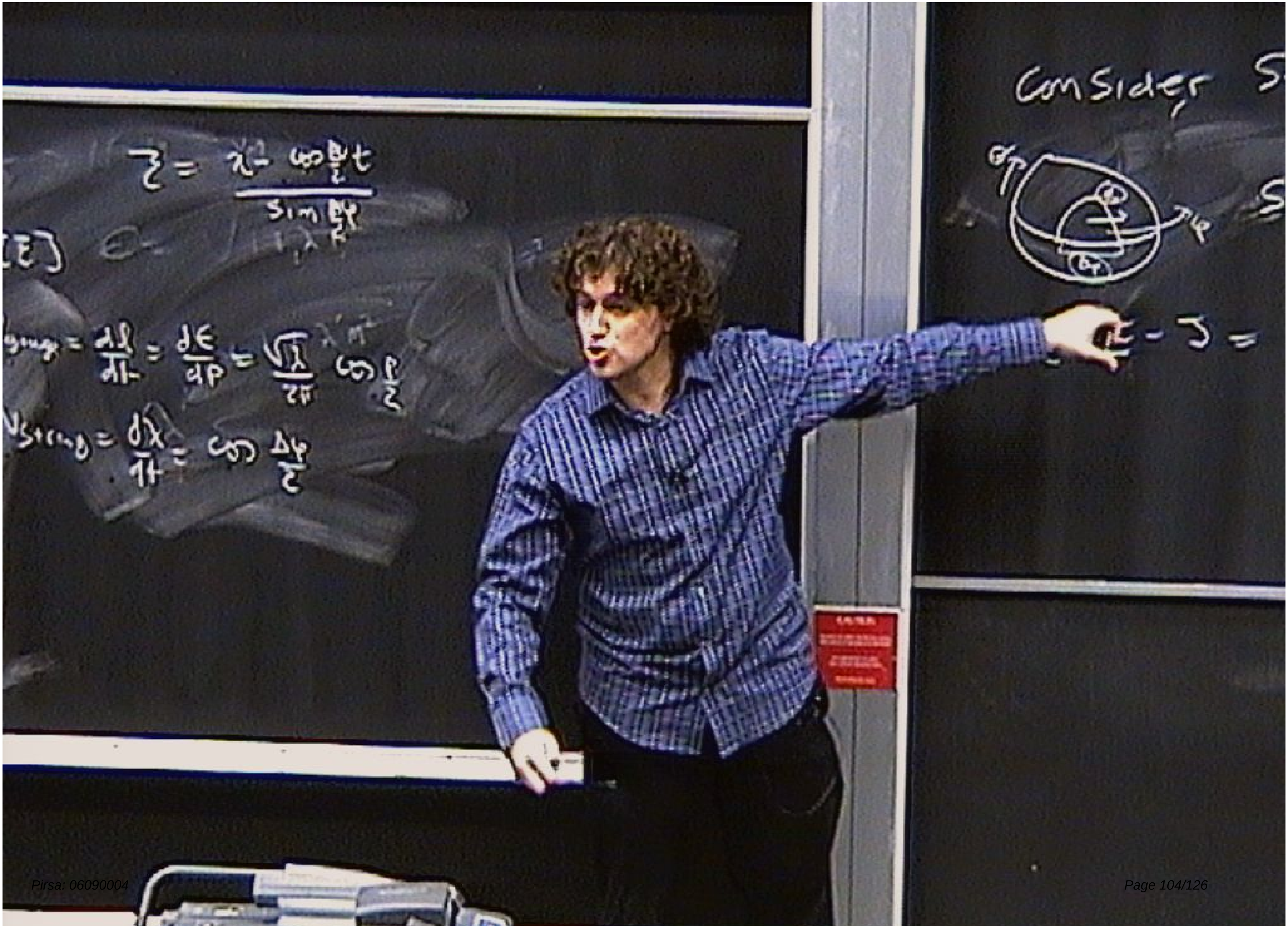
$$\sin\theta = \frac{\sin\theta_0}{\cos(\varphi - t)}$$

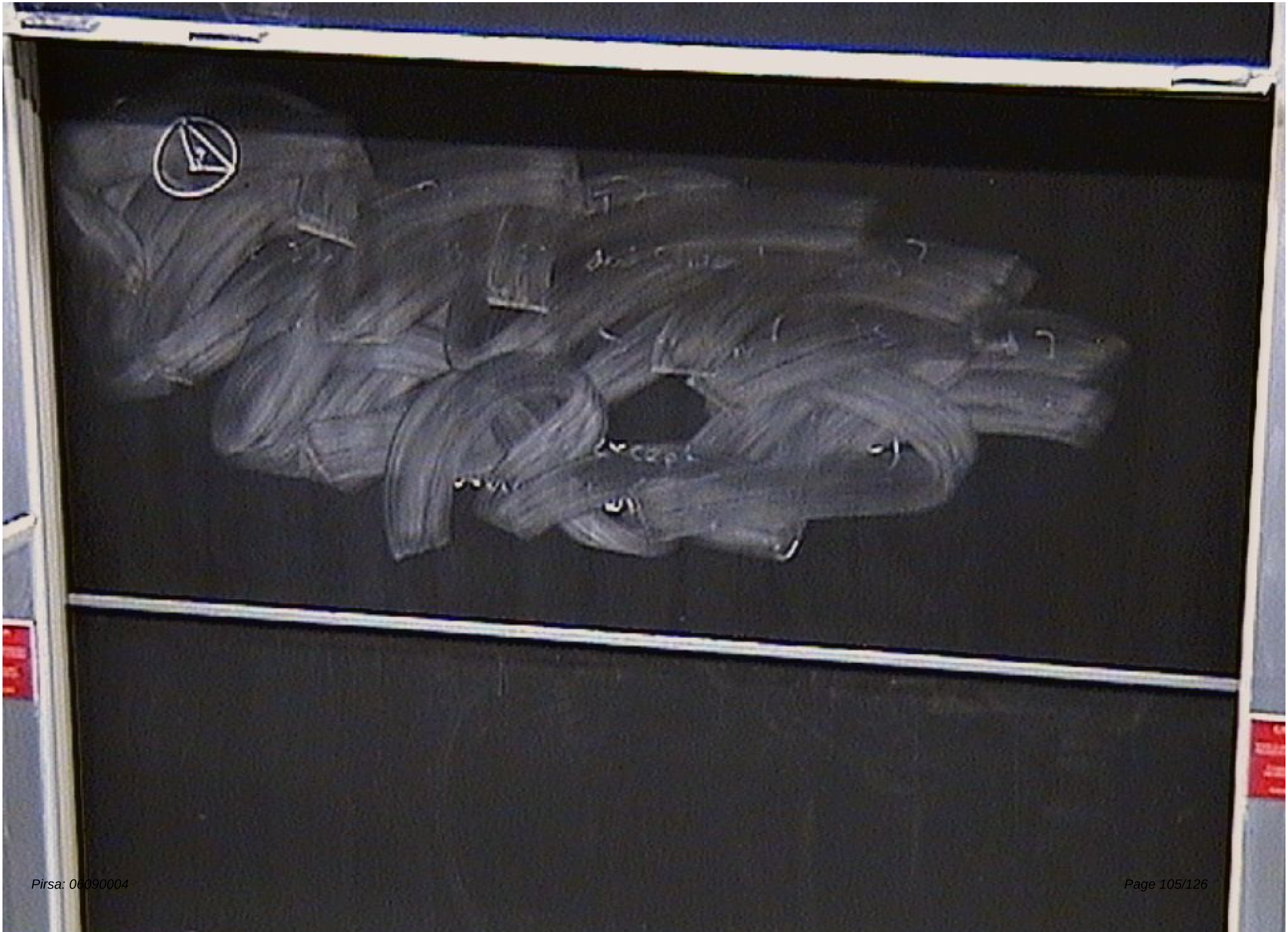
$$\epsilon = E - J = \frac{\sqrt{\lambda}}{\pi}\sin\frac{\Delta\varphi}{2} \longrightarrow \begin{cases} \Delta\varphi = p \\ \text{where's the } 1 \end{cases}$$

