Title: The Weird World of Quantum Physics - Part 4

Date: Jul 26, 2006 10:40 AM

URL: http://pirsa.org/06070074

Abstract:

- If we placed a series of ultra-sensitive light detectors in the wall, we would eventually see light coming in definite, localized clumps. (Kind of light particles of light.)
- i.e. light seems to be made up of tiny particles which we call photons
- But, as we shall see, things are much, much more complicated than this.

- In Newtonian physics, these are only two possible states of affairs. Surely, each photon must either take the first path or the second one. These appear to be the only two options.

But, this is quantum physics. As we saw earlier, each photon can take both paths.

How do we respresent this mathematically? Roughly speaking,

measuring it to be transmitted.

- Therefore, we need some mathematical way to represent these probabilities.

Probability(path 1) = |(1,1) element of matrix|^2

Probability(path 2) = |(2,1) element of matrix|^2

- What is the state after the photon has passed through the second half-silvered mirror?
- Check that the probabilities that this state gives for finding the photon in paths 1 and 2 are in agreement with the quantum probabilities for this experiment we saw earlier.

- The inverse of a matrix M is the matrix M^{-1} such that
  $M M^{-1} = I$ and $M^{-1} M = I$, where I is the identity matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

  i.e for 2x2 matrices

Question: Prove that the matrix that implements the state change at each half-silvered mirror is unitary.

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

Student activities

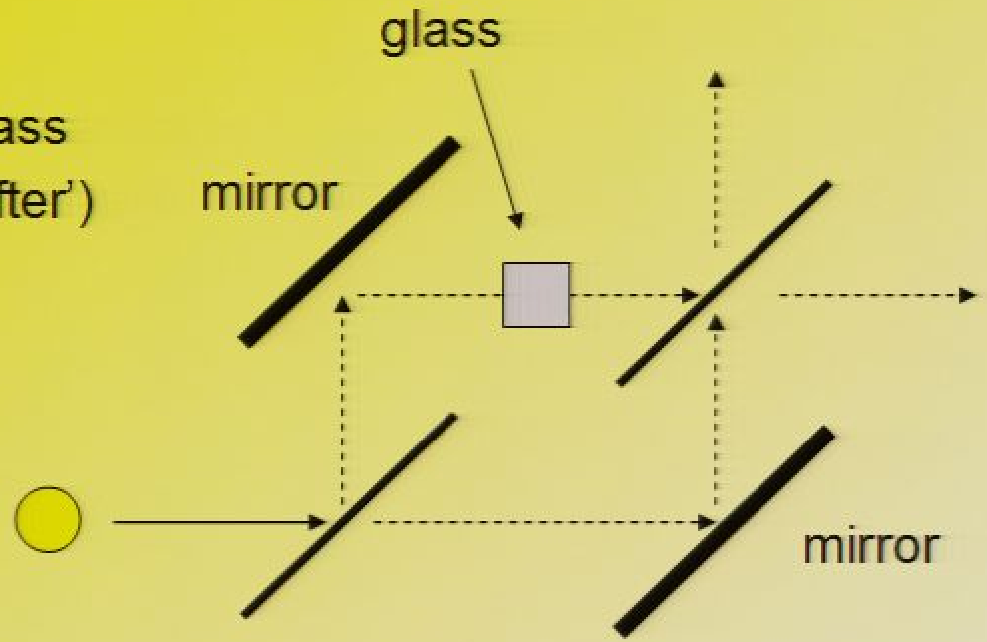
- 1. When $\phi=\pi$, what are the probabilities of finding the photon at detectors A and B.

- Bonus questions (harder):

  2. When $\phi=\pi/4$, what are the probabilities of finding the photon at detectors A and B.

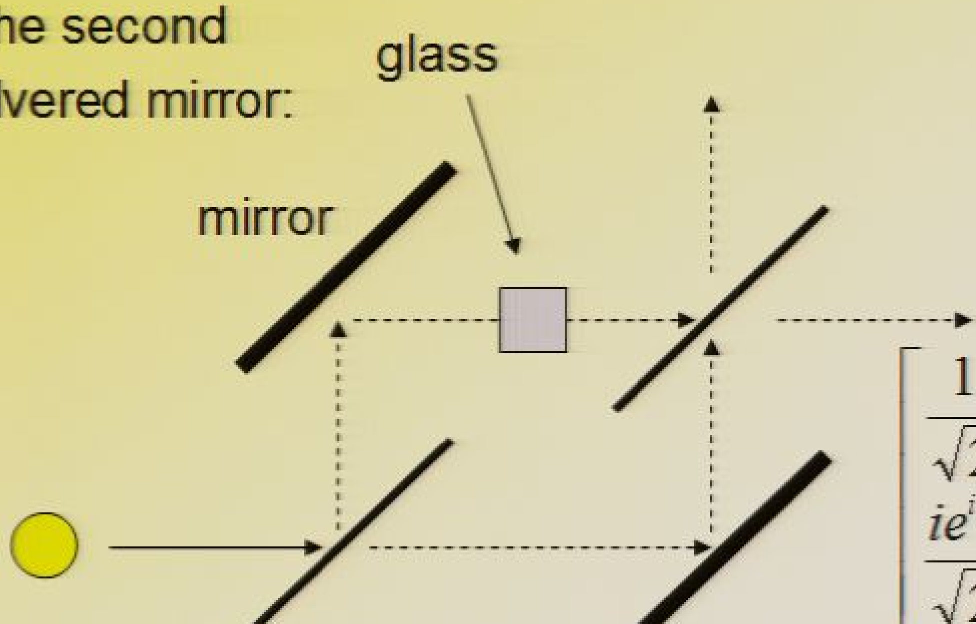
  3. Represent $\frac{1}{2}\begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$ in polar co-ordinates

- After the glass ('phase-shifter')

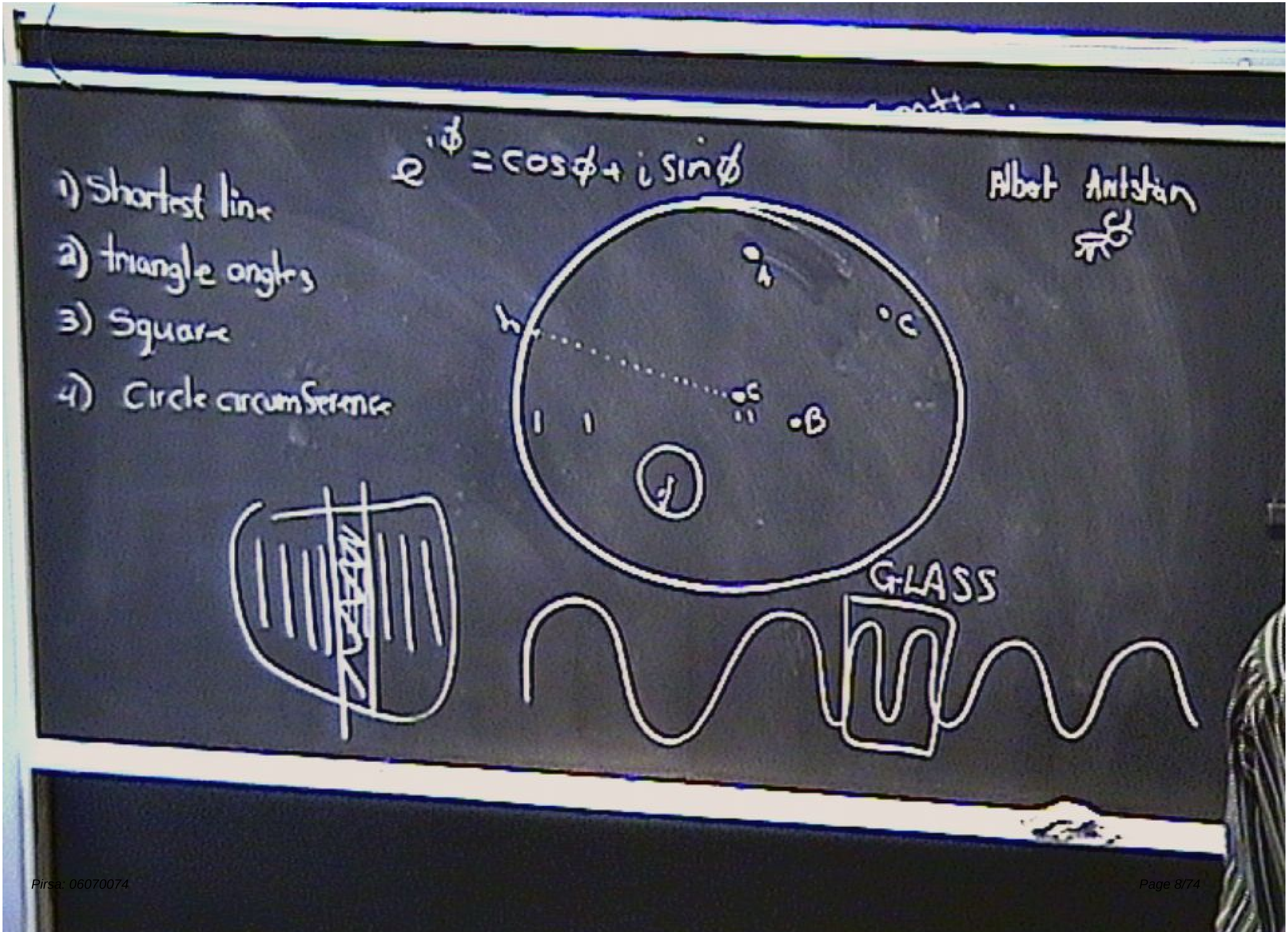


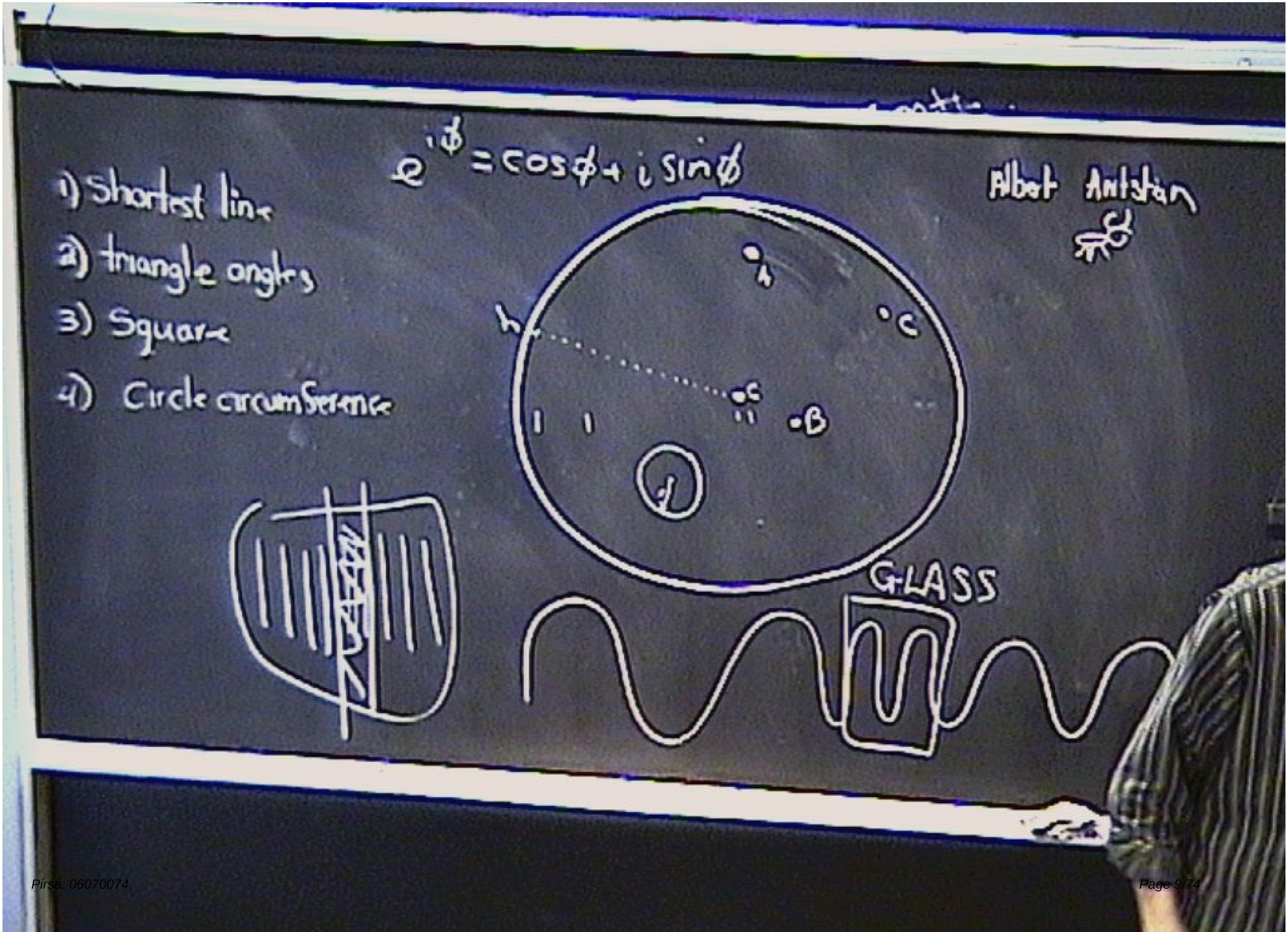
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \to \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix}$$

After the second half-silvered mirror:

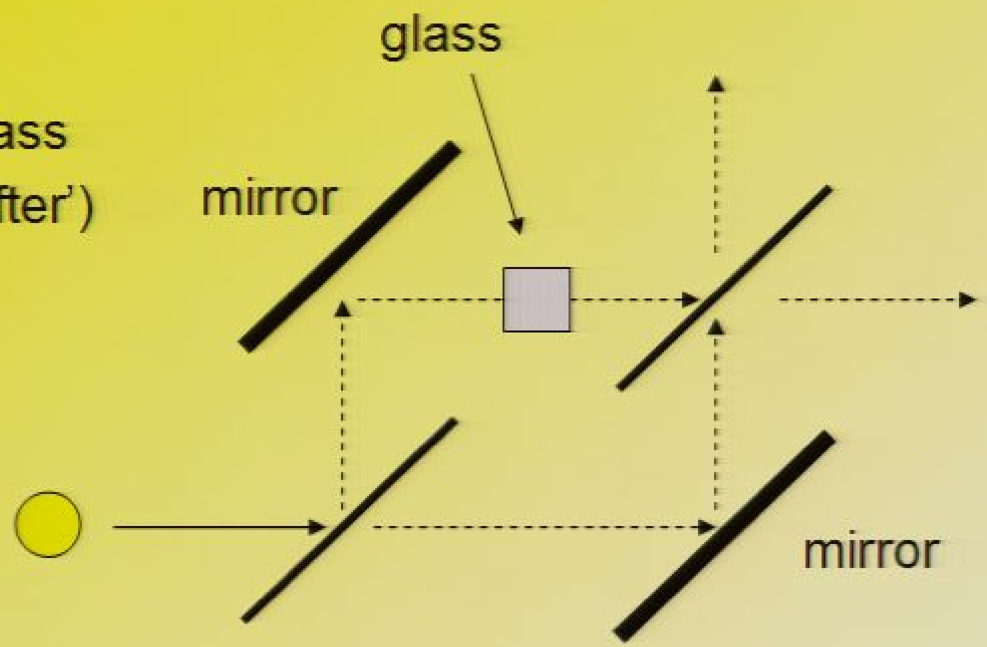


$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} \to \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$$


1) Shortest line
2) triangle angles
3) Square
4) Circle circumference
e^{iφ} = cosφ + i sinφ
GLASS

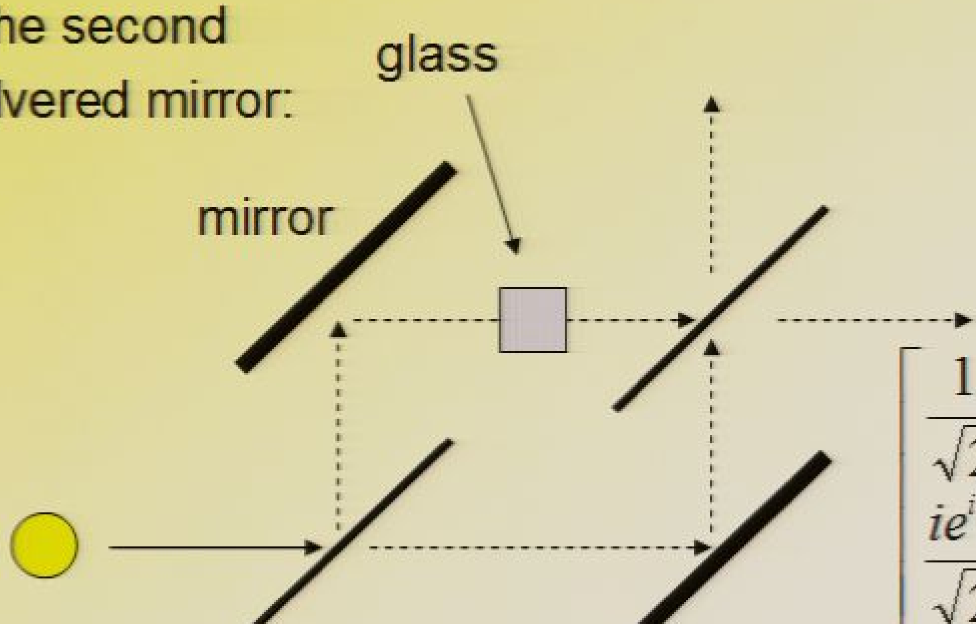

1) Shortest line
2) triangle angles
3) Square
4) Circle circumference
$e^{i\phi} = \cos\phi + i\sin\phi$
C
B
GLASS

- After the glass ('phase-shifter')



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix}$$

After the second half-silvered mirror:



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - e^{i\phi} \\ i(1 + e^{i\phi}) \end{bmatrix}$$

Student activities

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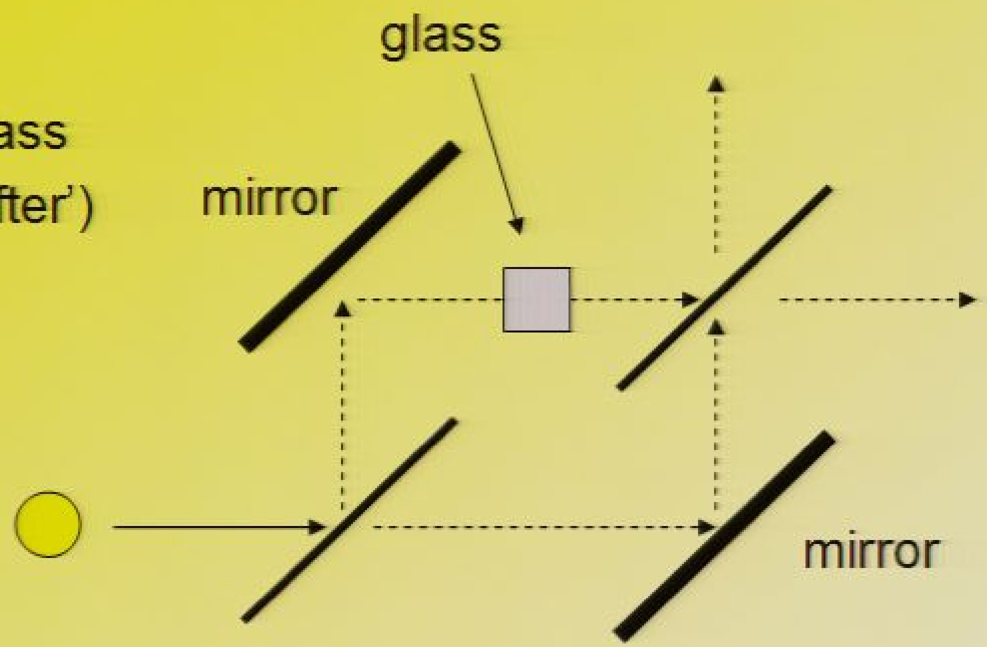
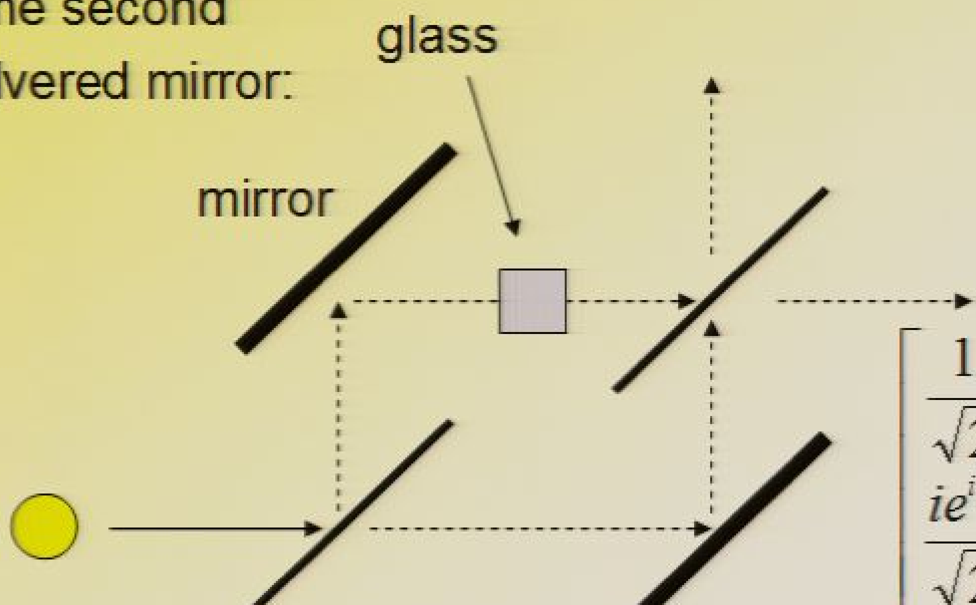
- After the glass ('phase-shifter')

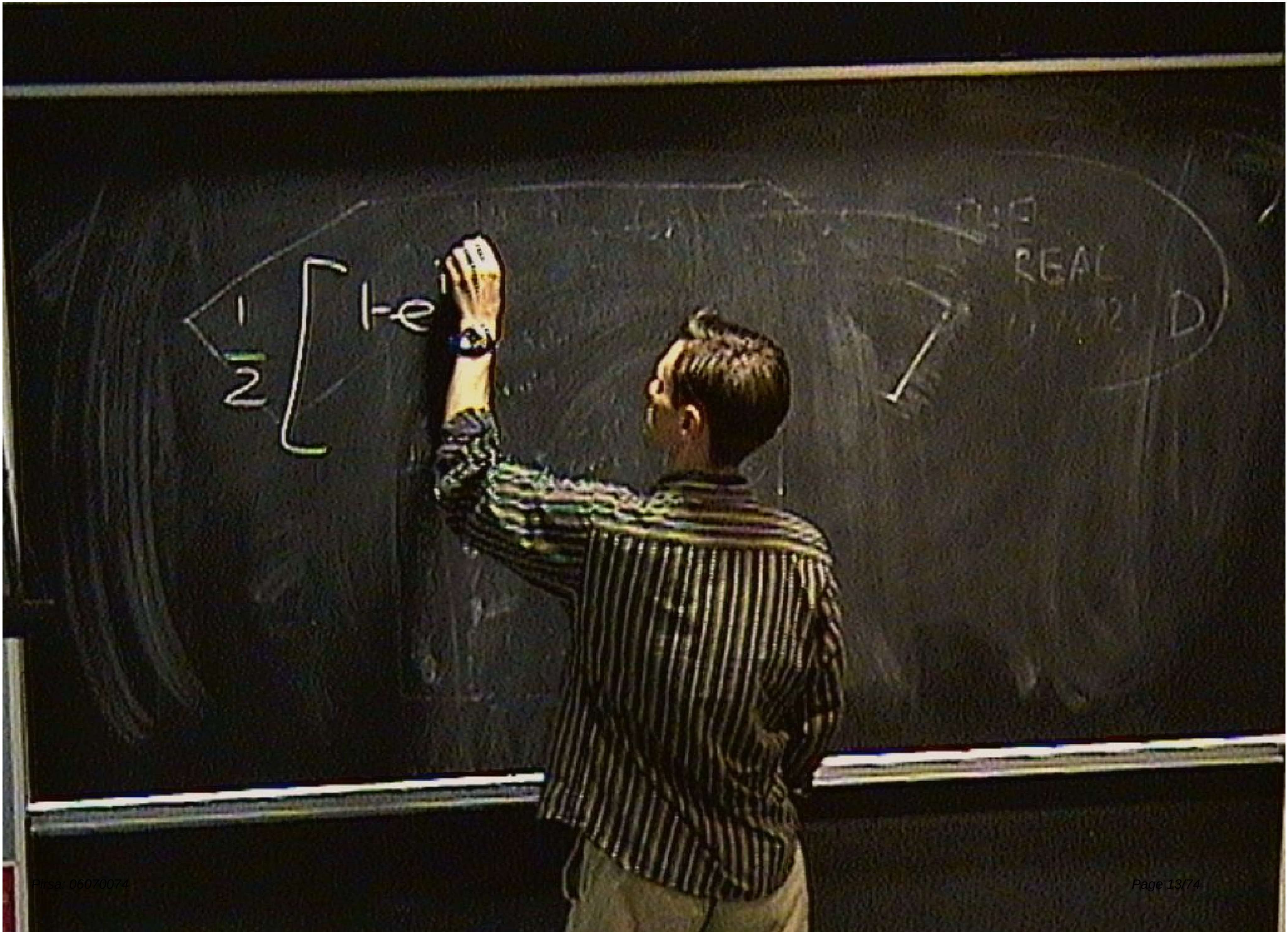


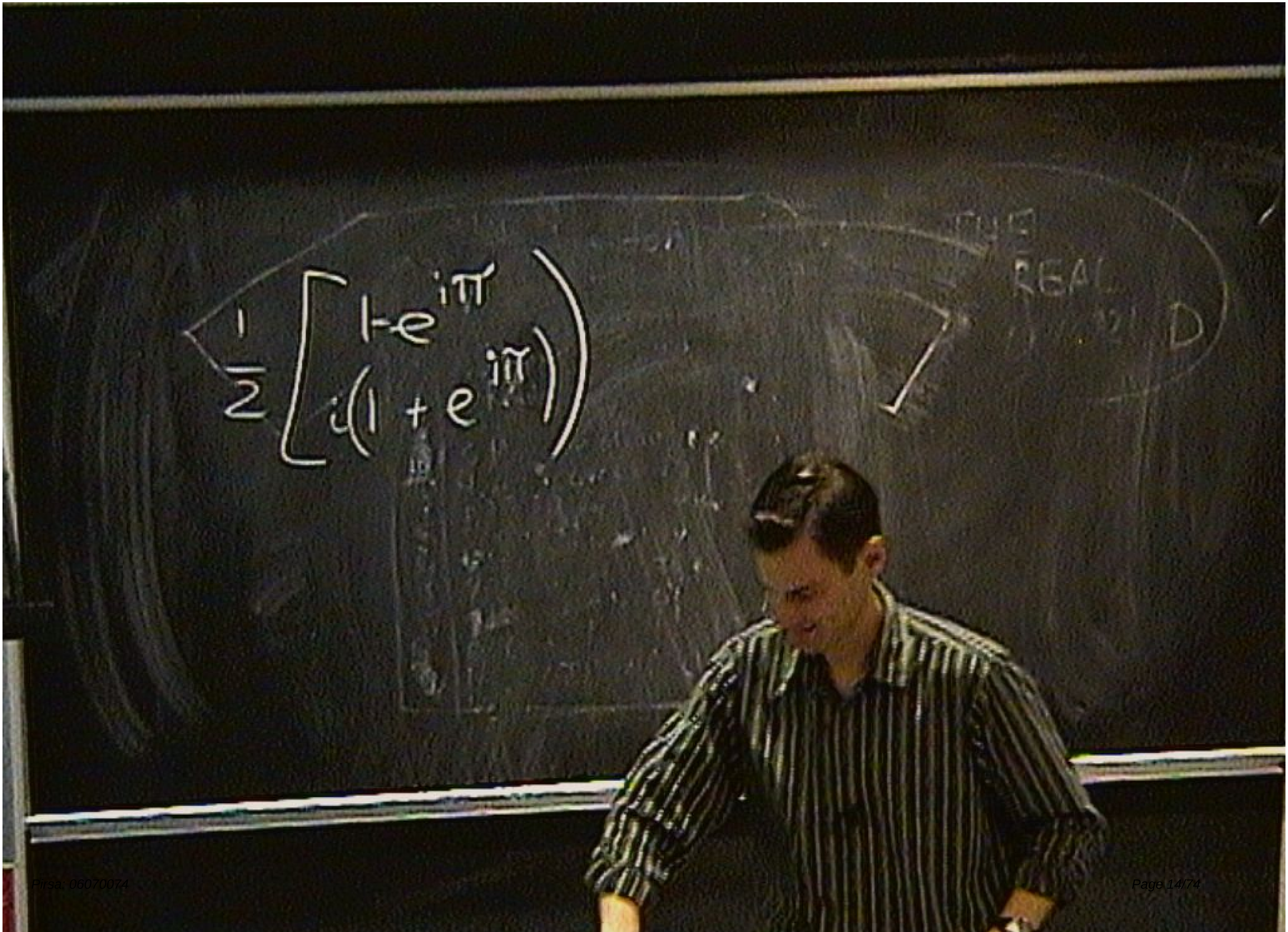
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \to \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix}$$

