

Title: The Weird World of Quantum Physics - Part 2

Date: Jul 25, 2006 01:00 PM

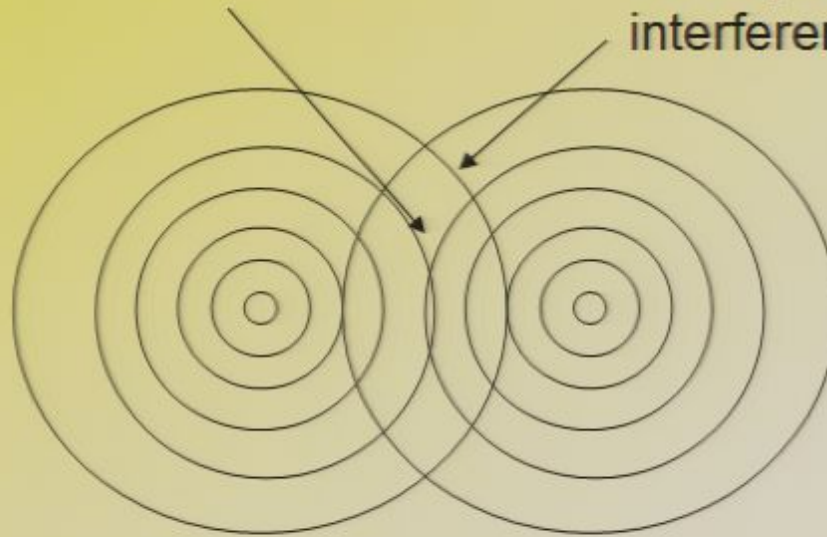
URL: <http://pirsa.org/06070070>

Abstract:

Compare with water waves

in phase: constructive interference

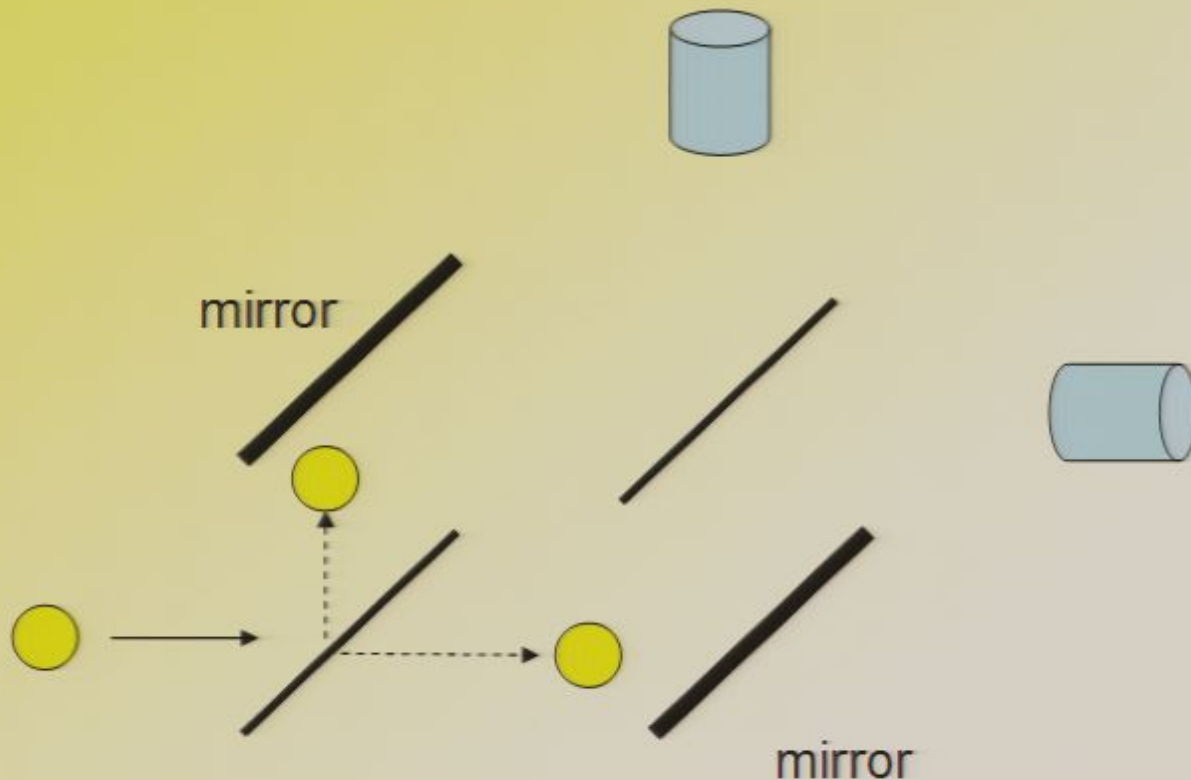
out of phase: destructive interference



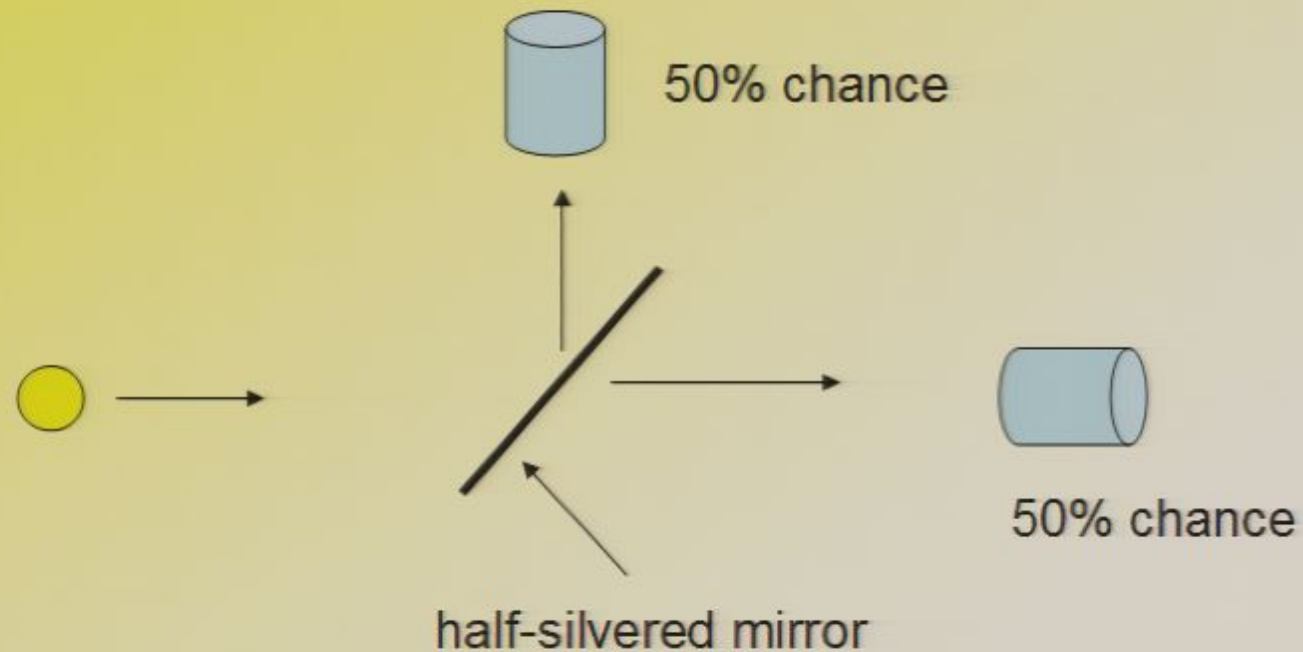
Ripples on a pond

Why?

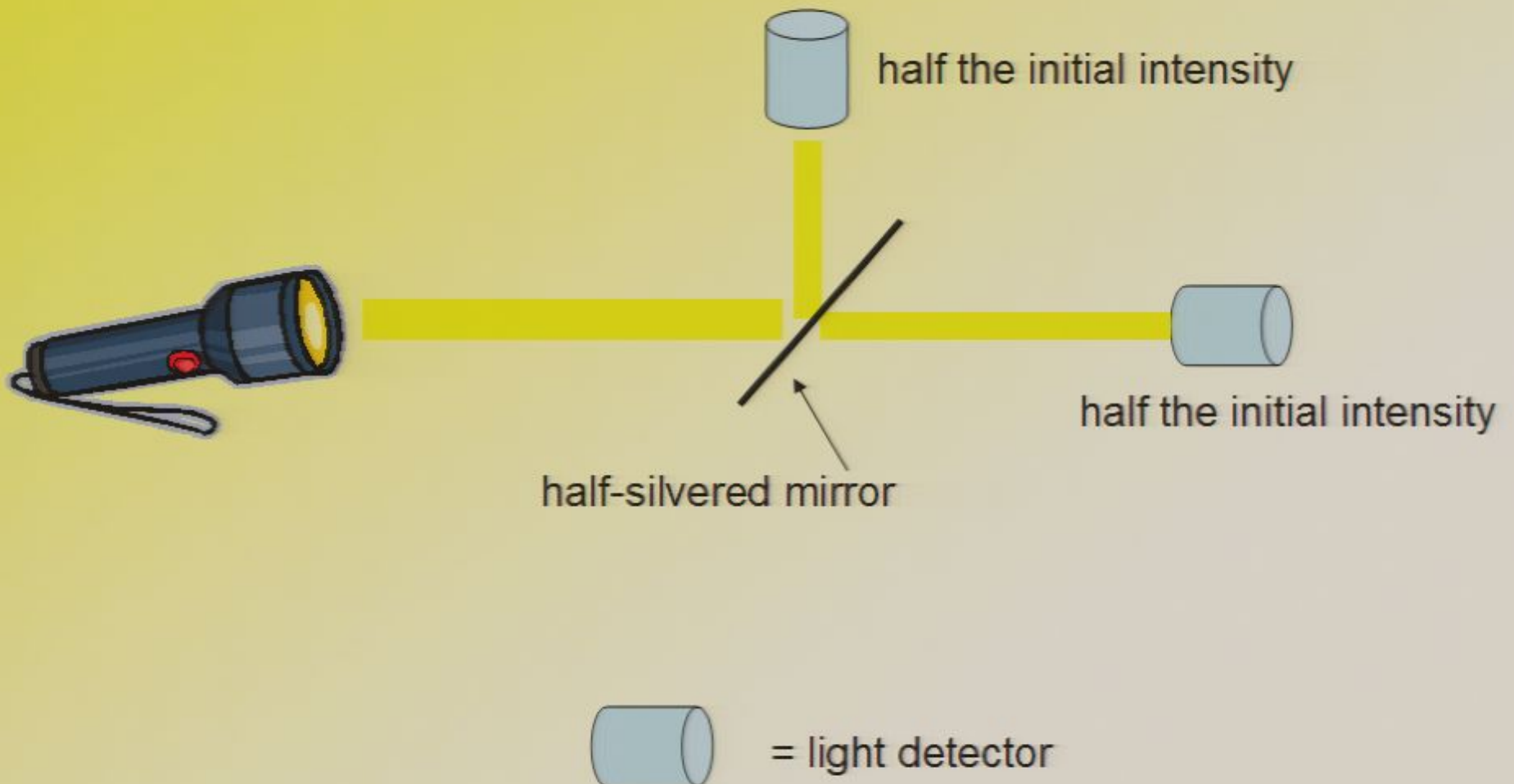
- Each individual photon takes *both* paths.
- Two possible paths to reach each detector
- Overlap between the two leads to constructive or destructive interference (helps to think of wave interference)



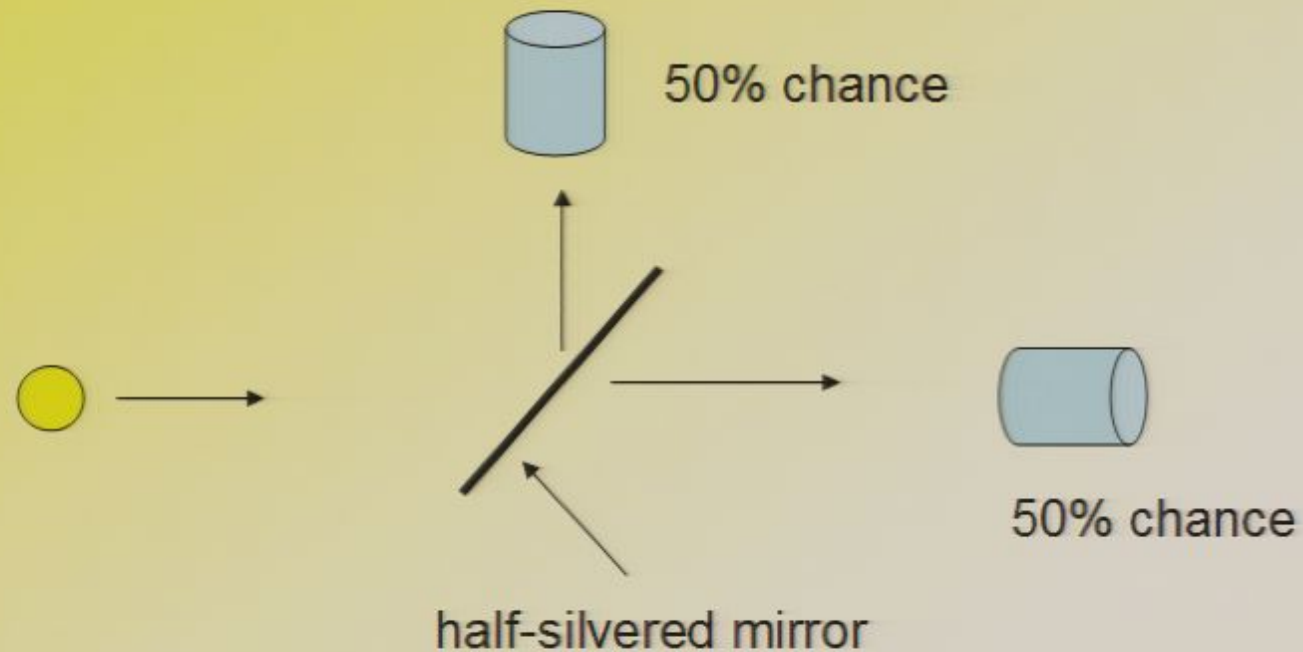
A single photon incident on a half-silvered mirror.



