Title: ISSYP 2006a - Student Presentations

Date: Aug 02, 2006 01:00 PM

URL: http://pirsa.org/06080006

Abstract:

Thermodynamics

How a Black Hole is formed

→ Cloud contracts due to gravity
~ Star is formed and burns, increasing the Tempera
~ Star cools
~ Black Hole is formed

The curved spacetime of a Black Hole.

Schwarzchild Radius

$$R = \frac{2GM}{c^2}$$

(where G and
are constants)

From this equation we can see that the distance from the singularity to the event horizon is directly proportional to t Law: Conservation of Energy

$$\Delta u = Q - W$$

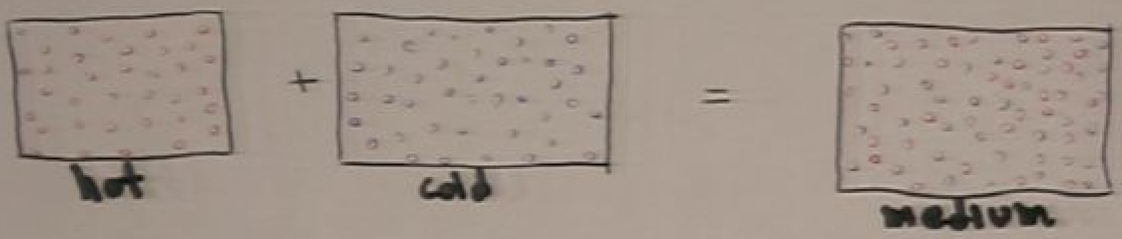
Δu ↑ change in internal energy

Q ↑ heat added to system

W ↖ work done by system

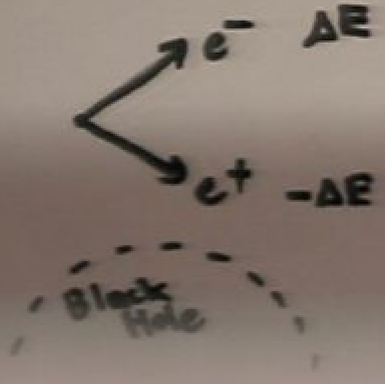
nd Law: Entropy

'Heat flows from hot to cold'



Hawking hypothesized that Black Holes may radiate particles.

According to Quantum Mechanics, particles and their corresponding anti-particles (eg. electrons and positrons) may spontaneously appear in a vacuum.



Total change in energy in system

$= $ Sum of energies formed

$= \Delta E + (-\Delta E)$

$= 0$

$$M_{BH} = M - \Delta E$$

Hawking suggested that black holes may be 'leaking' particles as the total mass decreases $(M_{BH} = M - \Delta E)$ and a particle is emitted.

From this assumption, and knowing that black holes radiate matter/energy, we came up with an equation for the lifespan of a black hole (how long it takes for it to radiate all its energy/mass away).

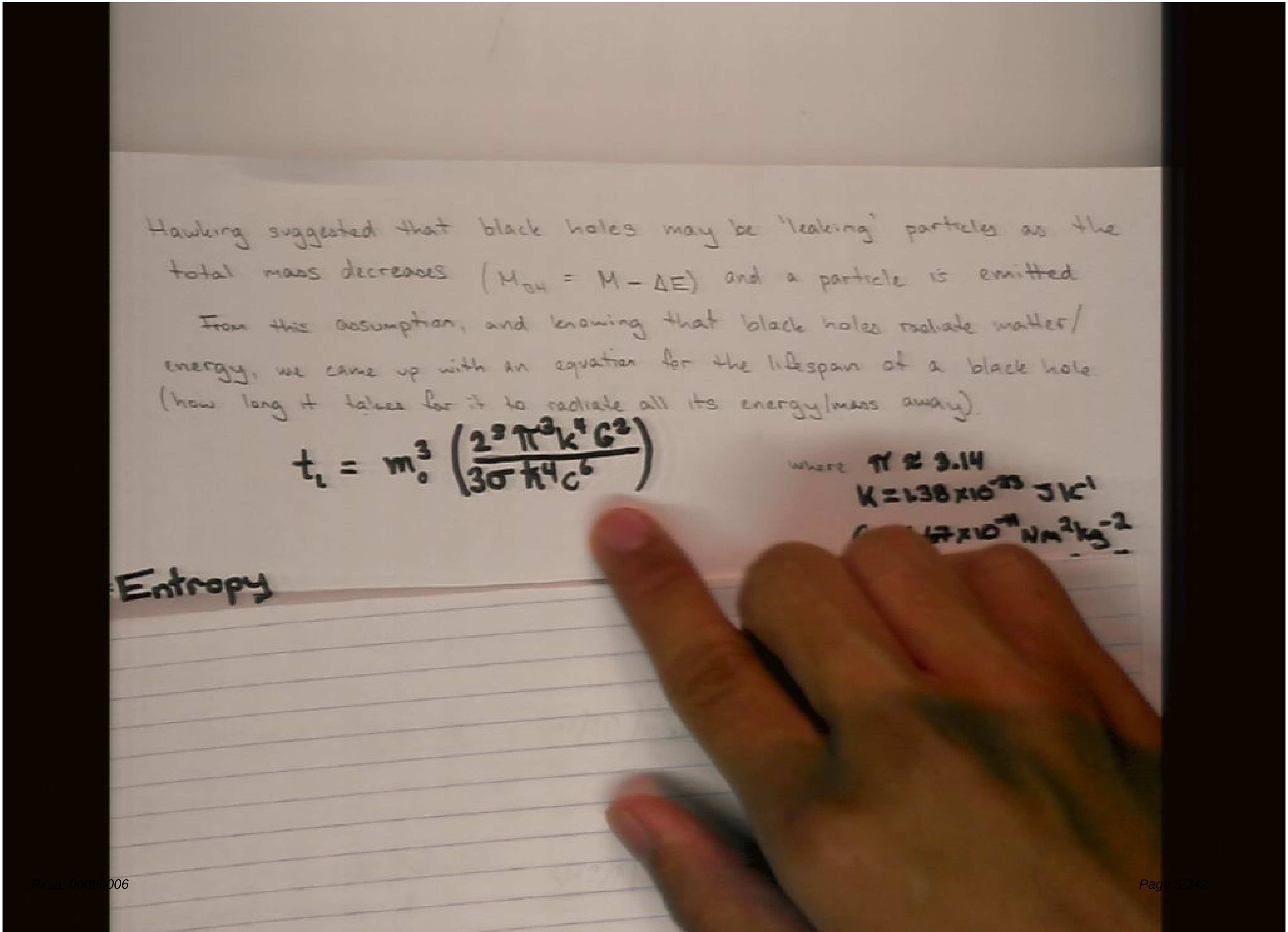
$$t_1 = m_0^3 \left(\frac{2^3 \pi^3 k^4 G^2}{3 \sigma \hbar^4 c^6} \right)$$

where $\pi \approx 3.14$

JK^{1}

$^{2}kg^{-2}$

$^{-4}$



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$$t_L = m_0^3 \left(\frac{2^8 \pi^3 k^4 G^2}{3 \sigma \hbar^4 c^6} \right)$$

where $\pi \approx 3.14$

$K = 1.38 \times 10^{-23}\ J K^{-1}$

G [illegible] $67 \times 10^{-11}\ N m^2 kg^{-2}$

Entropy

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*Entropy

- Measure of the order/disorder of a system
- Given as S

$$\Delta S = \frac{Q}{T}$$

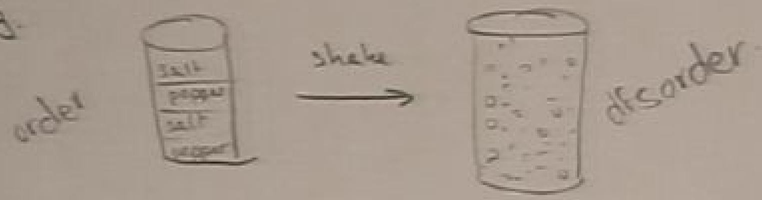
← heat added
← temperature

Dimensions of entropy are J per K
According to the second law of thermodynamics

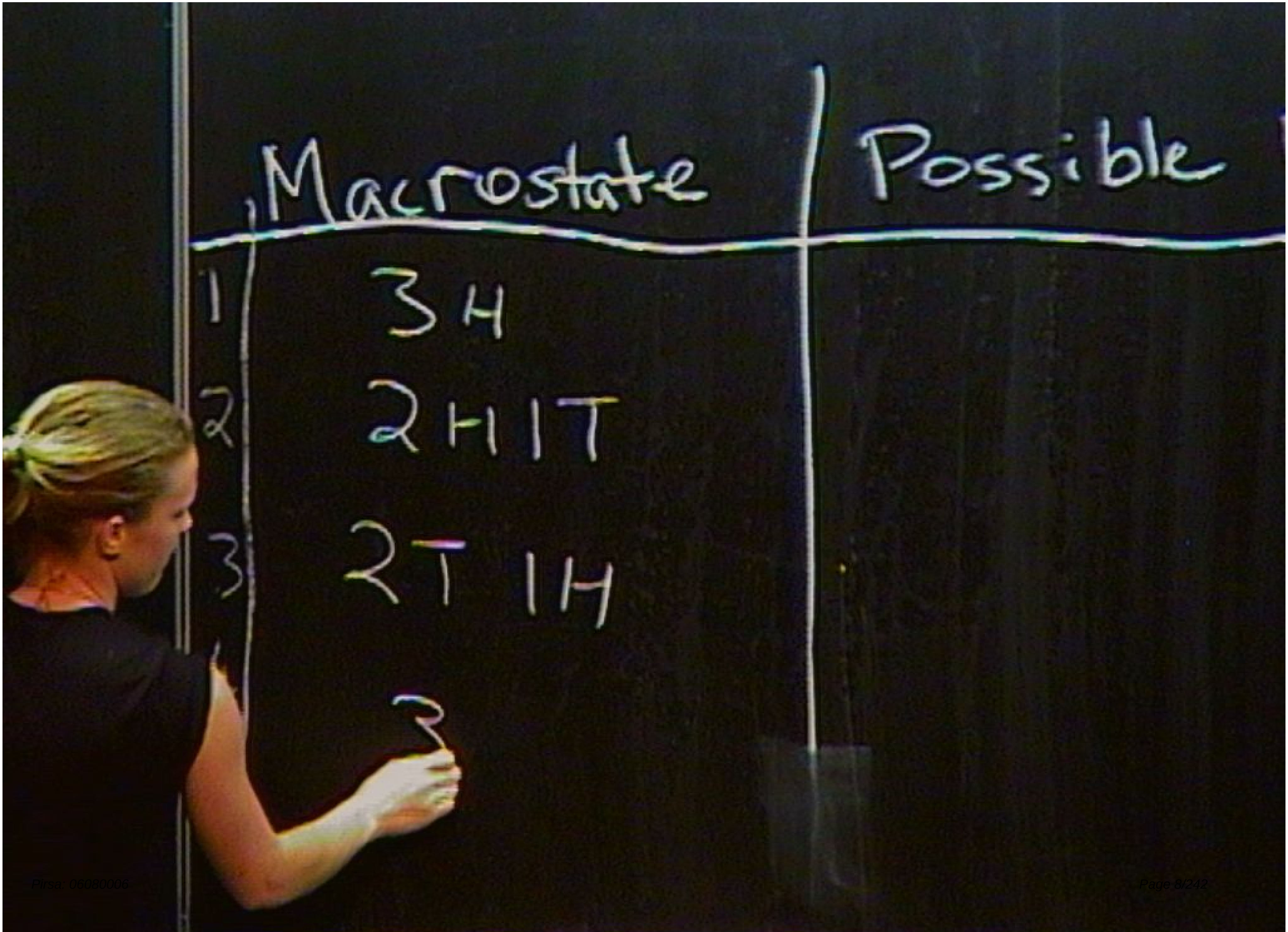
$$\Delta S \geq 0$$

Higher Entropy = Greater Disorder

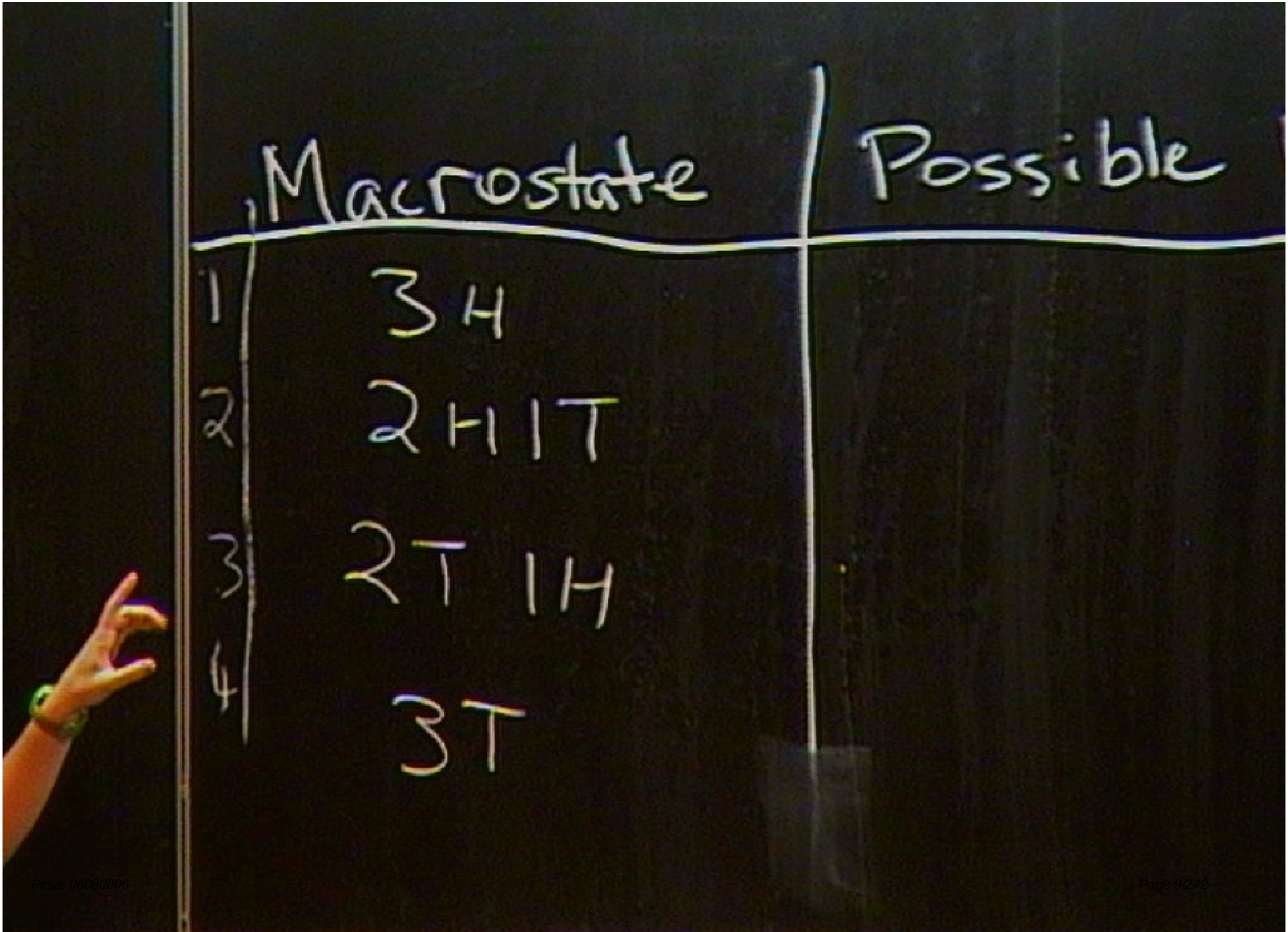
Eg.



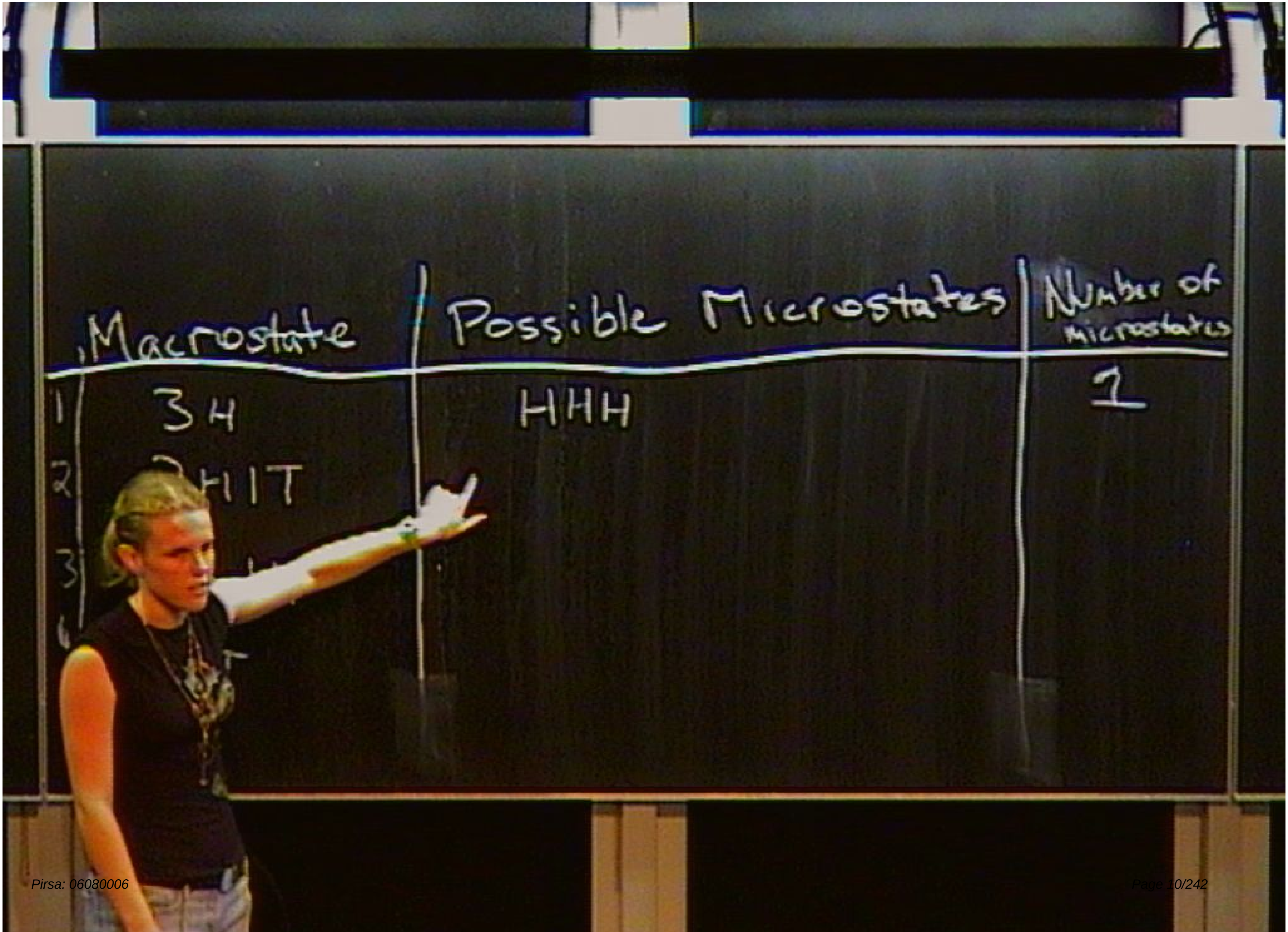
Entropy of systems increases over time



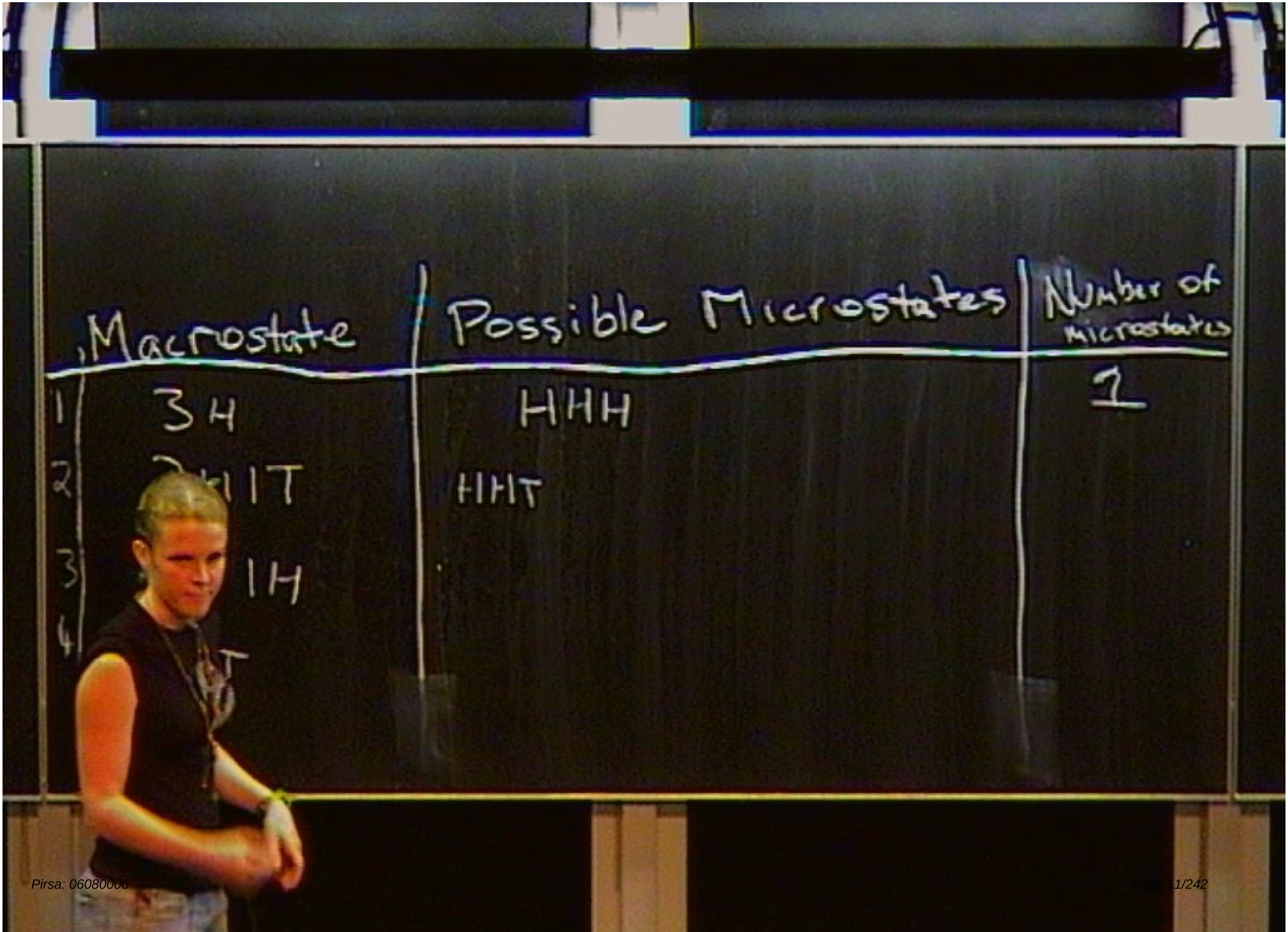
	Macrostate	Possible
1	3H	
2	2H 1T	
3	2T 1H	
	3	



	Macrostate	Possible
1	3H	
2	2H1T	
3	2T 1H	
4	3T	



	Macrostate	Possible Microstates	Number of microstates
1	3H	HHH	1
2	2H1T		
3	[illegible]		
[illegible]	[illegible]		



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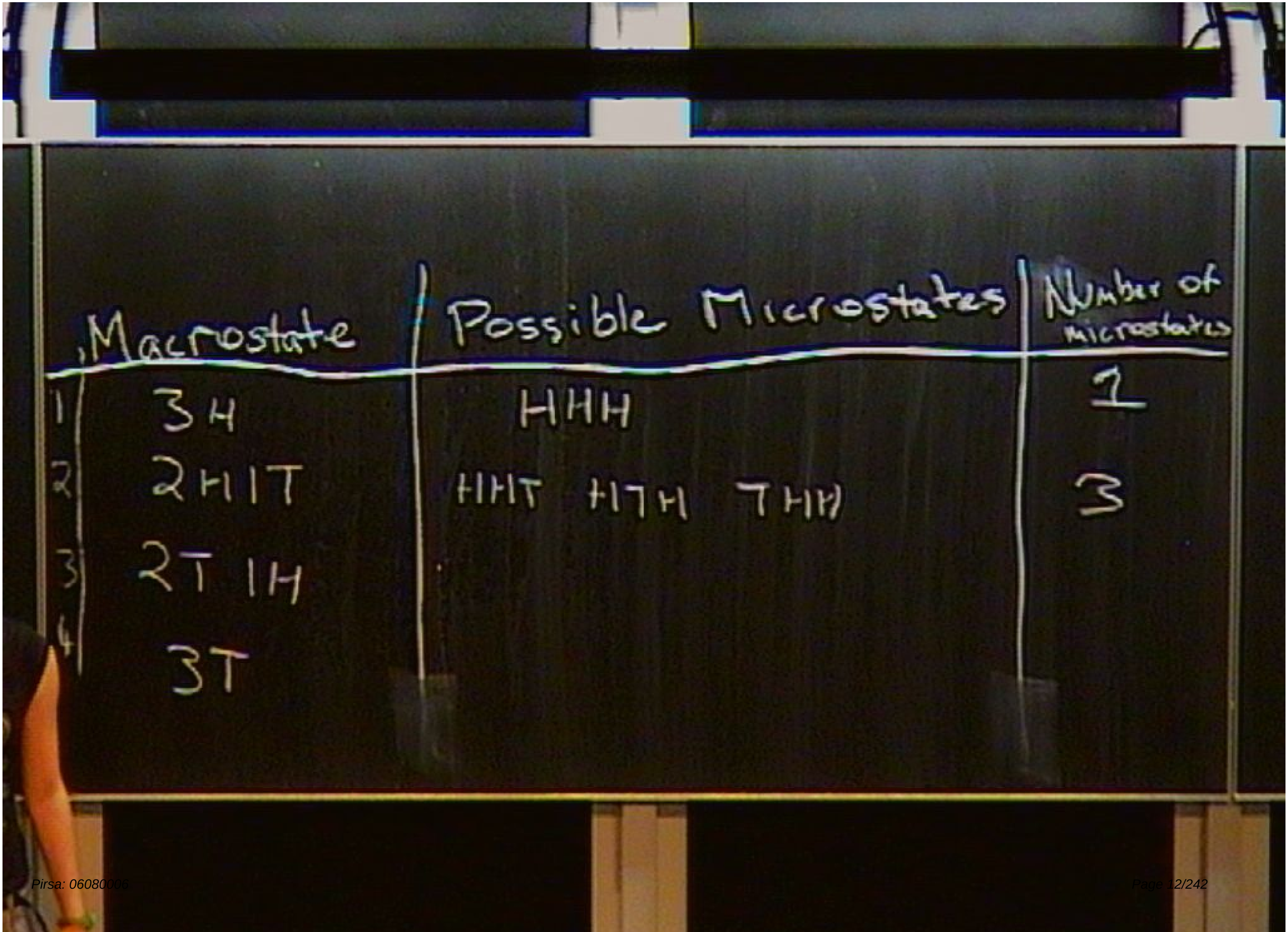
4	T		



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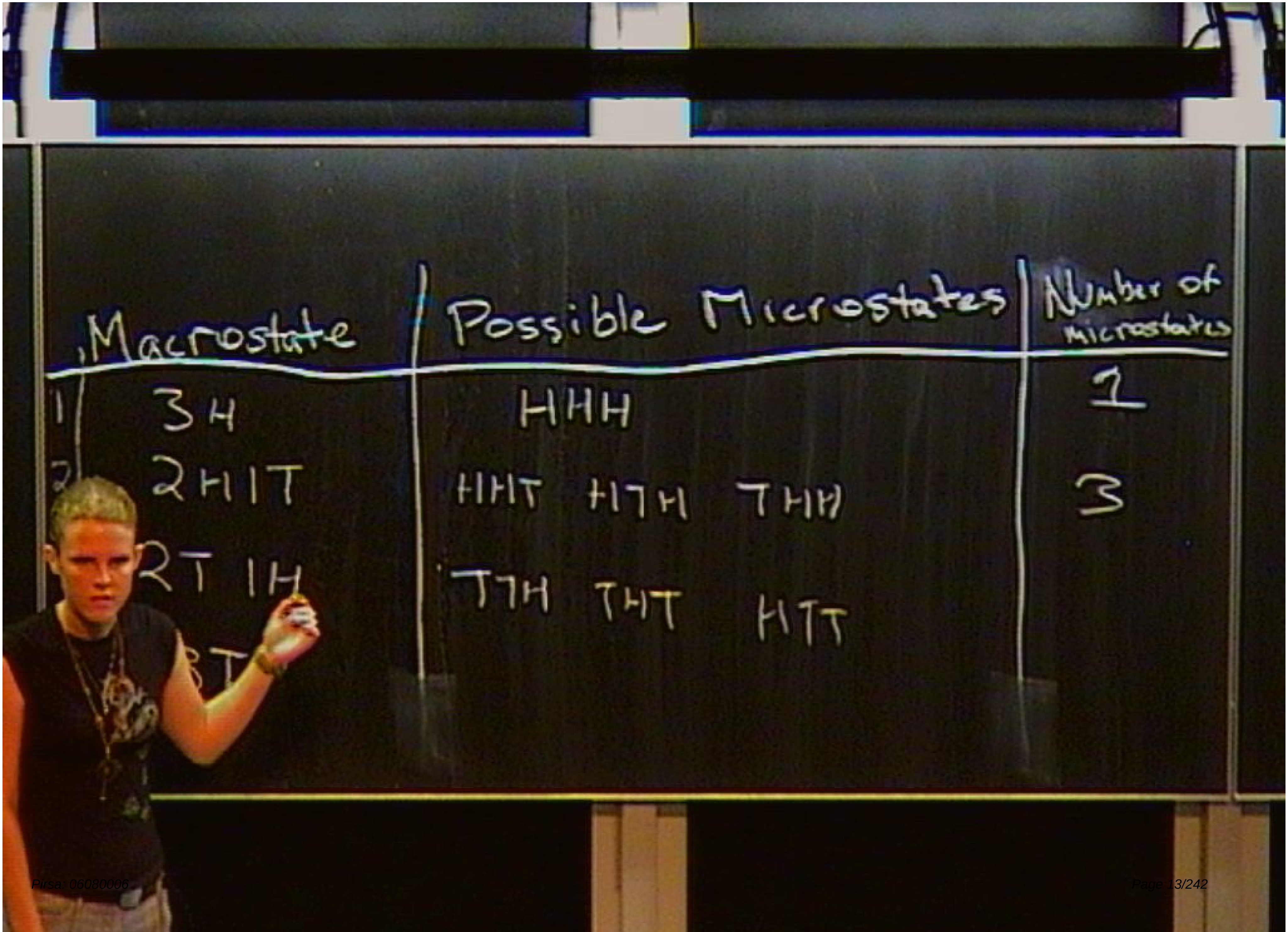
4	3T		



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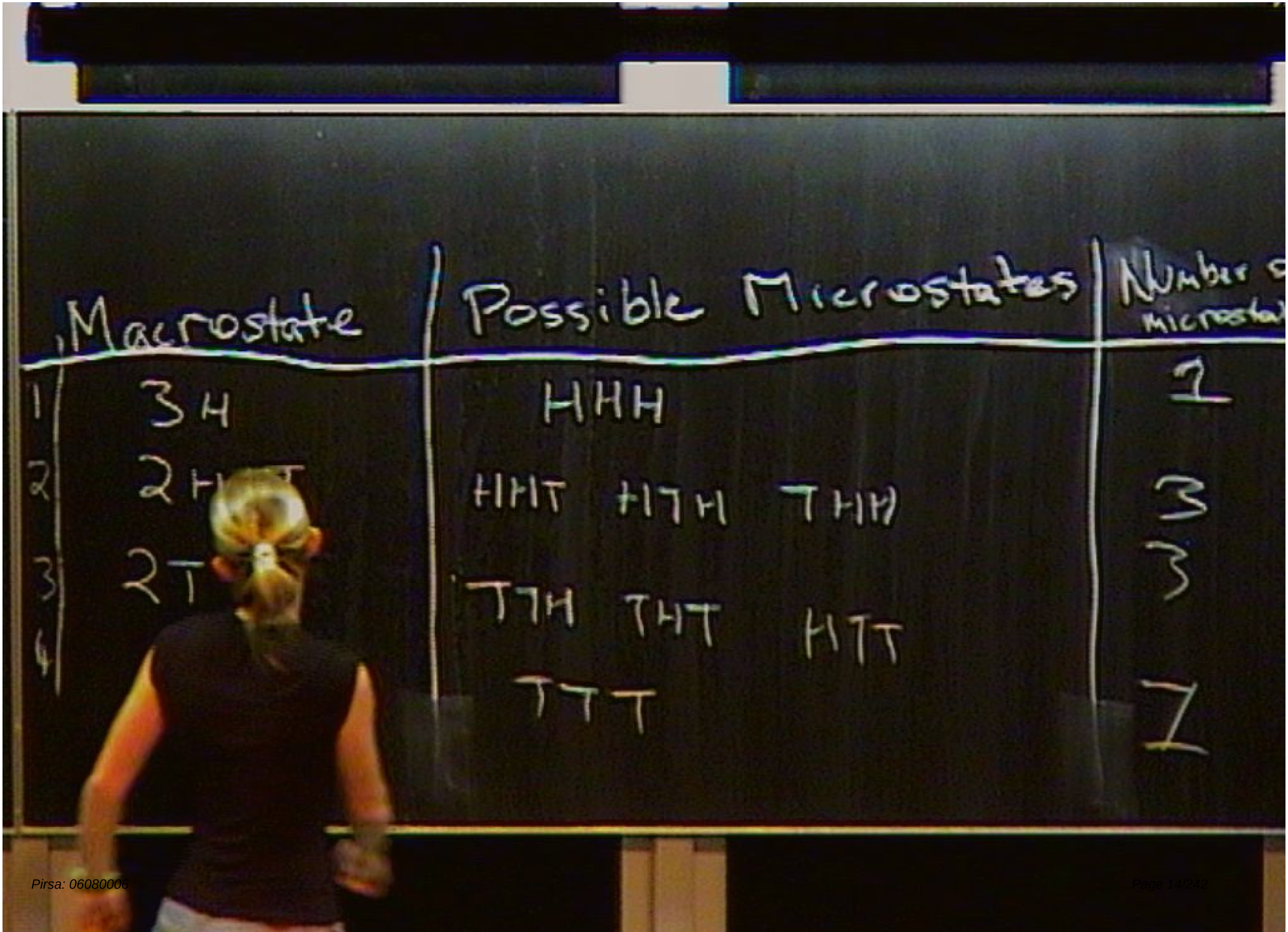
2	2H 1T	HHT HTH THH	3
	2T 1H	TTH THT HTT	
	3T		



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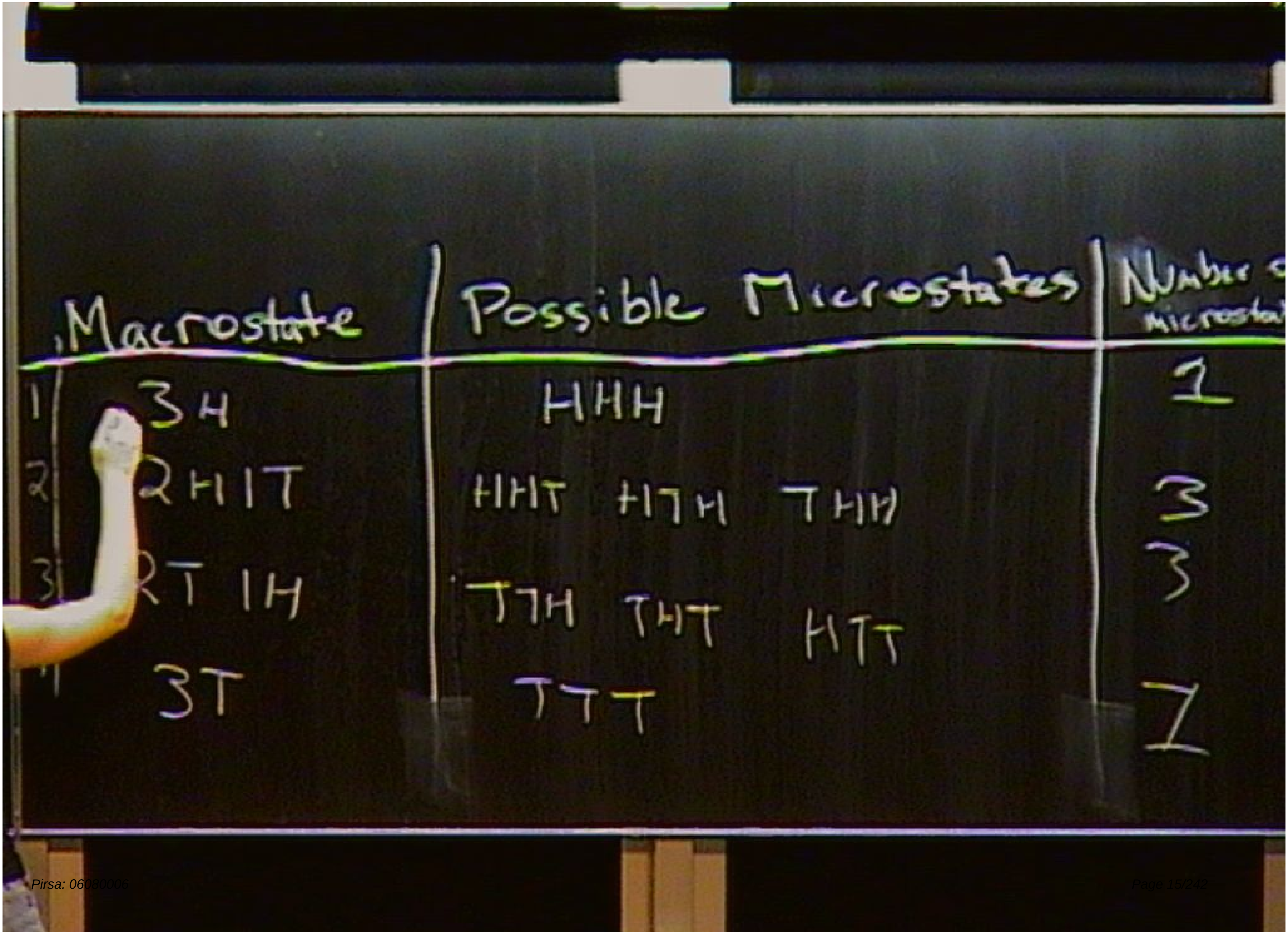
4		TTT	1



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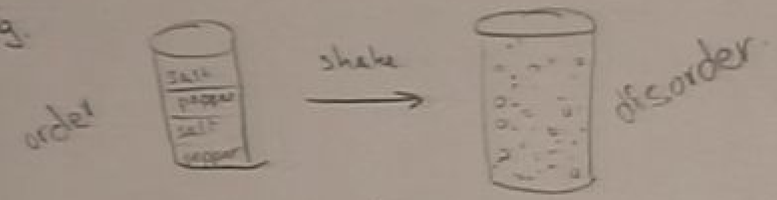

Entropy of systems increases over time

A system is made up of microstates and macrostates

Microstates - Specific information about systems like velocity & velocity

Macrostates - General information about systems like temperature or # of molecu

Consider a case where 3 coins are tossed

low entropy

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The least probable states are the most orderly and have lower entropy.

The reverse also applies.

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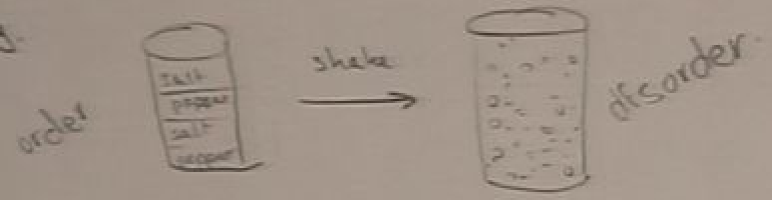

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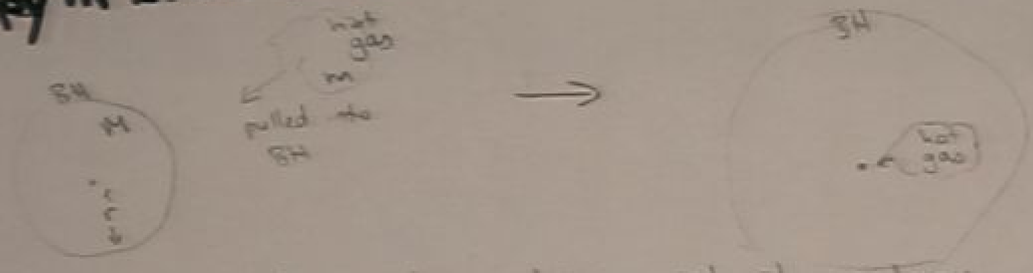
Entropy in Black Holes

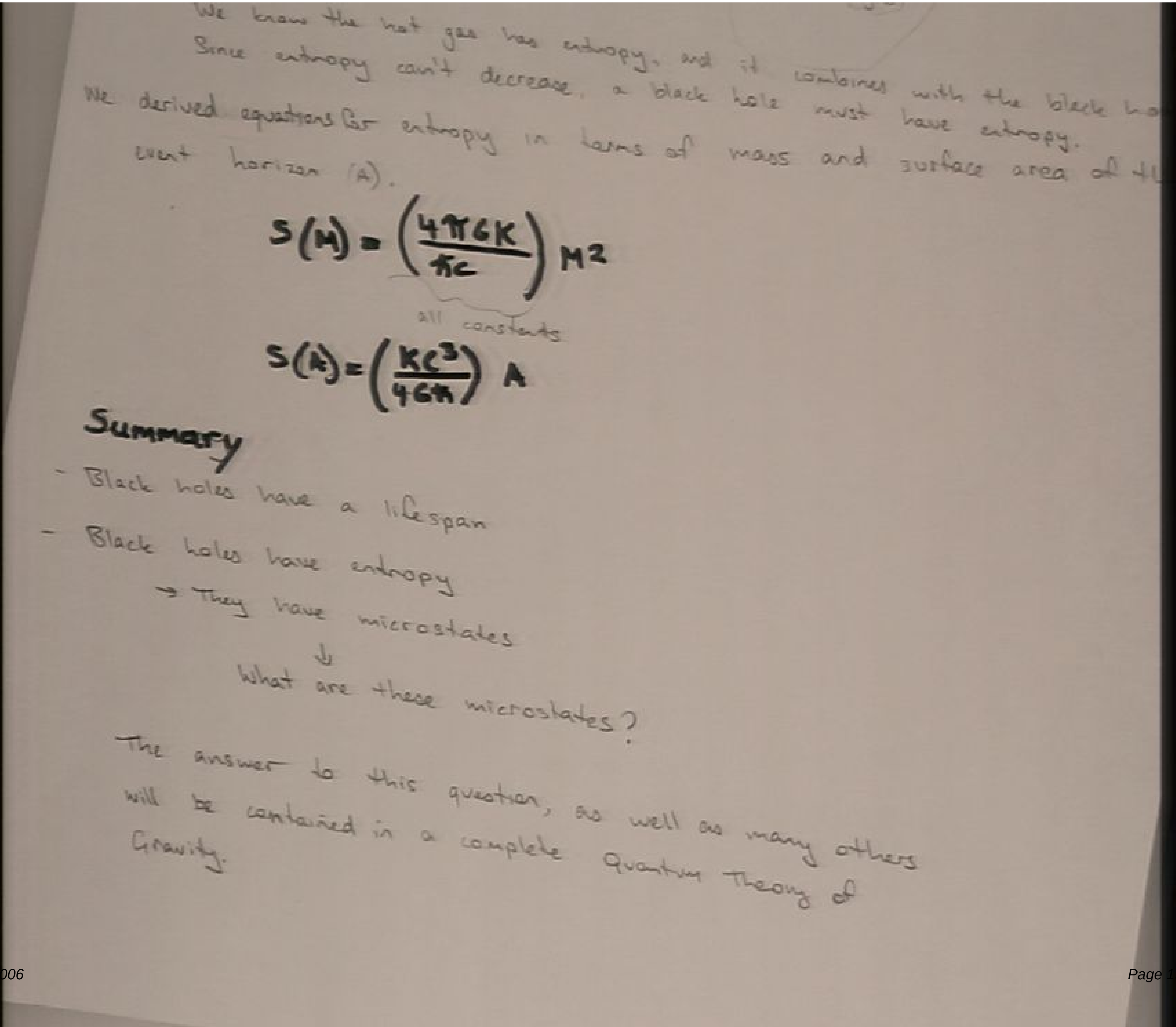


$M_{Final} = M + m$

=0

We know the hot gas has entropy, and it combines with the black hole. Since entropy can't decrease, a black hole must have entropy.

and surface area of the



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We derived equations for entropy in terms of mass and surface area of th

event horizon (A).

$$S(M) = \underbrace{\left(\frac{4\pi G K}{\hbar c}\right)}_{\text{all constants}} M^2$$

$$S(A) = \left(\frac{Kc^3}{4G\hbar}\right) A$$

Summary

- Black holes have a lifespan
- Black holes have entropy
  - → They have microstates

    ↓

    What are these microstates?

The answer to this question, as well as many others will be contained in a complete Quantum Theory of Gravity.

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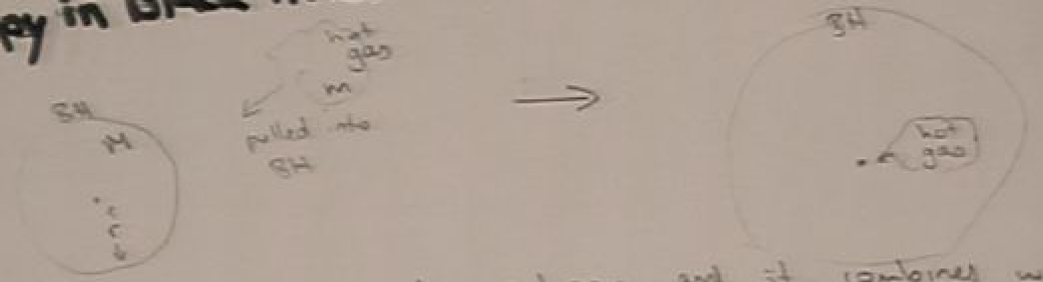

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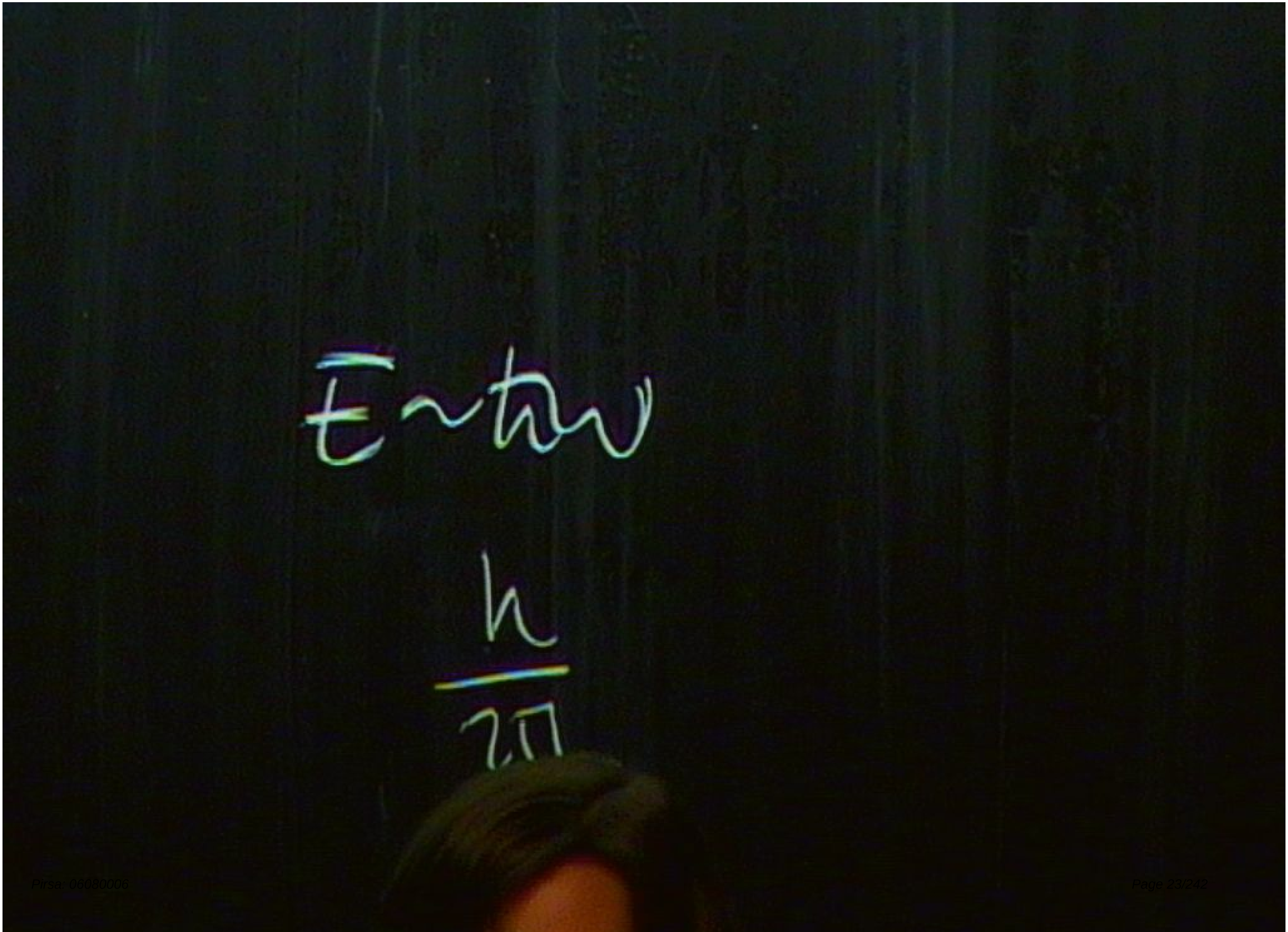
④



$$E \sim \hbar\omega$$

$$\frac{h}{2\pi}$$

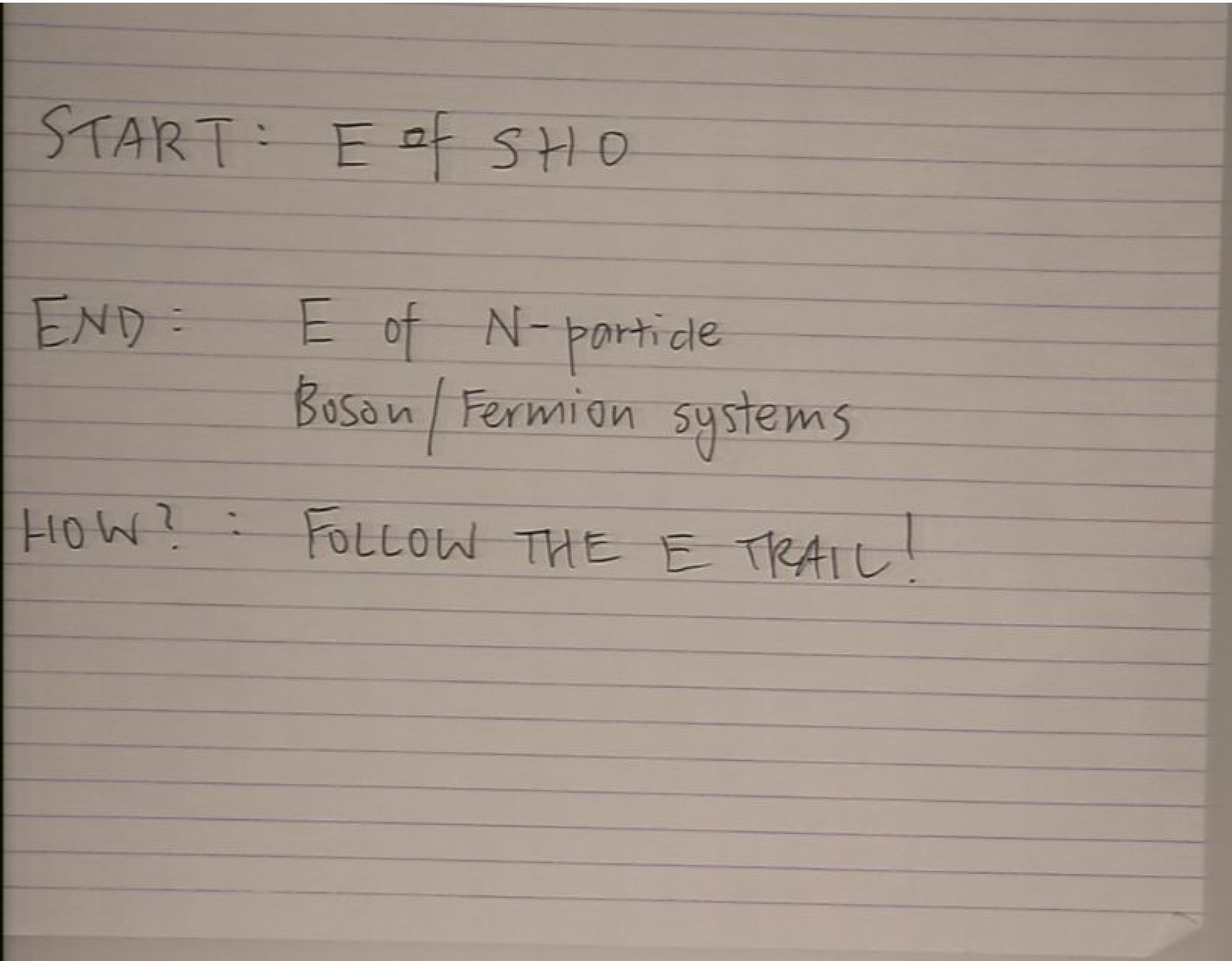
SCHRODINGER

wave function
partial diffentiation

VS

HEISENBERG.

matrices
eigenvalue/vectors



START: E of SHO

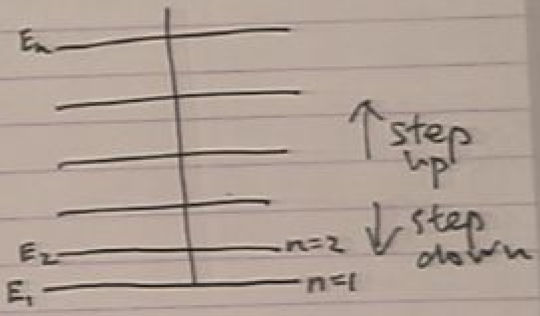
END: E of N-particle
Boson/Fermion systems

HOW? : FOLLOW THE E TRAIL!

$$\bar{z}z = \frac{m\omega^2}{2}\hat{x}^2 + \frac{\hat{p}^2}{2m}$$

$$z = \sqrt{\frac{m\omega^2}{2}}\,\hat{x} + \frac{i\hat{p}}{2m}$$

$\frac{1}{\sqrt{\hbar\omega}}z \Rightarrow \hat{a}$ annihilation operator



$\frac{1}{\sqrt{\hbar\omega}}\bar{z} \Rightarrow \hat{a}^+$ creation operator.

$\hat{a}^+ = \sqrt{n+1}\,|n+1\rangle$

$$\bar{z}z = \frac{m\omega^2}{2}\hat{x}^2 + \frac{\hat{p}^2}{2m}$$

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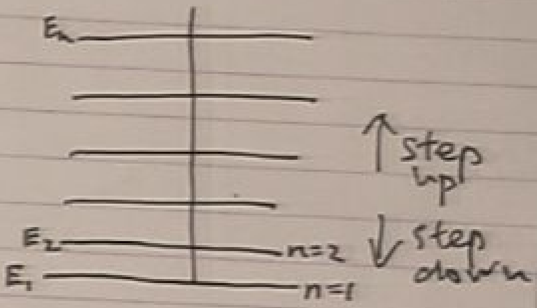
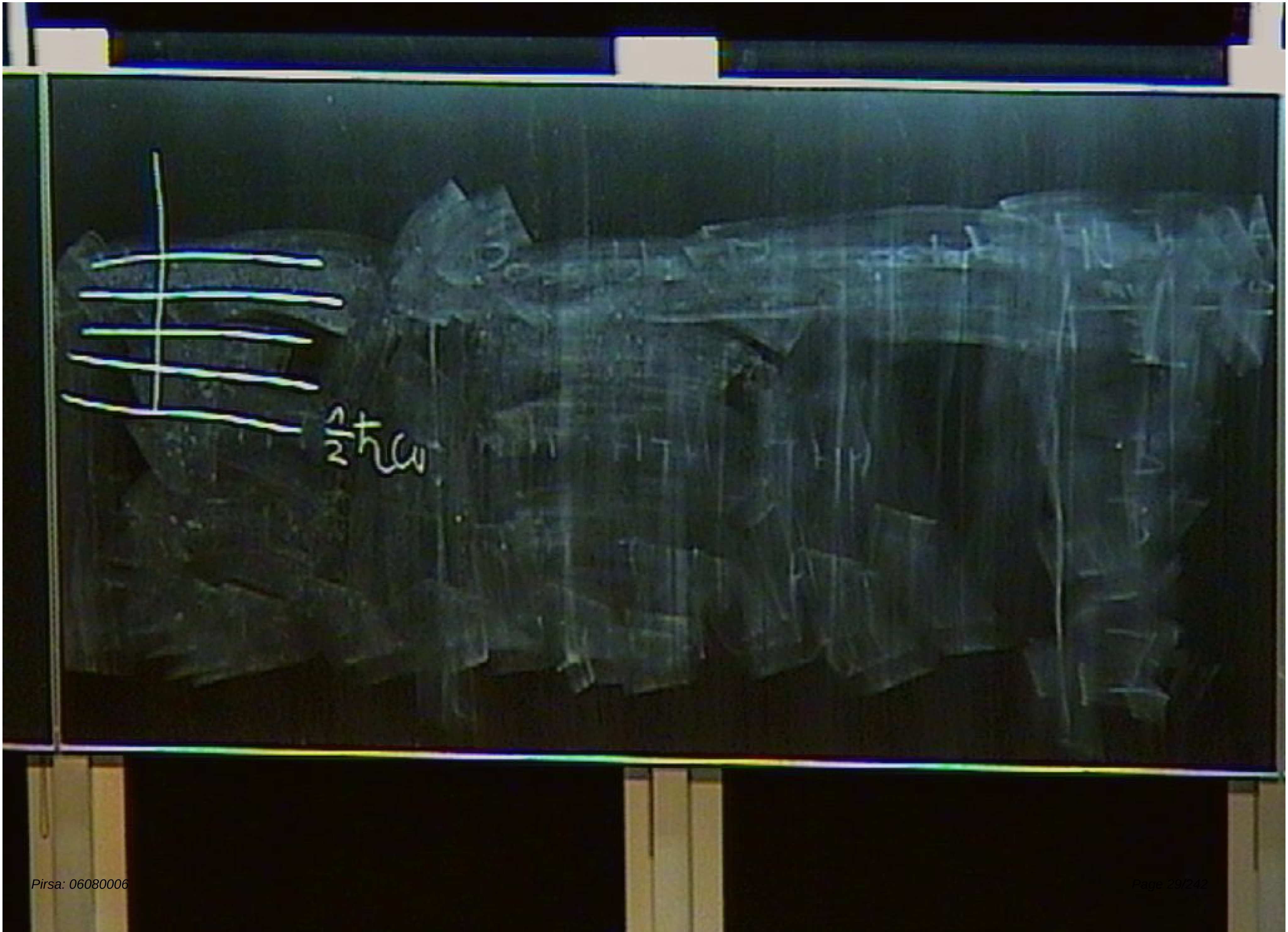
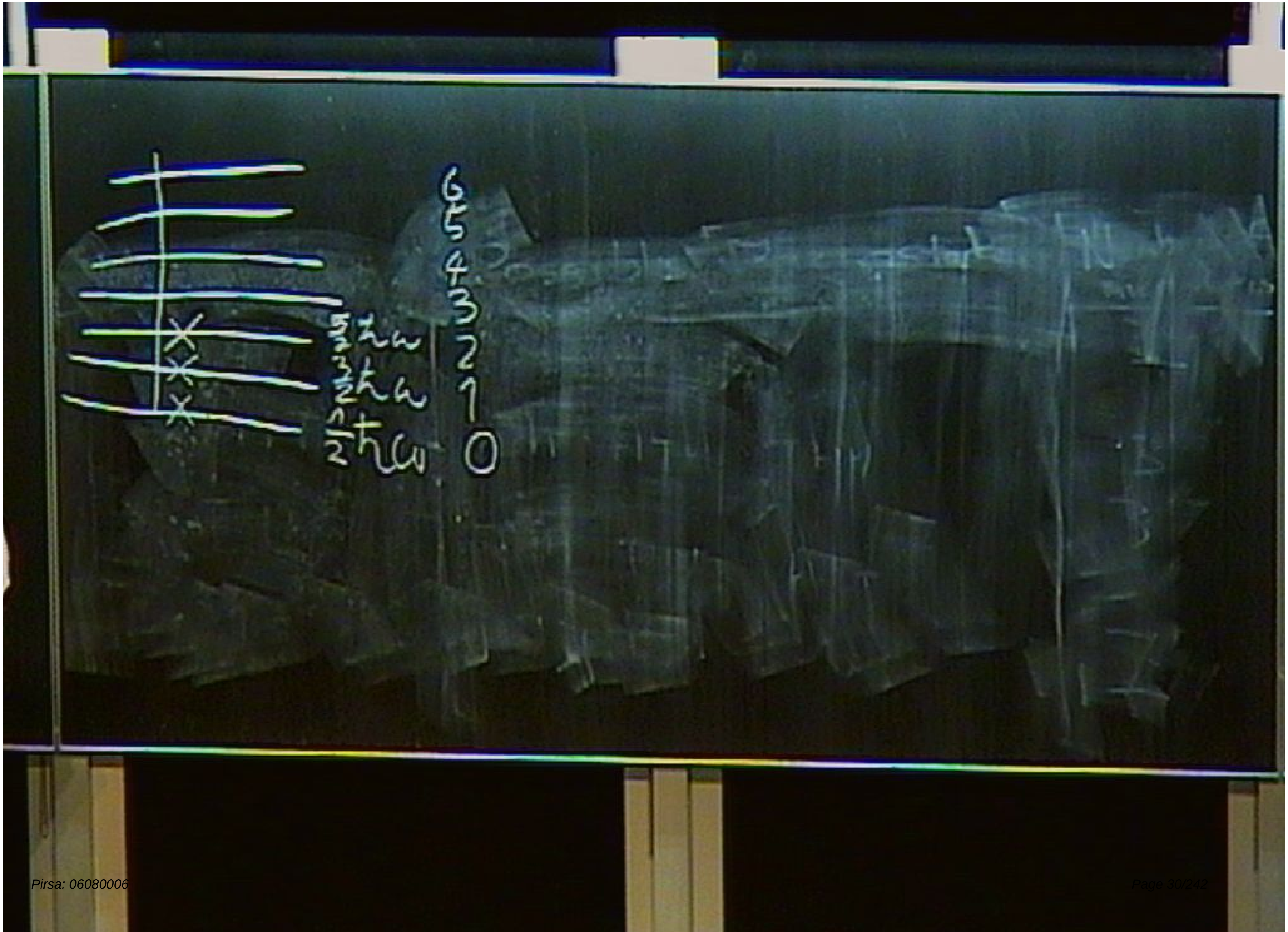


$$\frac{1}{\sqrt{\hbar\omega}}\bar{z} \Rightarrow \hat{a}^+$$ creation operator.

