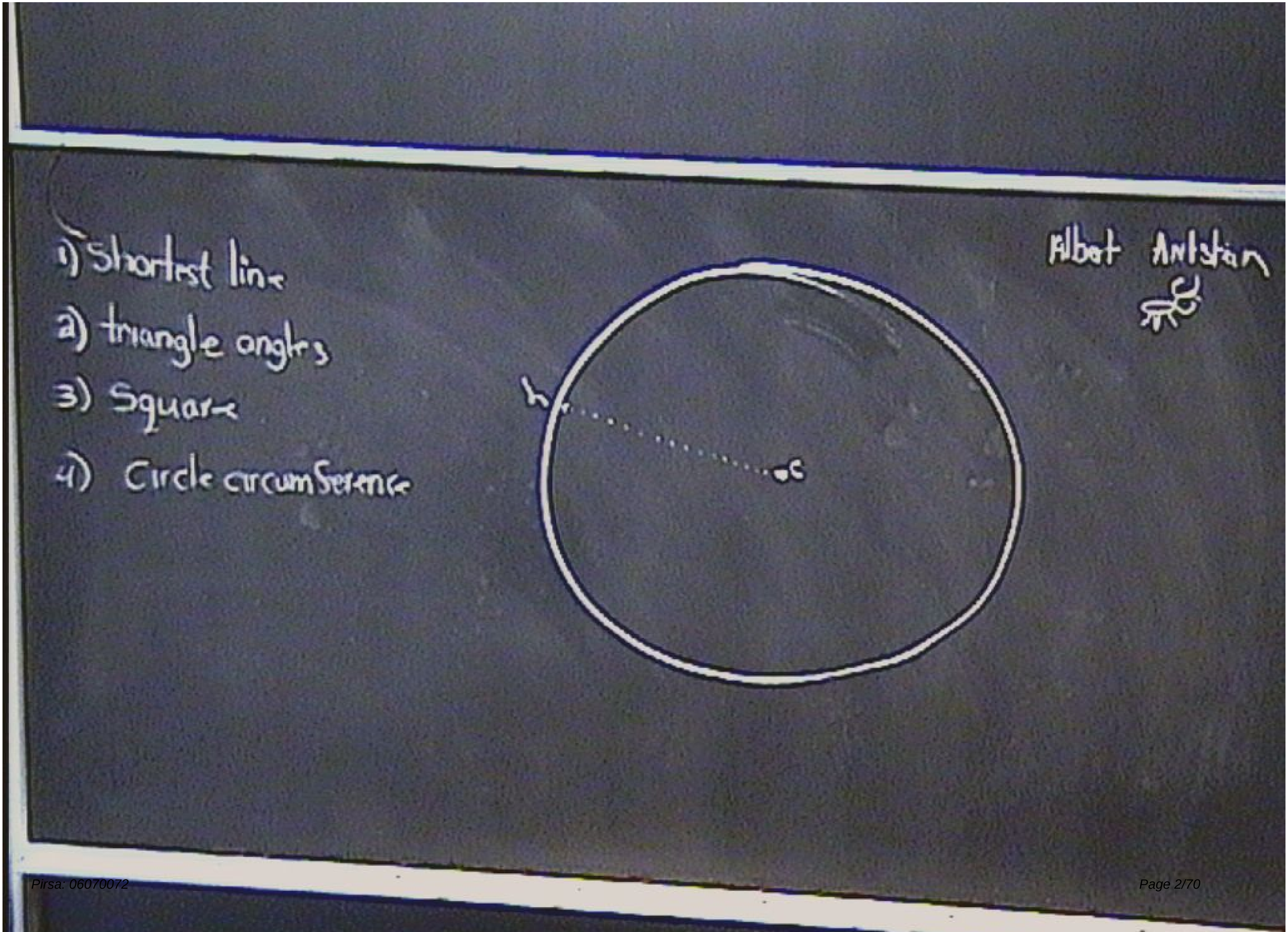
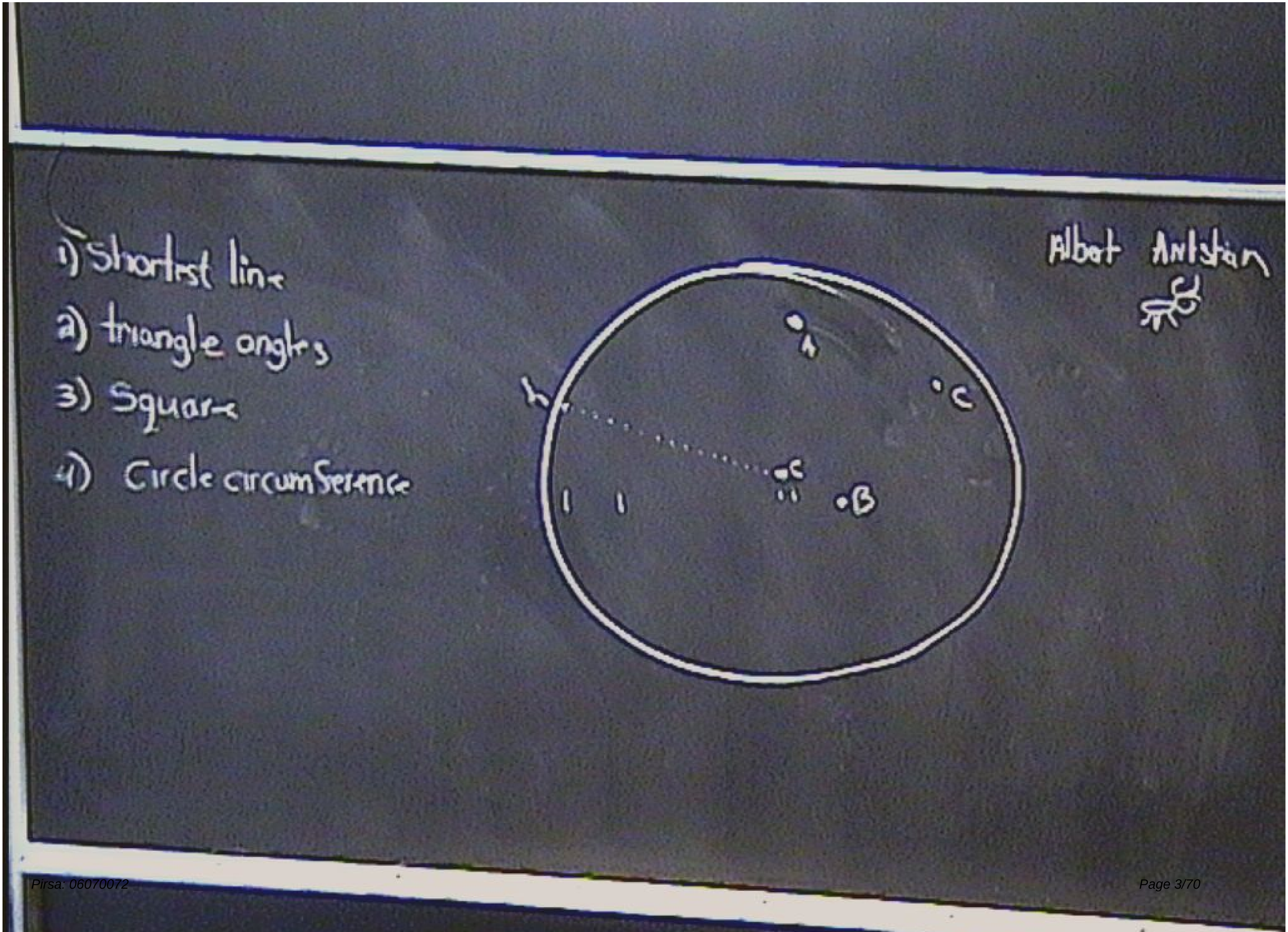
Title: The Weird World of Quantum Physics - Part 3

Date: Jul 25, 2006 03:45 PM

URL: http://pirsa.org/06070072

Abstract:


1) Shortest line
2) triangle angles
3) Square
4) Circle circumference
Albert Antstein
c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A
B
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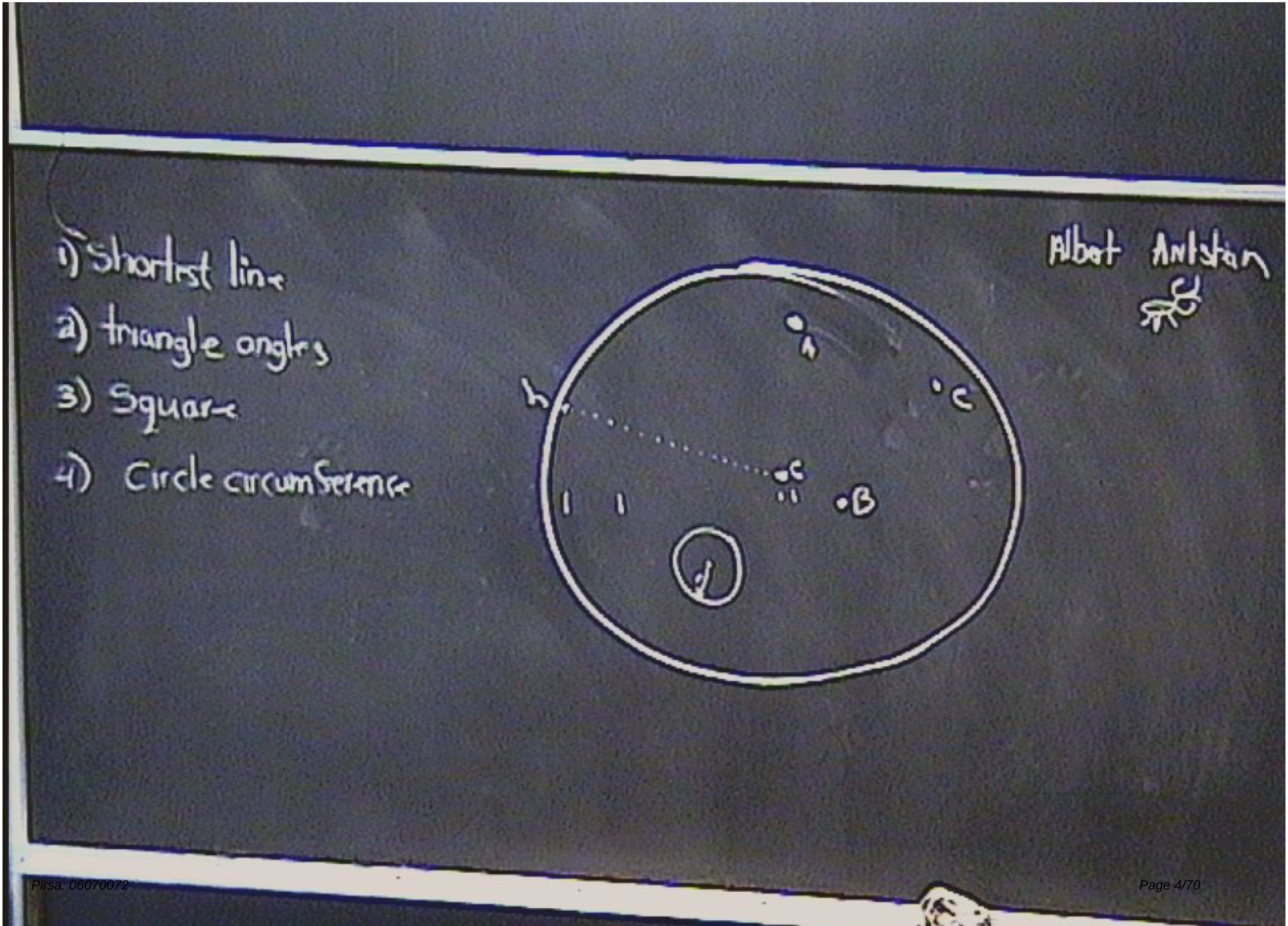
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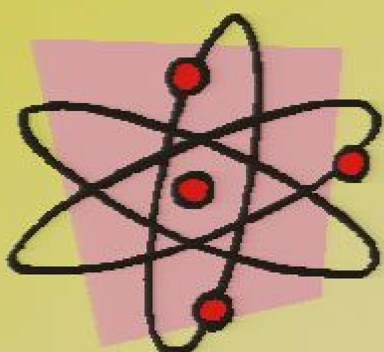
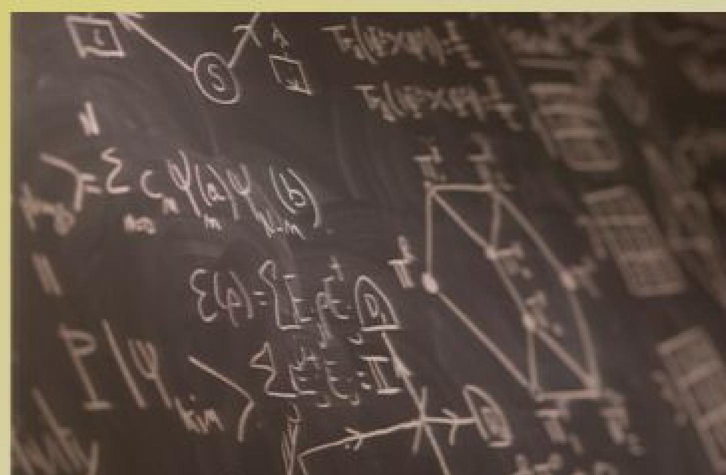
d

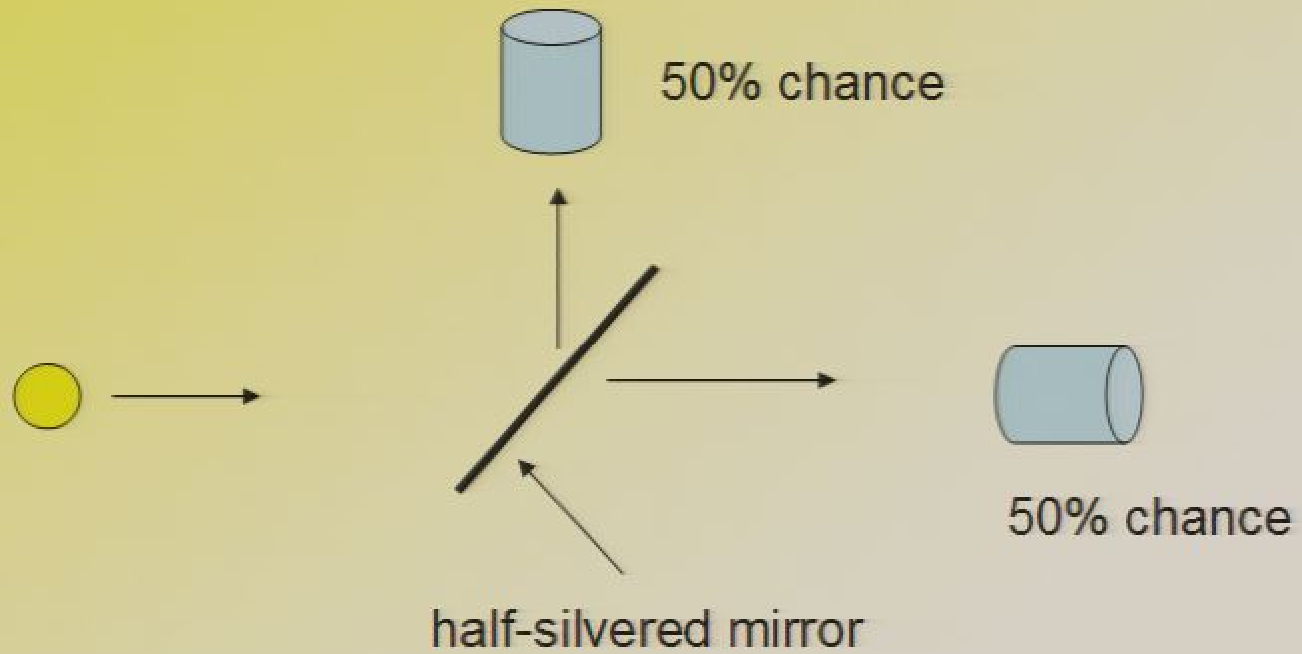
The Weird World of Quantum Physics



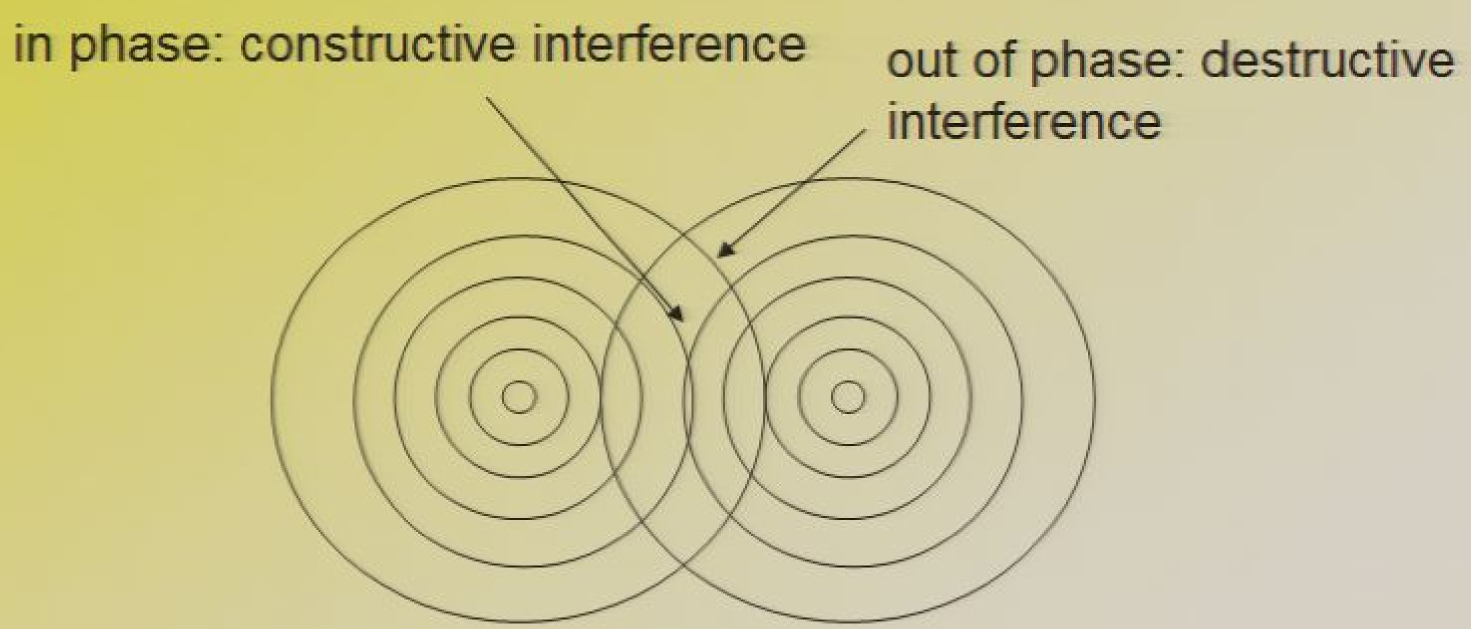


PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

A single photon incident on a half-silvered mirror.



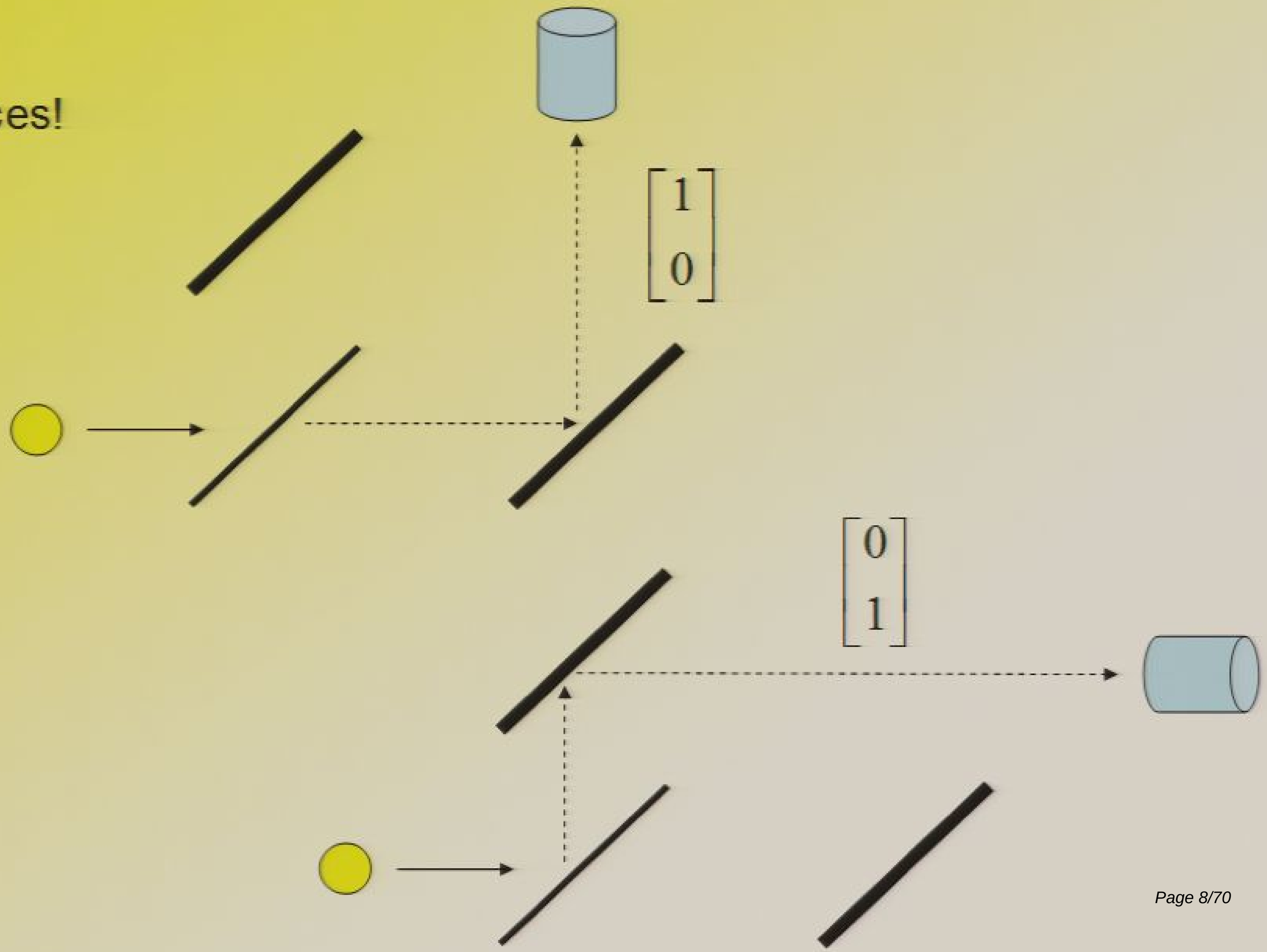
Compare with water waves



Ripples on a pond

Explaining things mathematically

- Matrices!