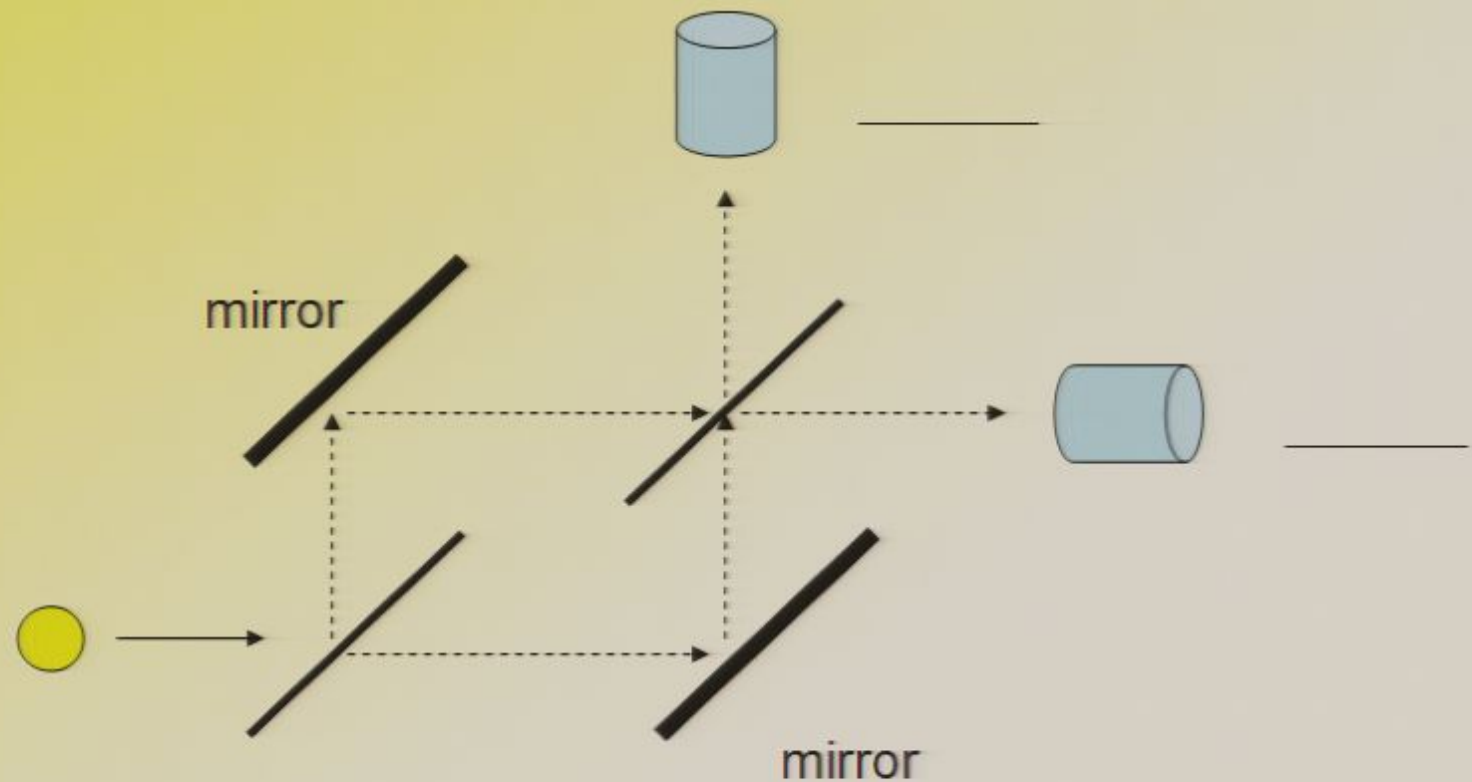
Light shining on a half-silvered mirror



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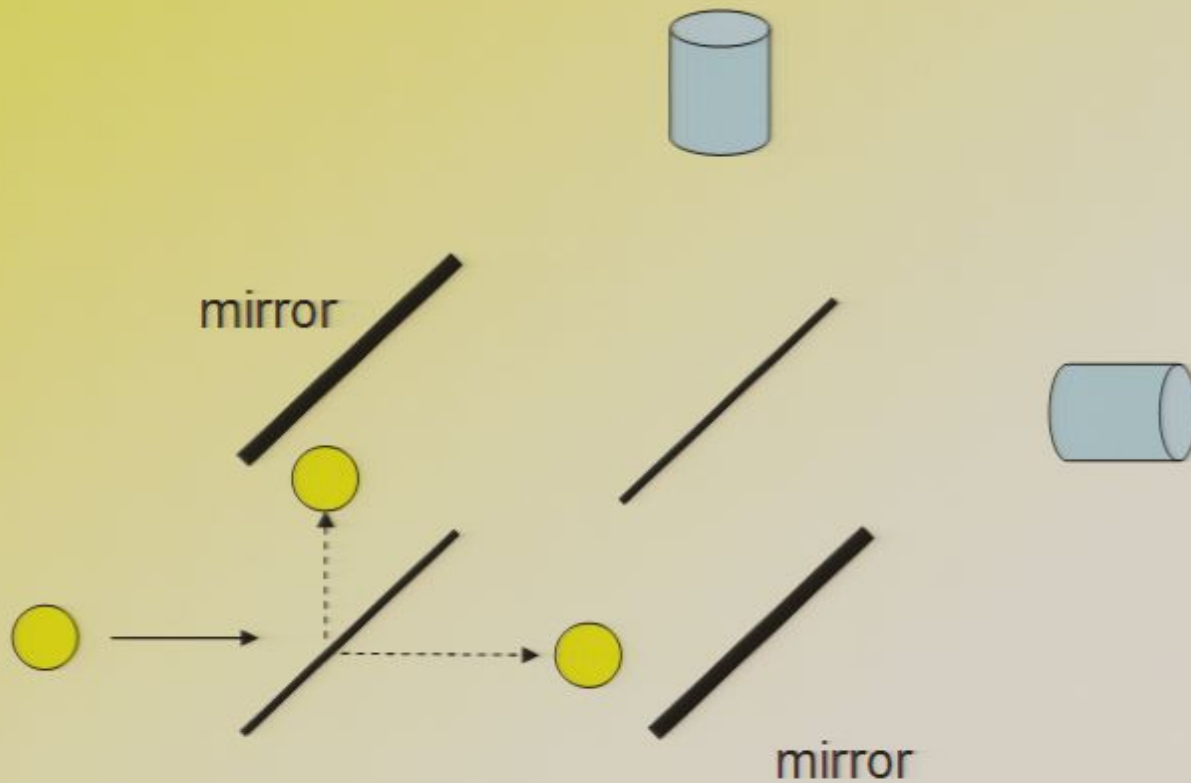


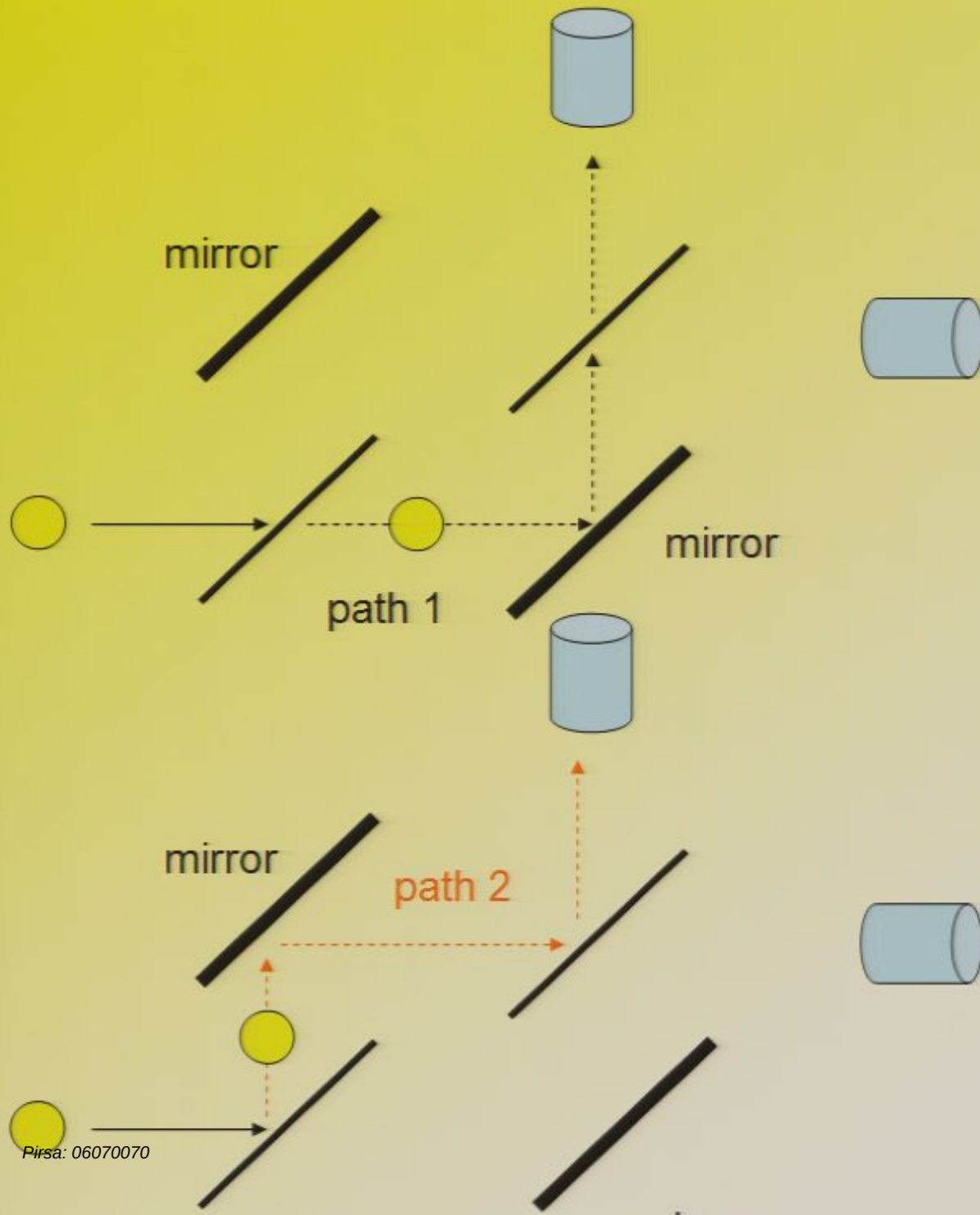
What does quantum theory give?



Why?

