

Title: Math Tutorial - Part 5

Date: Aug 08, 2006 01:00 PM

URL: <http://pirsa.org/06080017>

Abstract:

HOMEPLAY 2

A. $u = 4 + 2i$

$v = 3 + 2i$

represent u in polar notation

B. $(u+v)^{23}$

C. $(u+v)^{2006}$

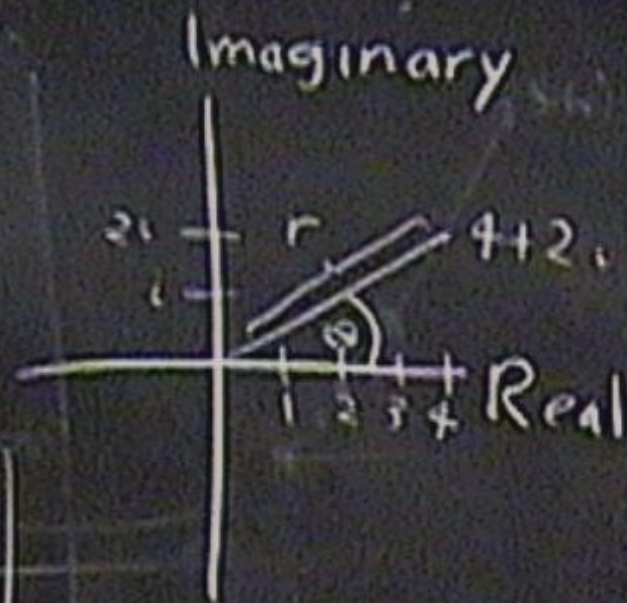
A



u (4, 2i)

or

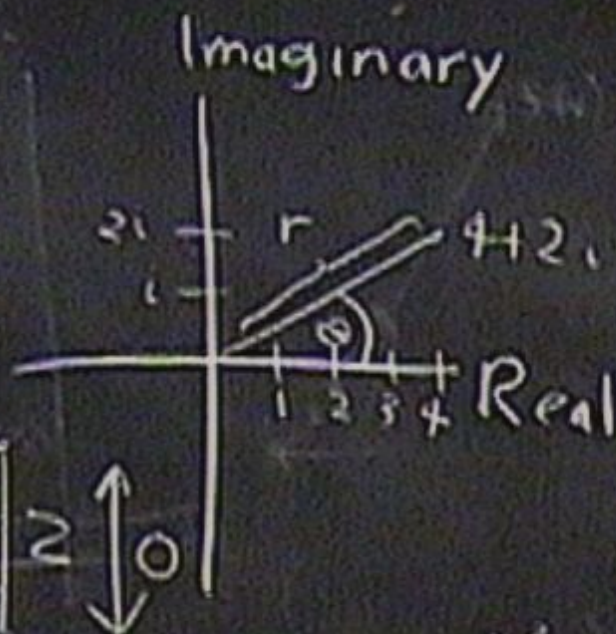
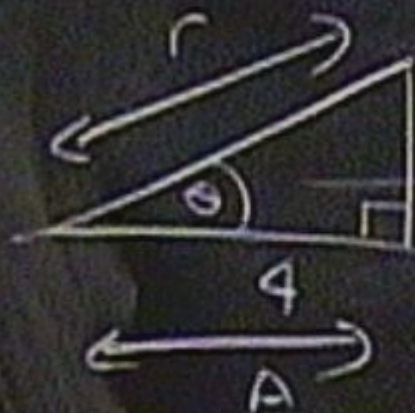
A



u (4, 2i)

r
θ

A



$\theta =$

slope = 2/4

u (4, 2i)