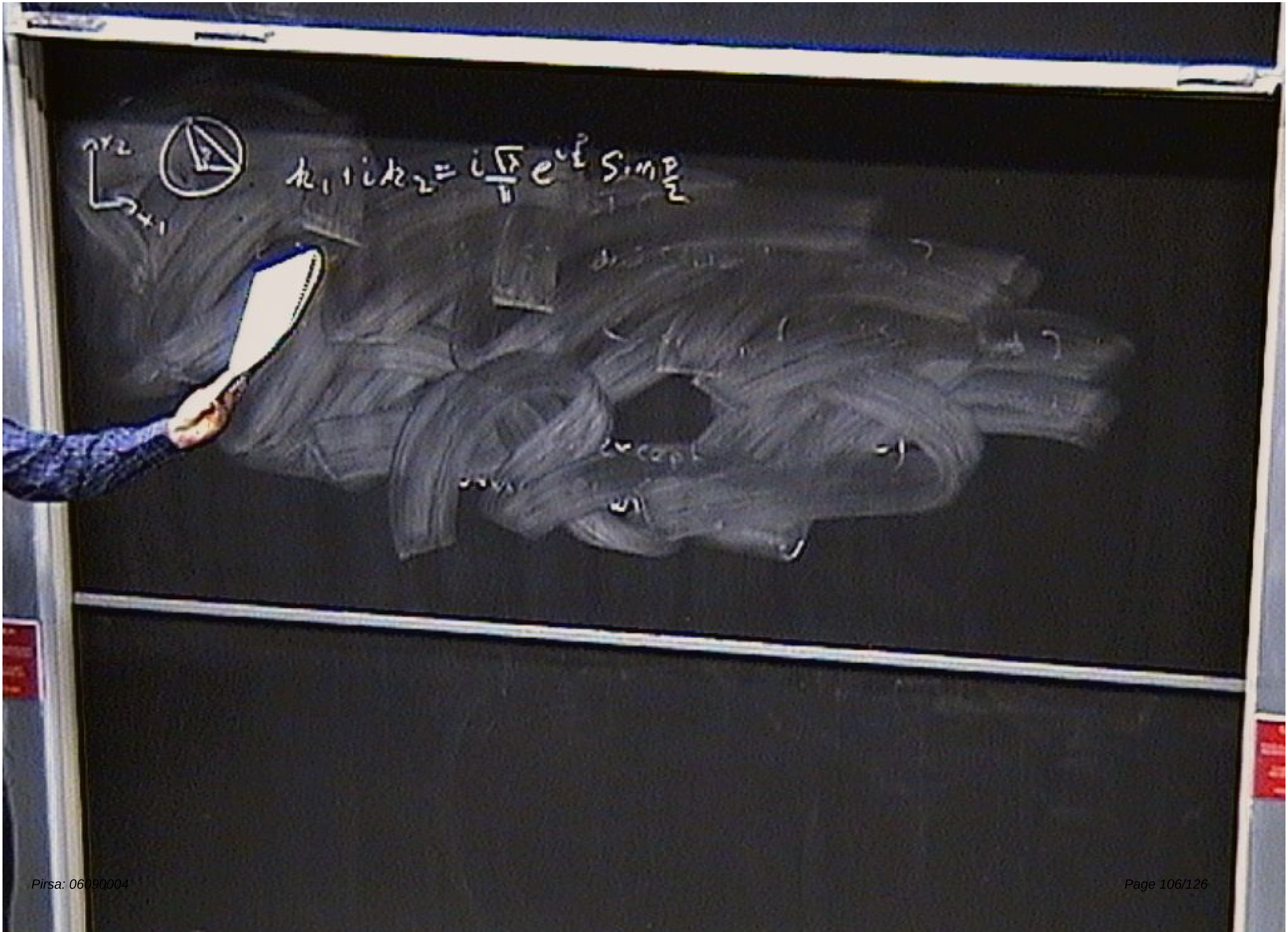

Questions

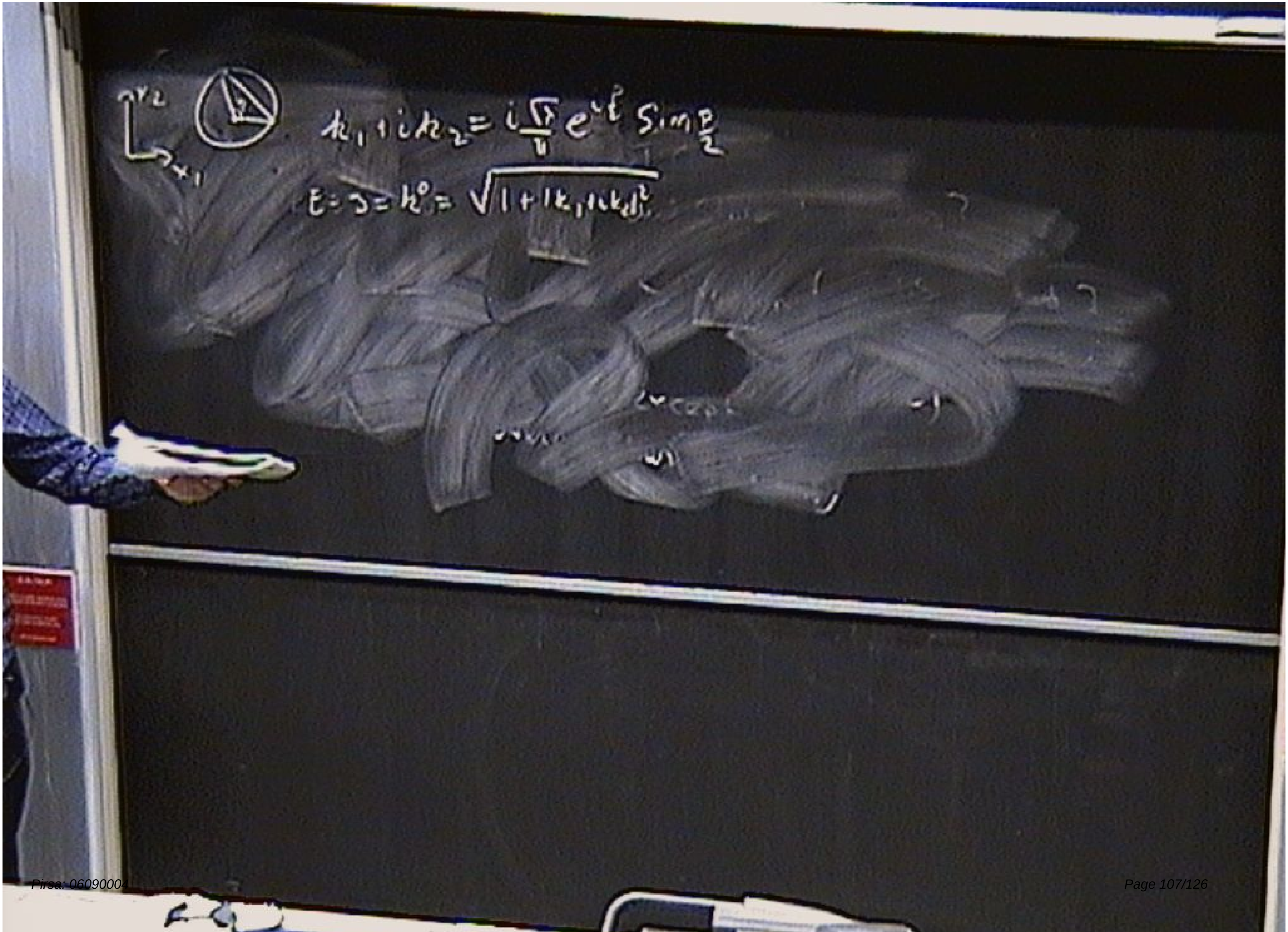

Questions
cos θ = sin p/2 / cosh[ξ]
tan(φ−t) = tan p/2 tanh[ξ]
ξ = x − cos p/2 t / sin p/2
V_group = dx/dt = dE/dp = √λ/2π cos p/2

