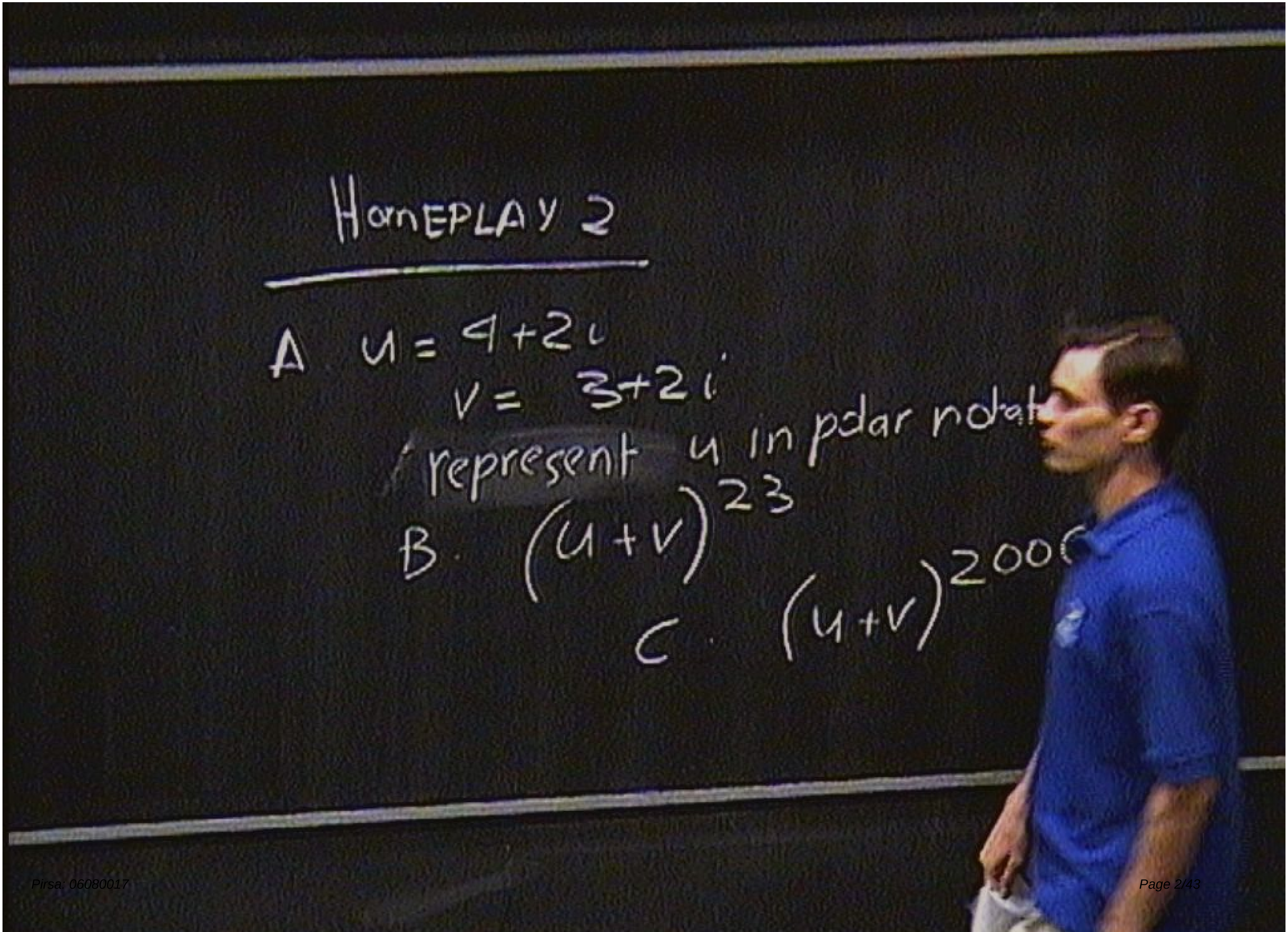
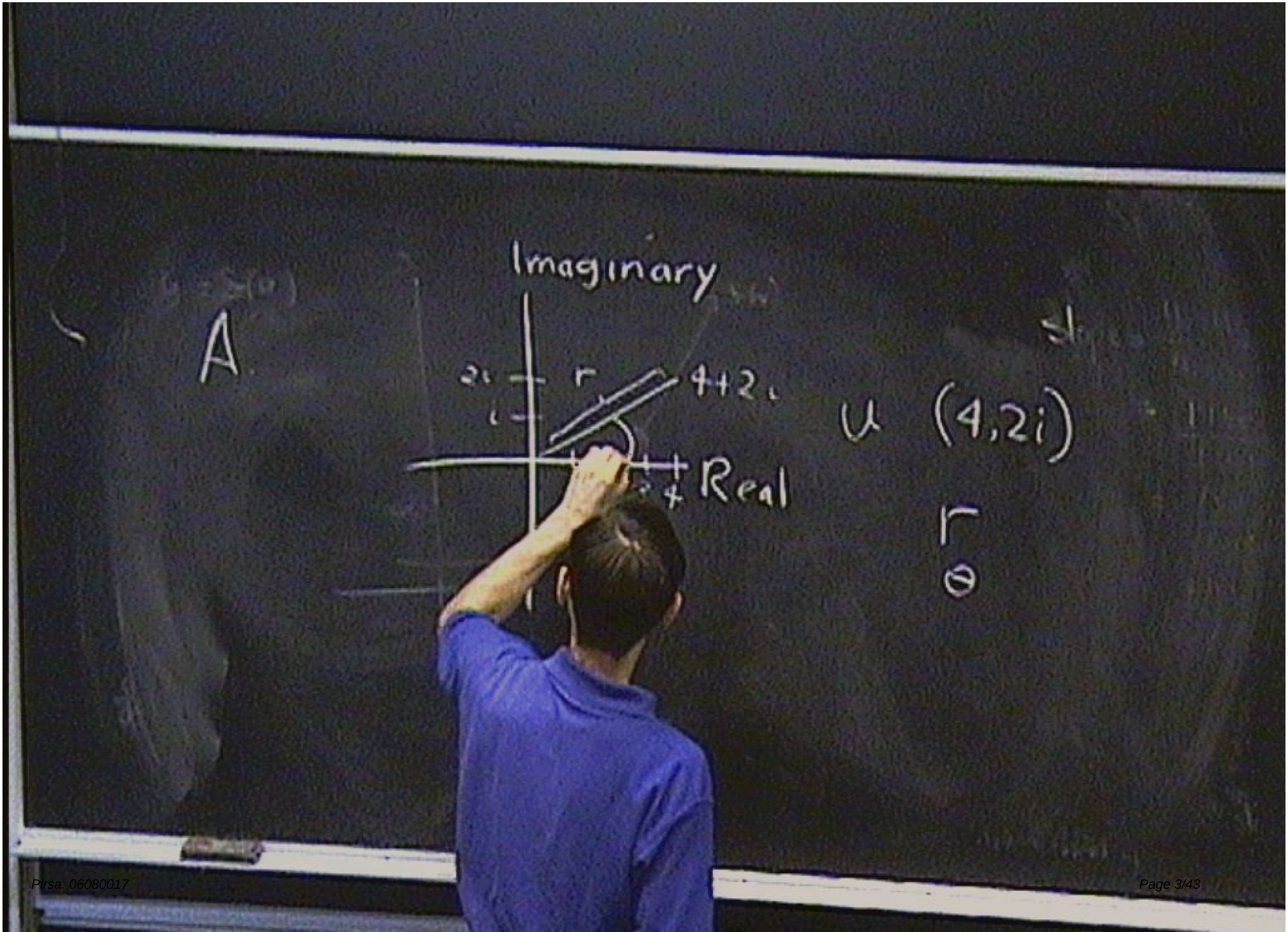
Title: Math Tutorial - Part 5

