Title: Algebraic Quantum Gravity

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Abstract: We introduce a new top down approach to canonical quantum gravity, called Algebraic Quantum Gravity (AQG): The quantum kinematics of AQG is determined by an abstract $*-$algebra generated by a countable set of elementary operators labelled by an algebraic graph. The quantum dynamics of AQG is governed by a single Master Constraint operator. While AQG is inspired by Loop Quantum Gravity LQG), it differs drastically from it because in AQG there is fundamentally no topology or differential structure. The missing information about the topology and differential structure of the spacetime manifold as well as about the background metric to be approximated is supplied by coherent states and is therefore only available in the semiclassical sector of the theory. Given such data, the corresponding coherent state defines a sector in the Hilbert space of AQG which can be identified with a usual QFT on the given manifold and background. Thus, AQG contains QFT on all curved spacetimes at once, possibly has something to say about topology change and provides the contact with the familiar low energy physics. We will show that AQG admits a semiclassical limit whose infinitesimal gauge symmetry generators agree with the ones of General Relativity.

Plan of the talk

- Part I: Conceptual Setup of AQG
  - Motivation
  - Status of the Semiclassical Limit of LQG
  - The Master Constraint Programme
  - Differences between LQG and AQG
- Part II: Semiclassical Analysis for AQG
- Part III: Semiclassical Perturbation Theory
- Conclusion and Outlook

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- Is General Relativity contained in the semiclassical sector of LQG?

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Uniqueness Theorem & finite Diffeos

- Use GNS-Construction in order to find representations of C*-algebra of LQG
- Assumption of Diff-invariance leads to uniqueness of representation of *-algebra LQG is based on
- Natural unitary action of Diff(σ) on $\mathcal{H}_{LQG}$
- Action not weakly continuous
- Consequence: Infinites, generators cannot be realised

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$$\hat{U}(\varphi) T_\gamma = T_{\varphi(\gamma)}$$

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$$\lim_{t \to 0} \langle T_\gamma, \hat{U}_t(\varphi) T_{\gamma'} \rangle = \langle T_\gamma, T_{\gamma'} \rangle \quad \forall T_\gamma, T_{\gamma'} \in \mathcal{H}_{LQG}$$

Choose $T_{\gamma'} = T_\gamma$, then $\langle T_\gamma, T_{\varphi_t(\gamma)} \rangle = 0 \neq 1 = \langle T_\gamma, T_\gamma \rangle$

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- Dirac Algebra $\mathfrak{D}$

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- No infinitesimal diff-generators — problem in representing $\mathfrak{D}$ on $\mathcal{H}_{LQG}$
- For first two exponentiated substitutes exist

$$U(\varphi)U(\varphi')U(\varphi)^{-1} = U(\varphi \circ \varphi' \circ \varphi^{-1}) \, , \, U(\varphi)\hat{C}(N)U(\varphi)^{-1} = \hat{C}(N \circ \varphi)$$

- Third relation — structure functions !

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Semiclassical Tools available

- Extremely difficult to define coherent state that approximate graph changing operators well
- Coherent states associated to one fixed graph ... $= 0$ trivially
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- **M** is weighted sum of single constraints

$$\mathbf{M} = \left\{ \int_\sigma d^3x \, \frac{\delta^{jk} C_j C_k + q^{ab} C_a C_b + C^2}{(\sqrt{\det(q)})^3}(x) \right\}$$

- **M** = is equivalent to $C_j = 0$ $C_a = 0$ $C = 0$
- **M** is spatially diffeo-invariant
- Weak Dirac Observables $\{O, \{O, \mathbf{M}\}\} \approx 0$
- Constraint algebra $\mathfrak{M}$ trivial $\{\mathbf{M}, \mathbf{M}\} = 0$
- Various examples: DID to get $\mathcal{H}_{phys}$, finite systems, $SL(2, \mathbb{R})$, free & interacting field theory (Dittrich, Thiemann)

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- Two possible ways of quantising **M**:
- 1.) Graph-changing: **M** spatially diff'-invariant
- Has to be defined on $\mathcal{H}_{Diff}$
- On $\mathcal{H}_{Diff}$ no semiclassical tools available today
  [work in progress Bahr, Neubürger, Thiemann]
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- 1.) Graph-changing: $\widehat{\mathbf{M}}$ spatially diff'-invariant
- Has to be defined on $\mathcal{H}_{\mathrm{Diff}}$
- On $\mathcal{H}_{diff}$ no semiclassical tools available today