"Combination"/twin-possibility state

More accurately, $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$

Where did the factors of $\frac{1}{\sqrt{2}}$ and i come from?

- When a photon passes through a half-silvered mirror then there is a 50% probability of us *measuring it* to be reflected and a 50% probability of us *measuring it* to be transmitted.

- Therefore, we need some mathematical way to represent these probabilities.

Probability(path 1) = $|(1,1)$ element of matrix$|^2$

Probability(path 2) = $|(2,1)$ element of matrix$|^2$

Student Activity

- 1. Verify that this rule works for the example on the previous page.
  i.e. that
  |(1,1) element of matrix|^2 = ½

  and

  |(2,1) element of matrix|^2 = ½

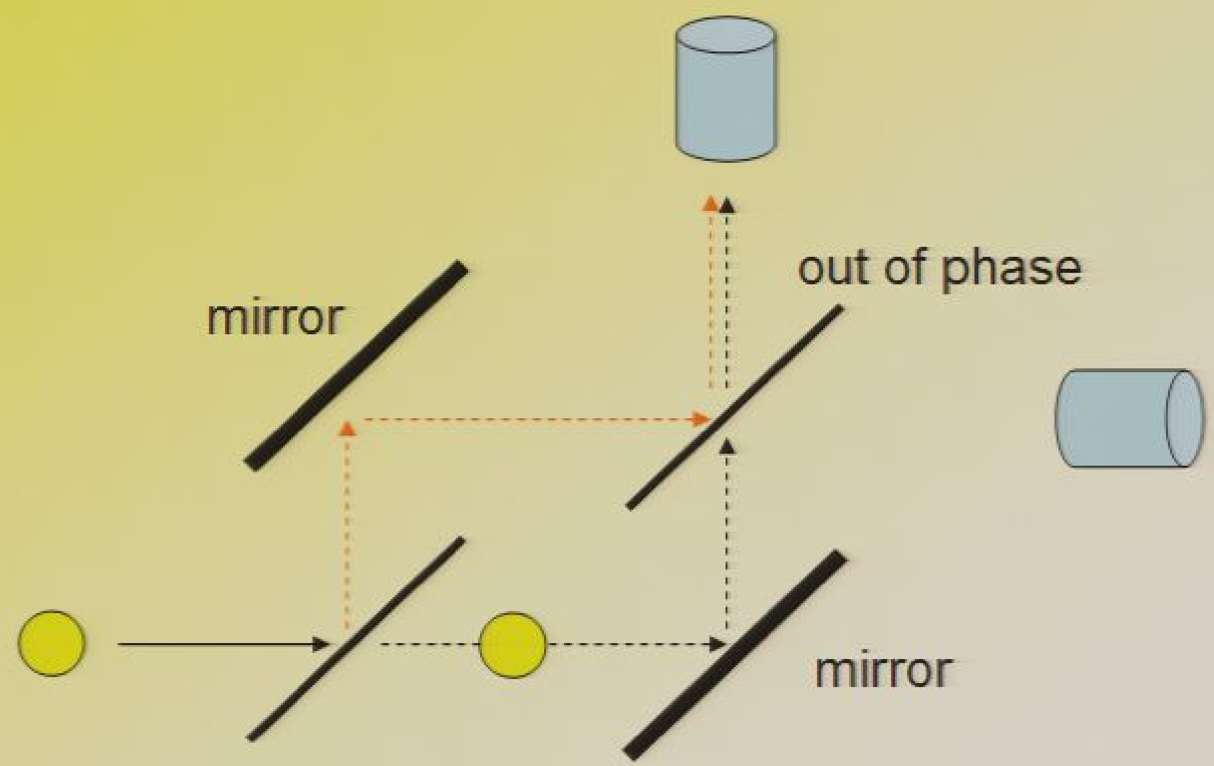
  2. Consider the matrix $\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} e^{i\pi/4} \end{bmatrix}$.

If it represents the state of affairs of a photon, what is the probability of finding the photon in paths 1 and 2?

- 3. State the condition that the (1,1) and (2,1) elements must, jointly, satisfy due to the fact that the probability of finding the photon somewhere is always one.

What about the 'i'?

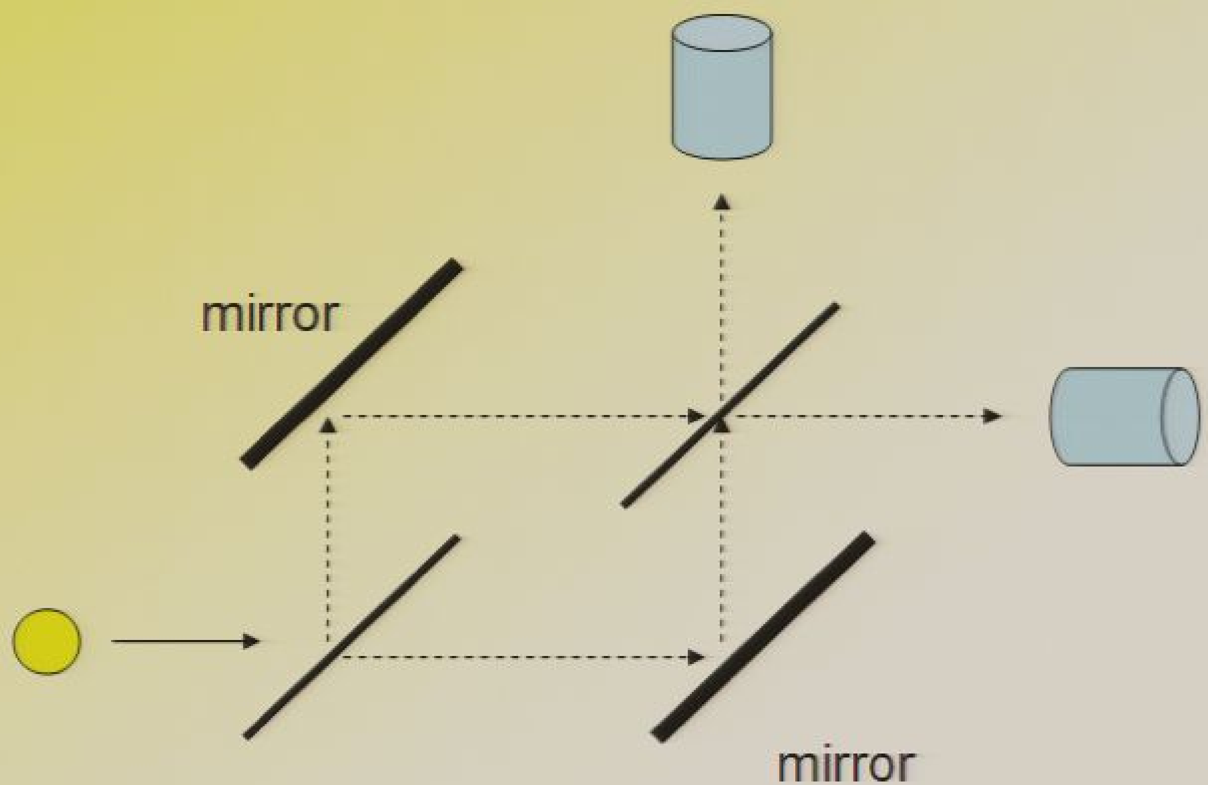
- Does not affect the probability as we take the magnitude.
- Affects how the two paths combine with each other when they overlap.
- i.e. constructive or destructive interference due to phase differences.
- We will see this in detail a little later …


mirror
out of phase
mirror

Two half-silvered mirrors

- STUDENT ACTIVITY:

  Determine the probabilities of finding a photon at each of the two detectors.



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Probability(path 1) = $|(1,1)$ element of matrix$|^2$

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- In Newtonian physics, these are only two possible states of affairs.
  Surely, each photon must either take the first path or the second one. These appear to be the only two options.

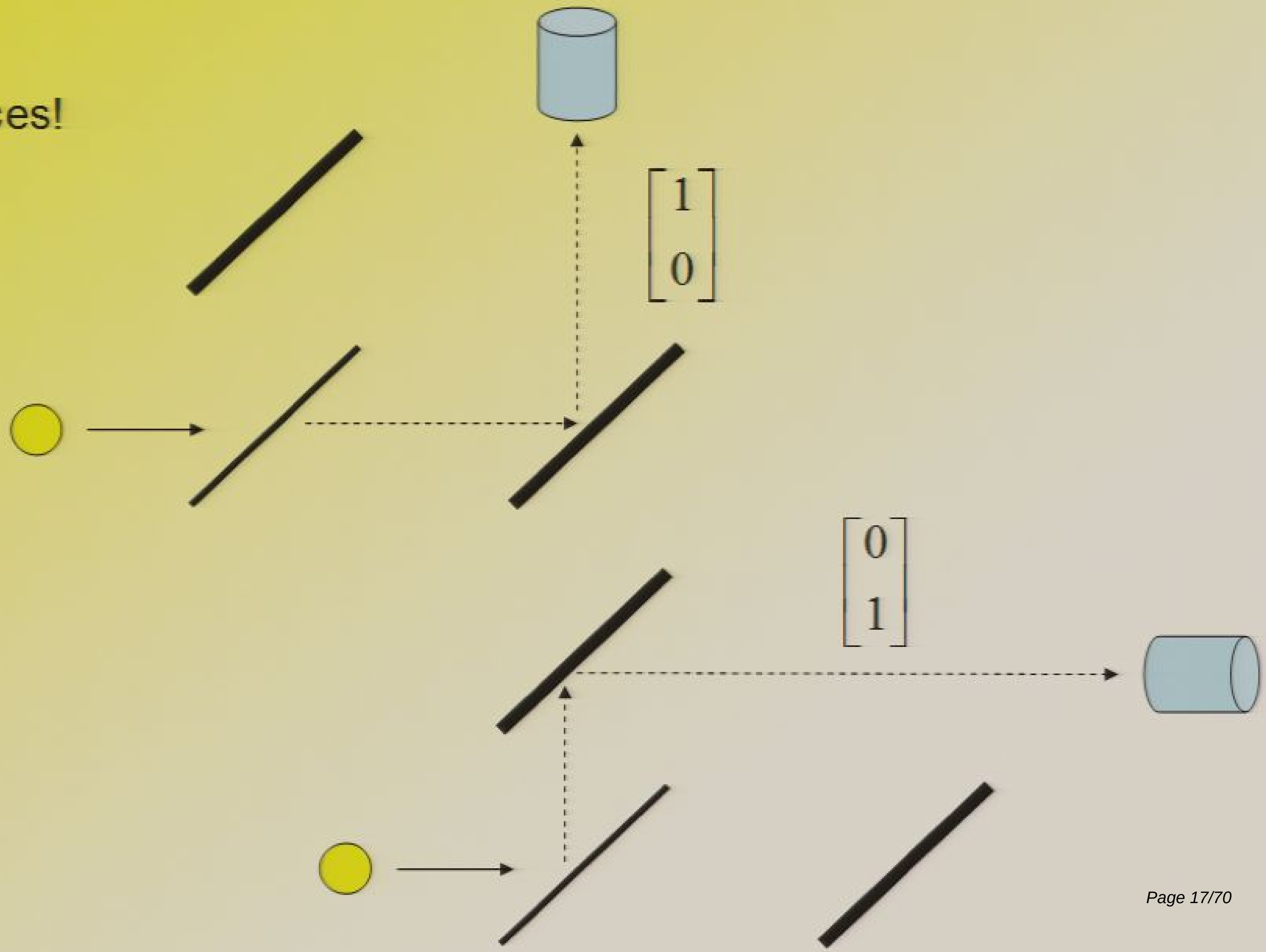
But, this is quantum physics. As we saw earlier, each photon can take both paths.

How do we respresent this mathematically? Roughly speaking,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Explaining things mathematically

- Matrices!



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- i.e. constructive or destructive interference due to phase differences.
- We will see this in detail a little later …

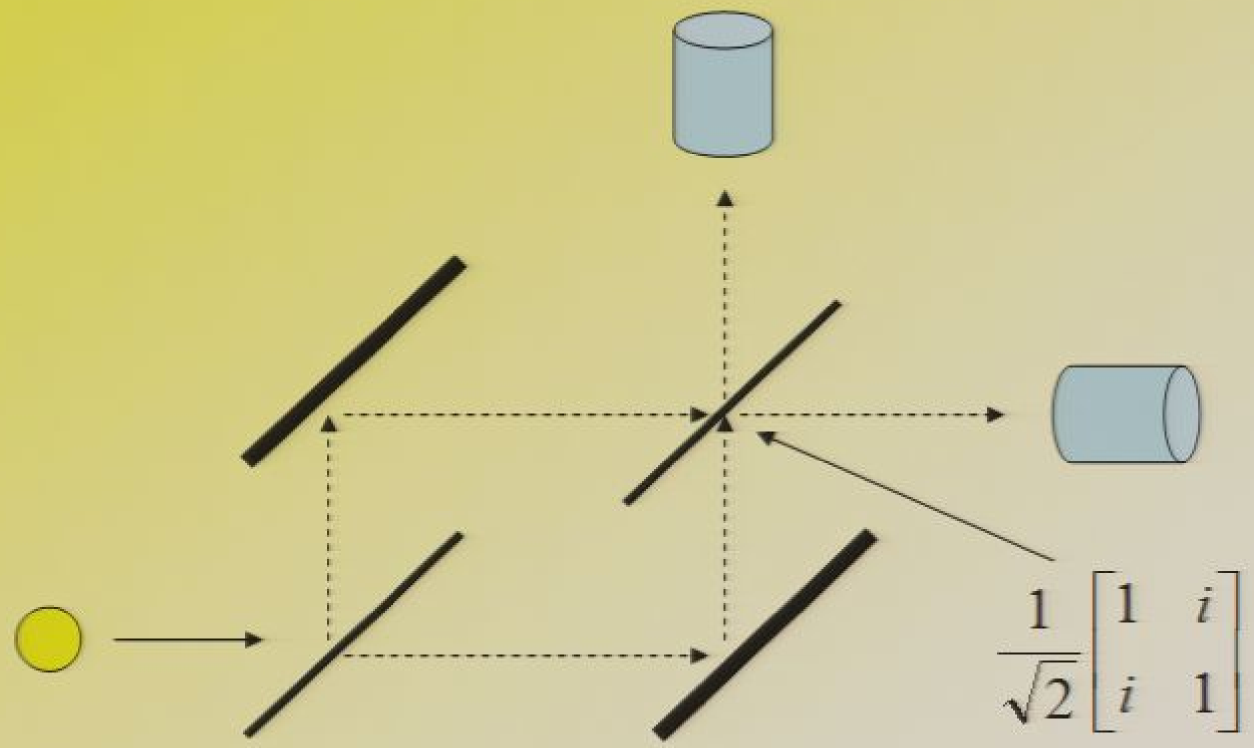
- What do we do with the second half-silvered mirror?

- Transforms from one 2x1 matrix to another.
  Eg.

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

- What sort of matrix does this?
- a) 2 x 2 matrix
  b) 3 x 3 matrix
  c) 1 x 2 matrix
  d) 2 x 1 matrix

Extra questions: i) Prove this. ii) Prove that none of the other three options work? iii) Is there any type of matrix that transforms one 2x1 matrix into another?



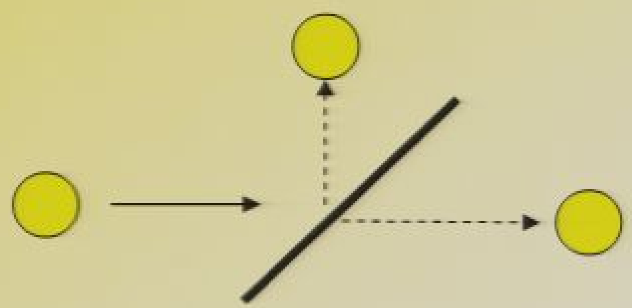
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=$$

Does the 2x2 matrix make sense?

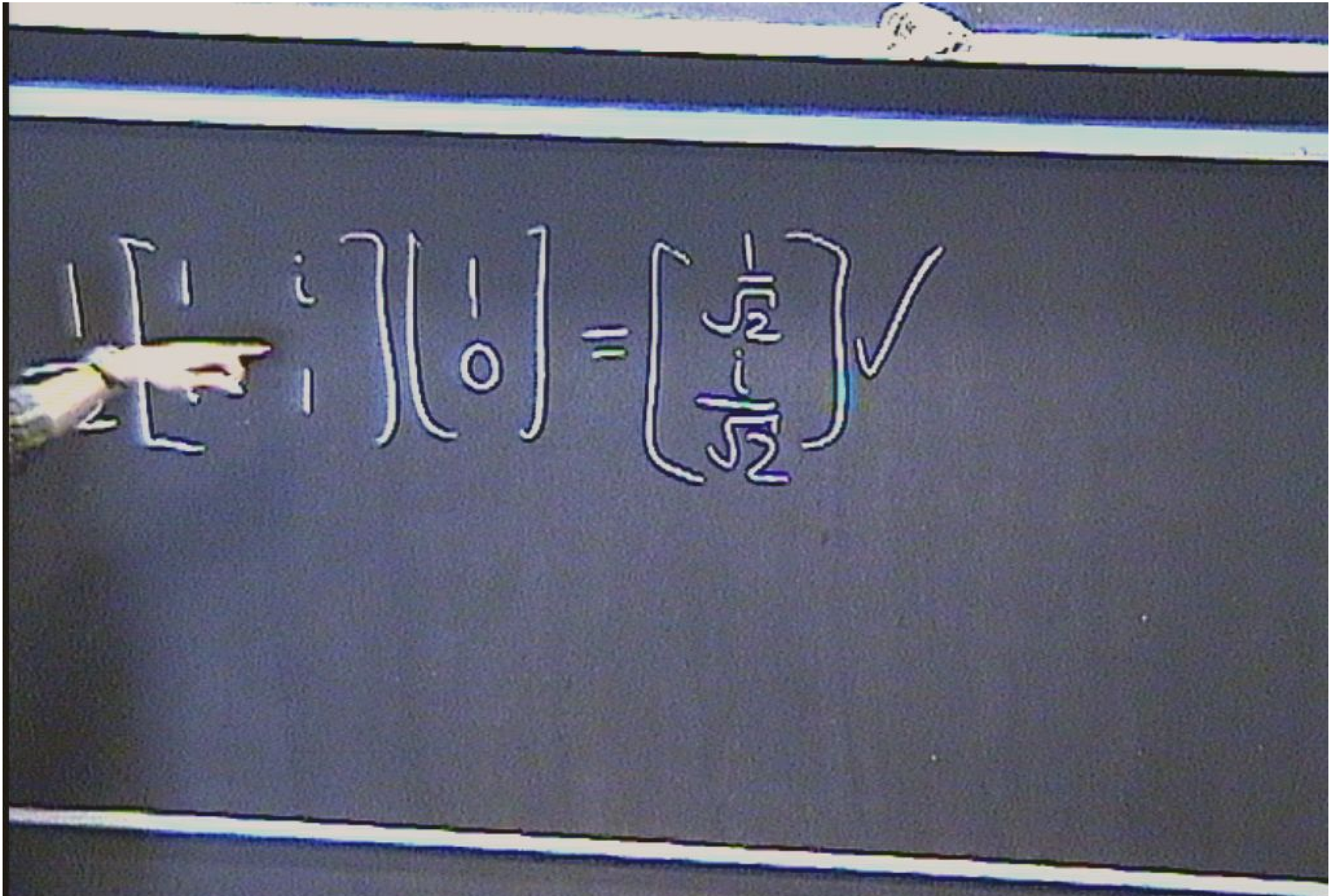
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

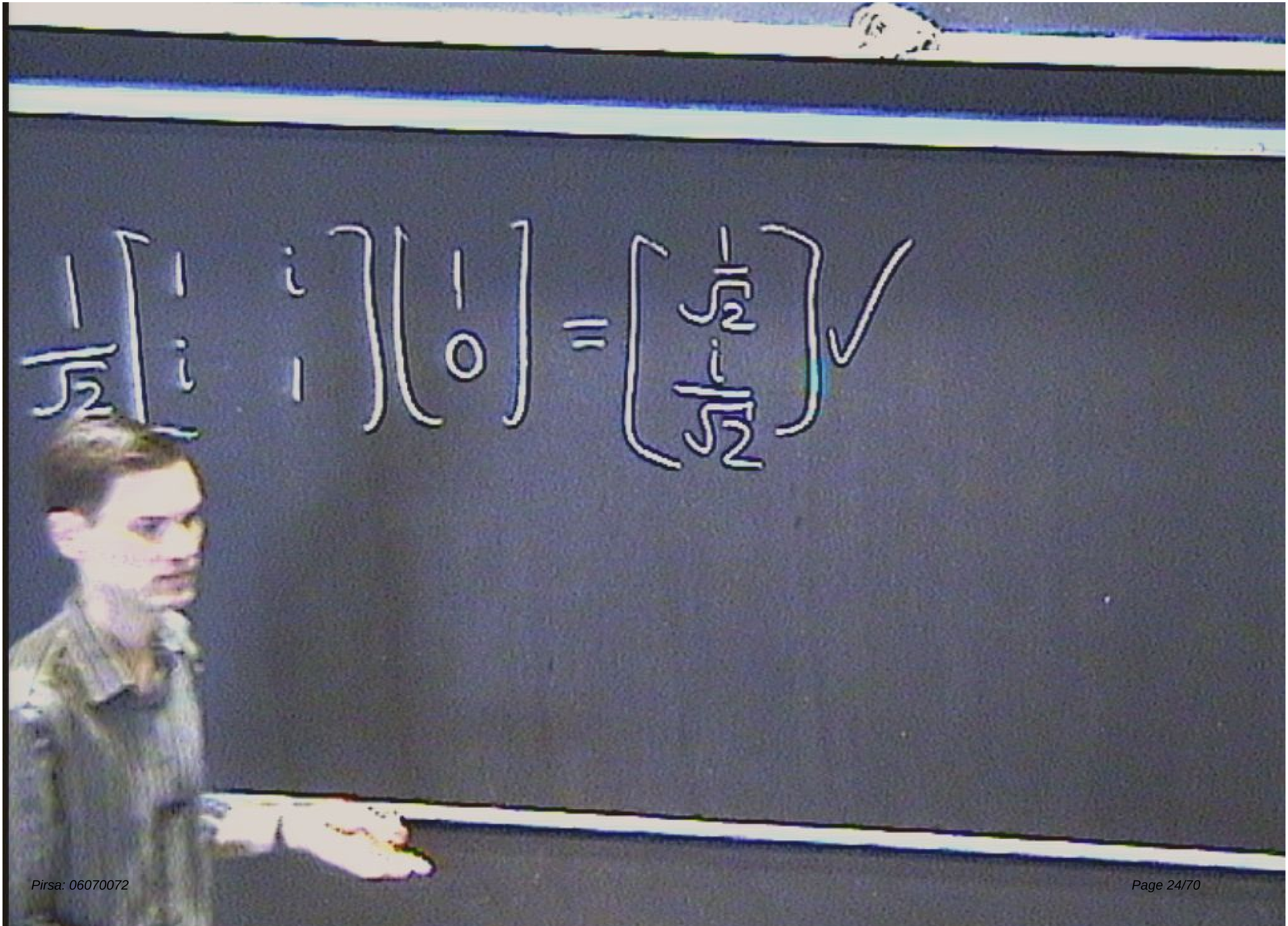
This is precisely the combination state of affairs we encountered before.



