Title: General Relativity - Part 1

Date: Aug 01, 2006 09:00 AM

URL: http://pirsa.org/06080000

Abstract:


Lecture 10
Black Holes


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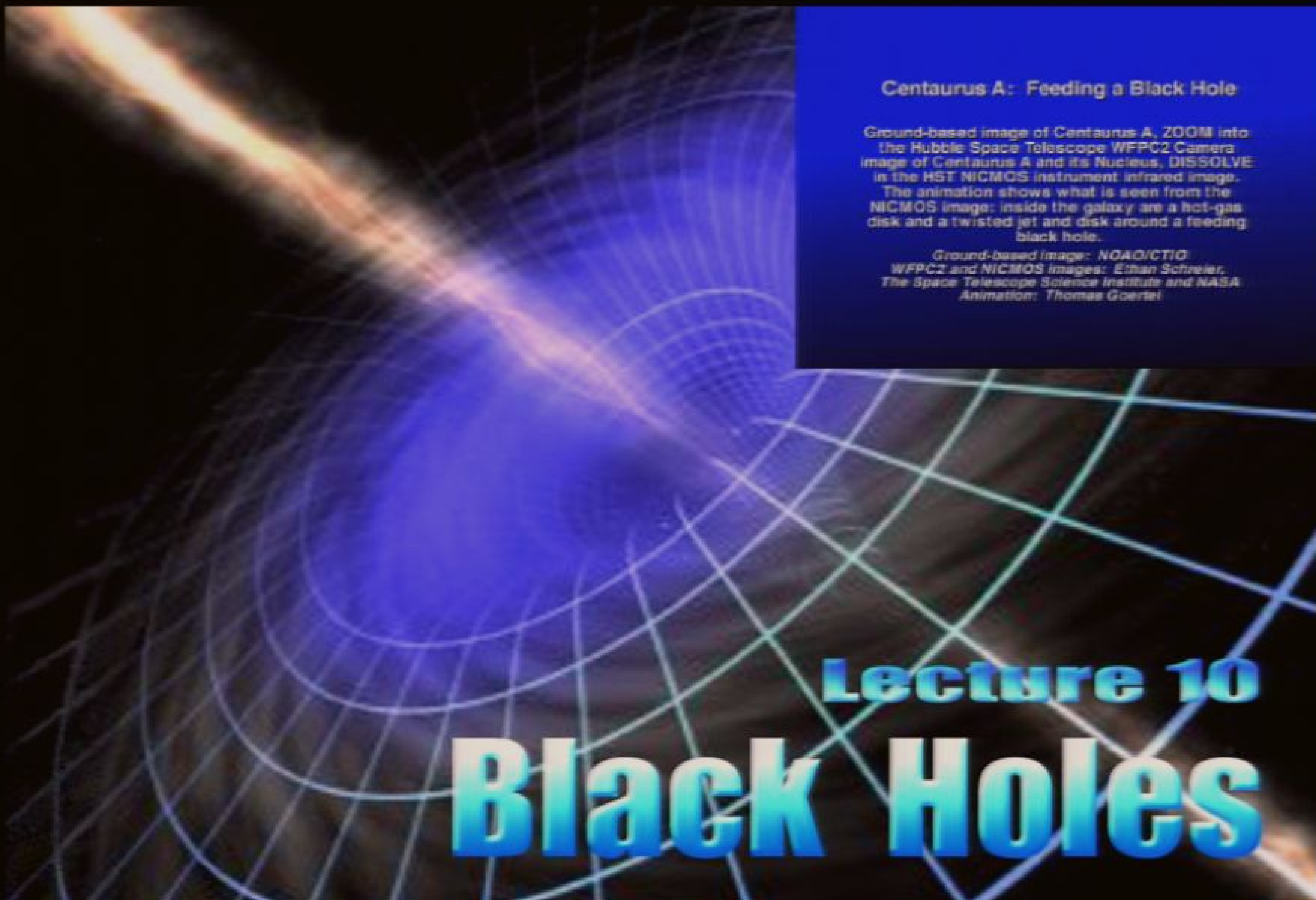
Centaurus A: Feeding a Black Hole
Ground-based image of Centaurus A, ZOOM into the Hubble Space Telescope WFPC2 Camera image of Centaurus A and its Nucleus, DISSOLVE in the HST NICMOS instrument infrared image. The animation shows what is seen from the NICMOS image: inside the galaxy are a hot-gas disk and a twisted jet and disk around a feeding black hole.
Ground-based image: NOAO/CTIO
WFPC2 and NICMOS images: Ethan Schreier, The Space Telescope Science Institute and NASA
Animation: Thomas Goertel
Lecture 10
Black Holes

Dark stars

- Rev. John Michell (1783)

  A British born “natural philosopher” dared to combine the corpuscular description of light with Newton’s gravitation laws to predict what large compact stars should look like.

- He showed that a star, that has the same density of the sun, but 500 time as big, would have such a gravity, that "All light emitted from such a body would be made to return towards it". He said we wouldn't be able to see such a body, but we sure will feel it's gravitational pull.

- We could fly close to this “Dark star” and look around and describe the features of the object.

- A novelty, world lost interest when light was shown to be waves in 1803 by Thomas Young.

Calculation of Escape Velocity

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

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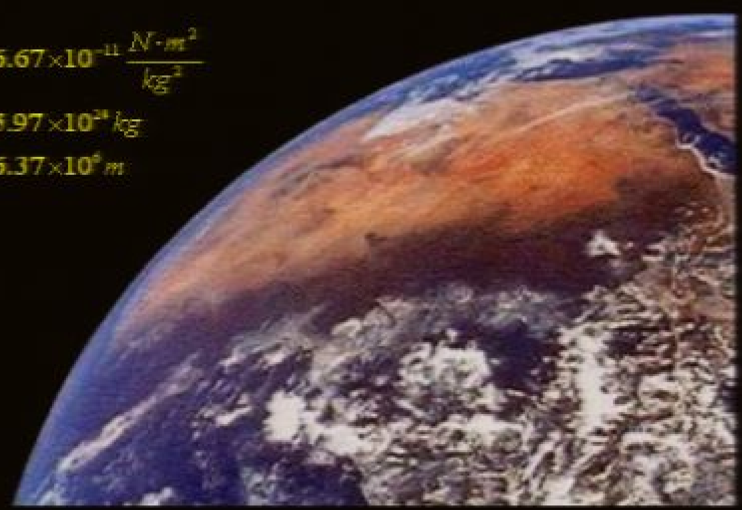
$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

$$M = 5.97 \times 10^{24} kg$$

$$r = 6.37 \times 10^{6} m$$



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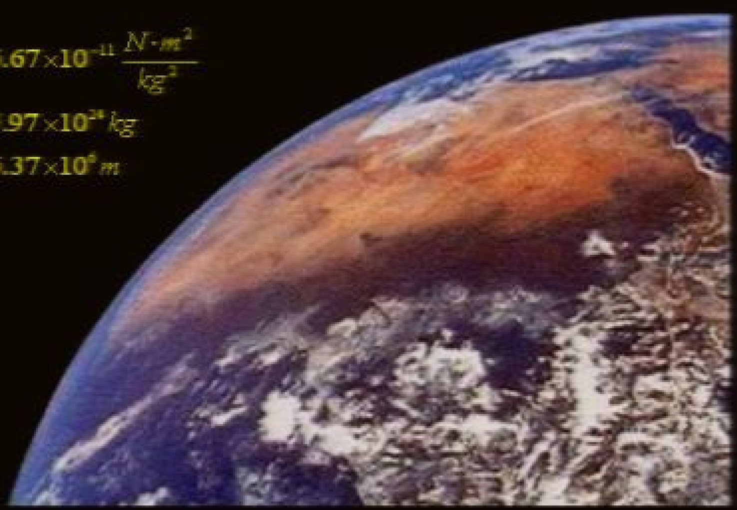

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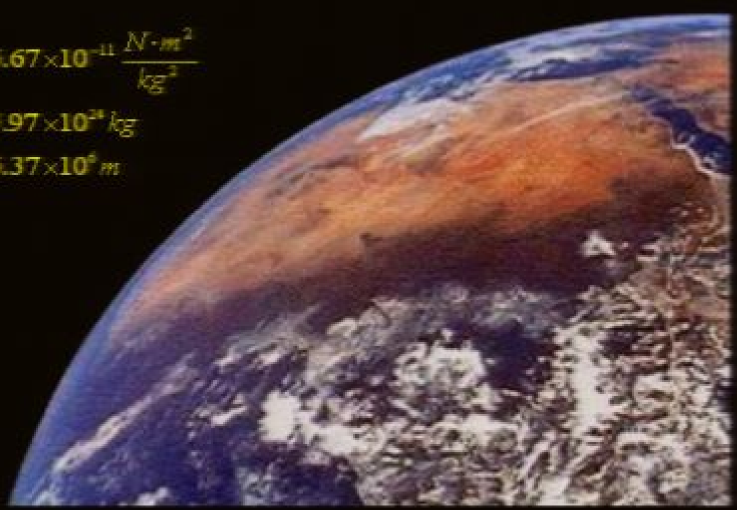

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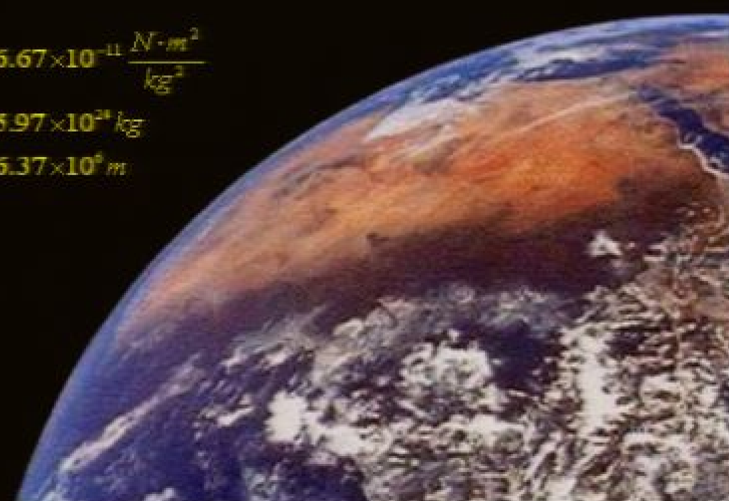

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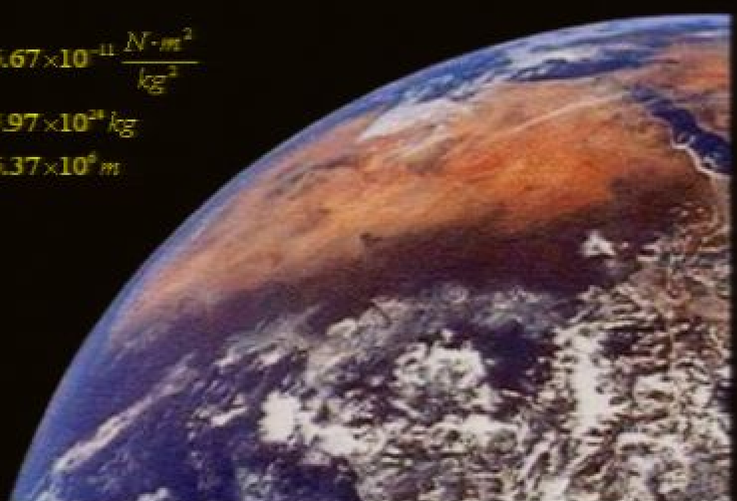

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Calculation of Escape Velocity

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$$\frac{1}{2}v^2 = \frac{GM}{r}$$

$$v = \sqrt{2\frac{(6.67\times10^{-11})(5.97\times10^{24})}{6.37\times10^{6}}}$$

$$v \approx 11181 m/s$$

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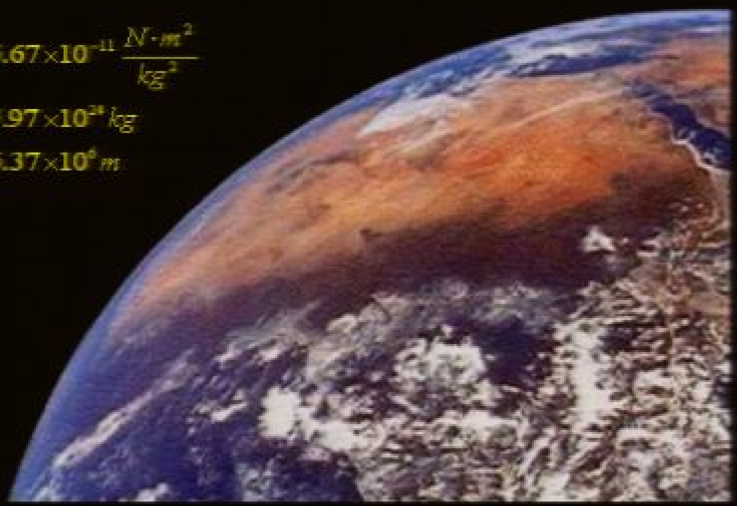

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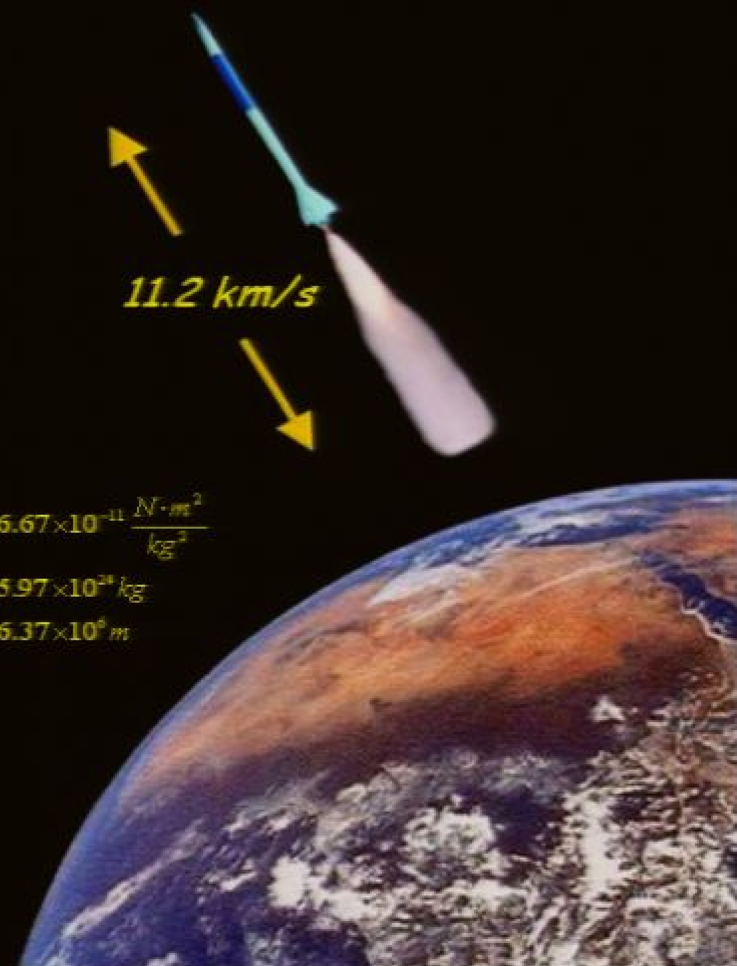

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What is r when v=3x10^8 m/s

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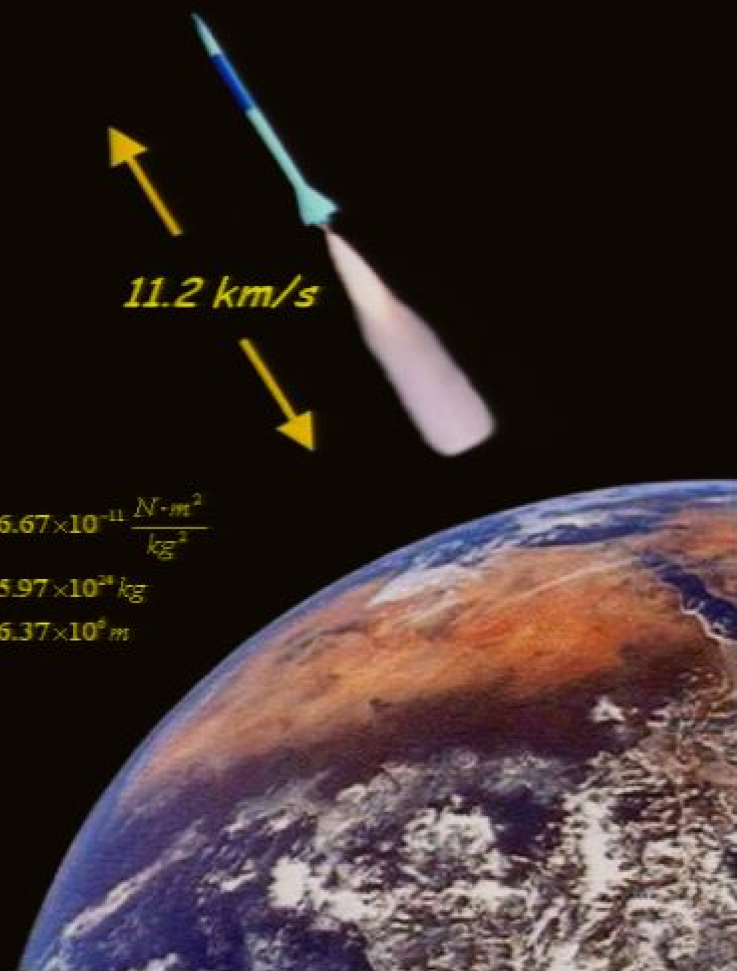

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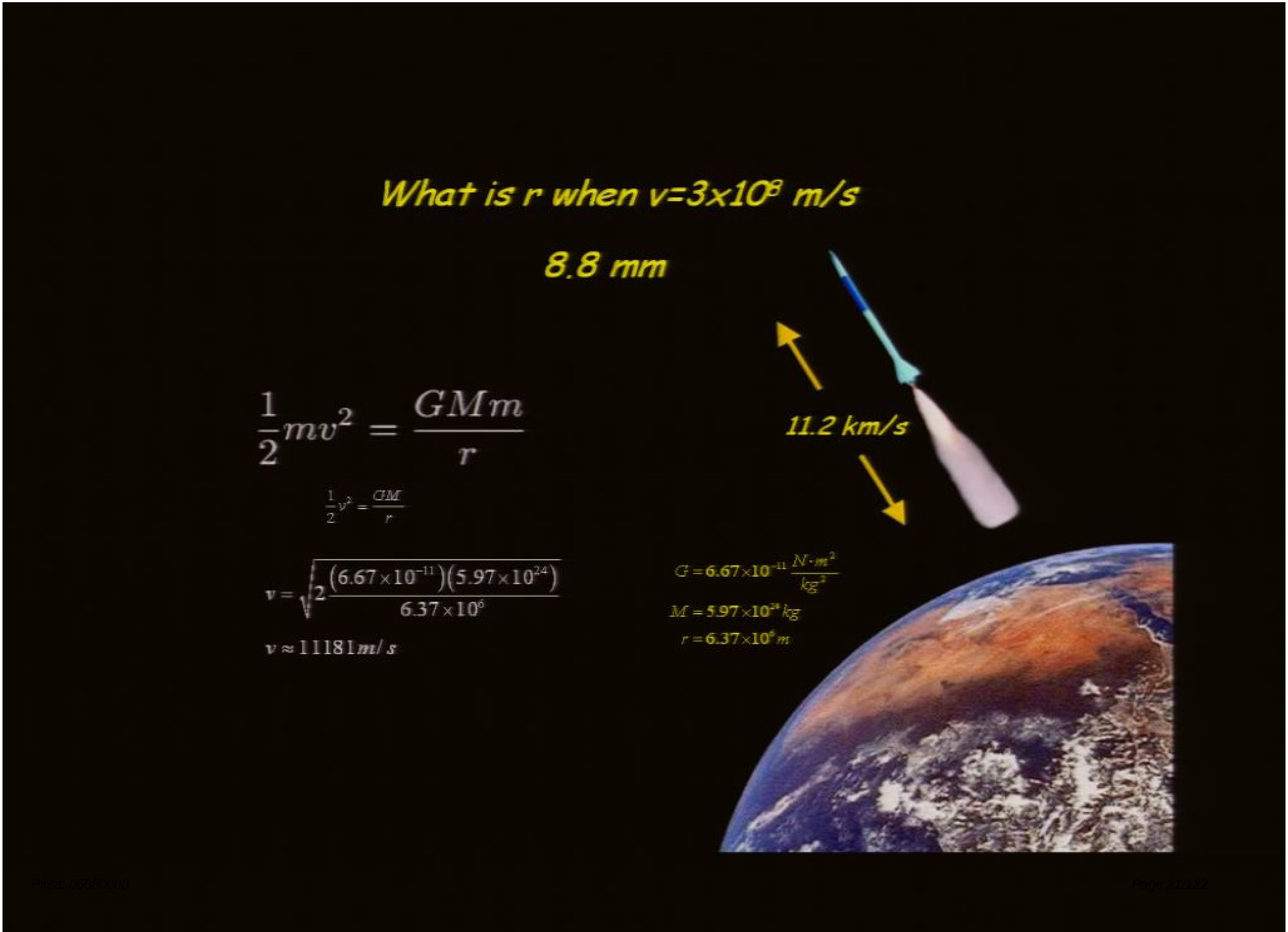
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11.2 km/s
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Einstein's Equivalence Principle

- There is no experiment that you can perform that will distinguish these two diagrams

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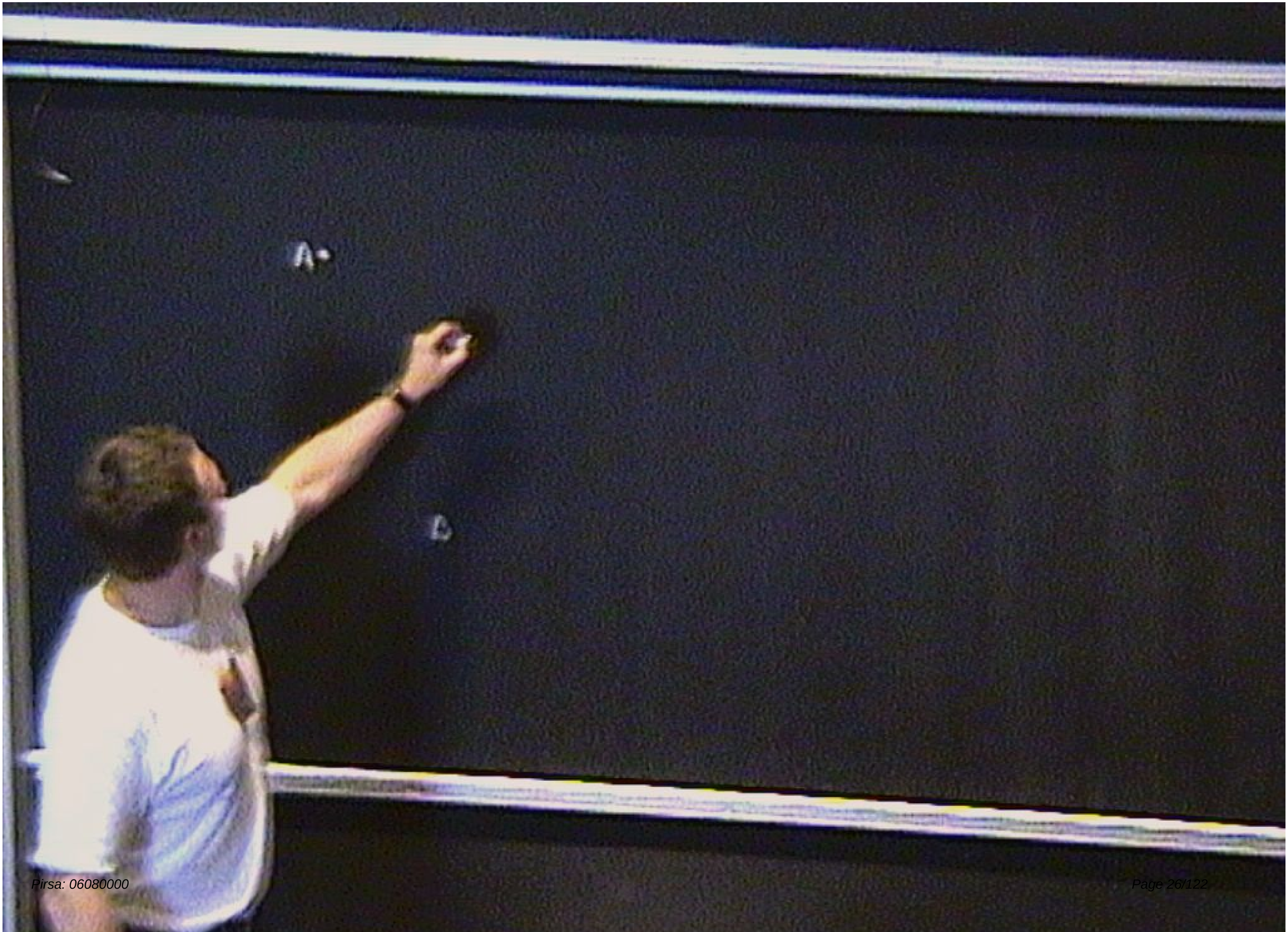
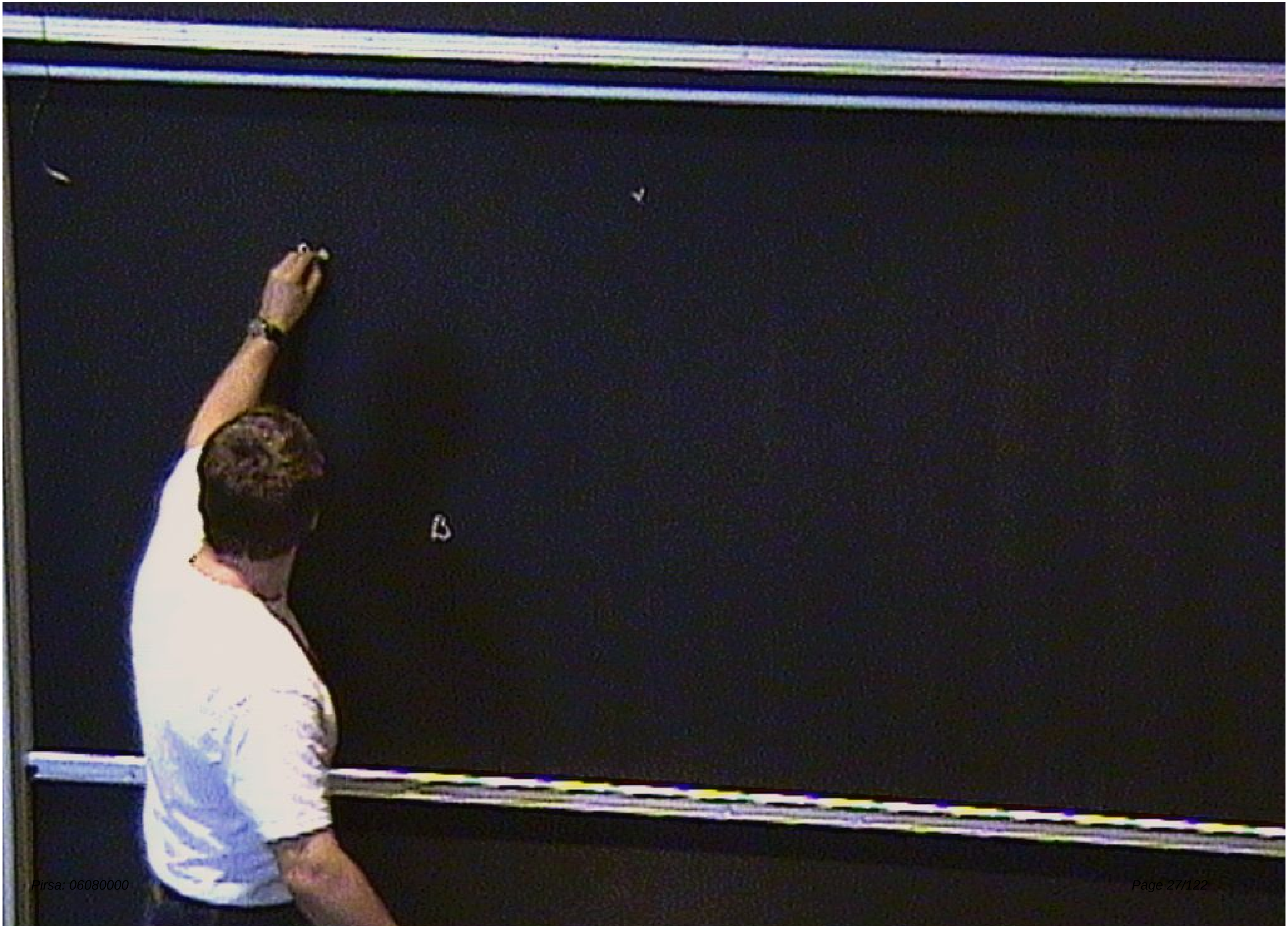
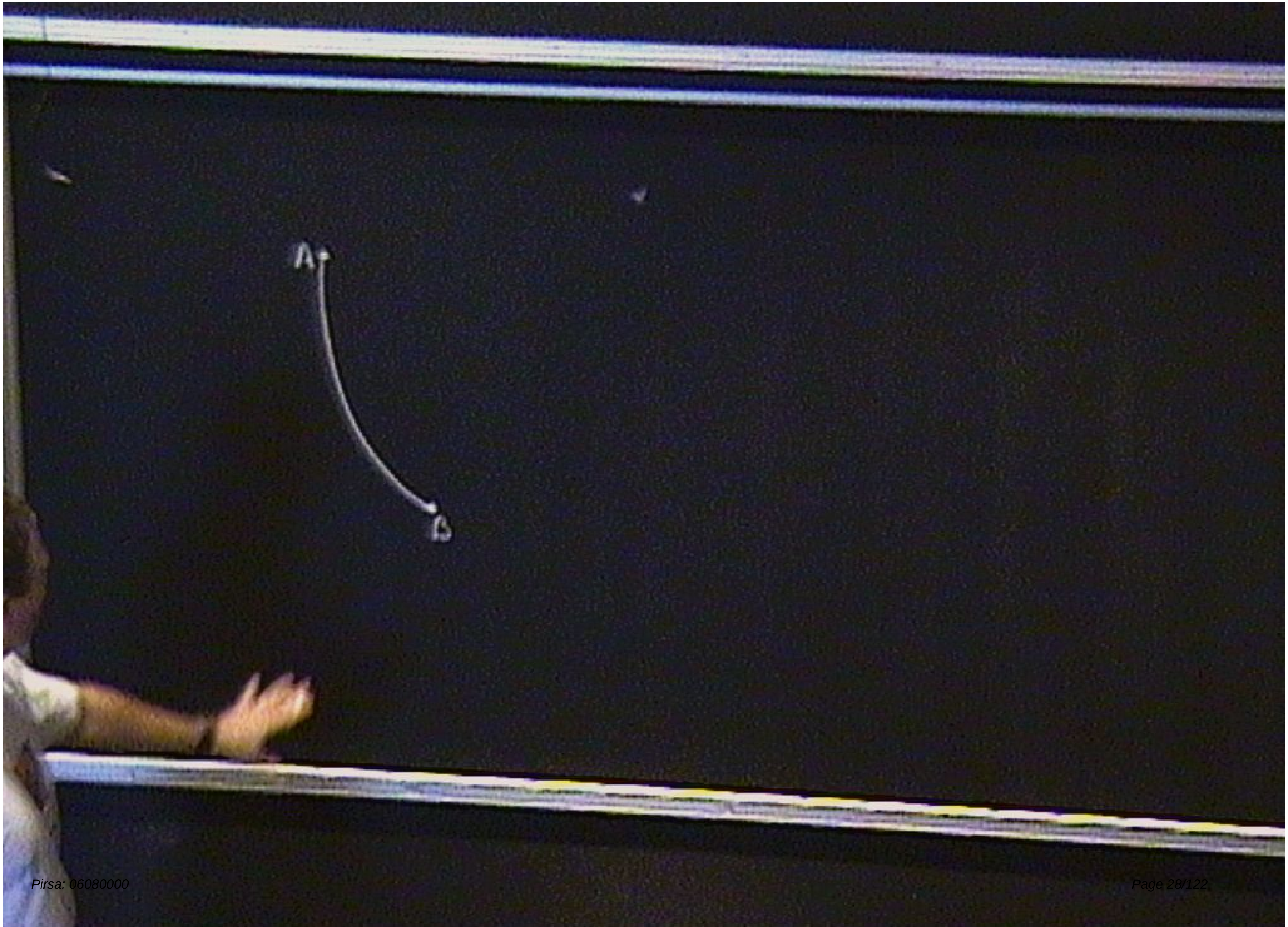
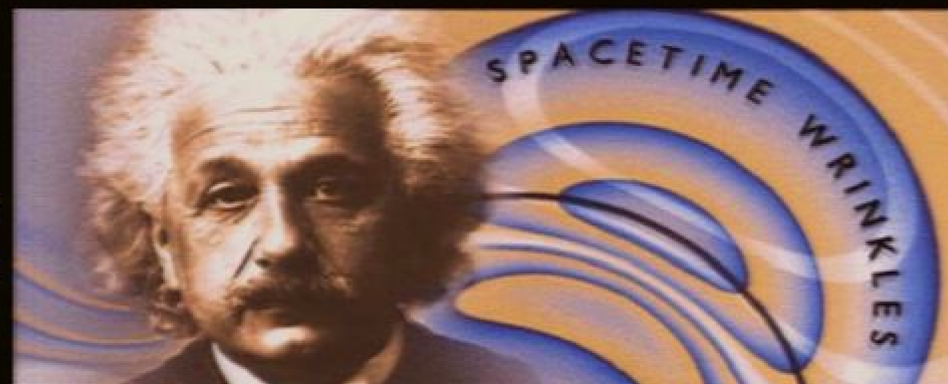