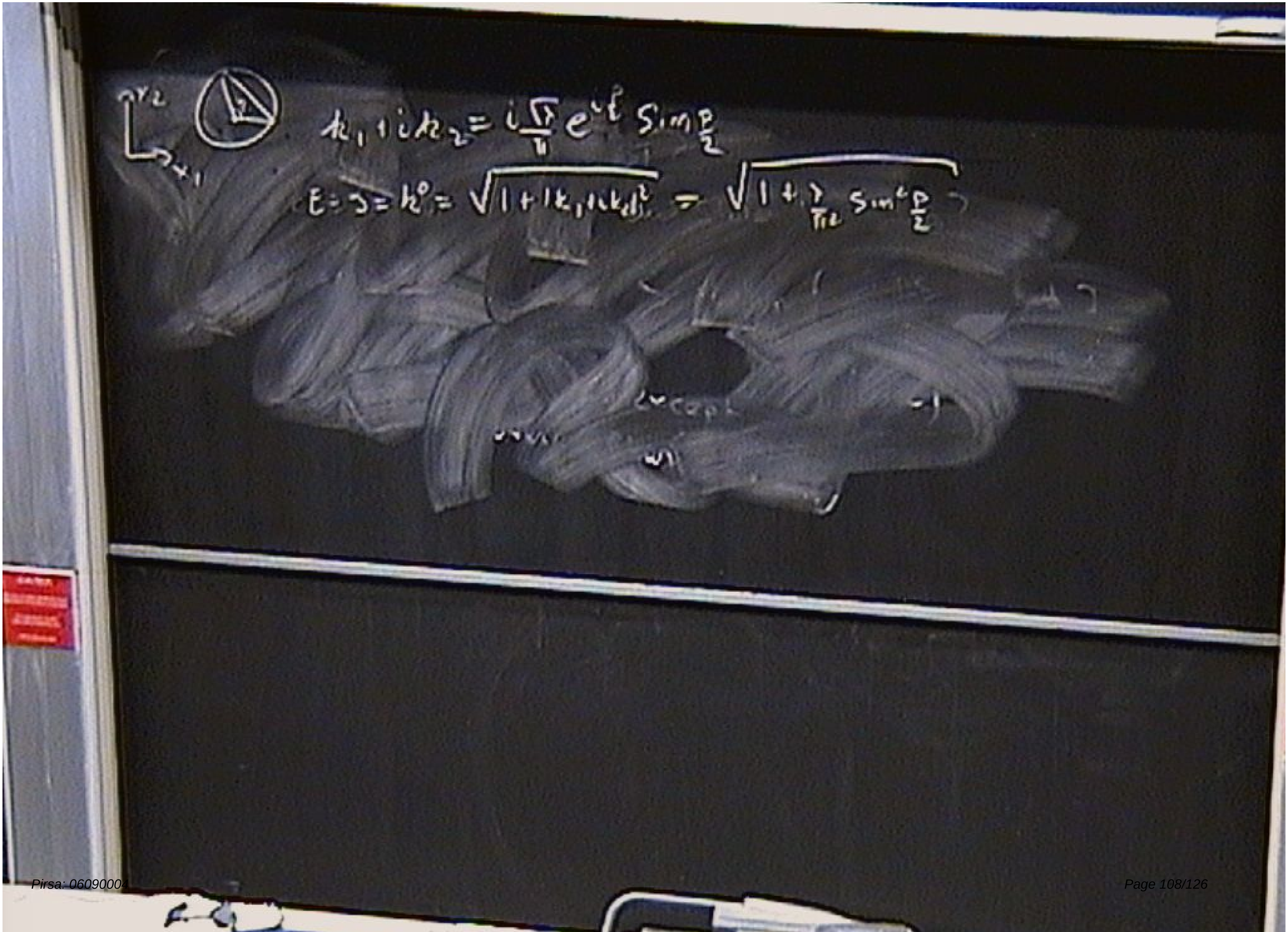

Questions
Vgroup
Vstring


Consider






x_2
x_1
$k_1 + ik_2 = i\frac{\sqrt{r}}{u} e^{i\varphi} \sin\frac{\theta}{2}$
$E = s = k^0 = \sqrt{1 + |k_1 + ik_2|^2}$



Consider Strings on $\mathbb{R}\times S^2$ $\left(SU(2) \hookrightarrow \mathbb{R}\times S^3\right)$

$$\sin\sigma = \frac{\sin\theta_0}{\cos(\varphi - t)}$$

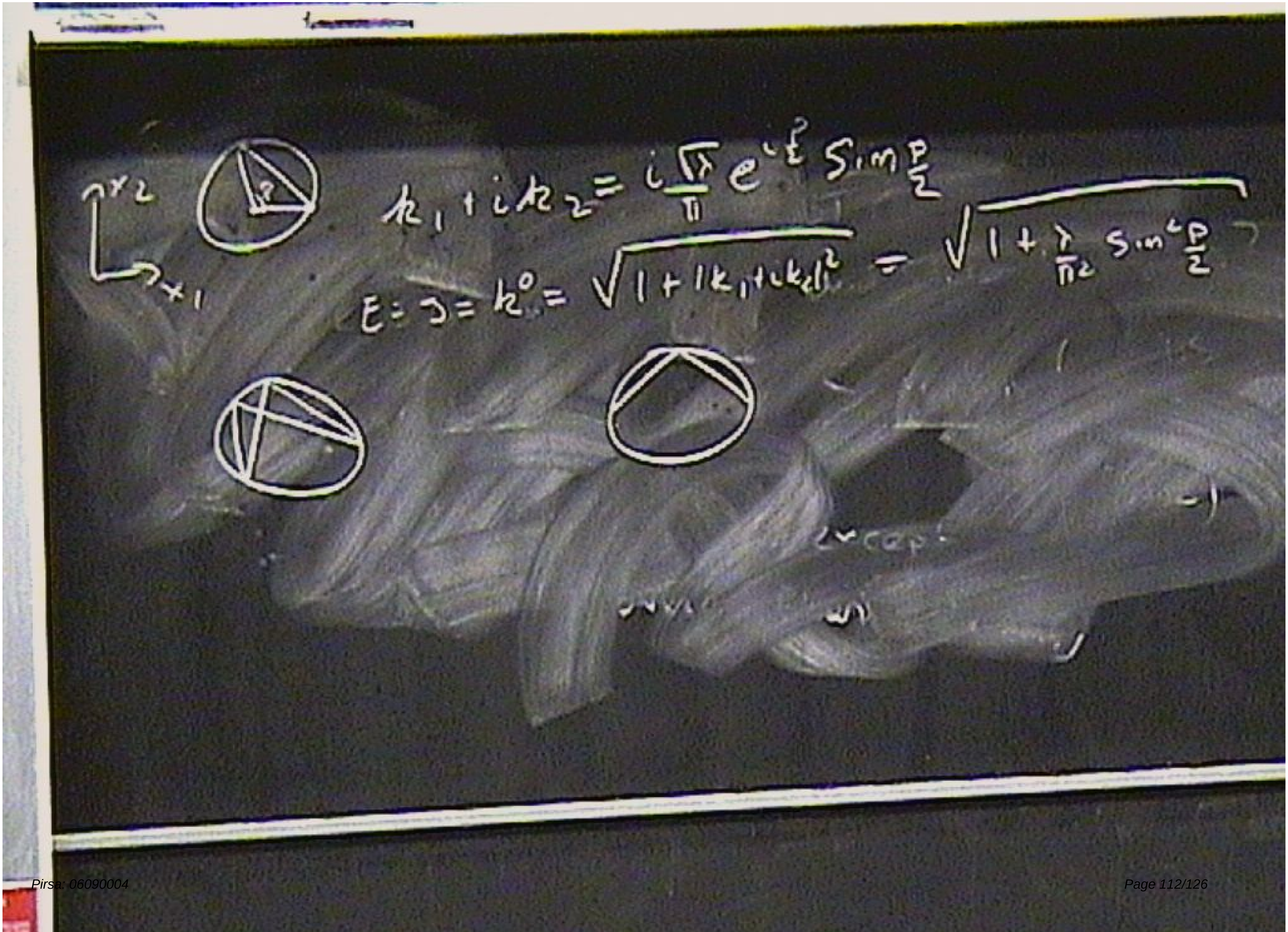
$$\epsilon = E - J = \frac{\sqrt{\lambda}}{\pi}\sin\frac{\Delta\varphi}{2} \longrightarrow \begin{cases} \Delta\varphi = P \\ \text{where's the 1} \end{cases}$$