$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Another "state" in which the photon is taking both paths.


$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\end{bmatrix}$ ✓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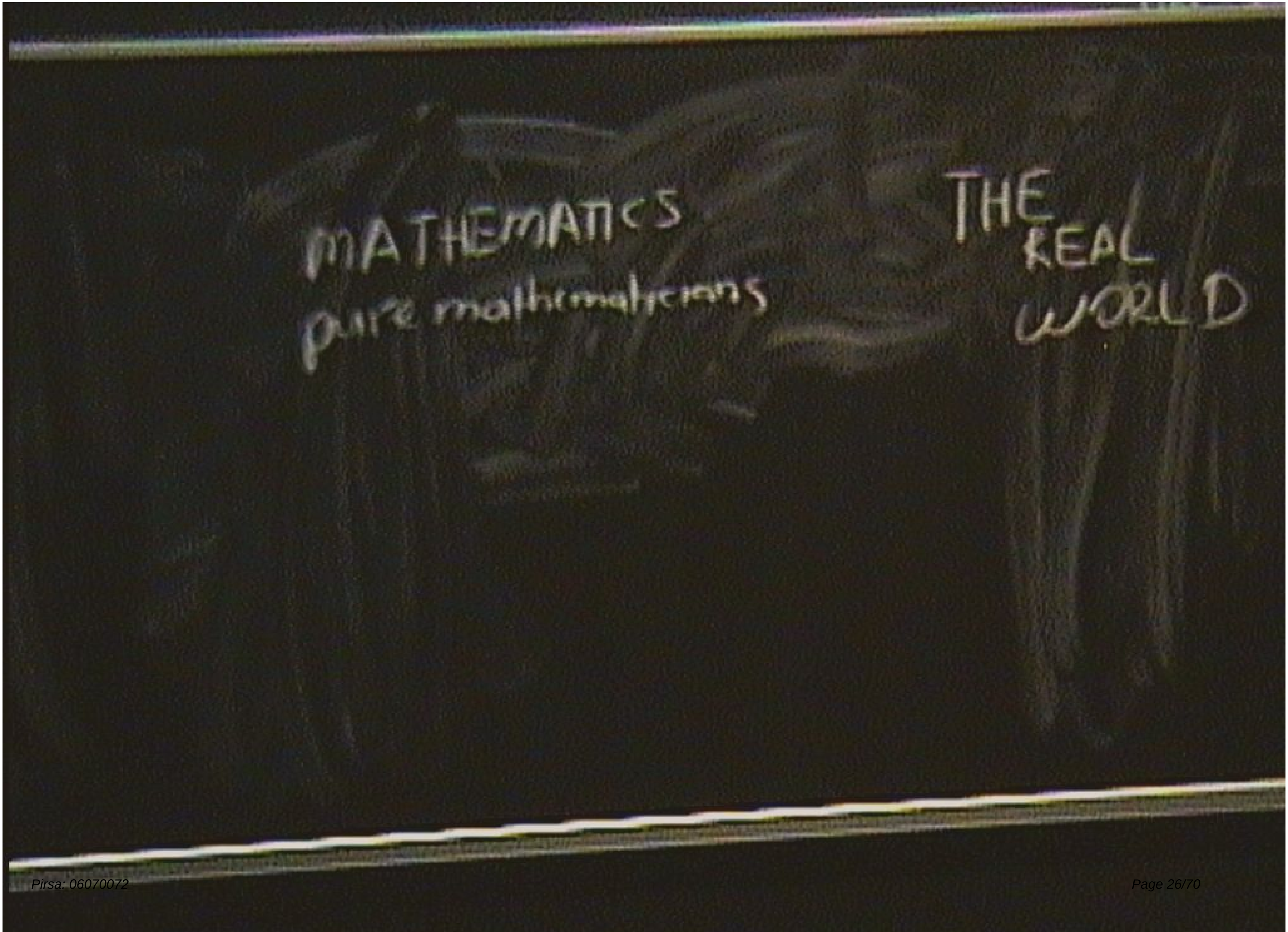
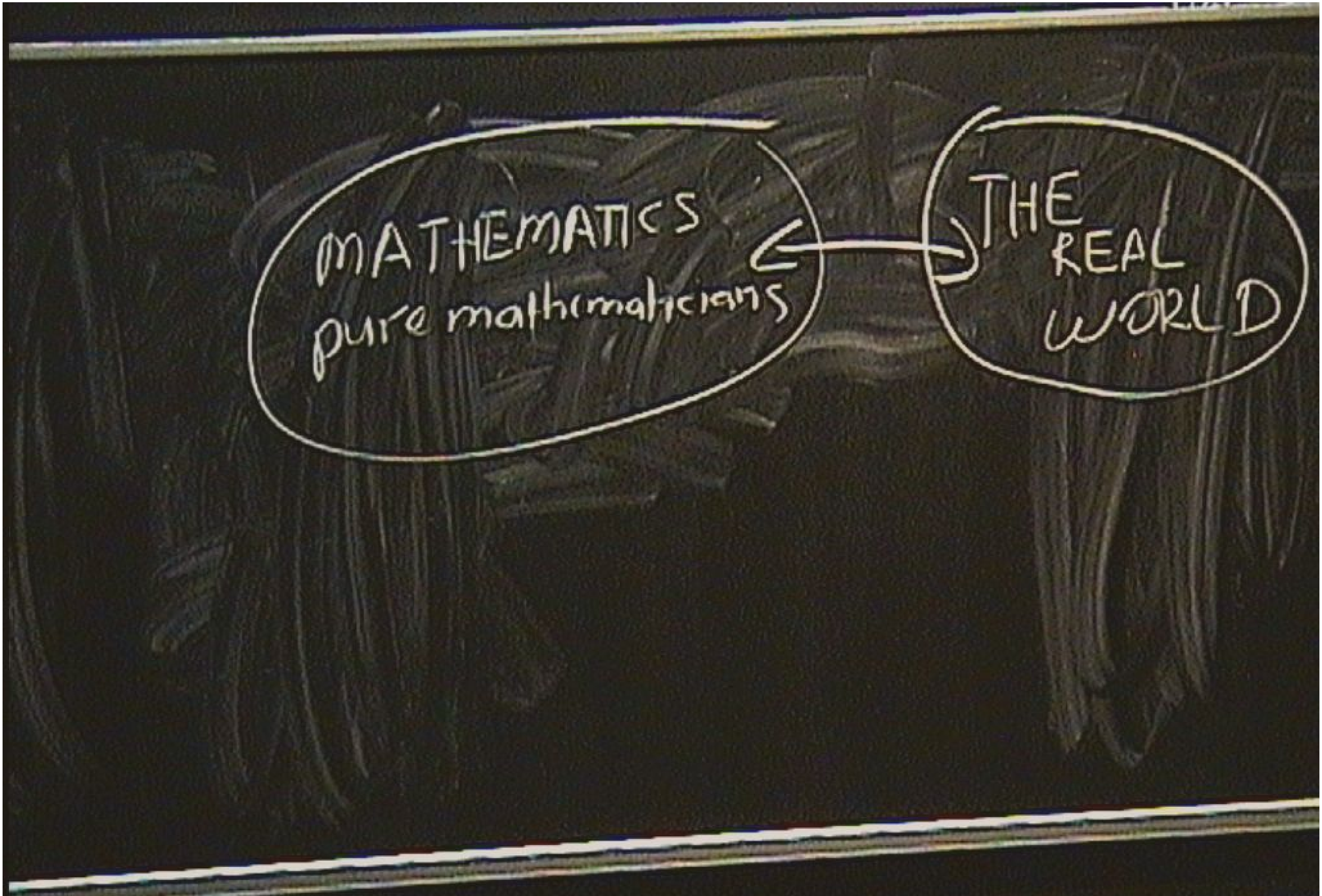
$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \checkmark$$


MATHEMATICS
pure mathematicians
THE REAL WORLD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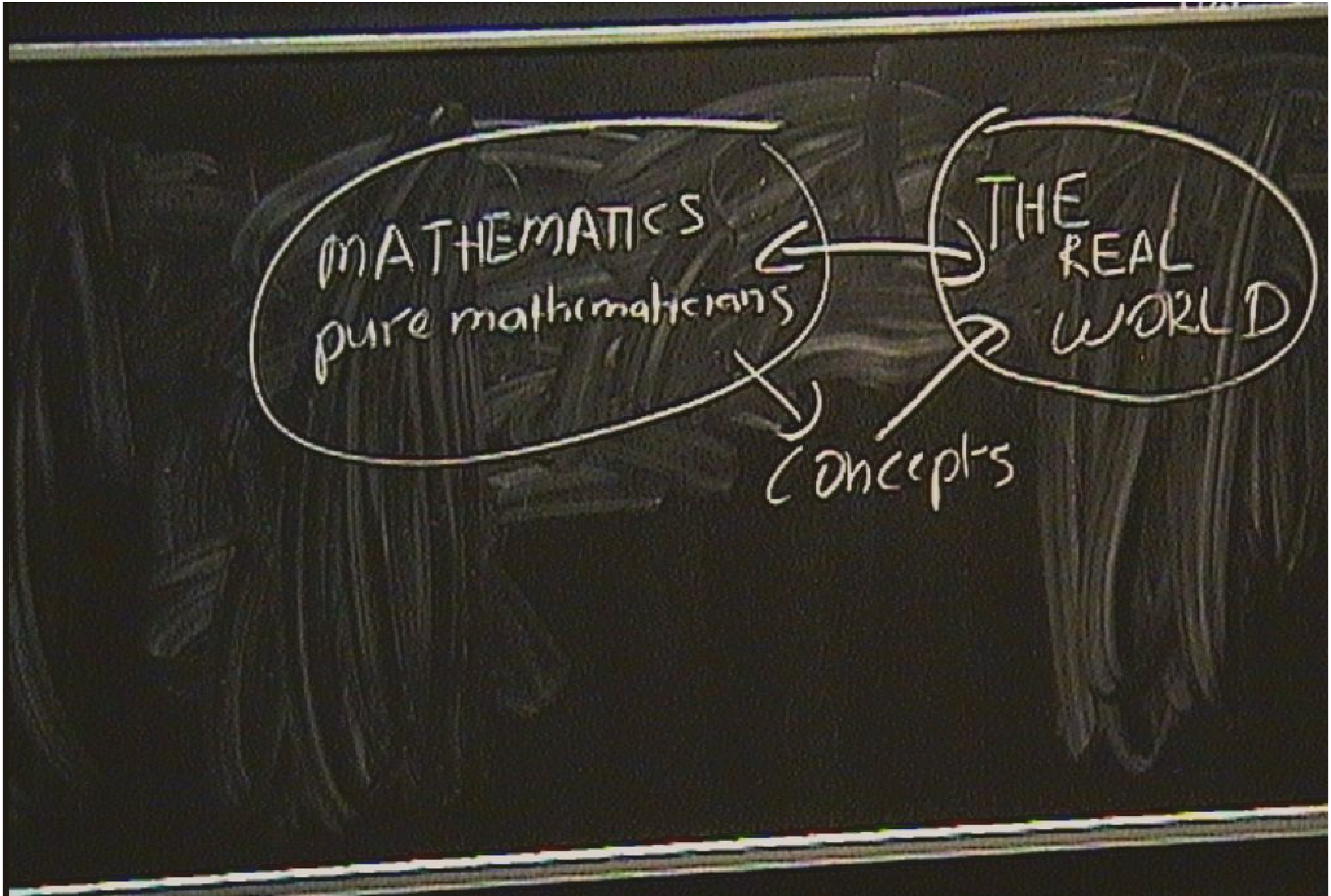
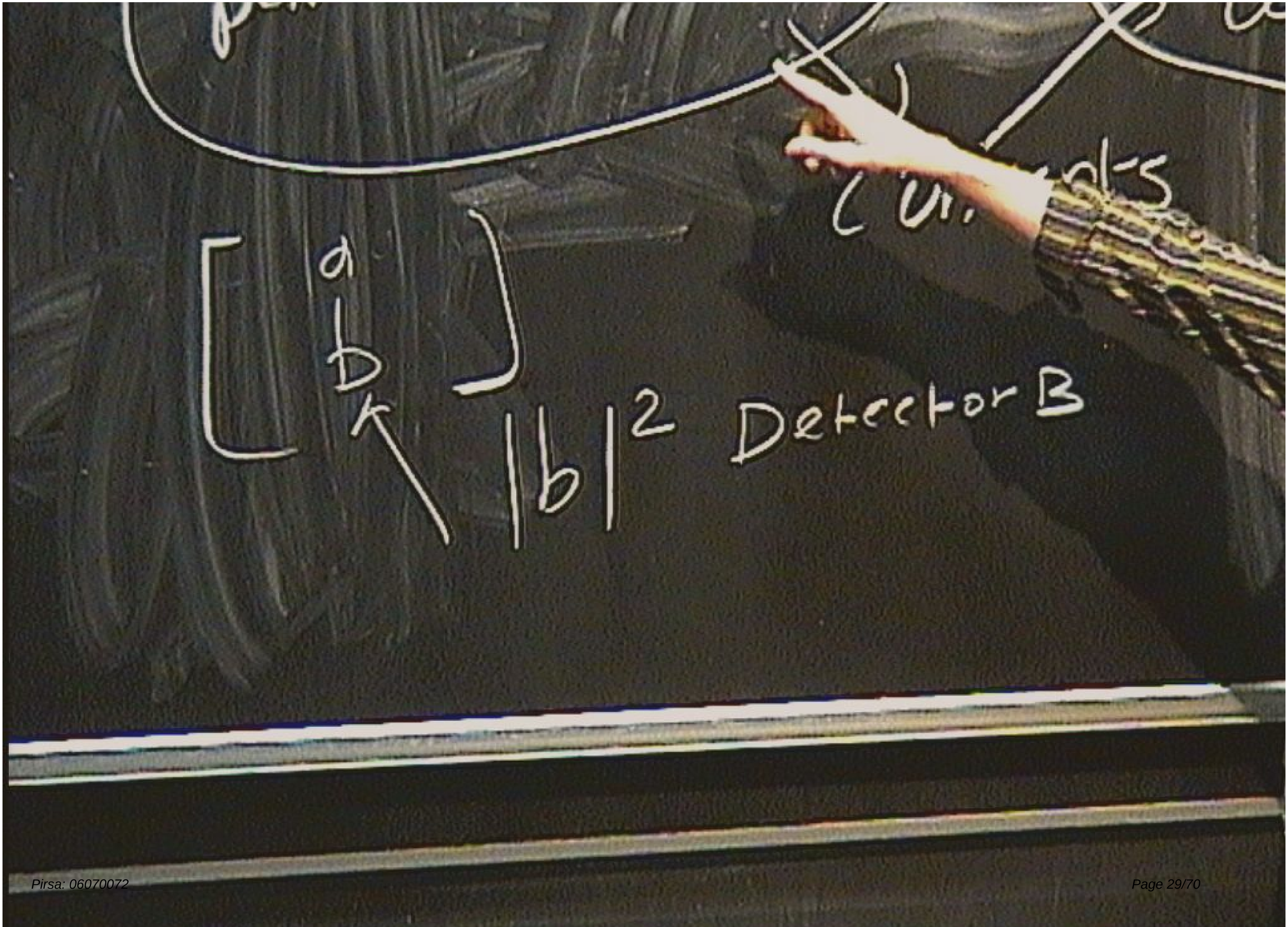
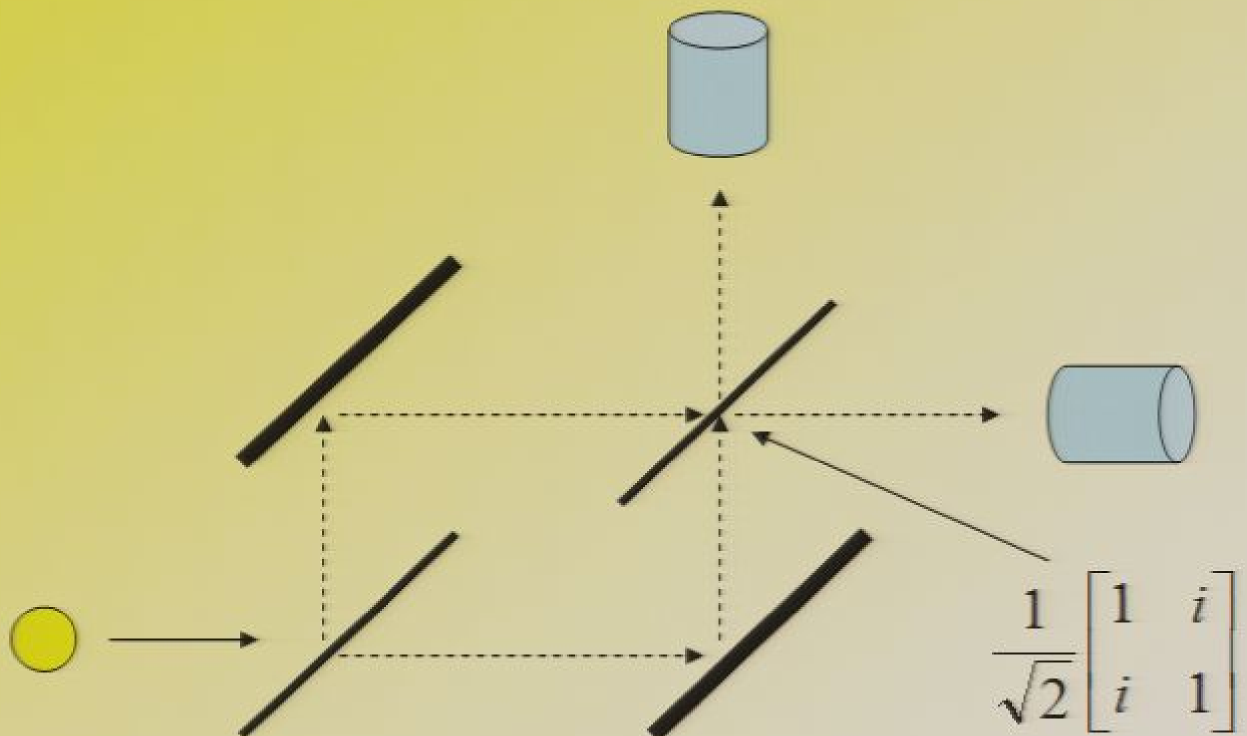

MATHEMATICS
pure mathematicians
THE REAL WORLD
concepts


a
b
|b|2
Detector B



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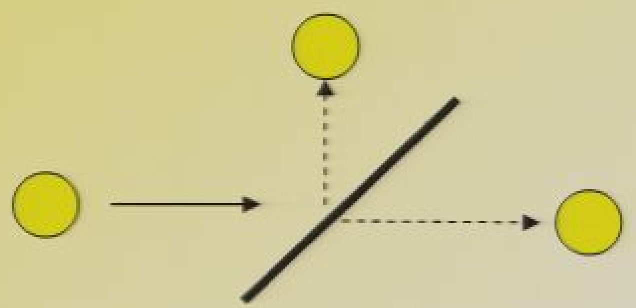


$$\begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}\frac{i}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$$

Another "state" in which the photon is taking both paths.