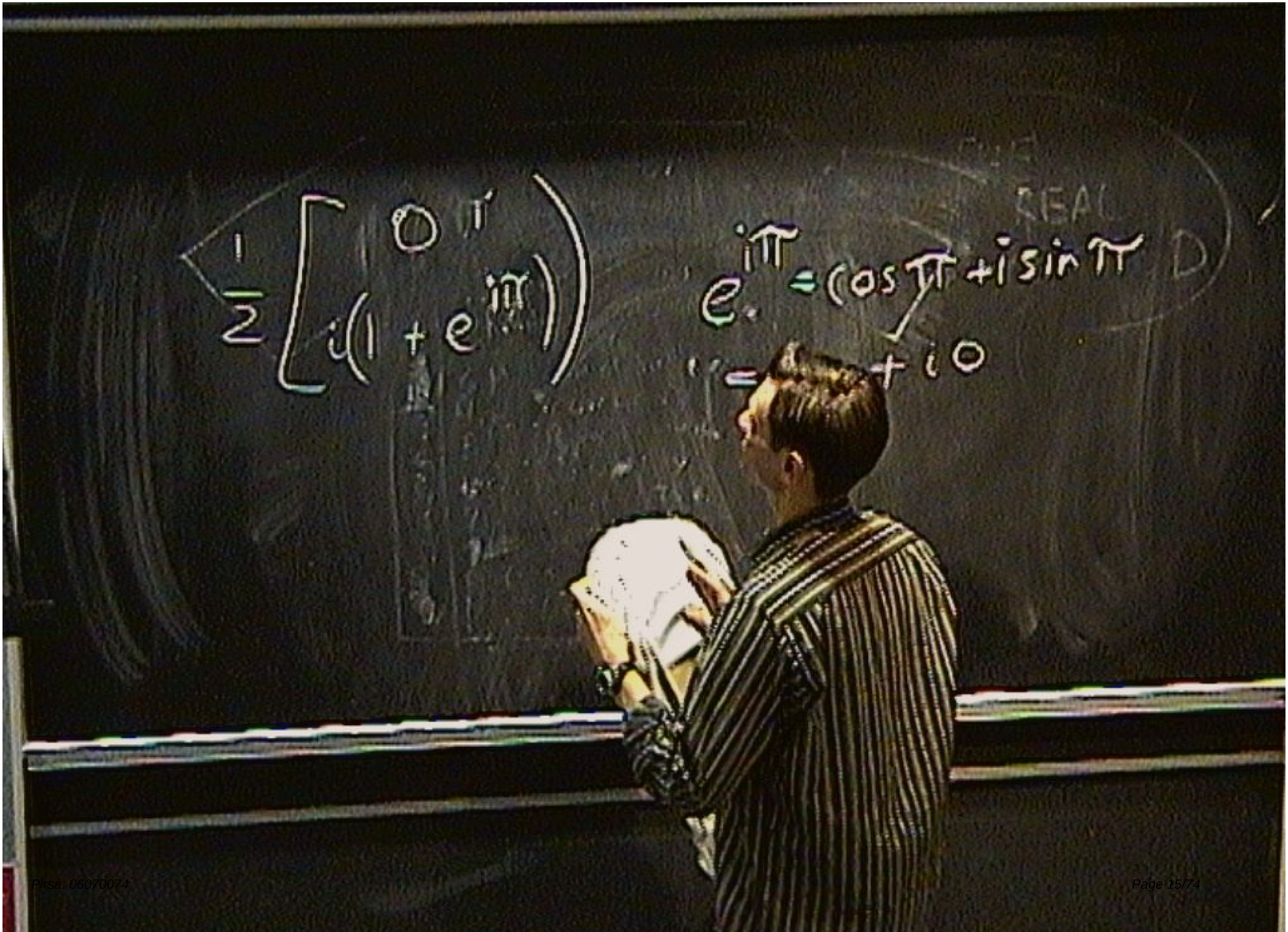
After the second half-silvered mirror:

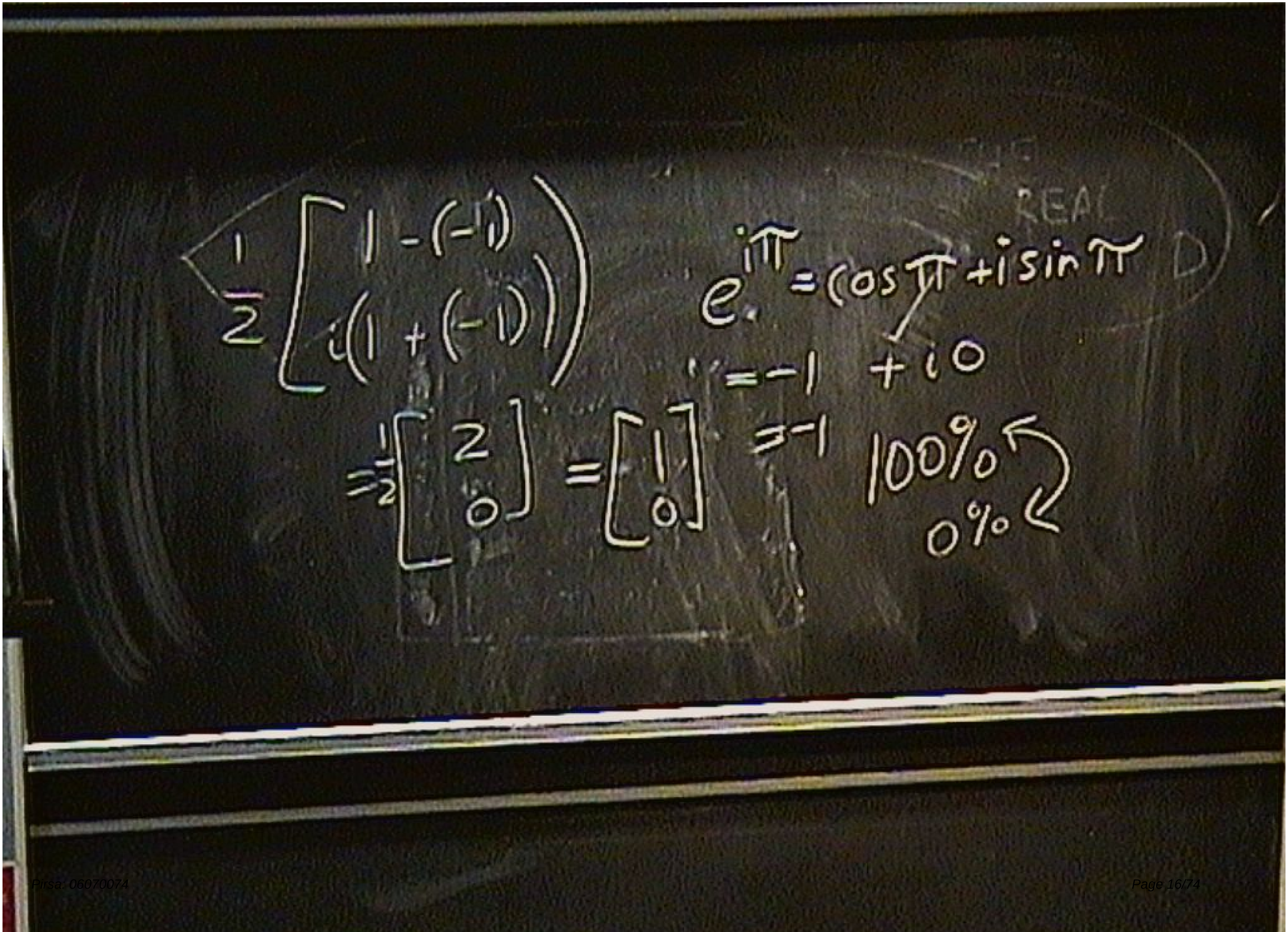


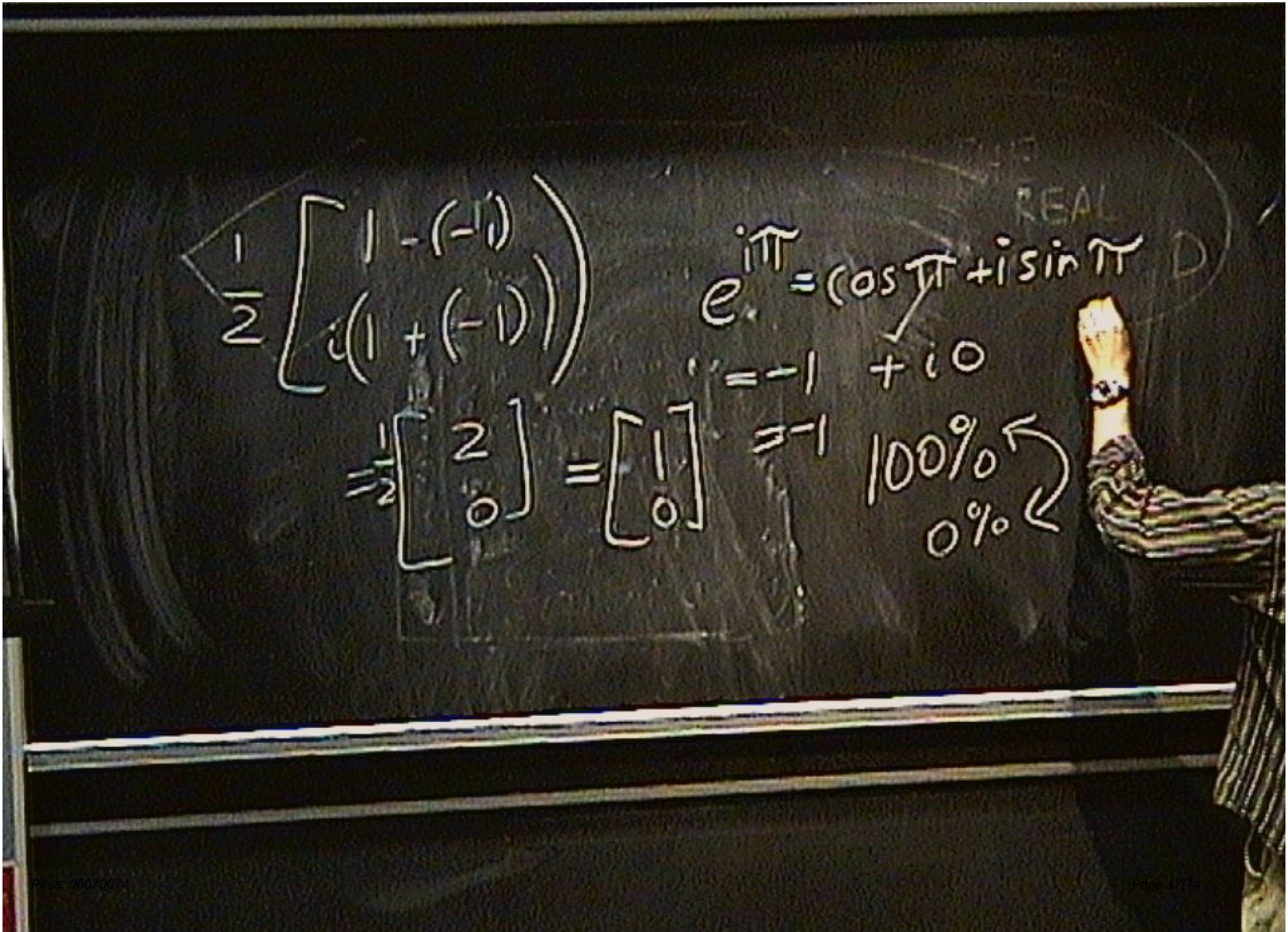
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} \to \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$$


1/2
[1+e
REAL


$\frac{1}{2}\left[\begin{matrix} 1+e^{i\pi} \\ i(1+e^{i\pi}) \end{matrix}\right)$
REAL
D


REAL


REAL
D
1/2 [(1 - (-1) ; i(1 + (-1)))]
e^iπ = cos π + i sin π
= -1 + i0
= 1/2 [2 ; 0] = [1 ; 0]
= -1
100%
0%


REAL
1/2 [(1 - (-1)) / i(1 + (-1))]
e^{iπ} = cos π + i sin π
= -1 + i0
= 1/2 [2 / 0] = [1 / 0]
= -1
100%
0%

$$\frac{1}{2}\begin{bmatrix} 1-(-1) \\ i(1+(-1)) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