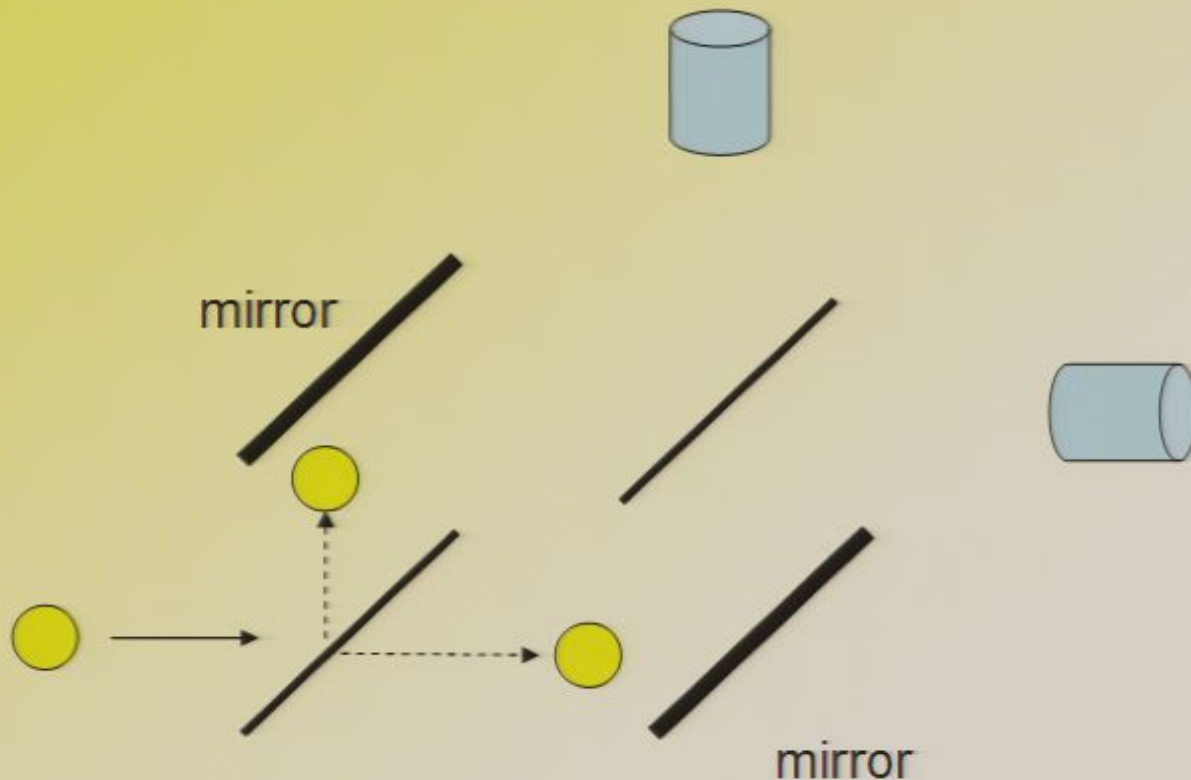
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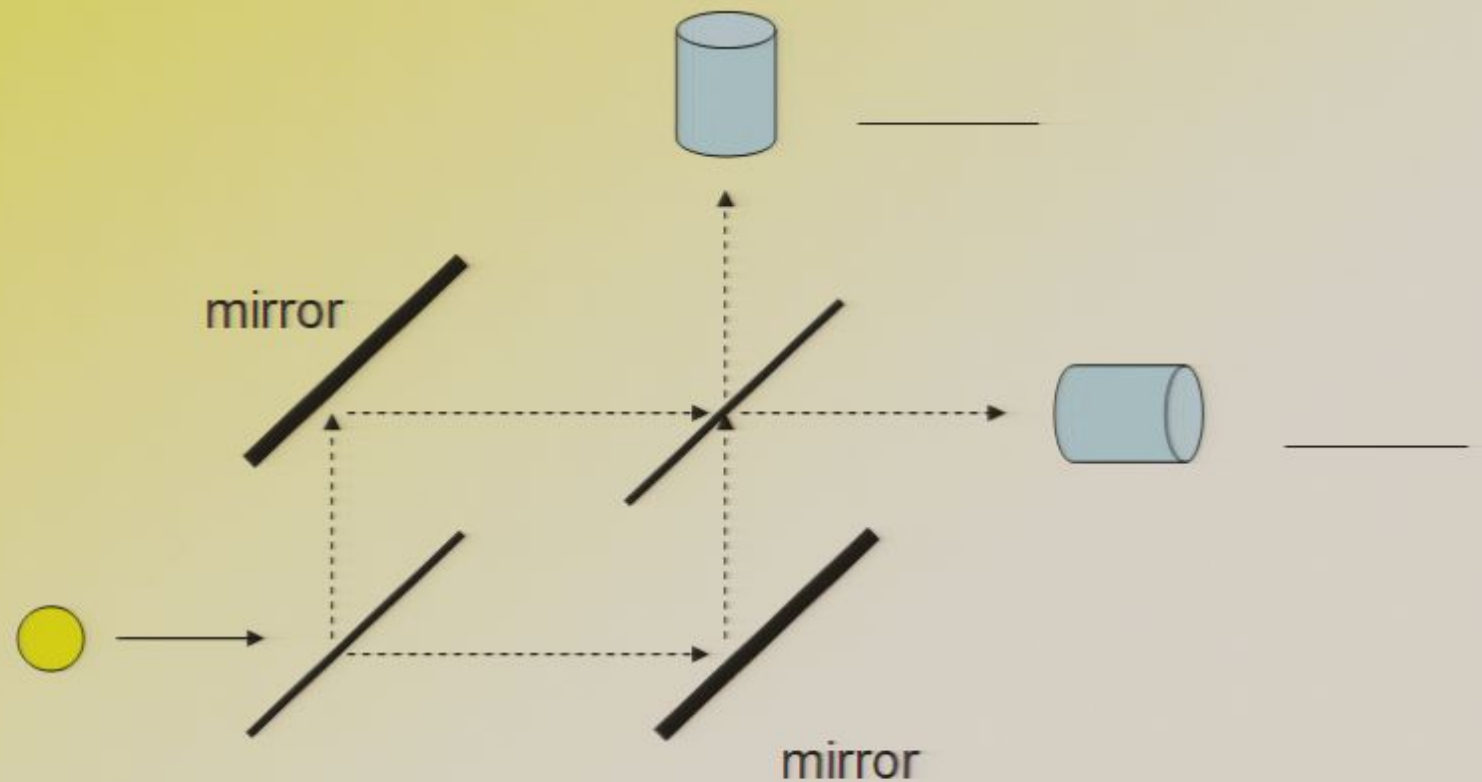


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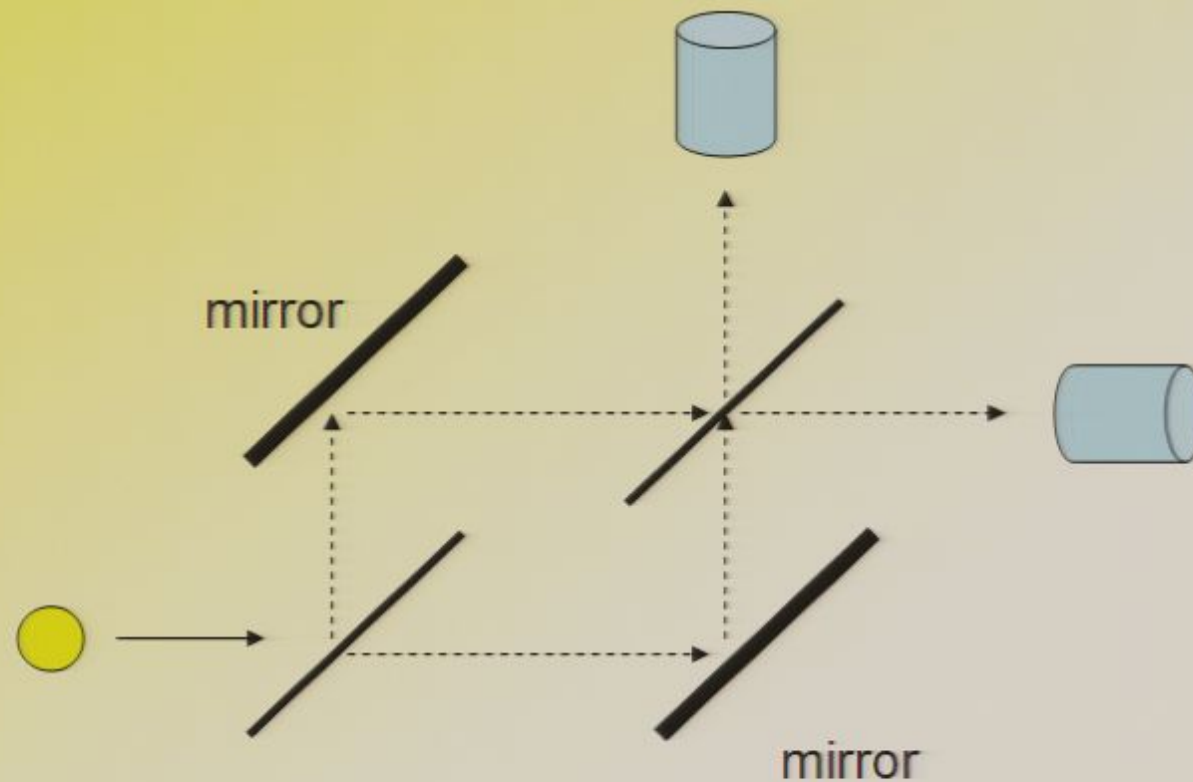
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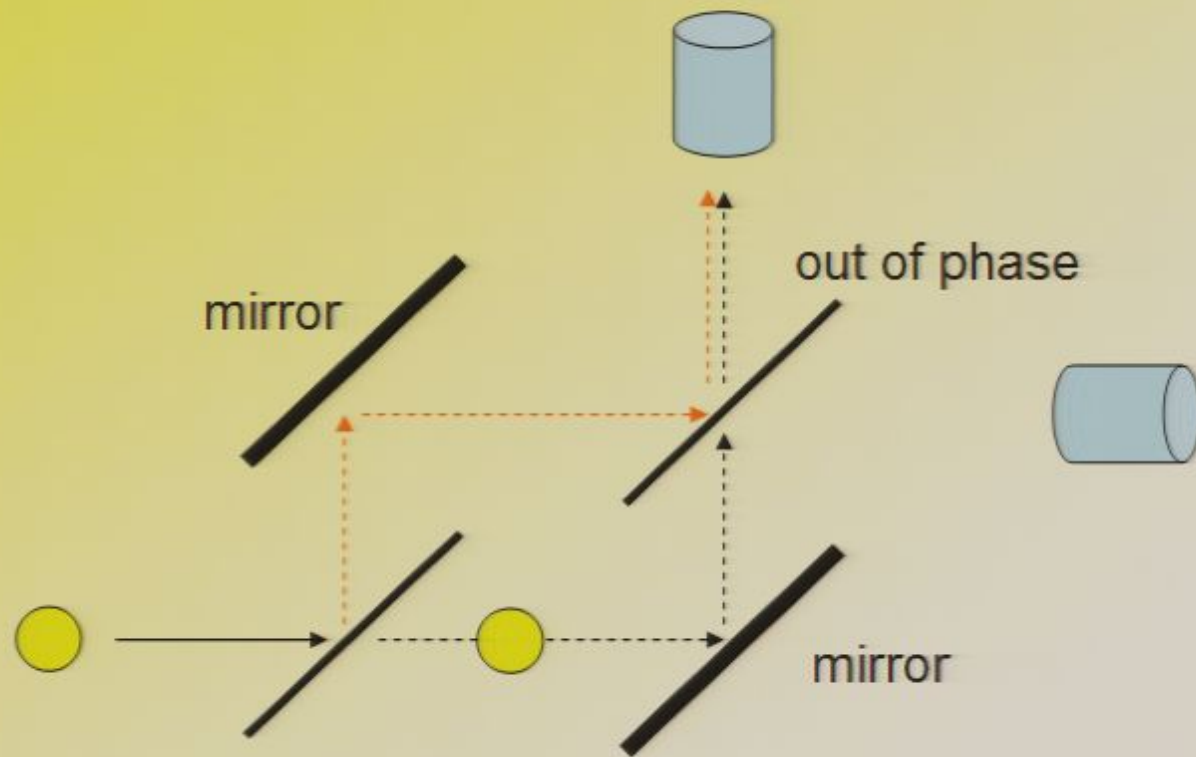