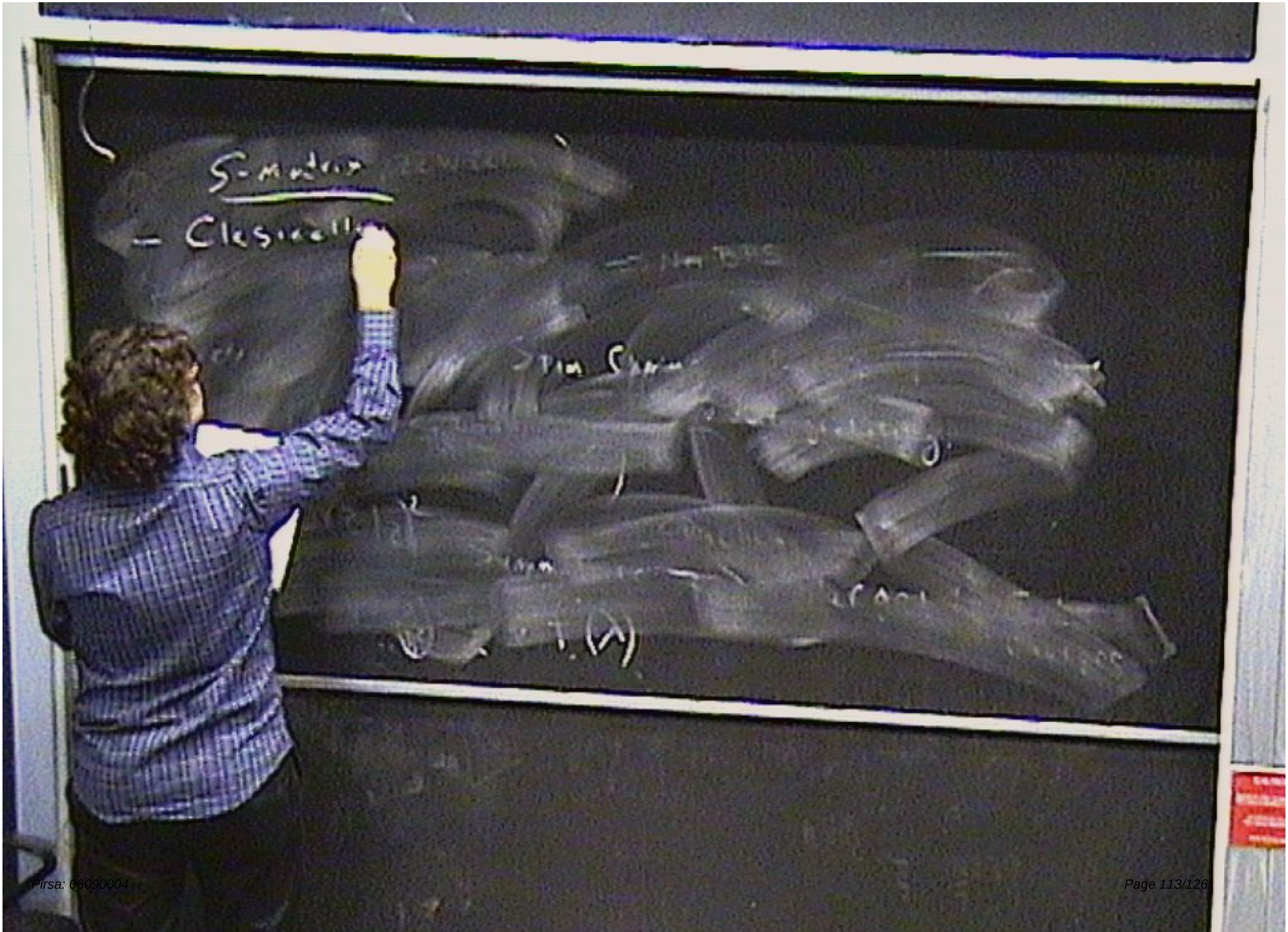
$$k_1 + i k_2 = i\frac{\sqrt{\lambda}}{\pi} e^{i\frac{\xi}{2}} \sin\frac{p}{2}$$

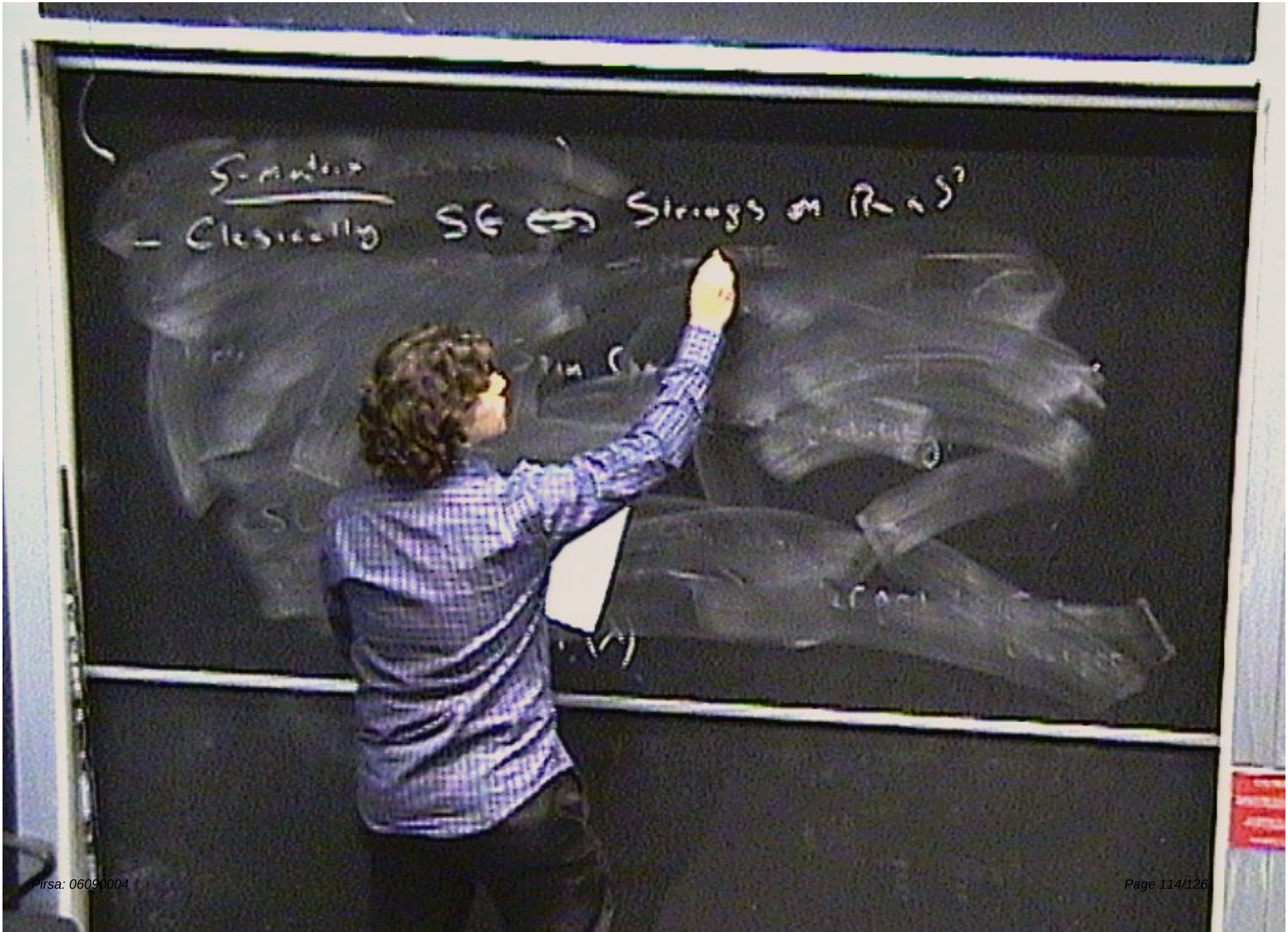
$$E = \gamma = k^0 = \sqrt{1 + |k_1 + i k_2|^2} = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$$