REAL

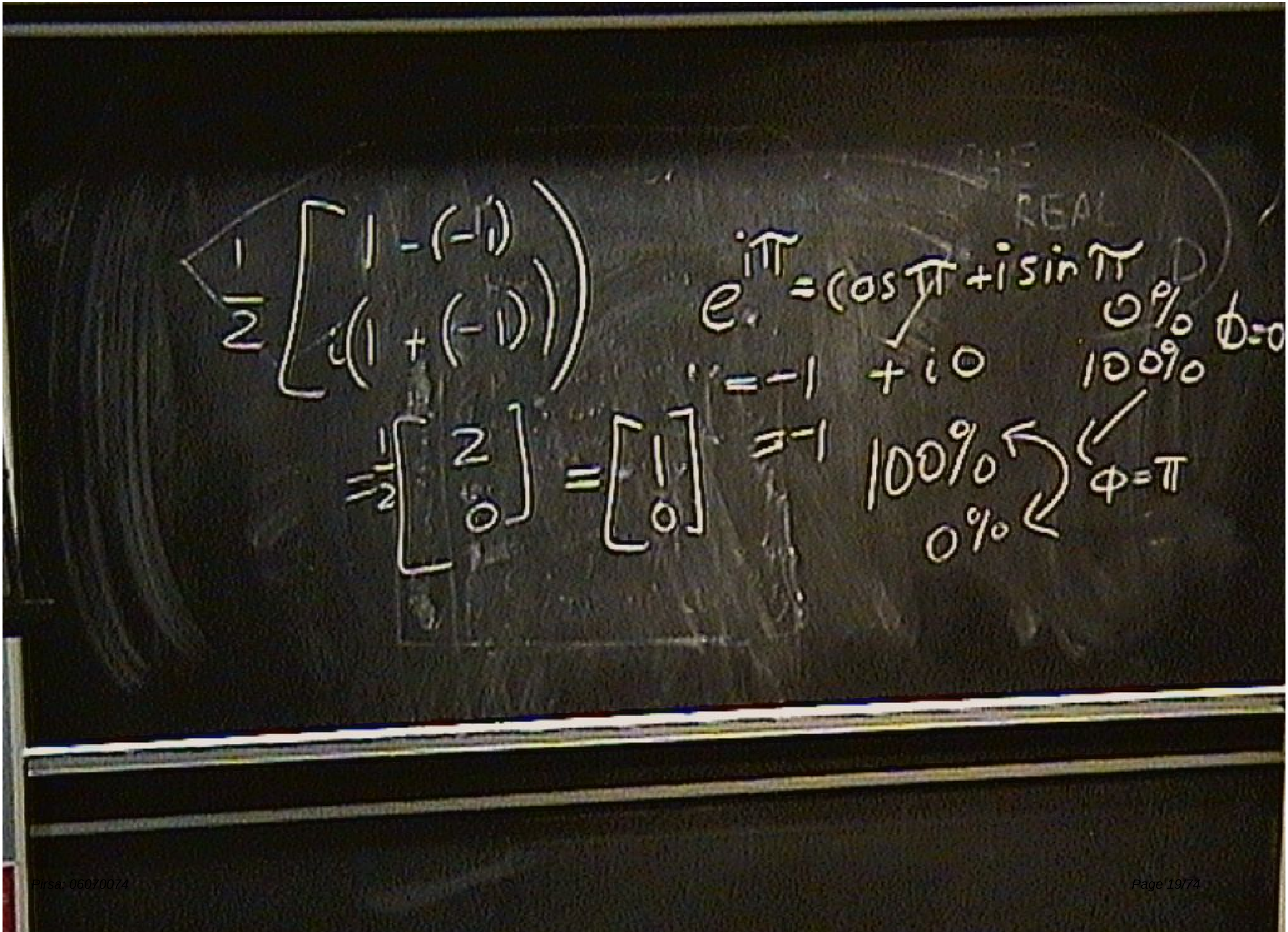
$$e^{i\pi} = \cos\pi + i\sin\pi$$

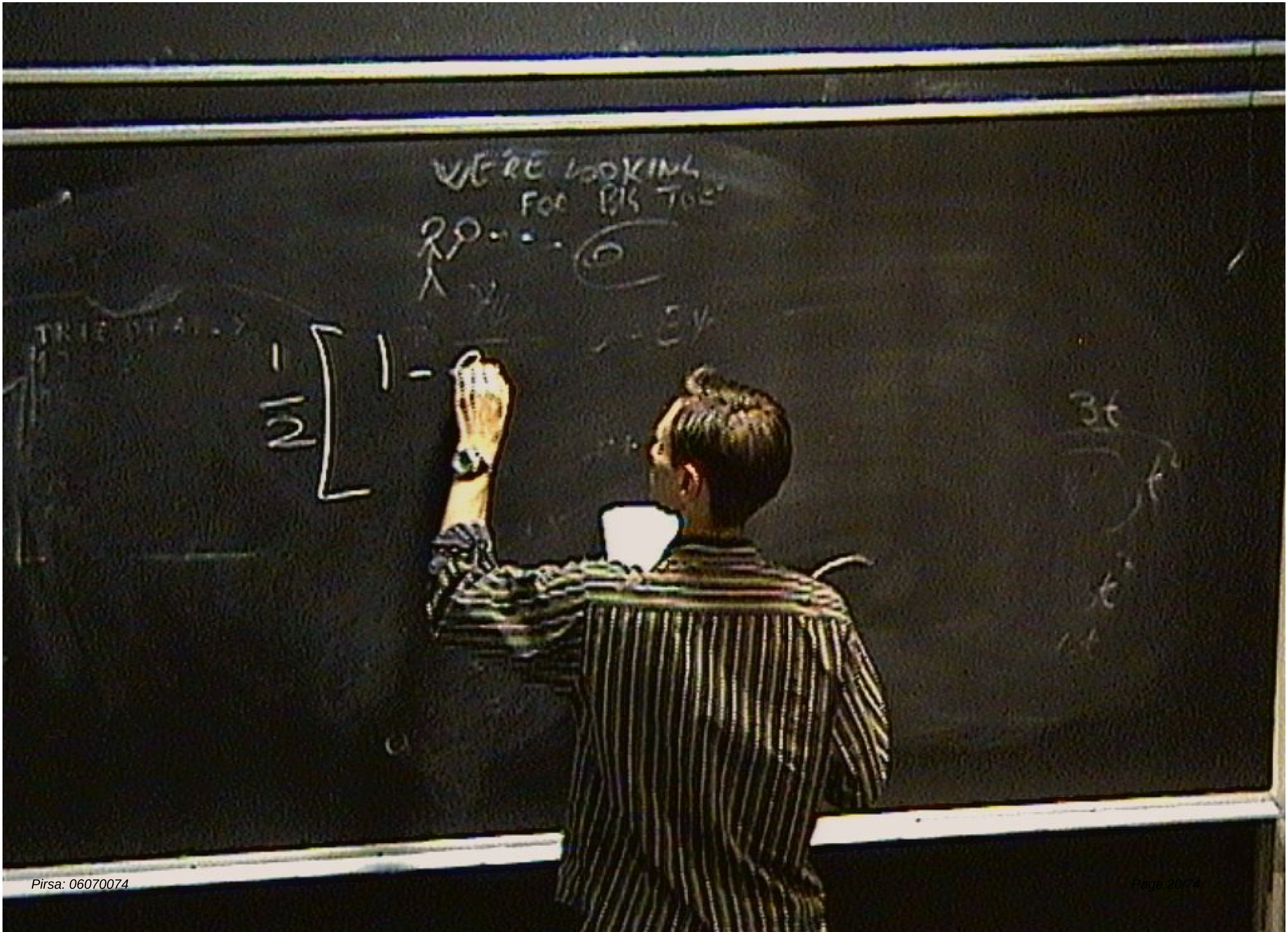
$$= -1 + i0$$

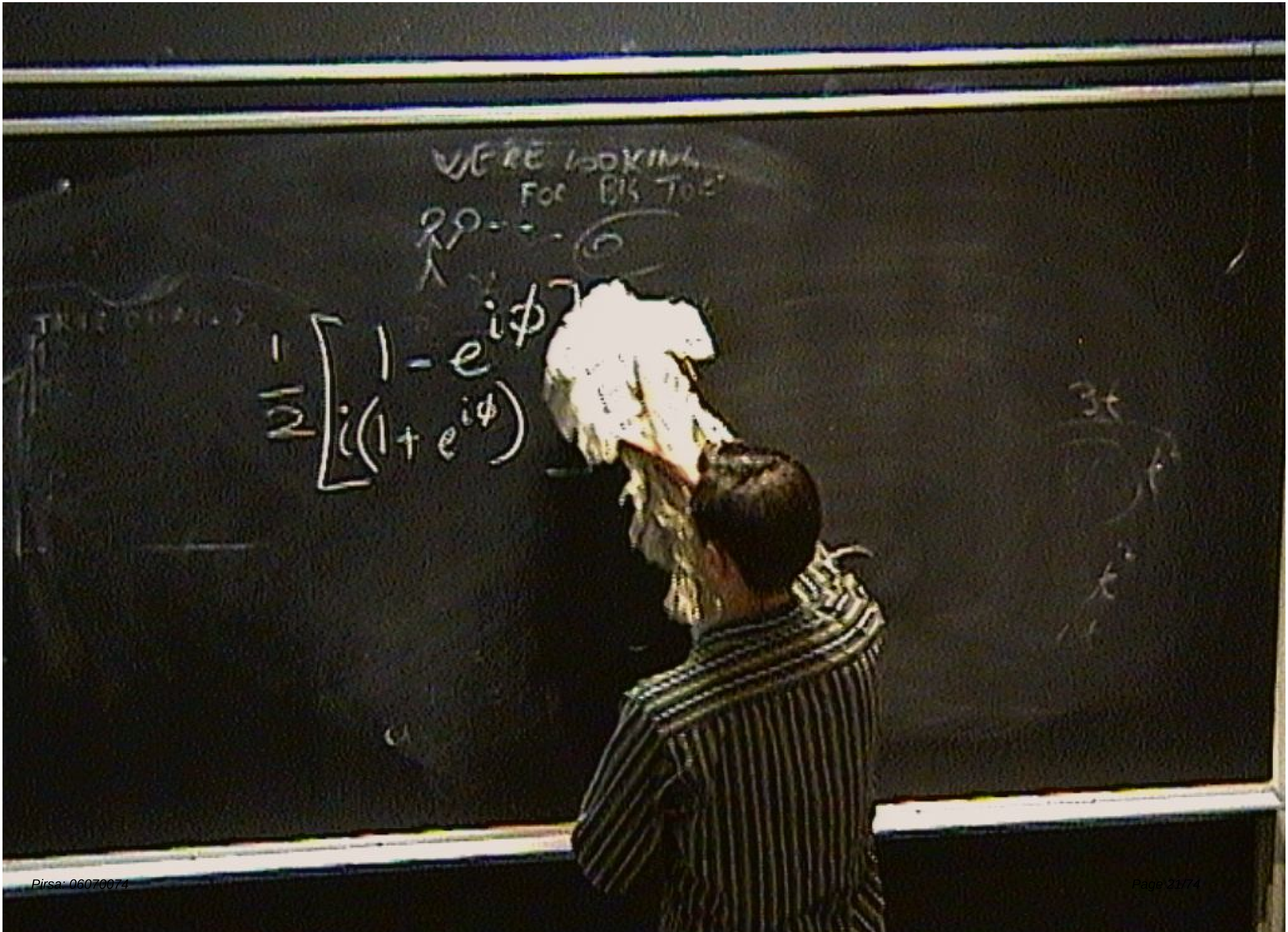
$= -1$

0% 100% $\phi = 0$

100% 0%


REAL
e^{iπ} = cos π + i sin π
= −1 + i0
0%
φ=0
100%
100%
0%
φ=π


WE'RE LOOKING
FOR BIG TOE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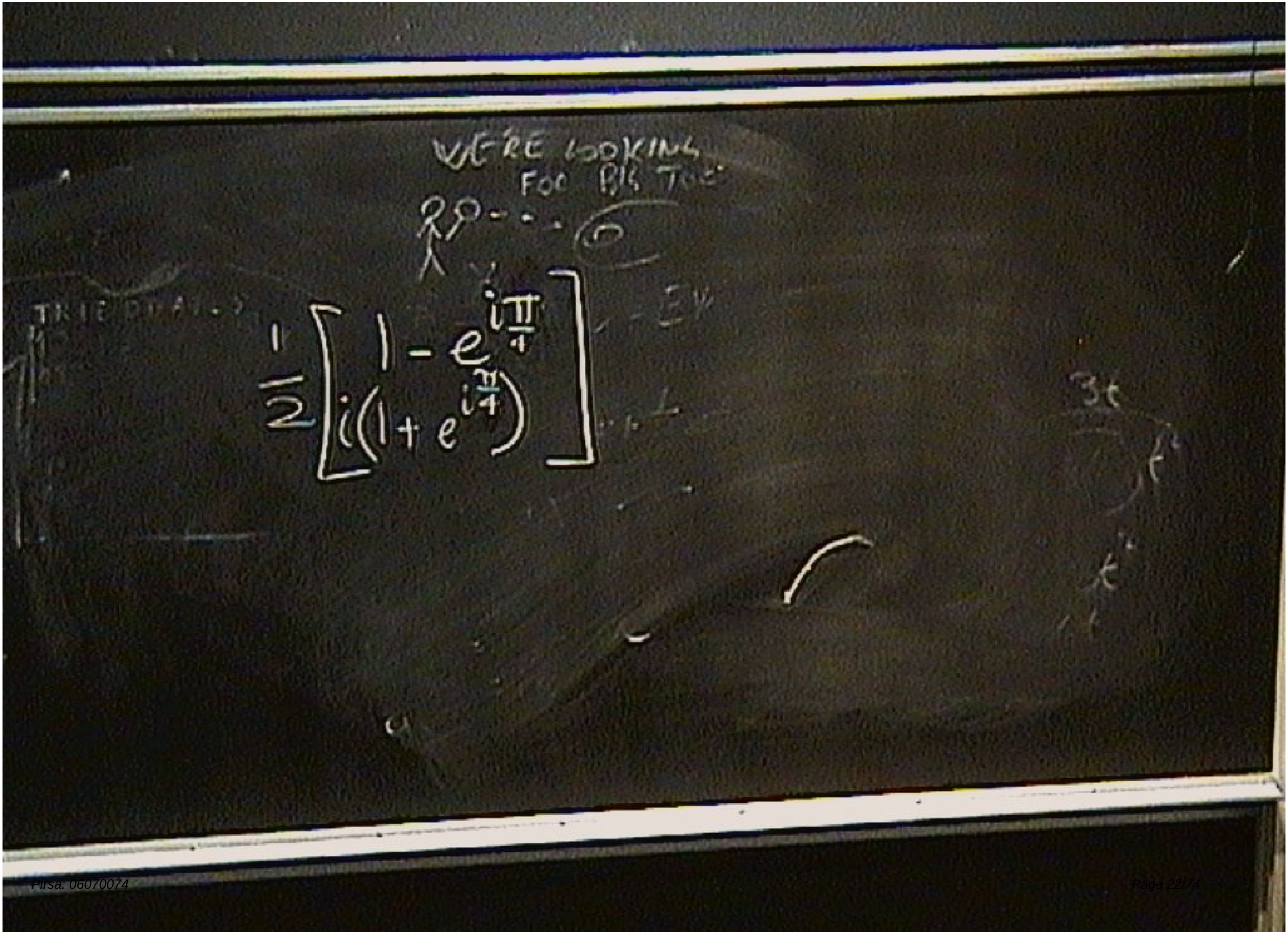
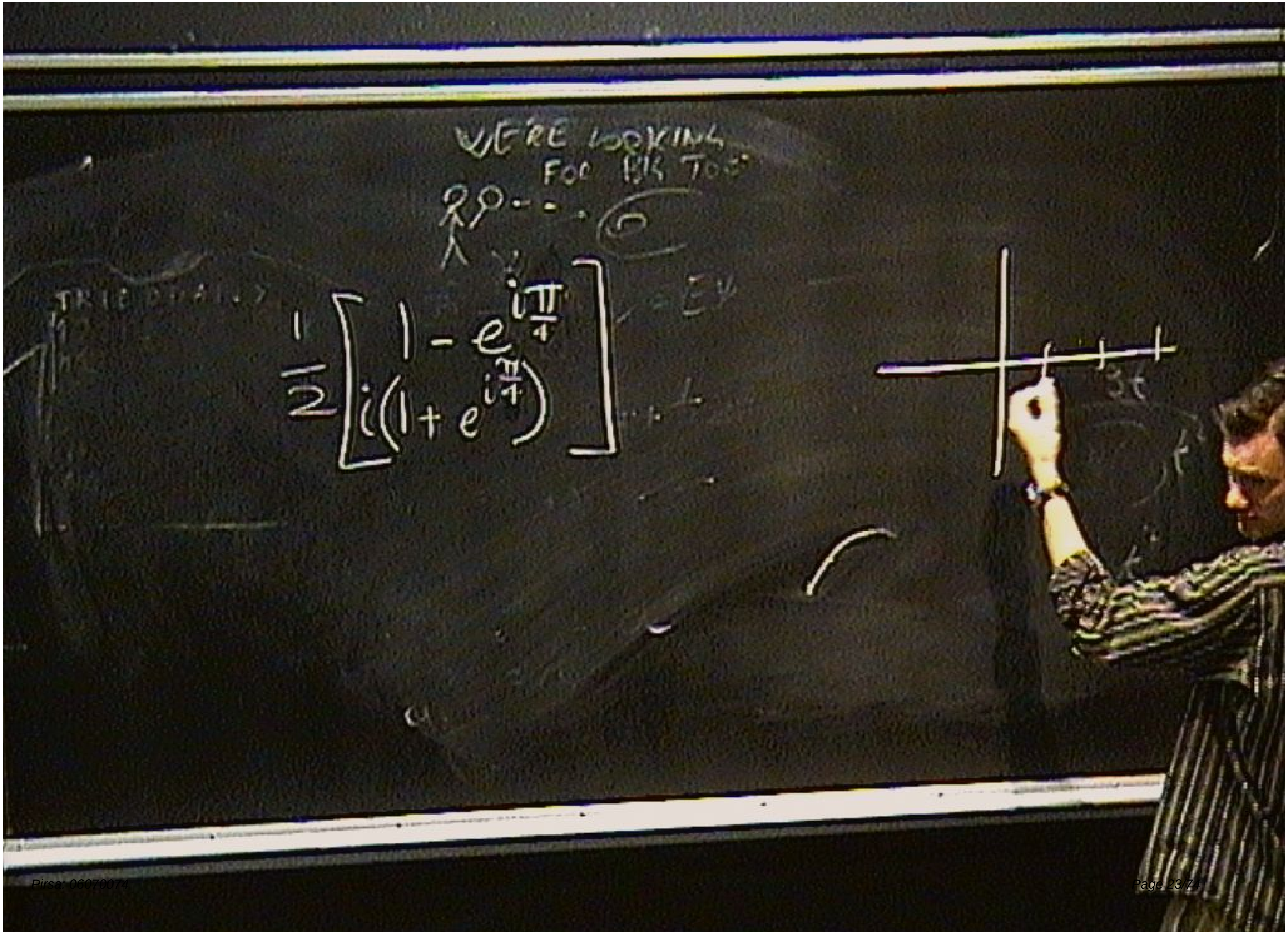
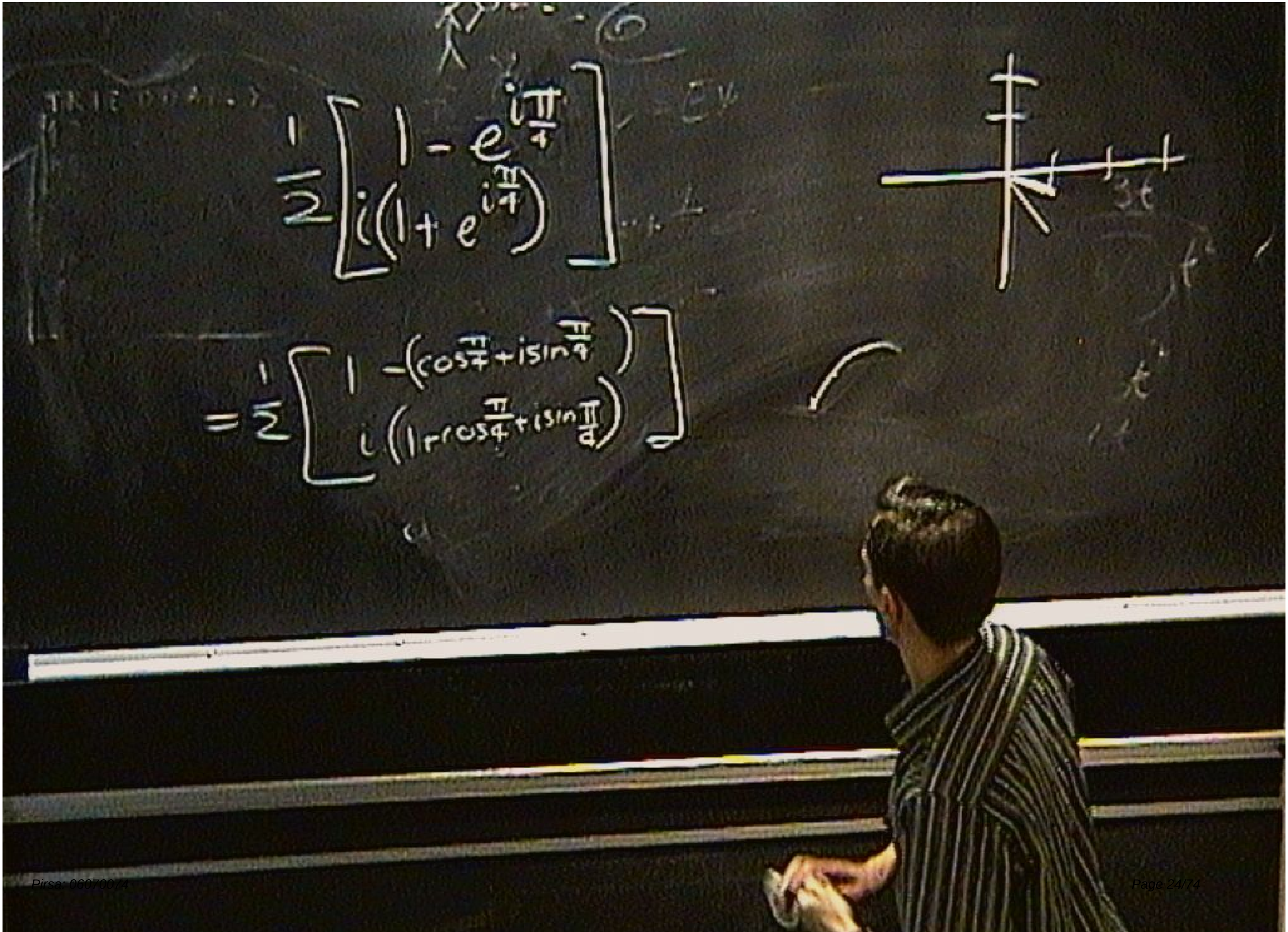
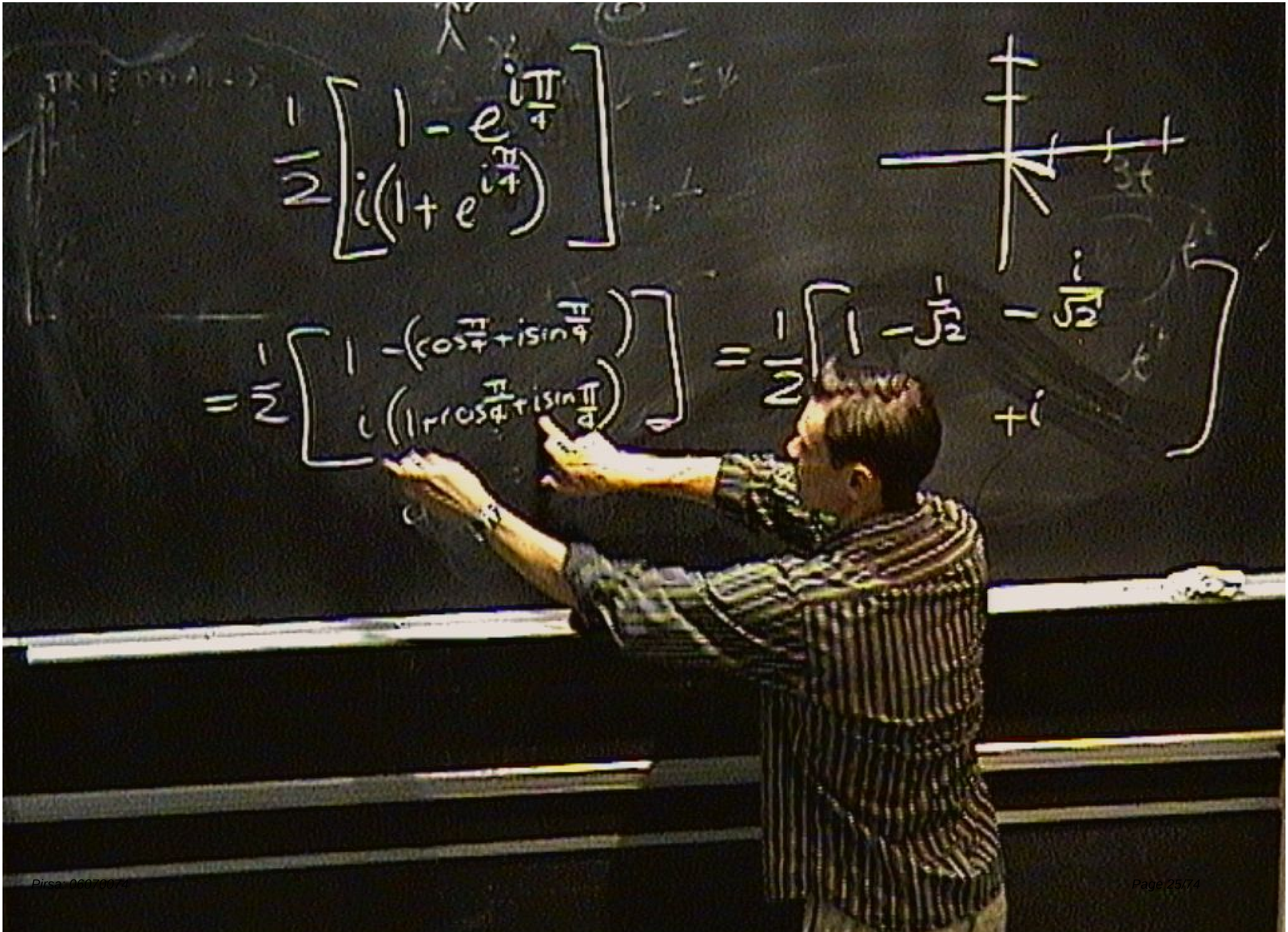
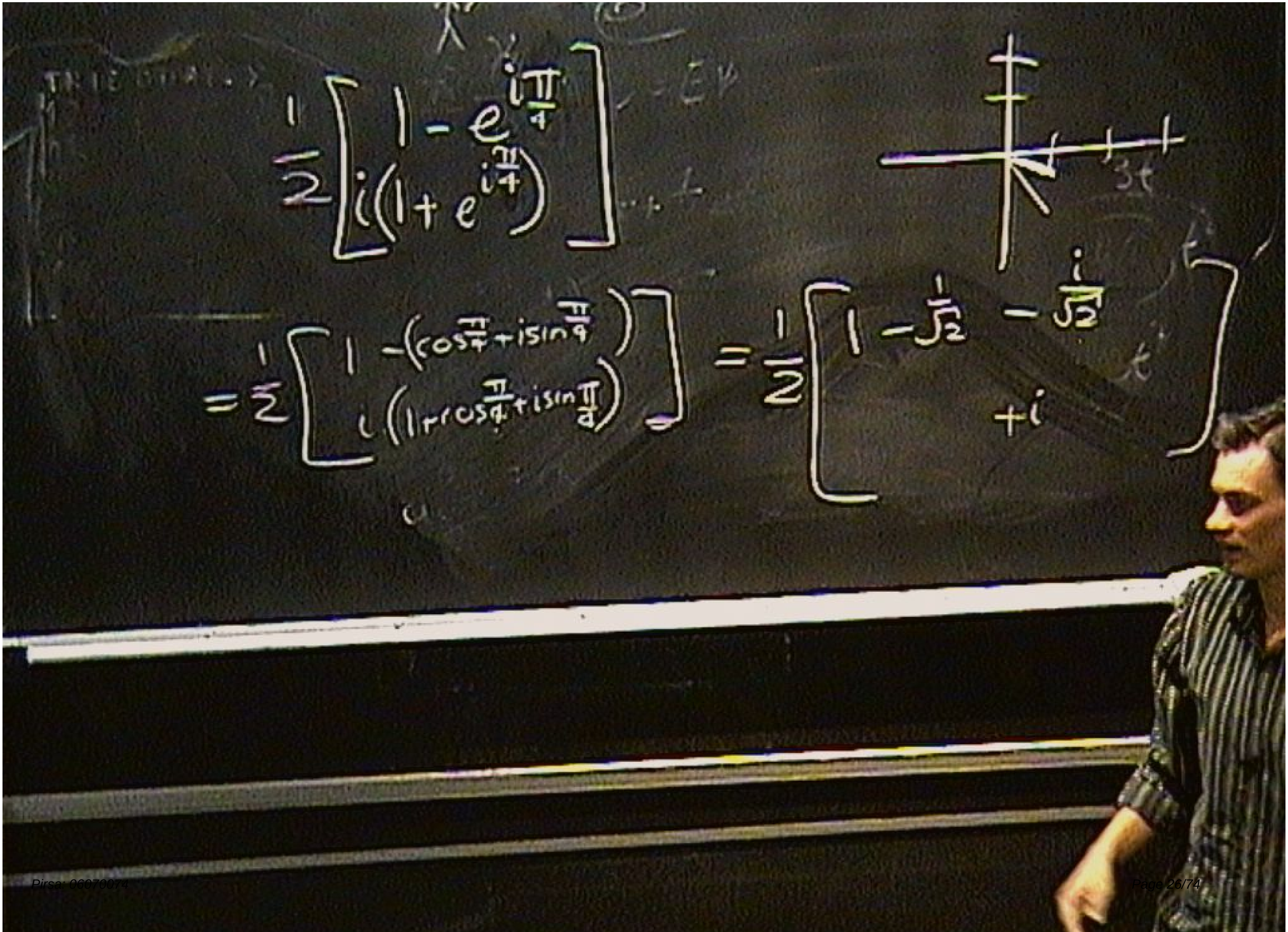
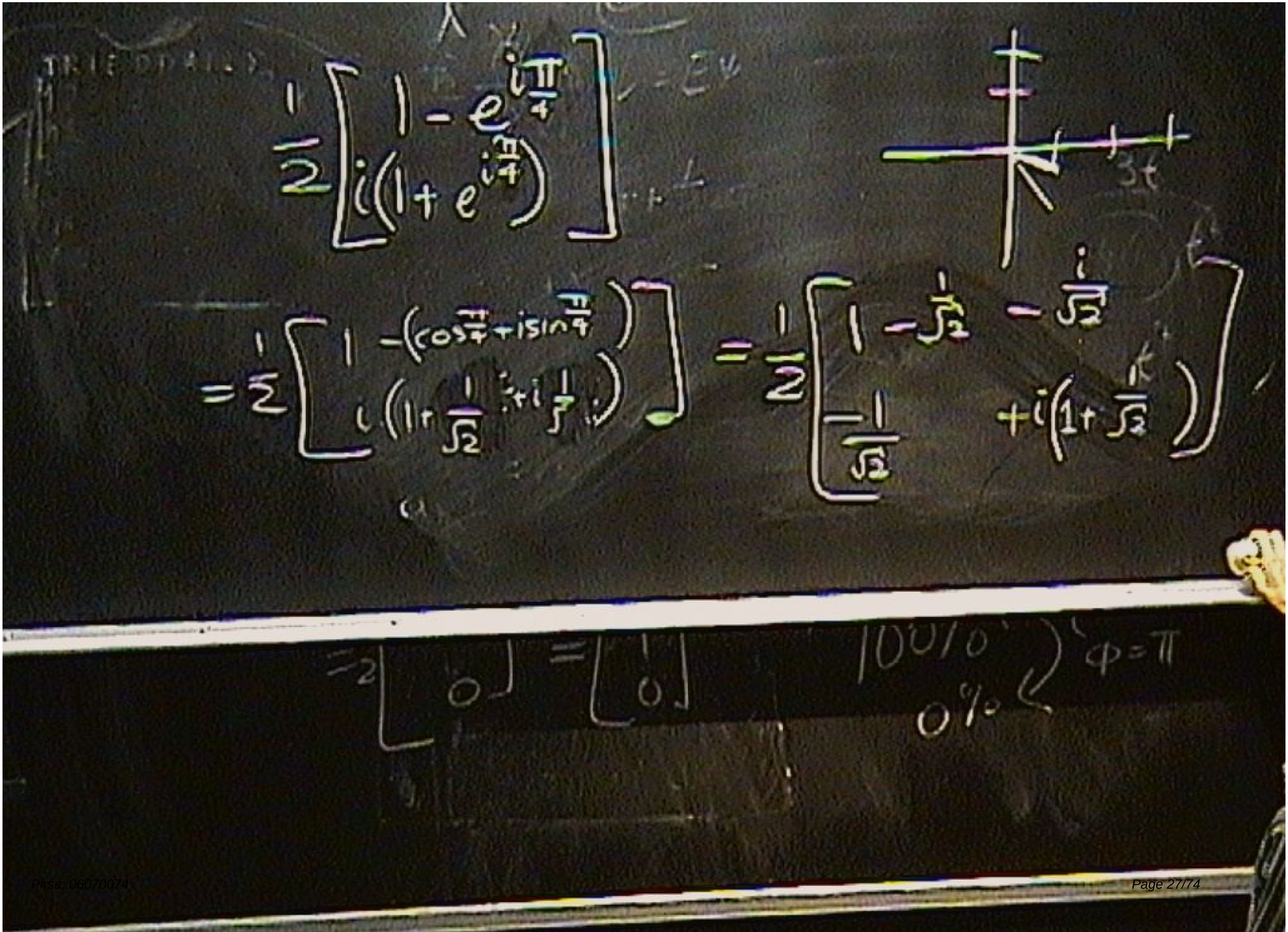

WE'RE LOOKING FOR BIG TOE
$\frac{1}{2}\begin{bmatrix} 1-e^{i\frac{\pi}{4}} \\ i(1+e^{i\frac{\pi}{4}}) \end{bmatrix}$

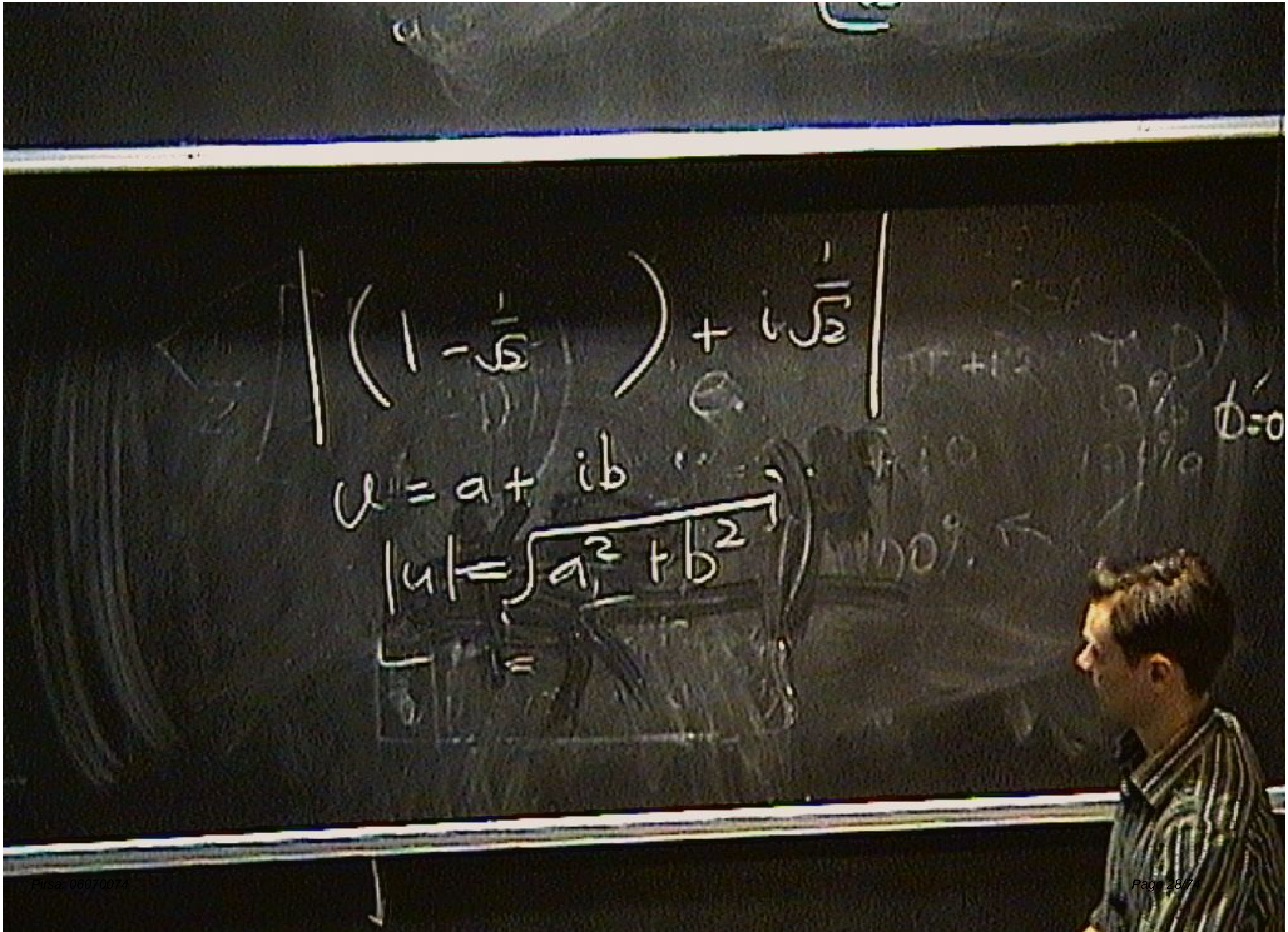

WE'RE LOOKING FOR BIG TOS
$\frac{1}{2}\begin{bmatrix} 1-e^{i\frac{\pi}{4}} \\ i(1+e^{i\frac{\pi}{4}}) \end{bmatrix}$

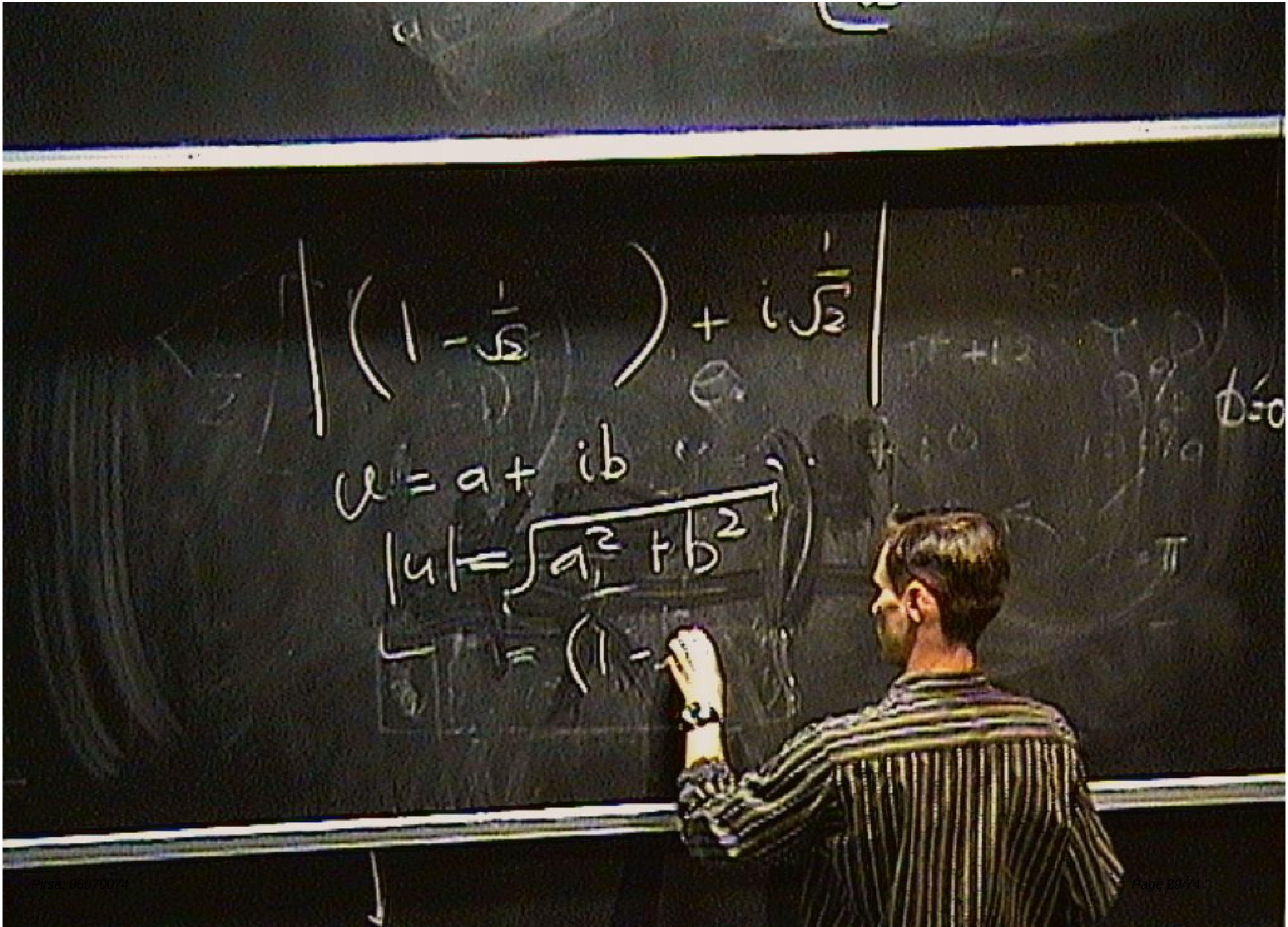

$\frac{1}{2}\begin{bmatrix} 1 - e^{i\frac{\pi}{4}} \\ i(1 + e^{i\frac{\pi}{4}}) \end{bmatrix}$
$= \frac{1}{2}\begin{bmatrix} 1 - (\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) \\ i(1 + \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) \end{bmatrix}$

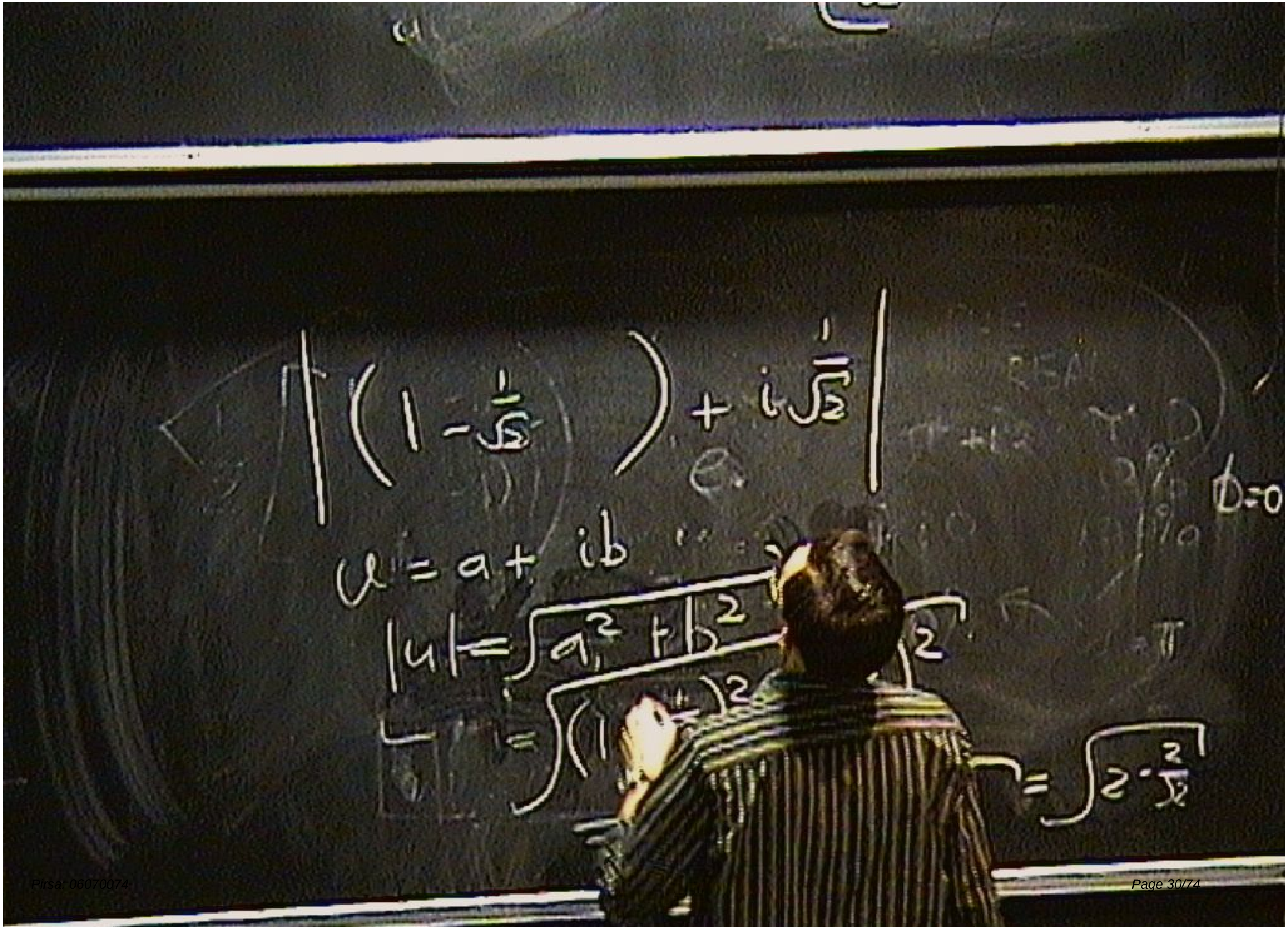

1/2 [1 - e^{iπ/4} ; i(1 + e^{iπ/4})]
= 1/2 [1 - (cos π/4 + i sin π/4) ; i (1 + cos π/4 + i sin π/4)]
= 1/2 [1 - 1/√2 - i/√2 ; + i]