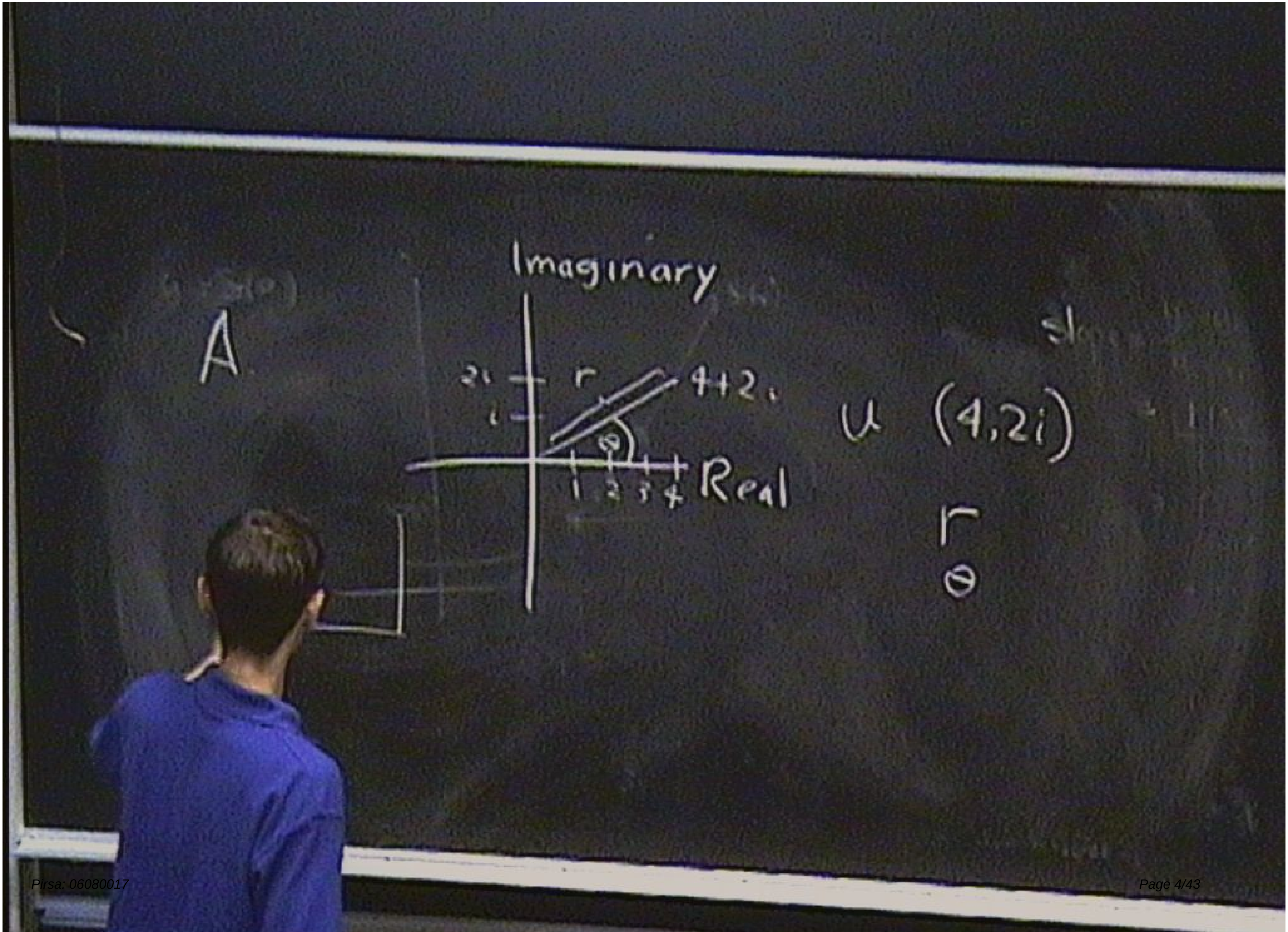
Date: Aug 08, 2006 01:00 PM

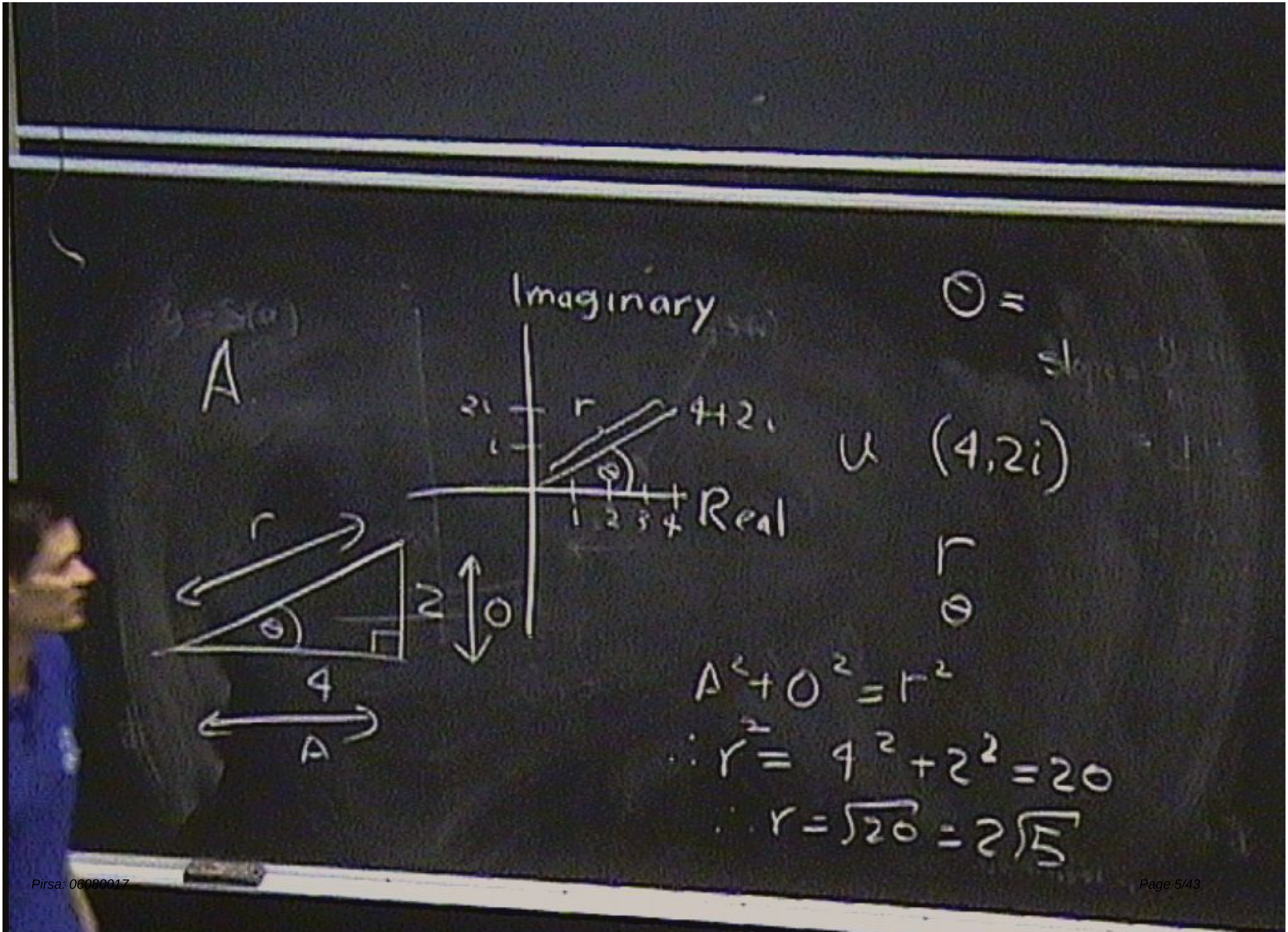
URL: http://pirsa.org/06080017

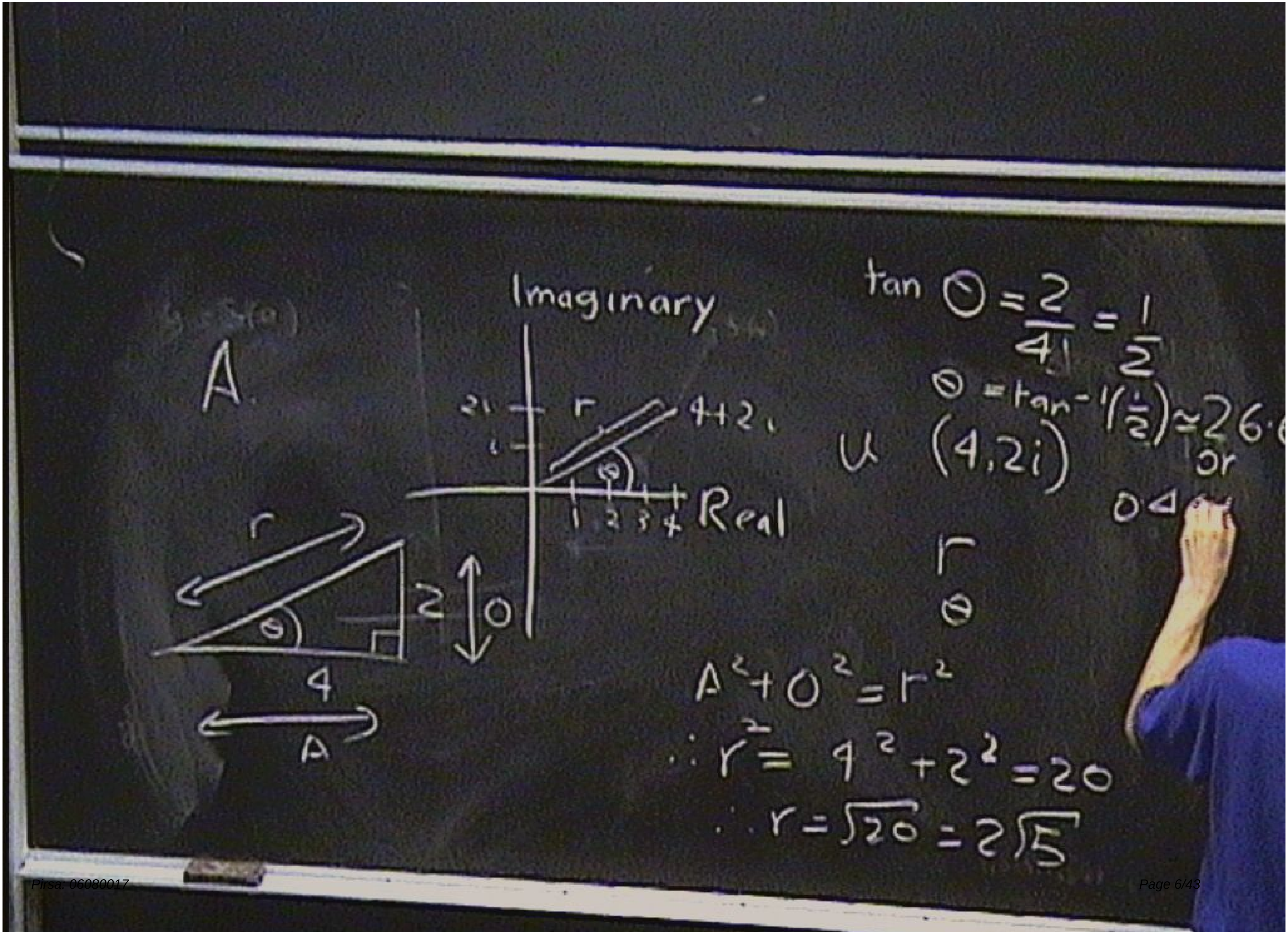
Abstract:

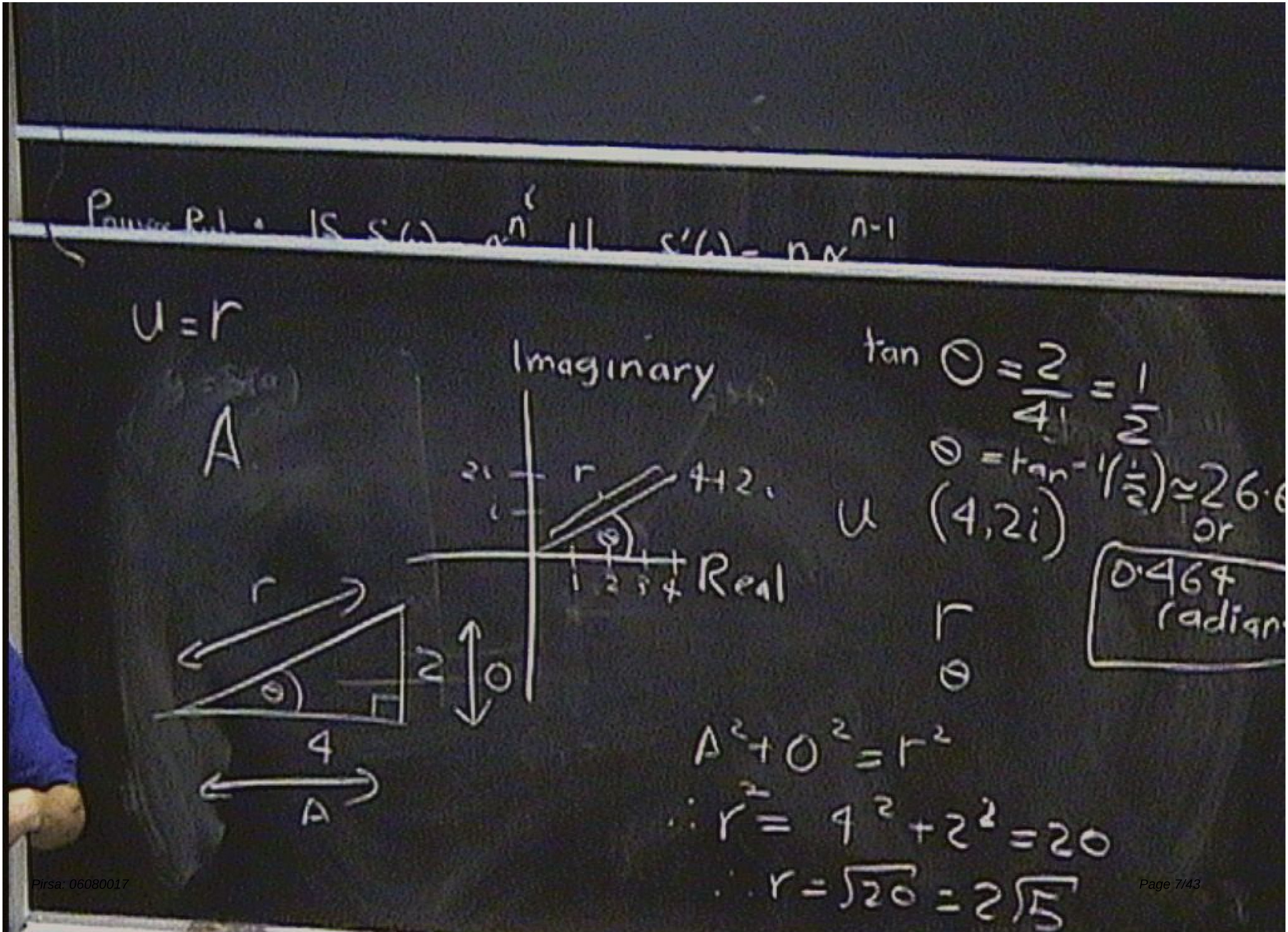

HomePLAY 2
A u = 4+2i
v = 3+2i
represent u in polar notat
B. (u+v)^23
C. (u+v)^200