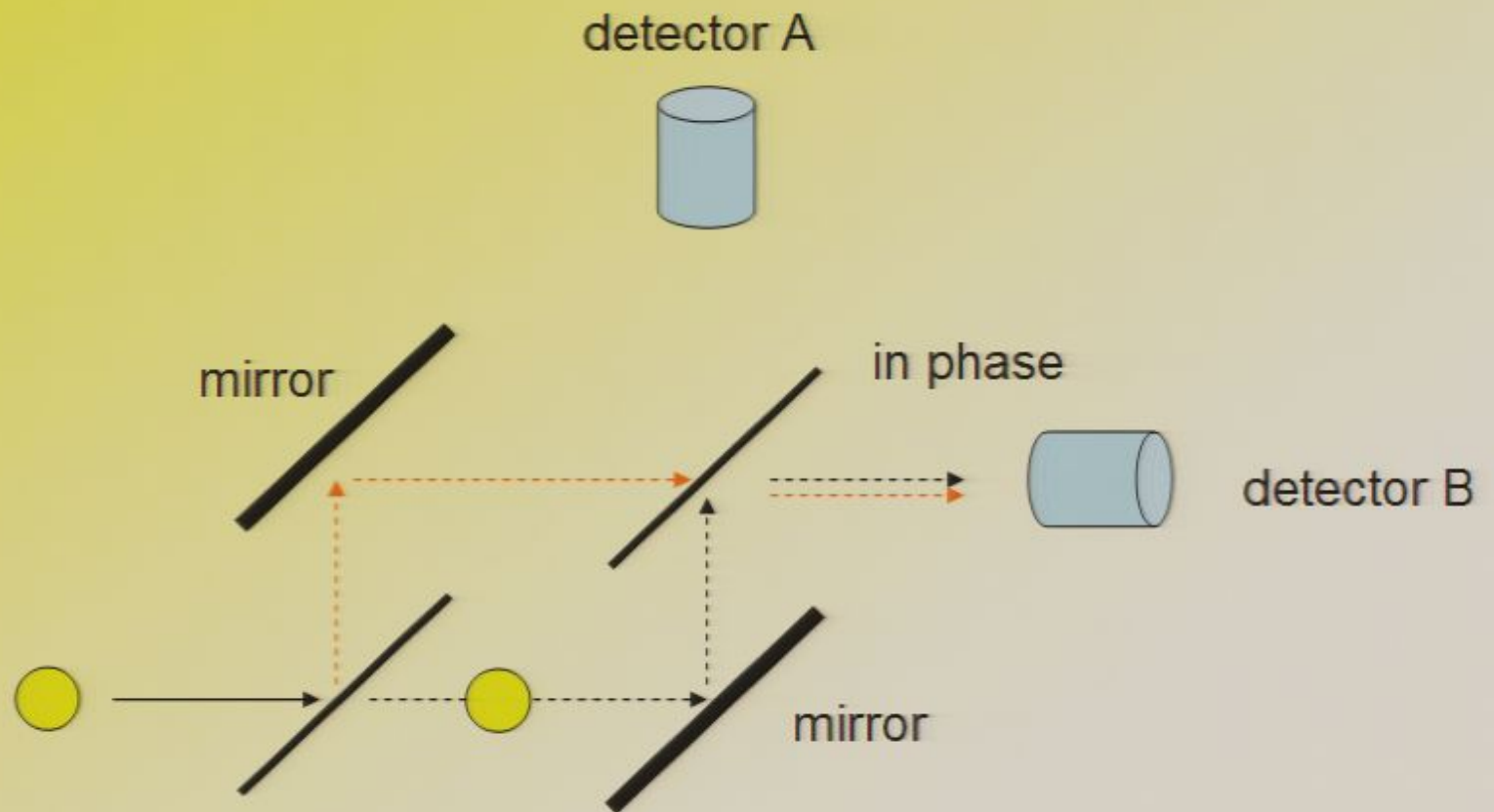
Two half-silvered mirrors

- STUDENT ACTIVITY:

Determine the probabilities of finding a photon at each of the two detectors.





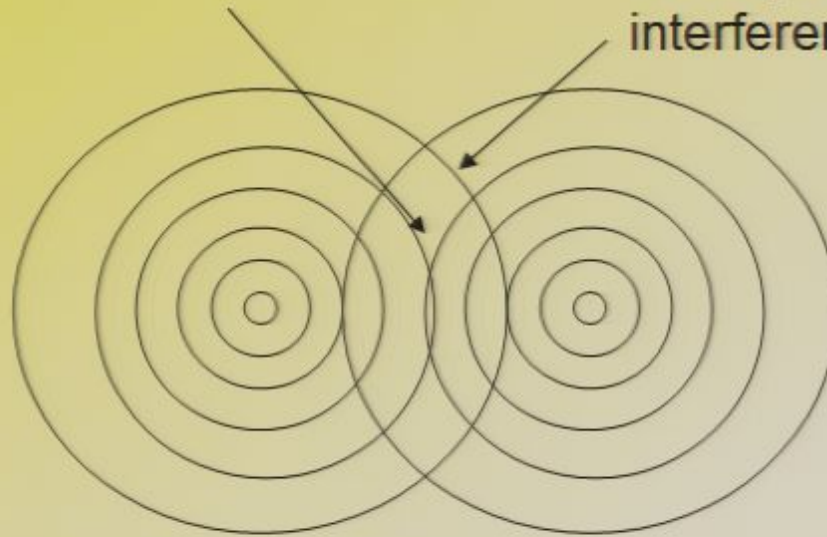


- N.B. We always detect or measure just one photon at detector B, not two as we only ever input one photon into the experiment.