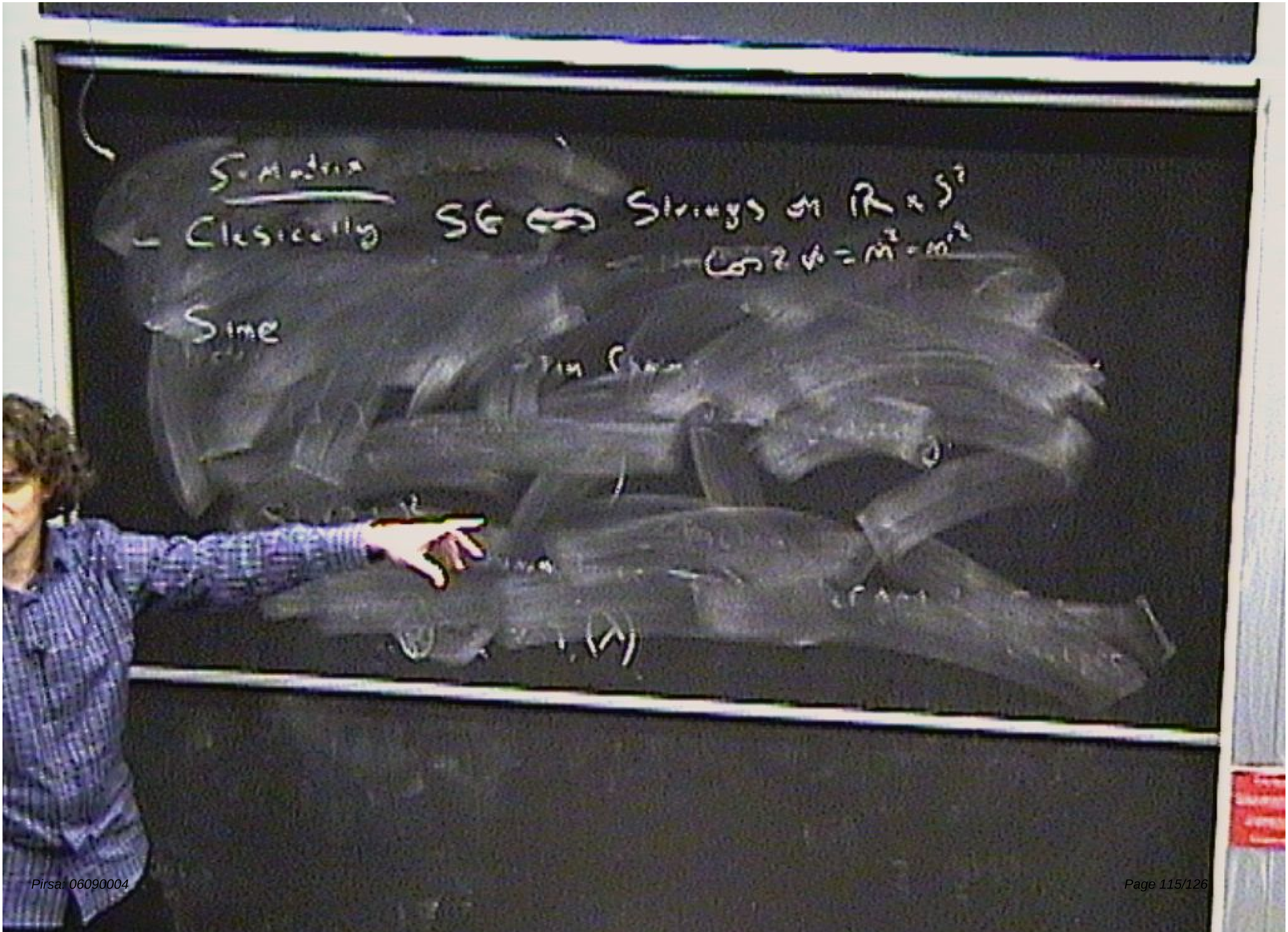
$$k_1 + i k_2 = i \frac{\sqrt{\lambda}}{\pi} e^{i\xi} \sin\frac{p}{2}$$

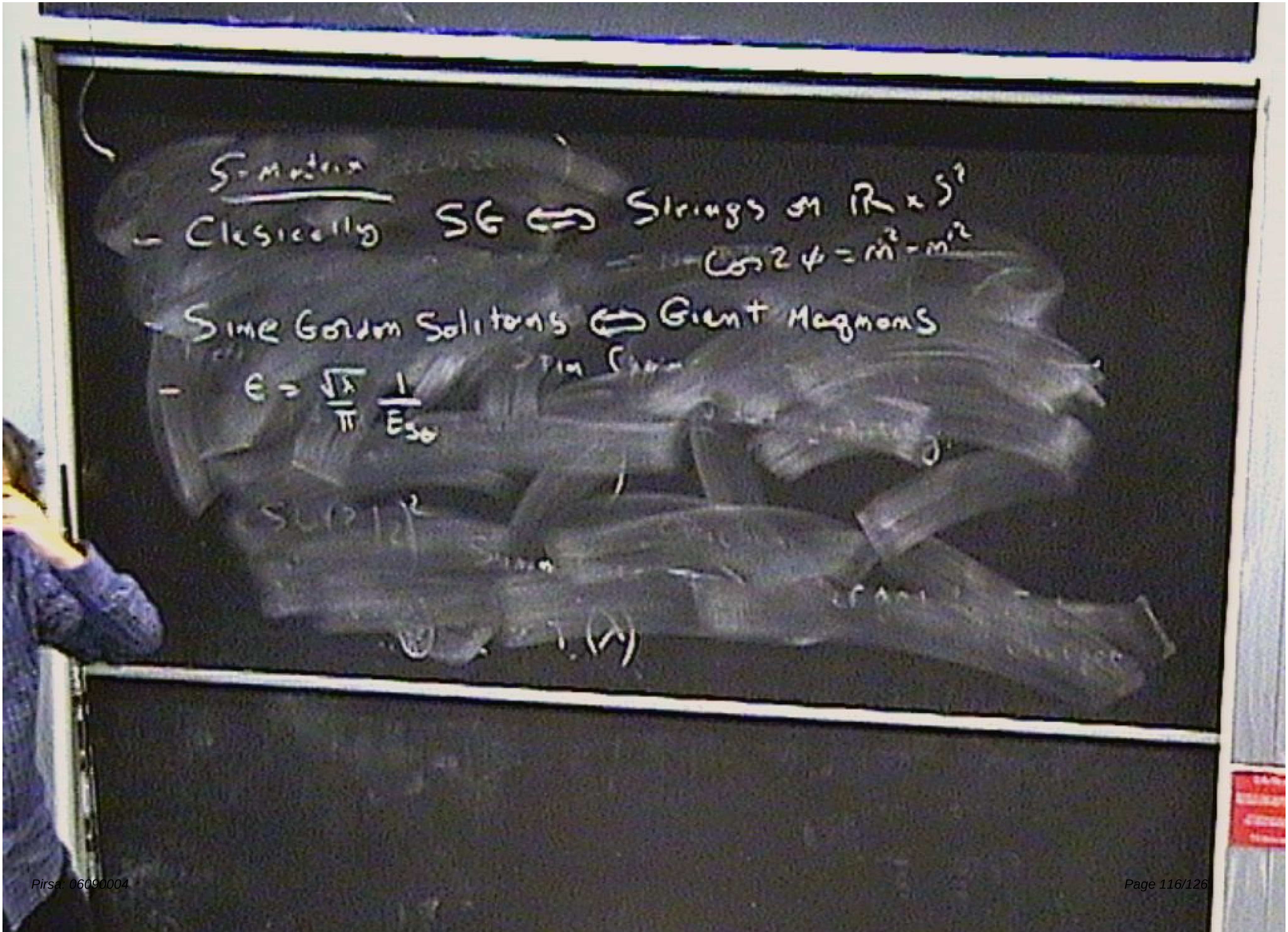
$$E = \gamma = k^0 = \sqrt{1 + |k_1 + i k_2|^2} = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2\frac{p}{2}}$$


x2
x1
k1 + ik2 = i √γ/π e^{iξ} sin p/2
E = γ = k0 = √(1 + |k1 + ik2|²) = √(1 + γ/π² sin² p/2)


S-Matrix


S-matrix
– Classically SG ⟺ Strings on R×S³


S-Matrix
- Classically SG ⟺ Strings on R × S³
- Sine


S-Matrix
- Classically SG ⟺ Strings on R x S
- Sine Gordon Solitons ⟺ Giant Magnons