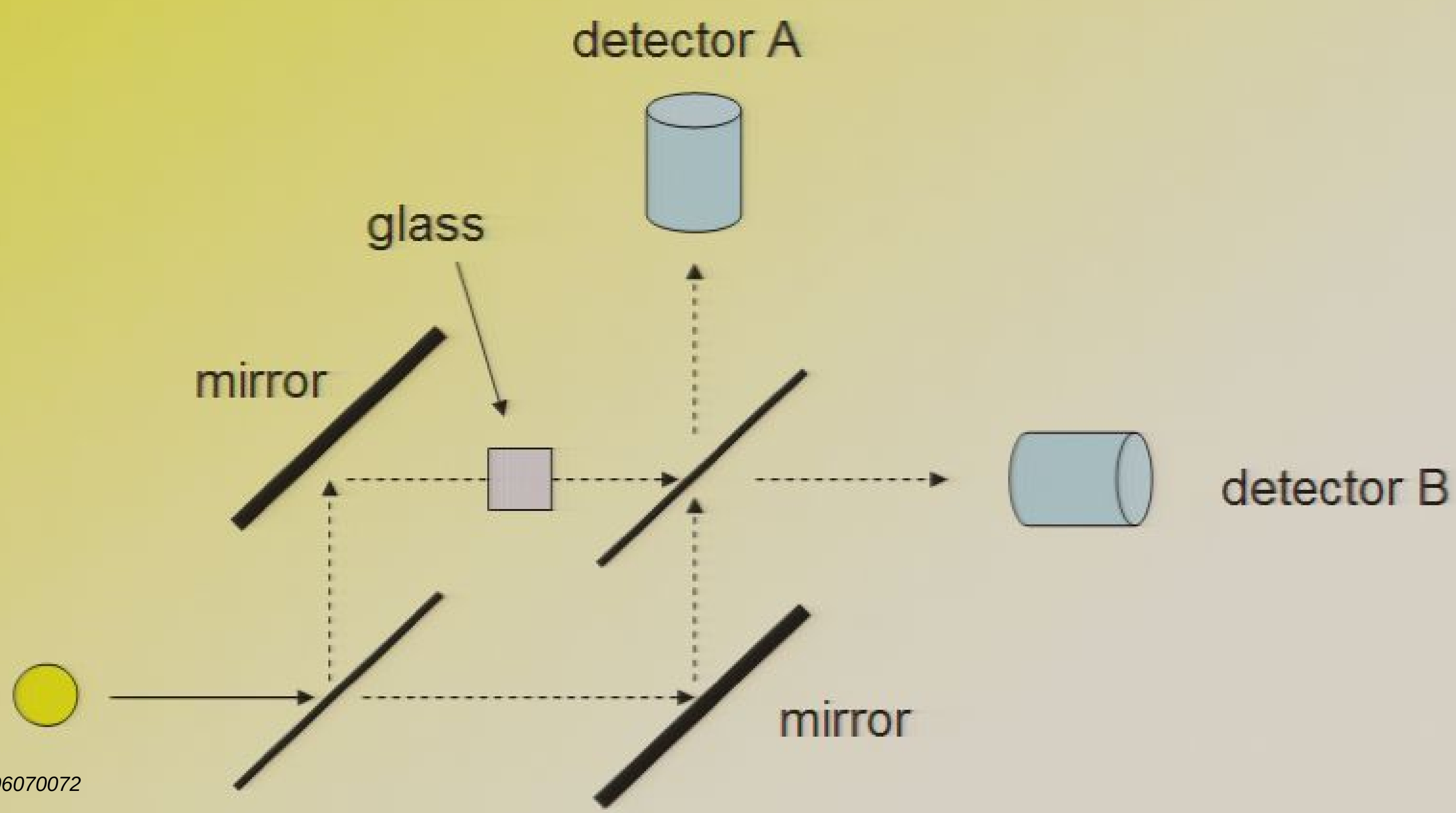
- What is the difference between $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$ *and* $\begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$?

- The probabilities for the photons to be measured in both paths are the same.

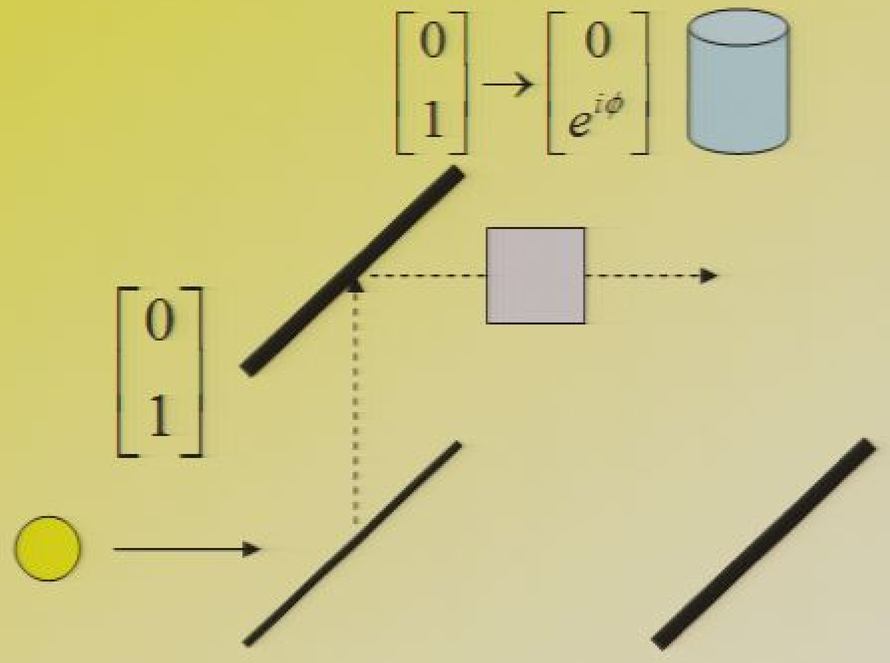
- As we shall soon see, they differ when we look at how different possibilities combine and overlap with each other.

Phases

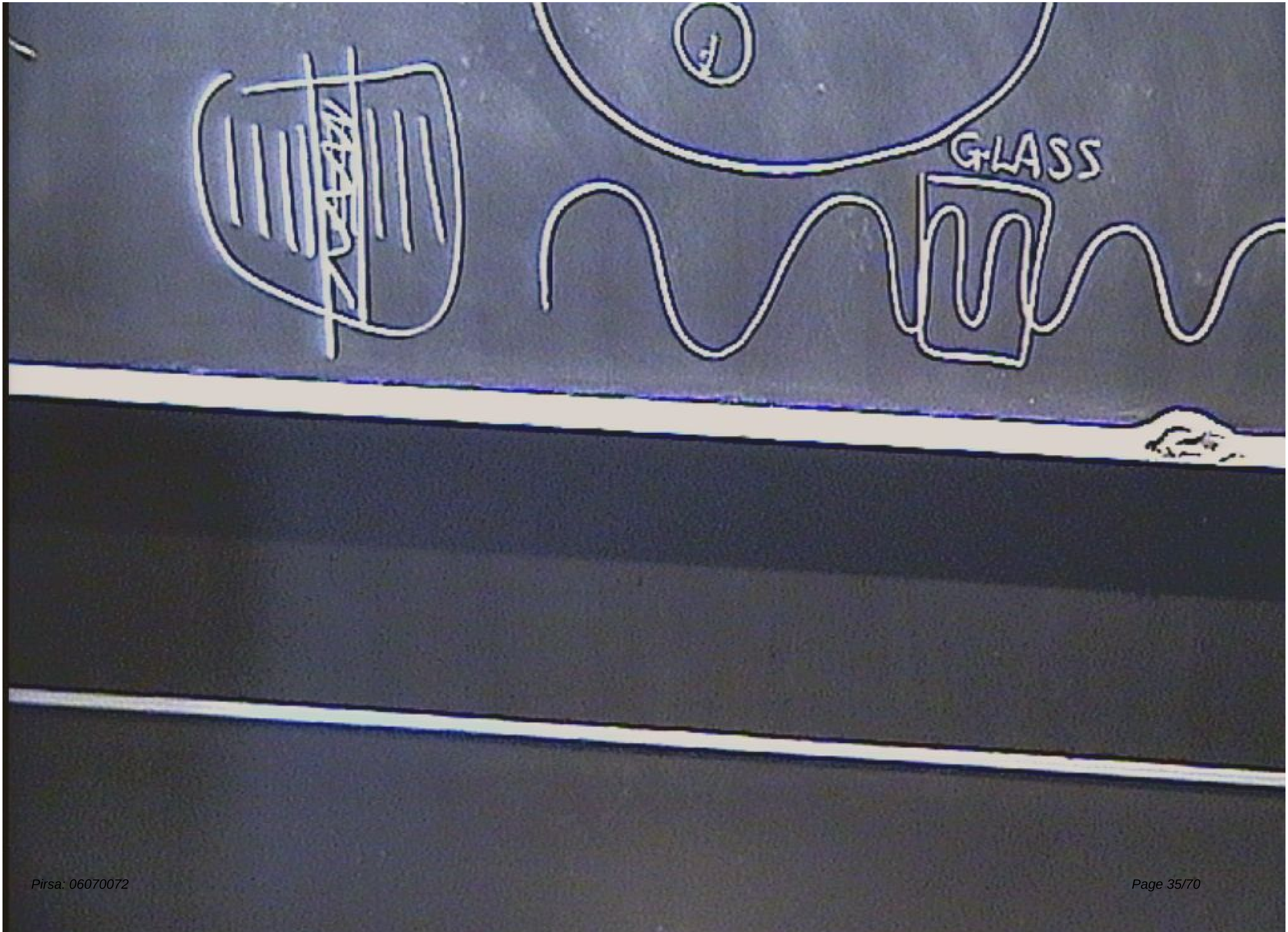
- Let us change the experiment slightly by adding a piece of glass that slows down the photon and so gives it an extra 'phase'

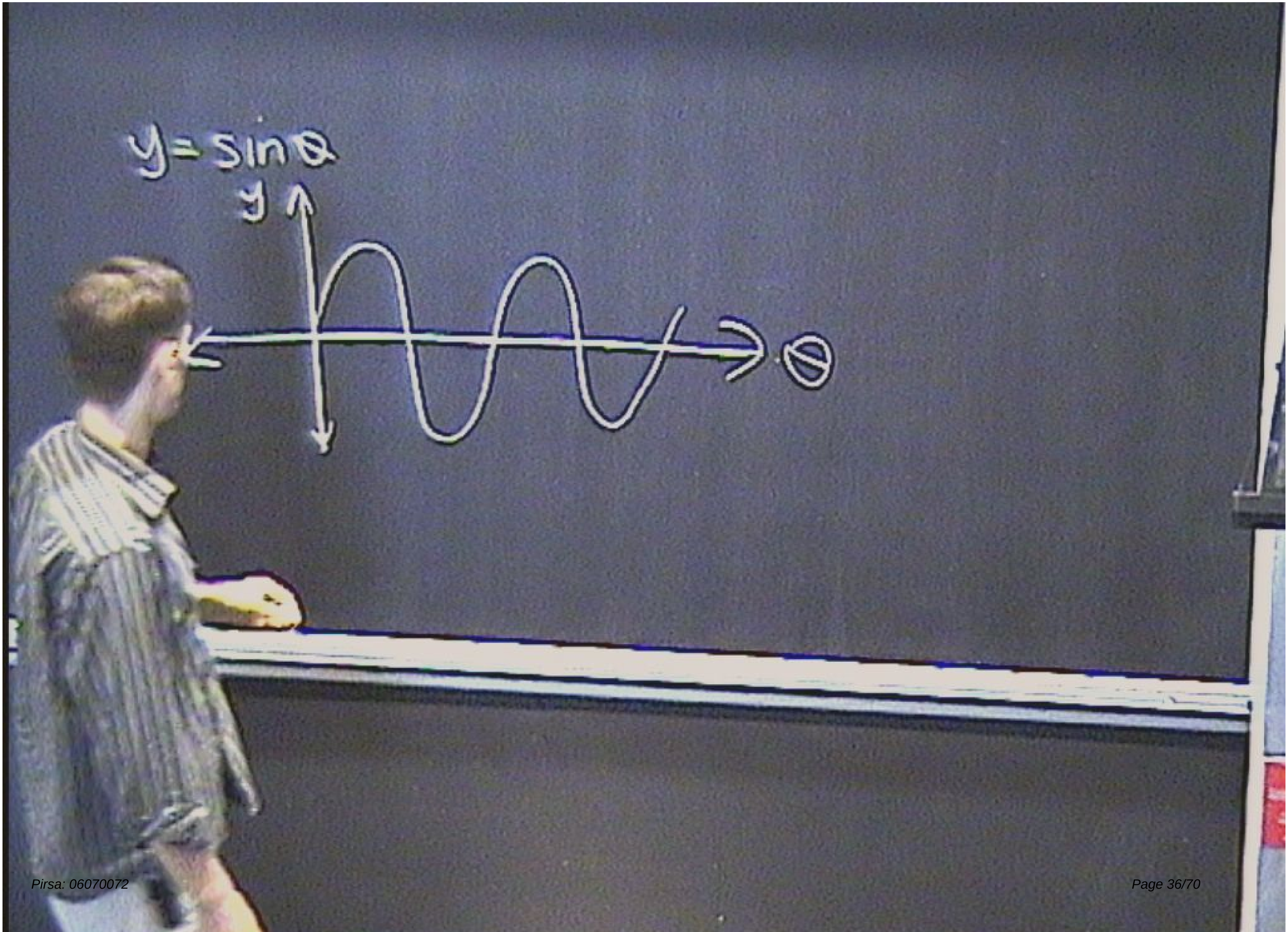


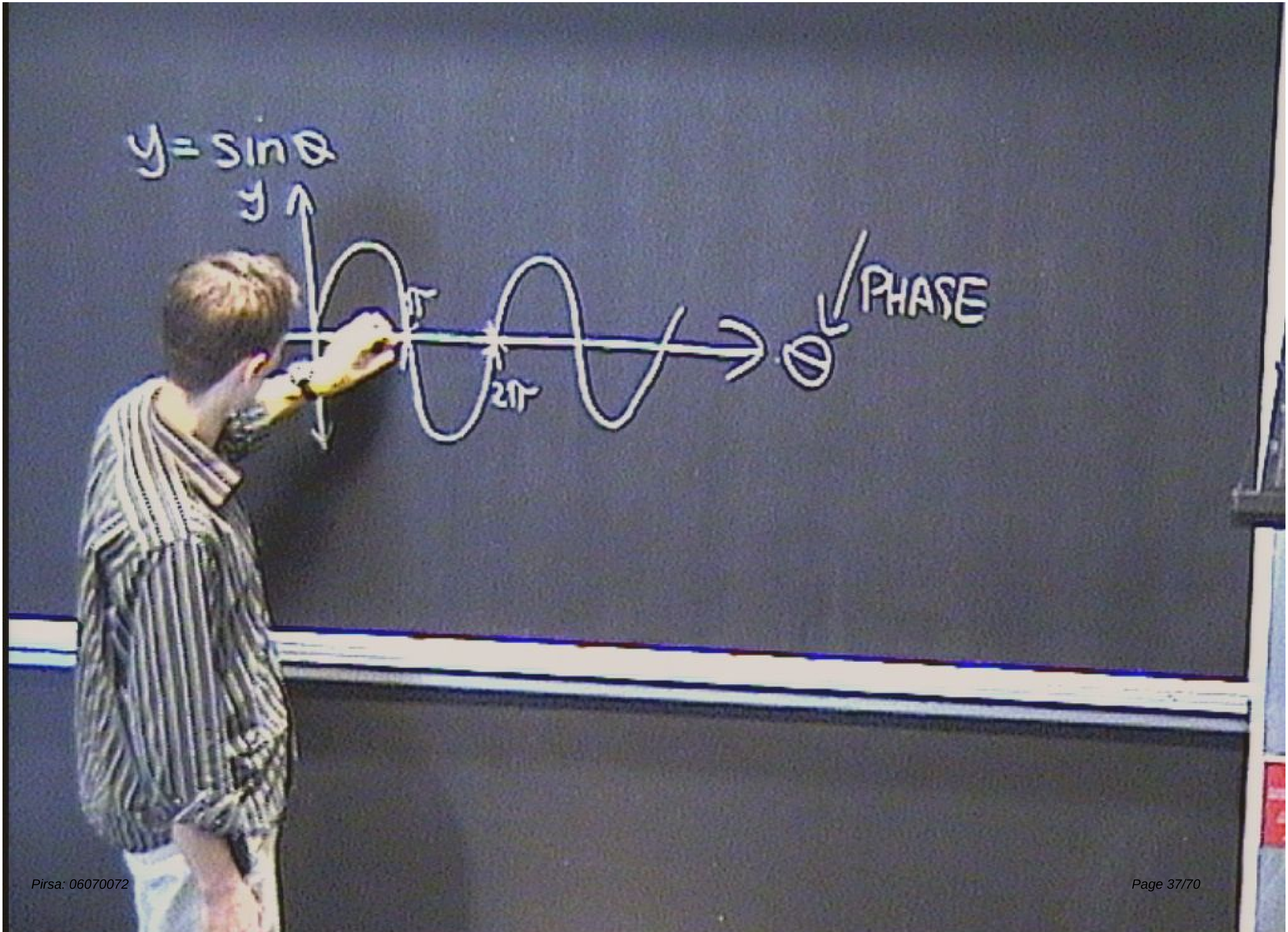
How does this affect the state?