Imaginary
A
2i
i
r
4+2i
Real
u
(4,2i)
r
θ

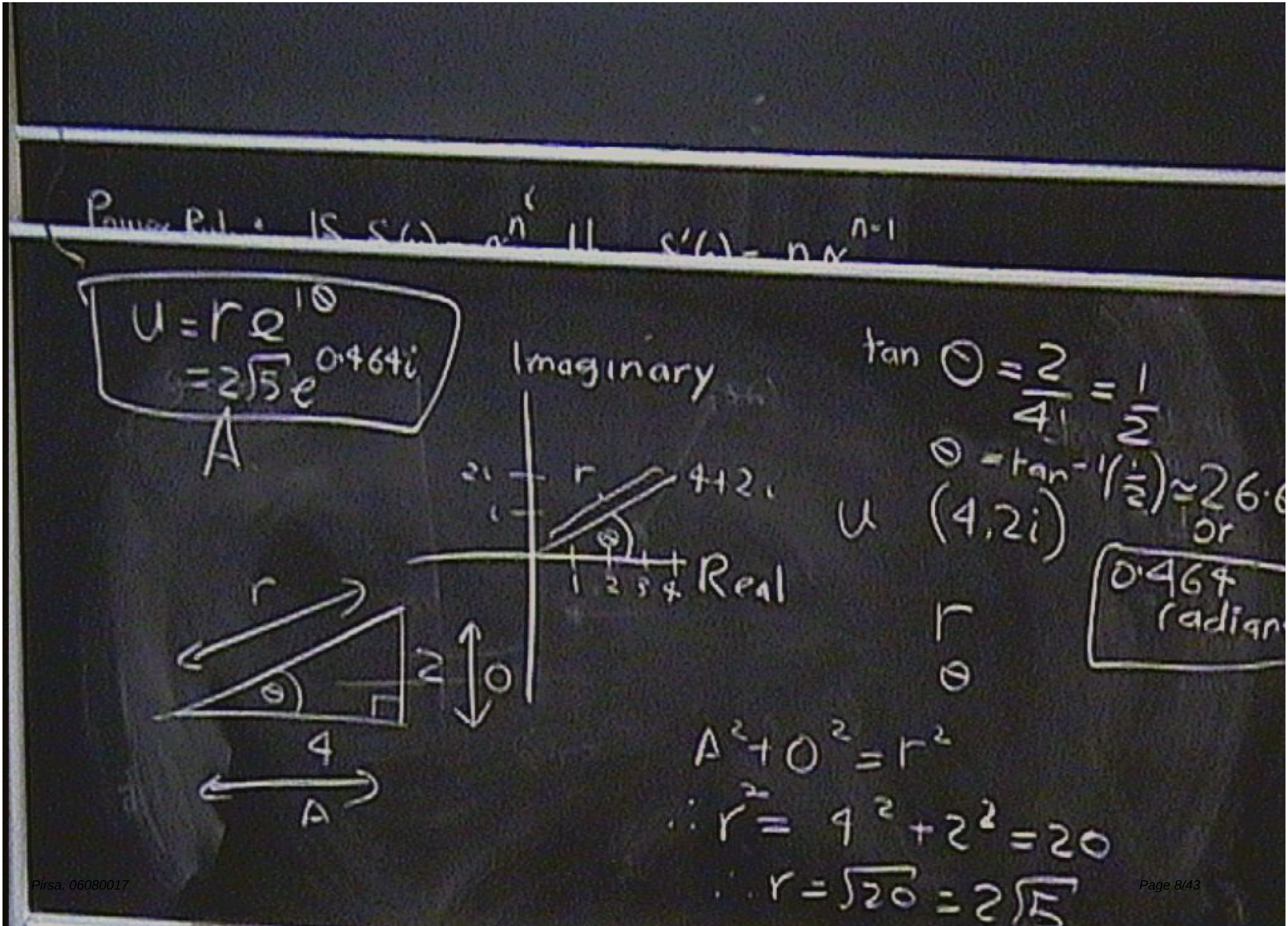

Imaginary
A
2i
i
r
4+2i
θ
1 2 3 4
Real
u (4,2i)
r
θ


Imaginary
Real
4+2i
2i
i
1 2 3 4
r
θ
A
4
2
O
θ =
u (4,2i)
r
θ
A²+O²=r²
∴ r² = 4²+2²=20
∴ r=√20 = 2√5

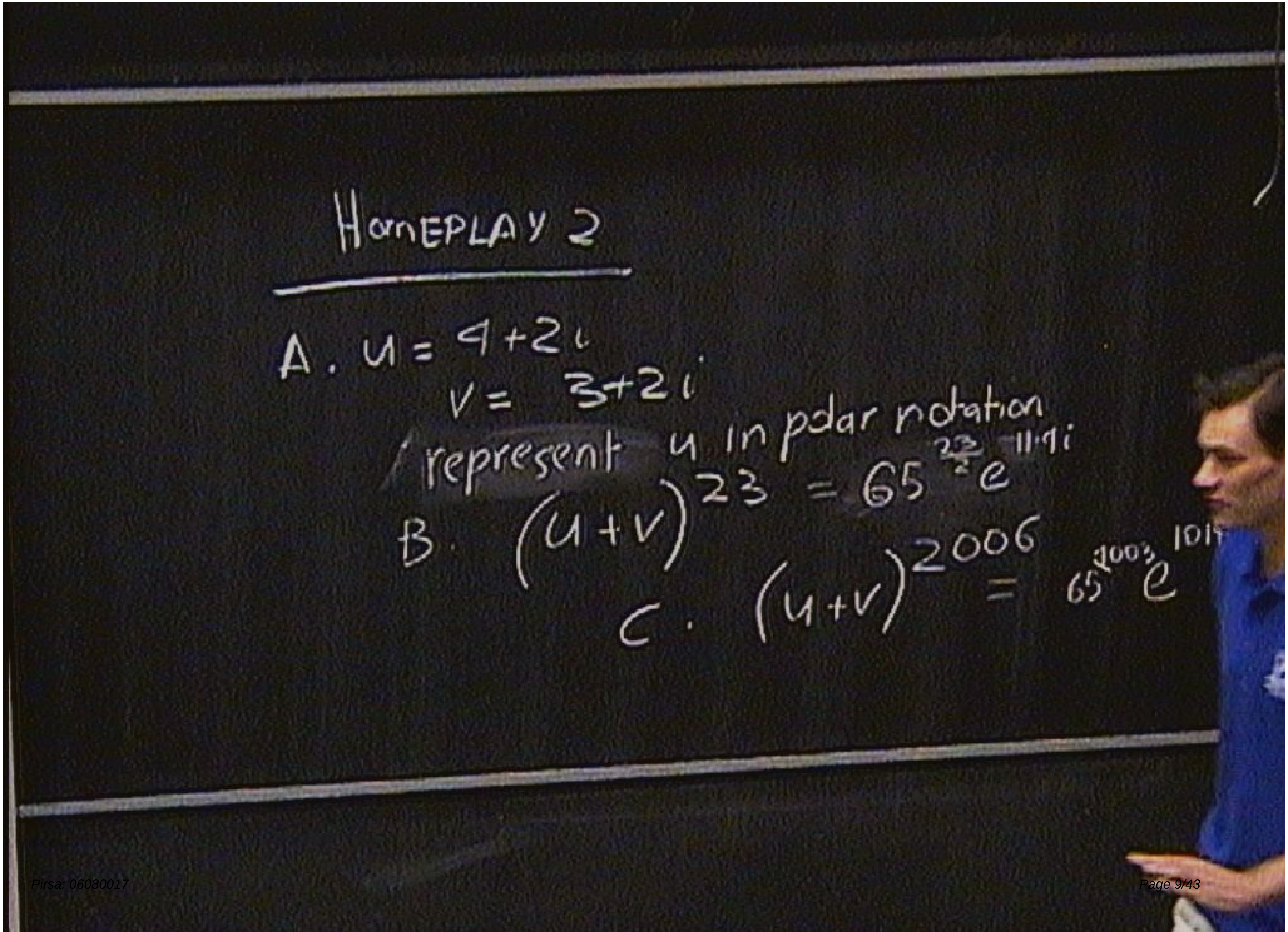

Imaginary
Real
A
r
2
O
4
A
2i
i
r
4+2i
1 2 3 4
tan θ = 2/4 = 1/2
θ = tan⁻¹(1/2) ≈ 26.6
or
u (4,2i)
r
θ
A² + O² = r²
∴ r² = 4² + 2² = 20
∴ r = √20 = 2√5



u = r
A
Imaginary
Real
r
4+2i
2i
i
1 2 3 4
2
O
4
A
tan θ = 2/4 = 1/2
θ = tan⁻¹(1/2) ≈ 26.6
or
0.464 radians
u (4,2i)
r
θ
A² + O² = r²
∴ r² = 4² + 2² = 20
∴ r = √20 = 2√5



u = re^{iθ}
= 2√5 e^{0.464i}
A
Imaginary
tan θ = 2/4 = 1/2
θ = tan⁻¹(1/2) ≈ 26.6
or
0.464 radian
4+2i
r
2i
i
1 2 3 4
Real
u (4, 2i)
r
θ
2
0
4
A
A² + O² = r²
∴ r² = 4² + 2² = 20
∴ r = √20 = 2√5


HOMEPLAY 2
A. $u = 4+2i$
$v = 3+2i$
represent u in polar notation
B. $(u+v)^{23} = 65^{\frac{23}{2}} e^{11.1i}$
C. $(u+v)^{2006} = 65^{1003} e^{1014}$