S-Matrix

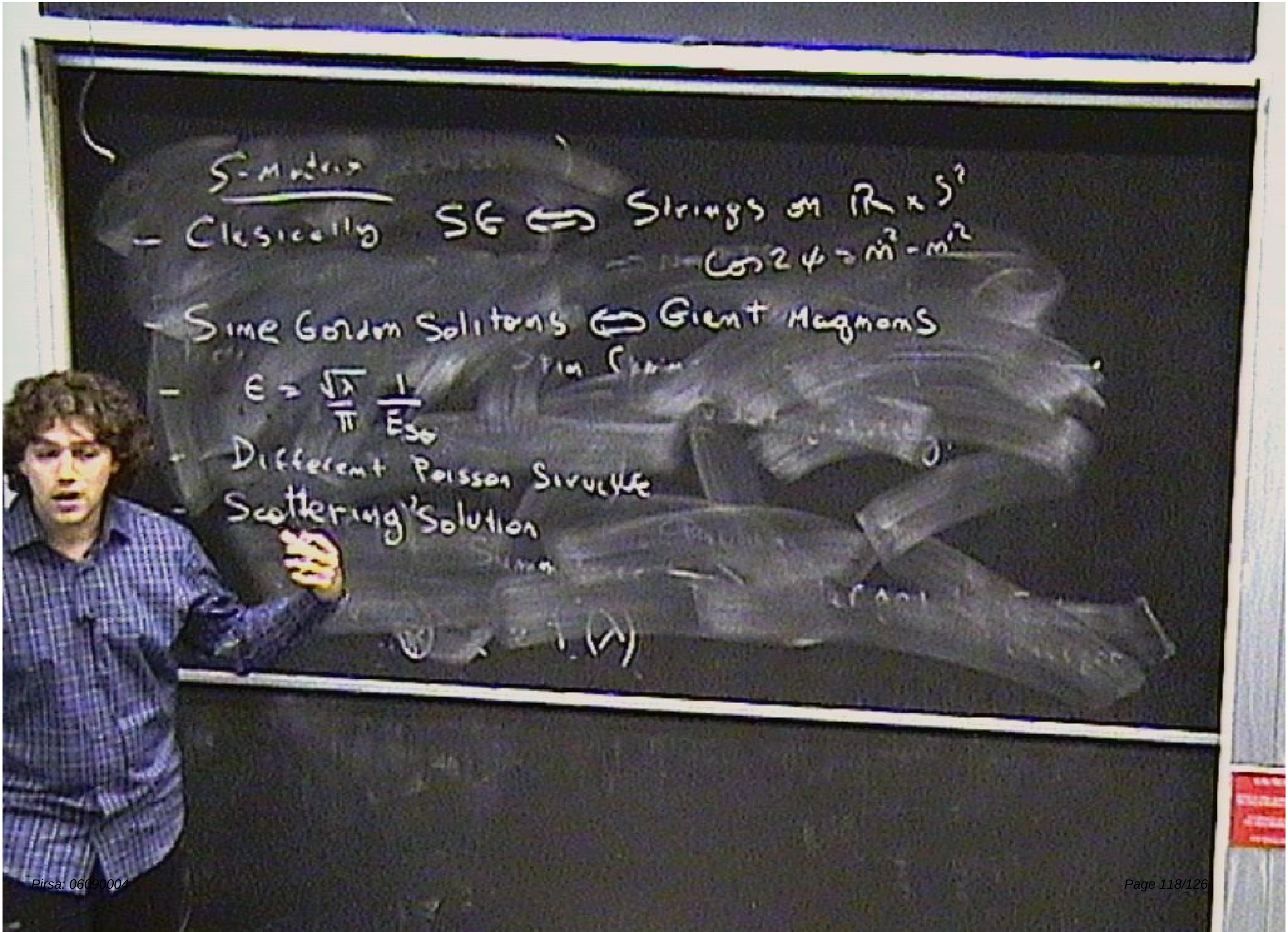
- Classically SG $\Longleftrightarrow$ Strings on $\mathbb{R} \times S^3$

