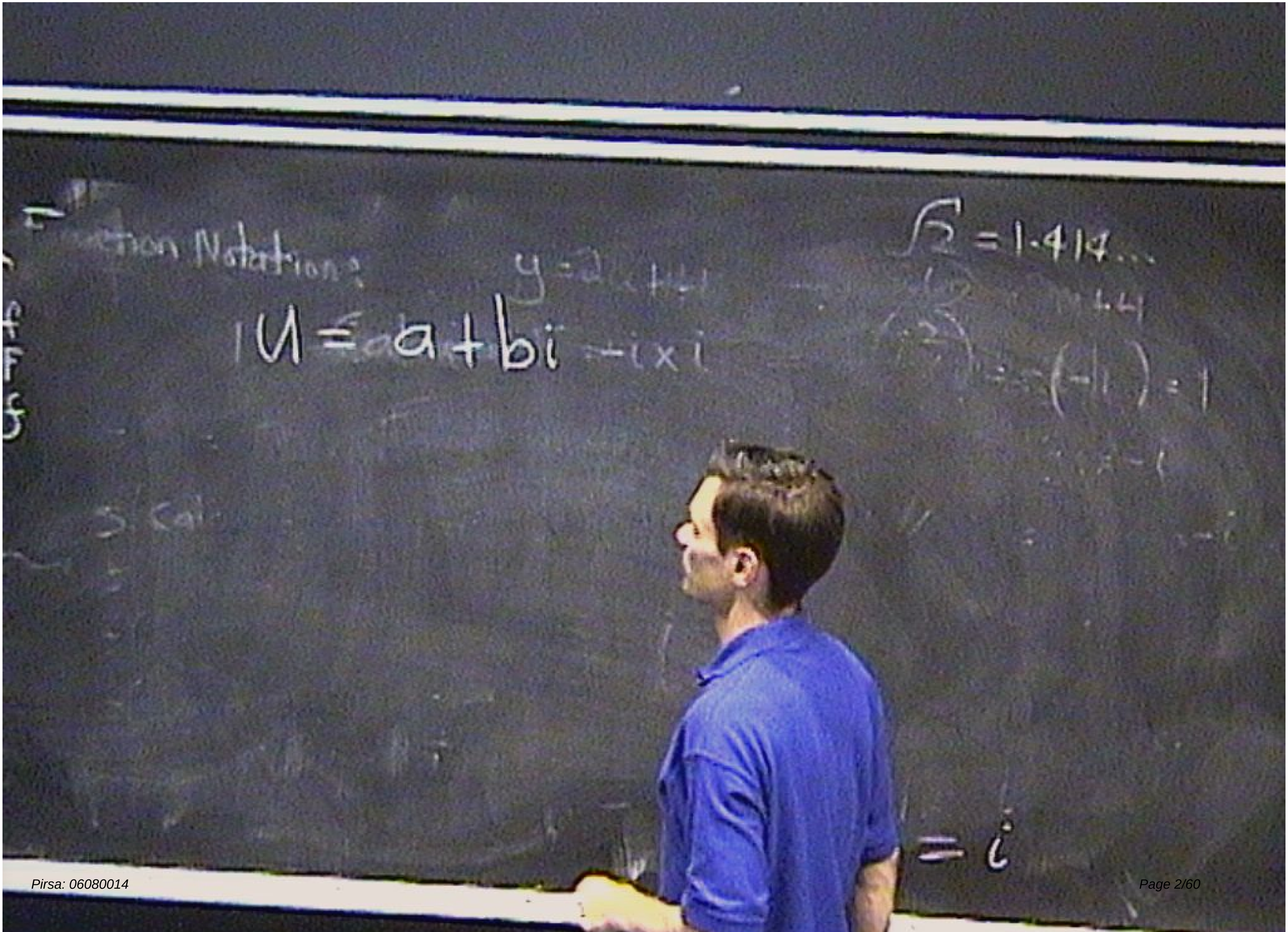
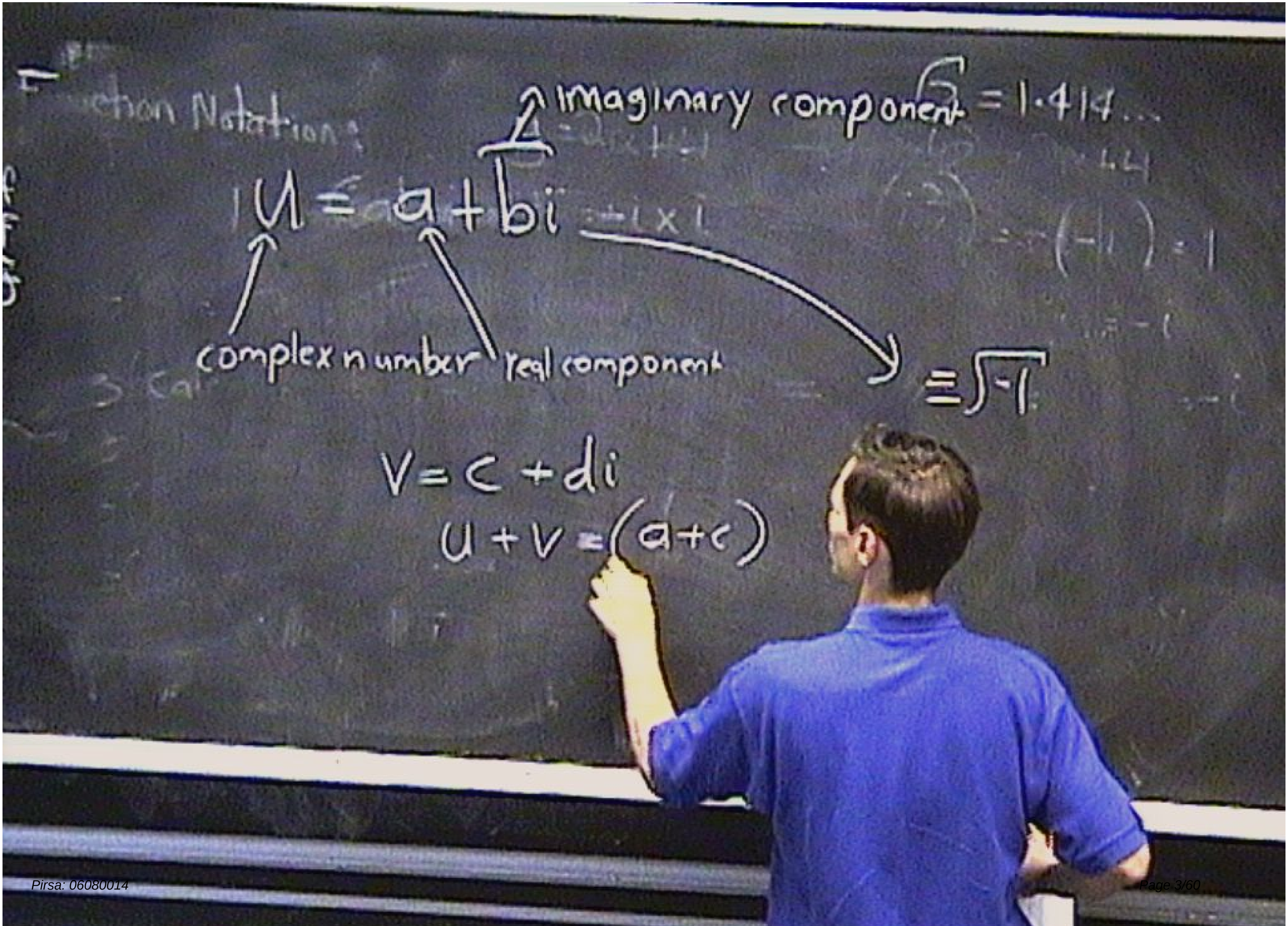
Title: Math Tutorial - Part 4

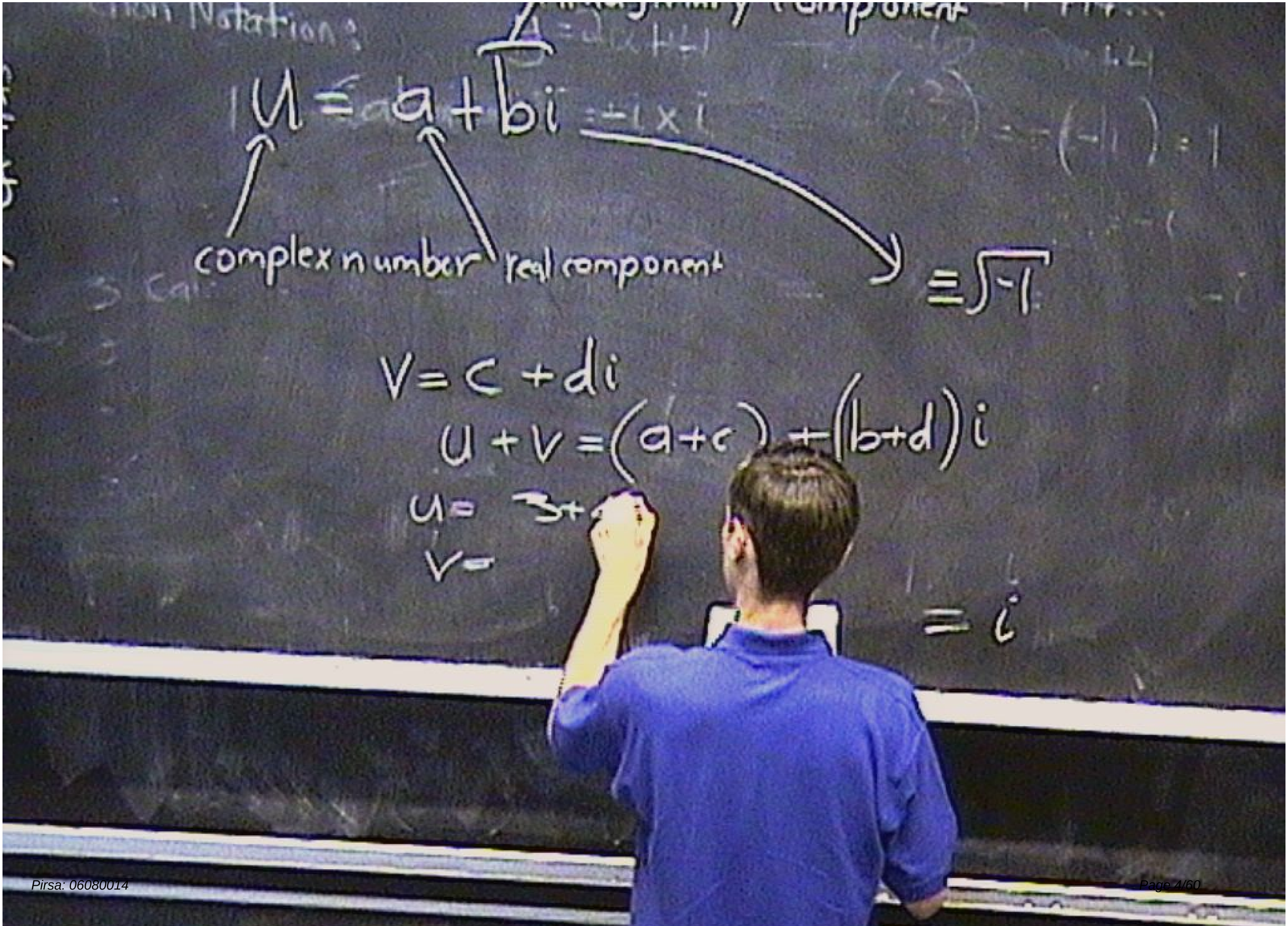
Date: Aug 07, 2006 03:30 PM

URL: http://pirsa.org/06080014

Abstract:


Notation:
u = a + bi
√2 = 1.414...
= i


imaginary component
√2 = 1.414...
u = a + bi
complex number
real component
= √-1
v = c + di
u + v = (a+c)


u = a + bi
complex number
real component
= √-1
v = c + di
u + v = (a+c) + (b+d)i
= i