A
B

Gravity as a curvature of Spacetime

The early 1900's changed the way gravity is looked at. Einstein didn't think of gravity as a force between objects, but as a curving of "straight lines" due to mass. Light always follows straight lines, but these may look curved near masses. Time also slows down near masses (space and time are different parts of "spacetime", which is what gets bent).



$$G_{uv} + \Lambda g_{uv} = 8\pi T_{uv}$$

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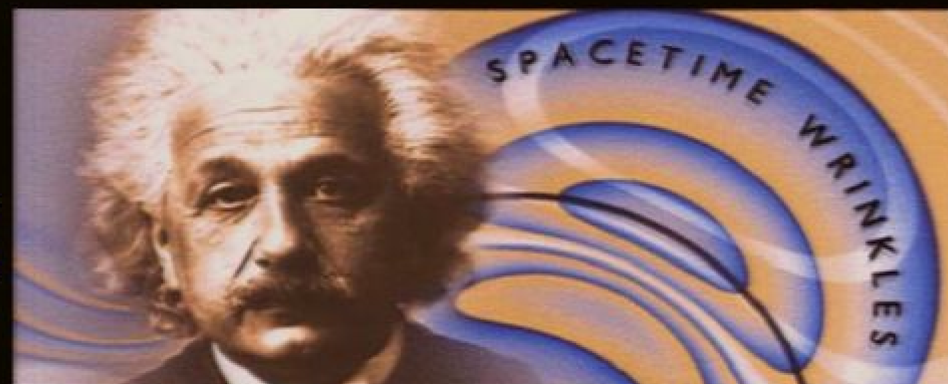

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Encodes geometry (curving) of Space Time

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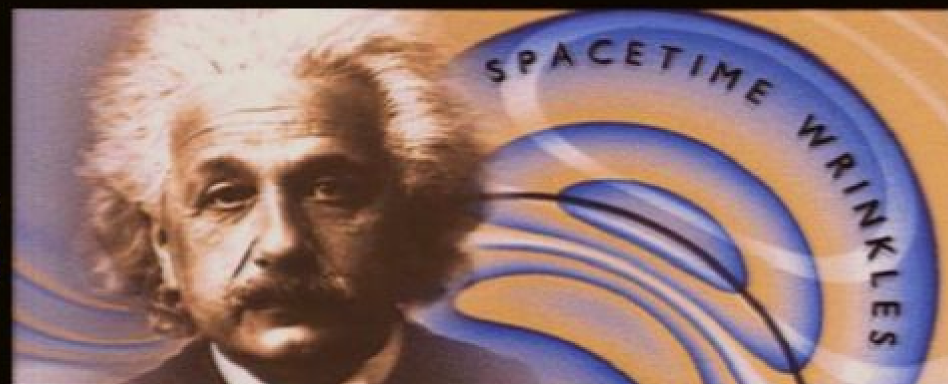

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Cosmological Constant

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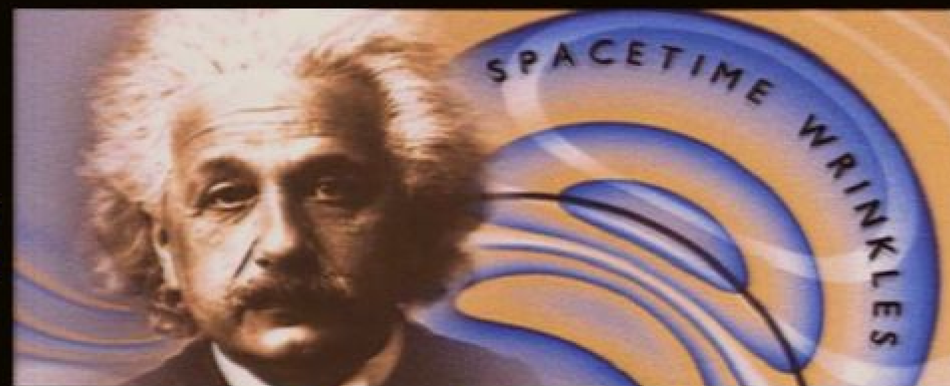

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Cosmological Constant

Encodes the distribution of matter and its motion in Space Time

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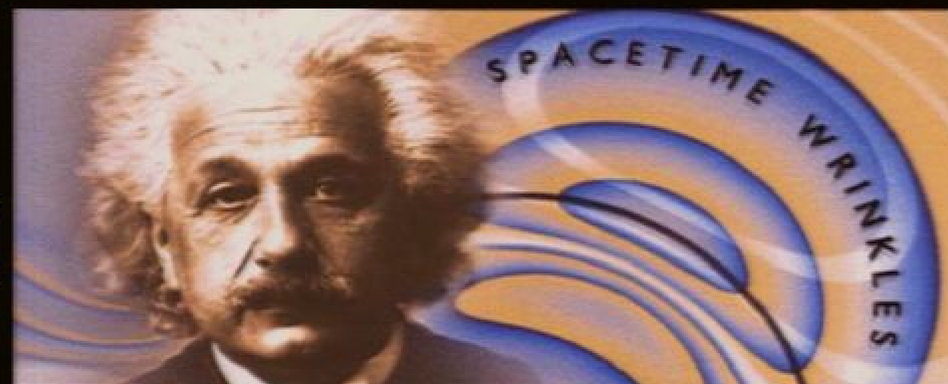

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$

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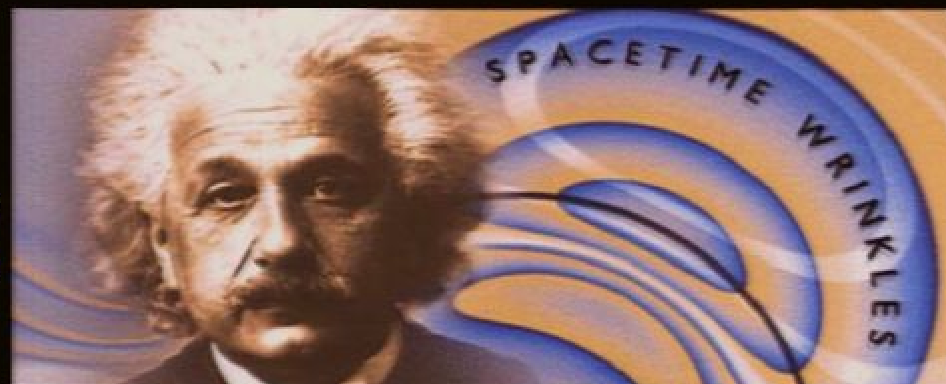

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Encodes geometry (curving) of Space Time

Encodes the distribution of matter and its motion in Space Time

Let's Review

Let's Review

Let’s Review

SPACETIME

Sp - cov

Space Diagram

Bob

Space Diagram

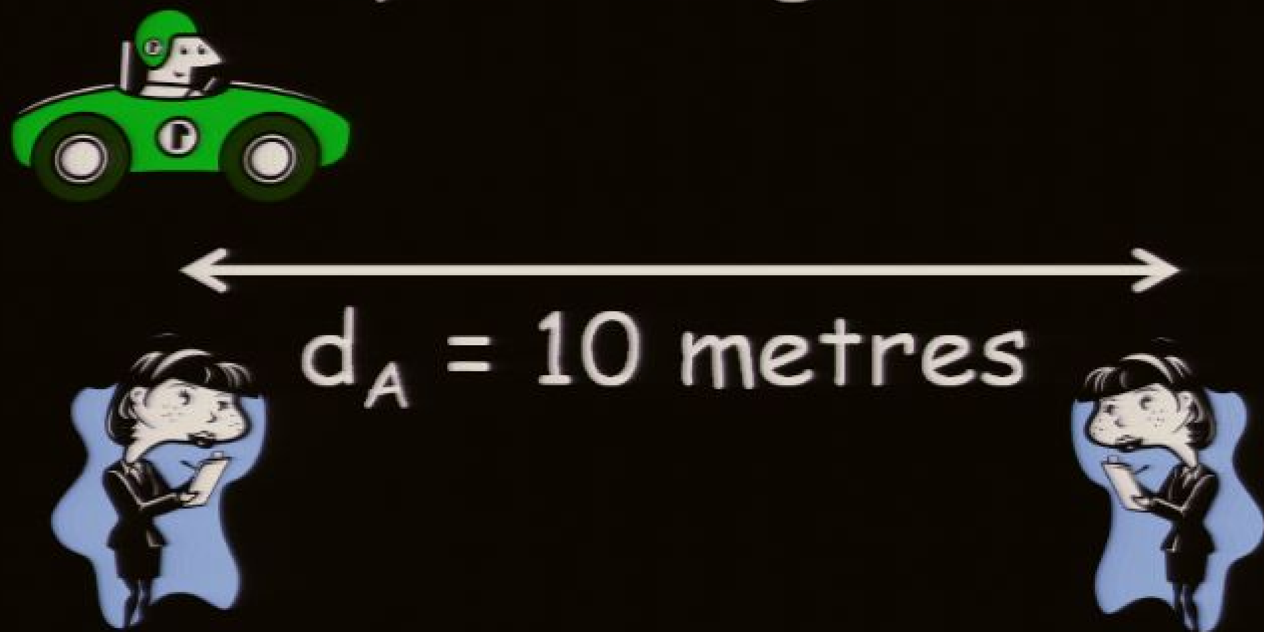
Space Diagram

Alice

Space Diagram

Alice's twin sister, Alice →

Space Diagram



Space Diagram

d_A = 10 metres

$t_A = 0$

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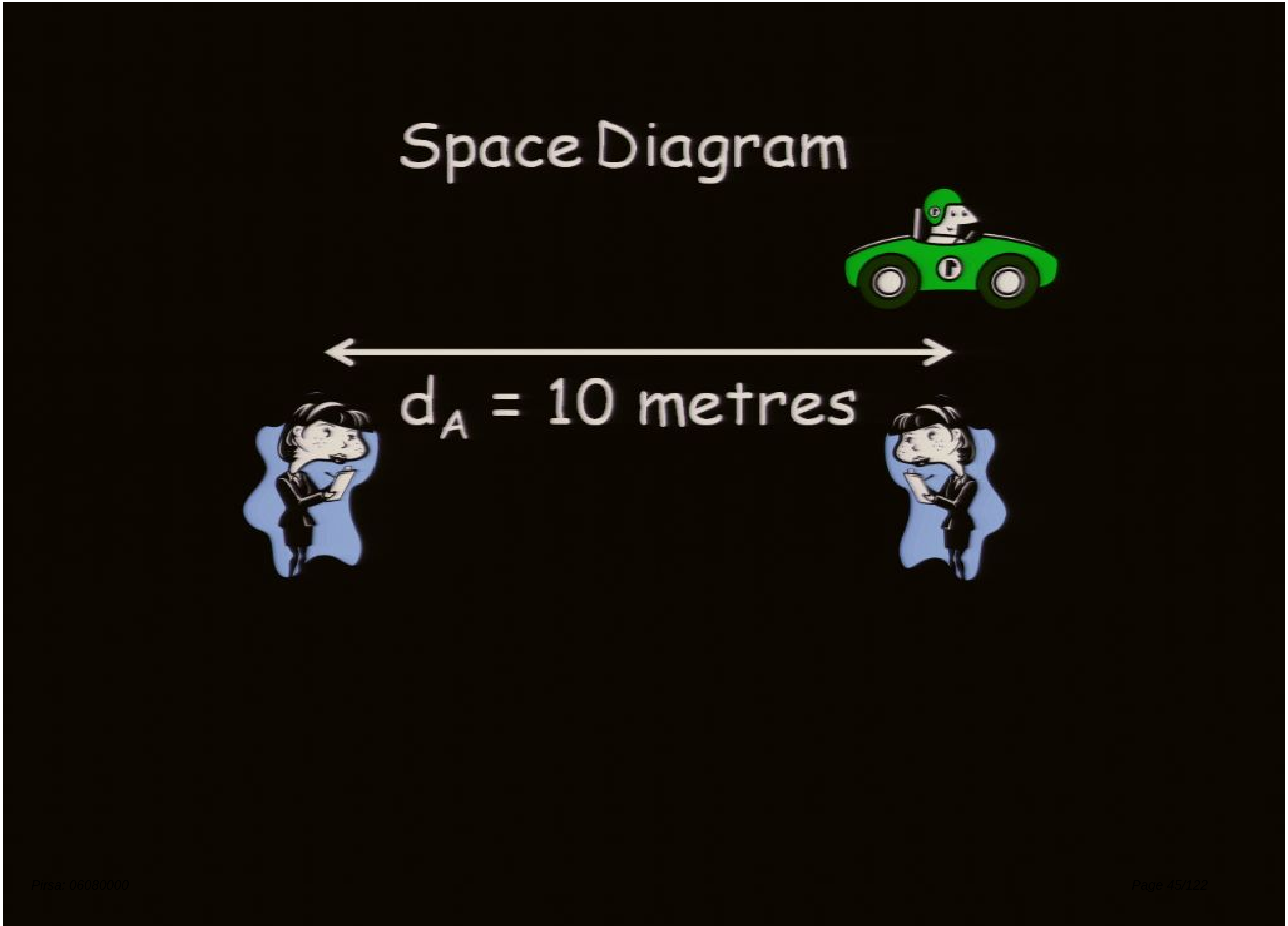
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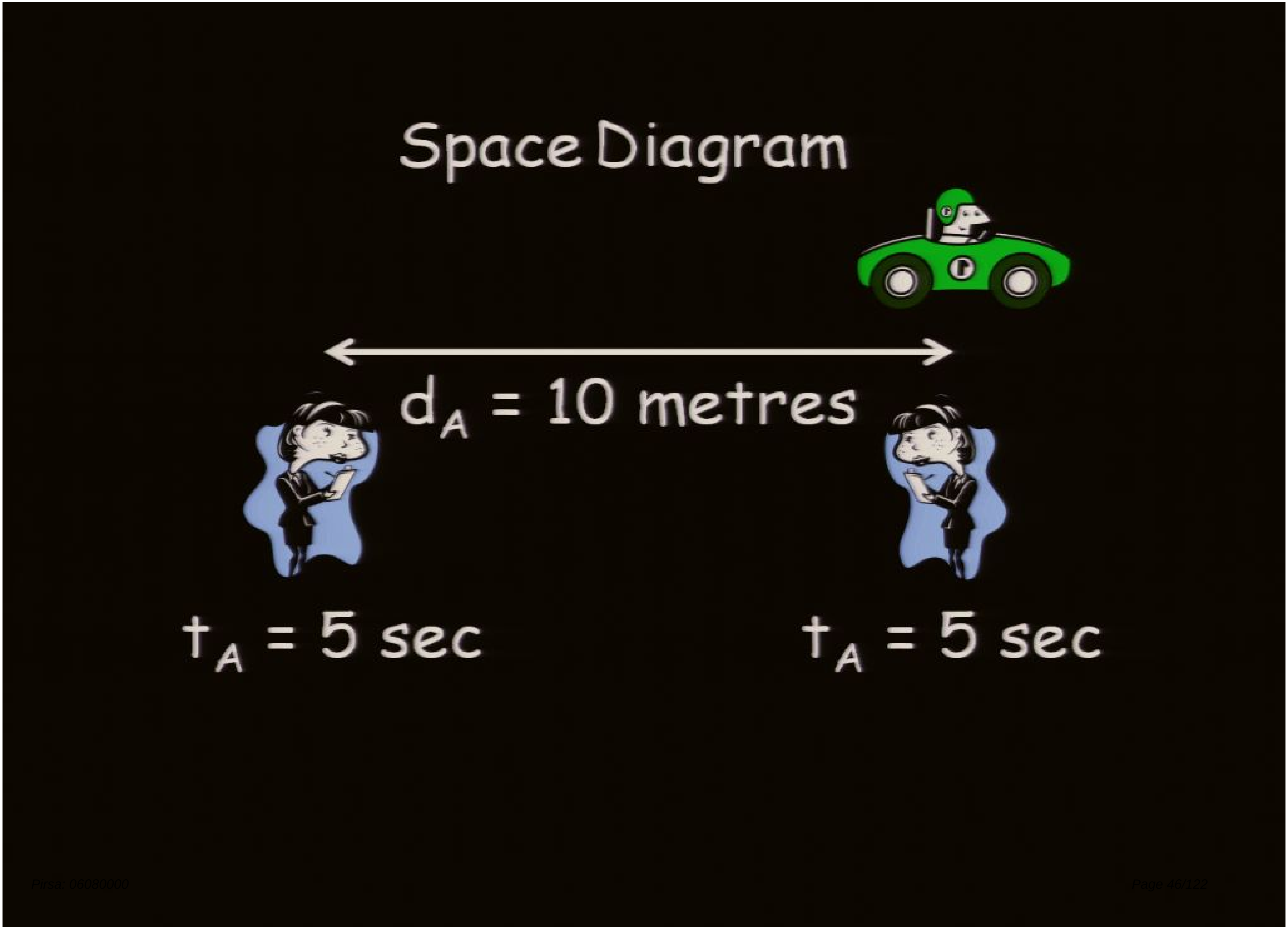
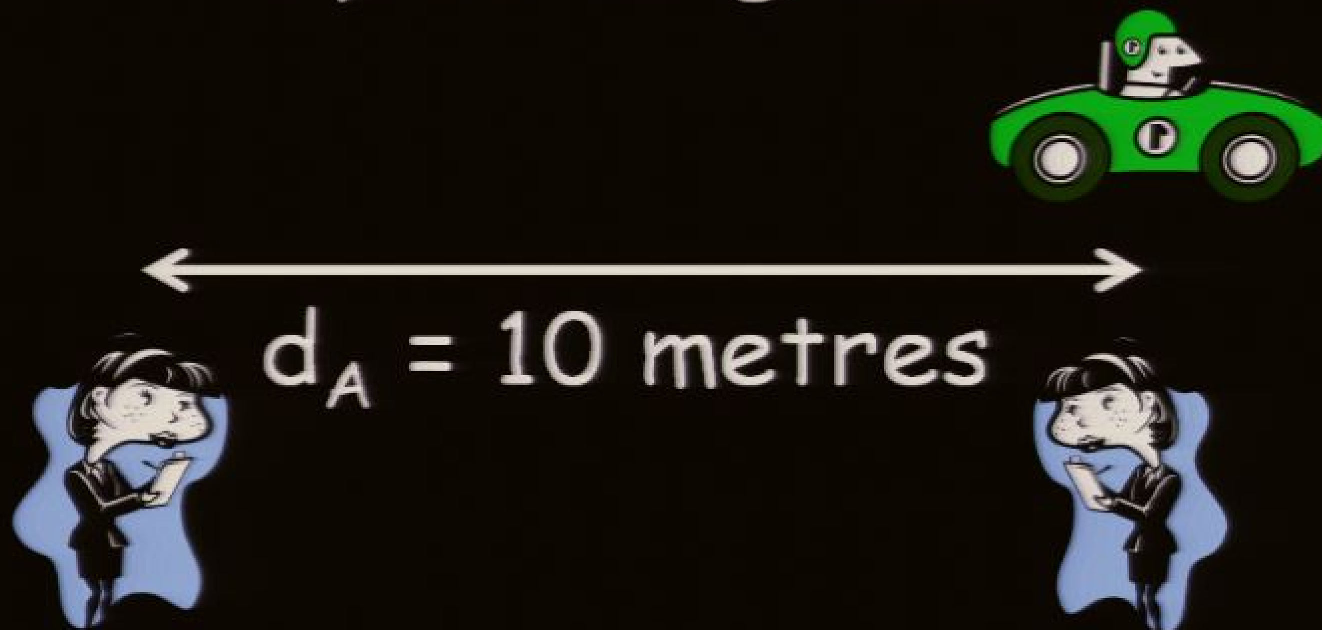

Space Diagram
d_A = 10 metres
t_A = 5 sec
t_A = 5 sec

Space Diagram



t_A = 5 sec t_A = 5 sec

Question: How much time has elapsed for Bob?

Draw a "Spacetime Diagram"

Draw a “Spacetime Diagram”

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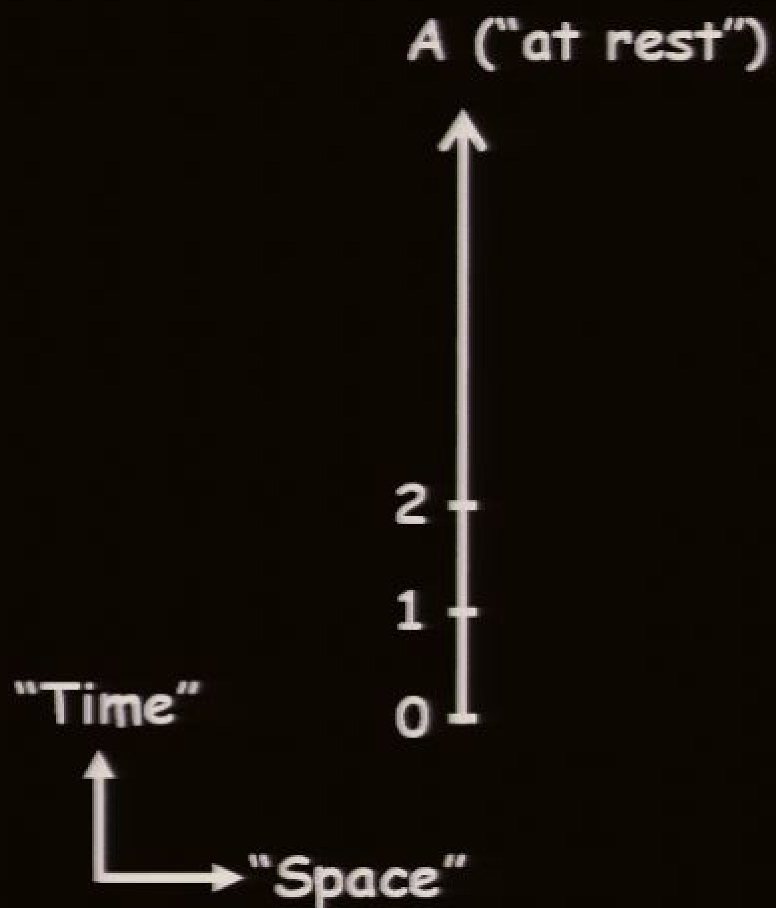
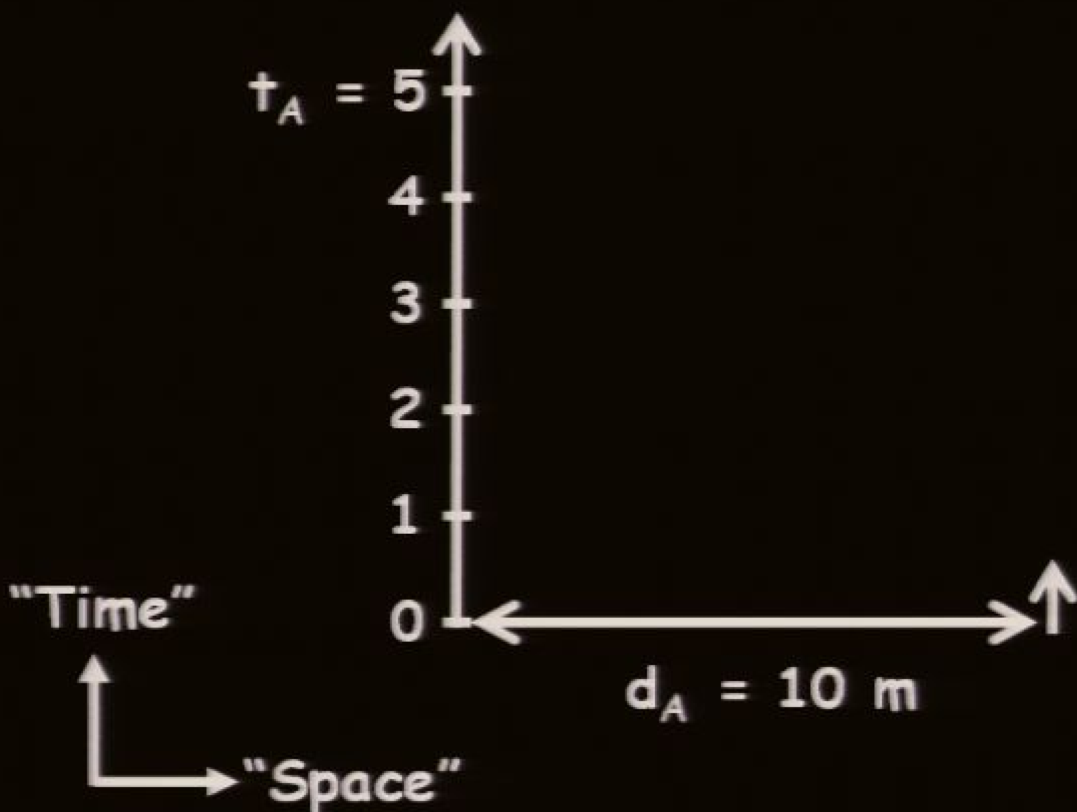


Draw a "Spacetime Diagram"

A ("at rest") A' (at rest relative to A)



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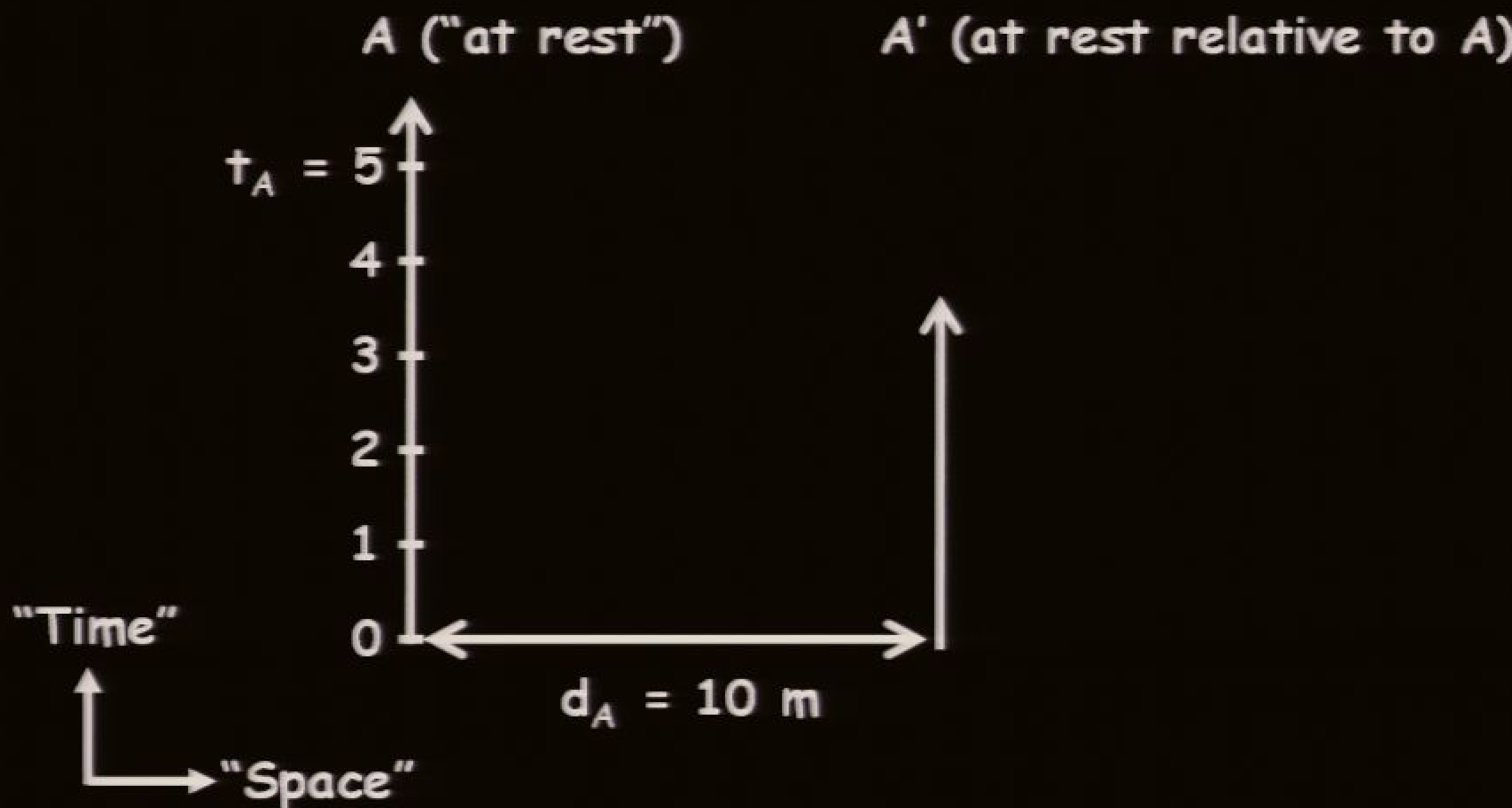