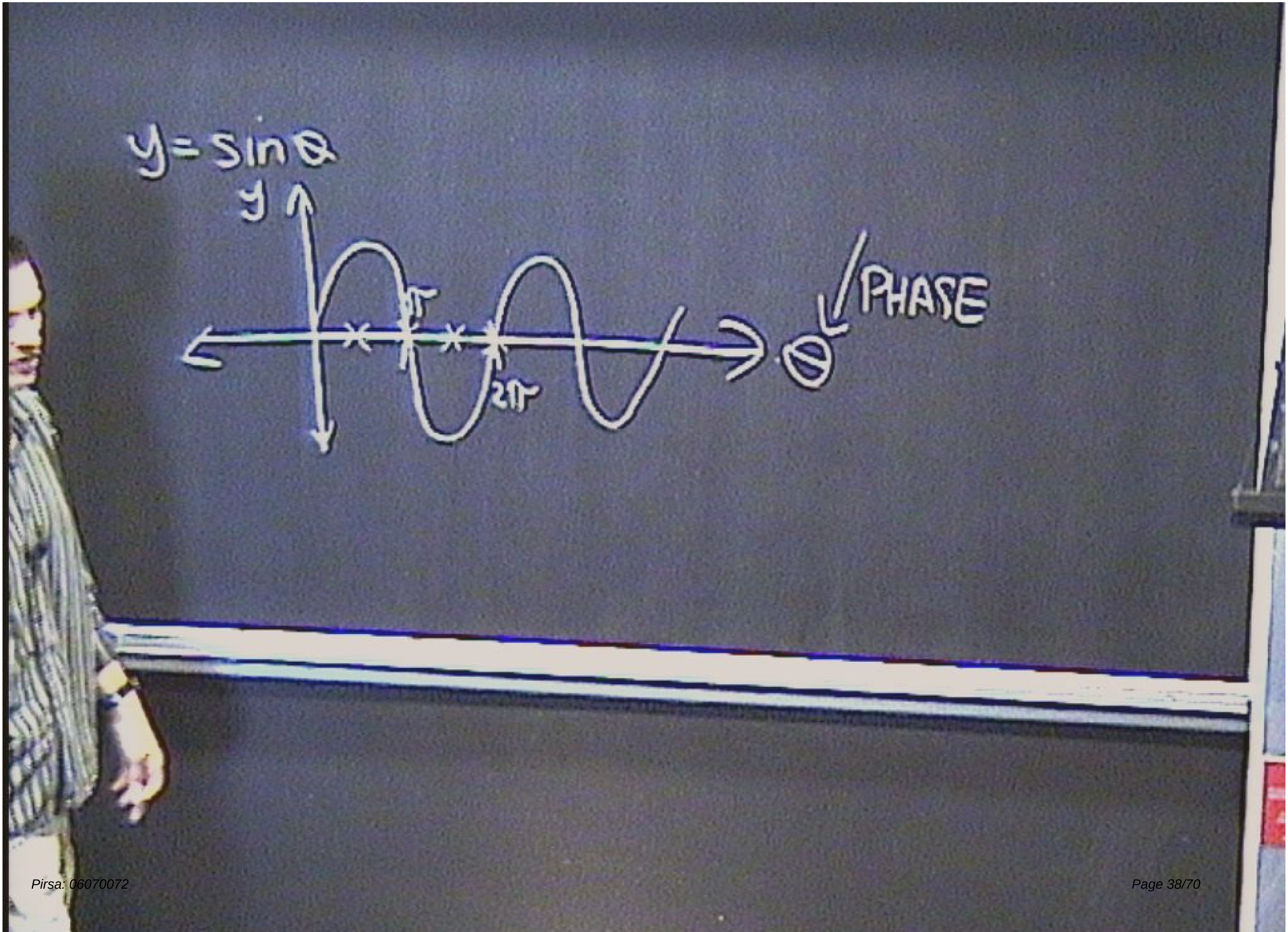


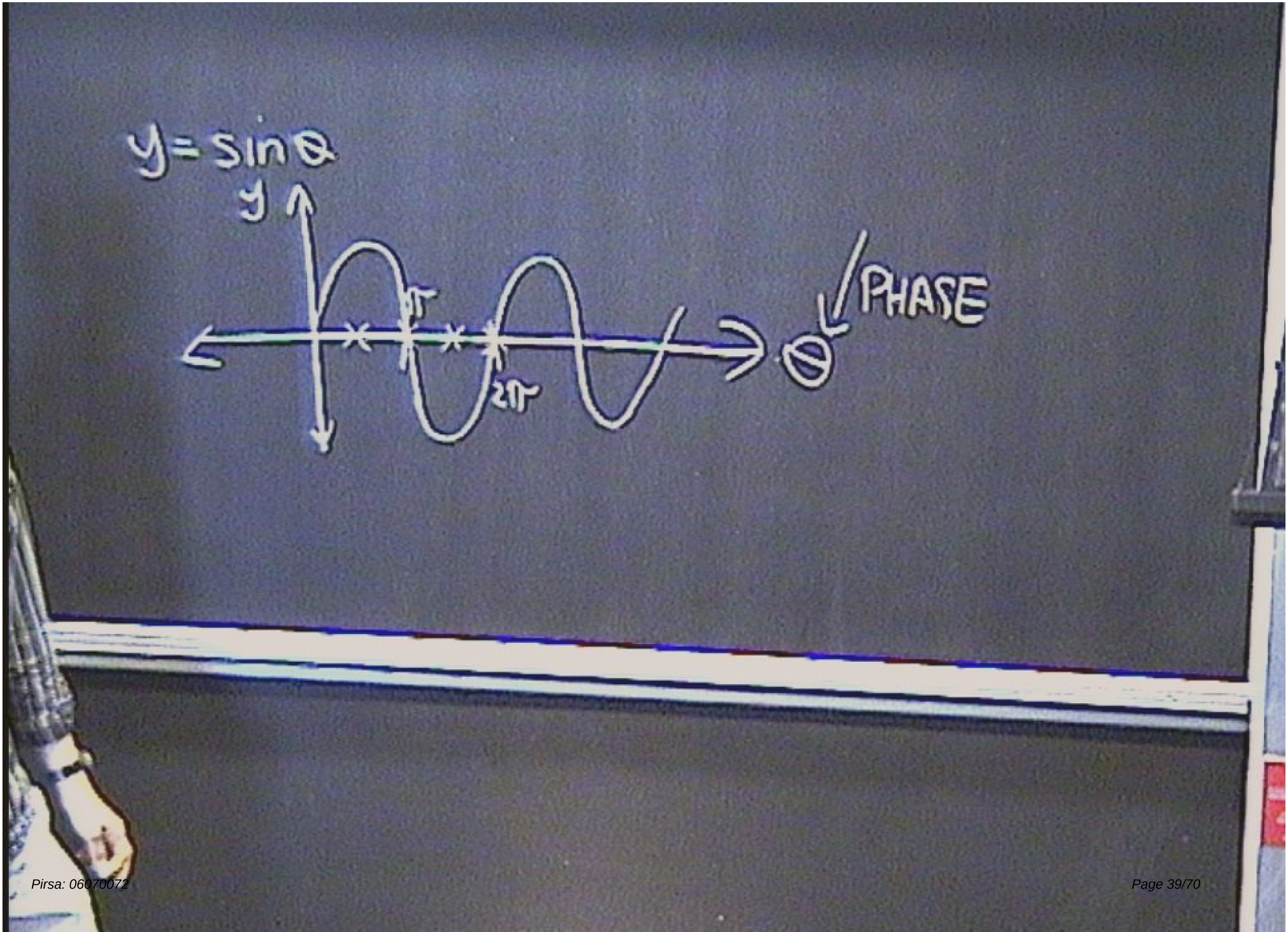
ϕ is related to the thickness of the glass. The thicker the glass, the greater the value of ϕ.


GLASS


y= sin θ
y
θ


y= sin θ
y
π
2π
θ
PHASE


y = sin θ
y
π
2π
θ
PHASE


y = sin θ
y
π
2π
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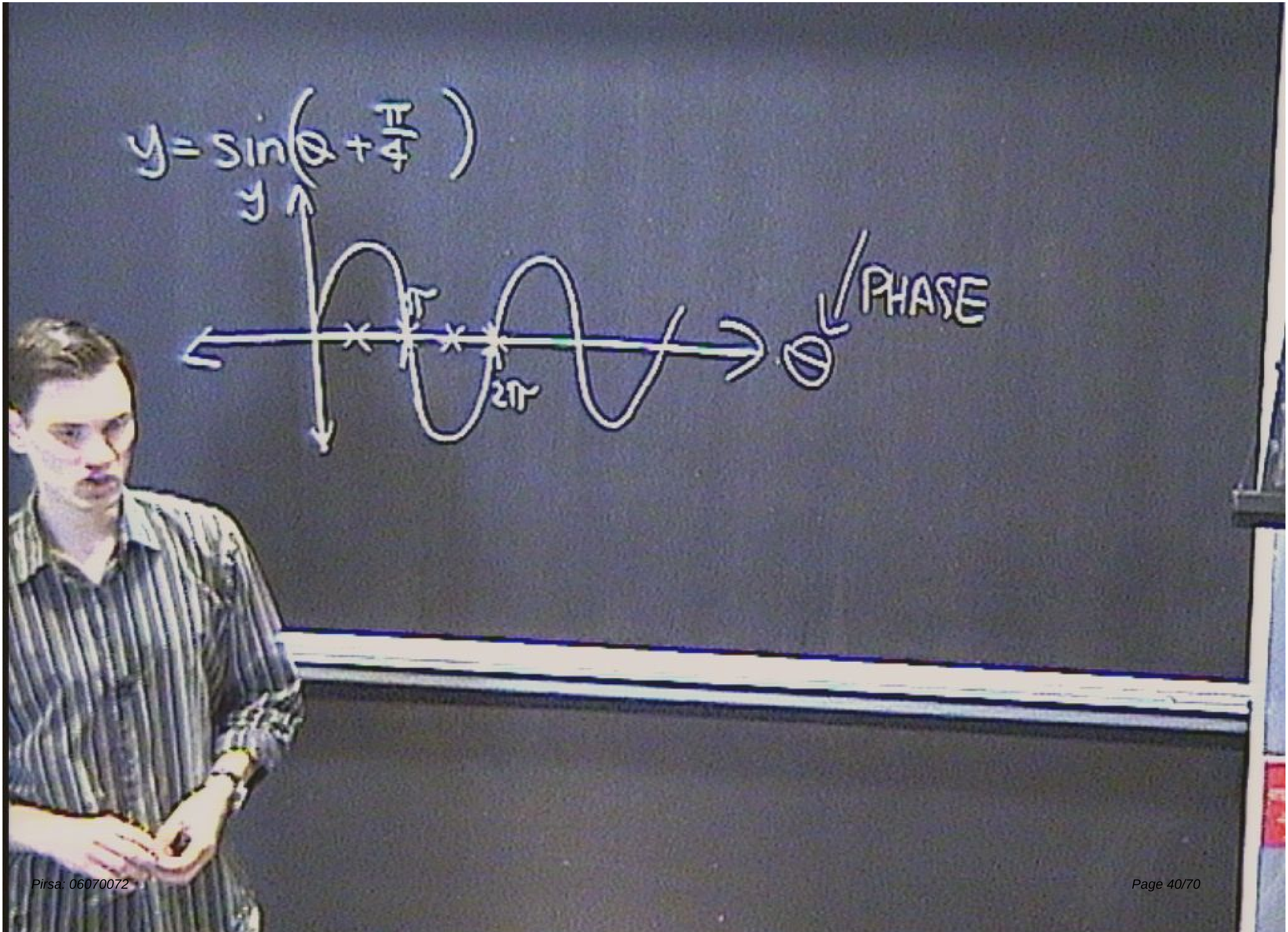
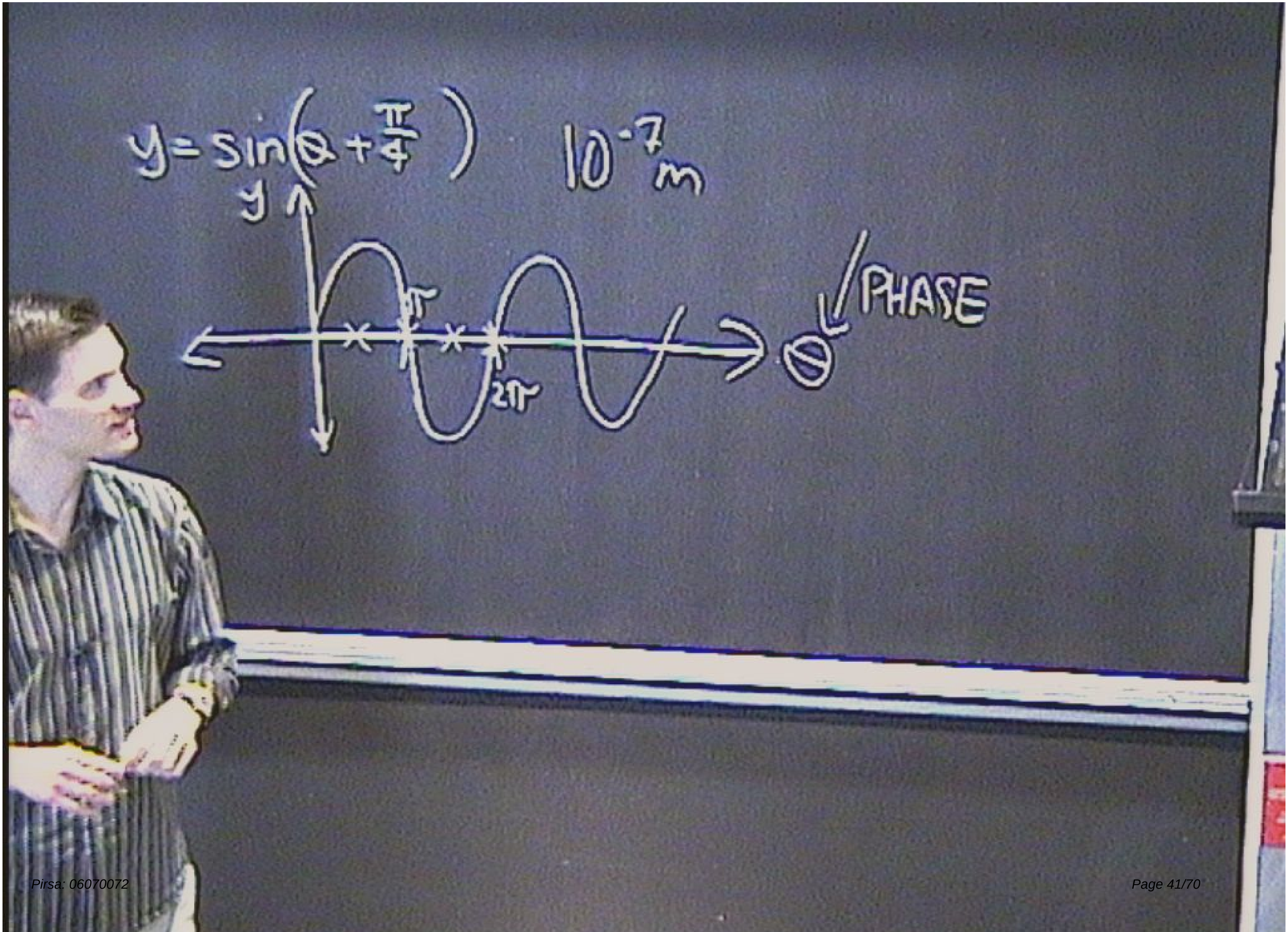
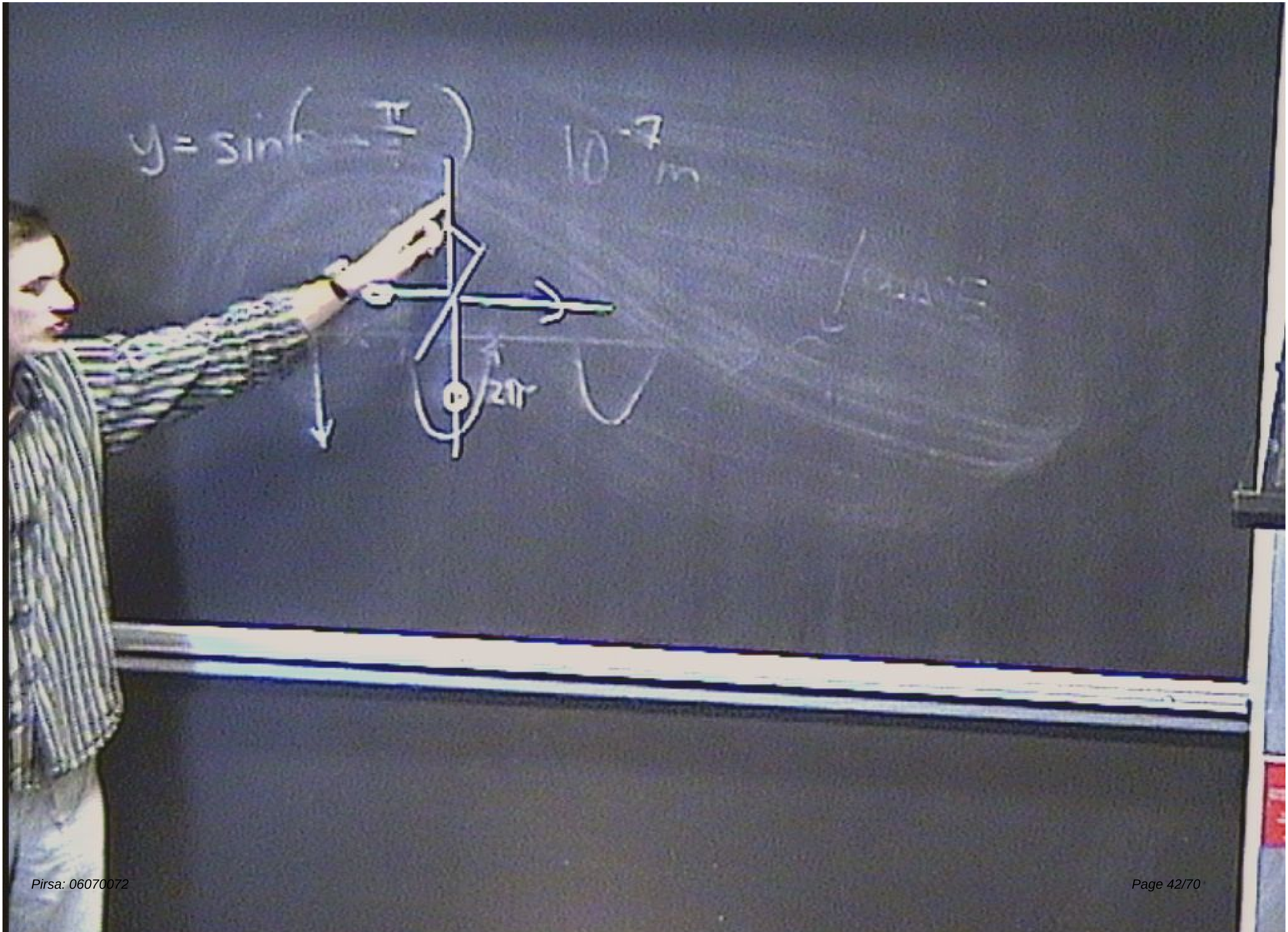
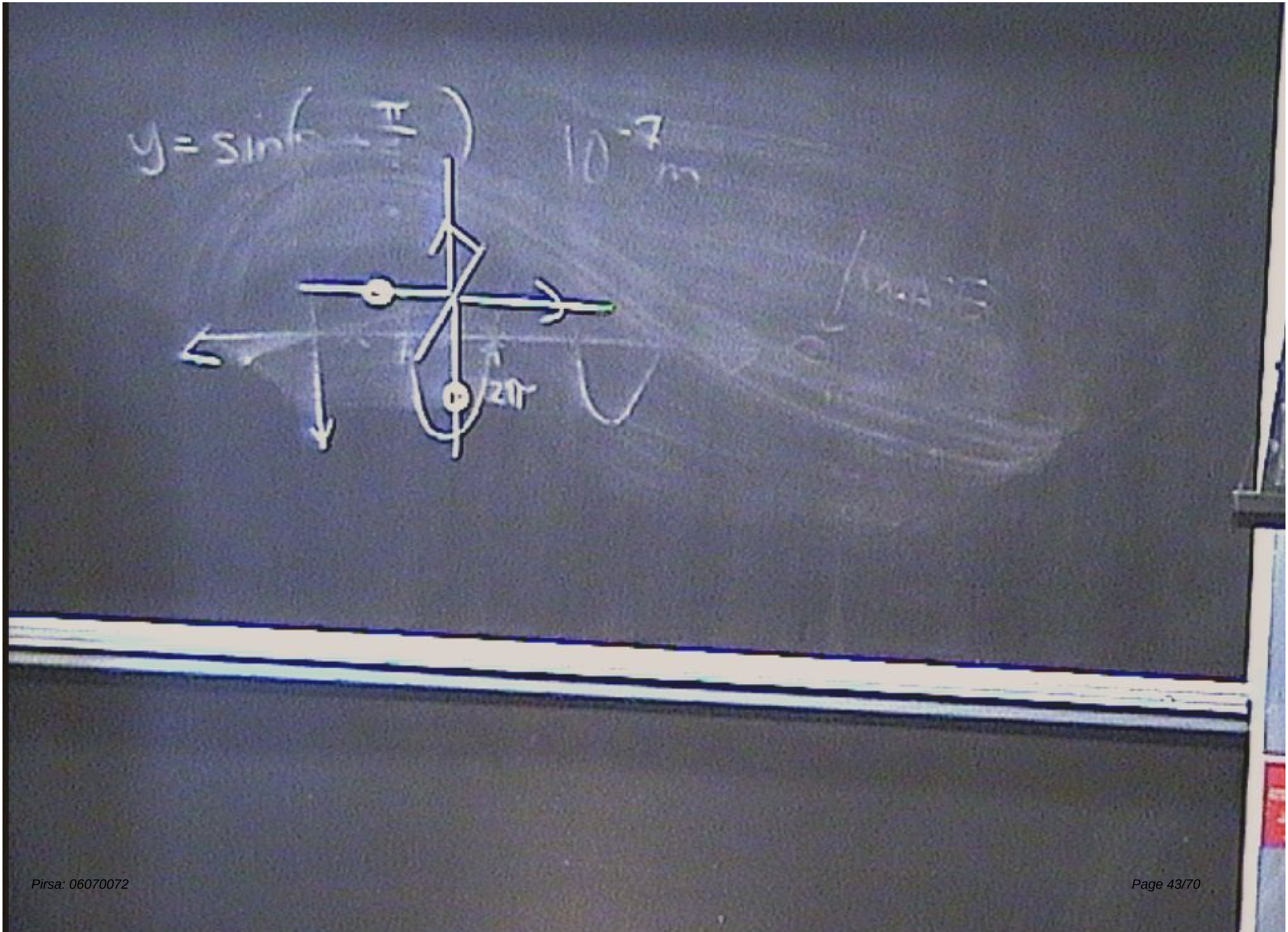
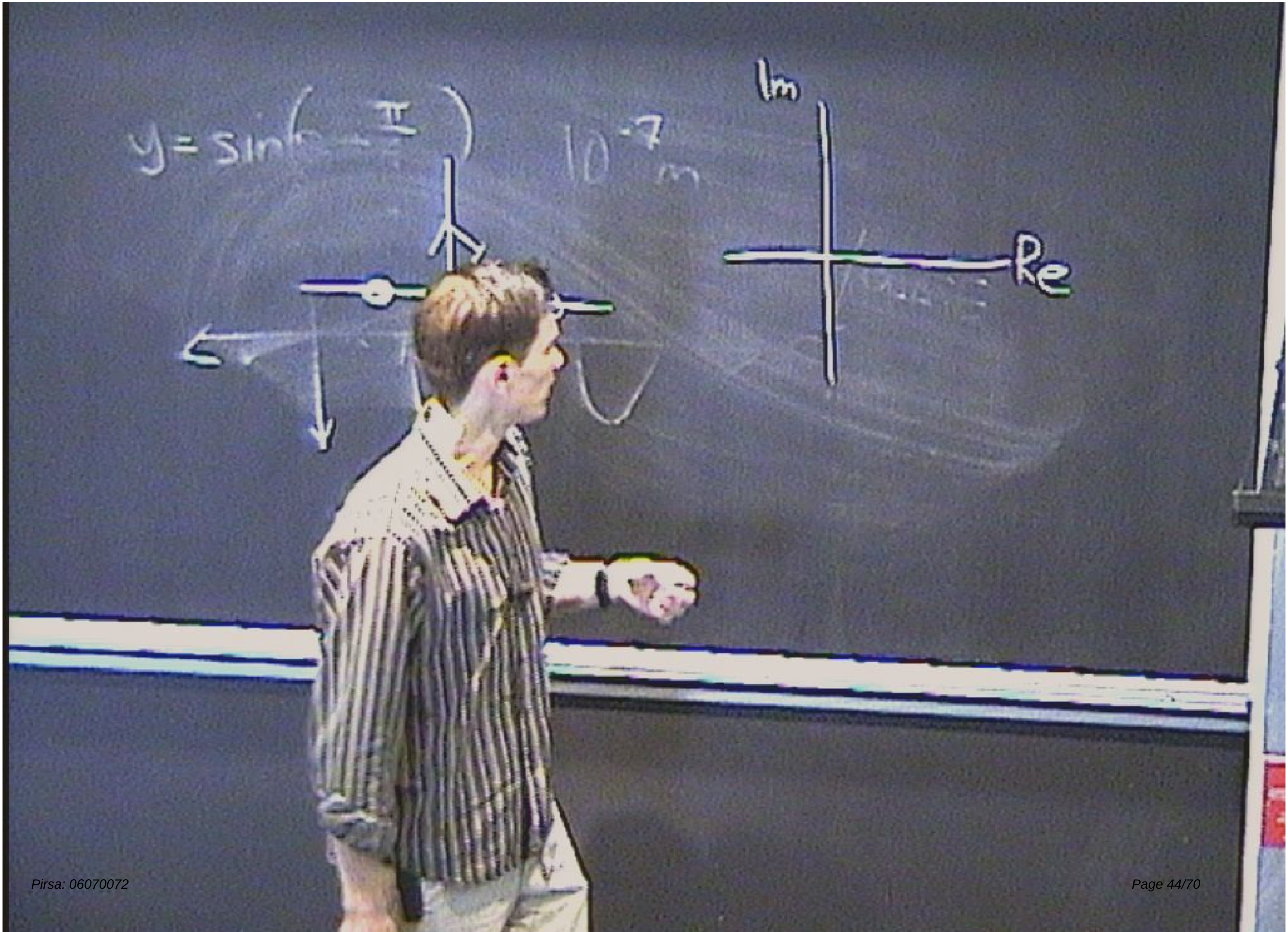
PHASE