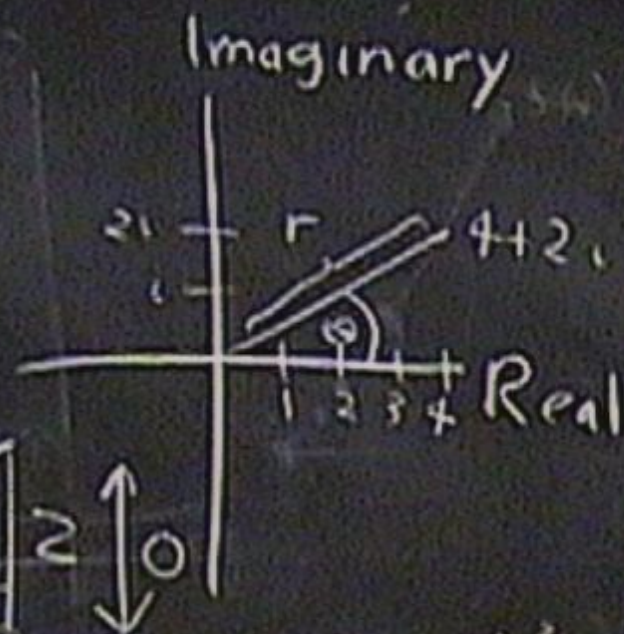
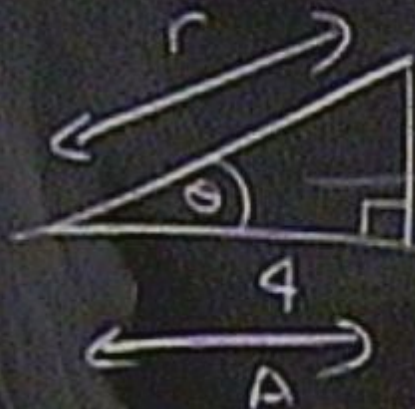
r
 θ

$$A^2 + 0^2 = r^2$$

$$\therefore r^2 = 4^2 + 2^2 = 20$$

$$\therefore r = \sqrt{20} = 2\sqrt{5}$$

A



$$\tan \theta = \frac{2}{4} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.6^\circ$$

or

r
 θ

$$A^2 + 0^2 = r^2$$

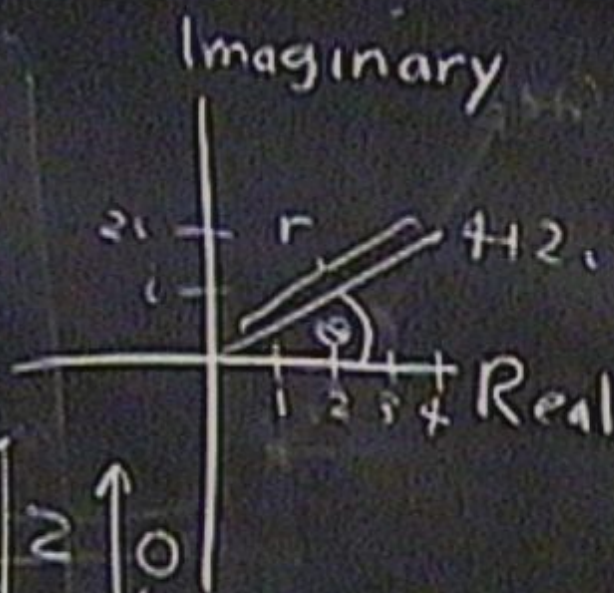
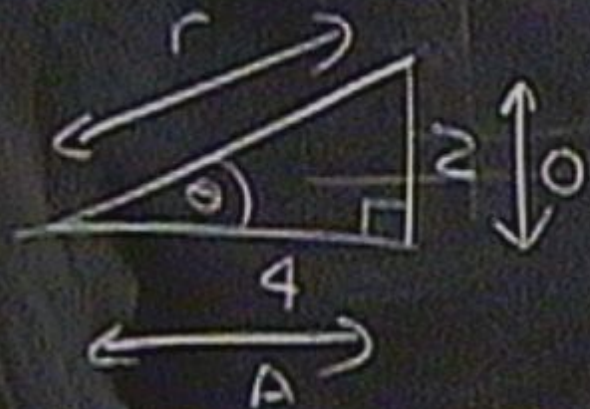
$$\therefore r^2 = 4^2 + 2^2 = 20$$

$$\therefore r = \sqrt{20} = 2\sqrt{5}$$

Power Rule: If $y = x^n$ then $y' = nx^{n-1}$

$u = r$

A



$\tan \theta = \frac{2}{4} = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.6^\circ$
or