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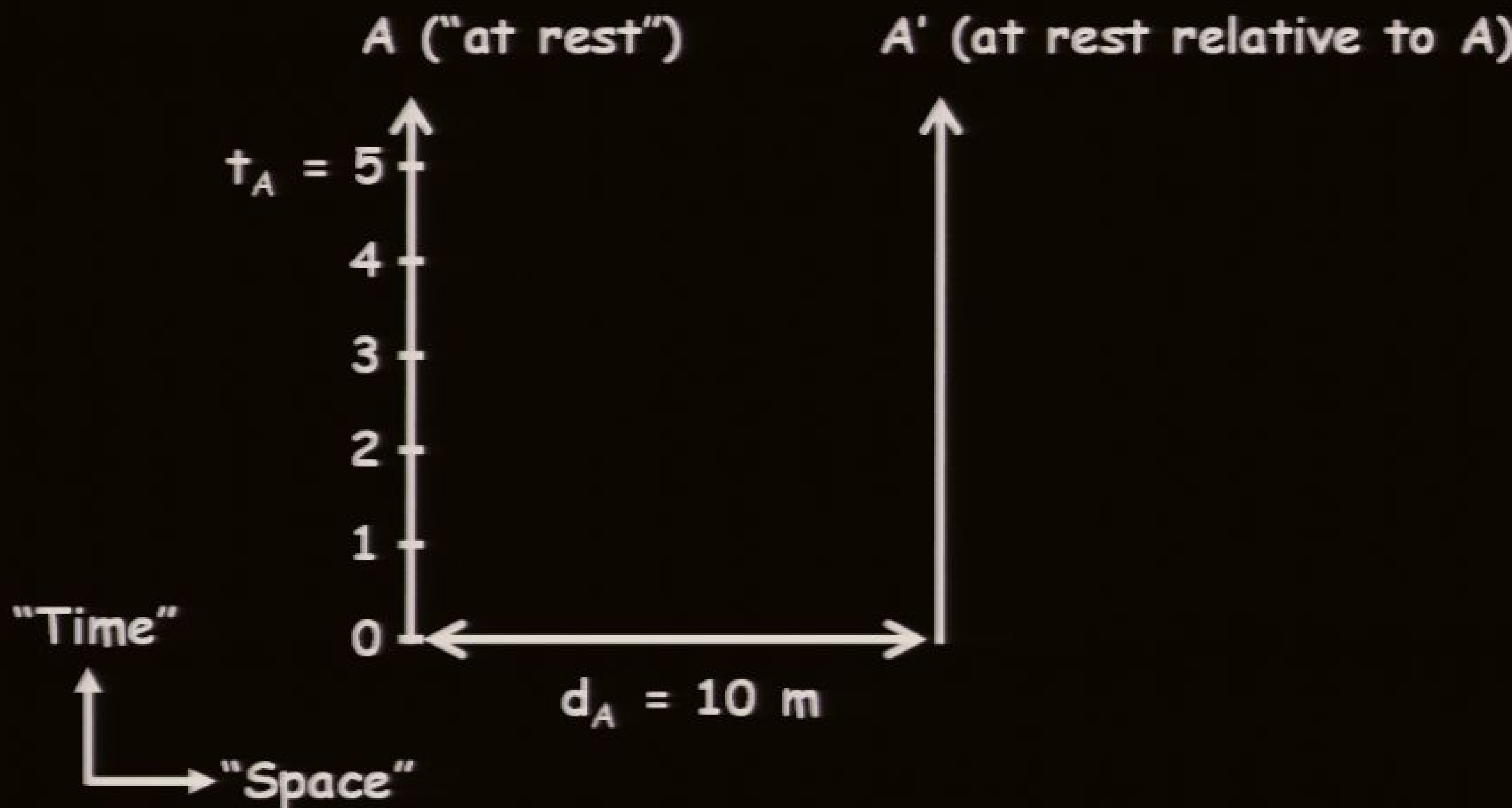

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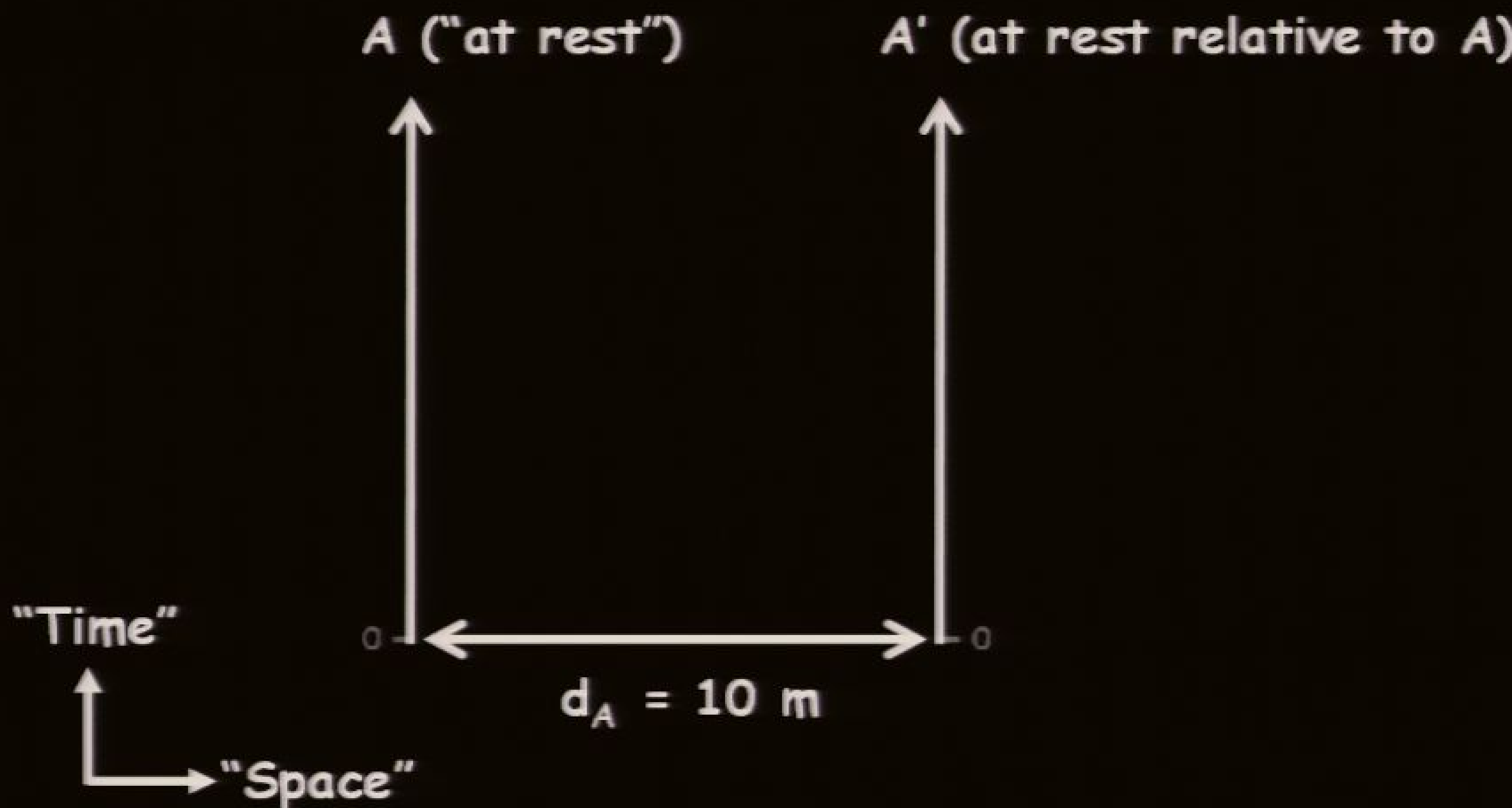

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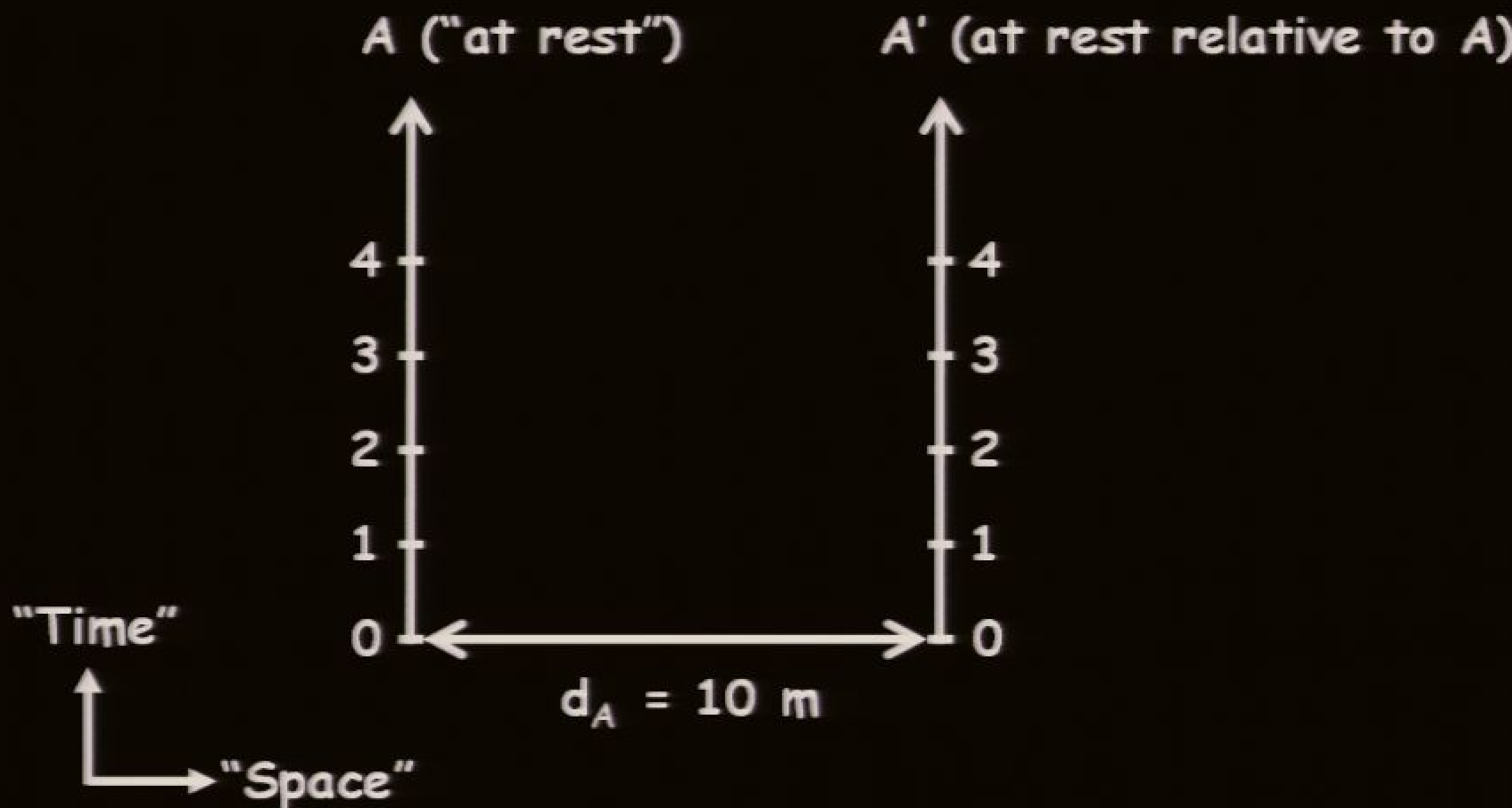

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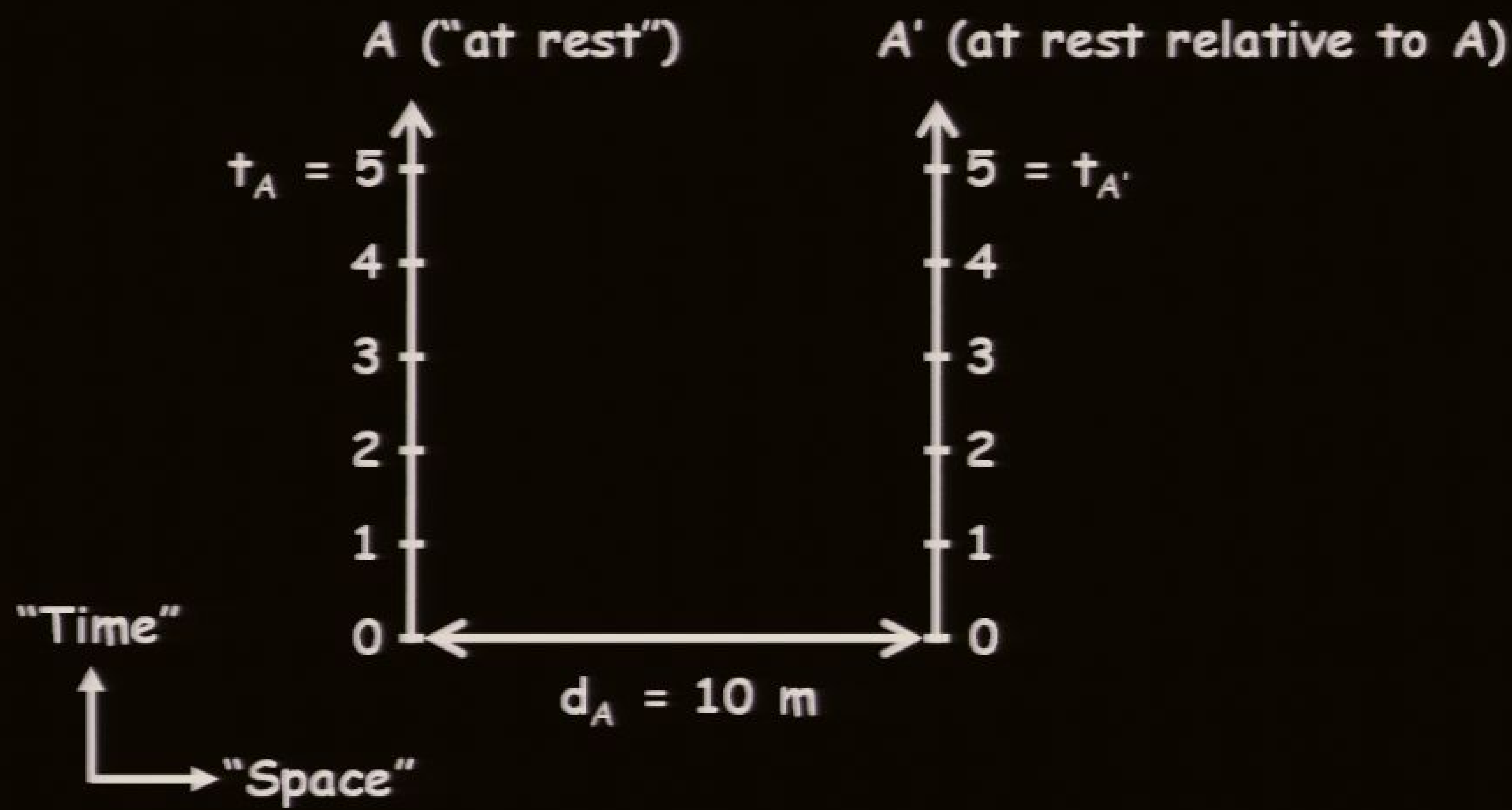

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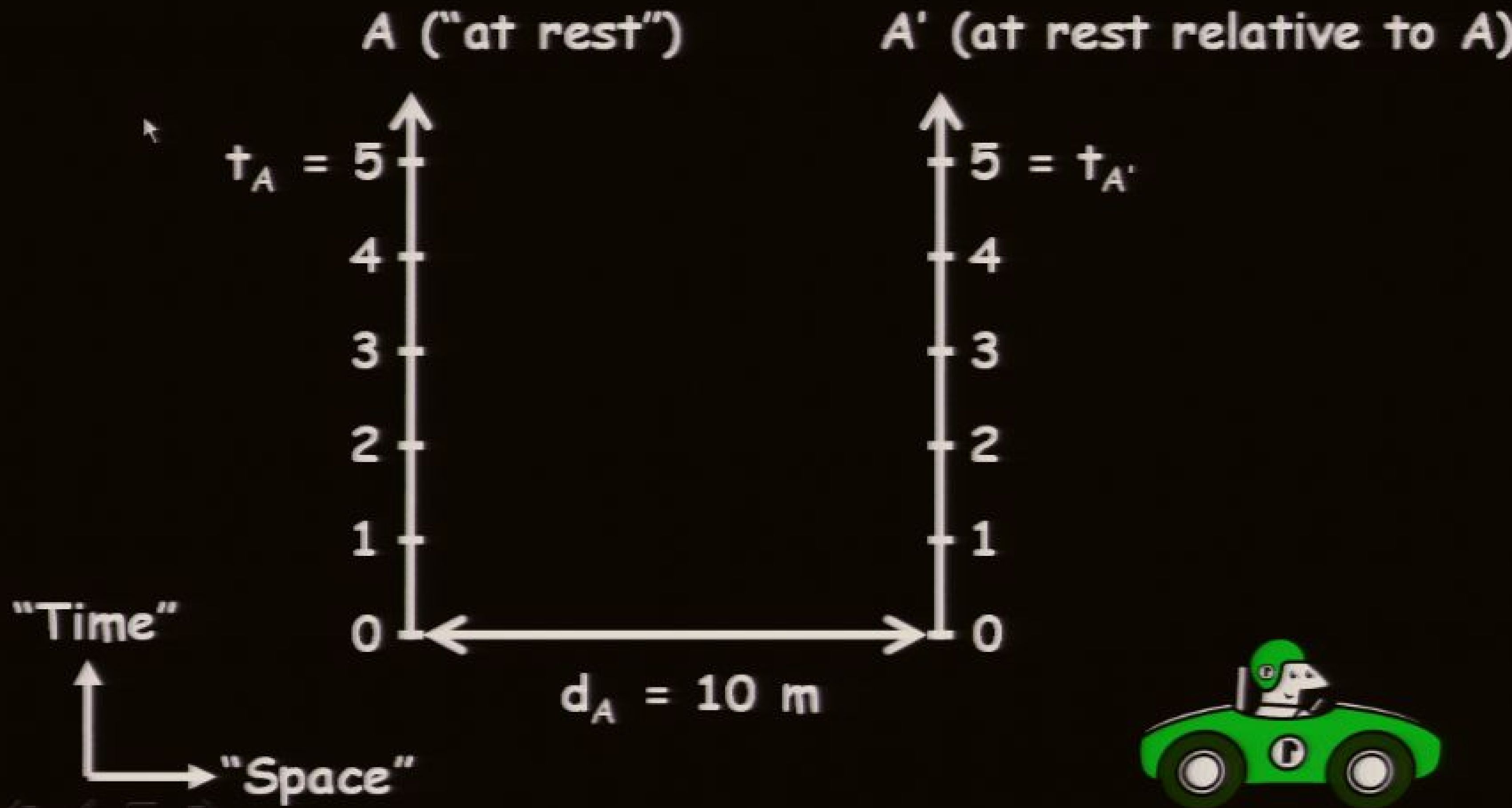

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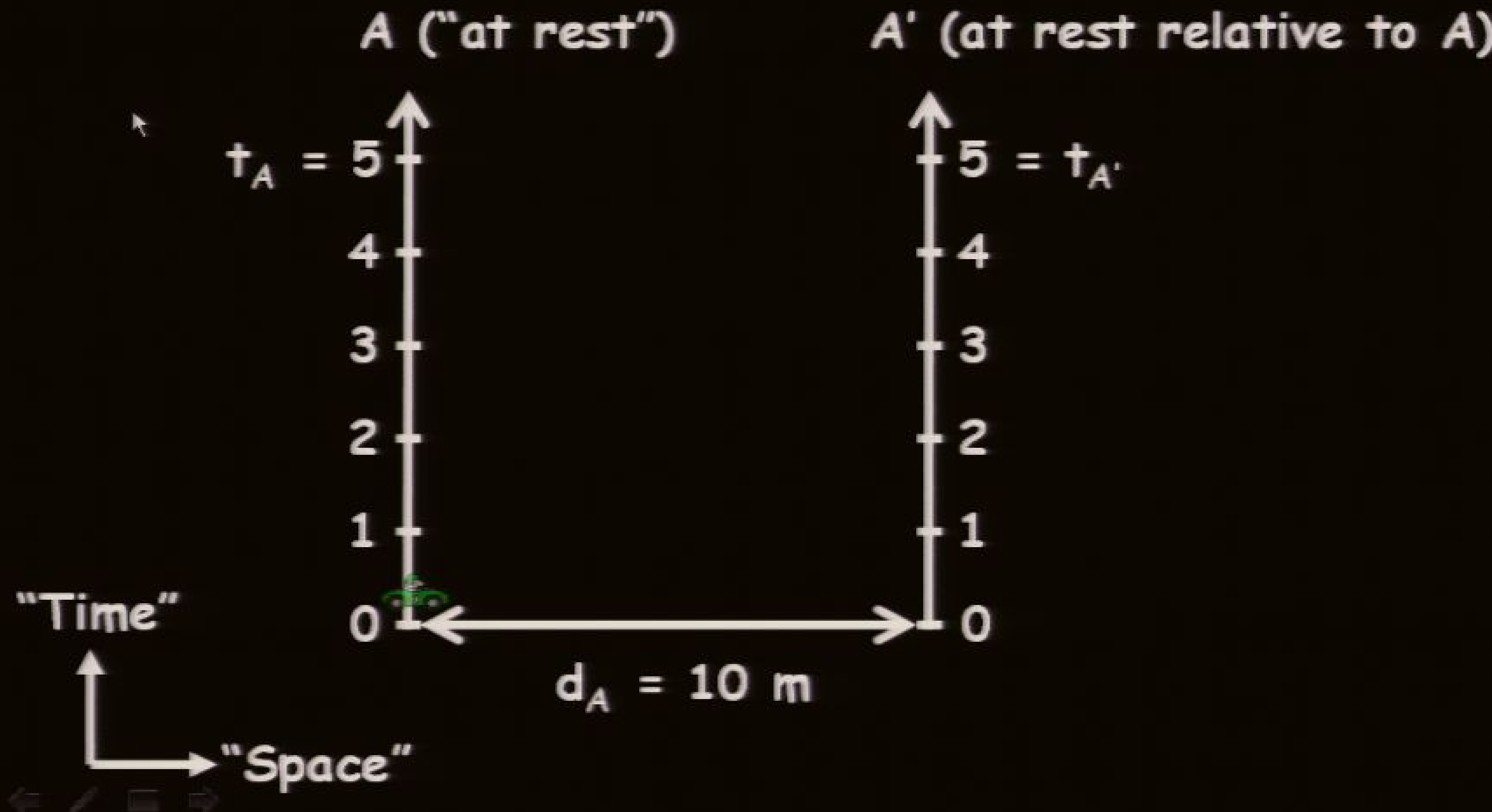

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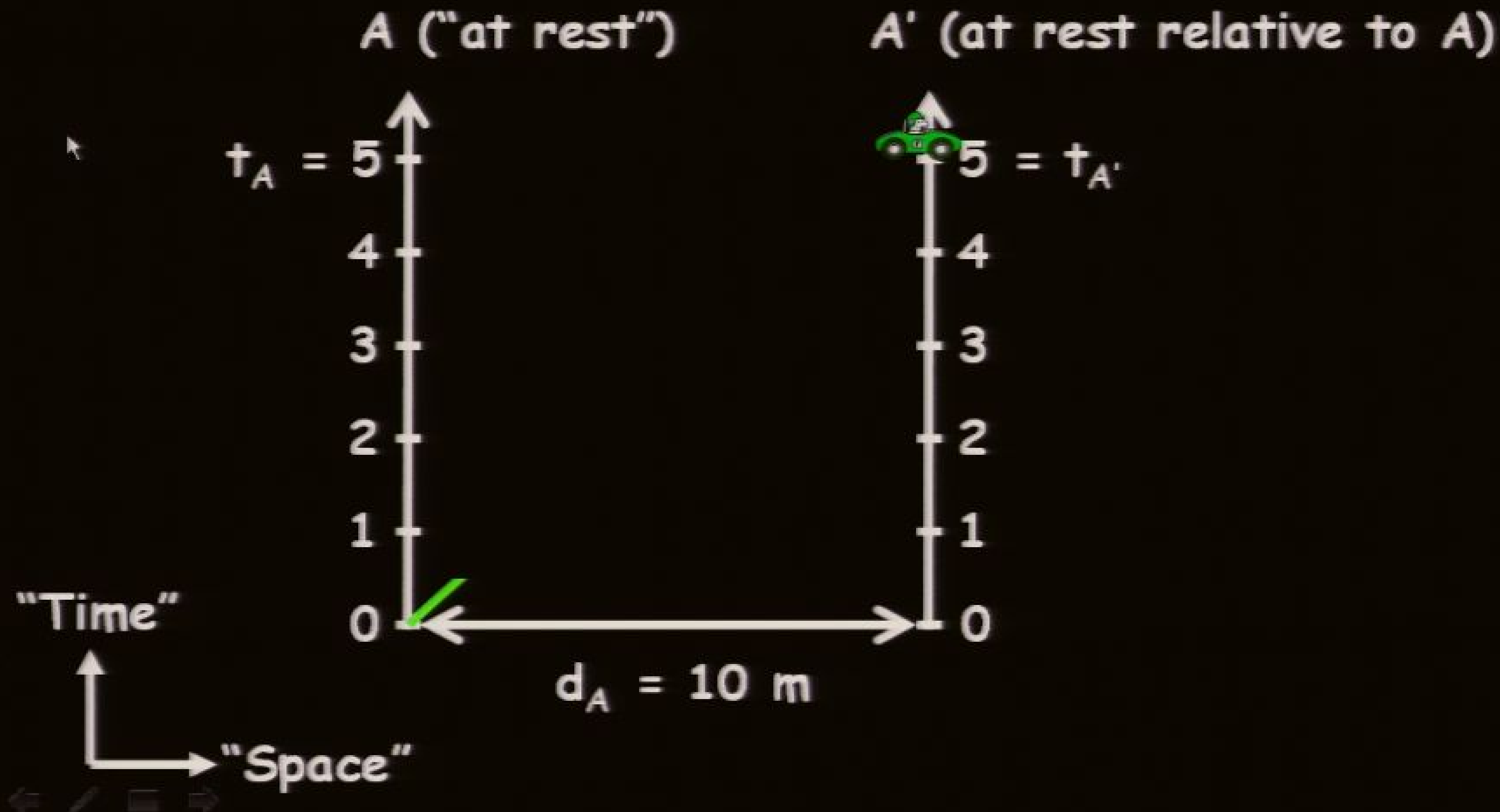

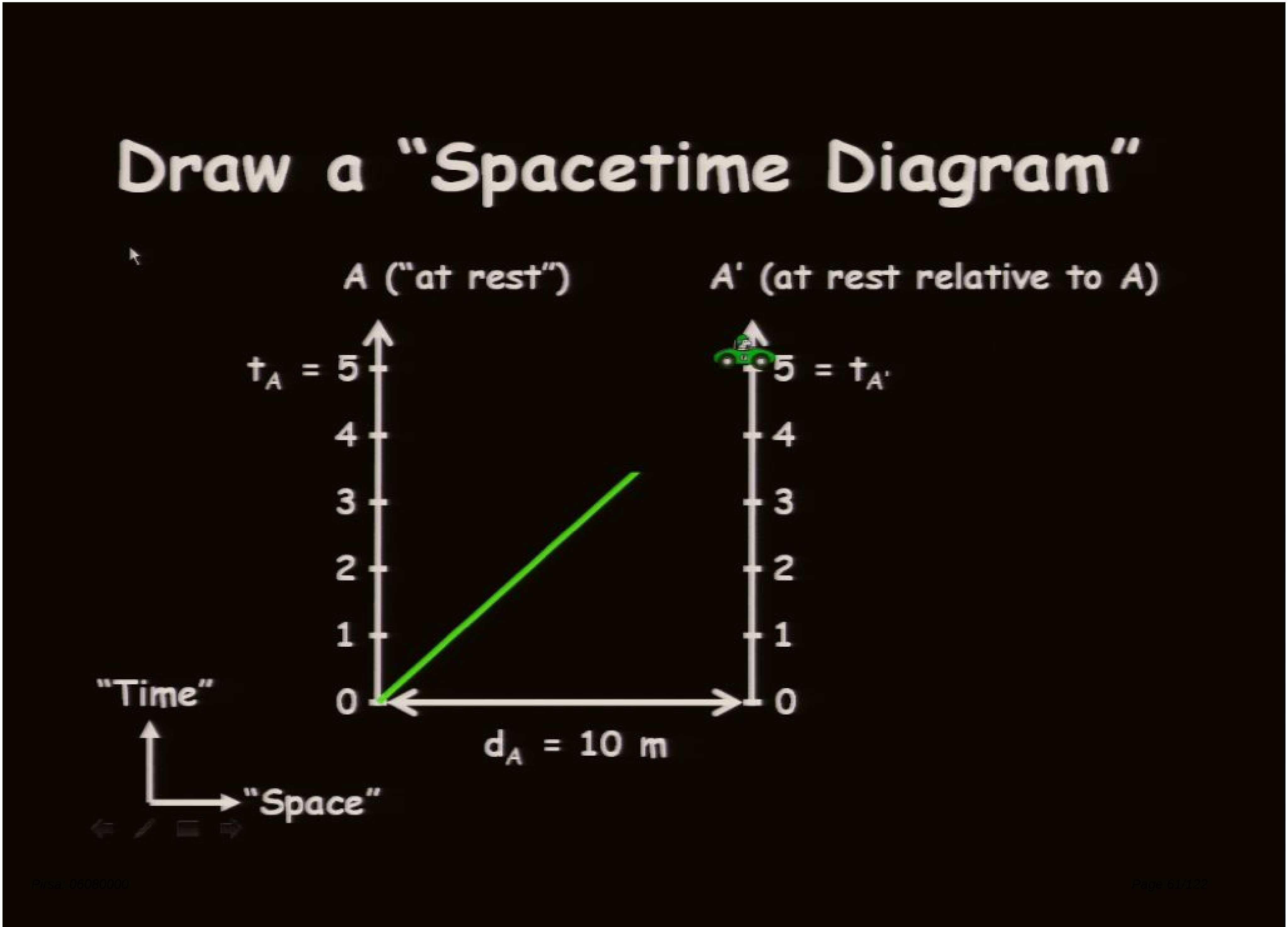
Draw a "Spacetime Diagram"
A ("at rest")
A' (at rest relative to A)
t_A = 5
4
3
2
1
0
5 = $t_{A'}$
4
3
2
1
0
"Time"
"Space"
d_A = 10 m