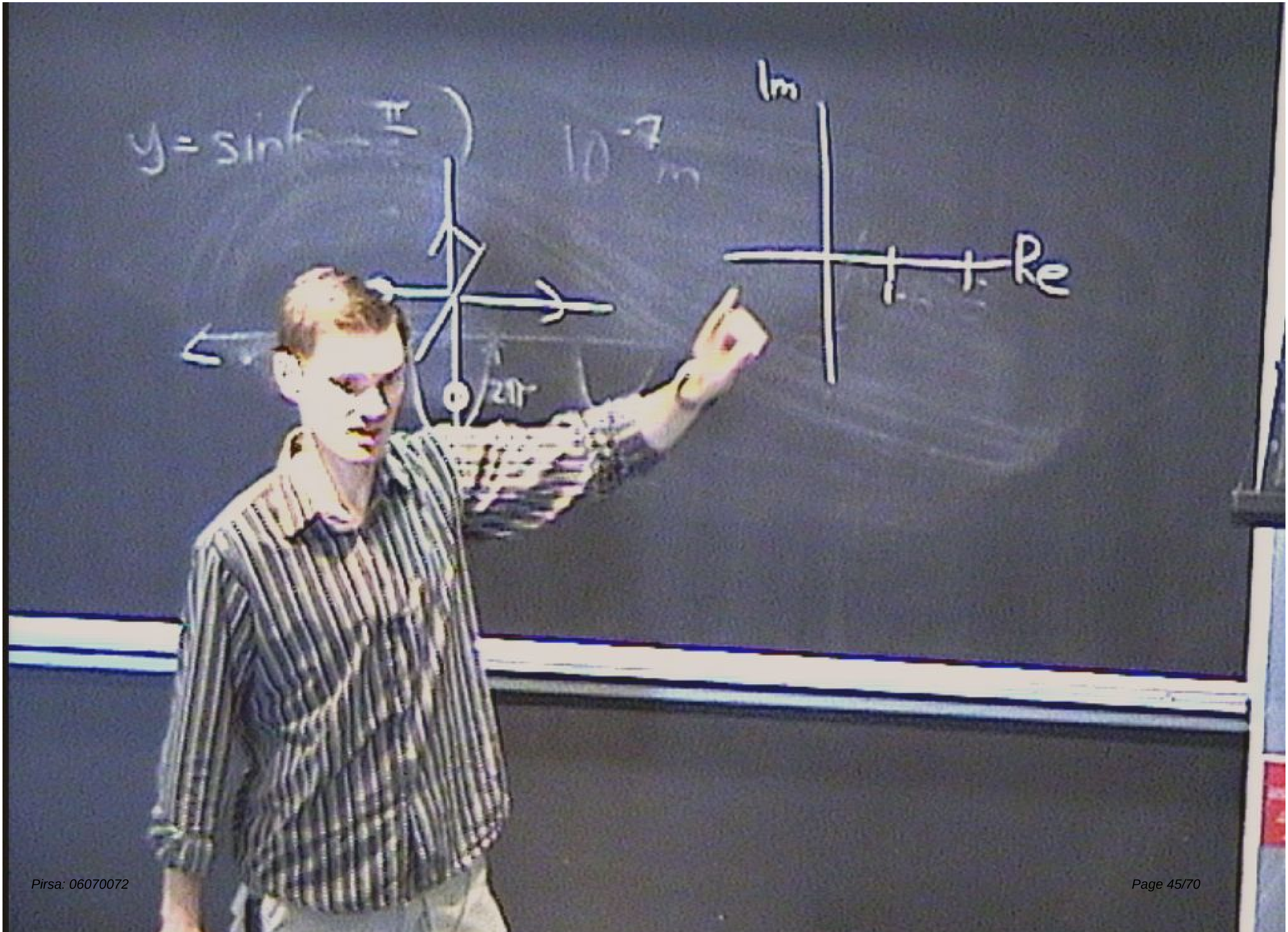

y = sin(θ + π/4)
y
π
2π
θ
PHASE


y = sin(θ + π/4)
10^-7 m
y
π
2π
θ
PHASE


y = sin
10^-7 m
2π r


y = sin(
10^-7 m
2π


Im
Re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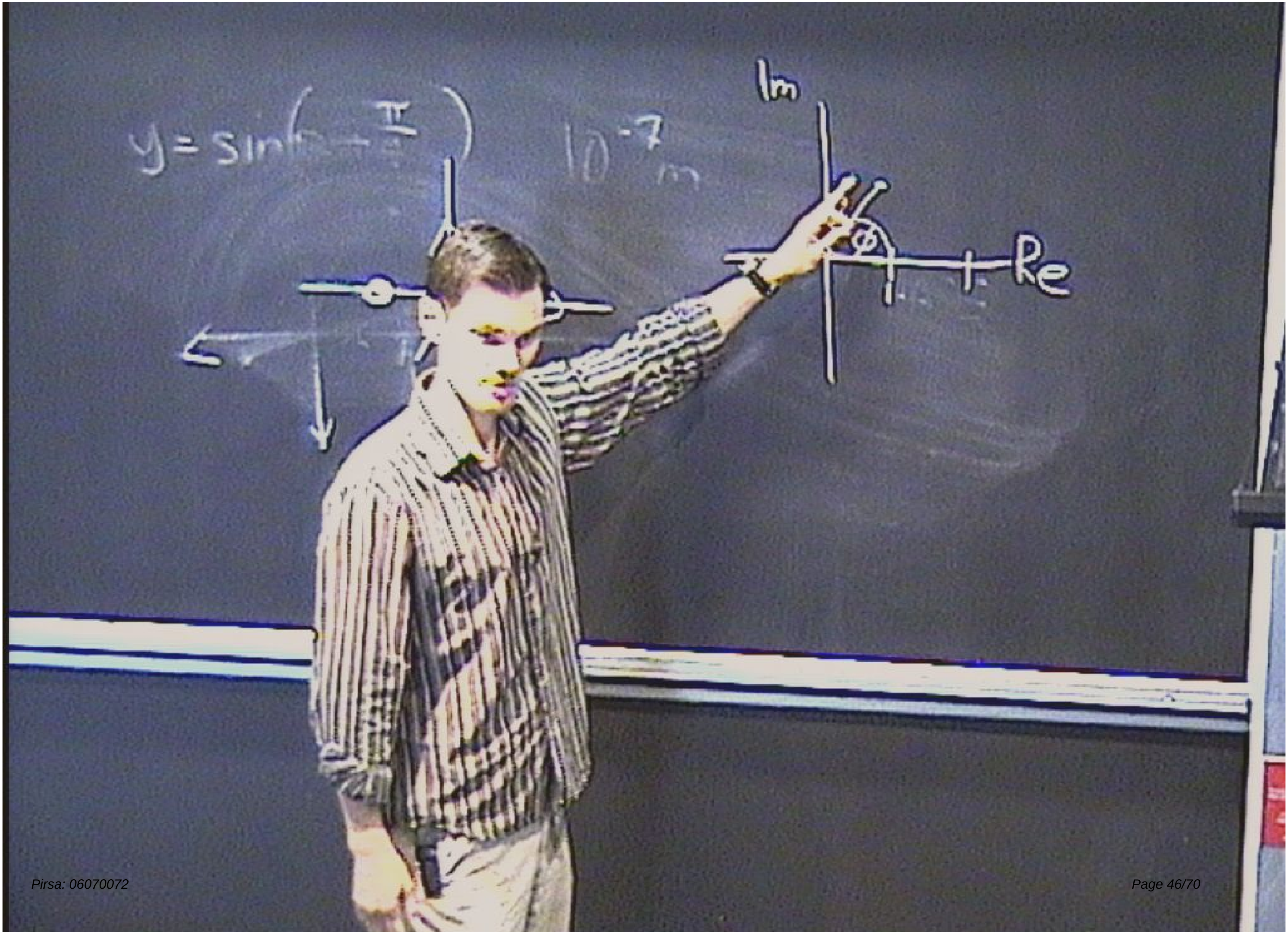
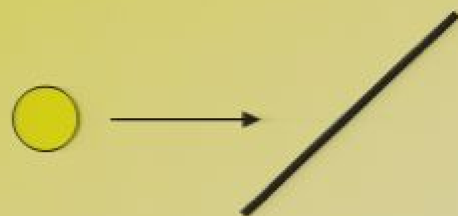
Re


Im
Re

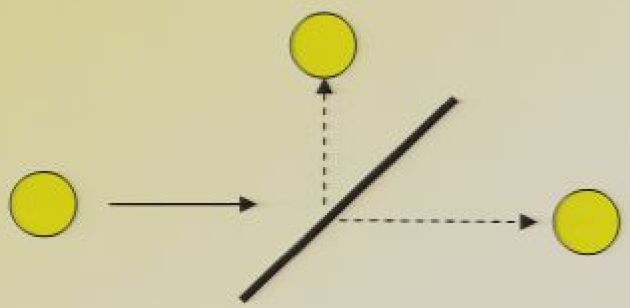
How does this change the probabilities for finding the photon at detectors A and B?

- Initial state:



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- After first half-silvered mirror:



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

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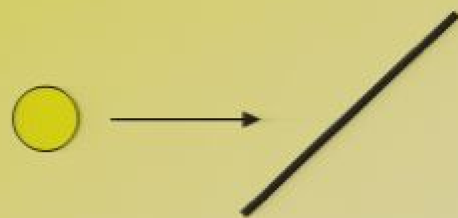

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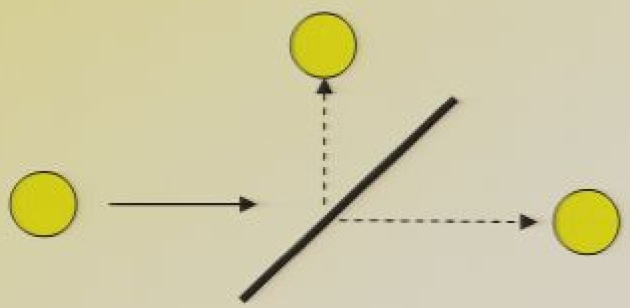

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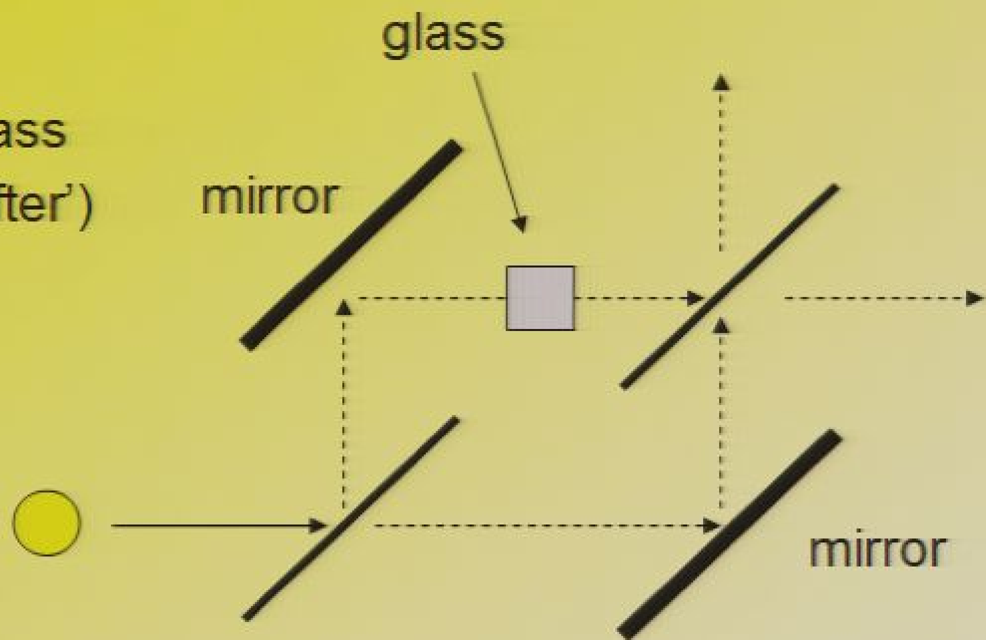
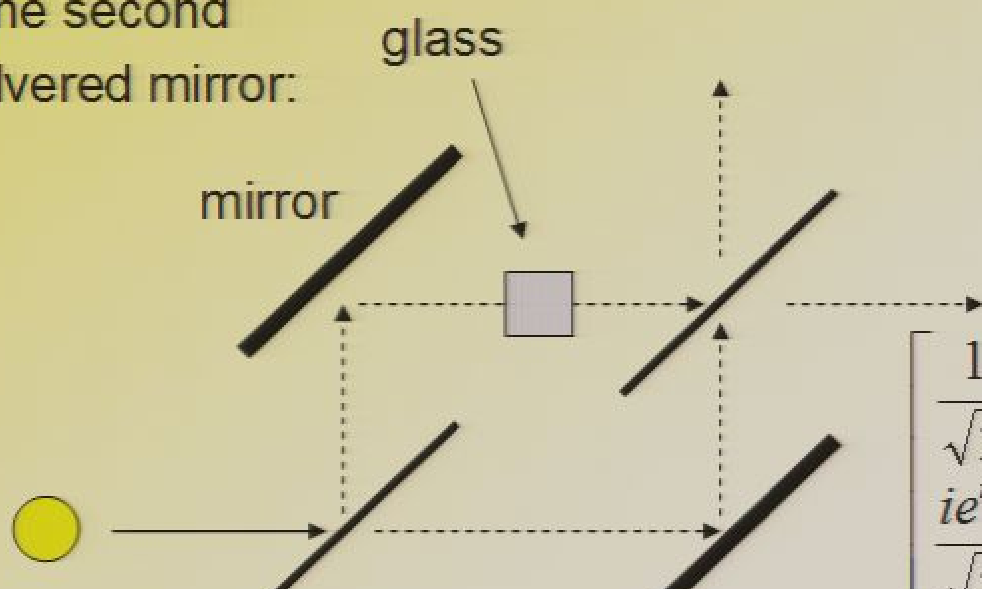
- After the glass ('phase-shifter')



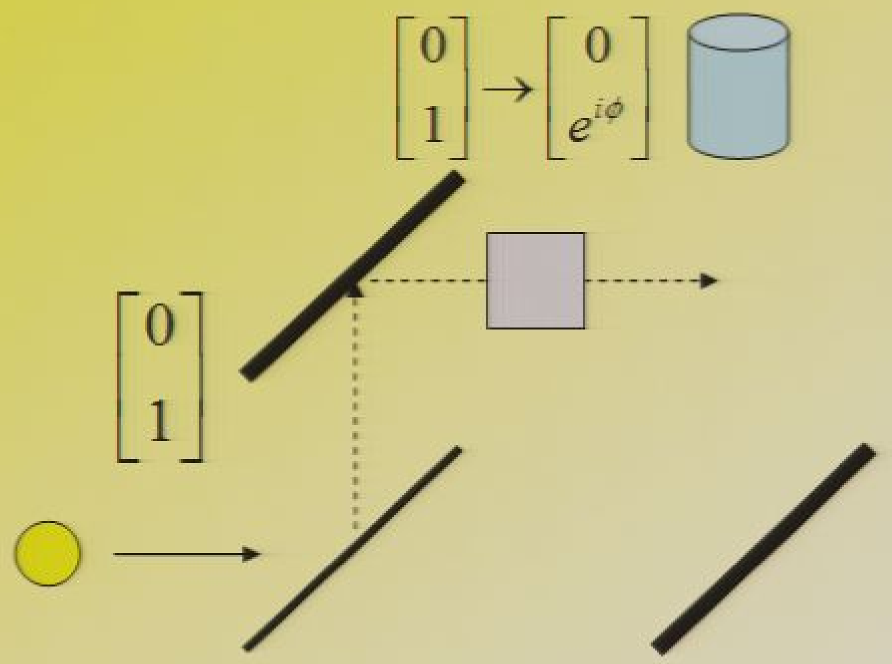
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix}$$

After the second half-silvered mirror:



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$$

How does this affect the state?



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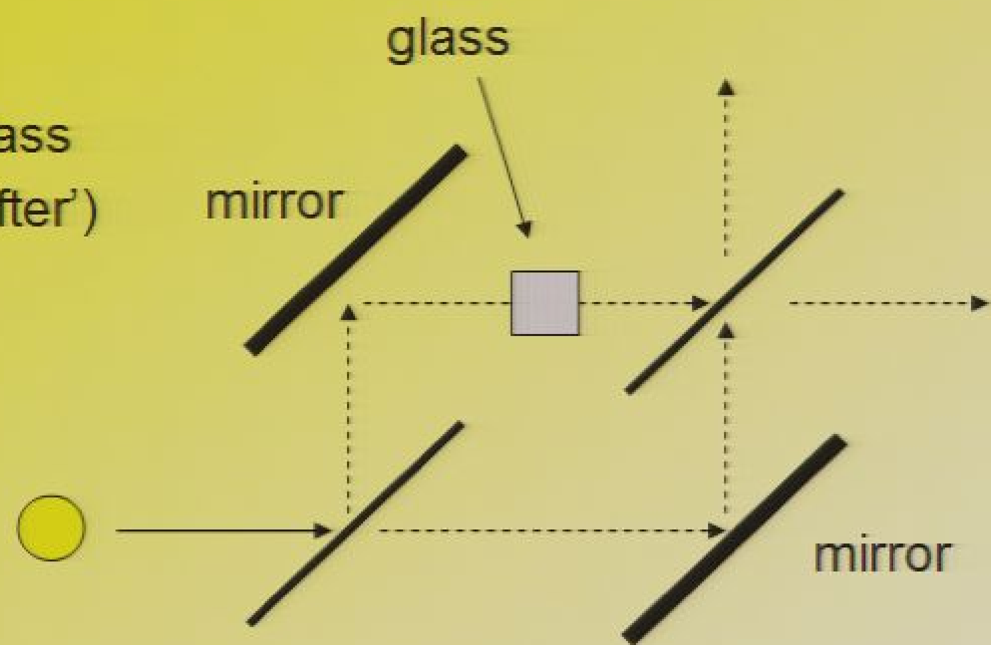

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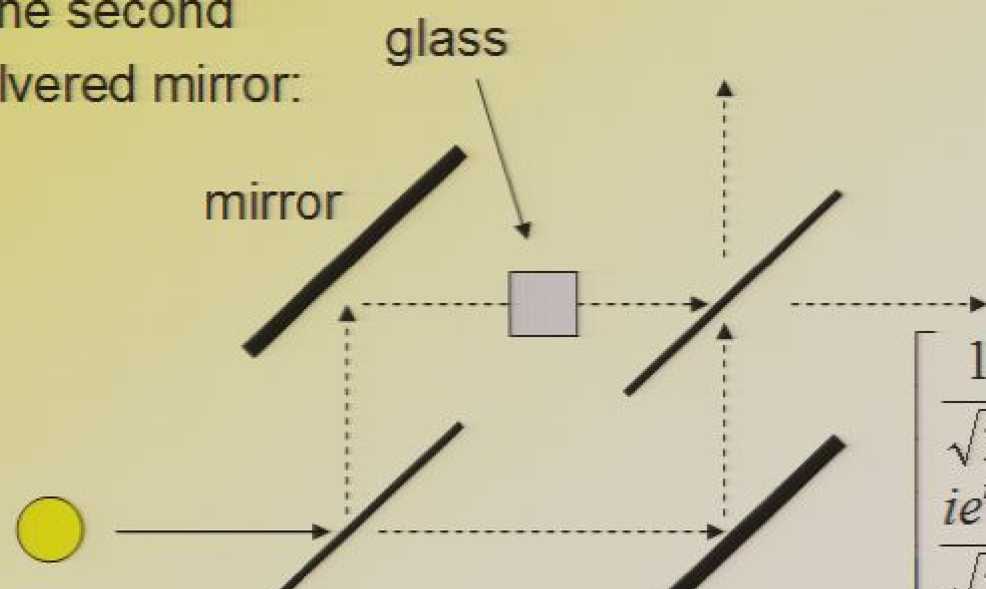

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Student activities

- 1. When $\phi=\pi$, what are the probabilities of finding the photon at detectors A and B.

- Bonus questions (harder):

  2. When $\phi=\pi/4$, what are the probabilities of finding the photon at detectors A and B.

  3. Represent $\frac{1}{2}\begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$ in polar co-ordinates

Homeplay question (hard).