0.464
radian

$A^2 + O^2 = r^2$

$\therefore r^2 = 4^2 + 2^2 = 20$

$\therefore r = \sqrt{20} = 2\sqrt{5}$

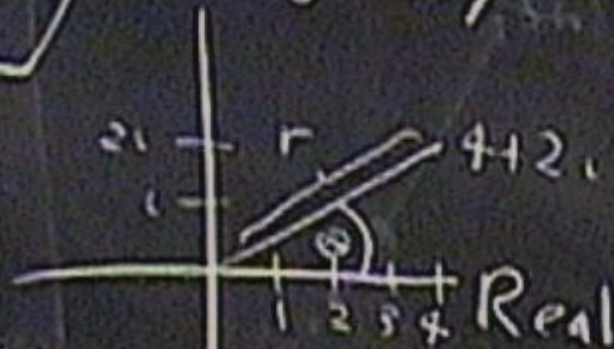
Power P.L. = $IS(s) = n^s \parallel s'(s) = n s^{n-1}$

$$U = r e^{j\theta}$$

$$= 2\sqrt{5} e^{j0.464}$$

A

Imaginary



$$\tan \theta = \frac{2}{4} = \frac{1}{2}$$

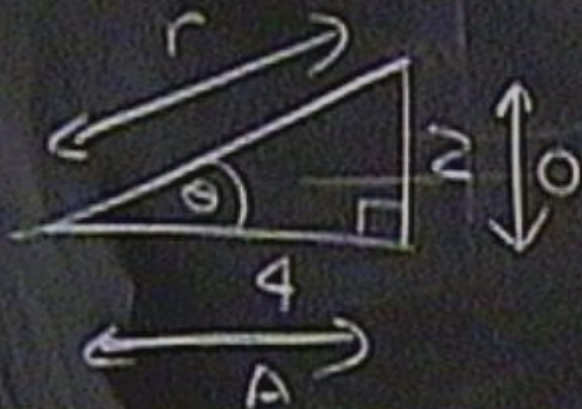
$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.6^\circ$$

or

$$U (4, 2j)$$

0.464
radian

r
θ



$$A^2 + 0^2 = r^2$$

$$\therefore r^2 = 4^2 + 2^2 = 20$$

$$\therefore r = \sqrt{20} = 2\sqrt{5}$$

HOMEPLAY 2

A. $u = 4 + 2i$

$v = 3 + 2i$

represent u in polar notation

B. $(u+v)^{23} = 65^{23} e^{11.9i}$

C. $(u+v)^{2006} = 65^{2006} e^{101i}$

HOMEPLAY 2

A. $u = 4 + 2i$

$v = 3 + 2i$

represent u in polar notation

B. $(u+v)^{23} = 65^{23} e^{11.4i}$

C. $(u+v)^{2006} = 65^{2006} e^{1014i}$

U = $e^{i\theta}$
Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos = 1 -$$

U = $e^{i\theta}$
Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

U = T2
prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$0 < \theta < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$e^x : \text{if } w(h) = e^{xh}, \text{ then } w'(h) = e^x$
 $\ln(h) : \text{if } w(h) = \ln(h) \text{ then } w'(h) = \frac{1}{h}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\theta \ll 1$$

$$1 - 0.20 = 0.80$$

$e^x : 15 \text{ whn } = e^{15}, \text{ then } w(h) = e^h$
 $\ln(h) : 15 \text{ whn } = \ln(h) \text{ then } w'(h) = \frac{1}{h}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\theta < 1$$

$$\cos\theta > 0$$

$$\sin\theta < 15$$

ing : If $w(t) = \sin t$ then $w'(t) = \cos t$
If $w(t) = \cos t$ then $w'(t) = -\sin t$
 e^x : If $w(t) = e^t$, then $w'(t) = e^t$
 $\ln(t)$: If $w(t) = \ln t$ then $w'(t) = \frac{1}{t}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\theta \ll 1$$