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle \ , \quad \hat{a}^+ = \sqrt{n+1}\,|n+1\rangle .$$


$\frac{1}{2}\hbar\omega$


6
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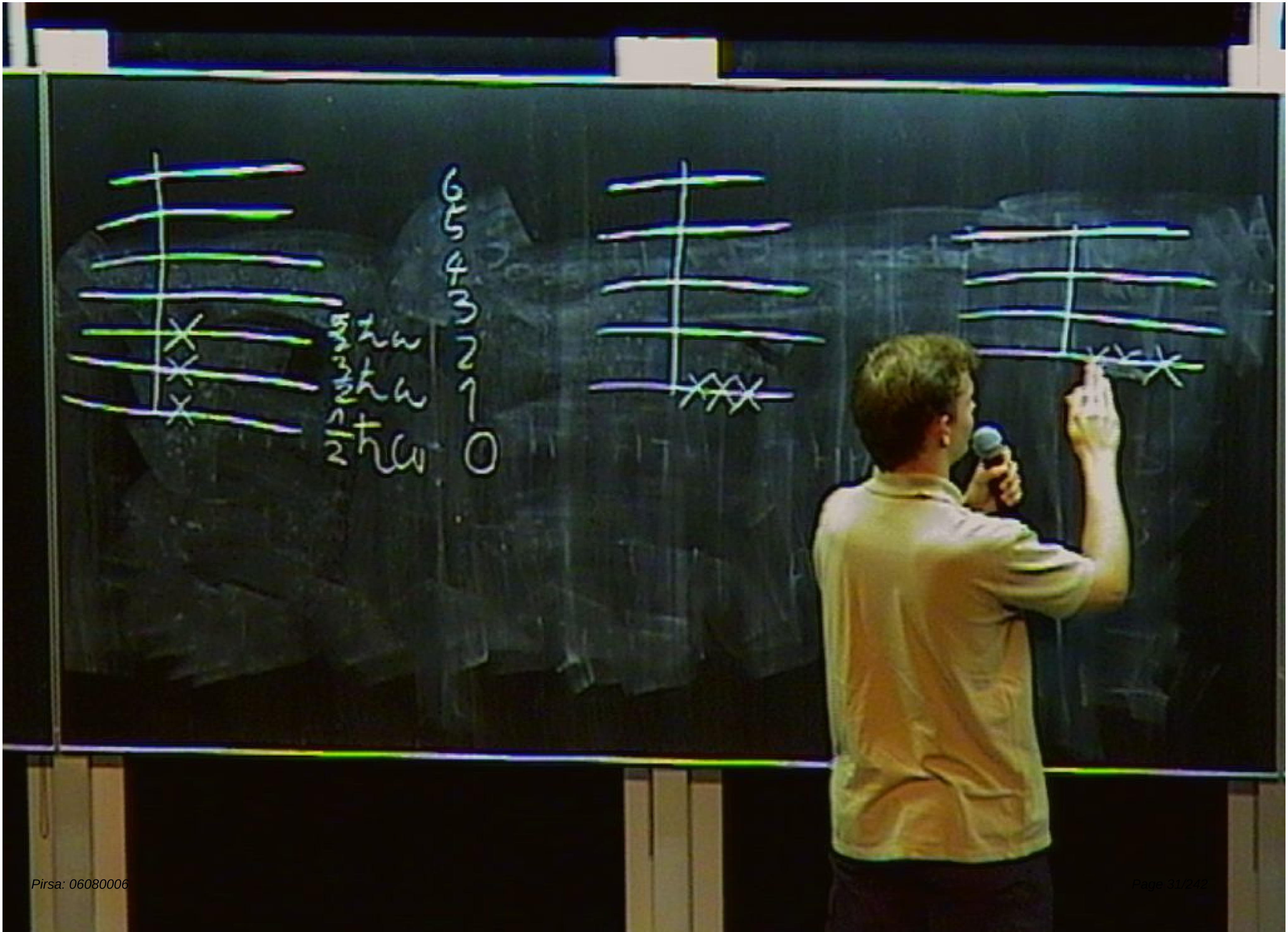
$\frac{3}{2}\hbar\omega$ 1
$\frac{1}{2}\hbar\omega$ 0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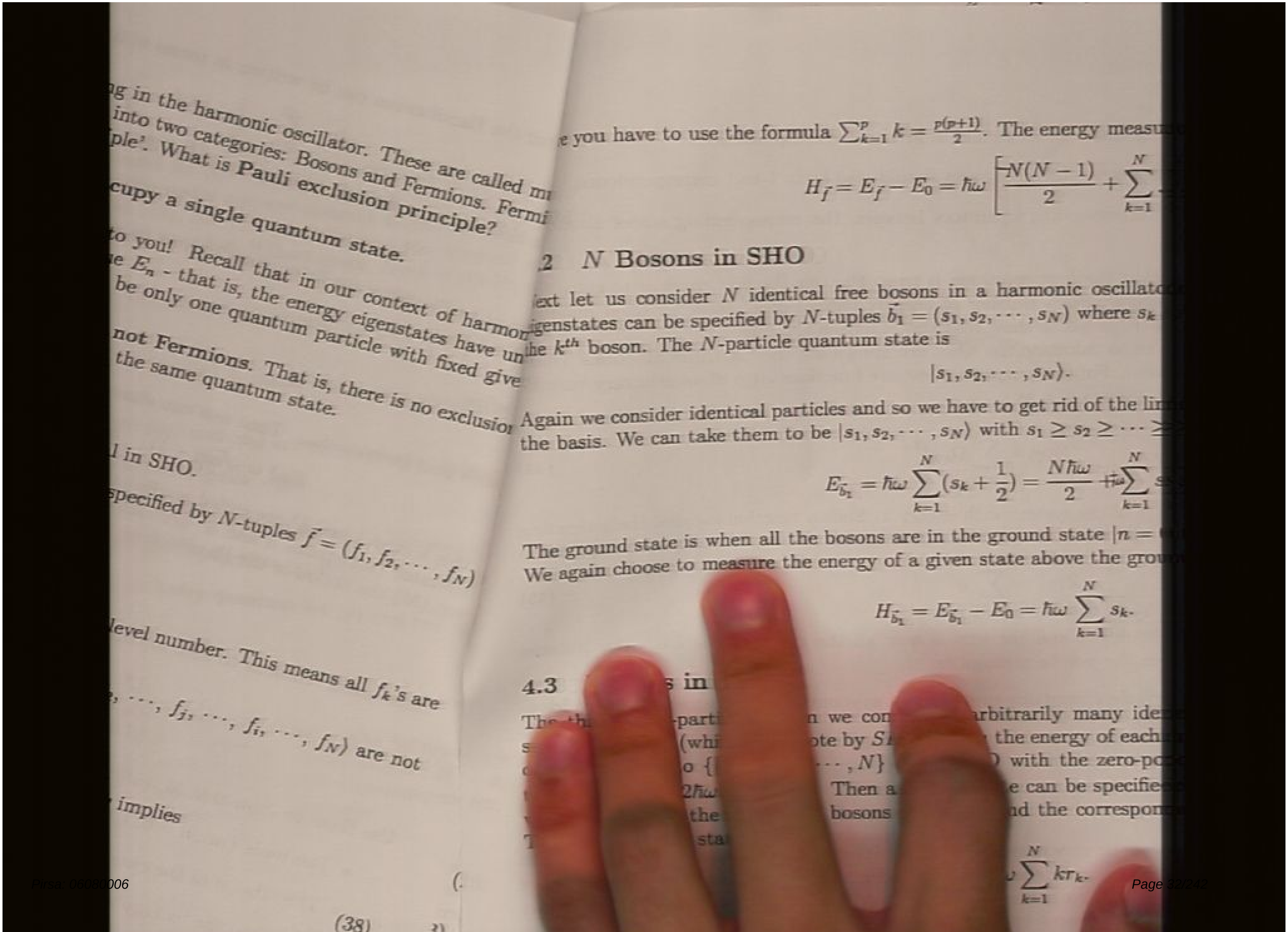
1/2 ħω 0



g in the harmonic oscillator. These are called m
into two categories: Bosons and Fermions. Fermi
ple[2]. What is **Pauli exclusion principle?**

cupy a single quantum state.

o you! Recall that in our context of harmon
e E_n - that is, the energy eigenstates have un
be only one quantum particle with fixed give

not Fermions. That is, there is no exclusio
the same quantum state.

l in SHO.

pecified by N-tuples $\vec{f} = (f_1, f_2, \cdots, f_N)$

level number. This means all f_k's are
$, \cdots, f_j, \cdots, f_i, \cdots, f_N)$ are not

implies

(38)

e you have to use the formula $\sum_{k=1}^{p} k = \frac{p(p+1)}{2}$. The energy measu

$$H_{\vec{f}} = E_{\vec{f}} - E_0 = \hbar\omega \left[\frac{N(N-1)}{2} + \sum_{k=1}^{N} \right.$$

2 N Bosons in SHO

ext let us consider N identical free bosons in a harmonic oscillat
igenstates can be specified by N-tuples $\vec{b_1} = (s_1, s_2, \cdots, s_N)$ where s_k
he k^{th} boson. The N-particle quantum state is

$$|s_1, s_2, \cdots, s_N\rangle.$$

Again we consider identical particles and so we have to get rid of the lin
the basis. We can take them to be $|s_1, s_2, \cdots, s_N\rangle$ with $s_1 \geq s_2 \geq \cdots \geq$

$$E_{\vec{b_1}} = \hbar\omega \sum_{k=1}^{N} (s_k + \frac{1}{2}) = \frac{N\hbar\omega}{2} + \hbar\omega \sum_{k=1}^{N} s$$

The ground state is when all the bosons are in the ground state $|n =$
We again choose to measure the energy of a given state above the grou

$$H_{\vec{b_1}} = E_{\vec{b_1}} - E_0 = \hbar\omega \sum_{k=1}^{N} s_k.$$

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Th [illegible] -parti [illegible] n we con [illegible] rbitrarily many ide
s [illegible] (whi [illegible] ote by S [illegible] the energy of each
o $\{$ [illegible] $\cdots, N\}$ [illegible]) with the zero-po
2$\hbar\omega$ [illegible] Then a [illegible] e can be specifie
the [illegible] bosons [illegible] nd the correspon
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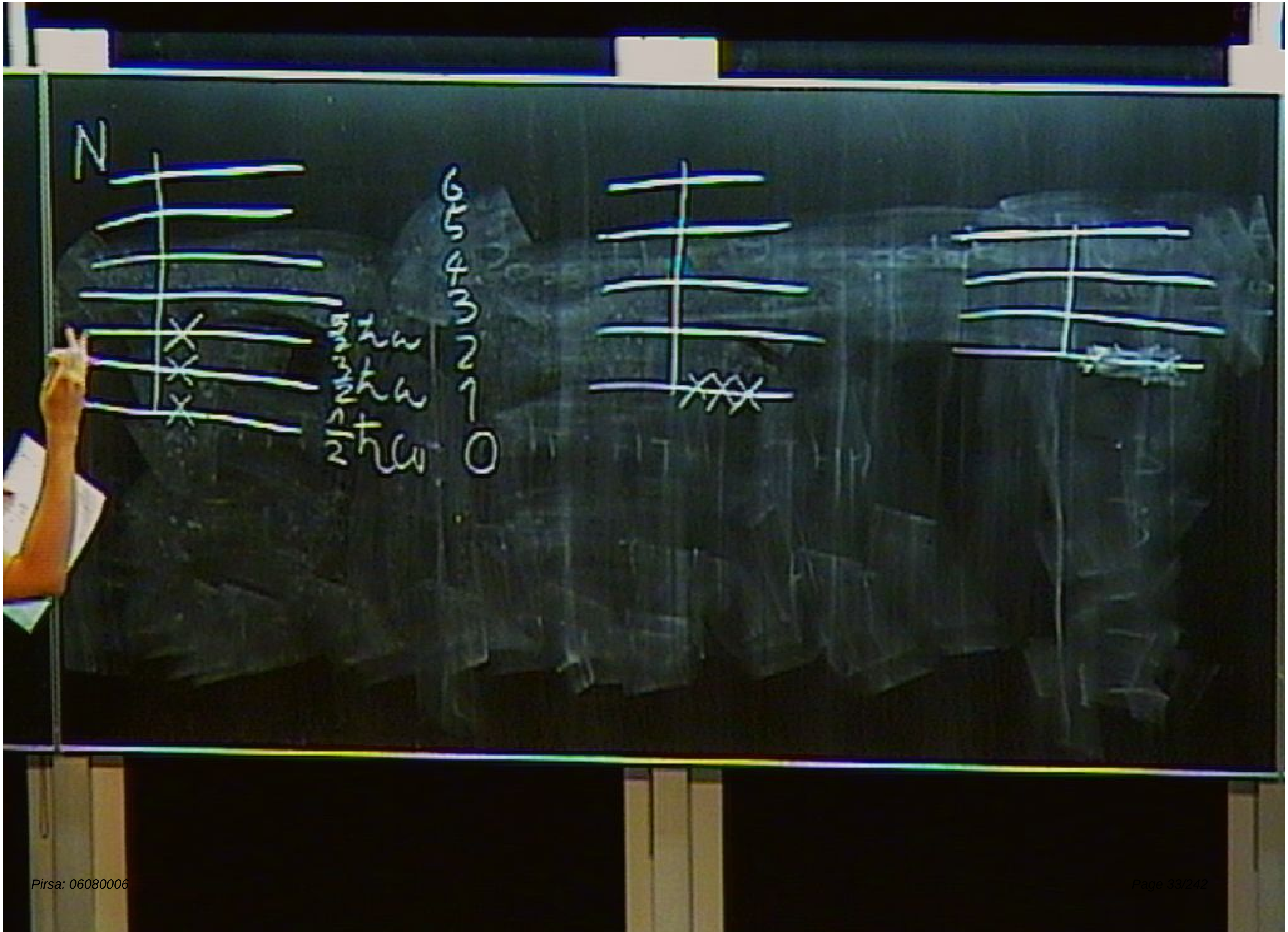
$$\sum_{k=1}^{N} k r_k.$$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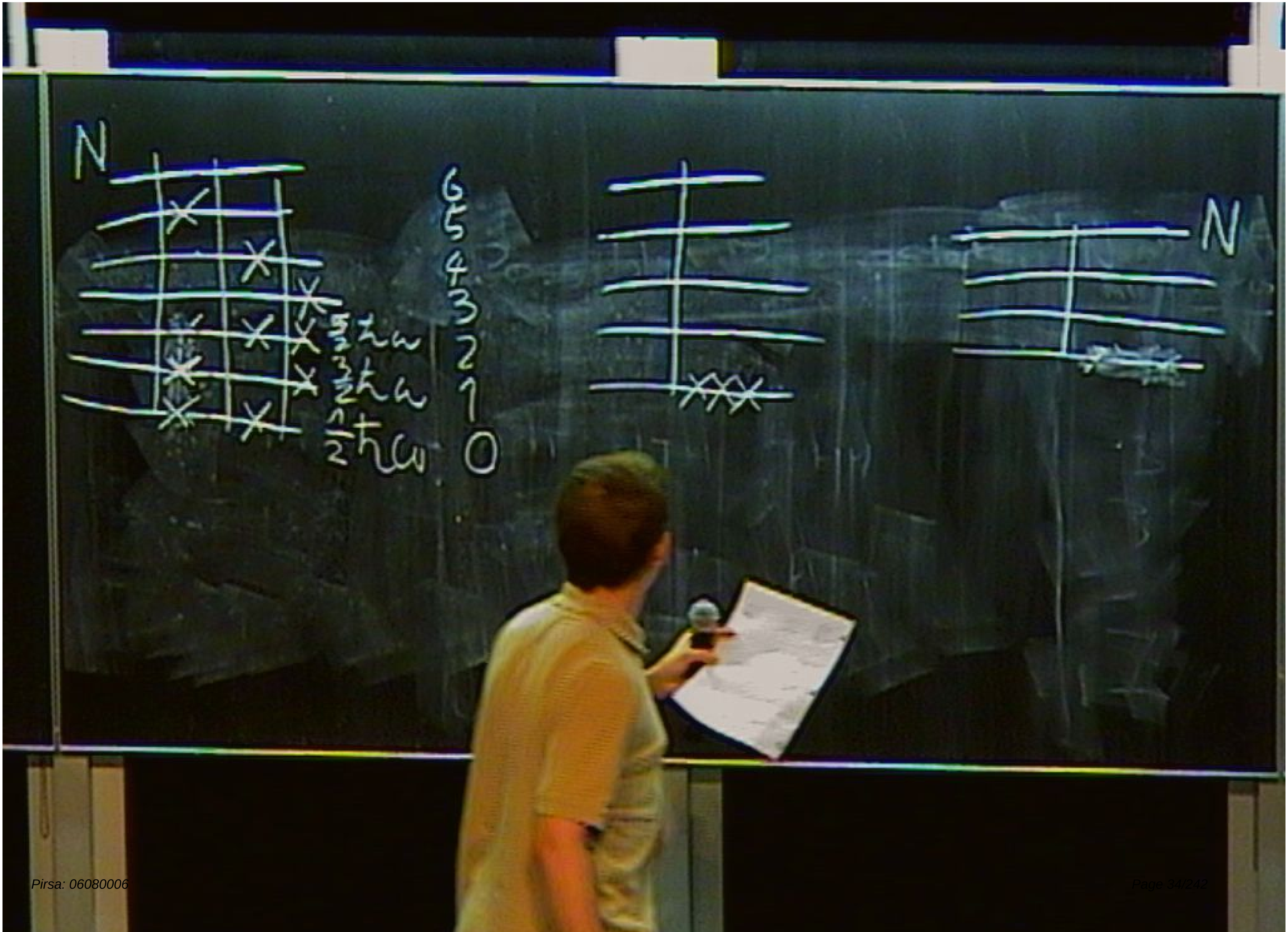
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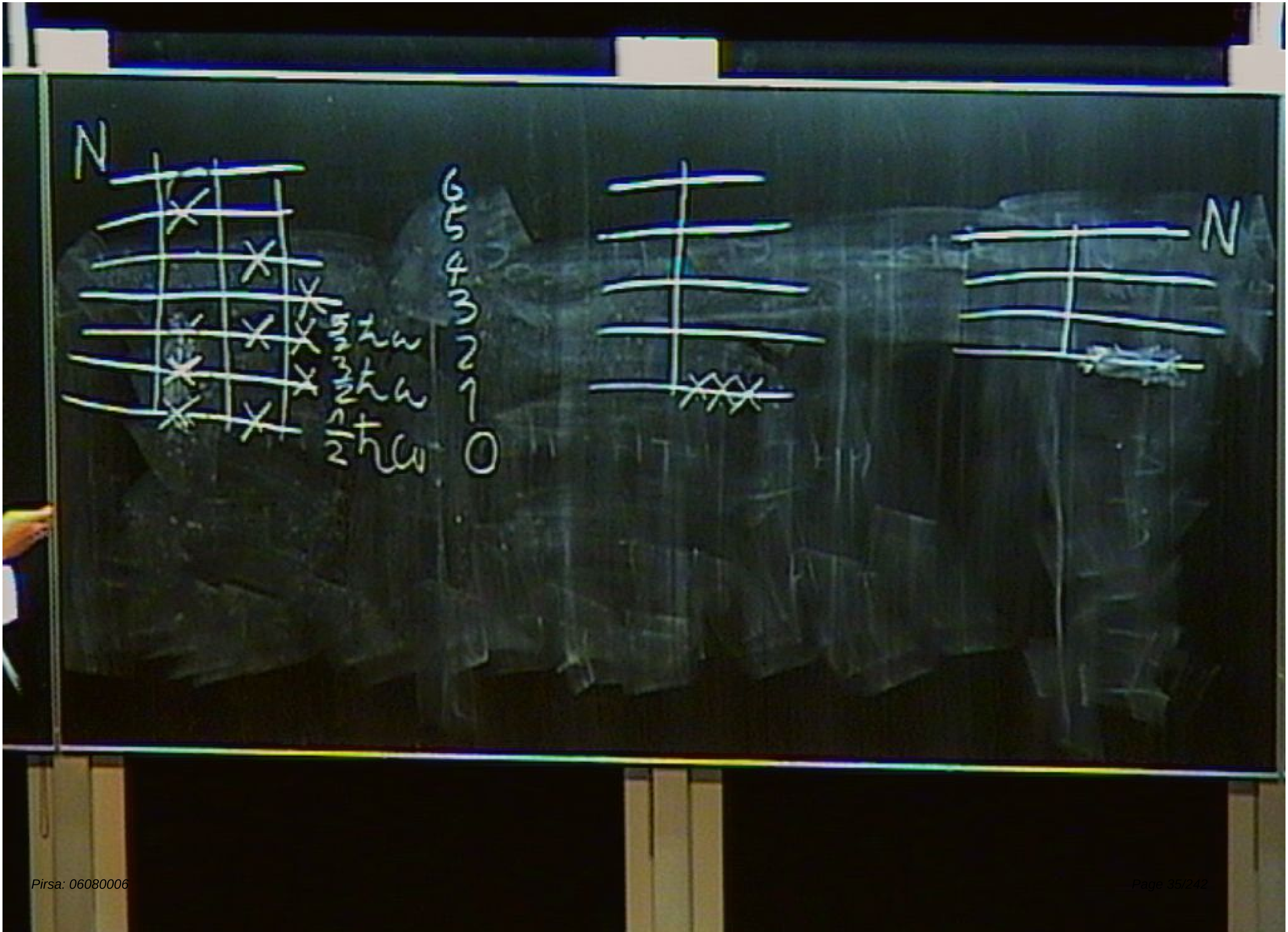
N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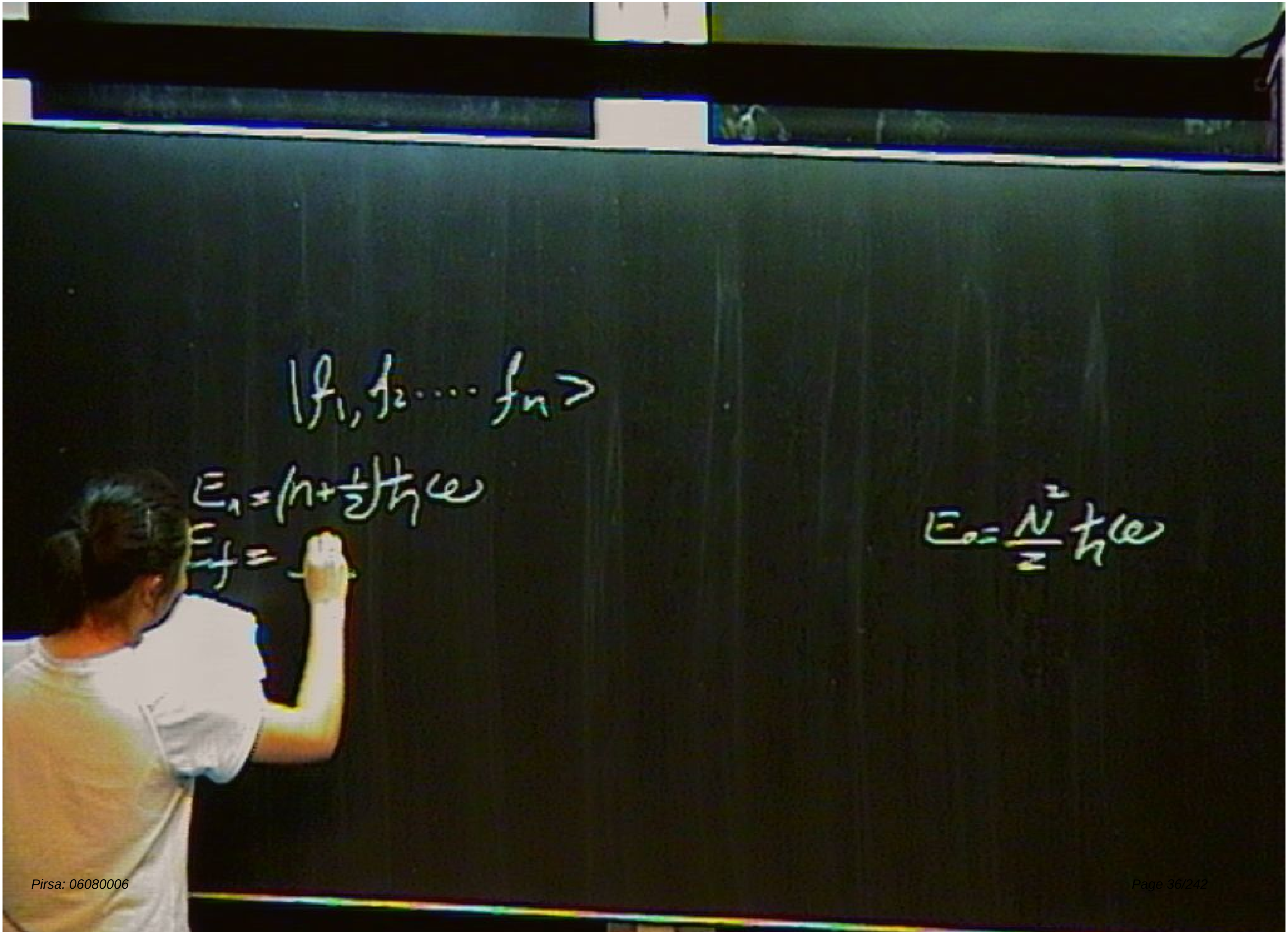
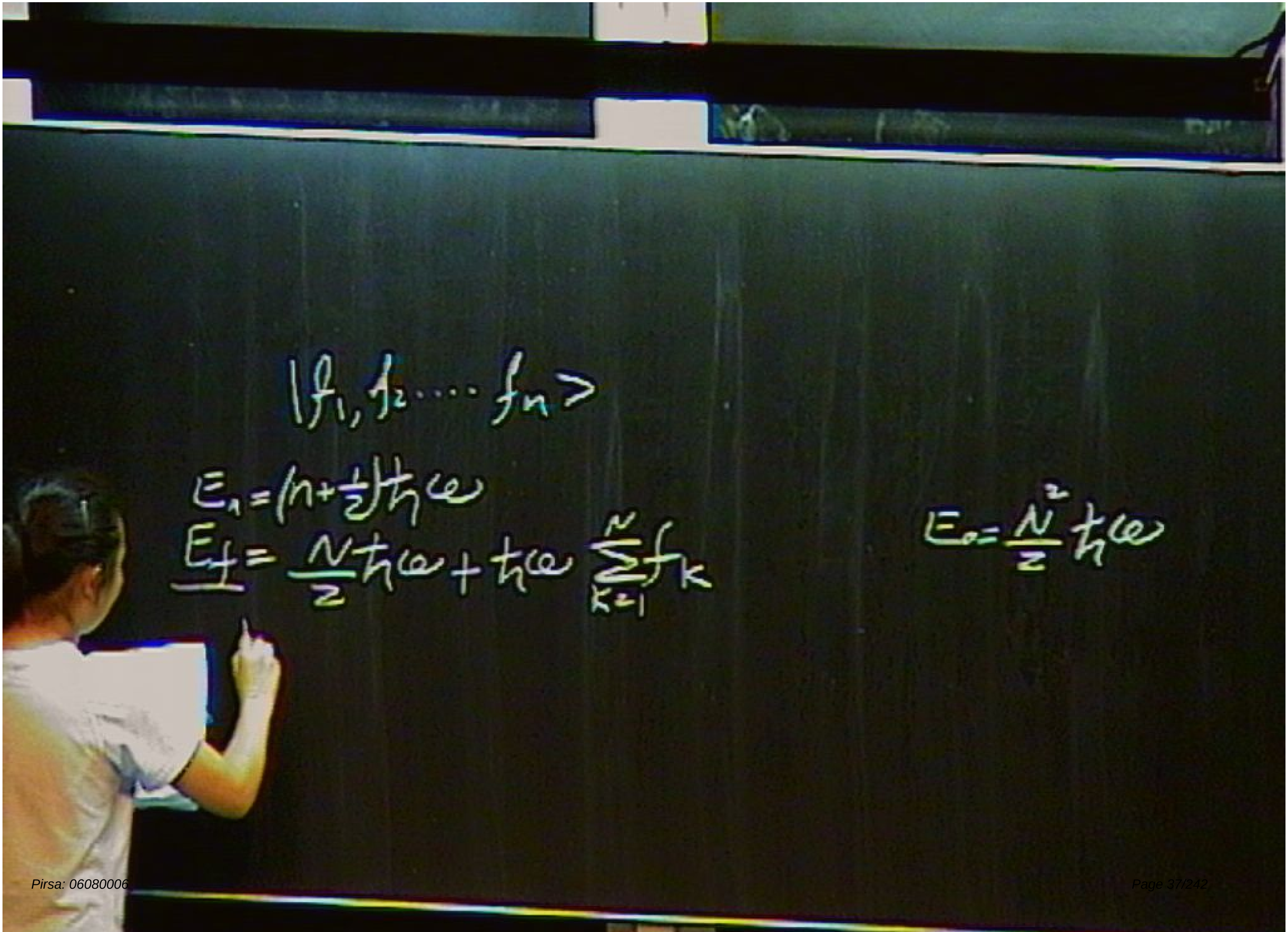


$|f_1, f_2 \cdots f_n\rangle$

$E_n = (n+\frac{1}{2})\hbar\omega$

$E_f =$

$E_0 = \frac{N^2}{2}\hbar\omega$



$$|f_1, f_2 \cdots\cdots f_n>$$

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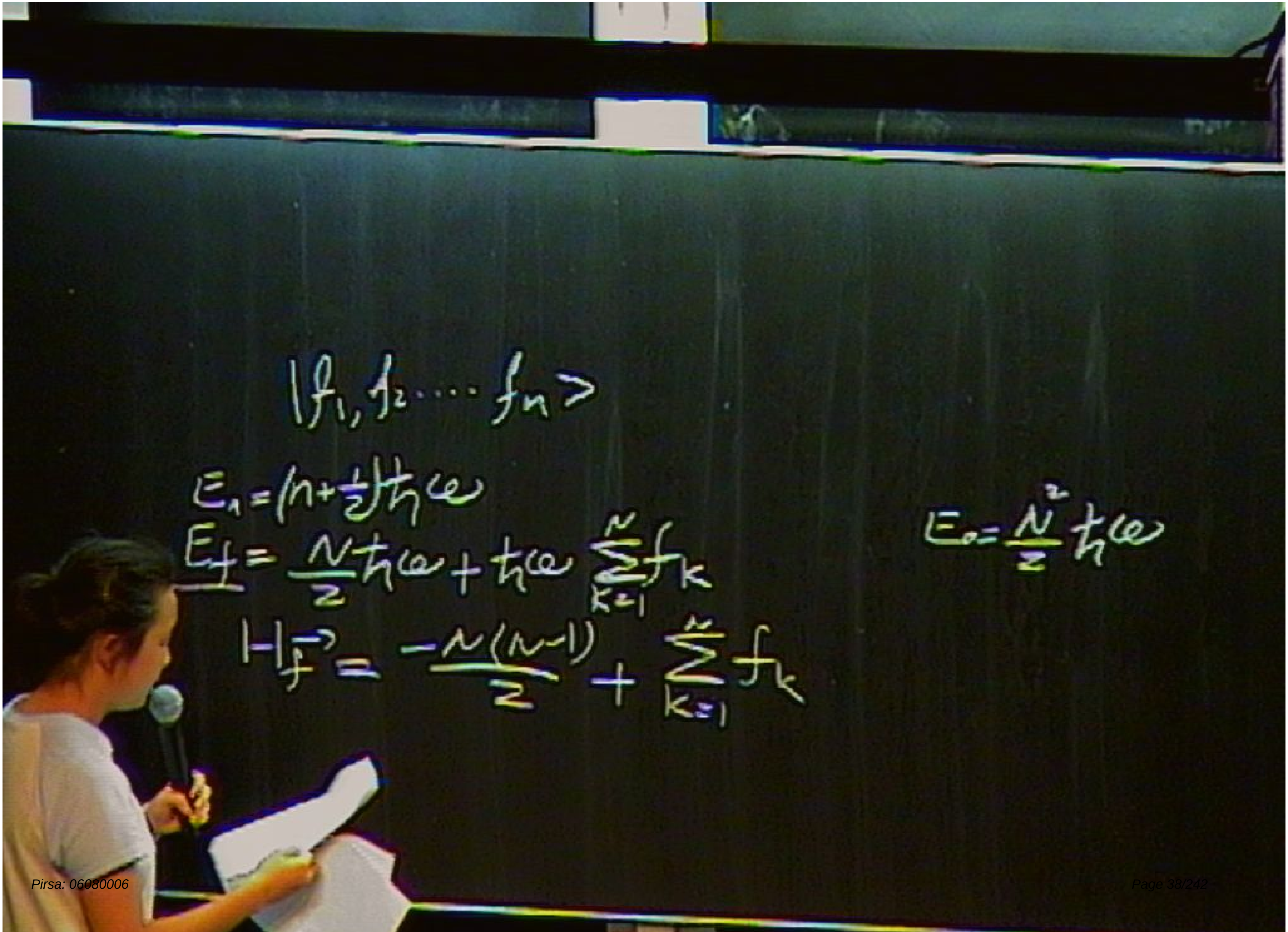

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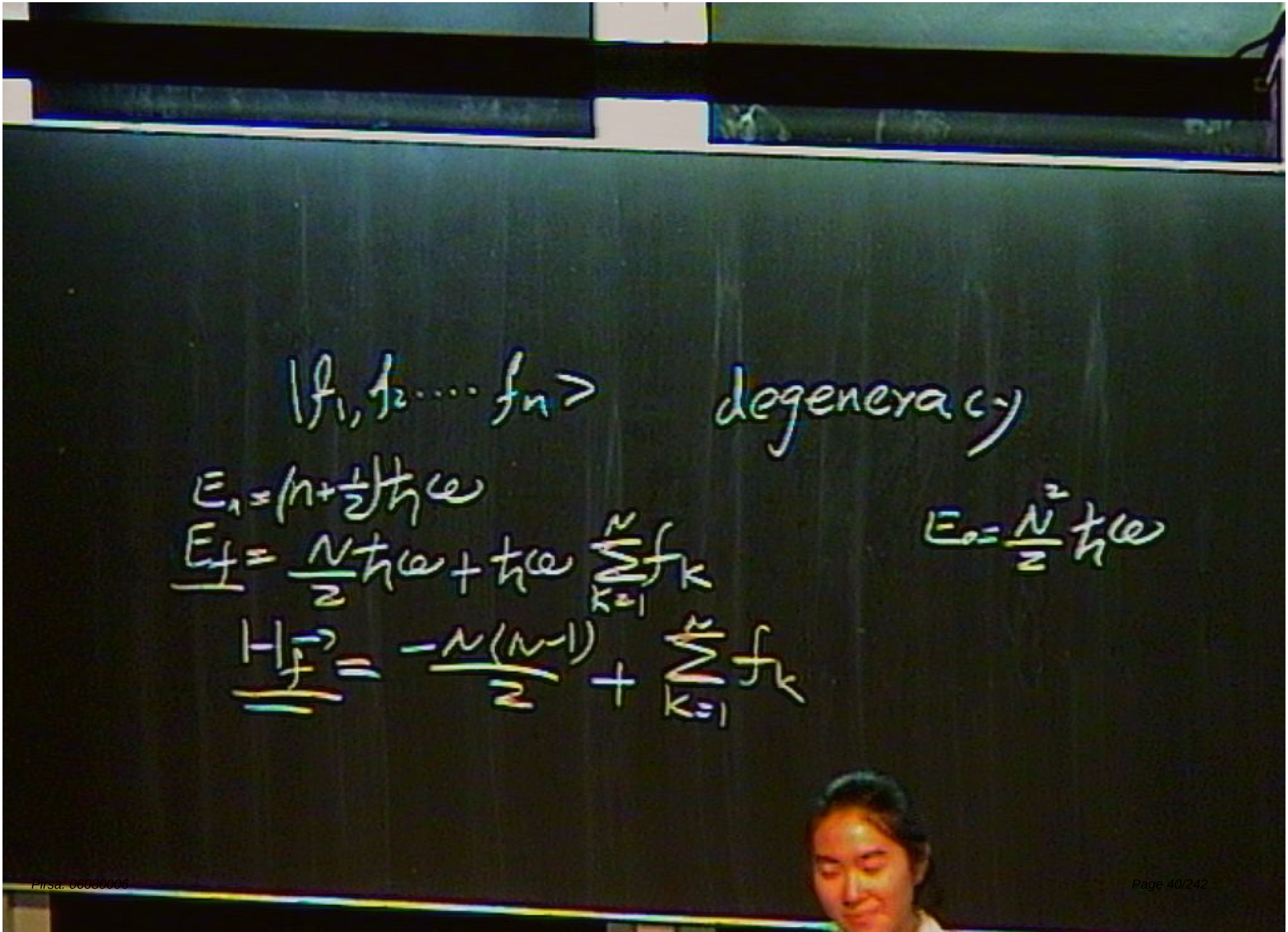


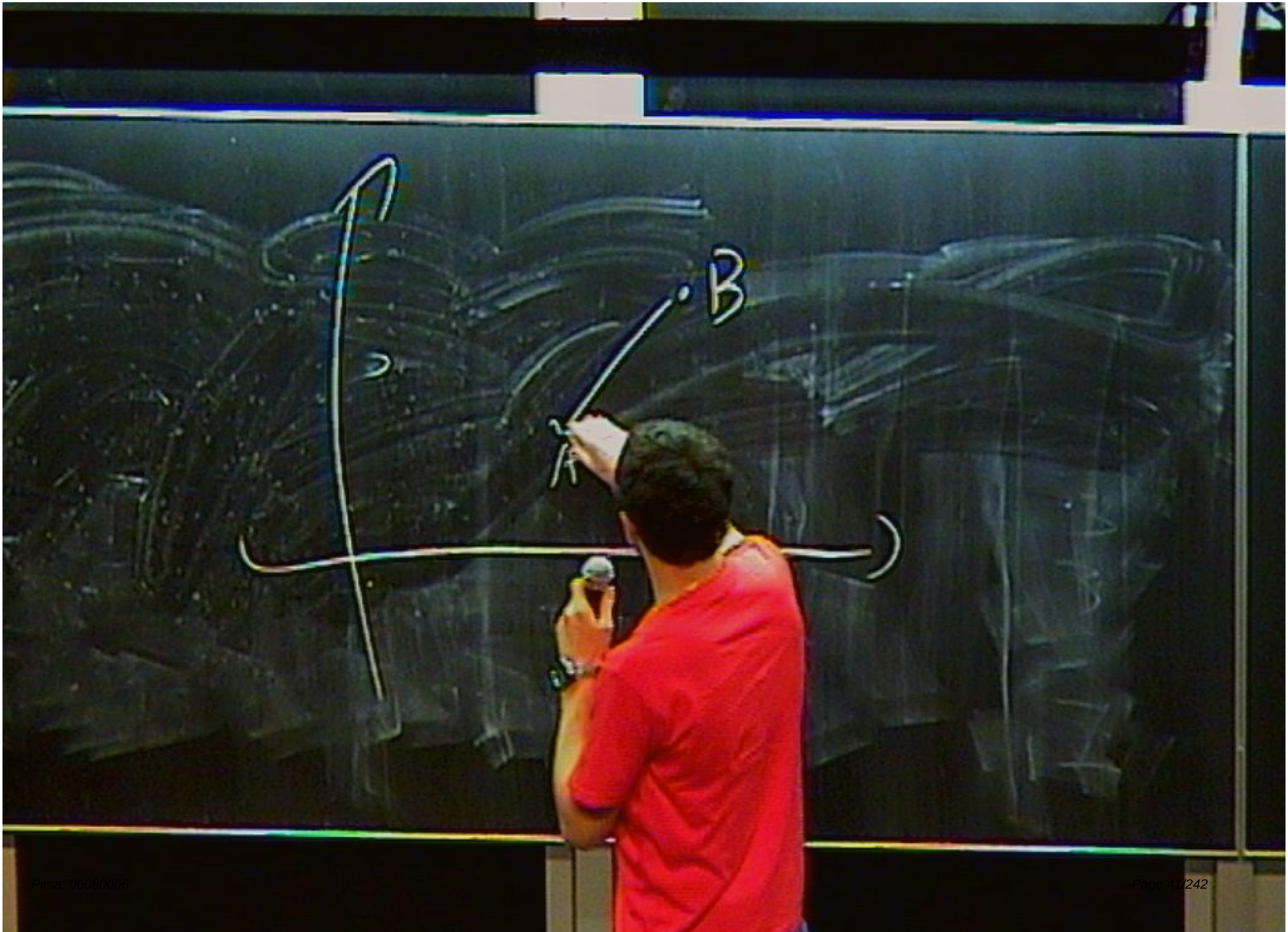
$|f_1, f_2 \cdots f_N\rangle$ degeneracy

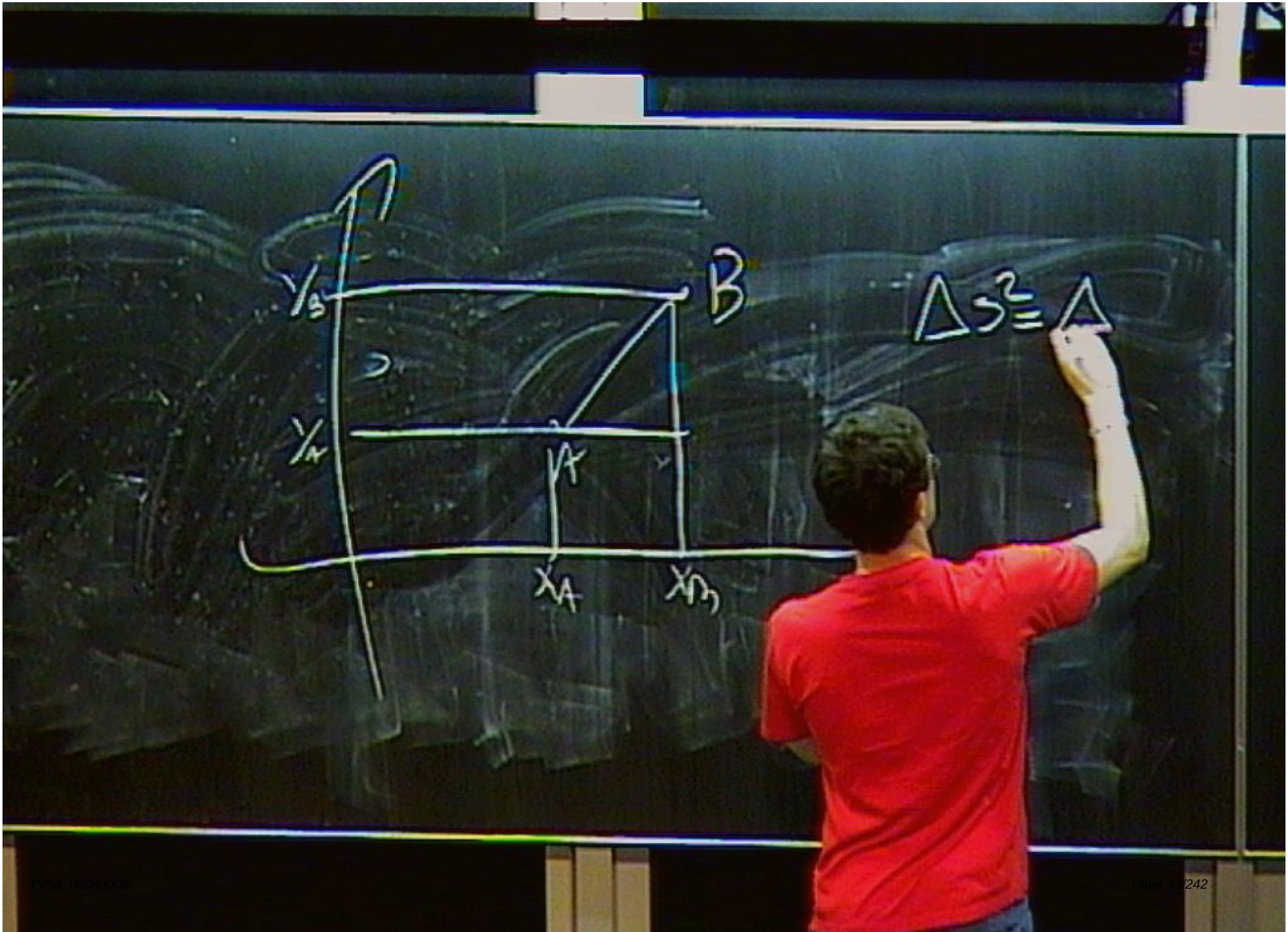
$E_n = (n+\frac{1}{2})\hbar\omega$

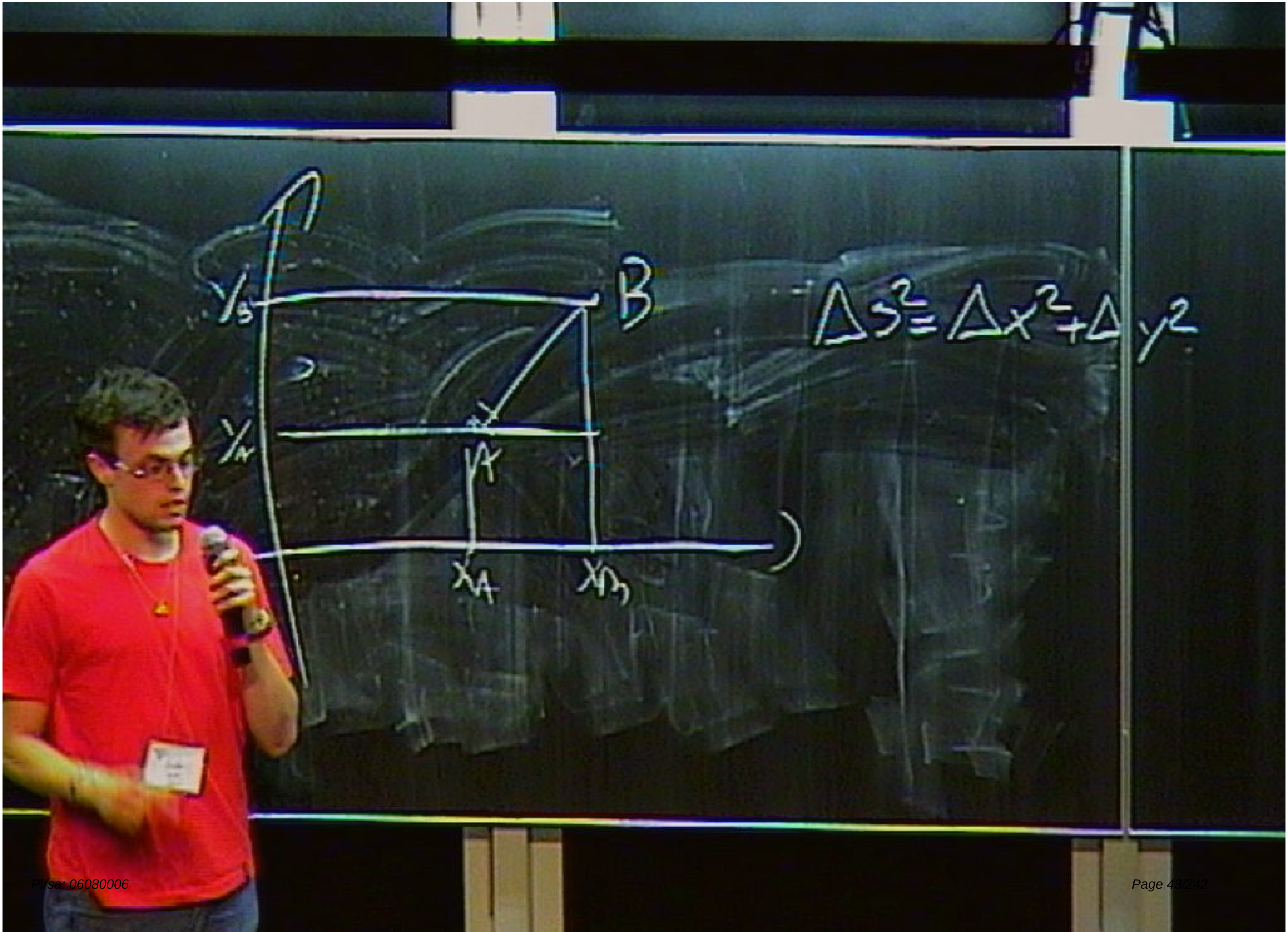
$\underline{E_f} = \frac{N}{2}\hbar\omega + \hbar\omega\sum_{k=1}^{N} f_k$

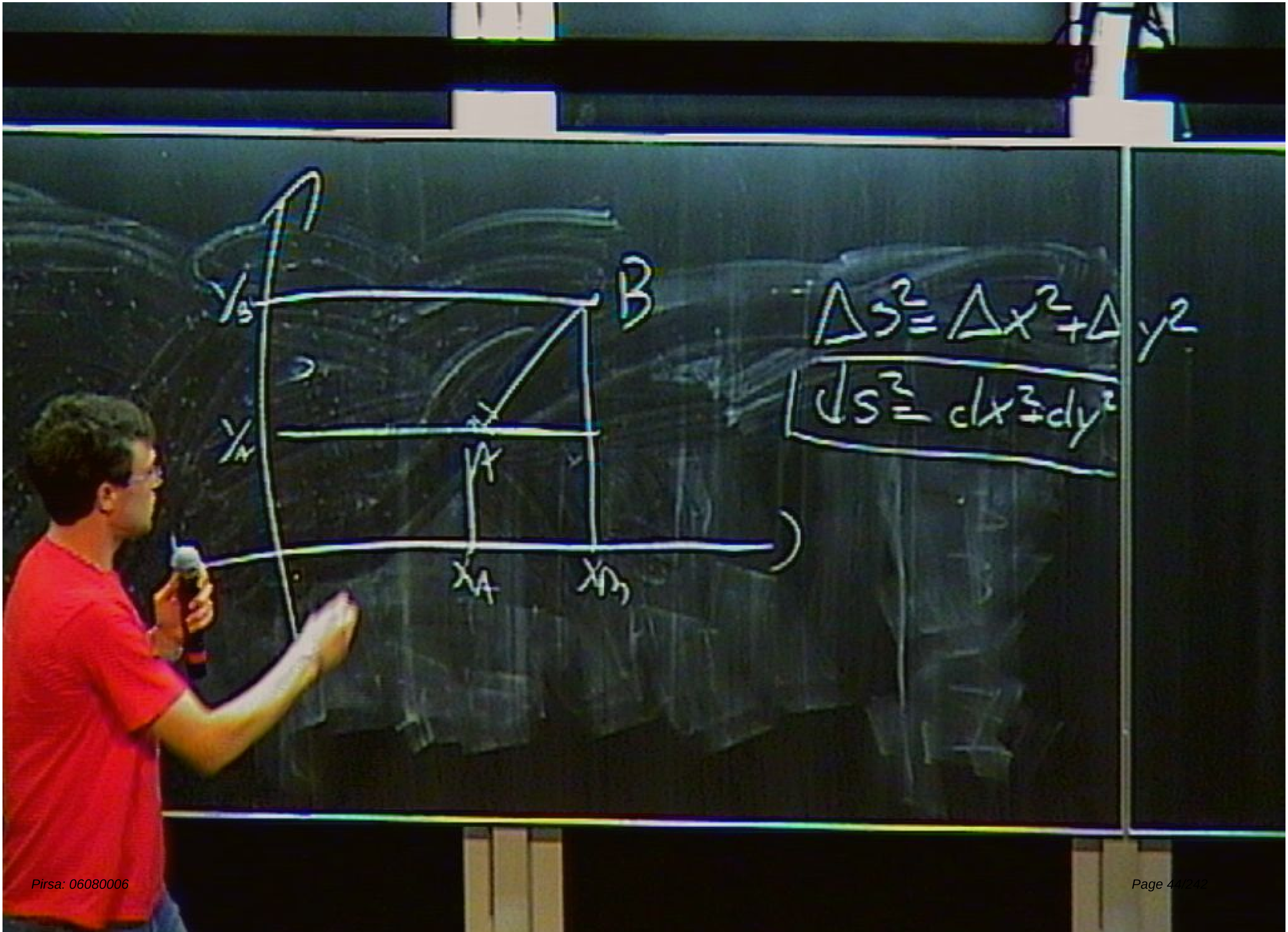
$E_0 = \frac{N^2}{2}\hbar\omega$

$\underline{\underline{H_{\vec{f}}}} = -\frac{N(N-1)}{2} + \sum_{k=1}^{N} f_k$


B
A

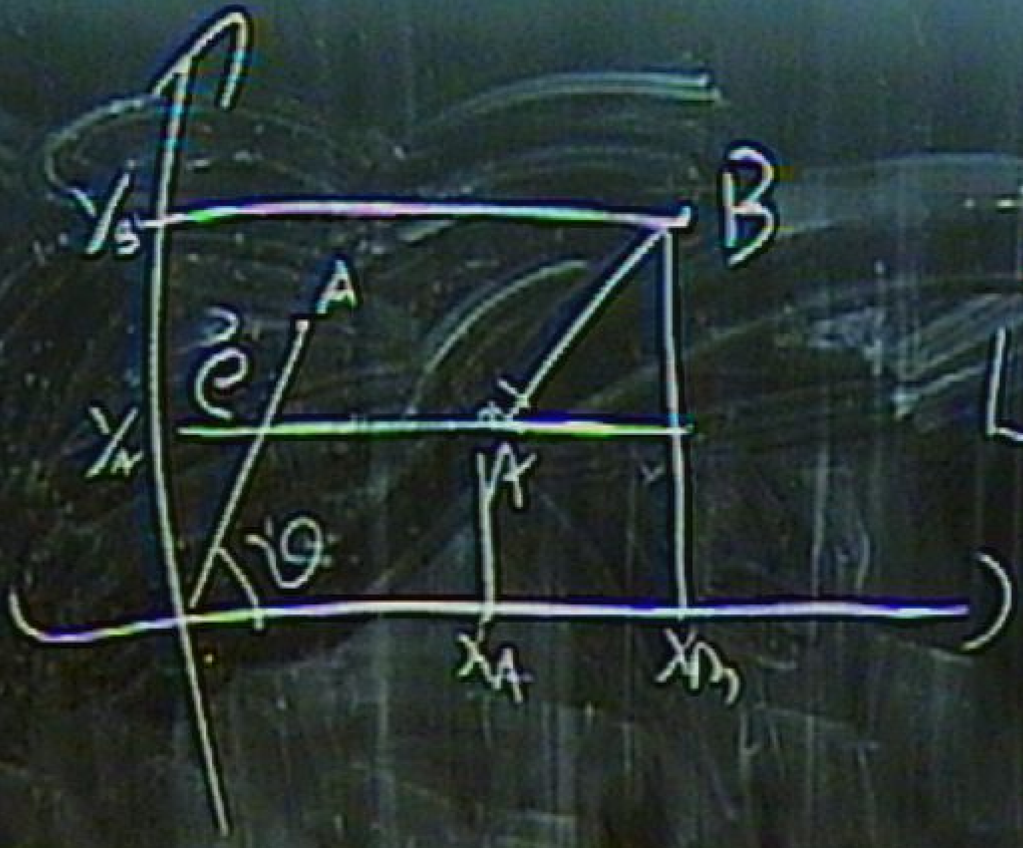

y_B
B
y_A
A
x_A
x_B
$\Delta s^2 = \Delta$


y_B
B
$\Delta s^2 = \Delta x^2 + \Delta y^2$
y_A
A
x_A
x_B



$$\Delta s^2 = \Delta x^2 + \Delta y^2$$

$$\boxed{ds^2 = dx^2 + dy^2}$$



$$\Delta s^2 = \Delta x^2 + \Delta y^2$$

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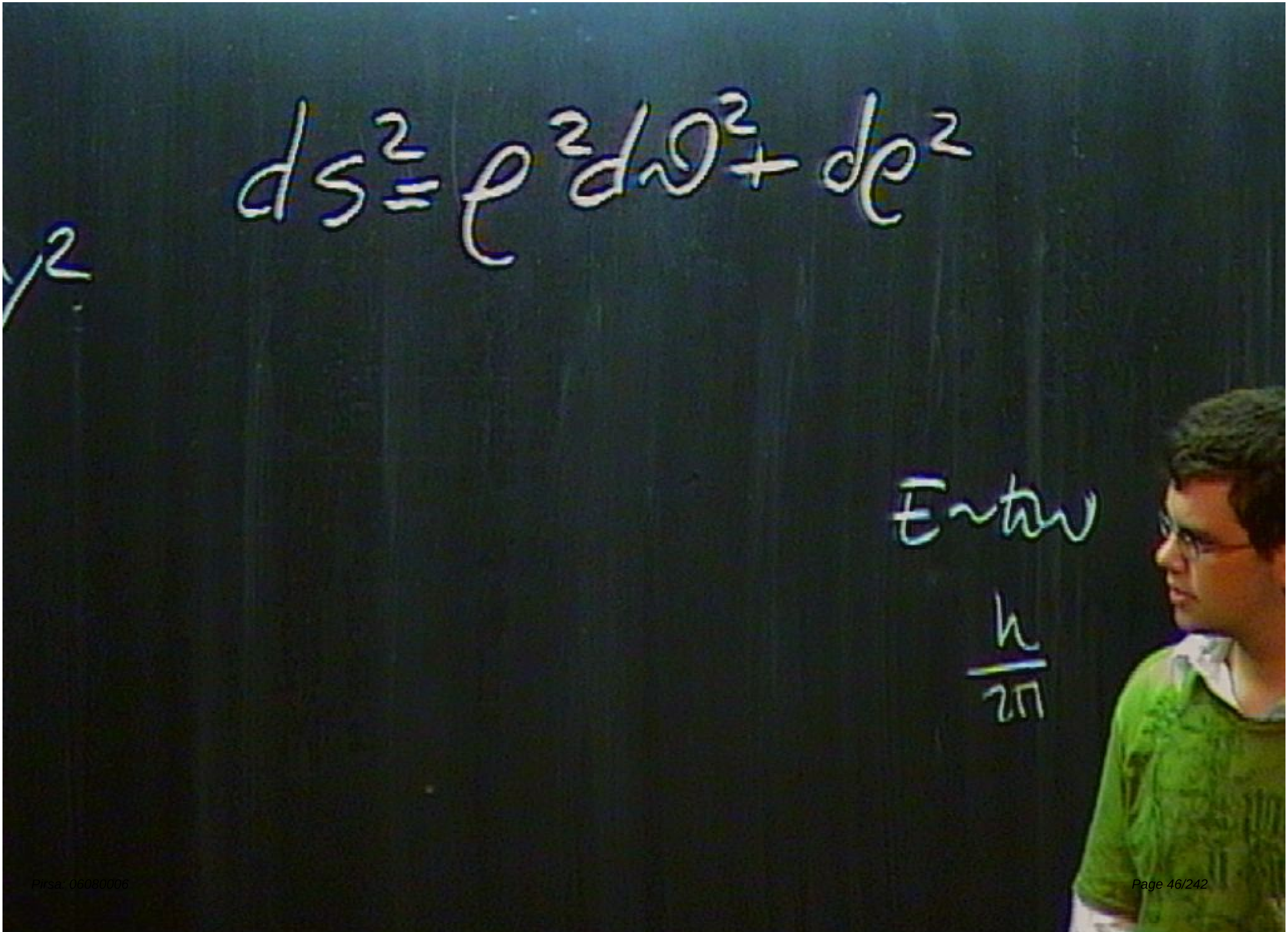