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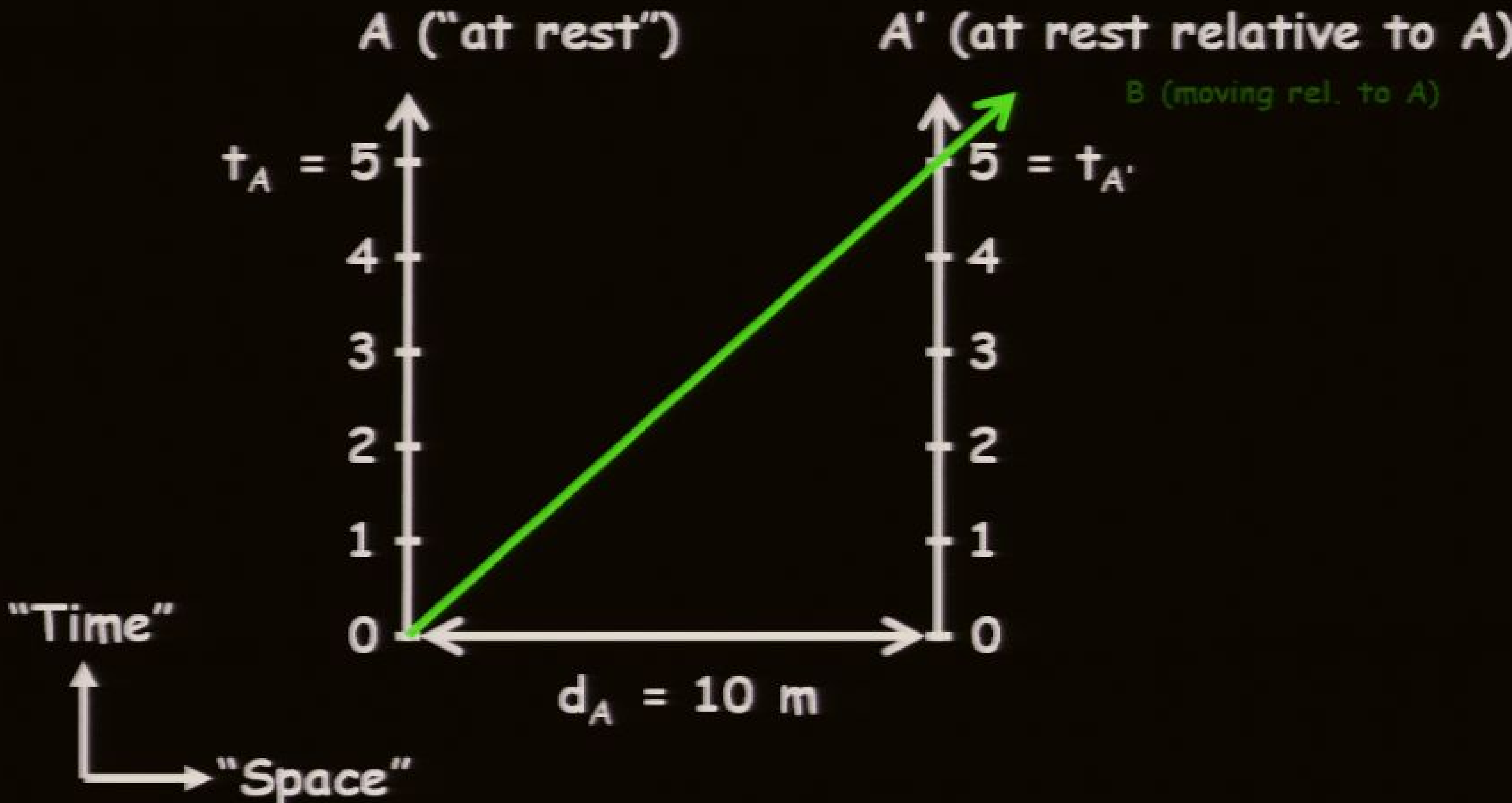

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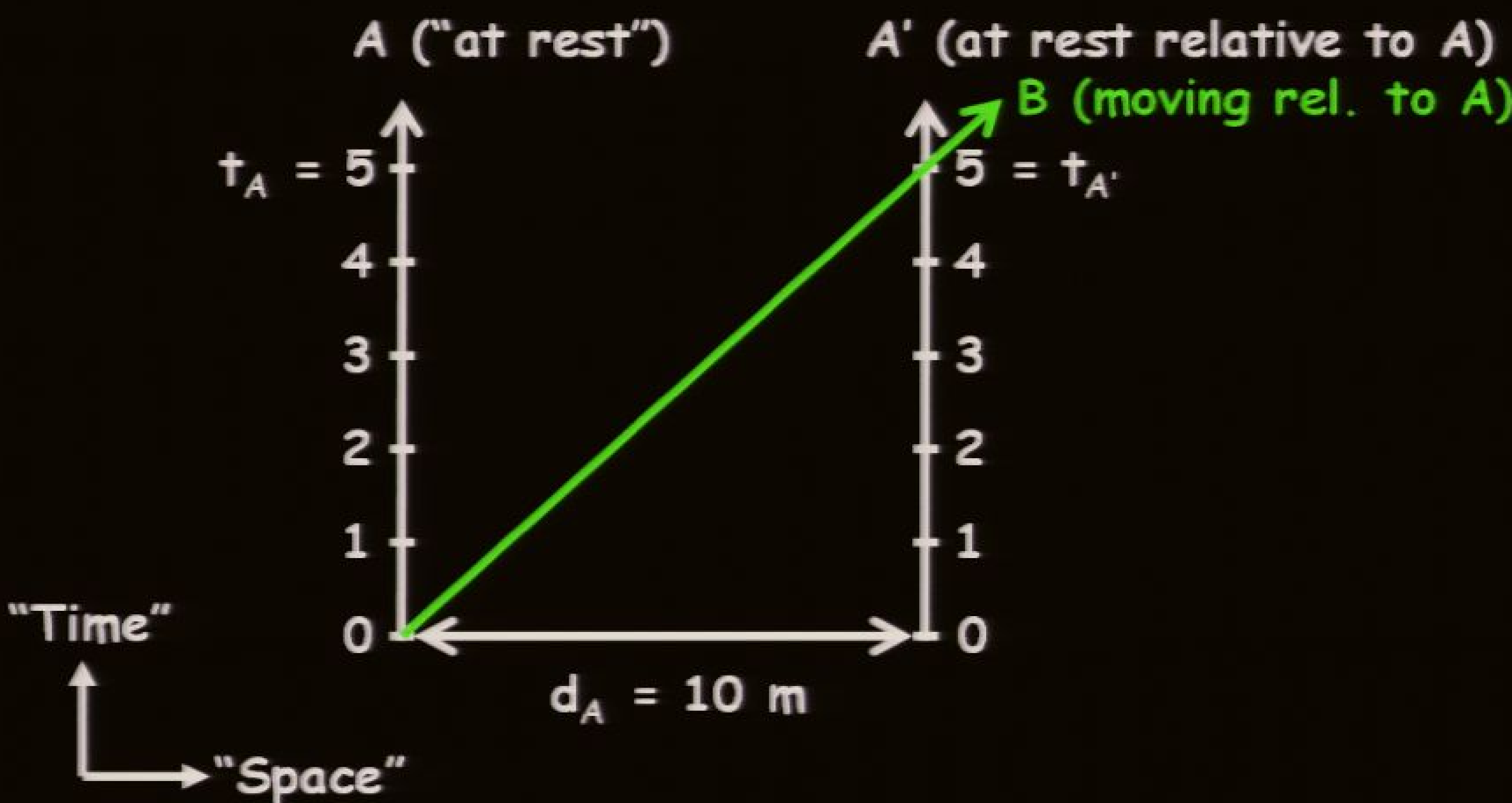

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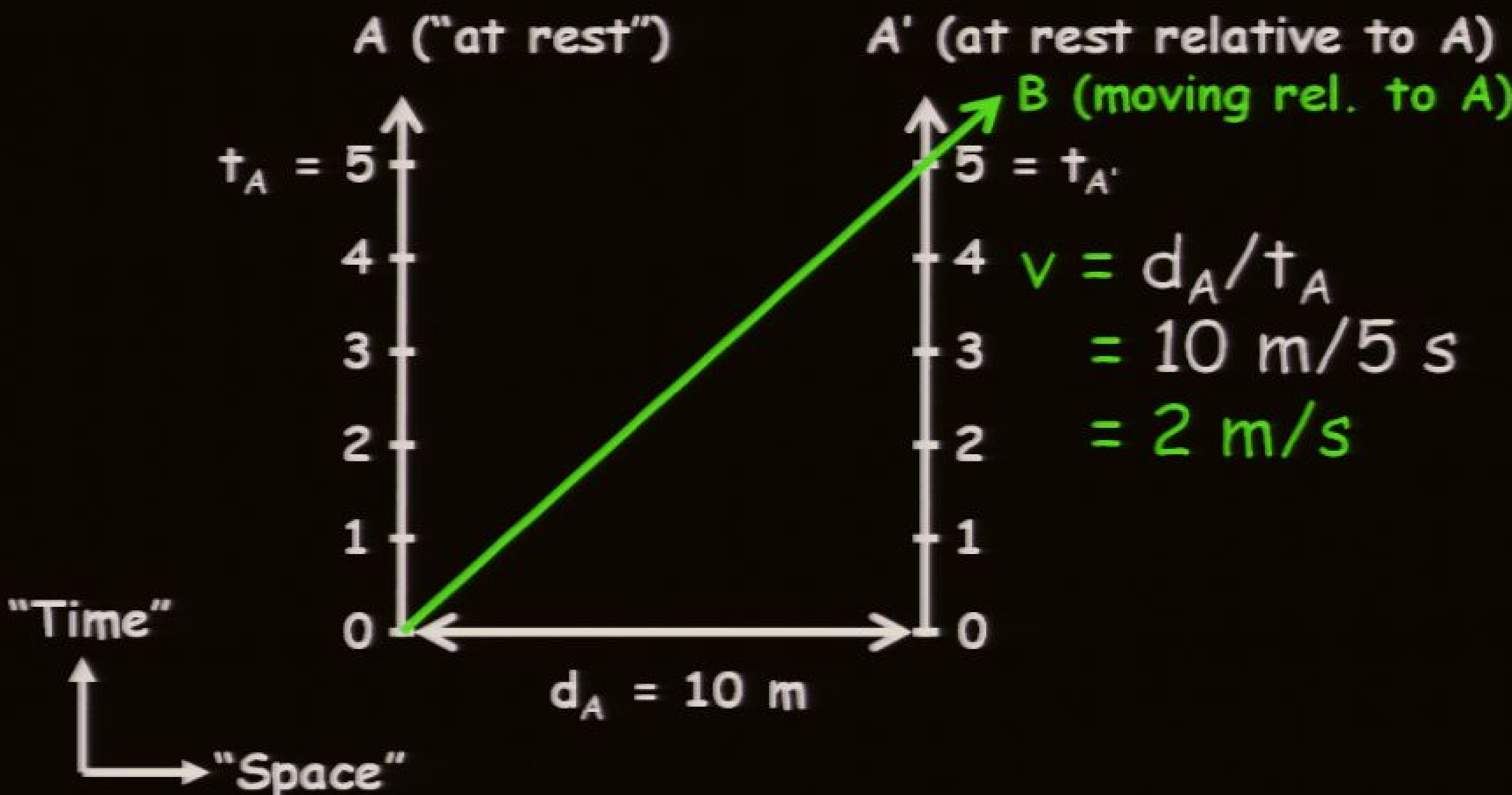

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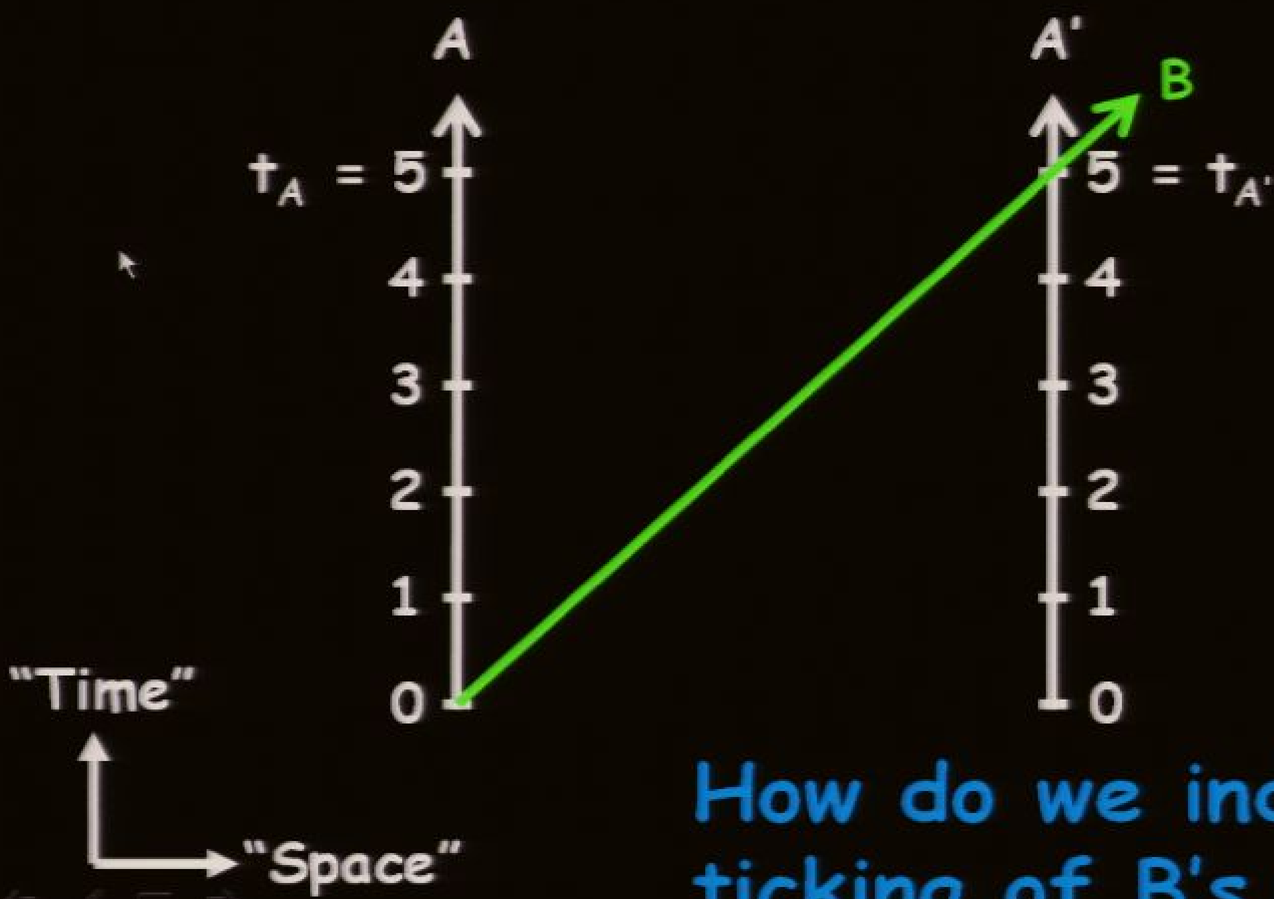
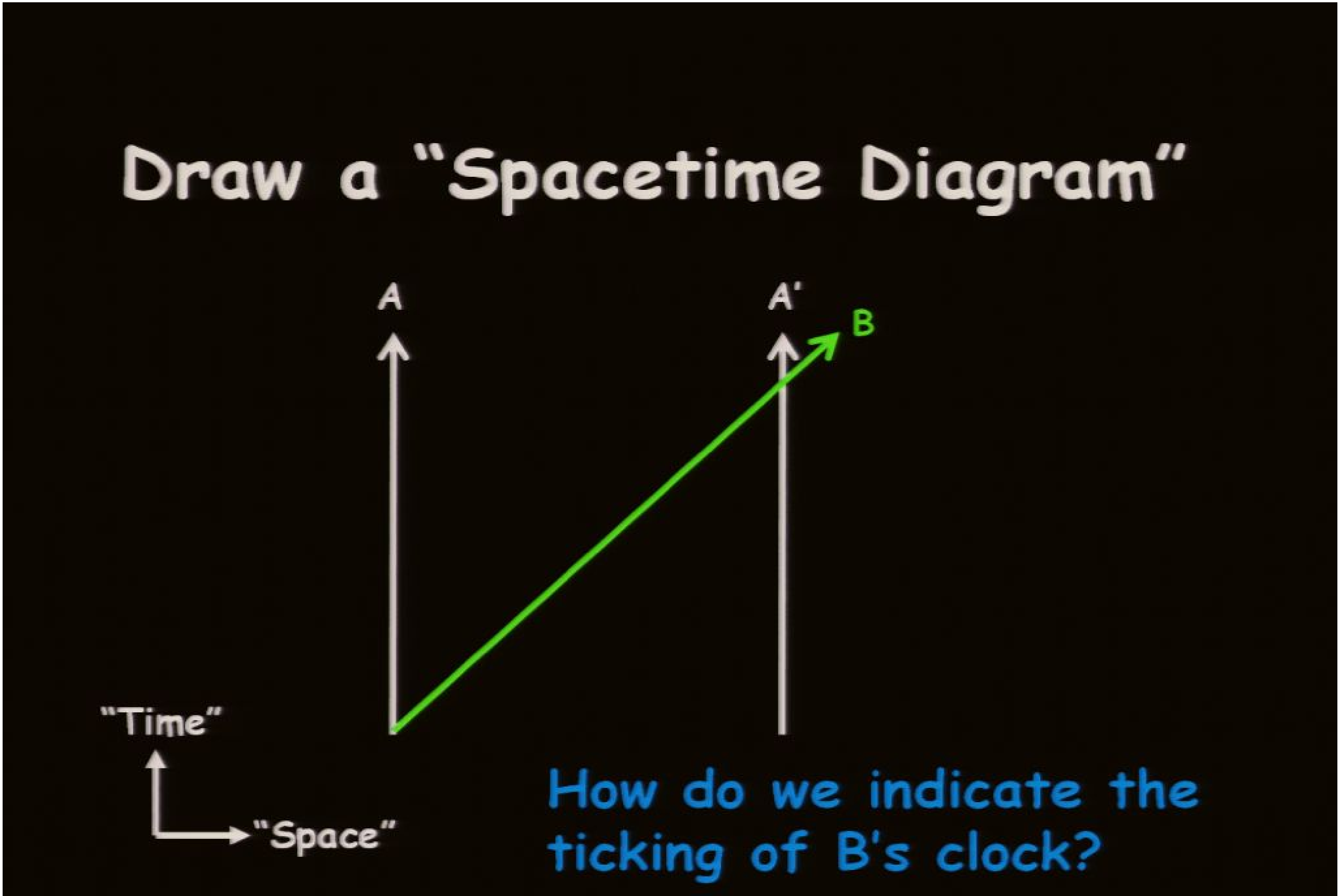


How do we indicate the ticking of B's clock?


Draw a "Spacetime Diagram"
A
A'
B
"Time"
"Space"
How do we indicate the
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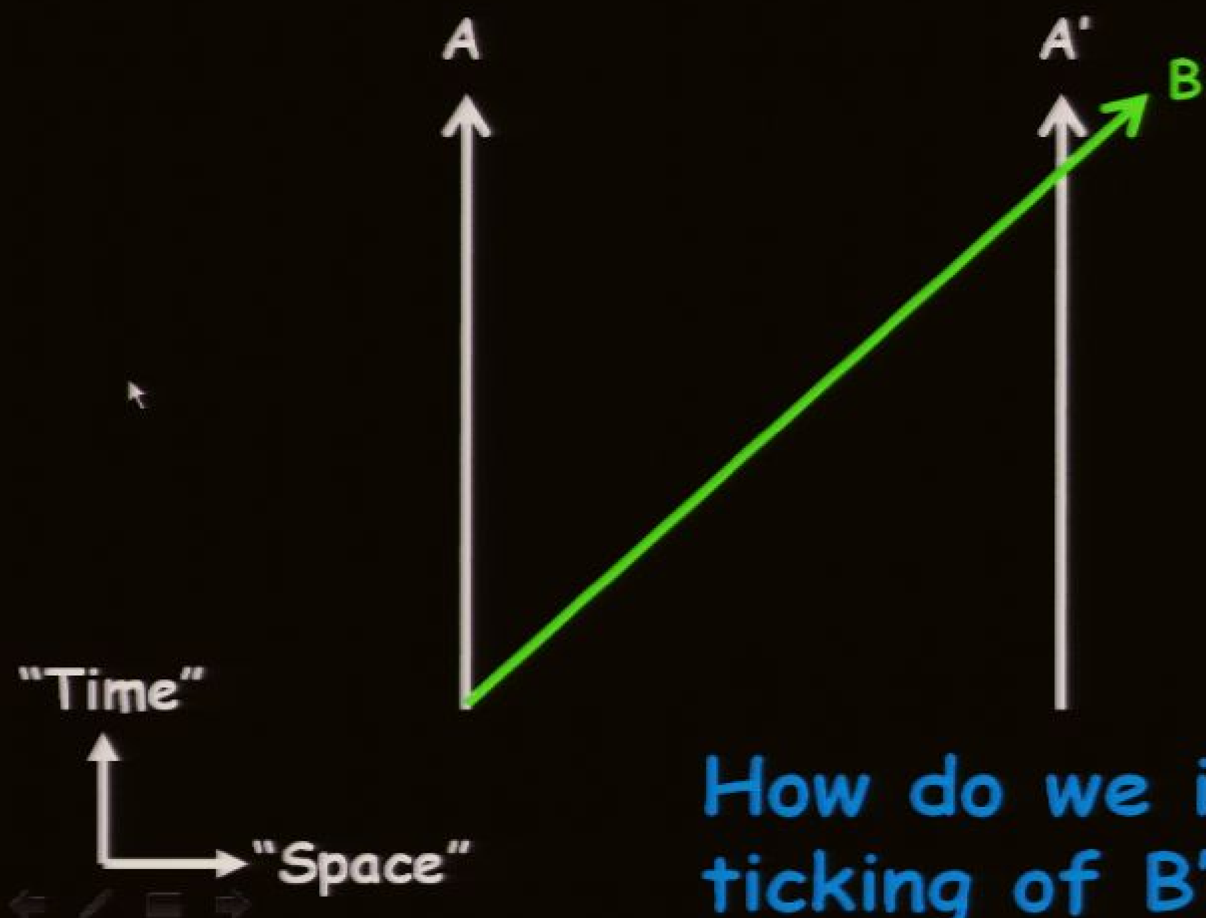

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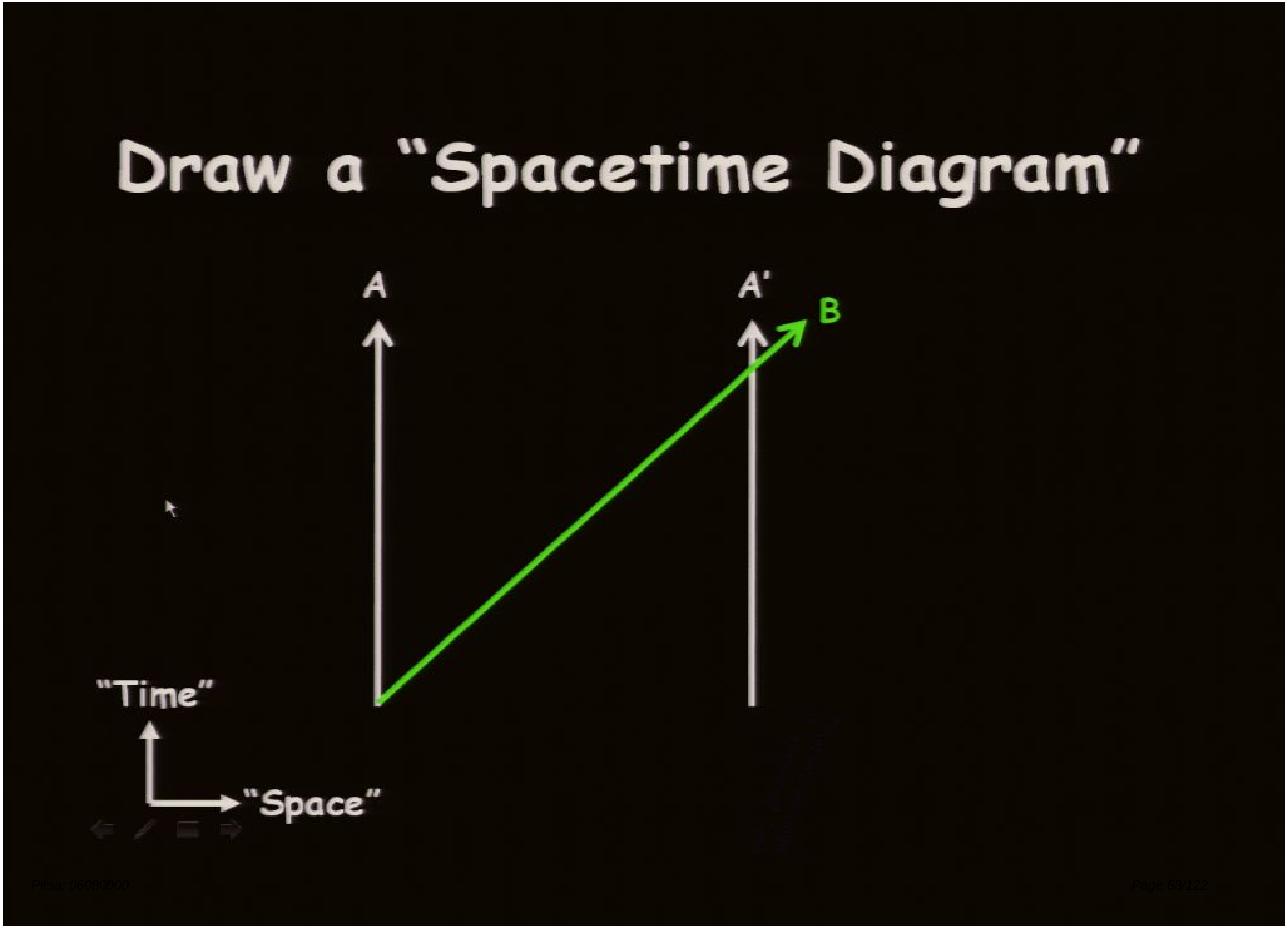
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A
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"Time"
"Space"

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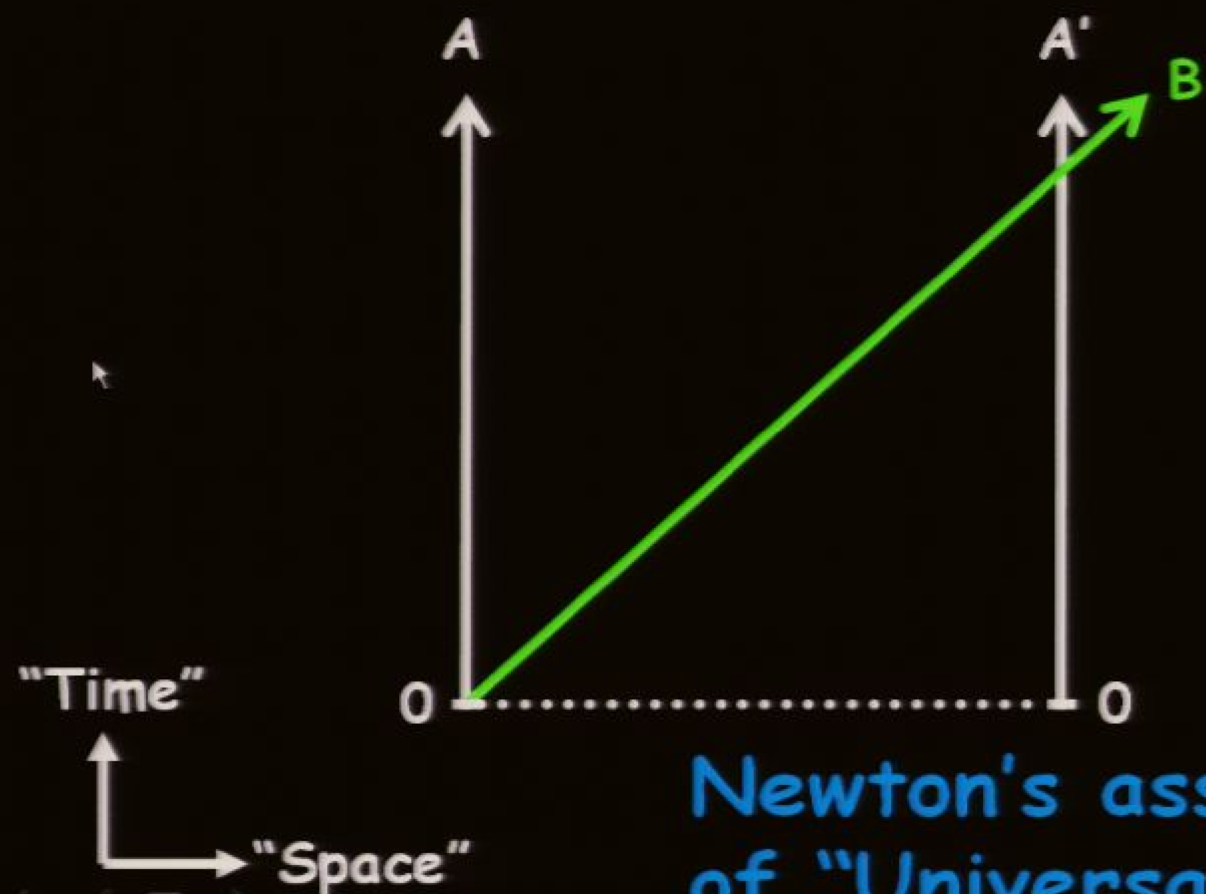