  [work in progress Bahr, Meusburger, Thiemann]
- Semiclassical limit cannot be investigated

Master Constraint Programme: Quantisation

- 2.) Graph-non-changing:
  - Can be defined on $\mathcal{H}_{LQG}$
  - Anomalous Hamiltonian in naive discretisation, zero not in $\sigma(\widehat{\mathbf{M}})$
  - $\widehat{\mathbf{M}}' = \widehat{\mathbf{M}} - \hbar\lambda_{min}$ 'normal ordered' [Dittrich, Thiemann]
  - Improve discretisation by lattice QFT techniques already developed
  - Due to weight function infinitesimal Diffeos can be implemented

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Graph-dependence of Semiclassical States

- Existing semiclassical tools in LQG
  - Pure state over single graph
  - Mixed states based on certain class of graphs [Bombelli, Winkler]
- Semiclassical states cannot be semiclassical for all dof of LQG
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Departure from LQG to AQG

- Discard notion of embedded graphs $\rightarrow$ one fundamental infinite (orientated) algebraic graph
  - labelling set consisting of abstract points (vertices) & inform. how many abstract arrows (edges) between points
  - Lost of information about topology & differential structure of spatial manifold σ
  - Algebraic graph can be embedded in all possible ways into σ
  - Embedding arbitrarily dense $\longrightarrow$ continuum limit built in
- All physical (diff-invariant) LQG-operators can be lifted to AQG
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Quantum Kinematics of AQG

- Given an algebraic graph α we associate with each of its edges e an element $A(e)$ of a compact, connected semisimple Lie group G and an element $E(e)$ of its Lie algebra Lie(G)

- Natural representation: infinite tensor product (ITP) Hilbert space

- $A(e)$ multiplication and $E(e)$ derivation operator

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  - Q plays role of coupling constant

$$[A(e), A(e')] = 0$$
$$[E_j(e), A(e')] = i\hbar Q^2 \delta_{e,e'} \tau_j / 2 A(e)$$
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Quantum Kinematics of AQG: ITP Hilbert space

- Properties of ITP:
  $\mathcal{H}^{\otimes} = span\{\otimes_e f_e; 0 < ||\otimes_e f_n|| := \prod_e ||f_n||_e < \infty\}$
- Decomposes into uncountably infinitely sum over separable $\mathcal{H}_\Omega$
- $\Omega = \otimes_{n=1}^{\infty} f_n$ with wlg $||f_n|| = 1$ and therefore $||\Omega|| = 1$
- $\mathcal{D} = \text{Poly}(A(e), E(e))\Omega$ dense in $\mathcal{H}_\Omega$
- $\mathcal{H}_{LQG} \longleftrightarrow \Omega = \otimes_{n=1}^{\infty} 1$
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Summary of Differences between LQG and AQG

Object	LQG	AQG
Topology	must be provided	absent
Differentiable structure	must be provided	absent
Hilbert space	$\mathcal{H}_{LQG} := \mathcal{H}_{AIL}$	$\mathcal{H}_{AQG} := \mathcal{H}^{\otimes}$
Separability	non – separable	non – separable
graphs	embedded	algebraic
$\#$ graphs	uncountably infinite	one
Structure of graphs	finite	countably infinite
Generating set of $^*-$algebra $\mathfrak{A}$	uncountably infinite	countably infinite

Semiclassical States in AQG

- In order to derive semiclassical limit we must provide following data
  - A 3-manifold σ
  - Initial data m (or equivalently a point in phase space)
  - An embedding of the graph (and a graph dual to it) into σ
- Out of these data coherent states can be constructed

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Semiclassical Analysis of $\widehat{\mathbf{M}}$

- We investigated the semiclassical limit of $\widehat{\mathbf{M}}$ wrt coherent states associated to a cubic graph
    - Substitution of $U(1)^3$ for $SU(2)$ — semiclassical perturbation theory
- For the considered algebraic graph of cubic symmetry