imaginary component

$$u = a + bi$$

complex number

real component

$$i = \sqrt{-1}$$

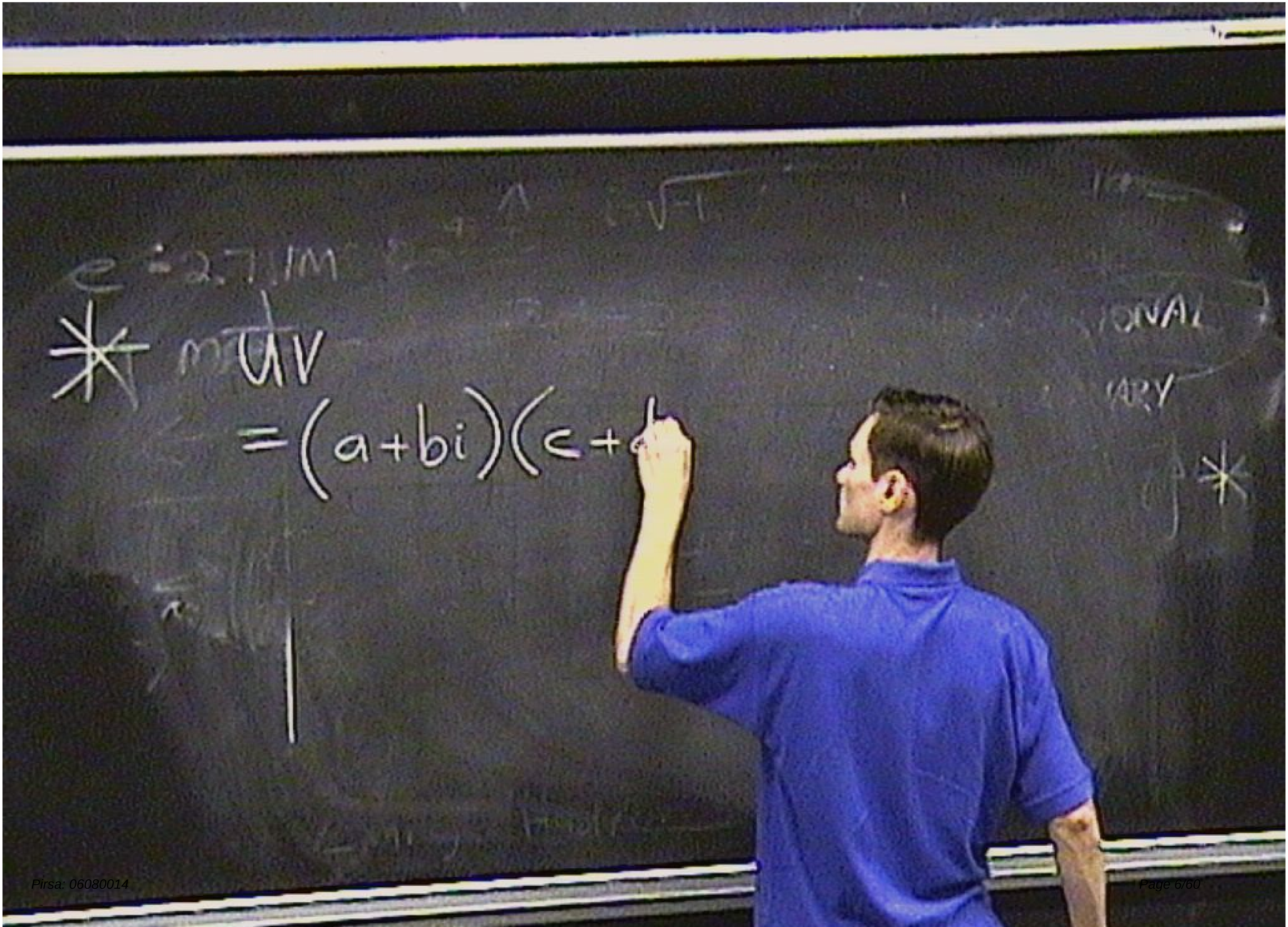
$$v = c + di$$

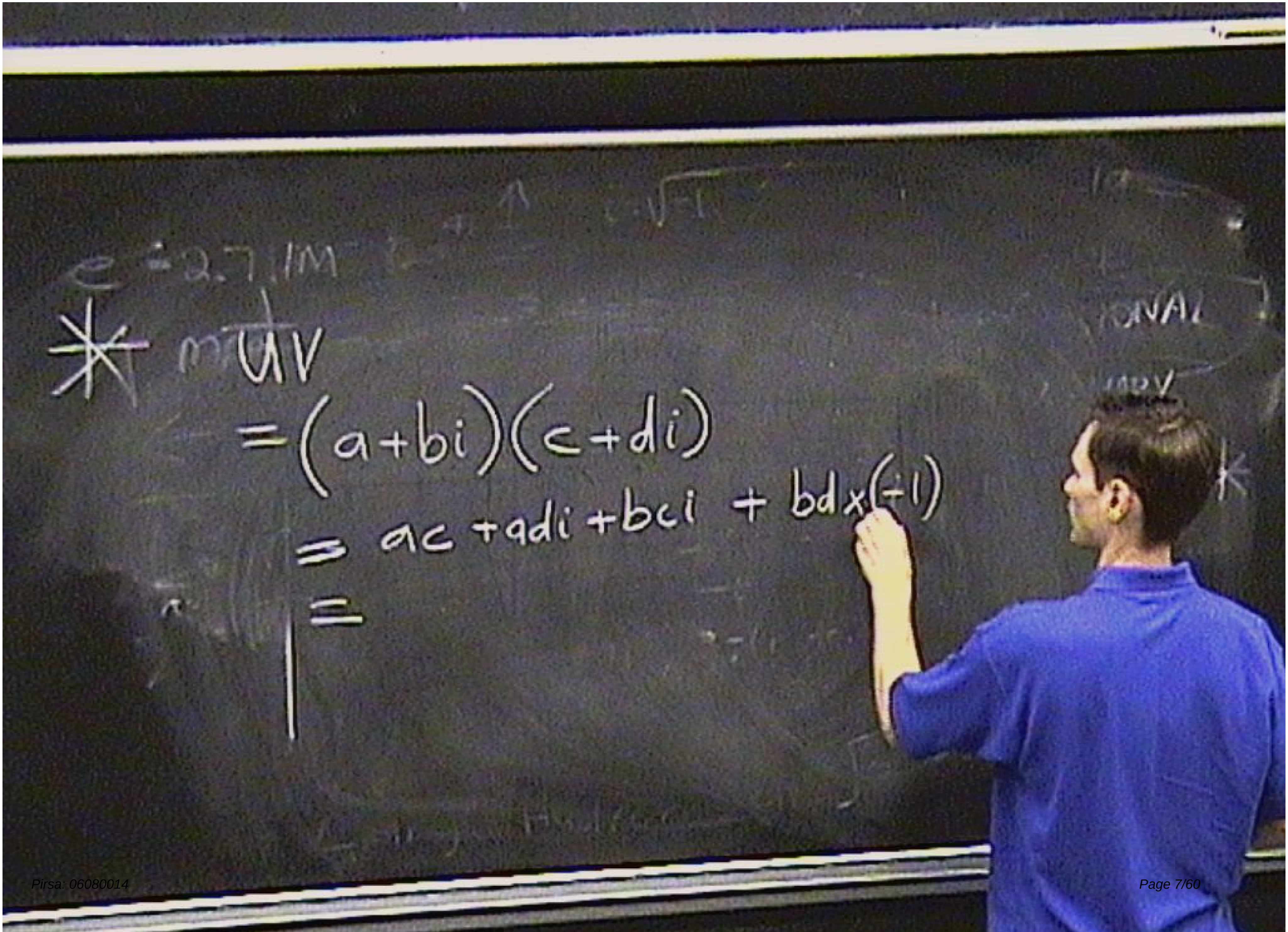
$$u + v = (a+c) + (b+d)i$$

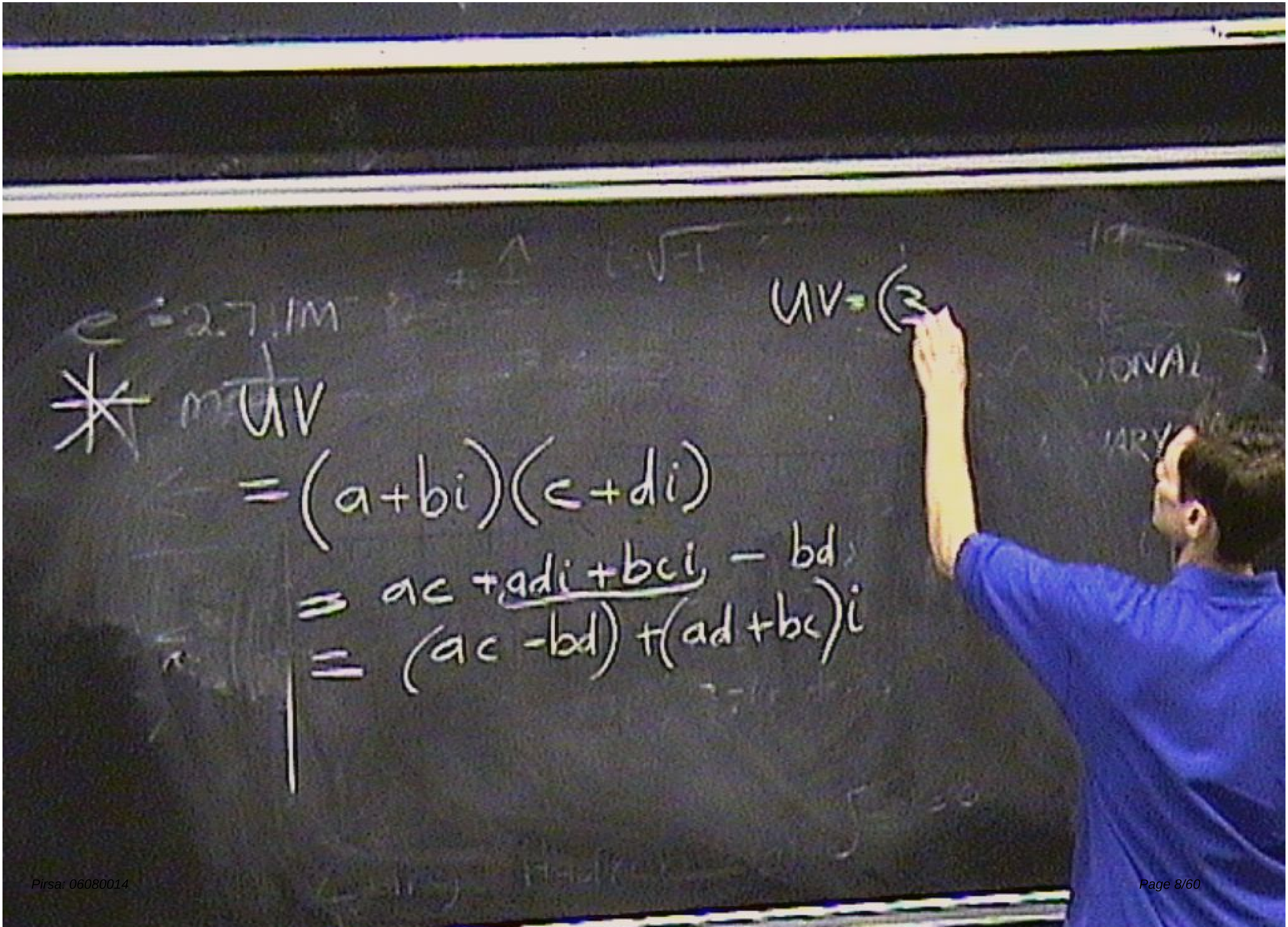
$$u = 3 + 4i$$

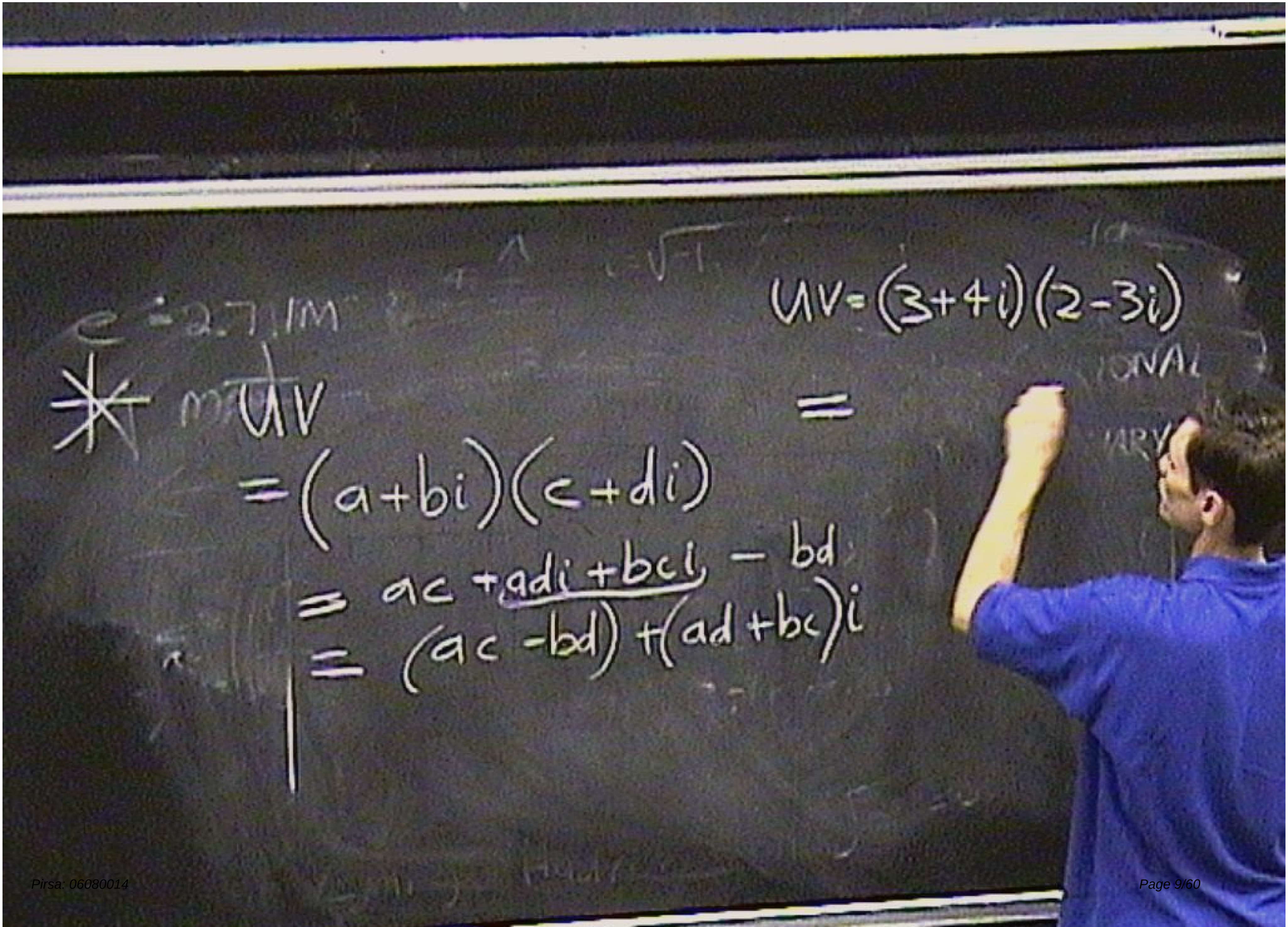
$$v = 2 - 3i$$

$$u + v = (3+2) + (4-3)i = 5 + i$$


uv
=(a+bi)(c+

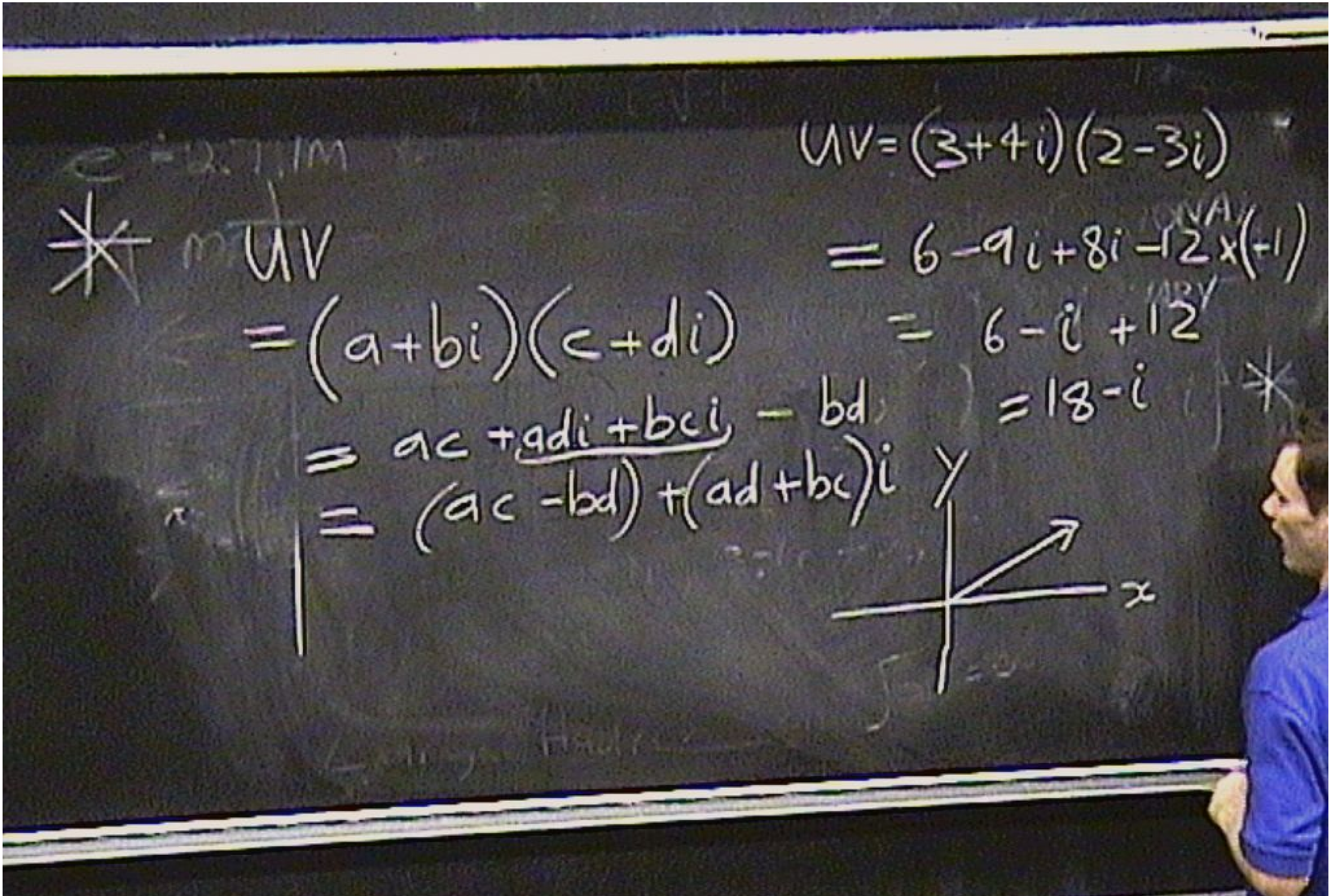

uv
= (a+bi)(c+di)
= ac + adi + bci + bd × (-1)
=


uv = (a
uv
= (a+bi)(c+di)
= ac + adi + bci − bd
= (ac − bd) + (ad + bc)i


uv = (3+4i)(2-3i)
=
uv
= (a+bi)(c+di)
= ac + adi + bci − bd
= (ac − bd) + (ad + bc)i

$$
\begin{aligned}
* \quad uv &= (a+bi)(c+di) \\
&= ac + \underline{adi + bci} - bd \\
&= (ac-bd) + (ad+bc)i
\end{aligned}
$$

$$
\begin{aligned}
uv &= (3+4i)(2-3i) \\
&= 6 - 9i + 8i - 12 \times (i^2) \\
&= 6 - 9
\end{aligned}
$$


UV = (3+4i)(2-3i)
= 6 - 9i + 8i - 12 x (-1)
= 6 - i + 12
= 18 - i
UV
= (a+bi)(c+di)
= ac + adi + bci - bd
= (ac - bd) + (ad + bc)i
y
x

$$uv = (3+4i)(2-3i)$$
$$= 6 - 9i + 8i - 12 \times (-1)$$
$$= 6 - i + 12$$
$$= 18 - i$$

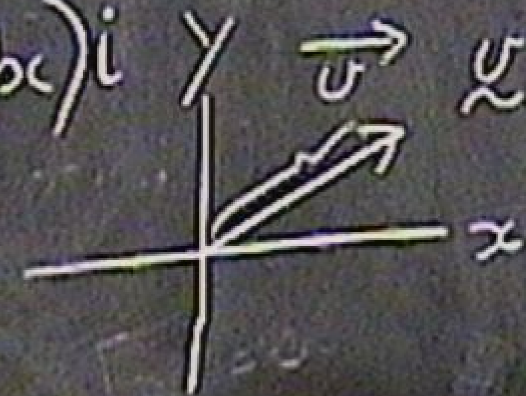
$$* \quad uv = (a+bi)(c+di)$$
$$= ac + adi + bci - bd$$
$$= (ac - bd) + (ad + bc)i$$

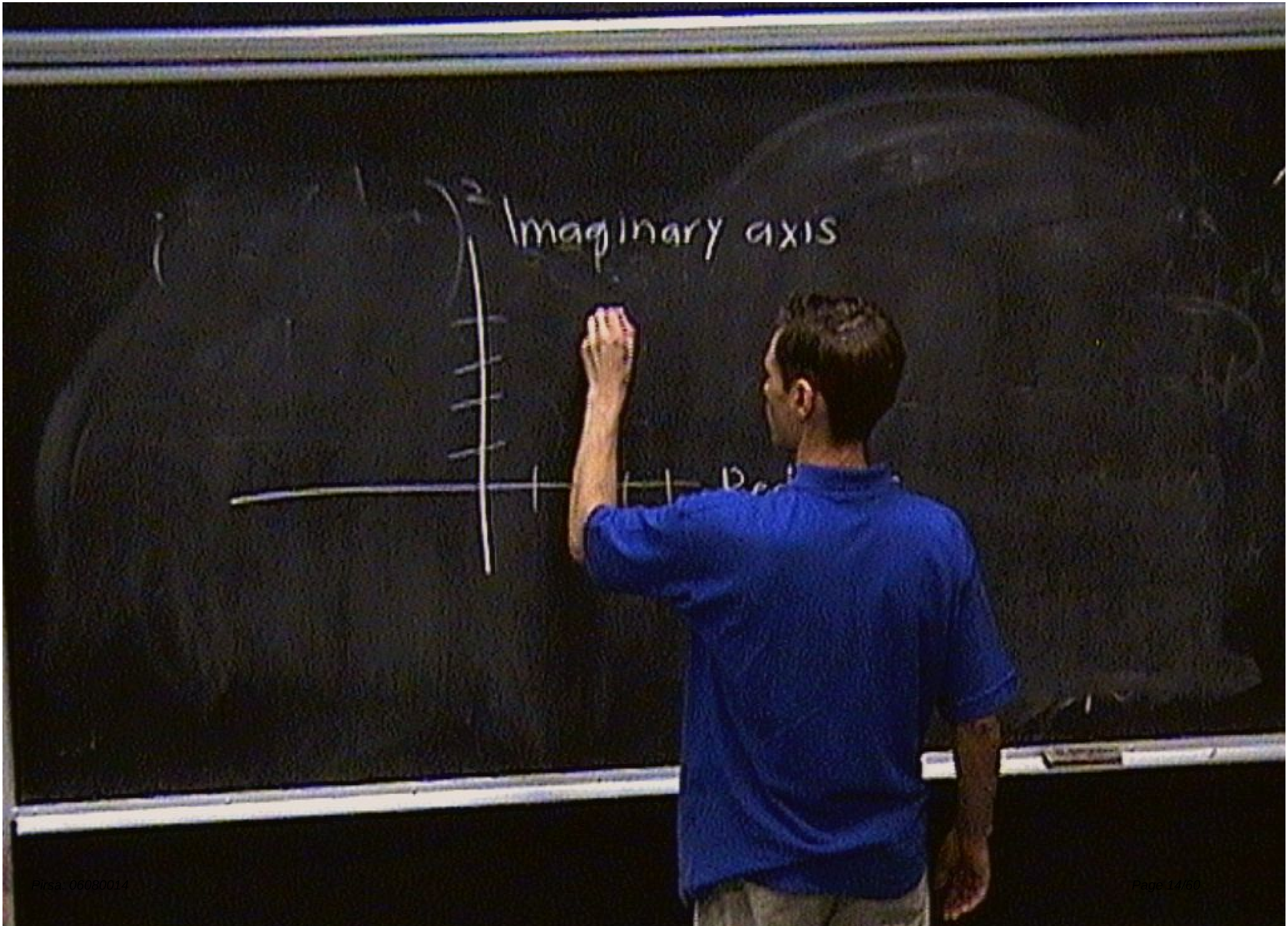
$\vec{v}$ $\underset{\sim}{v}$ $\tilde{v}$

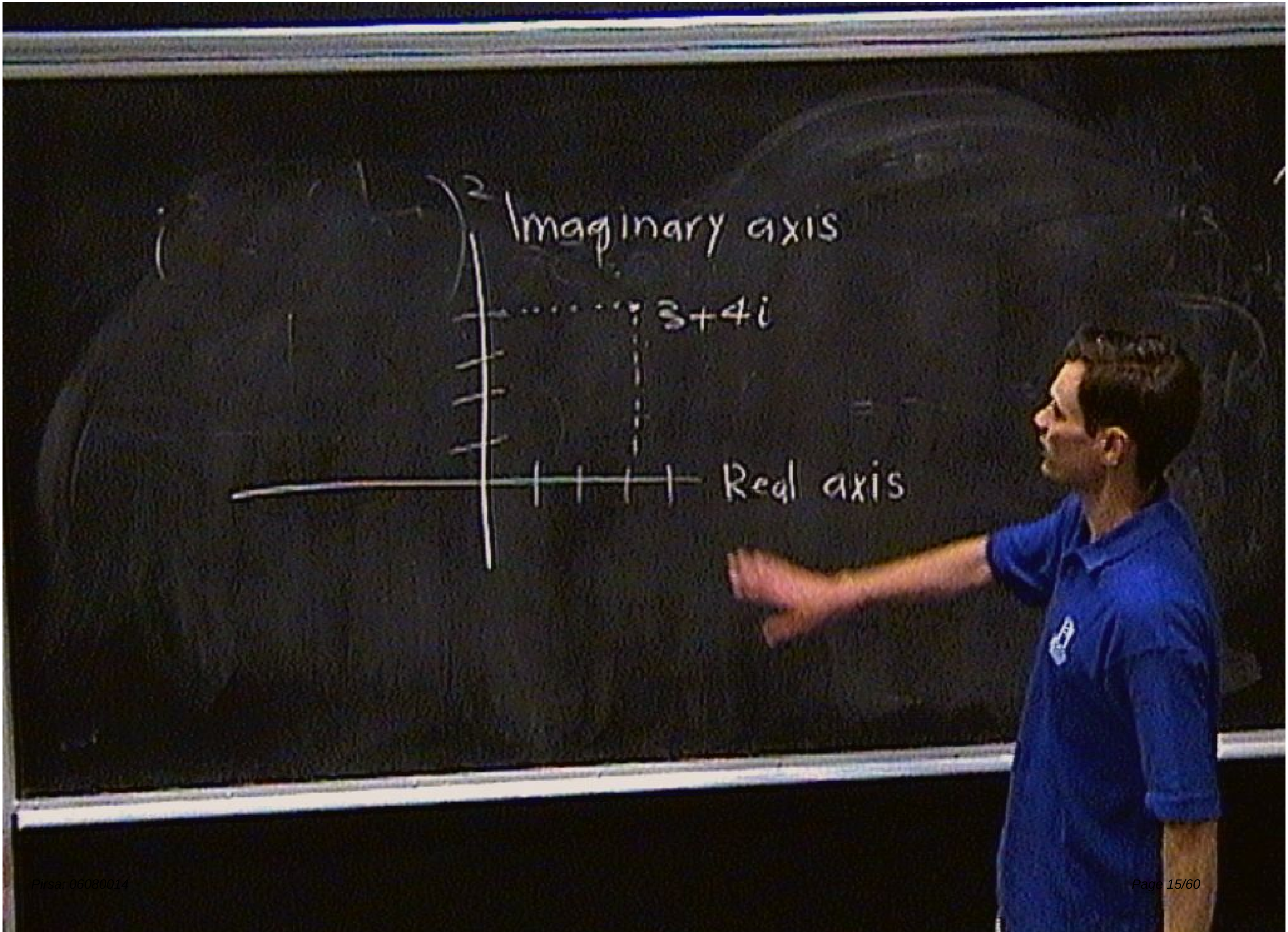
$$uv = (3+4i)(2-3i)$$
$$= 6 - 9i + 8i - 12 \times (-1)$$
$$= 6 - i + 12$$
$$= 18 - i$$

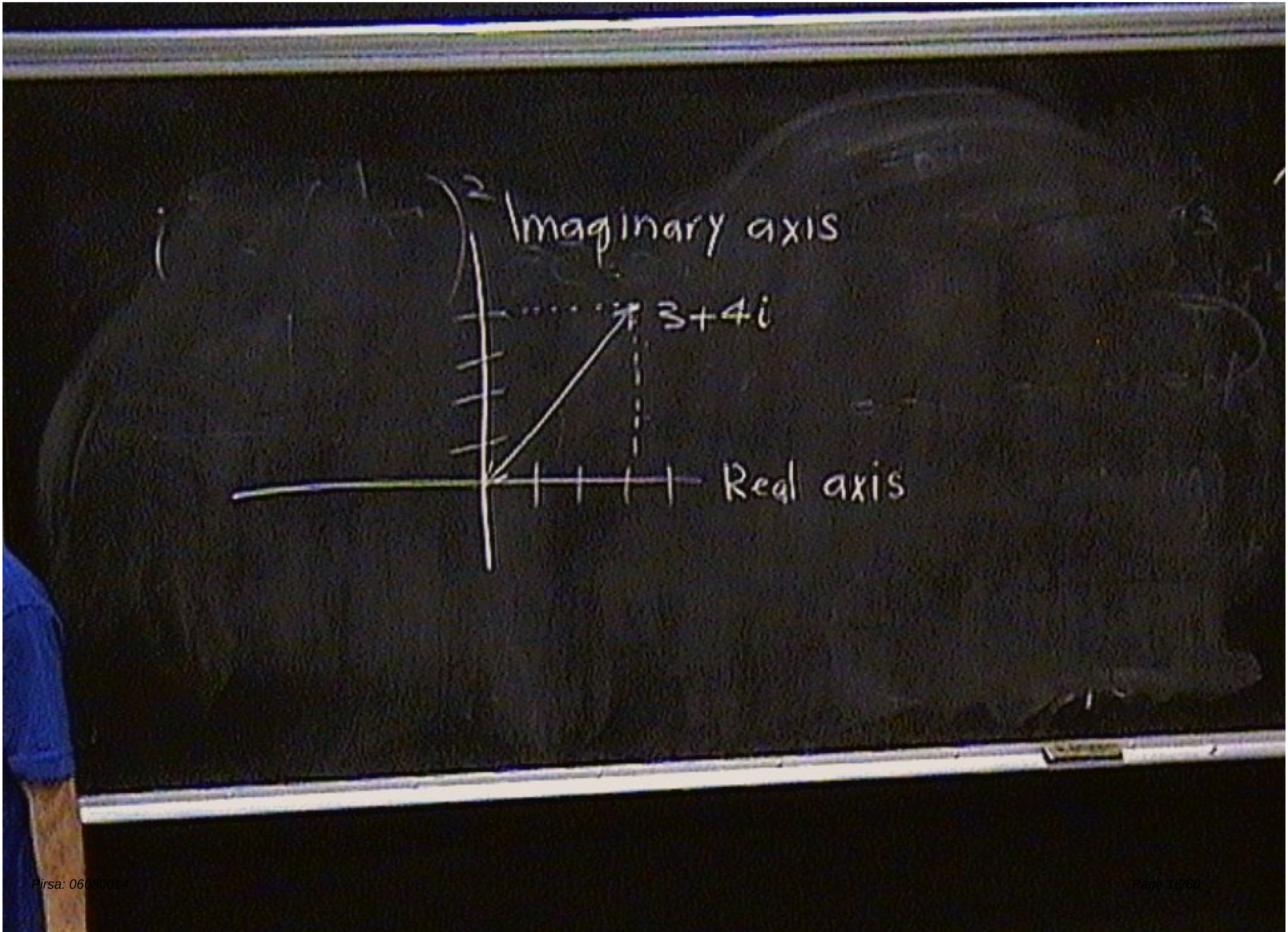
$$* \quad uv$$
$$= (a+bi)(c+di)$$
$$= ac + \underline{adi + bci} - bd$$
$$= (ac - bd) + (ad + bc)i$$
$$=$$

