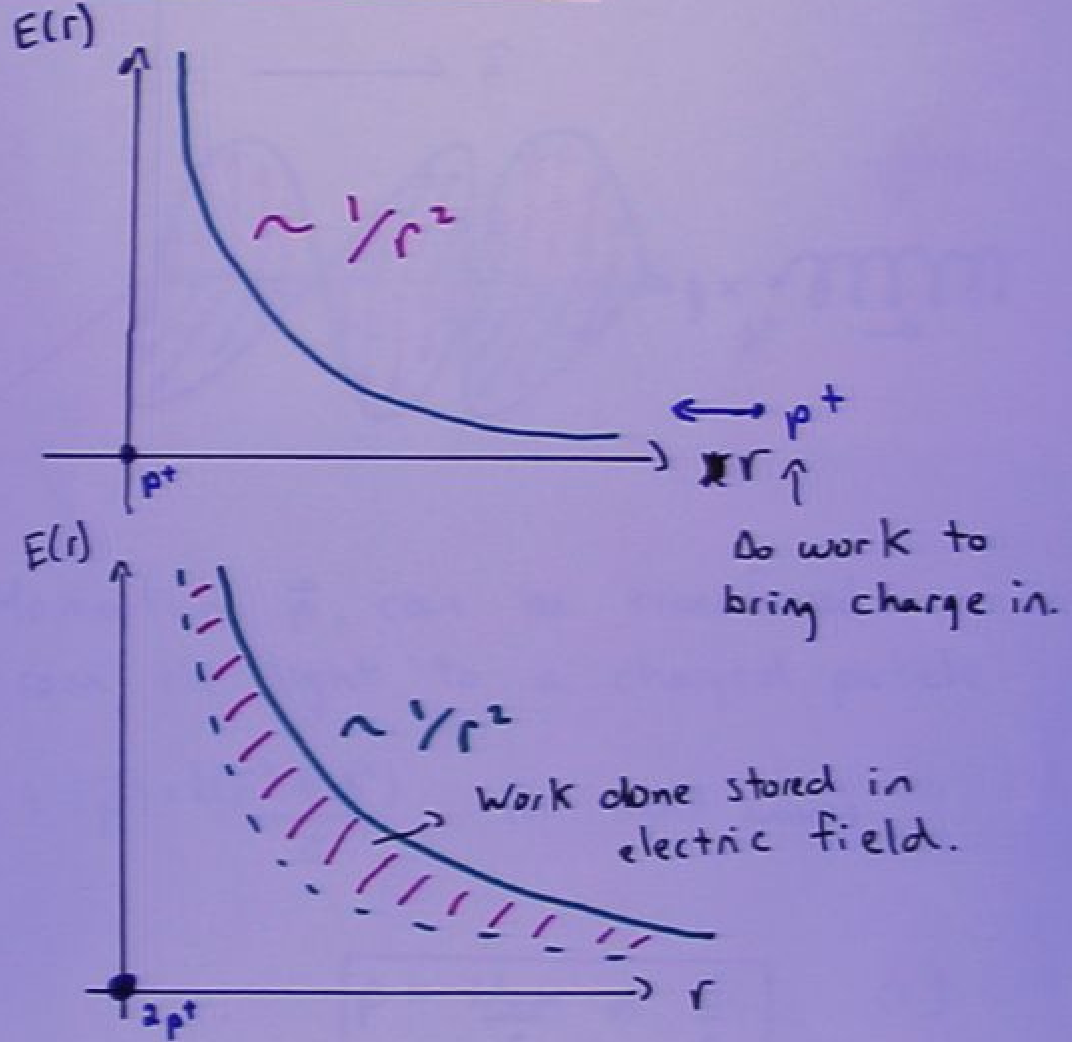
Title: Special Relativity - Part 1

Date: Jul 31, 2006 09:00 AM

URL: http://pirsa.org/06070082

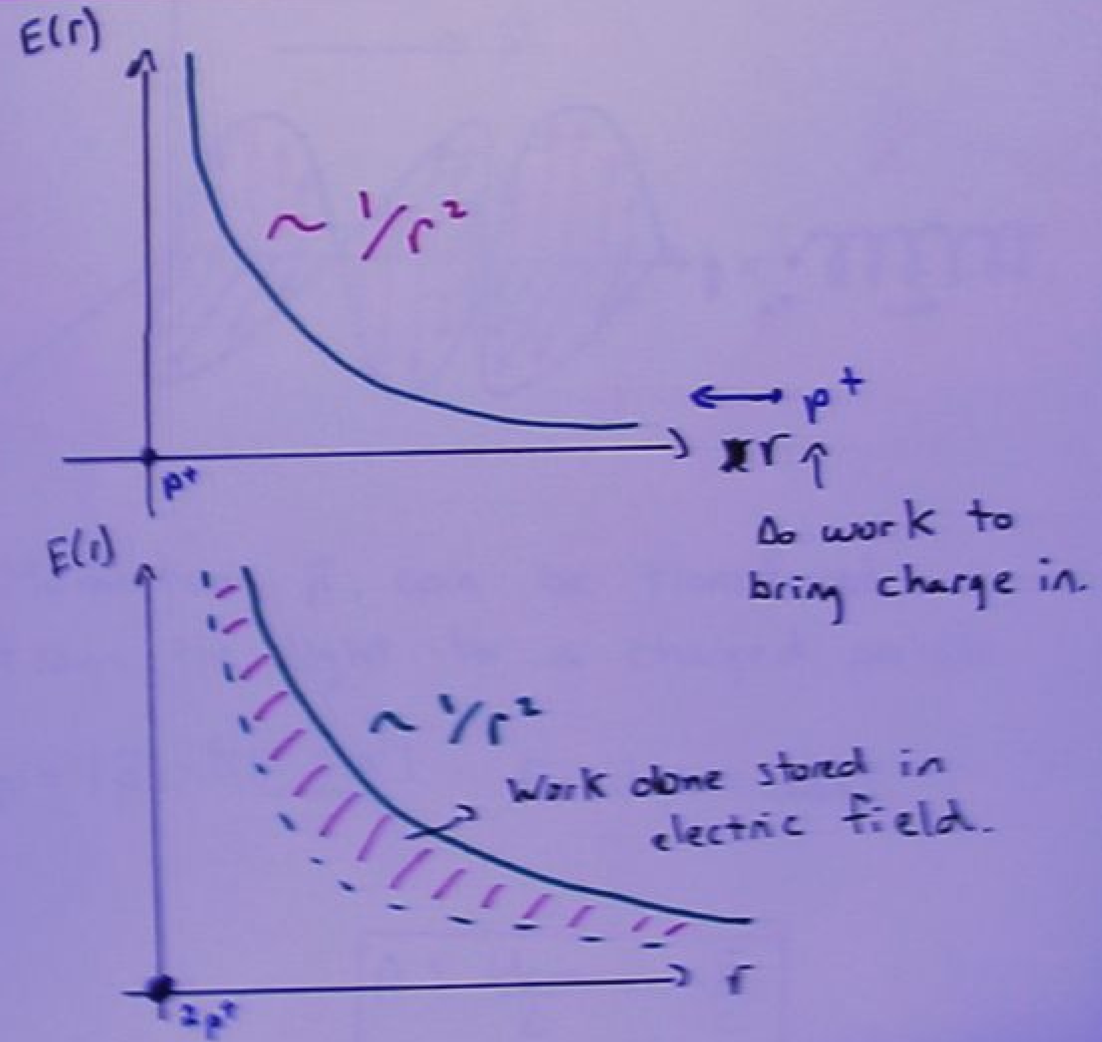
Abstract:

Maxwell:



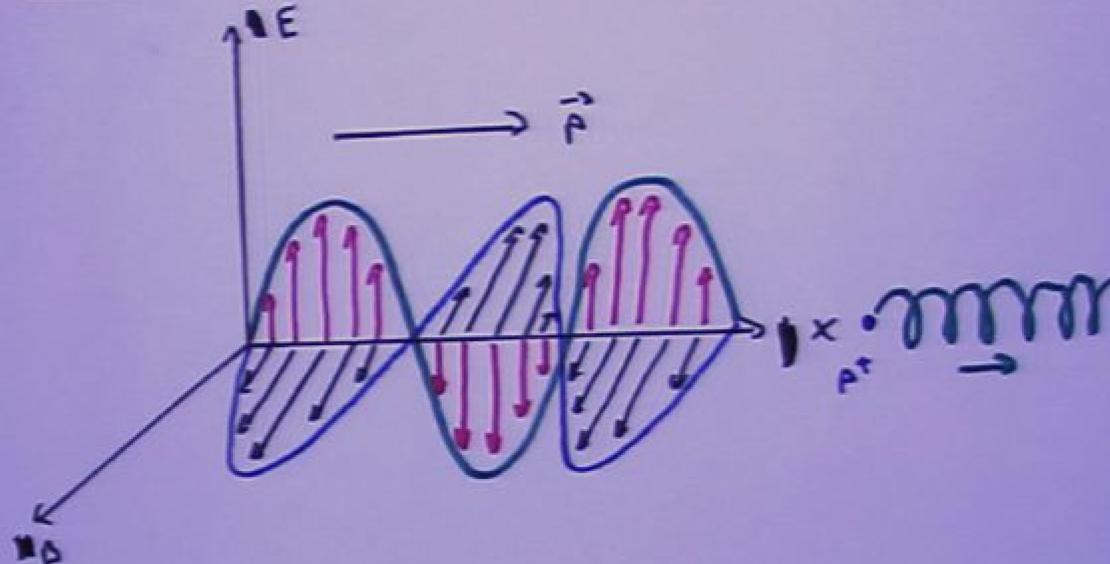
$U_e = \frac{1}{2}E^2 \;;\; U_m = \frac{1}{2}B^2 \;;\; \Rightarrow \boxed{U_{em} = \frac{1}{2}(E^2 + B^2)}$

Maxwell:



$U_e = \frac{1}{2}E^2$; $U_m = \frac{1}{2}B^2$; $\Rightarrow \boxed{U_{em} = \frac{1}{2}(E^2 + B^2)}$

Momentum:

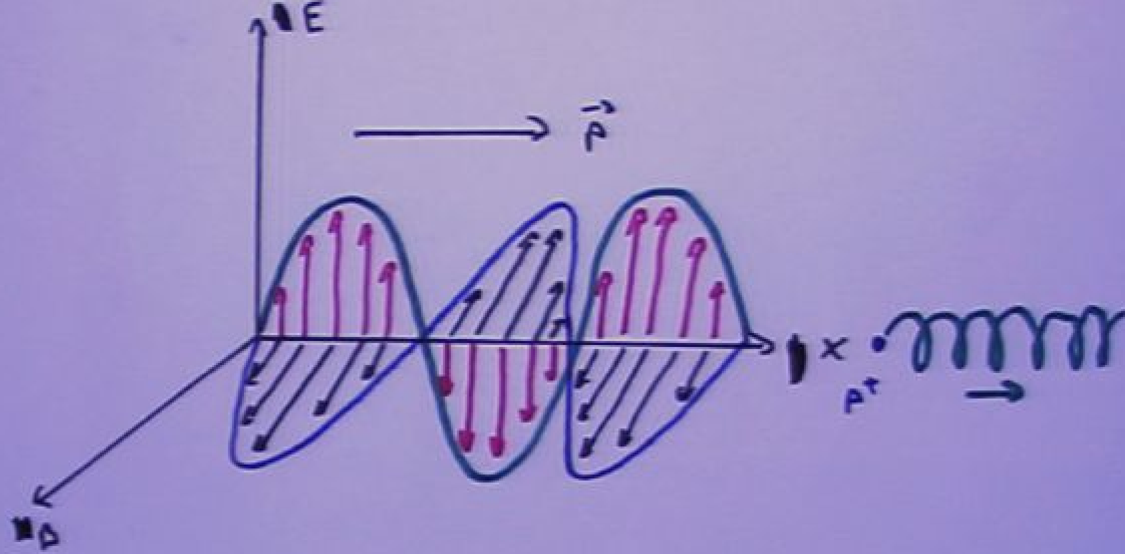


Momentum $\vec{p}$, can be transferred from the light to a charged particle.

$$p = |\vec{p}| = \frac{\frac{1}{2}(E^2 + B^2)}{c} \quad \text{for light waves.}$$

$$\therefore \quad \boxed{p = \frac{u_{em}}{c} \text{ or } \frac{E}{c}} \qquad (1)$$

Momentum:



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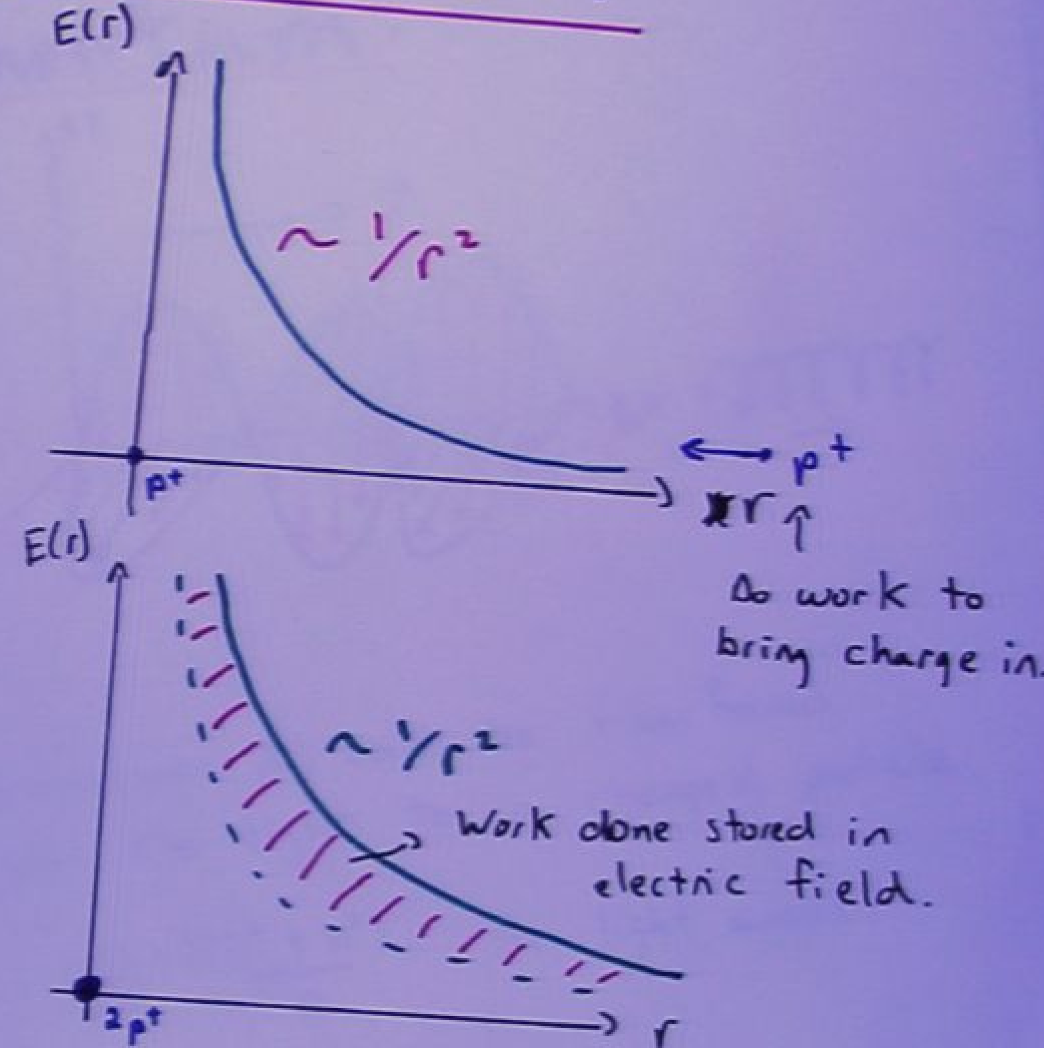
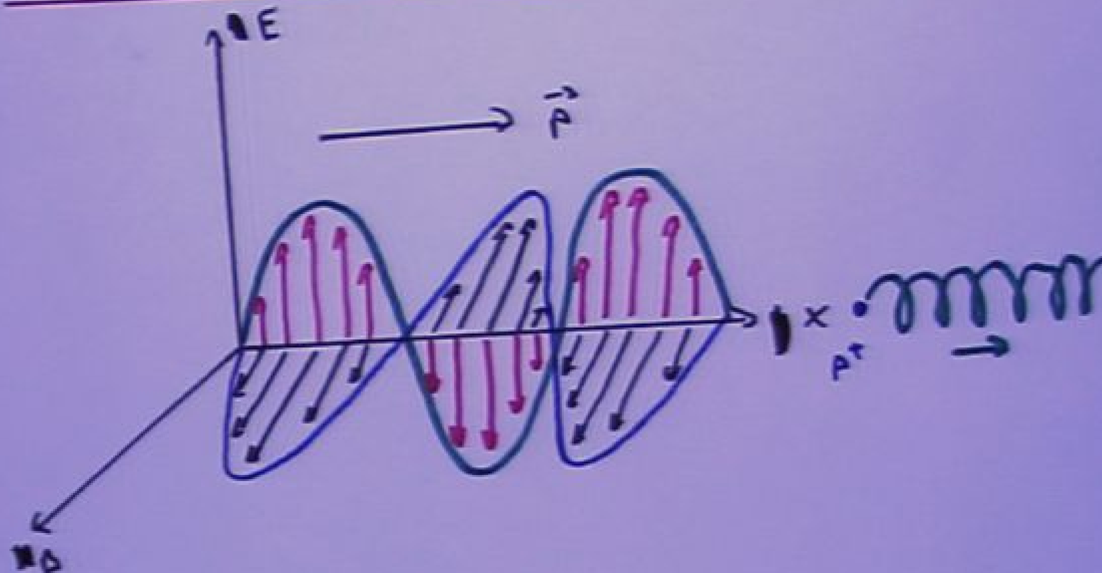


$$U_e = \tfrac{1}{2}E^2 \;;\; U_m = \tfrac{1}{2}B^2 \;;\; \Rightarrow \boxed{U_{em} = \tfrac{1}{2}(E^2 + B^2)}$$

Momentum:



Momentum $\vec{p}$, can be transferred from the light to a charged particle.

$$p = |\vec{p}| = \frac{\frac{1}{2}(E^2 + B^2)}{c} \quad \text{for light waves.}$$

$$\therefore \quad \boxed{p = \frac{u_{em}}{c} \text{ or } \frac{E}{c}} \qquad (1)$$