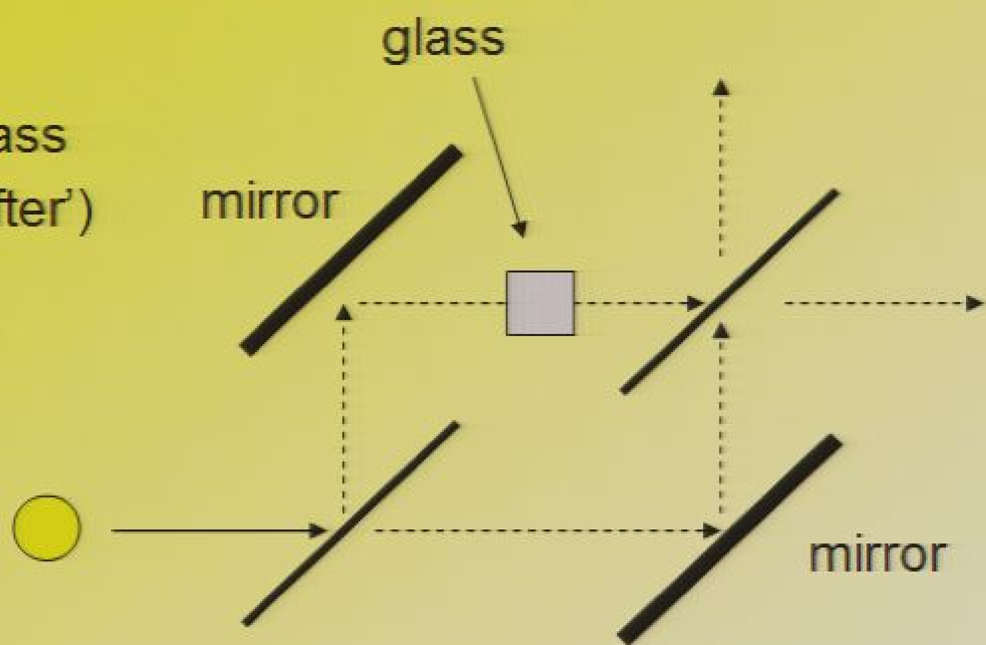
- A unitary matrix is any matrix U for which $U^{\dagger} = U^{-1}$
- The 'dagger' symbol † means that we take the transpose of U and then then complex conjugate of each element of the resulting matrix. It signifies what is called the *Hermitian conjugate* of U.

i.e. $$If \quad U = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$then \quad U^{+} = (U^{T})^{*} = \left(\begin{bmatrix} a & c \\ b & d \end{bmatrix} \right)^{*} = \begin{bmatrix} a^{*} & c^{*} \\ b^{*} & d^{*} \end{bmatrix}$$

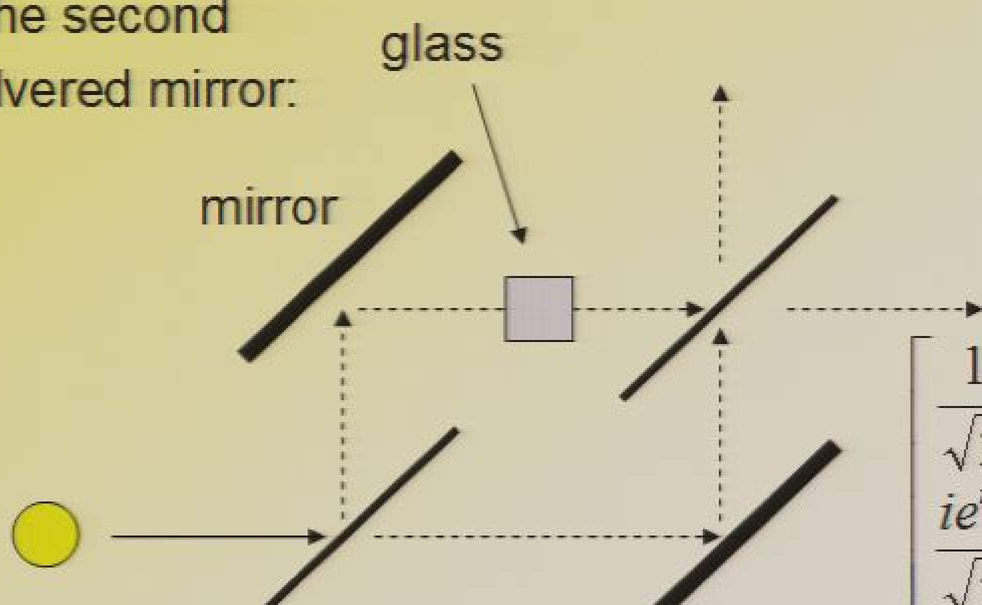
$$If \quad A = \begin{bmatrix} 4 & -2i \\ i & 3 \end{bmatrix}, what \quad is \quad A^{+}?$$

- After the glass ('phase-shifter')

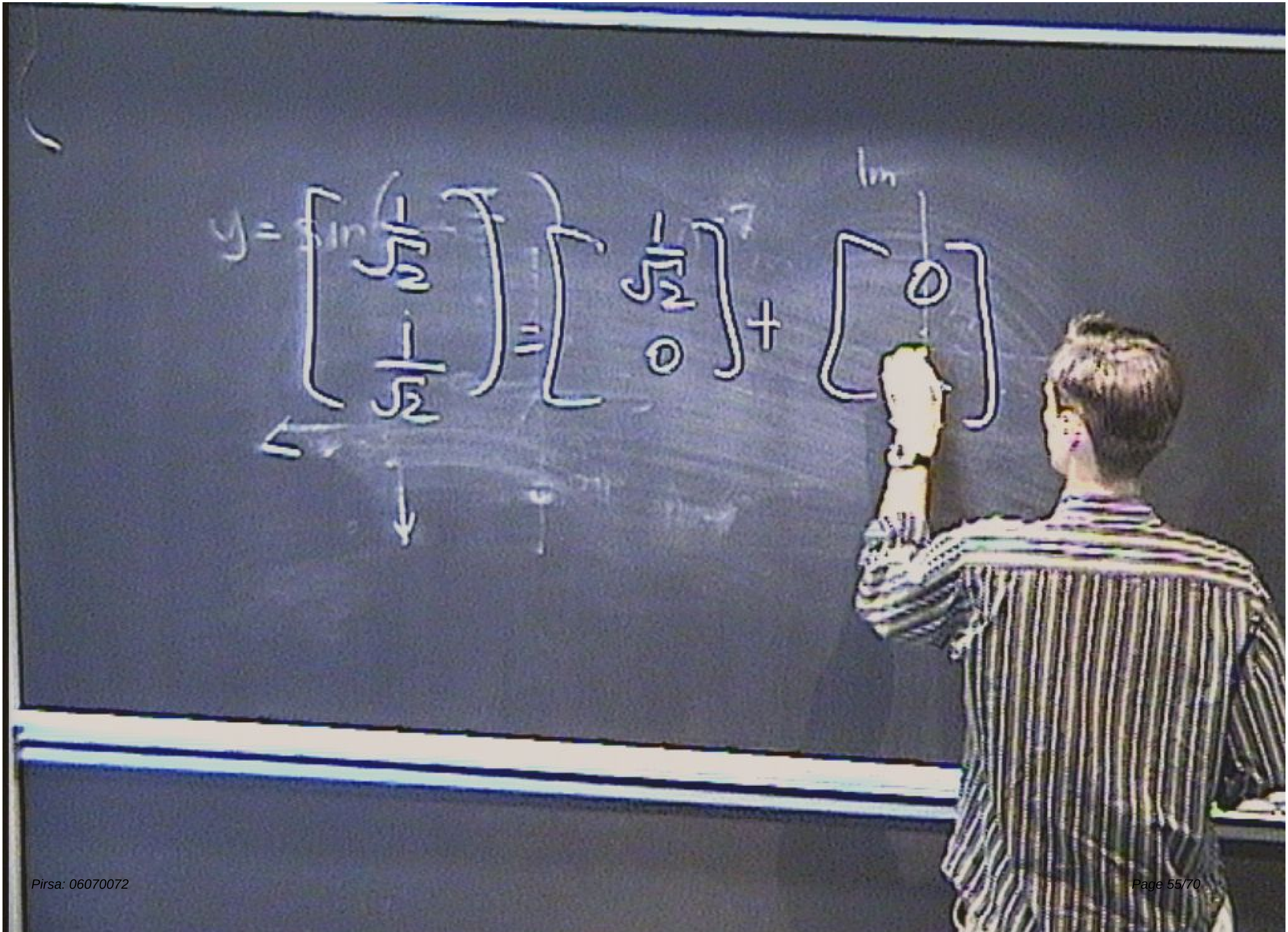


$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix}$$

After the second half-silvered mirror:



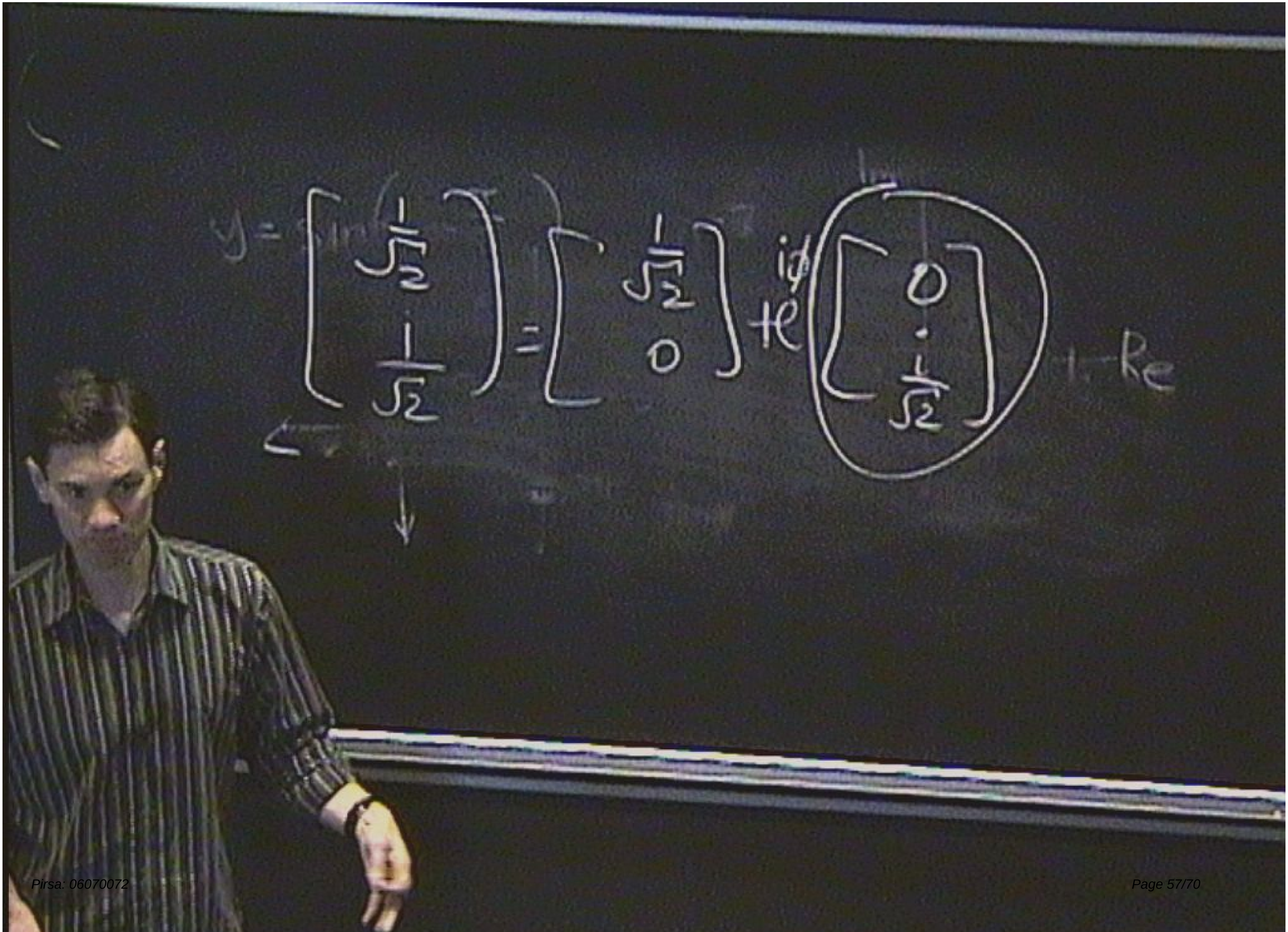
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{ie^{i\phi}}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$$


Im

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

Im

1 Re


Im
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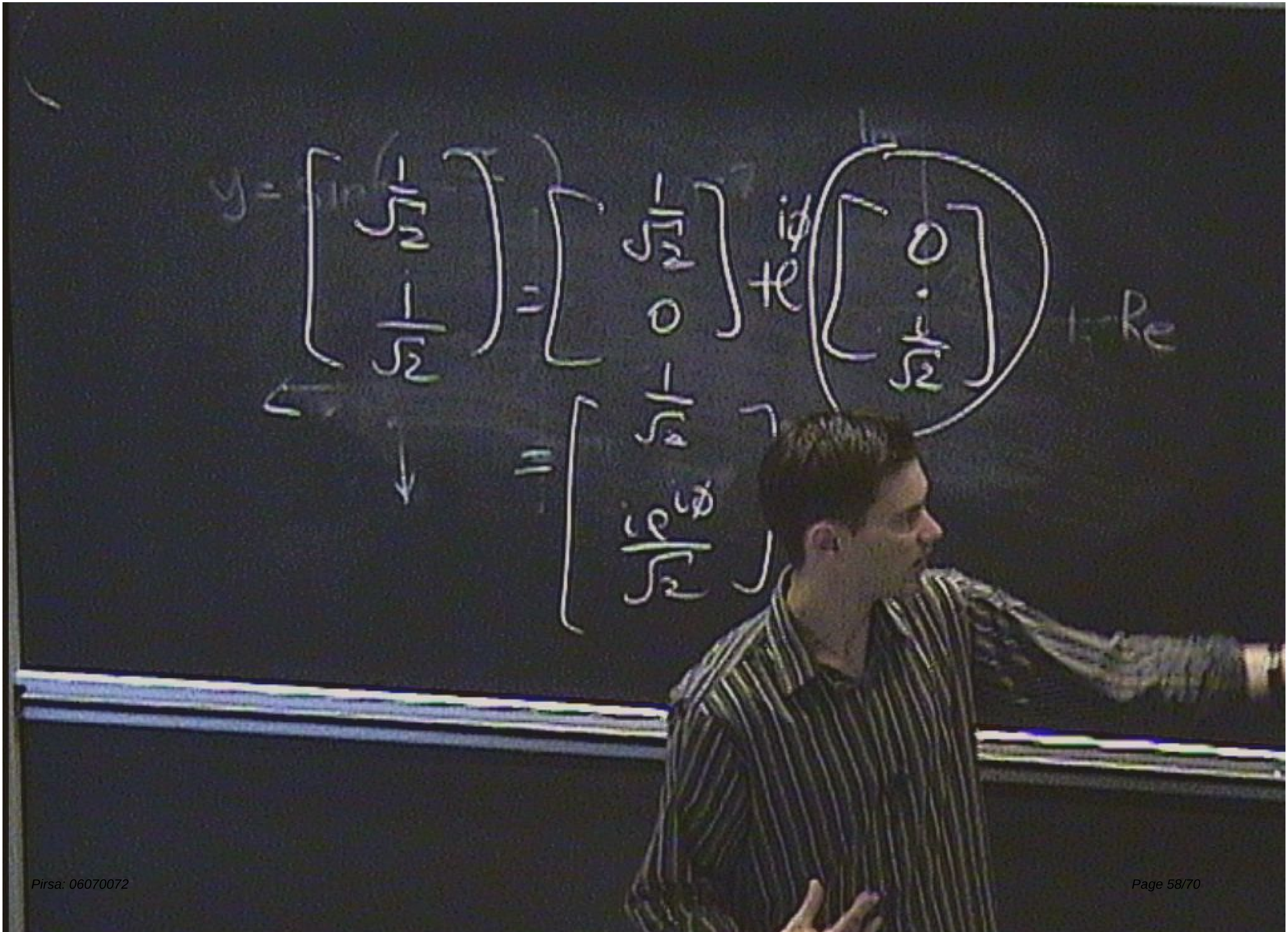
Im
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Student activities

- 1. When $\phi=\pi$, what are the probabilities of finding the photon at detectors A and B.

- Bonus questions (harder):

  2. When $\phi=\pi/4$, what are the probabilities of finding the photon at detectors A and B.

  3. Represent $\frac{1}{2}\begin{bmatrix} 1-e^{i\phi} \\ i(1+e^{i\phi}) \end{bmatrix}$ in polar co-ordinates

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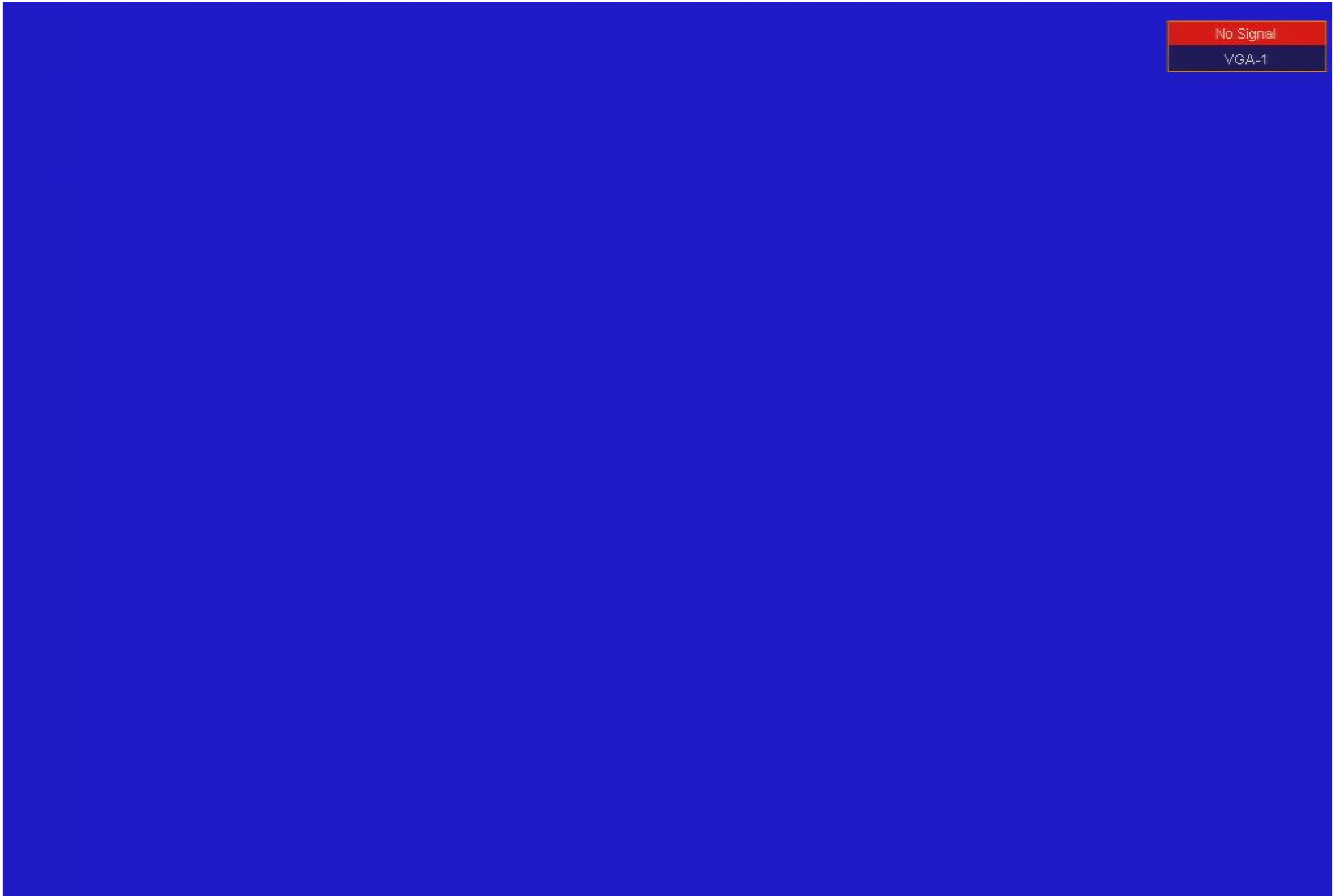
No Signal
VGA-1