$\vec{v}$ $\quad \underset{\sim}{v}$ $\quad \tilde{v}$



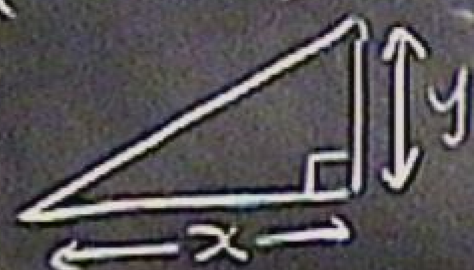

Imaginary axis


Imaginary axis
3+4i
Real axis

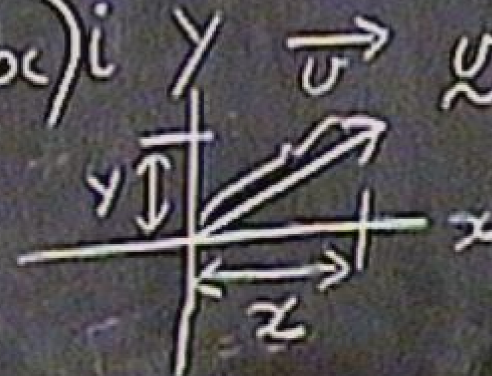

Imaginary axis
3+4i
Real axis

$$UV = (a+bi)(c+di)$$
$$= ac + \underline{adi + bci} - bd$$
$$= (ac - bd) + (ad+bc)i$$

$$= 6 - 9i + 8i - 12 \times (-1)$$
$$= 6 - i + 12$$
$$= 18 - i$$



$\vec{v}$ $\underset{\sim}{v}$ $\tilde{v}$



$= (a+bi)(c+di)$

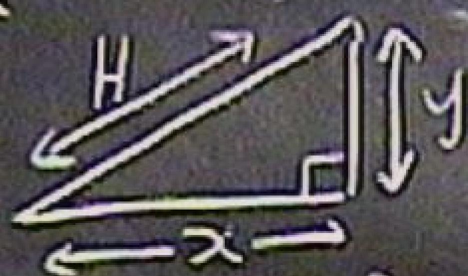
$= ac + adi + bci - bd$

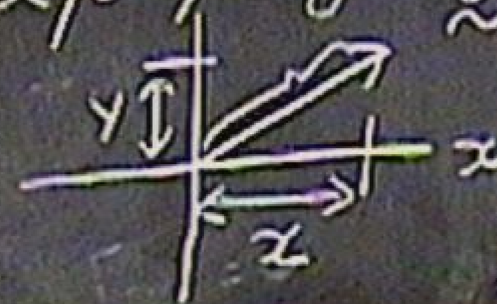
$= (ac - bd) + (ad + bc)i$

$= 6 - i + 12$

$= 18 - i$

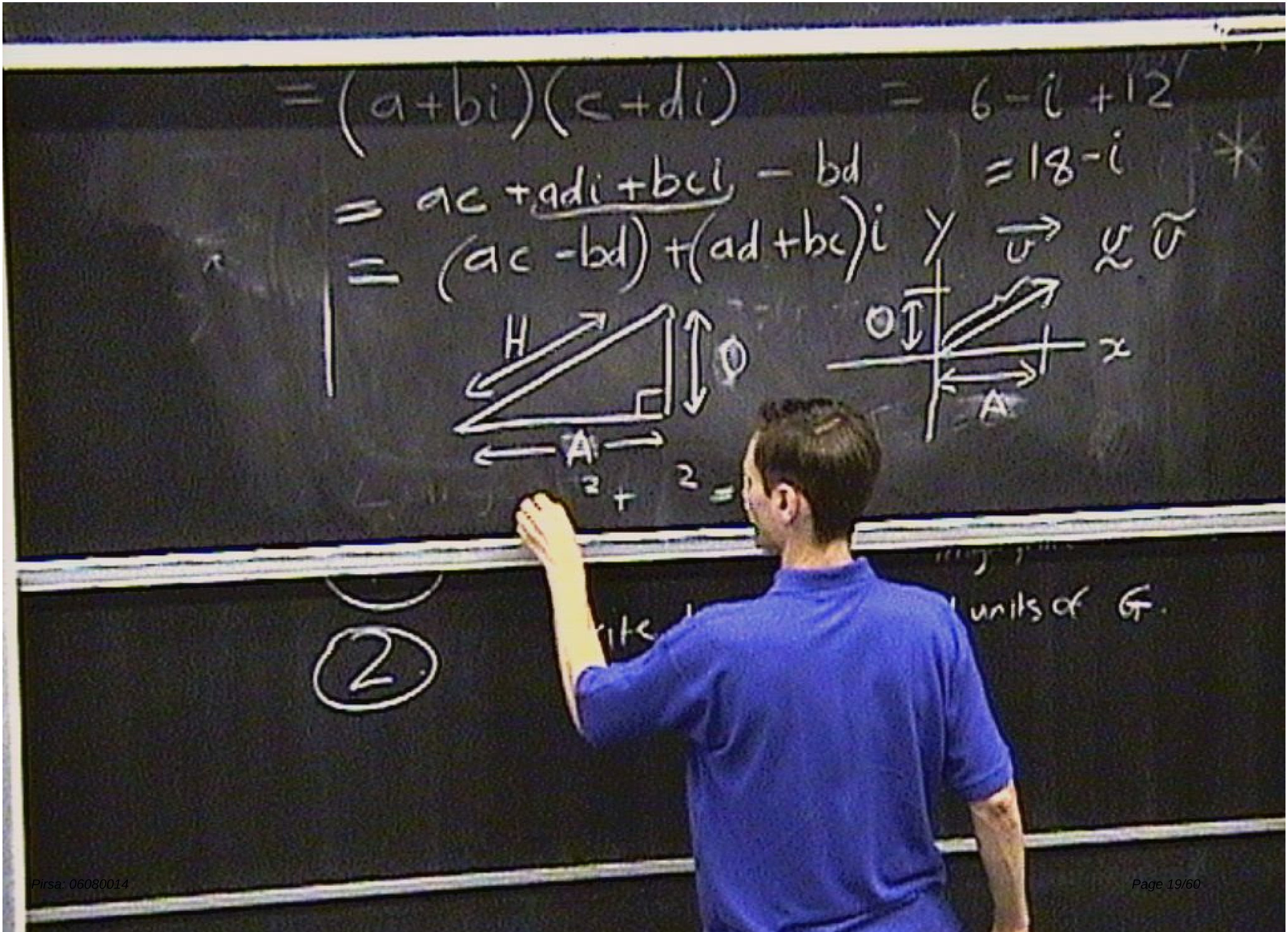
$\vec{v}$ $\underset{\sim}{v}$ $\tilde{v}$





$x^2 + y^2 = H^2$

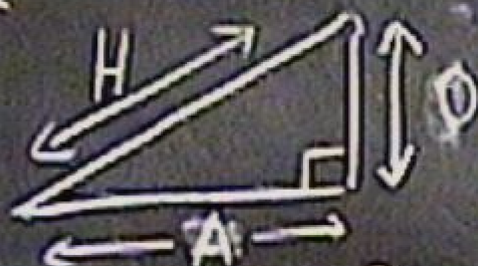
(2) Write down the standard units of G. (SI)

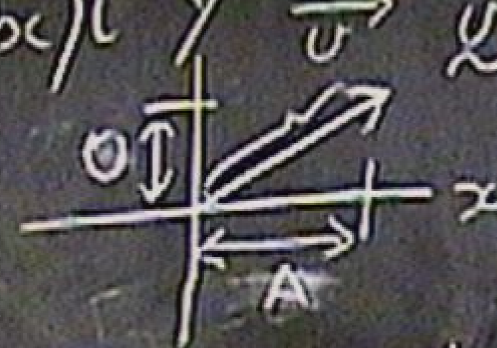

= (a+bi)(c+di)
= 6 - i + 12
= 18 - i
= ac + adi + bci - bd
= (ac - bd) + (ad + bc)i
y
x
H
O
A
units of G.
2

$$= (a+bi)(c+di) \qquad = 6 - i + 12$$

$$= ac + \underbrace{adi + bci} - bd \qquad = 18 - i$$

$$= (ac - bd) + (ad + bc)i \qquad \vec{v} \ \& \ \tilde{v}$$





$$\therefore H = \sqrt{O^2 + A^2}$$

$A^2 + O^2 = H^2$ Pythagorus' theorem

(2) Write down the standard units of G. (SI)