HOMEPLAY 2

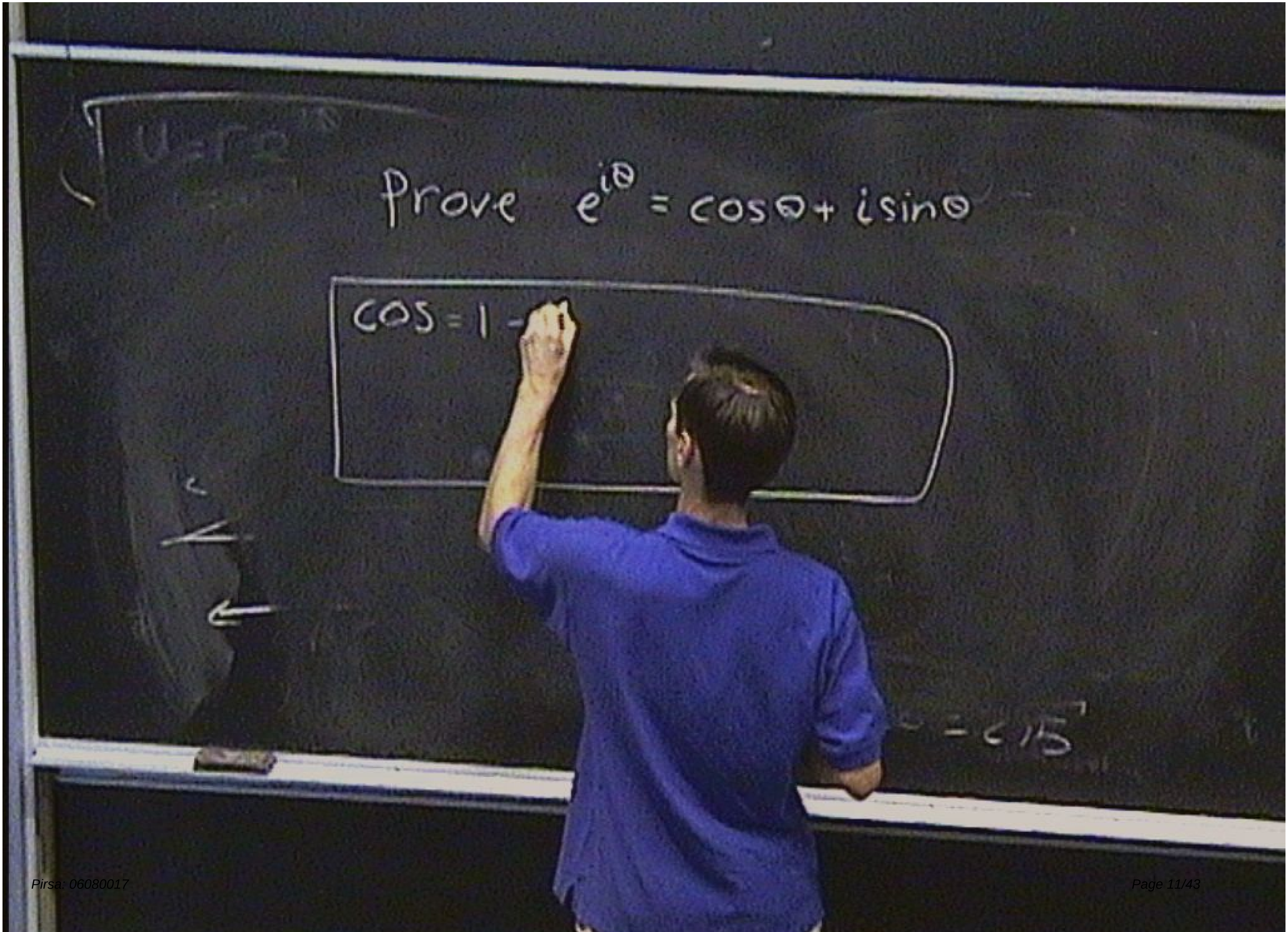
A. $u = 4 + 2i$

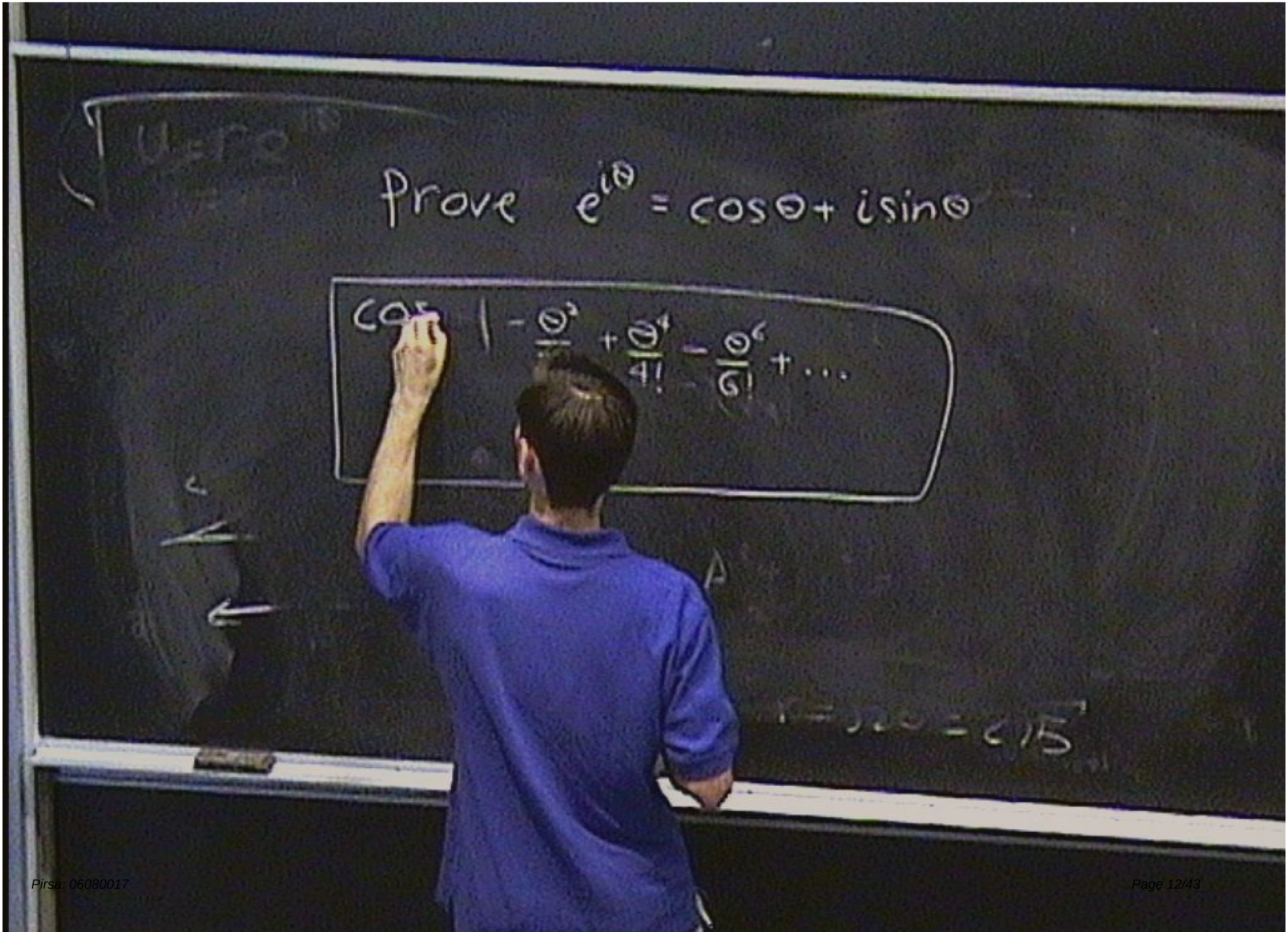
$v = 3 + 2i$

represent u in polar notation

B. $(u+v)^{23} = 65^{23/2} e^{11.9i}$

C. $(u+v)^{2006} = 65^{1003} e^{1014i}$


Prove $e^{i\theta} = \cos\theta + i\sin\theta$
cos = 1 -


Prove $e^{i\theta} = \cos\theta + i\sin\theta$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned} \cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \end{aligned}}$$

A

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned} \cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \end{aligned}}$$

$\theta \ll 1$

e^x : If $w(x) = e^x$, then $w'(x) = e^x$

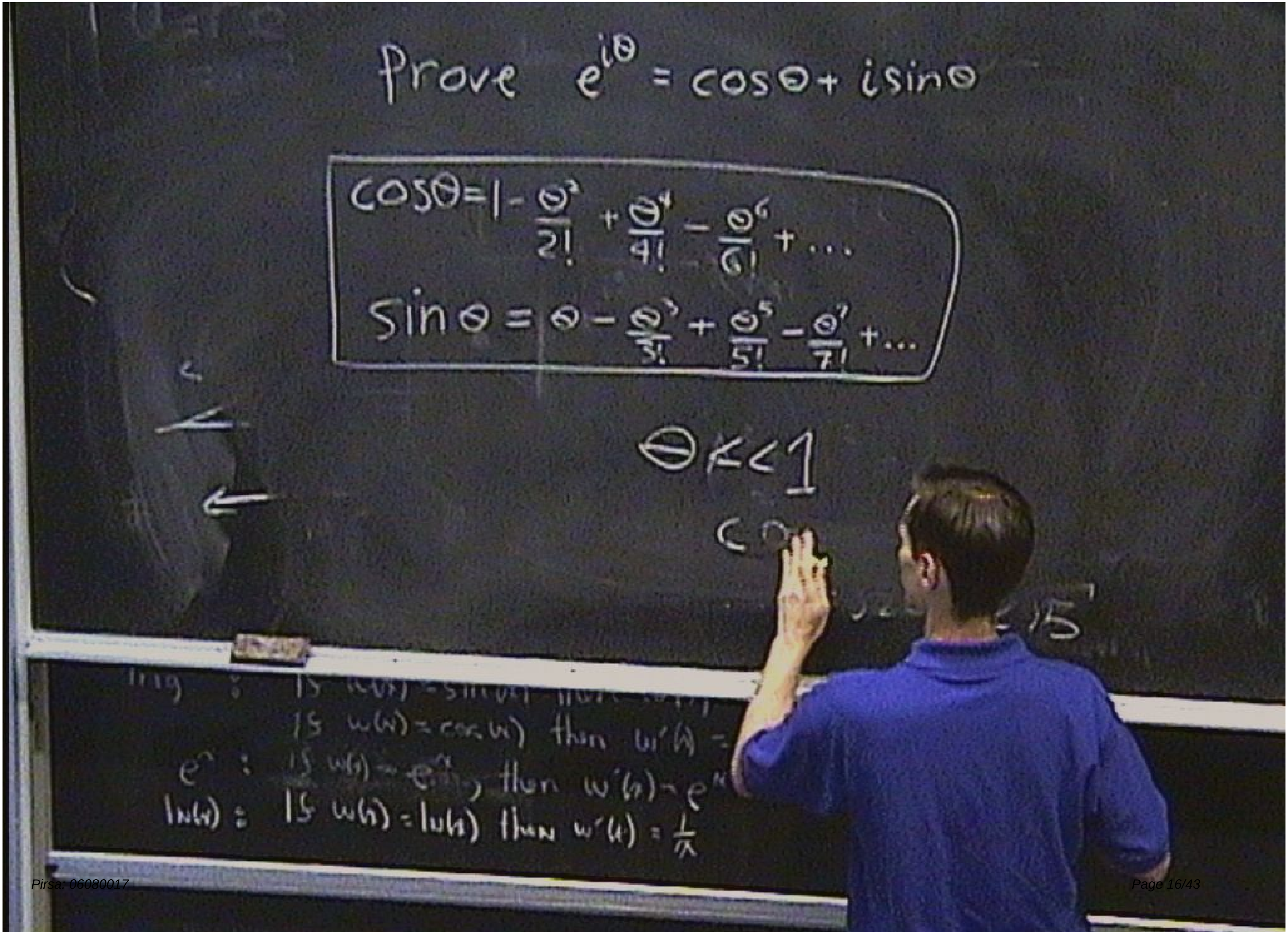
$\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned} \cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \end{aligned}}$$

$\theta \ll 1$

$\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$


Prove $e^{i\theta} = \cos\theta + i\sin\theta$
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$
$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$
$\theta \ll 1$
$\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned} \cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \end{aligned}}$$

$$\theta \ll 1$$

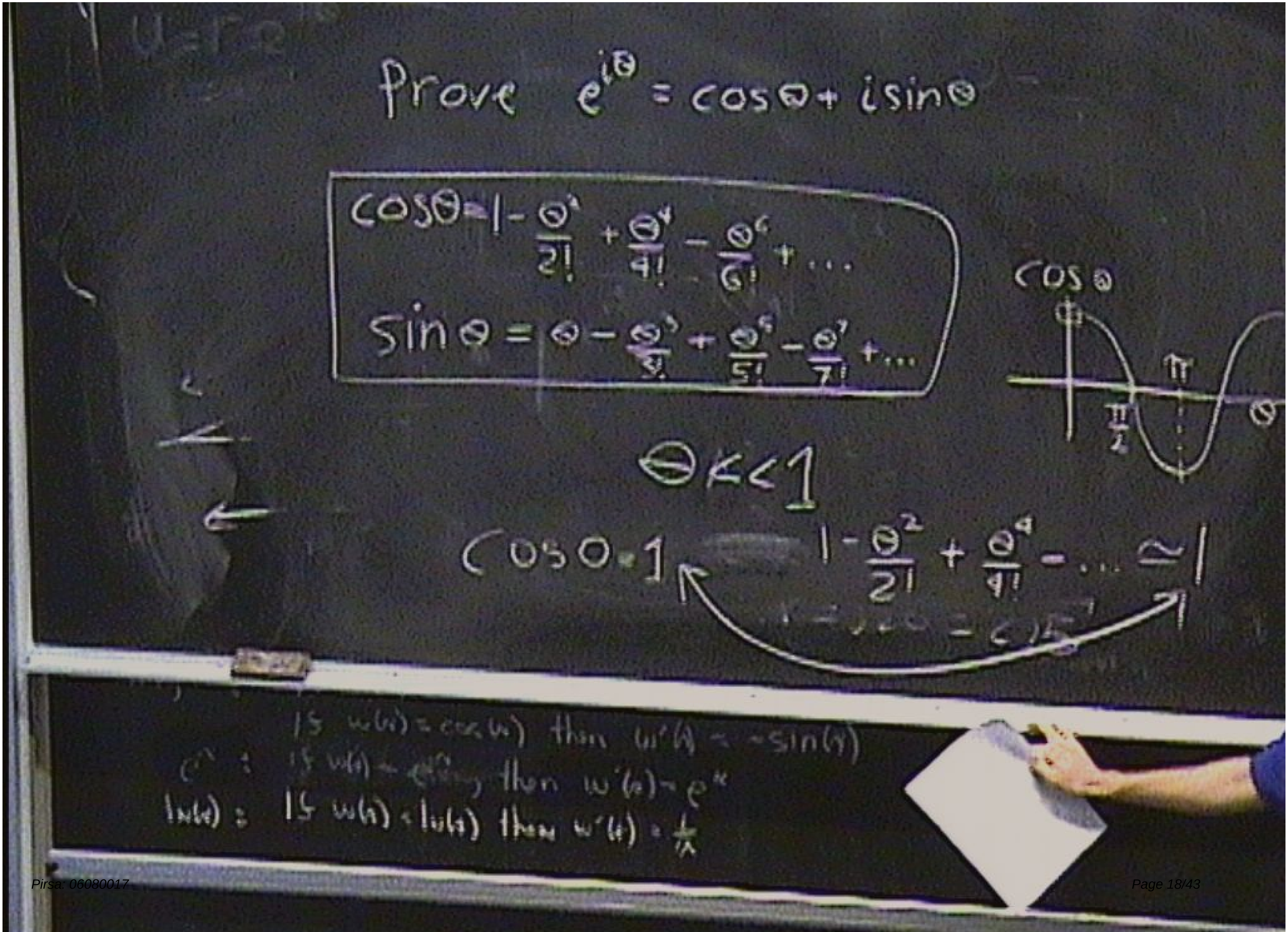
$$\cos 0 = 1 \qquad 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \simeq 1$$

Trig : If $w(x) = \sin(x)$ then $w'(x) = \cos(x)$

If $w(x) = \cos(x)$ then $w'(x) = -\sin(x)$

e^x : If $w(x) = e^x$, then $w'(x) = e^x$

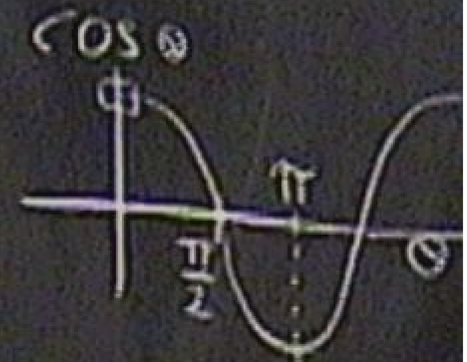
$\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$