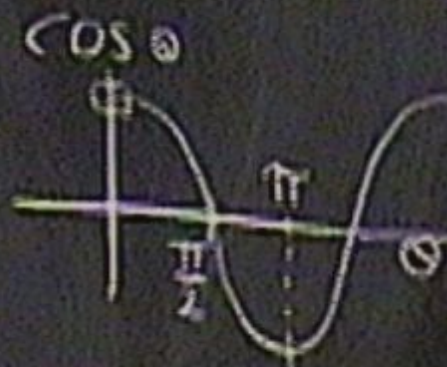
$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \approx 1$$

Trig : If $w(x) = \sin(x)$ then $w'(x) = \cos(x)$
 If $w(x) = \cos(x)$ then $w'(x) = -\sin(x)$
 e^x : If $w(x) = e^x$, then $w'(x) = e^x$
 $\ln(x)$: If $w(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$



$$\theta < 1$$

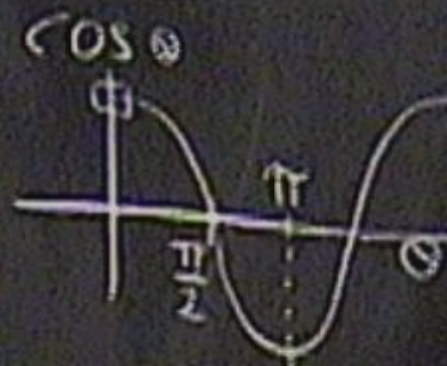
$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

If $w(t) = \cos(t)$ then $w'(t) = -\sin(t)$
 If $w(t) = e^{it}$ then $w'(t) = ie^{it}$
 If $w(t) = \frac{1}{t}$ then $w'(t) = -\frac{1}{t^2}$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$



$$\sin\theta \approx \theta$$

$$\theta \ll 1$$

$$\sin 0 = 0$$

$$\cos 0 = 1$$

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \approx 1$$

If $w(t) = \cos(t)$ then $w'(t) = -\sin(t)$

e^x : If $w(t) = e^{kt}$ then $w'(t) = e^k$

$\ln(t)$: If $w(t) = \ln(t)$ then $w'(t) = \frac{1}{t}$

$$(050=1) \quad \leftarrow \quad \frac{0}{2!} + \frac{0}{4!} - \dots = 1$$

Power rule: If $s(x) = x^n$ then $s'(x) = nx^{n-1}$

Const:

$$1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \dots$$

Sum:

$$\sum_{i=0}^n \frac{f^{(i)}(x) \cdot g^{(i)}(x)}{i!}$$

*

Summation notation

Try:

$$e^x: \ln(w) = \ln(x) \text{ then } w'(x) = \frac{1}{x}$$

$$(050=1) \quad \left(\frac{0}{2!} + \frac{0}{4!} - \dots = 1 \right)$$

Power rule

$$f(x) = x^n \quad \text{then } f'(x) = n x^{n-1}$$

Corollary

$$1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \dots = \sum_{i=0}^{\infty} (-1)^i \frac{0^{2i}}{(2i)!}$$

Sum

$$\sum_{i=0}^5$$

$$i = 0 + 1 + 2 + 3 + \dots$$

Summation notation

Trick

$$e^x : \ln(w) : \text{if } w = \ln(u) \text{ then } w'(u) = \frac{1}{u}$$

$$(050=1 \quad \leftarrow \quad 1 - \frac{0}{2!} + \frac{0}{4!} - \dots = 1$$

Power rule: If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Corollary

$$1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \dots = \sum_{i=0}^{\infty} (-1)^i \frac{0^{2i}}{(2i)!}$$

Sum

$$\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$$

*

Summation notation

Try

e^x :

$\ln(x) = 15$ then $w'(t) = \frac{1}{x}$

$$(050=1 \quad \leftarrow \quad 1 - \frac{0}{2!} + \frac{0}{4!} - \dots = 1$$

Power of e^x is e^x then $e'(x) = e^x$

Consider

$$1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \dots = \sum_{n=0,2,4,\dots} \frac{0^n}{n!}$$