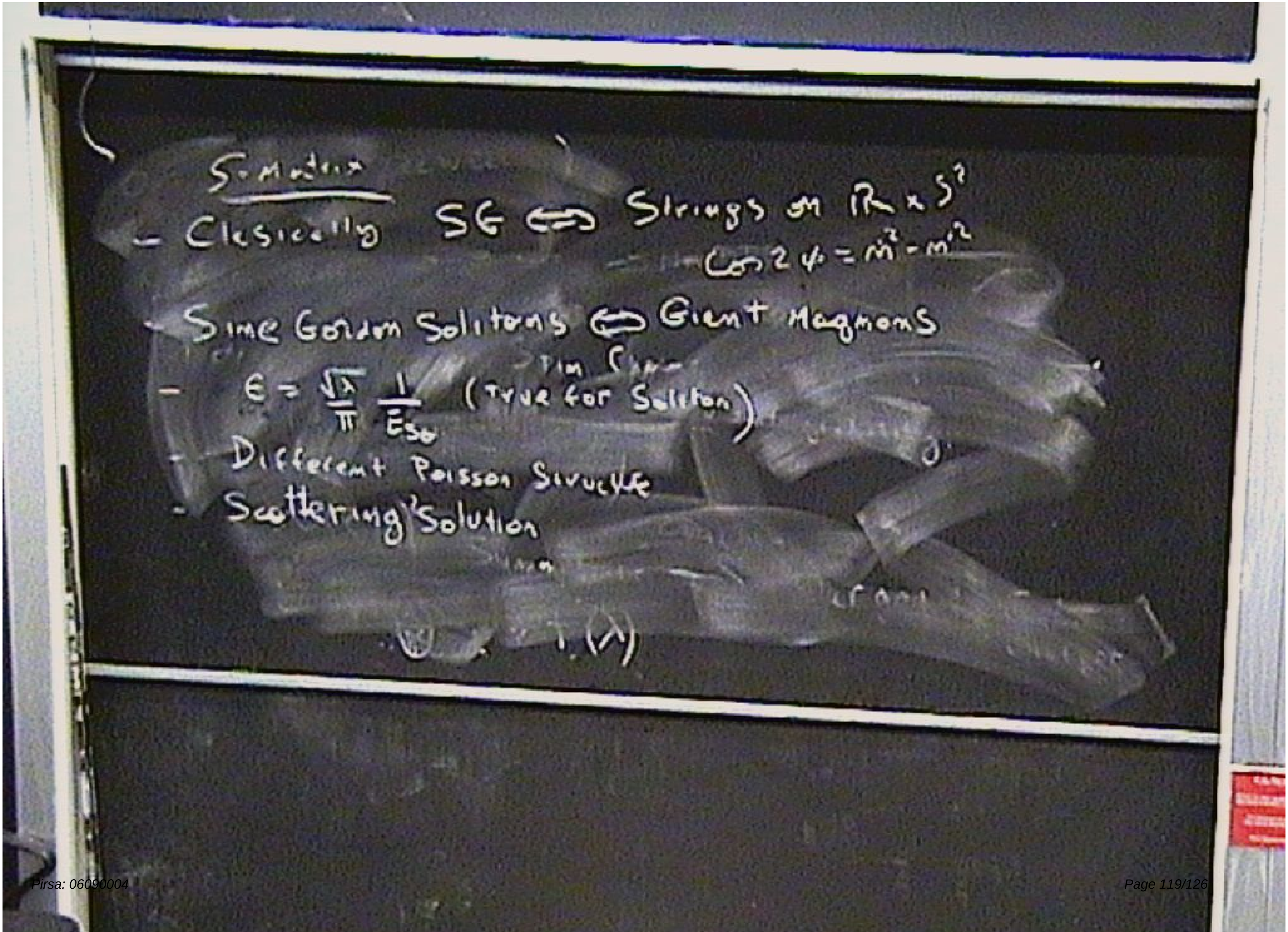
  $\cos 2\phi \to m^2 - m'^2$

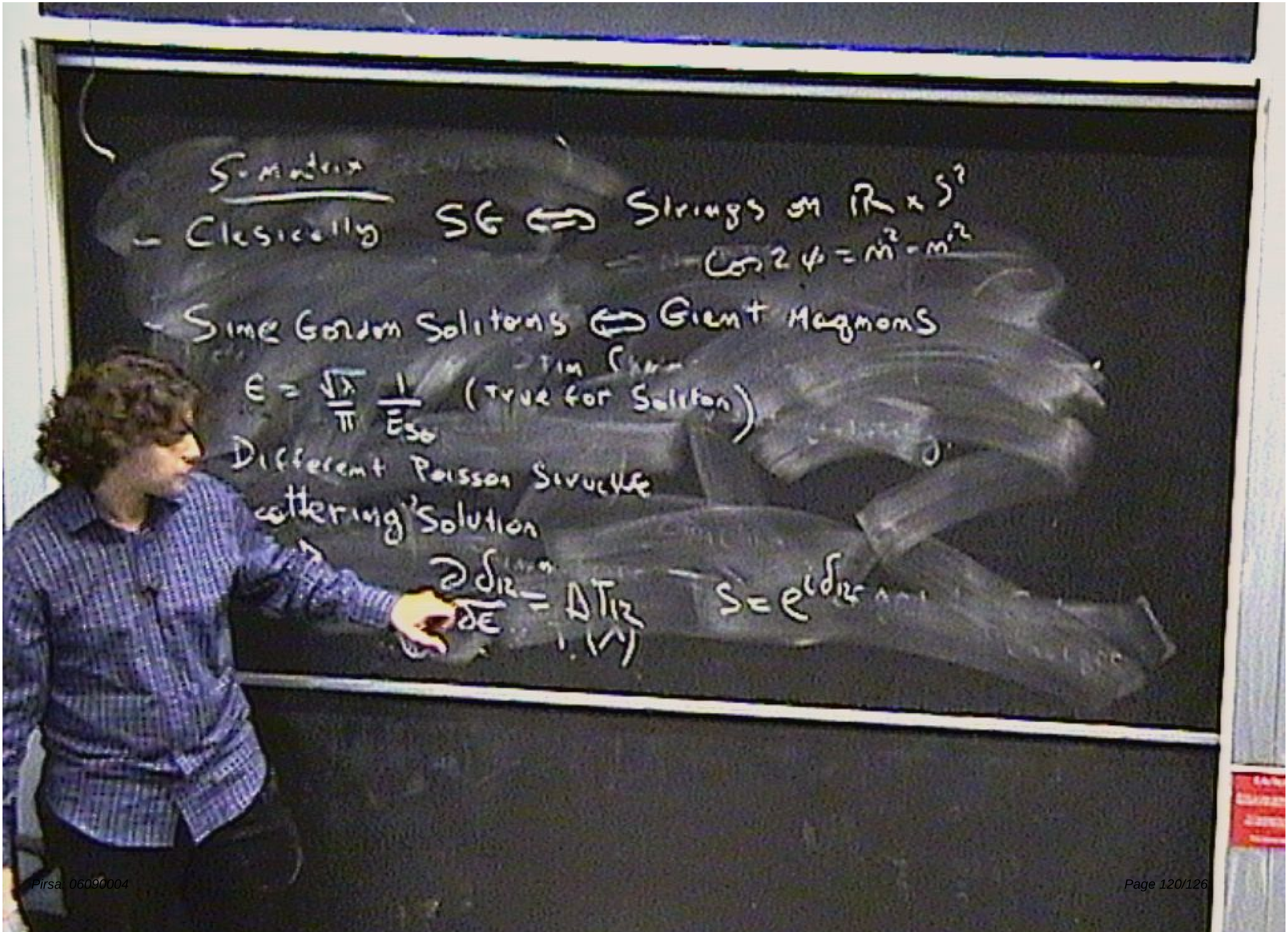
- Sine Gordon Solitons $\Longleftrightarrow$ Giant Magnons

- $\epsilon \to \frac{\sqrt{\lambda}}{\pi} \frac{1}{E_{SG}}$

- Different Poisson Structure


S-Matrix
- Classically SG ⟺ Strings on R x S3
- Sine Gordon Solitons ⟺ Giant Magnons
- Different Poisson Structure
- Scattering Solution


S-Matrix
- Classically SG ⟺ Strings on R × S^3
Cos 2φ = m^2 - m'^2
- Sine Gordon Solitons ⟺ Giant Magnons
- ε = √λ/π 1/E_SG (True for Soliton)
- Different Poisson Structure
- Scattering Solution


S-matrix
- Classically SG ⟺ Strings on R × S³
- Sine Gordon Solitons ⟺ Giant Magnons
(true for Soliton)
Different Poisson Structure
cattering Solution
S = e^{iδ_{12}}

S-Matrix

- Classically $SG \Leftrightarrow$ Strings on $\mathbb{R} \times S^2$

  $\cos 2\phi = m^2 - m'^2$

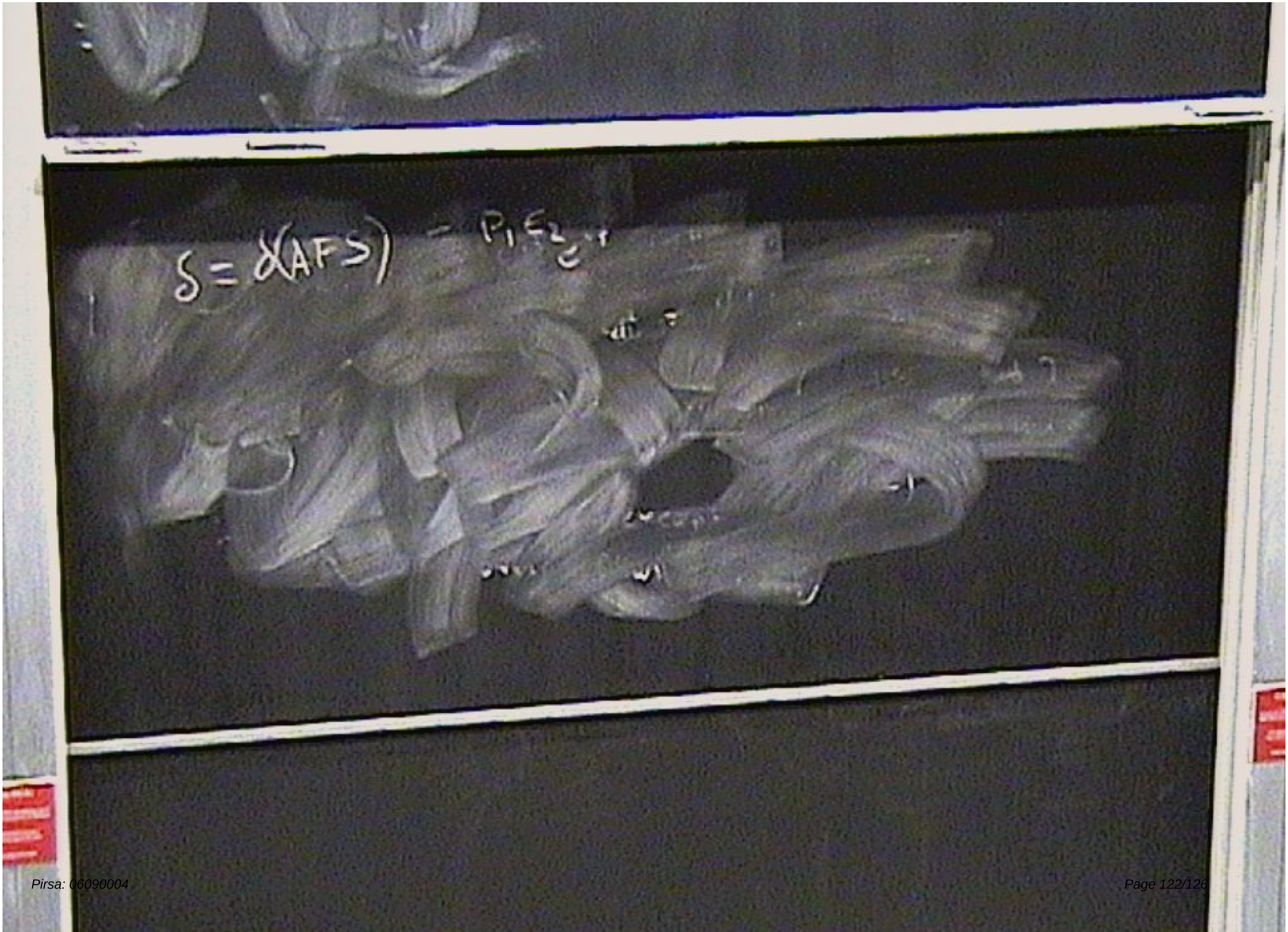
- Sine Gordon Solitons $\Leftrightarrow$ Giant Magnons