Prove $e^{i\theta} = \cos\theta + i\sin\theta$
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$
$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$
$\cos\theta$
π
$\frac{\pi}{2}$
θ
$\theta \ll 1$
$\cos\theta \approx 1$
$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \simeq 1$
If $w(t) = \cos(t)$ then $w'(t) = -\sin(t)$
If $w(t) = e^t$ then $w'(t) = e^t$
$\ln(t)$: If $w(t) = \ln(t)$ then $w'(t) = \frac{1}{t}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned}\cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots\end{aligned}}$$



$$\boxed{\sin\theta \simeq \theta} \qquad \boxed{\theta \ll 1}$$

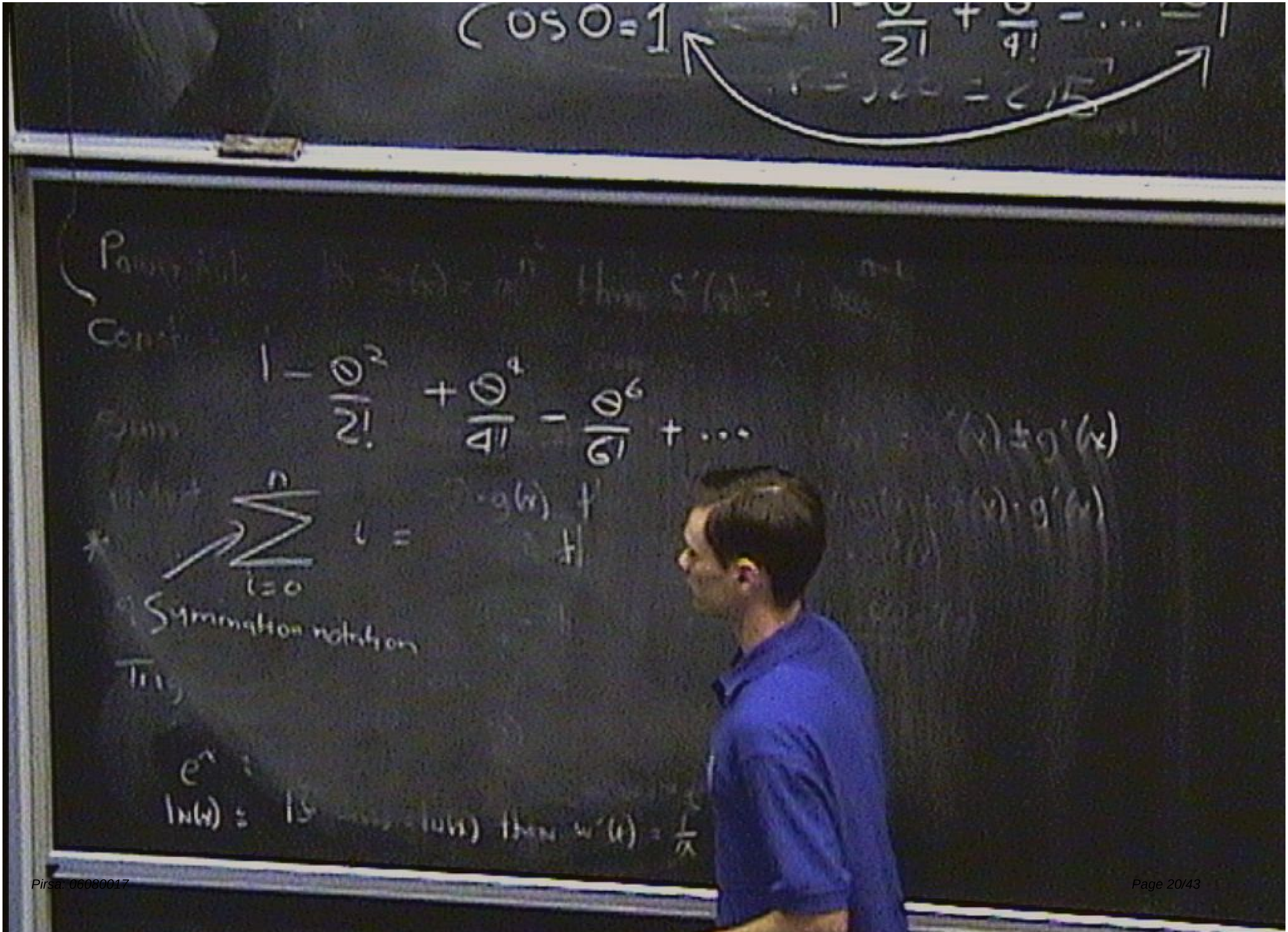
$\sin 0 = 0$

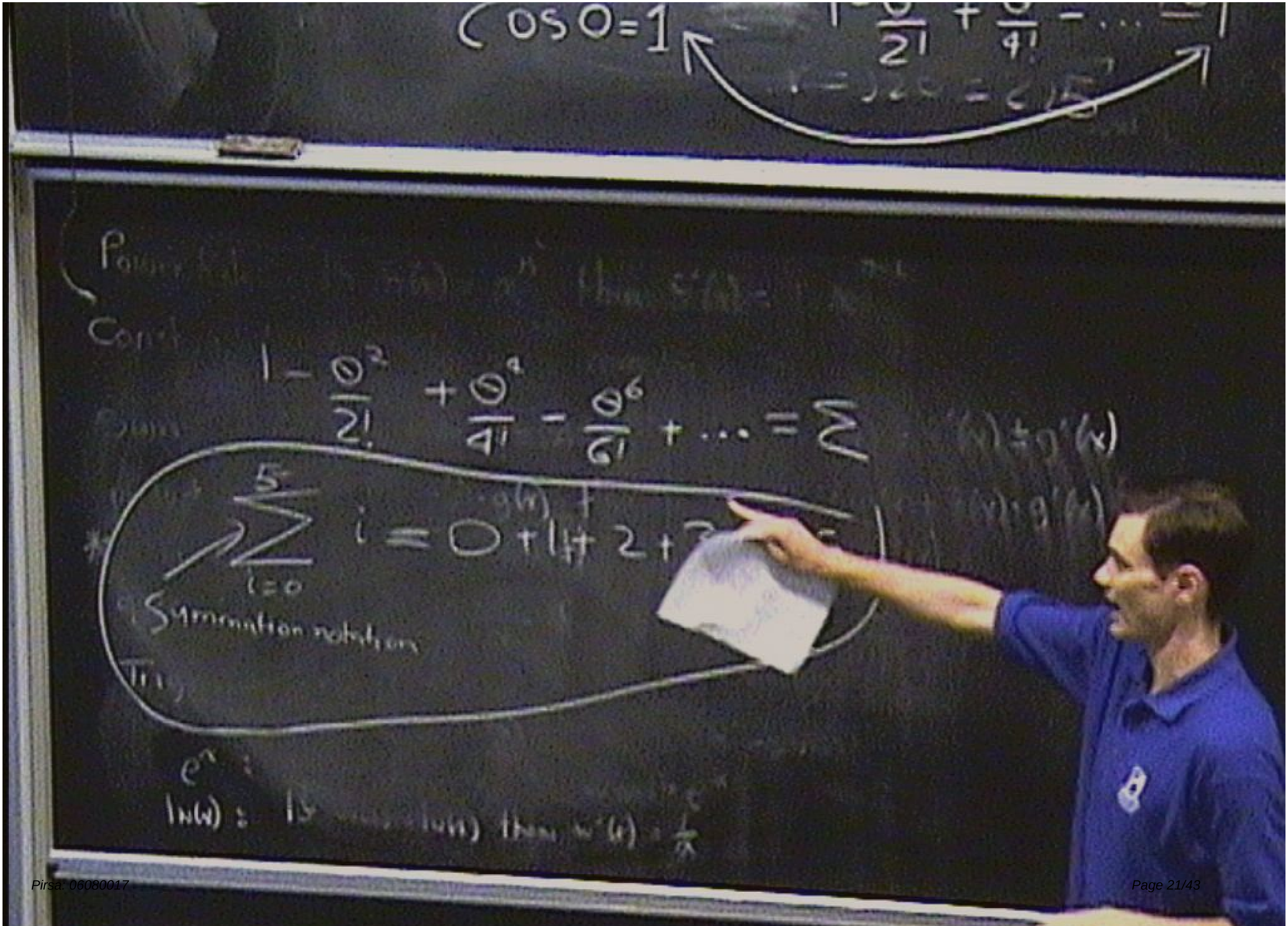
$\cos 0 = 1$ $\quad 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \simeq 1$

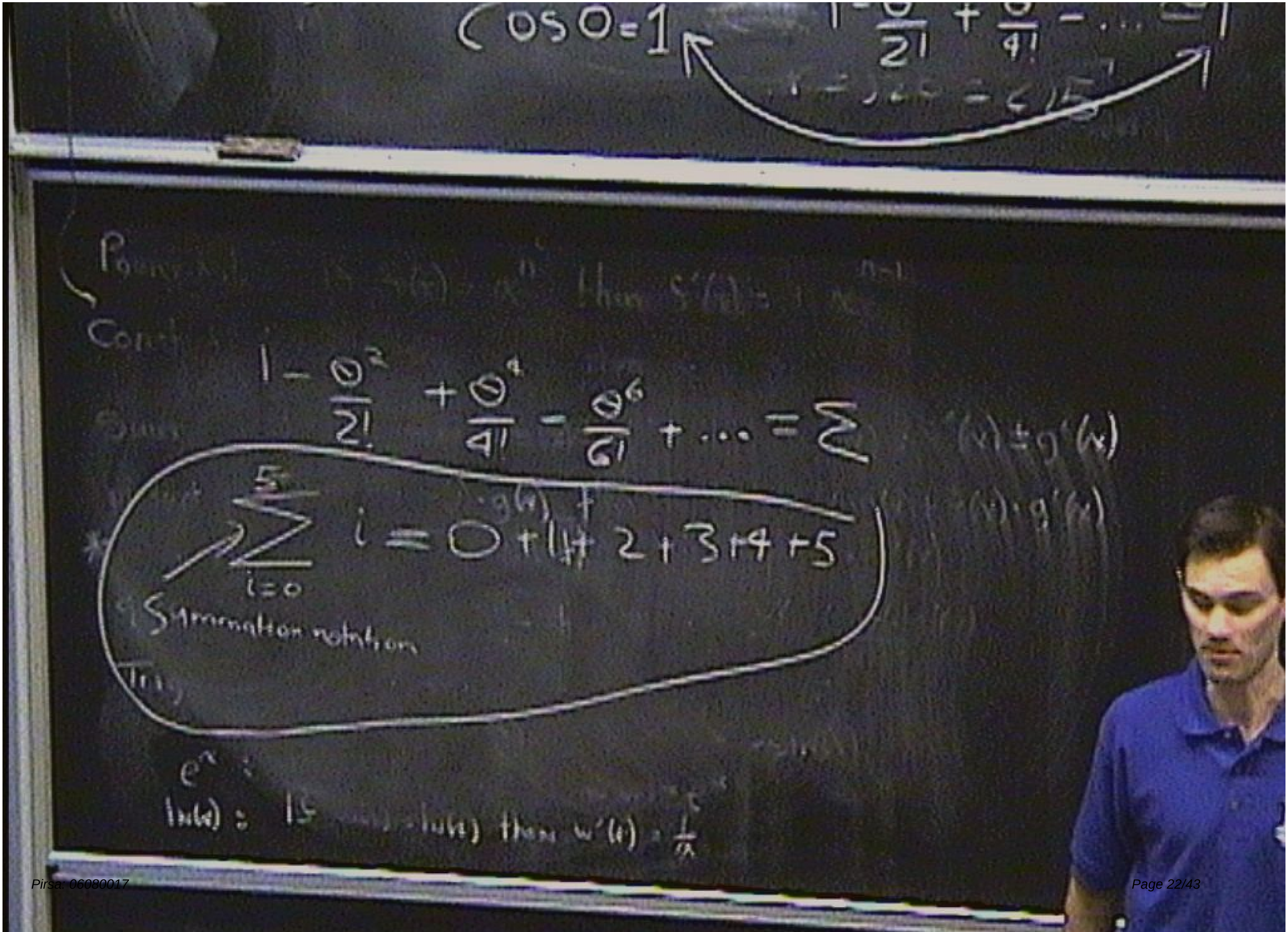
If $w(x) = \cos(x)$ then $w'(x) = -\sin(x)$

e^x : If $w(x) = e^x$ then $w'(x) = e^x$

$\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$


Cosθ=1
1 − θ²/2! + θ⁴/4! − θ⁶/6! + ...
Summation notation
Trig
ln(x) =


cos 0 = 1
1 − θ²/2! + θ⁴/4! − θ⁶/6! + ... = Σ
Σ_{i=0}^{5} i = 0 + 1 + 2 + 3
Summation notation


Cos θ = 1
1 − θ²/2! + θ⁴/4! − θ⁶/6! + … = Σ
Σ i = 0 + 1 + 2 + 3 + 4 + 5
Summation notation
ln(x) :
then