y^2

$$ds^2 = \rho^2 d\vartheta^2 + d\rho^2$$

$$E \sim \hbar\omega$$

$$\frac{h}{2\pi}$$

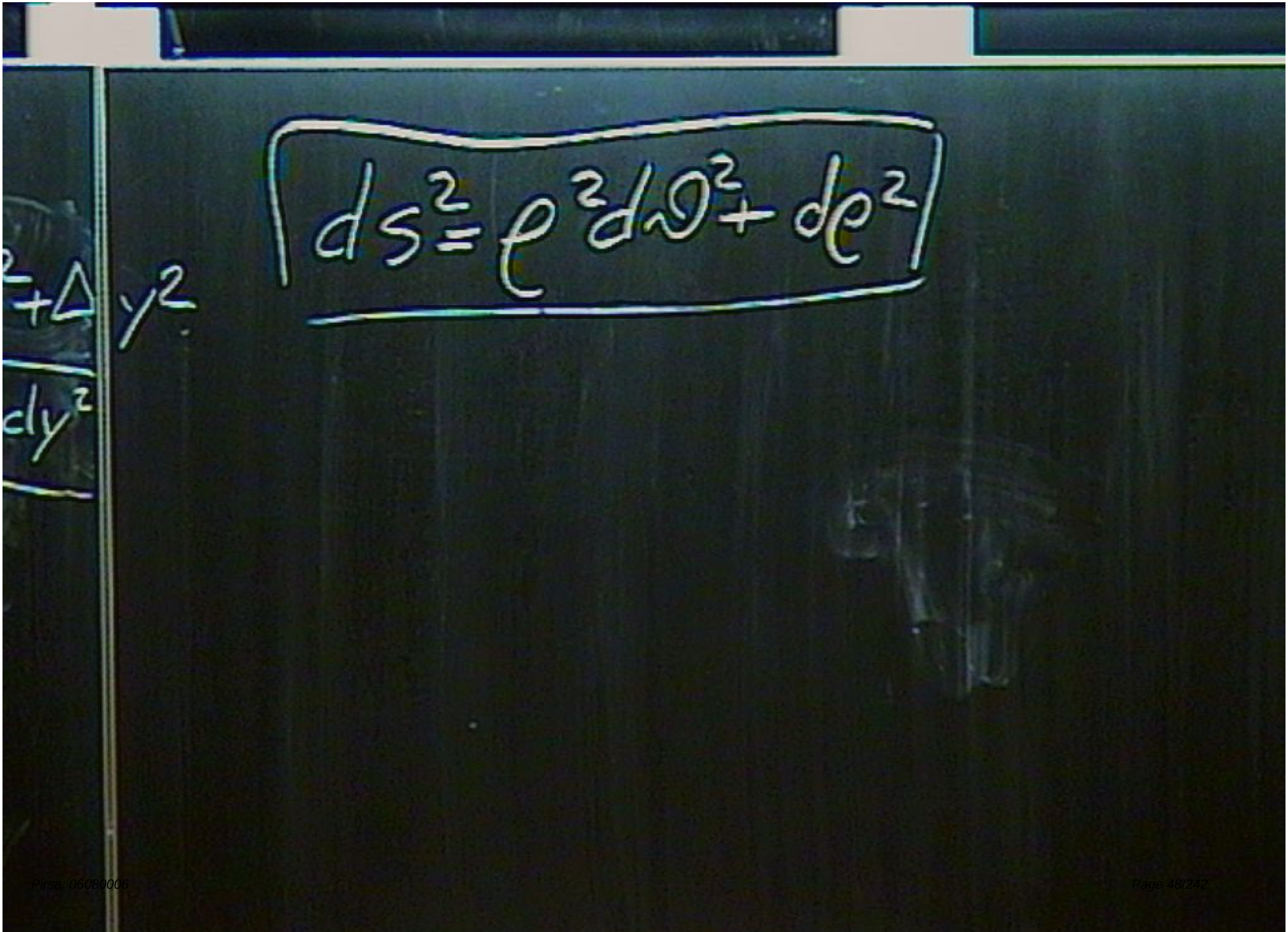
${}^2 + \Delta y^2$

dy^2

$$\boxed{ds^2 = \rho^2 d\vartheta^2 + d\rho^2}$$

$E \sim \hbar\omega$

$$\frac{h}{2\pi}$$

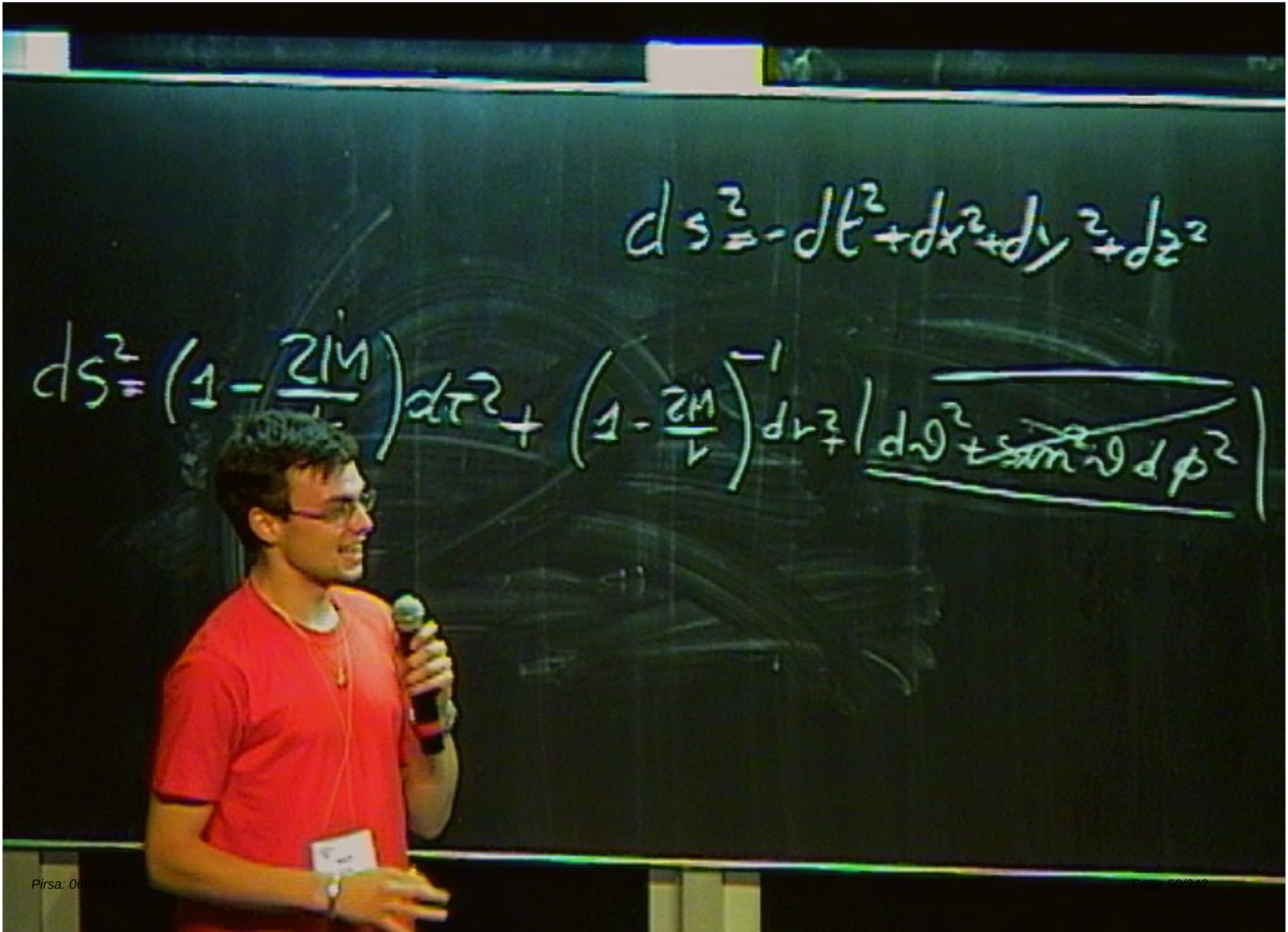


$^2+\Delta y^2$

dy^2

$$\boxed{ds^2 = \rho^2 d\vartheta^2 + d\rho^2}$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

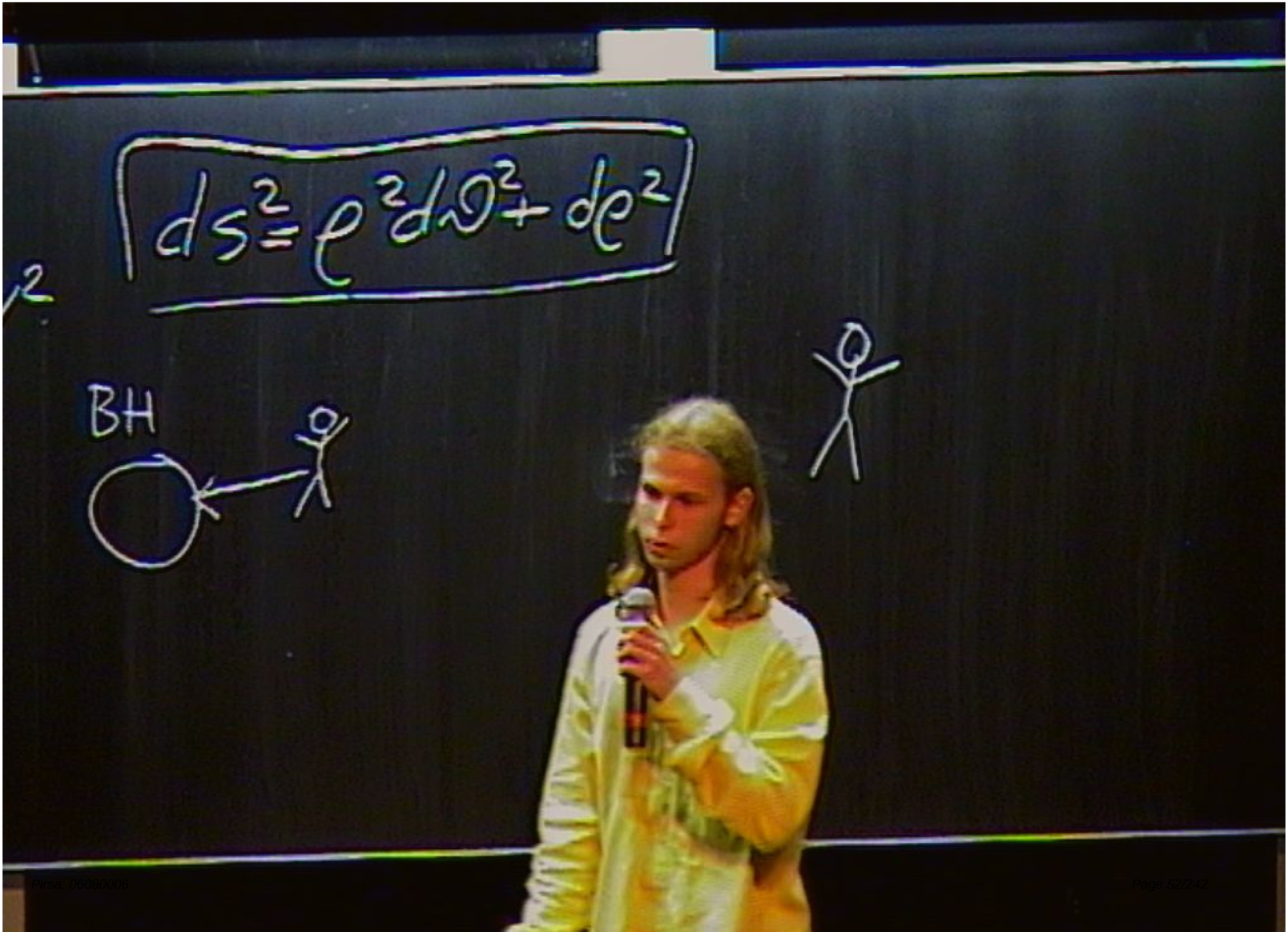
$$ds^2 = \left(1 - \frac{2M}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + \left| \overline{\underline{d\vartheta^2 + \cancel{\sin^2}\vartheta\, d\phi^2}} \right|$$

$c=1$

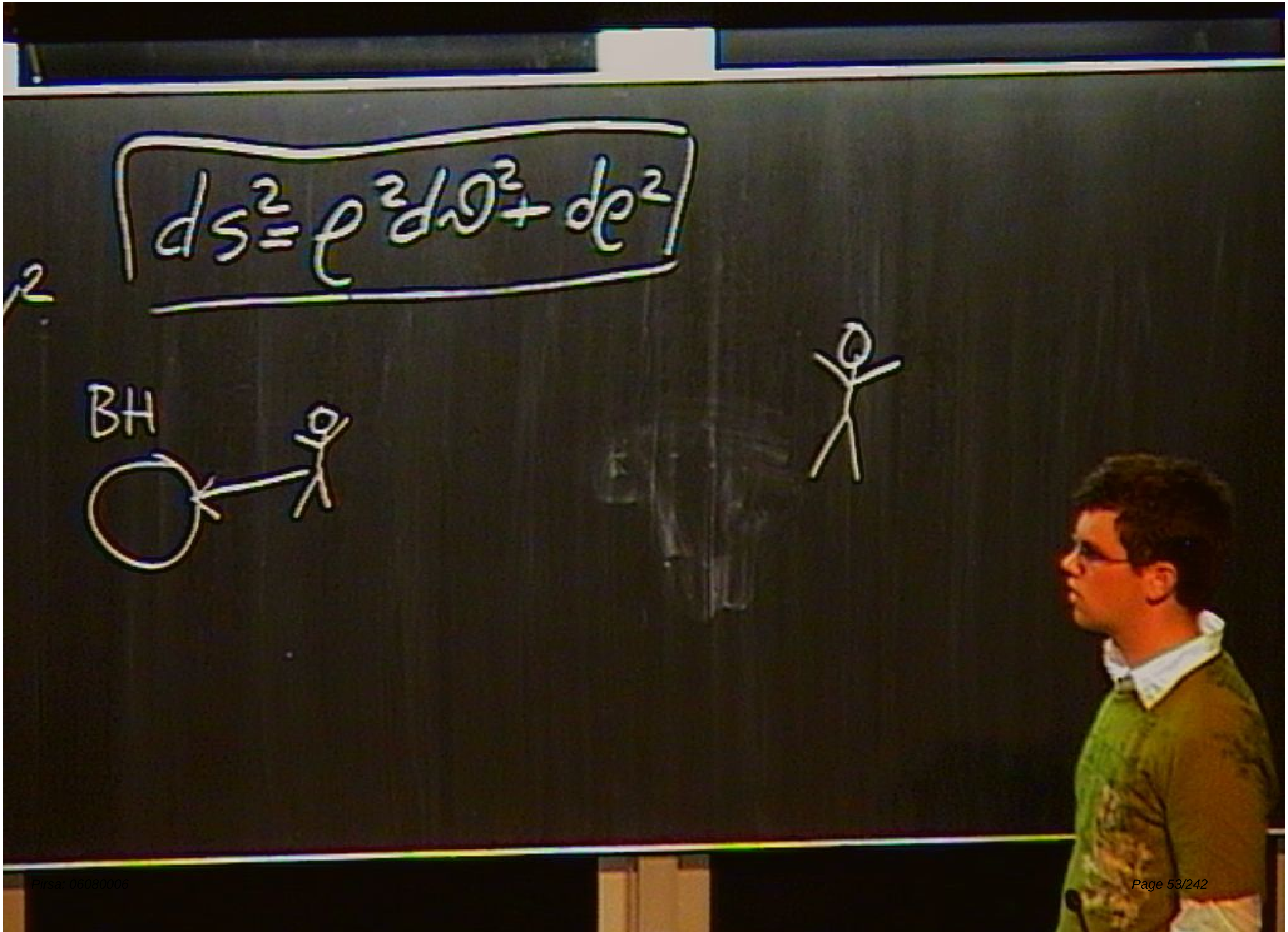
$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

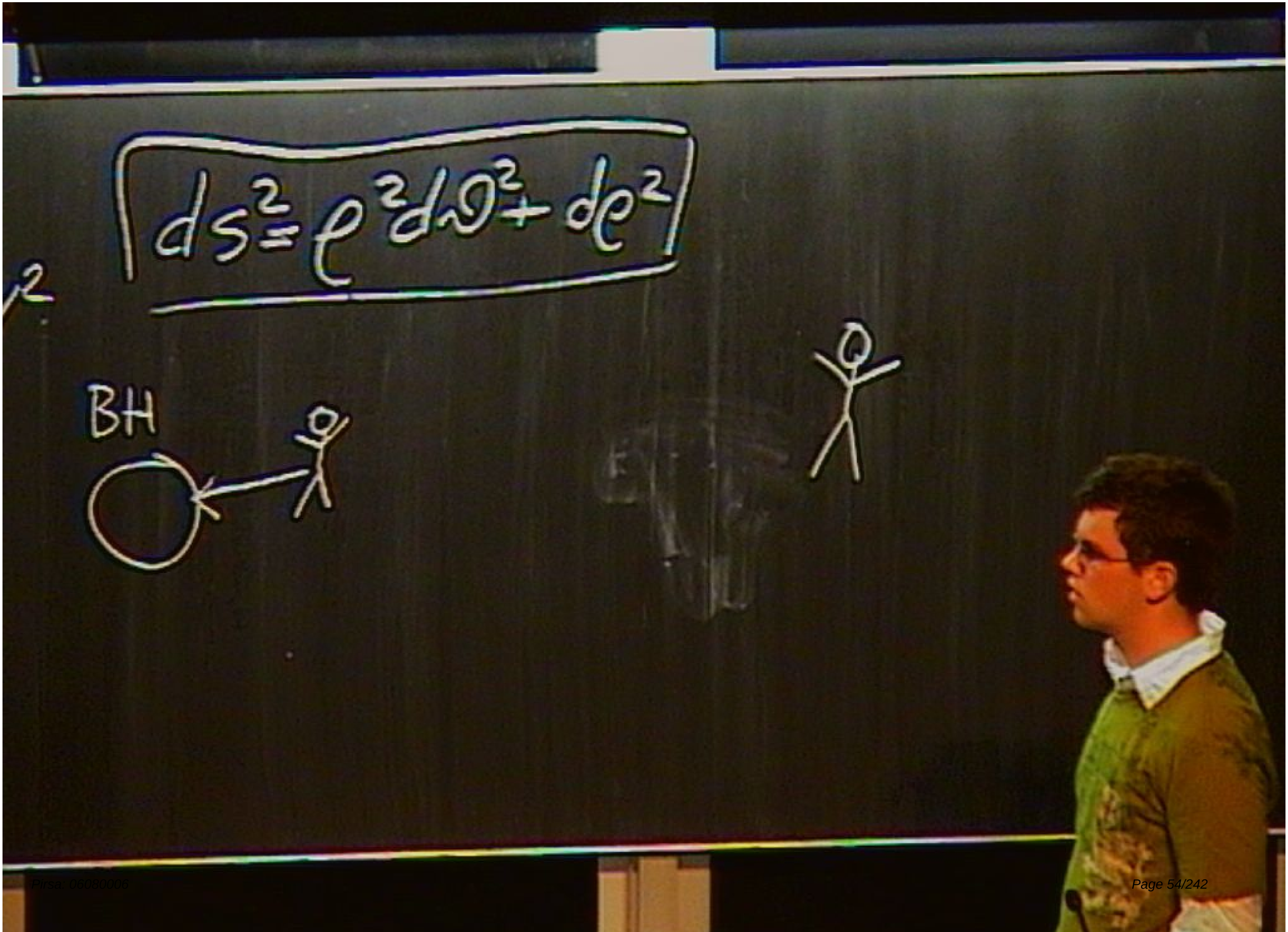
$$ds^2 = \left(1 - \frac{\boxed{2M}}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + \left|\, d\vartheta^2 + \cancel{\sin^2\vartheta\, d\phi^2} \,\right|$$



$$\boxed{ds^2 = \rho^2 d\vartheta^2 + d\rho^2}$$

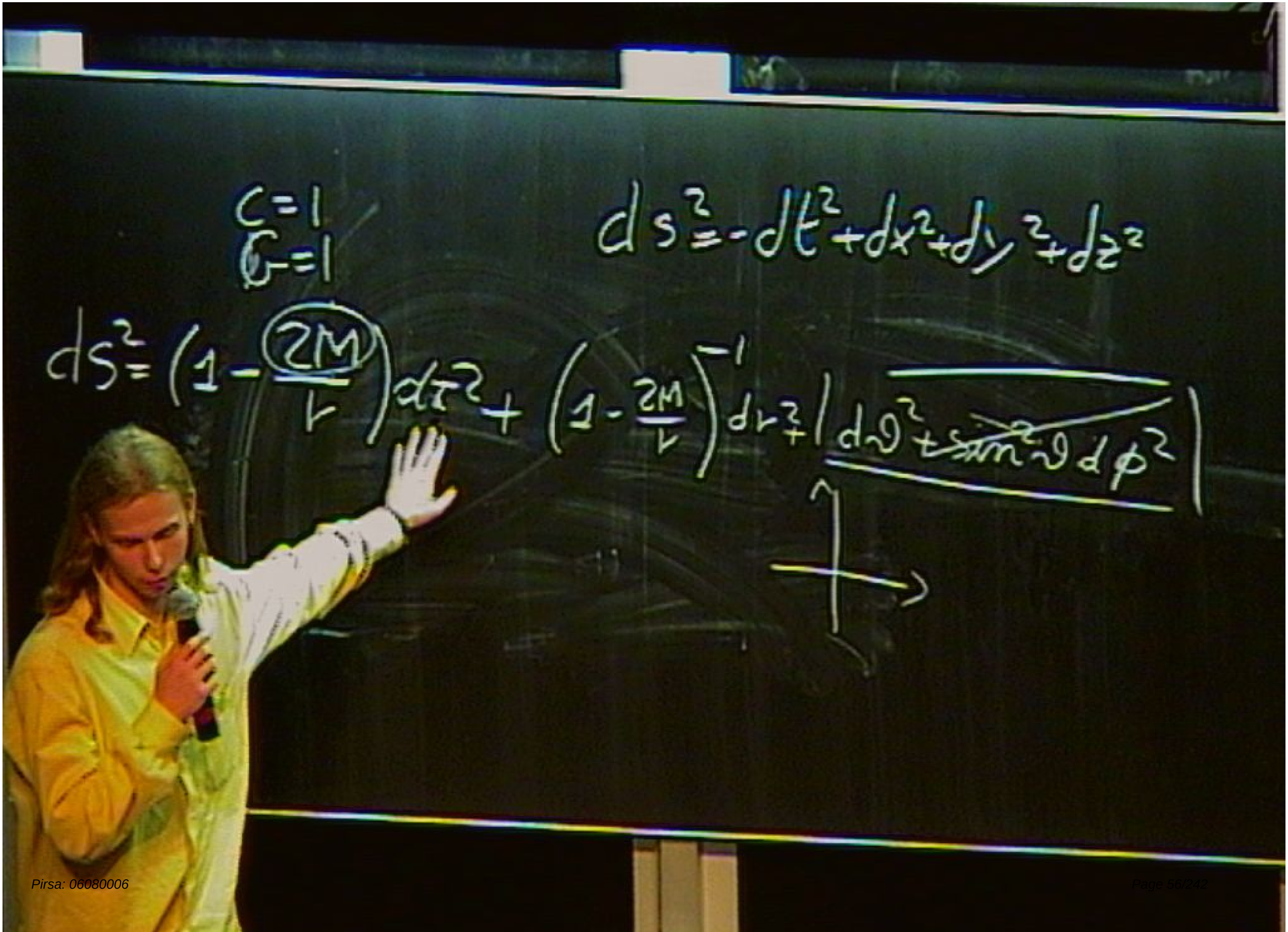


$$\boxed{ds^2 = \rho^2 d\Omega^2 + d\rho^2}$$



$$\boxed{ds^2 = \rho^2 d\Omega^2 + d\rho^2}$$

r	$1-\frac{2M}{r}$
$10M$	0,8
$8M$	0,75
$6M$	$\frac{2}{3}$
$4M$	0,5
$3M$	$\frac{1}{3}$
$2M$	0

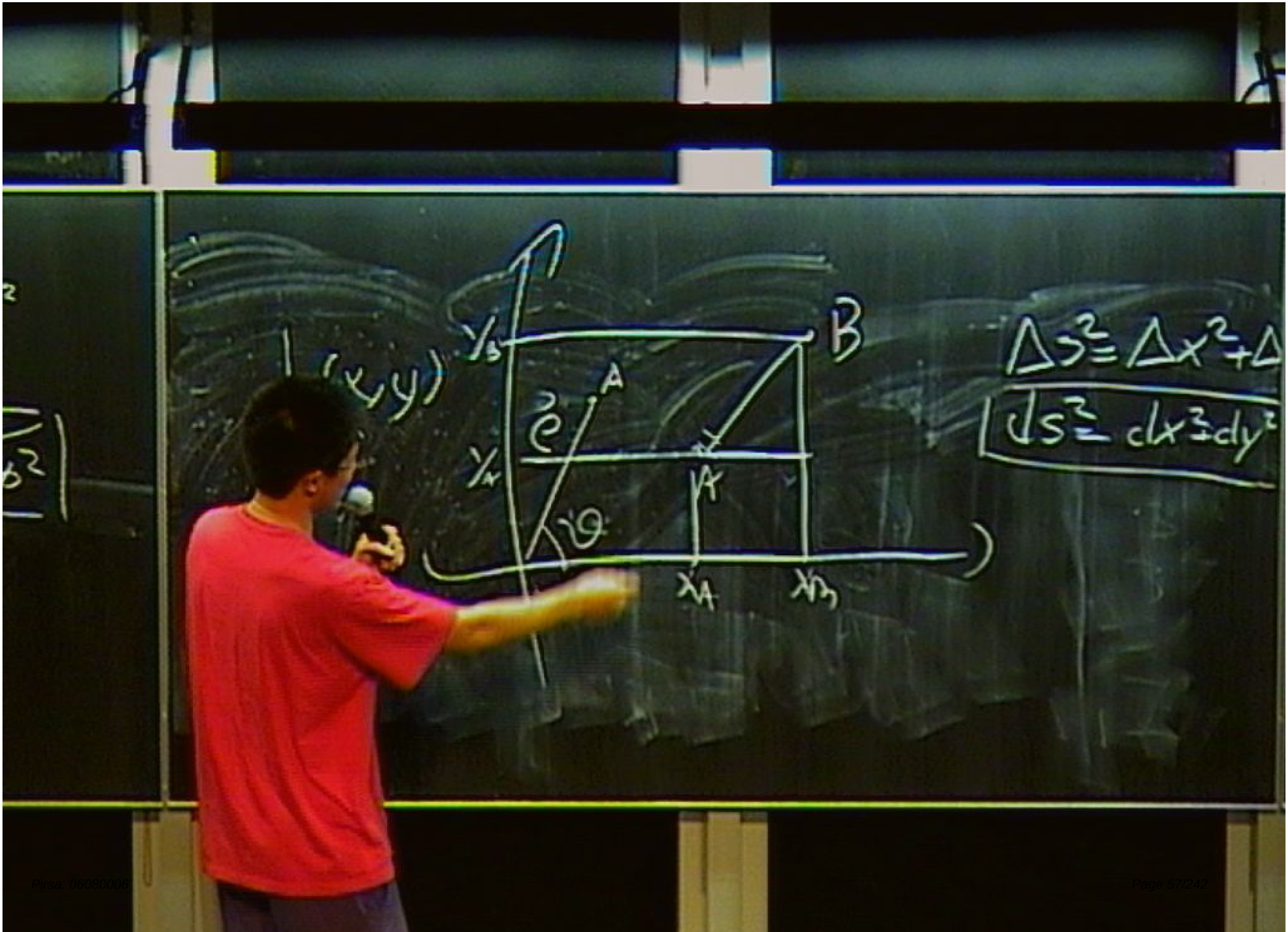


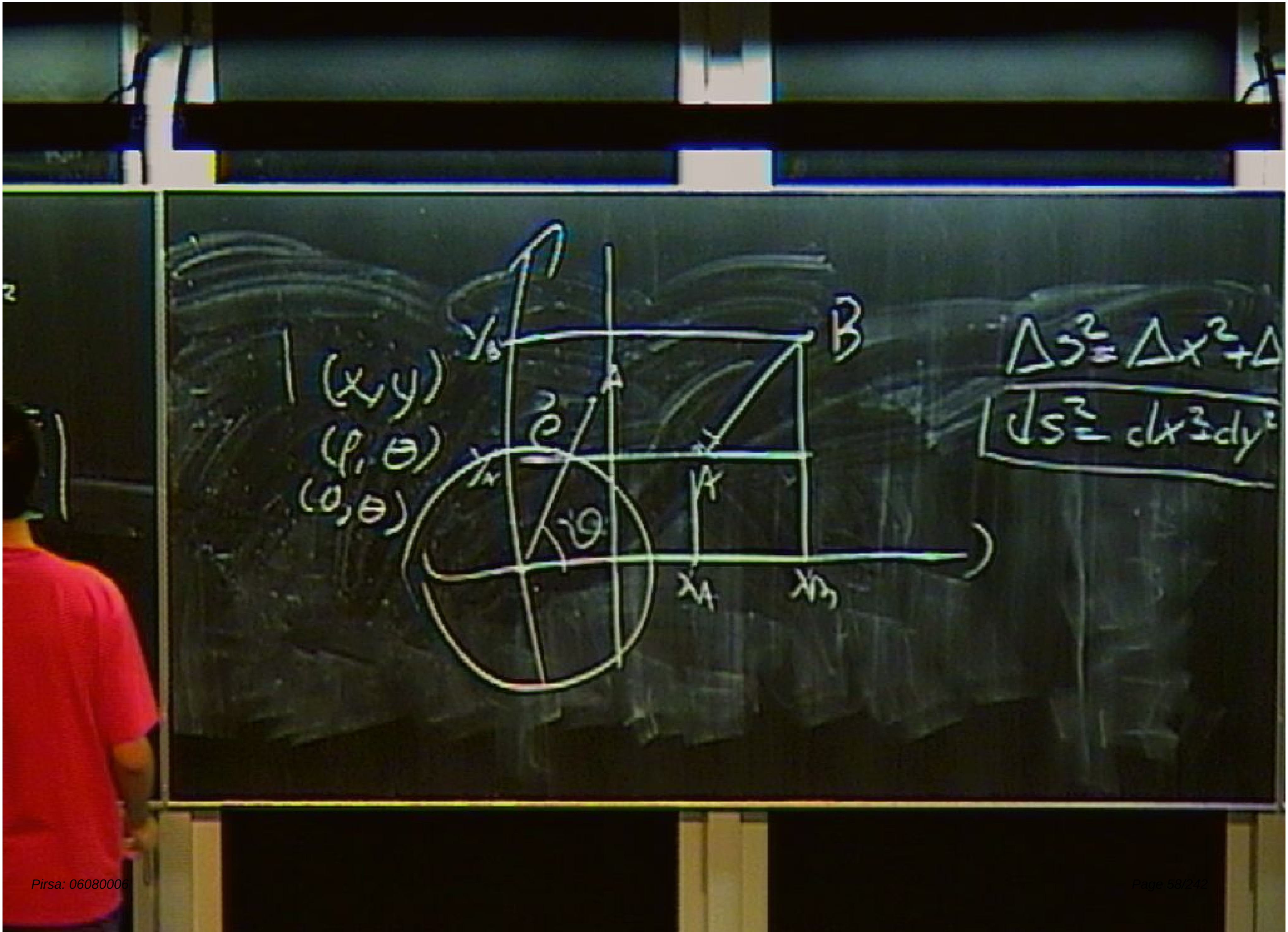
$c=1$

$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

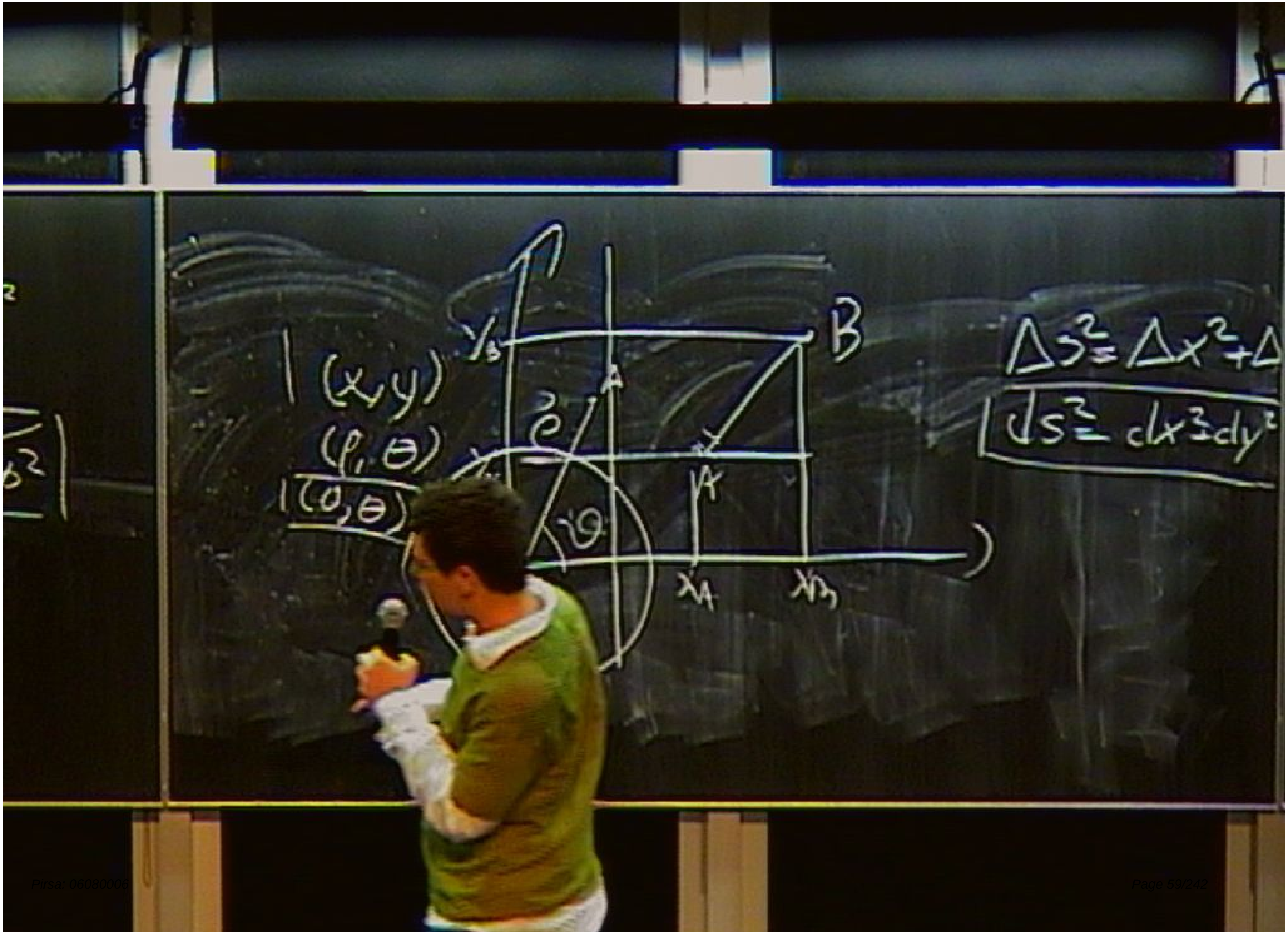
$$ds^2 = \left(1 - \frac{2M}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + \left| \overline{\underline{d\theta^2 \cancel{+ \sin^2\theta} \, d\phi^2}} \right|$$

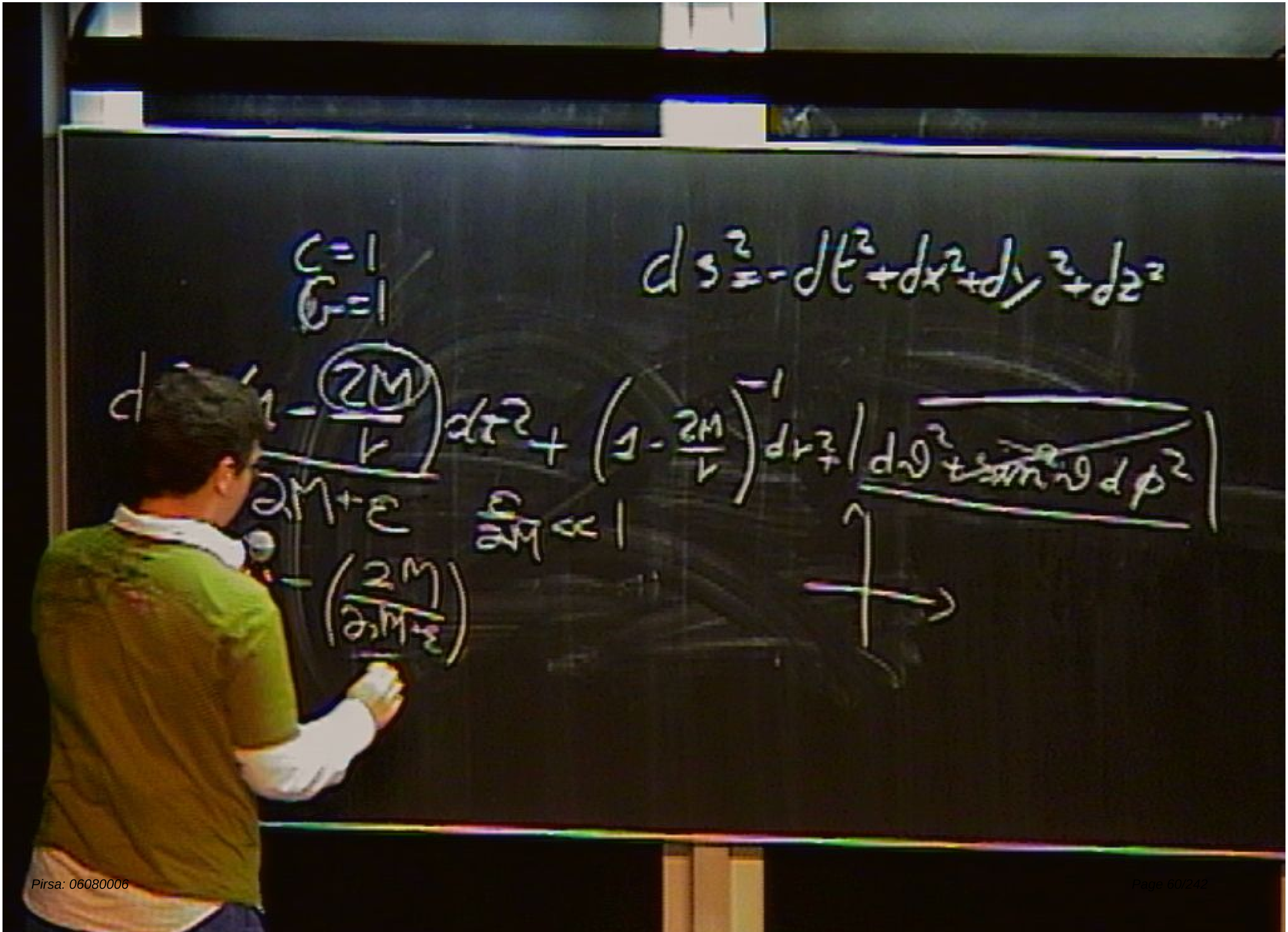

(x,y)
y_B
B
A
e
y_A
θ
x_A
x_B
Δs² = Δx² + Δ
ds² = dx² + dy²

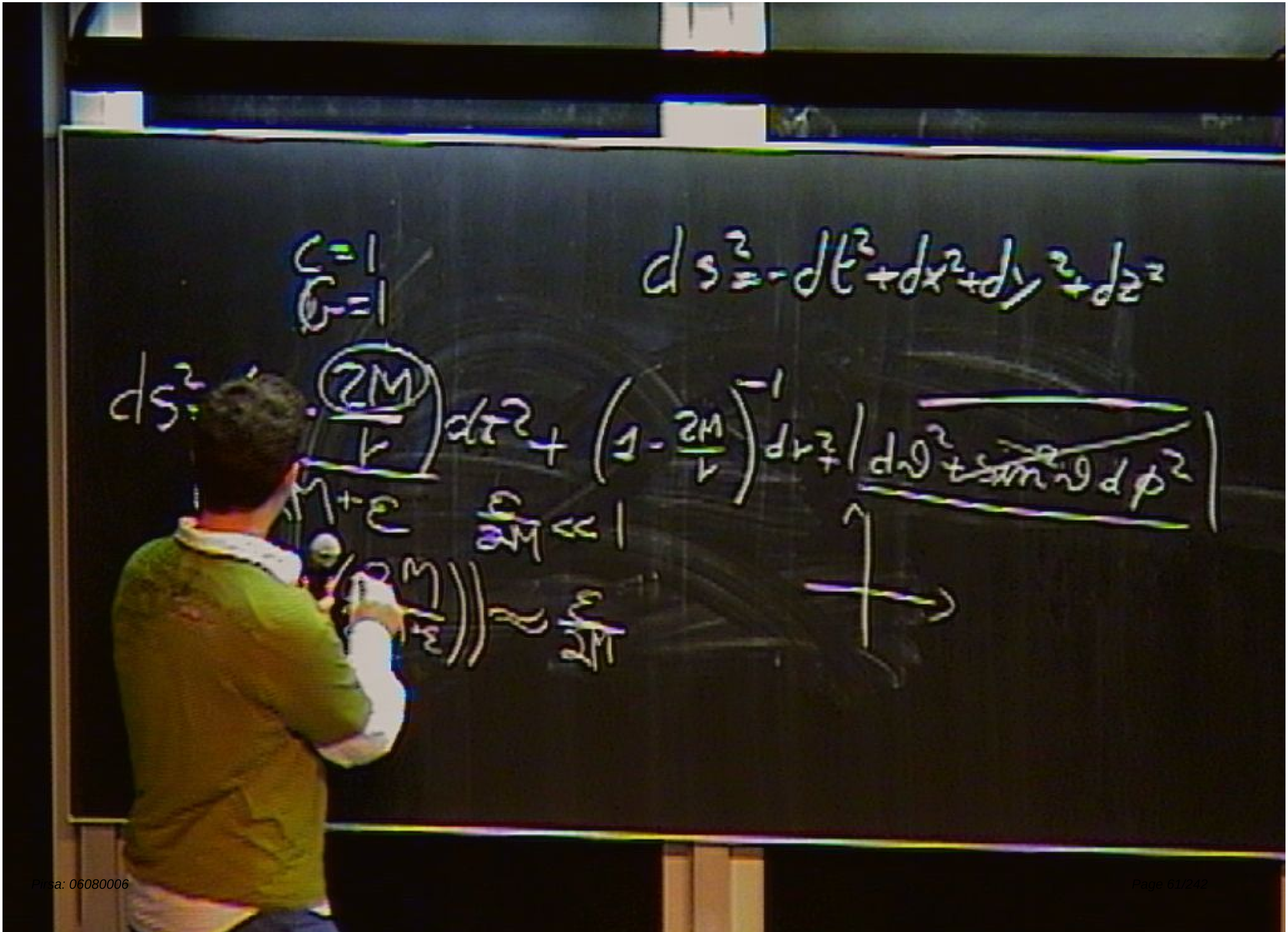


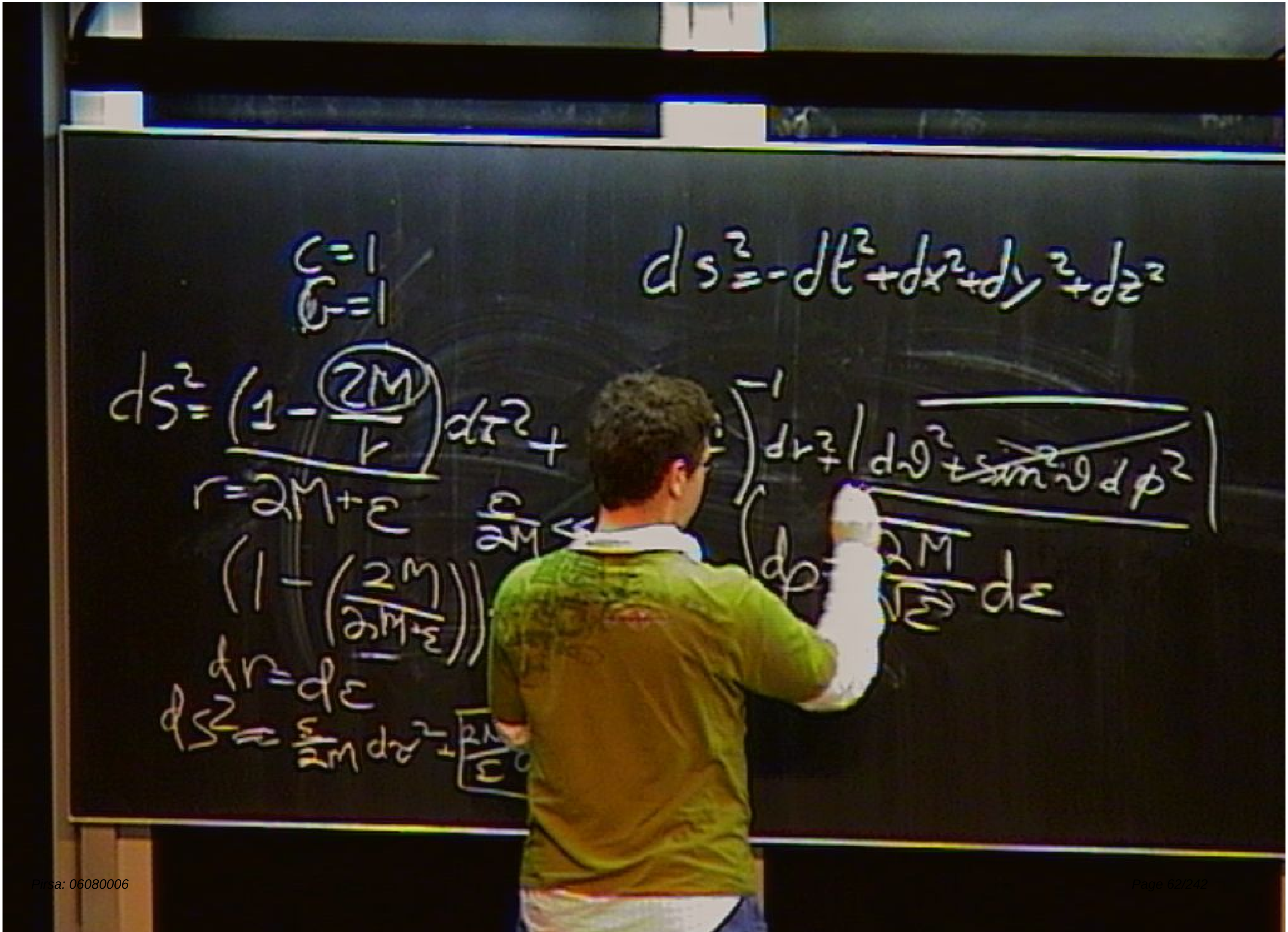
$\Delta s^2 = \Delta x^2 + \Delta$

$$\boxed{ds^2 = dx^2 + dy^2}$$


(x, y)
(ρ, θ)
(0, θ)
y_B
B
A
ρ
θ
x_A
x_B
ΔS² = Δx² + Δ
ds² = dx² + dy²


c=1
G=1
ds²=-dt²+dx²+dy²+dz²
d… (1 - 2M/r) dt² + (1 - 2M/r)⁻¹ dr² + [dθ² + sin²θ dφ²]
2M+ε
ε/2M ≪ 1
(2M/(2M+ε))





$c=1$

$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = \left(1-\frac{\boxed{2M}}{r}\right)d\tau^2 +$$

$$\Big)^{-1} dr^2 + \left[d\theta^2 \cancel{+ \sin^2\theta\, d\phi^2}\right]$$

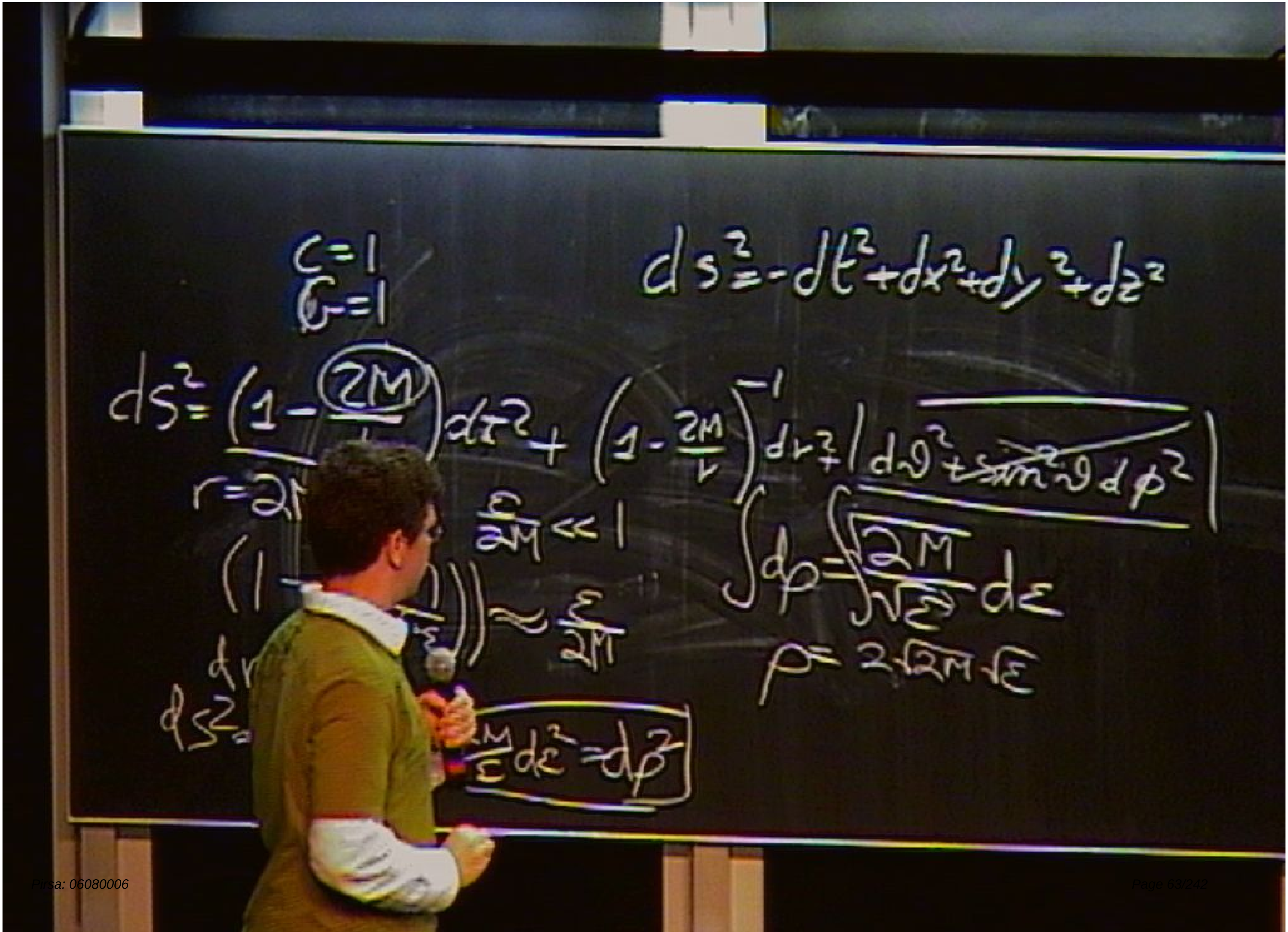
$$r = 2M + \varepsilon \qquad \frac{\varepsilon}{2M} \ll$$

$$\left(1 - \left(\frac{2M}{2M+\varepsilon}\right)\right)$$

$$dr = d\varepsilon$$

$$ds^2 = \frac{\varepsilon}{2M}\, d\tau^2 + \boxed{\frac{2M}{\varepsilon}}$$

$$\left(d\rho = \frac{\sqrt{2M}}{\sqrt{\varepsilon}}\, d\varepsilon\right.$$



$c=1$

$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + \left| d\theta^2 + \cancel{\sin^2\vartheta\, d\phi^2} \right|$$

$r = 2M$

$\frac{\varepsilon}{2M} \ll 1$

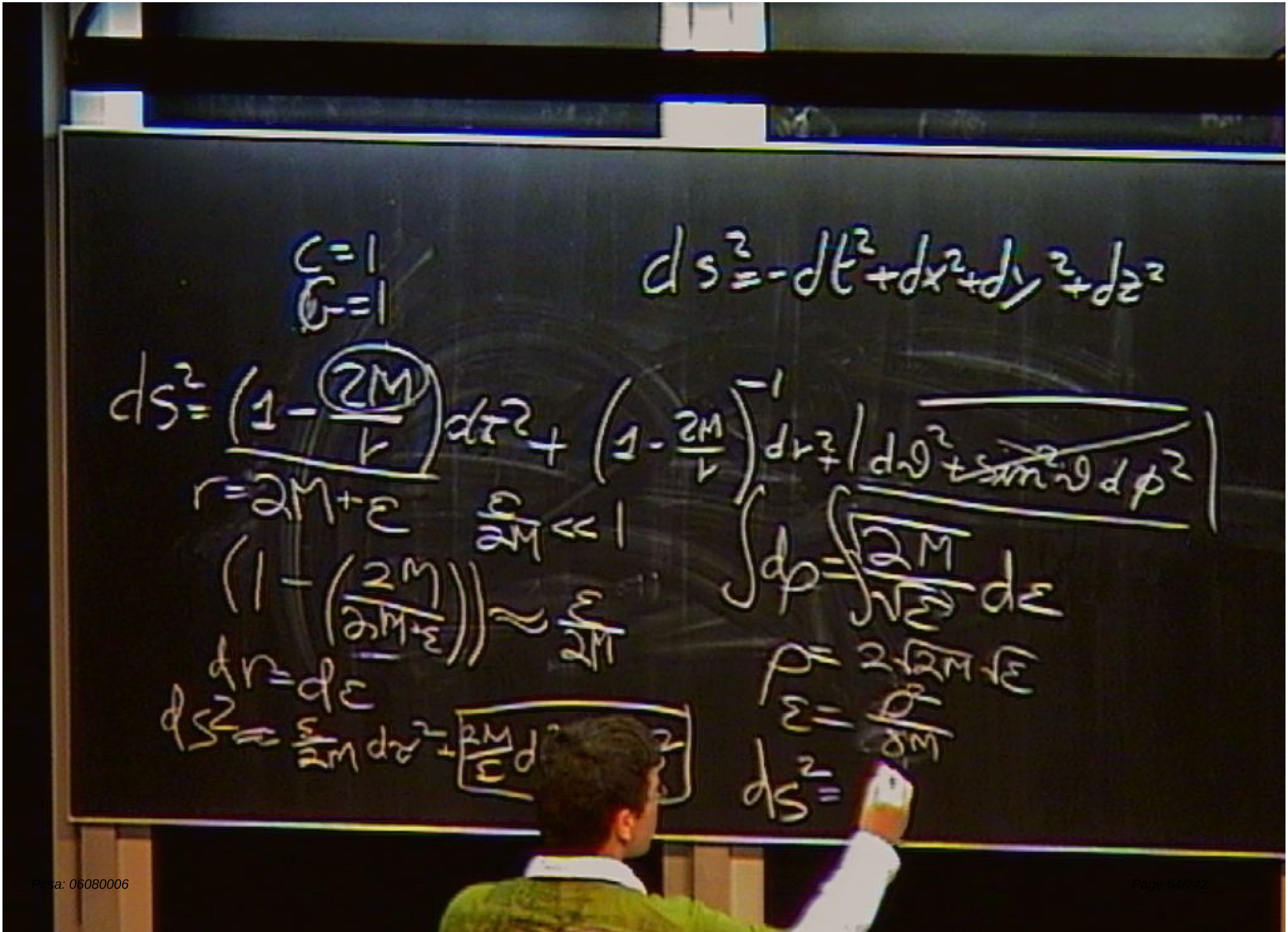
$\left(1 - \ldots\right) \approx \frac{\varepsilon}{2M}$

$$\int d\rho = \int \frac{\sqrt{2M}}{\sqrt{\varepsilon}} d\varepsilon$$

$$\rho = 2\sqrt{2M}\sqrt{\varepsilon}$$

dr

$$ds^2 = \ldots \boxed{\frac{2M}{\varepsilon} d\varepsilon^2 = d\rho^2}$$



$c=1$

$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = \left(1-\frac{2M}{r}\right) d\tau^2 + \left(1-\frac{2M}{r}\right)^{-1} dr^2 + \left|\overline{d\theta^2 \cancel{+\sin^2}\theta\, d\phi^2}\right|$$

$r = 2M + \varepsilon \qquad \frac{\varepsilon}{2M} \ll 1$

$$\left(1-\left(\frac{2M}{2M+\varepsilon}\right)\right) \approx \frac{\varepsilon}{2M}$$