Newton's assumption
of "Universal Time"

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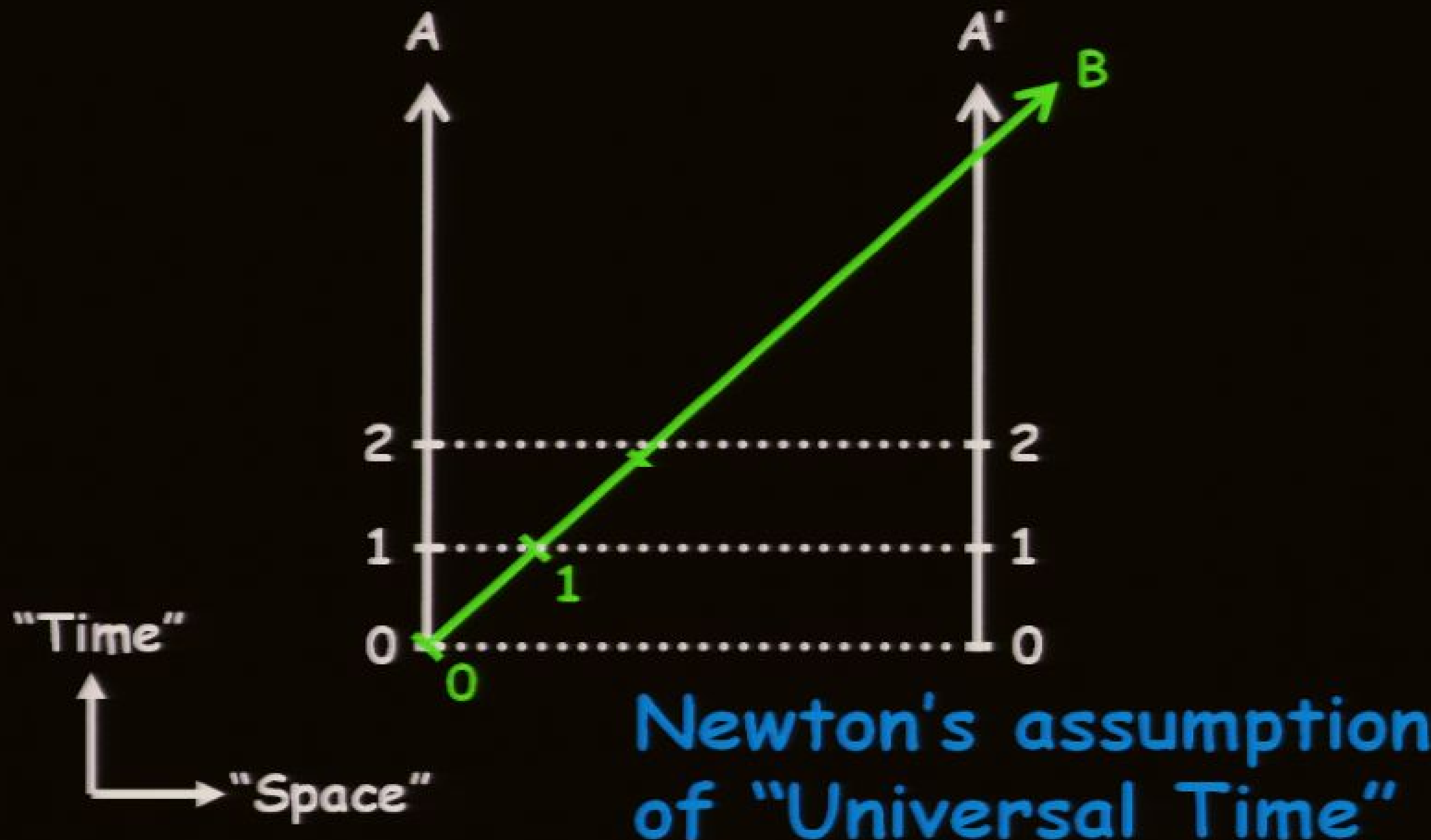

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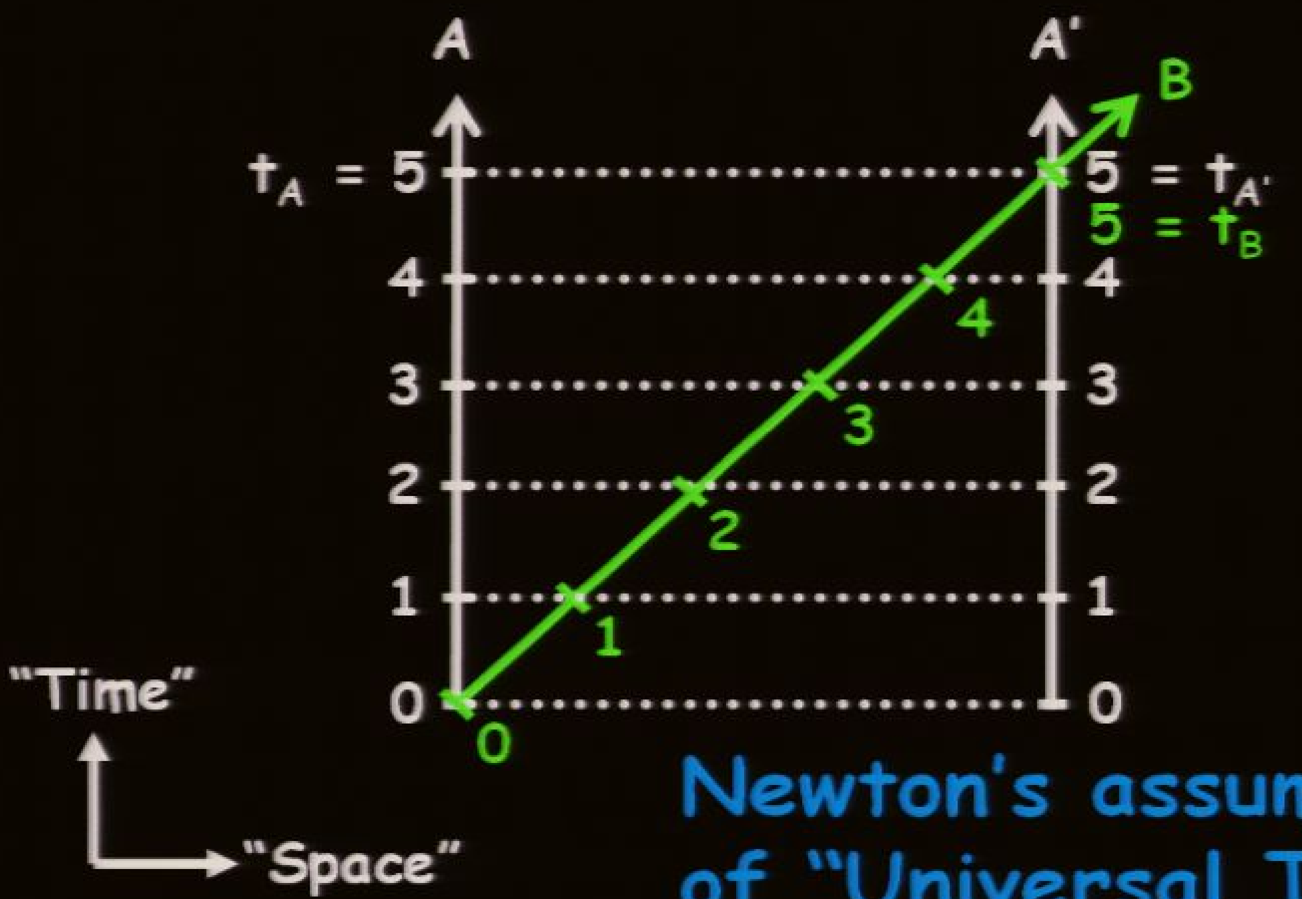


Newton's assumption of "Universal Time"

Let's Have Spacetime Fun!

Sketch spacetime diagrams for each:

t rest relative to Alice

tossing a baseball up

noving Fast

noving Slow

th revolving around the Sun

Let's Have Spacetime Fun!

Sketch spacetime diagrams for each:

1: Bob at rest relative to Alice
2: Alice tossing a baseball up
3: Bob moving Fast
4: Bob moving Slow
5: The Earth revolving around the Sun

Let's Have Spacetime Fun!

Sketch spacetime diagrams for each:

1: Bob at rest relative to Alice

2: Alice tossing a baseball up

3: Bob moving Fast

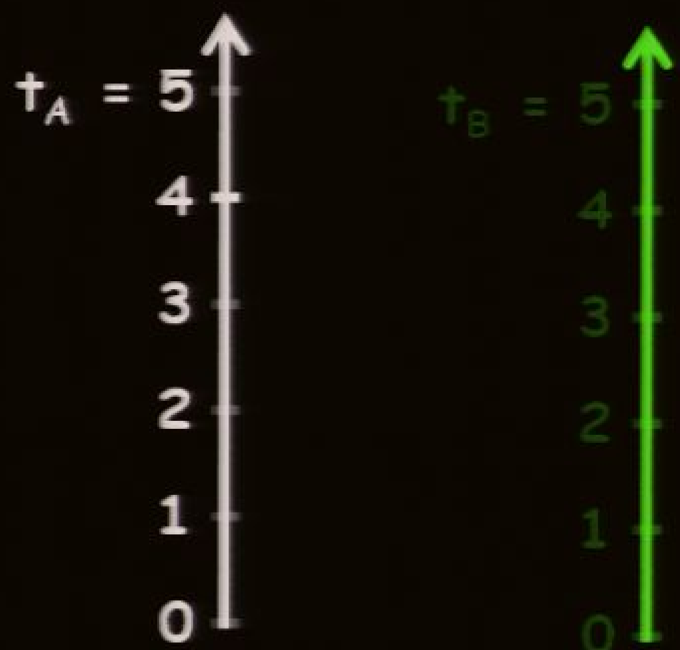
4: Bob moving Slow

5: The Earth revolving around the Sun

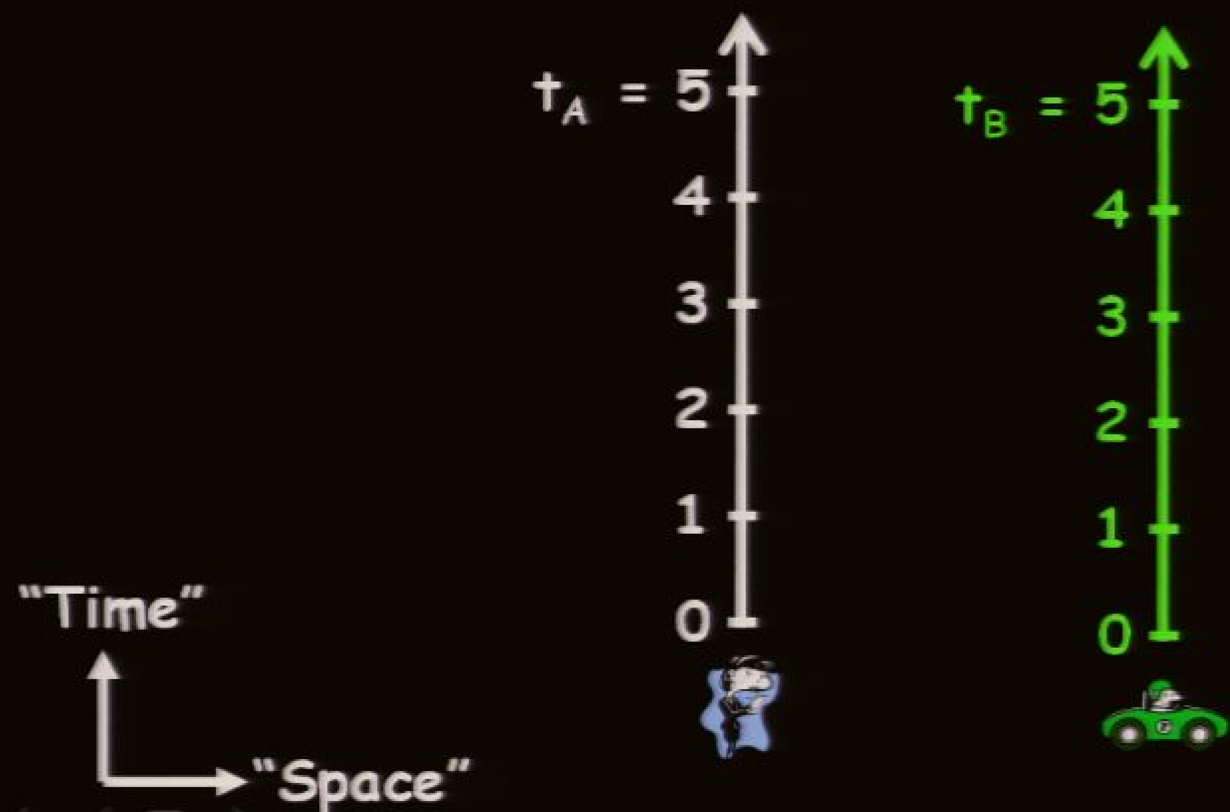
Bob at res

Bob at rest relative to Alice

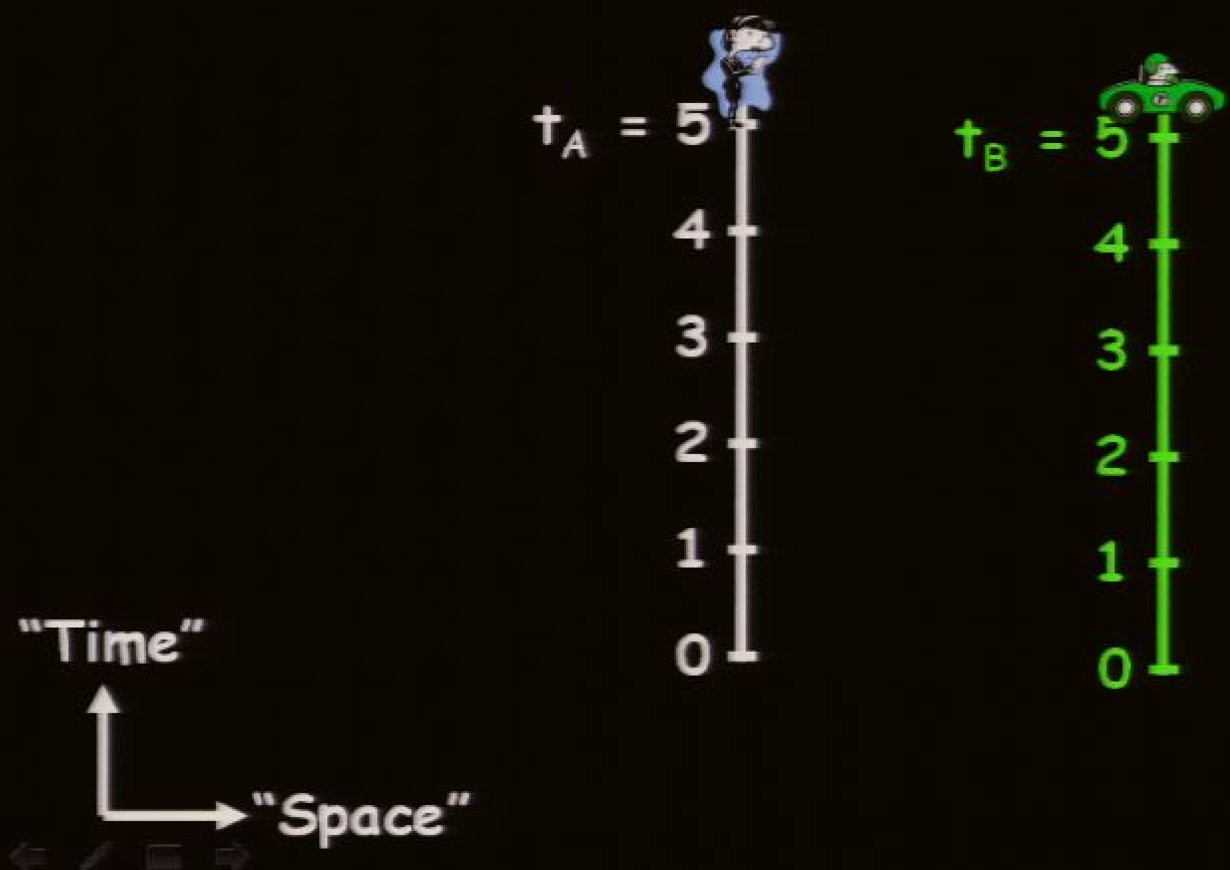
Bob at rest relative to Alice



Bob at rest relative to Alice

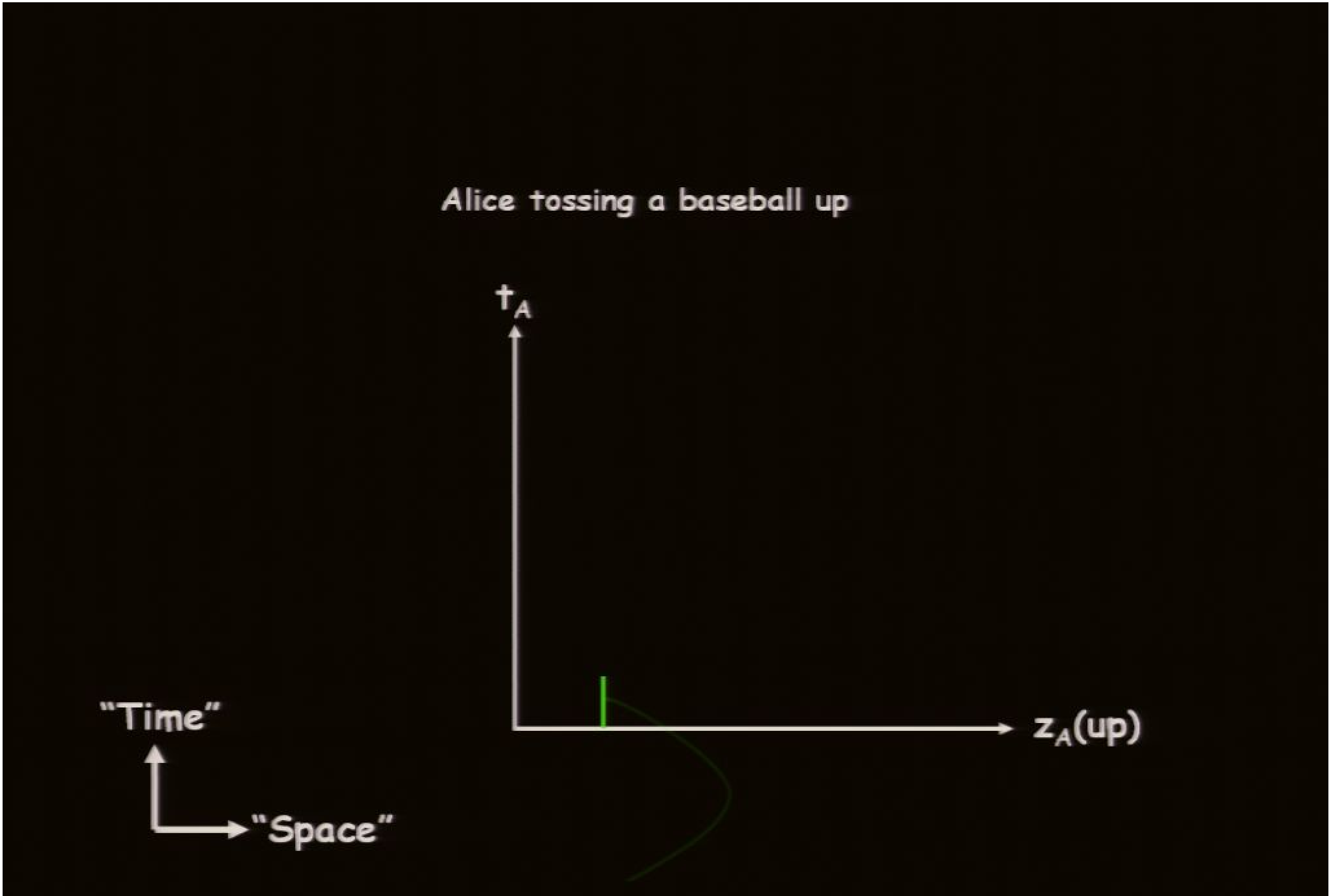


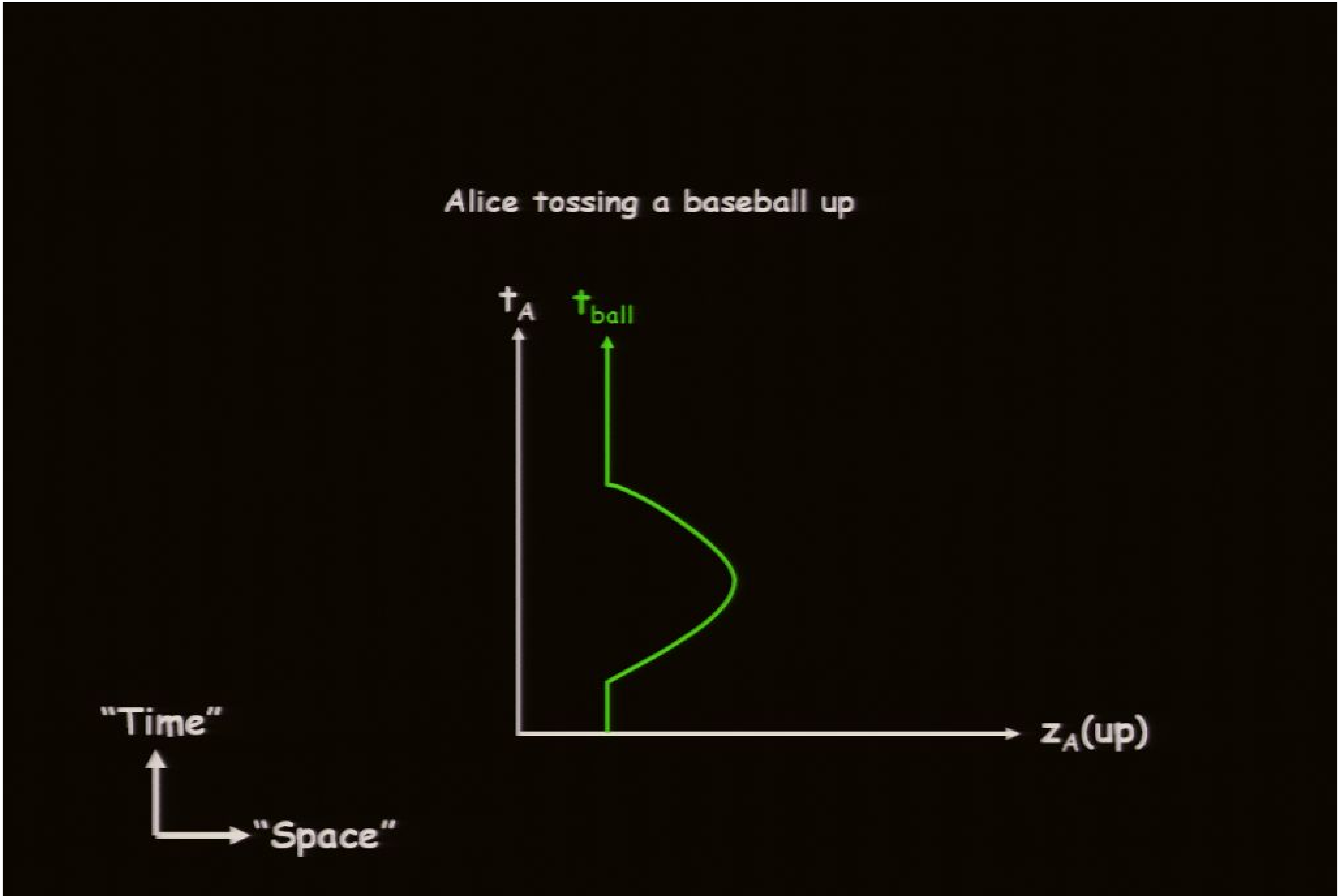
Bob at rest relative to Alice

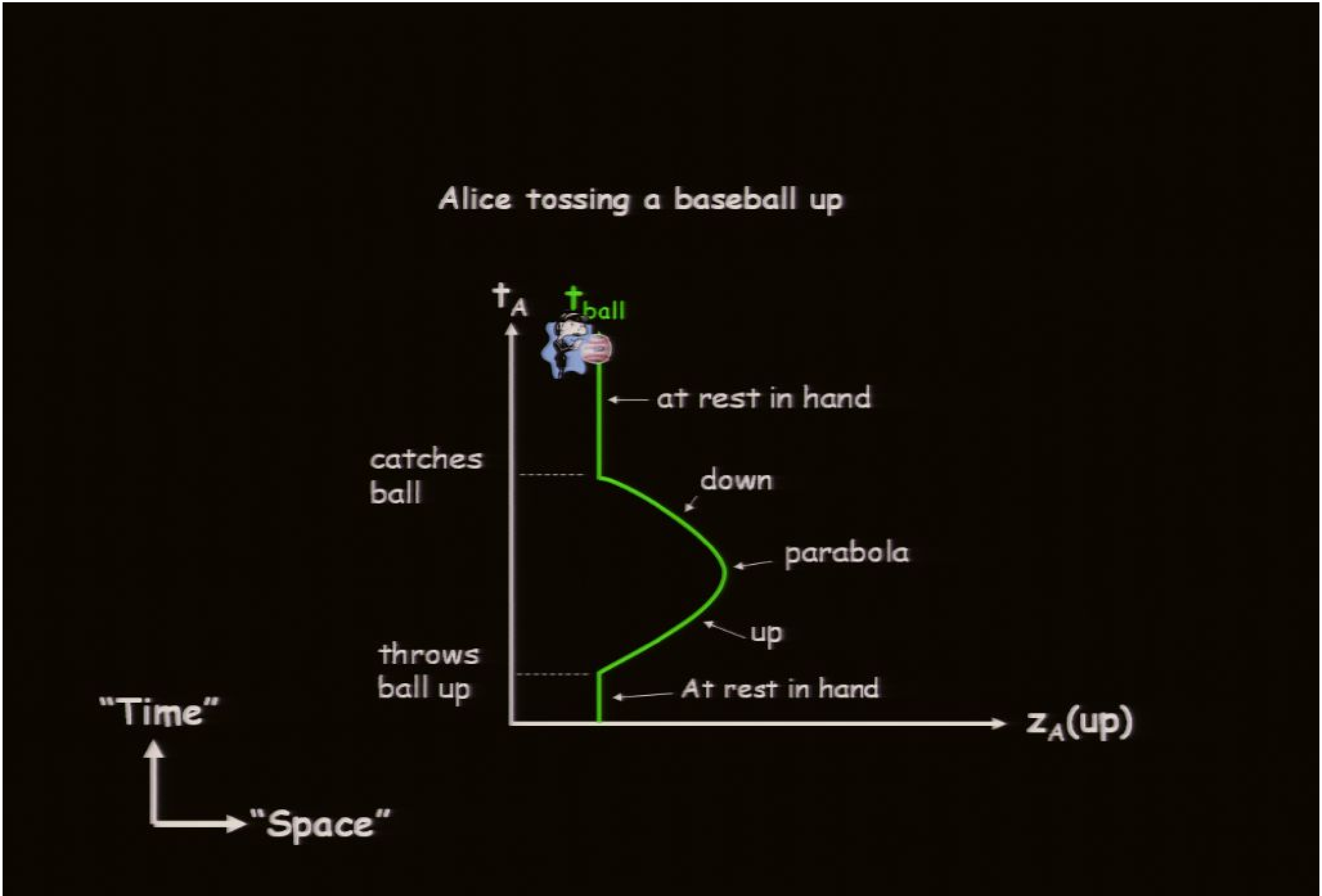


Alice tossing a baseball up

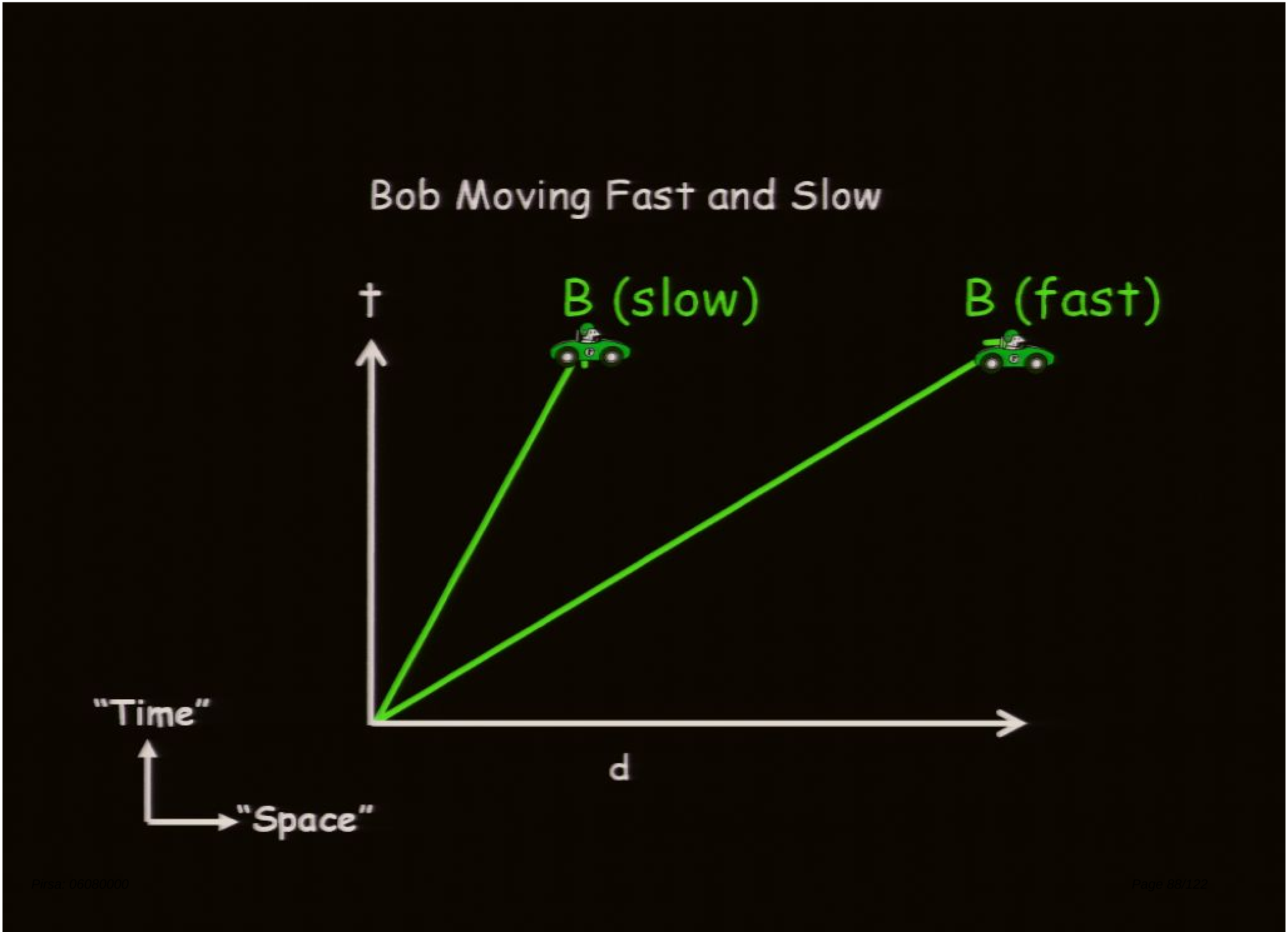
Alice tossing a baseball up


Alice tossing a baseball up
t_A
z_A(up)
"Time"
"Space"