$\cos 0 = 1$ $\quad 1 - \frac{0}{2!} + \frac{0}{4!} - \dots = 1$

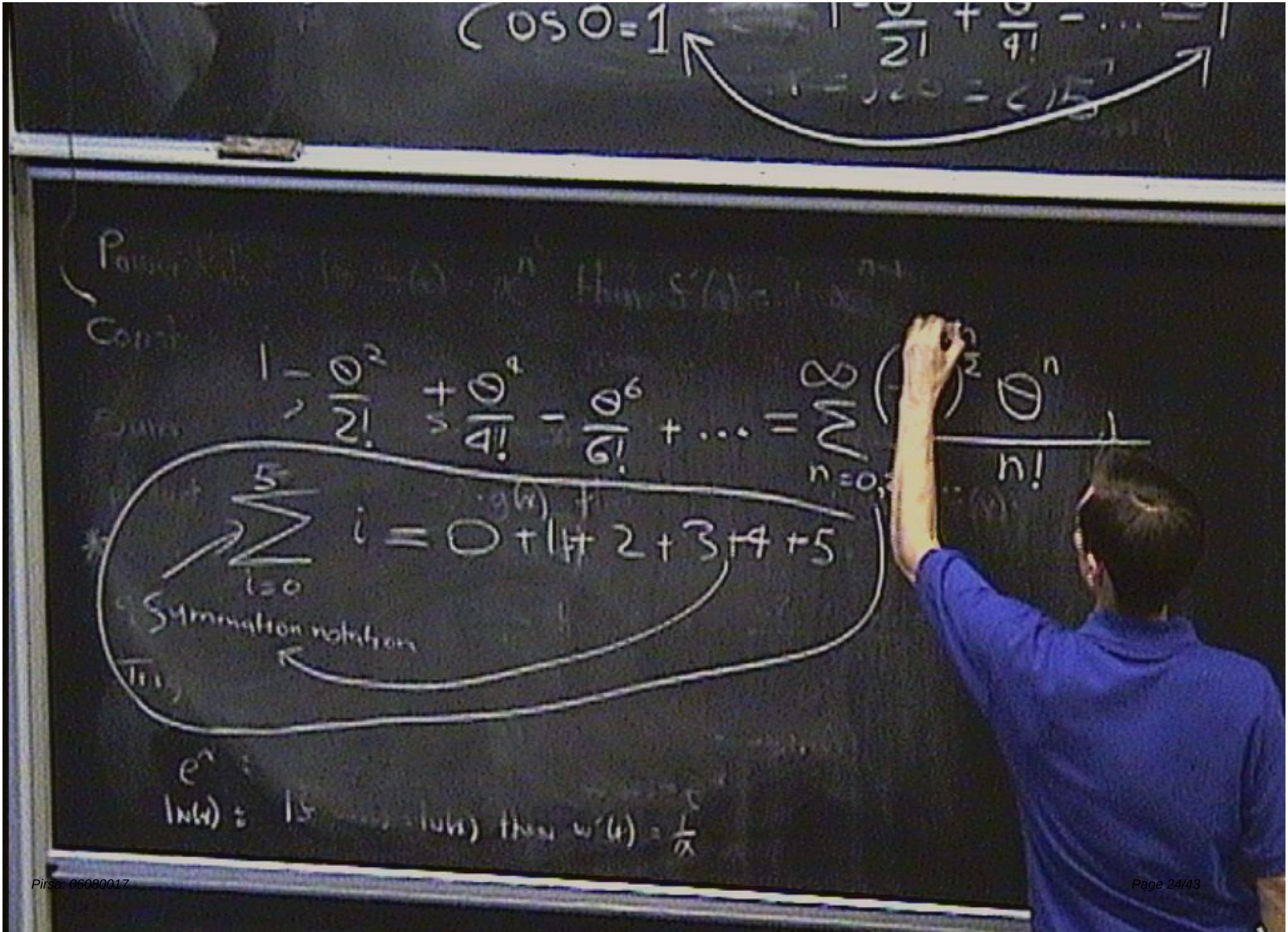
$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{n=0,2,4\dots}^{\infty} \frac{\theta^n}{n!}$$

$$\sum_{i=0}^{5} i = 0 + 1 + 2 + 3 + 4 + 5$$

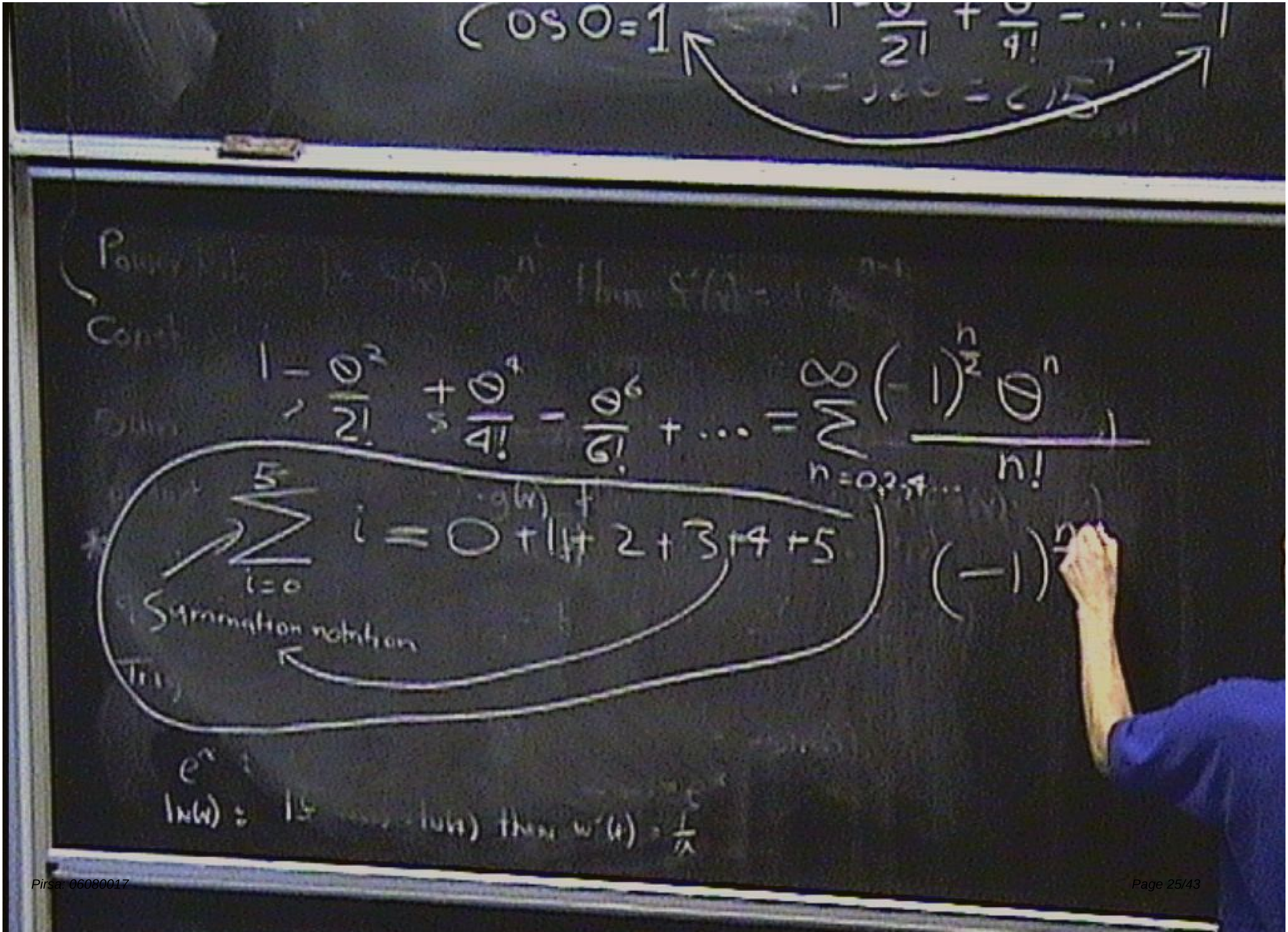
Summation notation

e^x :

$\ln(x)$: Is ... $\ln(x)$ then $w'(x) = \frac{1}{x}$



cos θ = 1
2! 4!
1 - θ²/2! + θ⁴/4! - θ⁶/6! + ... = Σ θⁿ/n!
n=0
Σ i = 0 + 1 + 2 + 3 + 4 + 5
i=0
Summation notation


cos 0 = 1
1 − θ²/2! + θ⁴/4! − θ⁶/6! + ... = Σ (−1)^(n/2) θⁿ/n!
n=0,2,4...
Σ i = 0 + 1 + 2 + 3 + 4 + 5
i=0
Summation notation

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{n=0,2,4,\dots}^{\infty} \frac{(-1)^{\frac{n}{2}}\theta^n}{n!}$$

$$\sum_{i=0}^{5} i = 0+1+2+3+4+5$$

Summation notation

(-1)

$\ln(x)$: If $\dots \ln(x)$ then $w'(x) = \frac{1}{x}$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{n=0,2,4\dots}^{\infty} \frac{(-1)^{\frac{n}{2}}\,\theta^n}{n!}$$

$$\sum_{i=0}^{5} i = 0+1+2+3+4+5$$

Summation notation

$$(-1)^{\frac{n=0}{2}} = (-1)^0 = +1$$

$$(-1)^{\frac{n=2}{2}} = (-1)$$

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$

Power Rule: If $f(x) = x^n$ then $f'(x) = n x^{n-1}$

Const

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{n=0,2,4,\dots}^{\infty} \frac{(-1)^{\frac{n}{2}}\theta^n}{n!} = \cos\theta$$