Compare with water waves

in phase: constructive interference

out of phase: destructive interference



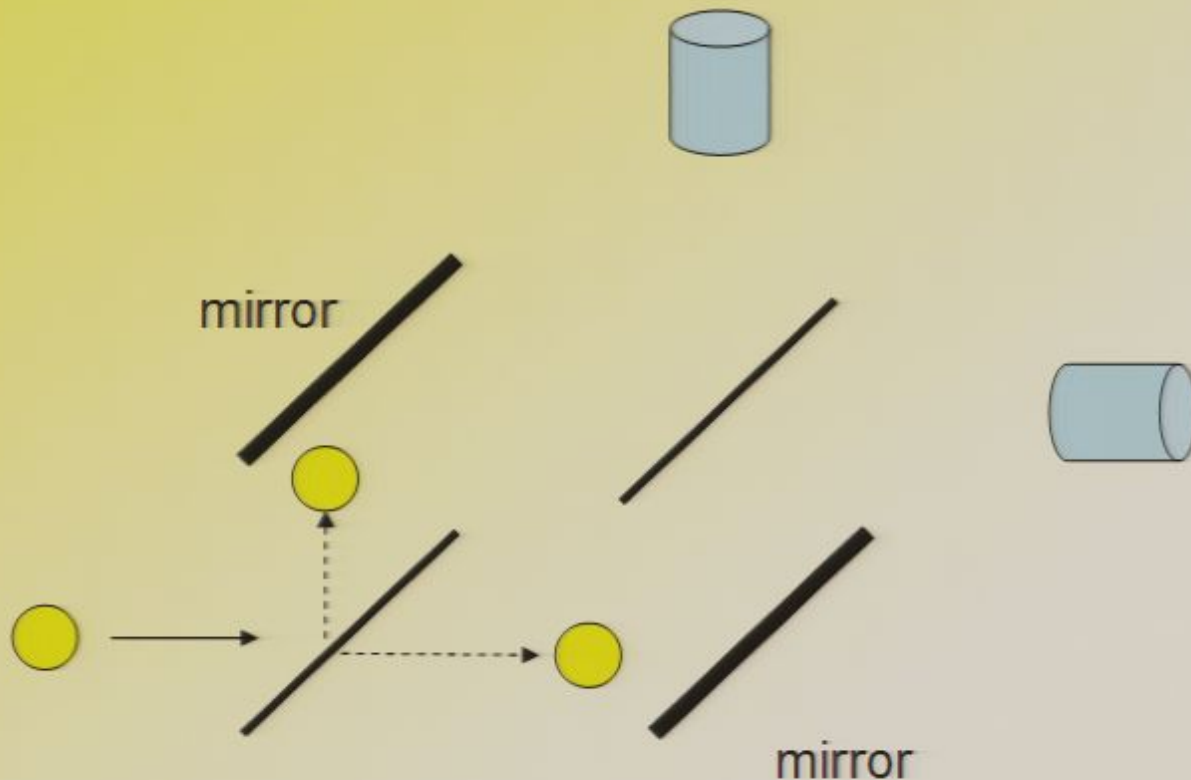
Ripples on a pond

Many worlds interpretation

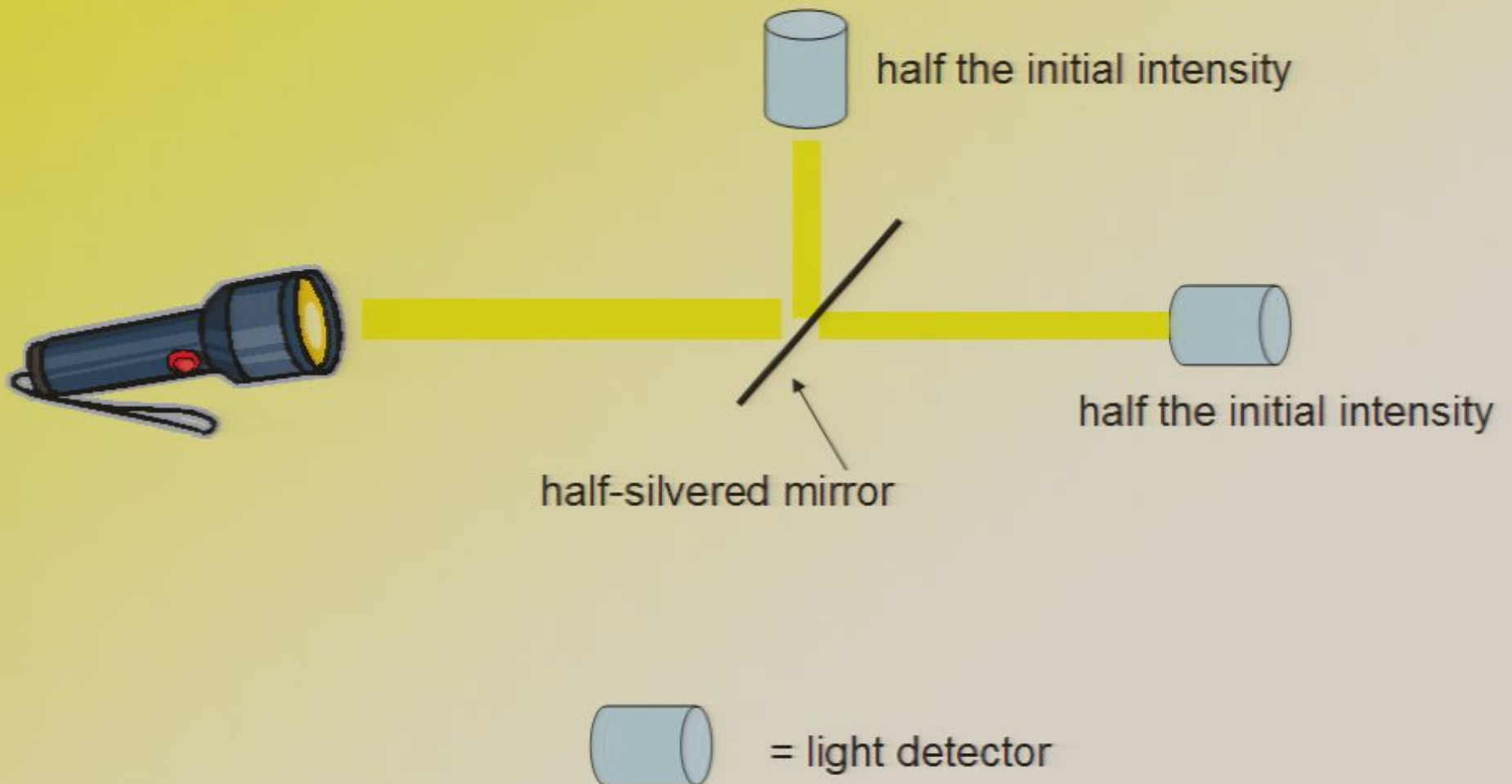


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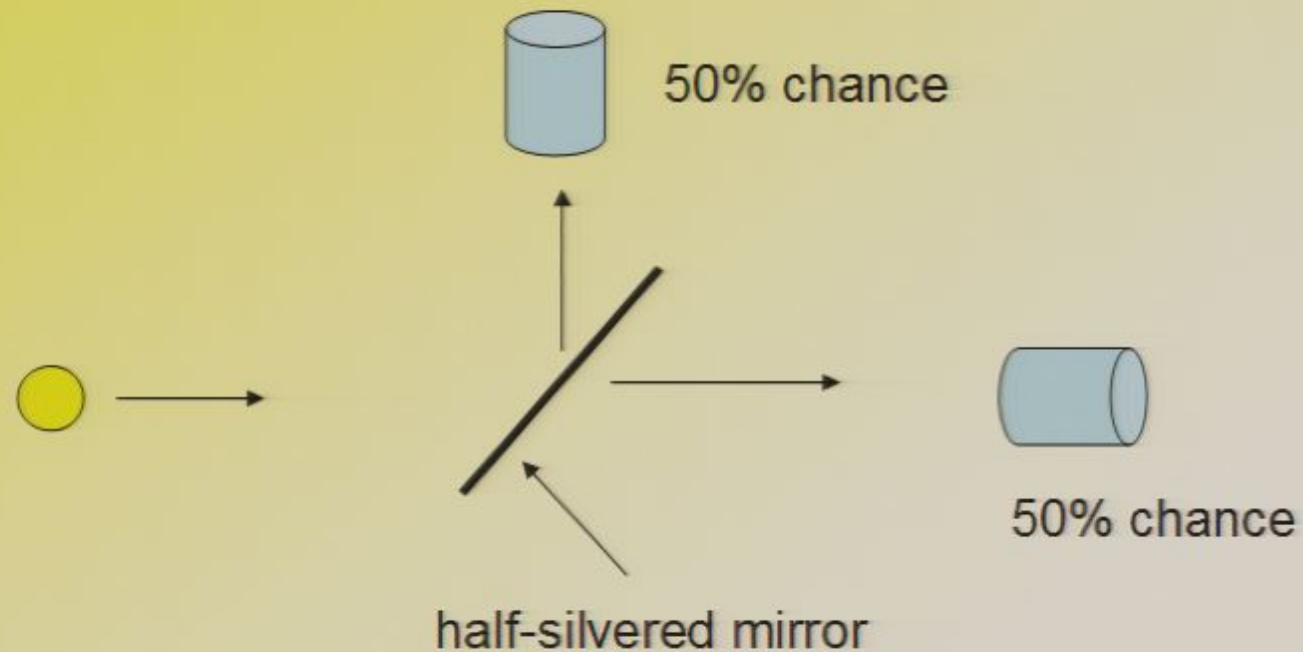
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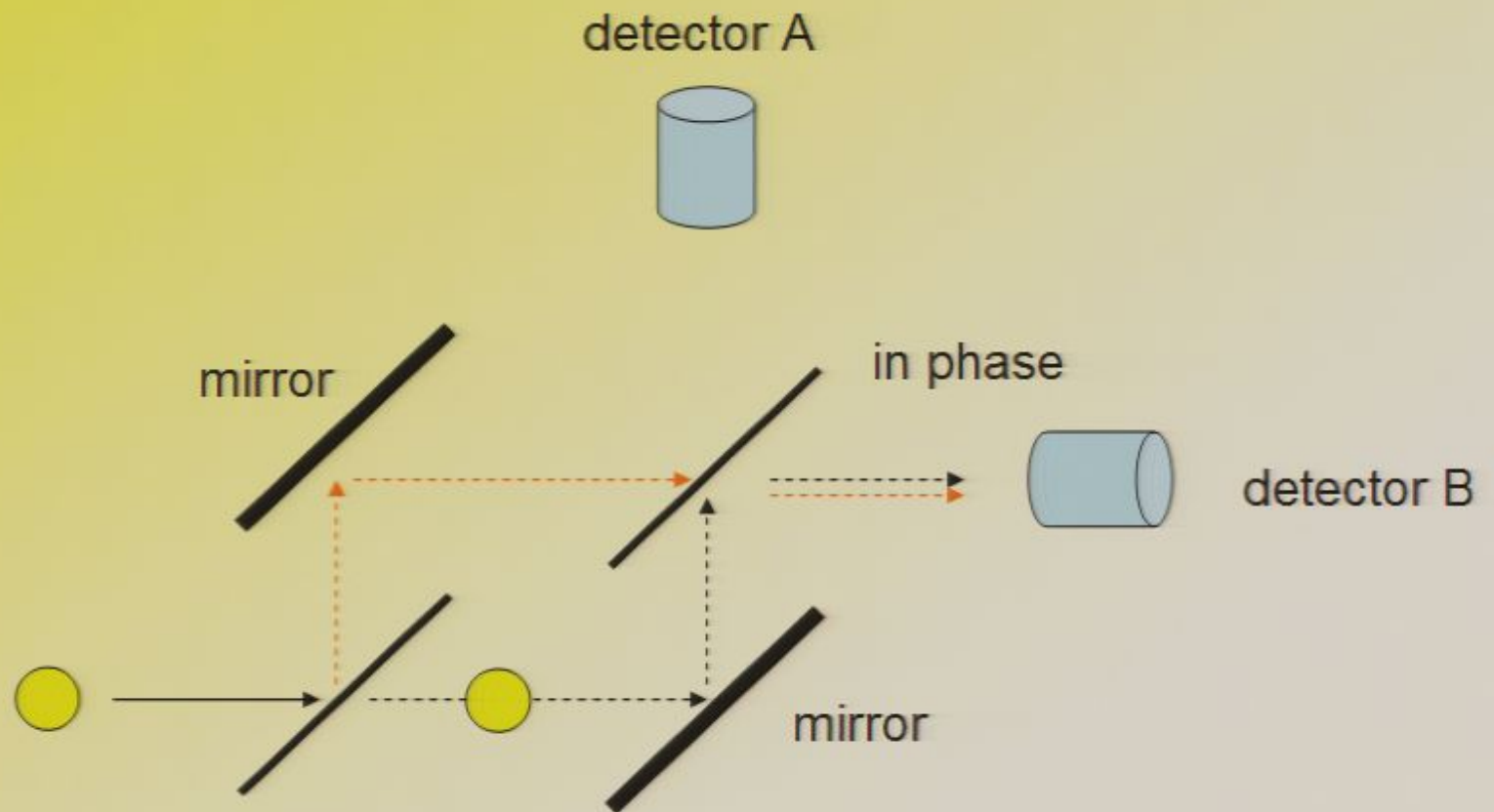


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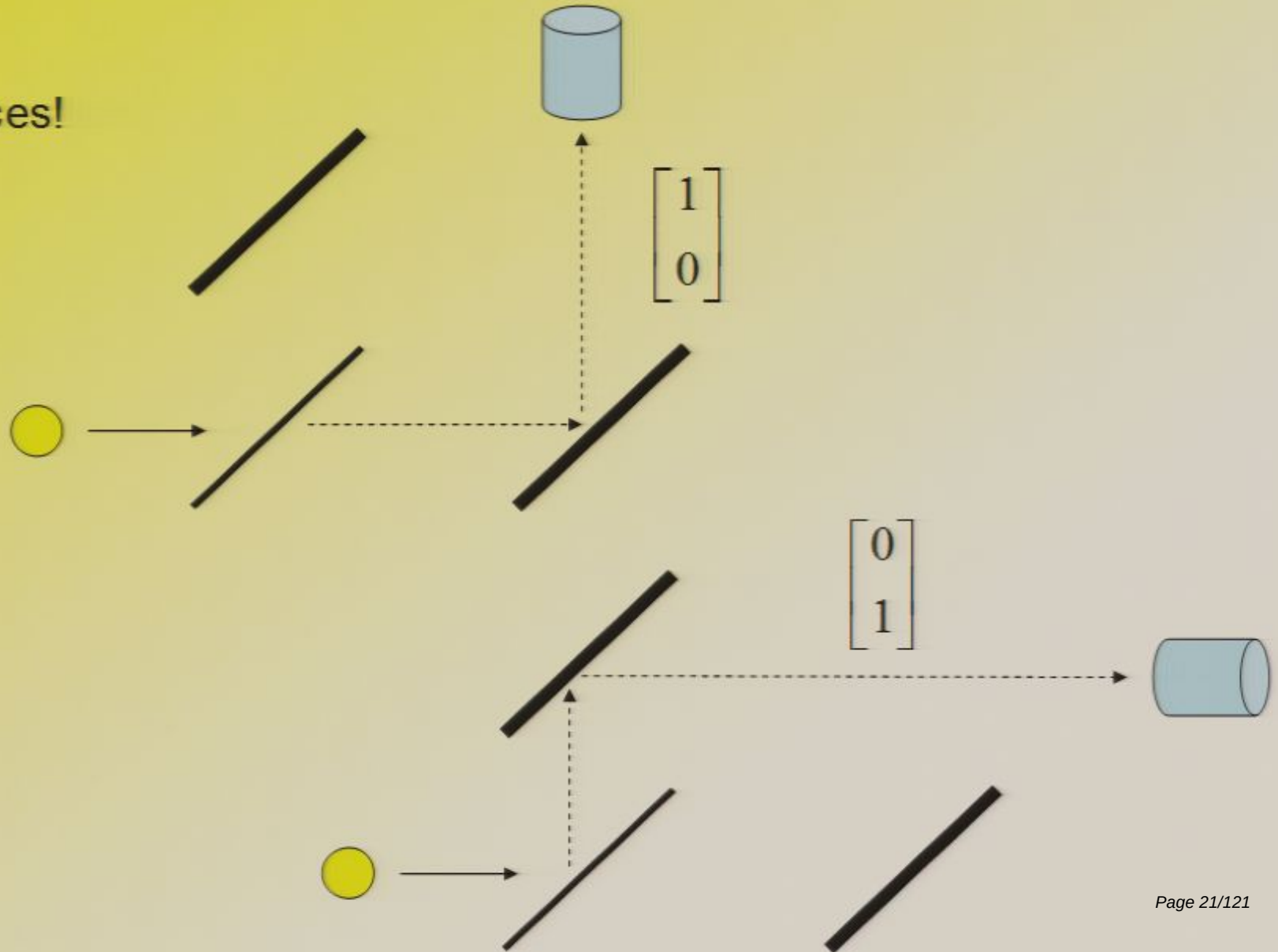




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Explaining things mathematically

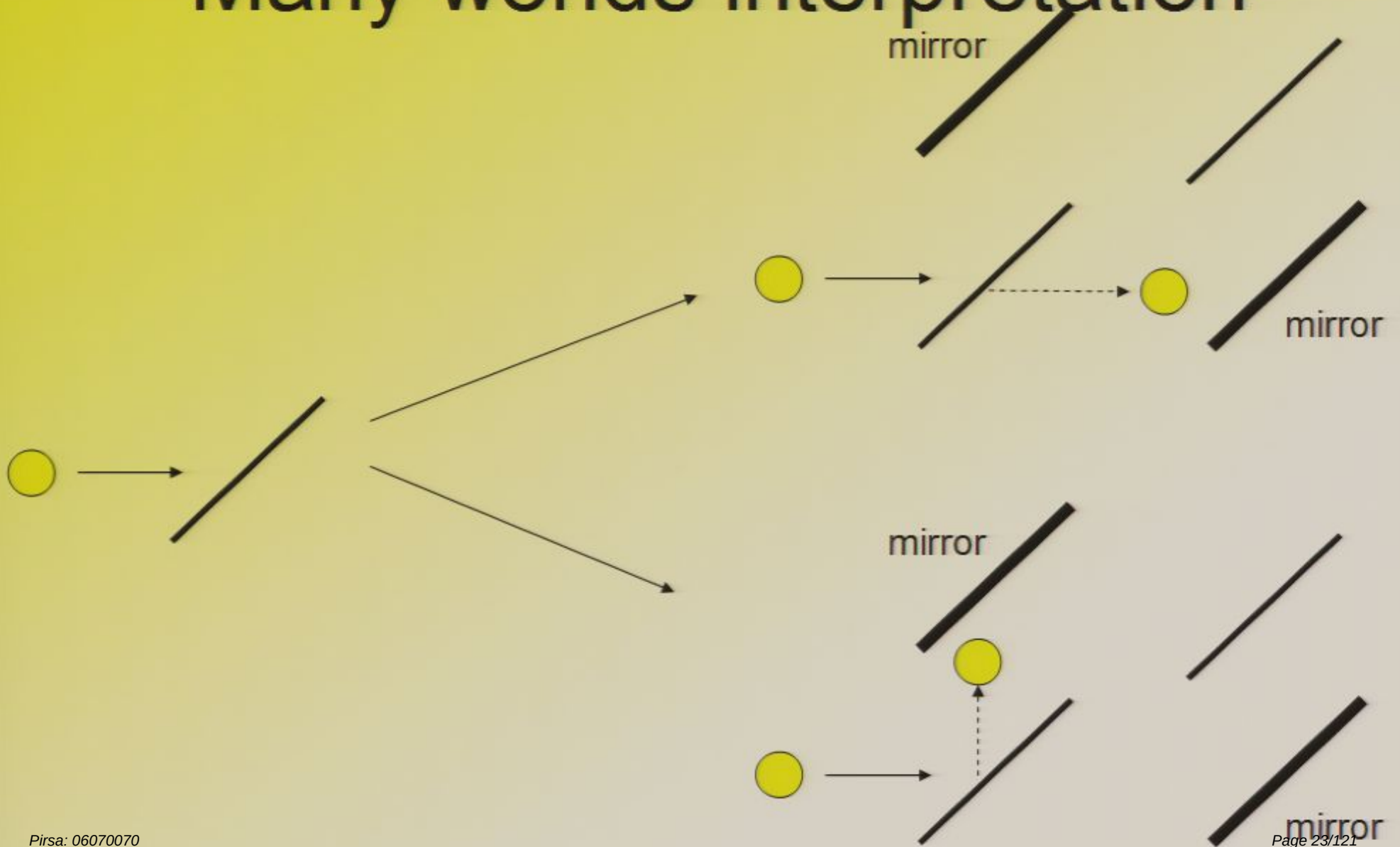
- Matrices!