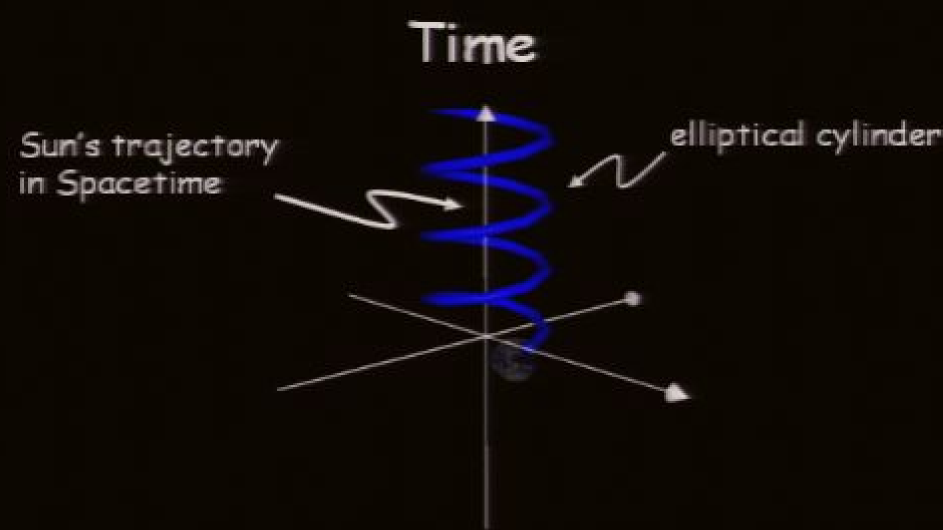

Alice tossing a baseball up
t_A
t_ball
z_A(up)
"Time"
"Space"


Alice tossing a baseball up
t_A
t_{ball}
at rest in hand
catches ball
down
parabola
up
throws ball up
At rest in hand
z_A(up)
"Time"
"Space"

Bob Moving Fast and Slow

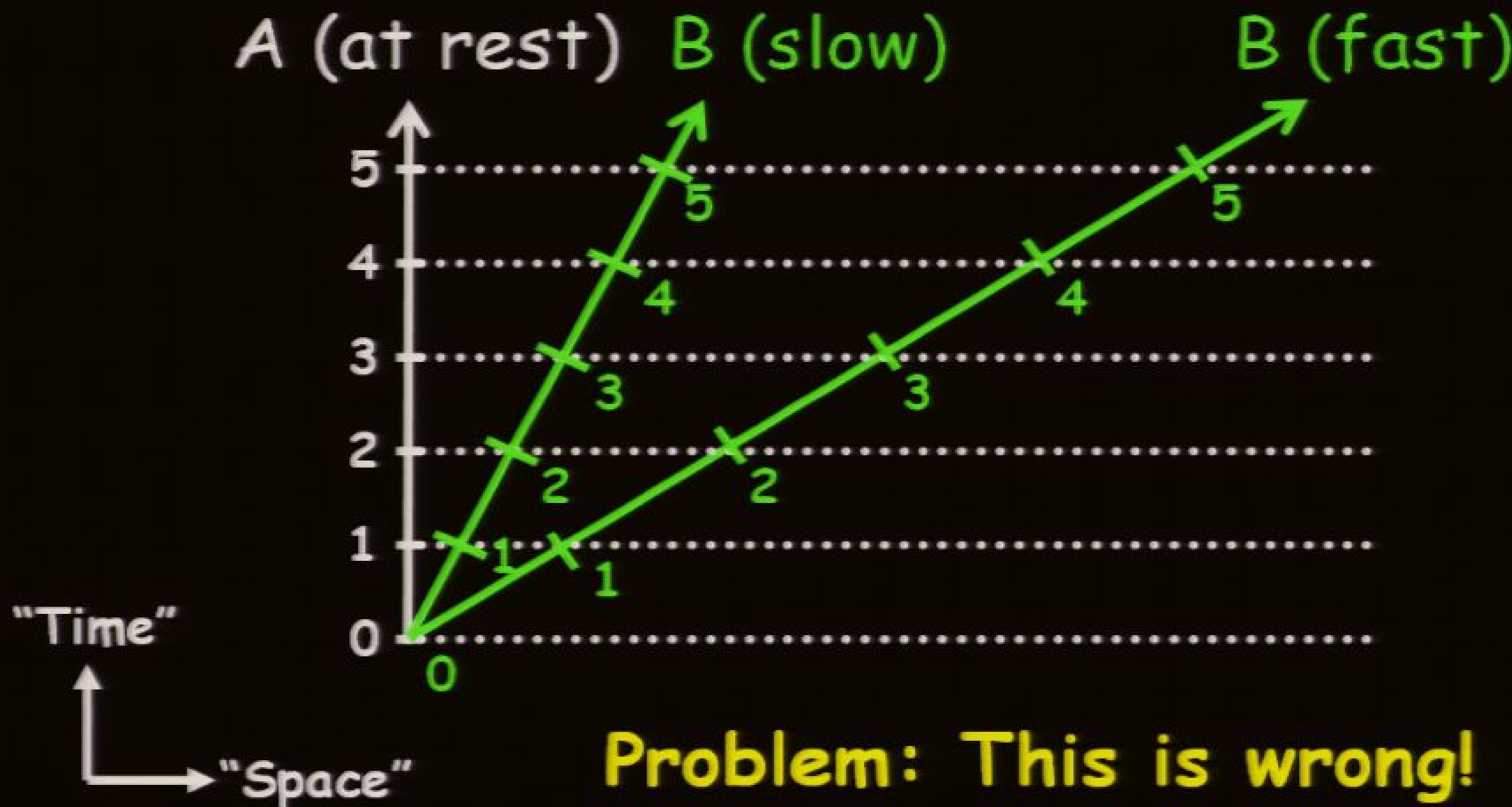

Bob Moving Fast and Slow
t
B (slow)
B (fast)
d
"Time"
"Space"

Earth Revolving Around Sun

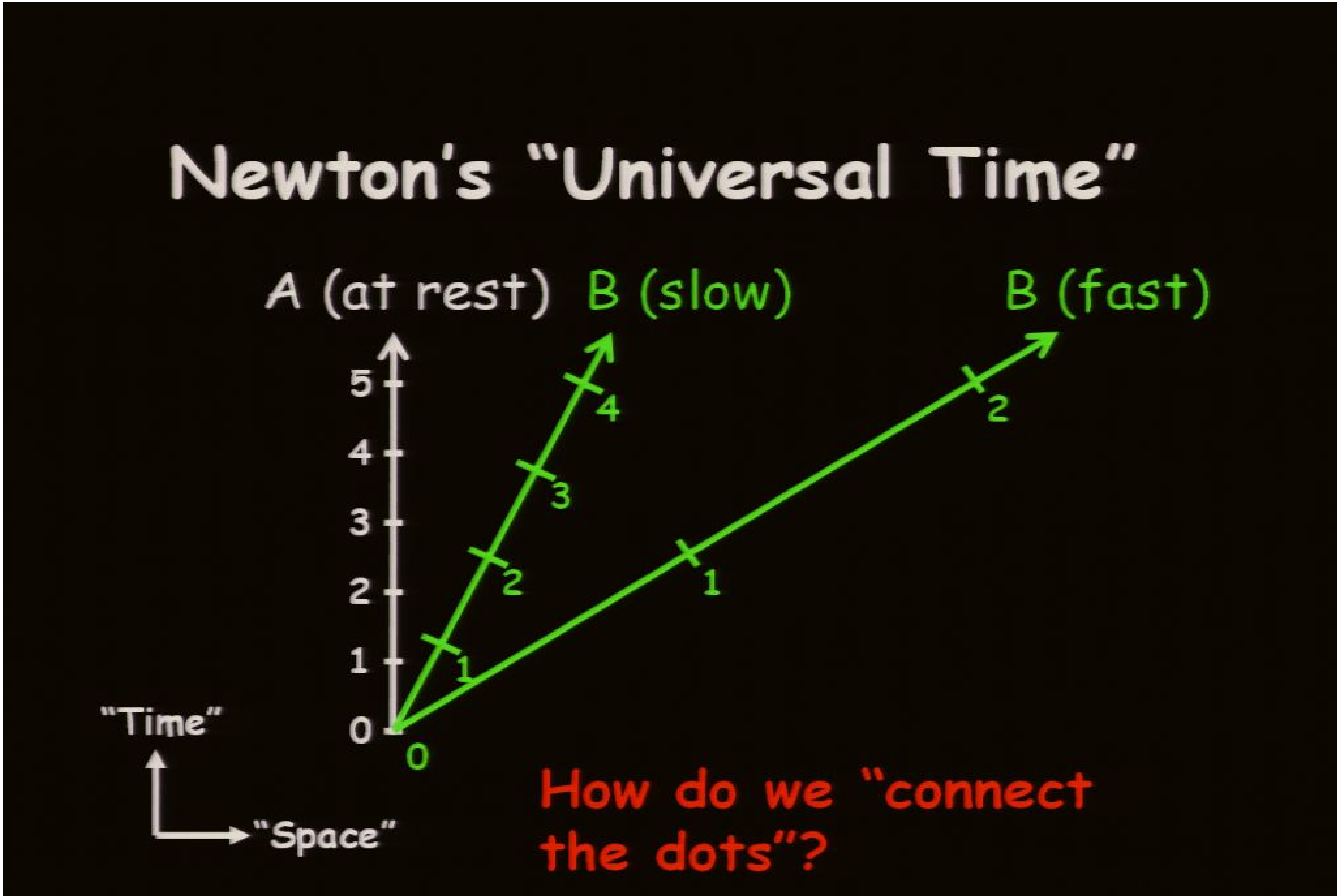


Earth's Trajectory

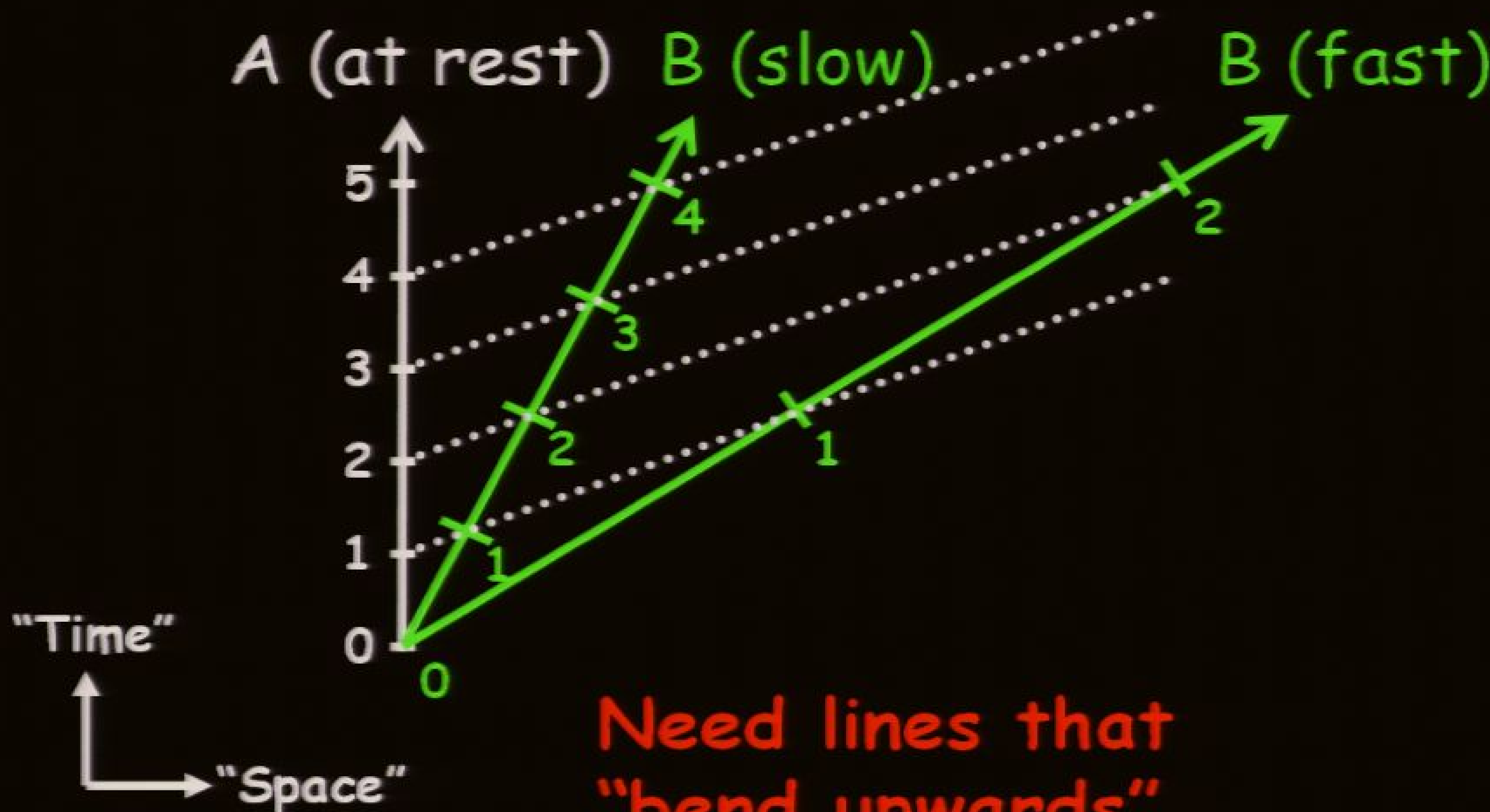
Newton's "Universal Time"



Problem: This is wrong!

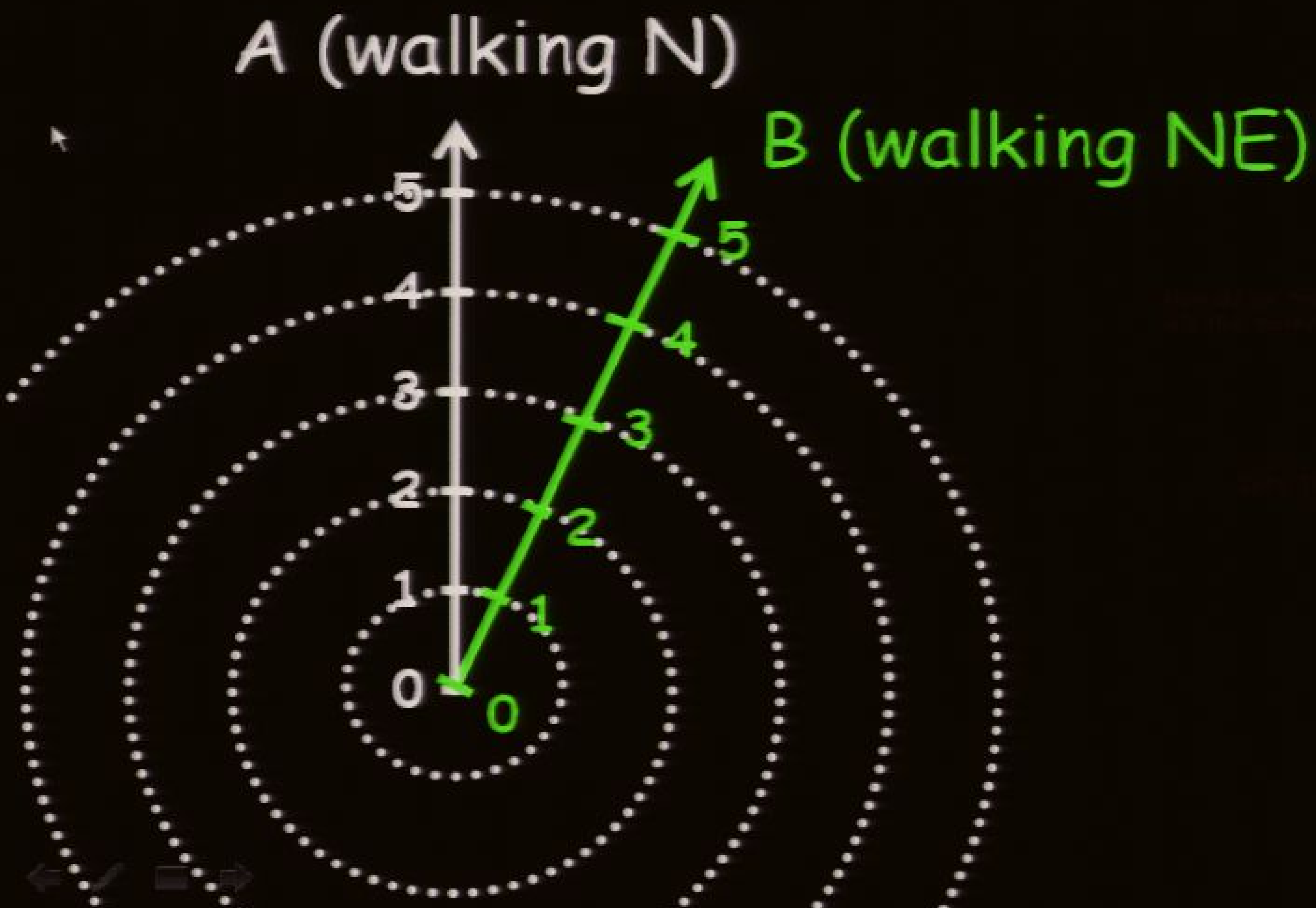

Newton's "Universal Time"
A (at rest)
B (slow)
B (fast)
5
4
3
2
1
0
1
2
3
4
1
2
0
"Time"
"Space"
How do we "connect the dots"?

Newton's "Universal Time"

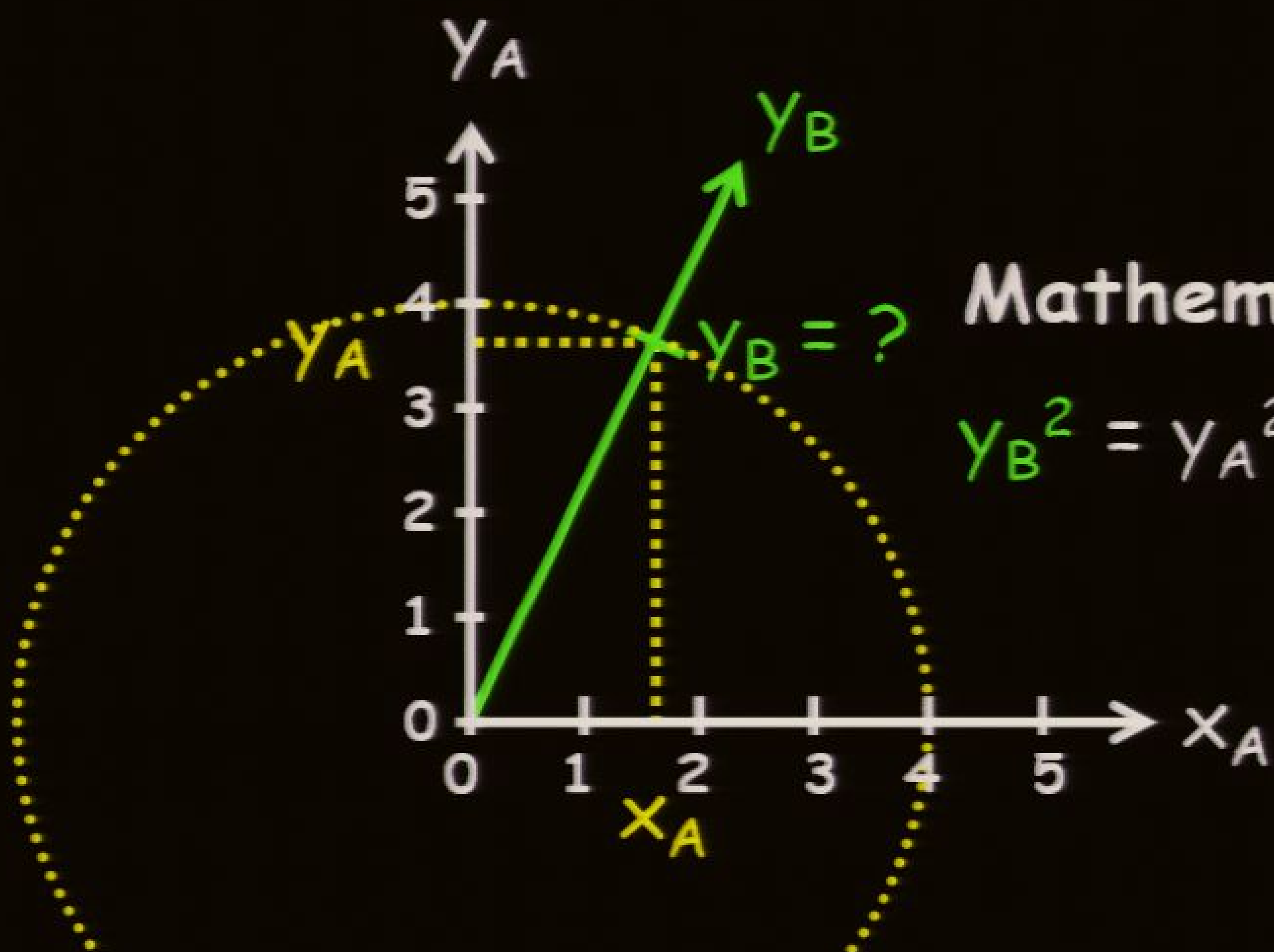


Need lines that
"bend upwards"

The Geometry of Space



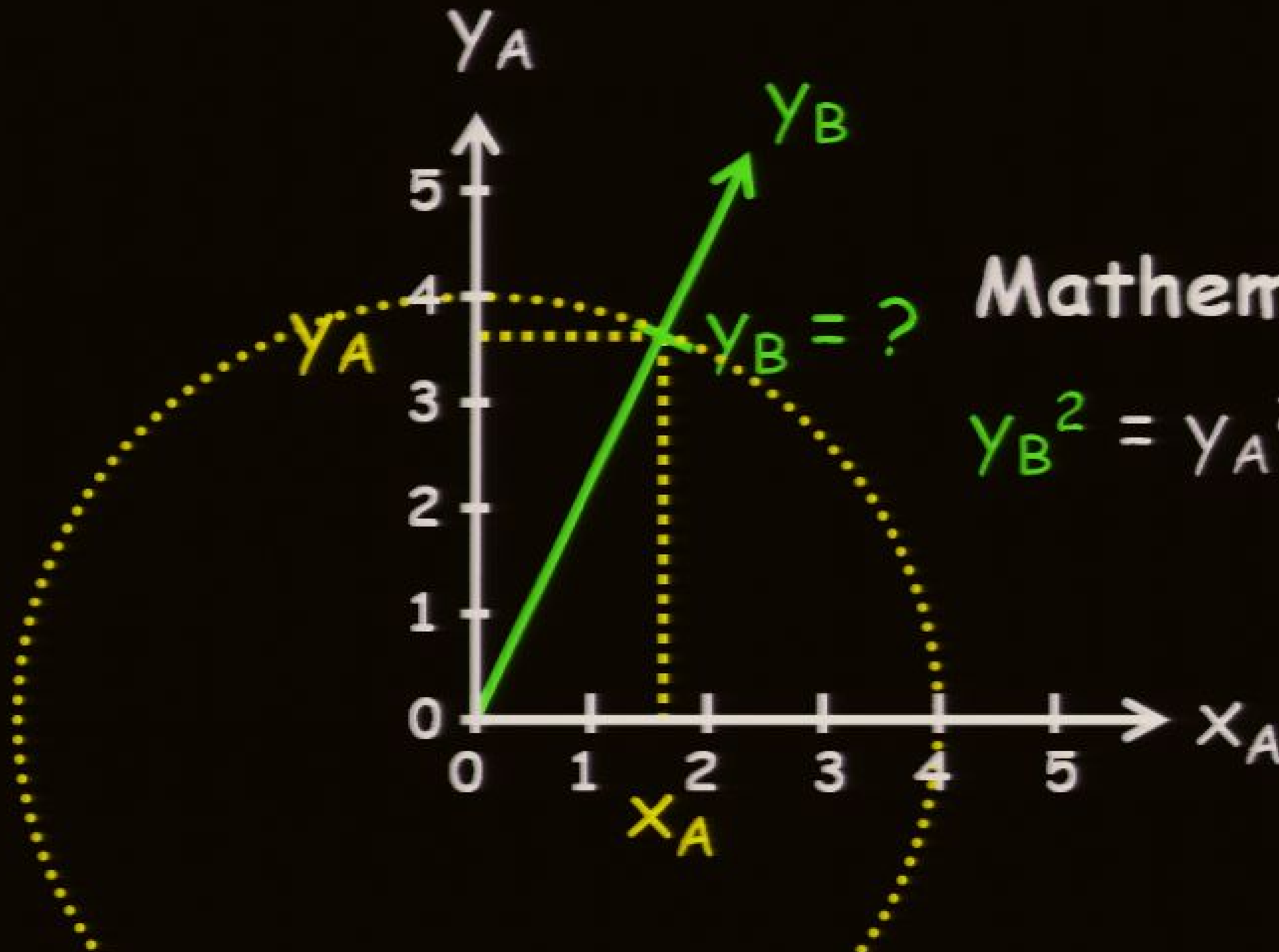
The Geometry of Space



Mathematically...

$$y_B^2 = y_A^2 + x_A^2$$

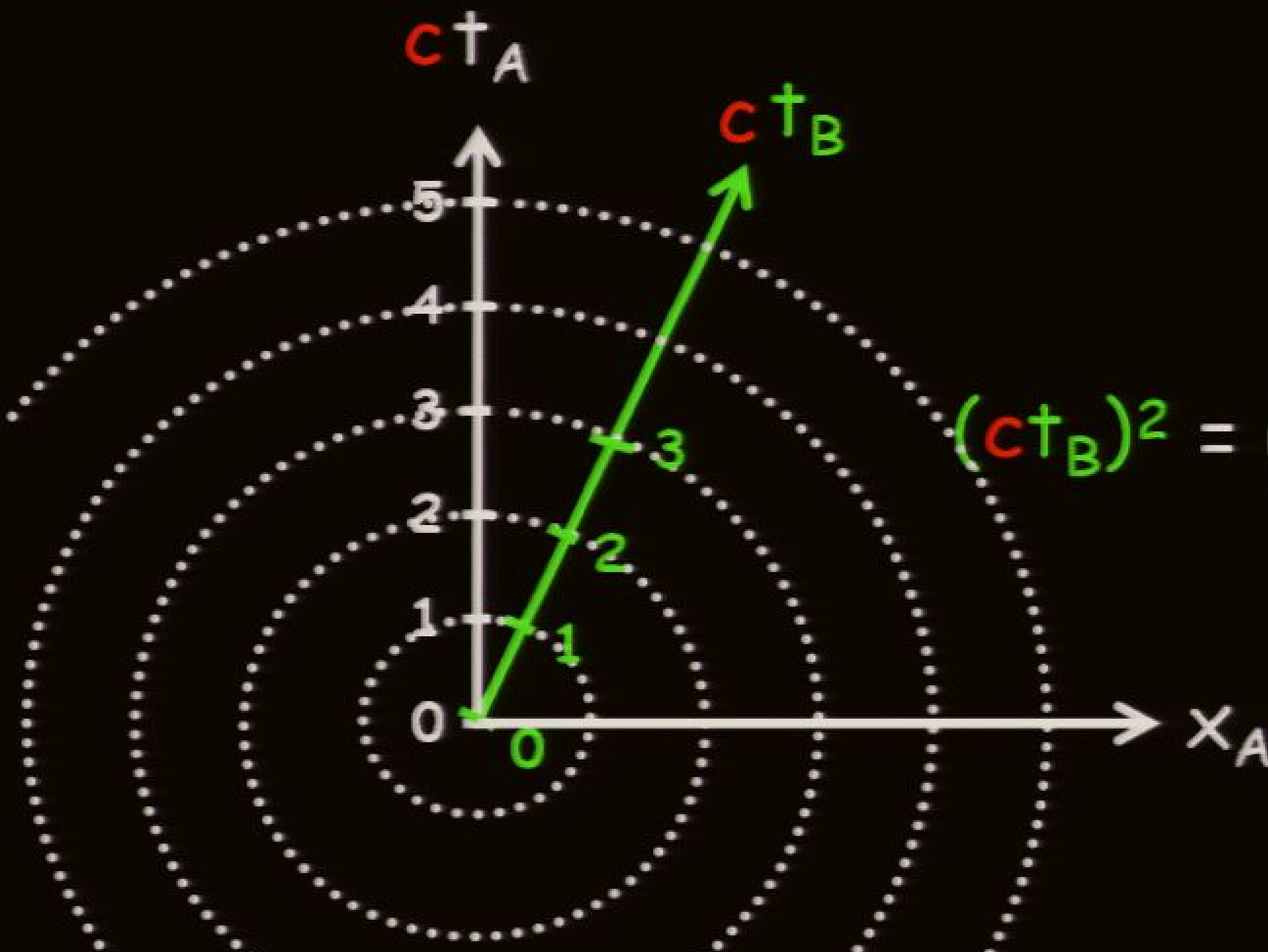
The Geometry of Space



Mathematically...