Sum

$$\sum_{i=0}^{5} i = 0 + 1 + 2 + 3 + 4 + 5$$

Summation notation

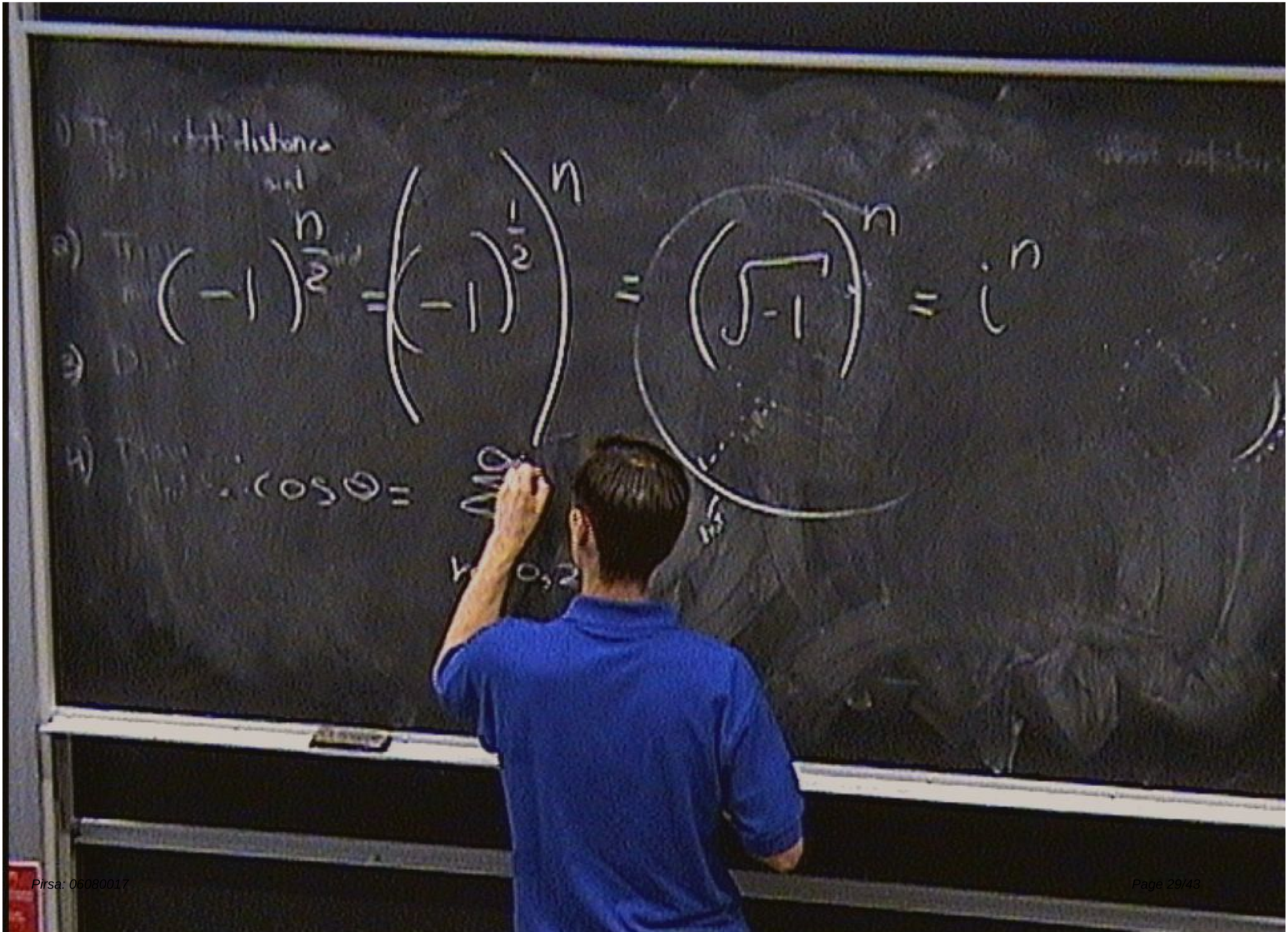
$(-1)^{\frac{n=0}{2}} = (-1)^0 = +1$

$(-1)^{\frac{n=2}{2}} = (-1)^1 = -$

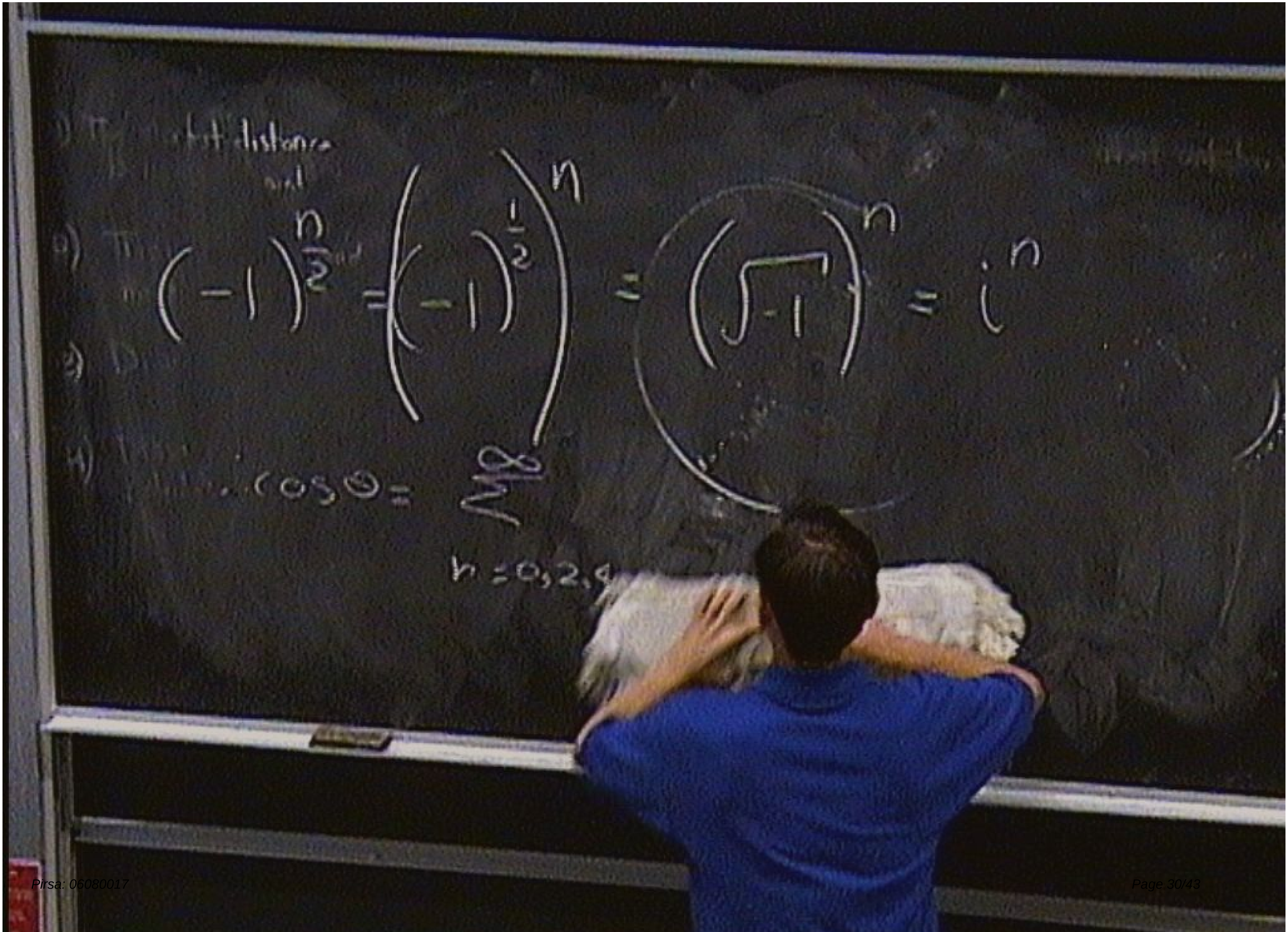
$(-1)^{\frac{4}{2}} = +1$

e^x:

$\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$



$(-1)^{\frac{n}{2}} = \left((-1)^{\frac{1}{2}}\right)^n = \left(\sqrt{-1}\right)^n = i^n$
$\cos\theta =$



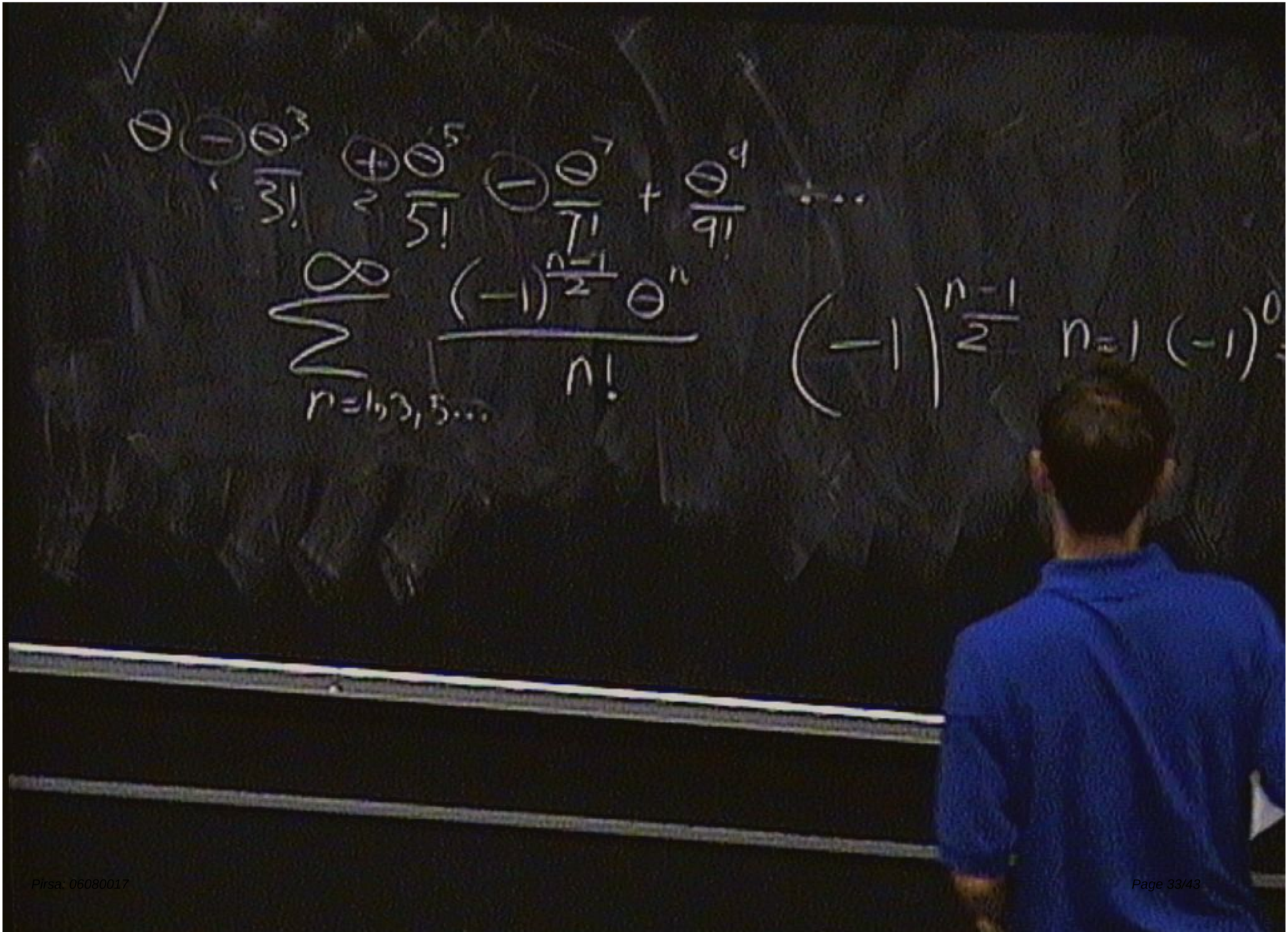
$(-1)^{\frac{n}{2}} = \left((-1)^{\frac{1}{2}}\right)^{n} = \left(\sqrt{-1}\right)^{n} = i^{n}$
$\cos\theta = \sum$
$n = 0, 2, 4$

$$(-1)^{\frac{n}{2}} = \left((-1)^{\frac{1}{2}}\right)^{n} = \left(\sqrt{-1}\right)^{n} = i^{n}$$

$$\boxed{\therefore \cos\theta = \sum_{n=0,2,4\ldots}^{\infty} \frac{(i\theta)^{n}}{n!}}$$

$$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \cdots$$

$$\sum_{n=1,3,5\ldots}^{\infty} \frac{\theta^n}{n!}$$


$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} + \dots$
$\sum_{n=1,3,5\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$
$(-1)^{\frac{n-1}{2}}$ $n=1$ $(-1)^0$

$$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} + \cdots$$

$$\sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}\,\theta^n}{n!}$$

$$(-1)^{\frac{n-1}{2}}$$

$n=1$ $(-1)^0$

$n=3$

$n=5$

$$+\theta \underset{1}{-} \frac{\theta^3}{3!} \underset{2}{+} \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} + \dots$$

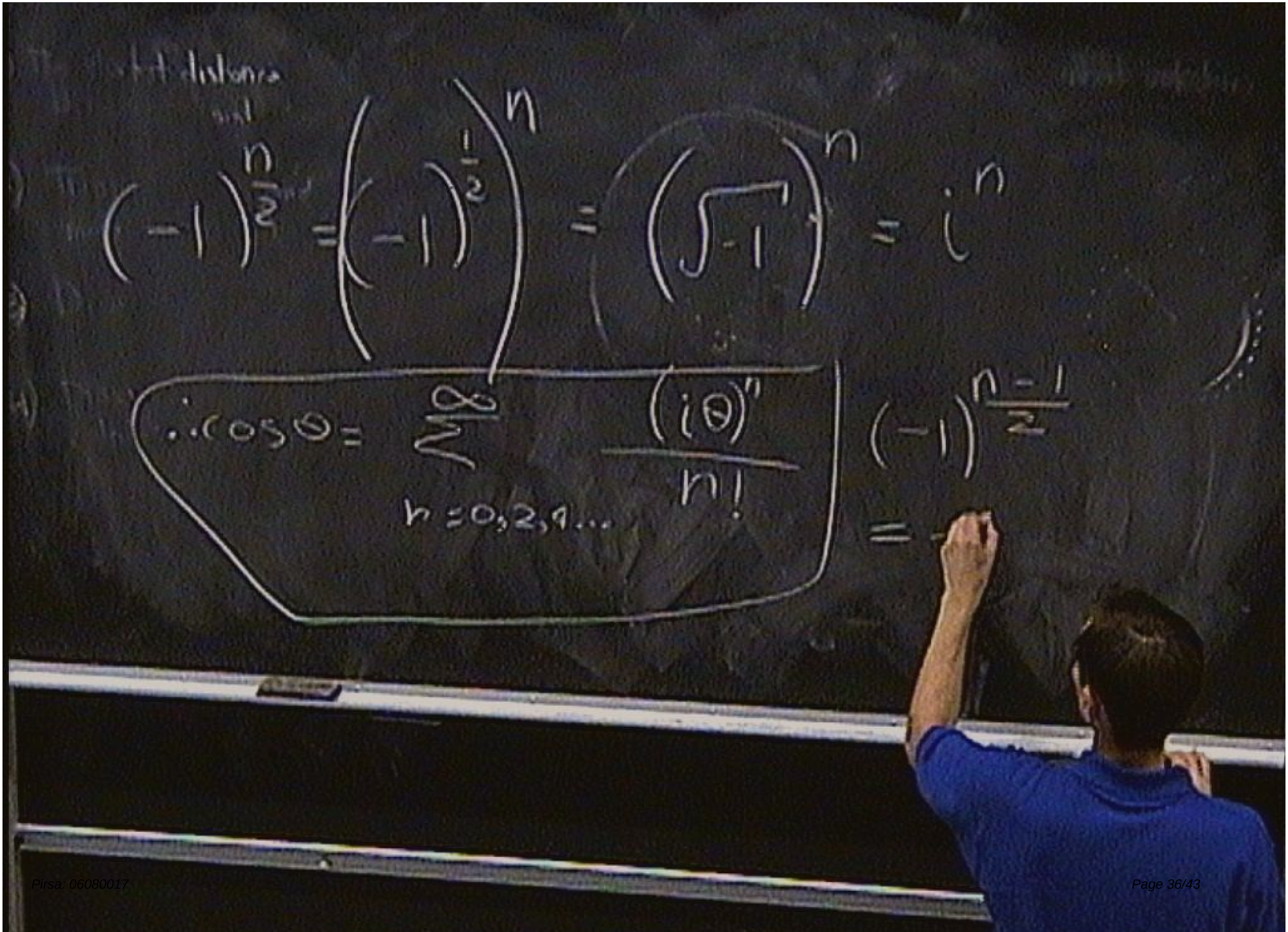
$$\sum_{n=1,3,5\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}\,\theta^n}{n!}$$