Many worlds interpretation

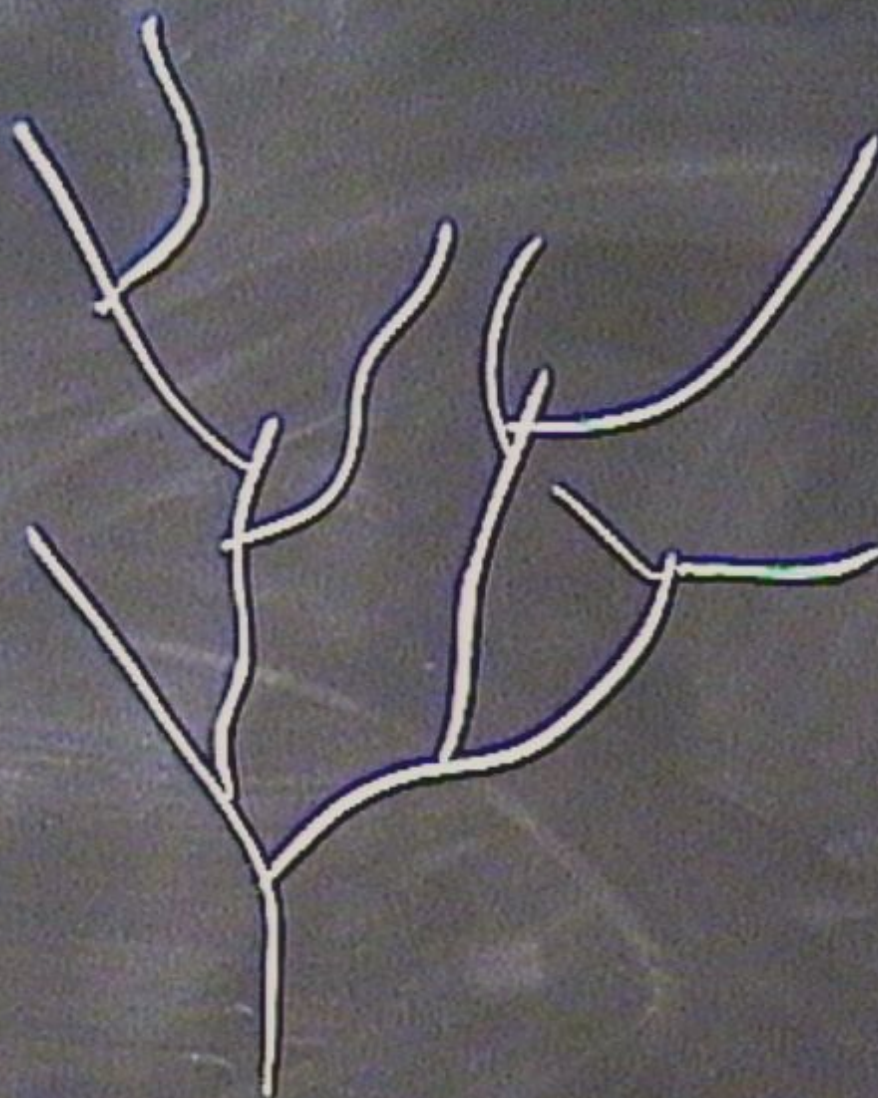


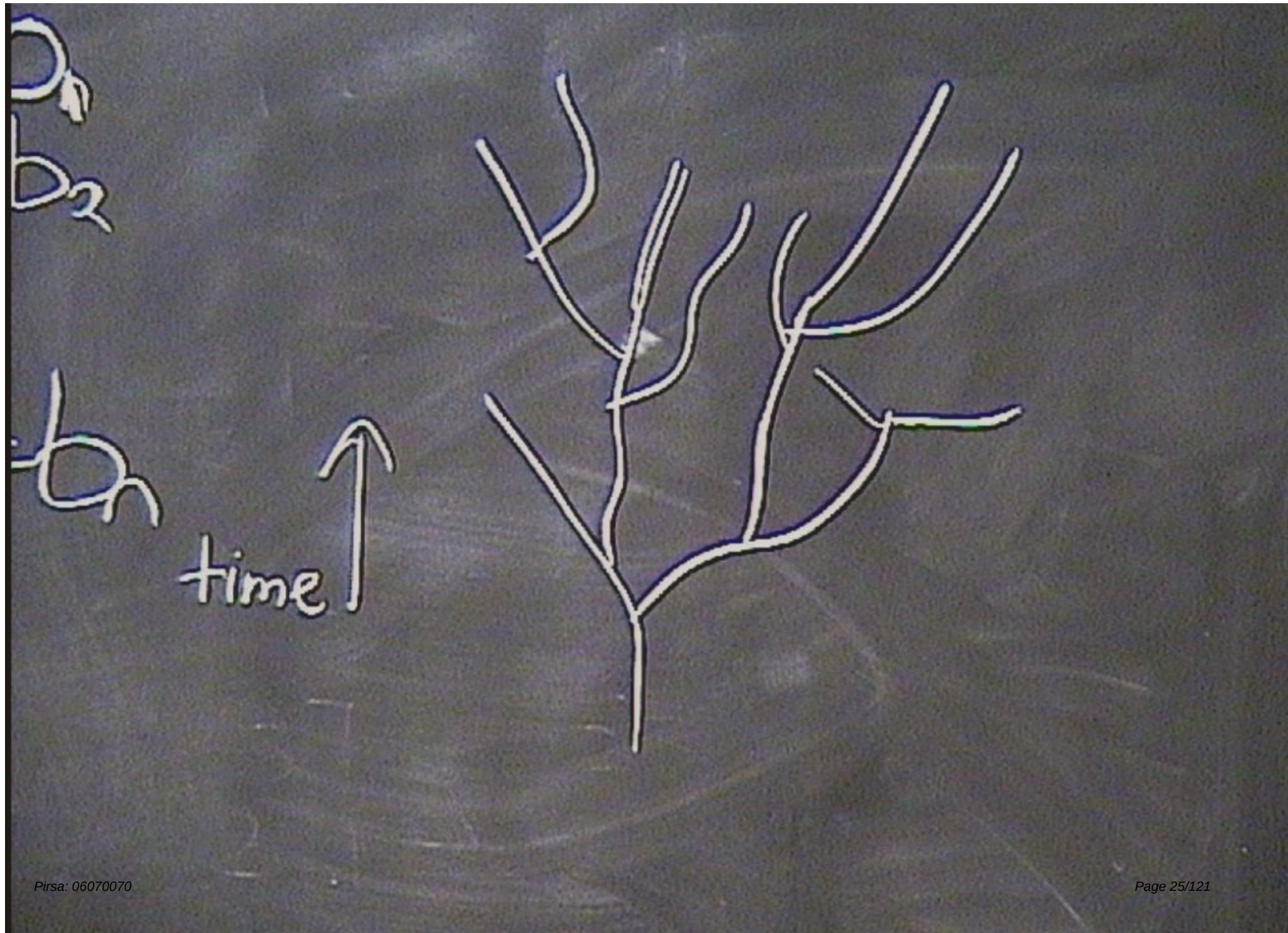
Many worlds interpretation



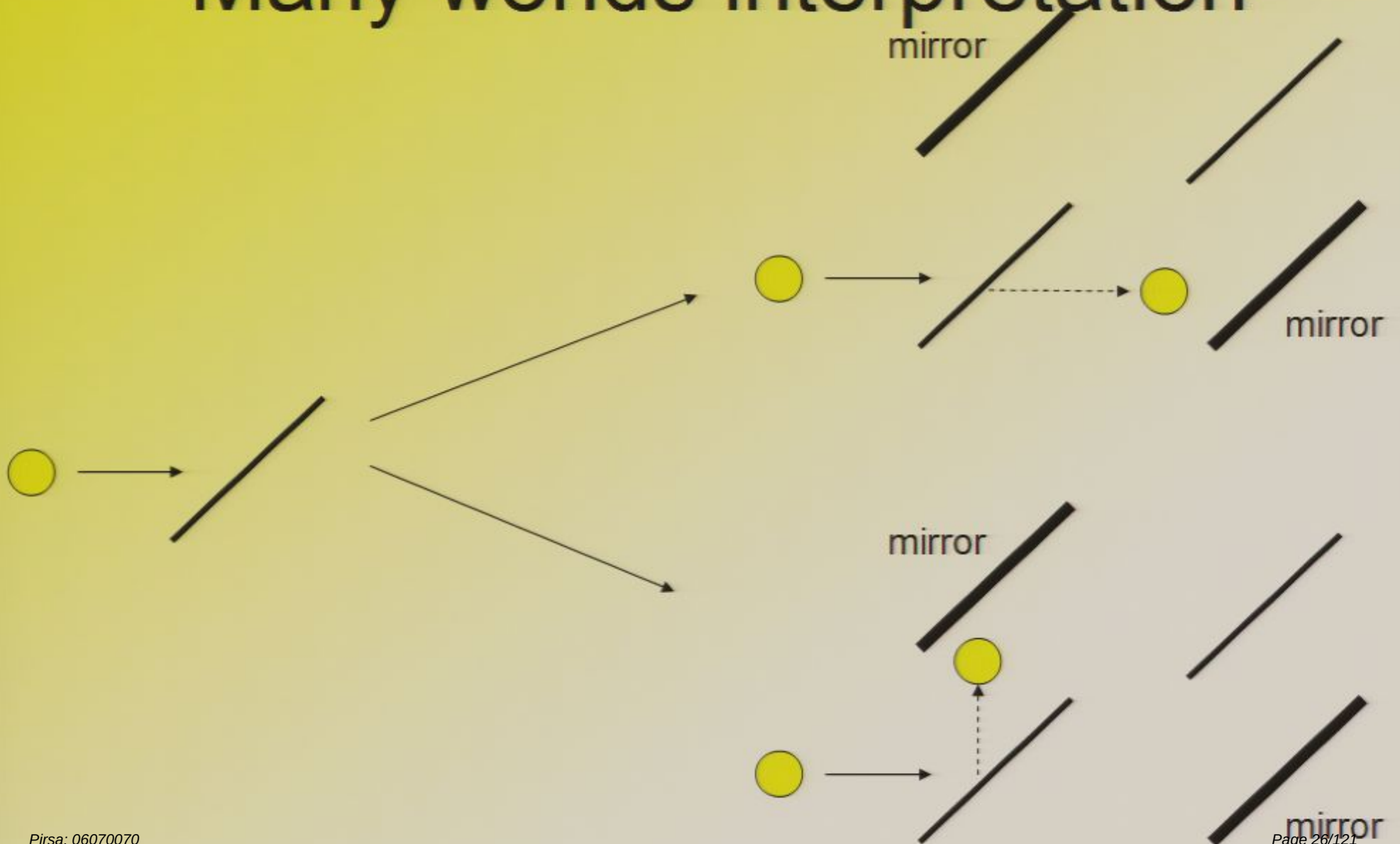
b_1
 b_2
 b_n

time ↑



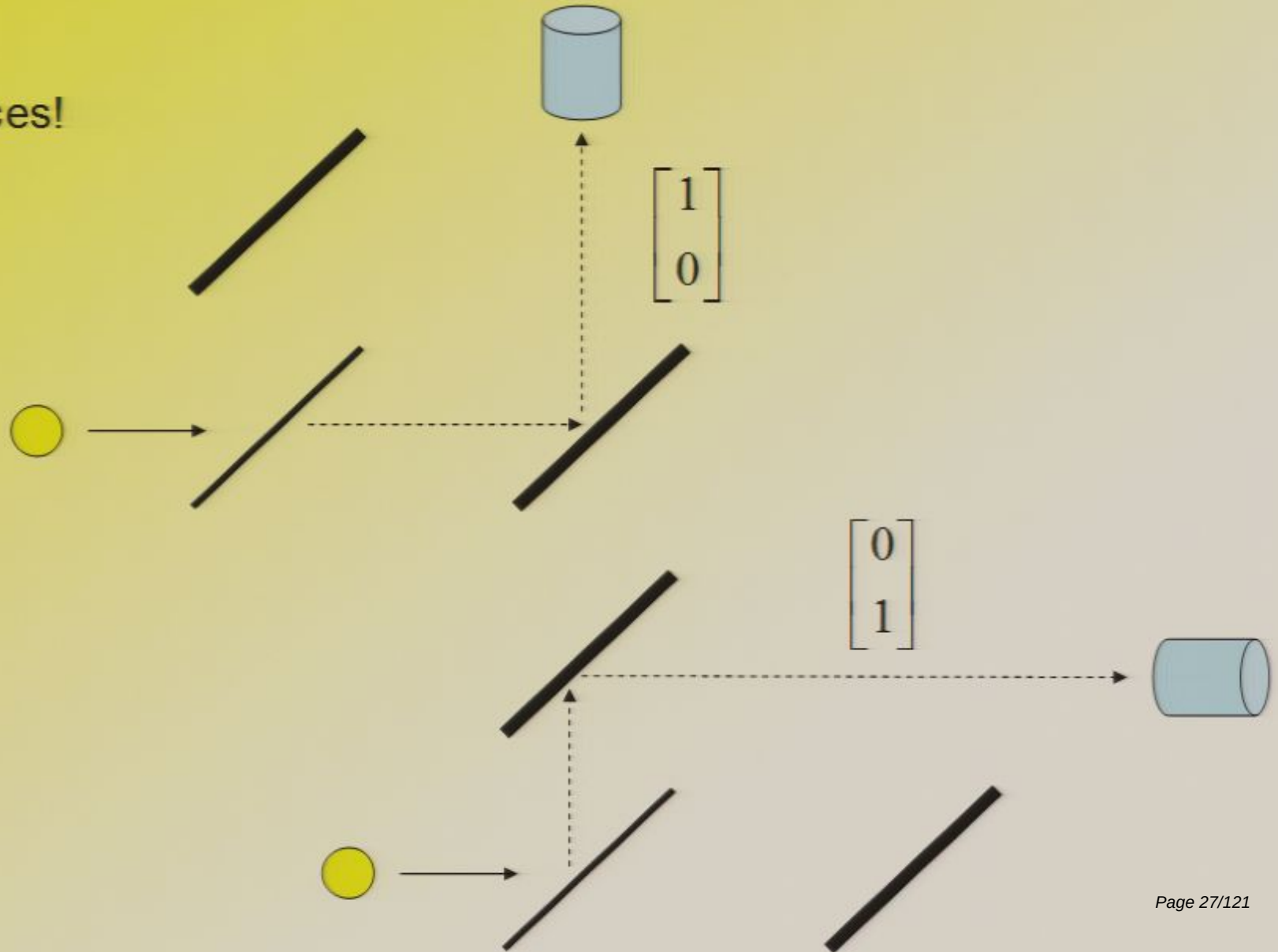


Many worlds interpretation



Explaining things mathematically

- Matrices!

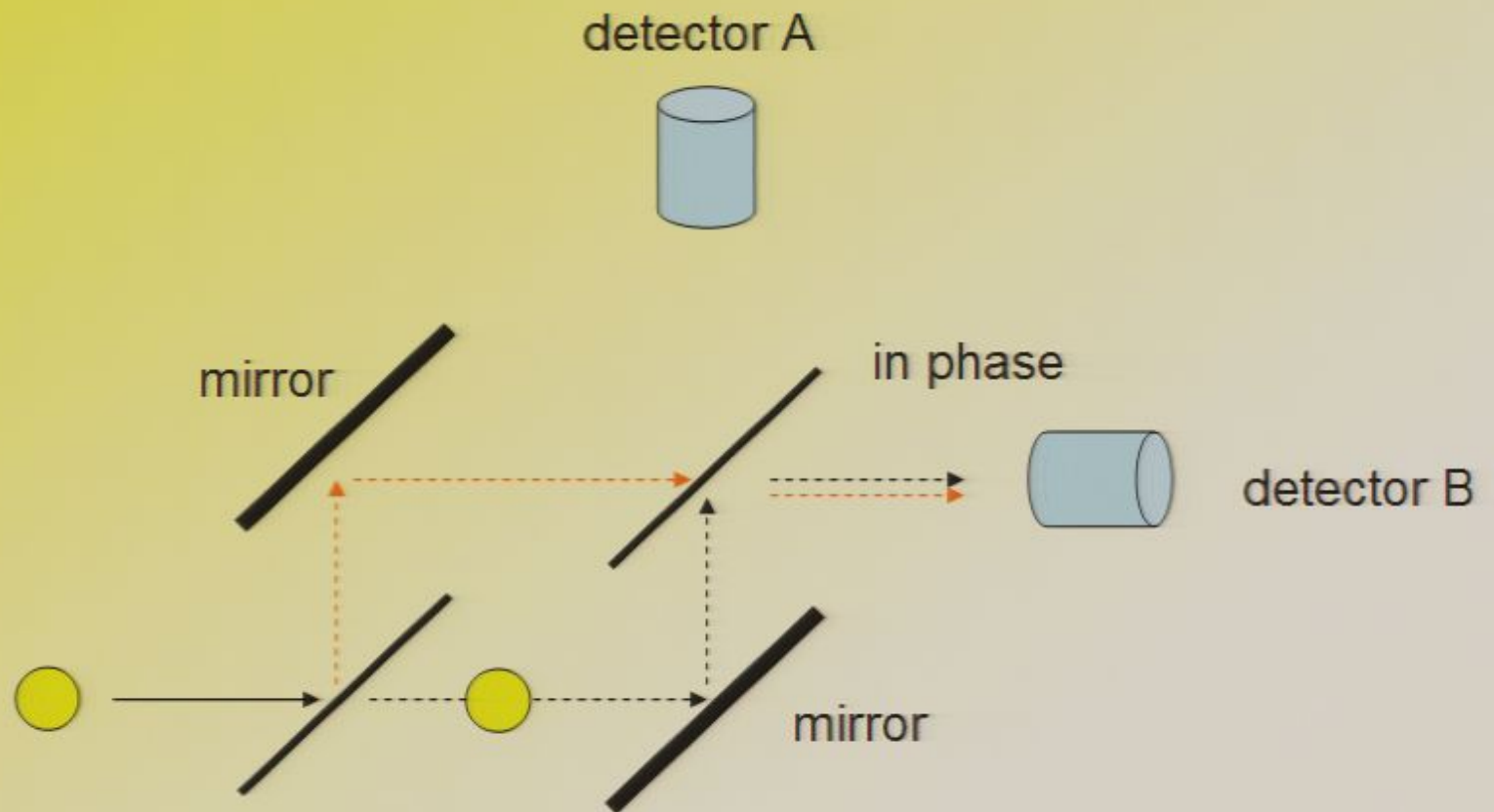