Results:

$$p = \frac{E}{c} \quad (1)$$

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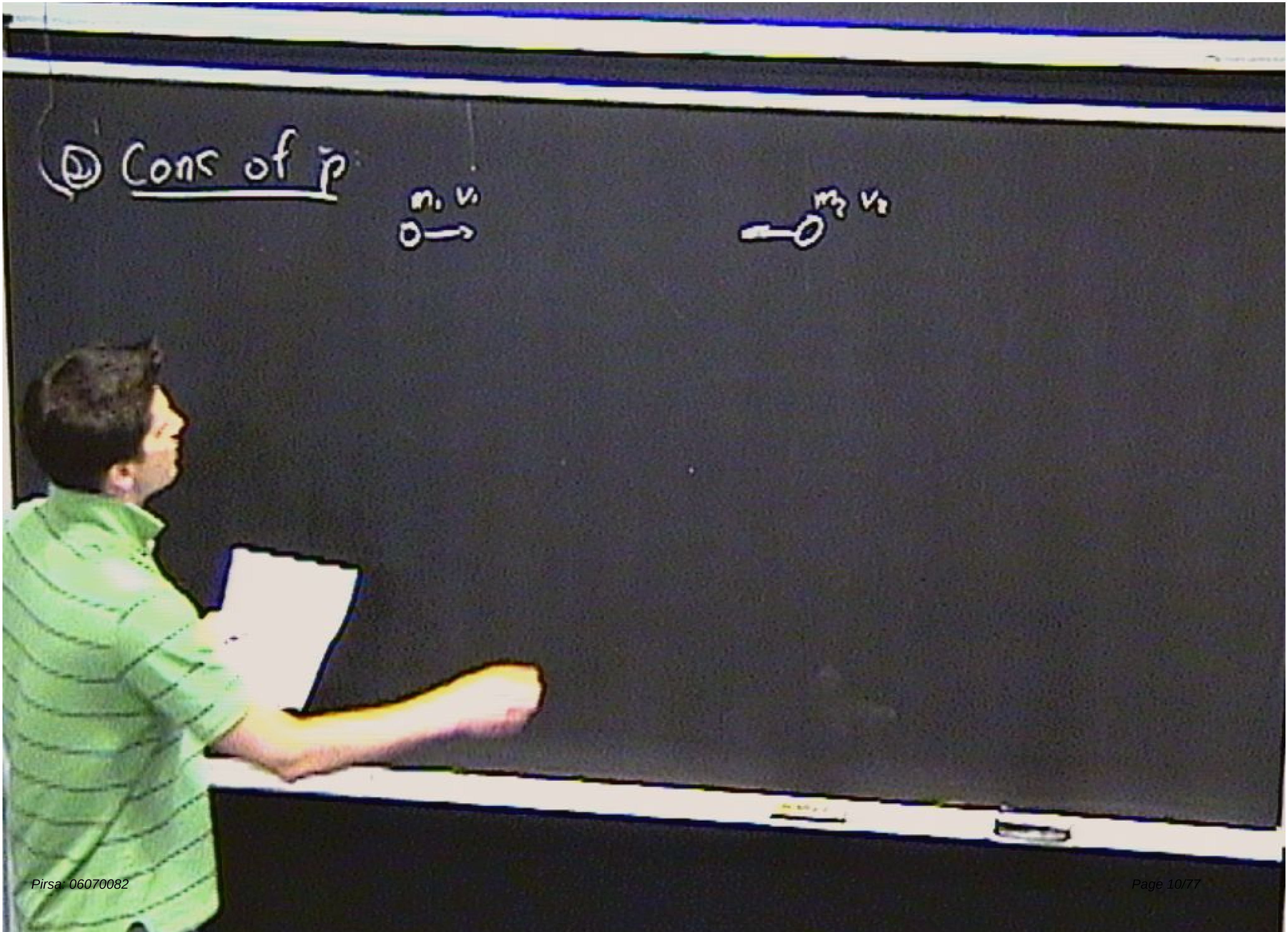

② Cons of p
m1 v1
m2 v2

② Cons of $\vec{p}$

② Cons of $\vec{p}$

$p = mv$; p_{total}

② Cons of $\vec{p}$

m_1 $v_{1,init}$

m_2 $v_{2,init}$

m_1 m_2

$v_{1,fin}$

$v_{2,fin}$

$p = mv$; $p_{tot} = m_1 v_1 + m_2 v_2$

② Cons of $\vec{p}$

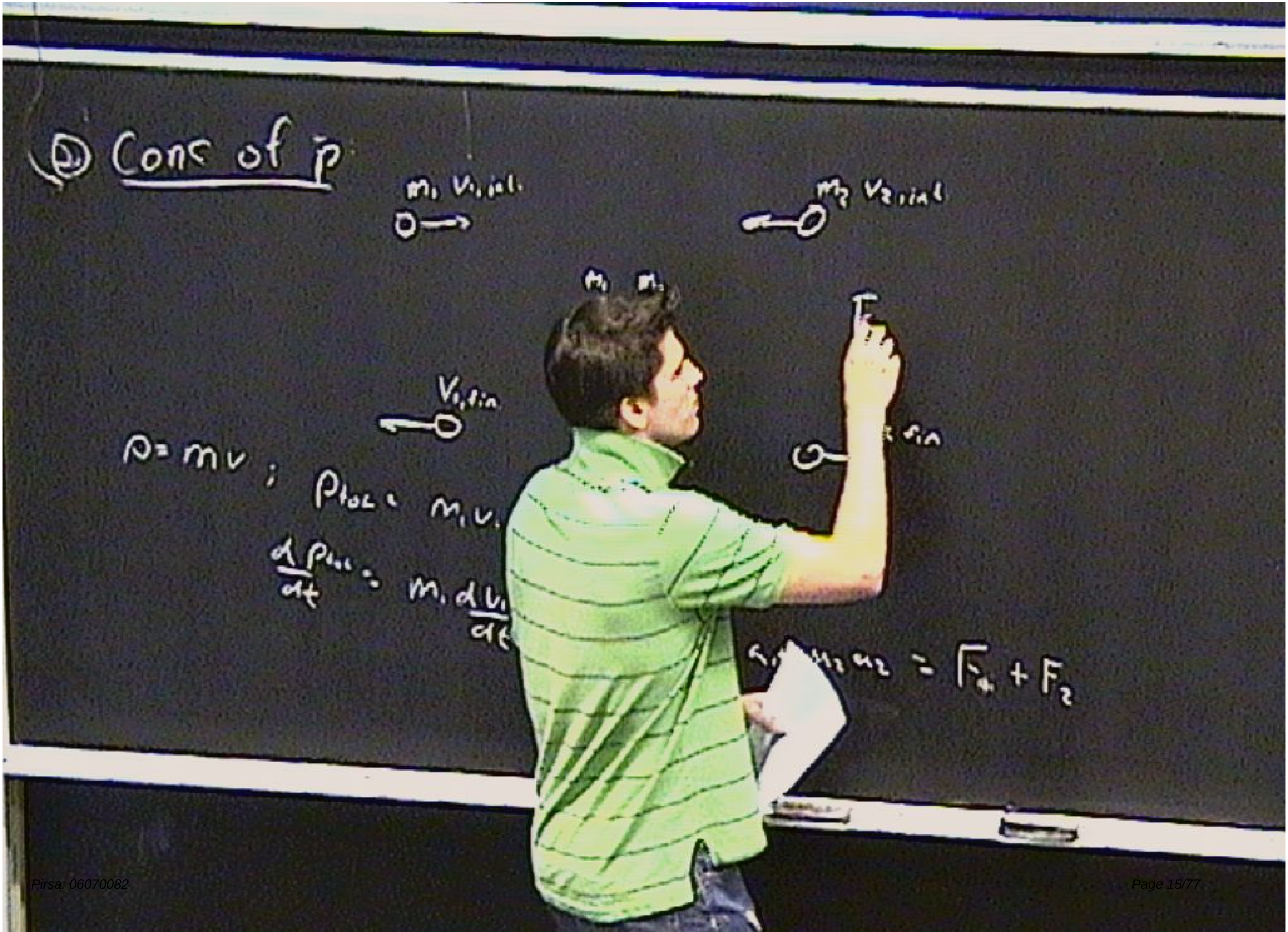
$m_1\ v_{1,init}$ $\qquad$ $m_2\ v_{2,init}$

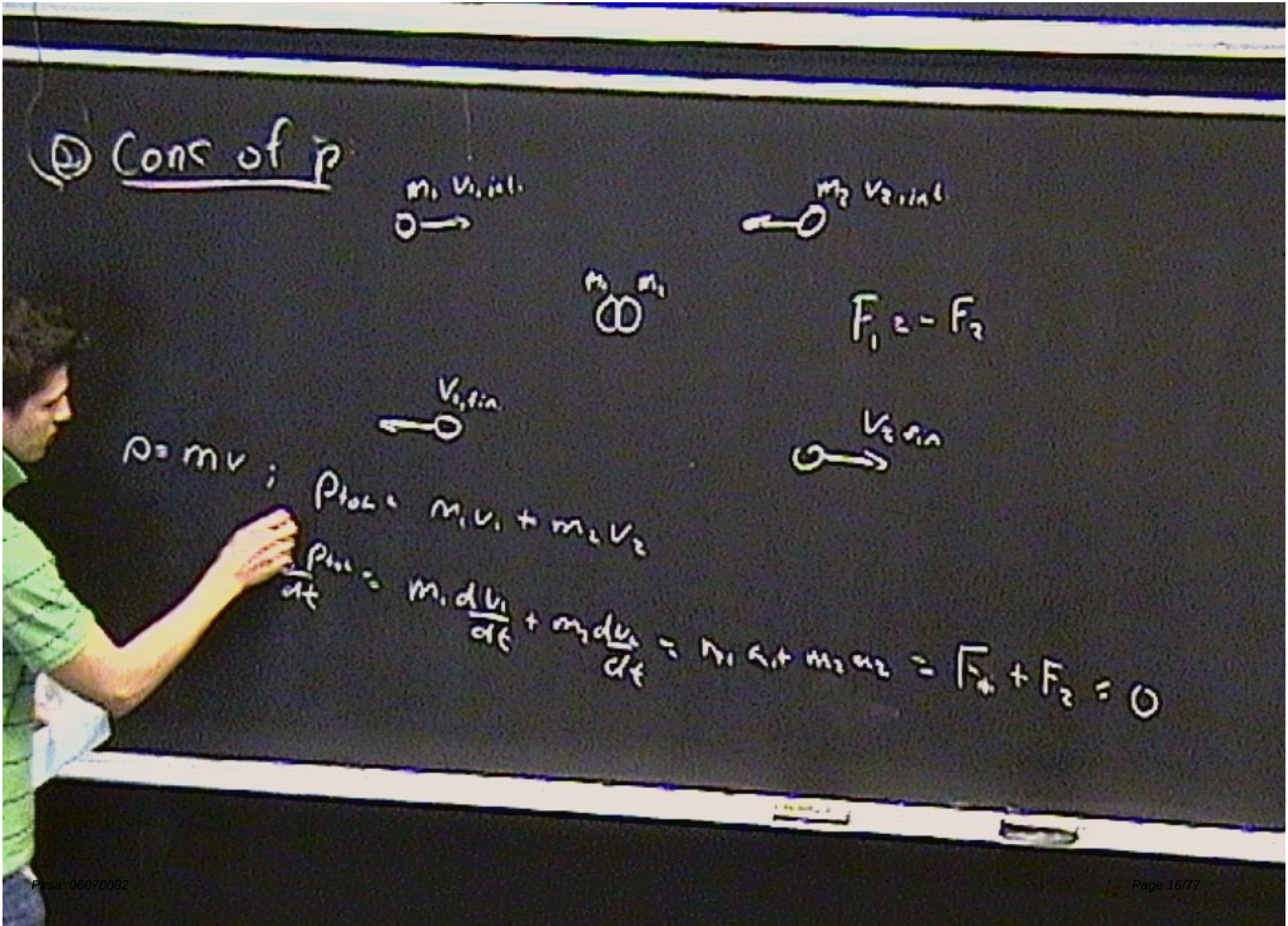
$m_1\ m_1$

$v_{1,fin}$ $\qquad$ $v_{2,fin}$

$p = mv$; $p_{tot} = m_1 v_1 + m_2 v_2$

$\frac{d p_{tot}}{dt} =$


Cons of p
m1 v1,init
m2 v2,init
v1,fin
p = mv ; ptot = m1v1
d ptot / dt = m1 d v1 / dt
= F1 + F2


② Cons of p
$m_1\ v_{1,ini}$
$m_2\ v_{2,ini}$
$F_1 = -F_2$
$v_{1,fin}$
$v_{2,fin}$
$p = mv$; $p_{tot} = m_1 v_1 + m_2 v_2$
$\frac{dp_{tot}}{dt} = m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = m_1 a_1 + m_2 a_2 = F_1 + F_2 = 0$

② Cons of p

$m_1 \; v_{1,\text{inl}}$ $\qquad$ $m_2 \; v_{2,\text{inl}}$

$m_1 \; m_1$

$F_1 = -F_2$

$v_{1,\text{fin}}$ $\qquad$ $v_{2,\text{fin}}$

$p = mv$; $p_{tot} = m_1 v_1 + m_2 v_2$

$$\boxed{\frac{d p_{tot}}{dt} = m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = m_1 a_1 + m_2 a_2 = F_1 + F_2 = 0}$$

② Cons of $\vec{p}$

$F_1 = -F_2$

$p = mv$; $p_{tot} = m_1 v_1 + m_2 v_2$

$$\boxed{\frac{d p_{tot}}{dt}} = m_1 \frac{d v_1}{dt} + m_2 \frac{d v_2}{dt} = m_1 a_1 + m_2 a_2 = F_1 + F_2 = \boxed{0}$$

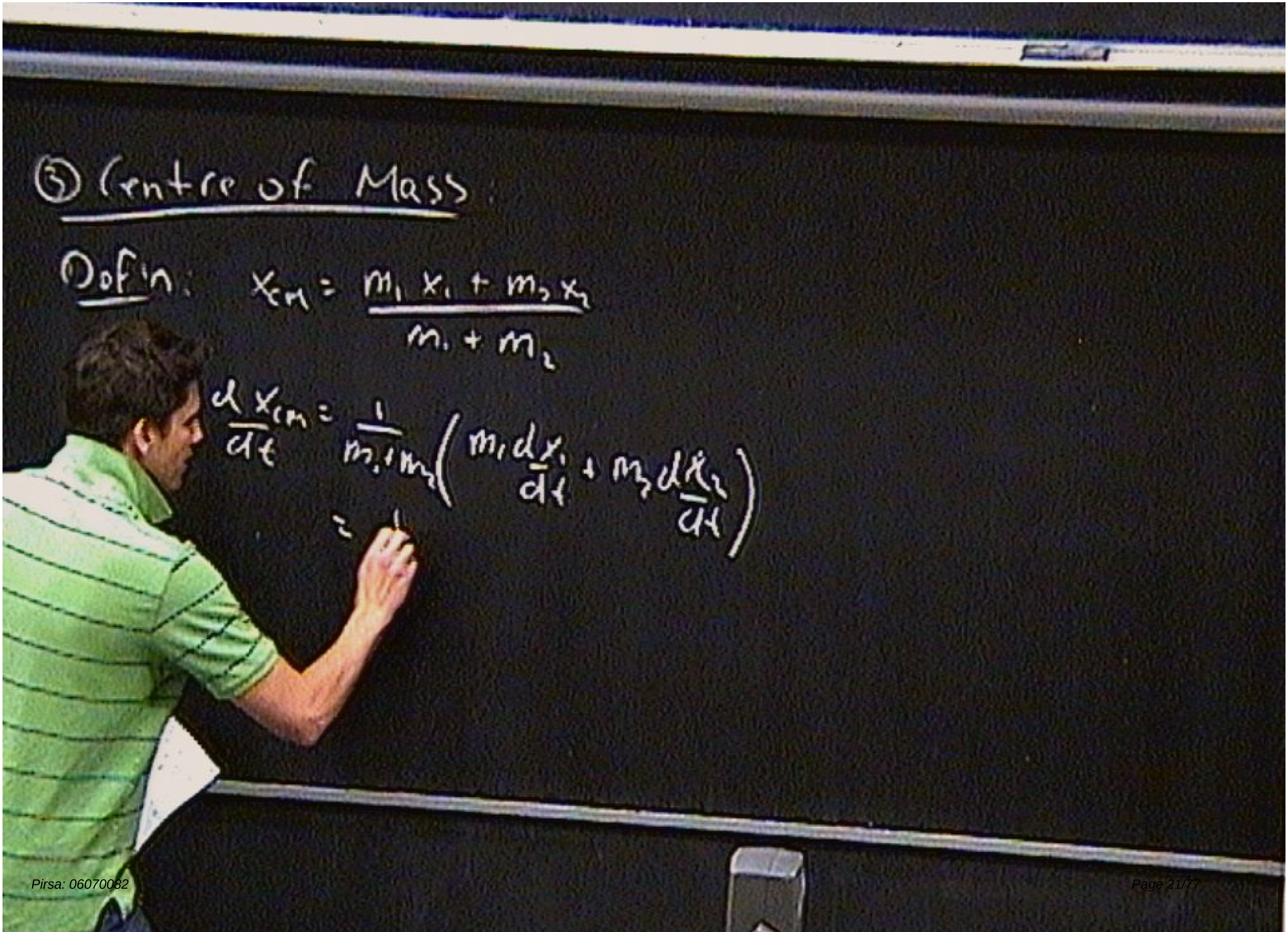
Results:

$$p = \frac{E}{c} \quad (1)$$

$$p_{ini} = p_{fin.} \quad (2)$$

③ Centre of Mass

① Def'n: $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$


③ Centre of Mass:
Def'n: $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$
$\frac{dx_{cm}}{dt} = \frac{1}{m_1+m_2}\left(m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt}\right)$
$= 1$