1/2 [1 - e^{iπ/4} ; i(1 + e^{iπ/4})]
= 1/2 [1 - (cos π/4 + i sin π/4) ; i(1 + cos π/4 + i sin π/4)]
= 1/2 [1 - 1/√2 - i/√2 ; + i]
3t


1/2 [1 - e^{iπ/4} ; i(1 + e^{iπ/4})]
= 1/2 [1 - (cos π/4 + i sin π/4) ; i(1 + 1/√2 + i 1/√2)]
= 1/2 [1 - 1/√2 - i/√2 ; -1/√2 + i(1 + 1/√2)]
φ = π


|(1 - 1/√2) + i 1/√2|
u = a + ib
|u| = √(a² + b²)
ϕ=0


u = a + ib
|u| = √(a² + b²)



|(1 - 1/√2) + i 1/√2|
u = a + ib
|u| = √(a² + b²)
= √(2 - 2/√2)
D=0

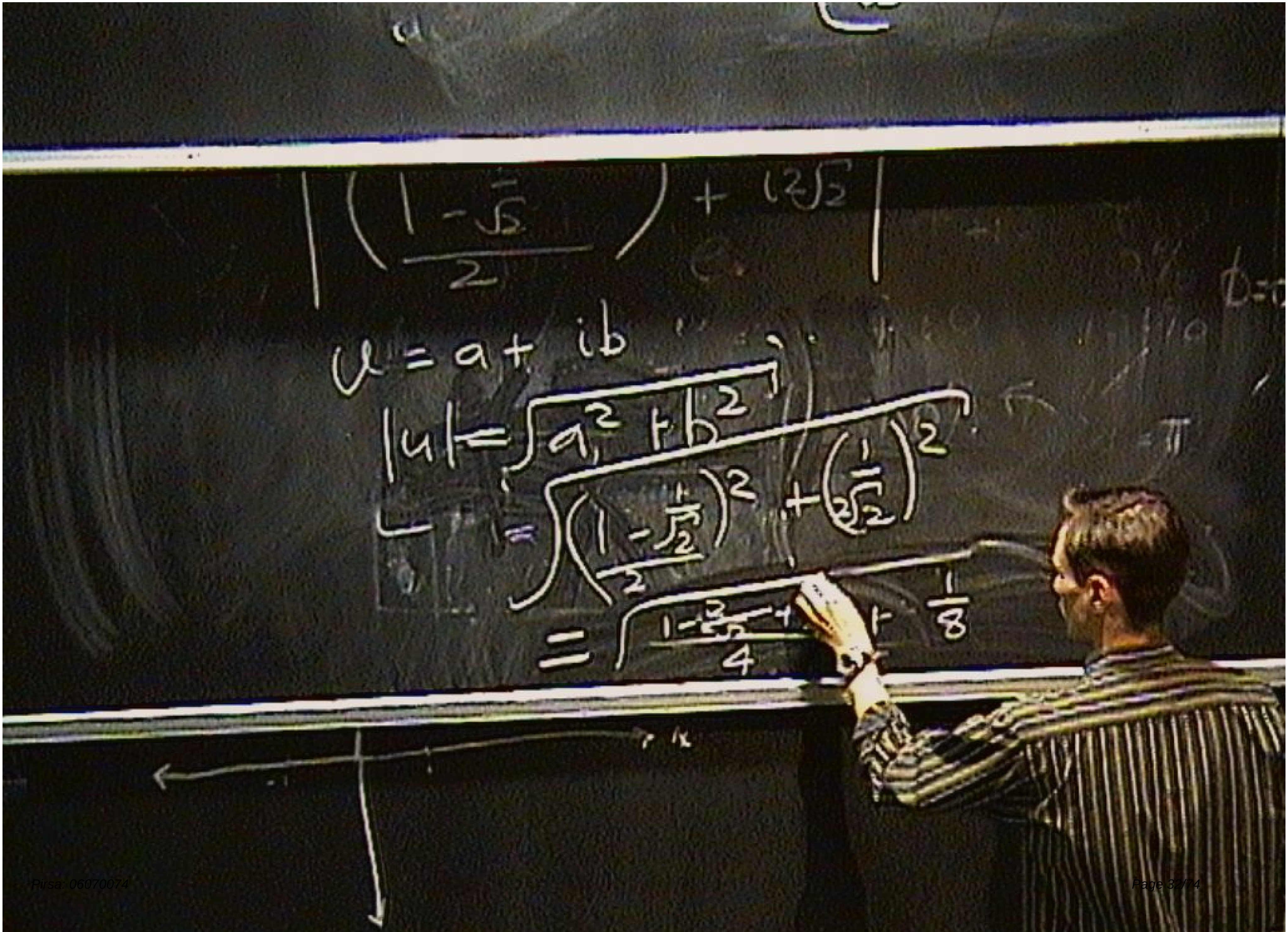
$$\left| \left(\frac{1 - \frac{1}{\sqrt{2}}}{2} \right) + i \frac{1}{2\sqrt{2}} \right|$$

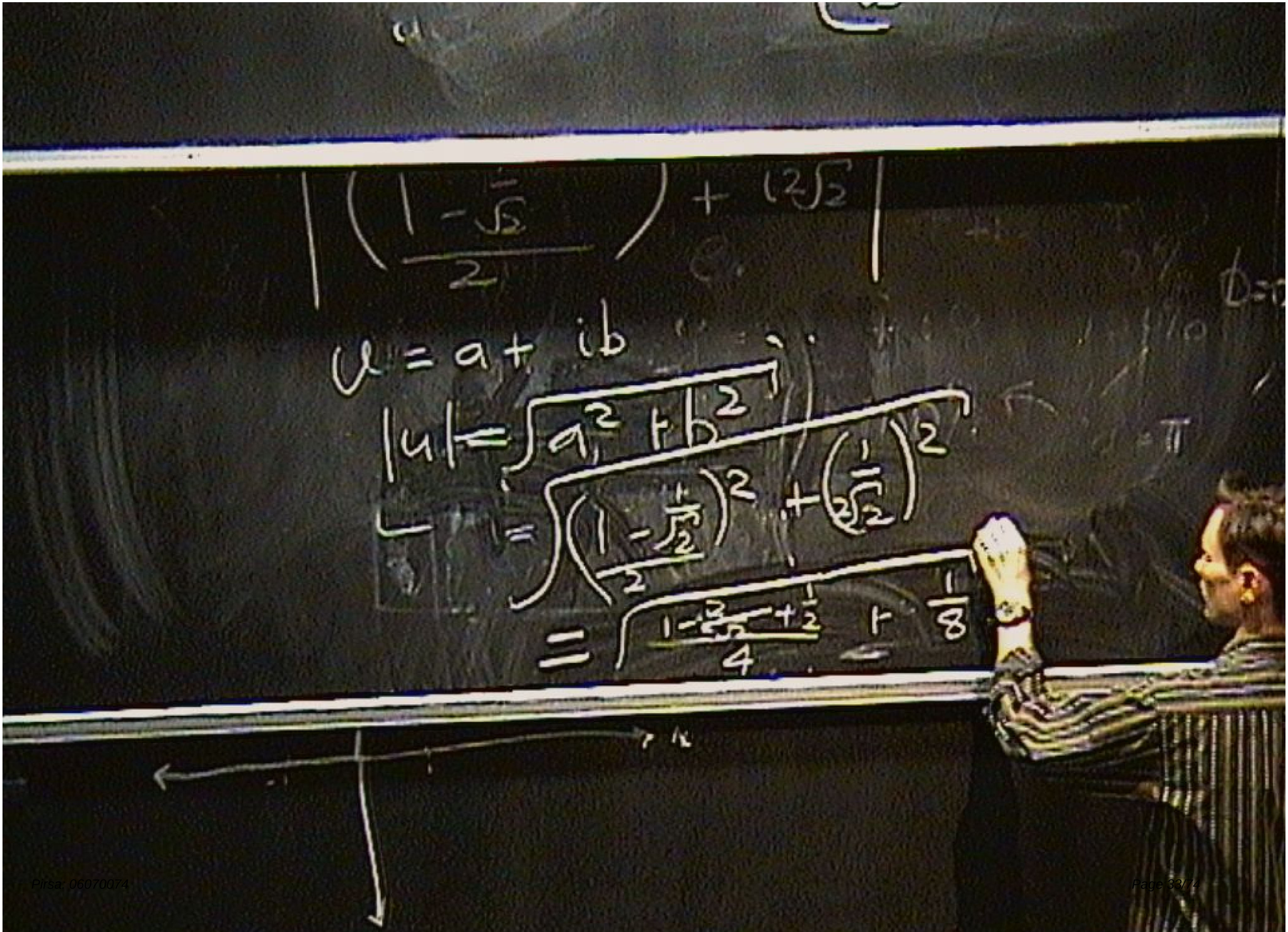
$$u = a + ib$$

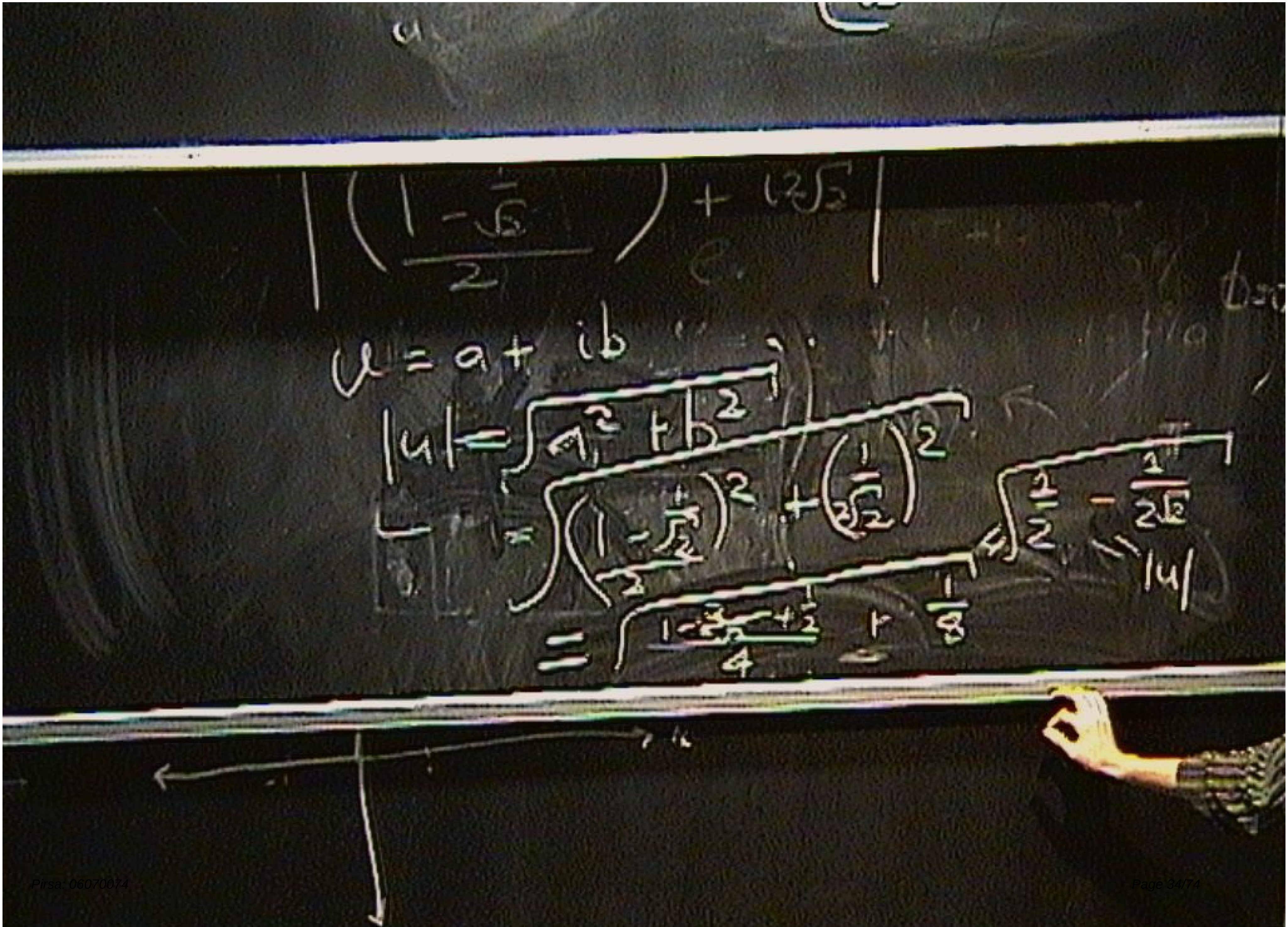
$$|u| = \sqrt{a^2 + b^2}$$

$$= \sqrt{\left(\frac{1 - \frac{1}{\sqrt{2}}}{2} \right)^2 + \left(\frac{1}{2\sqrt{2}} \right)^2}$$

$$= \sqrt{1 - \frac{2}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2}} = \sqrt{2 - \frac{2}{\sqrt{2}}}$$


u = a + ib
|u| = √(a² + b²)
1/8
4


u = a + ib
|u| = √(a² + b²)


u = a + ib
|u| = √(a² + b²)

$$\left| \left(\frac{1 - \frac{1}{\sqrt{2}}}{2}\right) + i\frac{1}{2\sqrt{2}} \right| \quad \therefore Pr = \frac{1}{2} - \frac{1}{2\sqrt{2}} \quad b>0$$

$$u = a + ib$$

$$|u| = \sqrt{a^2 + b^2}$$

$$= \sqrt{\left(\frac{1 - \frac{1}{\sqrt{2}}}{2}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2} \rightarrow \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}} \leftarrow |u|$$

$$= \sqrt{\frac{1 - \frac{2}{\sqrt{2}} + \frac{1}{2}}{4} + \frac{1}{8}}$$

$$\left|\left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right) + i\frac{1}{2\sqrt{2}}\right|$$

$$\therefore Pr = \frac{1}{2} - \frac{1}{2\sqrt{2}} \quad b>0$$

$$= 15\%$$

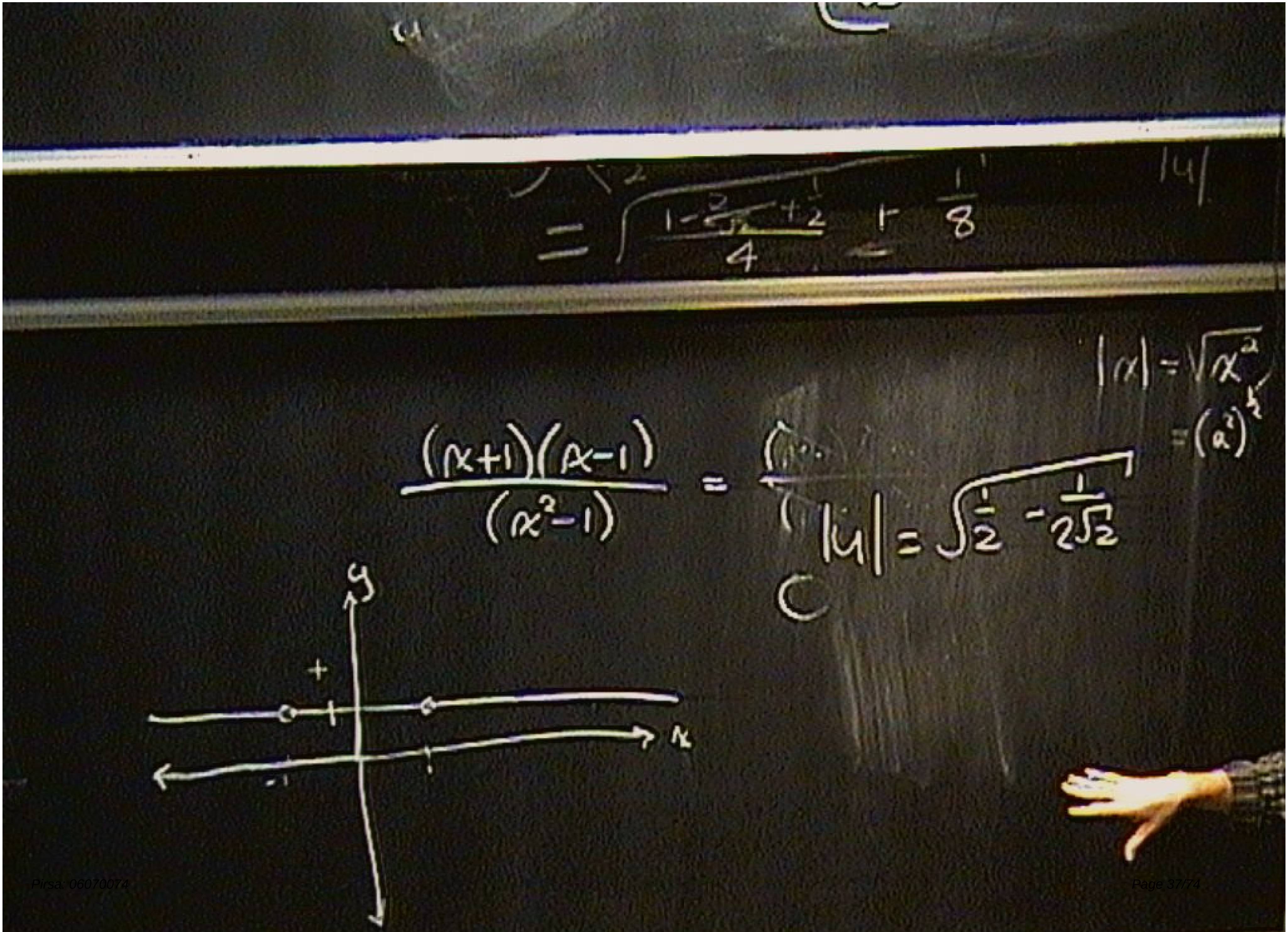
$$u = a + ib$$

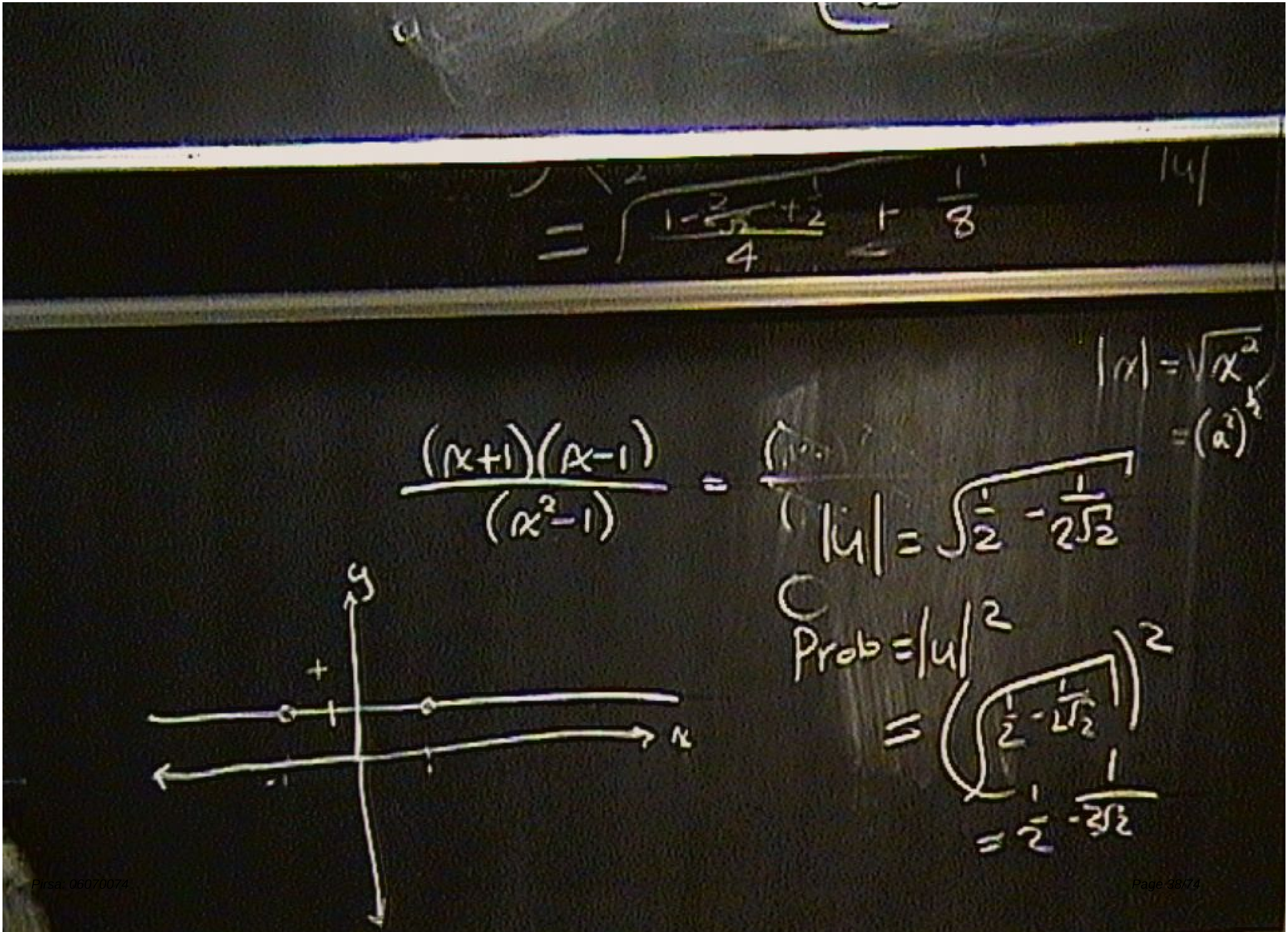
$$|u| = \sqrt{a^2 + b^2}$$

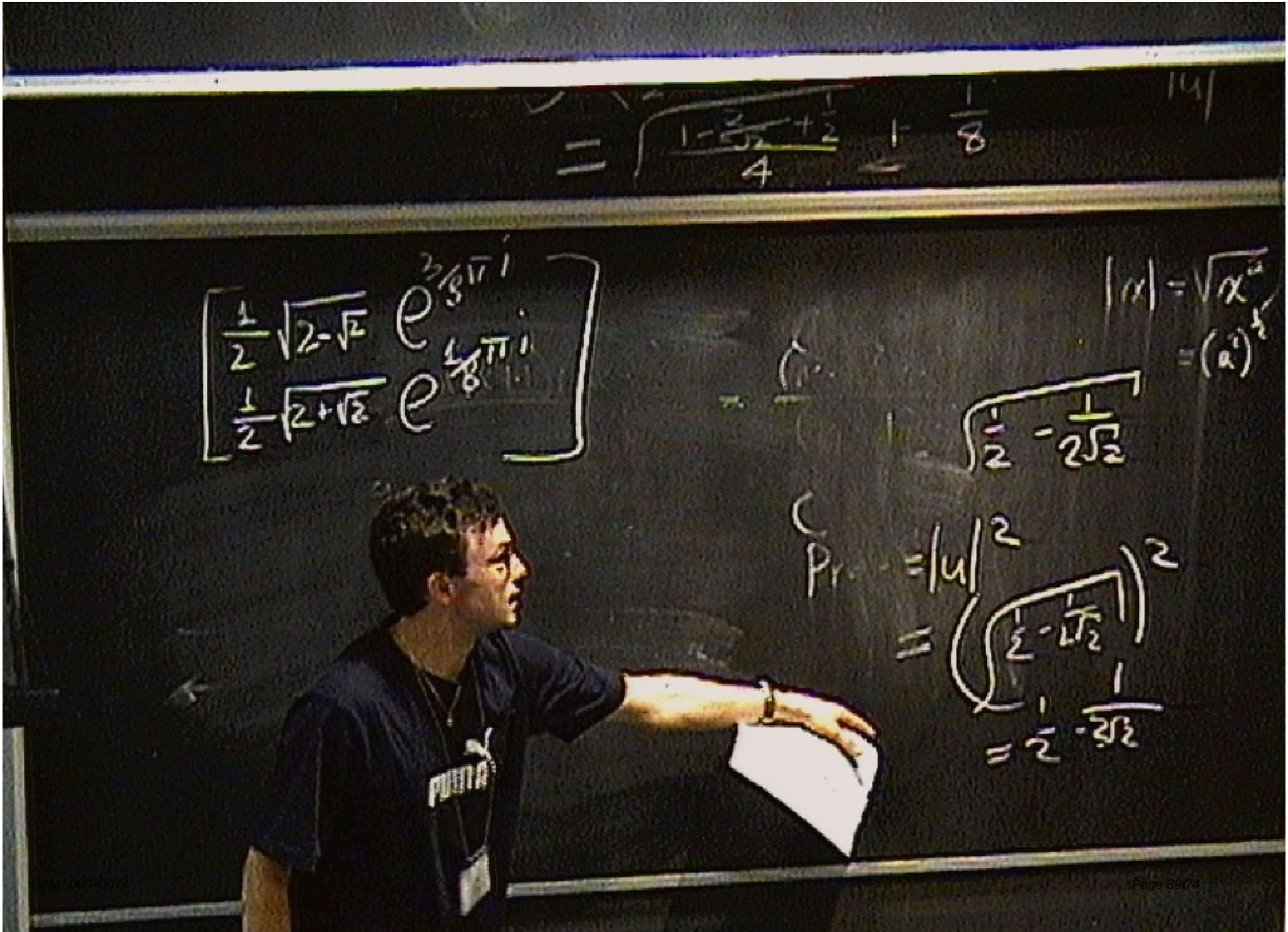
$$= \sqrt{\left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2}$$

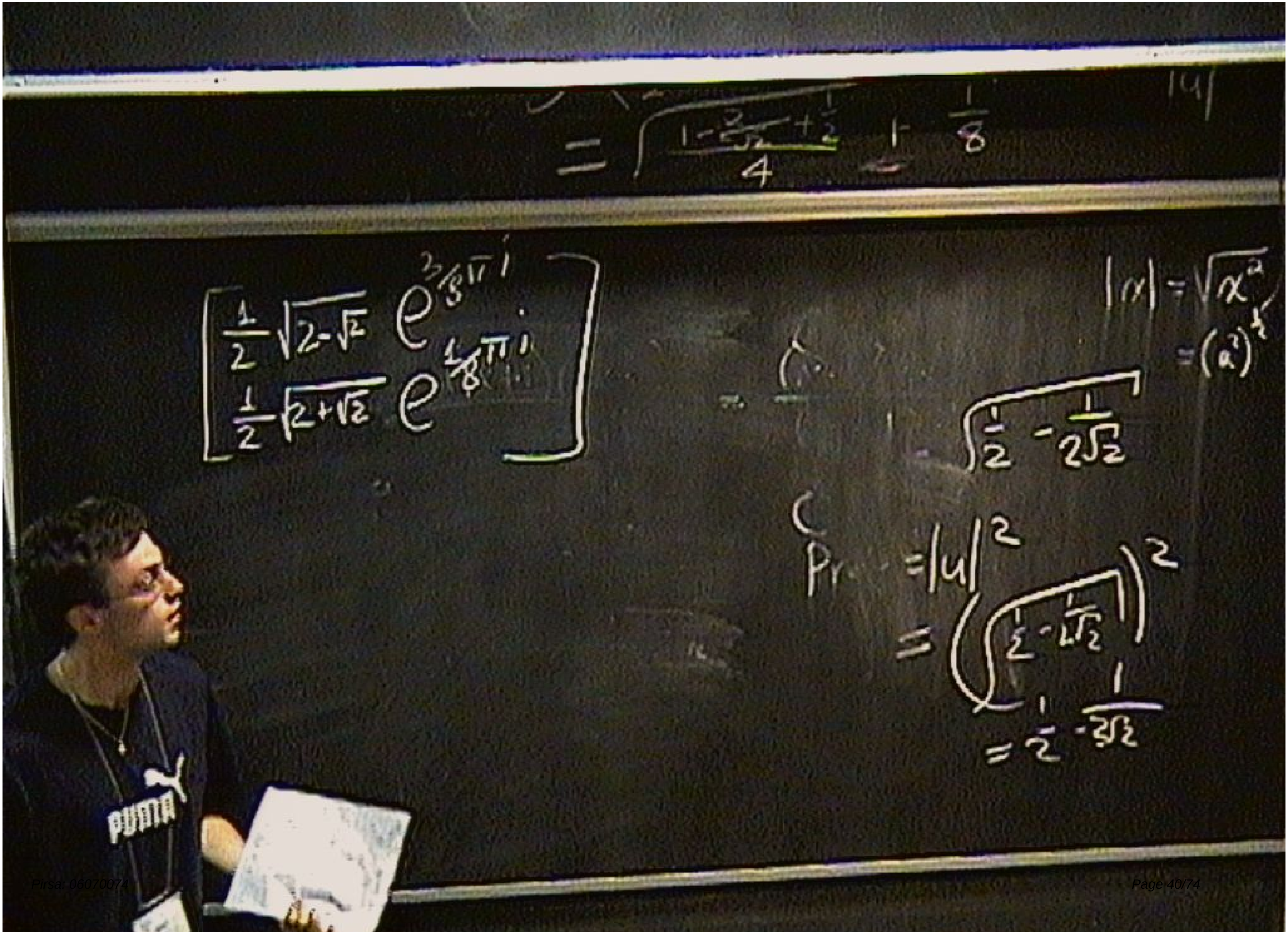
$$= \sqrt{\frac{1-\frac{2}{\sqrt{2}}+\frac{1}{2}}{4} + \frac{1}{8}}$$

$$\rightarrow \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}} \leftarrow |u|$$