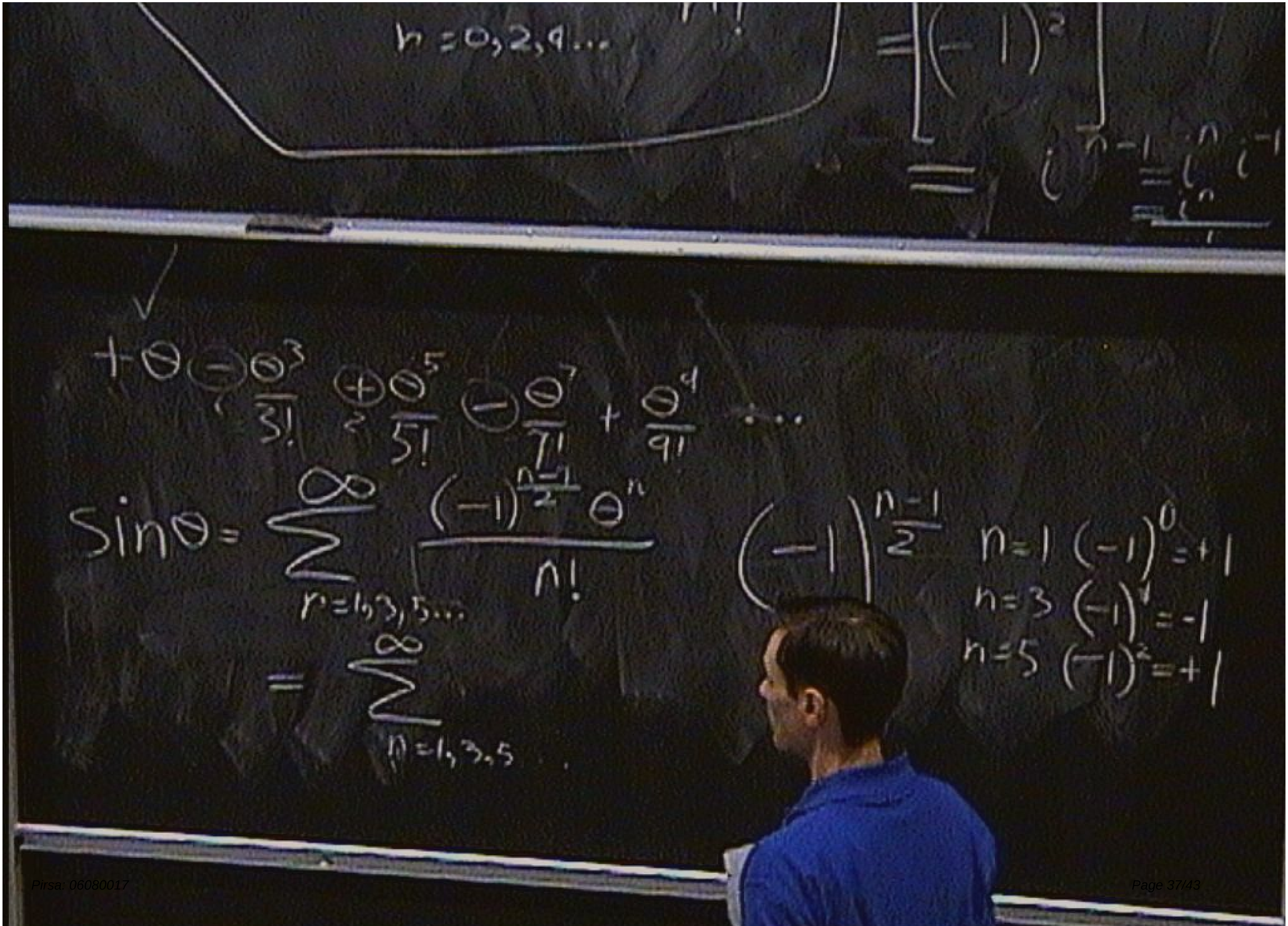
$$(-1)^{\frac{n-1}{2}}$$

$n=1 \quad (-1)^0 = +1$

$n=3 \quad (-1)^1 = -1$

$n=5 \quad (-1)^2 = +1$


$(-1)^{\frac{n}{2}} = \left((-1)^{\frac{1}{2}}\right)^n = \left(\sqrt{-1}\right)^n = i^n$
$\therefore \cos\theta = \sum_{n=0,2,4\ldots}^{\infty} \frac{(i\theta)^n}{n!}$
$(-1)^{\frac{n-1}{2}}$
$=$


$n = 0, 2, 4 \ldots$
$+\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \ldots$
$\sin\theta = \sum_{n=1,3,5\ldots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$
$= \sum_{n=1,3,5}^{\infty}$
$(-1)^{\frac{n-1}{2}}$
$n=1 \quad (-1)^0 = +1$
$n=3 \quad (-1)^1 = -1$
$n=5 \quad (-1)^2 = +1$

$\checkmark$

$$+\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \cdots$$

$$\sin\theta = \sum_{n=1,3,5\ldots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}\,\theta^n}{n!}$$

$$(-1)^{\frac{n-1}{2}}$$

$n=1 \quad (-1)^0 = +1$

$n=3 \quad (-1)^1 = -1$

$n=5 \quad (-1)^2 = +1$

$$\boxed{\sin\theta = -i \sum_{n=1,3,5}^{\infty} \frac{(\theta i)^n}{n!}}$$

$i^{n-1} = i^n i^{-1} = i^n \ldots$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned} \cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \end{aligned}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$\boxed{\sin\theta \simeq \theta}$ $\boxed{\theta \ll 1}$

$\sin 0 = 0$

$\cos 0 = 1$ $\quad 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \simeq 1$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\boxed{\begin{aligned}\cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \\ \sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots\end{aligned}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

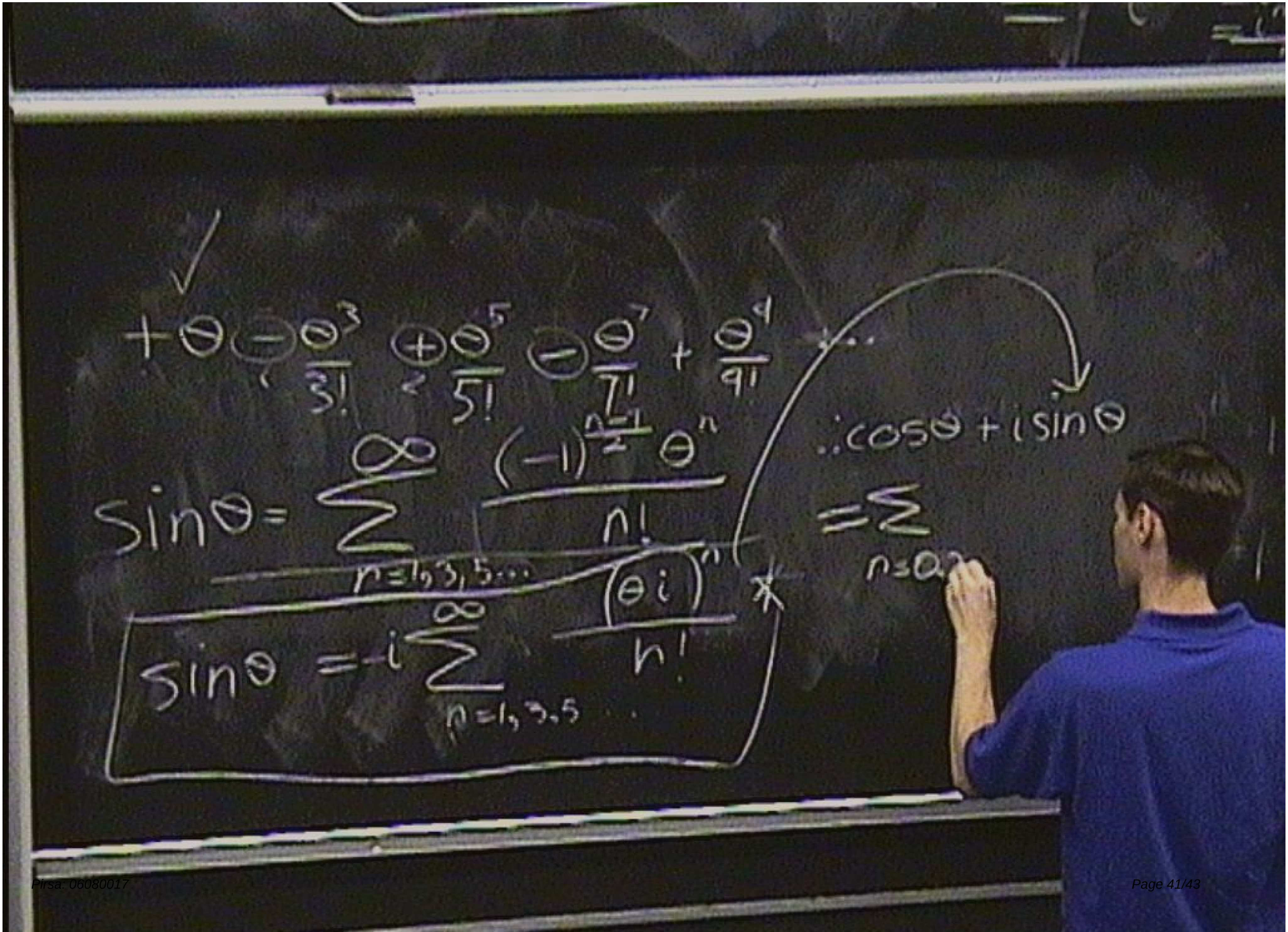
$\boxed{\sin\theta \simeq \theta}$

$\boxed{\theta \ll 1}$

$\sin 0 = 0$

$\cos 0 = 1$

$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \simeq 1$



+θ − θ^3/3! + θ^5/5! − θ^7/7! + θ^9/9! + ...
sinθ = Σ_{n=1,3,5...}^∞ (−1)^((n−1)/2) θ^n / n!
sinθ = −i Σ_{n=1,3,5...}^∞ (θi)^n / n!
∴ cosθ + i sinθ
= Σ

$$+\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} + \dots$$

$$\sin\theta = \sum_{n=1,3,5\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}\,\theta^n}{n!}$$

$$\boxed{\sin\theta = -i\sum_{n=1,3,5\dots}^{\infty} \frac{(\theta i)^n}{n!}}$$

$$\cos\theta + i\sin\theta = \sum_{n=0,2,4\dots}^{\infty} \frac{(i\theta)^n}{n!} + i(-1)\sum_n$$

$$+\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \dots$$

$$\sin\theta = \sum_{n=1,3,5\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}\,\theta^n}{n!}$$

$$\boxed{\sin\theta = -i\sum_{n=1,3,5\dots}^{\infty} \frac{(\theta i)^n}{n!}}$$

$$\therefore \cos\theta + i\sin\theta = \sum_{n=0,2,4\dots}^{\infty} \frac{(i\theta)^n}{n!} + \sum_{n=1,3,5\dots}^{\infty} \frac{(i\theta)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!}$$

$$=$$