- In Newtonian physics, these are only two possible states of affairs. Surely, each photon must either take the first path or the second one. These appear to be the only two options.

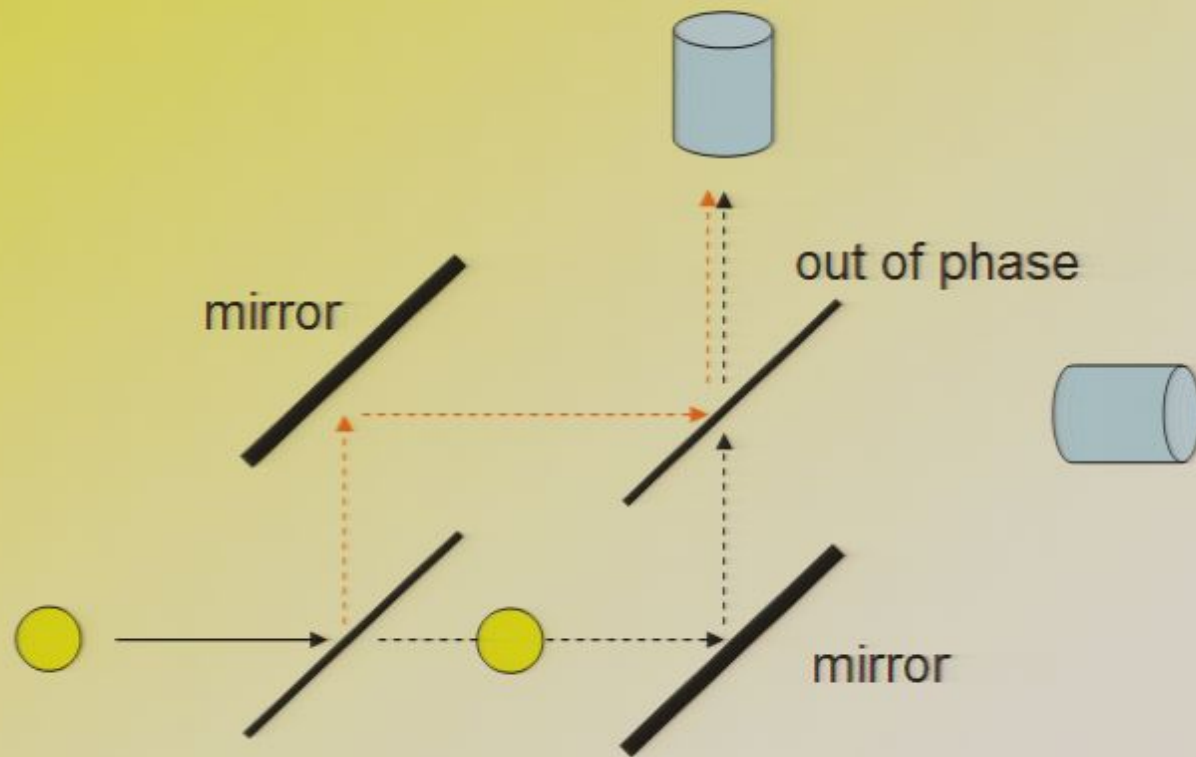
But, this is quantum physics. As we saw earlier, each photon can take both paths.

How do we represent this mathematically? Roughly speaking,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