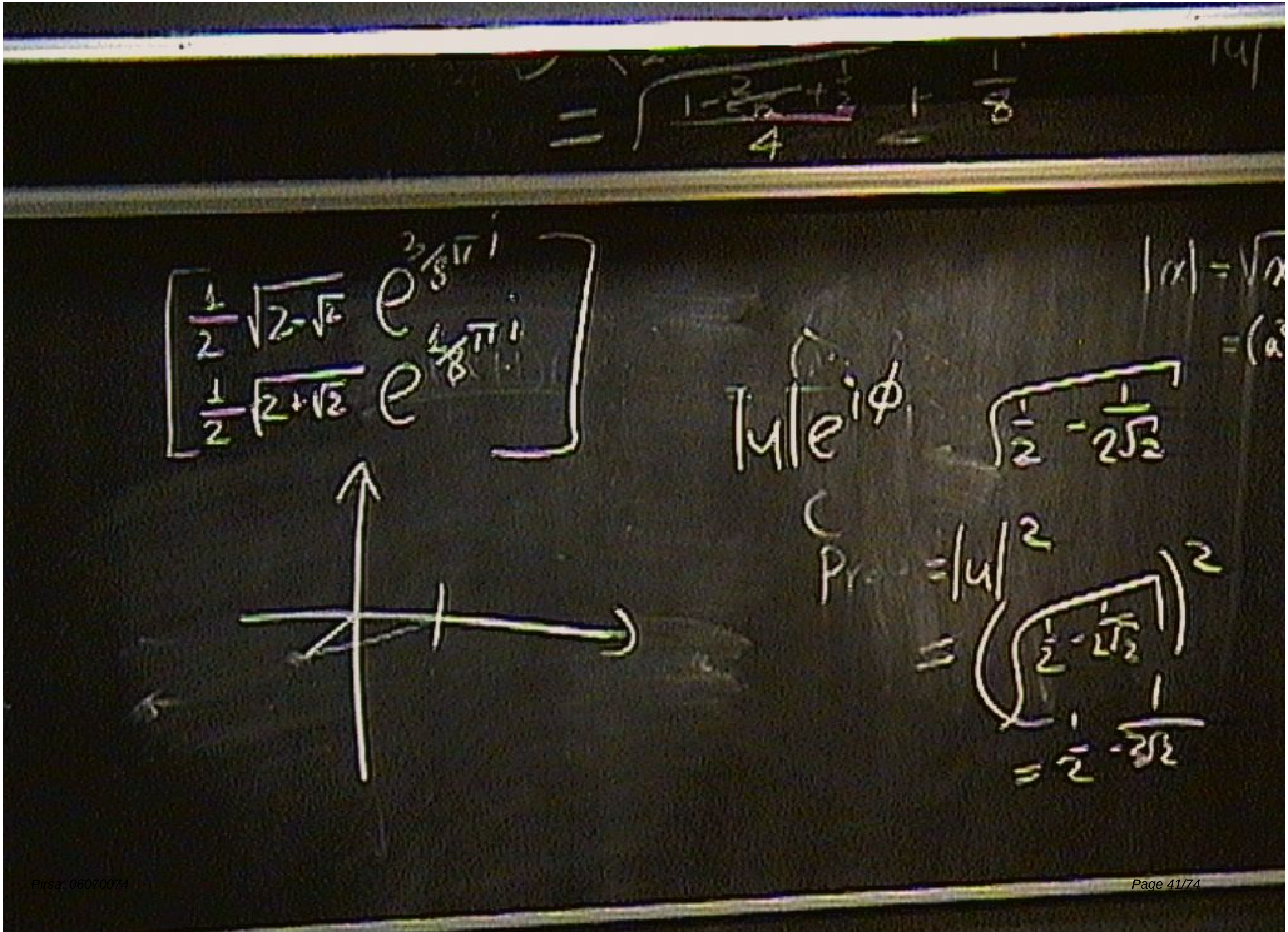

|u|
= √((1 − 2/√2 + 1/2)/4 + 1/8)
|x| = √(x²)
= (x²)^(1/2)
(x+1)(x−1) / (x²−1) =
|u| = √(1/2 − 1/(2√2))
y
x
+
-1
1


|u|
|x| = √x²
= (x²)^½
(x+1)(x−1) / (x²−1)
|u| = √(1/2 − 1/(2√2))
Prob = |u|²
= (√(1/2 − 1/(2√2)))²
= 1/2 − 1/(2√2)
y
x

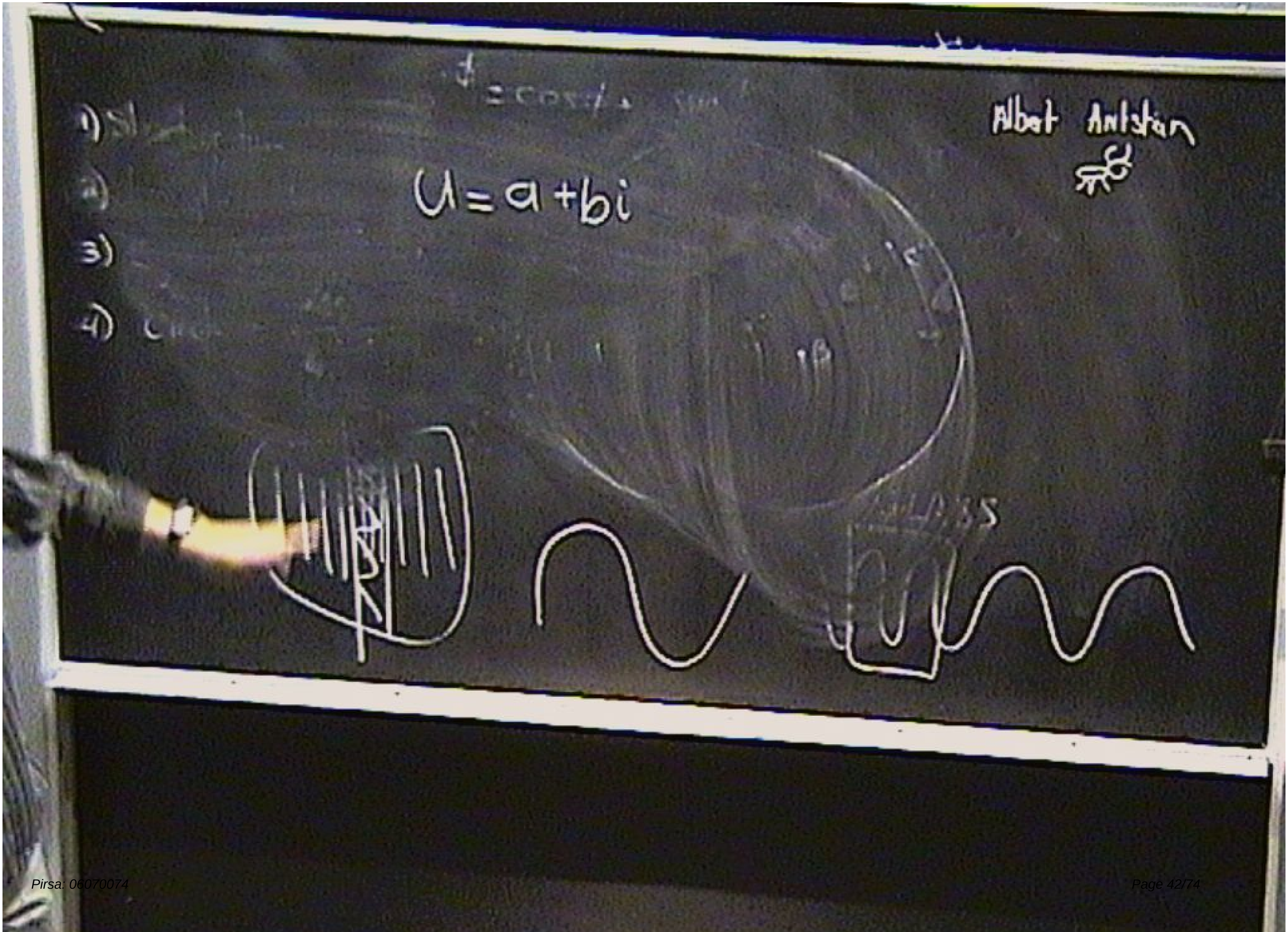




$= \sqrt{\frac{1-\frac{2}{\sqrt{2}}+\frac{1}{2}}{4}} \quad \frac{1}{8}$
|u|
$\frac{1}{2}\sqrt{2-\sqrt{2}}\, e^{\frac{3}{8}\pi i}$
$\frac{1}{2}\sqrt{2+\sqrt{2}}\, e^{\frac{1}{8}\pi i}$
$|x| = \sqrt{x^2}$
$= (x^2)^{\frac{1}{2}}$
$\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}}$
$Pr = |u|^2$
$= \left(\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}}\right)^2$
$= \frac{1}{2} - \frac{1}{2\sqrt{2}}$
PUMA



$= \sqrt{\frac{1-\frac{2}{\sqrt{2}}+\frac{1}{2}}{4} + \frac{1}{8}}$
$\begin{bmatrix} \frac{1}{2}\sqrt{2-\sqrt{2}}\, e^{\frac{3}{8}\pi i} \\ \frac{1}{2}\sqrt{2+\sqrt{2}}\, e^{\frac{4}{8}\pi i} \end{bmatrix}$
$|u|e^{i\phi}$
$\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}}$
$Pr = |u|^2$
$= \left(\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}}\right)^2$
$= \frac{1}{2} - \frac{1}{2\sqrt{2}}$


U=a+bi

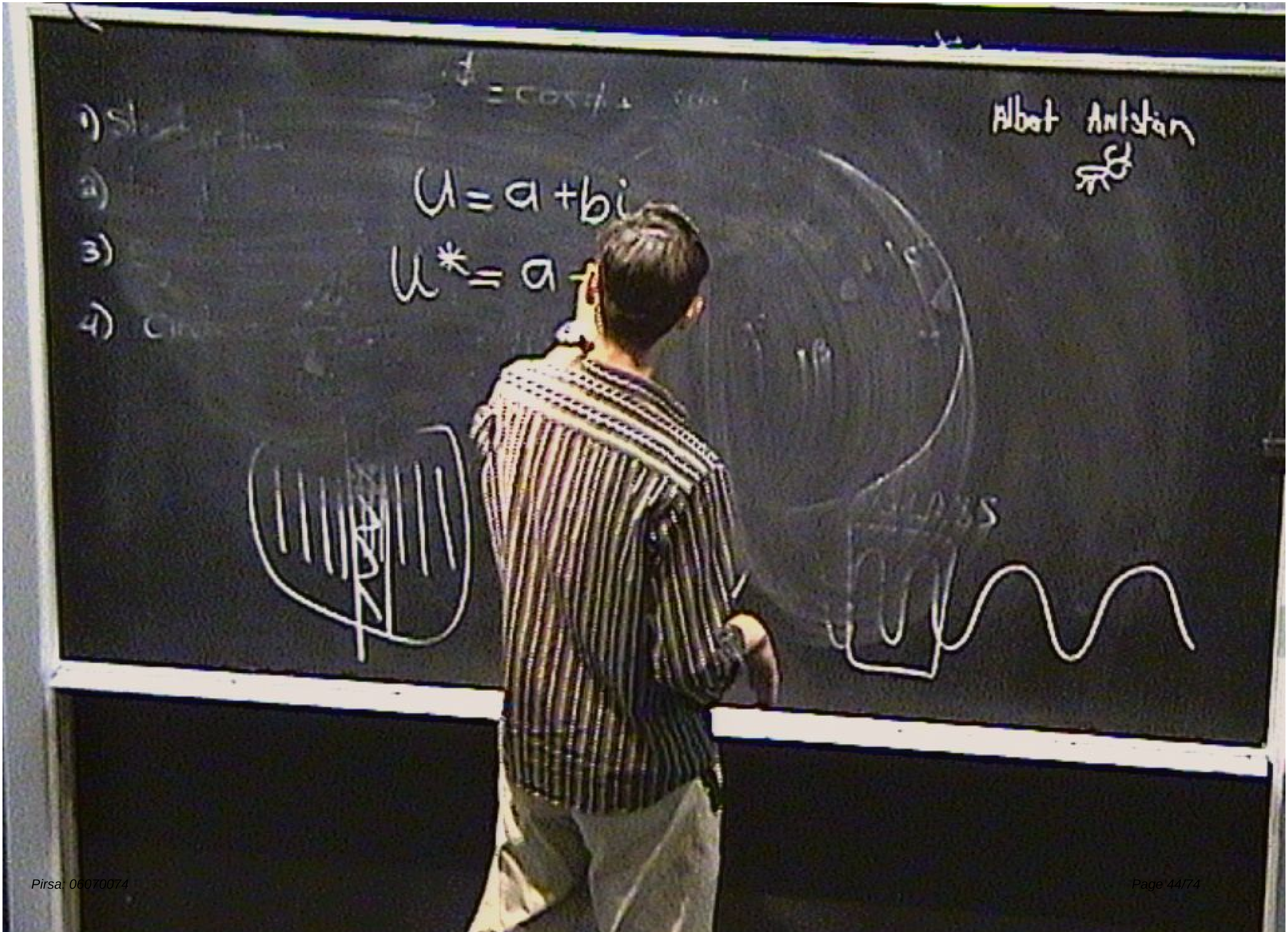
Homeplay question (hard).

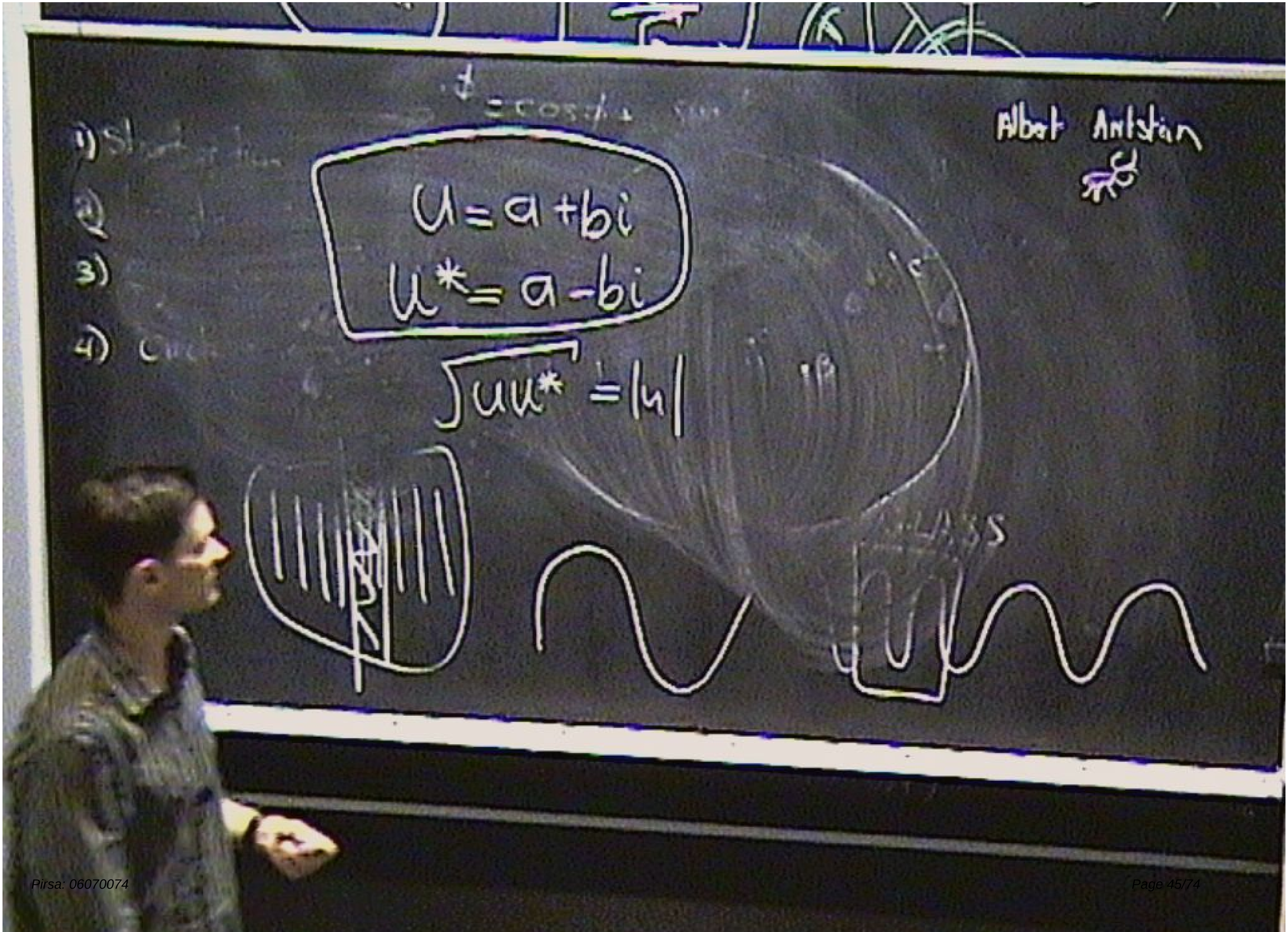
- A unitary matrix is any matrix U for which $U^{\dagger} = U^{-1}$
- The 'dagger' symbol † means that we take the transpose of U and then then complex conjugate of each element of the resulting matrix. It signifies what is called the *Hermitian conjugate* of U.

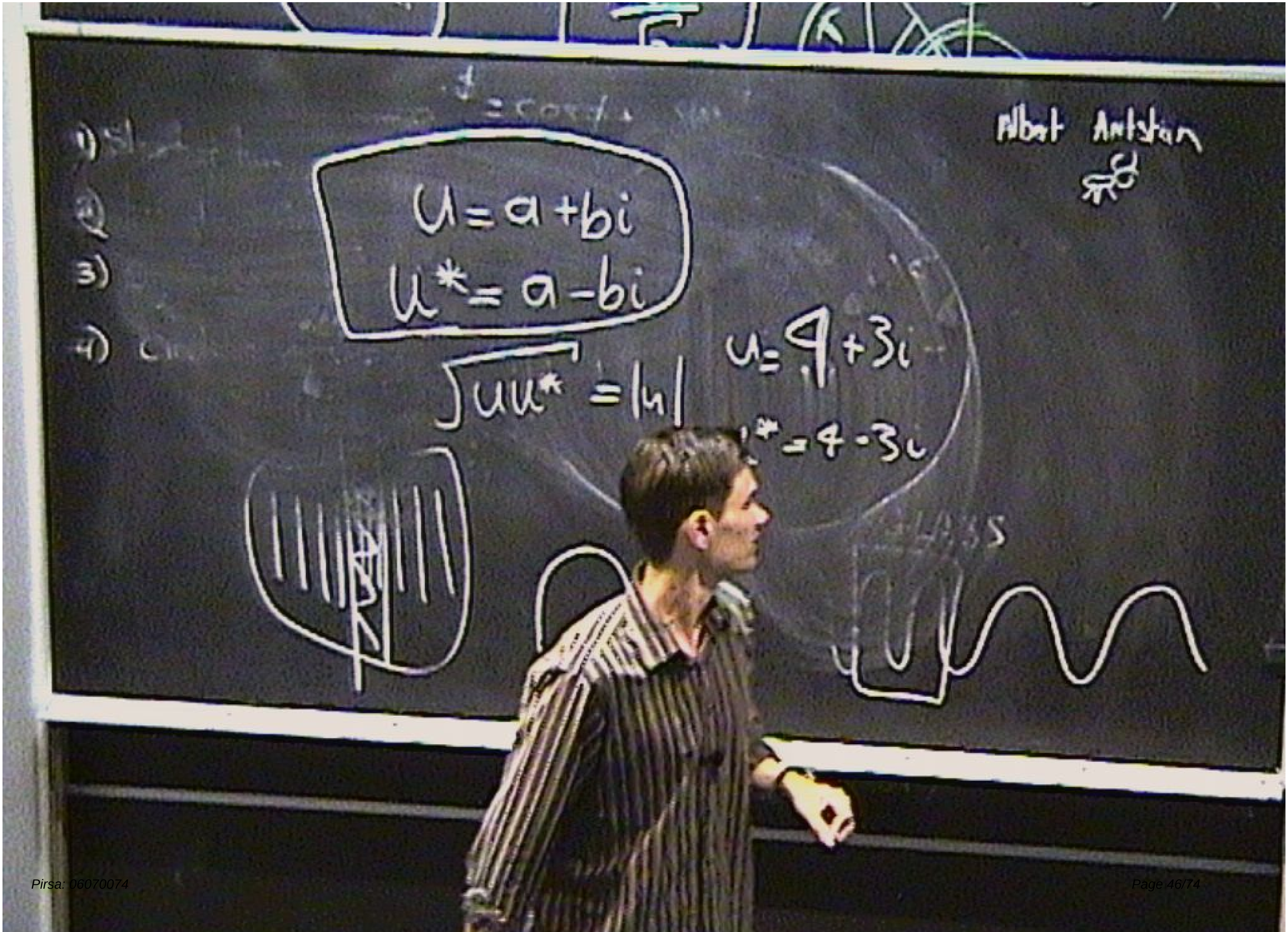
i.e.
$$If \quad U = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

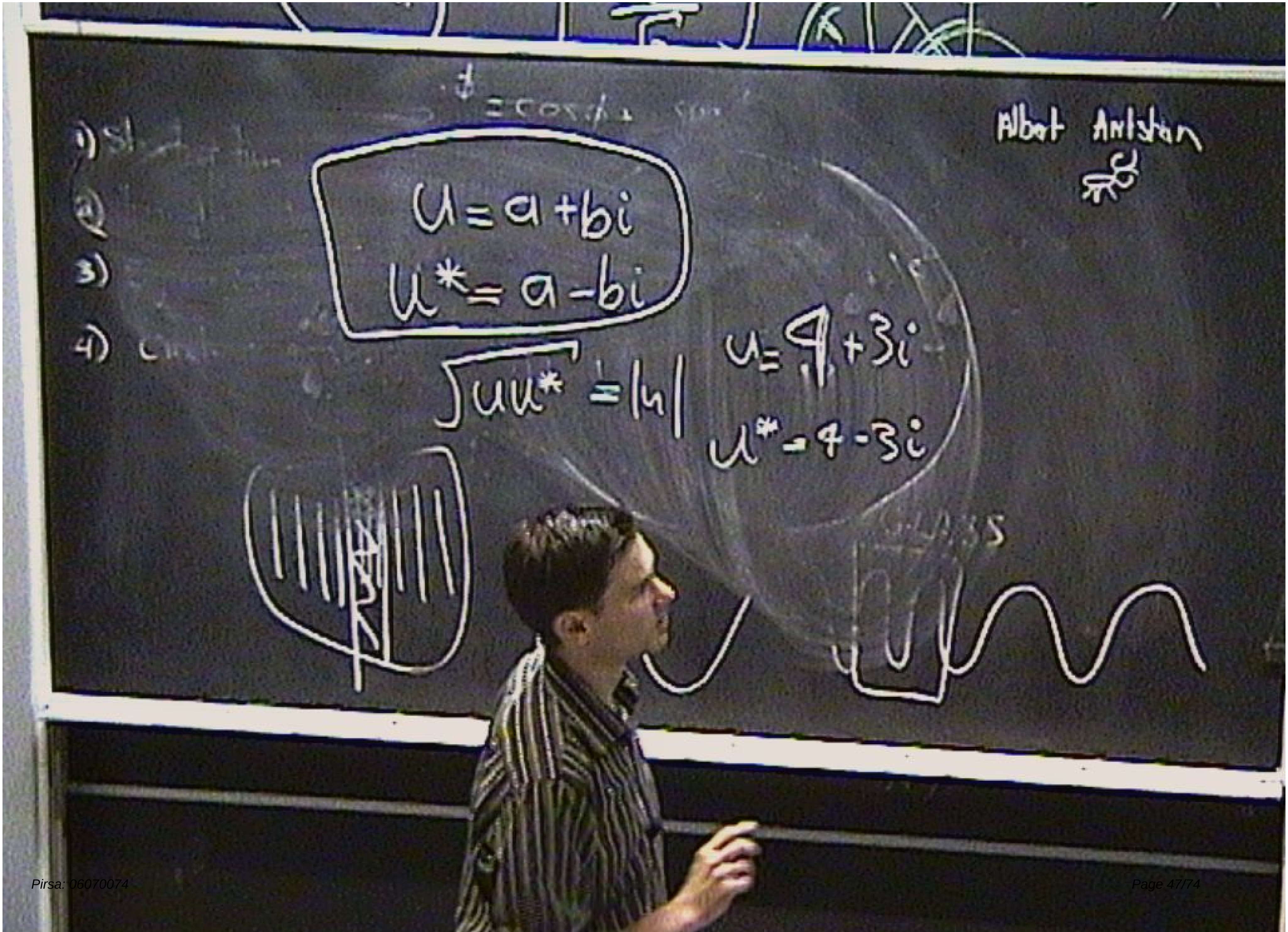
$$then \quad U^{+} = (U^{T})^{*} = \left(\begin{bmatrix} a & c \\ b & d \end{bmatrix} \right)^{*} = \begin{bmatrix} a^{*} & c^{*} \\ b^{*} & d^{*} \end{bmatrix}$$