$dr = d\varepsilon$

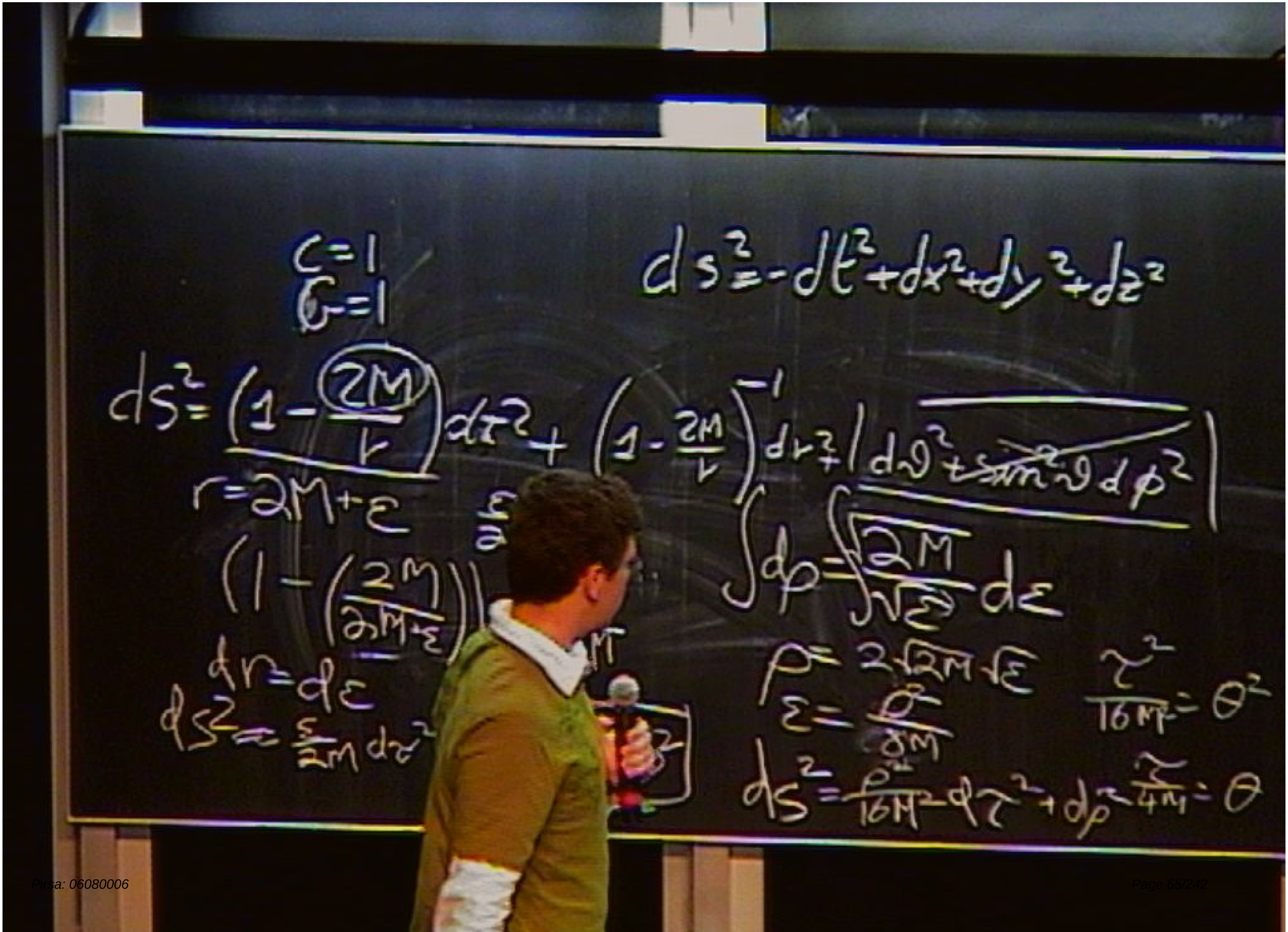
$$ds^2 \approx \frac{\varepsilon}{2M} d\tau^2 + \boxed{\frac{2M}{\varepsilon} d}$$

$$\int d\rho = \int \frac{\sqrt{2M}}{\sqrt{\varepsilon}} d\varepsilon$$

$\rho = 2\sqrt{2M}\sqrt{\varepsilon}$

$\varepsilon = \frac{\rho^2}{8M}$

$ds^2 =$



$c=1$

$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = \underline{\left(1-\boxed{\frac{2M}{r}}\right)} d\tau^2 + \left(1-\frac{2M}{r}\right)^{-1} dr^2 + \left|\cancel{d\theta^2 + \sin^2\theta\, d\phi^2}\right|$$

$r = 2M + \varepsilon$

$\frac{\varepsilon}{2}$

$\left(1-\left(\frac{2M}{2M+\varepsilon}\right)\right)$

$dr = d\varepsilon$

$ds^2 = \frac{\varepsilon}{2M} d\tau^2$

$$\int d\rho = \int \frac{\sqrt{2M}}{\sqrt{\varepsilon}} d\varepsilon$$

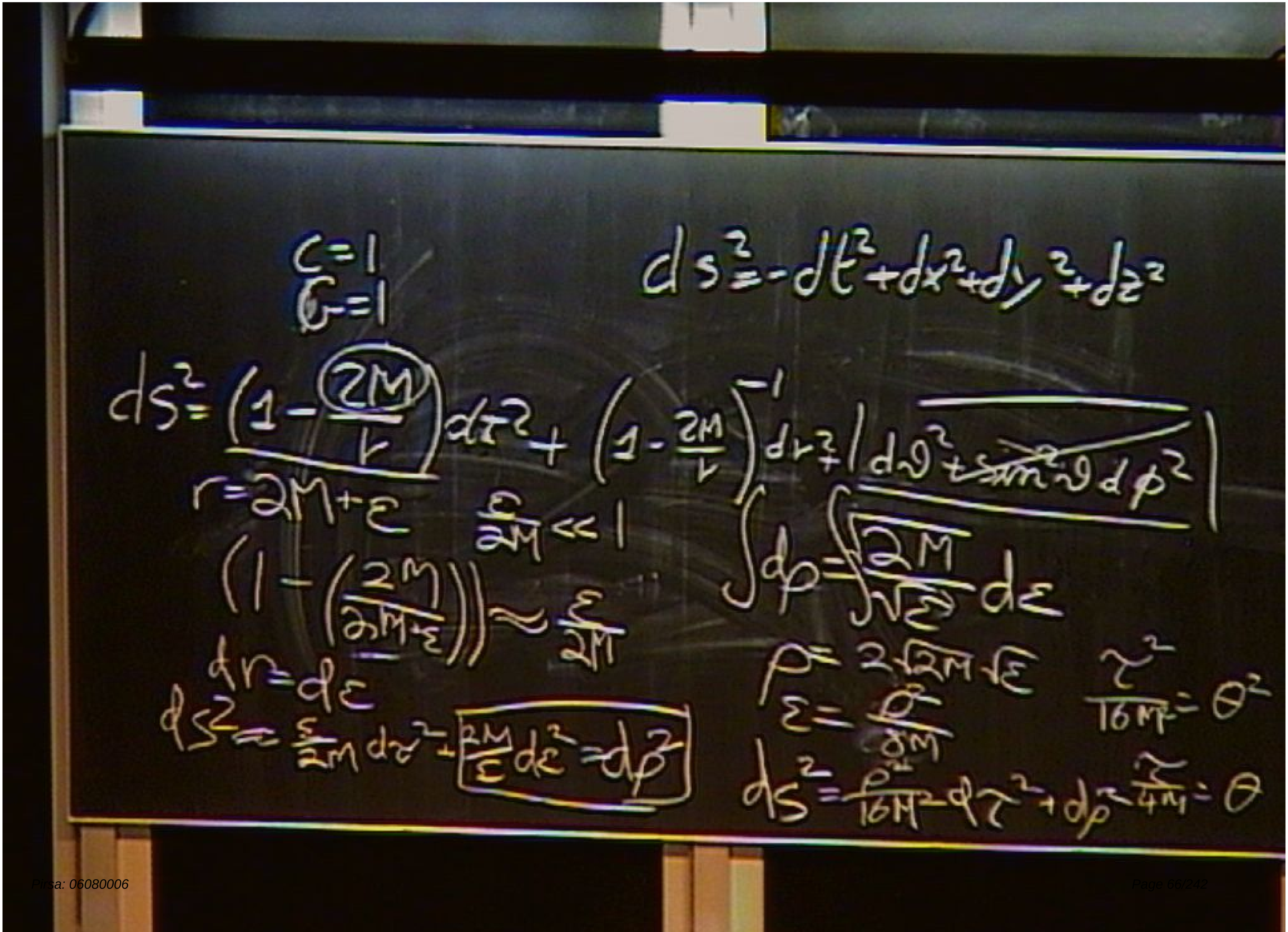
$\rho = 2\sqrt{2M}\sqrt{\varepsilon}$

$\varepsilon = \frac{\rho^2}{8M}$

$ds^2 = \frac{\rho^2}{16M^2} d\tau^2 + d\rho^2$

$\frac{\tau^2}{16M^2} = \theta^2$

$\frac{\tau}{4M} = \theta$



$c=1$

$G=1$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = \underline{\left(1-\boxed{\frac{2M}{r}}\right)} d\tau^2 + \left(1-\frac{2M}{r}\right)^{-1} dr^2 + \left| \overline{\underline{d\theta^2 + \cancel{\sin^2\theta}\, d\phi^2}} \right|$$

$r = 2M + \varepsilon$ $\quad \frac{\varepsilon}{2M} \ll 1$

$$\left(1-\left(\frac{2M}{2M+\varepsilon}\right)\right) \approx \frac{\varepsilon}{2M}$$

$dr = d\varepsilon$

$$ds^2 \approx \frac{\varepsilon}{2M} d\tau^2 + \boxed{\frac{2M}{\varepsilon} d\varepsilon^2 = d\rho^2}$$

$$\int d\rho = \int \frac{\sqrt{2M}}{\sqrt{\varepsilon}} d\varepsilon$$

$\rho = 2\sqrt{2M}\sqrt{\varepsilon}$ $\qquad \frac{\tau^2}{16M^2} = \theta^2$

$\varepsilon = \frac{\rho^2}{8M}$

$$ds^2 = \frac{\rho^2}{16M^2} d\tau^2 + d\rho^2 \qquad \frac{\tau}{4M} = \theta$$

$$\boxed{ds^2 = \rho^2 d\vartheta^2 + d\rho^2}$$

Quantum Memory

Quantum Memory

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Kristin Flowers
Franziskuss Benedict Moritz Konstantin Hillerbrand

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Tim Horton
Rachel Mahnke
Richard McNamara
David Nissimoff
Michael Perssons

Outline

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- Classical Memory
    - Energy and Spin
    - Dimensions and the Ising Model
    - Decay
    - Canonical Ensemble

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- Quantum Memory
  - Energy, Spin, Qubits
  - Entanglement
  - Major Problems
  - Self-Healing Memory
  - Possible Solutions
  - Quest for 3-D and Control of Entanglement Patterns
  - Future of Quantum Memory

Classical Memory

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- Lifetime: roughly 50 years
- Examples include magnetic tape, hard disk drives, CDs, DVDs

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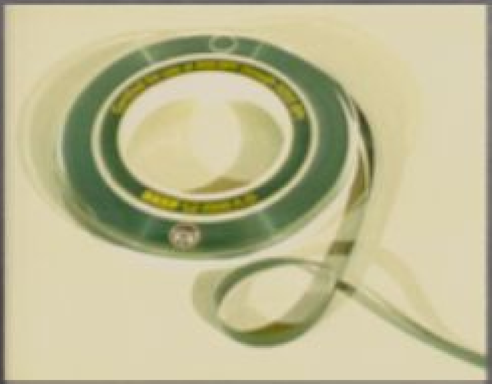

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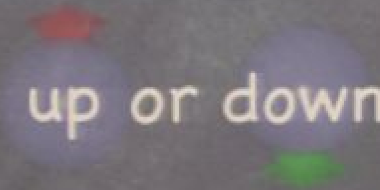
Energy and Spin

Energy and Spin

- Hamiltonian Equation for the energy of a state:

$$E = E_0 \bullet (\#\text{ of nearest neighbors})$$



- Spin states: up or down
- Uses millions of spins to represent one bit.
- Spins tend to follow their neighbors.

Dimensions and the Ising Model

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- 0-D: spins isolated

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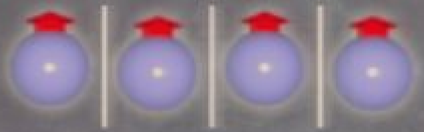

Dimensions and the Ising Model

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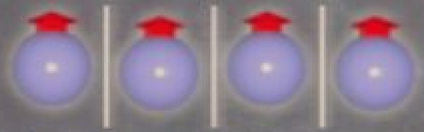

- 1-D: spins connected in a line

Dimensions and the Ising Model

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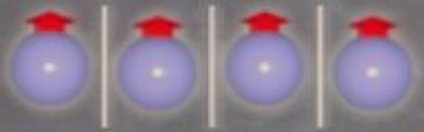

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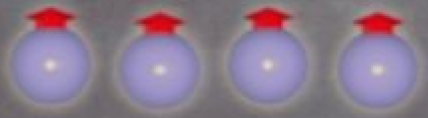

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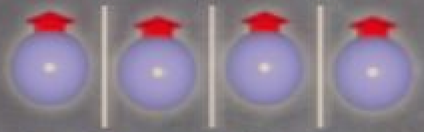
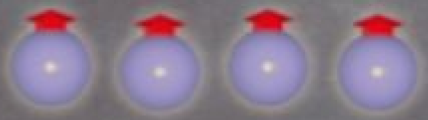


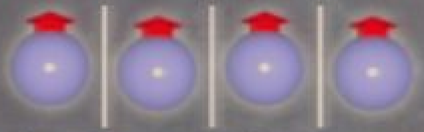
- 1-D: spins connected in a line



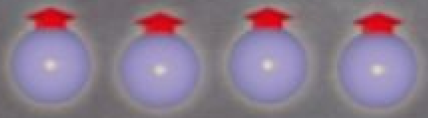
- 2-D: spins aligned in a grid

Dimensions and the Ising Model

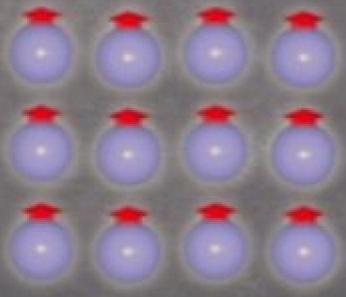
- 0-D: spins isolated



- 1-D: spins connected in a line



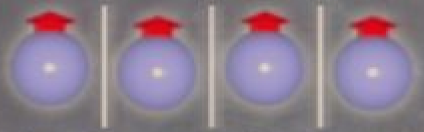
- 2-D: spins aligned in a grid



- ∞-D: all spins interconnected

Dimensions and the Ising Model

- 0-D: spins isolated



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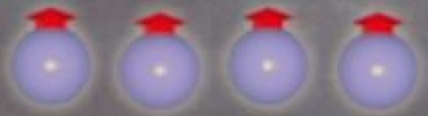
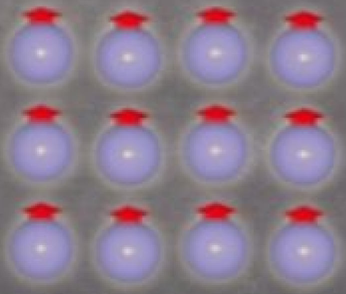


- 2-D: spins aligned in a grid



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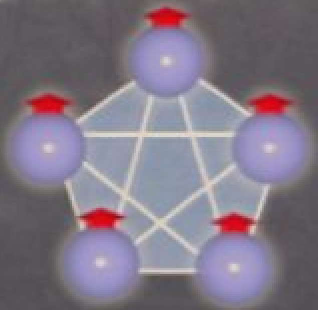
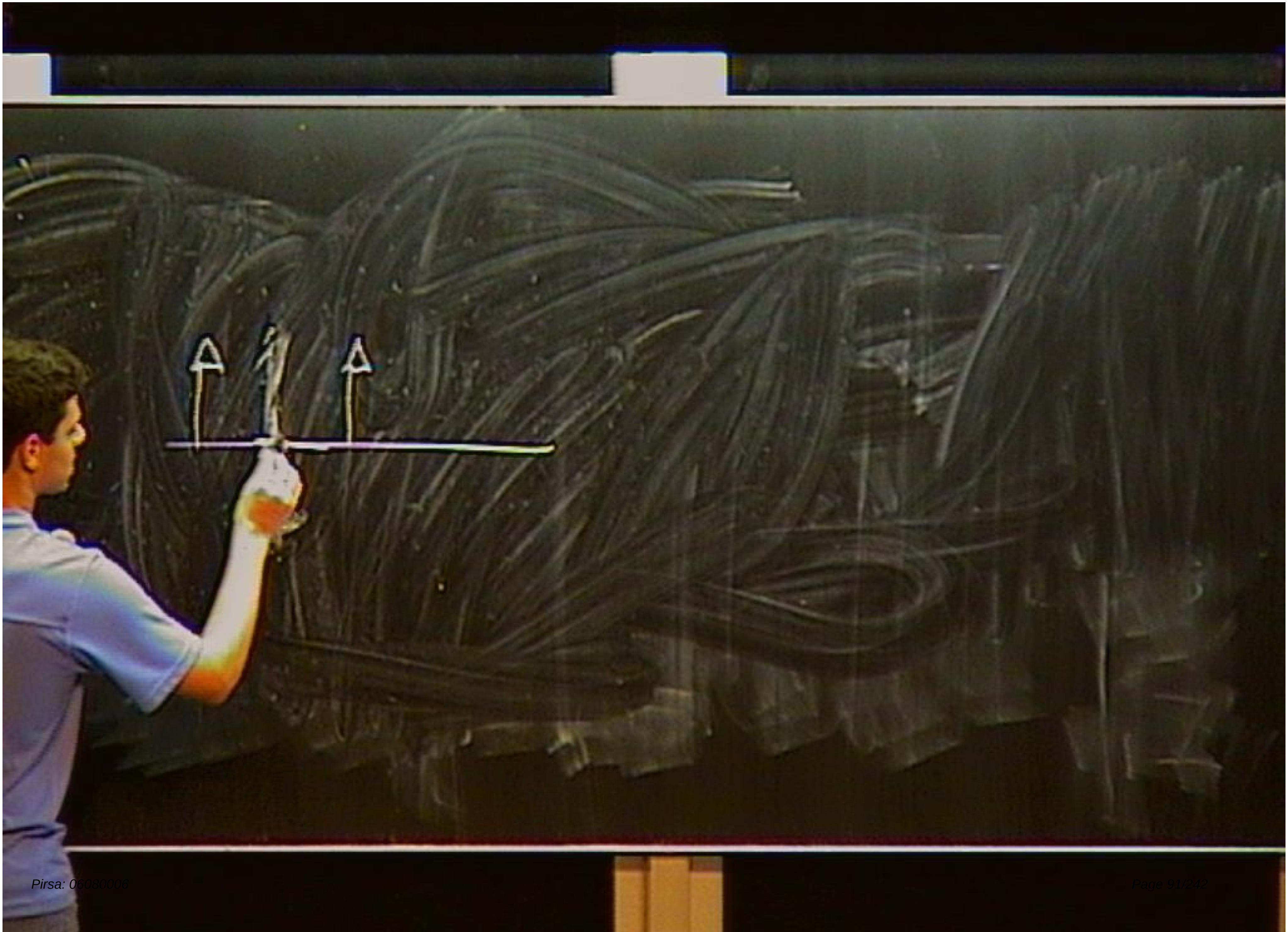
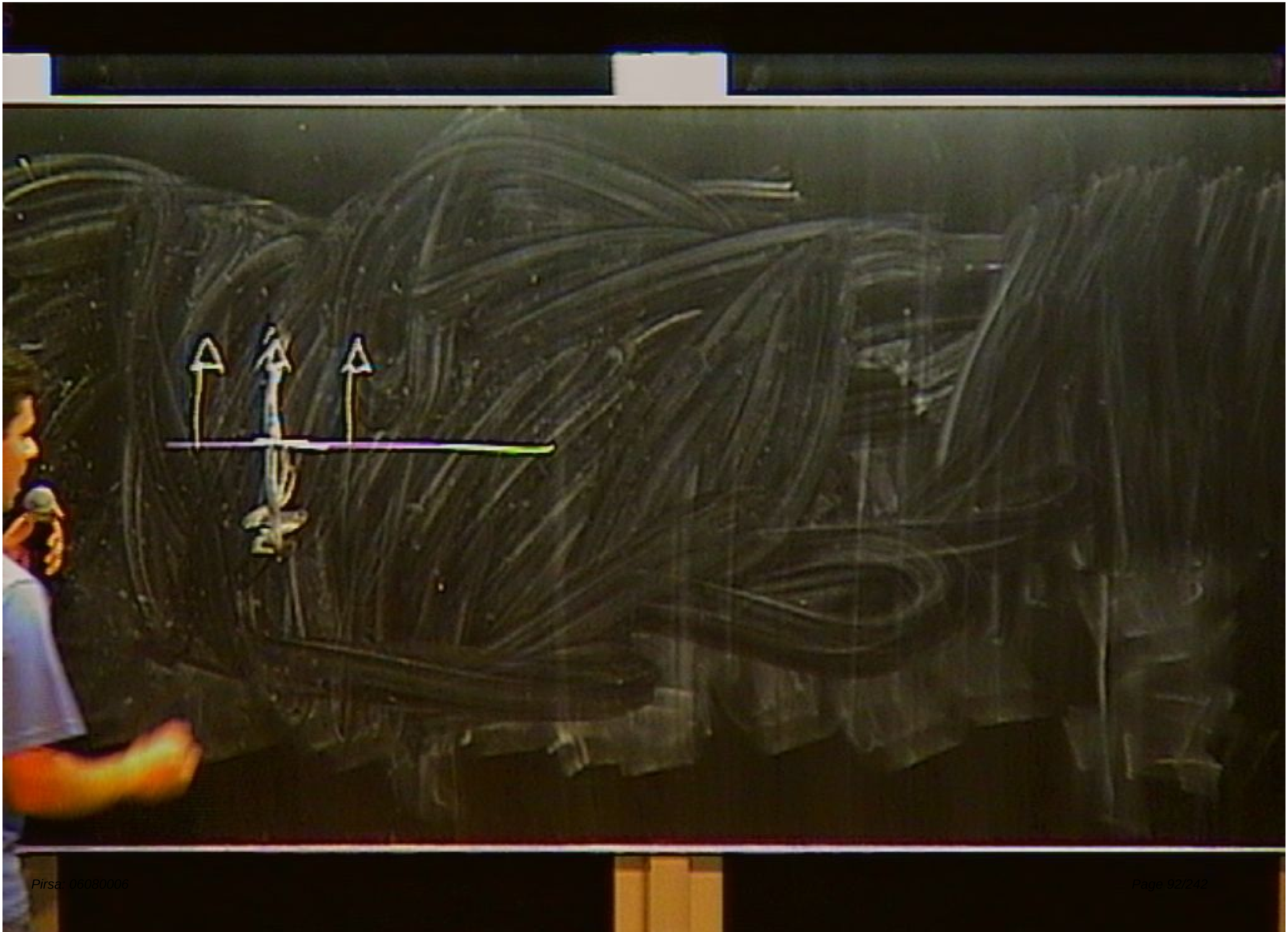
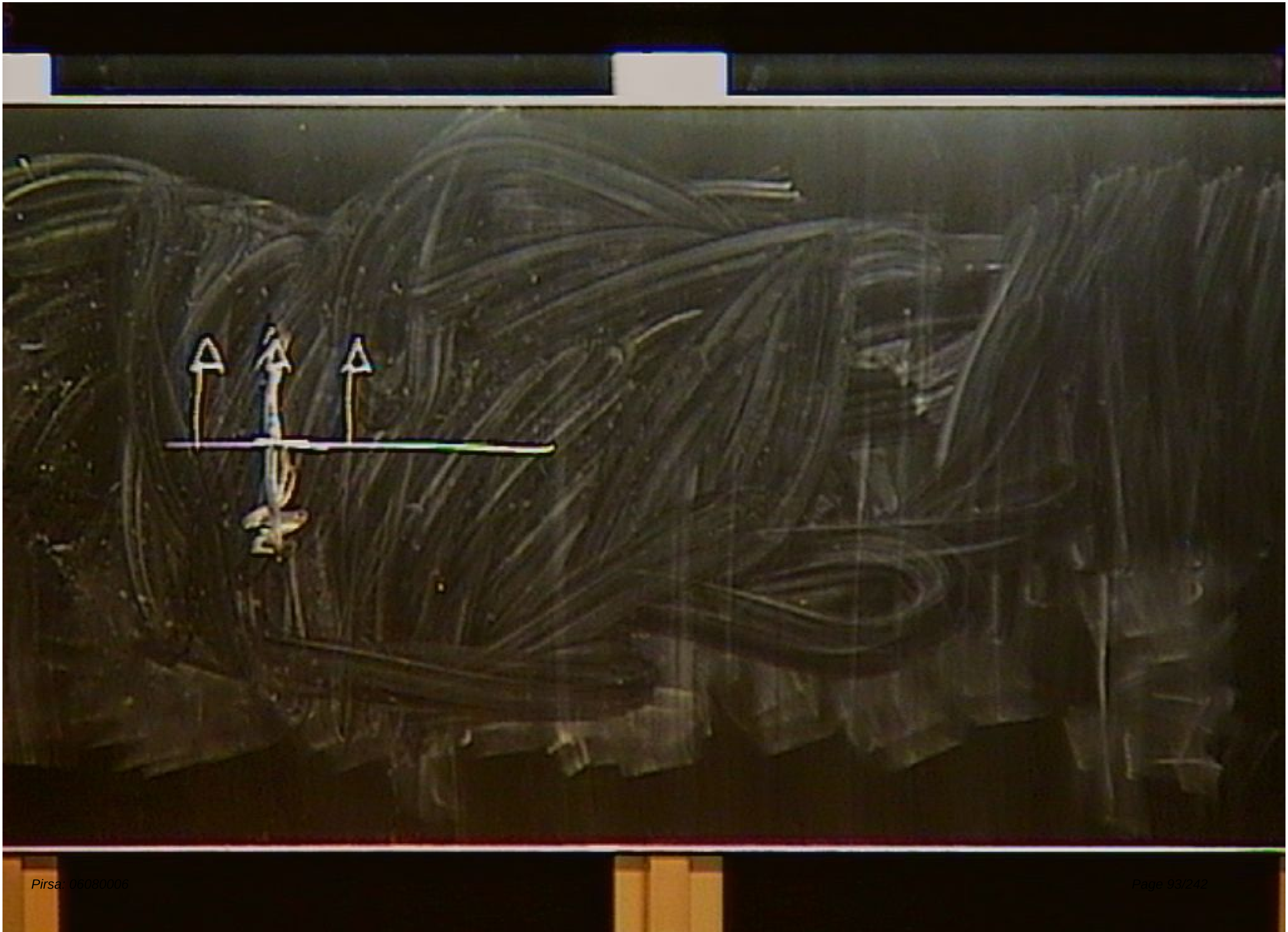
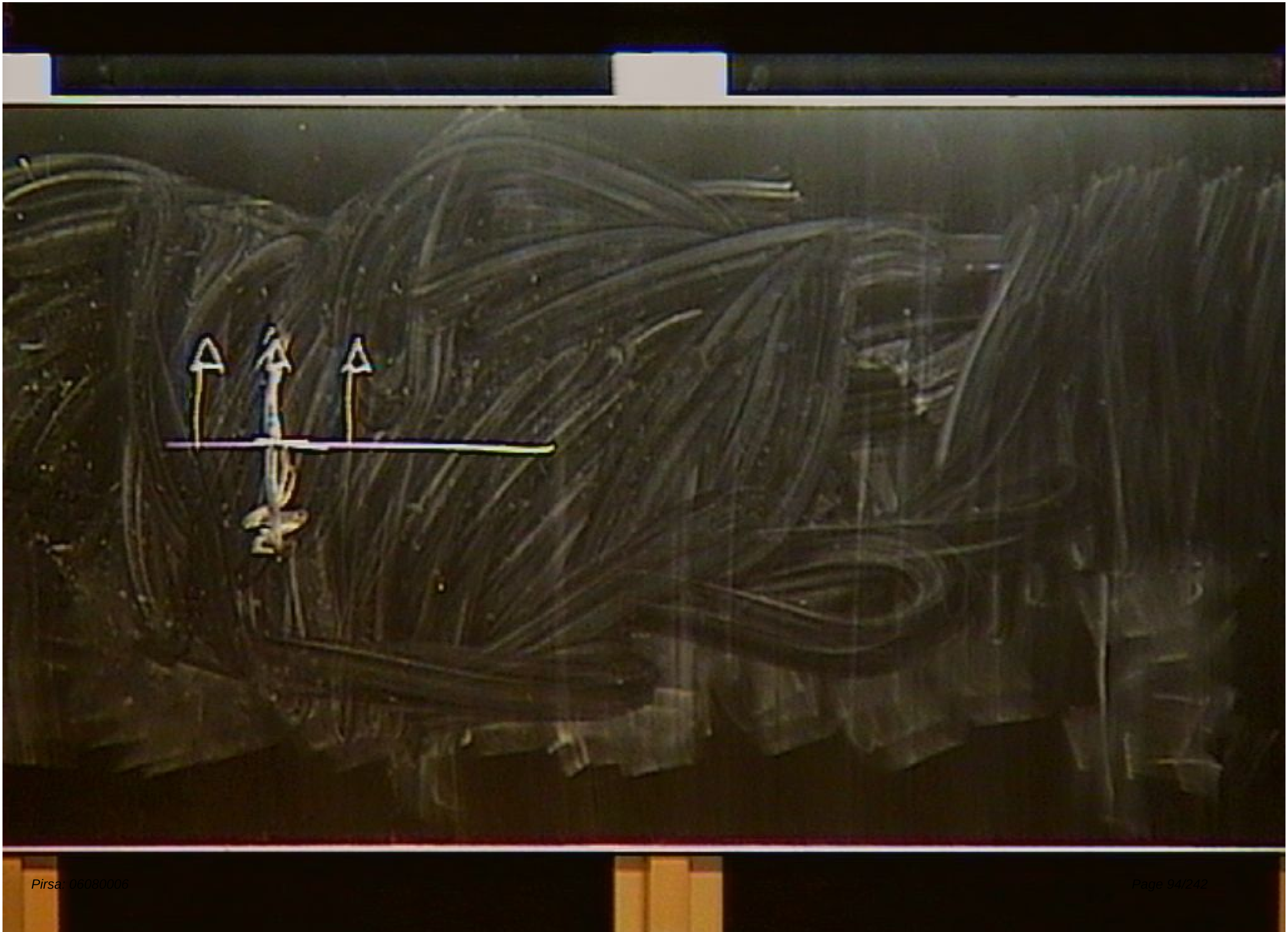
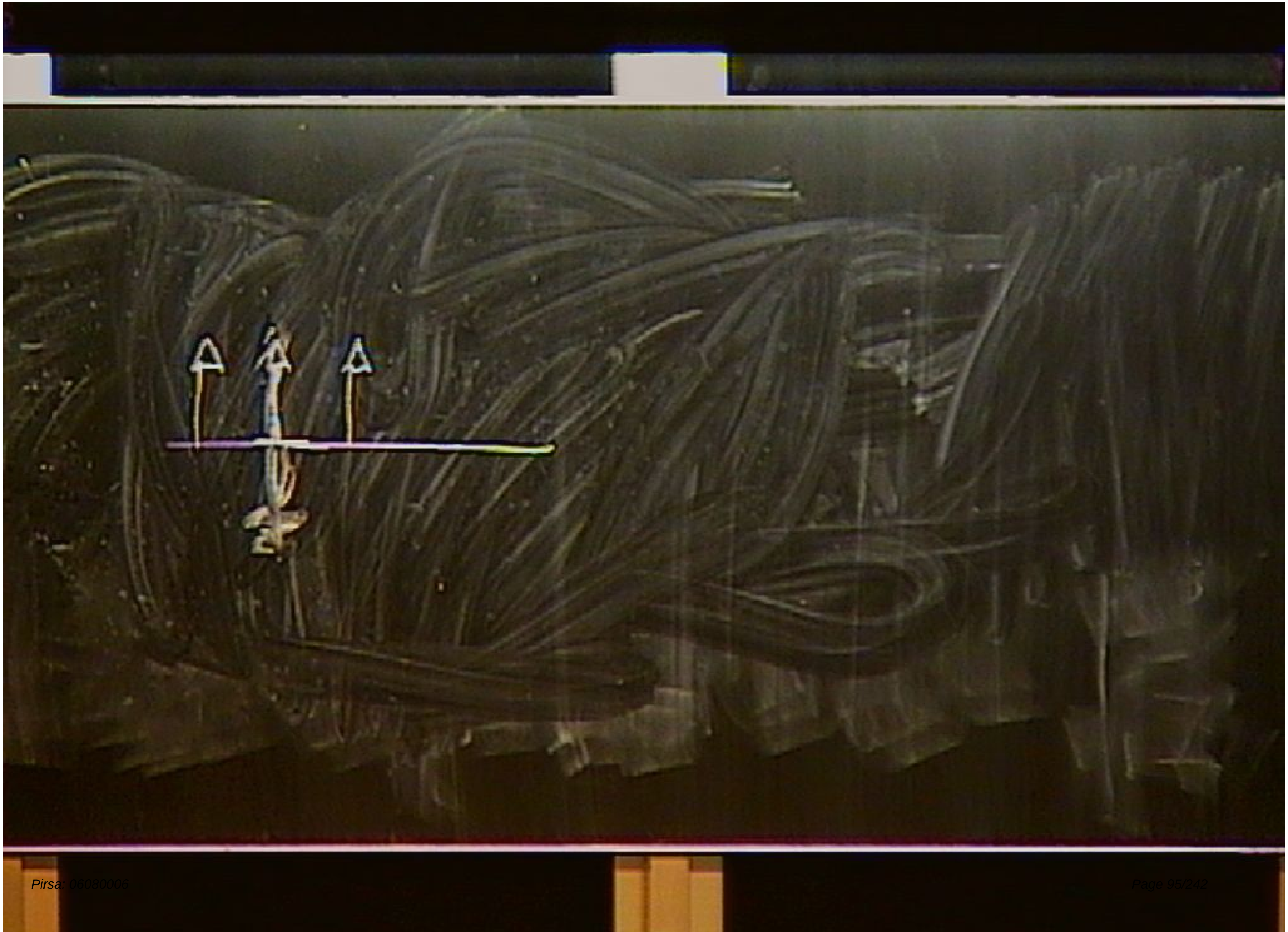






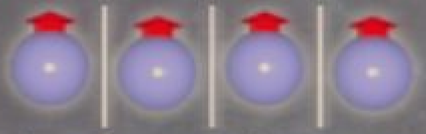




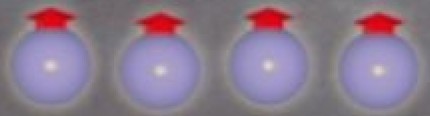


Dimensions and the Ising Model

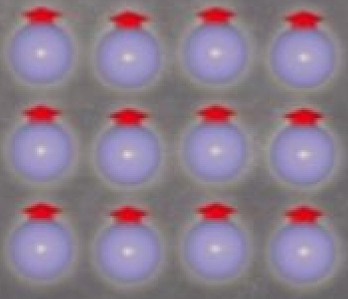
- 0-D: spins isolated



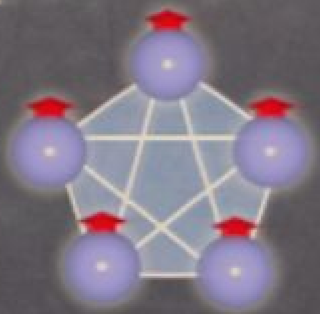
- 1-D: spins connected in a line



- 2-D: spins aligned in a grid



- ∞-D: all spins interconnected



Decay

- All memory systems undergo decay over time, mainly due to collisions with external particles.

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- This decay can be modeled mathematically, as will be demonstrated momentarily.

Chance of a Single Bit Persisting in its Original State over Time

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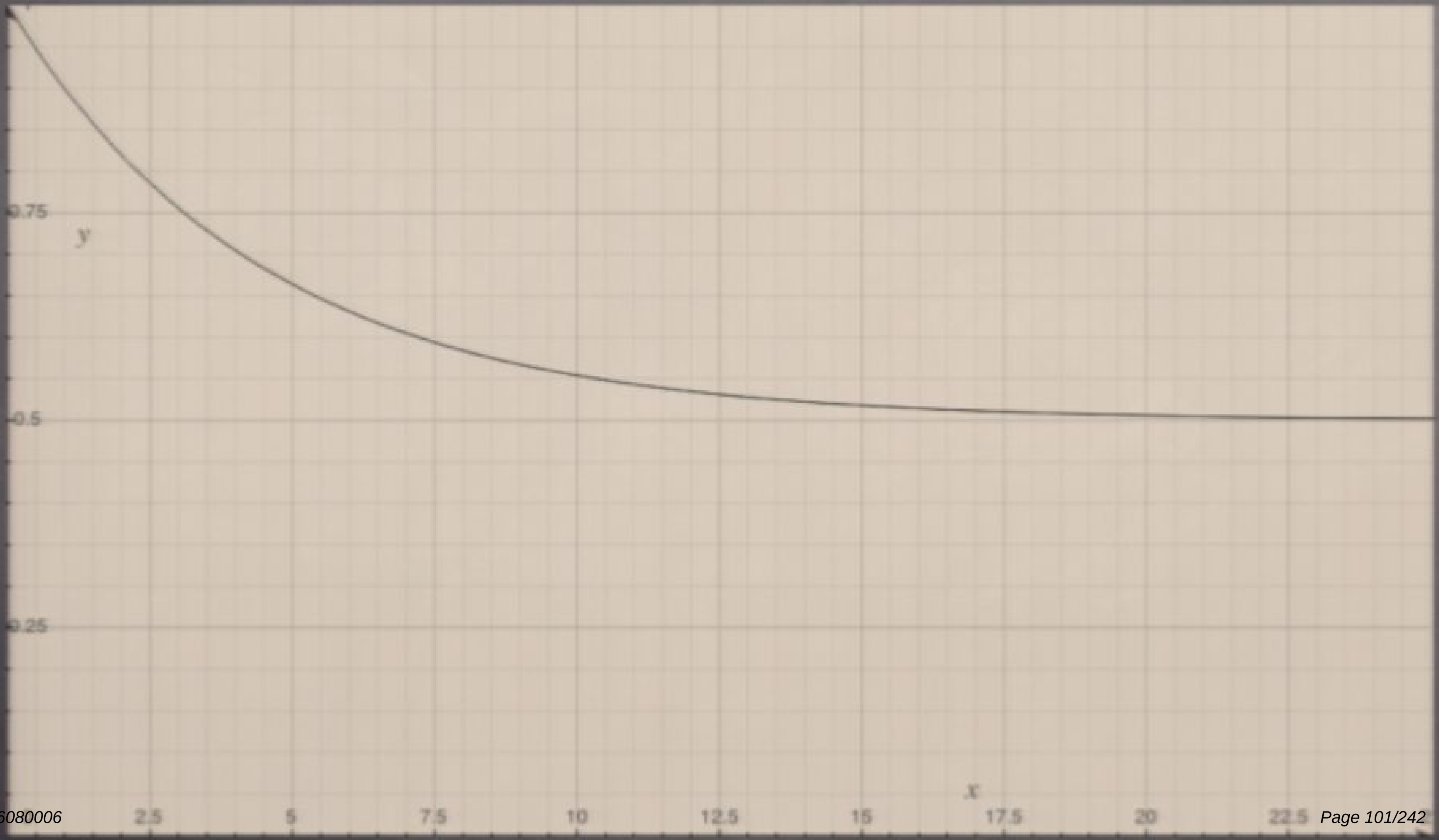

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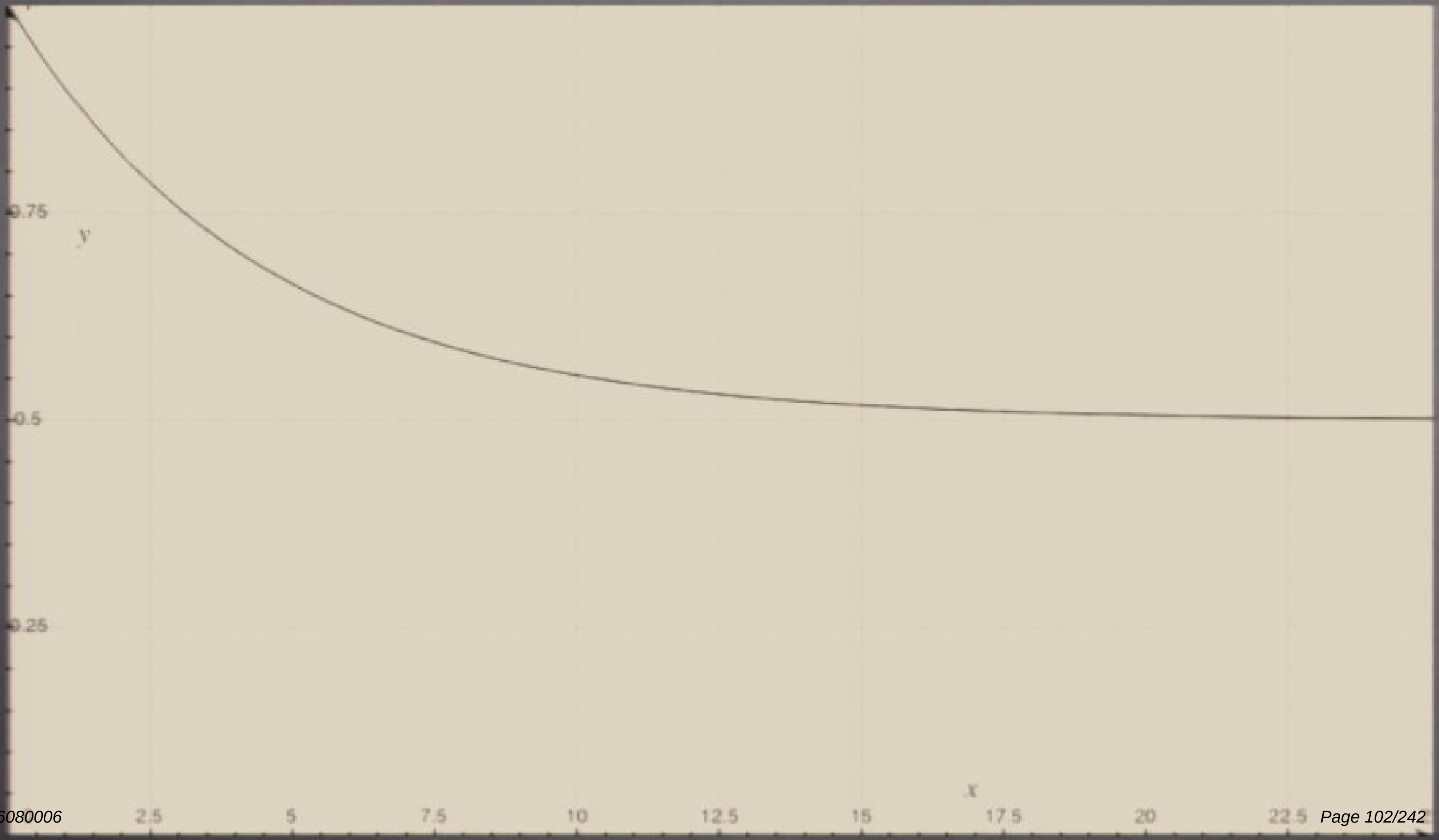

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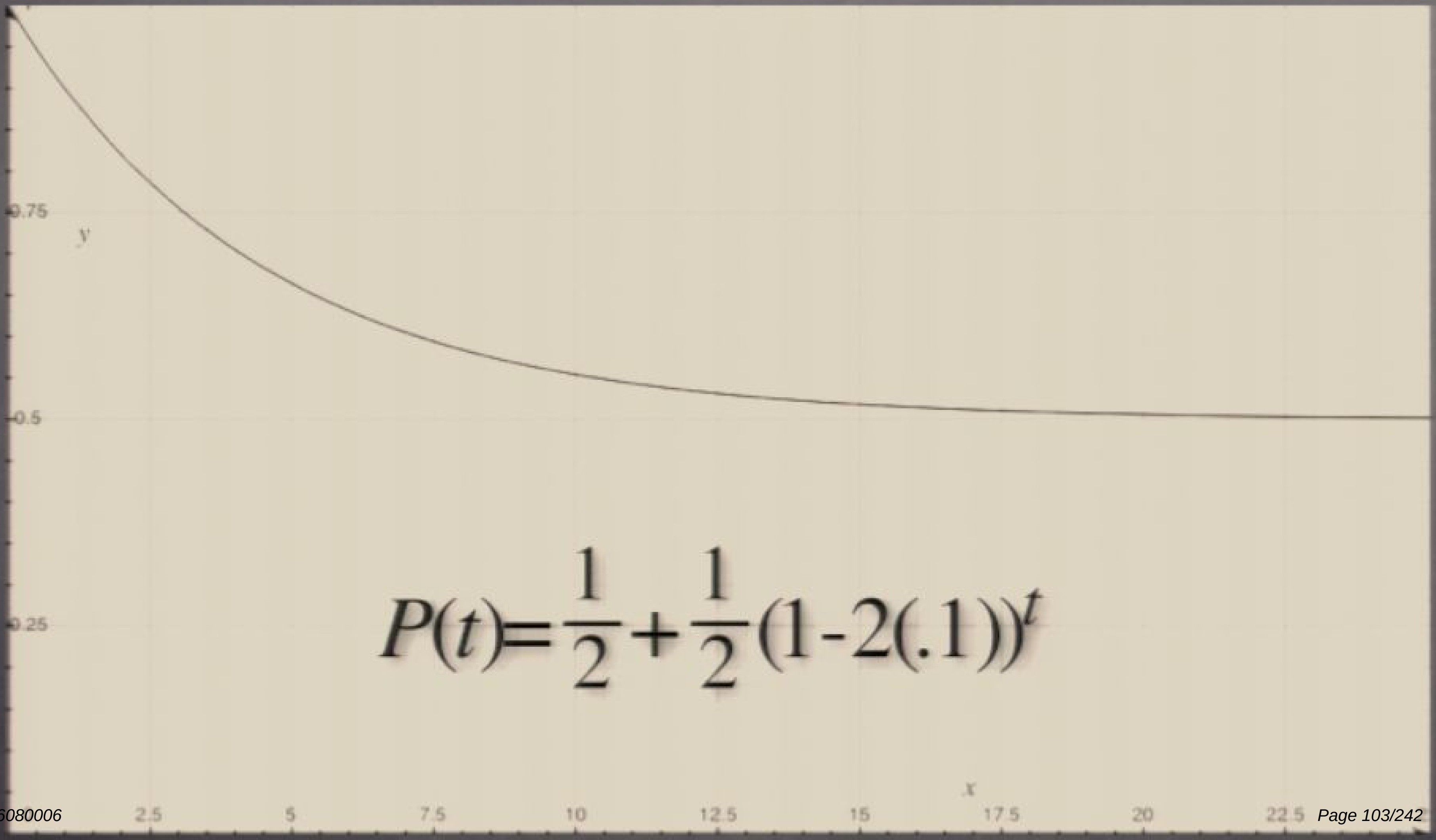

$$P(t)=\frac{1}{2}+\frac{1}{2}(1-2(.1))^t$$

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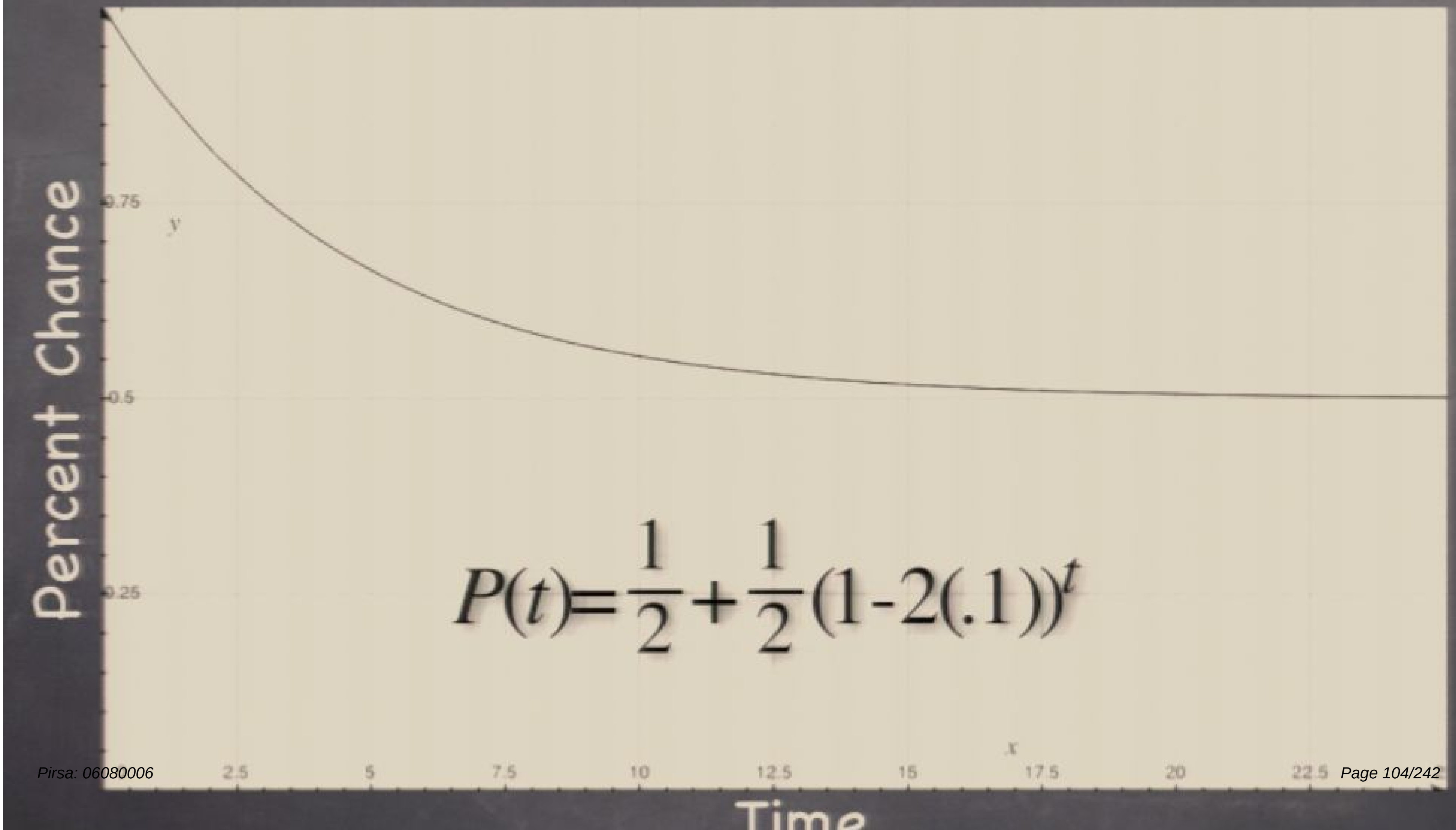

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Canonical Ensemble

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- Determines the probability of a system being in a certain state at a certain temperature and energy.

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- Determines the probability of a system being in a certain state at a certain temperature and energy.
- Therefore, the lower the energy of a state, the more likely it is to exist in that state, potentially causing an error.

antum Memo

Quantum Memory

Quantum Memory

- Standard unit of memory: qubit.
- Qubit is based on the quantum state of one particle, e.g. the polarization of a photon, the spin of an electron, etc.
- Current quantum memory systems hold information for about 1 second.
- One goal of quantum information is to achieve reliable and high capacity memory systems.

Entanglement

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- The key to faster computations with quantum memory.
- Allows for teleportation, superdense coding, solving "hard" mathematical problems (factoring), etc.
- Without entanglement, the qubit would be equivalent to classical bits.

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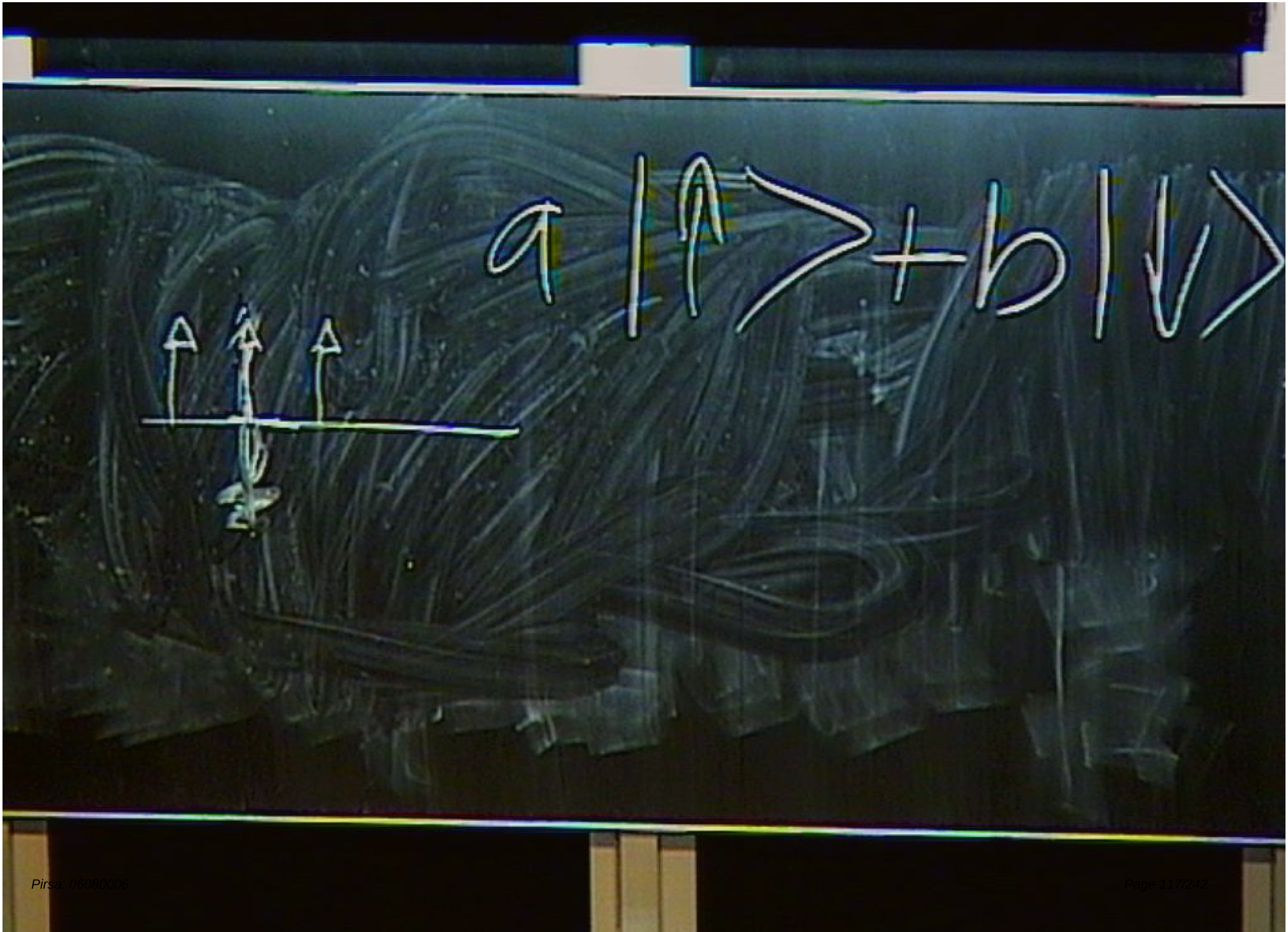

$$a|\uparrow\rangle + b|\downarrow\rangle$$

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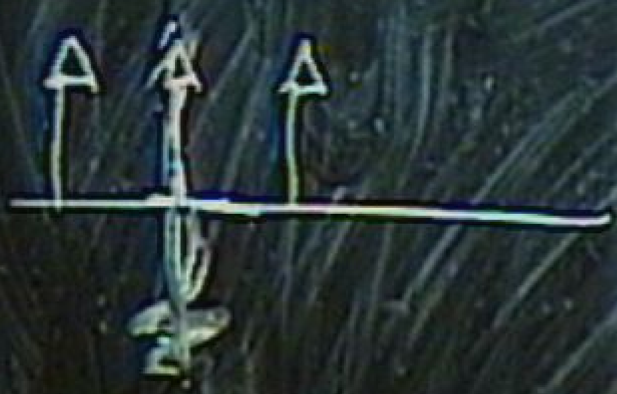

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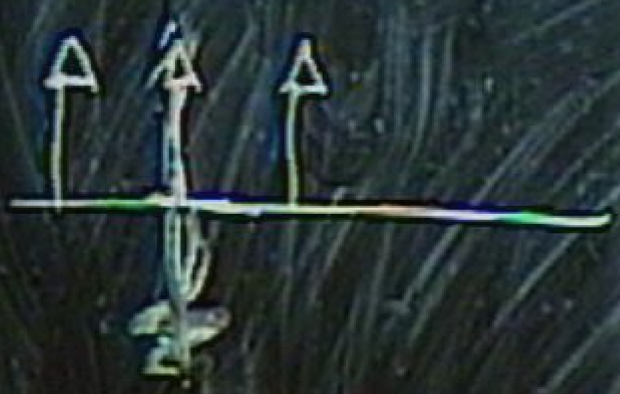

$$a|\uparrow\rangle + b|\downarrow\rangle$$

Major Problems

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- The genuinely random nature of subatomic particles creates many possibilities for error.
- X Error
- Y Error
- Z Error
- Difficulty in defining meaningful entanglement patterns.