③ Centre of Mass

Def'n: $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$$\frac{d x_{cm}}{dt} = \frac{1}{m_1+m_2}\left(m_1 \frac{d x_1}{dt} + m_2 \frac{d x_2}{dt}\right)$$

$$= \frac{1}{m_1+m_2} p_{tot}$$

③ Centre of Mass

Def'n: $$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\frac{dx_{cm}}{dt} = \frac{1}{m_1+m_2}\left(m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt}\right)$$

$$= \frac{1}{m_1+m_2}\, p_{tot}$$

$$\frac{d^2x_{cm}}{dt^2} = \frac{1}{m_1+m_2}\frac{dp_{tot}}{dt} = 0$$

Results:

$$p = \frac{E}{c} \quad (1)$$

$$p_{ini} = p_{fin.} \quad (2)$$

$$V_{cm,int} = V_{cm,fin} \quad (3)$$

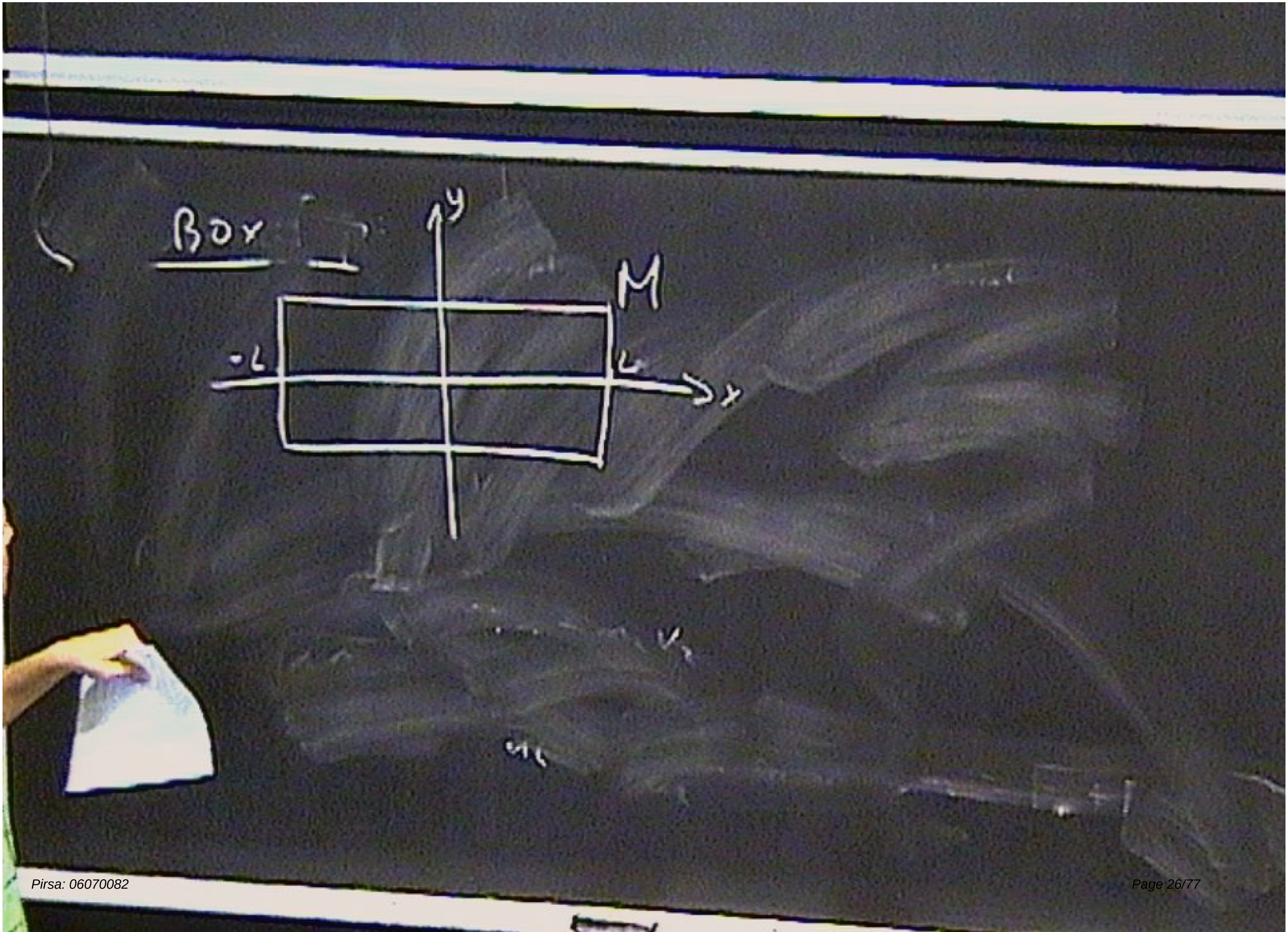
Results:

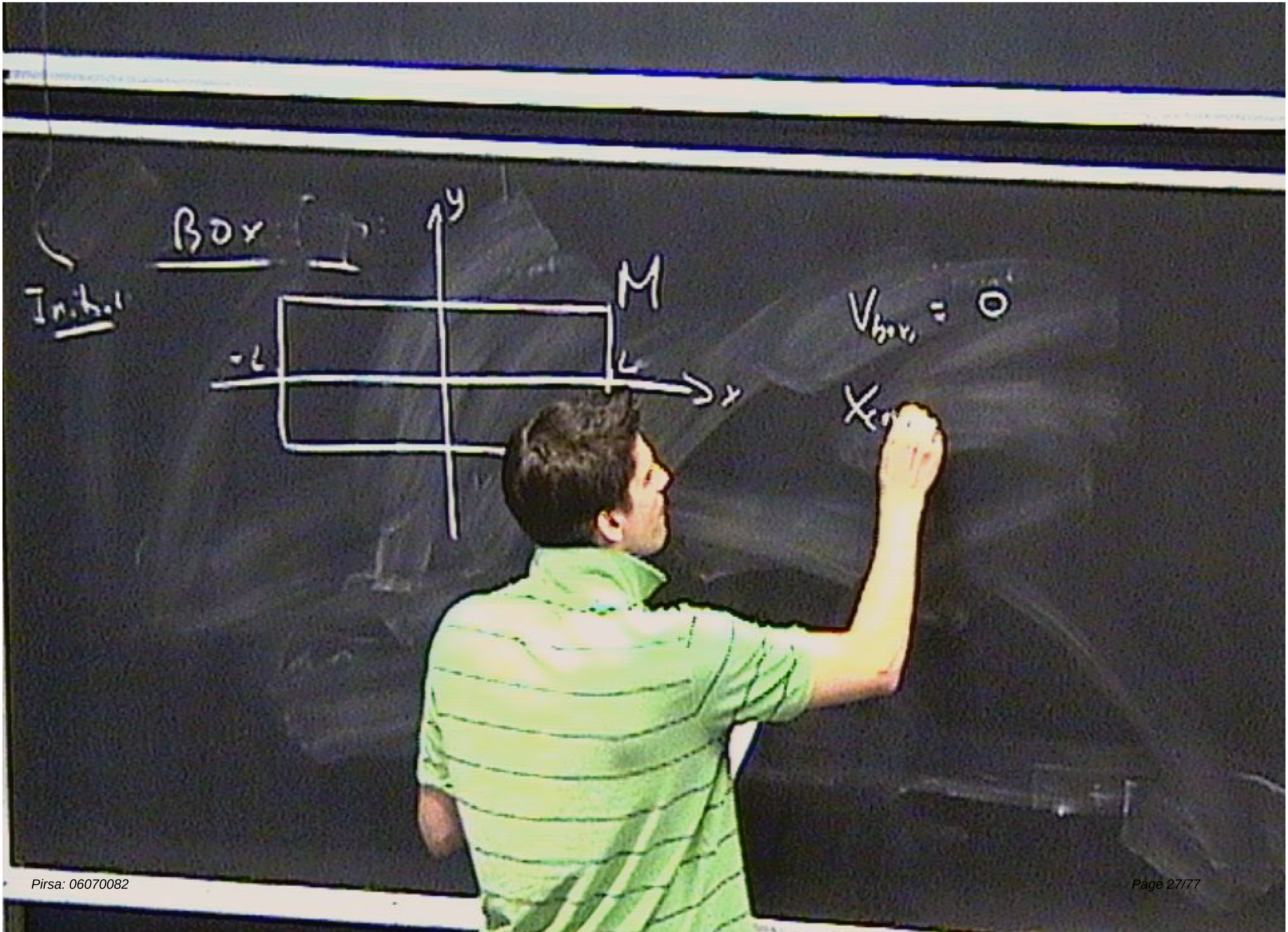
$$p = \frac{E}{c} \quad (1)$$

$$p_{ini} = p_{fin.} \quad (2)$$

$$V_{cm,ini} = V_{cm,fin} \quad (3)$$

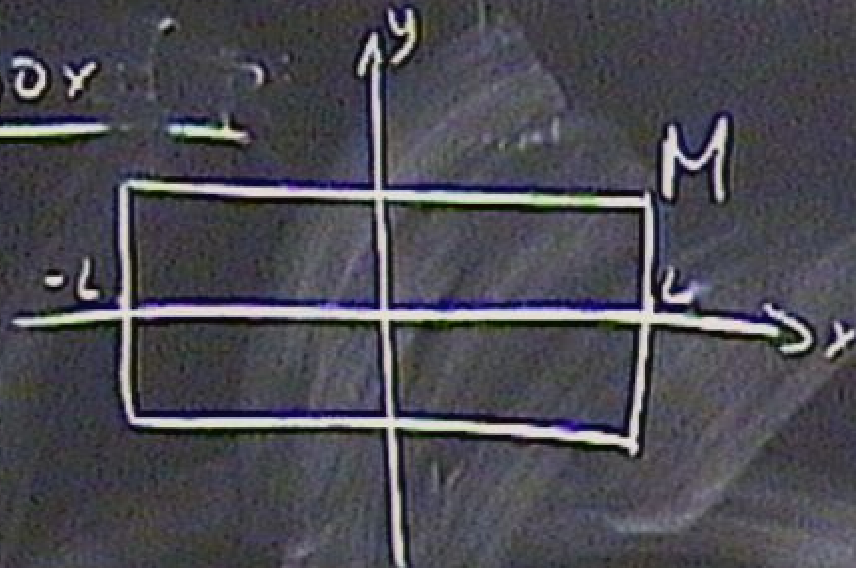
$$0 = 0$$


Box I:
y
M
-L
L
x


Box
Initial
y
M
-L
L
x
$V_{box} = 0$

Box

Initial



$V_{box} = 0$; $p_{in} = 0$

$X_{cm,M} = 0$

Results:

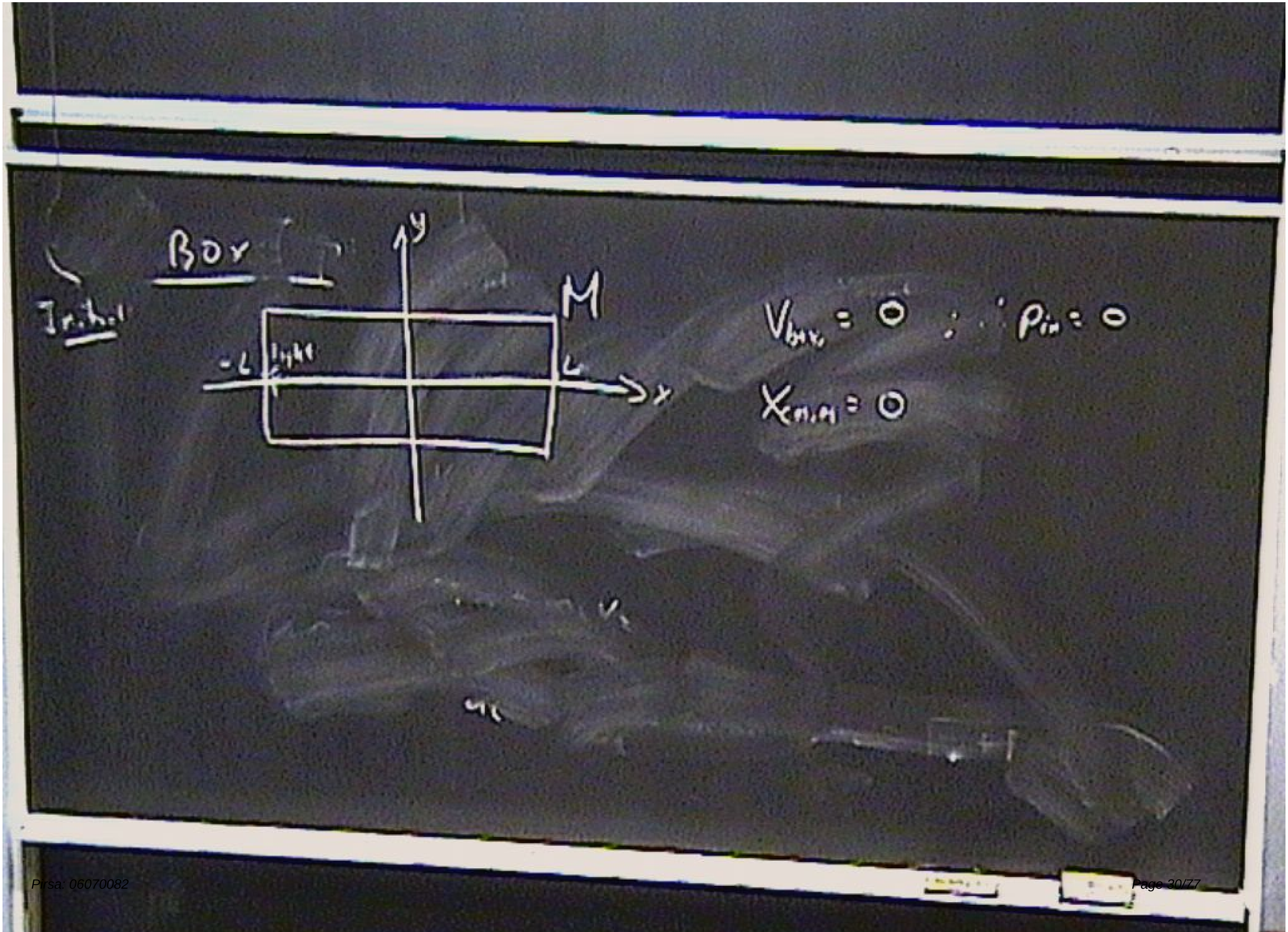
$$p = \frac{E}{c} \quad (1)$$

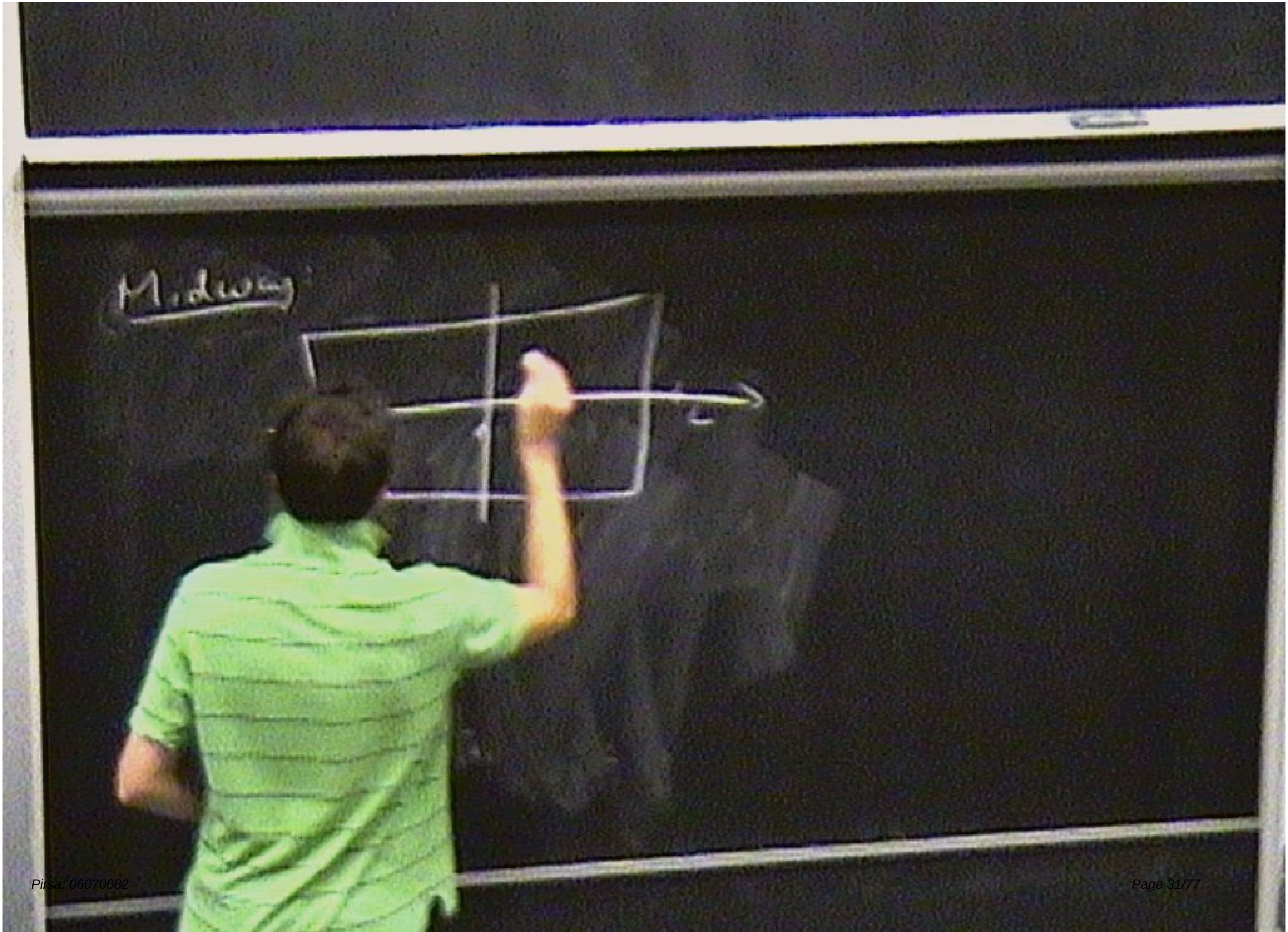
$$p_{ini} = p_{fin.} \quad (2)$$

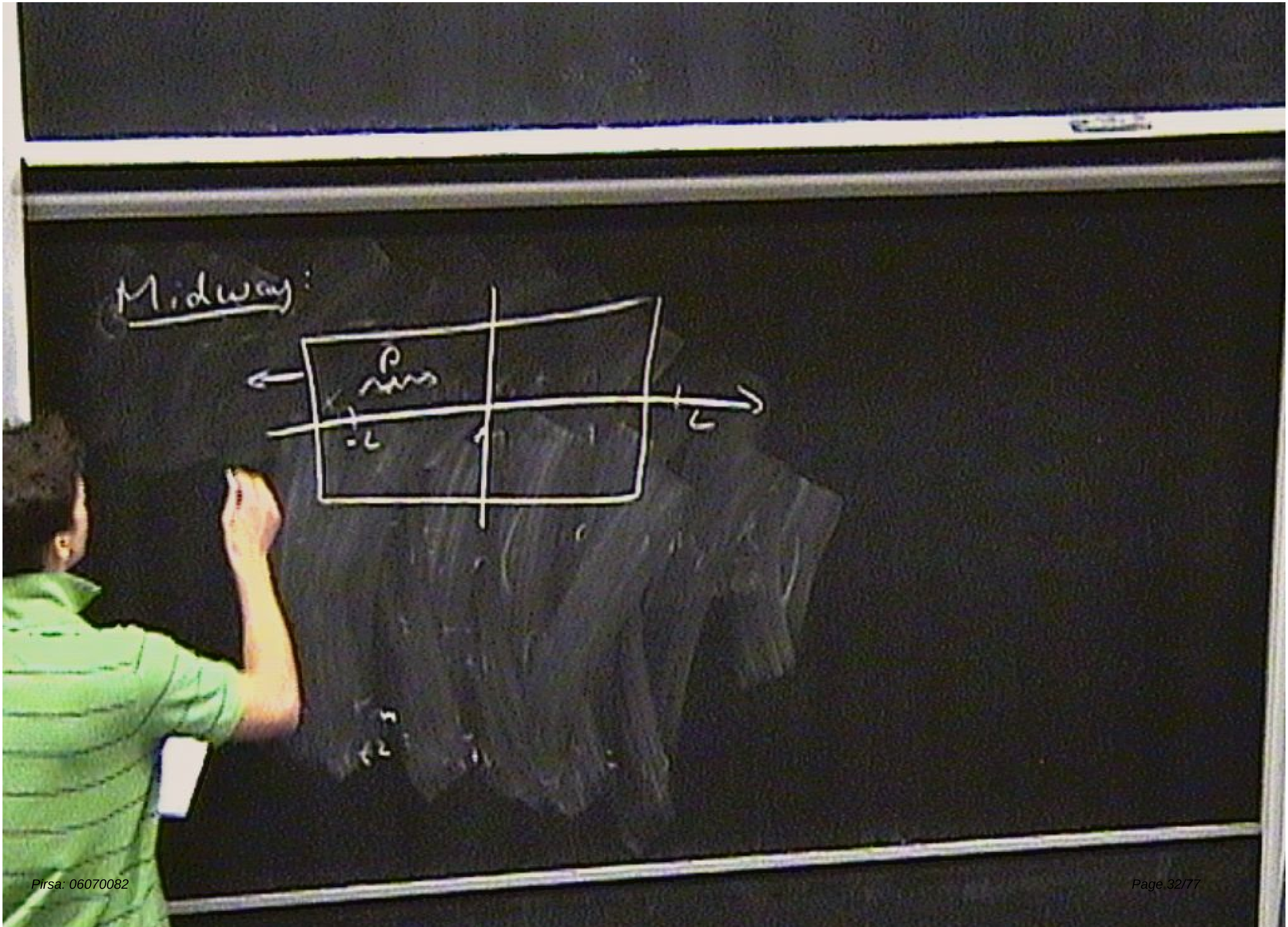
$$V_{cm,int} = V_{cm,fin} \quad (3)$$

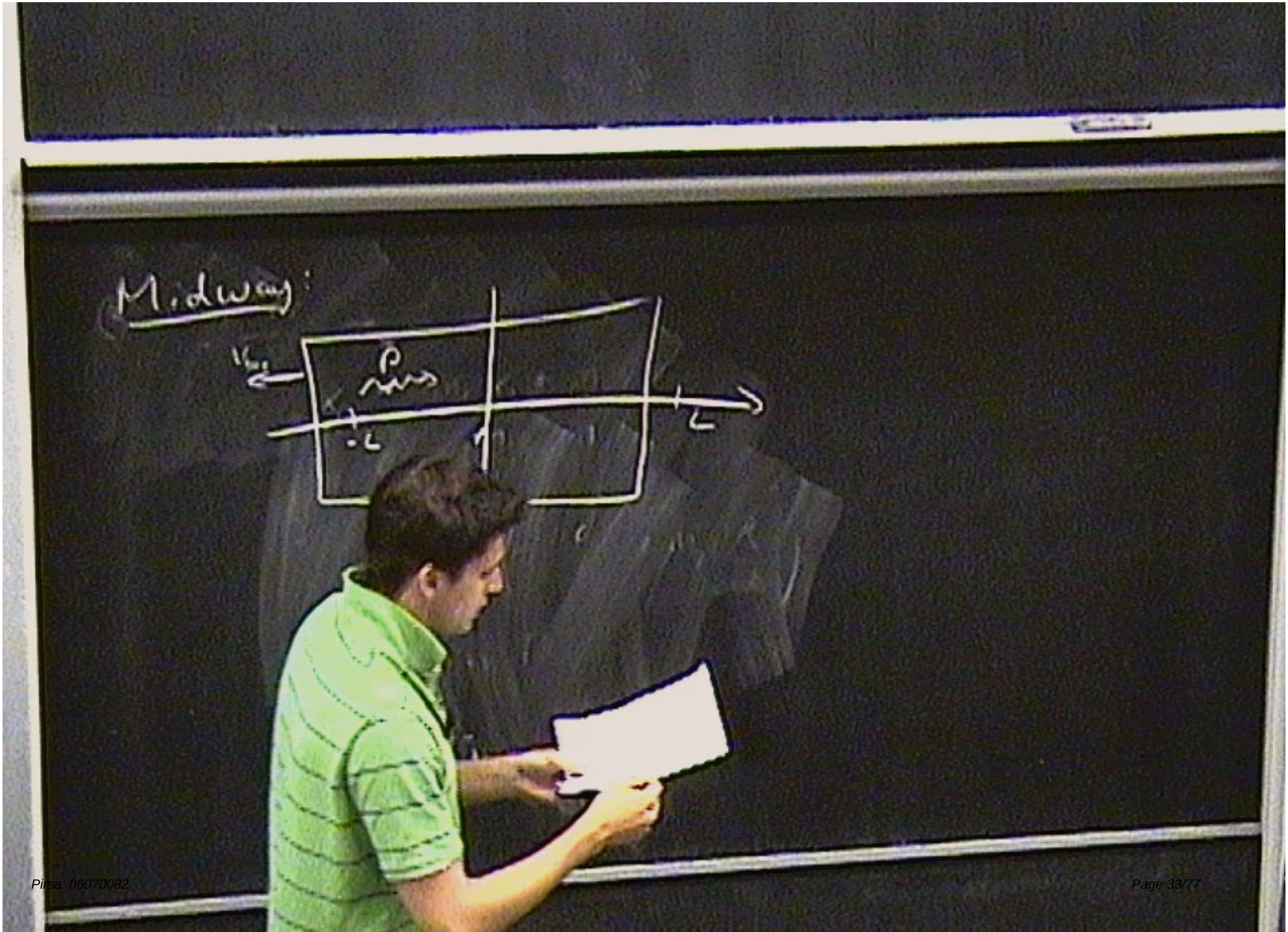
$$0 = 0$$

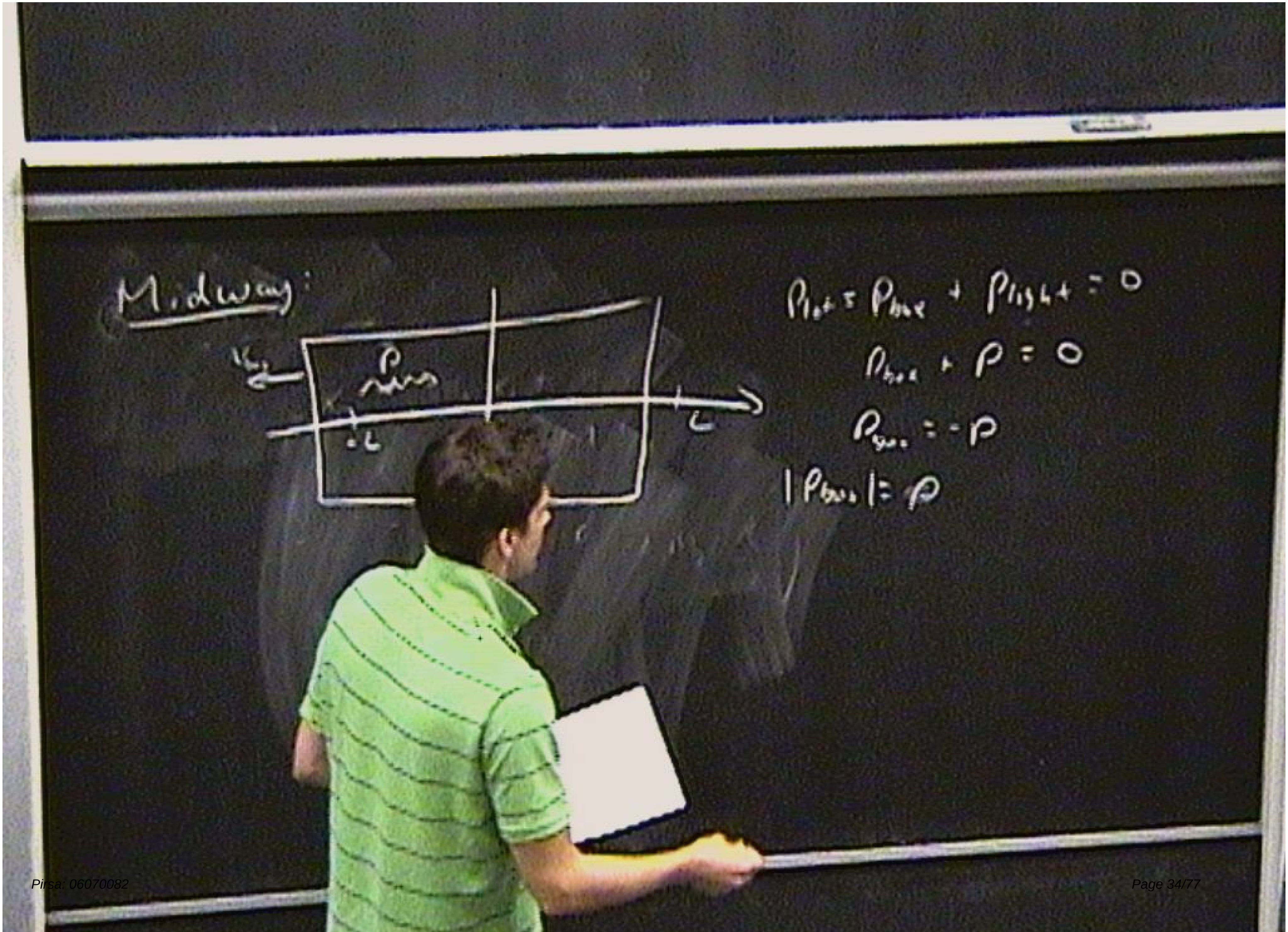
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (4)$$

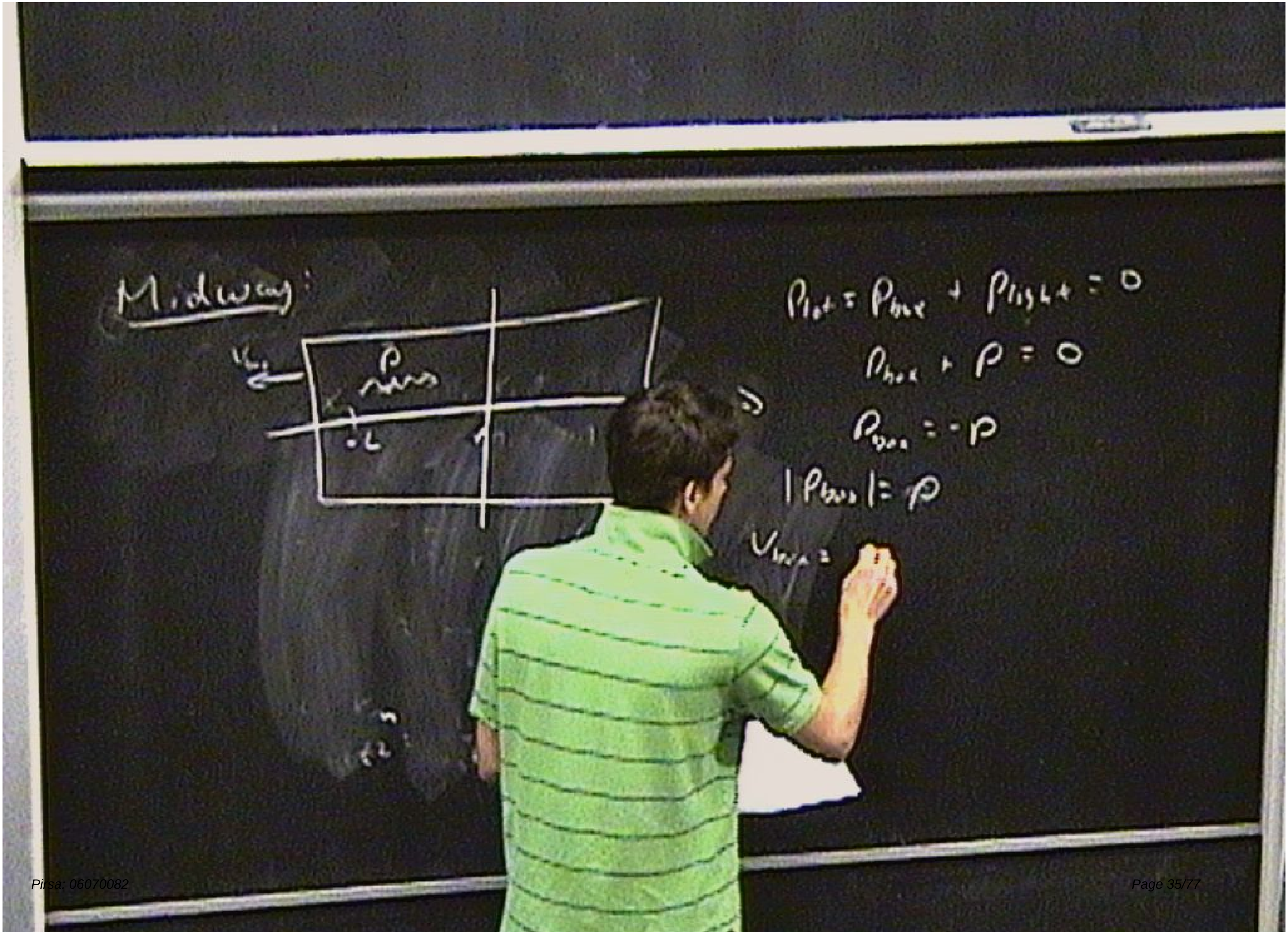

Box (I)
Initial
y
M
-L
light
L
x
V_box = 0 ; ∴ p_in = 0
X_c.m. = 0


Midway

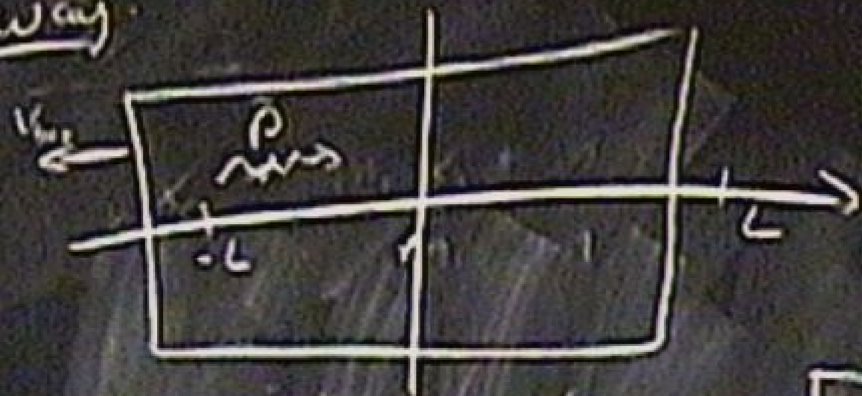

Midway:
-L
L


Midway:


Midway:
ρtot = ρbox + ρlight = 0
ρbox + ρ = 0
ρbox = -ρ
|ρbox| = ρ
ρ
-L
L


Midway:
p_tot = p_box + p_light = 0
p_box + p = 0
p_box = -p
|p_box| = p
v_box =

Midway:



$$p_{tot} = p_{box} + p_{light} = 0$$

$$p_{box} + p = 0$$

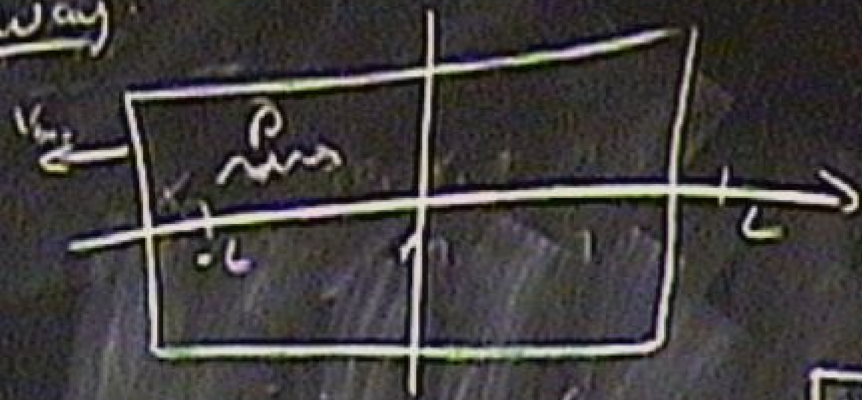
$$p_{box} = -p$$

$$|p_{box}| = p$$

$$\boxed{v_{box} = \frac{p}{M}}$$

$x_{cm} = ?$

Midway:



$P_{tot} = P_{box} + P_{light} = 0$

$P_{box} + P = 0$

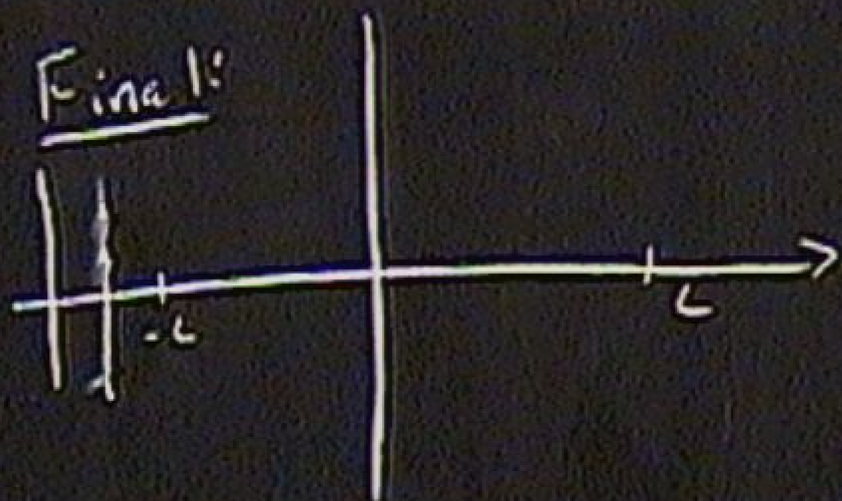
$P_{box} = -P$

$|P_{box}| = P$

$\boxed{V_{box} = \frac{P}{M}}$

$X_{CM} = ?$

Final:



Results:

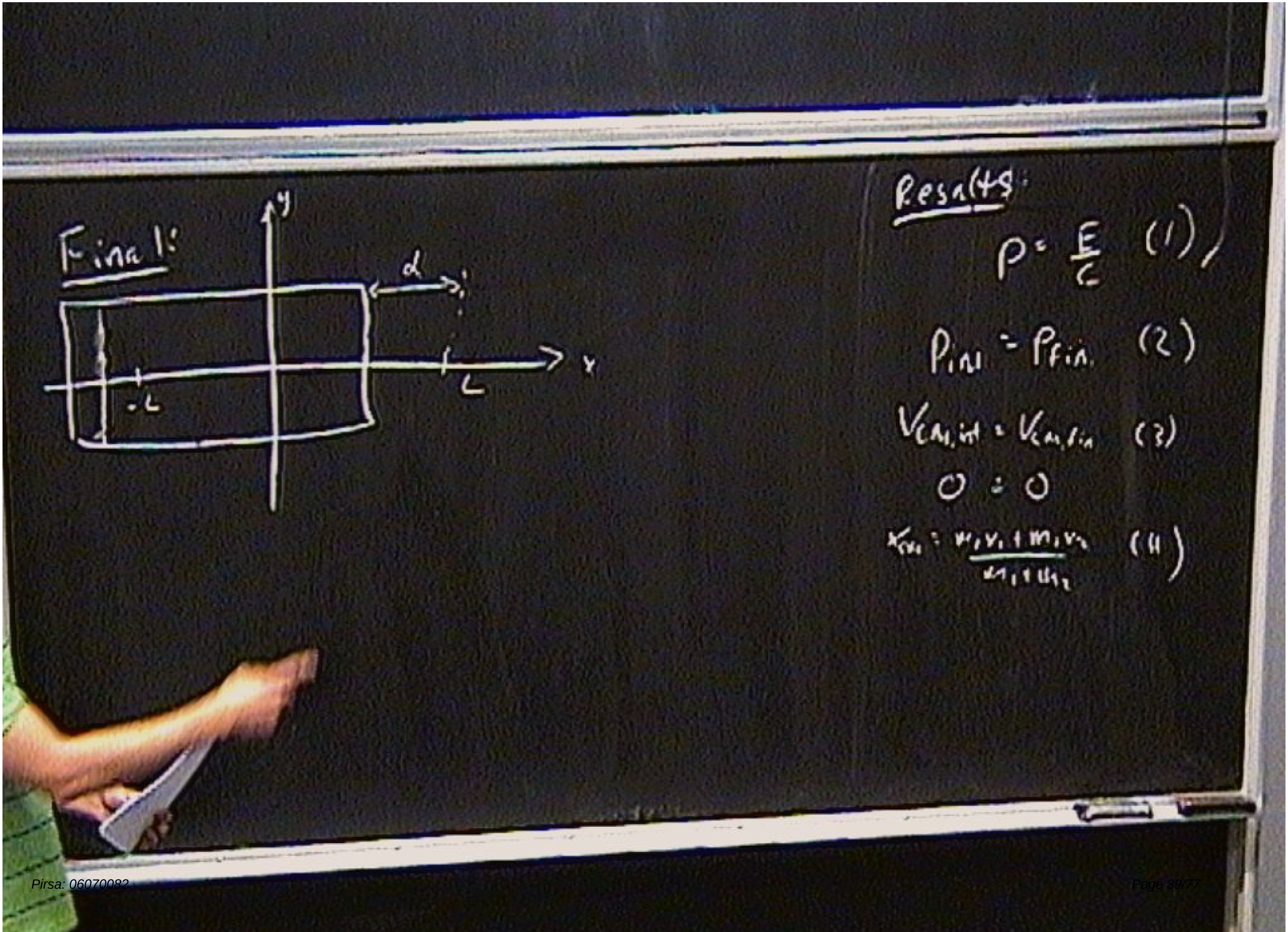
$$p = \frac{E}{c} \quad (1)$$

$$P_{ini} = P_{fin} \quad (2)$$

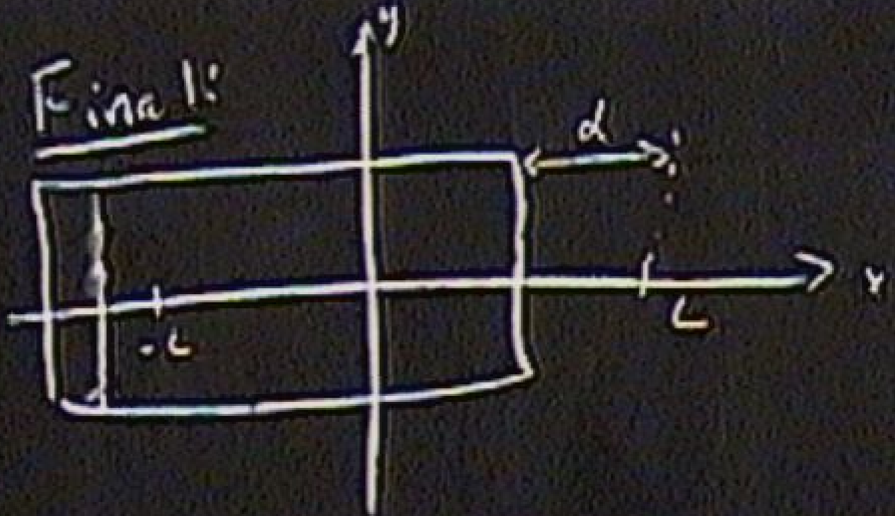
$$V_{cm,ini} = V_{cm,fin} \quad (3)$$

$$0 = 0$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (4)$$


Final:
y
d
x
-L
L
Results:
$p = \frac{E}{c}$ (1)
$P_{ini} = P_{fin}$ (2)
$V_{cm,int} = V_{cm,fin}$ (3)
$0 = 0$
$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ (4)

Final:



Results:

$$p = \frac{E}{c} \quad (1)$$

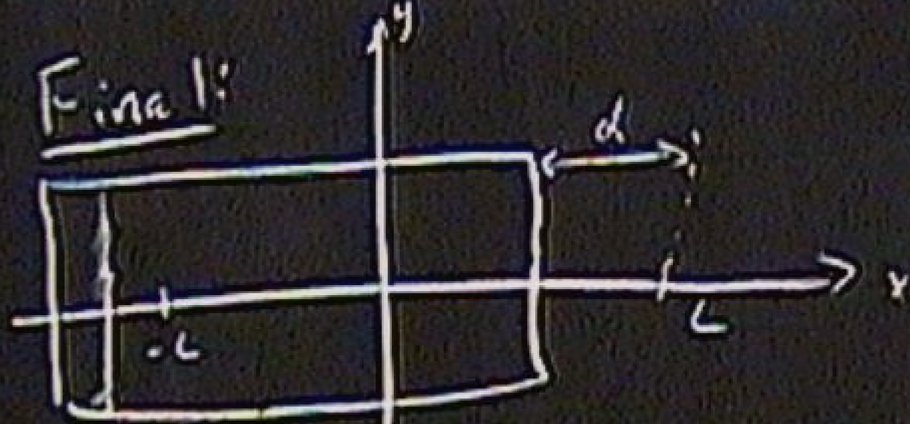
$$p_{ini} = p_{fin} \quad (2)$$

$$V_{cm,ini} = V_{cm,fin} \quad (3)$$

$$0 = 0$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (4)$$

Final:



$$T = \frac{2L}{c}$$

Results:

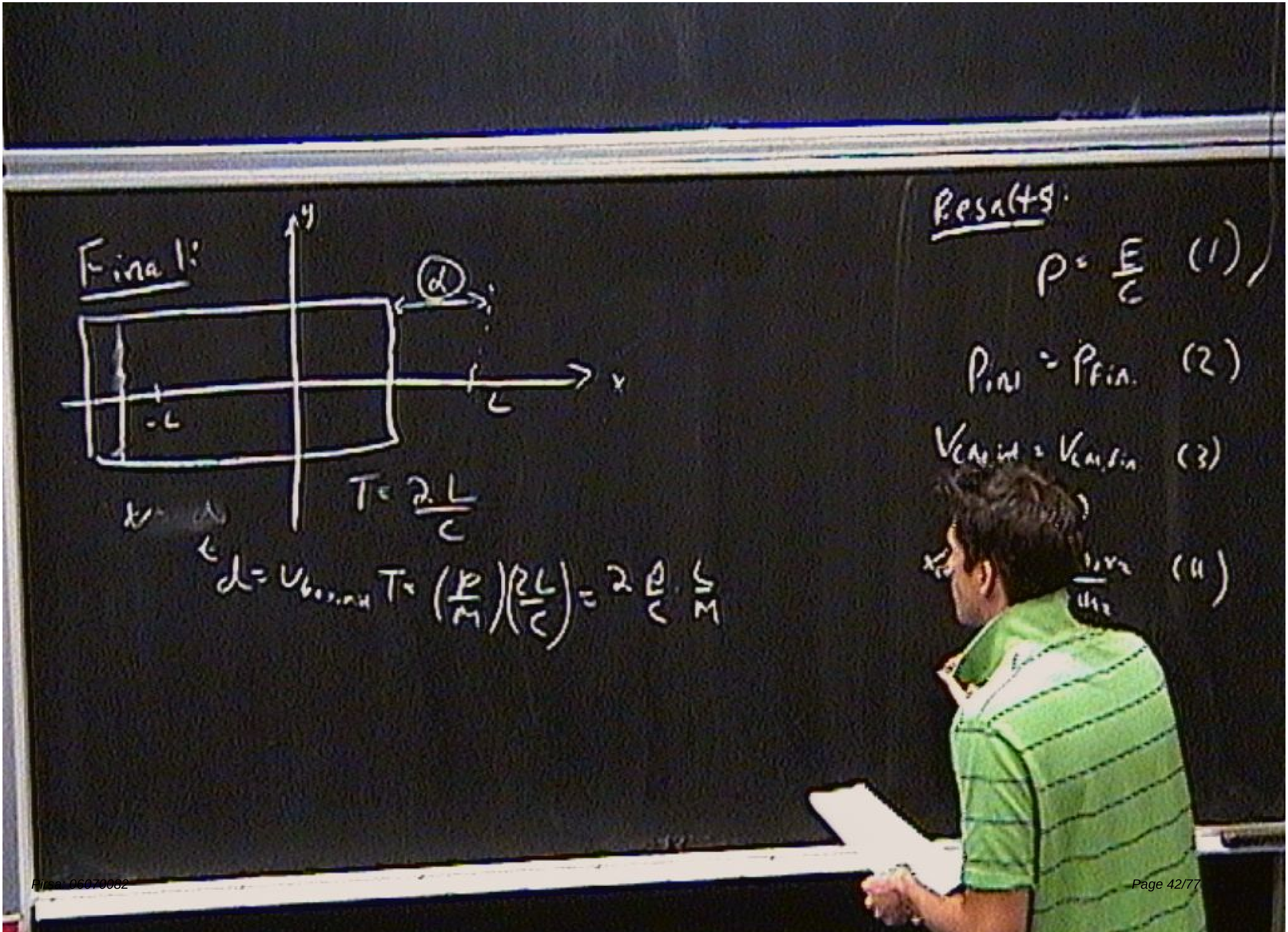
$$p = \frac{E}{c} \quad (1)$$

$$p_{ini} = p_{fin} \quad (2)$$

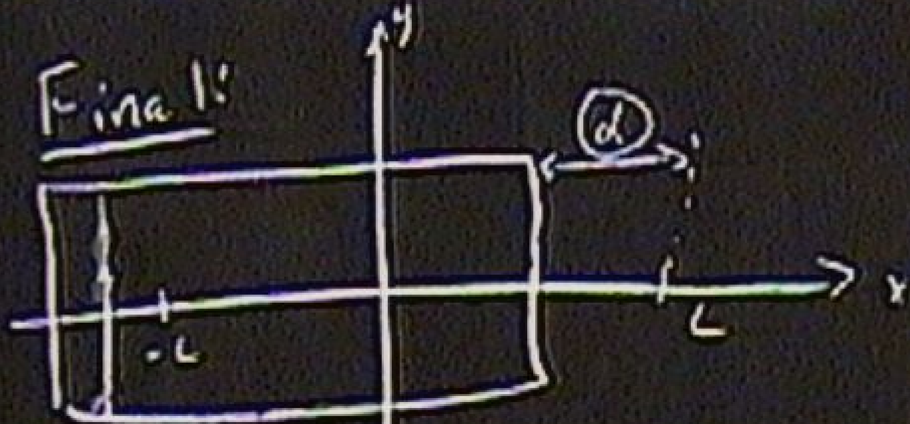
$$V_{CM,ini} = V_{CM,fin} \quad (3)$$

$$0 = 0$$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (4)$$


Final:
Results:
p = E/c (1)
P_ini = P_fin (2)
V_CM,ini = V_CM,fin (3)
T = 2L/c
d = v_box,ini T = (p/M)(2L/c) = 2 p/c · L/M

Final:



$$T = \frac{2L}{c}$$

$$d = V_{box,final} T = \left(\frac{p}{M}\right)\left(\frac{2L}{c}\right) = 2\frac{p}{c}\cdot\frac{L}{M}$$

Results:

$$p = \frac{E}{c} \quad (1)$$

$$P_{ini} = P_{fin.} \quad (2)$$

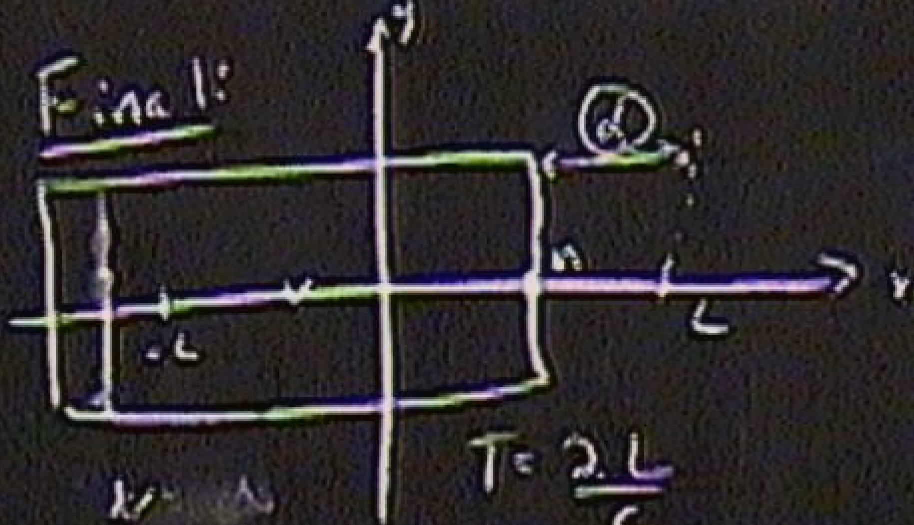
$$V_{CM,ini} = V_{CM,fin} \quad (3)$$

$$0 = 0$$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (4)$$

$$d = \frac{2pL}{cM} \quad (5)$$

Final:



$T = \frac{2L}{c}$

$V_{CM} = \cdot d$

$d = v_{box} T = \left(\frac{p}{M}\right)\left(\frac{2L}{c}\right) = 2\,\frac{p}{c}\cdot\frac{L}{M}$

Results:

$p = \frac{E}{c}$ (1)

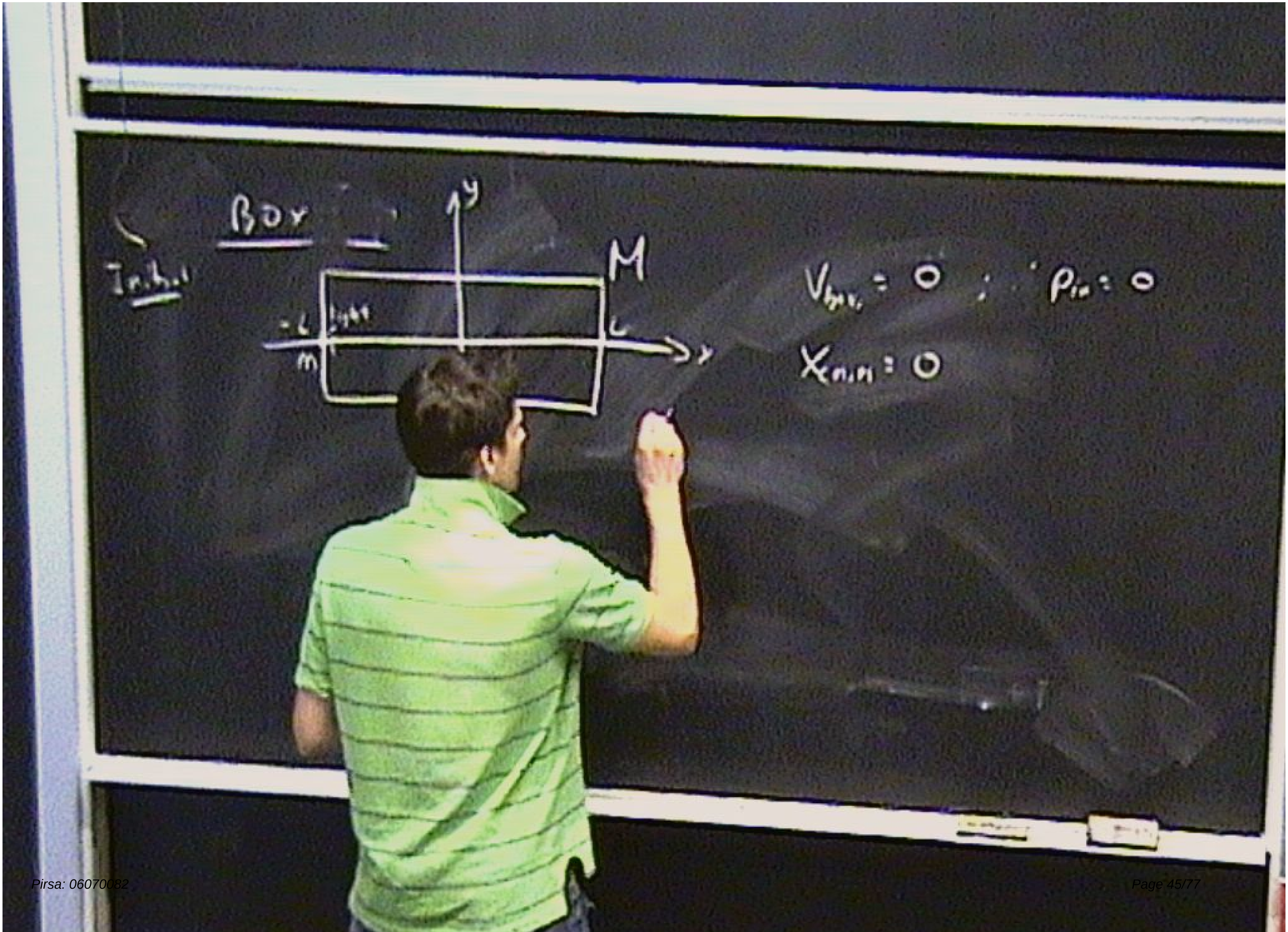
$P_{ini} = P_{fin}$ (2)

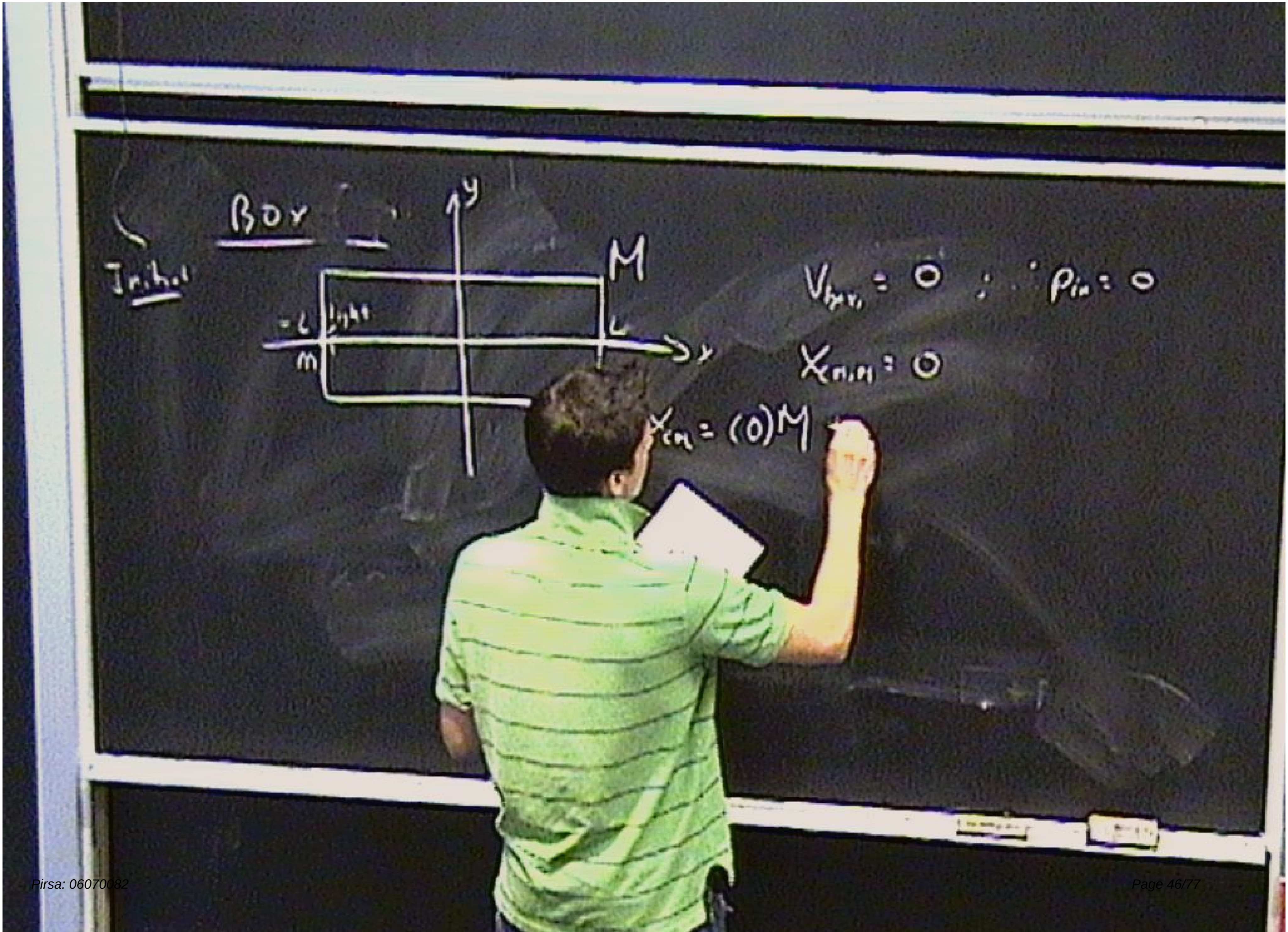
$V_{CM,ini} = V_{CM,fin}$ (3)