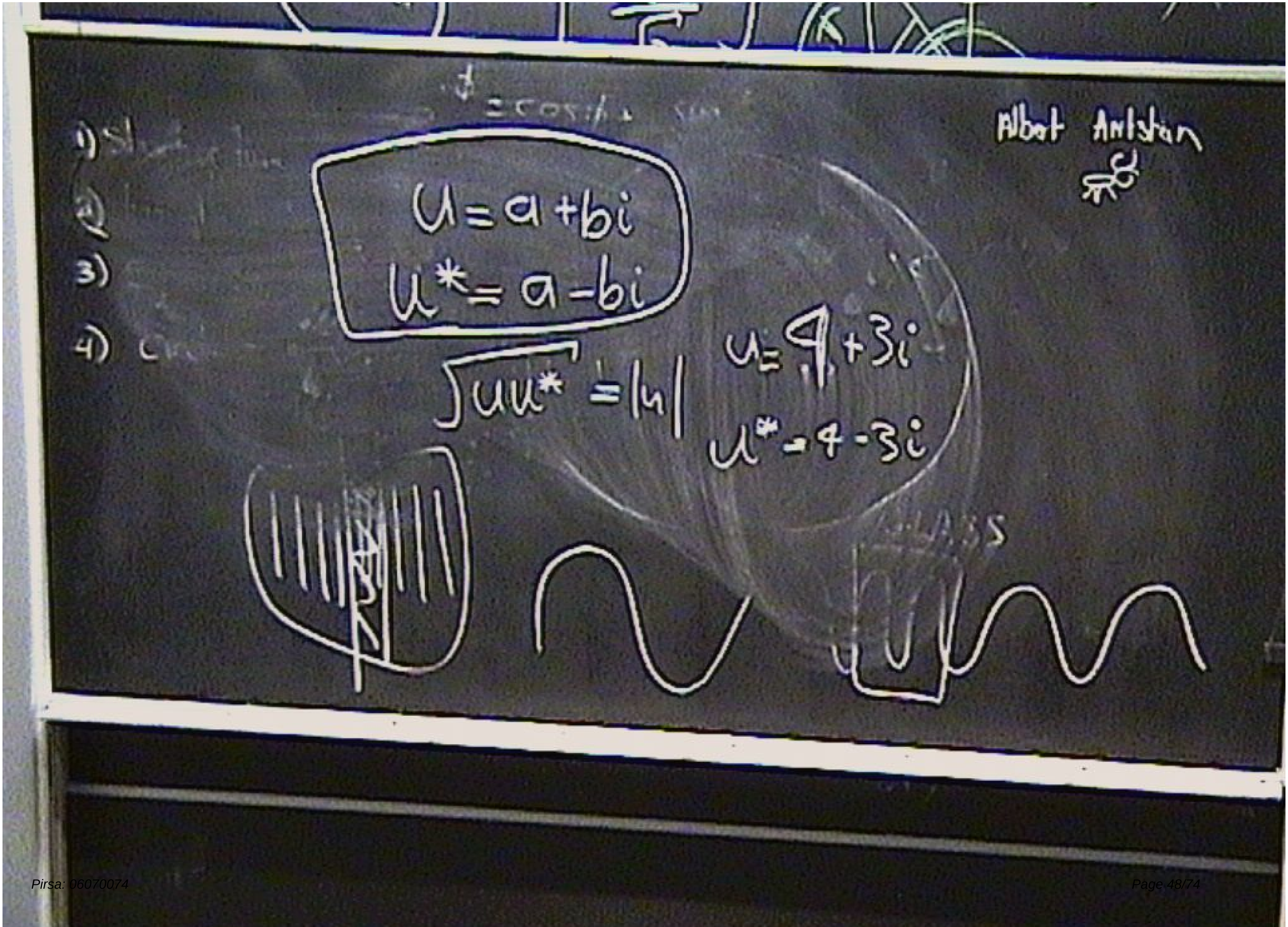
$$If \quad A = \begin{bmatrix} 4 & -2i \\ i & 3 \end{bmatrix}, what \quad is \quad A^{+}?$$


Albat Antstan
u = a + bi
u* = a
1)
2)
3)
4)


1)
2)
3)
4)
u = a + bi
u* = a − bi
√(uu*) = |u|
Albet Antstin


1)
2)
3)
4)
u = a + bi
u* = a − bi
√(uu*) = |u|
u = 4 + 3i
u* = 4 − 3i

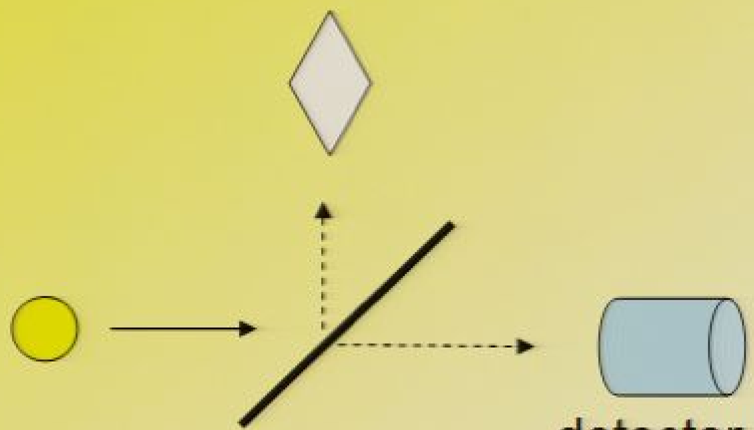

Albert Anlstein
u = a + bi
u* = a - bi
√(uu*) = |u|
u = 4 + 3i
u* = 4 - 3i
GLASS



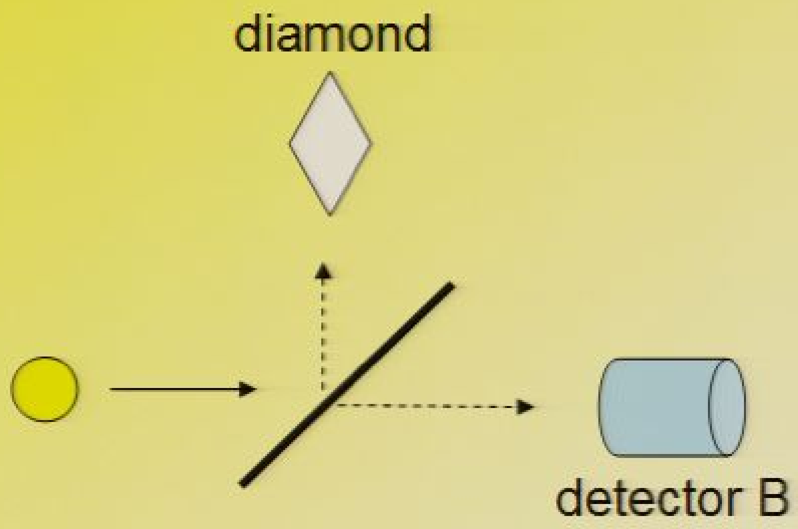
u = a + bi
u* = a - bi
√(uu*) = |u|
u = 4 + 3i
u* = 4 - 3i

Can you 'see' a safely secured precious diamond without looking at it?

- Consider a multi-million dollar diamond kept safe by an unusual security system in a large room in a museum.
- The museum's owners are so protective that they would rather see it destroyed than being stolen.
- Your challenge is to try to figure out a way to steal it.
- You don't know exactly where it is and so you need to locate it. You are only allowed to use light to do this (i.e. photons).
- Is there anywhere you can tell whether or not the diamond is in a particular location using only light without destroying it?
- You cannot simply look for it as by the time you've seen it, you've already destroyed it.
- Is there any way of using what you've learnt so far about quantum physics to find out where it is without setting off the security system?
  (Even with a certain probability? eg. 25%)

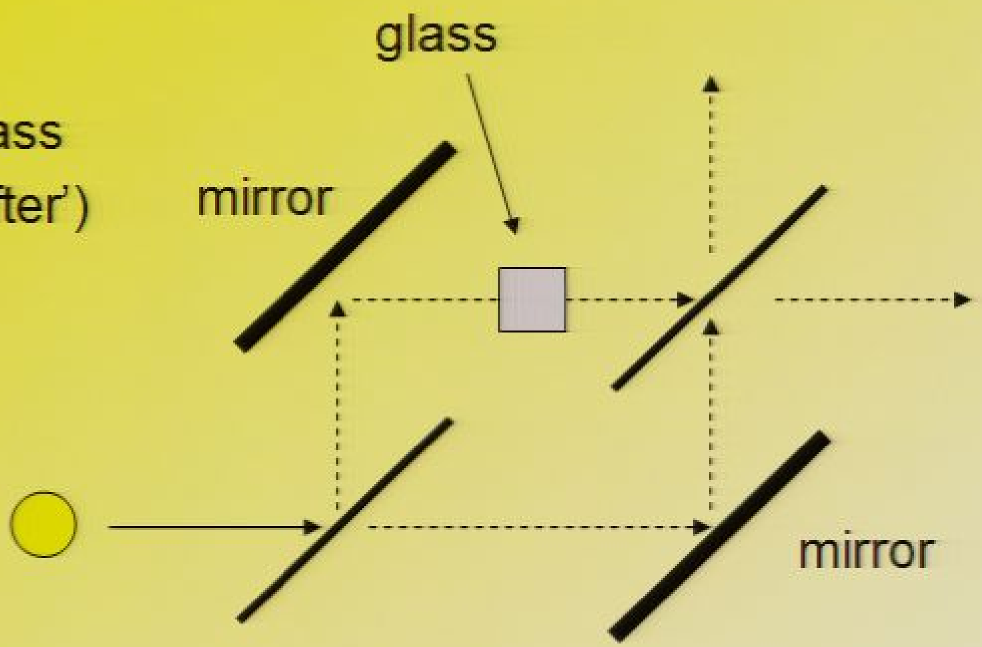


- 50% of the time, we detect a photon at detector B
- 50% of the time, the photons hits the diamond, destroying the diamond.



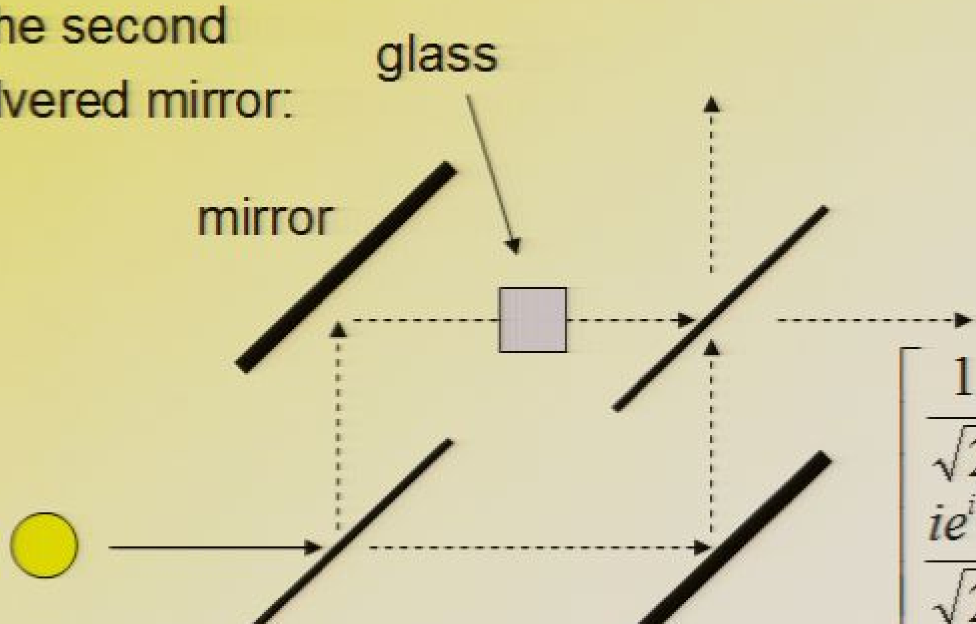
- 50% of the time, we detect a photon at detector B
- 50% of the time, the photons hits the diamond, destroying the diamond.

- After the glass ('phase-shifter')



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix}$$

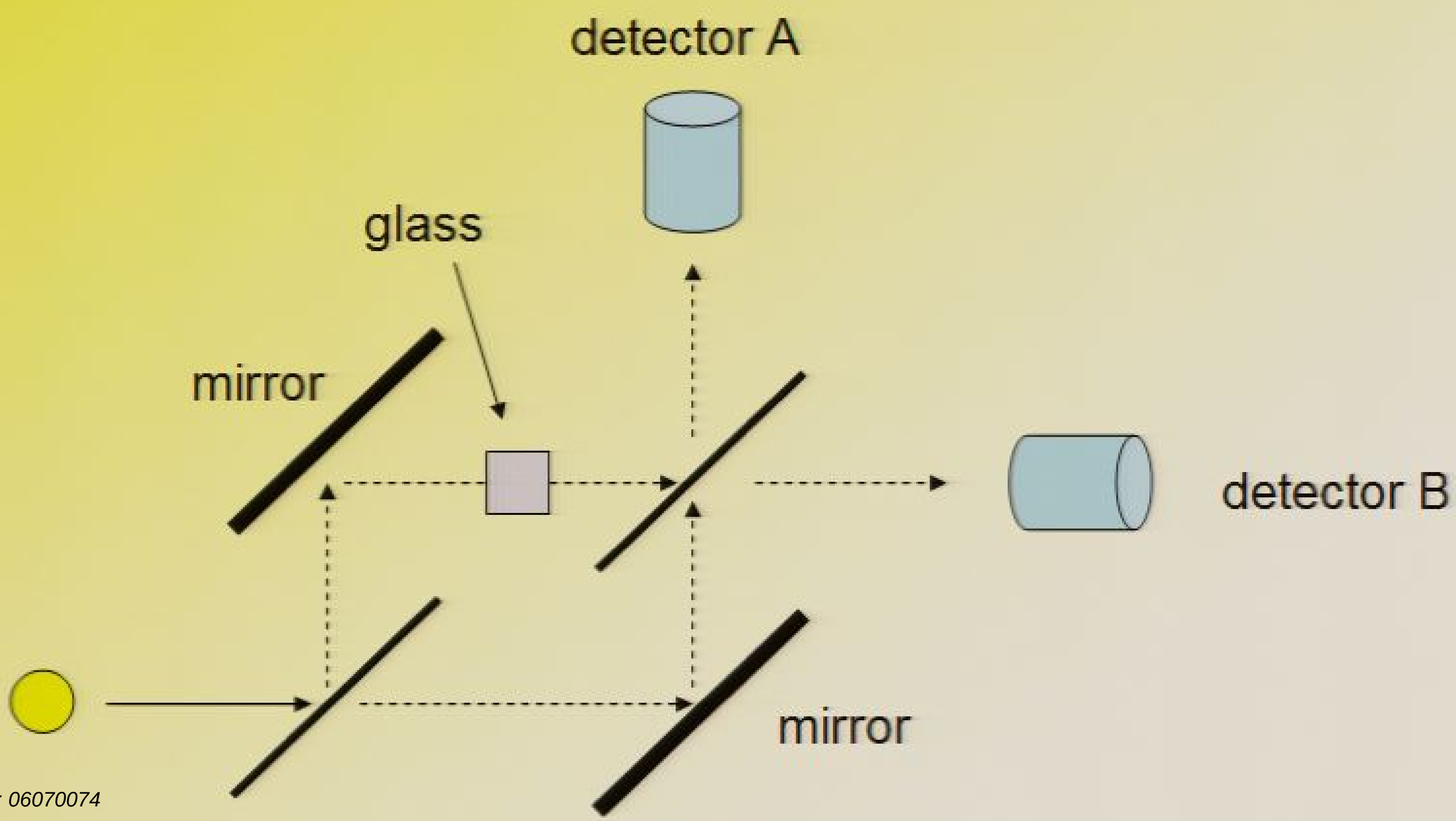
After the second half-silvered mirror:

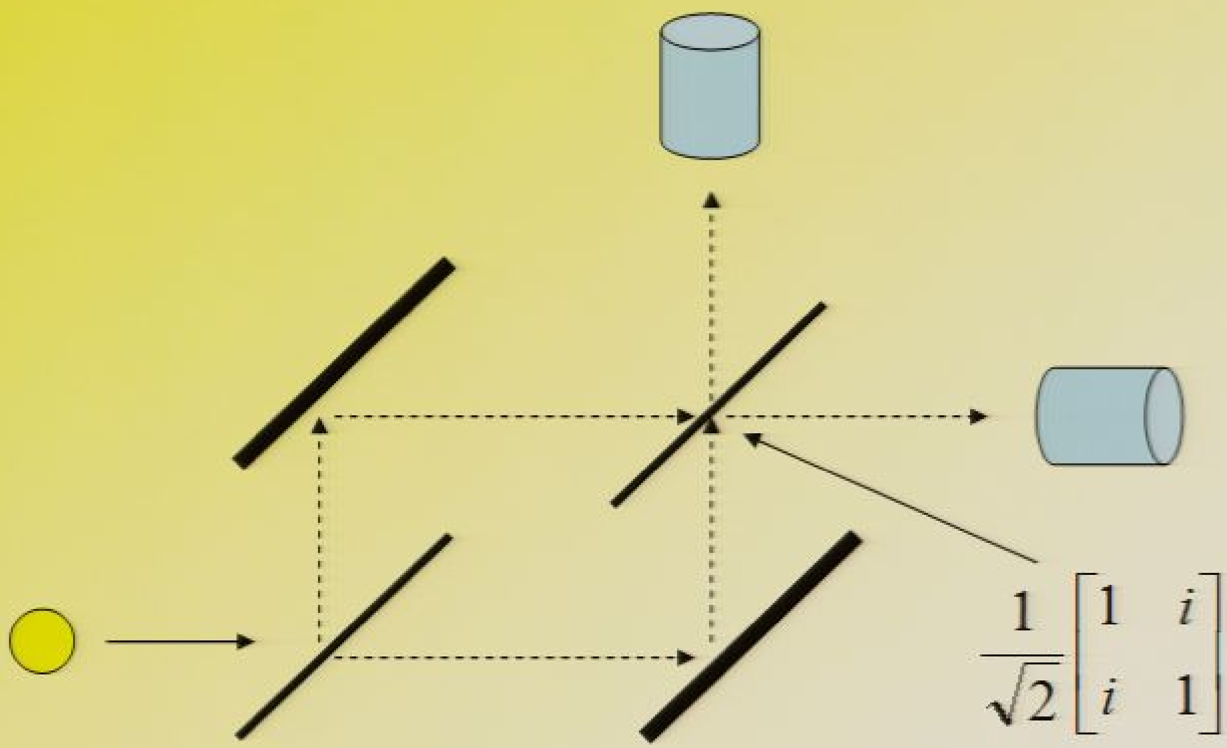


$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$$

Phases

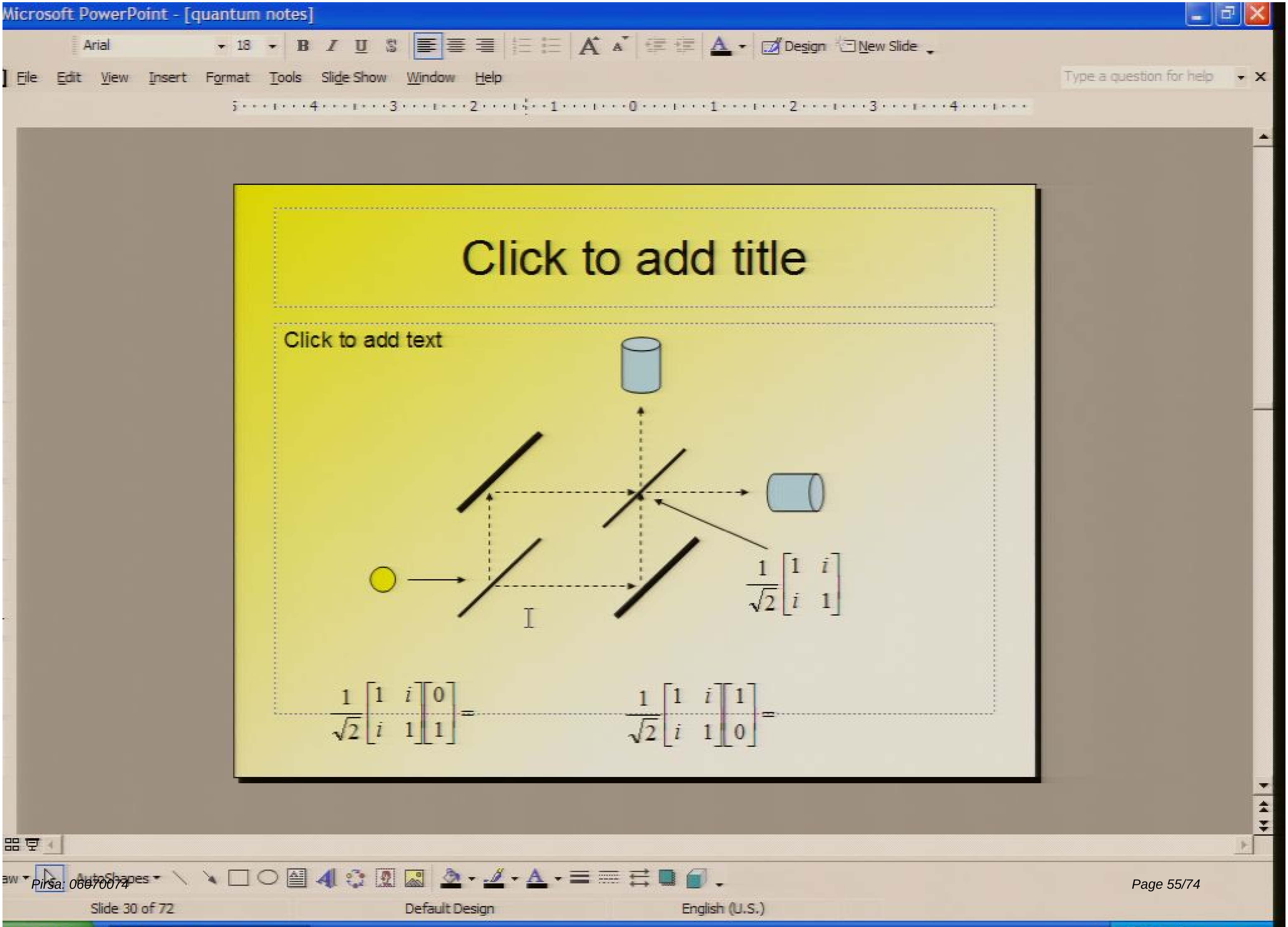
- Let us change the experiment slightly by adding a piece of glass that slows down the photon and so gives it an extra ‘phase’

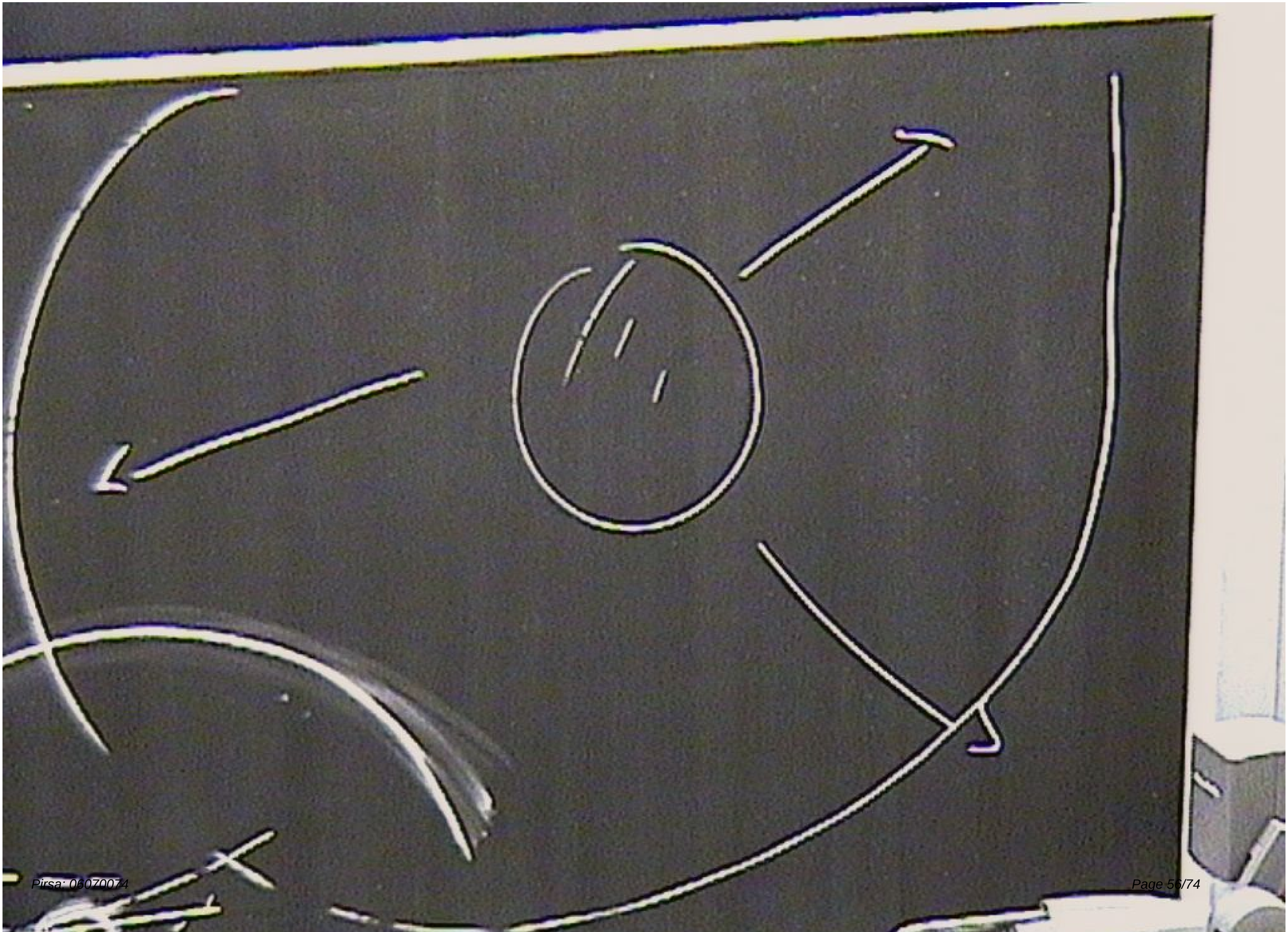


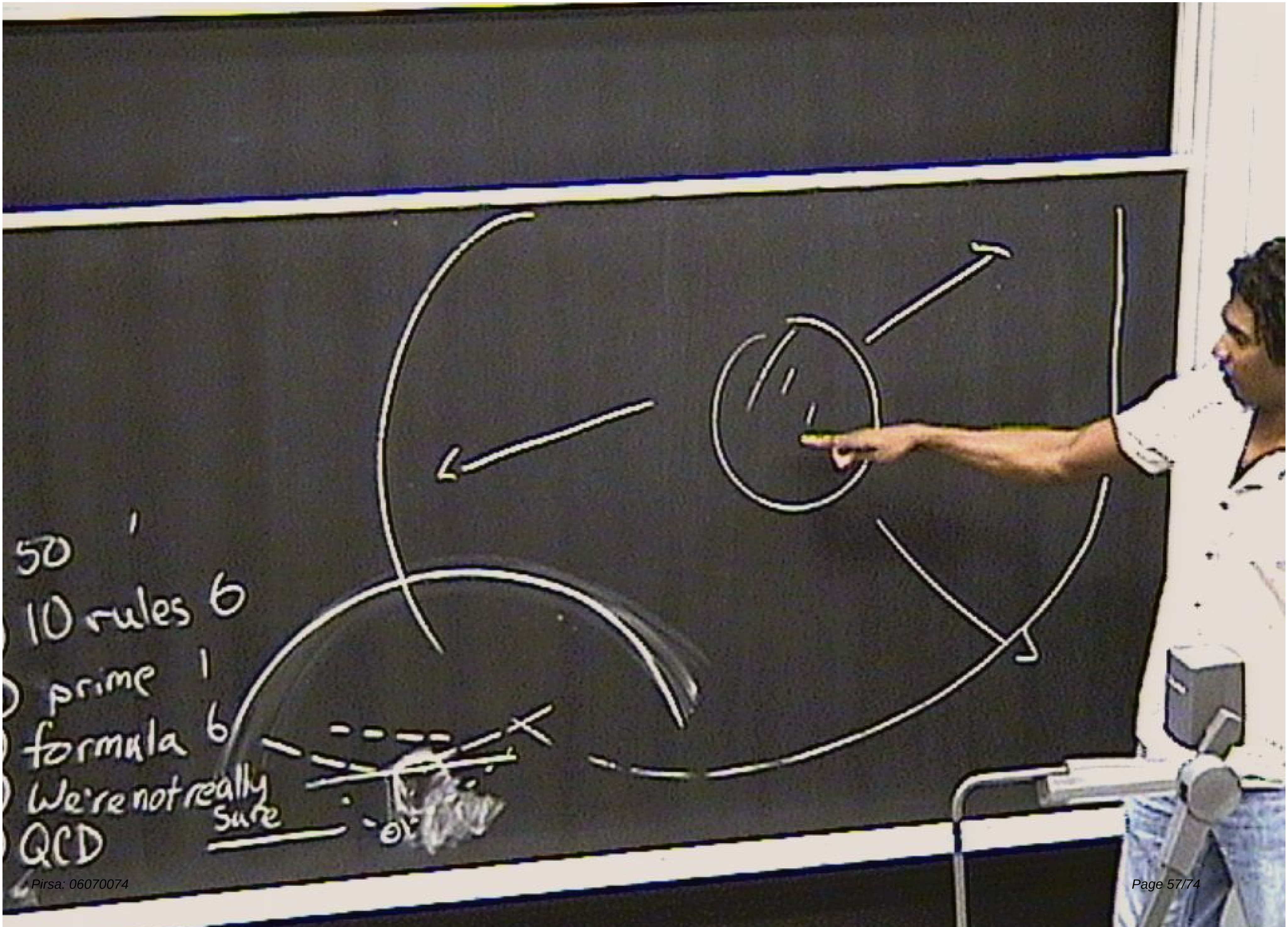


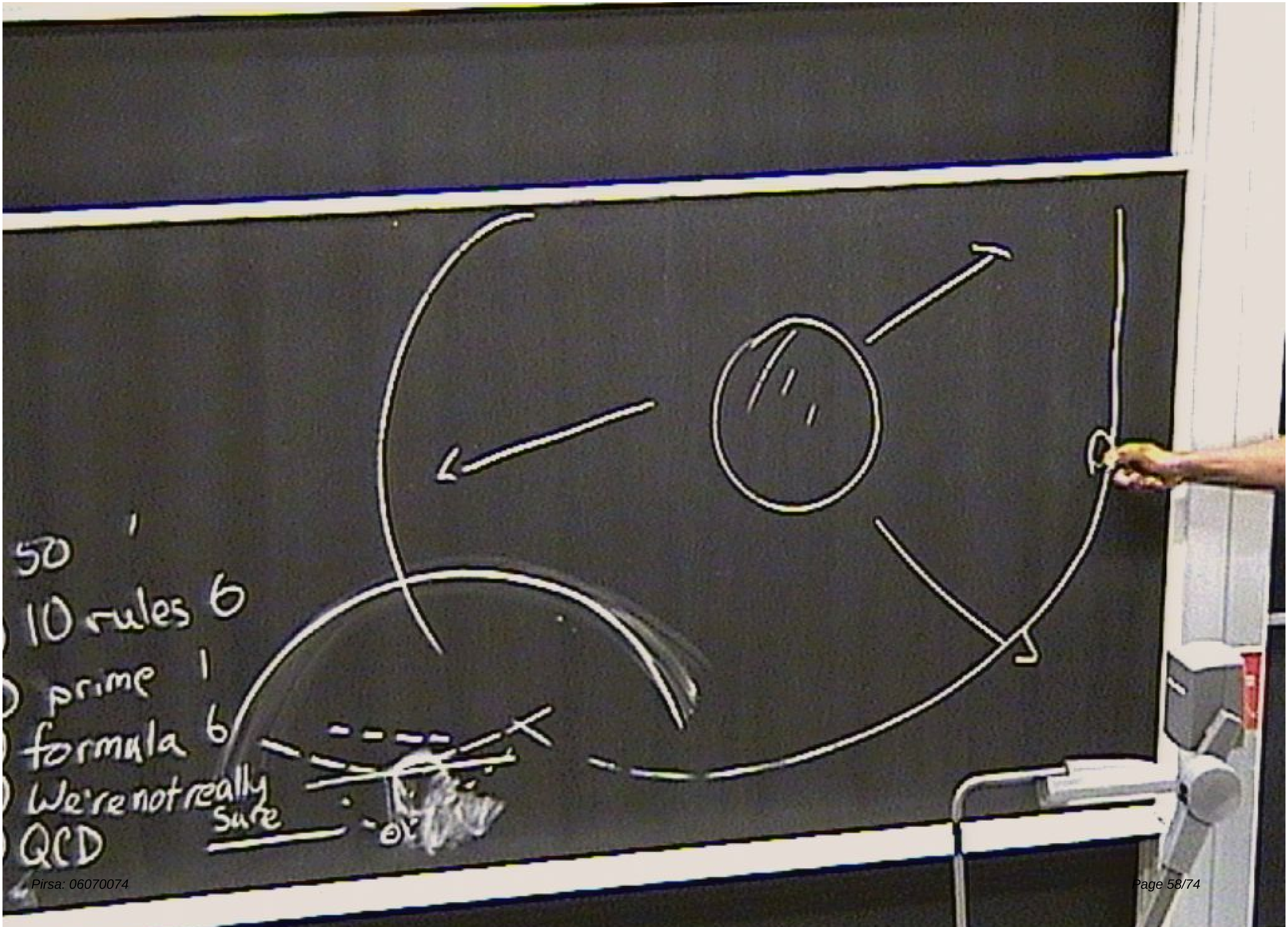
$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