magnitude

$$* \quad |uv|$$

$$= (a+bi)(c+di)$$

$$= ac + \underline{adi + bci} - bd$$

$$= (ac-bd) + (ad+bc)i$$

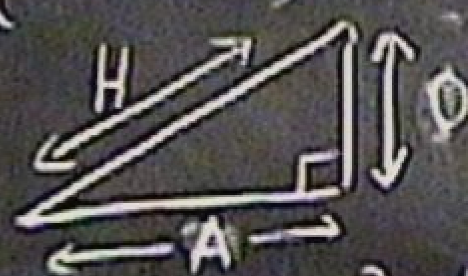
$$uv = (3+4i)(2-3i)$$

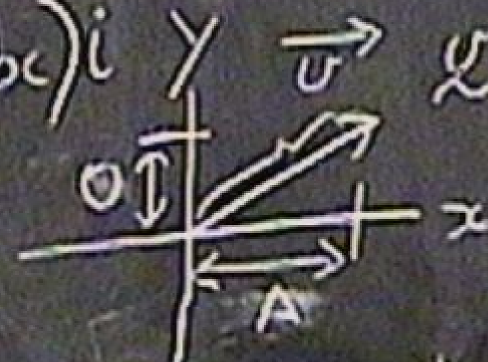
$$= 6 - 9i + 8i - 12 \times (-1)$$

$$= 6 - i + 12$$

$$= 18 - i$$

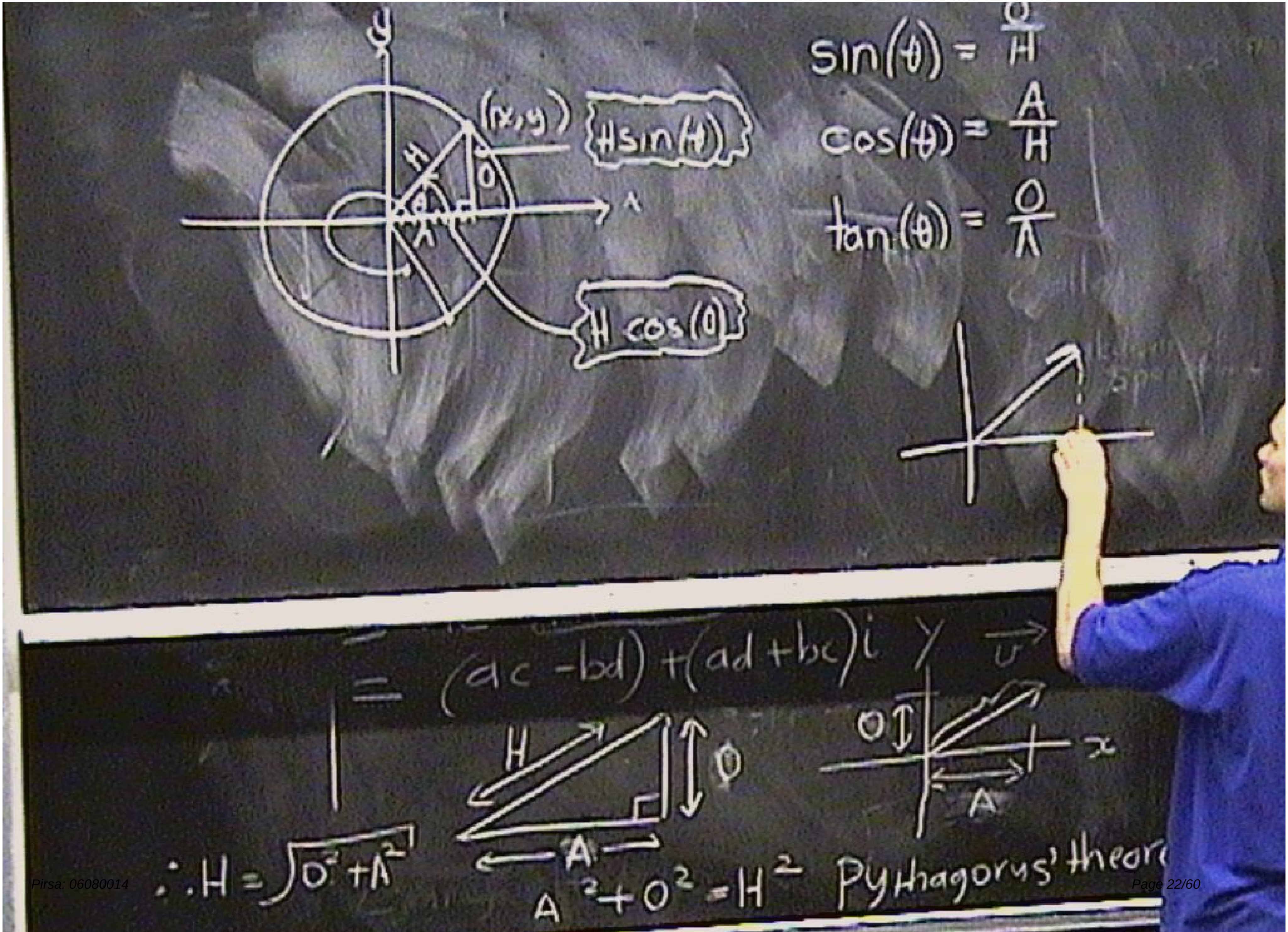
$\vec{u}$ & $\vec{v}$

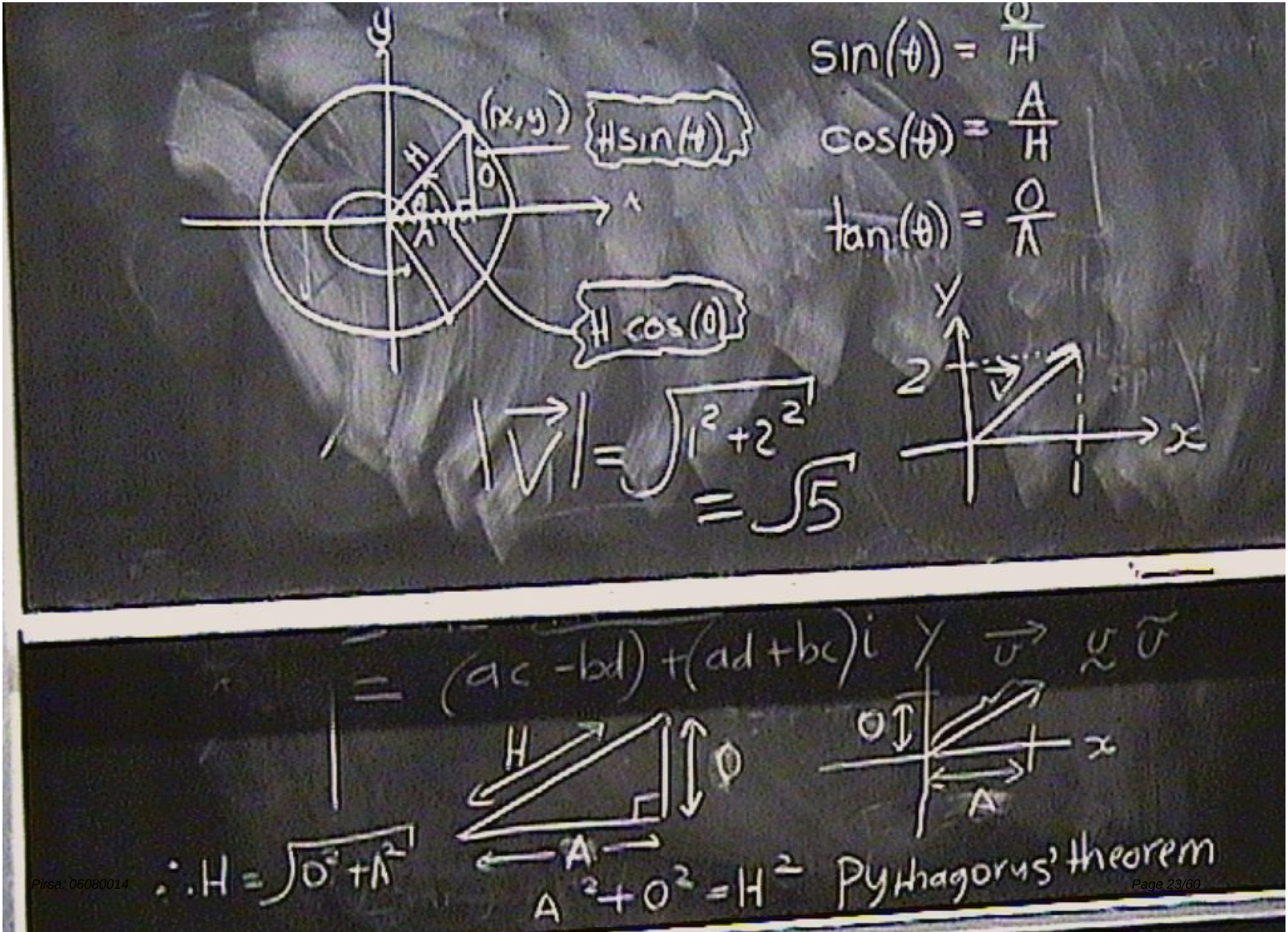


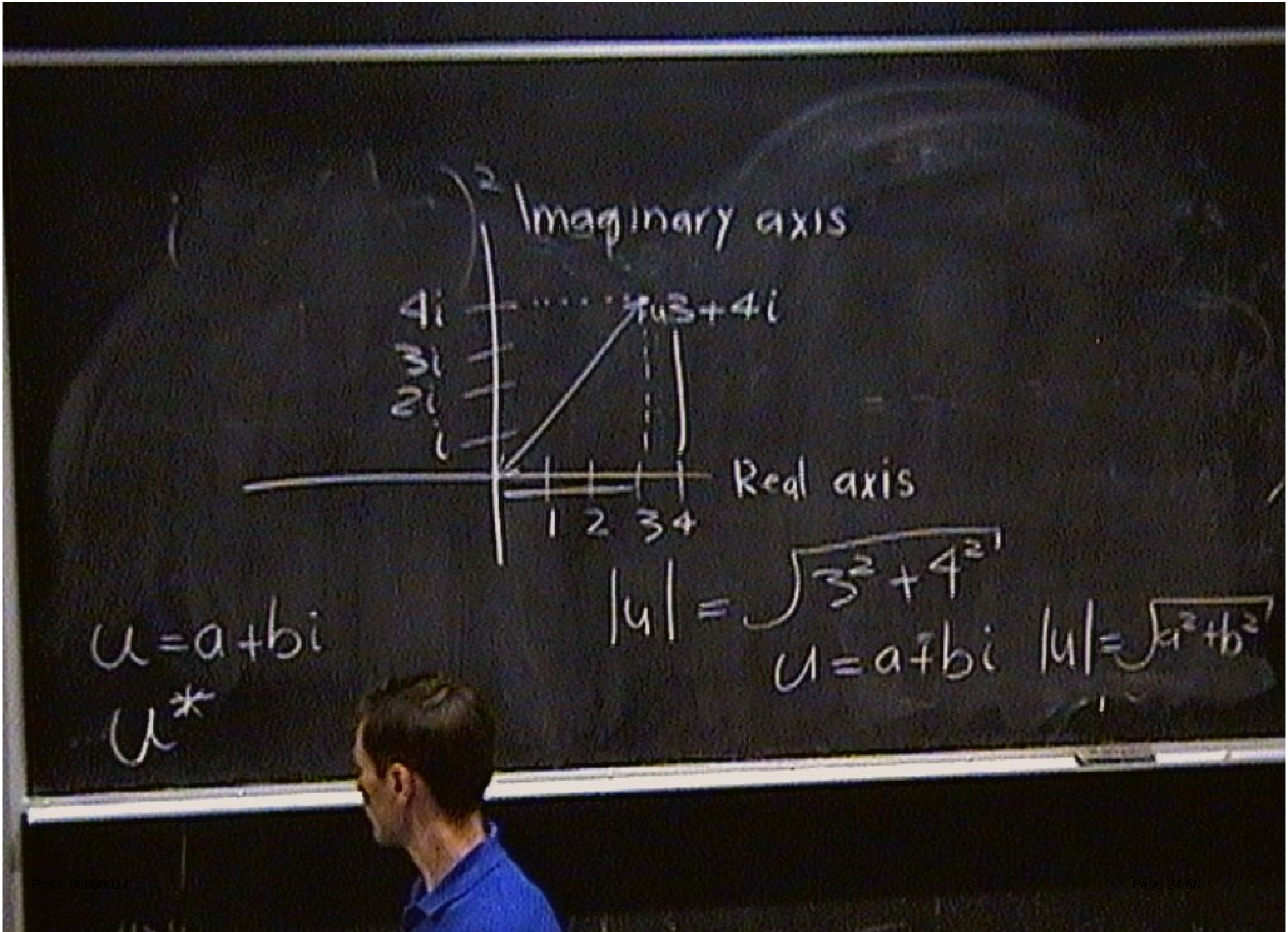


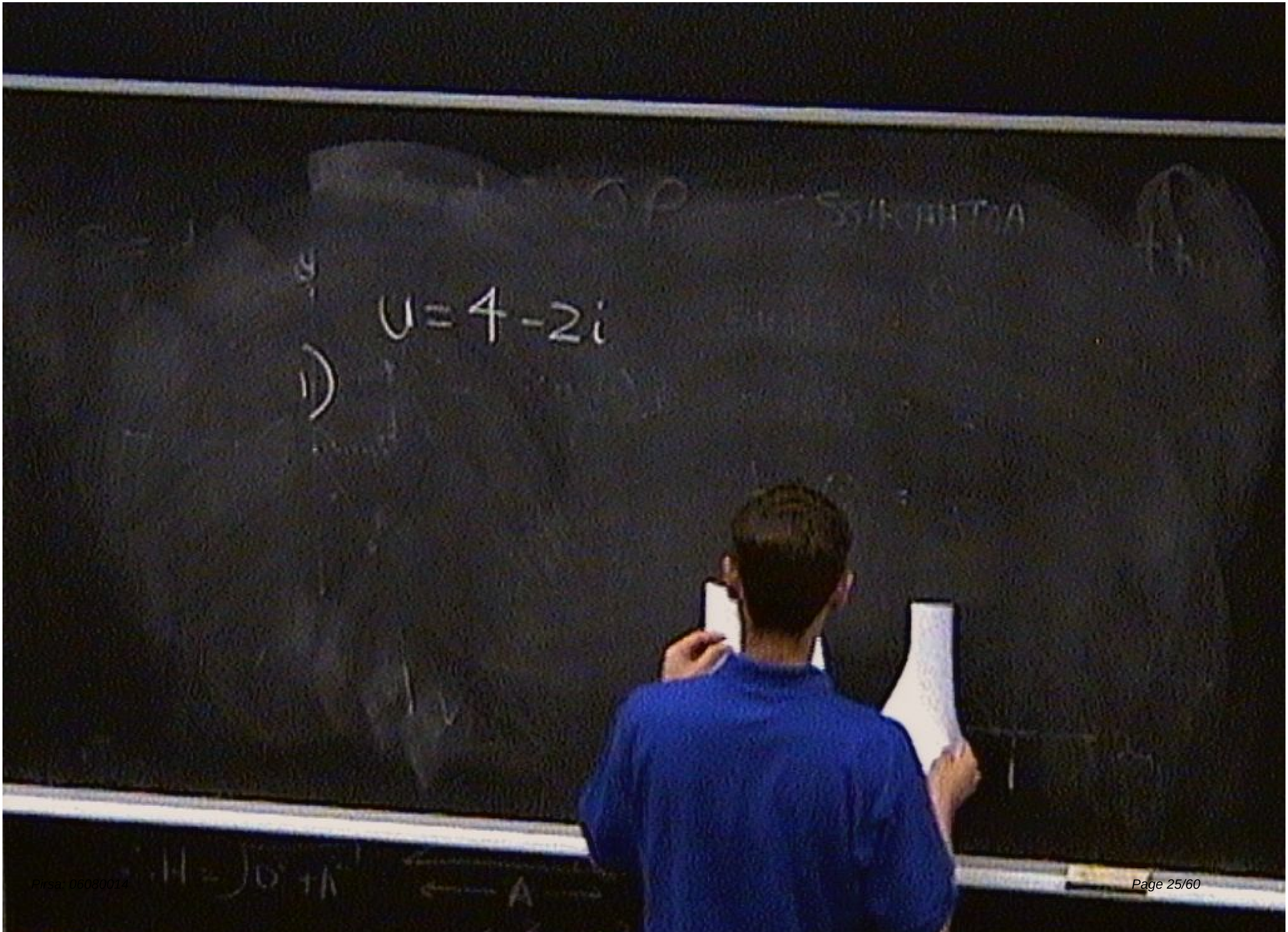
$$\therefore H = \sqrt{O^2 + A^2}$$

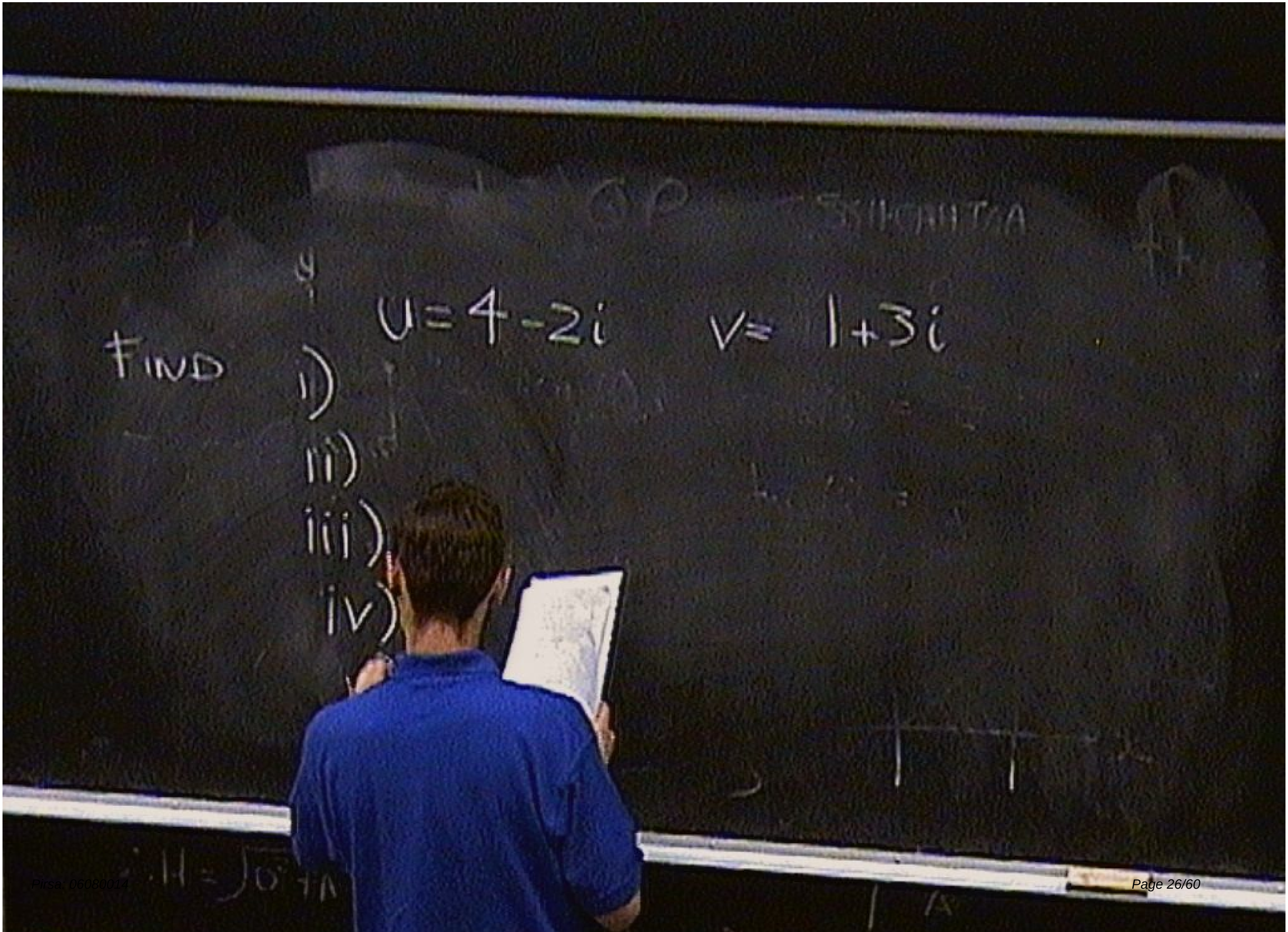
$$A^2 + O^2 = H^2$$ Pythagorus' theorem

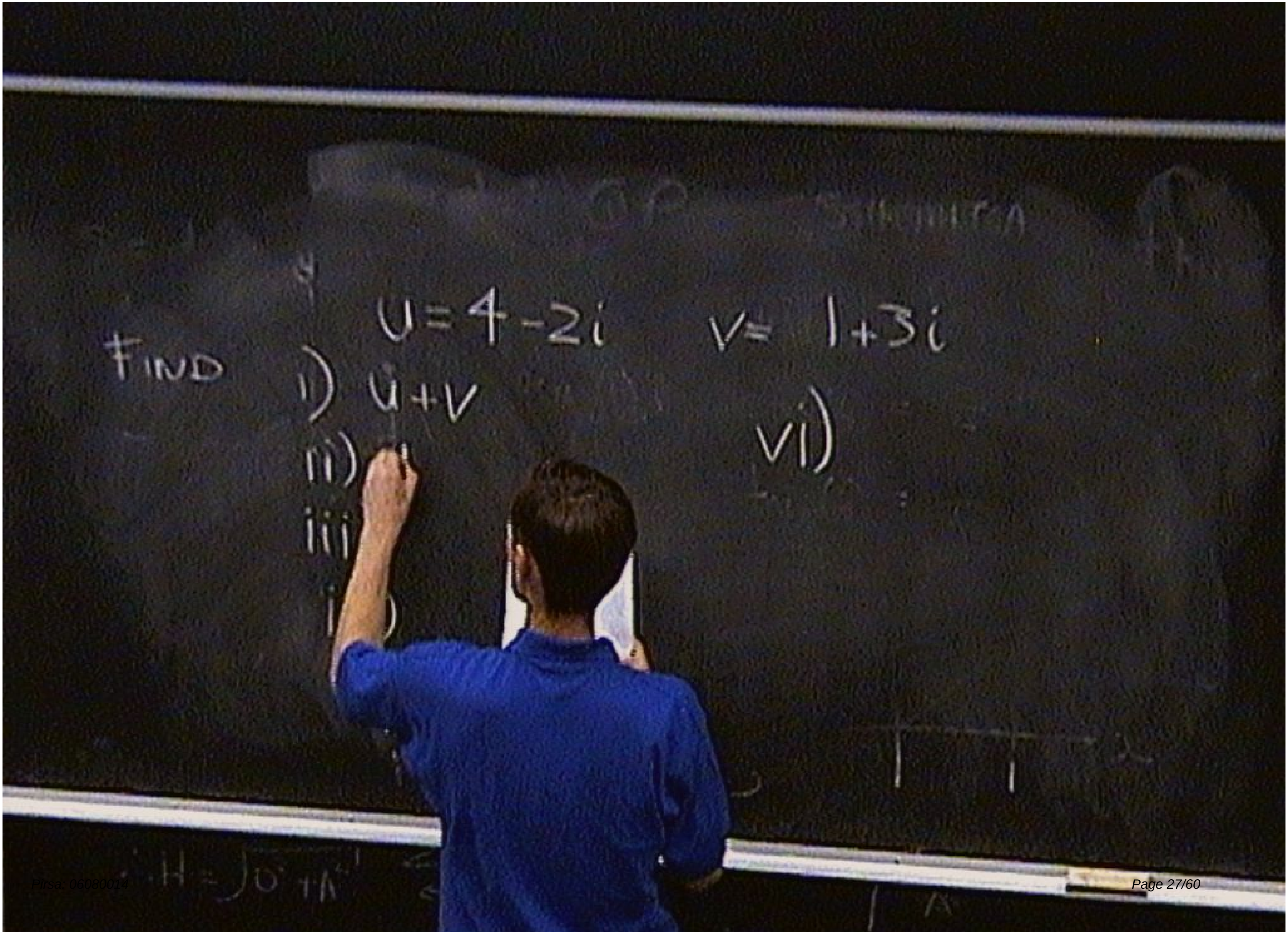

y
(x,y)
H sin(θ)
H cos(θ)
H
O
A
θ
sin(θ) = O/H
cos(θ) = A/H
tan(θ) = O/A
= (ac - bd) + (ad + bc)i
y
x
∴ H = √(O² + A²)
A² + O² = H²
Pythagorus' theor


sin(θ) = O/H
cos(θ) = A/H
tan(θ) = O/A
(x,y)
H sin(θ)
H cos(θ)
|v| = √(1² + 2²)
= √5
(ac - bd) + (ad + bc)i
∴ H = √(O² + A²)
A² + O² = H²
Pythagorus' theorem


Imaginary axis
4i
3i
2i
i
u 3+4i
Real axis
1 2 3 4
|u| = √(3² + 4²)
u=a+bi
|u|=√(a²+b²)
u=a+bi
u*


u = 4 - 2i


u=4-2i
v= 1+3i
FIND
i)
ii)
iii)
iv)


u=4-2i
v= 1+3i
FIND
i) u+v
ii)
iii
vi)

$u = 4 - 2i \qquad v = 1 + 3i$

FIND

i) $u + v$

ii) uv

iii) $|u|$

iv) $|v|$

v) u^*

vi) v^*

$$\boxed{\frac{|uv^*| \times v^* \times |(u+v)|}{|u + v^*|}}$$

$u = 4 - 2i \qquad v = 1 + 3i$

FIND

i) $u + v$

ii) uv

iii) $|u|$

iv) $|v|$

v) u^*

vi) v^*

EXTRA:

$$\frac{|uv^*| \times u^* \times |(u+v)|}{|u + v^*|}$$

$u = 4 - 2i \qquad v = 1 + 3i$

FIND

i) $u + v = 5 + i$ ✓

ii) $uv =$

iii) $|u|$

iv) $|v|$

v) u^*

vi) v^*

EXTRA:

$$\boxed{\frac{|u(v^*)| \times (v^*) \times |(u+v)|}{|u + v^*|}}$$

$u = 4 - 2i \qquad v = 1 + 3i$ ✓

FIND

i) $u + v = 5 + i$ ✓

ii) $uv = 10 + 10i$ ✓

iii) $|u| = 2\sqrt{5} = \sqrt{20}$ ✓

iv) $|v| = \sqrt{10}$ ✓

v) $u^* = 4 + 2i$

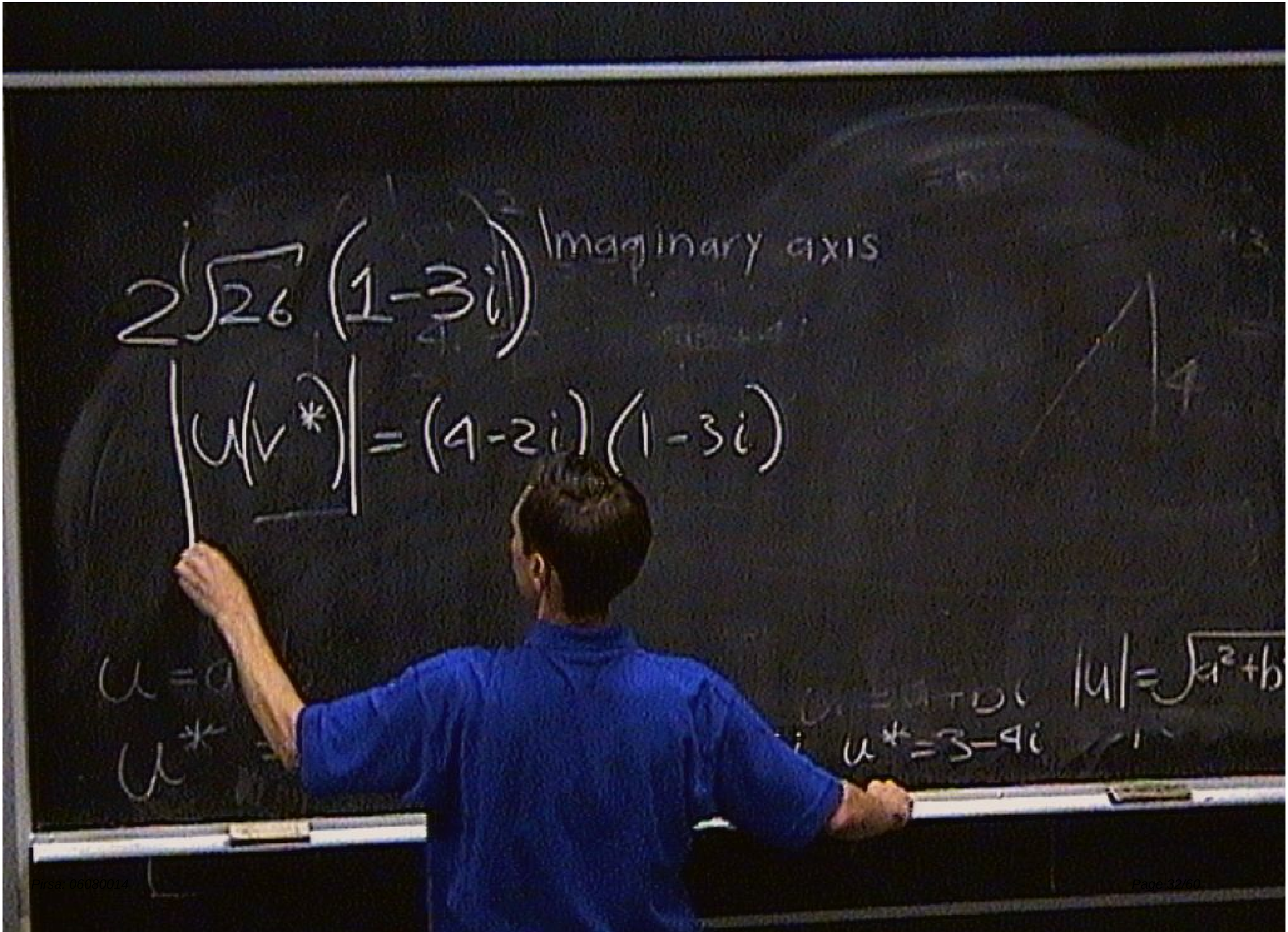
vi) $v^* = 1 - 3i$ ✓

EXTRA:

$$\frac{|u(v^*)| \times (v^*) \times |(u+v)|}{|u + v^*|}$$

$= (ac - bd) + (ad + bc)i \qquad \vec{v} \quad \underset{\sim}{v} \quad \tilde{v}$

$H \qquad \theta \qquad x$


2√26 (1-3i)
Imaginary axis
|u(v*)| = (4-2i)(1-3i)
|u| = √a²+b
u* = 3-4i

$$\boxed{2\sqrt{26}\,(1-3i)}$$

imaginary axis

$$v^* = 1-3i$$

$$|u+v|$$
$$= |5+i|$$
$$= \sqrt{5^2+1^2}$$
$$= \sqrt{26}$$

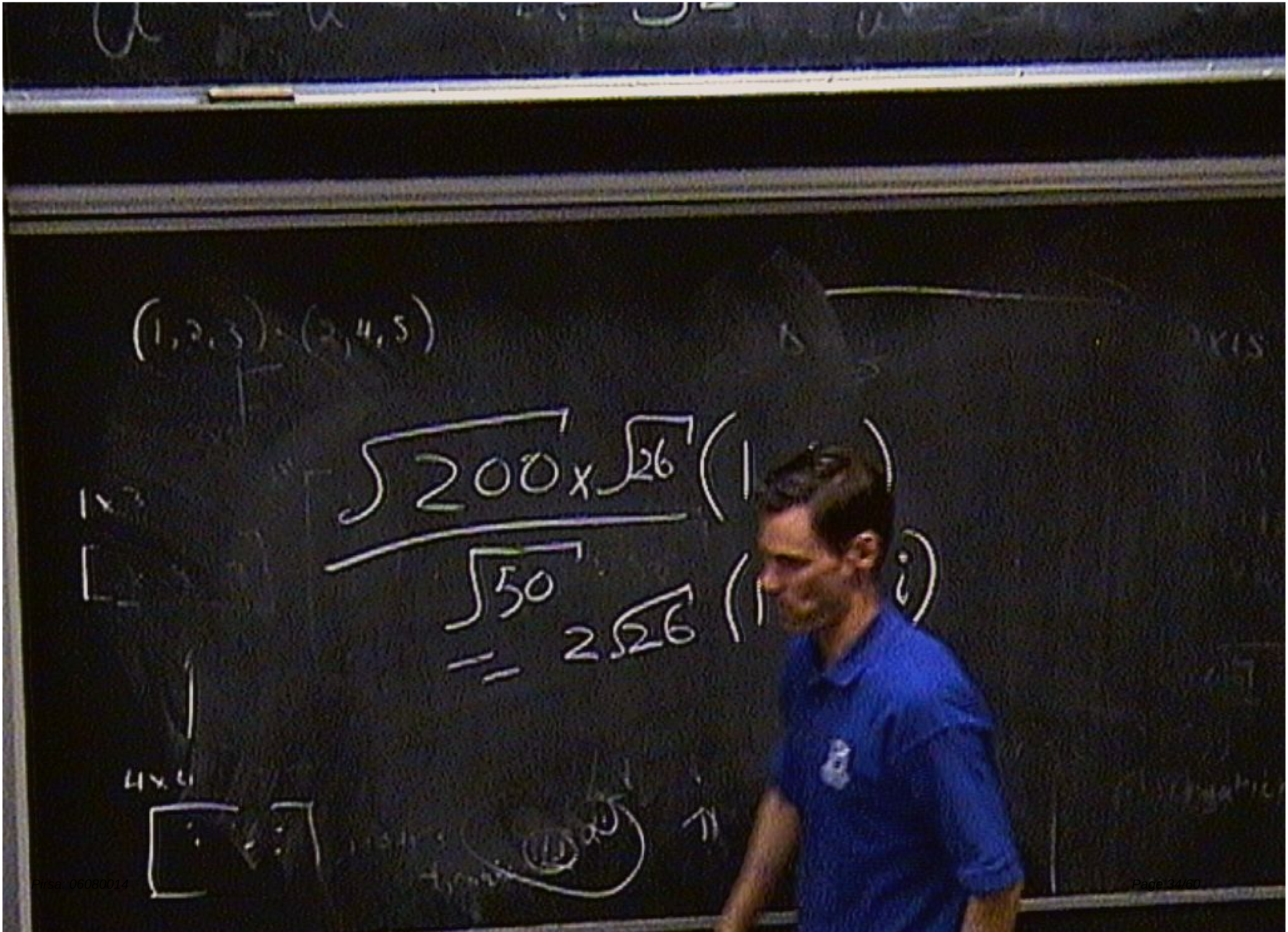
$$|u(v^*)| = |(4-2i)(1-3i)|$$
$$= |4-6-14i|$$
$$= |-2-14i|$$
$$= \sqrt{2^2+14^2} = \sqrt{200}$$