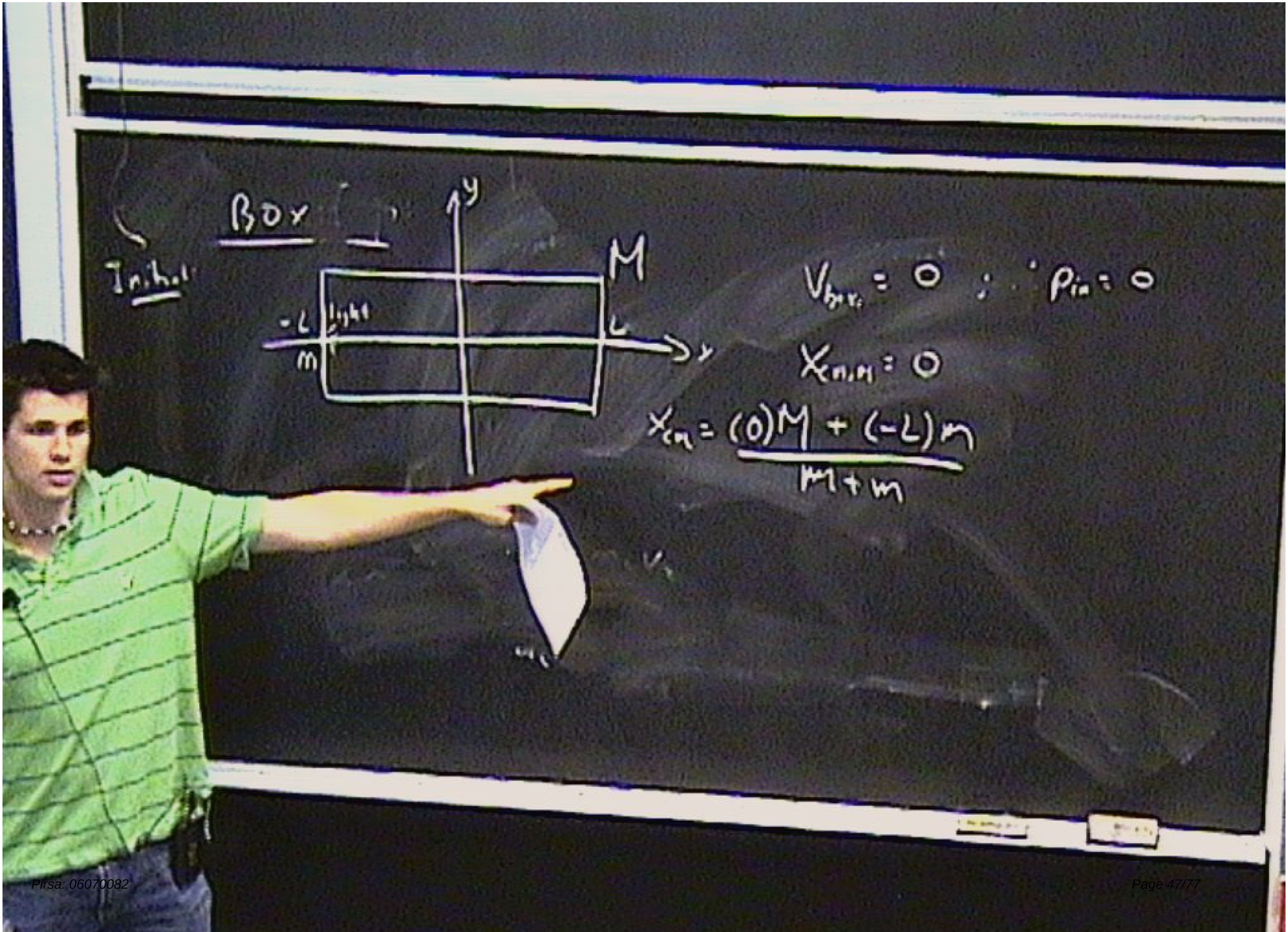
$0 = 0$

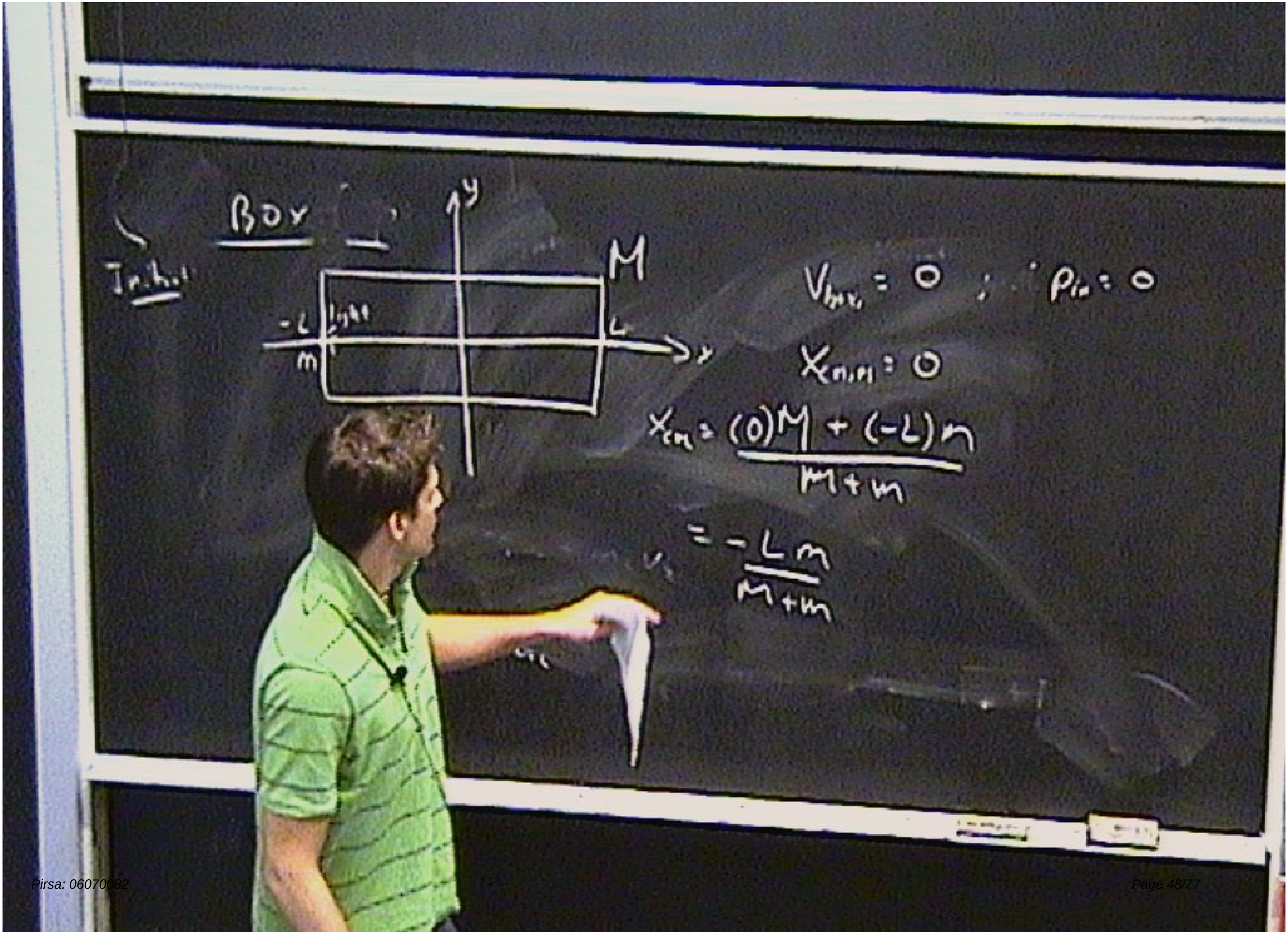
$V_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$ (4)

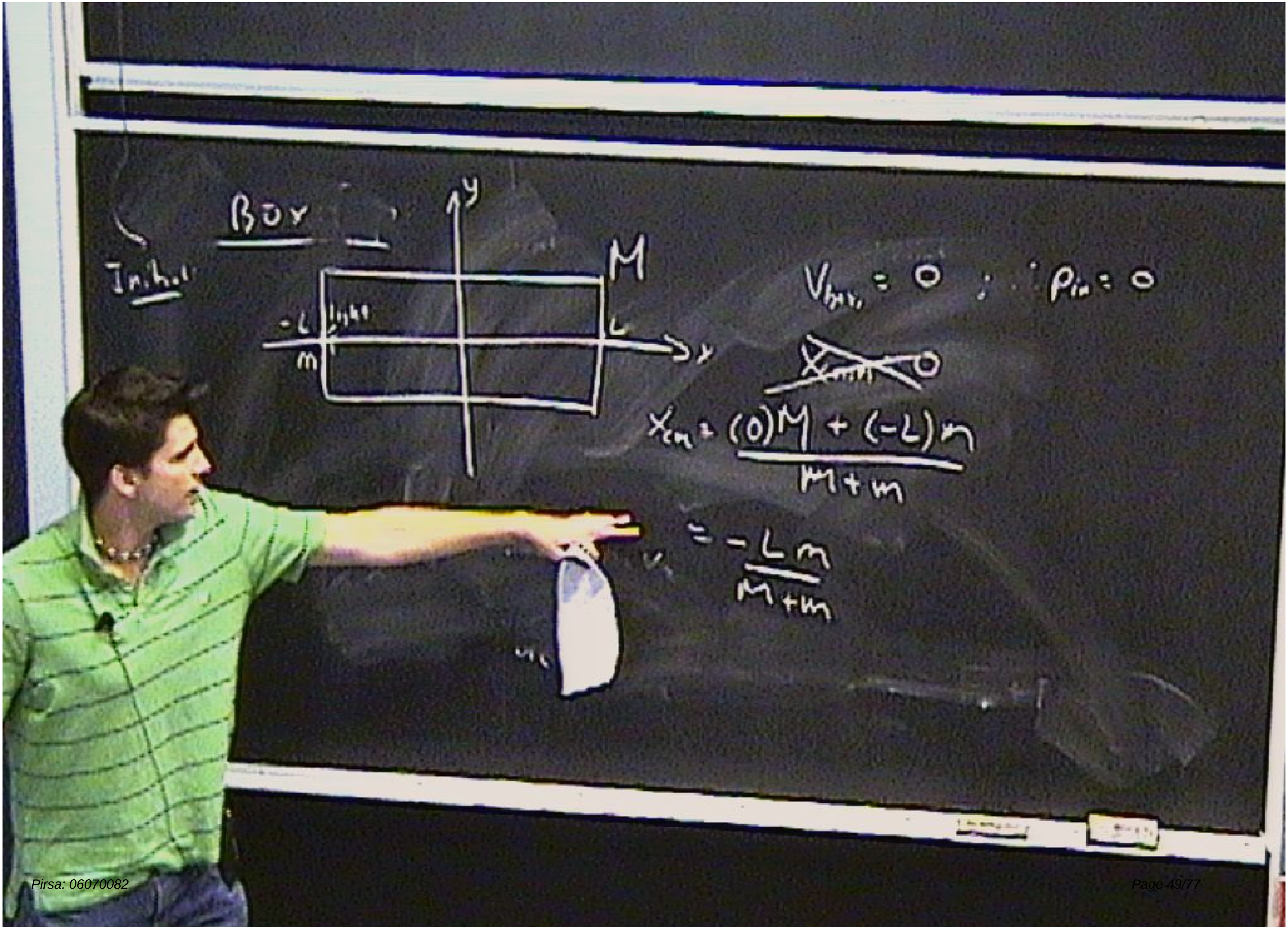
$d = \frac{2pL}{cM}$ (5)


Box
Initial
M
m
light
-L
L
y
x
V_box = 0
P_in = 0
X_min = 0


Box
Initial
y
M
-L
light
L
m
x
$V_{box,i} = 0$; $p_{in} = 0$
$x_{cm,i} = 0$
$x_{cm} = (0)M$

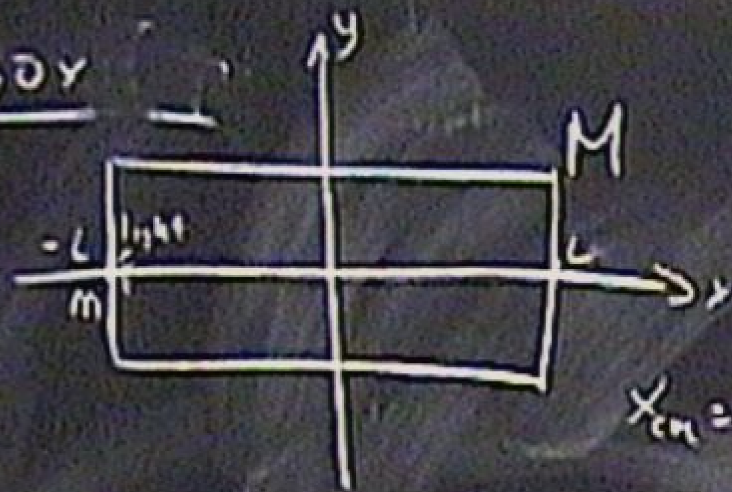

Box
Initial
y
M
-L
light
L
m
x
$V_{box} = 0$; $p_{in} = 0$
$X_{cm,i} = 0$
$X_{cm} = \frac{(0)M + (-L)m}{M+m}$


Box
Initial
y
M
-L
light
L
m
x
Vbox,i = 0 ; pli = 0
Xcm,i = 0
Xcm = (0)M + (-L)m / M+m
= -Lm / M+m


Box
Initial
y
M
-L
light
m
L
x
$V_{box} = 0$; $p_{in} = 0$
$X_{cm} = \frac{(0)M + (-L)m}{M+m}$
$= \frac{-Lm}{M+m}$

Box

Initial



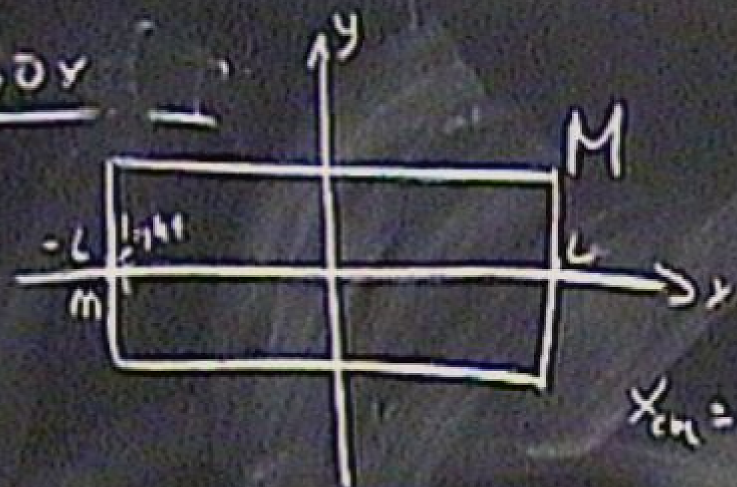
$V_{box,i} = 0$; $p_{in} = 0$

$$X_{cm} = \frac{(0)M + (-L)m}{M+m}$$

$$= \frac{-Lm}{M+m}$$

Initial

Box



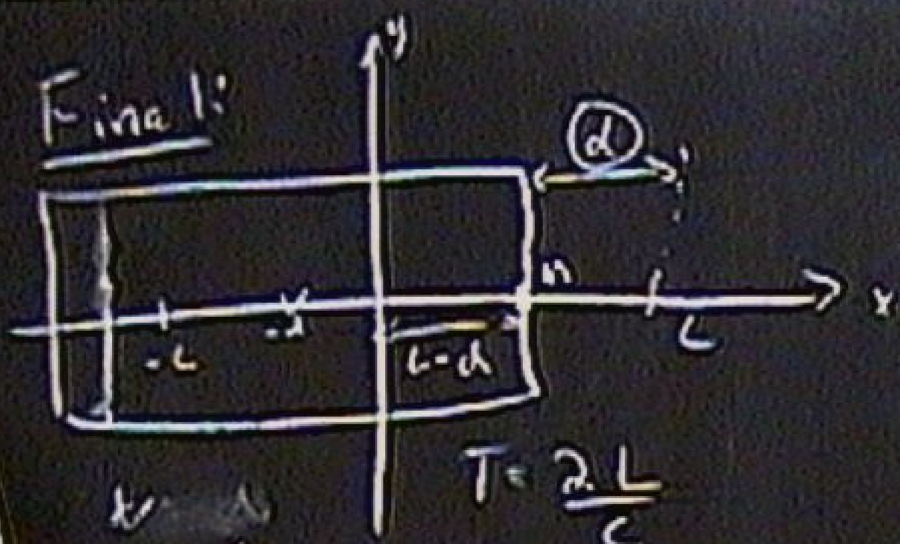
$V_{box,i} = 0$; $p_{in} = 0$

$\cancel{X_{cm} = 0}$

$$X_{cm} = \frac{(0)M + (-L)m}{M+m}$$

$$= \frac{-Lm}{M+m}$$

Final 1:



$x_{cm,M} = -d$

$T = \frac{2L}{c}$

$d = v_{box,max} T = \left(\frac{p}{M}\right)\left(\frac{2L}{c}\right) = 2\frac{E}{c}\cdot\frac{L}{M}$

$x_{cm} = M(-d)$

Results:

$p = \frac{E}{c}$ (1)

$p_{ini} = p_{fin}$ (2)