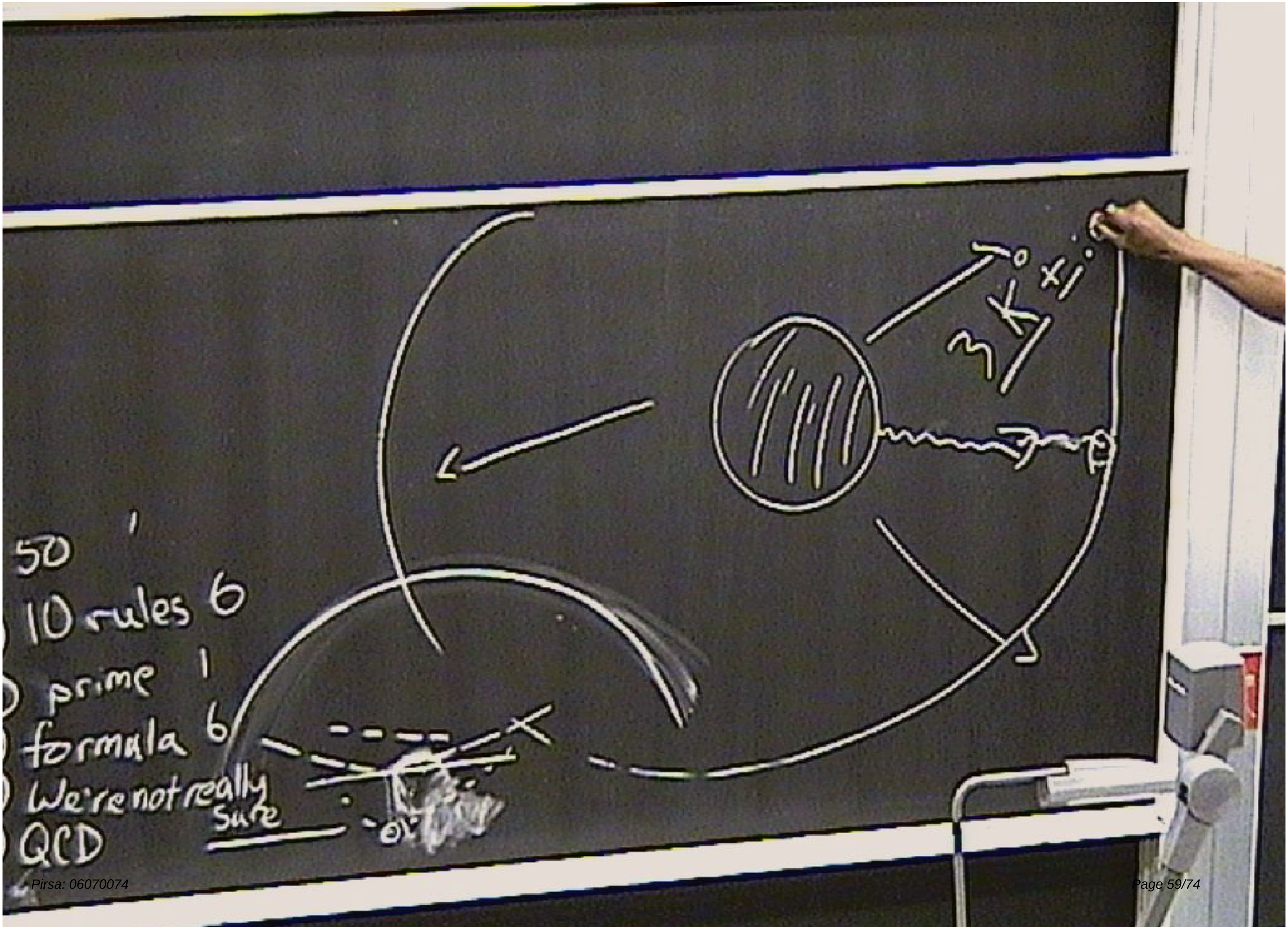

Microsoft PowerPoint - [quantum notes]
File Edit View Insert Format Tools Slide Show Window Help
Type a question for help
Click to add title
Click to add text
$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}$
$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=$
$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=$
Slide 30 of 72
Default Design
English (U.S.)



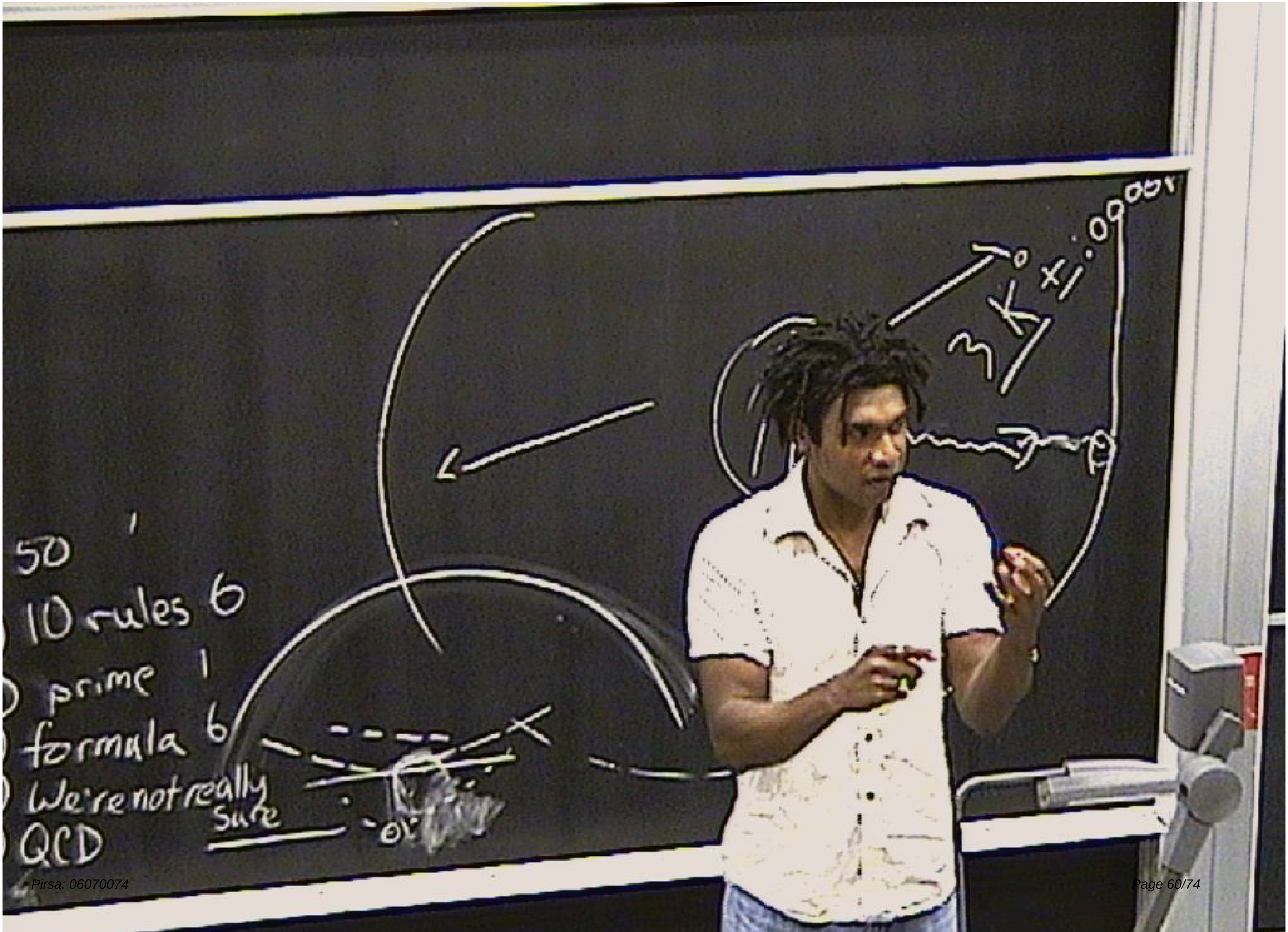

50
10 rules 6
prime 1
formula 6
We're not really sure
QCD



50 1
10 rules 6
prime 1
formula 6
We're not really sure
QCD



50
10 rules 6
prime 1
formula 6
We're not really sure
QCD
3K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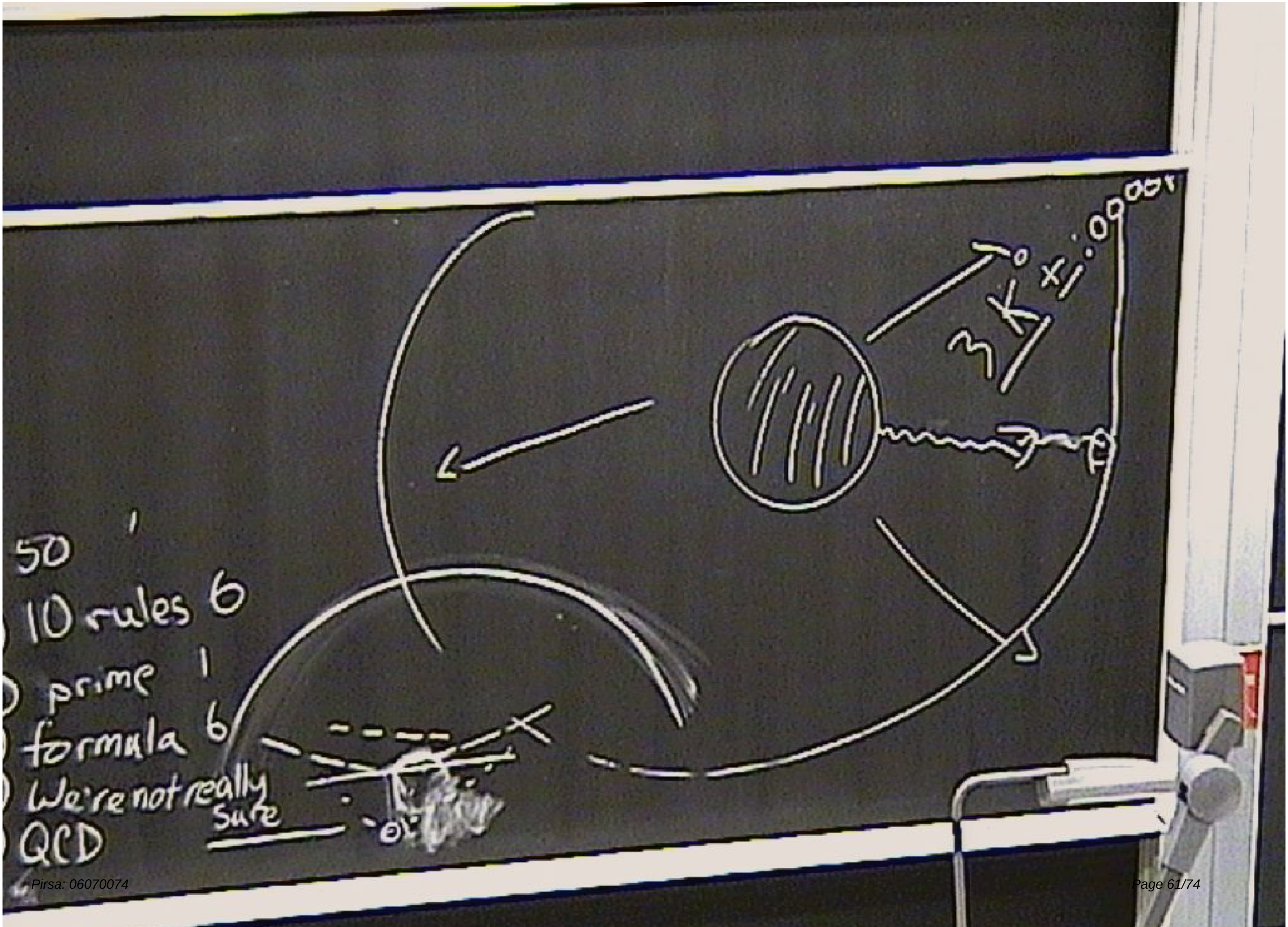
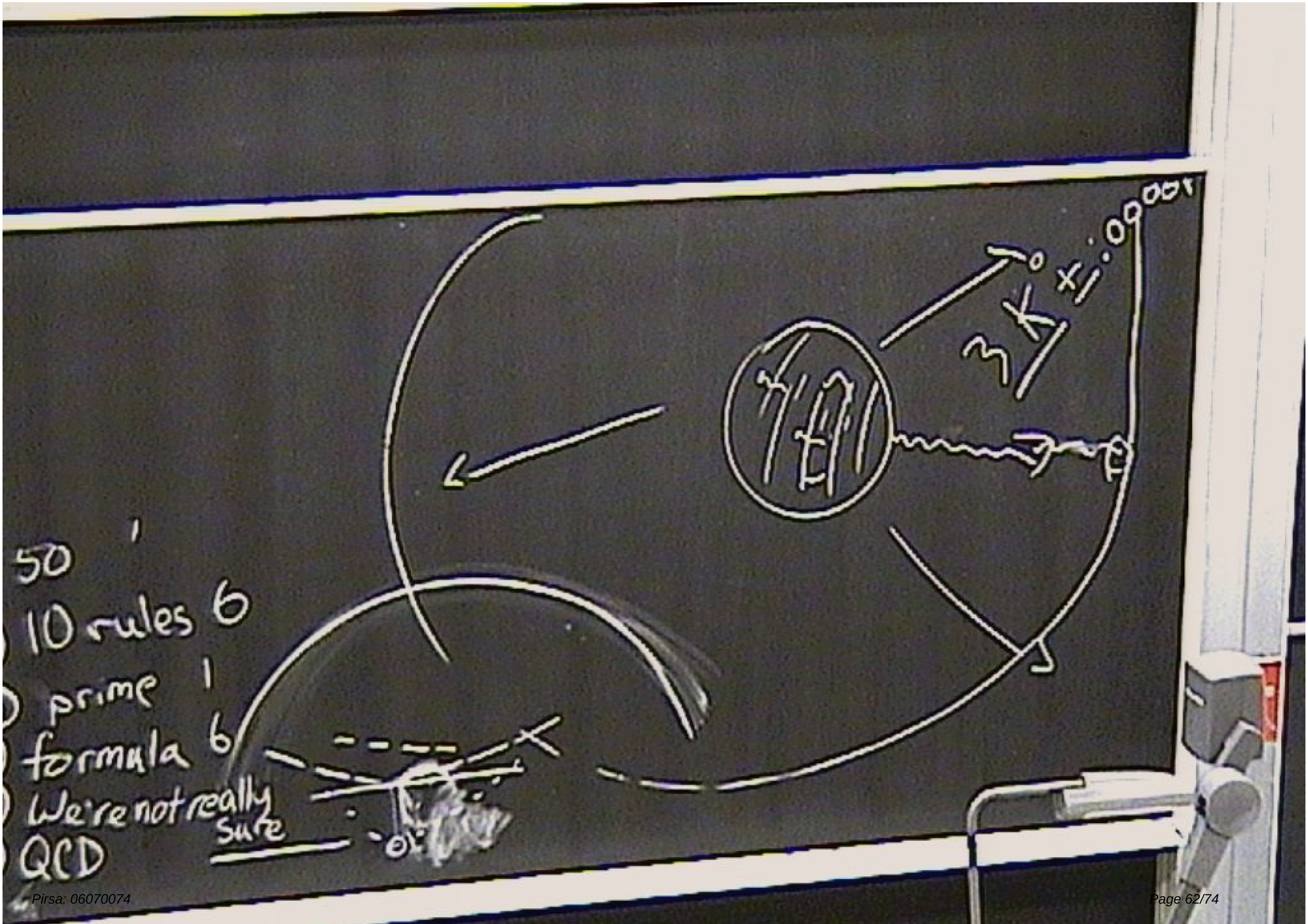
10 rules 6
prime 1
formula 6
We're not really sure
QCD
3K
.09008


50
10 rules 6
prime 1
formula 6
We're not really sure
QCD
3K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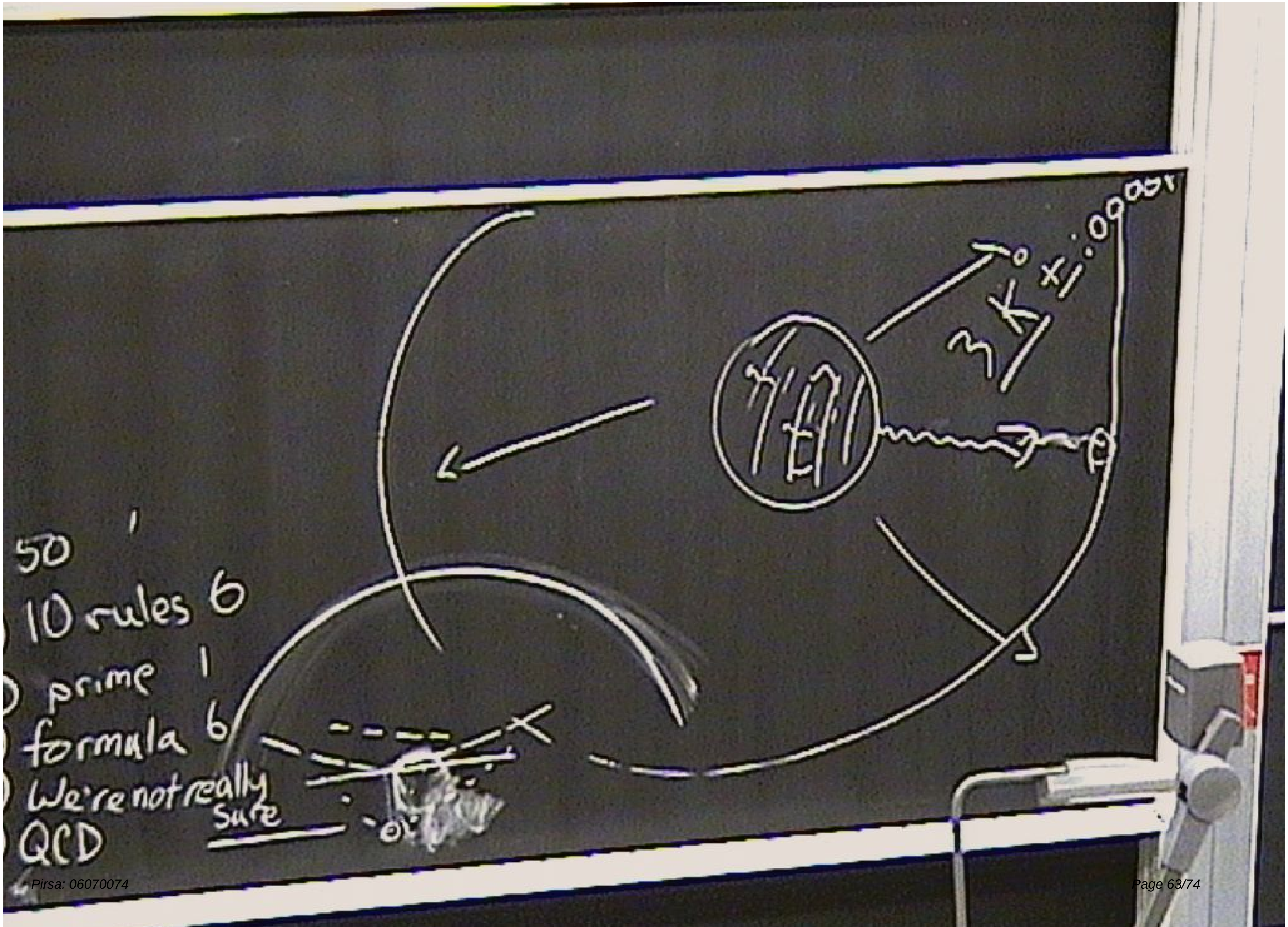
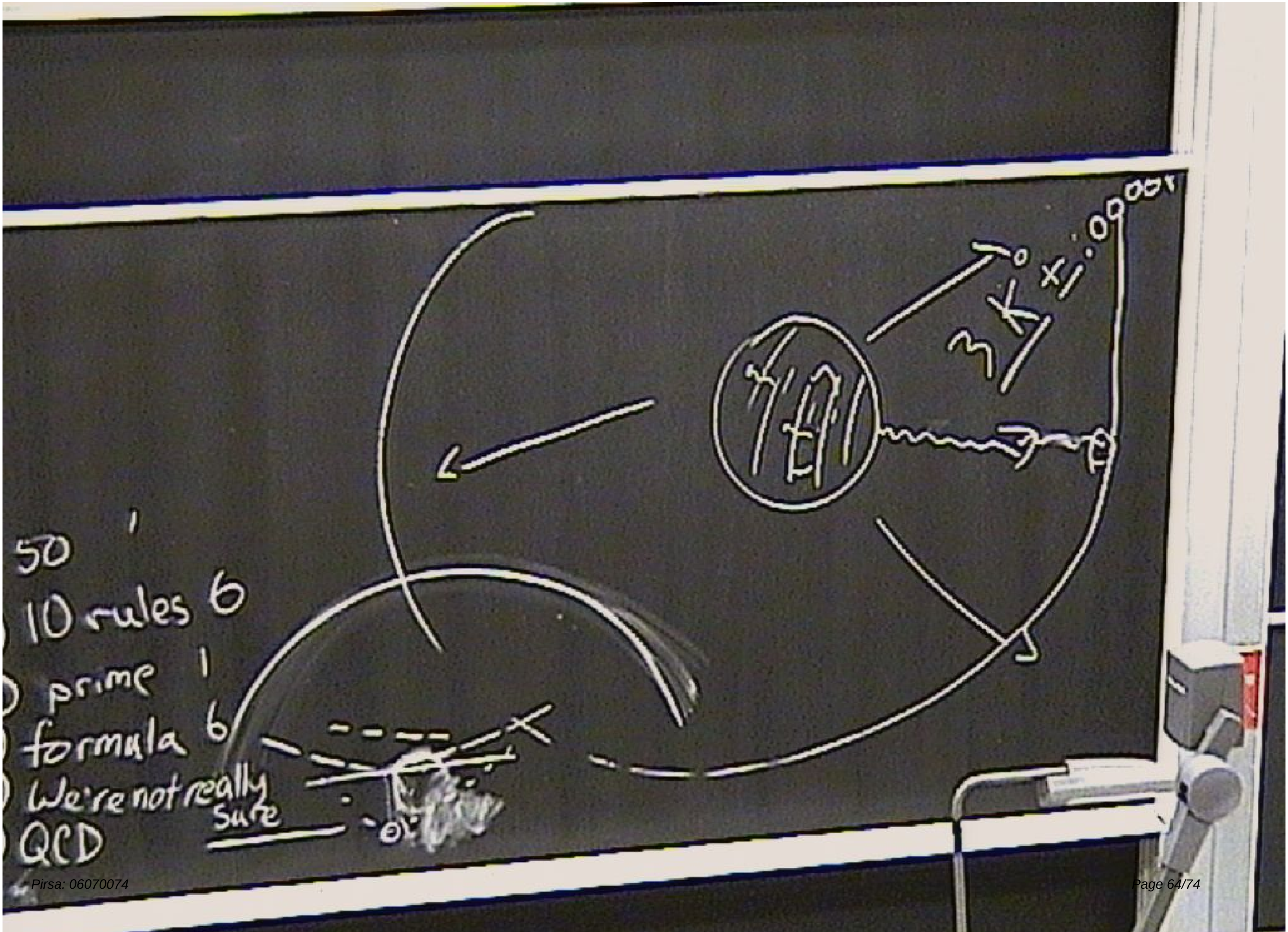
10 rules 6
prime 1
formula 6
We're not really sure
QCD
3K