$u = a + \ldots$

$u^* = \bar{u} = \ldots$

$|u+v^*|$

$= |\ldots$


(1,2,3)·(2,4,5)
√200 x √26
√50
2√26

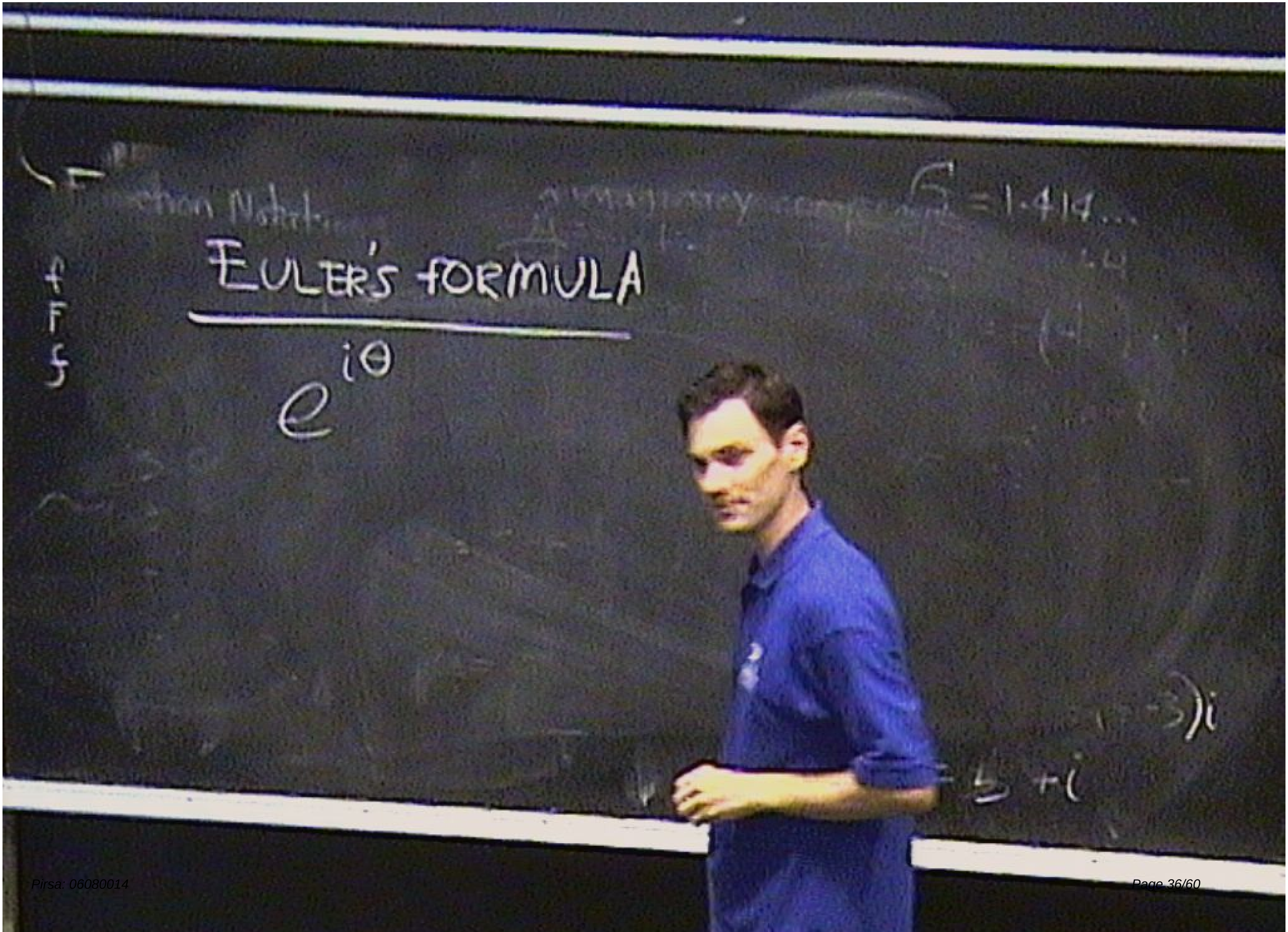
$(1,2,3)\cdot(2,4,5)$

1×3

$$\frac{\sqrt{200}\times\sqrt{26}}{\sqrt{50}}(1-3i)$$

$$= 2\sqrt{26}\,(1-3i)$$

4×4


EULER'S FORMULA
e^{iθ}

EULER'S FORMULA

$$e^{i\theta} = \cos\theta + i\sin\theta$$

EXERCISE : homeplay

$= 5 + i$

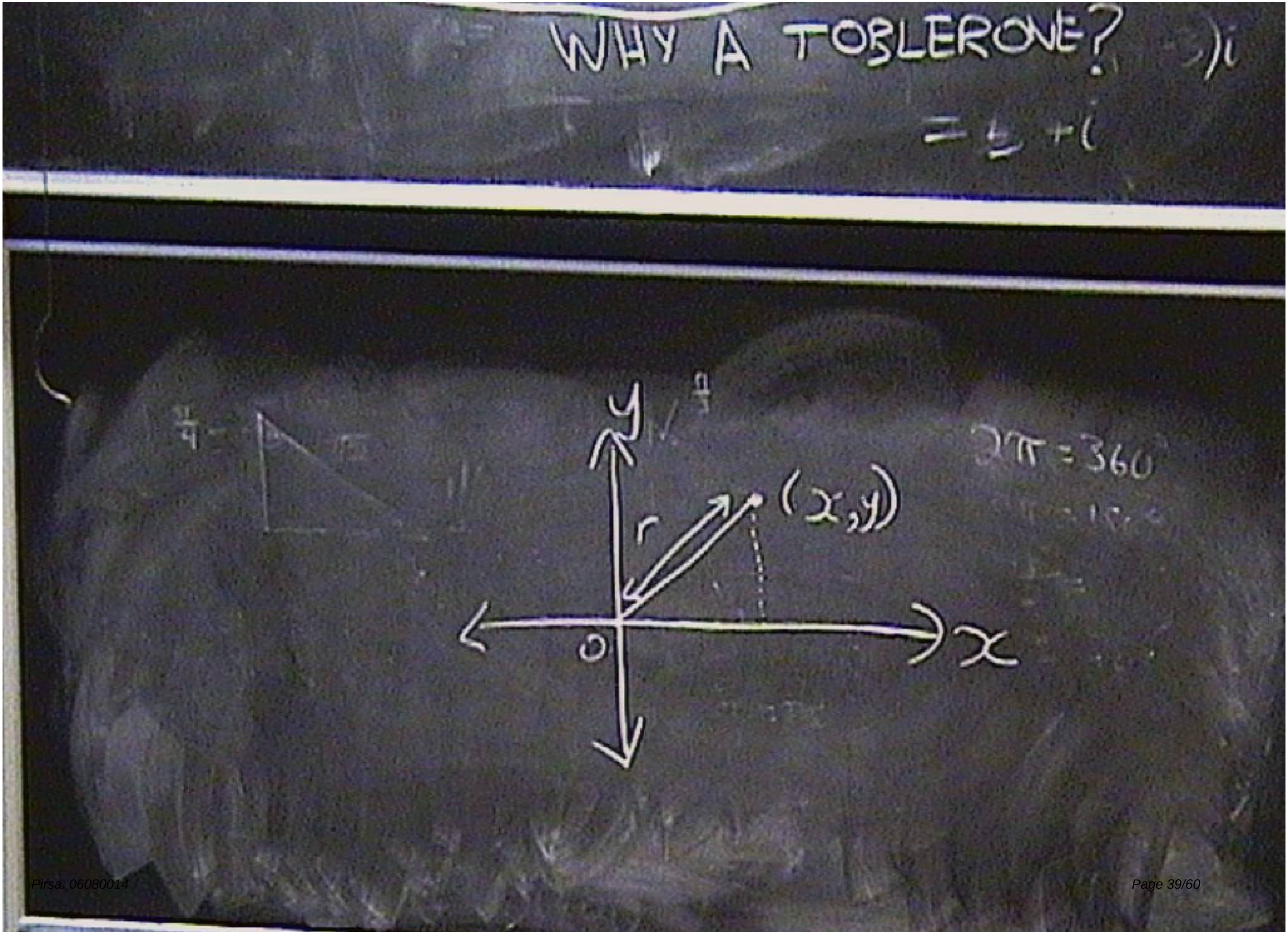
EULER'S FORMULA

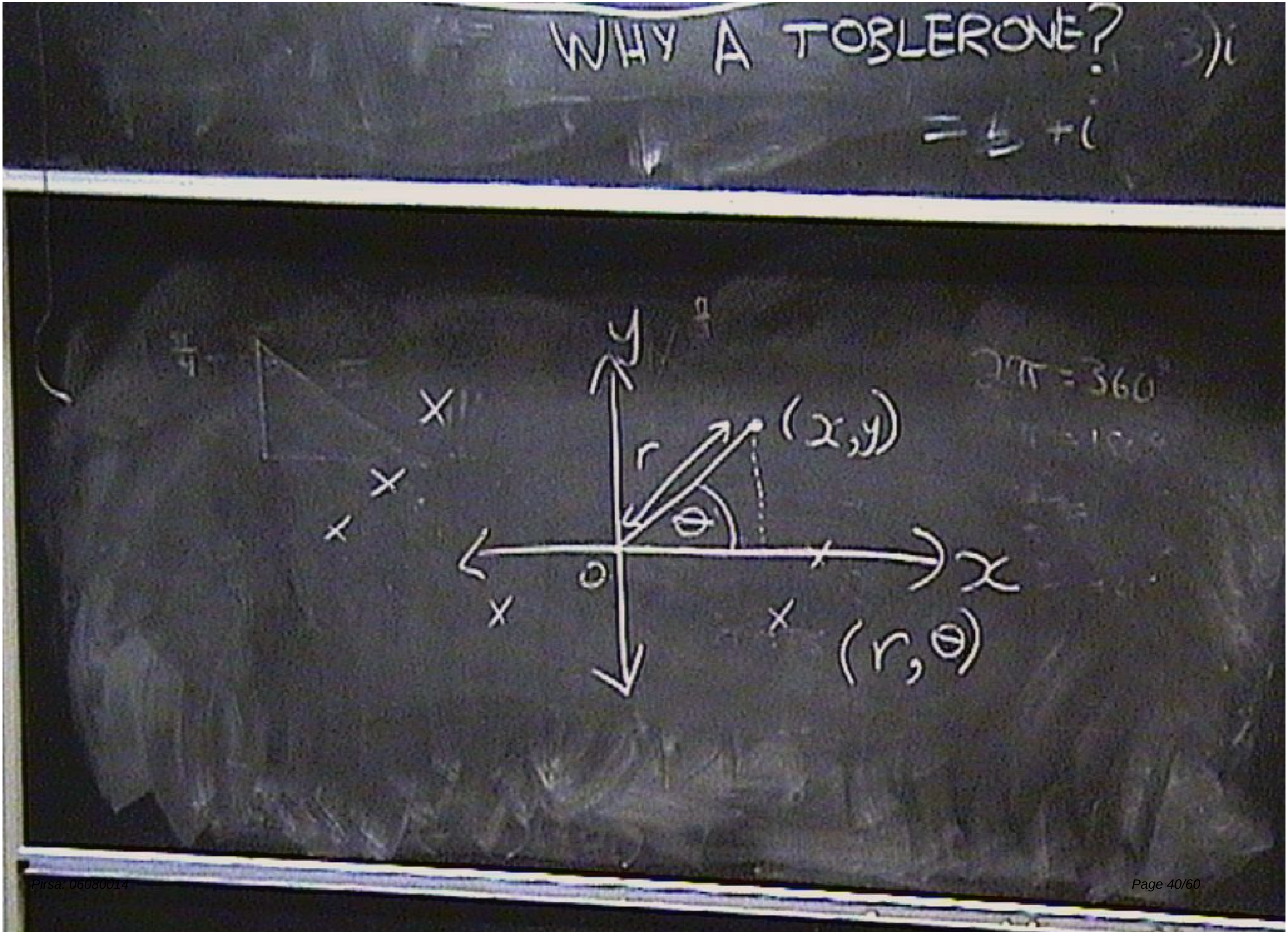
$$e^{i\theta} = \cos\theta + i\sin\theta$$

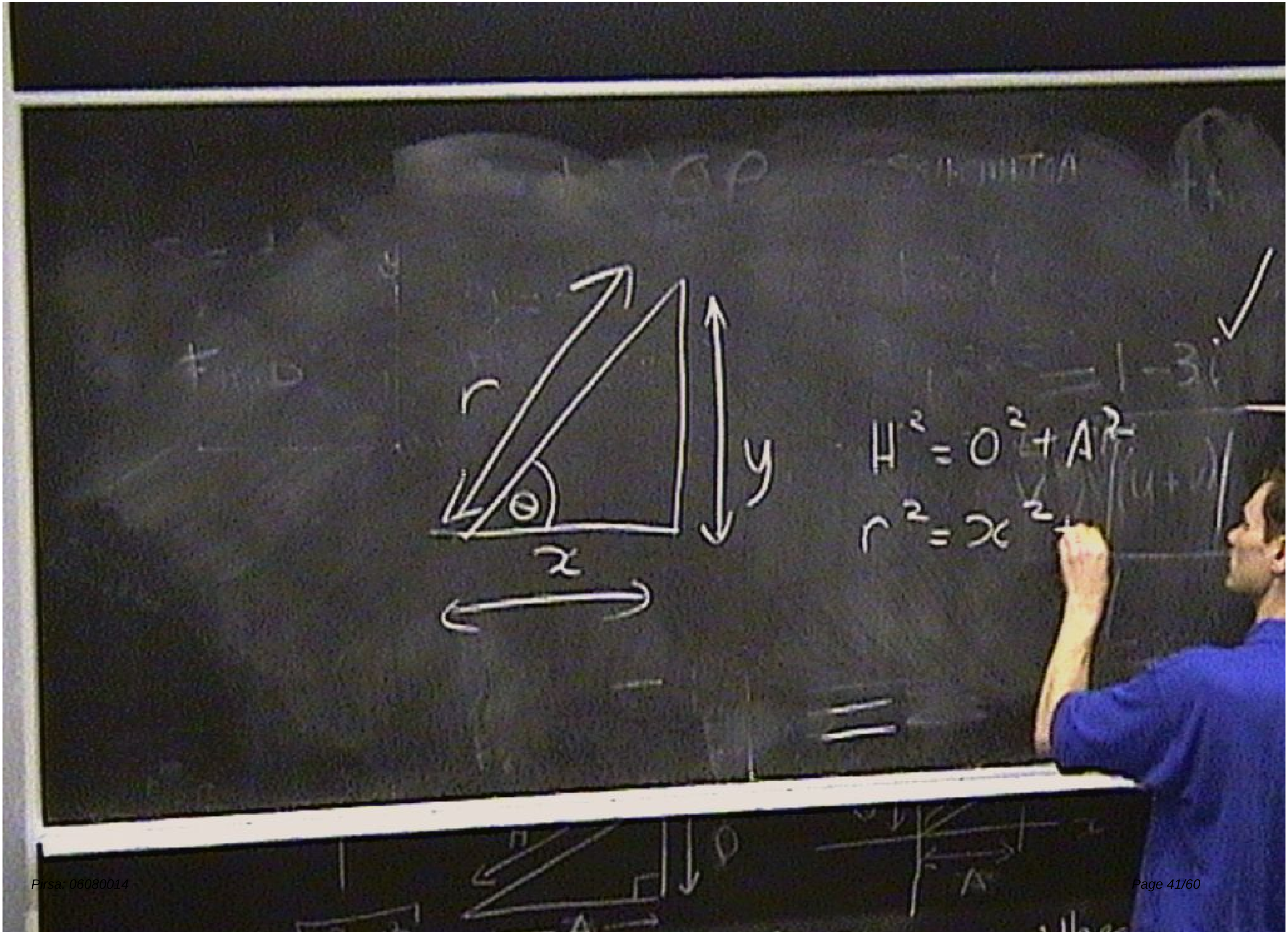
EXERCISE : homeplay

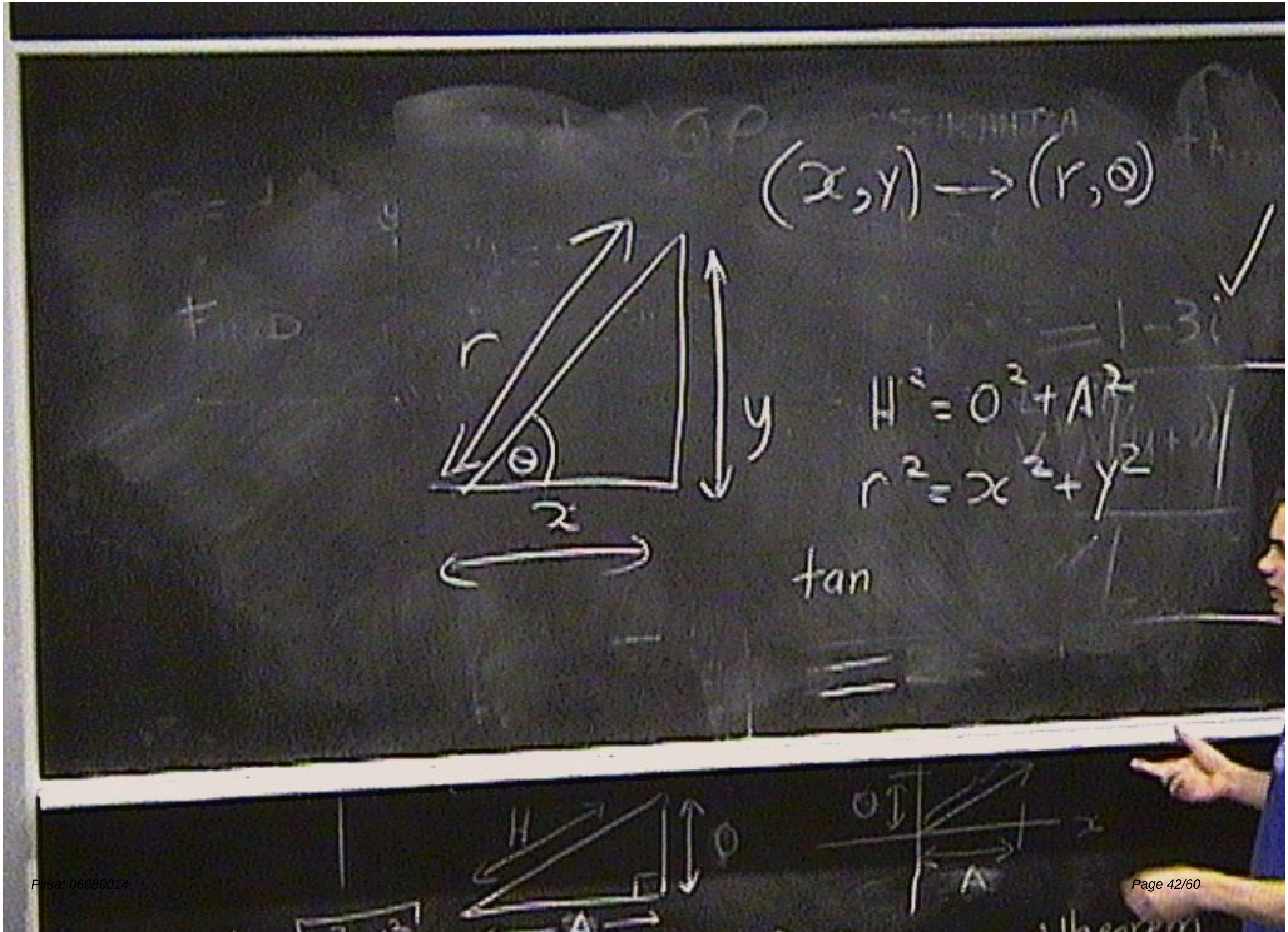
PROVE EULER'S FORMULA

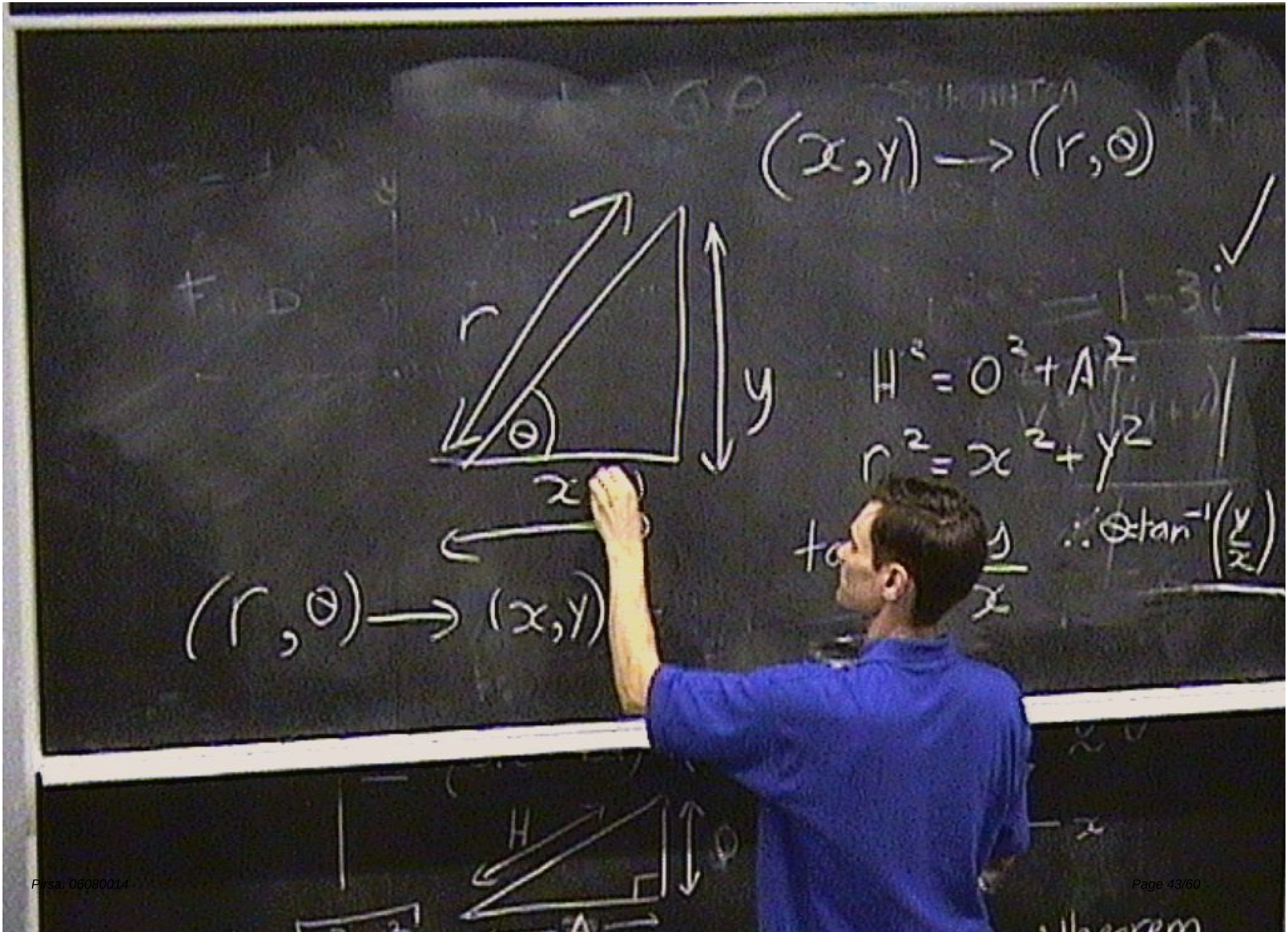
WHY A TOBLERONE?