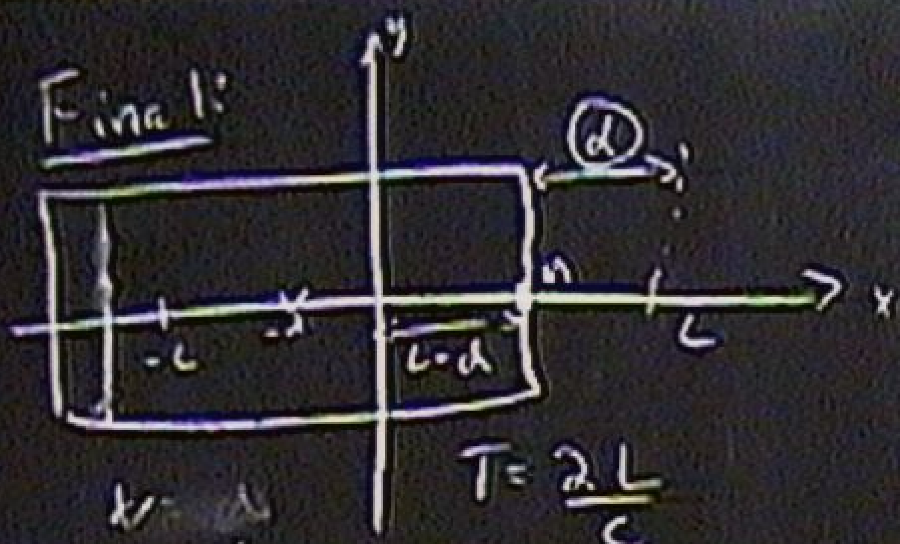
$v_{cm,ini} = v_{cm,fin}$ (3)

$0 = 0$

$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ (4)

$d = \frac{2pL}{cM}$ (5)

Final:



$x_{cm,i} = -d$

$T = \frac{2L}{c}$

$d = v_{box,final}\,T = \left(\frac{p}{M}\right)\left(\frac{2L}{c}\right) = 2\,\frac{p}{c}\cdot\frac{L}{M}$

$$x_{cm} = \frac{M(-d) + (L-d)m}{M + m}$$

Results:

$p = \frac{E}{c}$ (1)

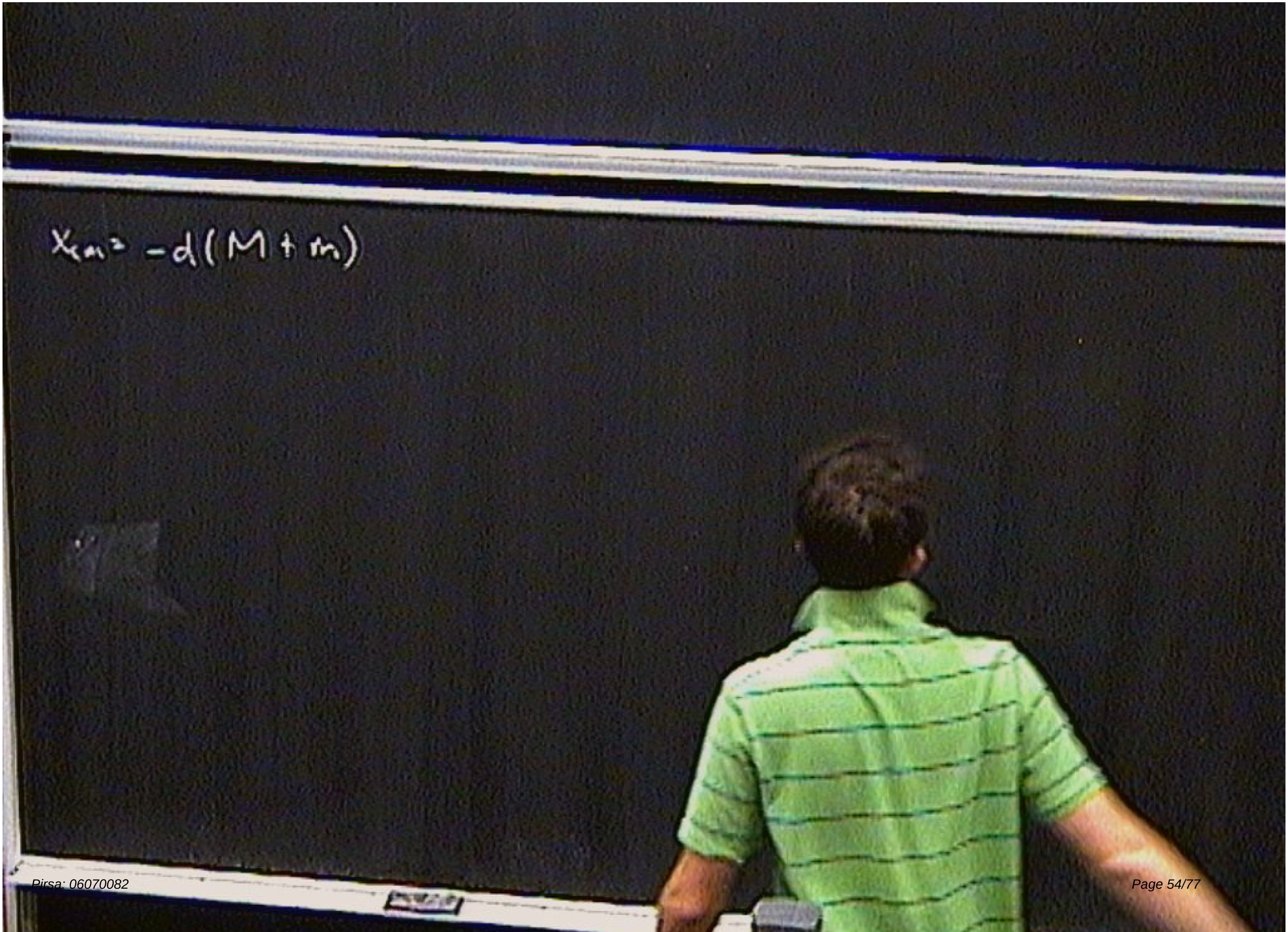
$p_{ini} = p_{fin}$ (2)

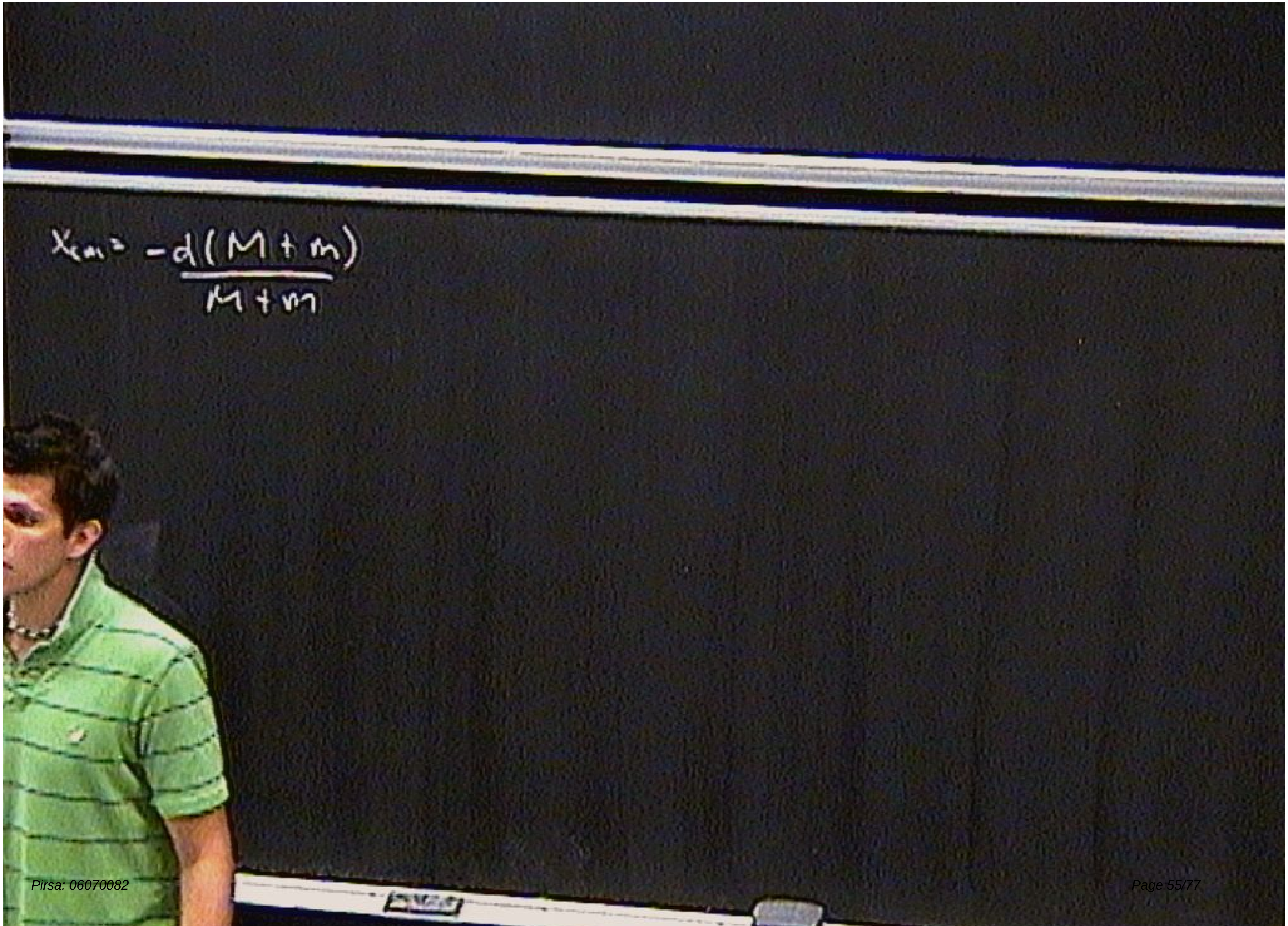
$V_{cm,ini} = V_{cm,fin}$ (3)

$0 = 0$

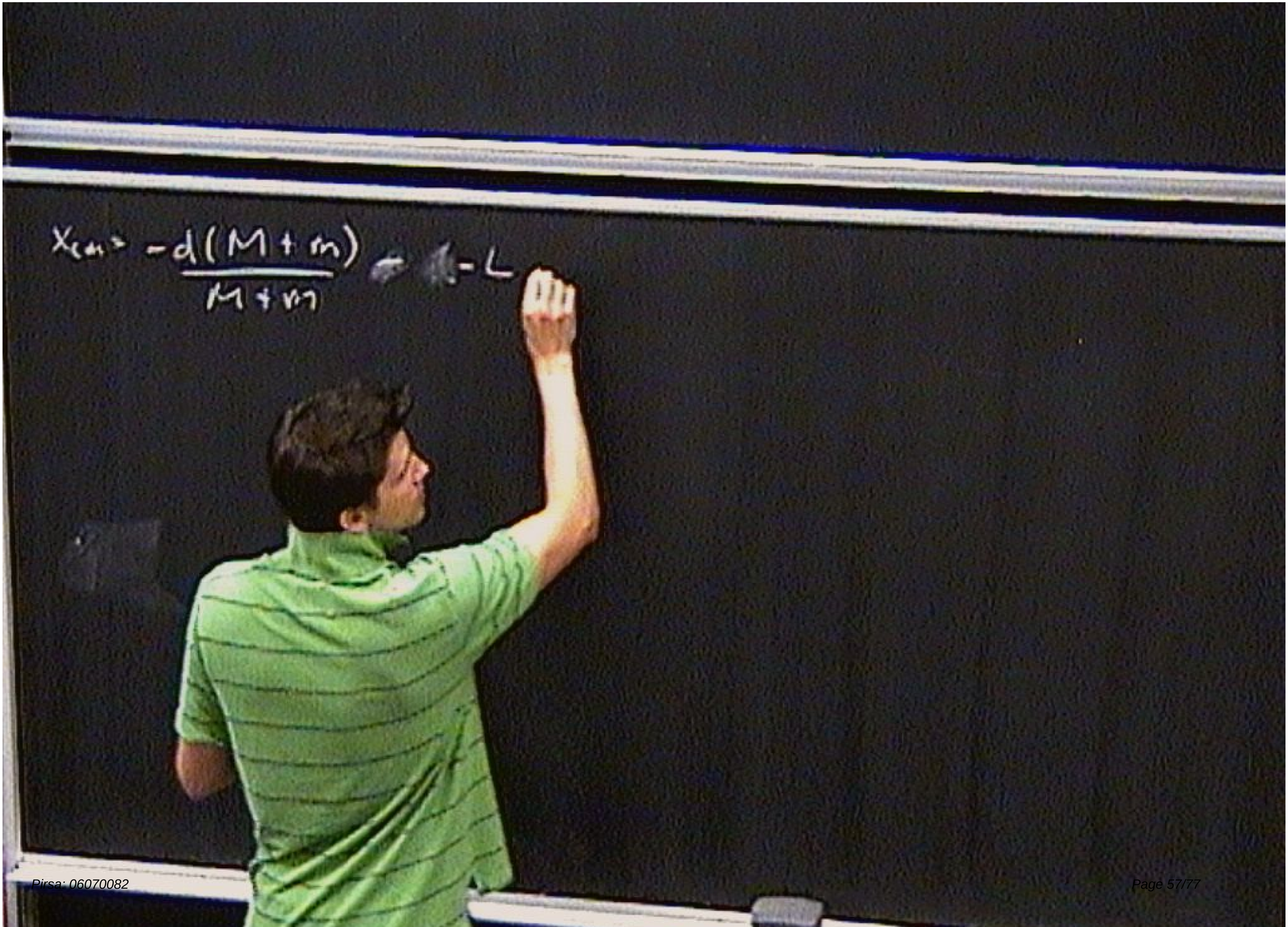
$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ (4)

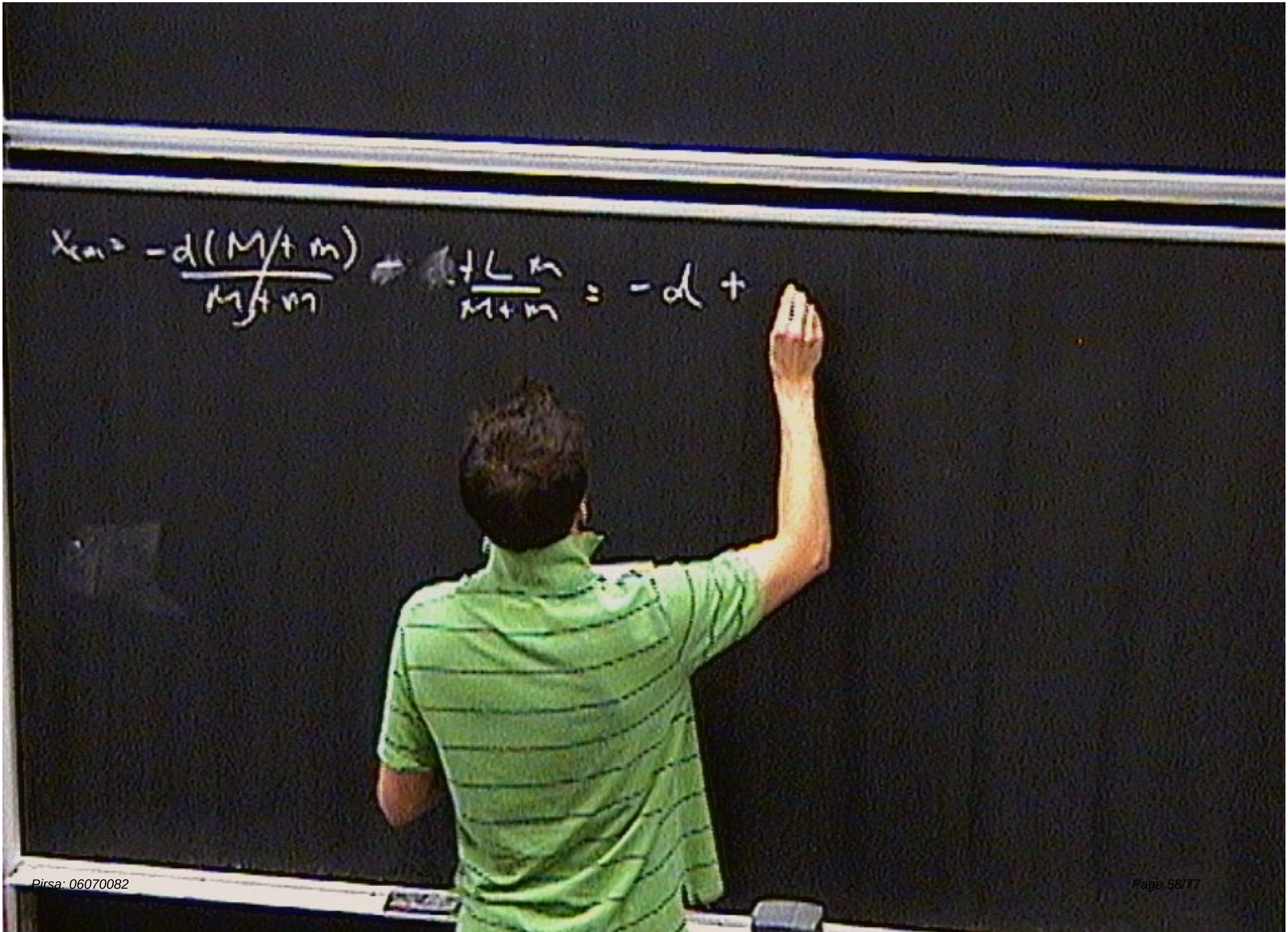
$d = \frac{2pL}{cM}$ (5)