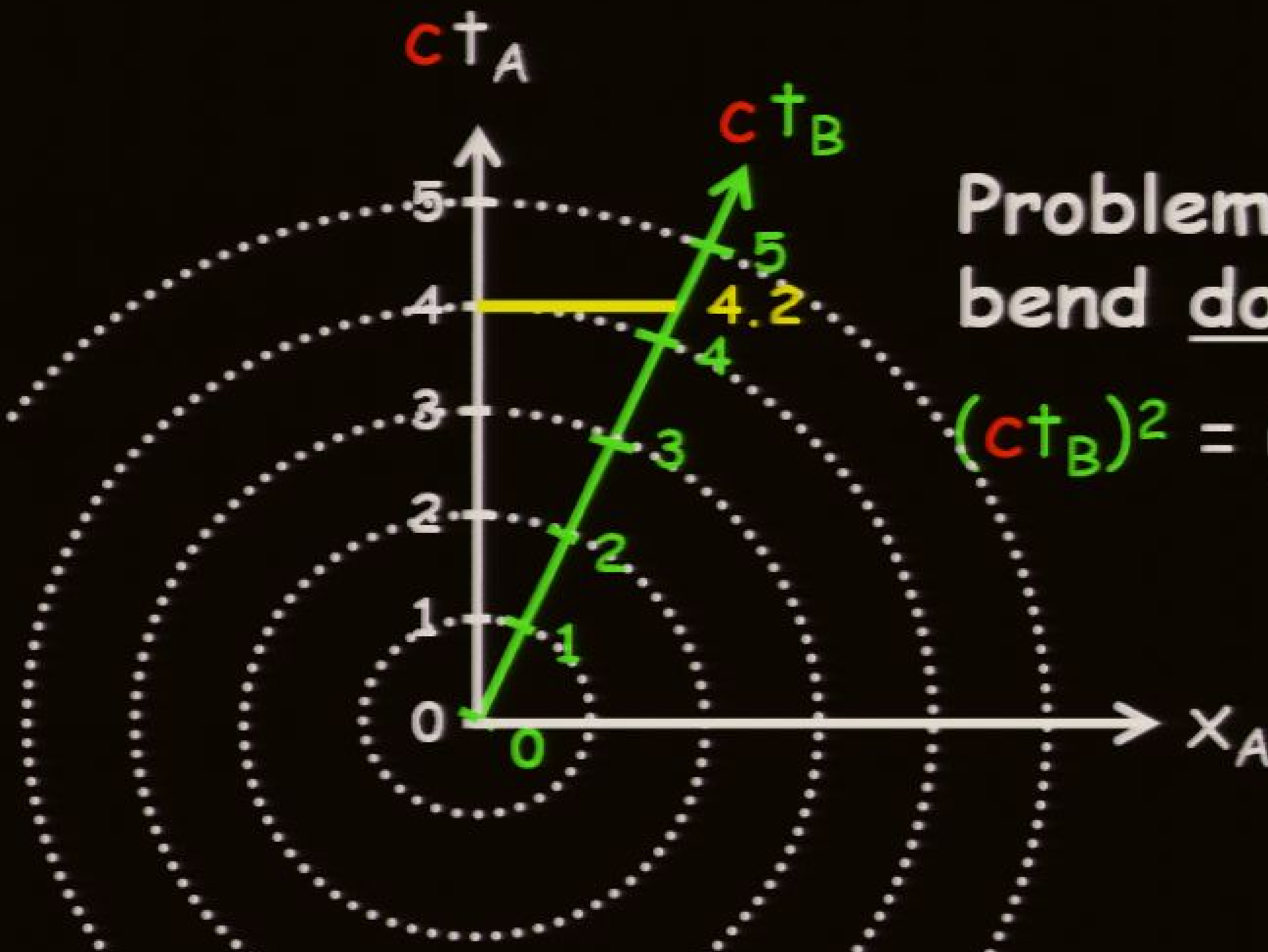
$$y_B^2 = y_A^2 + x_A^2$$

The Geometry of Spacetime



$$(ct_B)^2 = (ct_A)^2 + x_A{}^2$$

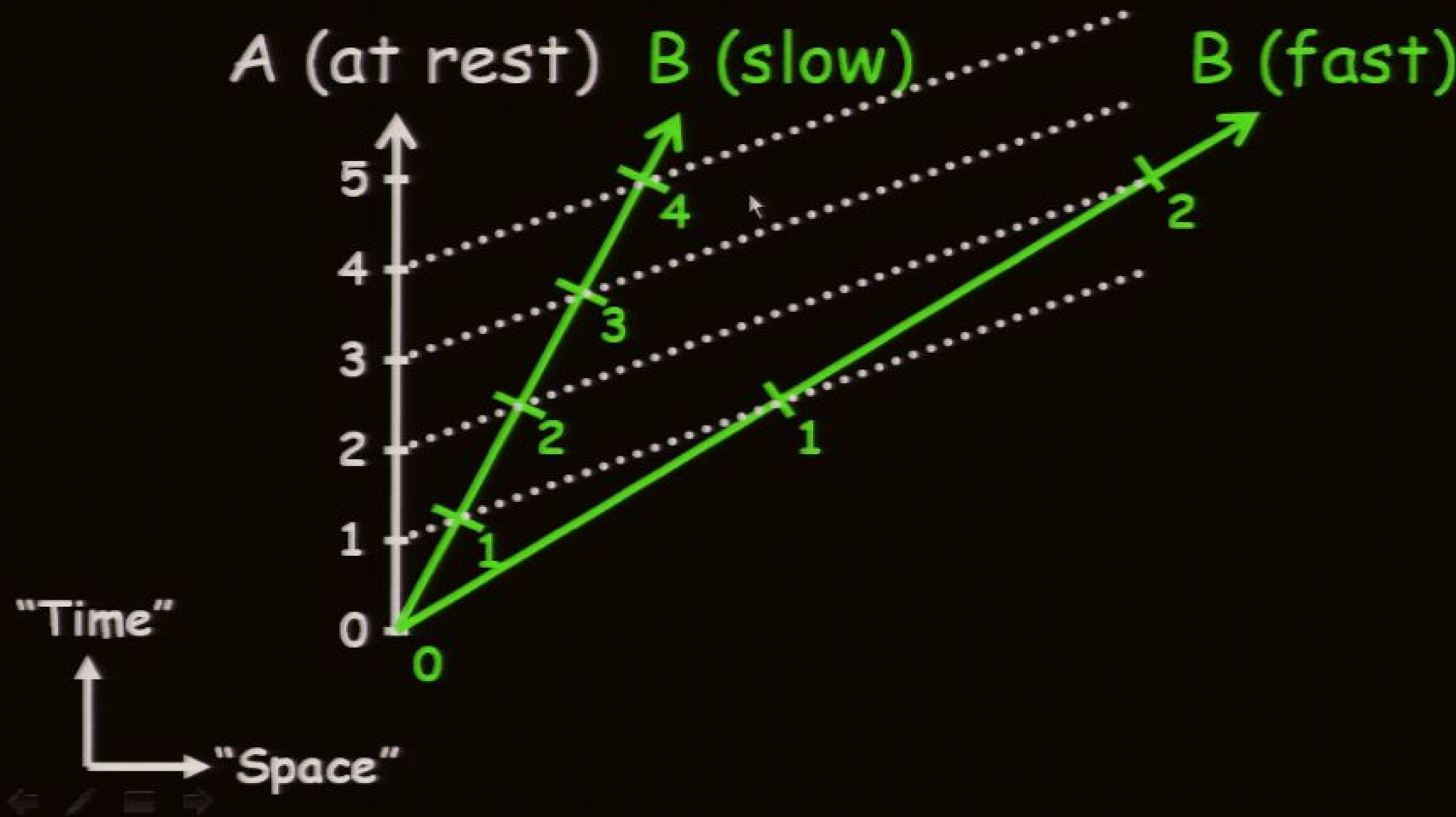
The Geometry of Spacetime



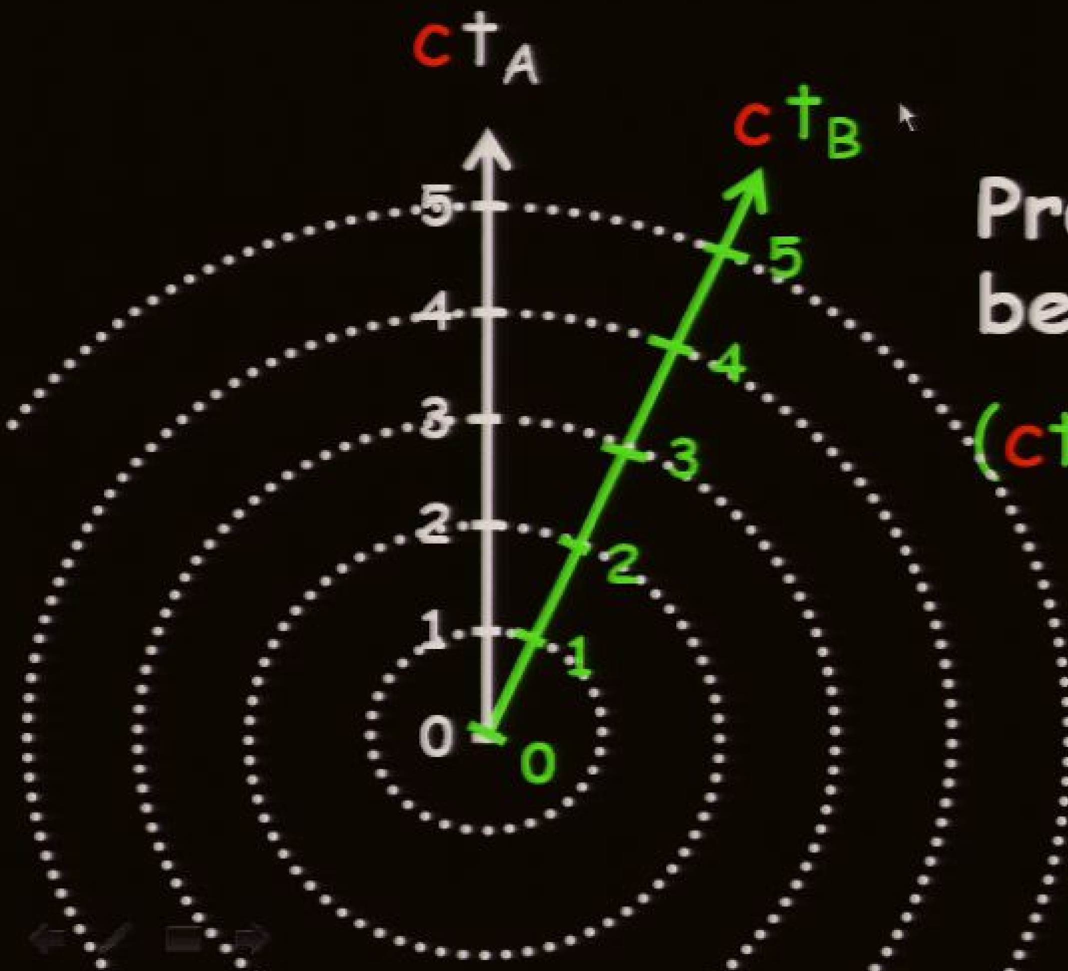
Problem 2: Curves bend down not up

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

Experimental Data:



The Geometry of Spacetime



Problem 2: Curves bend down not up

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

The Geometry of Spacetime

Try <u>hyperbolas</u> instead of circles:

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

The Geometry of Spacetime

Try <u>hyperbolas</u> instead of circles:

$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

Minus sign

The Geometry of Spacetime

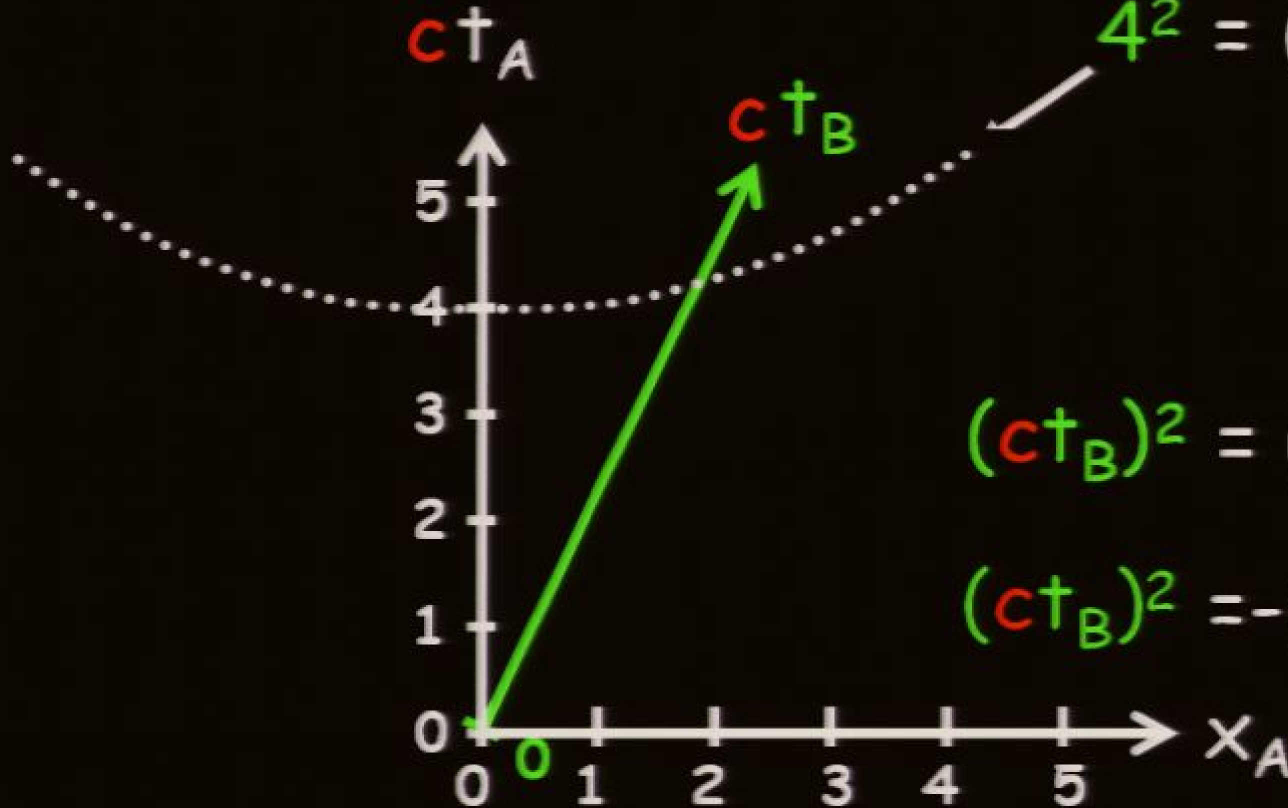
Try <u>hyperbolas</u>
instead of circles:

$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

Minus sign

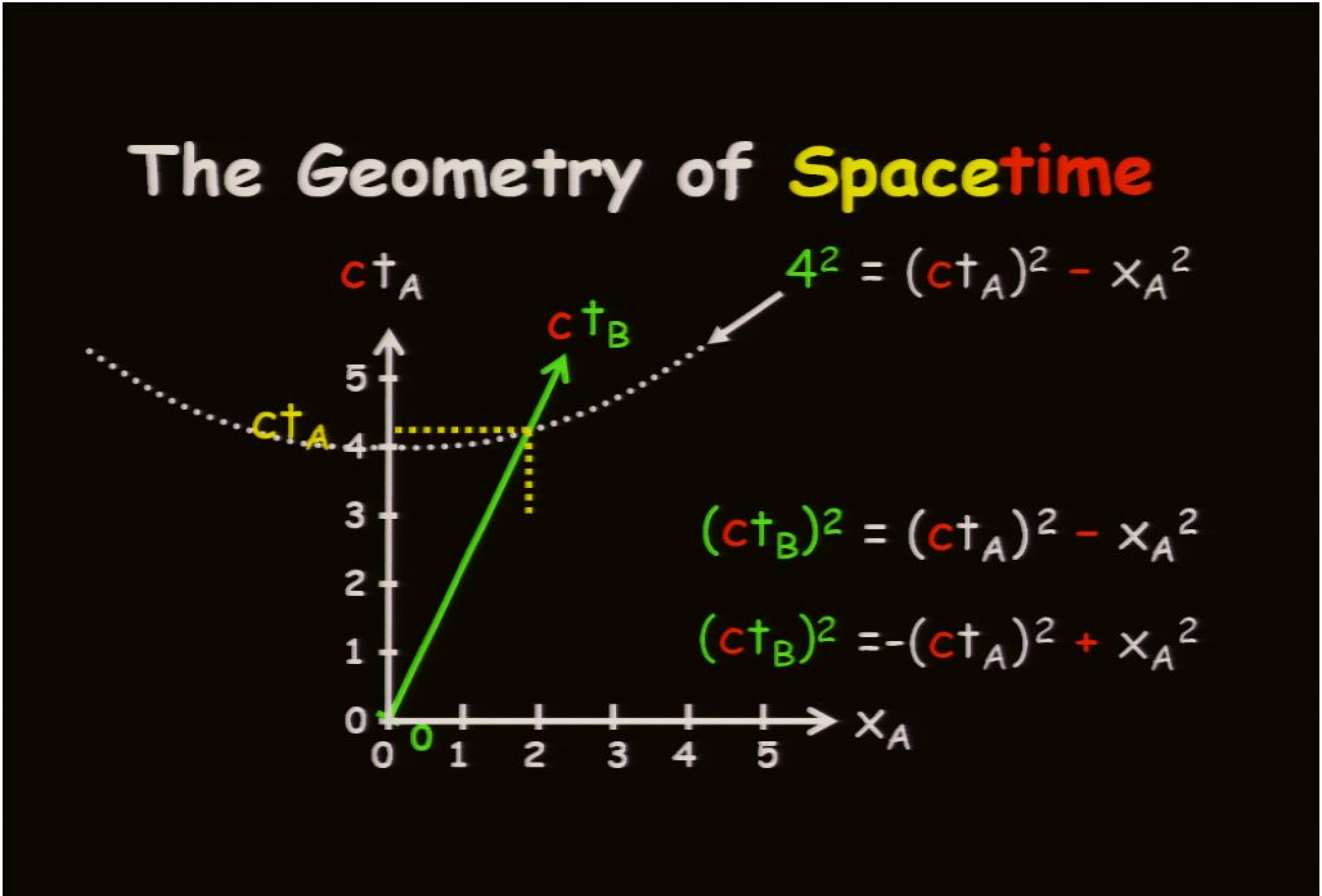
The Geometry of Spacetime

$$4^2 = (ct_A)^2 - x_A^2$$

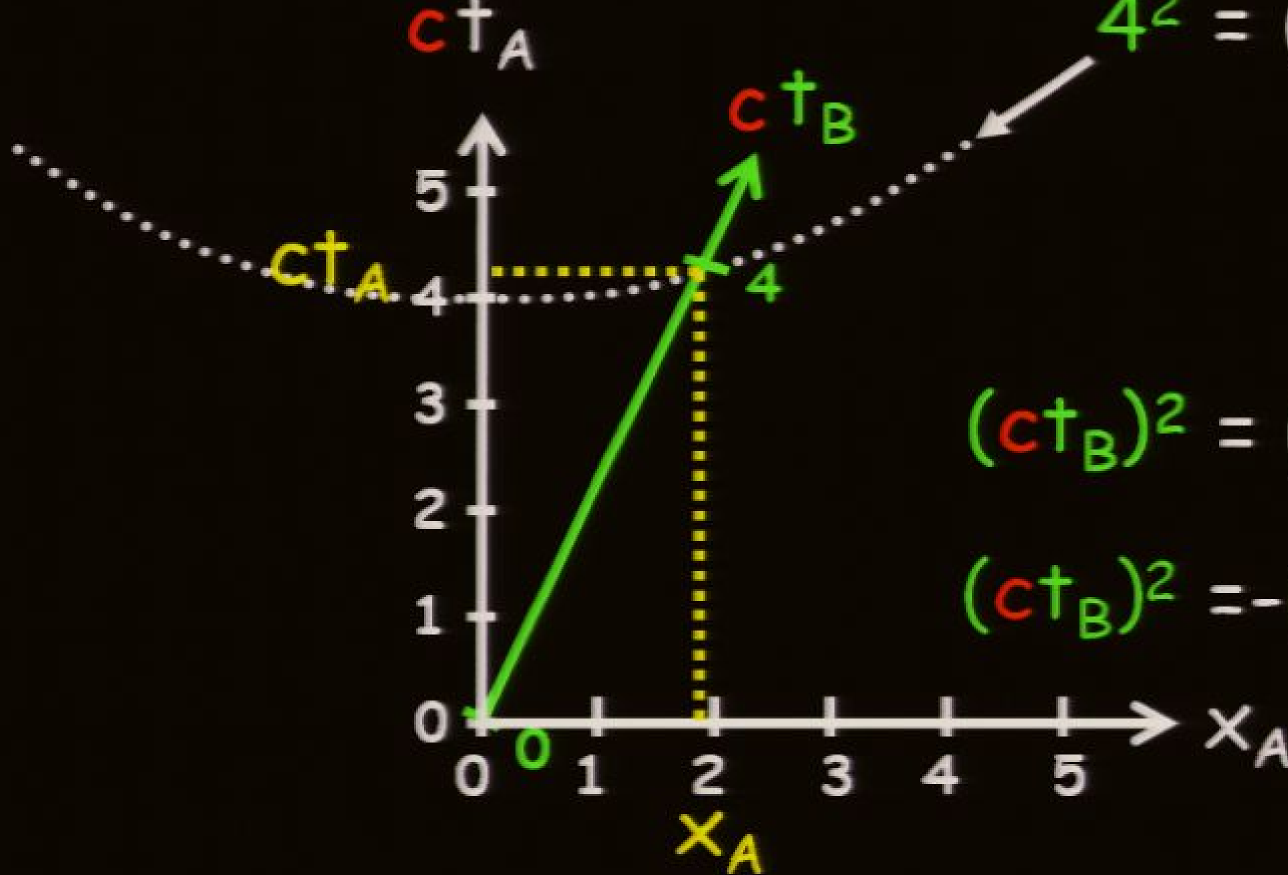


$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$


The Geometry of Spacetime
ct_A
ct_B
$4^2 = (ct_A)^2 - x_A^2$
ct_A
$(ct_B)^2 = (ct_A)^2 - x_A^2$
$(ct_B)^2 = -(ct_A)^2 + x_A^2$
0 1 2 3 4 5
0 1 2 3 4 5
0
x_A

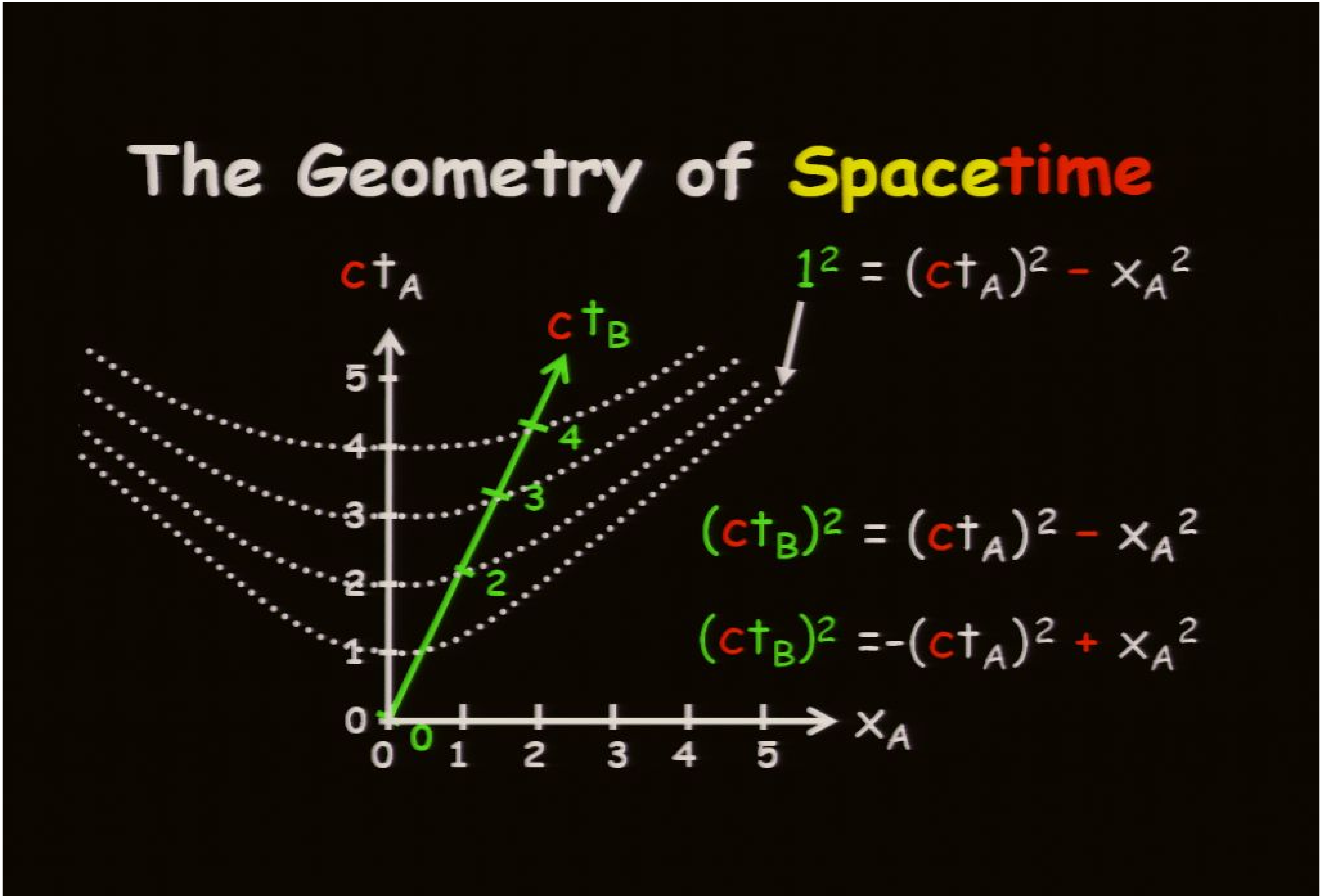
The Geometry of Spacetime



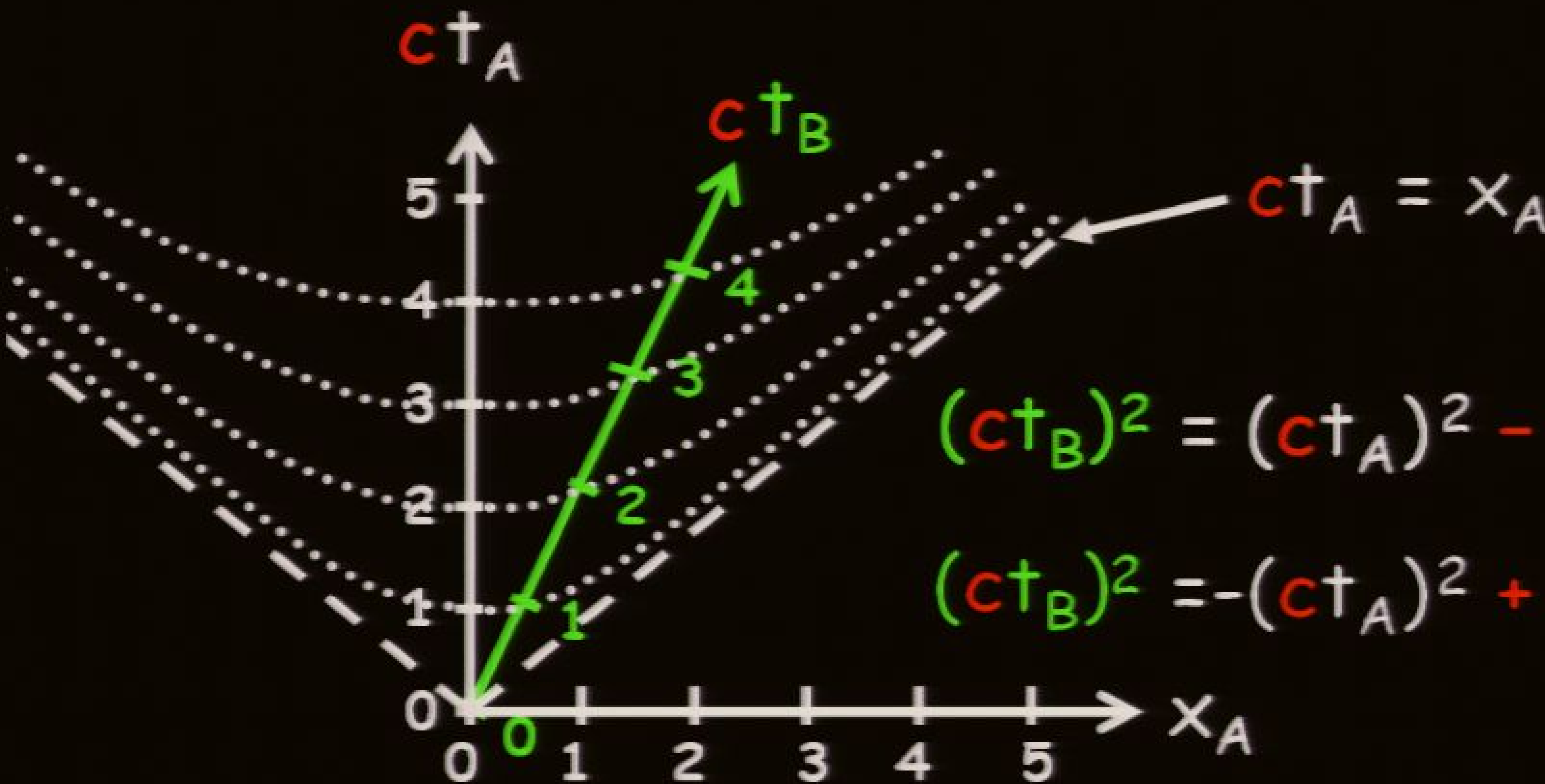
$$4^2 = (ct_A)^2 - x_A{}^2$$

$$(ct_B)^2 = (ct_A)^2 - x_A{}^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A{}^2$$