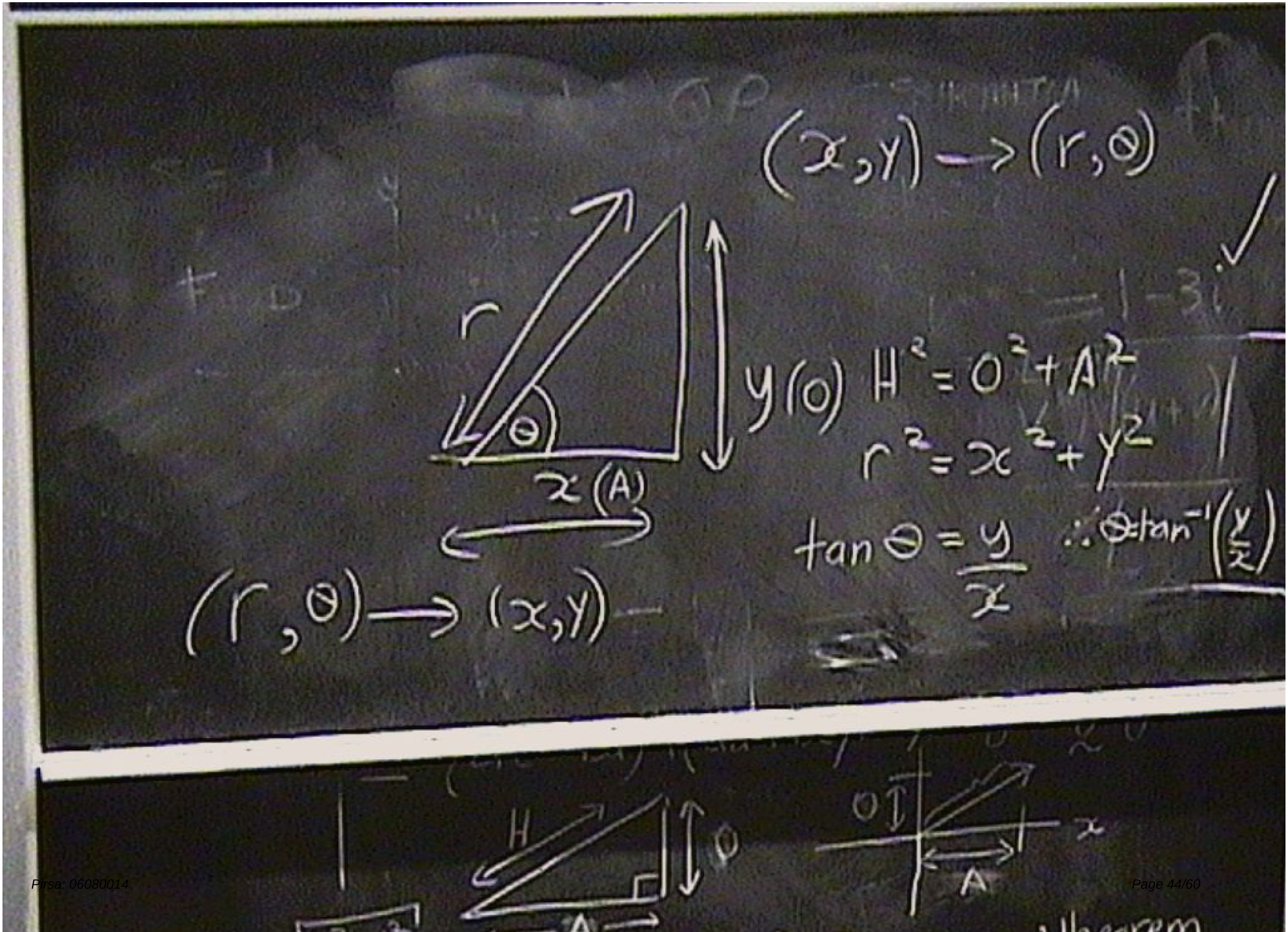

WHY A TOBLERONE?
y
(x,y)
r
0
x
2π = 360

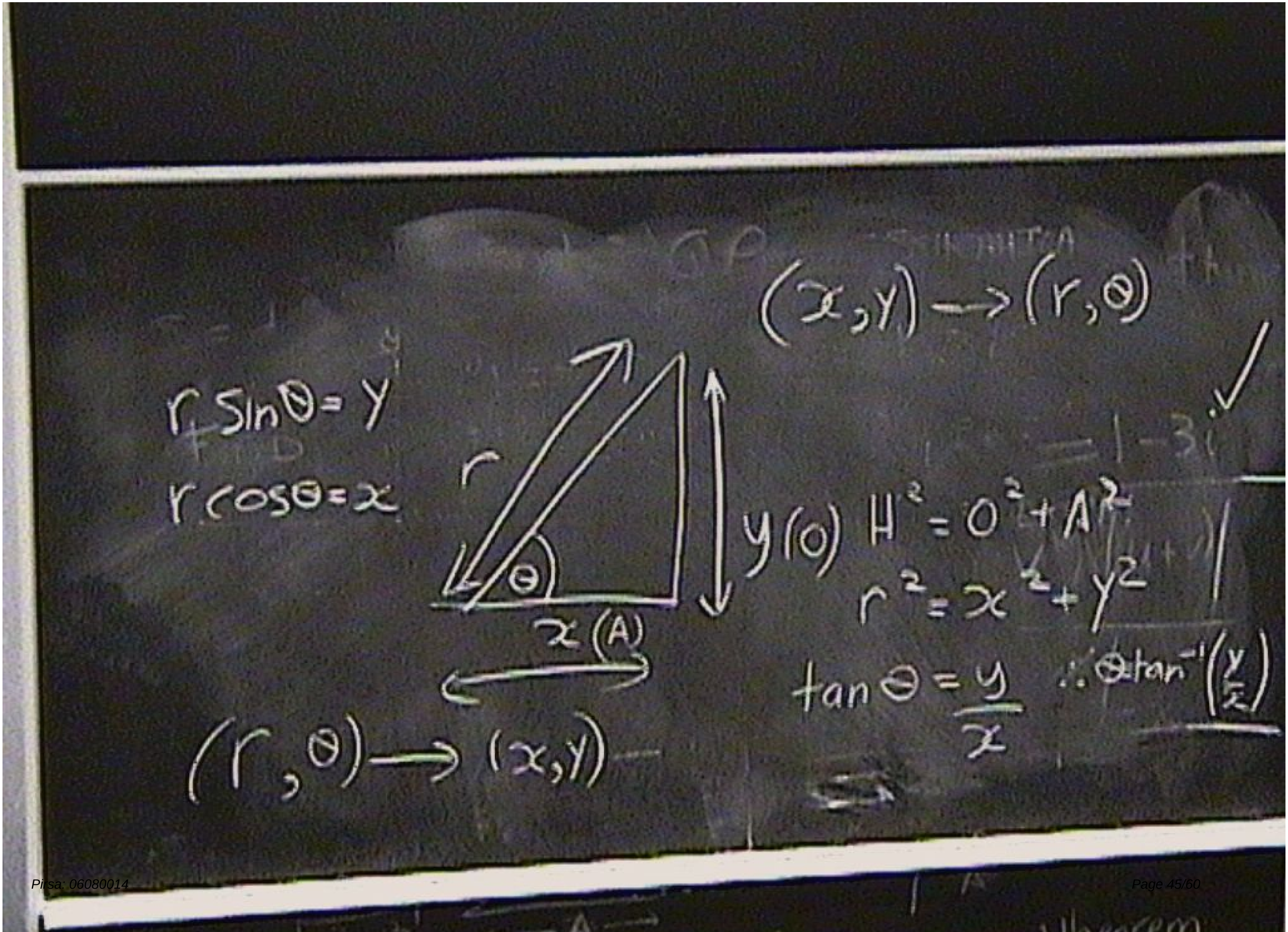

WHY A TOBLERONE?
y
(x,y)
r
θ
0
x
(r,θ)
2π = 360

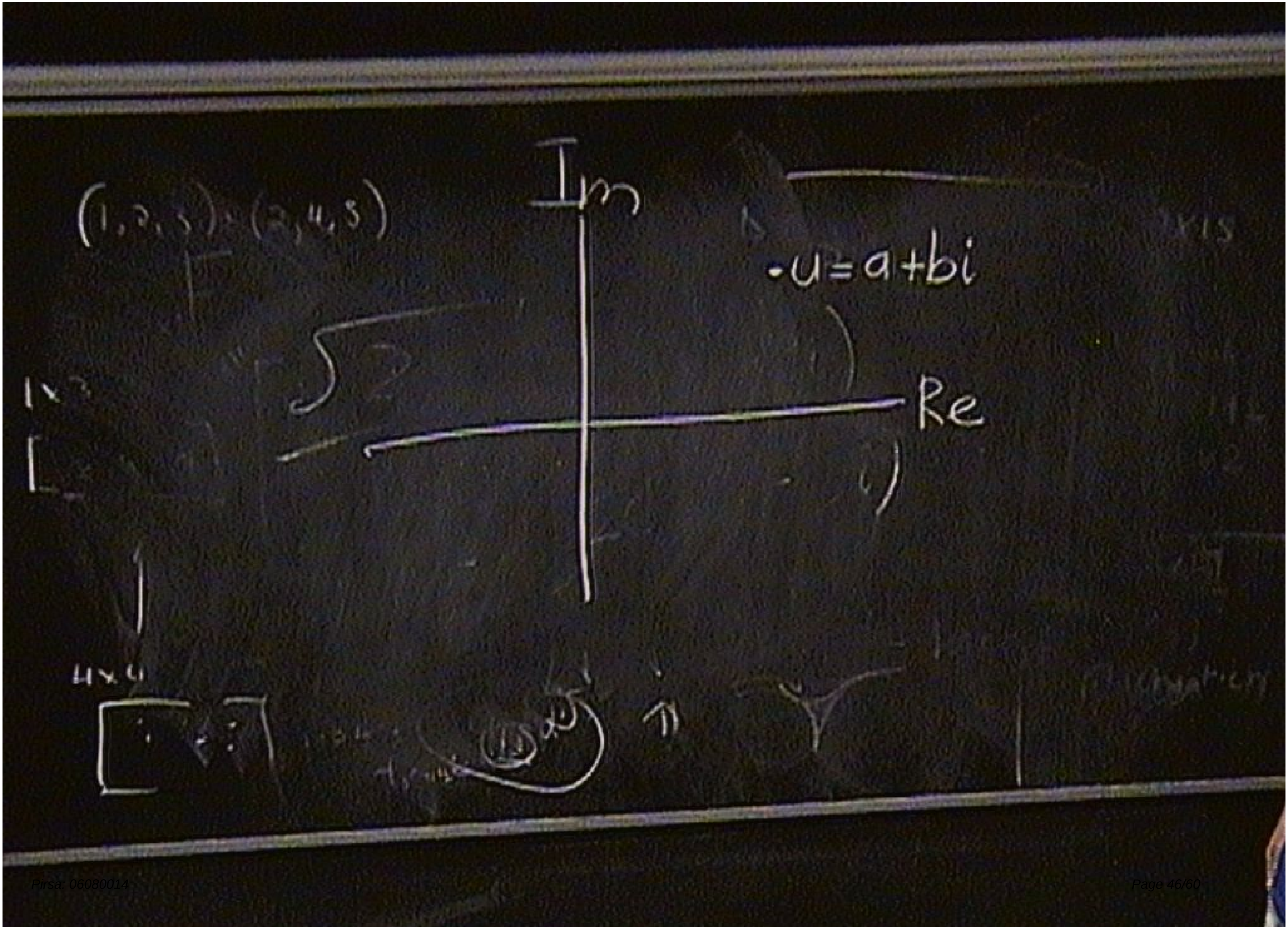

r
θ
y
x
$H^2 = O^2 + A^2$
$r^2 = x^2 +$

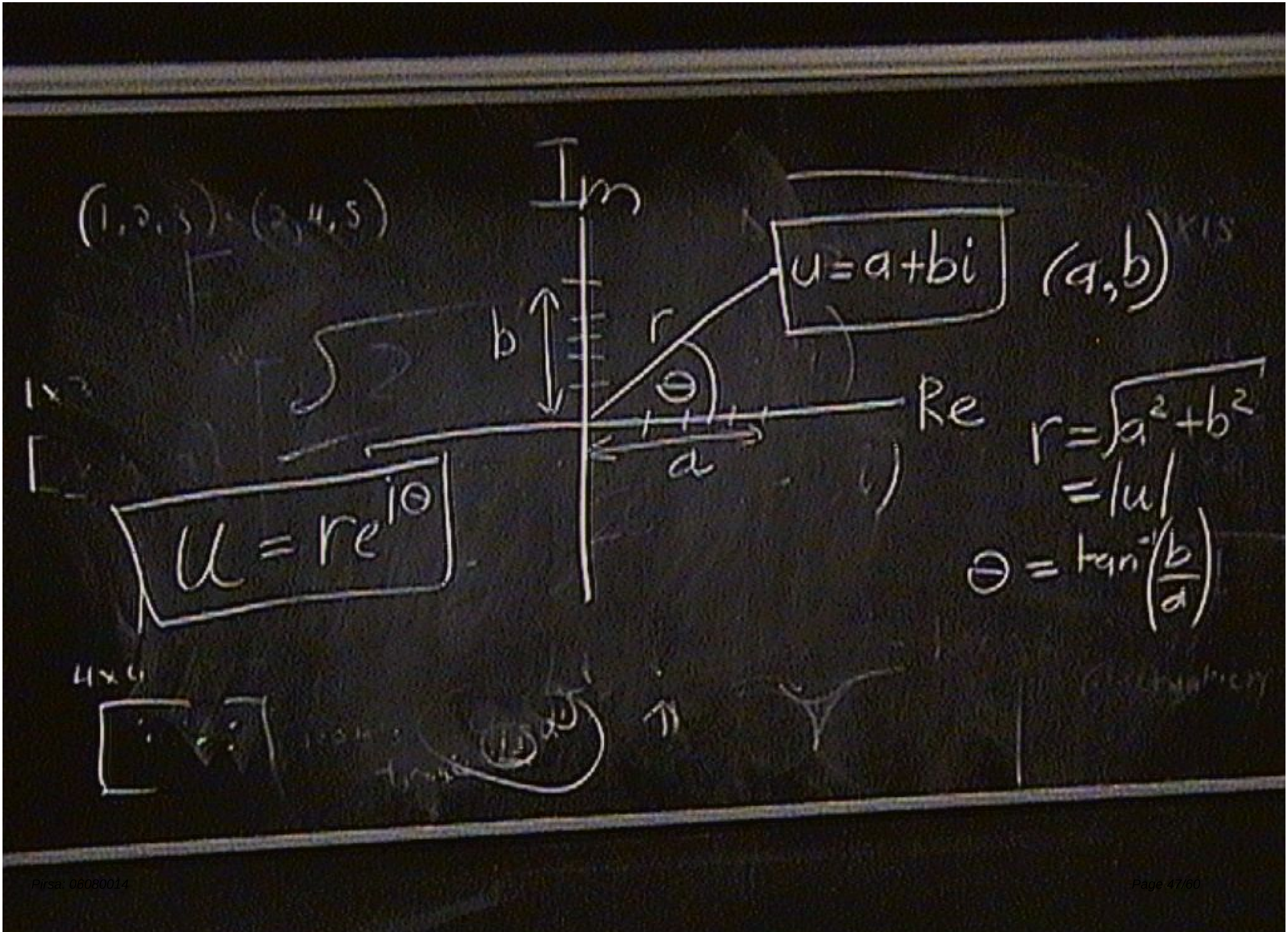

(x,y) → (r,θ)
r
θ
y
x
H² = O² + A²
r² = x² + y²
tan
H
O
A

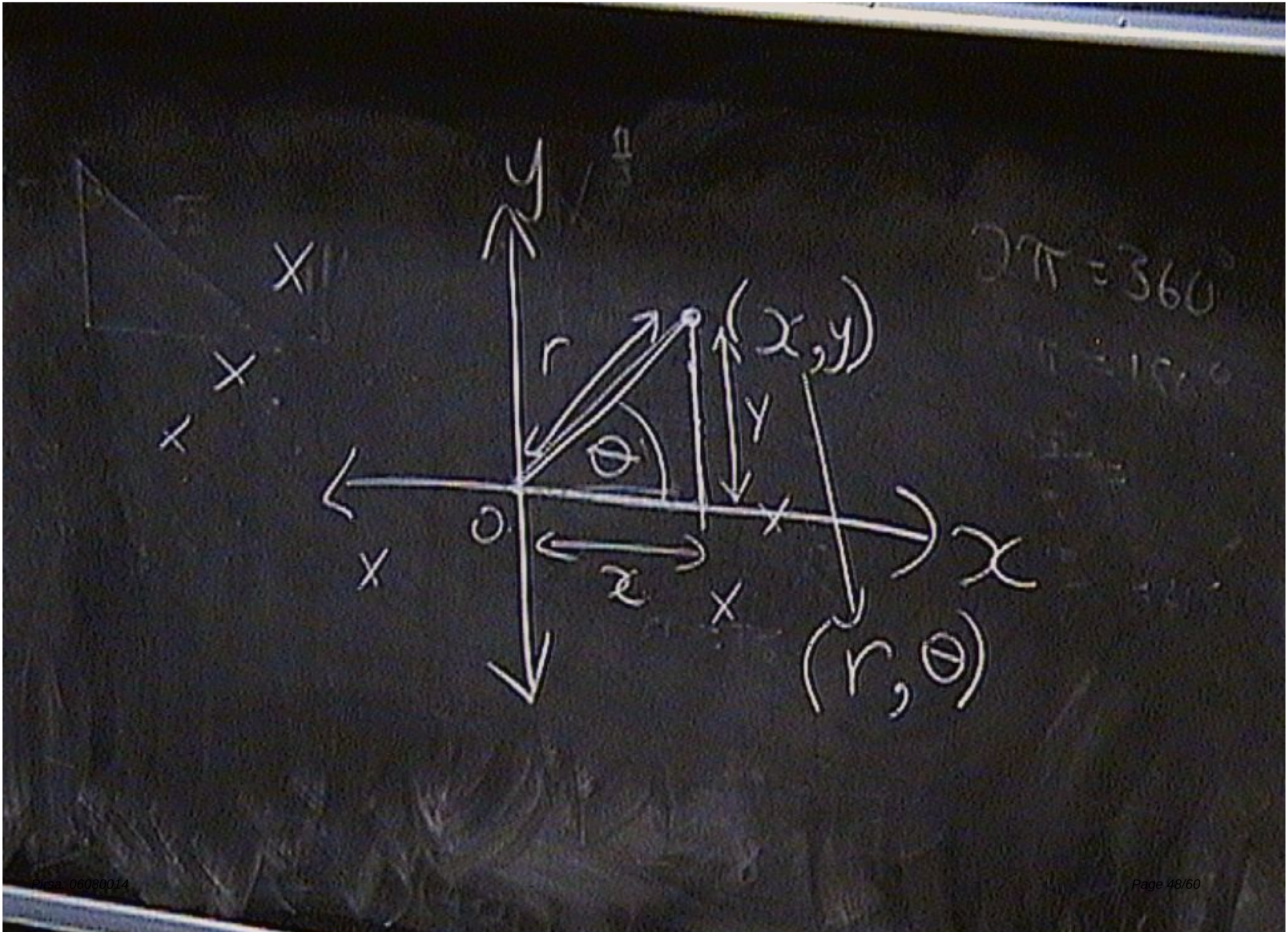

(x, y) → (r, θ)
r
θ
y
x
H² = O² + A²
r² = x² + y²
∴ θ = tan⁻¹(y/x)
(r, θ) → (x, y)