$x_{cm} = -d(M+m)$


$x_{cm} = \frac{-d(M+m)}{M+m}$

$$X_{cm} = -\frac{d(M+m)}{M+m}$$


$x_{cm} = \frac{-d(M+m)}{M+m} = -L$



$x_{cm} = \frac{-d(M+m)}{M+m} + \frac{L\,m}{M+m} = -d +$

$$x_{cm} = \frac{-d\cancel{(M+m)}}{\cancel{M+m}} + \frac{L\,m}{M+m} = -d + \frac{L\,m}{M+m}$$

$$x_{cm} = -\frac{d(M + m)}{M + m} + \frac{+Lm}{M+m} = -d + \frac{Lm}{M+m}$$

$$x_{cm} = \frac{-d(\cancel{M}+m)}{\cancel{M}+m} + \frac{L m}{M+m} = -d + \frac{L m}{M+m}$$

$$-\frac{L m}{m+M}$$

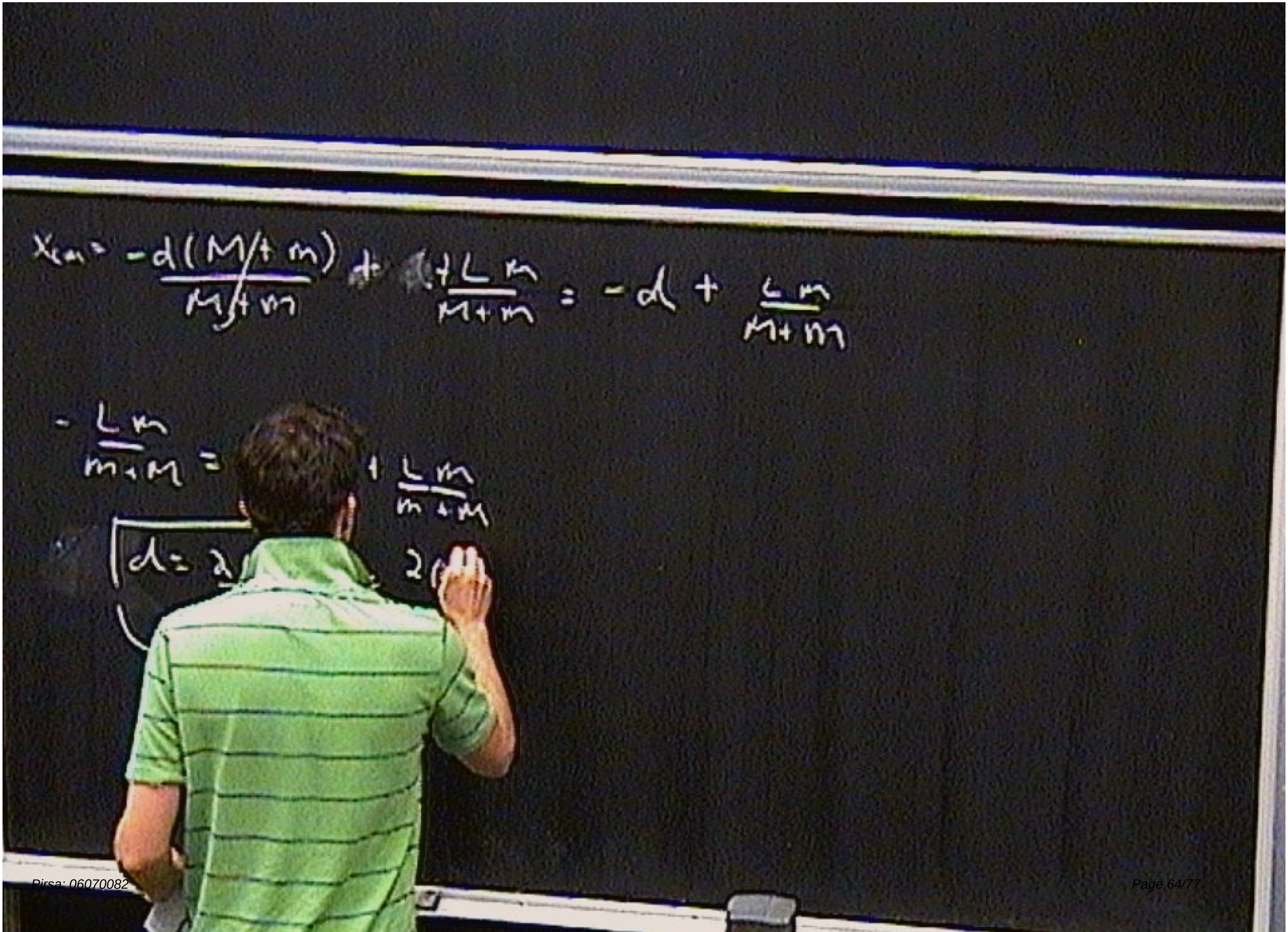
$$x_{cm} = \frac{-d(M+m)}{M+m} + \frac{+Lm}{M+m} = -d + \frac{Lm}{M+m}$$

$$-\frac{Lm}{m+M} = -d + \frac{Lm}{m+M}$$

$$x_{cm} = -\frac{d(\cancel{M}+m)}{\cancel{M}+m} + \frac{L\,m}{M+m} = -d + \frac{L\,m}{M+m}$$

$$-\frac{L\,m}{m+M} = -d + \frac{L\,m}{m+M}$$

$$\boxed{d = \frac{2Lm}{M+m}}$$


$x_{cm} = -\frac{d(\cancel{M+m})}{\cancel{M+m}} + \frac{+Lm}{M+m} = -d + \frac{Lm}{M+m}$
$-\frac{Lm}{m+M} =$
$+\frac{Lm}{m+M}$
$d = 2$
$2($

$$x_{cm} = \frac{-d(\cancel{M+m})}{\cancel{M+m}} + \frac{Lm}{M+m} = -d + \frac{Lm}{M+m}$$

$$-\frac{Lm}{m+M} = -d + \frac{Lm}{m+M}$$

$$\boxed{d = \frac{2Lm}{M+m}} = \frac{2pL}{cM}$$

$$x_{cm} = -\frac{d(\cancel{M}+m)}{\cancel{M}+m} + \frac{+Lm}{M+m} = -d + \frac{Lm}{M+m}$$

$$-\frac{Lm}{m+M} = -d + \frac{Lm}{m+M}$$

$$\boxed{d = \frac{2Lm}{M+m}} = \frac{2pL}{cM}$$

$$x_{cm} = -\frac{d\,(\cancel{M} + m)}{\cancel{M} + m} + \frac{L m}{M + m} = -d + \frac{L m}{M + m}$$

$$-\frac{L m}{m + M} = -d + \frac{L m}{m + M}$$

$$\boxed{d = 2 L m} = \frac{E L}{c M} \approx \frac{E}{c} \cdot \frac{L}{M}$$

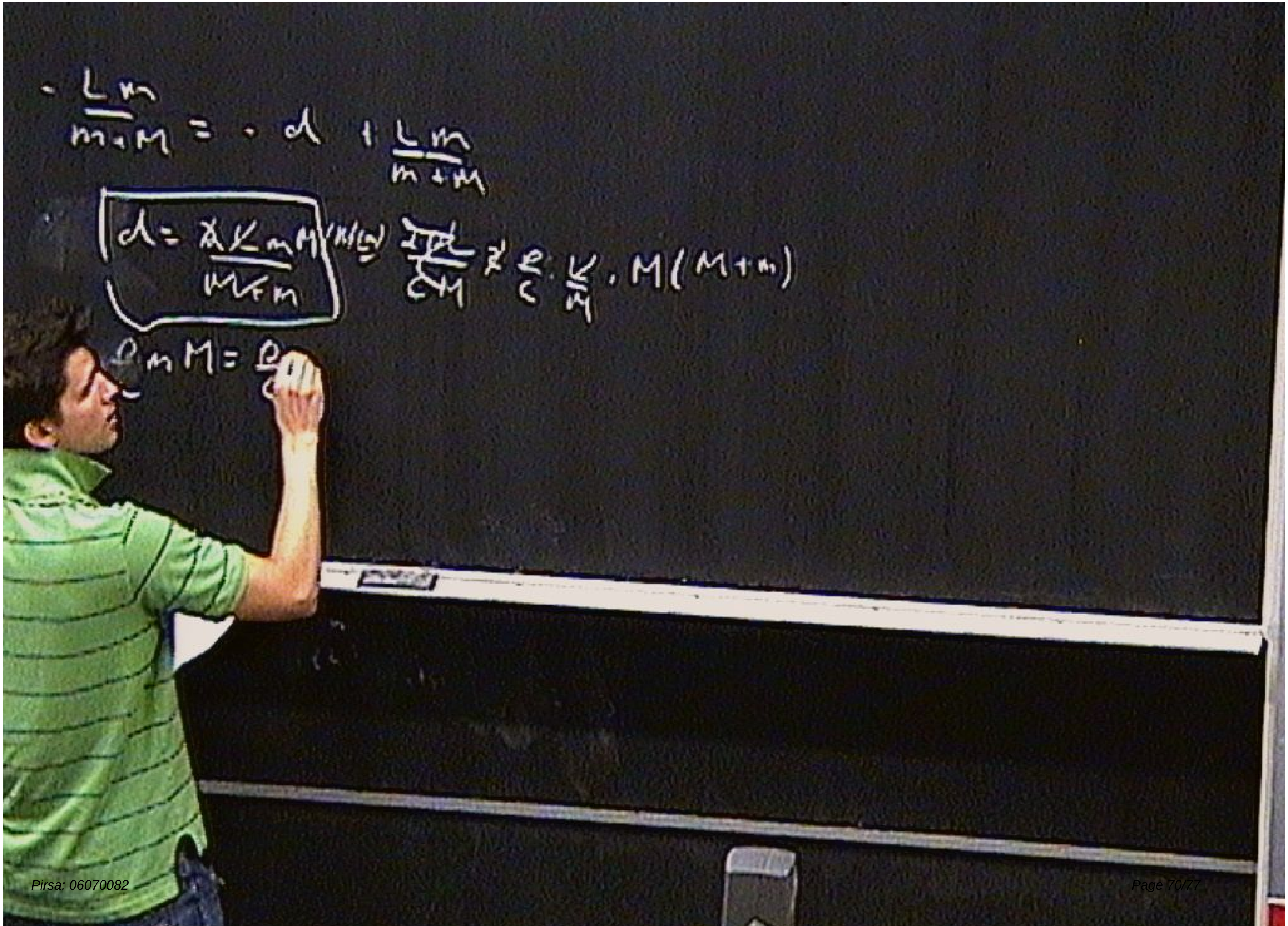
$$x_{cm} = \frac{-d(\cancel{M}+m)}{\cancel{M}+m} + \frac{L\,m}{M+m} = -d + \frac{L\,m}{M+m}$$

$$-\frac{L\,m}{m+M} = -d + \frac{L\,m}{m+M}$$

$$\boxed{d = \frac{2Lm}{M+m}} = \frac{\cancel{[illegible]}}{cM} \, 2 \, \frac{E}{c} \cdot \frac{L}{M}$$

$$-\frac{Lm}{m+M} = -d + \frac{Lm}{m+M}$$

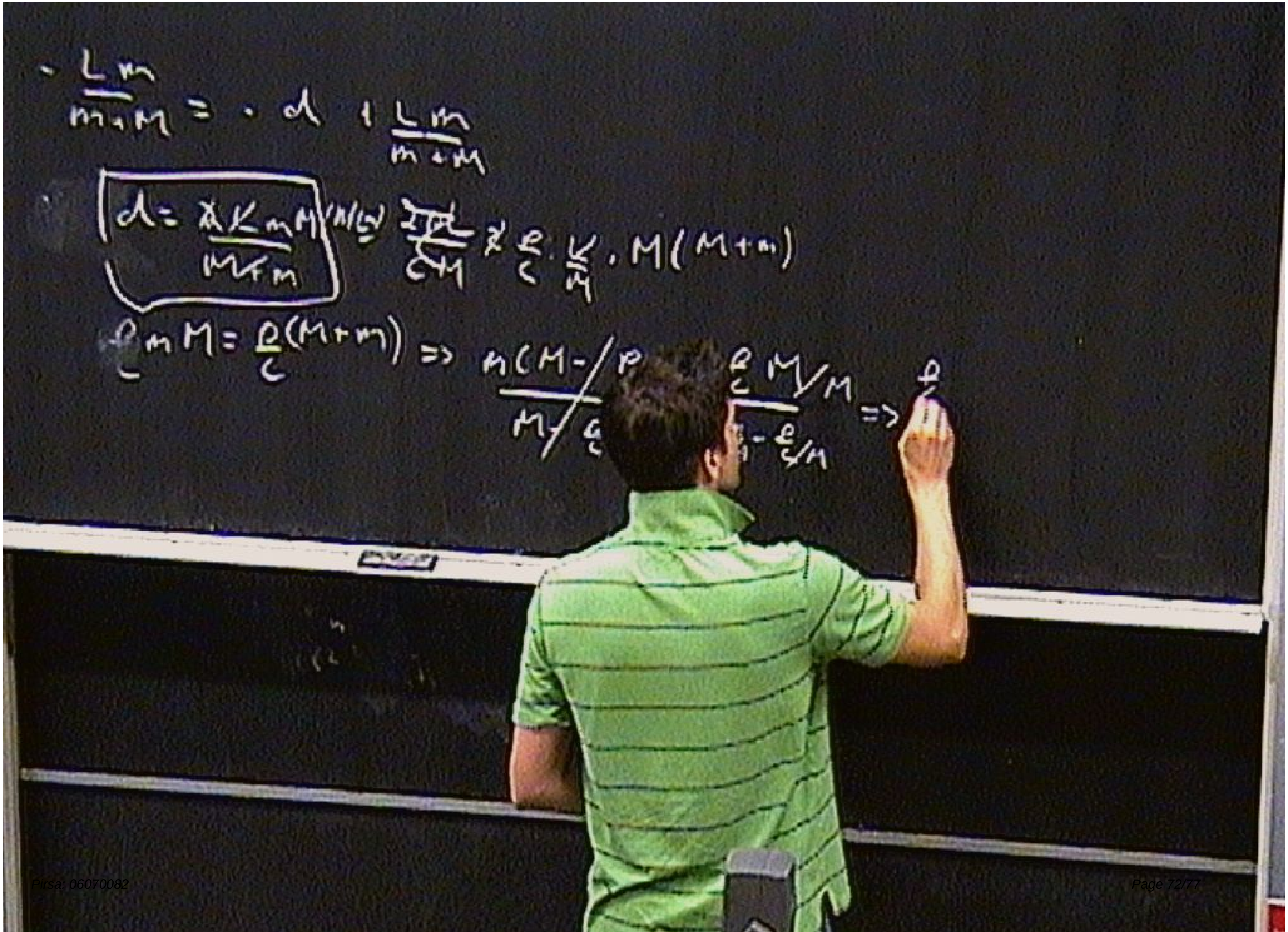
$$\boxed{d = \frac{2Lm}{M+m}} = \frac{2EL}{cM} \approx \frac{e}{c}\cdot\frac{L}{M}$$



$$-\frac{Lm}{m+M} = -\alpha + \frac{Lm}{m+M}$$

$$\boxed{\alpha = \frac{2 K m M}{M+m}} \approx \frac{\ell}{c} \cdot \frac{K}{M} \cdot M(M+m)$$

$$\frac{\ell}{c} m M = \frac{\ell}{c}(M+m) \Rightarrow \frac{m\left(M - \frac{\ell}{c}\right)}{M-\frac{\ell}{c}} = \frac{\frac{\ell}{c} M}{M-\frac{\ell}{c}}$$

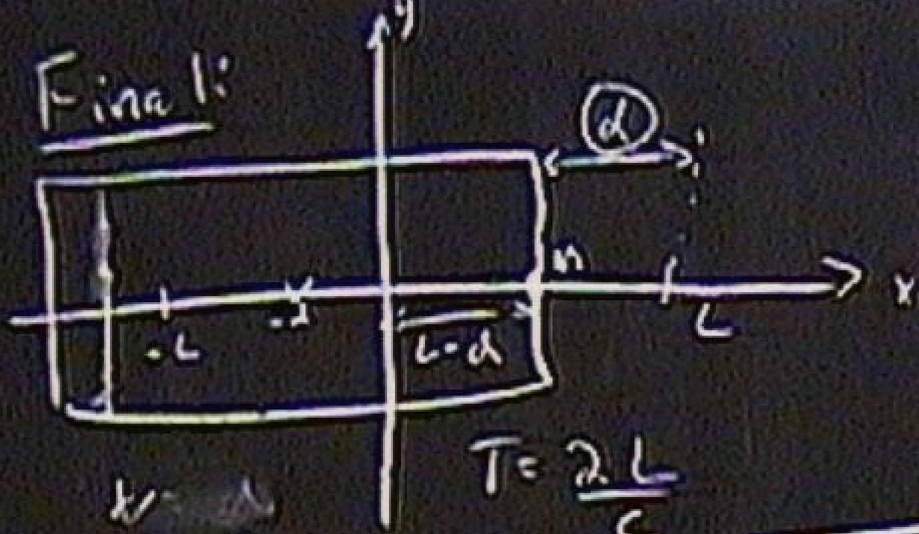

$-\frac{Lm}{m+M} = -d + \frac{Lm}{m+M}$
$d = \frac{2KmM}{M+m}$
$\cdot M(M+m)$
$\frac{\rho}{c} mM = \frac{\rho}{c}(M+m) \Rightarrow$
$\Rightarrow$

$$-\frac{Lm}{m+M} = -\alpha + \frac{Lm}{m+M}$$

$$\boxed{\alpha = \frac{\kappa K m M}{M+m}} \qquad \frac{\kappa K}{cM} \approx \frac{e}{c}\cdot\frac{K}{M}\cdot M(M+m)$$

$$\frac{e}{c} m M = \frac{e}{c}(M+m) \Rightarrow \frac{\boxed{m}(M - \frac{e}{c})}{M - \frac{e}{c}} = \frac{\frac{e}{c}M/M}{M/M - \frac{e}{c}/M} \Rightarrow \boxed{\frac{e}{c}\cdot\left(\frac{1}{1-\frac{e}{cM}}\right)}$$

Final:



$x_{cm,M} = -d$

$T = \frac{2L}{c}$

$V_{box} << c$

$\frac{V_{box}}{c} << 1$

$\frac{E}{Mc} << 1$

Results:

$p = \frac{E}{c}$ (1)

$p_{ini} = p_{fin}$ (2)

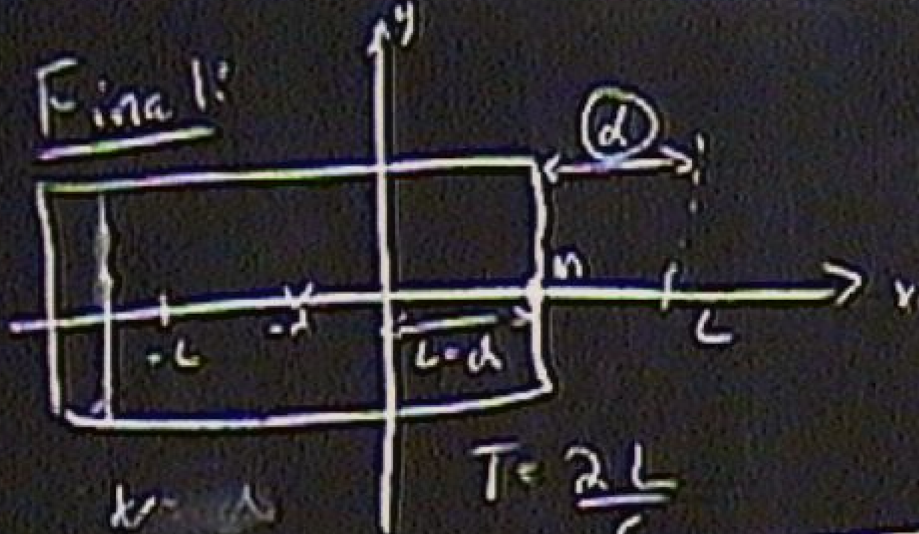
$V_{cm,int} = V_{cm,fin}$ (3)

$0 = 0$

$x_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$ (4)

$d = \frac{2pL}{cM}$ (5)

Final:



$x_{cm,H} = -d$

$T = \frac{2L}{c}$

$V_{box} \ll c$

$\frac{V_{box}}{c} \ll 1$

$\frac{p}{Mc} \ll 1$

$$m \approx \frac{p}{c} = \left(\frac{E}{c}\right)\frac{1}{c} = \frac{E}{c^2}$$

$$\boxed{E = mc^2} \quad !!!$$

Results:

$p = \frac{E}{c}$ (1)

$p_{ini} = p_{fin}$ (2)

$V_{cm,ini} = V_{cm,fin}$ (3)

$0 = 0$

$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ (4)

$d = \frac{2pL}{cM}$ (5)

$E = mc^2$ (6)

$$E = \pm\sqrt{m^2c^4 + p^2c^2}$$

$$\sqrt{E^2} = \pm\sqrt{m^2c^4 + p^2c^2}$$

$$E = -\hbar \frac{\partial}{\partial t}$$

$$p = -i\hbar \frac{\partial}{\partial x}$$