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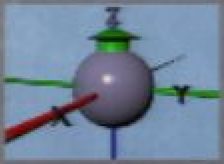
- X Error 

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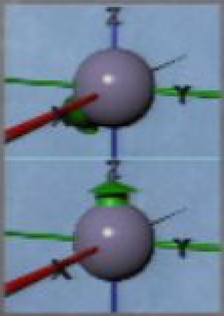

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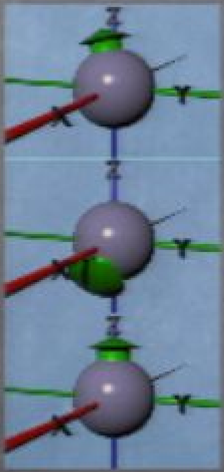

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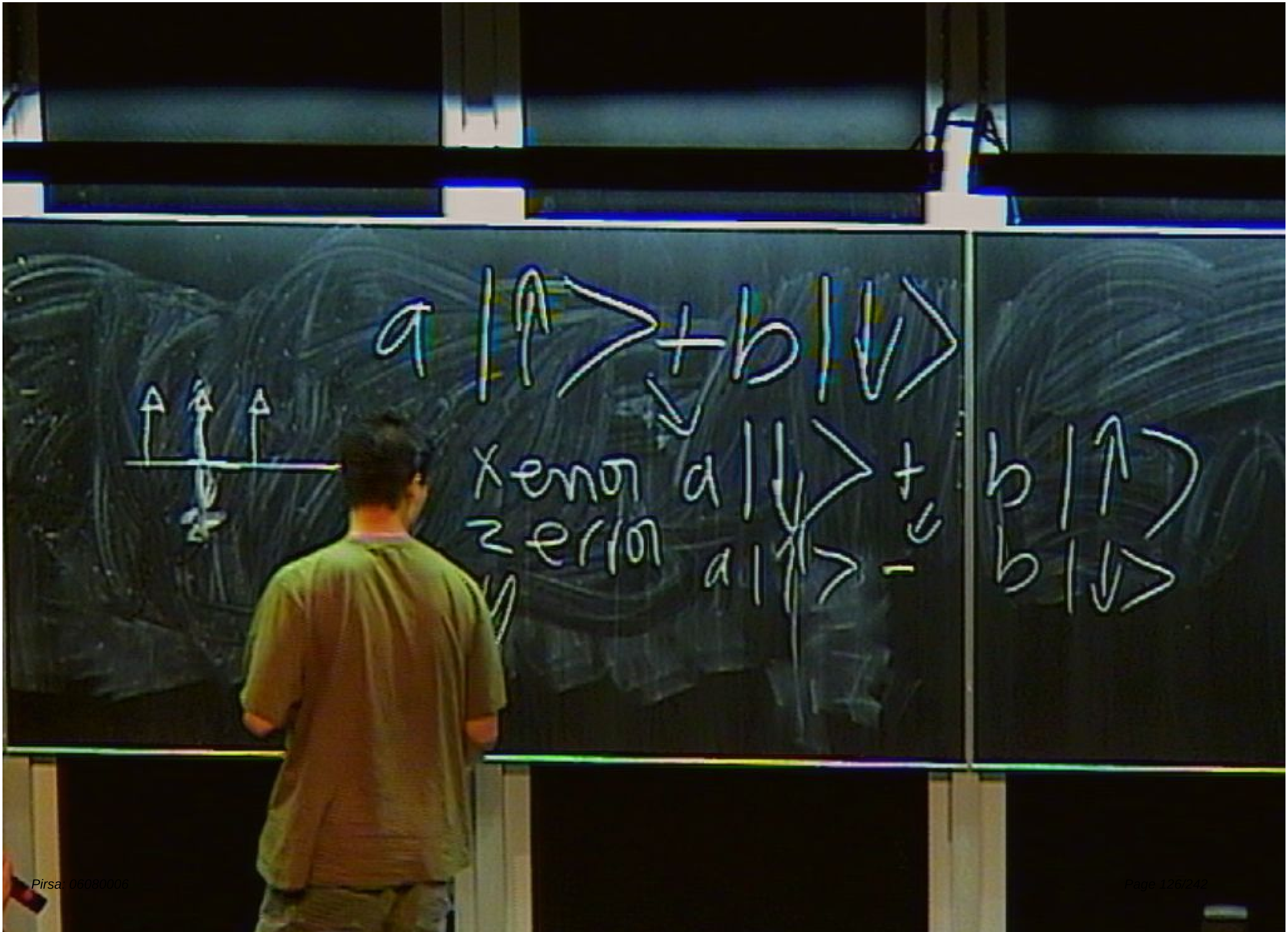
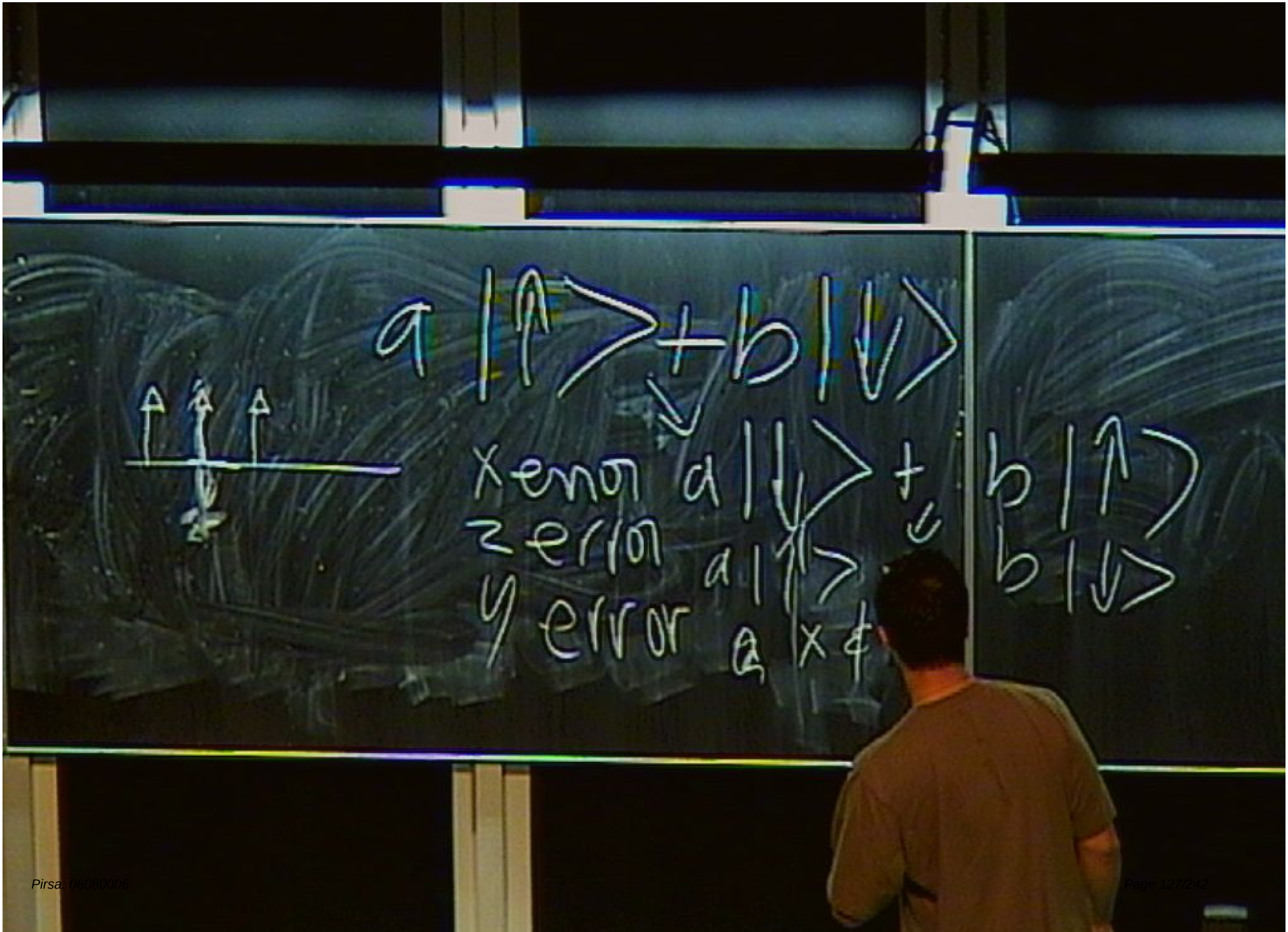


$a|\uparrow\rangle + b|\downarrow\rangle$

$\downarrow$

xenon $a|\downarrow\rangle \overset{+}{\downarrow} b|\uparrow\rangle$

zeron $a|\uparrow\rangle - b|\downarrow\rangle$



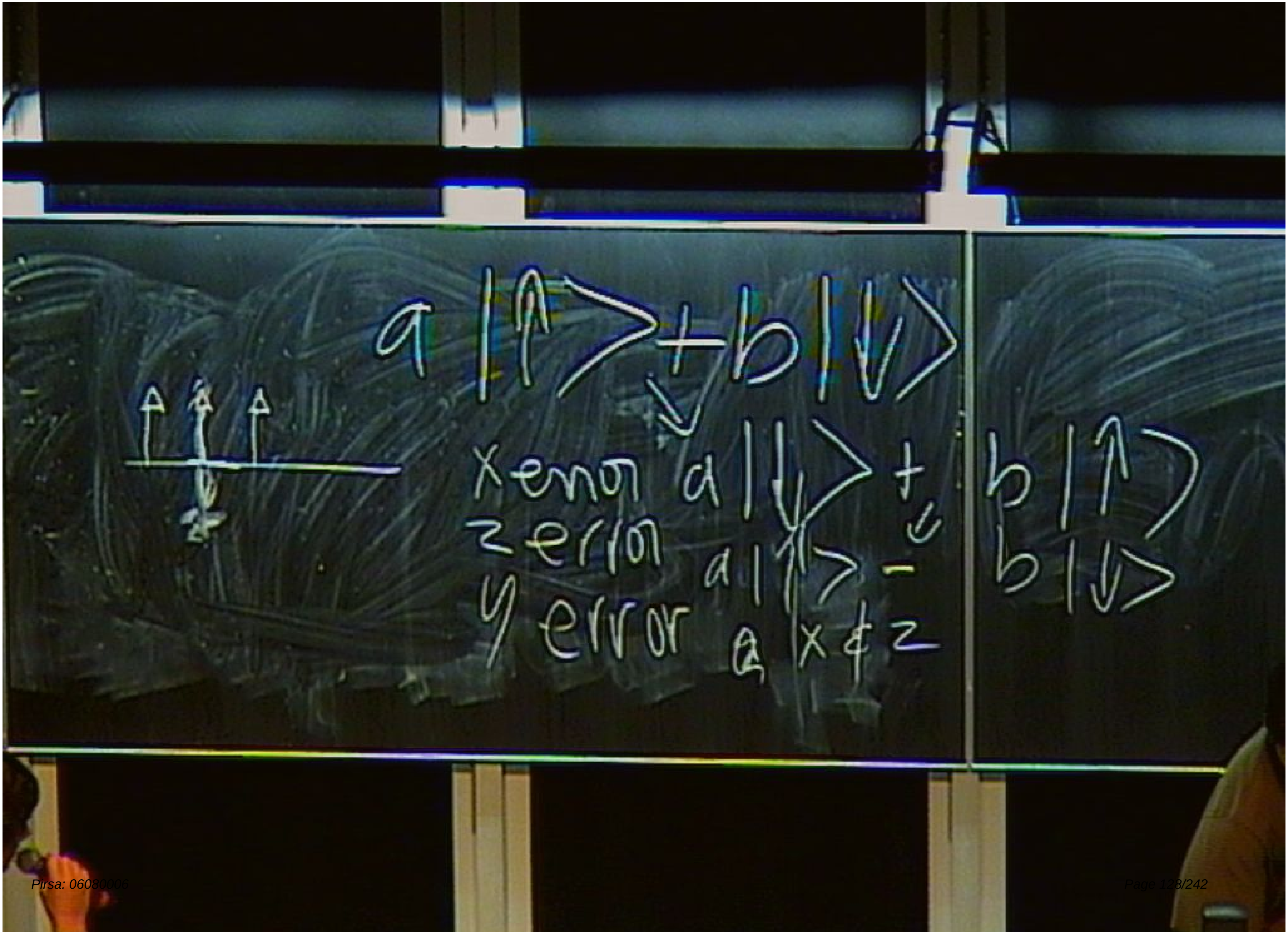
$a|\uparrow\rangle + b|\downarrow\rangle$

x error $a|\downarrow\rangle + b|\uparrow\rangle$

z error $a|\uparrow\rangle \quad b|\downarrow\rangle$

y error $a|\downarrow\rangle$

a x +



$a|\uparrow\rangle + b|\downarrow\rangle$

x error $a|\downarrow\rangle + b|\uparrow\rangle$

z error $a|\uparrow\rangle - b|\downarrow\rangle$

y error

x & z

Self-Healing Memory

- Passive vs. Active error correction: using nature or man-made algorithms
- Current passive healing rate is much slower than the rate of errors
- If we can raise the energy required to maintain erratic states, the probability of errors decreases.

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$$a\left(\frac{|\uparrow\uparrow\uparrow> + |\downarrow\downarrow\downarrow>}{\sqrt{2}}\right)\left(\frac{|\uparrow\uparrow\uparrow> + |\downarrow\downarrow\downarrow>}{\sqrt{2}}\right)\left(\frac{|\uparrow\uparrow\uparrow> + |\downarrow\downarrow\downarrow>}{\sqrt{2}}\right)+$$

Current Progress

- 9-Qubit Code

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Current Progress

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- Toric Code

Current Progress

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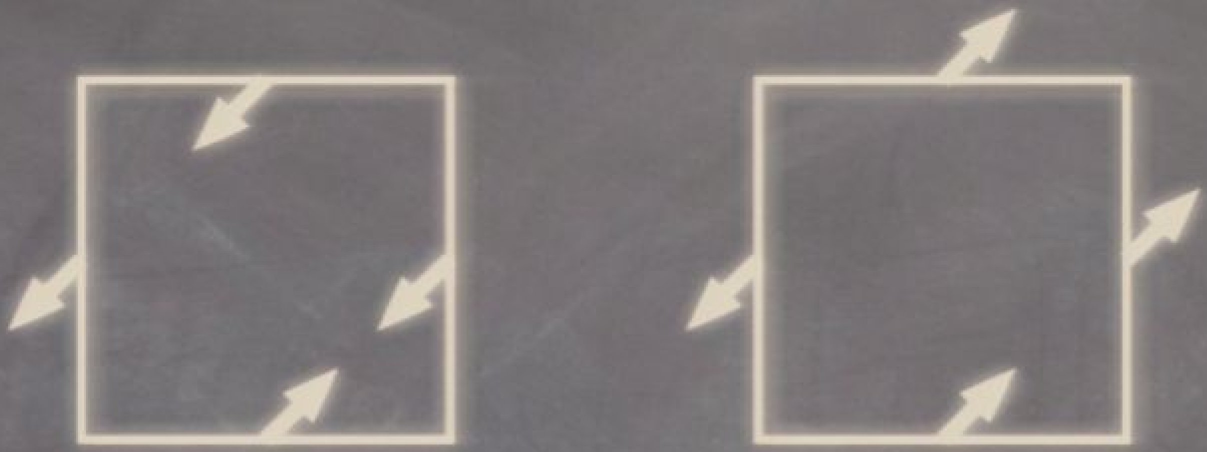
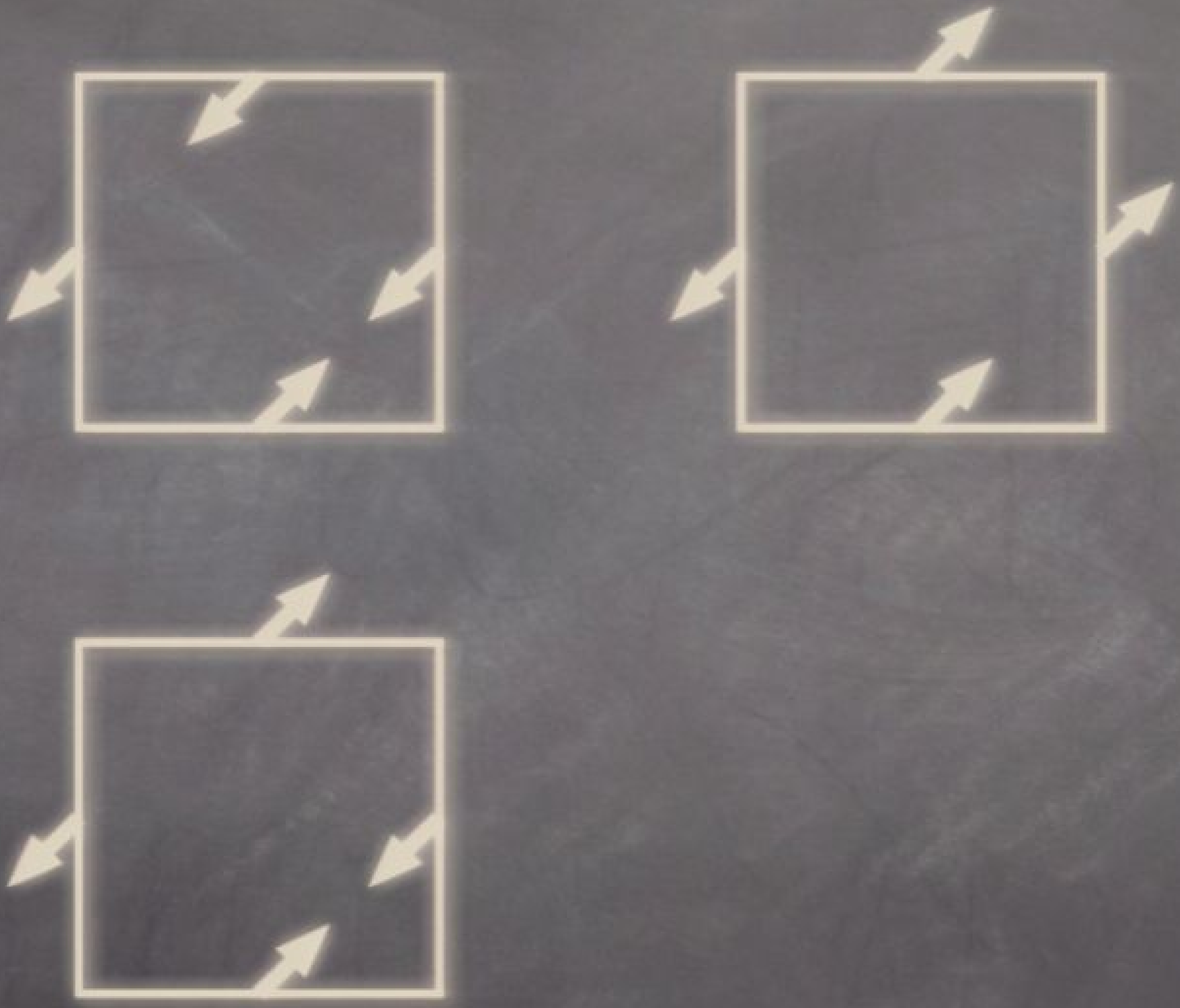
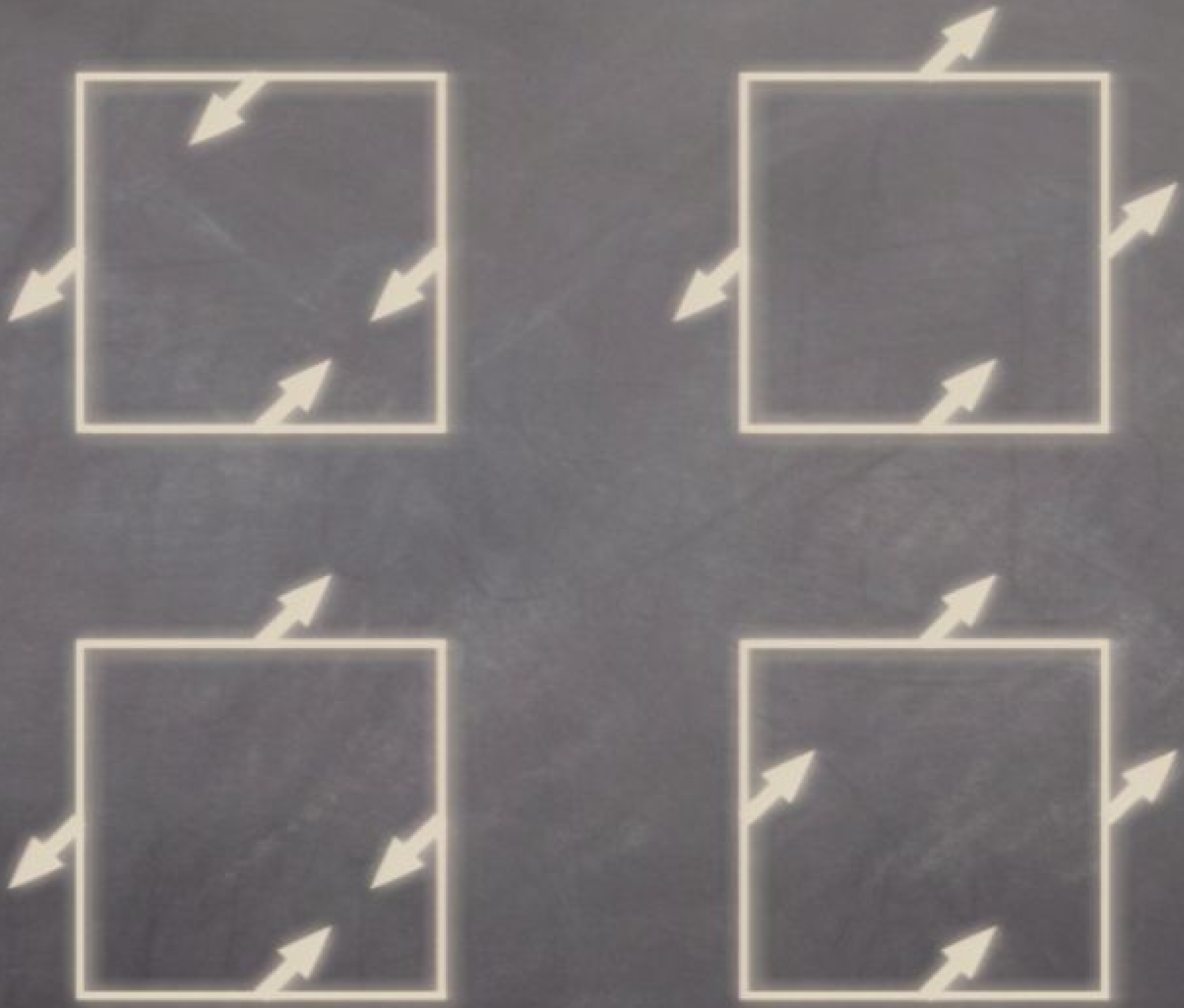
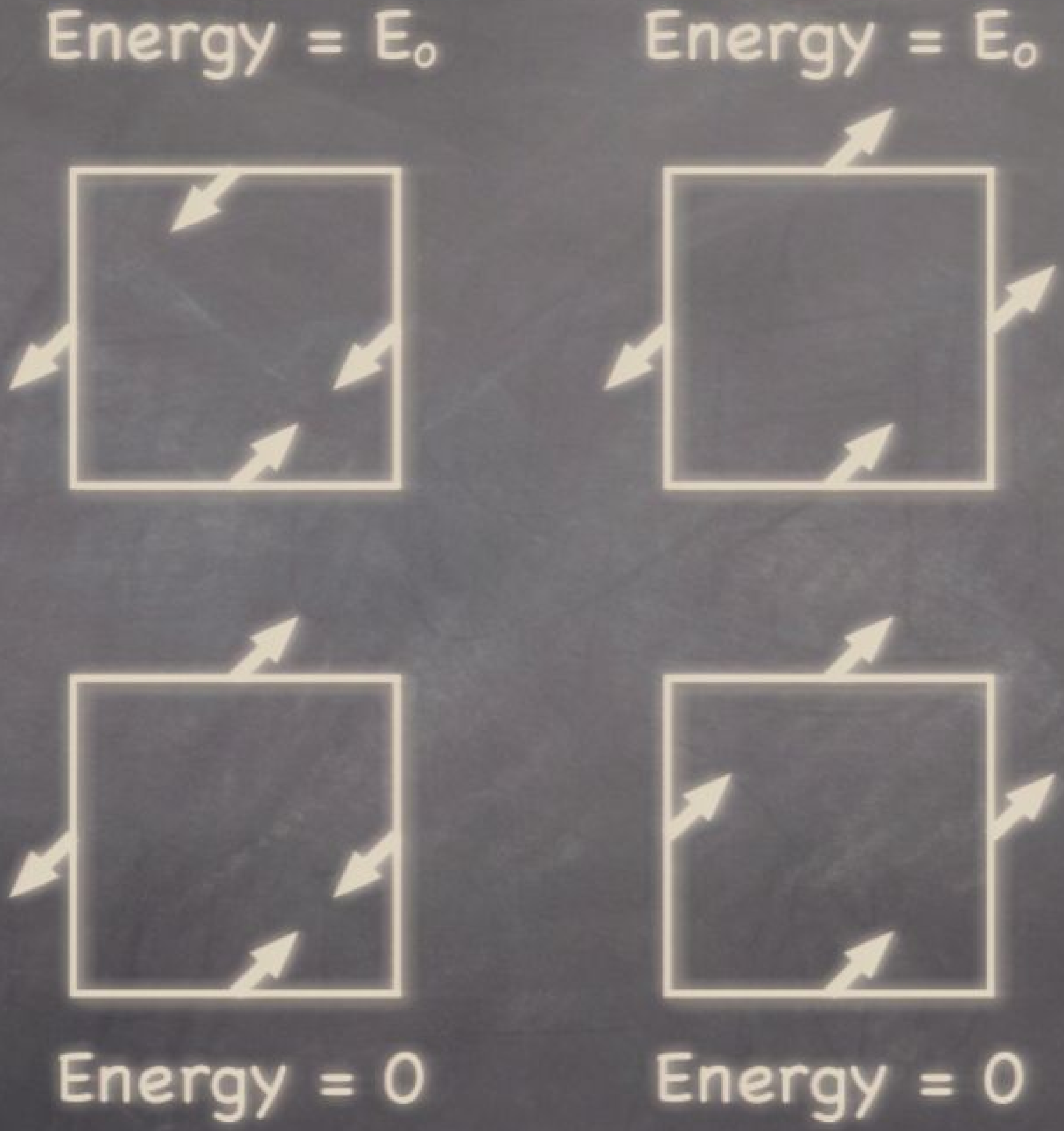
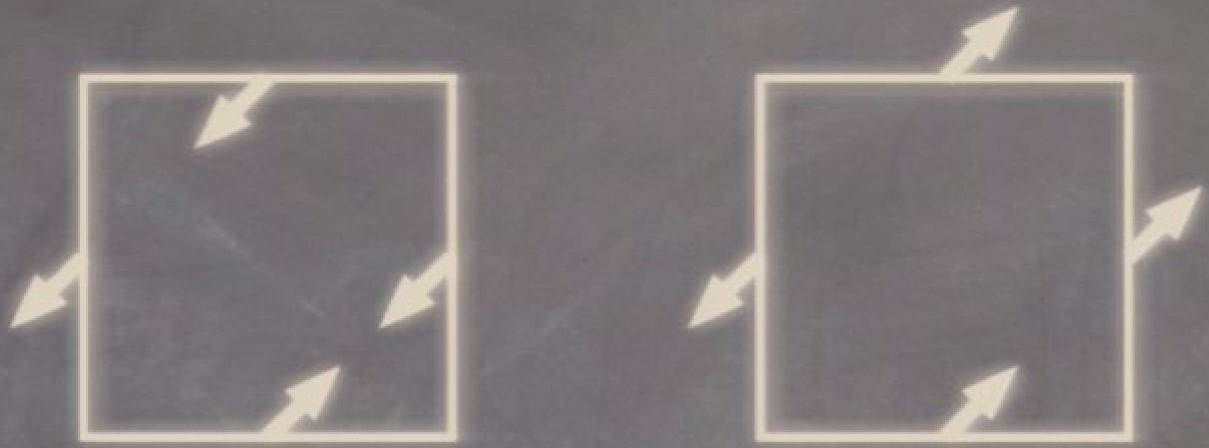






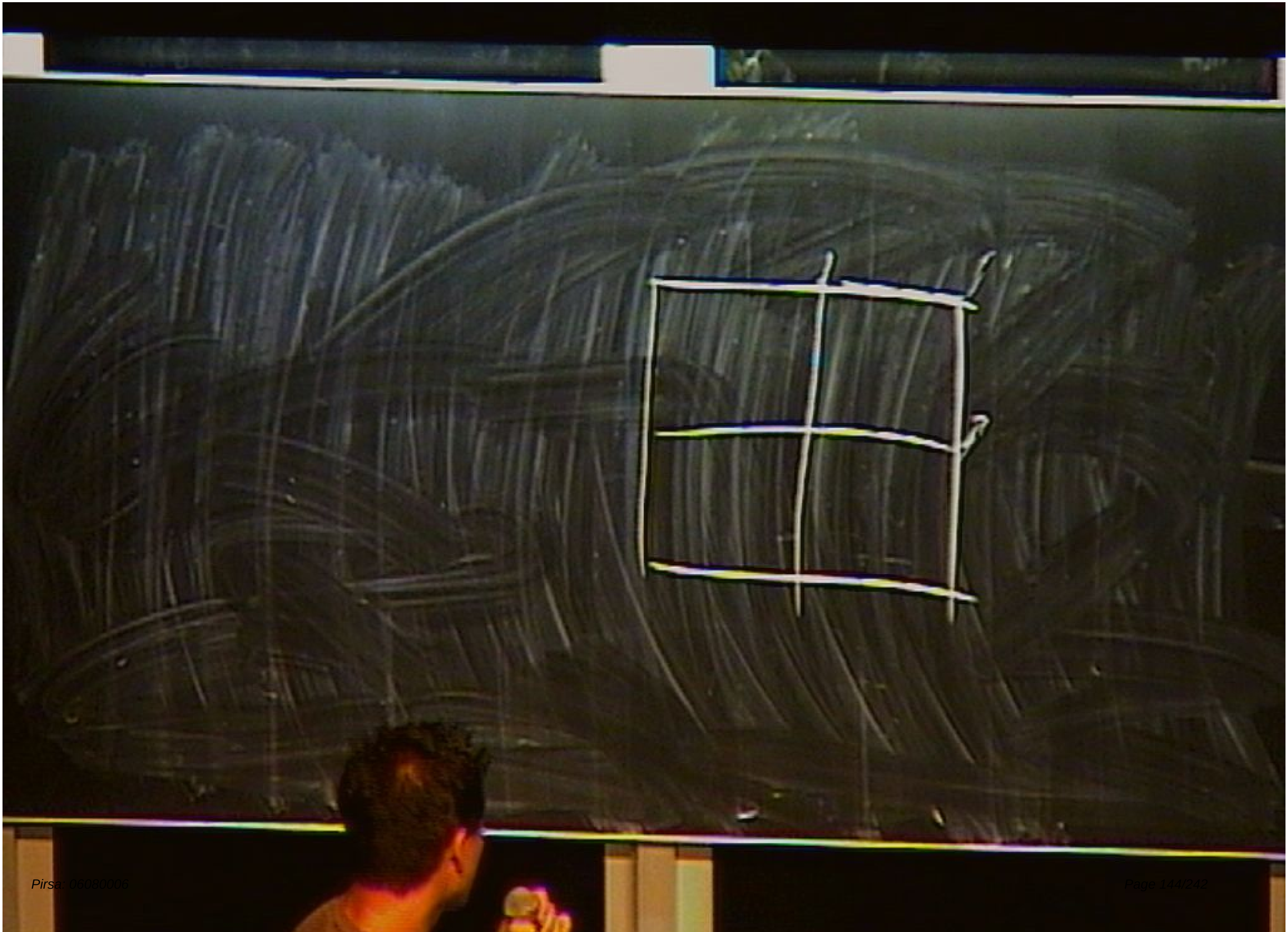

Energy = E_0
Energy = E_0
Energy = 0
Energy = 0

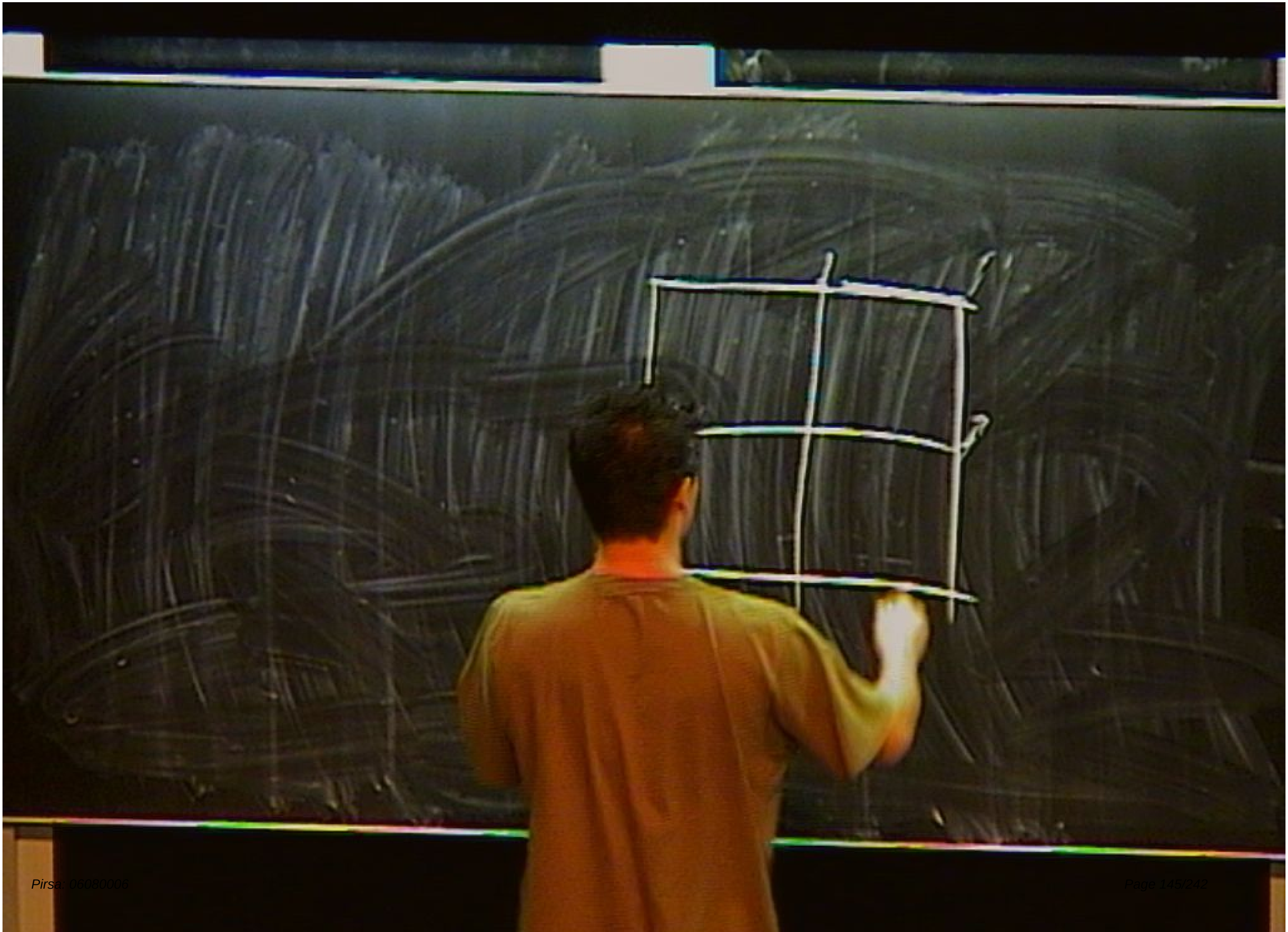
Energy = E_0 Energy = E_0

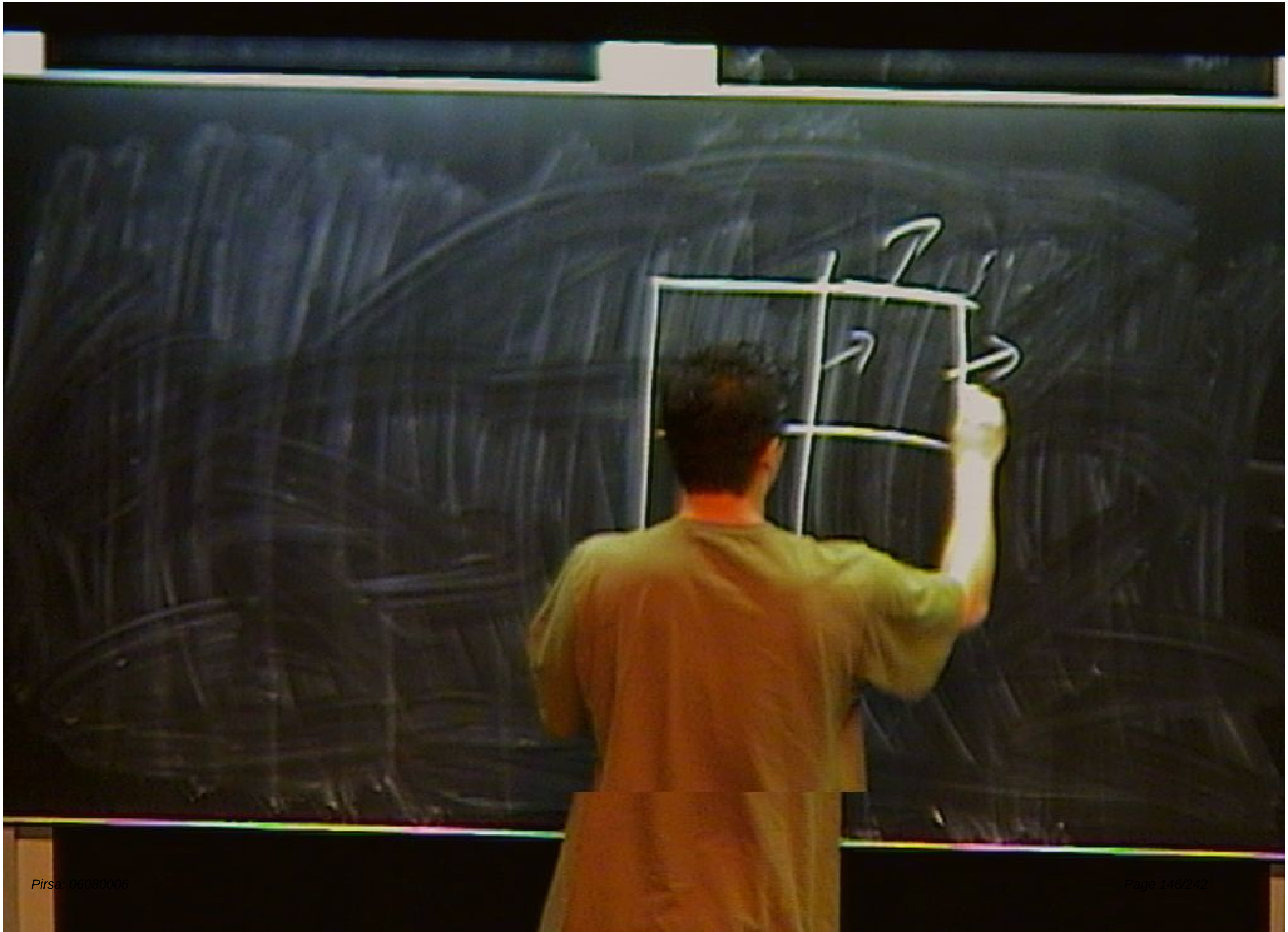


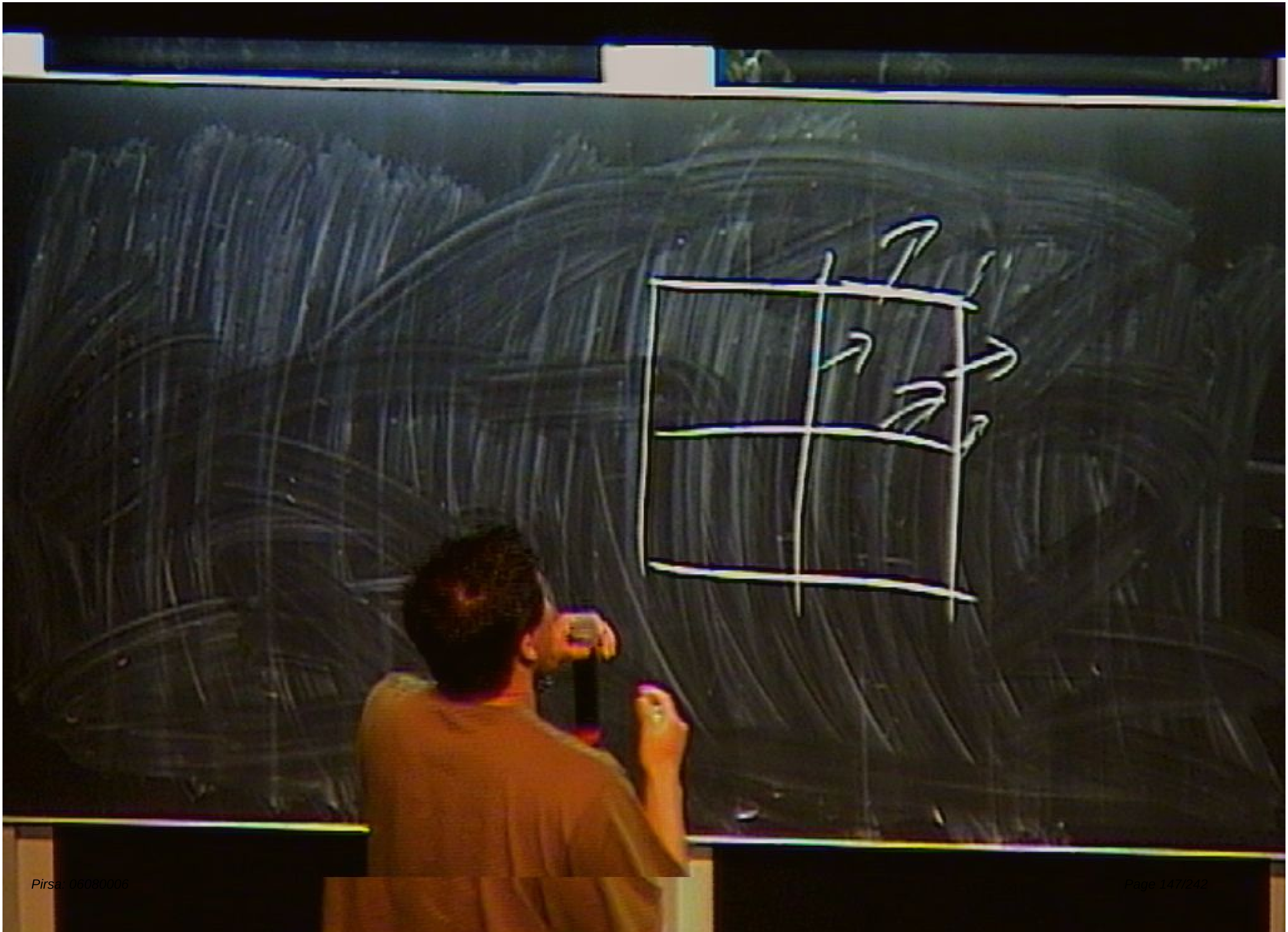
(Count of UP Spins) MOD 2

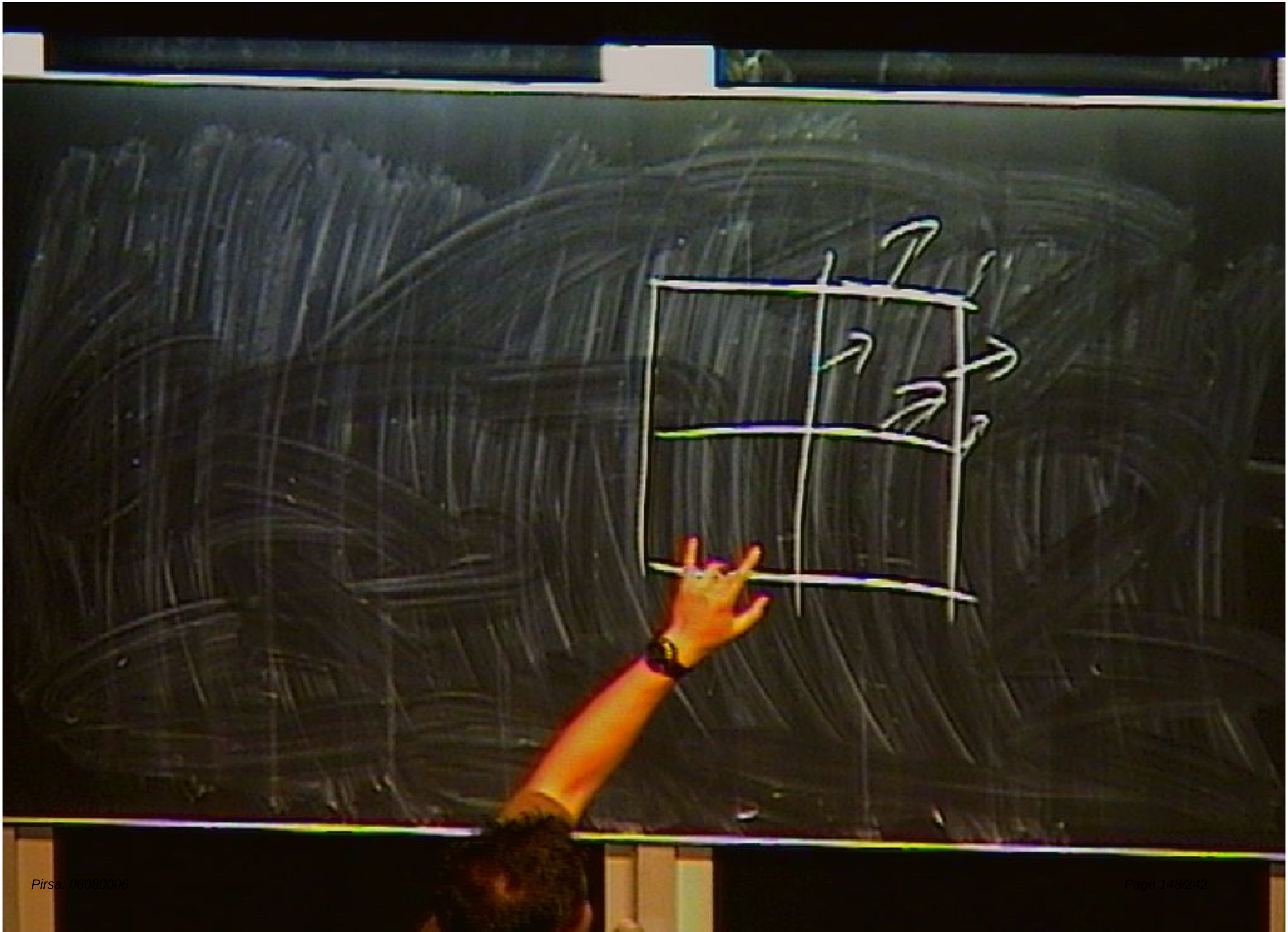
Energy = 0 Energy = 0

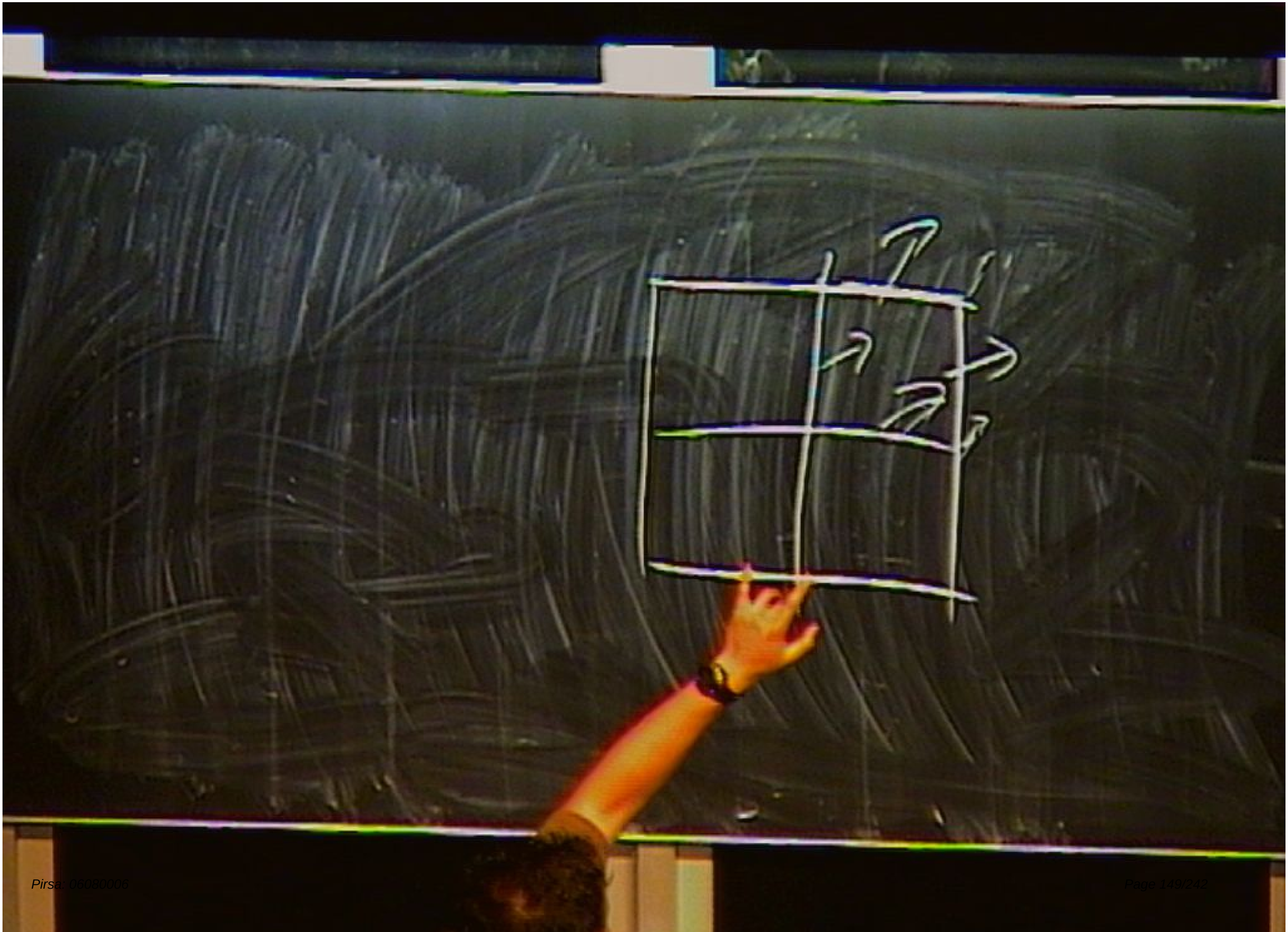












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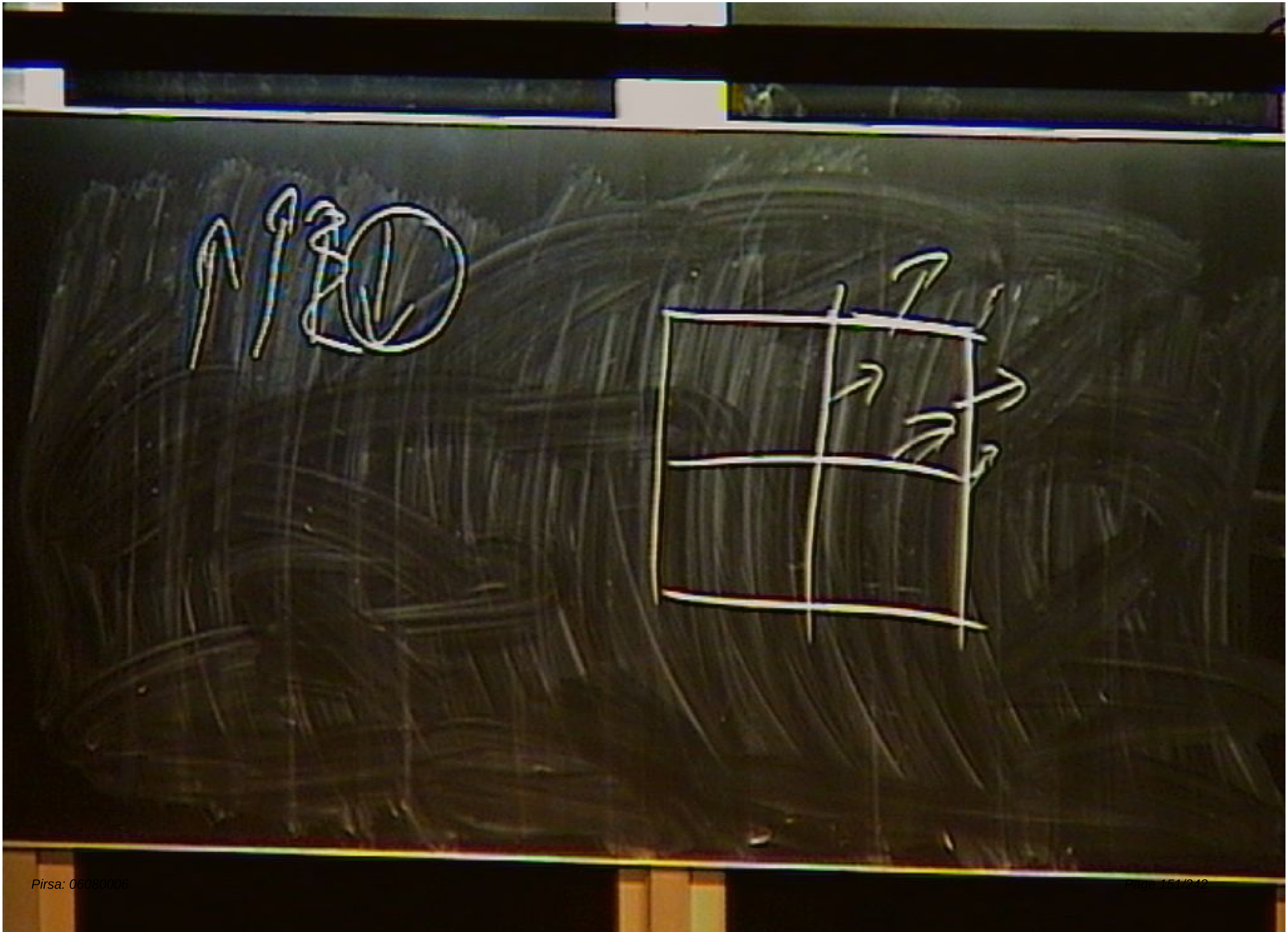

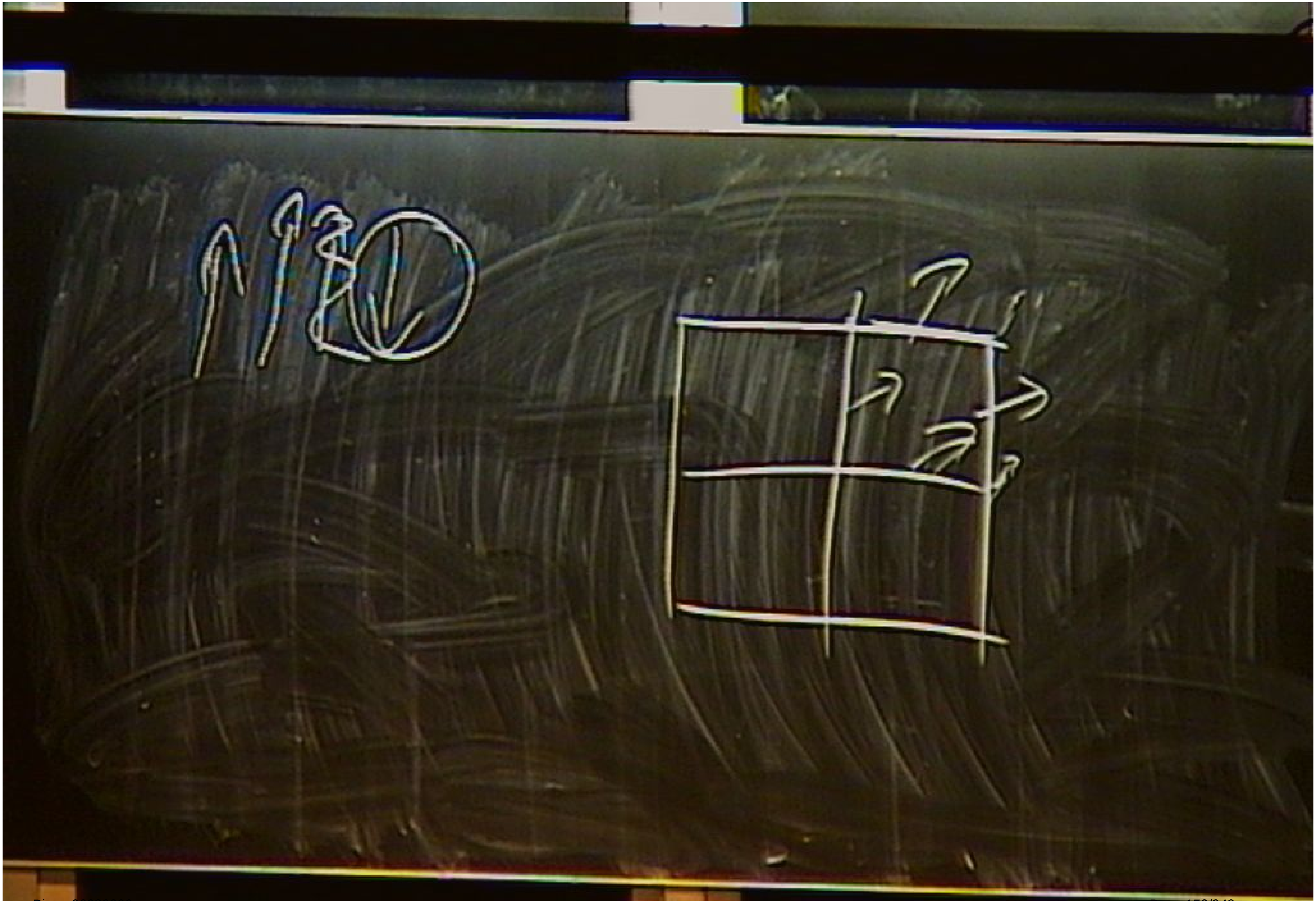

Quest for 3-D Toric Code and Deterministic Entanglement

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- Flaws in 2-D Toric code

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- C
- Flaws in 2-D Toric code
- ?
  4-D Toric code is o equivalent to 2-D Ising Model

3 r c

t e a i d

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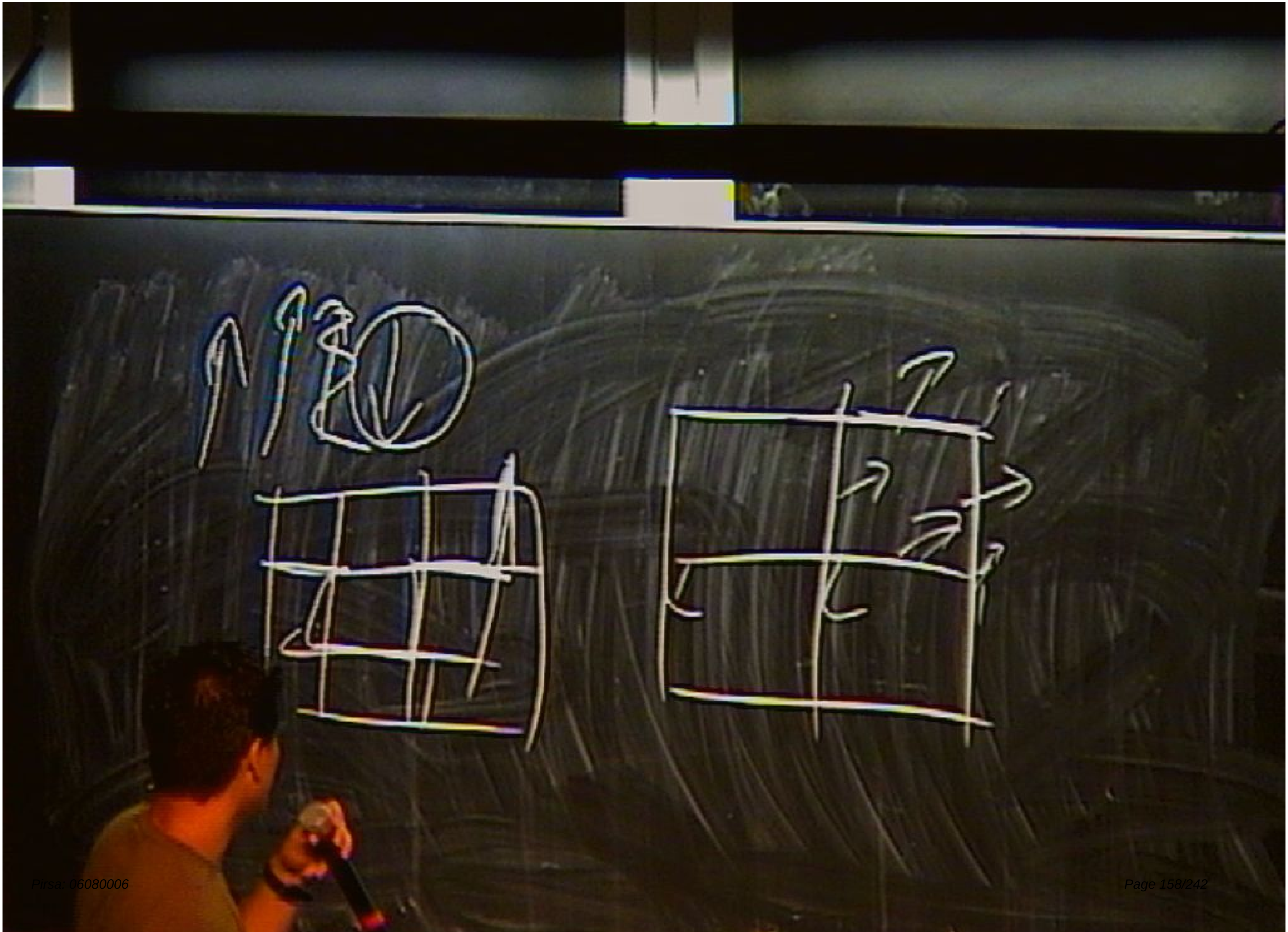
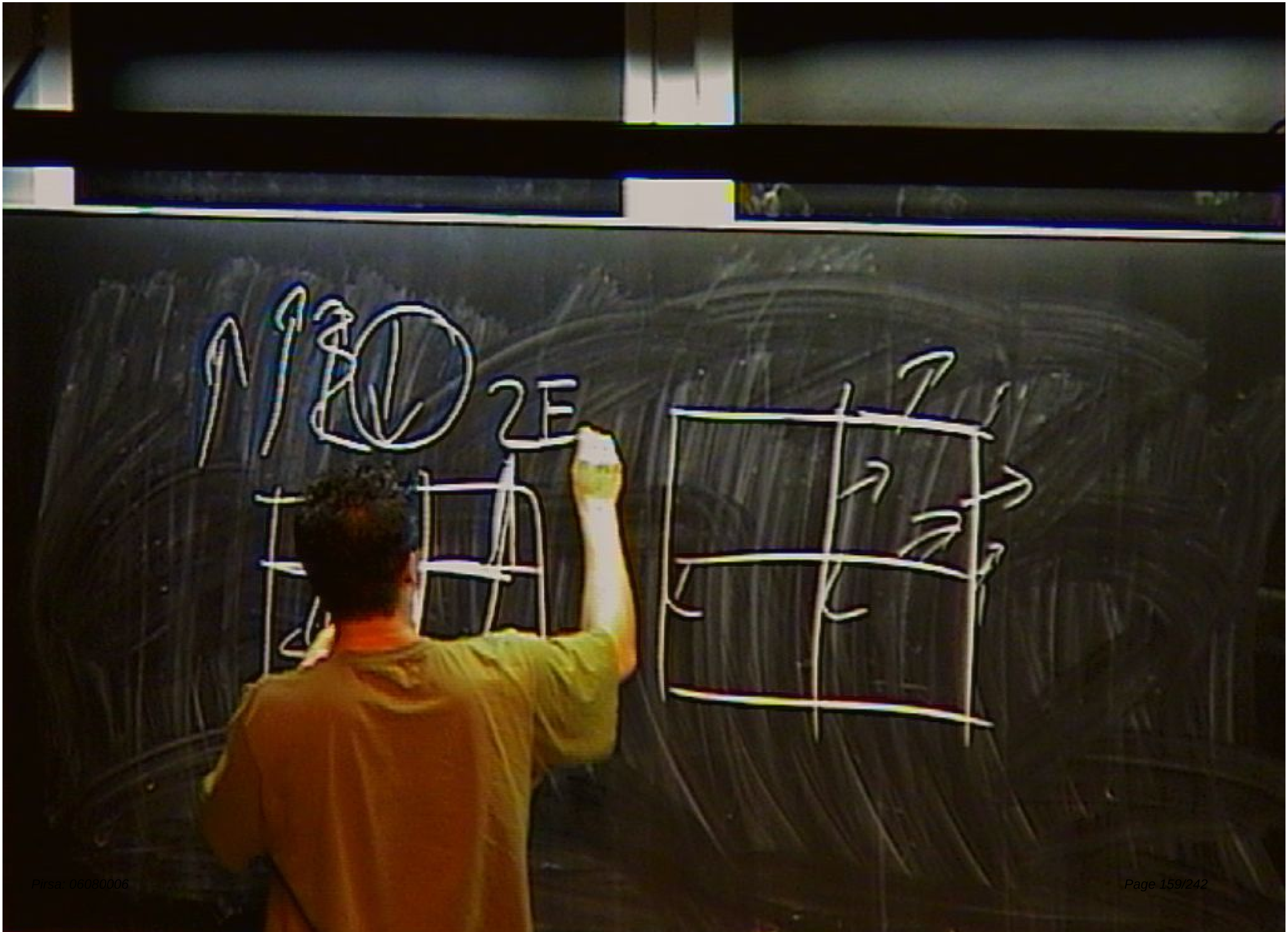
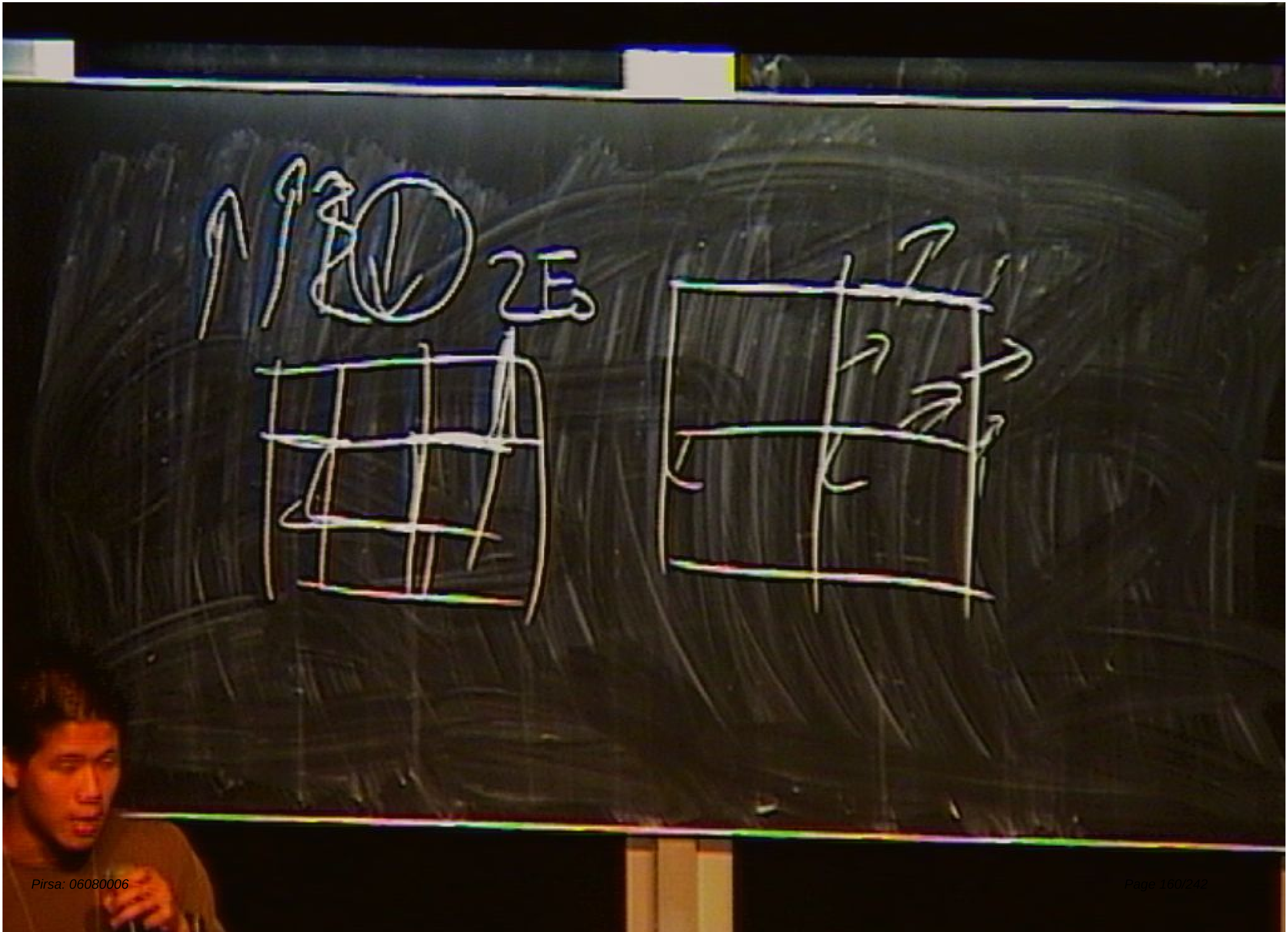
- Is there a 3-D Toric Code?
- Can we create and maintain useful entanglement patterns?