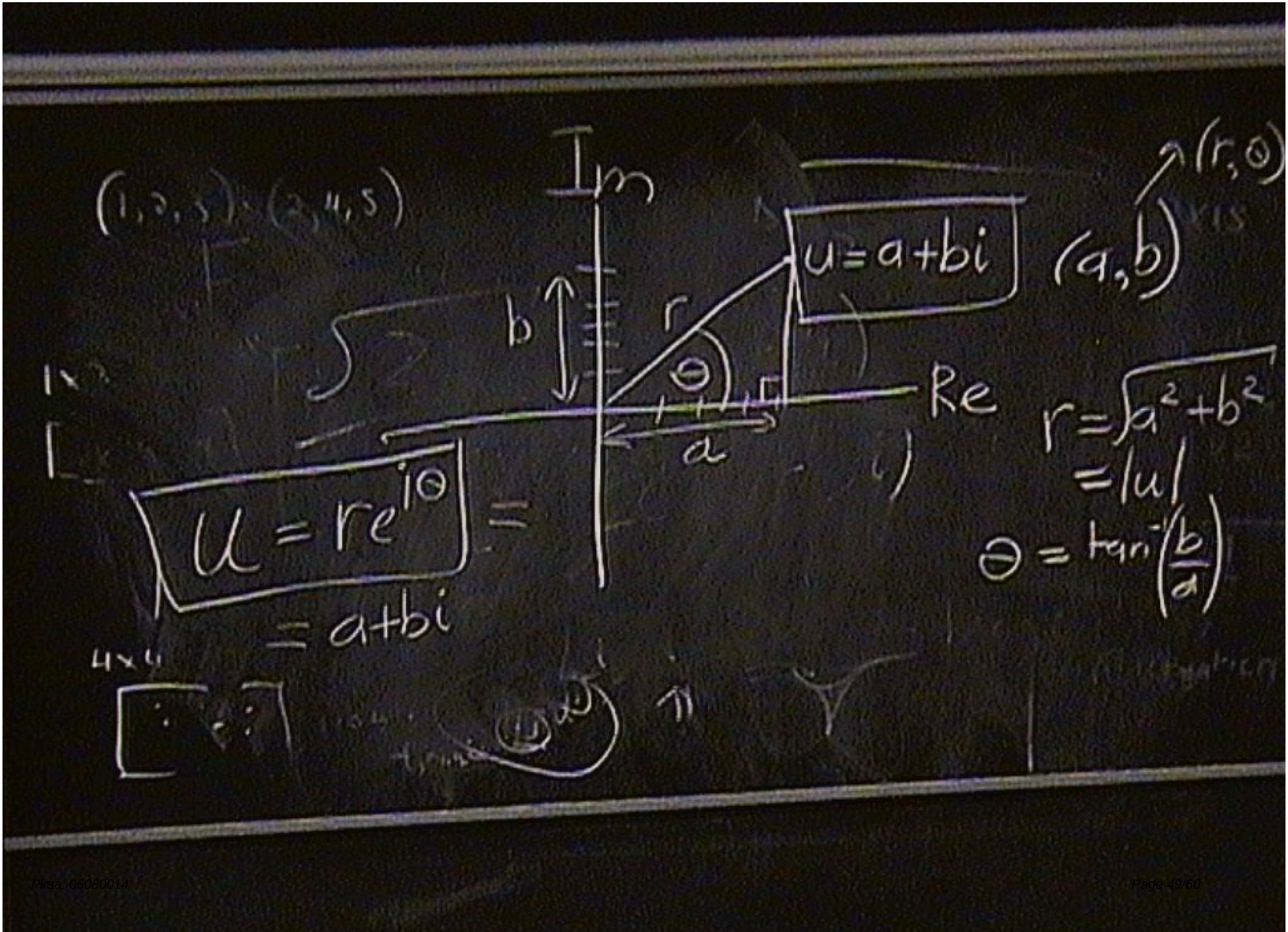

(x, y) → (r, θ)
r
θ
y (O)
x (A)
H² = O² + A²
r² = x² + y²
tan θ = y/x
∴ θ = tan⁻¹(y/x)
(r, θ) → (x, y)
1 − 3i
H
O
A
x

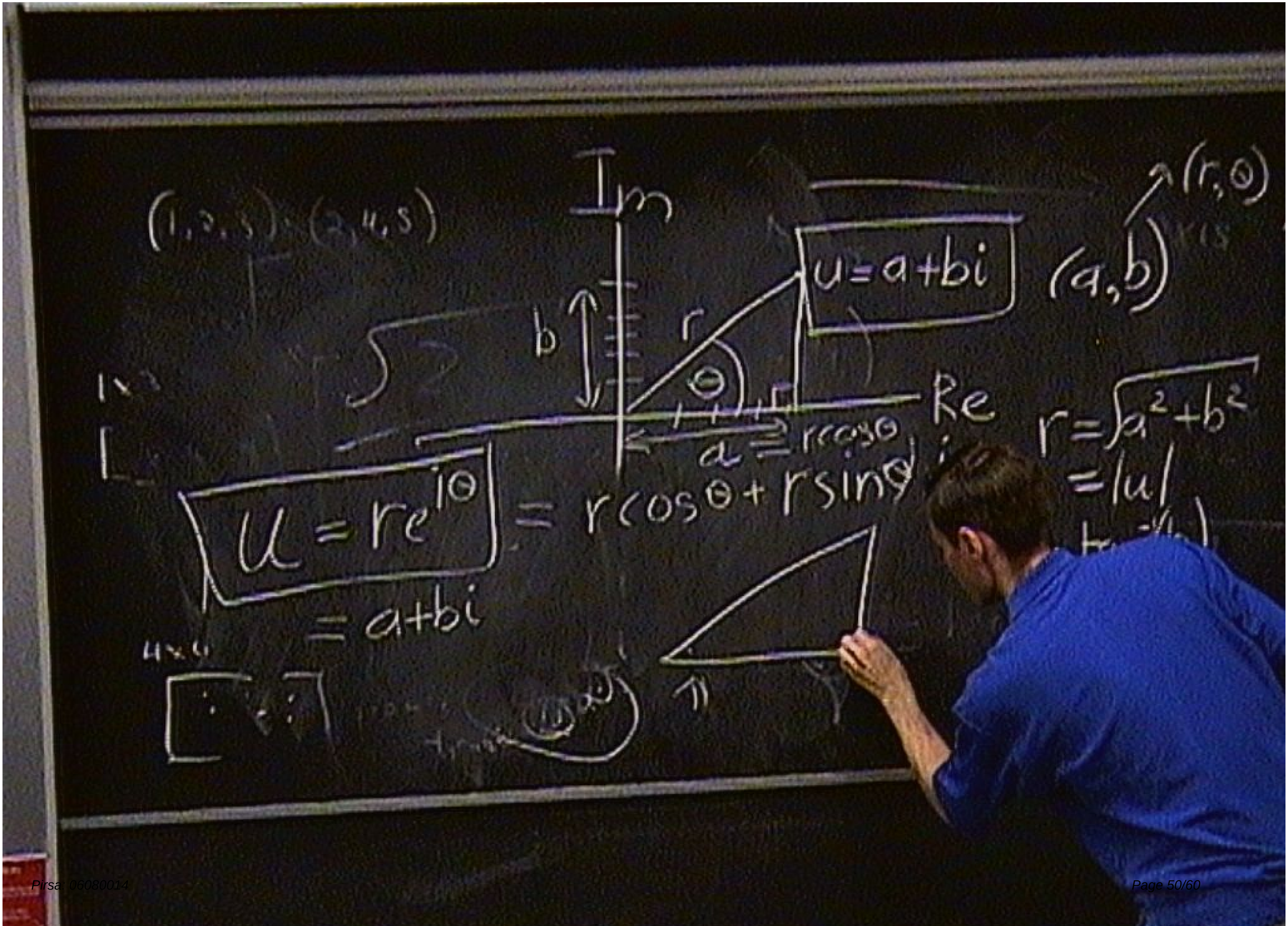

(x,y) → (r,θ)
r sinθ = y
r cosθ = x
r
θ
y (O)
H² = O² + A²
r² = x² + y²
x (A)
tan θ = y/x
∴ θ = tan⁻¹(y/x)
(r,θ) → (x,y)


(1,2,3)+(2,4,5)
Im
u=a+bi
Re

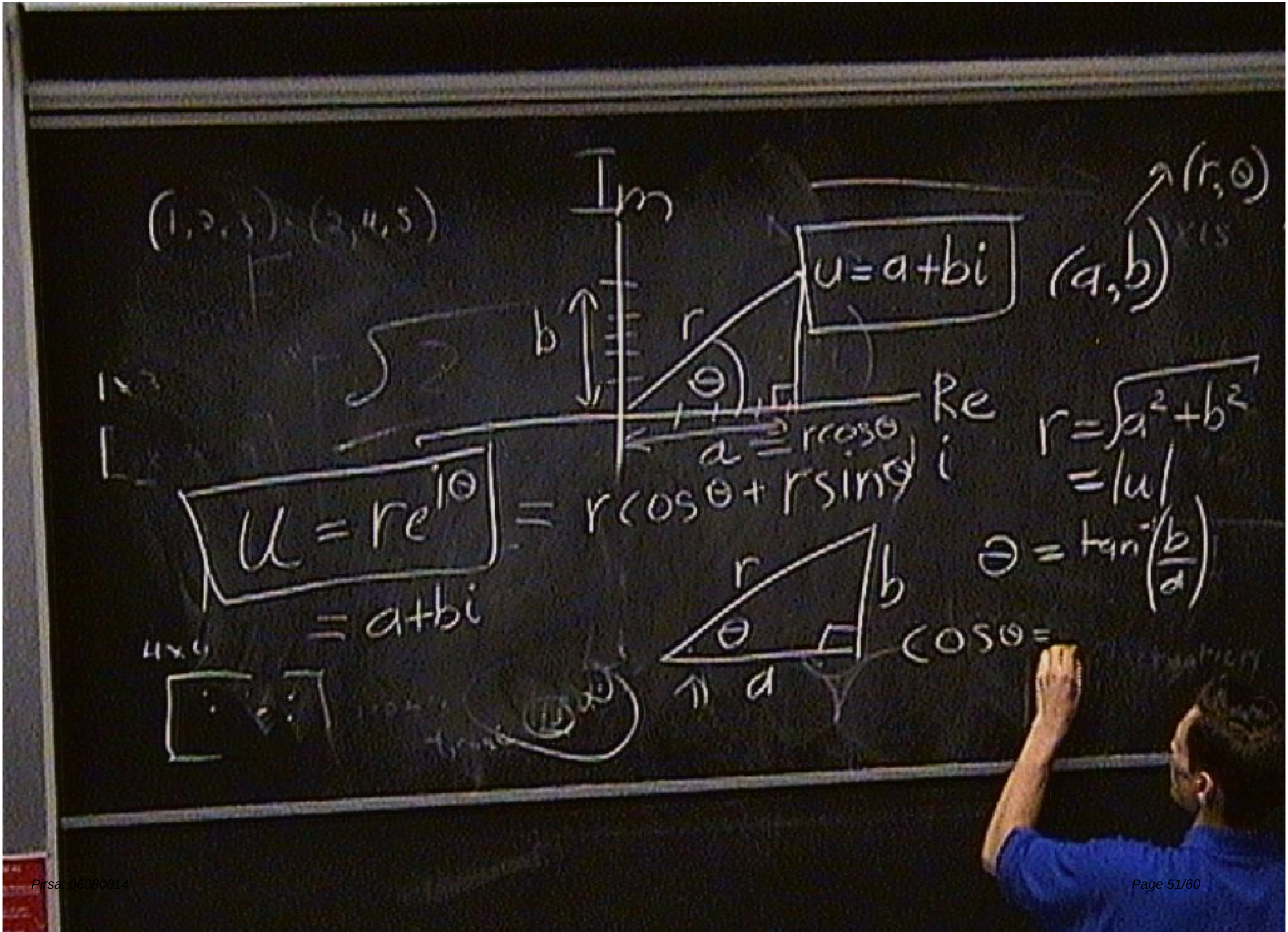

(1,2,3)·(2,4,5)
1×3
4×4
Im
Re
b
a
r
θ
u=a+bi
(a,b)
u=re^iθ
r=√(a²+b²)
=|u|
θ=tan⁻¹(b/a)


y
(x,y)
r
y
θ
O
x
x
(r,θ)
2π = 360°

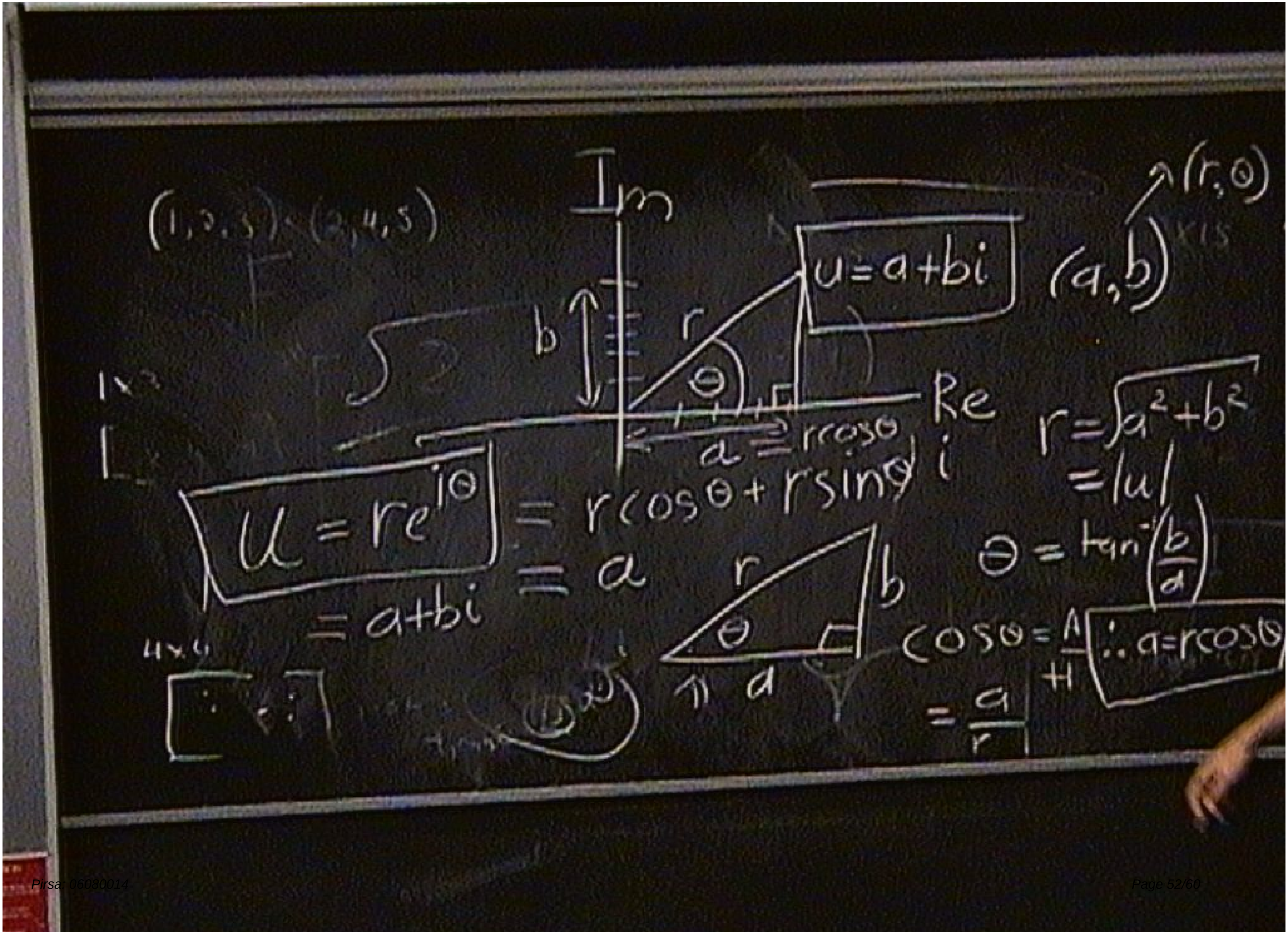

Im
Re
u=a+bi
(a,b)
(r,θ)
r
θ
b
a
u=re^{iθ}
=a+bi
r=√(a²+b²)
=|u|
θ=tan⁻¹(b/a)

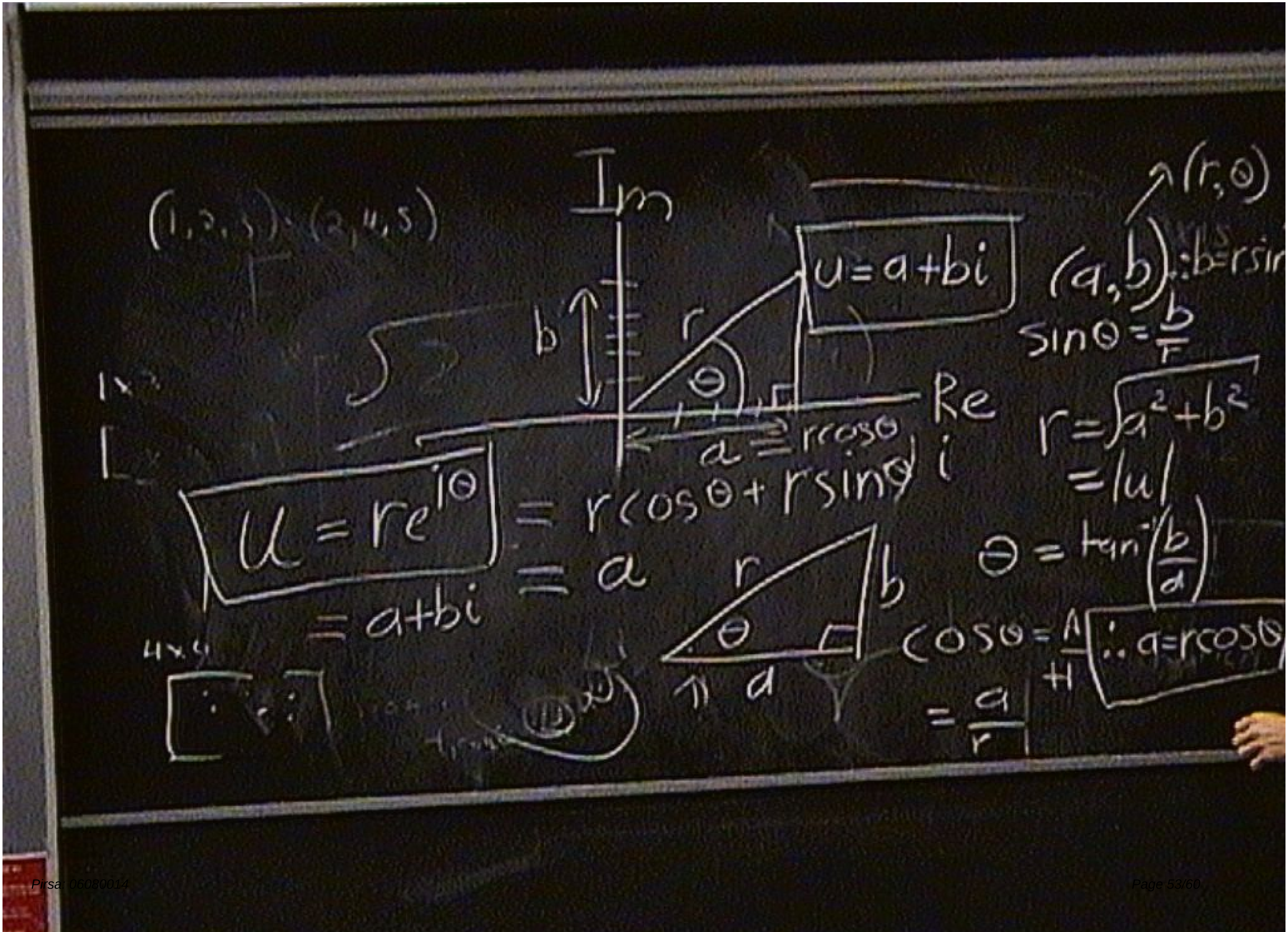


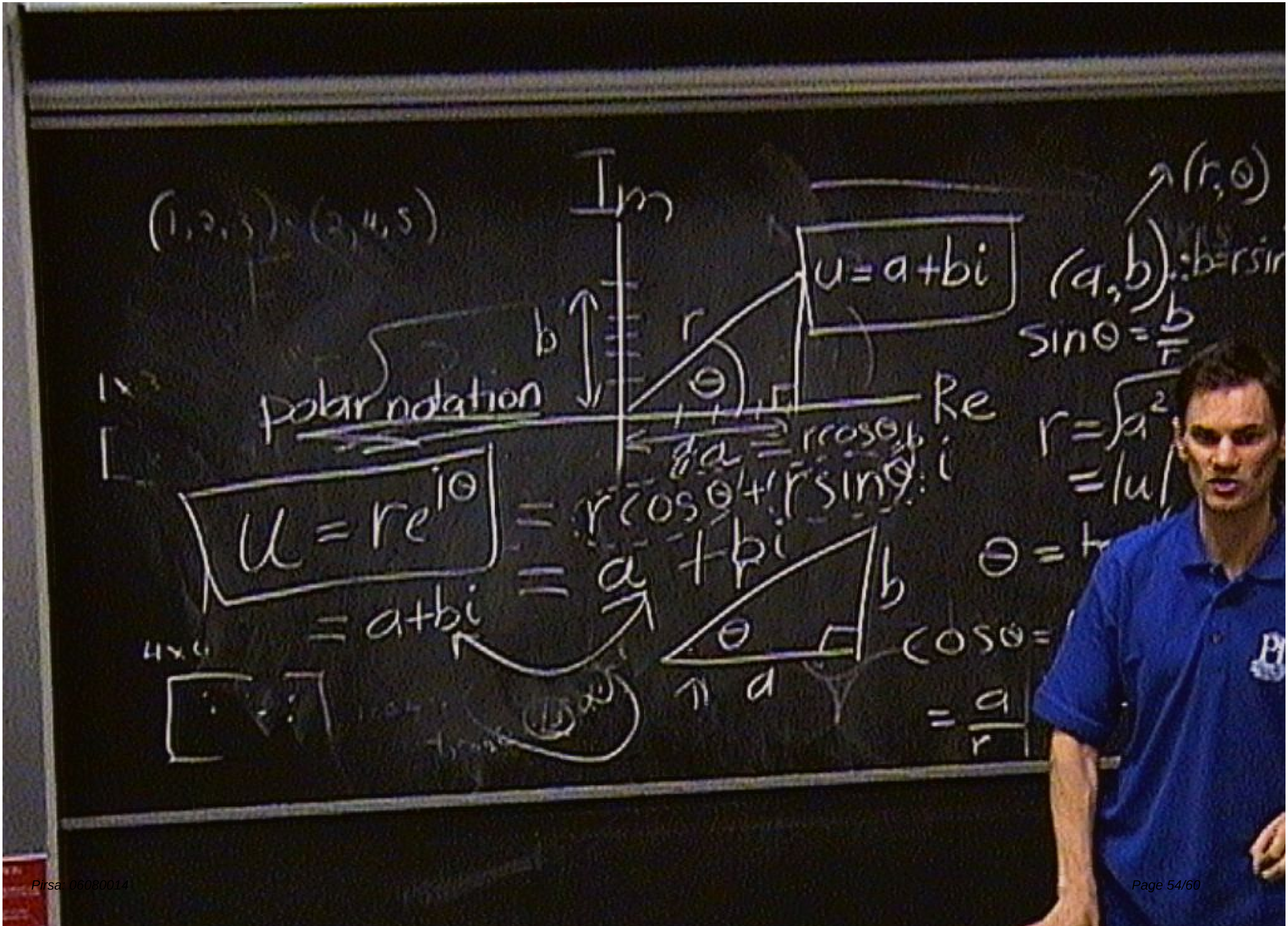
Im
Re
u=a+bi
(a,b)
(r,θ)
b
r
θ
a = r cos θ
r = √(a²+b²)
= |u|
u = re^{iθ} = r cos θ + r sin θ i
= a+bi
(1,2,3)·(2,4,5)