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$$\widehat{\mathbf{M}} = \sum_{v\in V(\gamma)} \widehat{\mathbf{M}}_v, \quad \widehat{\mathbf{M}}_v = \sum_{\ell=0}^{3} \widehat{C}^\dagger_{\ell,v}\widehat{C}_{\ell,v}$$

$$\widehat{C}_{0,v} = \sum_{IJK}\sum_{\sigma=+,-}\sum_{\sigma'=+,-}\sum_{\sigma''=+,-} \frac{i}{2}\epsilon^{IJK}\widehat{h}_{\alpha_{I\sigma' J\sigma''}\ell v}\widehat{h}_{K\sigma\ell v}\frac{1}{i\hbar}\left[\widehat{h}^{-1}_{K\sigma\ell v}, \widehat{V}^{\frac{1}{2}}_{\gamma,v}\right]$$

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Semiclassical Analysis of $\widehat{\mathbf{M}}$

- Result in leading order

$$\frac{\langle \Psi^t_{\alpha,m} \mid \widehat{\mathbf{M}} \mid \Psi^t_{\alpha,m} \rangle}{||\Psi^t_{\alpha,m}||^2} = \sum_{v \in V(\alpha)} \frac{\langle \psi_{m,\sigma,X} \mid \widehat{\mathbf{M}}_v \mid \psi_{m,\sigma,X} \rangle}{||\psi_{m,\sigma,X}||^2} \underset{\substack{\lim \\ t \longrightarrow 0}}{=} \mathbf{M}^{cubic}[m] \underset{\substack{\lim \\ \epsilon \longrightarrow 0}}{=} \mathbf{M}[m]$$

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- NLO: quantum fluctuations are finite

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Problem

- Semiclassical Calculations in $SU(2)$:
  We want to calculate
  $\langle \psi, \widehat{\mathbf{M}}_v \, \psi \rangle = \langle h_\alpha h[h^{-1}, \sqrt{V_v}]\psi, h_\alpha h[h^{-1}, \sqrt{V_v}]\psi \rangle$

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- Volume operator

$$V_v := \ell_p^3 \sqrt{\left| \frac{1}{48} \sum_{e_1 \cap e_2 \cap e_3 = v} \epsilon_v(e_1, e_2, e_3) \epsilon^{ijk} E_i(e_1) E_j(e_2) E_k(e_3) \right|}$$

- $V_v = \sqrt[4]{Q_v^2}$ and thus $\sqrt{V_v} = (Q_v^2)^{\frac{1}{8}}$ general $F_i(V_v) = (Q_v^2)^q$
- Problem: We cannot calculate $(Q_v^2)^q$ analytically

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- Power expansion of $t \mapsto f(t) = (1+t)^q, \; -1 \leq t < \infty$

$$f(t) := 1 + \sum_{n=1}^{\infty} \begin{pmatrix} q \\ n \end{pmatrix} t^n, \quad \begin{pmatrix} q \\ n \end{pmatrix} = (-1)^{n+1}\frac{q(1-q)..(n-1+q)}{n!}$$

Spectral Theorem

- Use the spectral theorem for operator valued function $f_l(x_l)$

$$
\begin{aligned}
f_l(x_l) &= \int_{-1}^{\infty} f_l(t)dE_l(t) = \int_{-1}^{\infty}[1+\sum_{n=1}^{\infty}\binom{q}{n}t^n]\,dE_l(t)\\
&= [1+\sum_{n=1}^{\infty}\binom{q}{n}x_l^n]
\end{aligned}
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where E_l is the projection valued measure associated with x_l.

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- Of course, the second equality is wrong if $t \in (-1, 1)$!
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- For each $k \geq 0$ there exists $0 < \beta_k < \infty$ such that

$$f_l^- := f_{2k+1}(t) - \beta_k t^{2k+2} \leq f(t) \leq f_{2k+1}(t) =: f_l^+$$

  where $f_k(t)$ denotes the partial Taylor series of $f(t) = (1+t)^q$, $0 < q \leq \frac{1}{4}$ up to to order t^k.

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- AQG provides a platform to analyse the dynamics of the theory semiclassically
- $\widehat{\mathbf{M}}$ reproduces in LO the correct infinitesimal generators of GR
- Quantum fluctuation are finite: semiclassical perturbation theory
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