2E


2E₀

Future of Quantum Memory

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Many thanks to Dr. Carlos Mochon

Major props to
STIMAC


aqua cool *

No Signal
VGA-1

No Signal

VGA-1

No Signal

VGA-1

No Signal

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Schroedinger's equation:

$$i\hbar \cdot \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x) \cdot \psi(x,t)$$

take : $\hbar = 1 \Rightarrow$

$\Rightarrow$ wave function:

$$i \frac{\partial}{\partial t} \psi(x,t) = -\frac{1}{2m} \cdot \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x) \cdot \psi(x,t)$$

$+\overset{\nabla}{\circ} \Rightarrow$ general formula for waves:

$$\psi(x,t) = R(x,t)\, e^{iS(x,t)}$$

Movement formula:

$$\frac{\partial x}{\partial t}(t) = \frac{1}{m} \frac{\partial S}{\partial x}(x,t)$$ (by laziness) $\Rightarrow$

$$\Rightarrow \quad m \cdot \frac{\partial^2 x}{\partial t^2} = -\frac{\partial}{\partial x}\left(V(x) + Q(x)\right)$$

Initial momentum is determined by the incident wave function,

$$p = \partial s/\partial z = p_0$$

In practice, however, we do not control the initial location (so, though it goes through a definite slit, we cannot predict which slit it will be.)

Acted on by "Quantum-Mechanical" potential; or U.

$$U = (-h^2/2m)\nabla^2 R/R$$ ← case-simplified equation

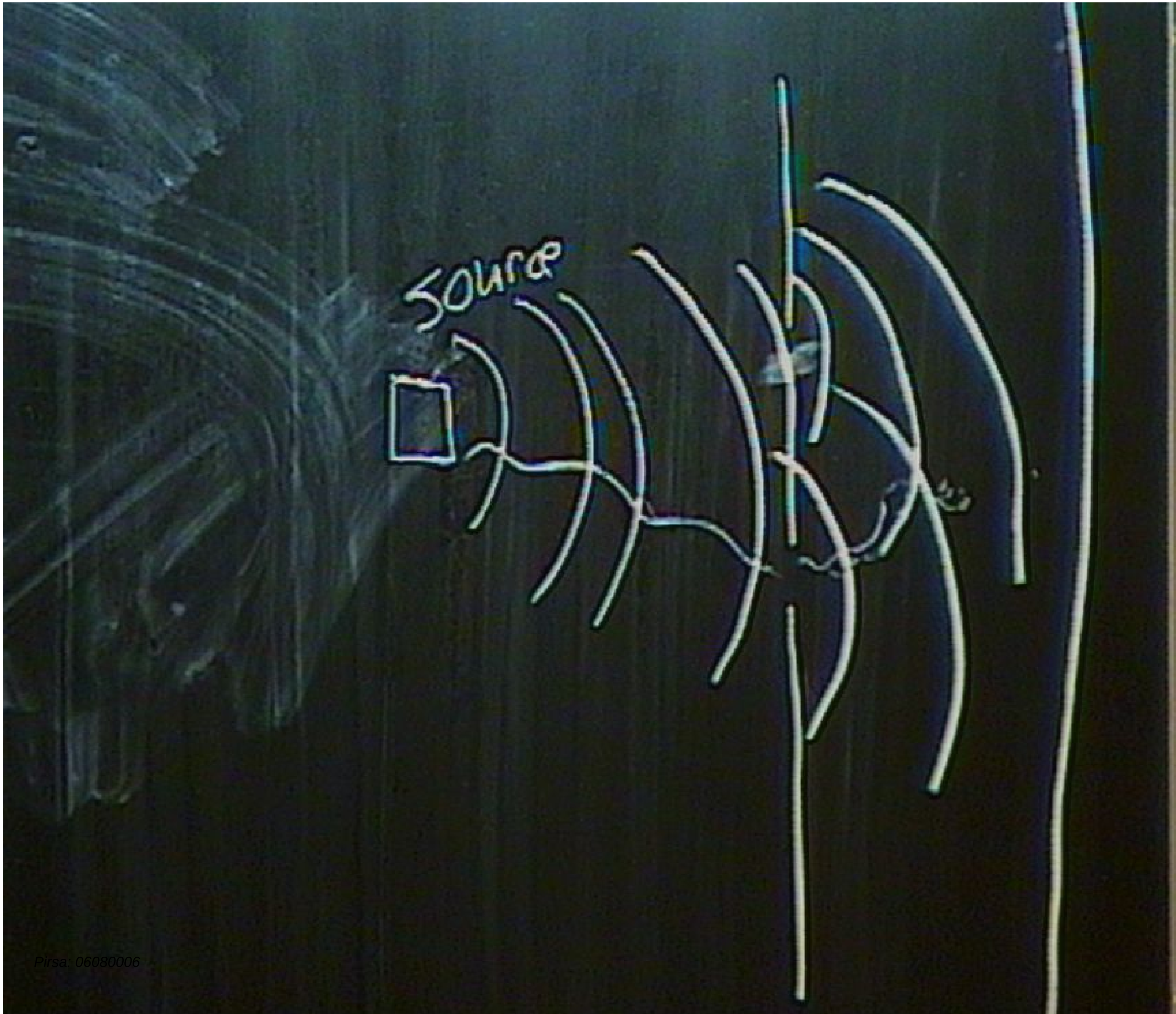
Because U becomes infinite when R becomes zero (making a repelling force) we have now got a simple and accurate model of why particles are never found where the wave-function vanishes.

If one slit is closed, the ψ-field is changed and so the particle can now reach some places where it could not go when both slits were open.

THEREFORE the slit can only affect the motion of the particle indirectly, through an effect on the Schroedinger ψ-field.

[redacted]

∴ $U(x)$ acting on a particle depends on a wave intensity, $P(x)$; also numerically equal to a probability density.. so the wave function can be interpreted both as a force and a probability density.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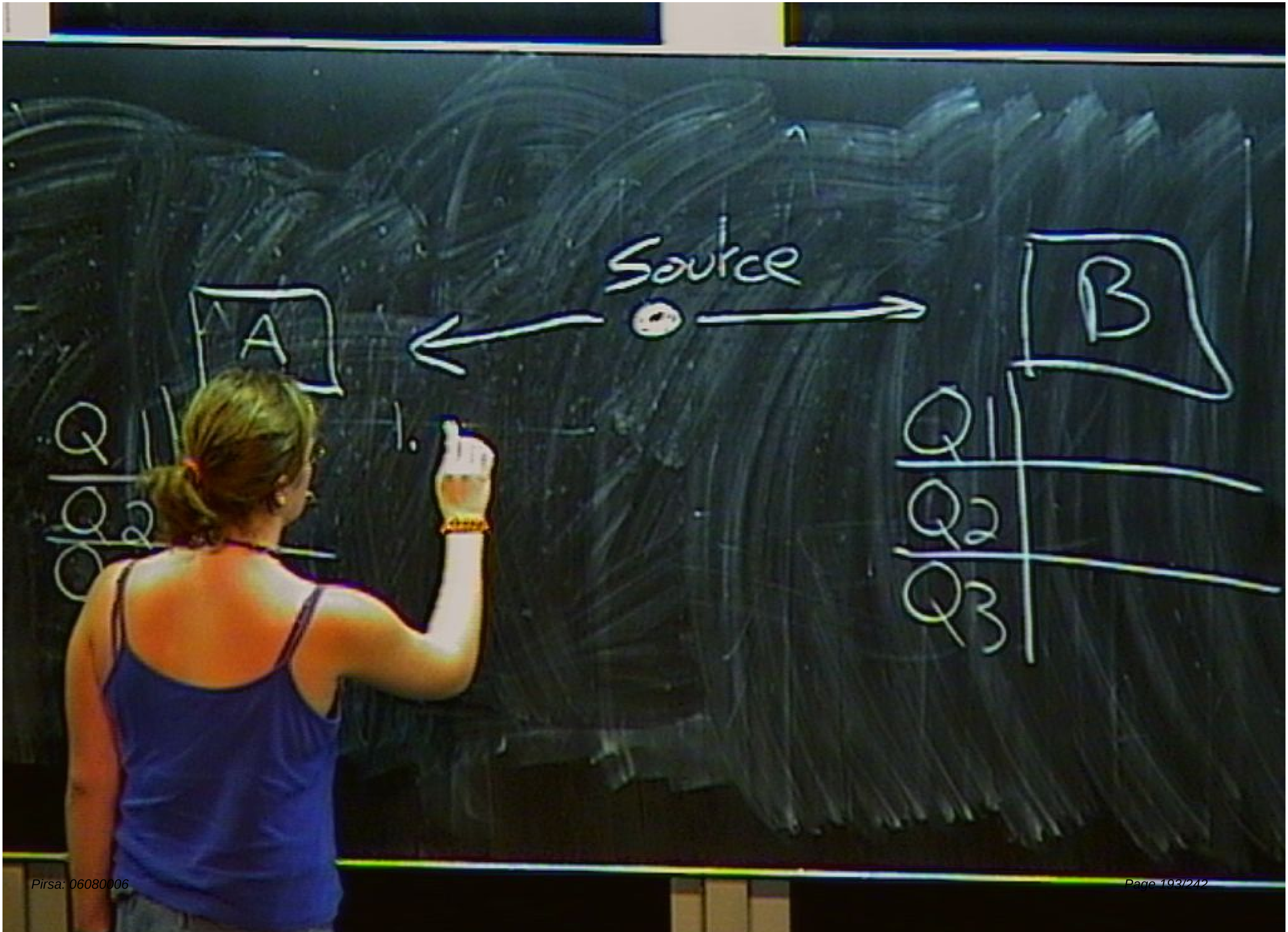
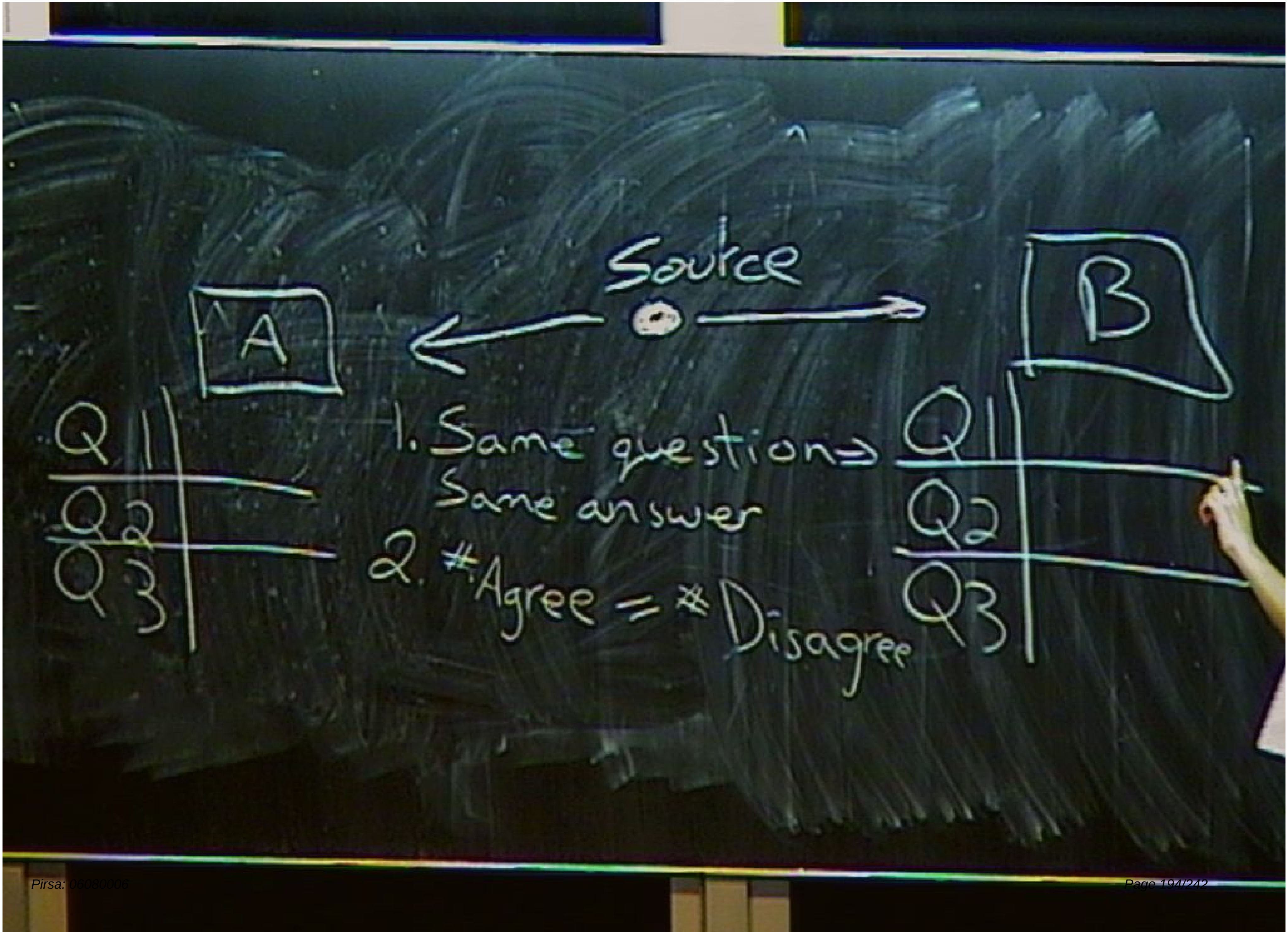
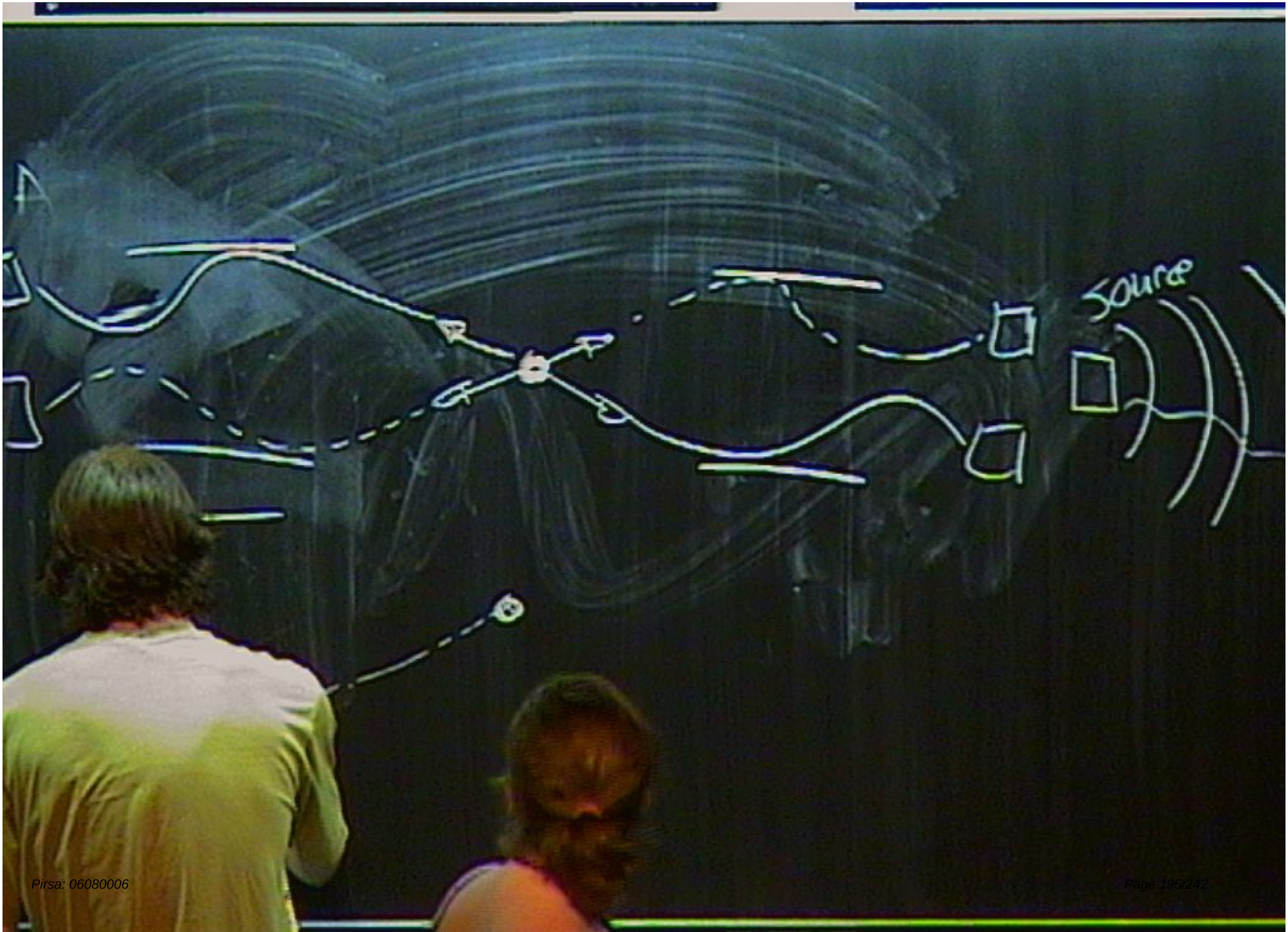

Source
A
B
Q1
Q2
Q
1.
Q1
Q2
Q3



1. Same question ⇒ Same answer

2. # Agree = # Disagree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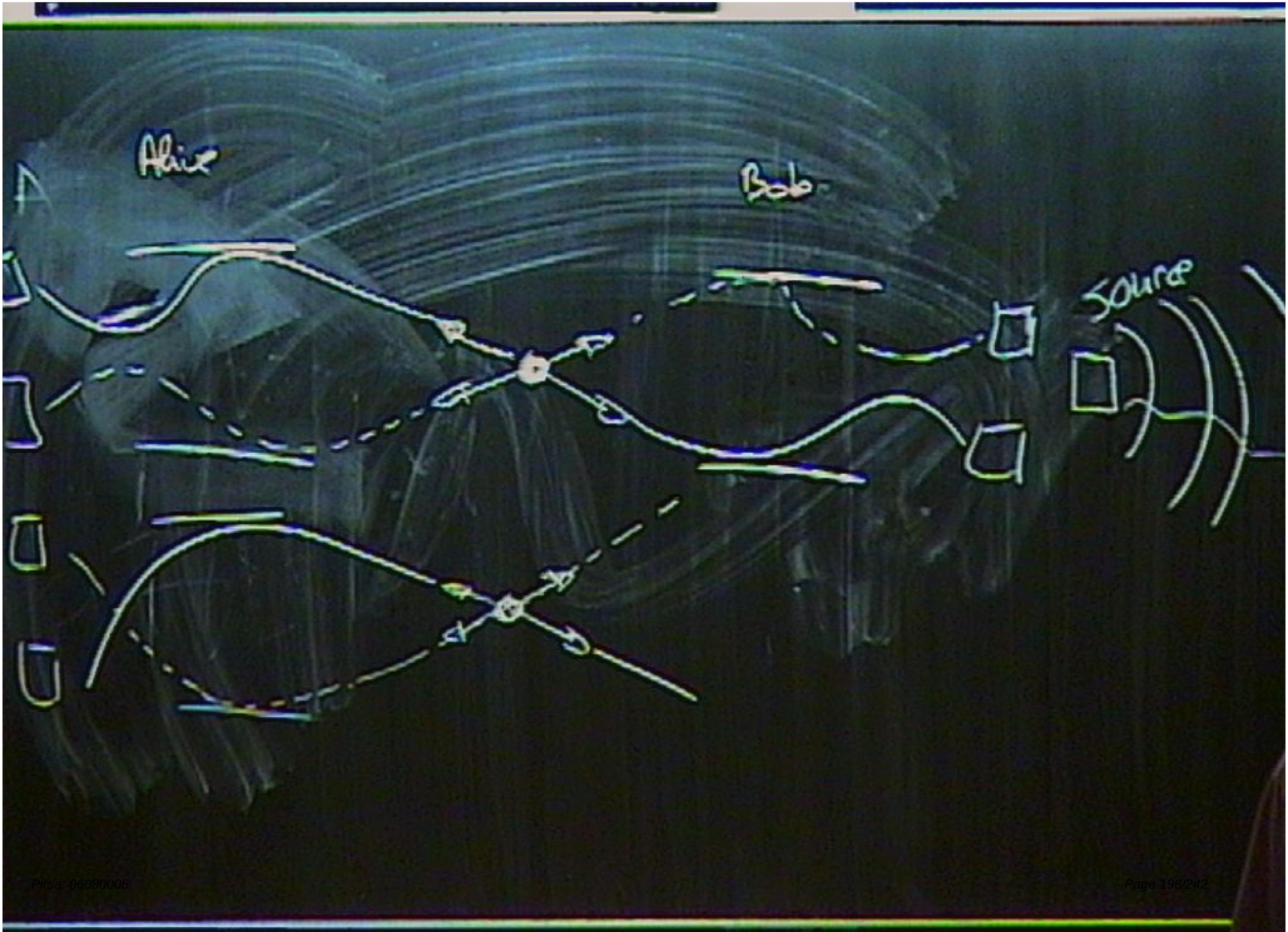
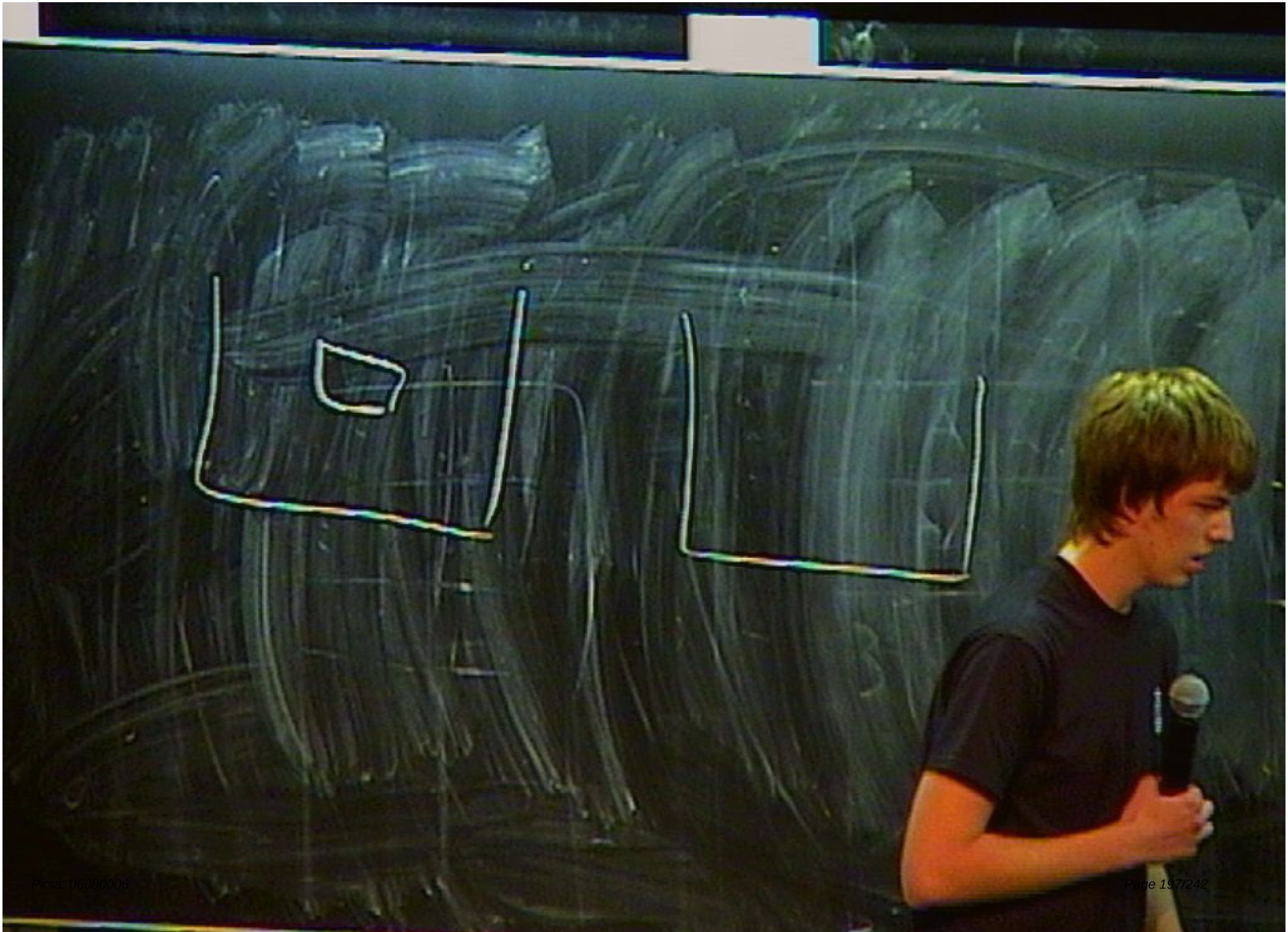
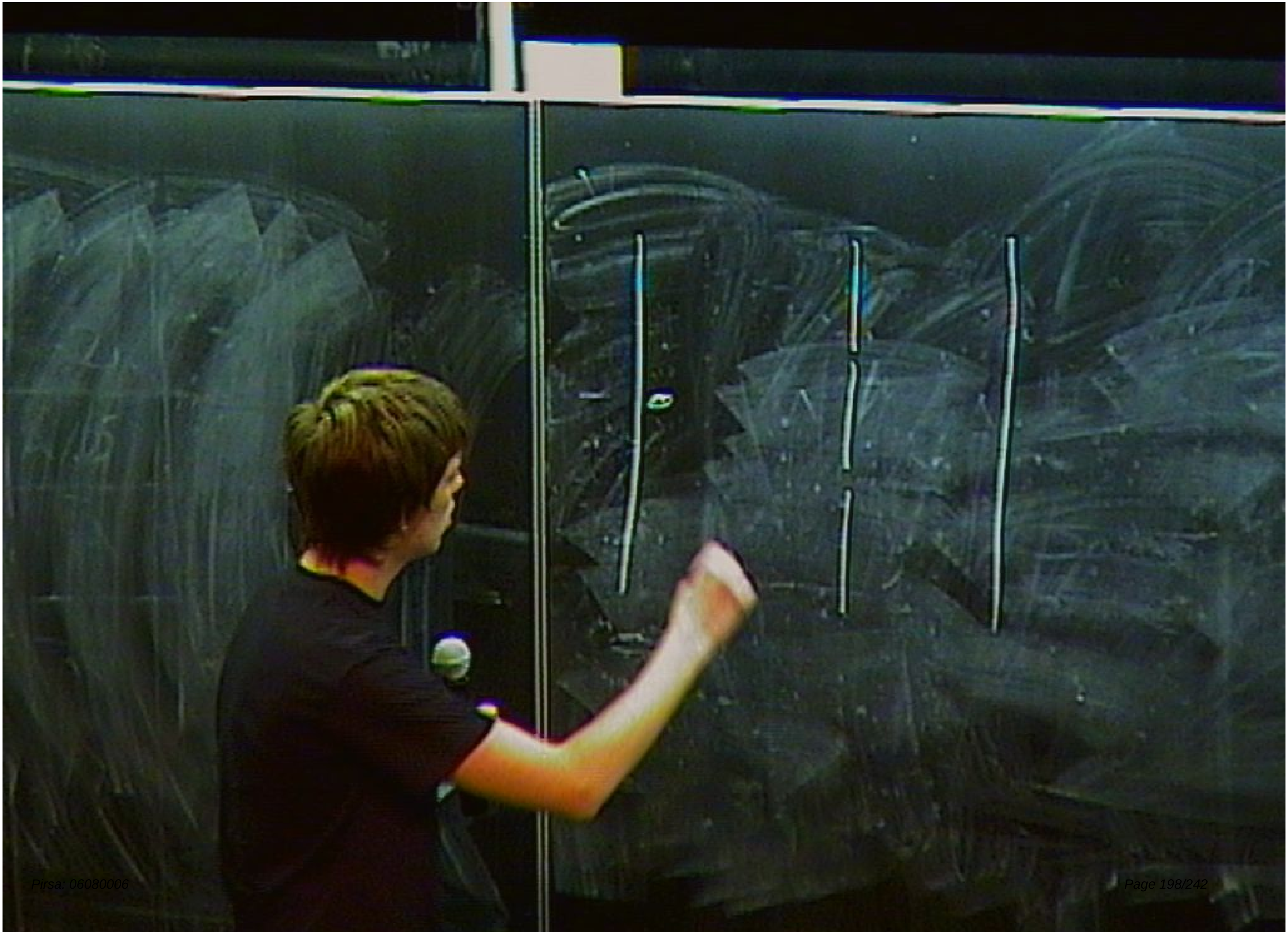
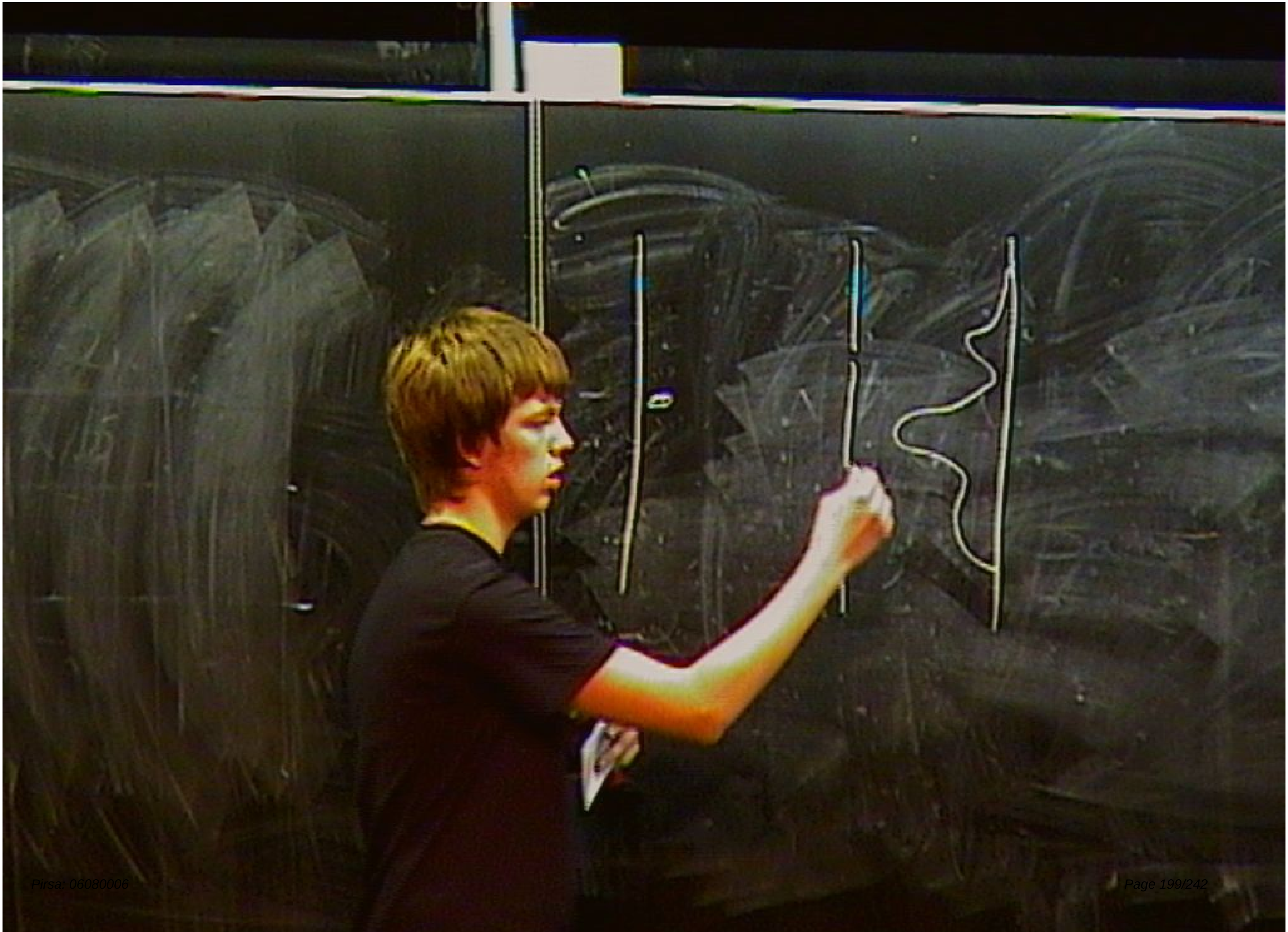
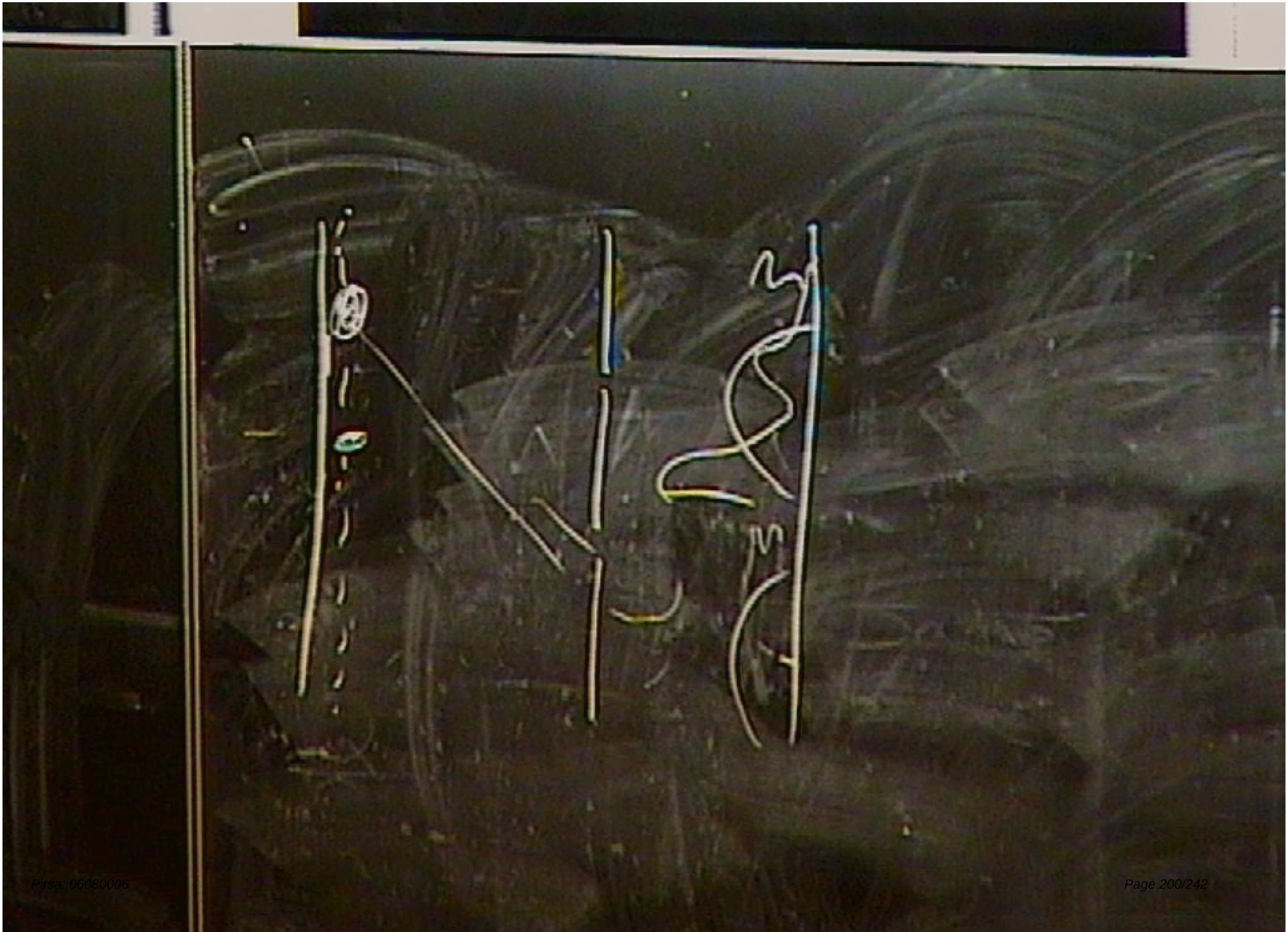
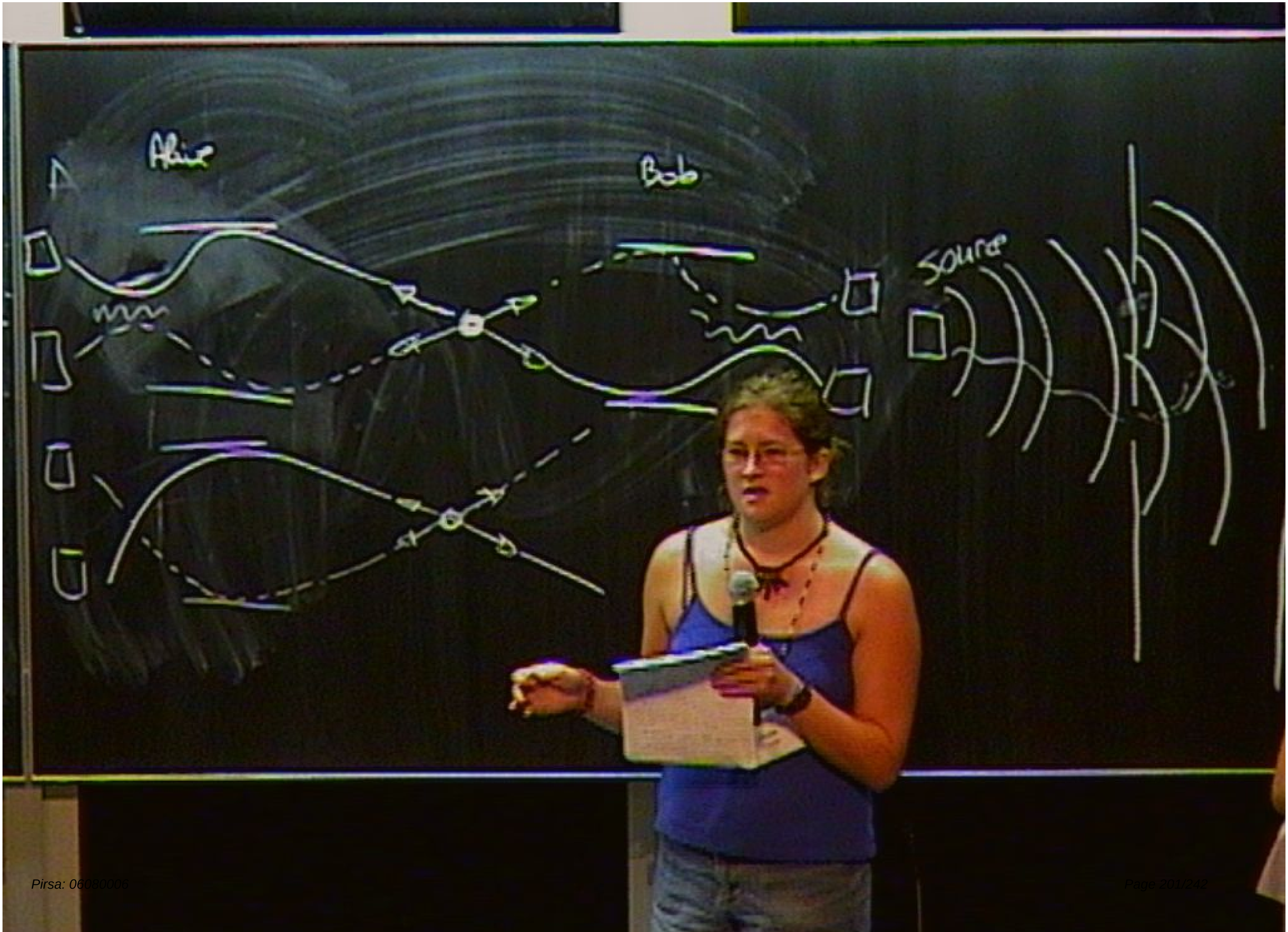

Alice
Bob
Source

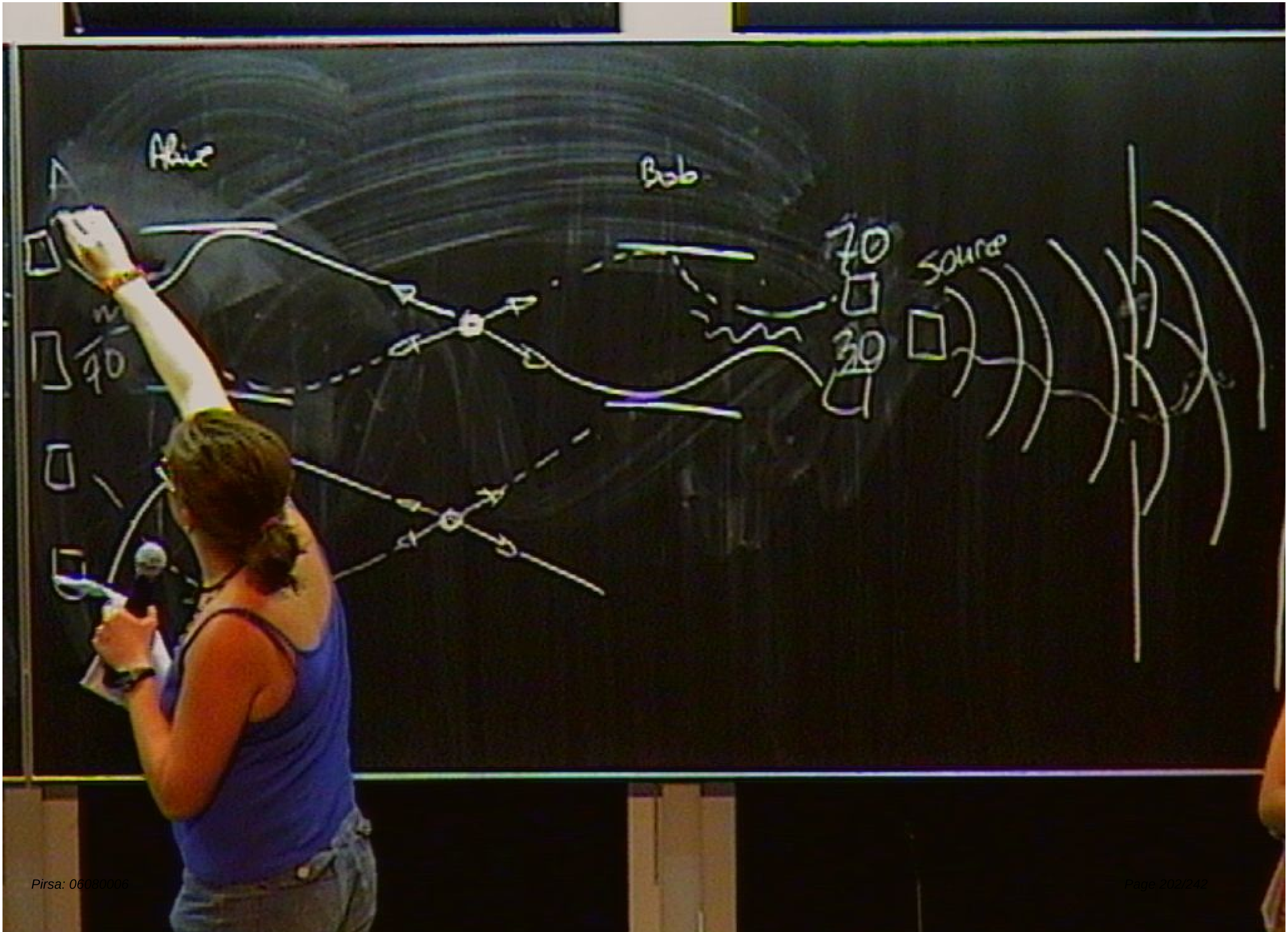




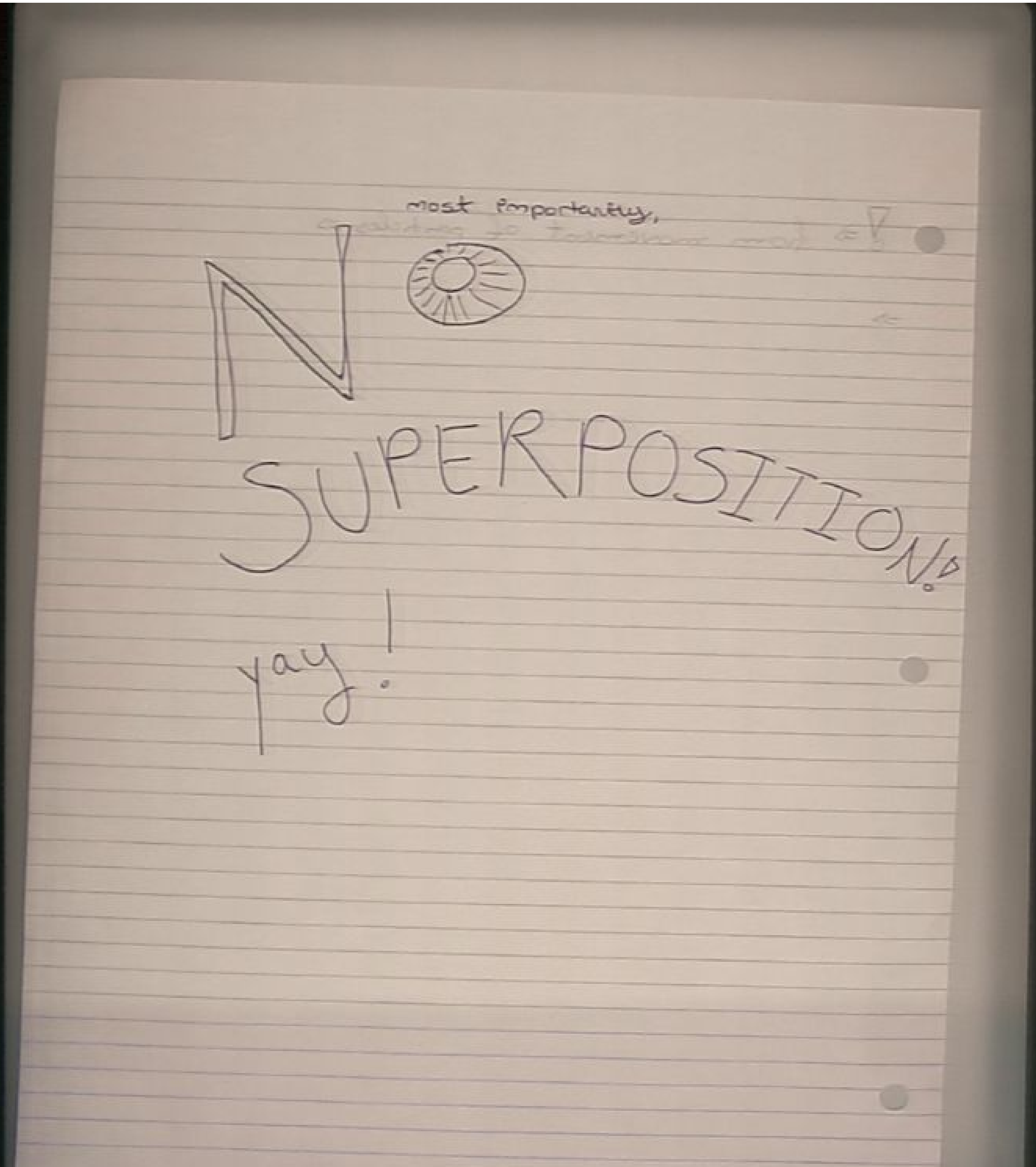





A
Alice
Bob
Source


A
Alice
Bob
70
source
30
70


Alice
Bob
30
70
70
30
Source

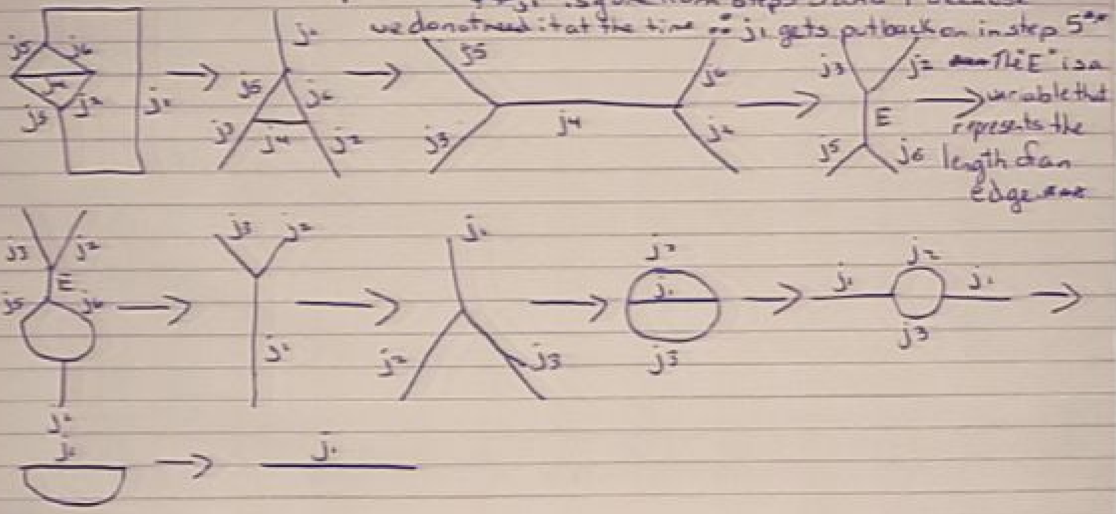


most importantly,

NO SUPERPOSITION!

yay!

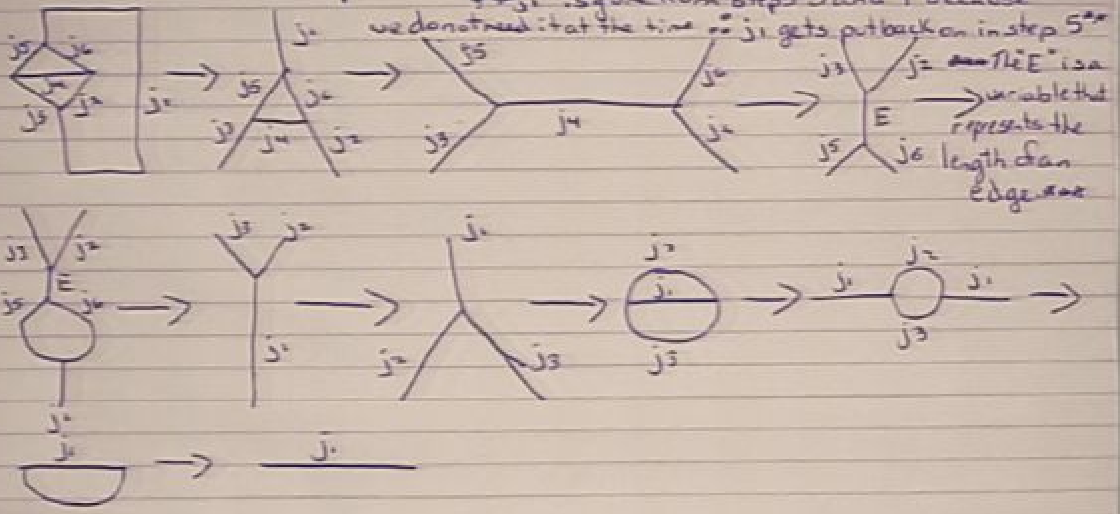
6 J Symbols



j1 is gone from steps 3 and 4 because we do not need it at the time ** j1 gets put back on in step 5

The "E" is a variable that represents the length of an edge.