* $\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$

Summation notation

e^x :
 $\ln(x) = 15$ then $w'(x) = \frac{1}{x}$

$$(050=1 \quad \leftarrow \quad \frac{0}{2!} + \frac{0}{4!} - \dots = 1 \quad \rightarrow \quad 1 - 0.20 = 0.8$$

Power: $f(x) = e^x$ Then $f'(x) = e^x$

Constr:

Sum:

$$1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n 0^n}{n!}$$

$$\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$$

* Symmation notation

Tru

$$e^x: \ln(x) \pm 15 \dots \ln(x) \text{ then } w'(x) = \frac{1}{x}$$

$$(050=1) \quad \leftarrow \quad \left(\frac{0}{2!} + \frac{0}{4!} - \dots \right)$$

Power series $f(x) = e^x$ then $f'(x) = e^x$

Const

$$1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \frac{0^6}{6!} + \dots = \sum_{n=0,2,4,\dots} \frac{(-1)^{\frac{n}{2}} 0^n}{n!}$$

Sum

* $\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$

Summation notation

$$(-1)^{\frac{n}{2}}$$

e^x

$\ln(w) = 15$ then $w'(t) = \frac{1}{\lambda}$

$$(050=1) \quad \leftarrow \quad \frac{0}{2!} + \frac{0}{4!} - \dots = 1$$

Power Series: $y = f(x) = x^n$ then $f'(x) = 1 \cdot x^{n-1}$

Con:

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{n=0,2,4,\dots} \frac{(-1)^{n/2} \theta^n}{n!}$$

Sum:

* $\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$

Summation notation

$$(-1)^{n/2}$$

e^x
 $\ln(w) = 15$ then $w'(t) = \frac{1}{x}$

$$(050=1) \quad \frac{0}{2!} + \frac{0}{4!} - \dots = 1$$

Power rule: If $y = x^n$ then $y' = nx^{n-1}$

Const:

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0,2,4,\dots} \frac{(-1)^{n/2} x^n}{n!}$$

* $\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$

Summation notation

$$\begin{aligned} (-1)^{\frac{n=0}{2}} &= (-1)^0 = +1 \\ (-1)^{\frac{n=2}{2}} &= (-1)^1 = -1 \end{aligned}$$

ex: $\ln(x) = \ln(x)$ then $w'(x) = \frac{1}{x}$

$$\cos 0 = 1 \quad \leftarrow \quad 1 - \frac{0^2}{2!} + \frac{0^4}{4!} - \dots = 1$$

Power rule: $y = x^n$ then $y' = nx^{n-1}$

Const

Sum

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{n=0,2,4,\dots} \frac{(-1)^{\frac{n}{2}} \theta^n}{n!} = \cos \theta$$

* $\sum_{i=0}^5 i = 0 + 1 + 2 + 3 + 4 + 5$

Summation notation

$$\begin{aligned} (-1)^{\frac{n=0}{2}} &= (-1)^0 = +1 \\ (-1)^{\frac{n=2}{2}} &= (-1)^1 = -1 \\ (-1)^{\frac{n=4}{2}} &= +1 \end{aligned}$$

e^x : $\ln(x) = 1/x$ then $w'(x) = \frac{1}{x}$

1) The shortest distance
is

a)
$$(-1)^{\frac{n}{2}} = \left((-1)^{\frac{1}{2}}\right)^n = \left(\sqrt{-1}\right)^n = i^n$$

b) $\cos \theta = \frac{1}{2}$
 $\theta = 0, 2\pi$

1) π path distance
and

$$b) \quad \pi_m \left(-1 \right)^{\frac{n}{2}} = \left(-1 \right)^{\frac{n-1}{2}} = \left(\sqrt{-1} \right)^n = i^n$$