No Signal

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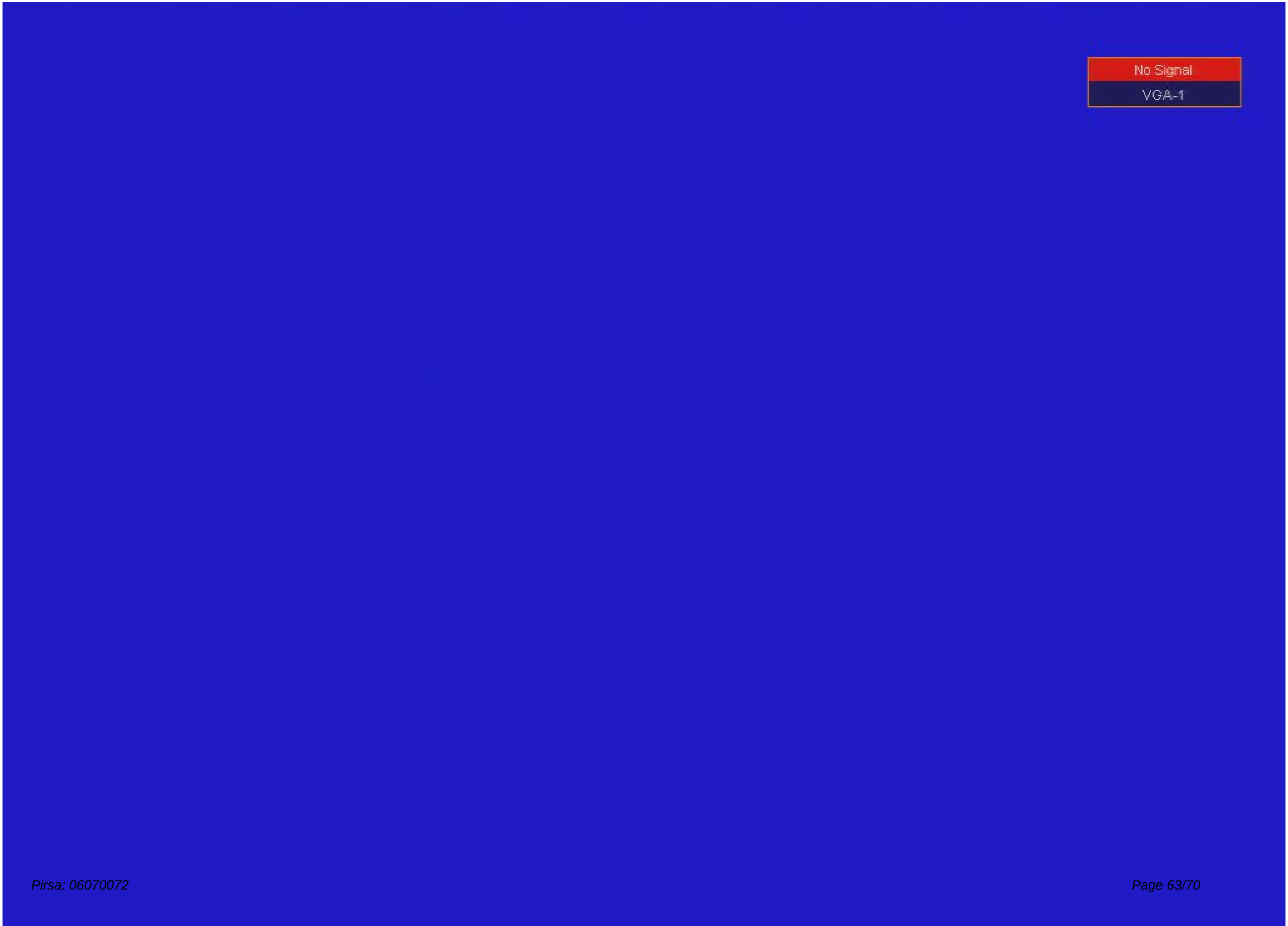
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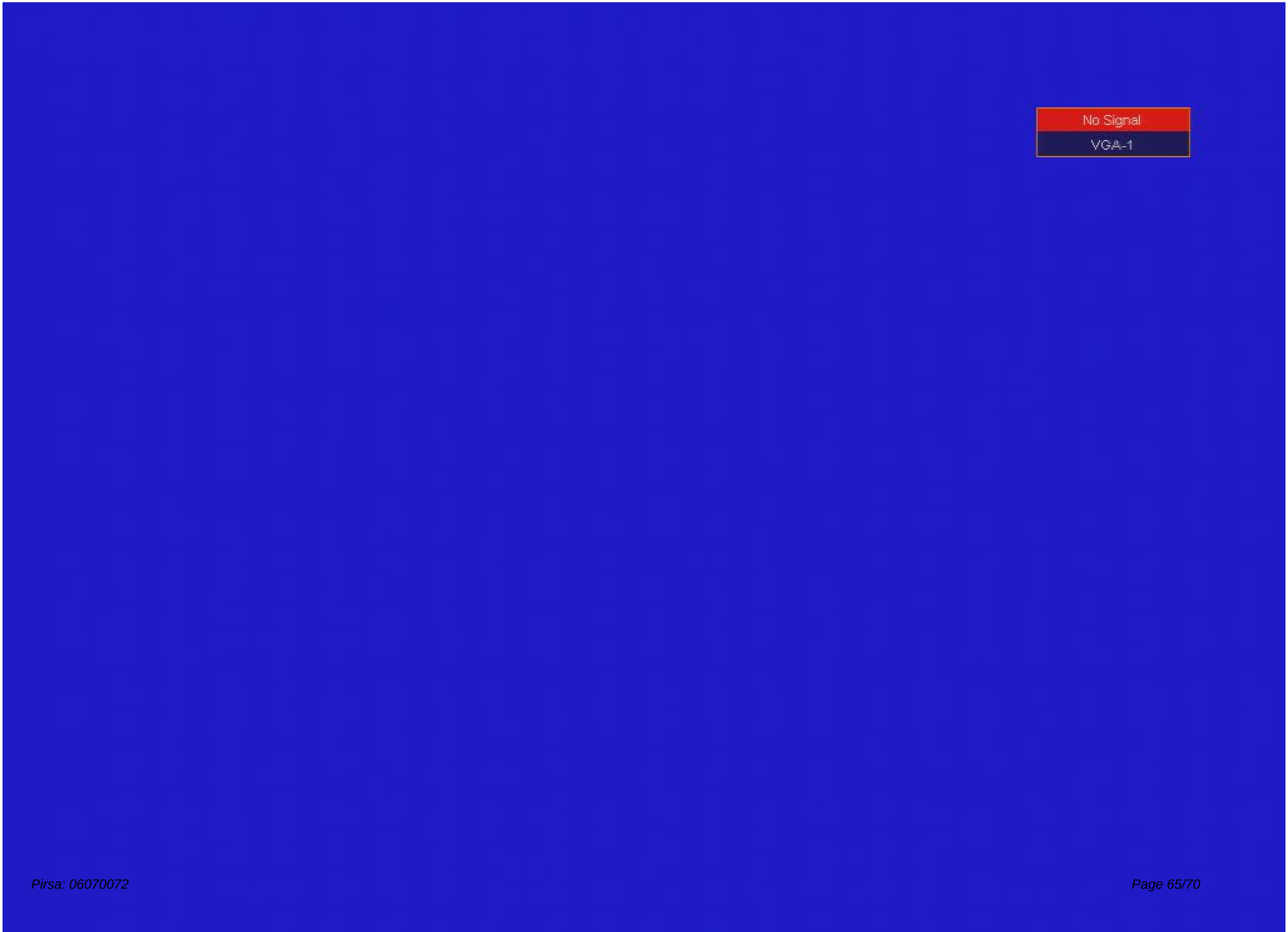
No Signal
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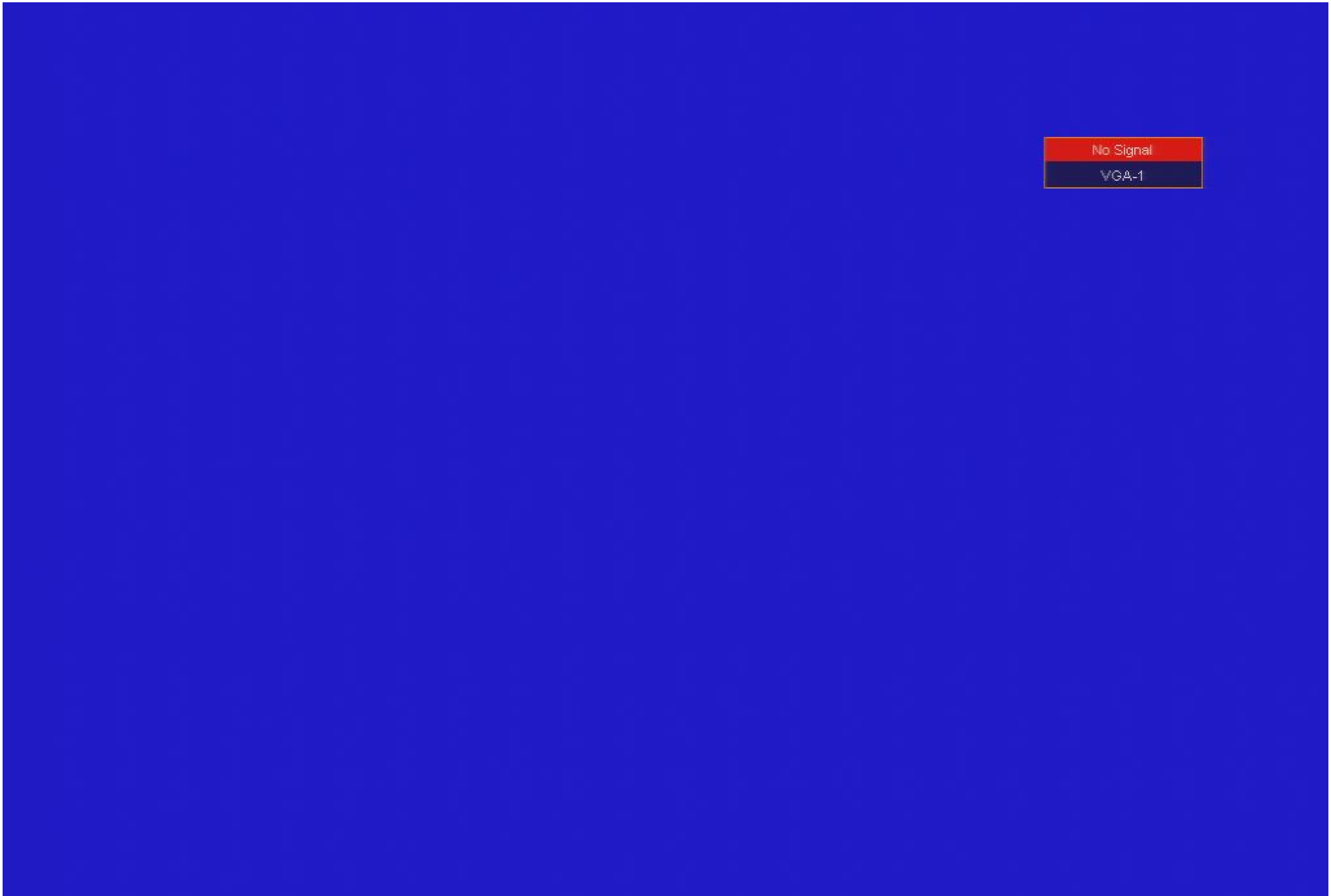
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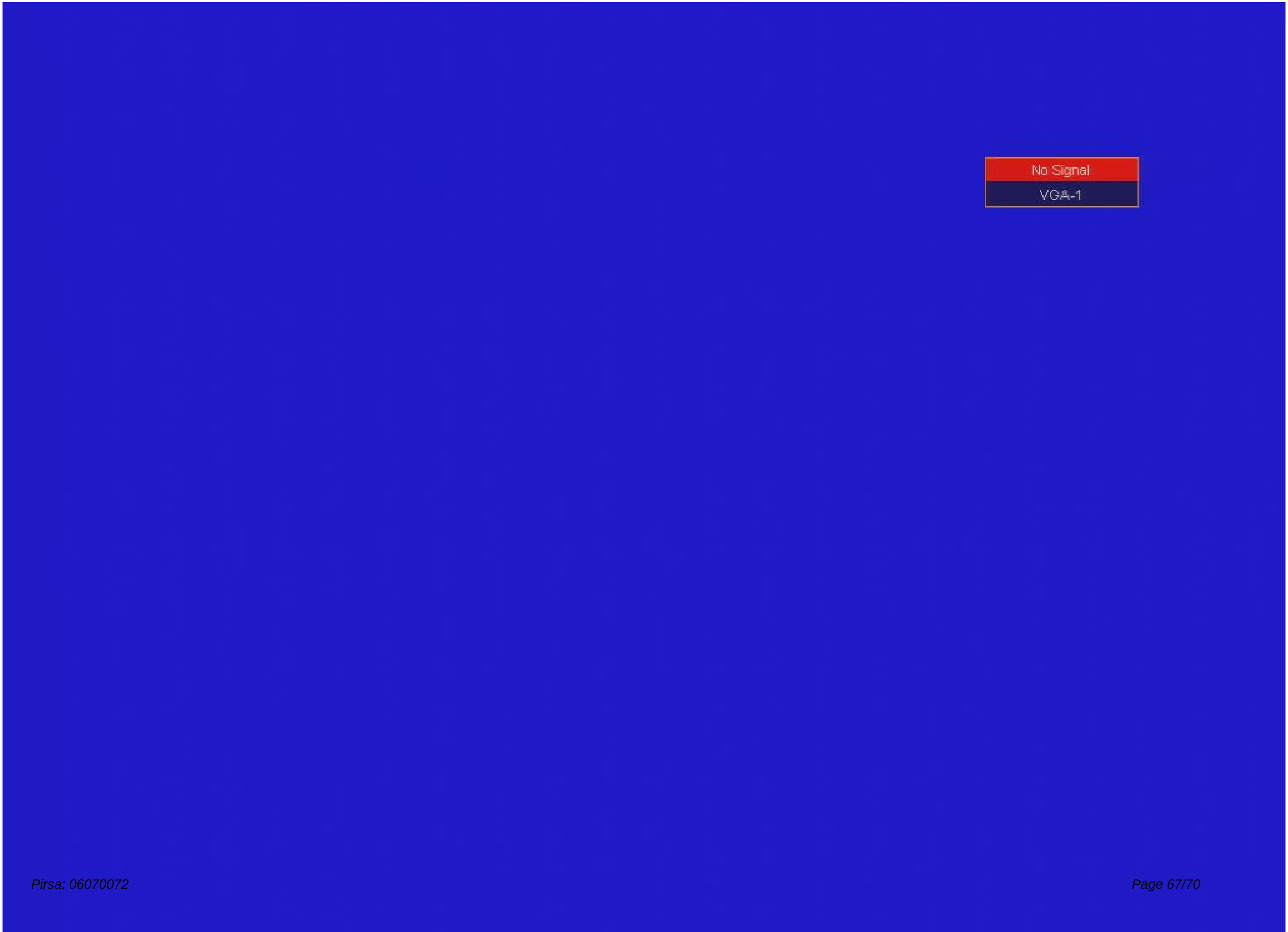
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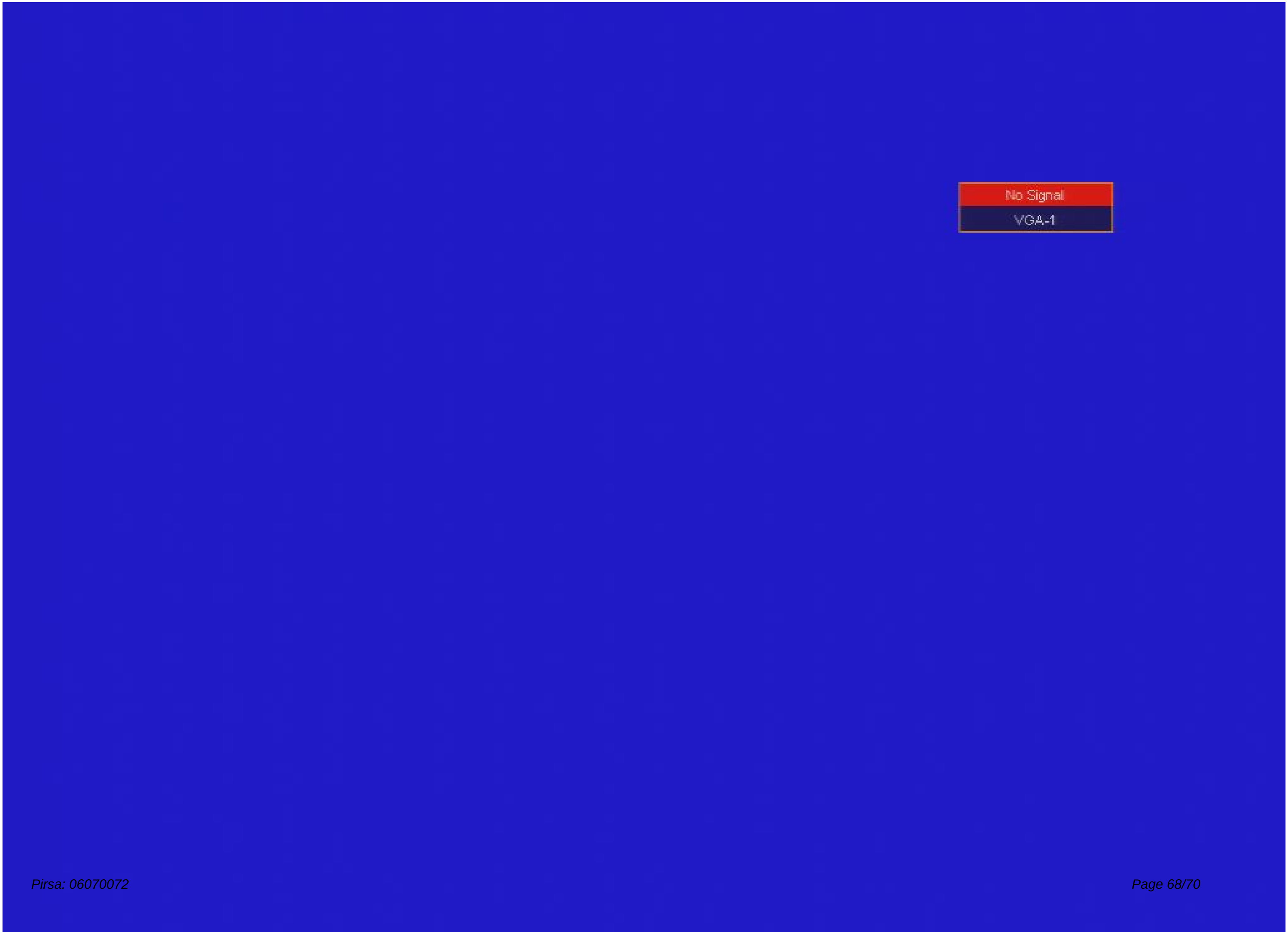
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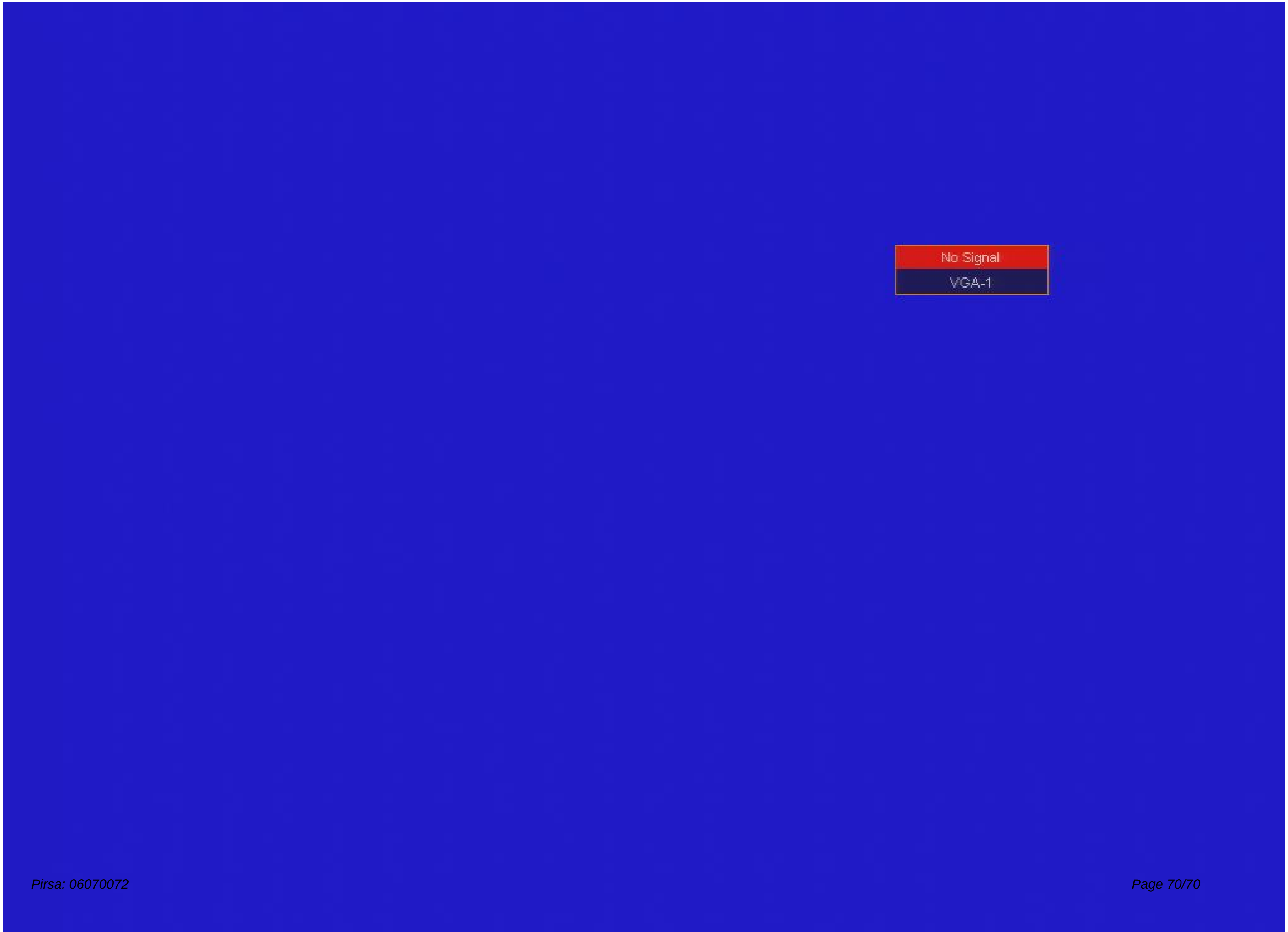
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