The Geometry of Spacetime
$1^2 = (ct_A)^2 - x_A{}^2$
$(ct_B)^2 = (ct_A)^2 - x_A{}^2$
$(ct_B)^2 = -(ct_A)^2 + x_A{}^2$
ct_A
ct_B
x_A
0
1
2
3
4
5

The Geometry of Spacetime



$$(ct_B)^2 = (ct_A)^2 - x_A^{\,2}$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^{\,2}$$

Einstein's Spacetime

- *Define a metric that handles both Space and Time P(t, x, y, z)*
- *Example two dimensional Euclidean Space*

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2$$ *Cartesian coordinates*

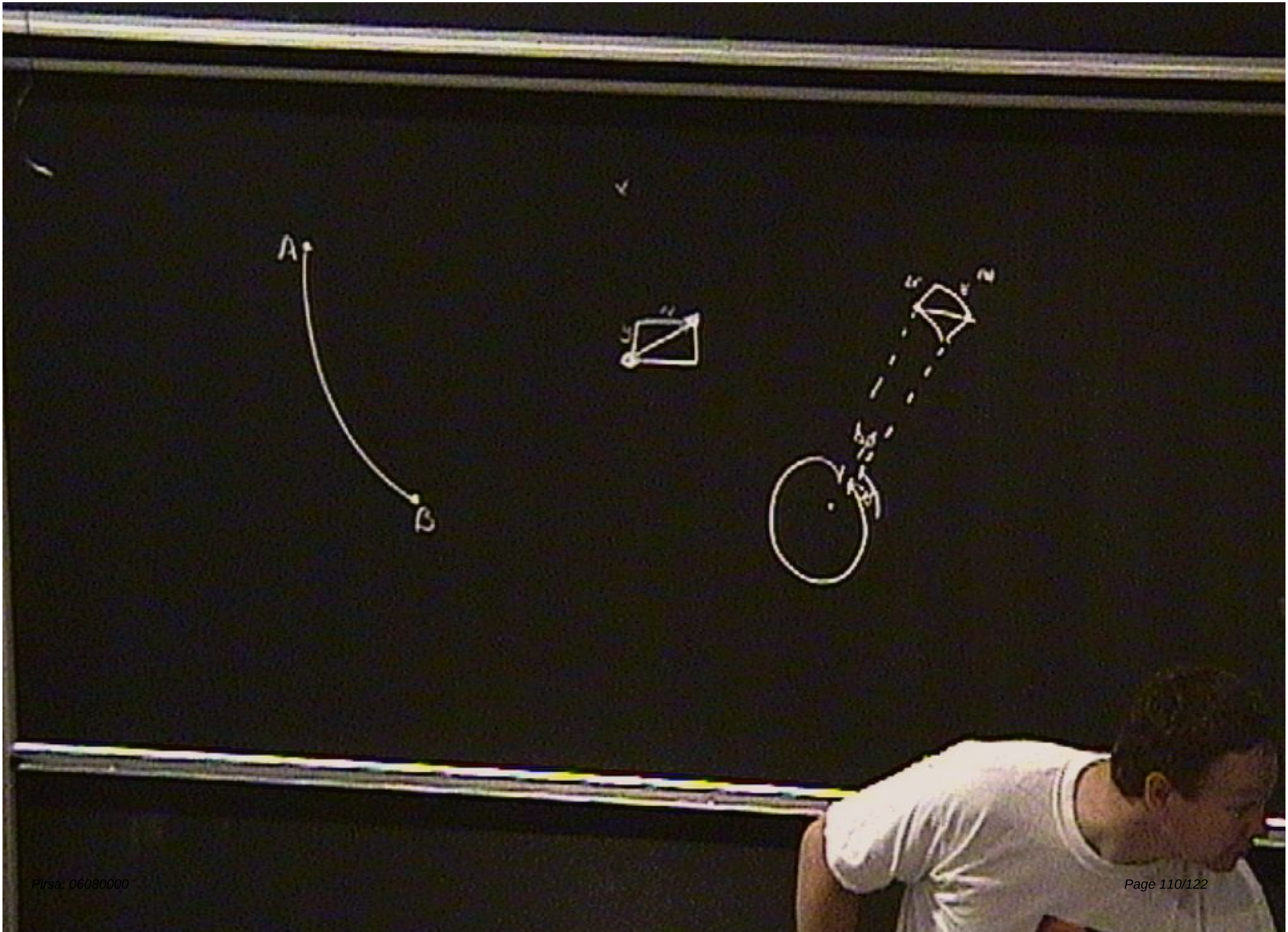
$$(\Delta s)^2 = (\Delta r)^2 + r^2 (\Delta \phi)^2$$ *Polar coordinates*

- *Example of two dimensional Minkowski Space*

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$$ *Usual representation*

$$(\Delta s)^2 = -(\Delta t)^2 + t^2 (\Delta \phi)^2$$ *Milne representation*


A
B


A
B

Einstein's Spacetime

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- *Example of two dimensional Minkowski Space*

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$$ *Usual representation*

$$(\Delta s)^2 = -(\Delta t)^2 + t^2 (\Delta \phi)^2$$ *Milne representation*

The Einstein field equation (EFE) is usually written in the form

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

Here R_{ab} is the Ricci tensor, R is the Ricci scalar, g_{ab} is the metric tensor, Tab is the stress-energy tensor, and the constants are π , G (the gravitational constant) and c (the speed of light). The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. It is written here using the abstract index notation. Each tensor has 10 independent components. Given the freedom of choice of the four spacetime coordinates, the independent equations reduce to 6 in number

$$R_{\eta\eta} = -\frac{2a^2 \frac{\partial\psi}{\partial\theta}\cot\theta}{\delta\,\psi} + \frac{2ac\frac{\partial\psi}{\partial\eta}\cot\theta}{\delta\,\psi} + \frac{a\frac{\partial c}{\partial\eta}\cot\theta}{\delta} - \frac{\frac{\partial a}{\partial\eta}c\cot\theta}{2\delta} - \frac{a\frac{\partial a}{\partial\theta}\cot\theta}{2\delta} - \frac{2a^2\frac{\partial^2\psi}{\partial\theta^2}}{\delta\,\psi}$$

$$-\frac{2a^2(\frac{\partial\psi}{\partial\theta})^2}{\delta\,\psi^2} + \frac{4ac\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\,\psi^2} - \frac{a^2\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta\,d\,\psi} + \frac{ac\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\,d\,\psi} + \frac{2a\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\,\psi} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta\,\psi}$$

$$-\frac{3a\frac{\partial a}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta\,\psi} - \frac{2a^2c\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\,\psi} + \frac{2a^2b\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta^2\,\psi} - \frac{a^2\frac{\partial b}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta^2\,\psi} - \frac{a\frac{\partial a}{\partial\eta}bc\frac{\partial\psi}{\partial\theta}}{\delta^2\,\psi} + \frac{a^3\frac{\partial b}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\,\psi}$$

$$+\frac{a^2\frac{\partial a}{\partial\theta}b\frac{\partial\psi}{\partial\theta}}{\delta^2\,\psi} - \frac{2ab\frac{\partial^2\psi}{\partial\eta^2}}{\delta\,\psi} - \frac{2\frac{\partial^2\psi}{\partial\eta^2}}{\psi} + \frac{4ac\frac{\partial^2\psi}{\partial\eta\partial\theta}}{\delta\,\psi} - \frac{2ab(\frac{\partial\psi}{\partial\eta})^2}{\delta\,\psi^2} + \frac{6(\frac{\partial\psi}{\partial\eta})^2}{\psi^2}$$

$$+\frac{ac\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta\,d\,\psi} - \frac{ab\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\,d\,\psi} - \frac{2c\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\,\psi} + \frac{\frac{\partial a}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta\,\psi} - \frac{2a\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\,\psi} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial\psi}{\partial\eta}}{\delta\,\psi}$$

$$+\frac{2a^2b\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta^2\,\psi} - \frac{2abc\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\,\psi} - \frac{a^2\frac{\partial b}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta^2\,\psi} - \frac{a\frac{\partial a}{\partial\theta}bc\frac{\partial\psi}{\partial\eta}}{\delta^2\,\psi} + \frac{a^2b\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\,\psi} + \frac{a\frac{\partial a}{\partial\eta}b^2\frac{\partial\psi}{\partial\eta}}{\delta^2\,\psi}$$

$$+\frac{a\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\theta}}{2\delta\,d} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial d}{\partial\theta}}{4\delta\,d} - \frac{a\frac{\partial a}{\partial\theta}\frac{\partial d}{\partial\theta}}{4\delta\,d} - \frac{\frac{\partial^2 d}{\partial\eta^2}}{2d} + \frac{(\frac{\partial d}{\partial\eta})^2}{4d^2} - \frac{c\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\eta}}{2\delta\,d}$$

$$+\frac{\frac{\partial a}{\partial\theta}c\frac{\partial d}{\partial\eta}}{4\delta\,d} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial d}{\partial\eta}}{4\delta\,d} + \frac{a\frac{\partial^2 c}{\partial\eta\partial\theta}}{\delta} - \frac{a\frac{\partial^2 b}{\partial\eta^2}}{2\delta} - \frac{a\frac{\partial^2 a}{\partial\theta^2}}{2\delta} + \frac{ac\frac{\partial c}{\partial\eta}\frac{\partial c}{\partial\theta}}{\delta^2}$$

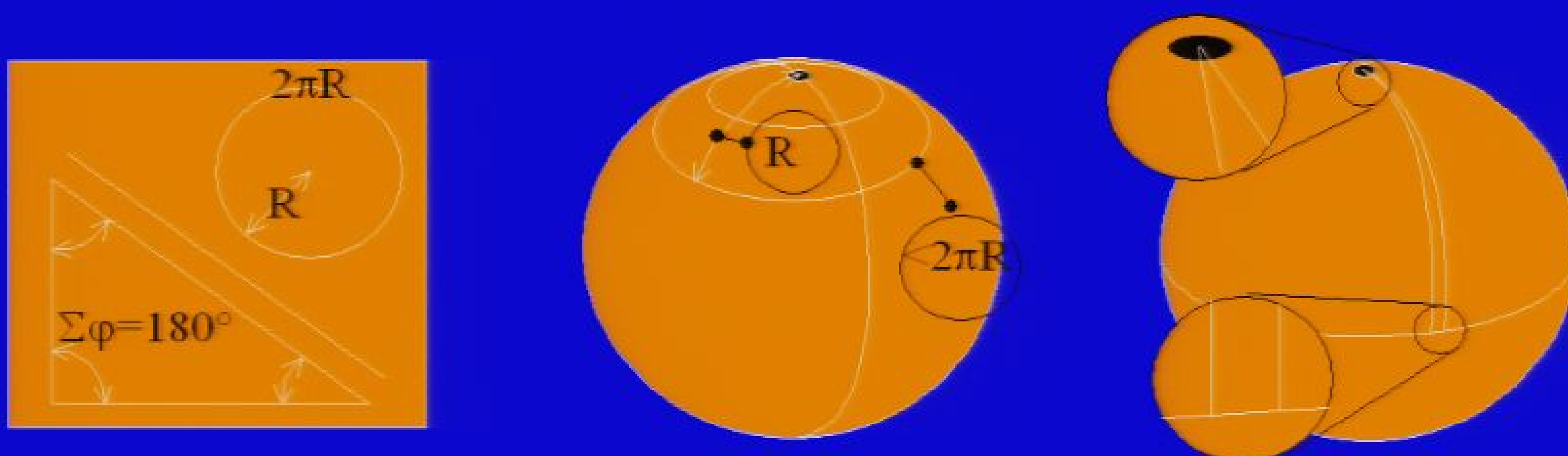
Curvature in 2D…

- In a curved space, Euclidean geometry does not apply:

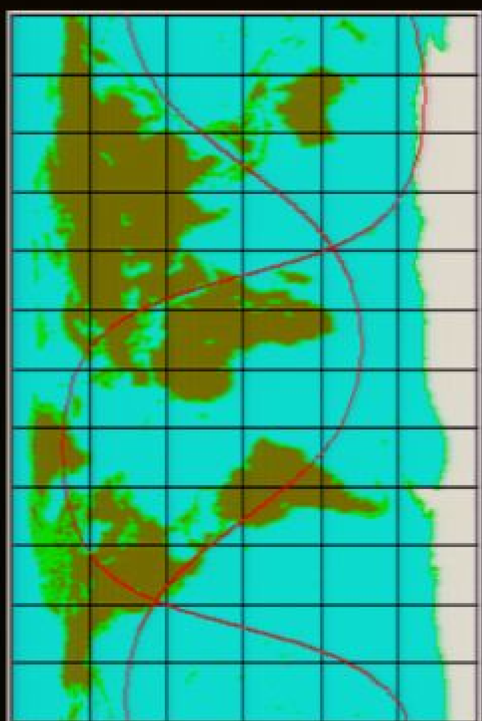
  - circumference ≠ 2π R

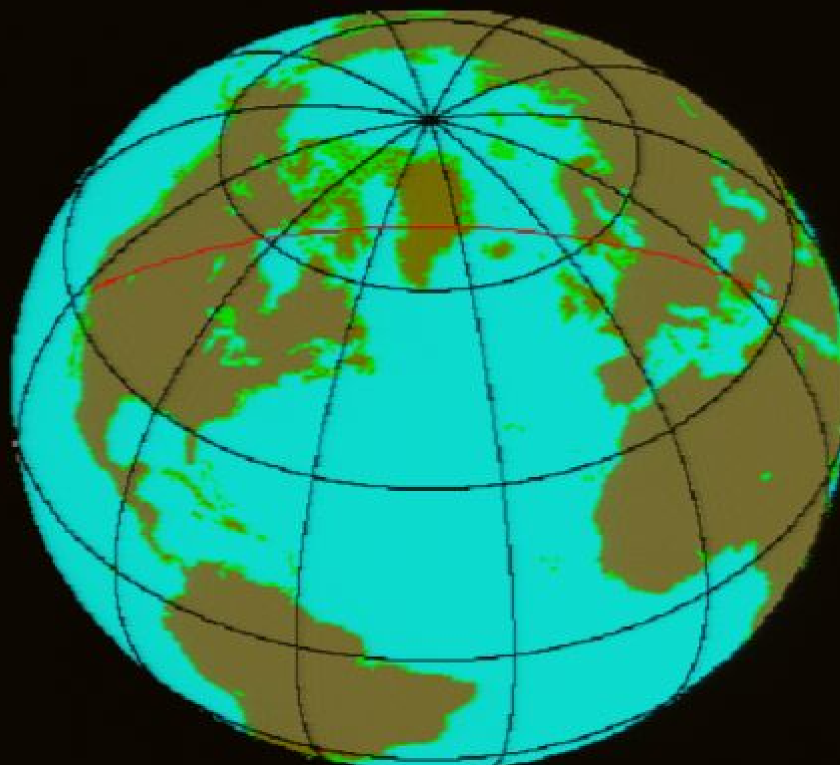
  - triangles ≠ 180°

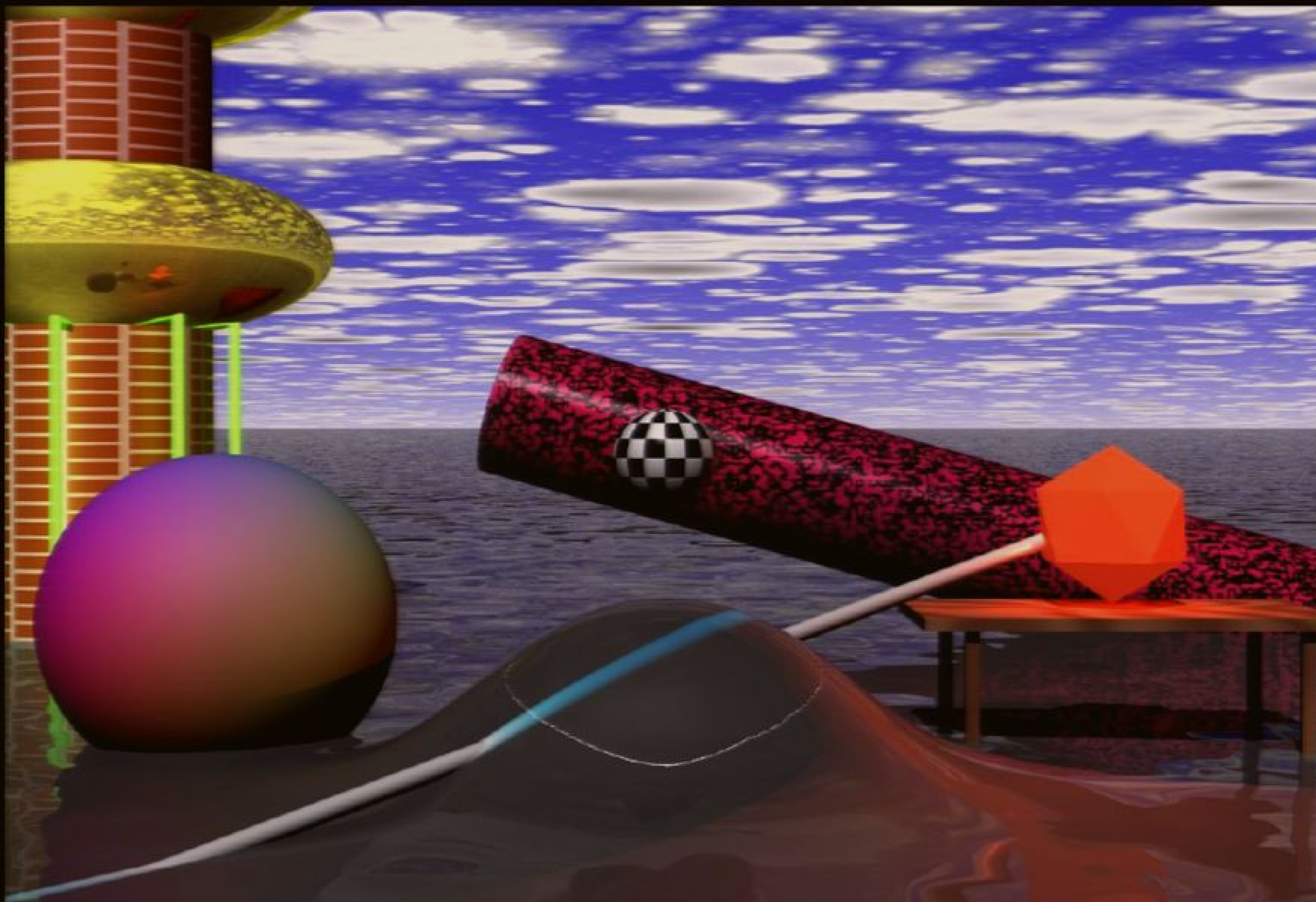
  - parallel lines don't stay parallel

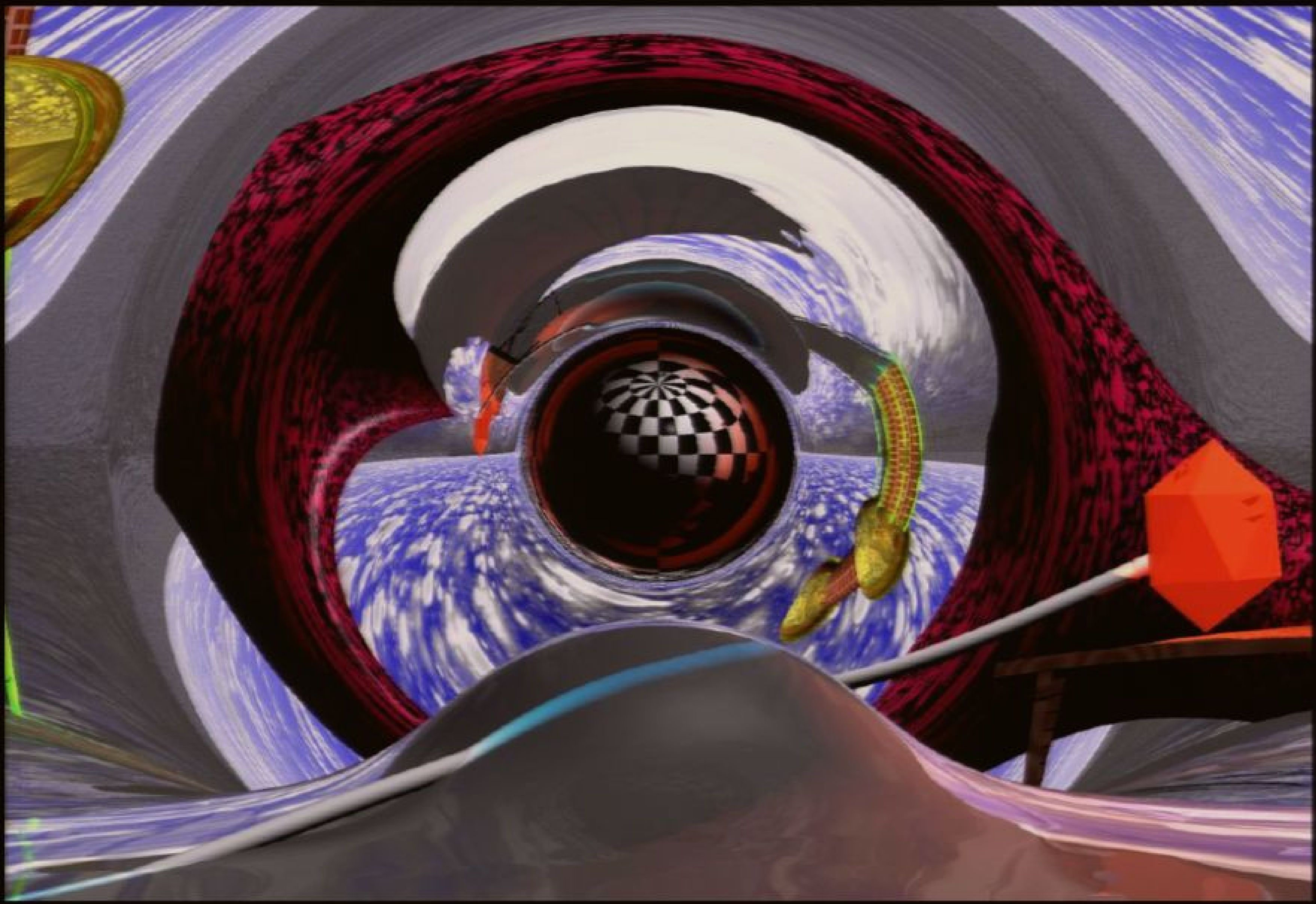


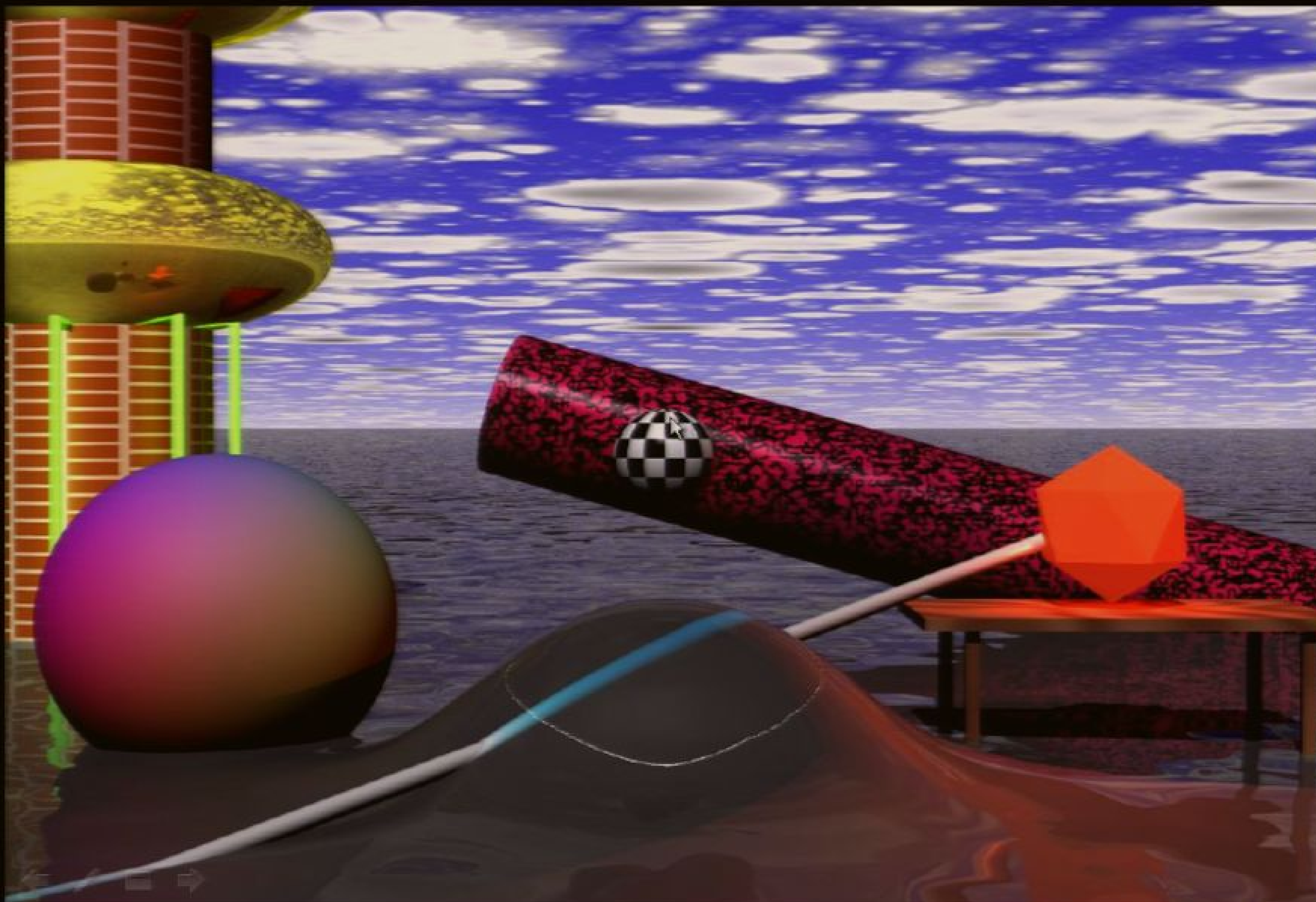
Working with a Curved Geometry

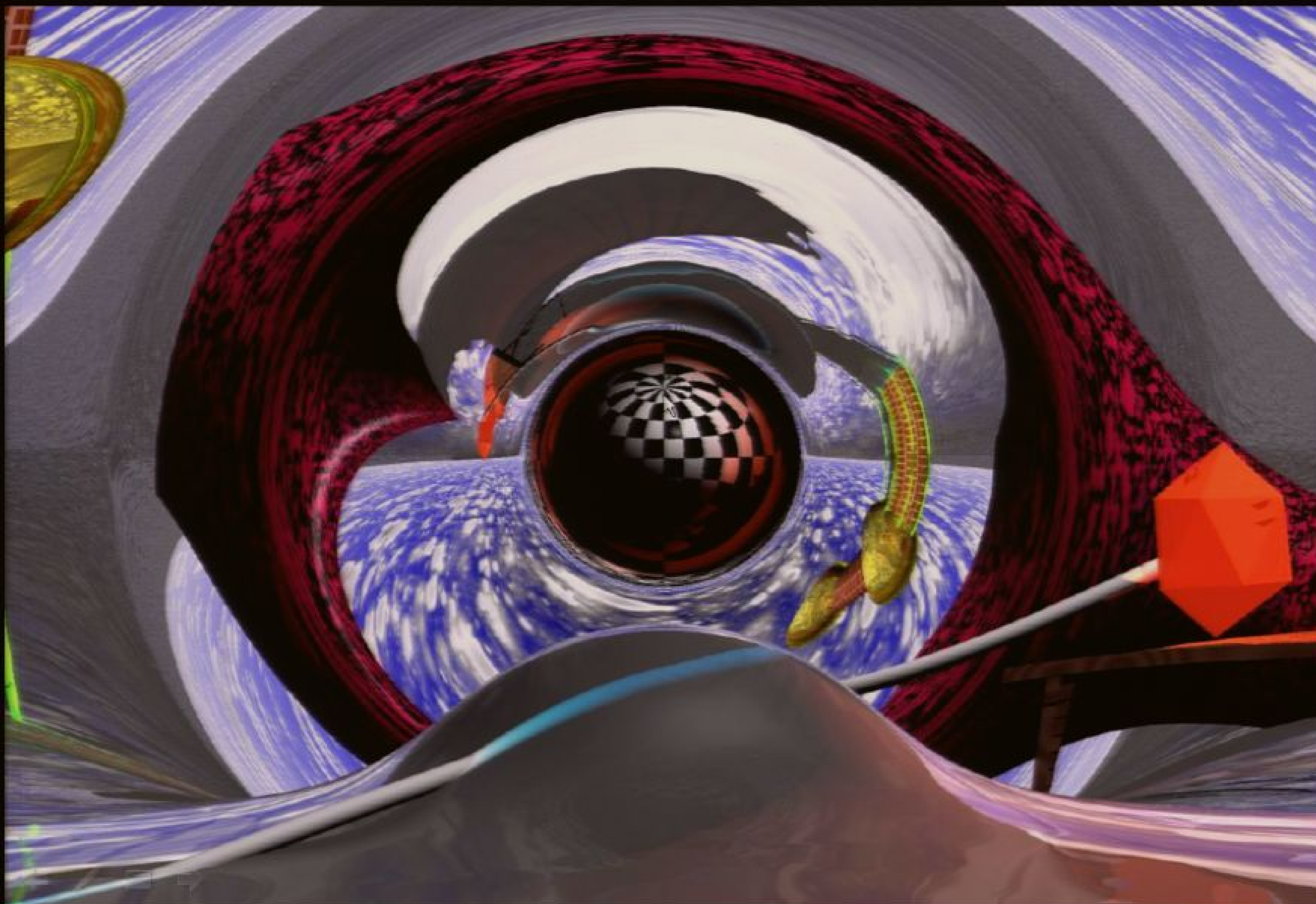




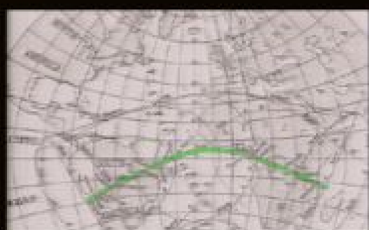








1919 Verification



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.