6 J Symbols

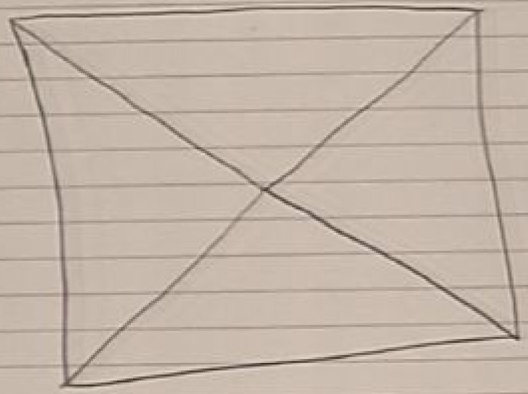


j_1 is gone from steps 3 and 4 because we do not need it at the time ** j_1 gets put back on in step 5

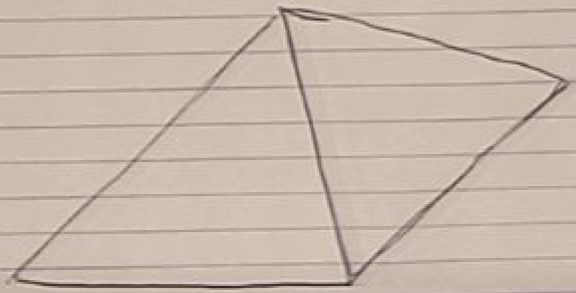
** The "E" is a variable that represents the length of an edge **

Tetrahedron

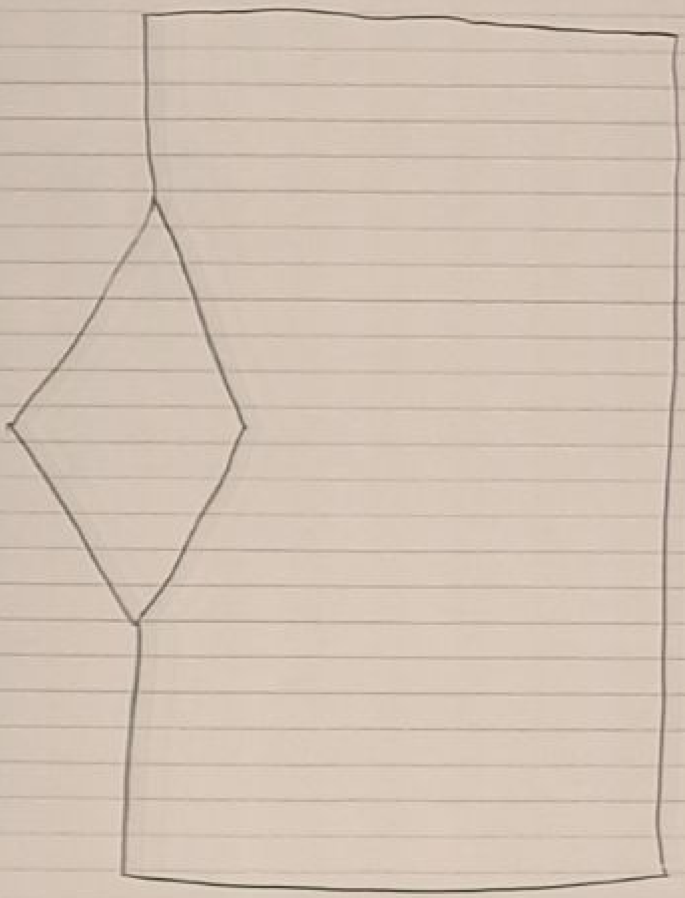
Top



Side



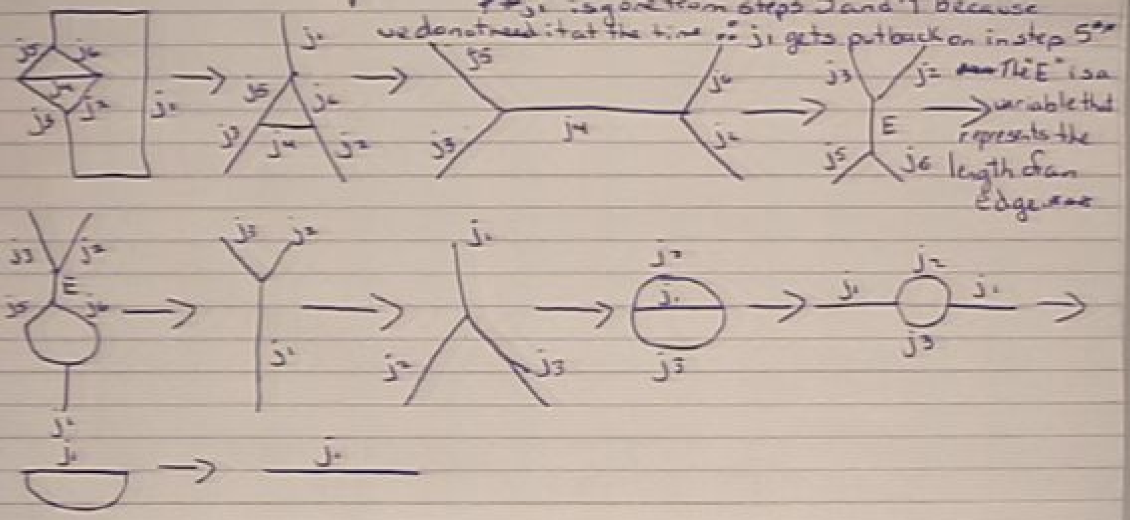
Projection of a tetrahedron

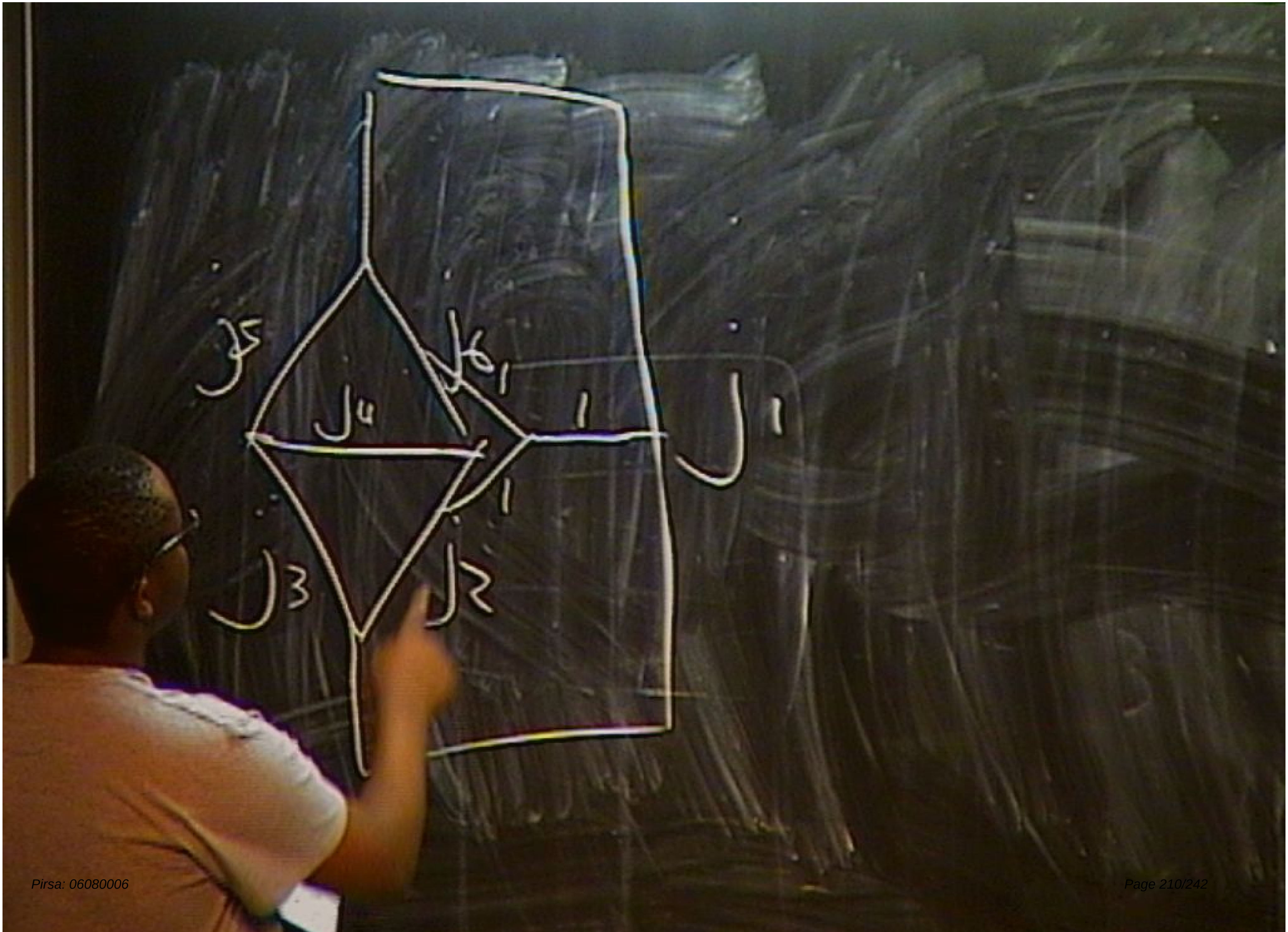


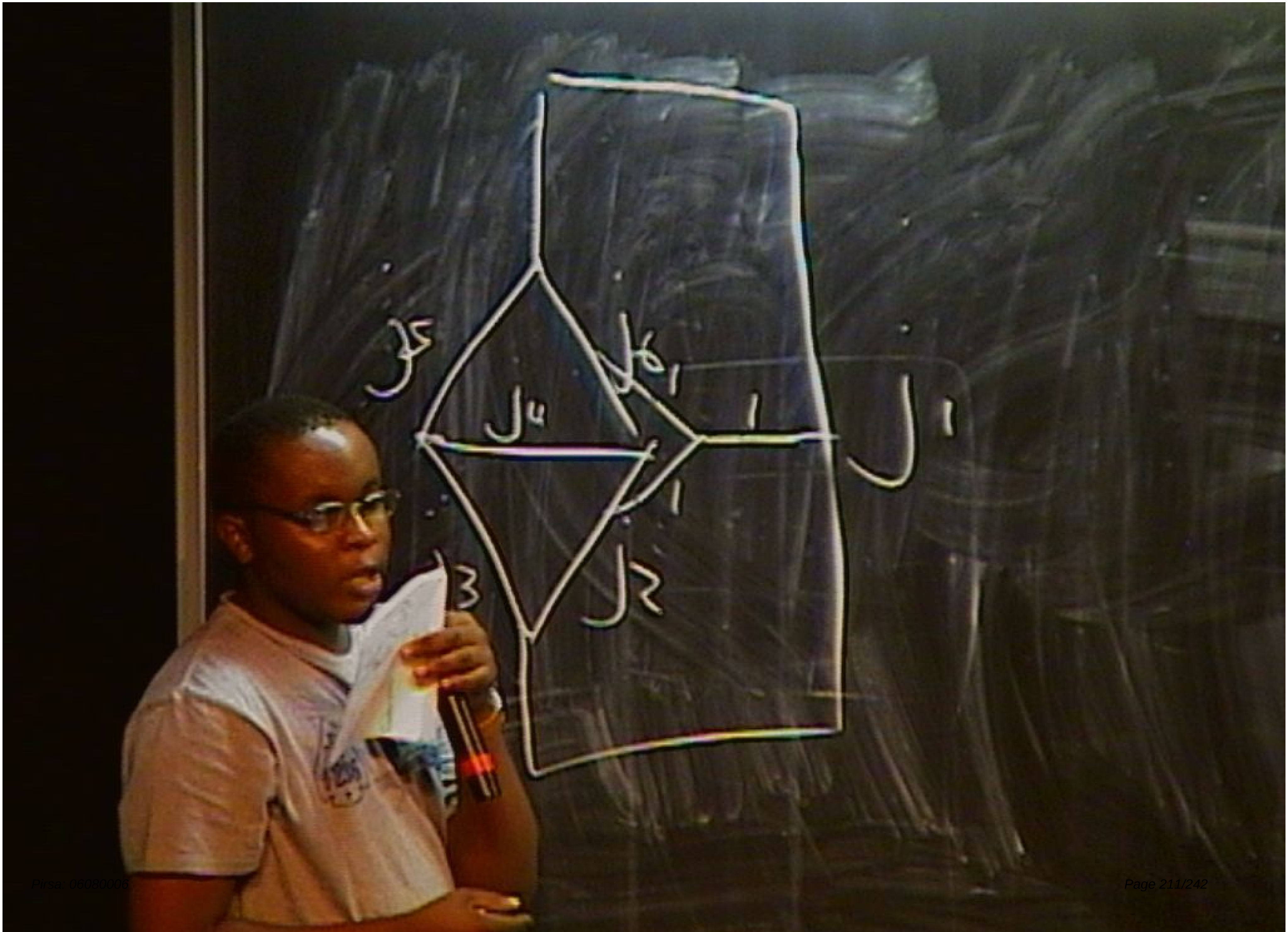
6 J Symbols

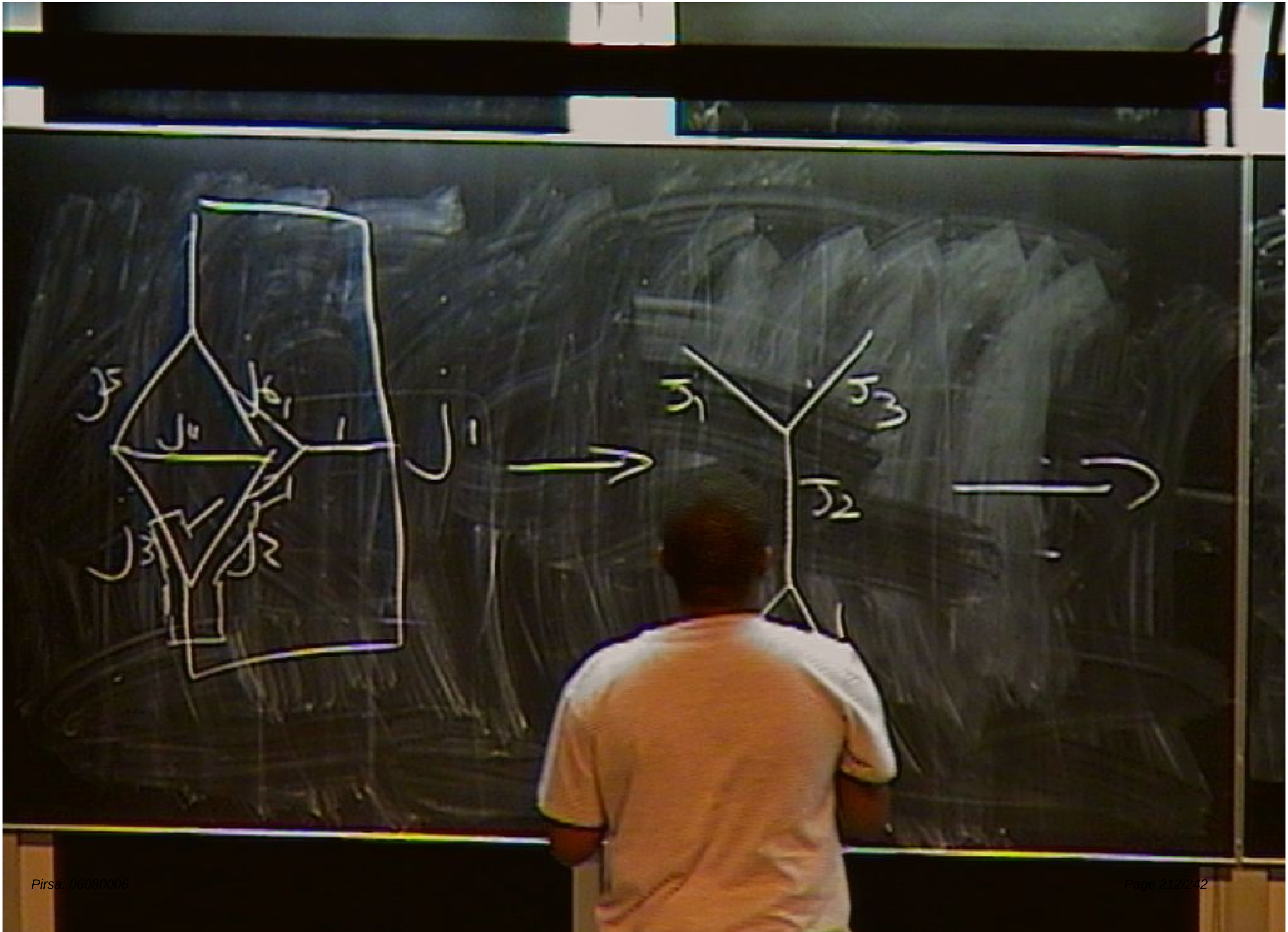
j1 is gone from steps 3 and 4 because we do not need it at the time ** j1 gets put back on in step 5

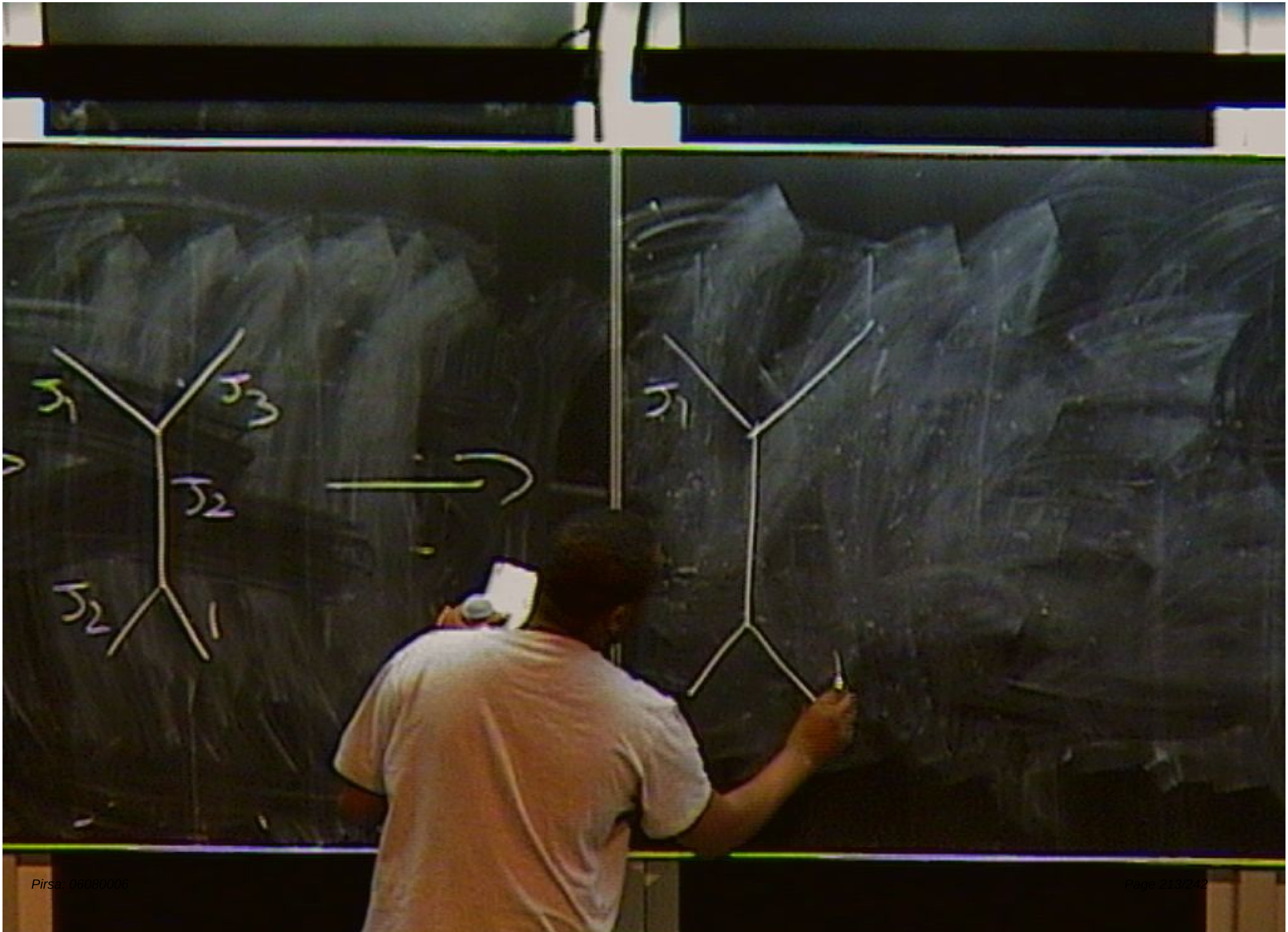
The "E" is a variable that represents the length of an Edge

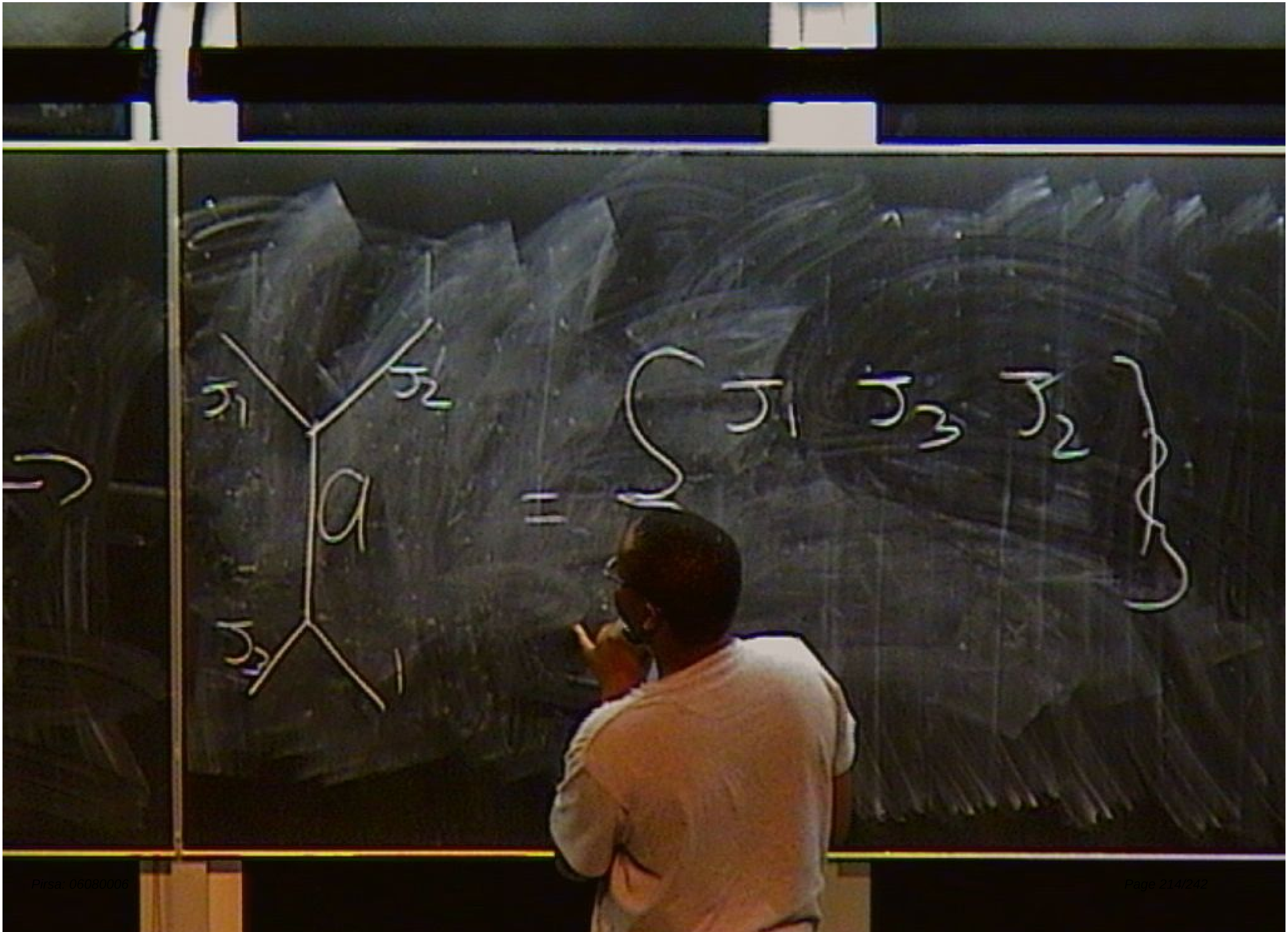


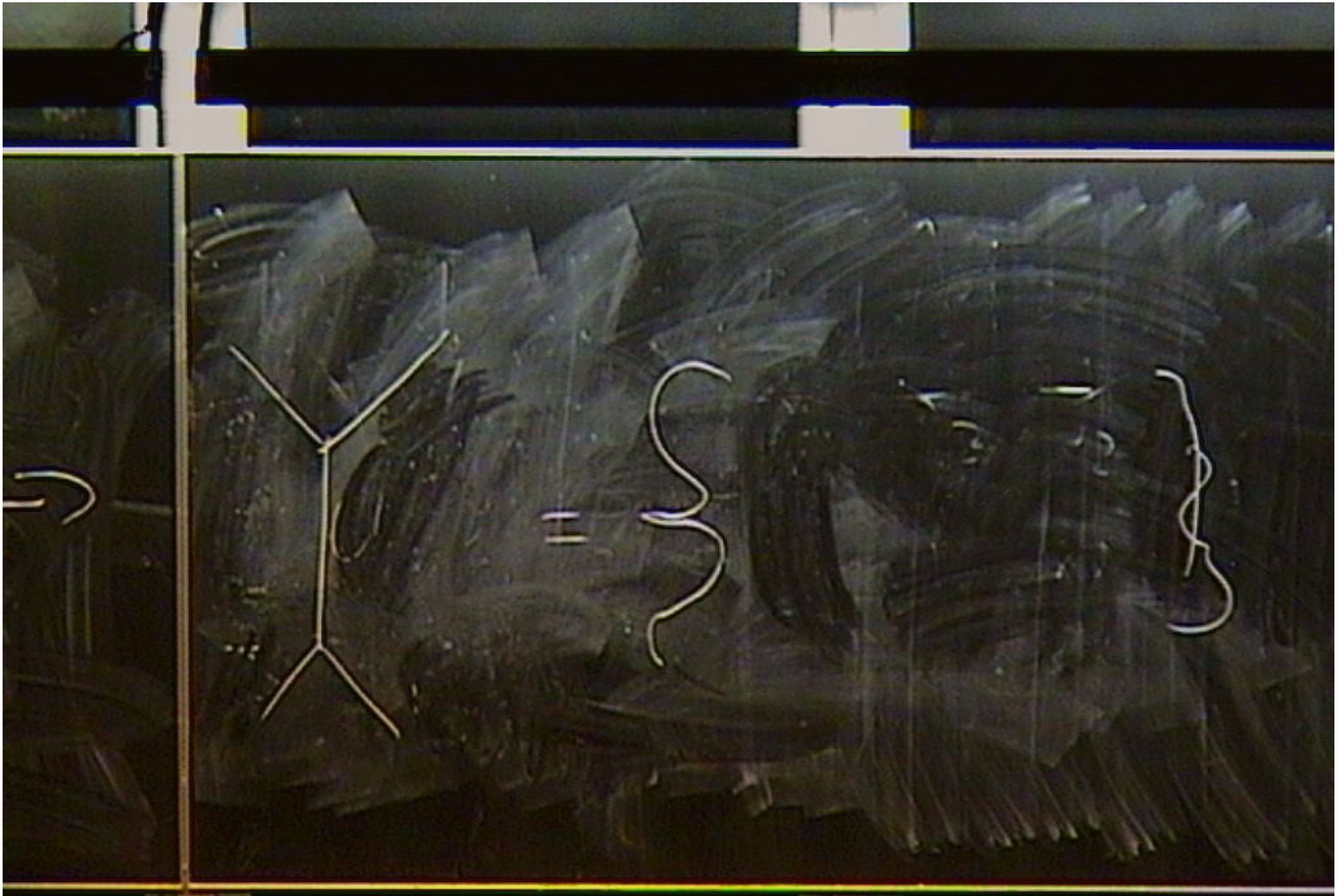

j5
j6
1
j4
1
j1
1
j3
j2

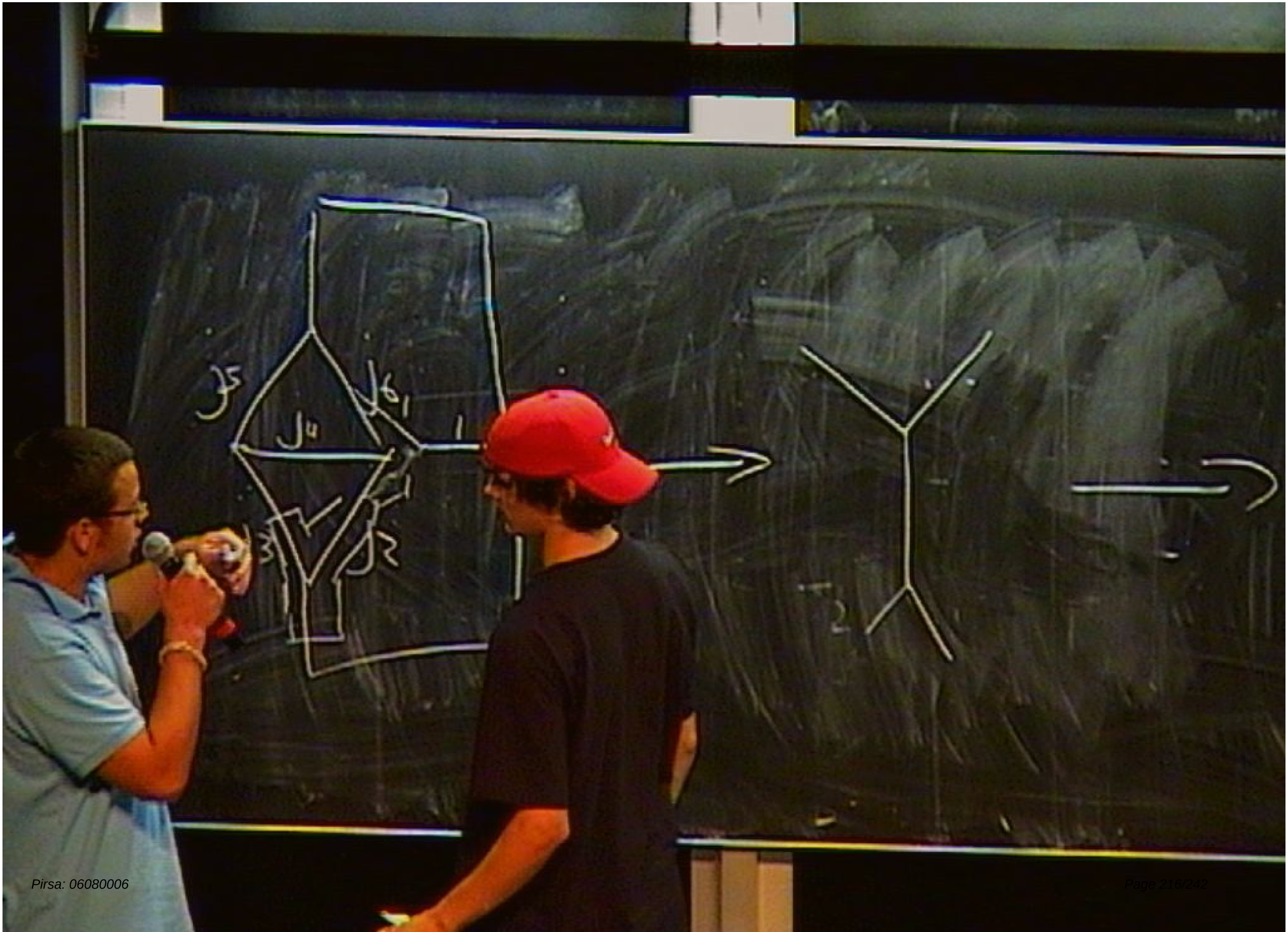

j5
j6
1
j4
1
j1
1
j3
j2

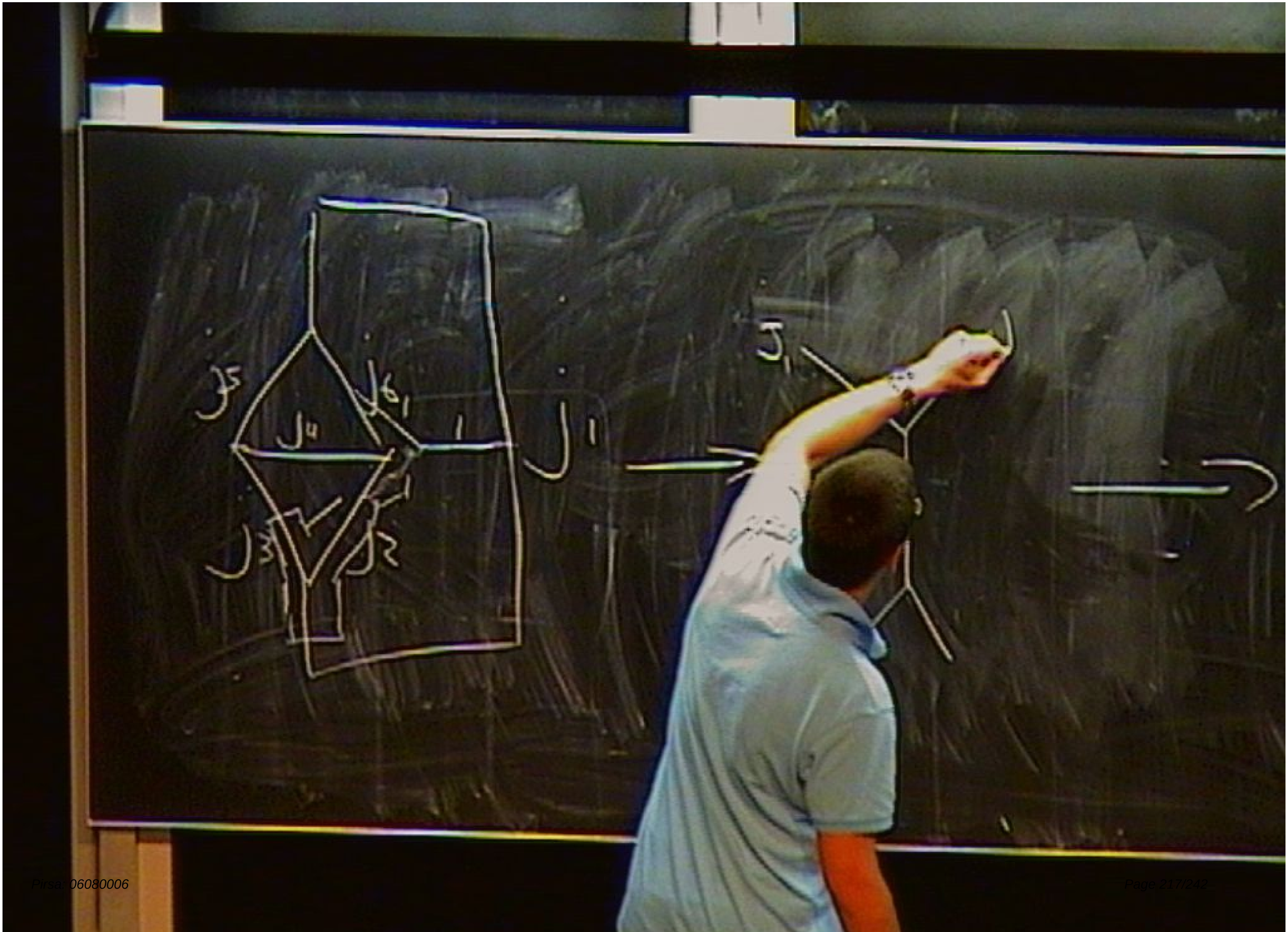

j5
j6, 1
j4
1
j1
j3
j2
j1
j3
j2
1

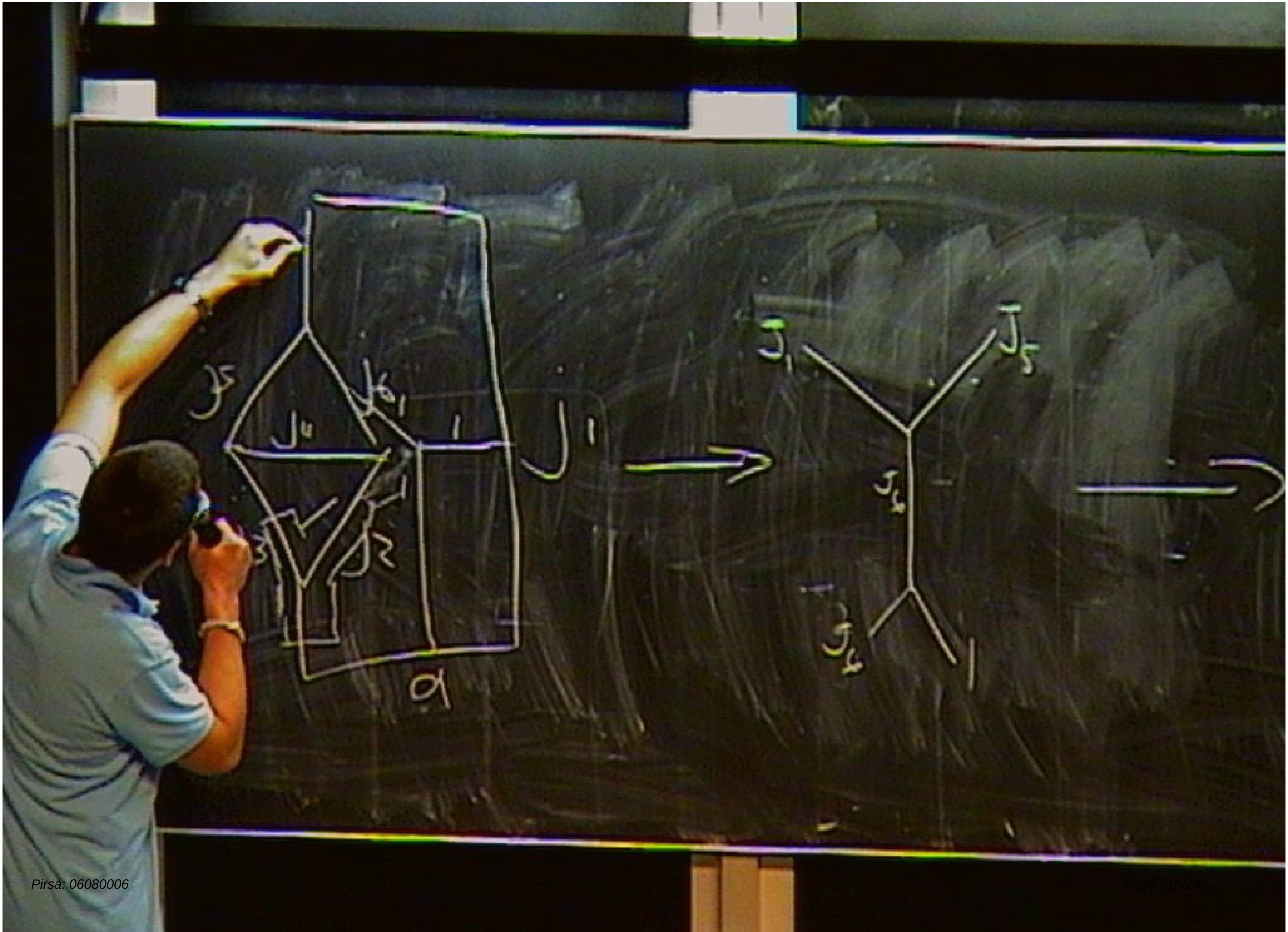

j_1
j_3
j_2
j_2
1
j_1

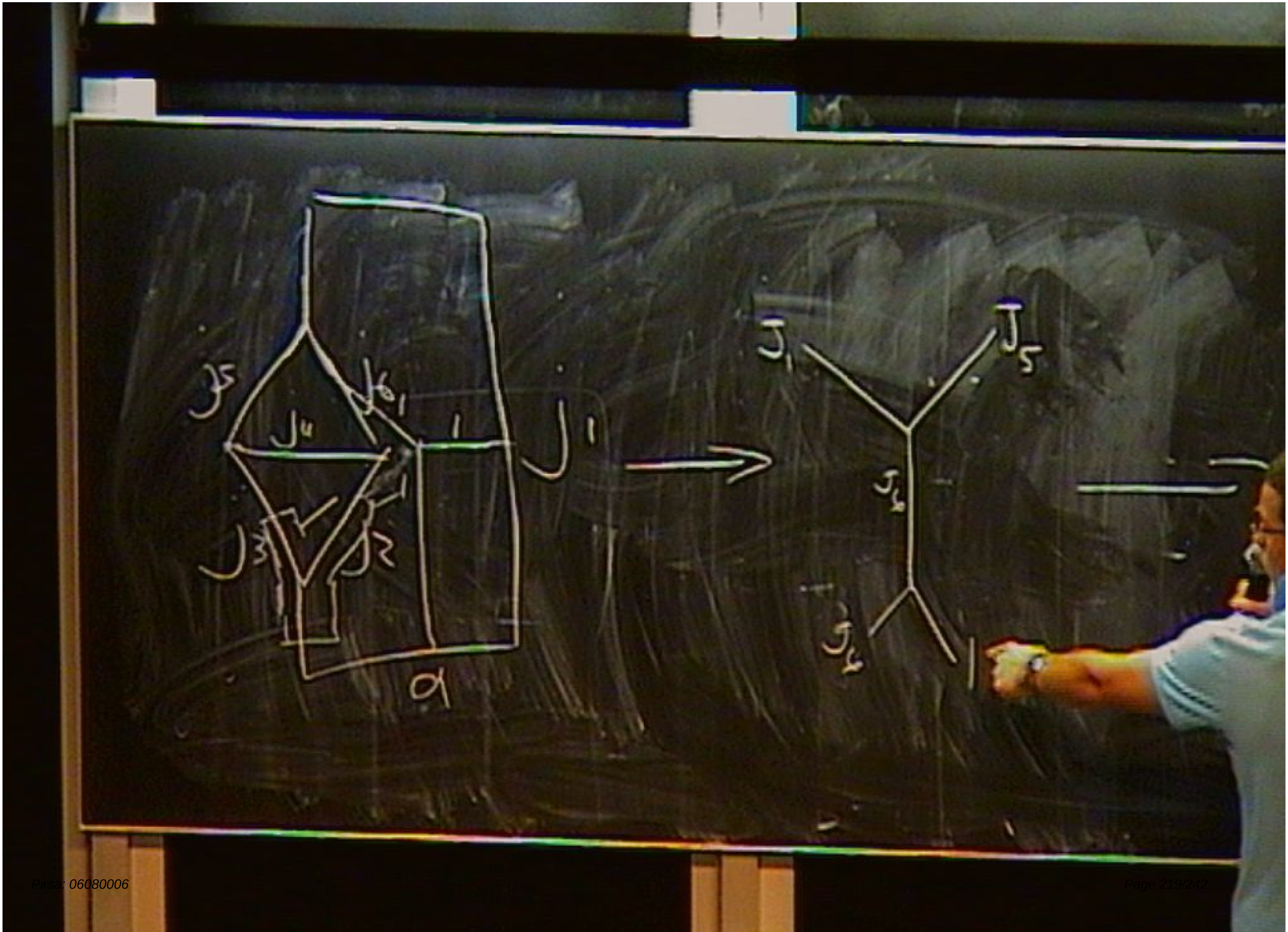

j1
j2
a
j3
1
=
{ j1 j3 j2 }

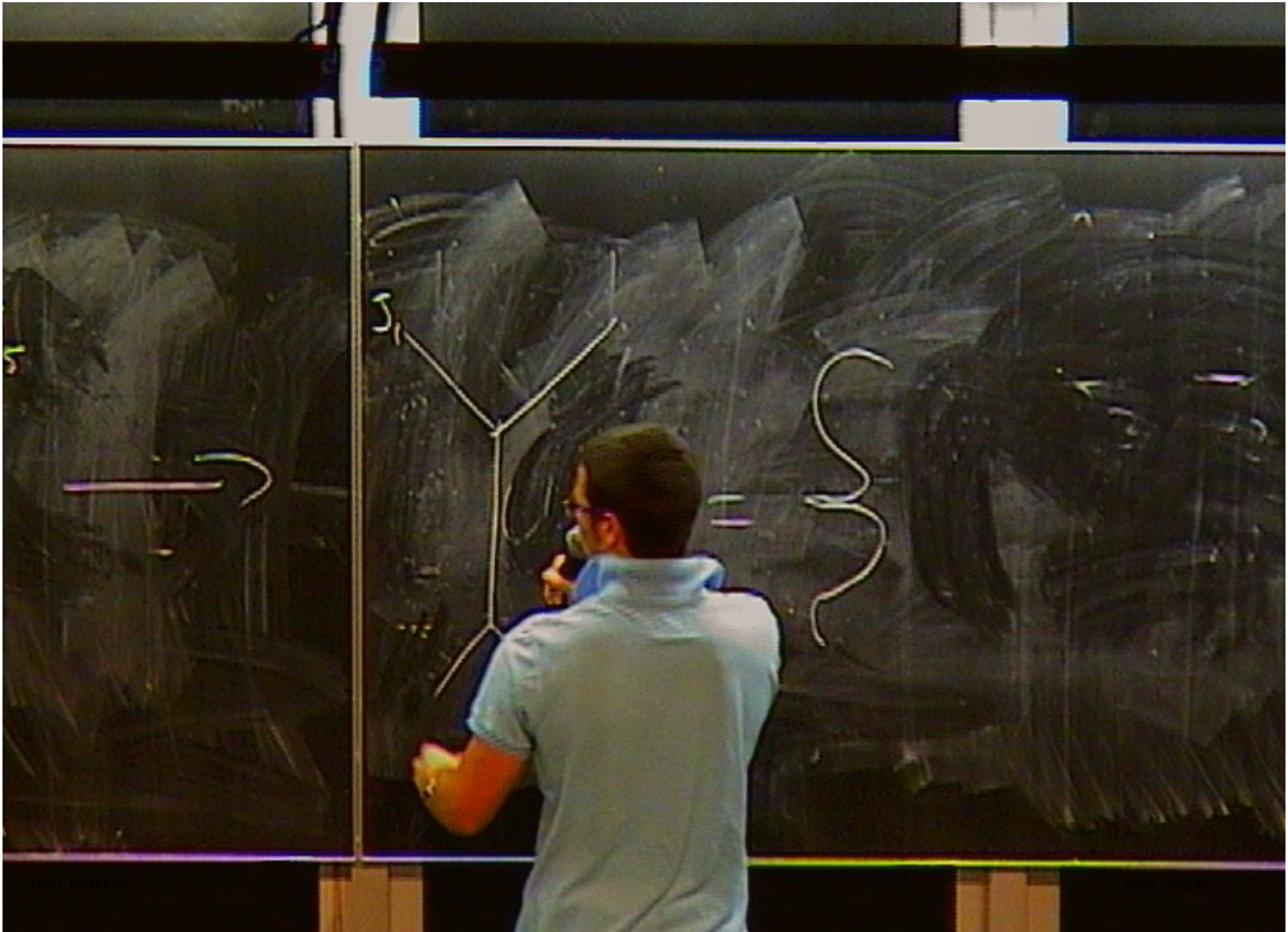


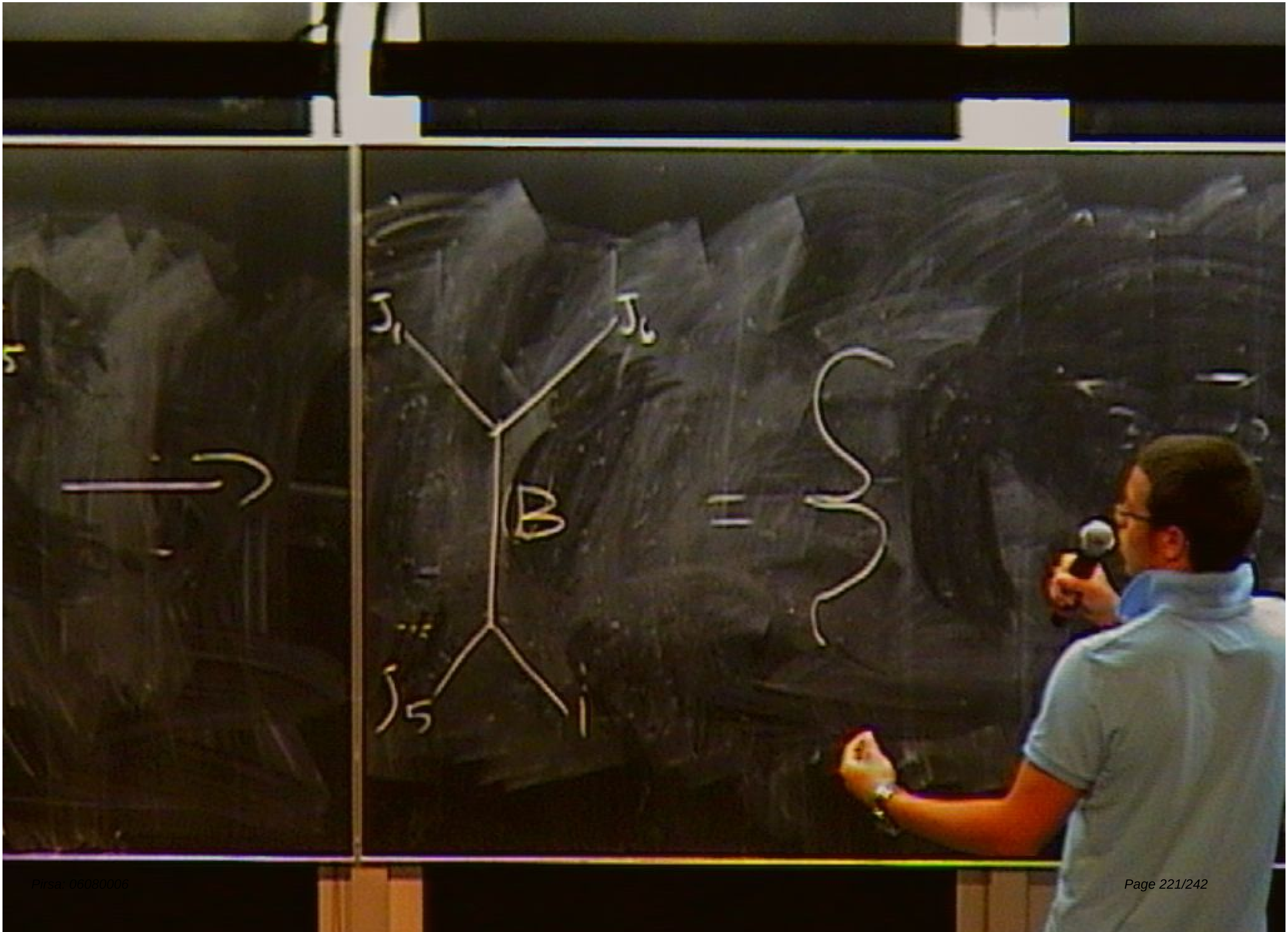

j5
j6
1
j4
j3
j2
2


j5
j6
j4
1
j1
j3
j2
j1

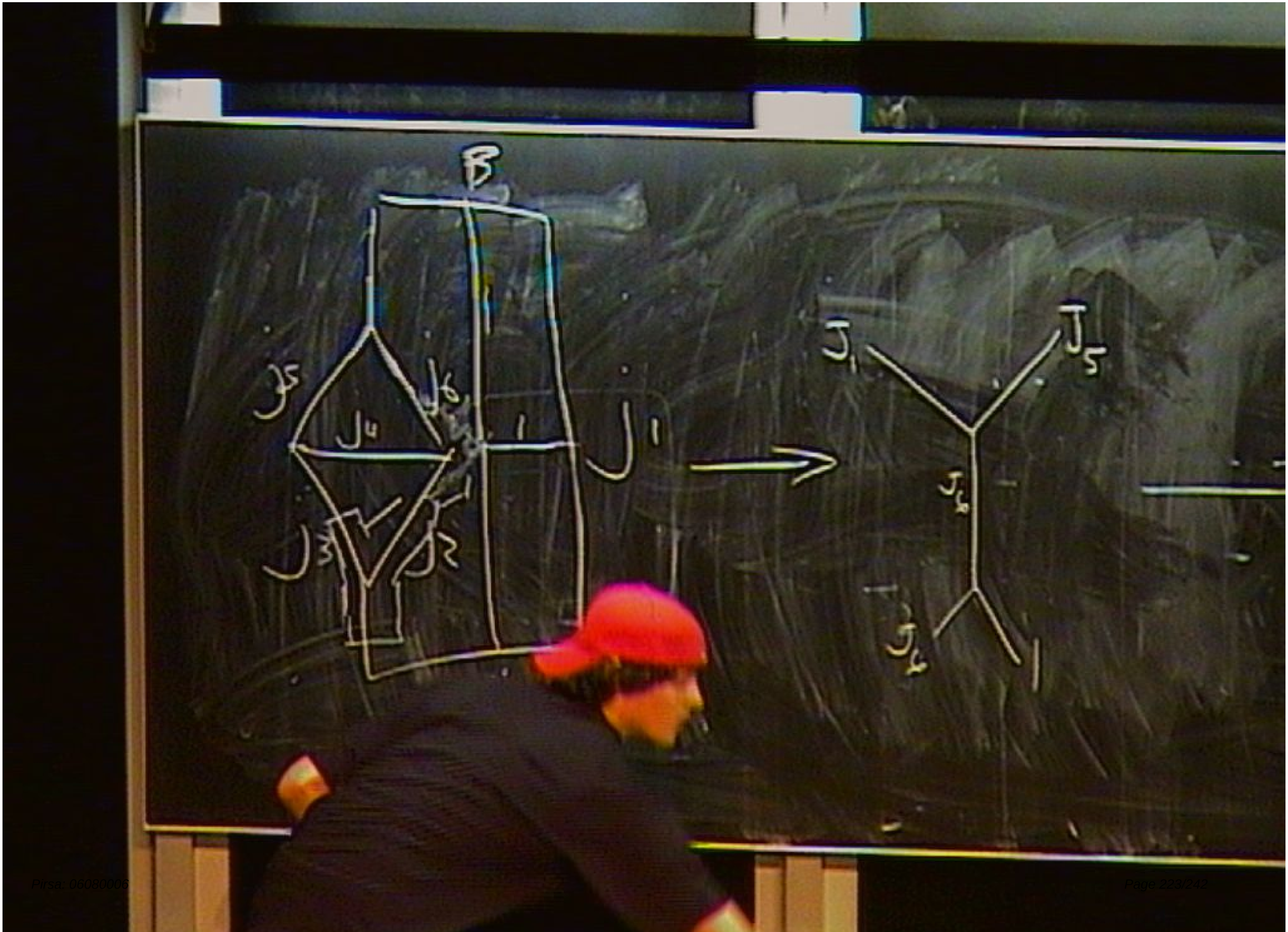

j5
j6, 1
j4
1
j1
j3
j2
a
j1
j5
j4
j6
1

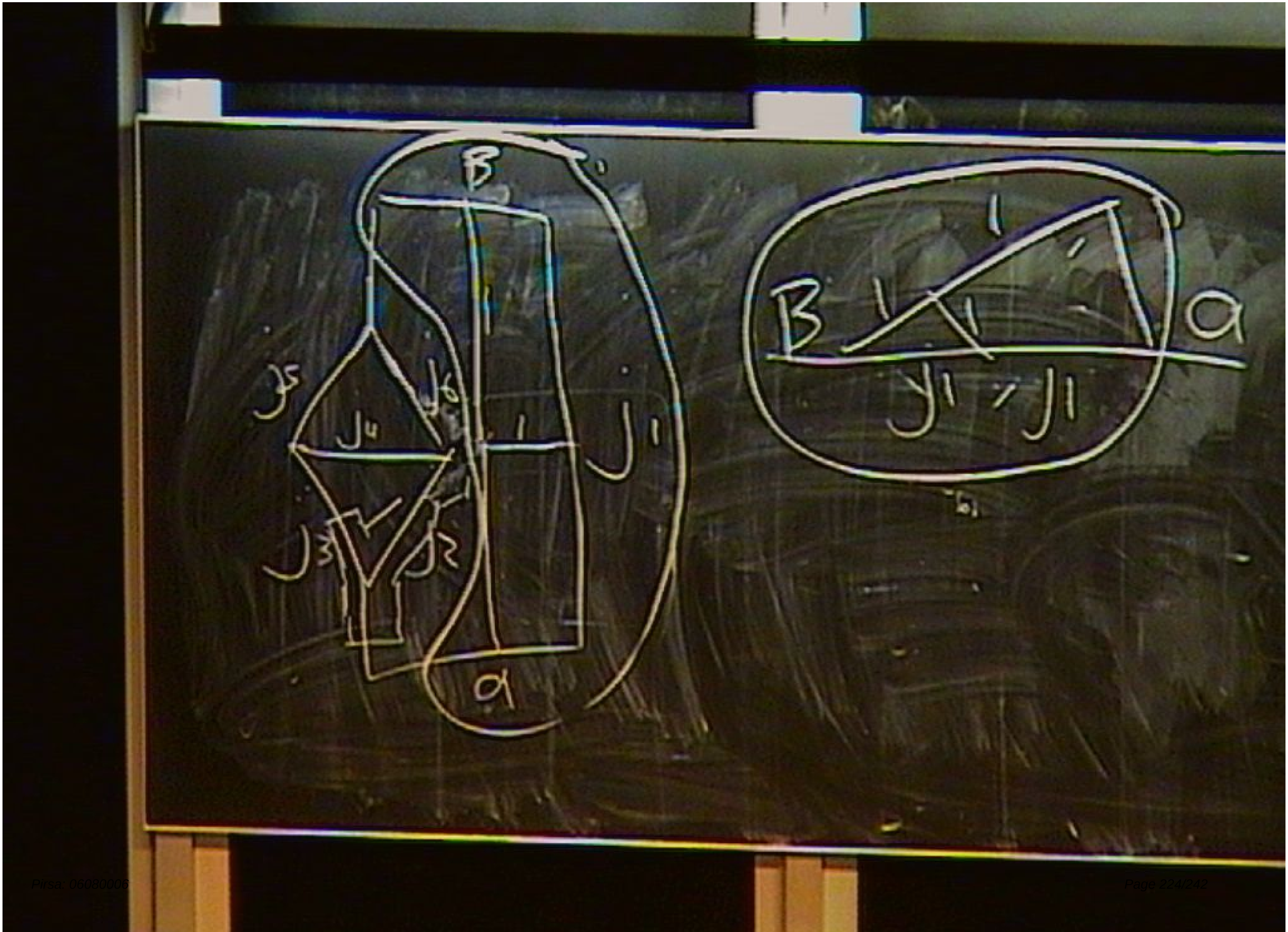

j5
j6, 1
1
j4
j1
j3
j2
α
j1
j5
j6
j6
1

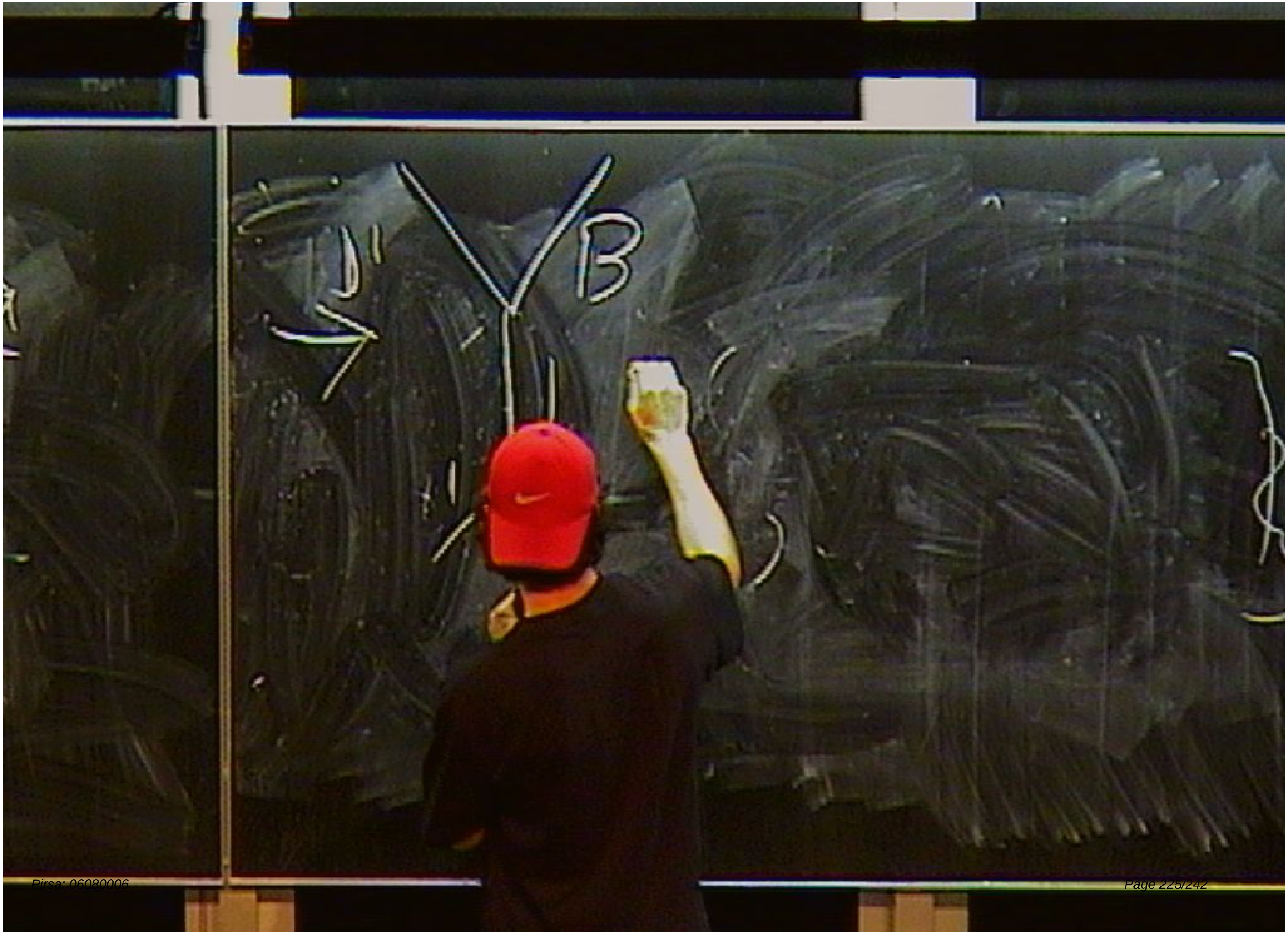

J1

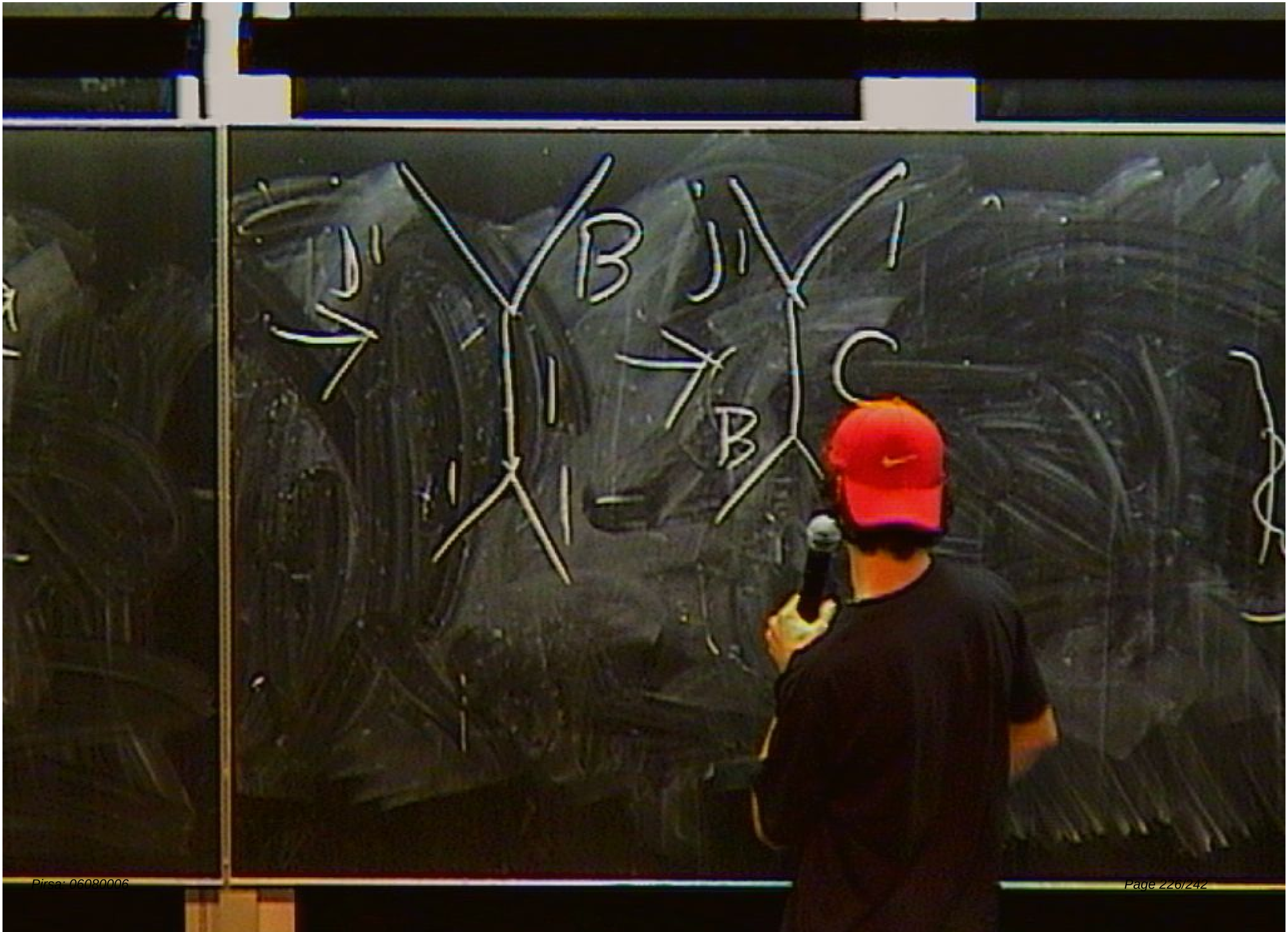

j1
j6
B
=
j5
i

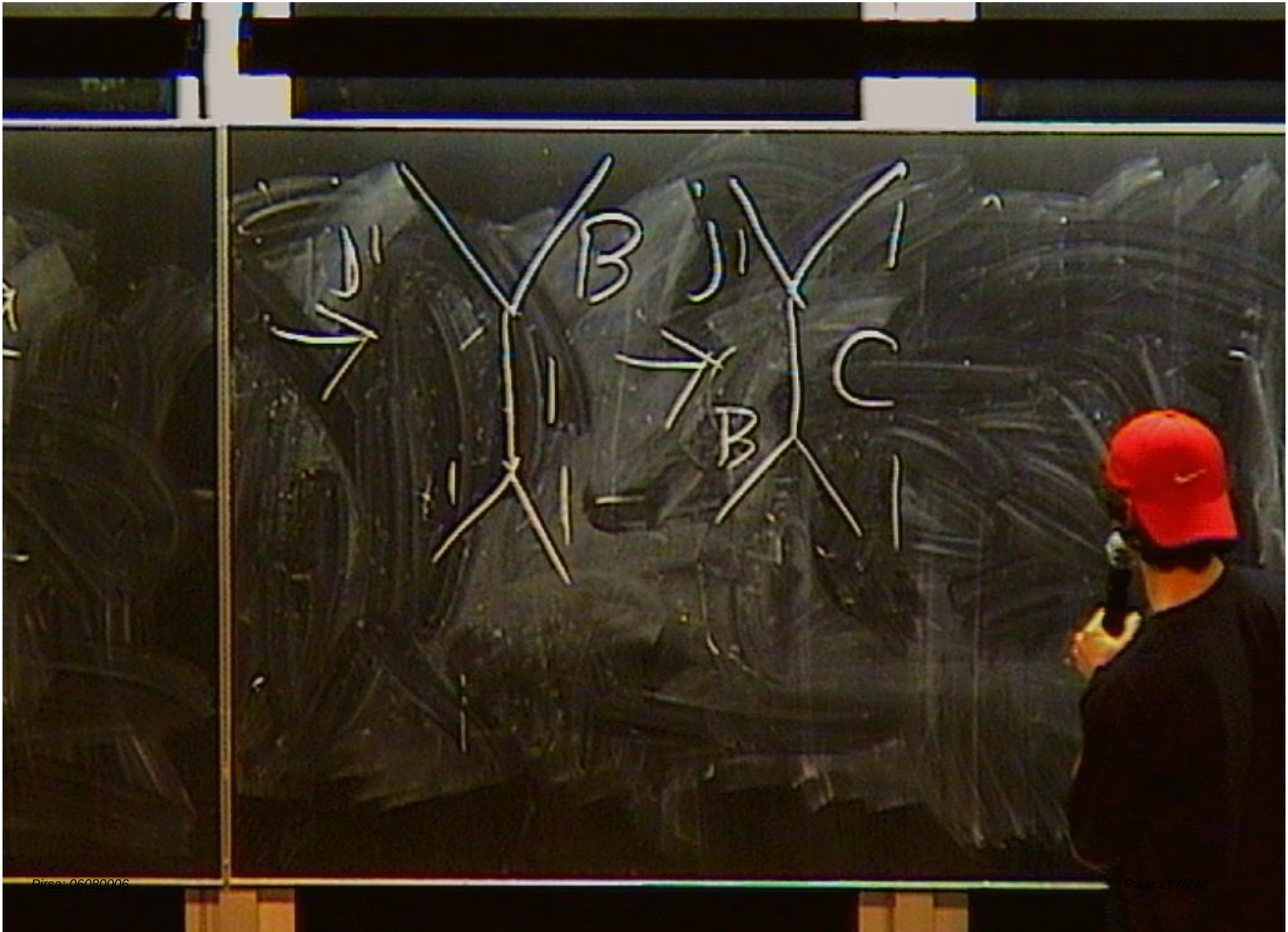
$$= \left\{ \begin{matrix} J_1 & J_5 & J_6 \\ J_6 & 1 & B \end{matrix} \right\}$$