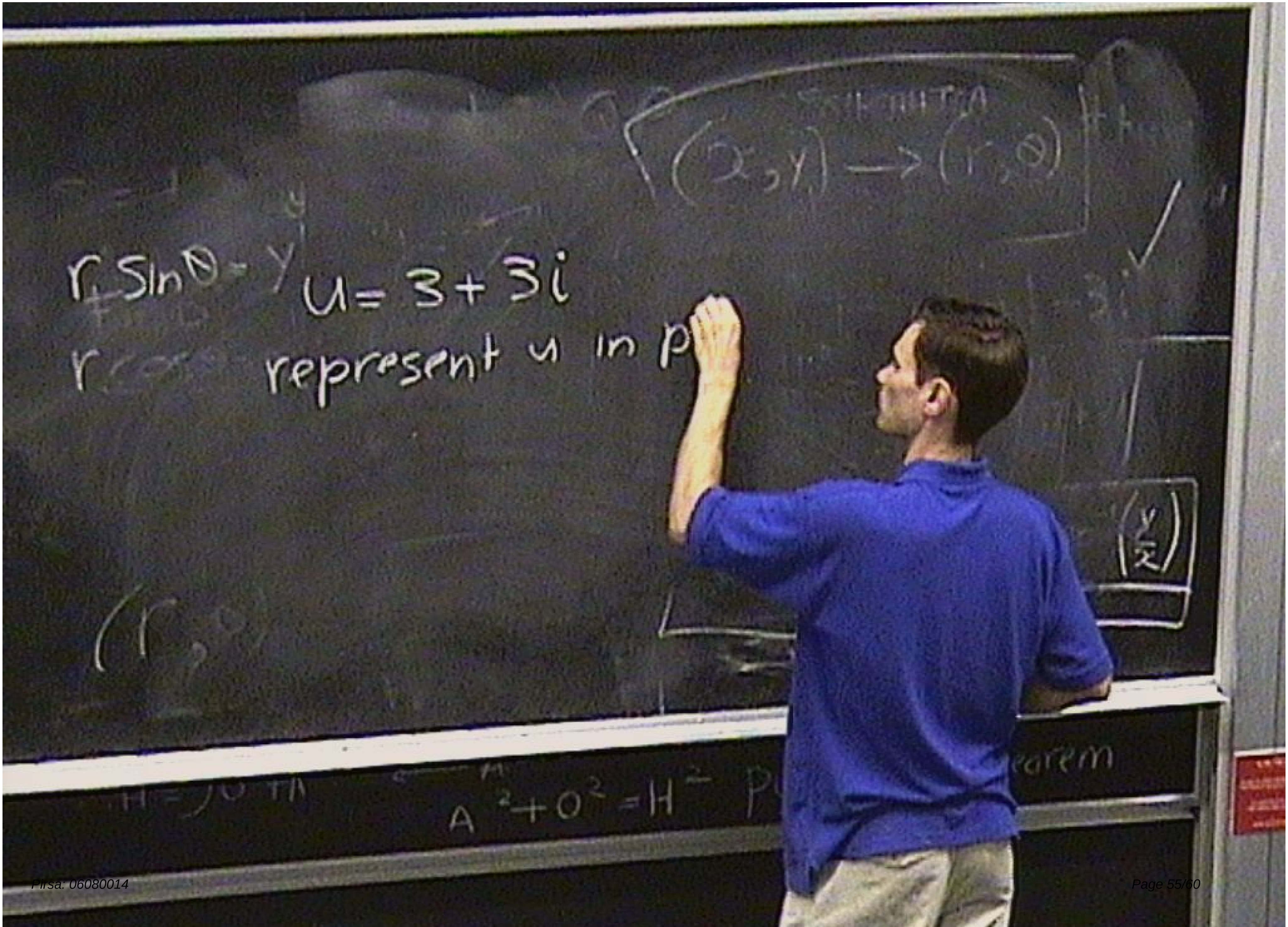


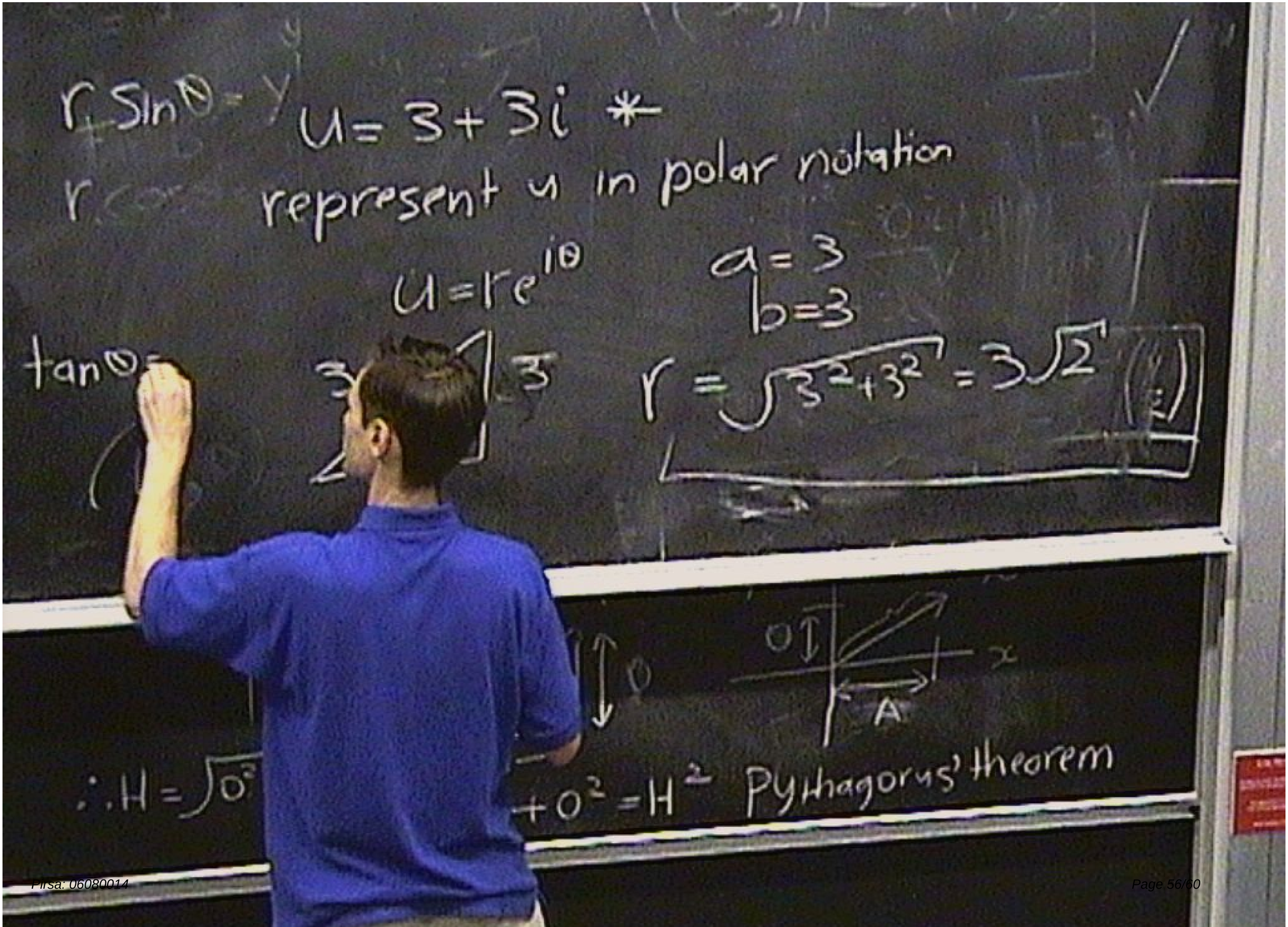
Im
(r,θ)
u=a+bi
(a,b)
b
r
θ
Re
a = rcosθ
r=√(a²+b²)
=|u|
u=re^{iθ} = rcosθ + rsinθ i
= a+bi
θ = tan⁻¹(b/a)
r
b
θ
a
cosθ=

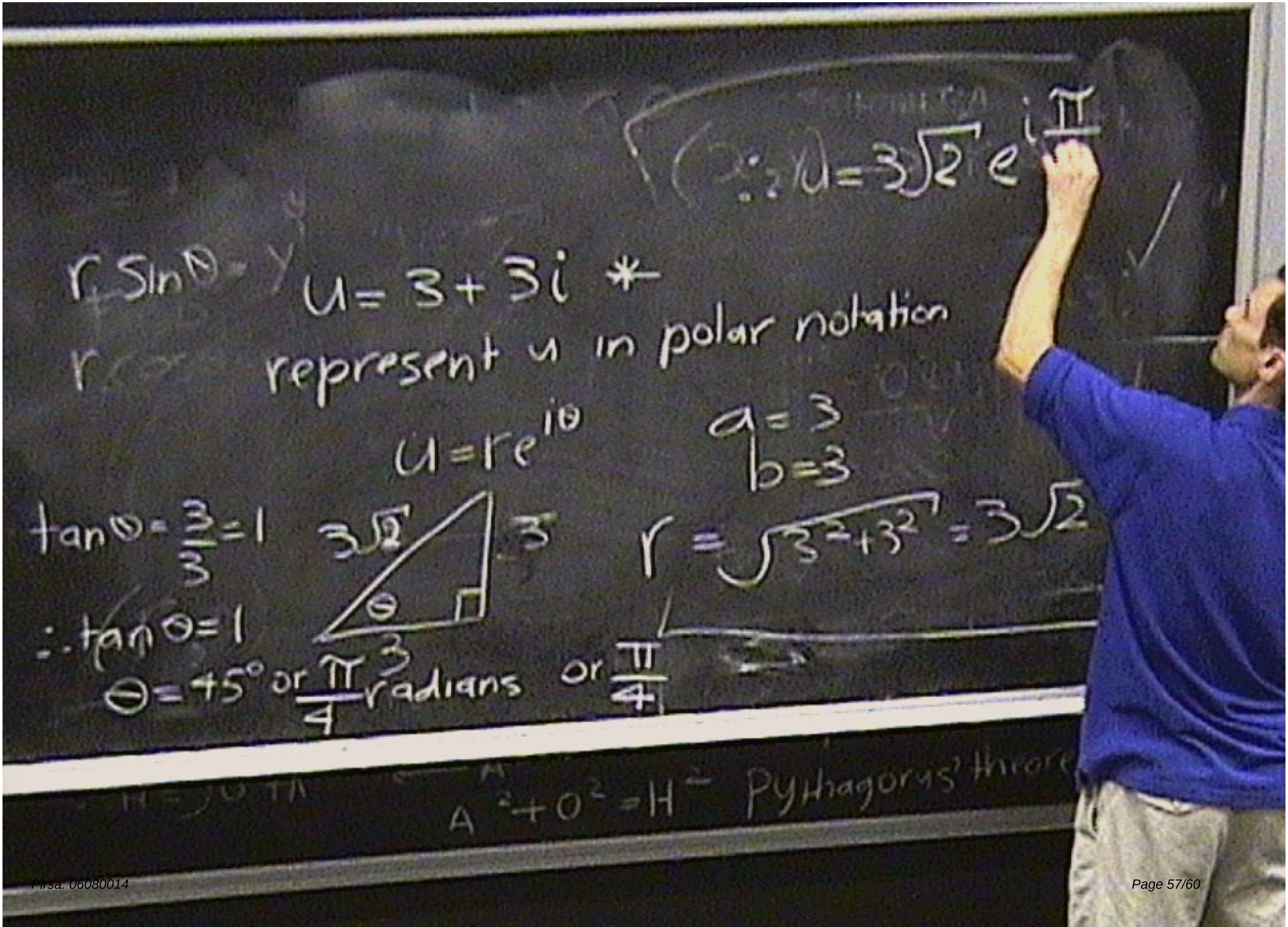

Im
Re
u=a+bi
(a,b)
(r,θ)
r=√(a²+b²)
=|u|
θ=tan⁻¹(b/a)
a=rcosθ
u=re^{iθ}
=rcosθ+rsinθ i
=a
=a+bi
cosθ=A/H
=a/r
∴ a=rcosθ


(1,2,3)·(2,4,5)
Im
Re
(r,θ)
u=a+bi
(a,b)
sinθ = b/r
r = √(a²+b²)
= |u|
a = rcosθ
u = re^iθ = rcosθ + rsinθ i
= a+bi
θ = tan⁻¹(b/a)
cosθ = A/H
= a/r
∴ a=rcosθ


(1,2,3)·(2,4,5)
Im
Re
u = a+bi
(r,θ)
(a,b)
sinθ = b/r
r
b
θ
polar notation
u = re^{iθ}
= r cosθ + r sinθ · i
= a + bi
= a+bi
a
cosθ =
= a/r
θ =


(x, y) → (r, θ)
r sinθ = y
u = 3 + 3i
represent u in p
A² + O² = H²


r sinθ = y
u = 3 + 3i *
represent u in polar notation
u = re^{iθ}
a = 3
b = 3
tanθ =
r = √(3² + 3²) = 3√2
∴ H = √O²
+ O² = H²
Pythagorus' theorem
x
A


$\therefore u = 3\sqrt{2}\, e^{i\frac{\pi}{4}}$
$r \sin\theta = y$
$u = 3 + 3i$ *
represent u in polar notation
$u = re^{i\theta}$
$a = 3$
$b = 3$
$\tan\theta = \frac{3}{3} = 1$
$3\sqrt{2}$
3
θ
3
$r = \sqrt{3^2 + 3^2} = 3\sqrt{2}$
$\therefore \tan\theta = 1$
$\theta = 45°$ or $\frac{\pi}{4}$ radians or $\frac{\pi}{4}$
$A^2 + O^2 = H^2$ Pythagoras' theore

$$\therefore u = 3\sqrt{2}\, e^{i\frac{\pi}{4}}$$

$r \sin\theta = y$

$u = 3 + 3i$ *

r

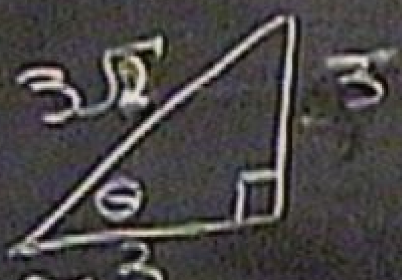
represent u in polar notation

$u = re^{i\theta}$

$a = 3$

$b = 3$

$\tan\theta = \frac{3}{3} = 1$

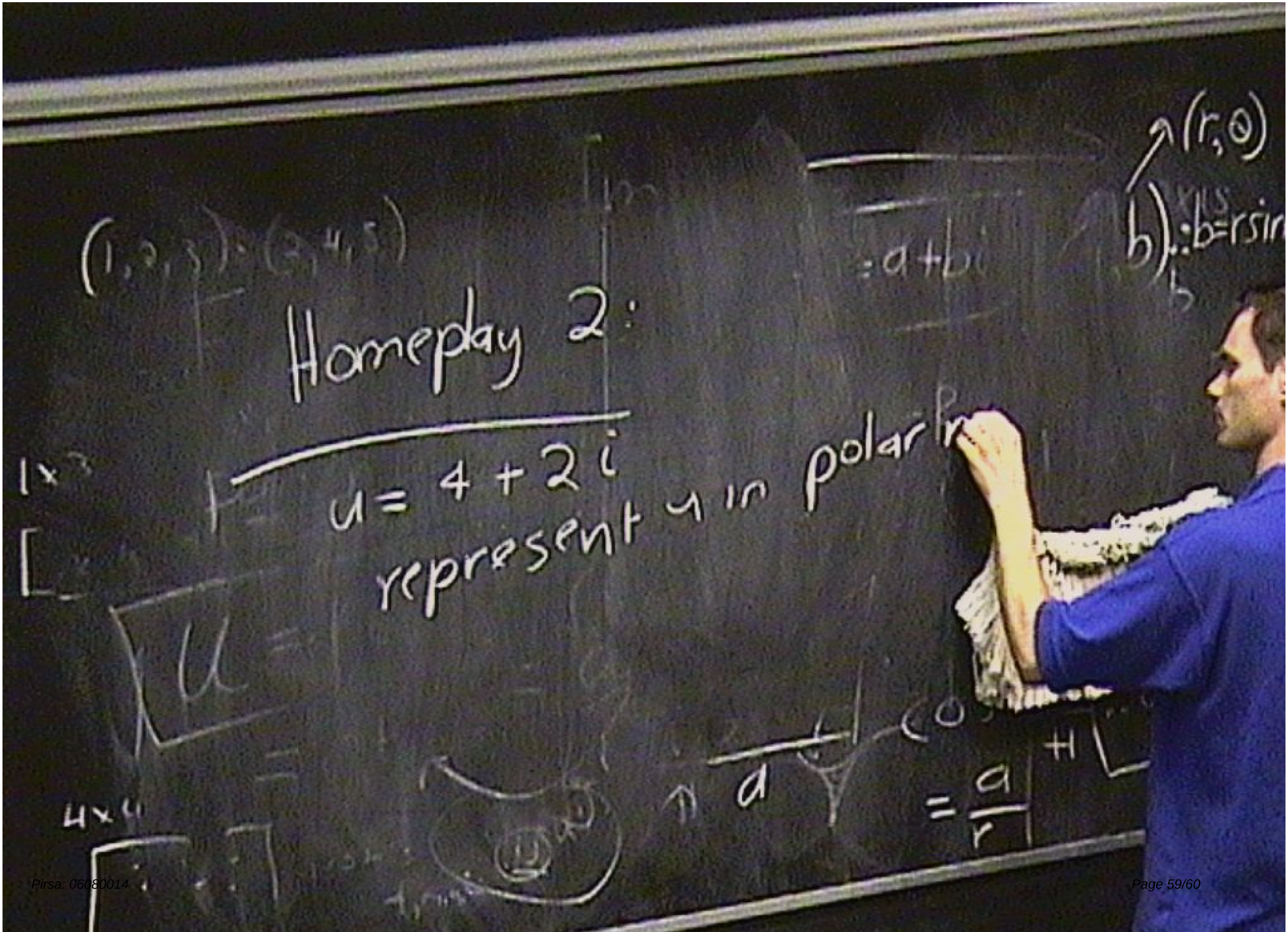


$r = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$\therefore \tan\theta = 1$

$\theta = 45^\circ$ or $\frac{\pi}{4}$ radians or $\frac{\pi}{4}$

$A^2 + O^2 = H^2$ Pythagoras' theorem


Homeplay 2:
u = 4 + 2i
represent u in polar
(r,θ)
= a+bi
= a/r

Homeplay 2:

$u = 4 + 2i$

A. represent u in polar notation

$v = 3 + 2i$

B. Calculate $(u+v)^{23}$