c) D_m

$$h) \quad \cos \theta = \frac{8h\nu}{\dots}$$

$$n = 0, 2, 4$$

$$(-1)^2 = (-1)^2 = (\sqrt{-1})^2 = -1$$

$$\therefore \cos \theta = \sum_{n=0,2,4,\dots}^{\infty} \frac{(i\theta)^n}{n!}$$

$$\theta = \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots$$

$$\sum_{n=1,3,5,\dots}^{\infty} \frac{\theta^n}{n!}$$

$$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots$$

$$\sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$(-1)^{\frac{n-1}{2}} \quad n=1 \quad (-1)^0$$

$$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots$$

$$\sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$\left(-1\right)^{\frac{n-1}{2}} \quad \begin{array}{l} n=1 \quad (-1)^0 \\ n=3 \\ n=5 \end{array}$$

$$+ \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots$$

$$\sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$\left(-1 \right)^{\frac{n-1}{2}}$$

$n=1 \quad (-1)^0 = +1$
 $n=3 \quad (-1)^1 = -1$
 $n=5 \quad (-1)^2 = +1$

The shortest distance
and

$$(-1)^{\frac{n}{2}} = \left((-1)^{\frac{1}{2}} \right)^n = \left(\sqrt{-1} \right)^n = i^n$$

$$\therefore \cos \theta = \sum_{n=0,2,4,\dots} \frac{(i\theta)^n}{n!} (-1)^{\frac{n-1}{2}}$$

$$n = 0, 2, 4, \dots$$

$$= (-1)^2$$

$$+\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots$$

$$\sin \theta = \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$= \sum_{n=1,3,5,\dots}^{\infty}$$

$$\left(-1\right)^{\frac{n-1}{2}}$$

$$\begin{aligned} n=1 & (-1)^0 = +1 \\ n=3 & (-1)^1 = -1 \\ n=5 & (-1)^2 = +1 \end{aligned}$$

$$+ \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \dots$$

$$\sin \theta = \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$\boxed{\sin \theta = -i \sum_{n=1,3,5,\dots}^{\infty} \frac{(\theta i)^n}{n!}}$$

$$\left(-1 \right)^{\frac{n-1}{2}} \quad \begin{array}{l} n=1 \quad (-1)^0 = +1 \\ n=3 \quad (-1)^1 = -1 \\ n=5 \quad (-1)^2 = +1 \end{array}$$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin\theta \geq 0$$

$$\sin 0 = 0$$

$$\theta < 1$$

$$\cos 0 = 1$$

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

$$\ln(x) = \ln\left(\frac{1}{1-x}\right) = -\ln(1-x)$$

Prove $e^{i\theta} = \cos\theta + i\sin\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin\theta \approx \theta$$

$$\sin 0 = 0$$

$$\theta \ll 1$$

$$\cos 0 = 1$$

$$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \approx 1$$

$\ln(w) = 15$ then $w(t) = \frac{1}{\lambda}$

$$+ \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots$$

$$\sin \theta = \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$\boxed{\sin \theta = -i \sum_{n=1,3,5,\dots}^{\infty} \frac{(\theta i)^n}{n!}}$$

$$\therefore \cos \theta + i \sin \theta$$

$$= \sum_{n=0}^{\infty}$$

$$+ \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \dots$$

$$\sin \theta = \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$\sin \theta = -i \sum_{n=1,3,5,\dots}^{\infty} \frac{(\theta i)^n}{n!} *$$

$$\therefore \cos \theta + i \sin \theta = \sum_{n=0,2,4,\dots}^{\infty} \frac{(i\theta)^n}{n!} + i \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$+ \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \dots$$

$$\sin \theta = \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \theta^n}{n!}$$

$$\boxed{\sin \theta = -i \sum_{n=1,3,5,\dots}^{\infty} \frac{(i\theta)^n}{n!}} \quad *$$

$$\begin{aligned} \therefore \cos \theta + i \sin \theta &= \sum_{n=0,2,4,\dots}^{\infty} \frac{(i\theta)^n}{n!} + \sum_{n=1,3,5,\dots}^{\infty} \frac{(i\theta)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!} \\ &= \end{aligned}$$