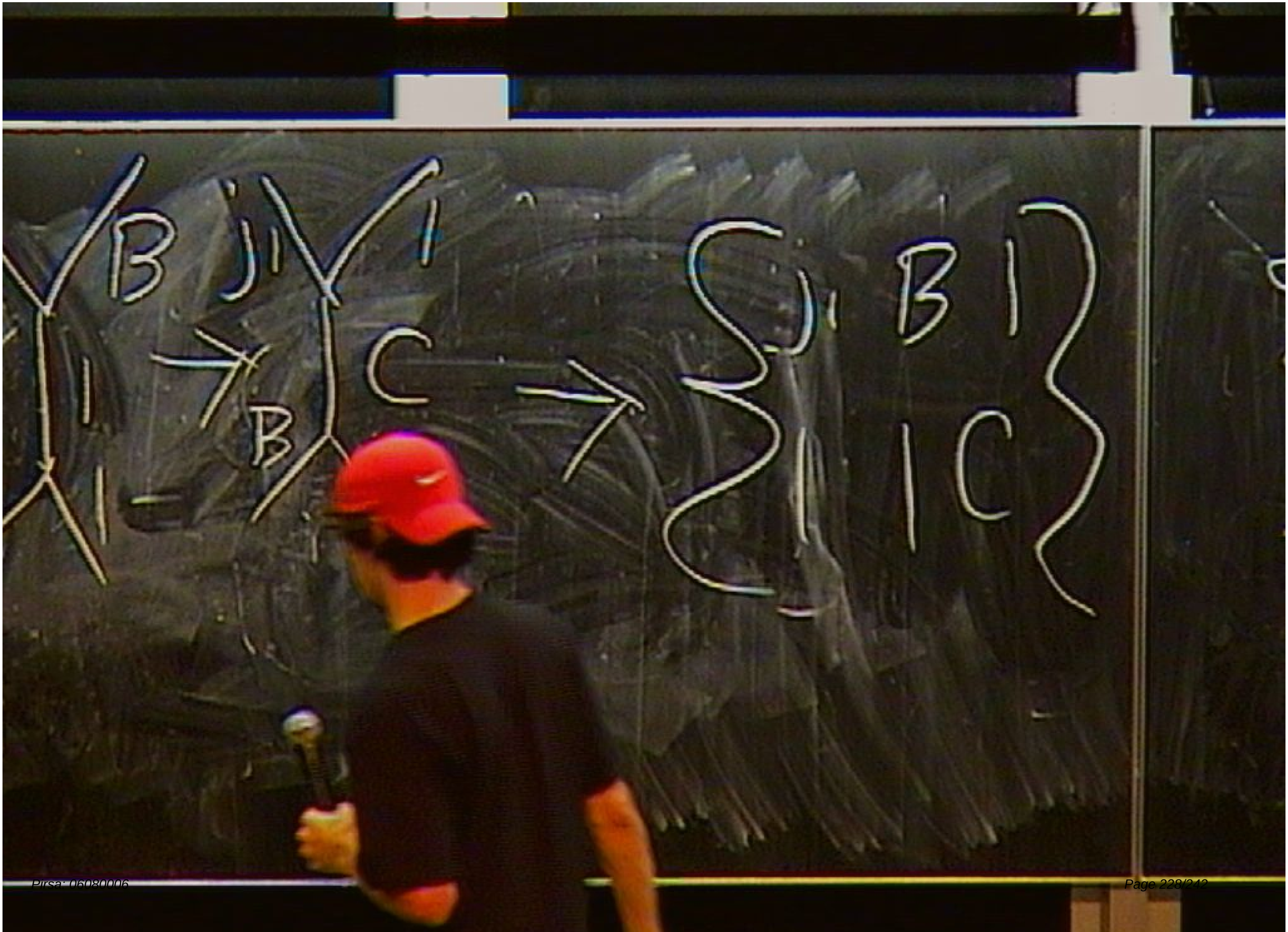

B
J5
J6
J4
1
J1
J3
J2
J1
J5
J6
J2
1


B
a
j5
j6
j4
1
j1
j3
j2
B
1
a
j1
j1


B


j'
B
i
i
i
j'
i
B
C


j'
B
1
1
1
j'
1
C
B
1

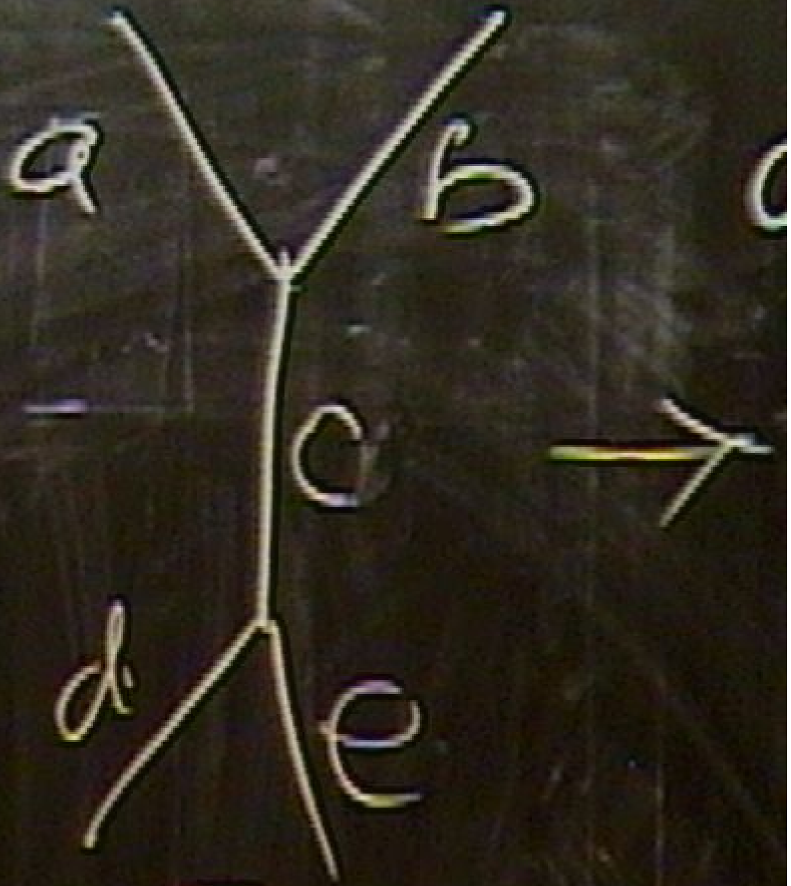

B
j
i
B
C
B
1
1
C

$$\begin{Bmatrix} j_1 & j_3 & j_{12} \\ j_2 & l & a \end{Bmatrix}$$

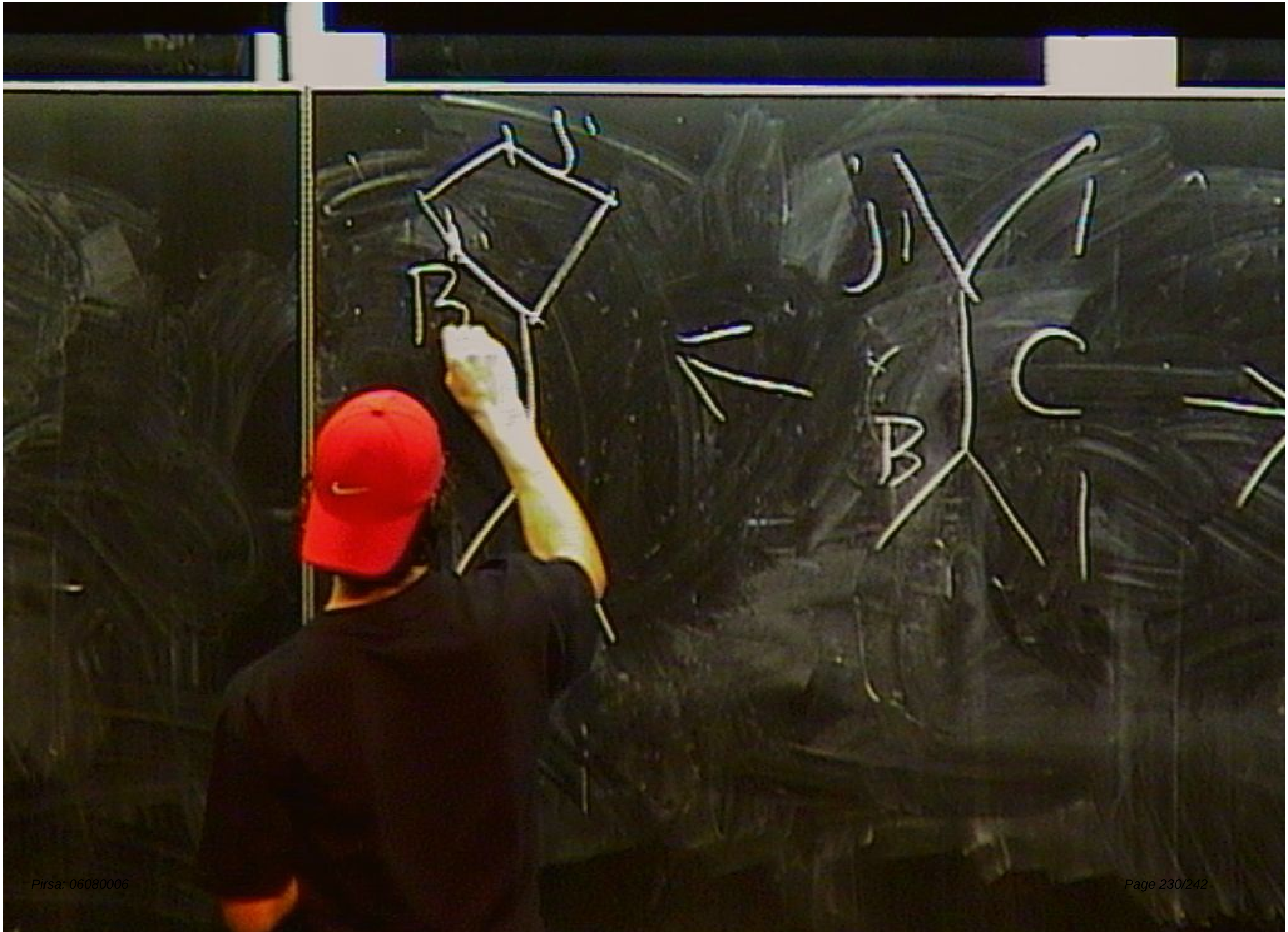
l

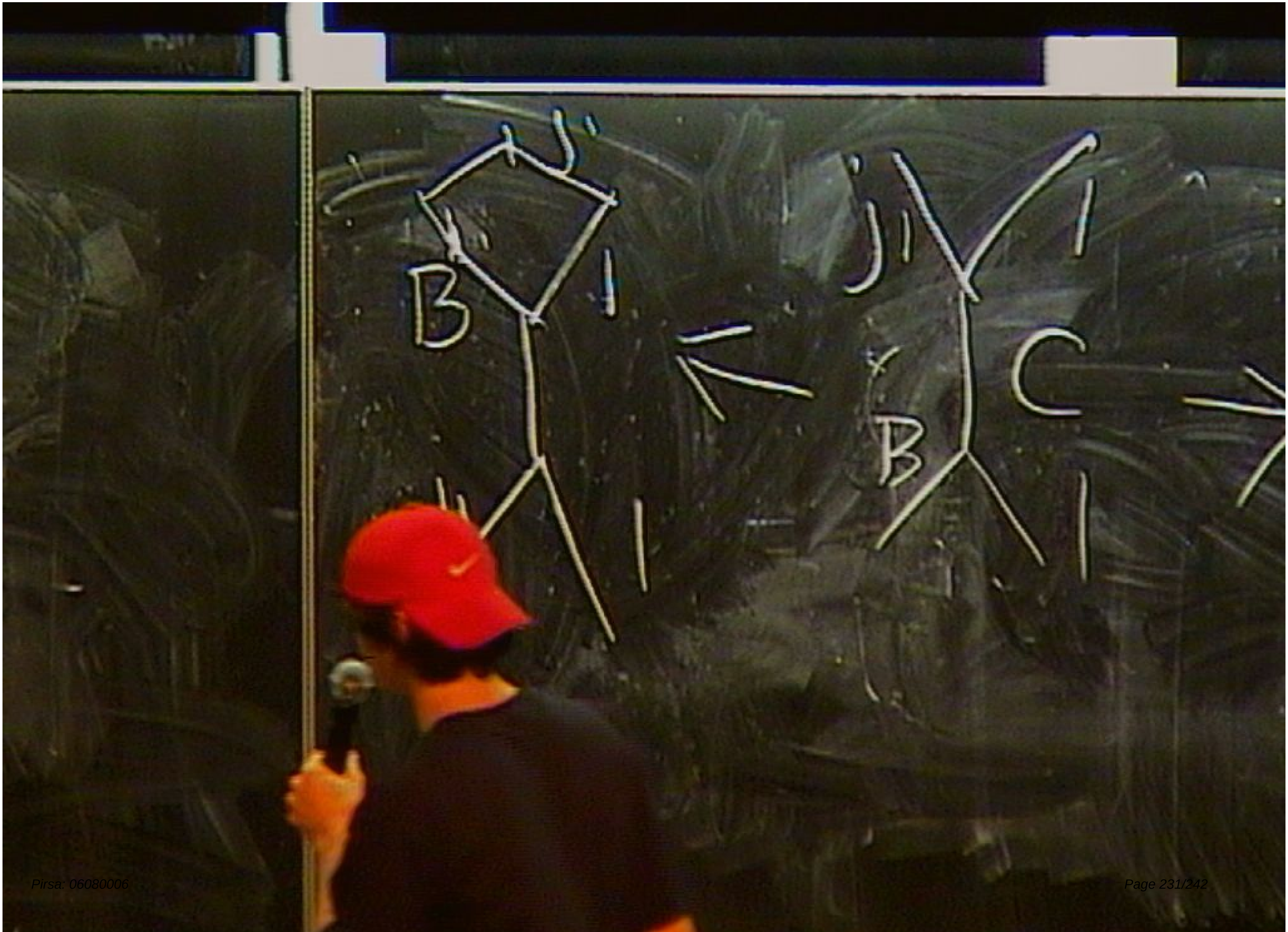
$$\begin{Bmatrix} j_1 & j_5 & j_6 \\ j_6 & l & B \end{Bmatrix}$$

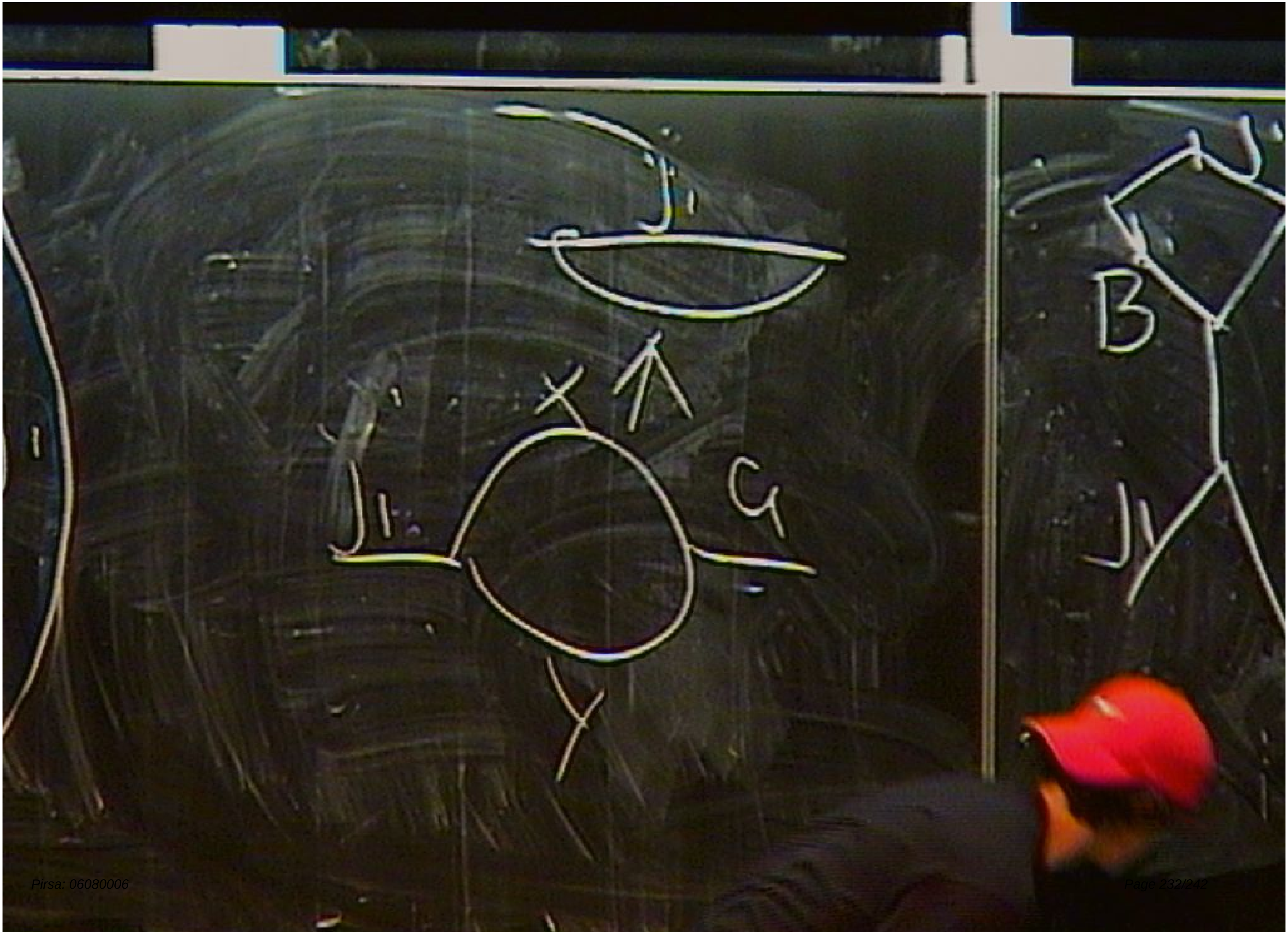
$$\begin{Bmatrix} J_1 & B & l \\ l & l & C \end{Bmatrix}$$

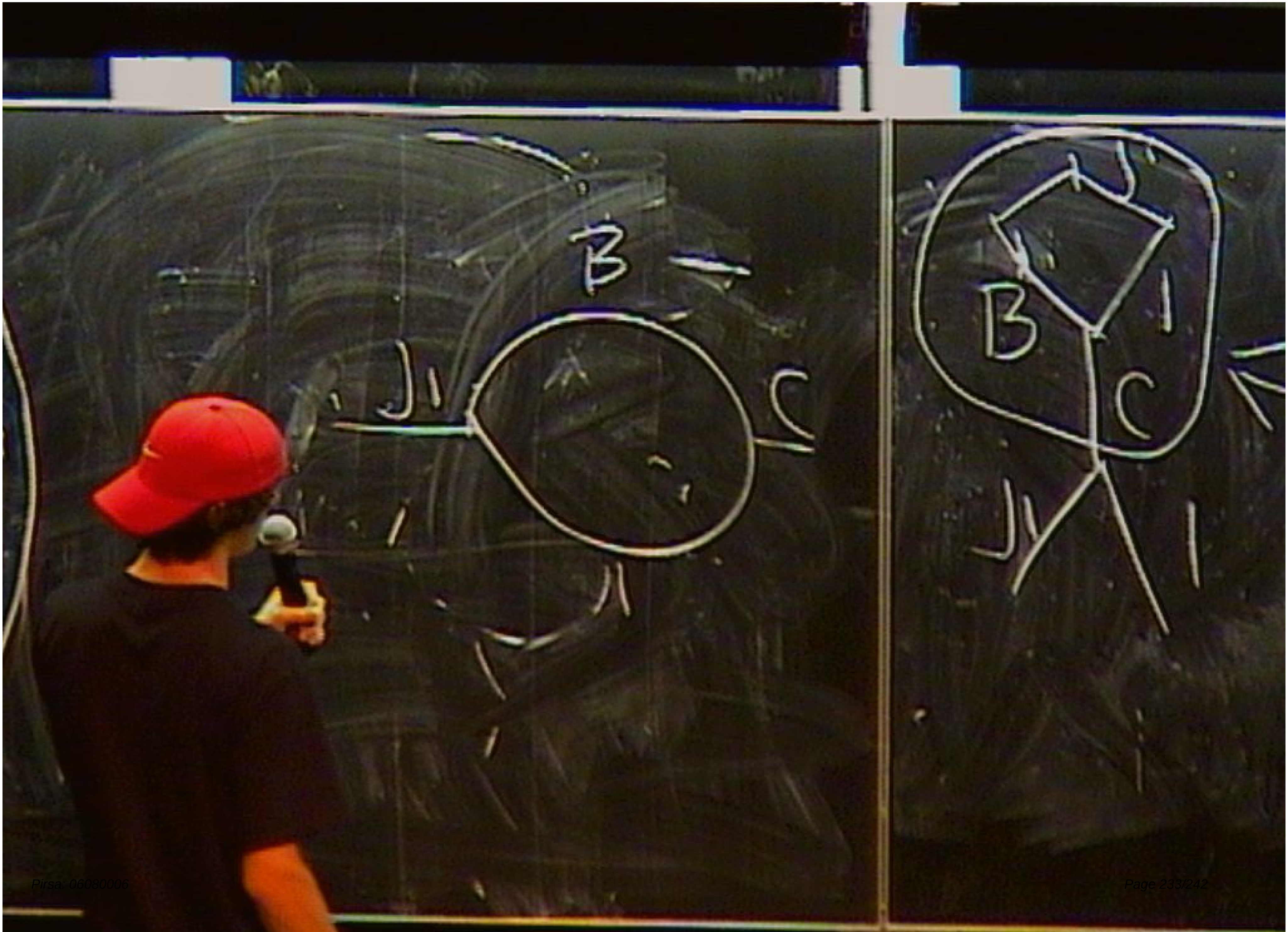


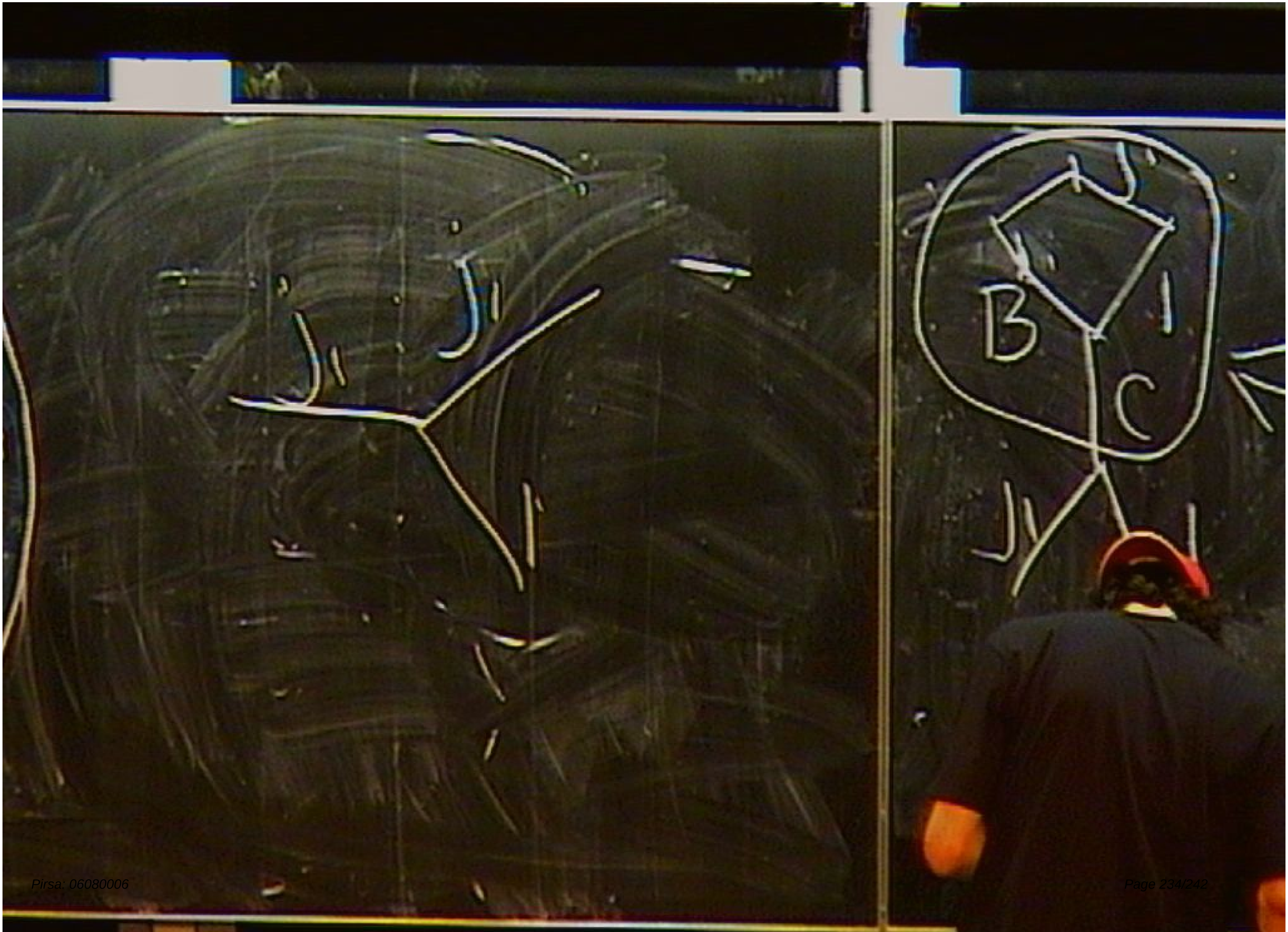
$$\begin{Bmatrix} a & b & c \\ d & e & f \end{Bmatrix}$$

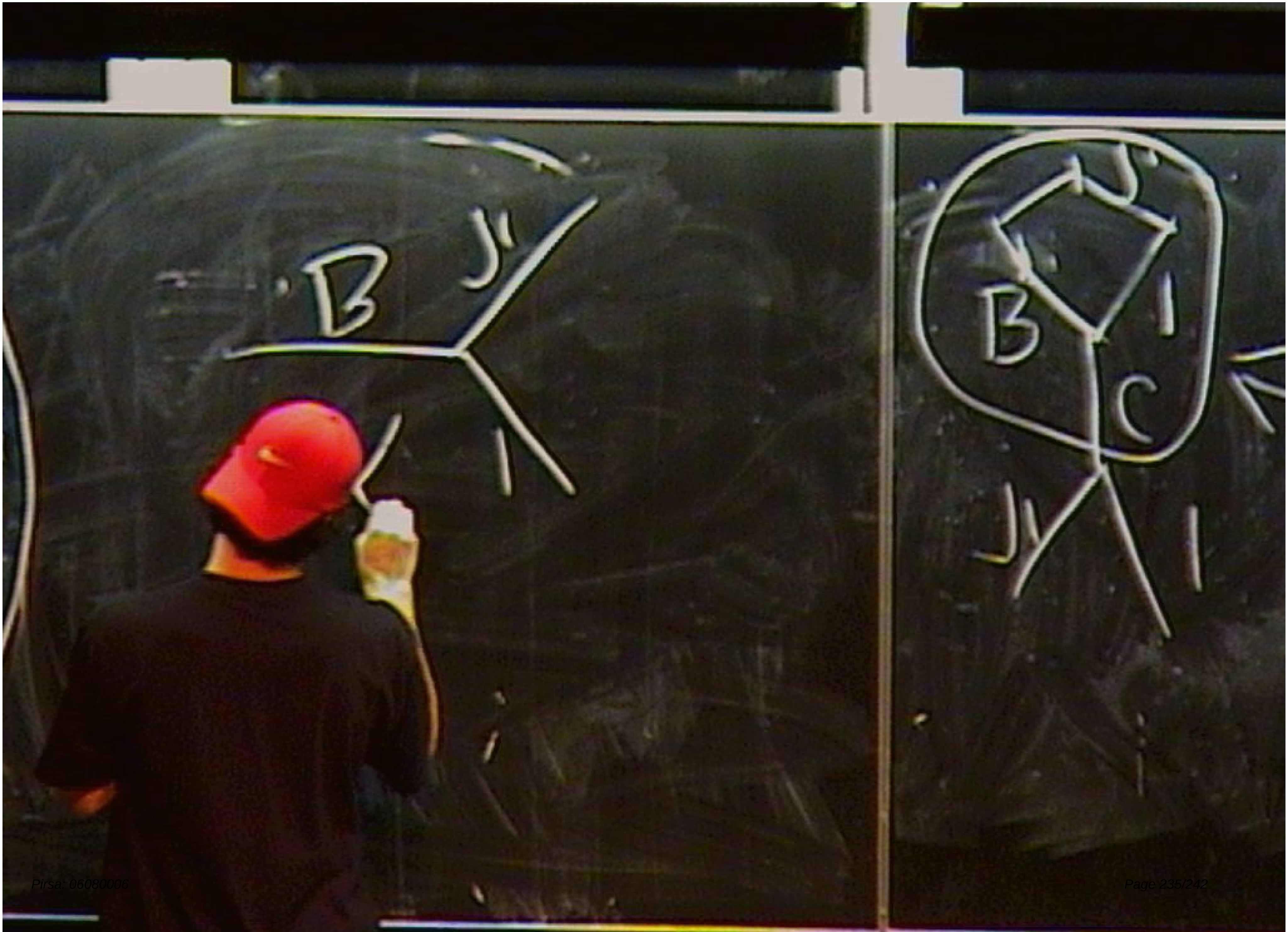

j'
B
j'
B
C

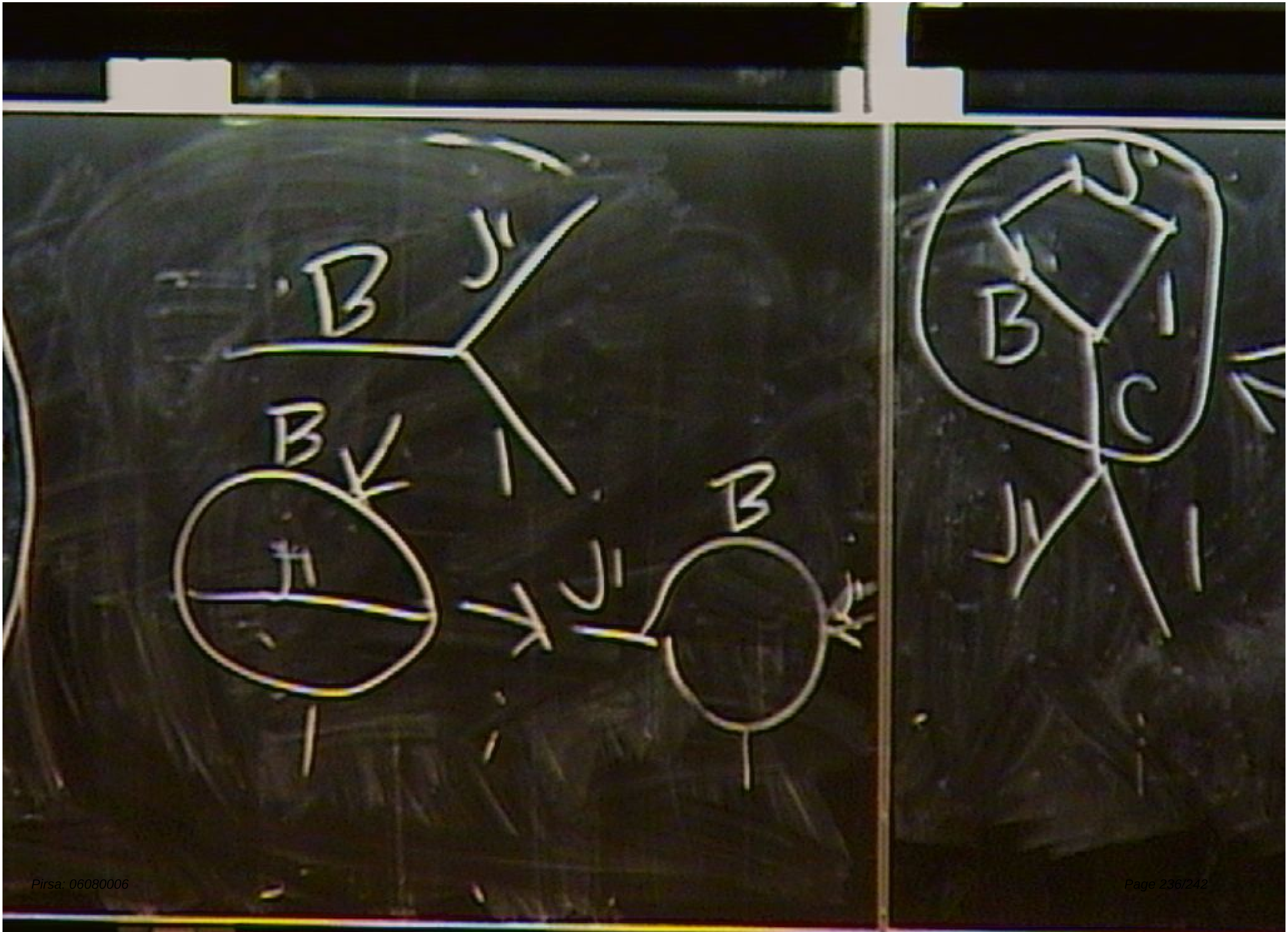

j'
j
B
i
j
j'
i
C
B
i

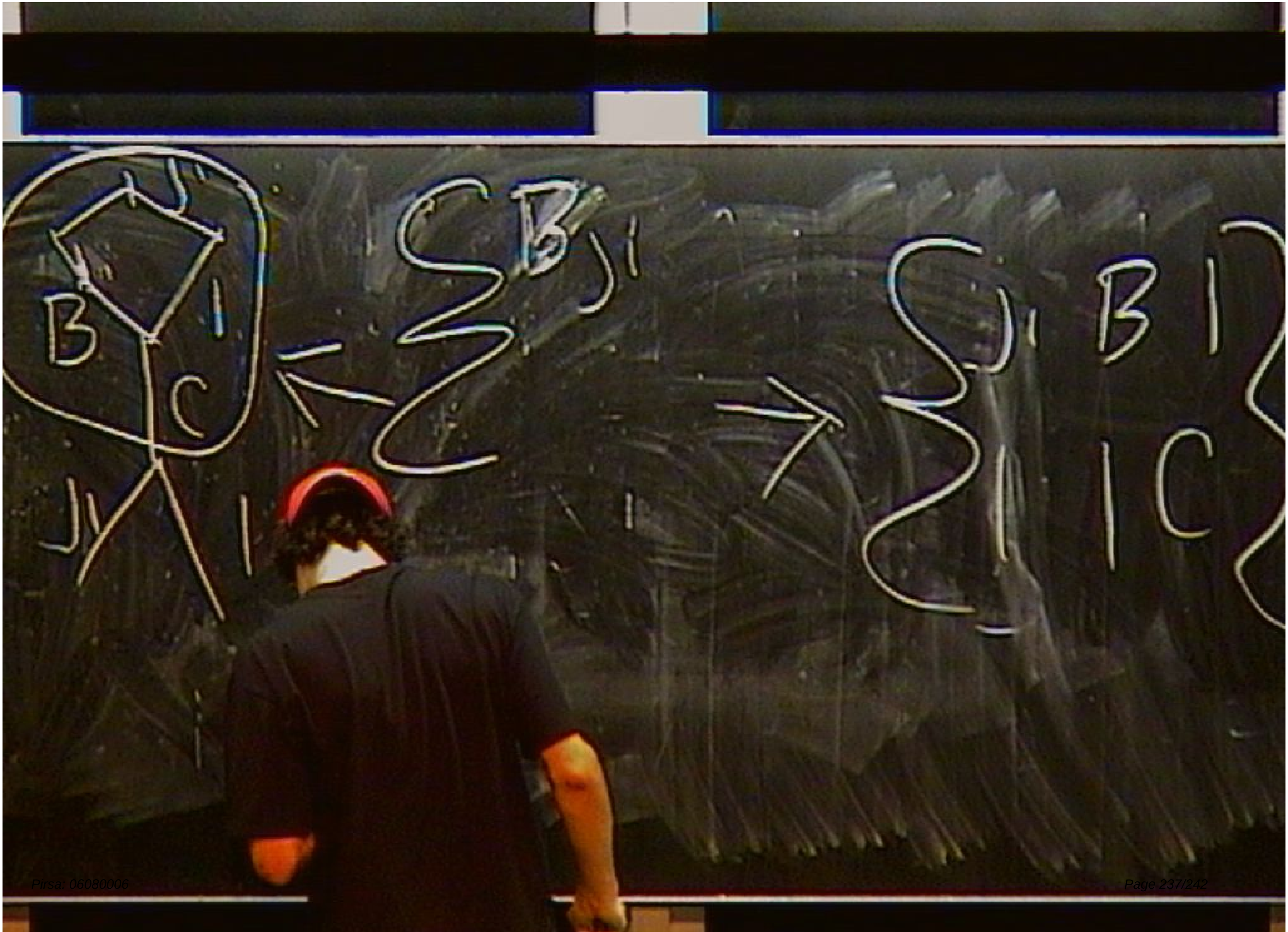

j₁
x
G
j₁
y
B
j₁
1

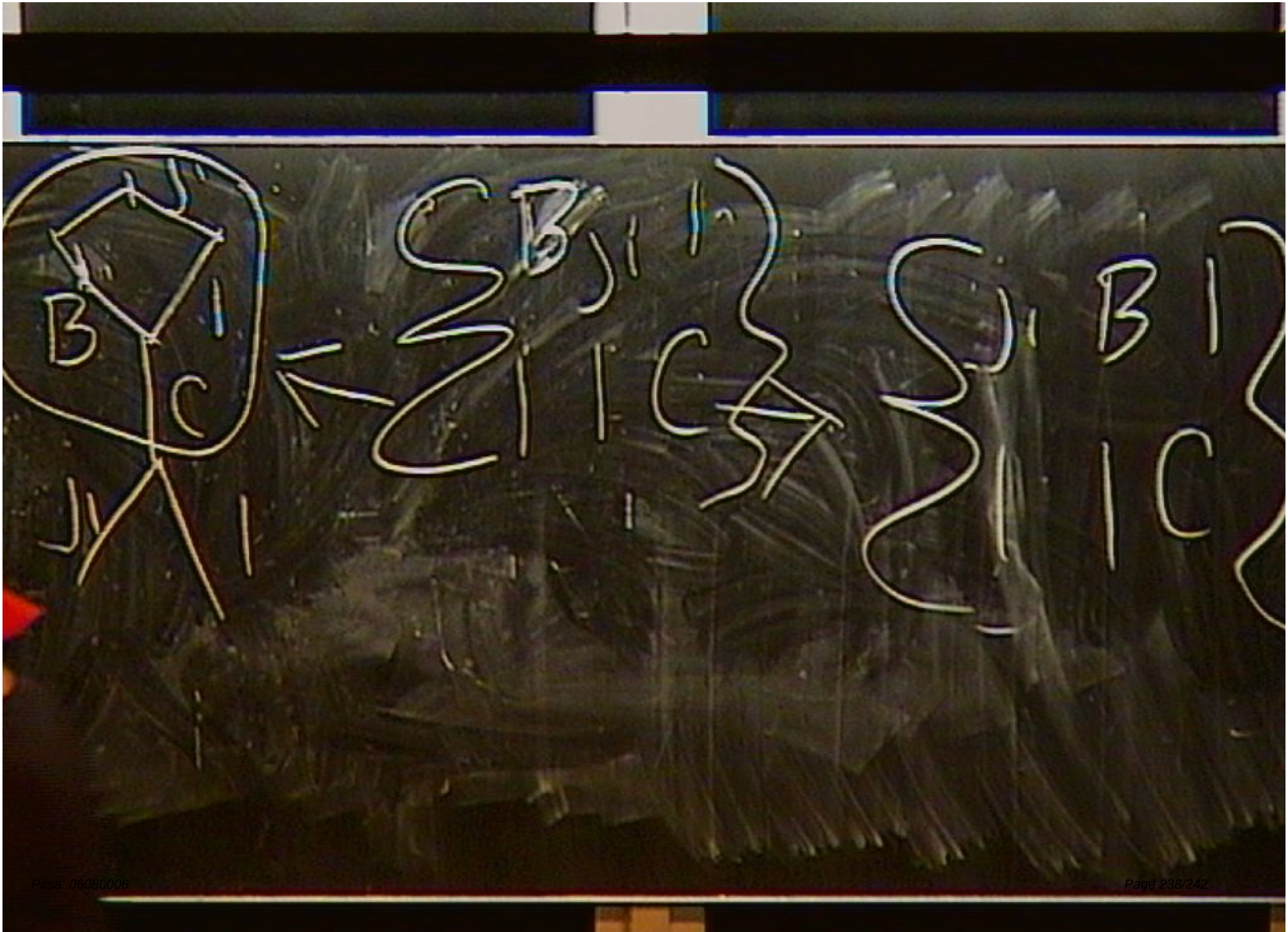

B
J1
C
J1
J1
B
1
C
J1
1

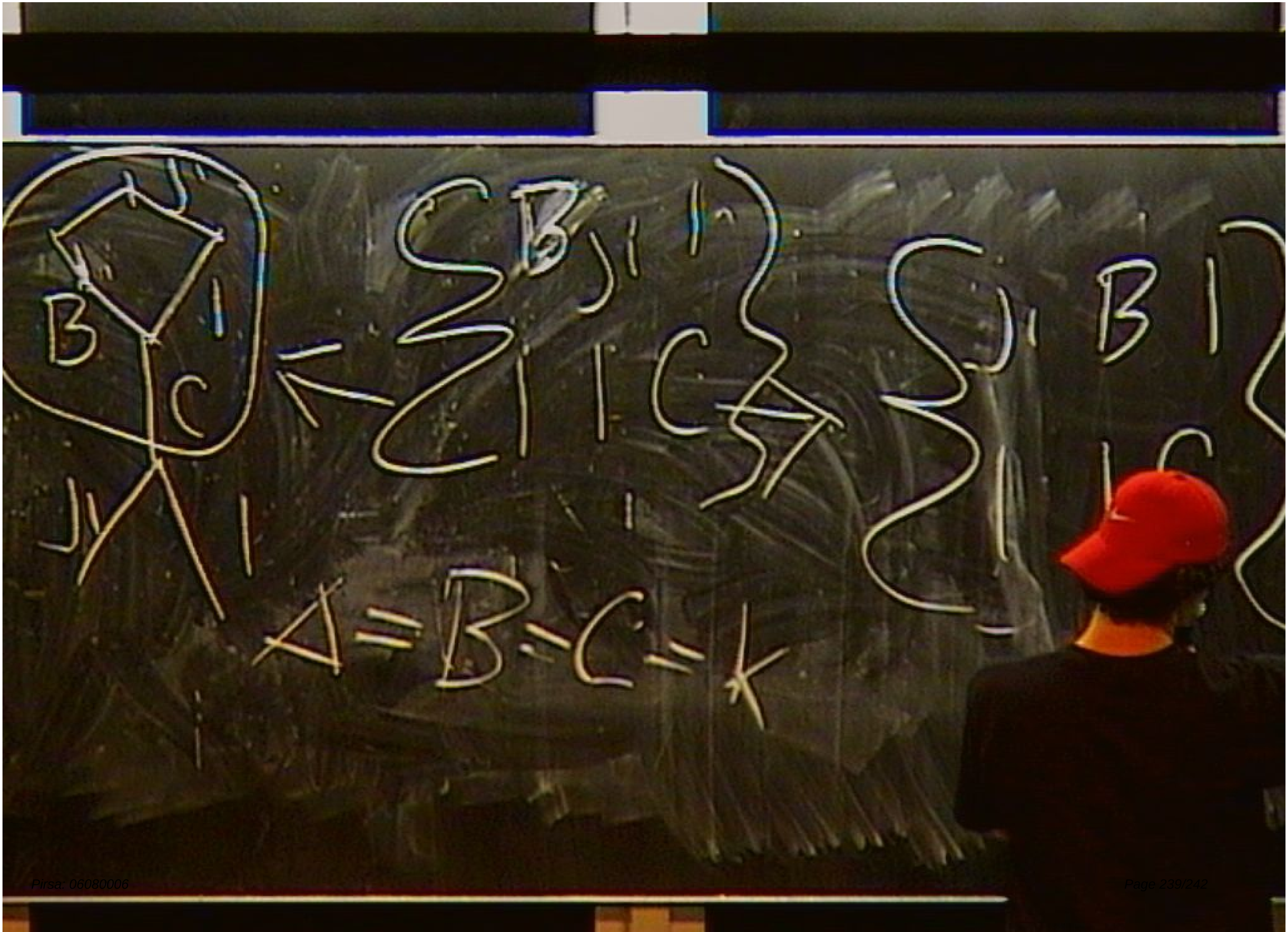

j
j
j
j
B
C
j

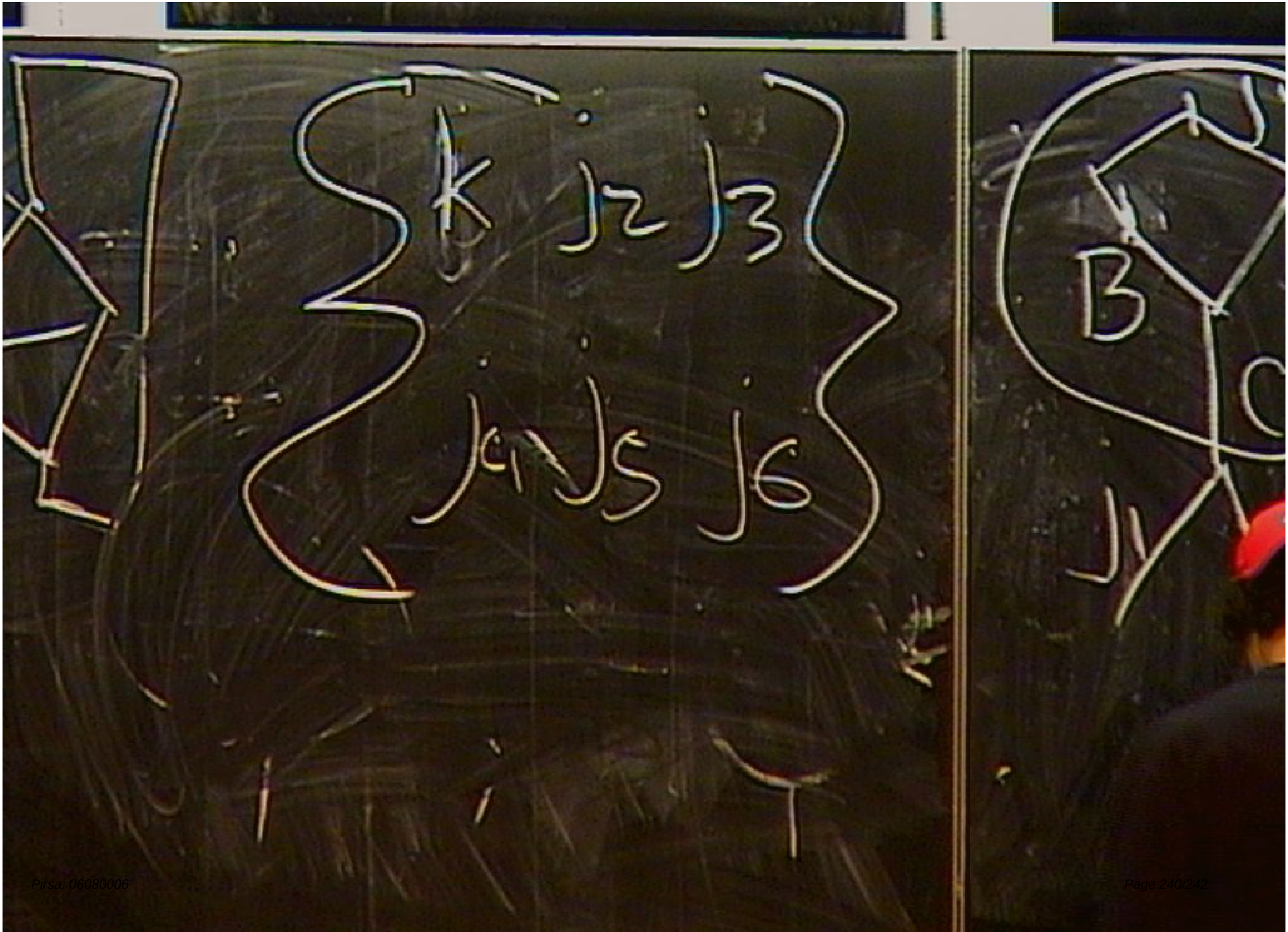

B
j'
1
j'
B
1
C
j'
1


B
J'
1
B
J'
B
J'
B
1
C
J'
1

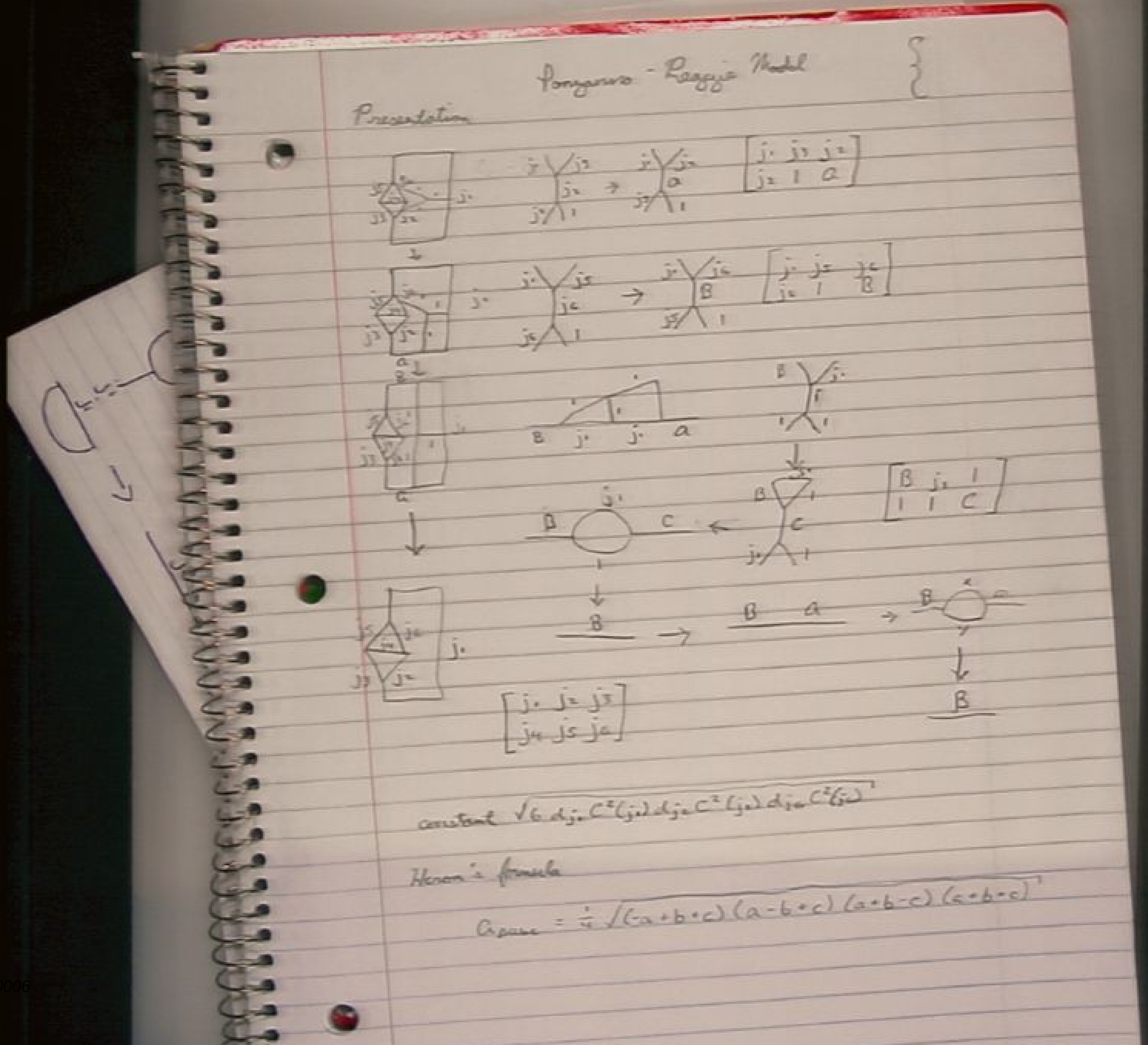

B
C
J
S
B
J
S
B
C


J
B
I
C
J
I
B
I
C
B
I
C


B
C
1
B
1
C
1
B
1
C
A=B=C=K



$$\begin{Bmatrix} k & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$



Ponzano - Regge Model

Presentation

$\begin{bmatrix} j_1 & j_3 & j_2 \\ j_2 & 1 & a \end{bmatrix}$

$\begin{bmatrix} j_1 & j_2 & j_6 \\ j_4 & 1 & B \end{bmatrix}$

$\begin{bmatrix} B & j_1 & 1 \\ 1 & 1 & C \end{bmatrix}$

$\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix}$

constant $\sqrt{6\, d_{j_a} C^2(j_a)\, d_{j_a} C^2(j_a)\, d_{j_a} C^2(j_a)}$

Heron's formula

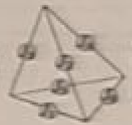
$A_{space} = \frac{1}{4}\sqrt{(-a+b+c)(a-b+c)(a+b-c)(a+b+c)}$

Ponzano - Regge Model {

$$\begin{Bmatrix} j_1-1 & j_1 & 1 \\ 1 & 1 & j_1 \end{Bmatrix} = \frac{(-1)^{-2j_1+1}(j_1+1)}{\sqrt{6C^2(j_1)\Delta_{j_1}}} \tag{E11}$$

$$\begin{Bmatrix} j_1+1 & j_1 & 1 \\ 1 & 1 & j_1 \end{Bmatrix} = \frac{(-1)^{-2j_1}j_1}{\sqrt{6C^2(j_1)\Delta_{j_1}}} \tag{E12}$$

$${}^{K}\frac{\delta}{\delta J^I_{e_1}}\frac{\delta}{\delta J^J_{e_2}}\frac{\delta}{\delta J^K_{e_6}}\Big)$$



$$\Big|_{J=0}$$

$$= \sqrt{6\, d_{j_1} C^2(j_1)\, d_{j_2} C^2(j_2)\, d_{j_6} C^2(j_6)} \sum_k \dim k \begin{Bmatrix} 1 & 1 & 1 \\ j_1 & k & j_1 \end{Bmatrix} \begin{Bmatrix} 1 & j_2 & j_2 \\ j_3 & j_1 & k \end{Bmatrix} \begin{Bmatrix} j_1 & j_5 & j_6 \\ j_6 & 1 & k \end{Bmatrix} \begin{Bmatrix} k & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \tag{E13}$$

, the simplified evaluations of the 6-j symbols containing values of 1 we get a result of

$$\begin{aligned}
\delta_1\delta_2\delta_6 \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} = & \frac{-(j_1+1)}{4j_1(2j_1+1)}\sqrt{(j_1+j_2-j_3)(j_1-j_2+j_3)(1-j_1+j_2+j_3)(1+j_1+j_2+j_3)} \\
& \sqrt{(j_1+j_5-j_6)(j_1-j_5+j_6)(1-j_1+j_5+j_6)(1+j_1+j_5+j_6)} \begin{Bmatrix} j_1-1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \\
& + \frac{(C^2(j_1)+C^2(j_2)-C^2(j_3))(C^2(j_1)+C^2(j_6)-C^2(j_5))}{4j_1(j_1+1)} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \\
& + \frac{j_1}{4(j_1+1)(2j_1+1)}\sqrt{(1+j_1+j_2-j_3)(1+j_1-j_2+j_3)(-j_1+j_2+j_3)(2+j_1+j_2+j_3)} \\
& \sqrt{(1+j_1+j_5-j_6)(1+j_1-j_5+j_6)(-j_1+j_5+j_6)(2+j_1+j_5+j_6)} \begin{Bmatrix} j_1+1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}
\end{aligned} \tag{E14}$$