50
10 rules 6
prime 1
formula 6
We're not really sure
QCD
3K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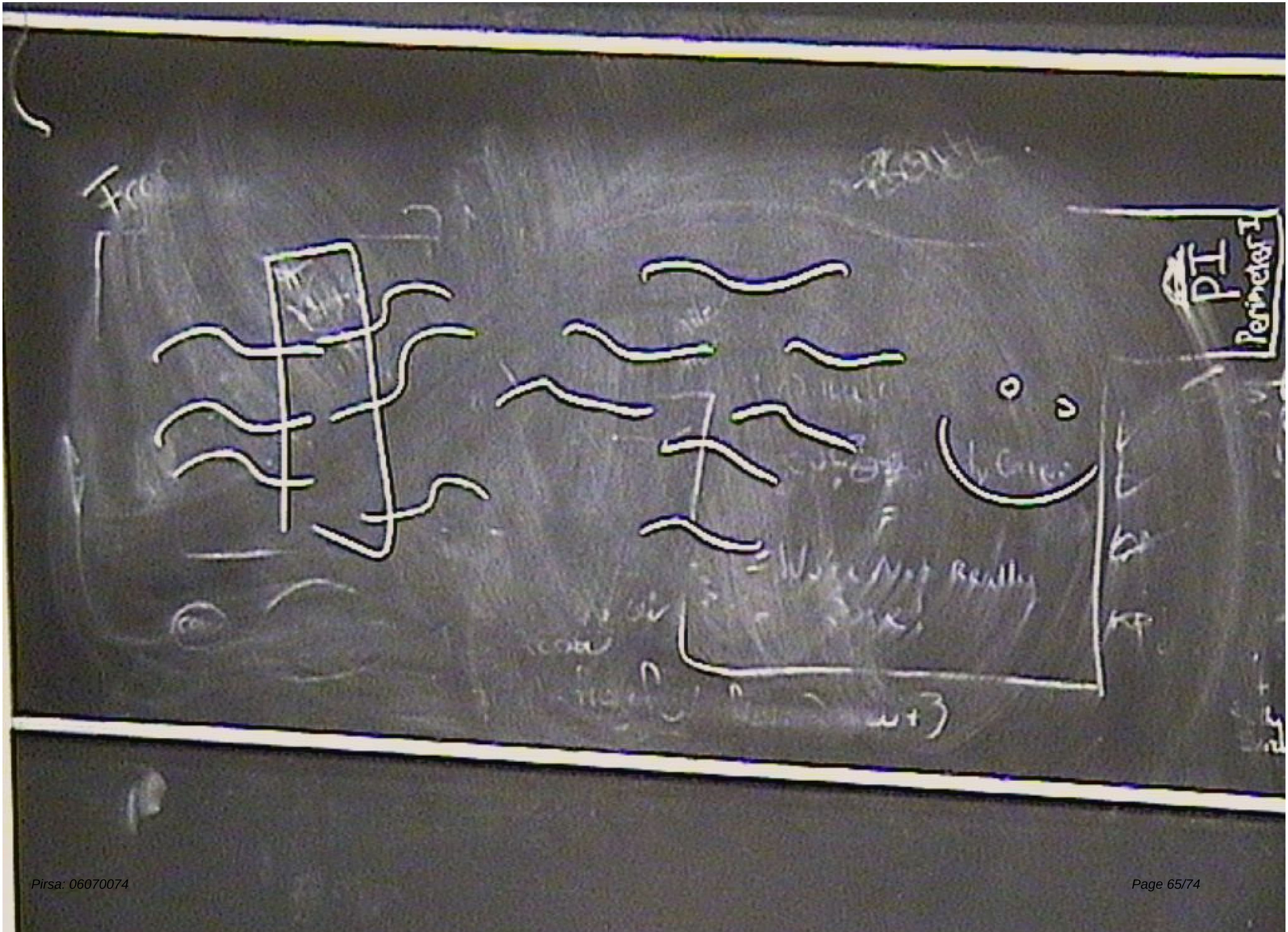
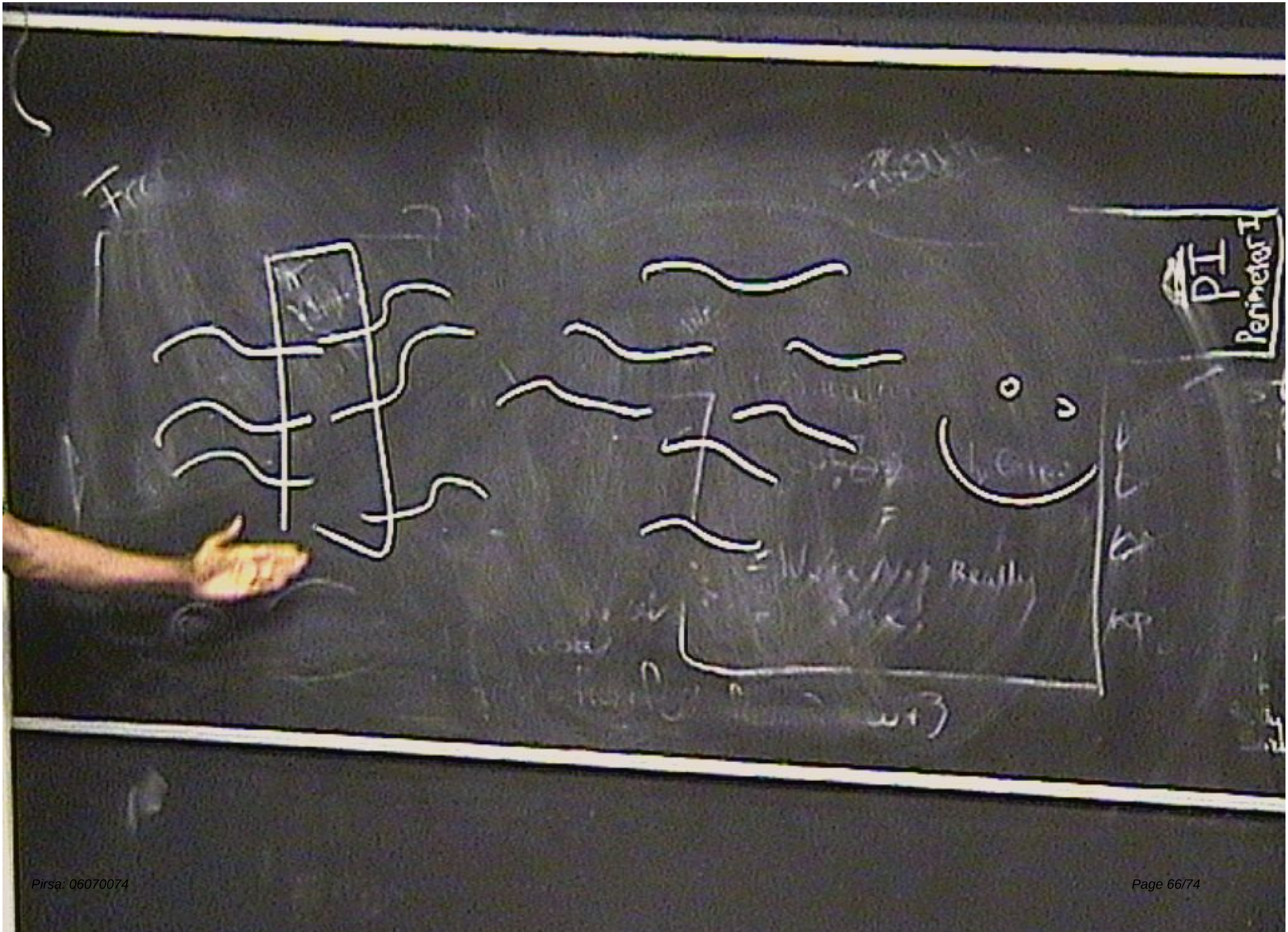
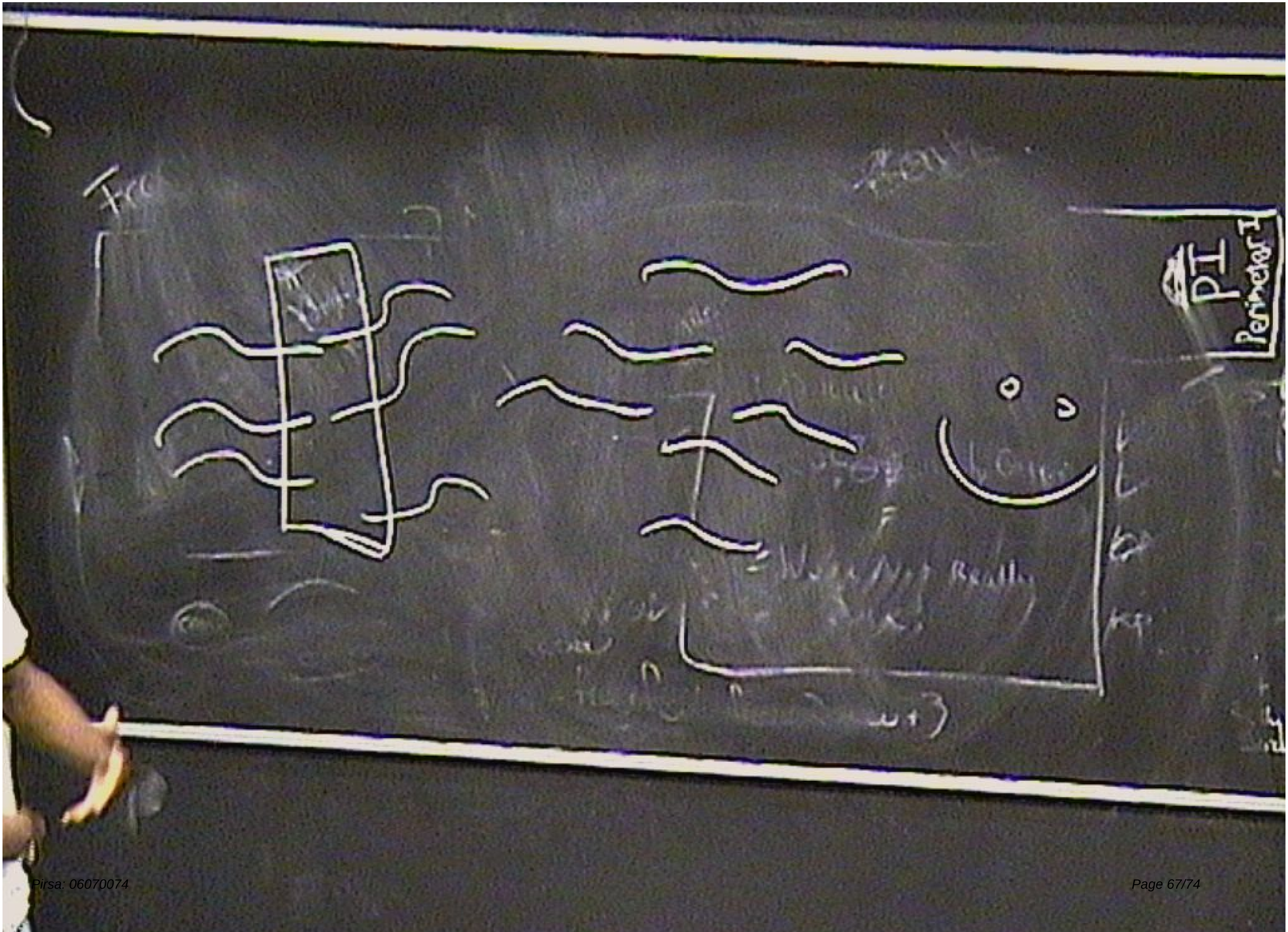
10 rules 6
prime 1
formula 6
We're not really sure
QCD
3K



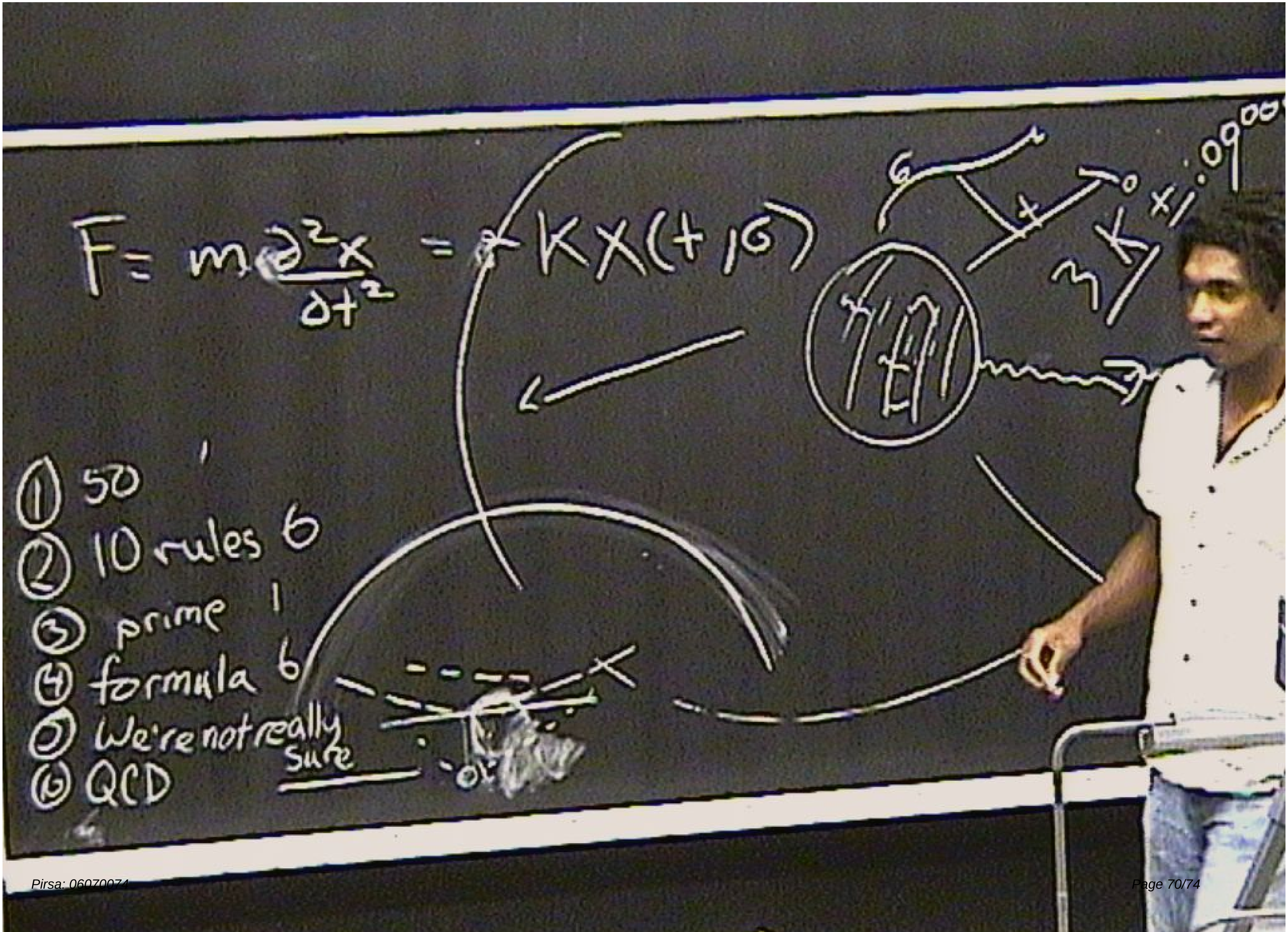
PI
Perimeter I
Not Really

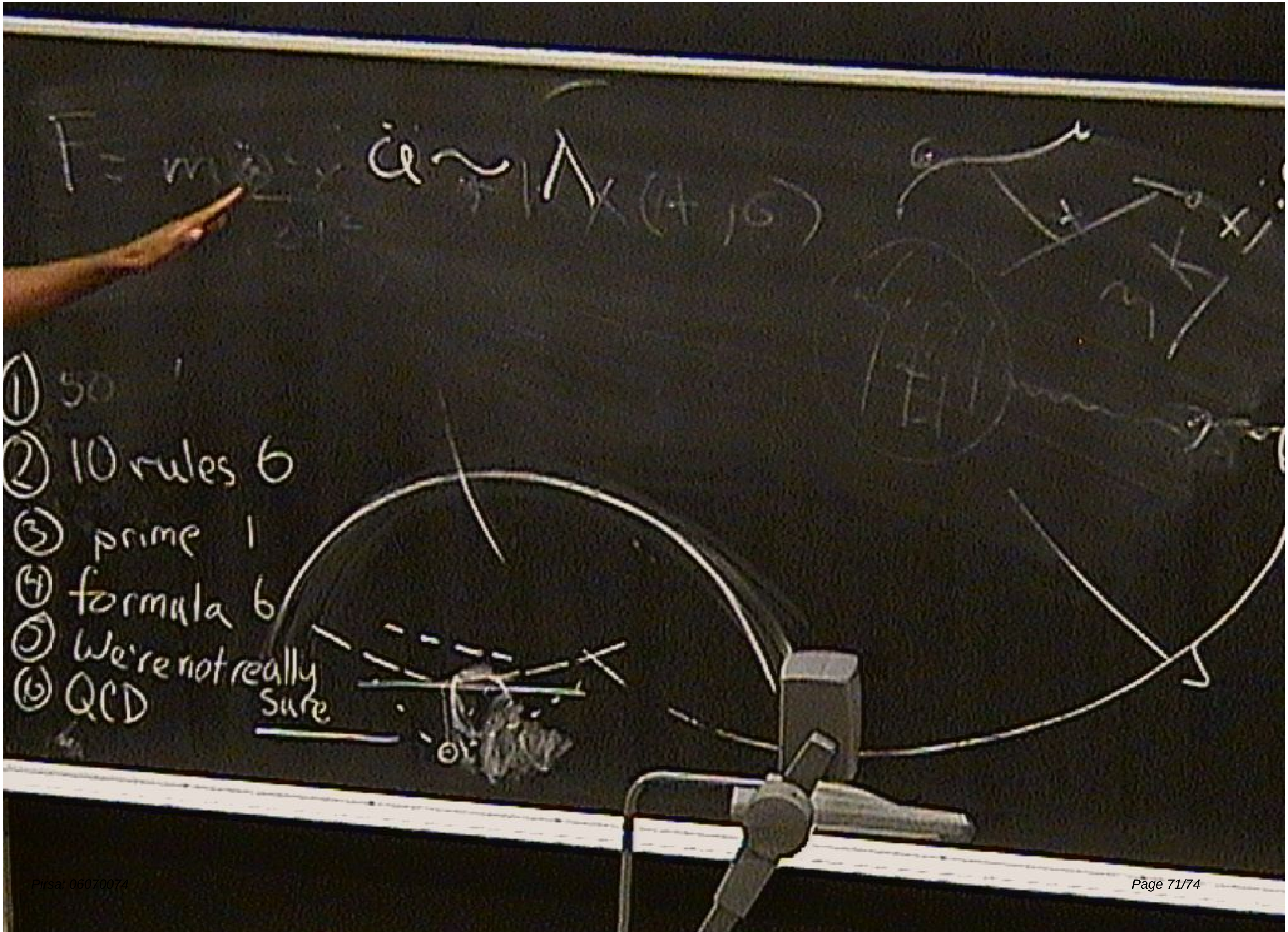

PI


PI

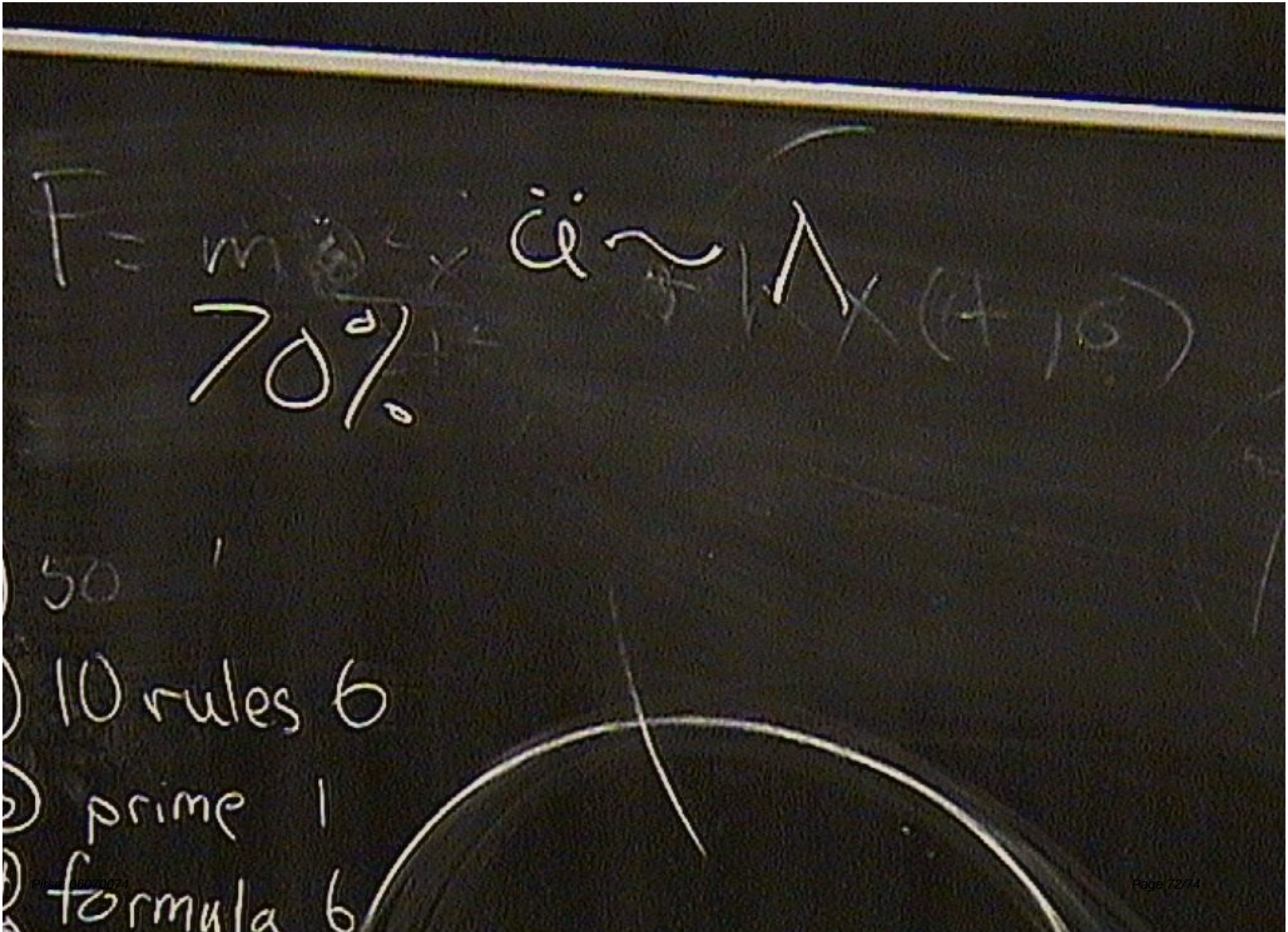
$$F = m\frac{\partial^2 x}{\partial t^2} = 0$$

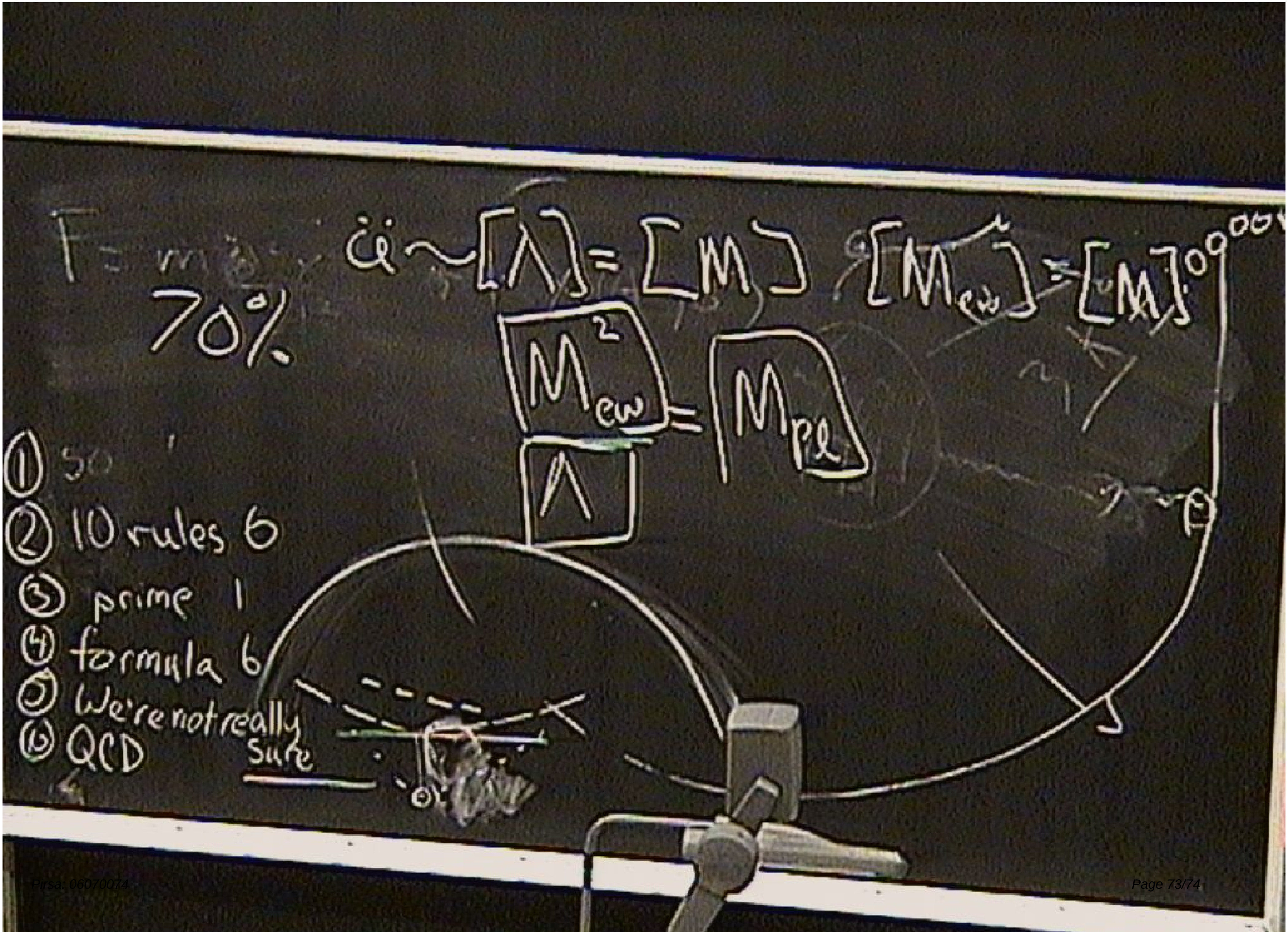
$$F = m\frac{\partial^2 x}{\partial t^2} = 0$$

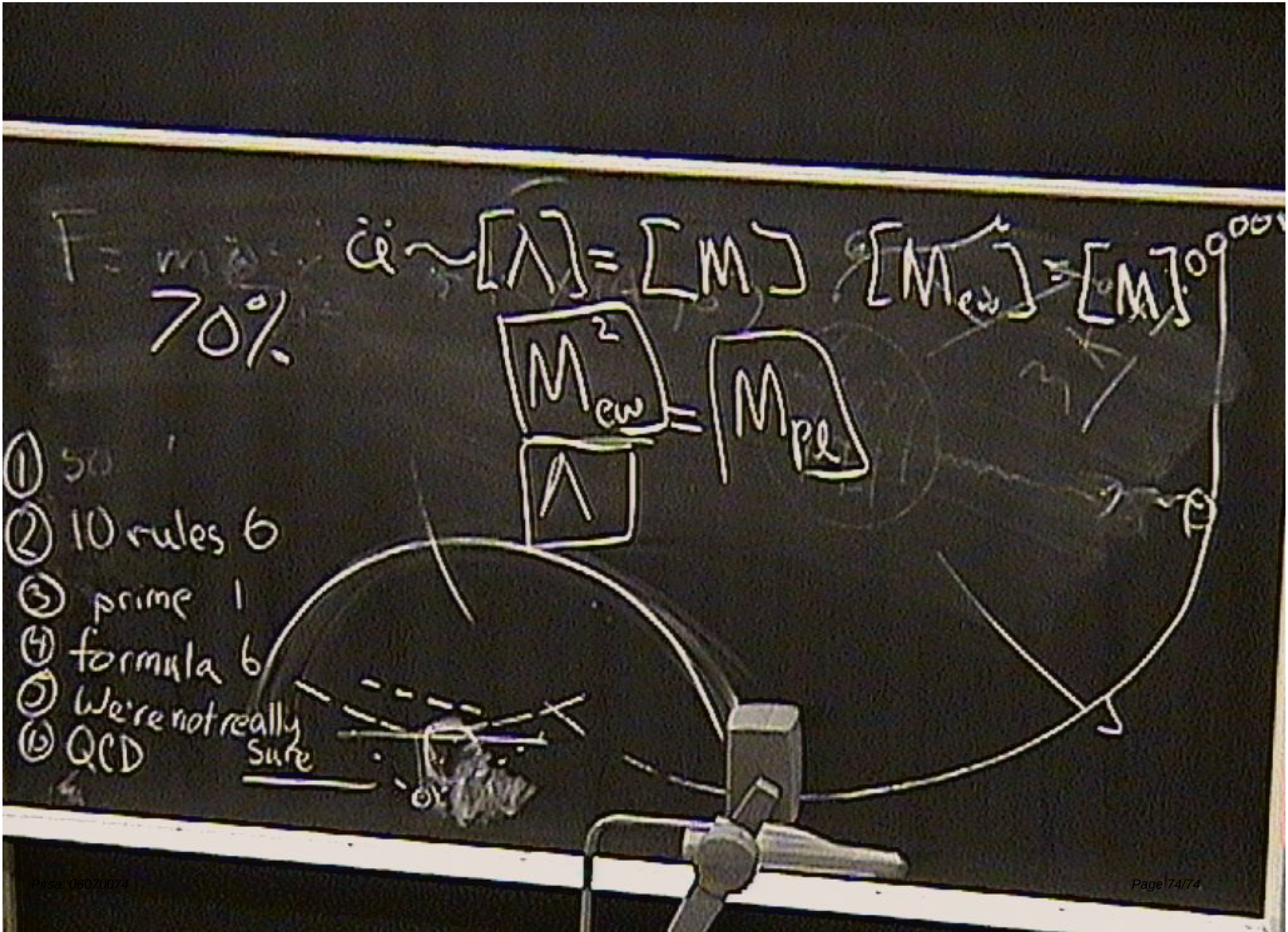

F= m∂²x/∂t² = -KX(t,σ)
① 50
② 10 rules 6
③ prime 1
④ formula 6
⑤ We're not really sure
⑥ QCD



F = ma
① SO
② 10 rules 6
③ prime 1
④ formula 6
⑤ We're not really sure
⑥ QCD


70%
50
10 rules 6
prime 1
formula 6


70%
[Λ] = [M]
M_ew
Λ
M_pl
① so
② 10 rules 6
③ prime 1
④ formula 6
⑤ We're not really sure
⑥ QCD


70%
$\frac{M_{ew}^2}{\Lambda} = M_{pl}$
1 50
2 10 rules 6
3 prime 1
4 formula 6
5 We're not really sure
6 QCD