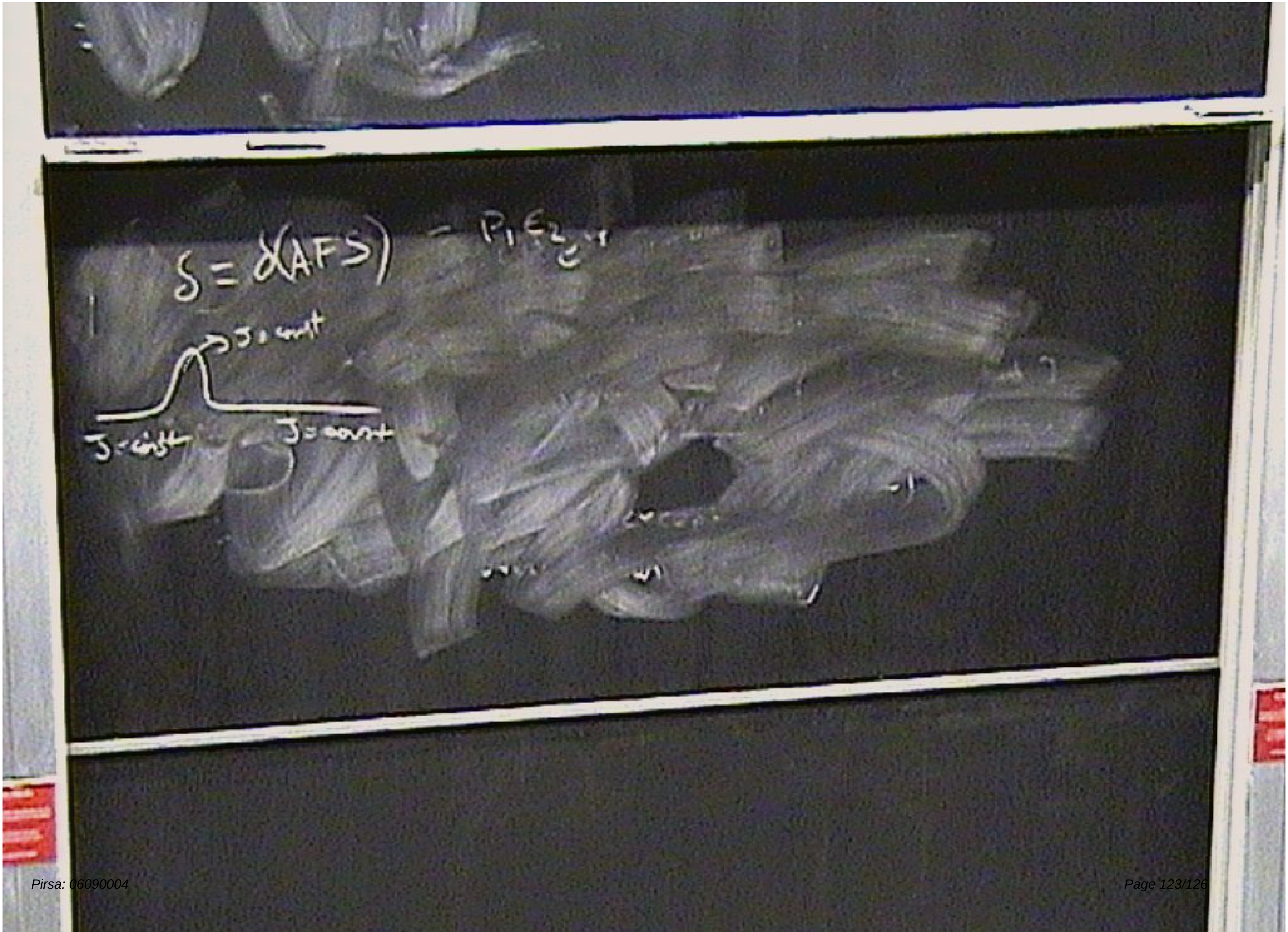
- $\epsilon = \frac{\sqrt{\lambda}}{\pi} \frac{1}{E_{SG}}$ (True for Soliton)

- Different Poisson Structure

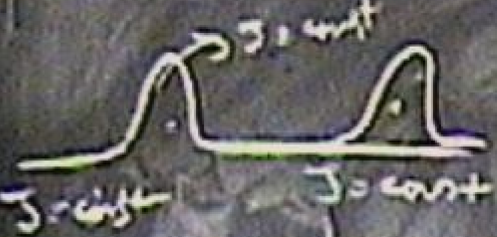
- Scattering Solution

- WKB $\frac{\partial \delta_{12}}{\partial E} = \Delta T_{12}$ $\quad S = e^{i\delta_{12}}$


δ = δ(AFS)


δ = δ(AFS)
J = const
J = const
J = const

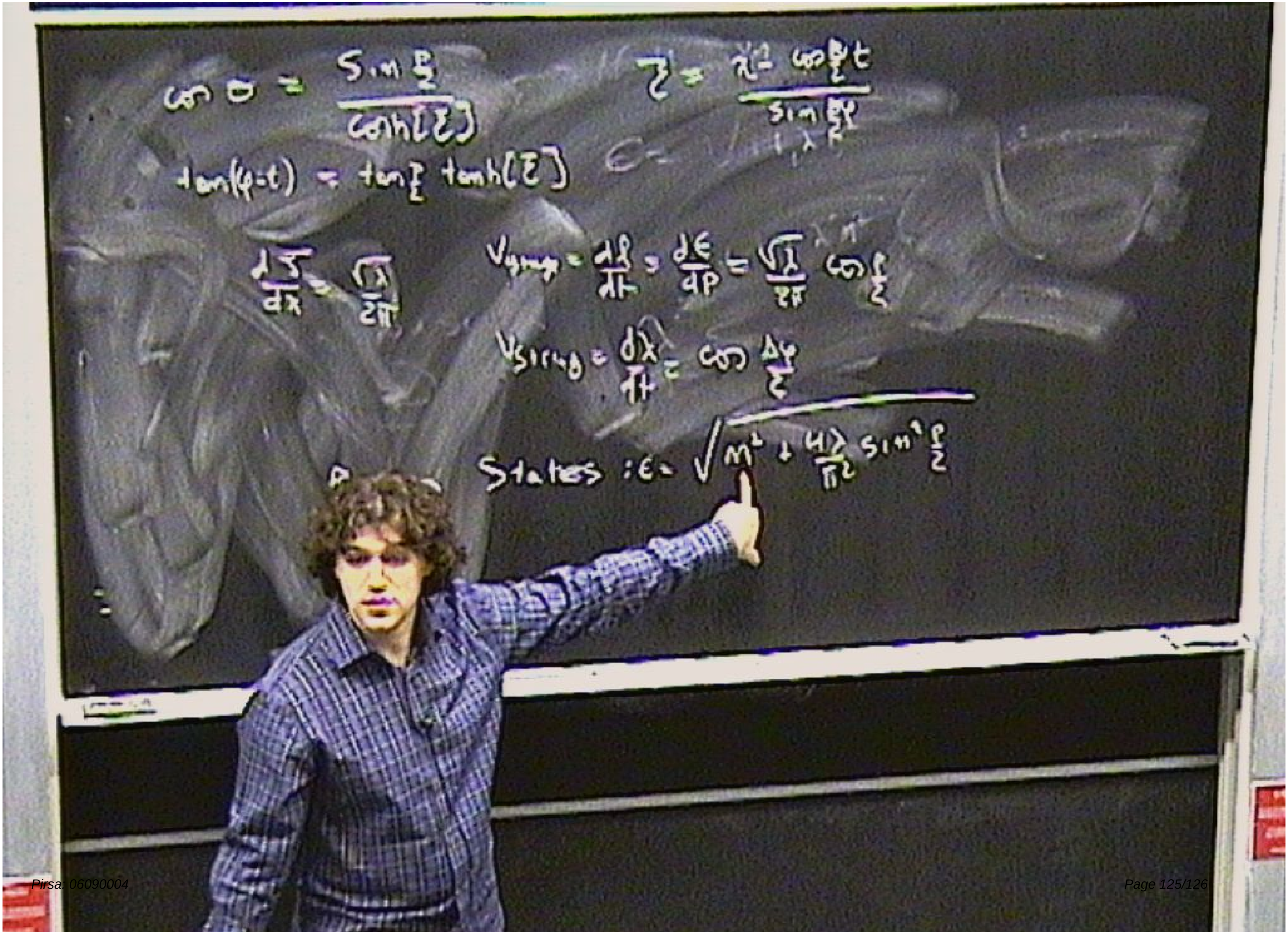
$$\delta = \delta(AFS) \quad - P_1 \epsilon_2 \ldots$$



$$\Delta l = \int dx \frac{dJ}{dx} = \int dx \frac{dE}{dx} - \left(\frac{dE}{dx} - \frac{dJ}{dx}\right)$$

$$= \frac{2\pi}{\sqrt{\lambda}} \Delta x - \epsilon$$

$$e^{iPl} = \sum e^{iP\epsilon}$$


States

$$\cos\theta = \frac{\sin\frac{p}{2}}{\cosh[\xi]} \qquad \xi = \frac{x^1 - \cos\frac{p}{2}\,t}{\sin\frac{p}{2}}$$

$$\tan(\varphi \cdot t) = \tan\frac{p}{2}\,\tanh[\xi]$$

$$\frac{d\xi}{dx} = \frac{\sqrt{\lambda}}{2\pi} \qquad V_{group} = \frac{dx}{dt} = \frac{dE}{dp} = \frac{\sqrt{\lambda}}{2\pi}\cos\frac{p}{2}$$

$$V_{string} = \frac{dx}{dt} = \cos\frac{\Delta\varphi}{2}$$

Bound States : $E = \sqrt{M^2 + \frac{4\lambda}{\pi^2}\sin^2\frac{p}{2}}$