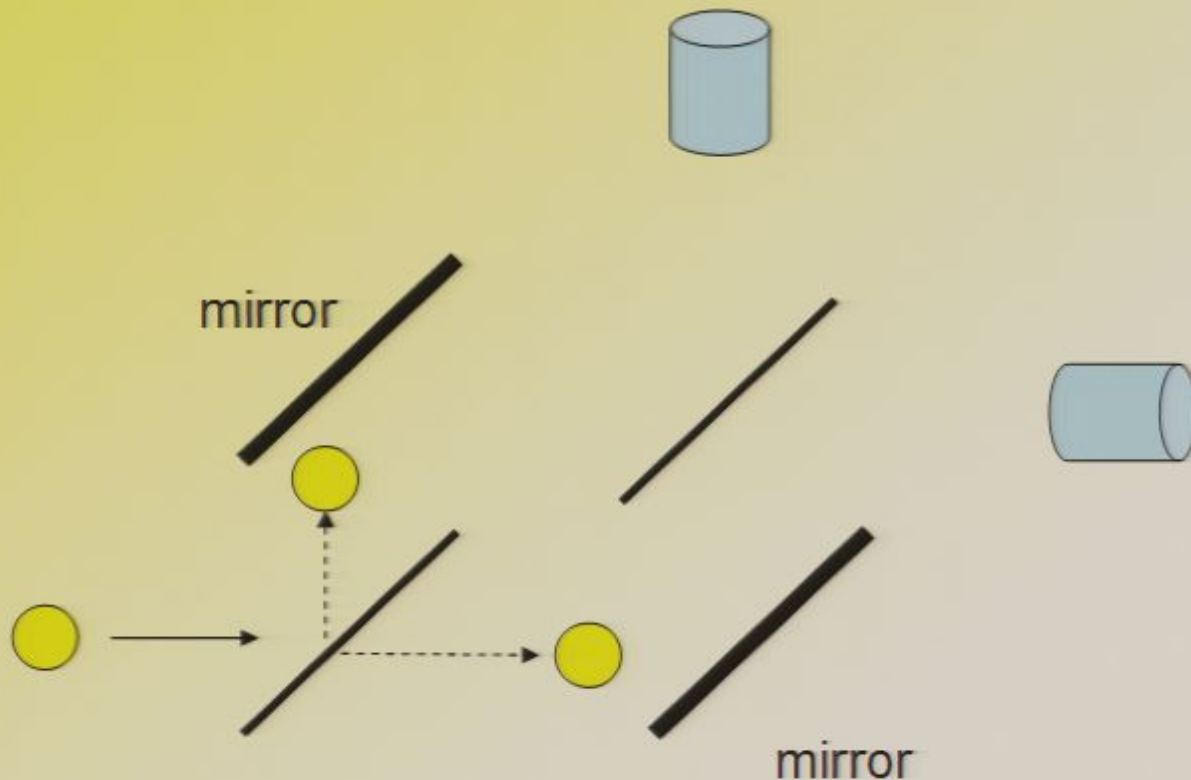


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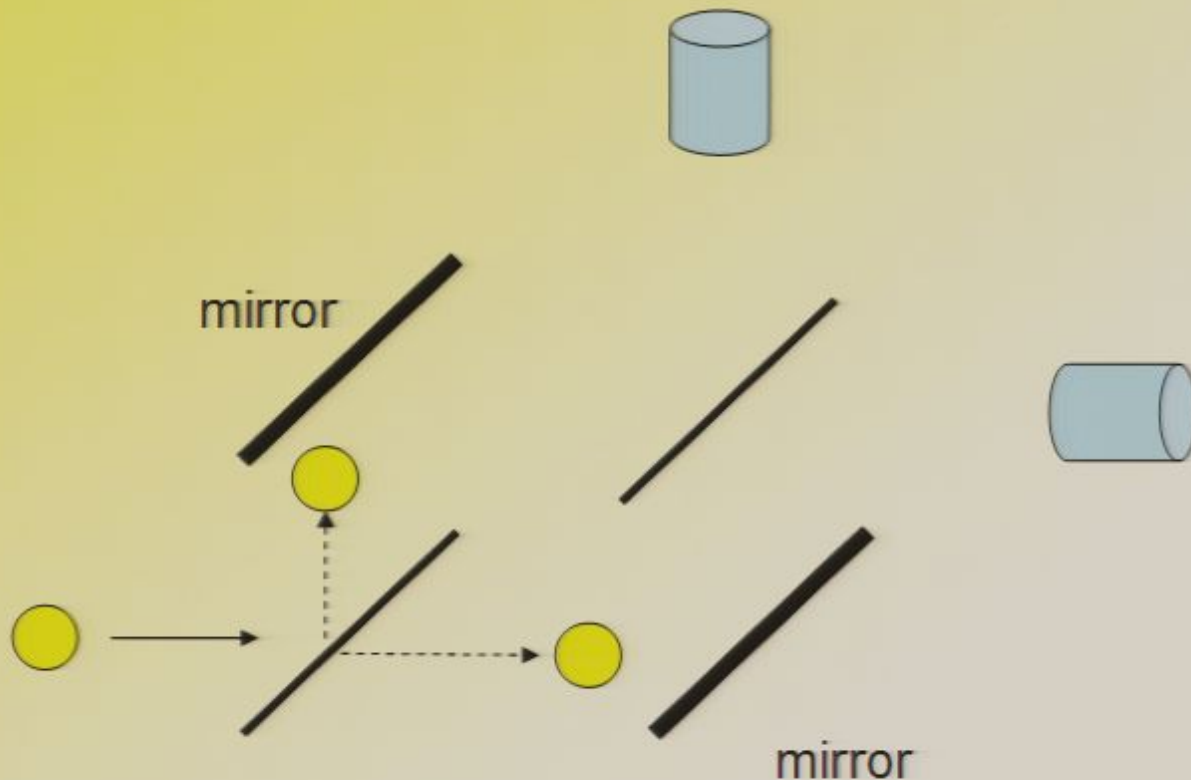
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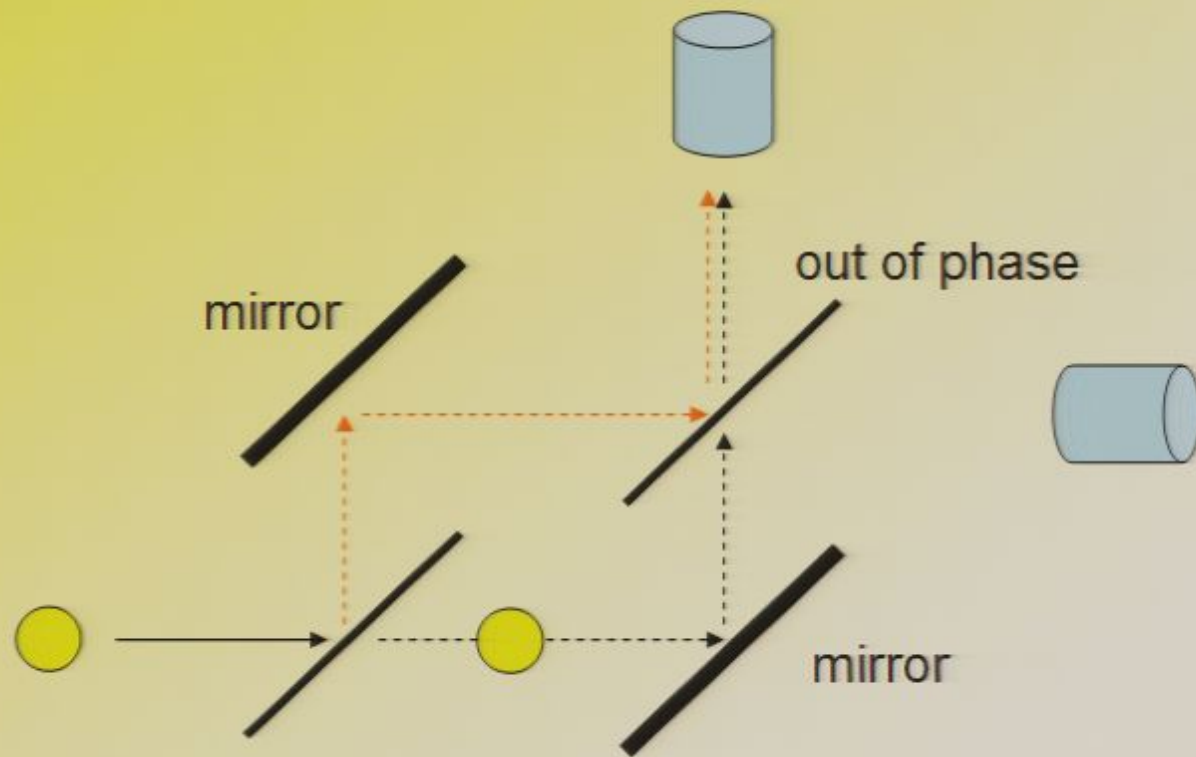
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“Combination”/twin-possibility state

More accurately,

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ i \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Where did the factors of $\frac{1}{\sqrt{2}}$ and i come from?

- When a photon passes through a half-silvered mirror then there is a 50% probability of us *measuring it* to be reflected and a 50% probability of us *measuring it* to be transmitted.
- Therefore, we need some mathematical way to represent these probabilities.

$$\text{Probability}(\text{path 1}) = |(1,1) \text{ element of matrix}|^2$$

$$\text{Probability}(\text{path 2}) = |(2,1) \text{ element of matrix}|^2$$

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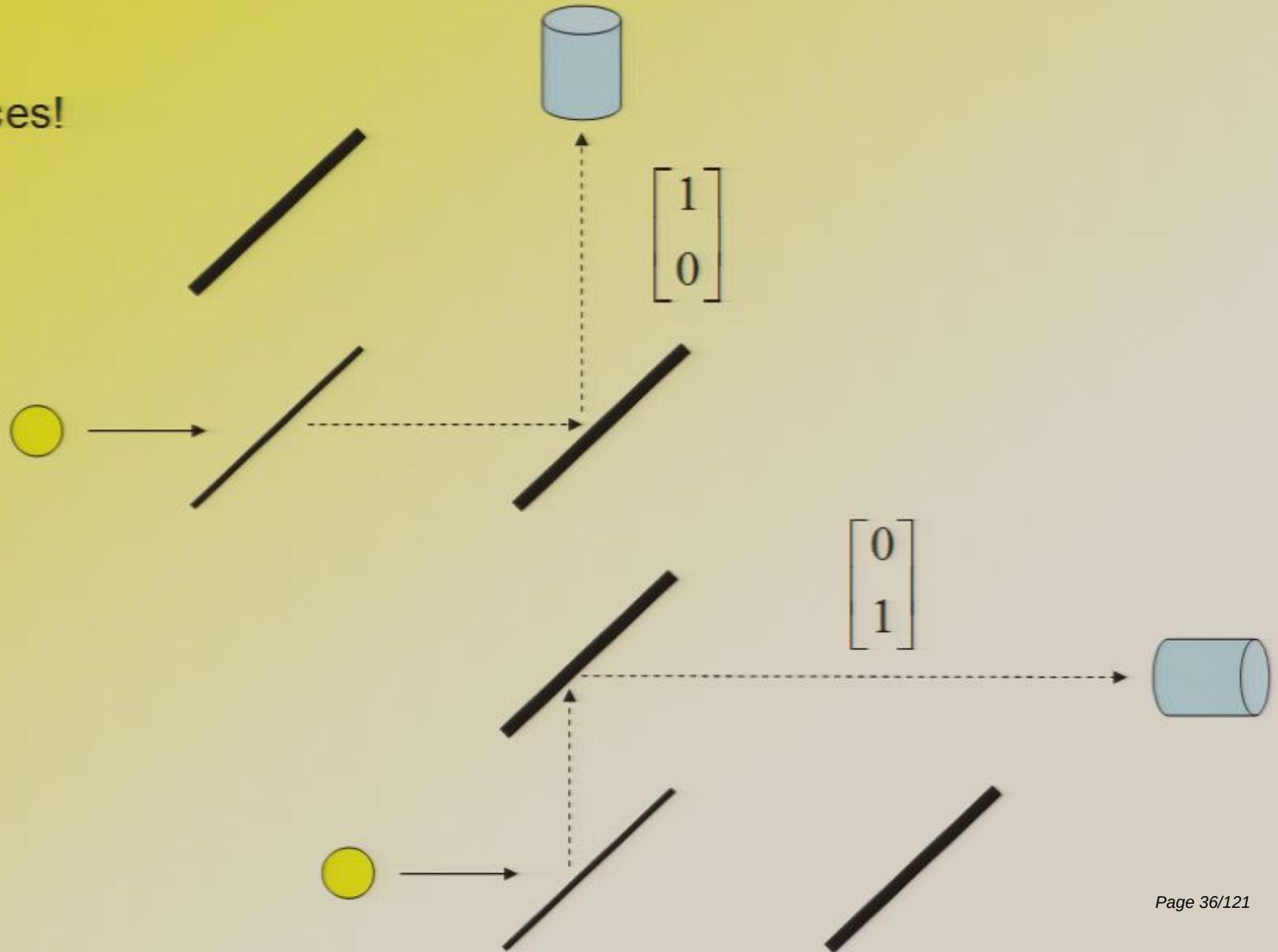
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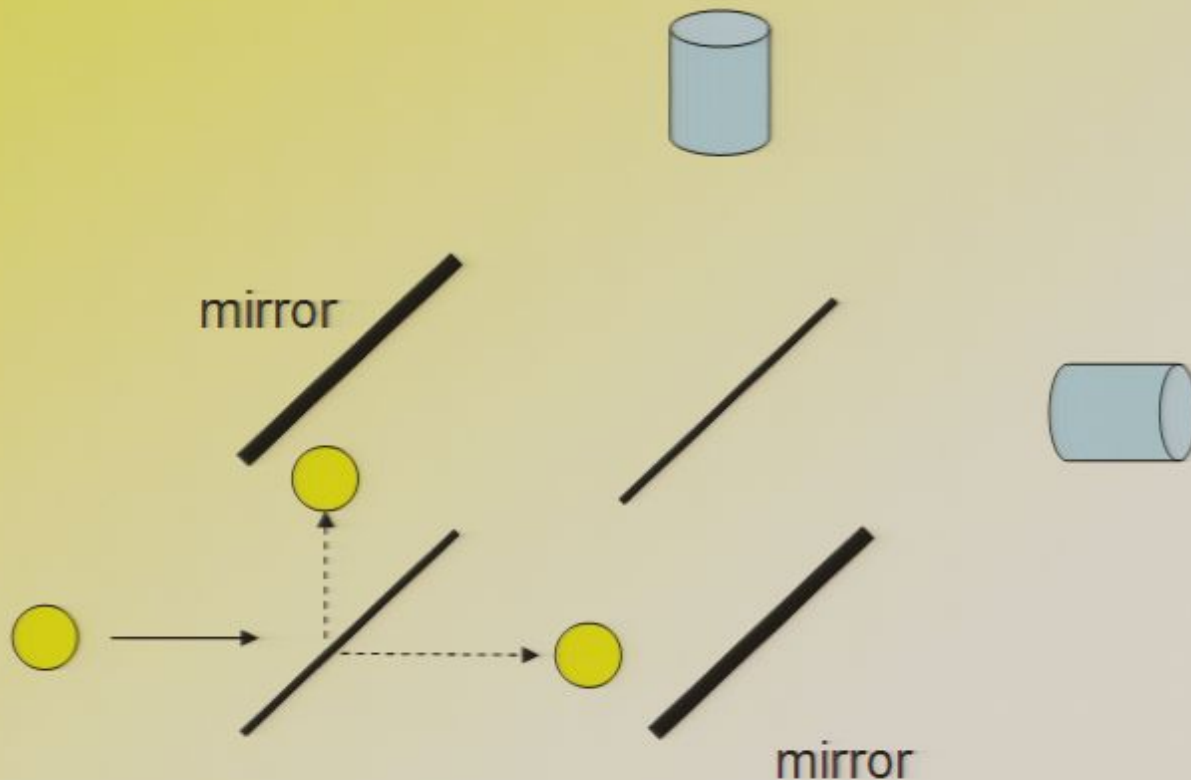
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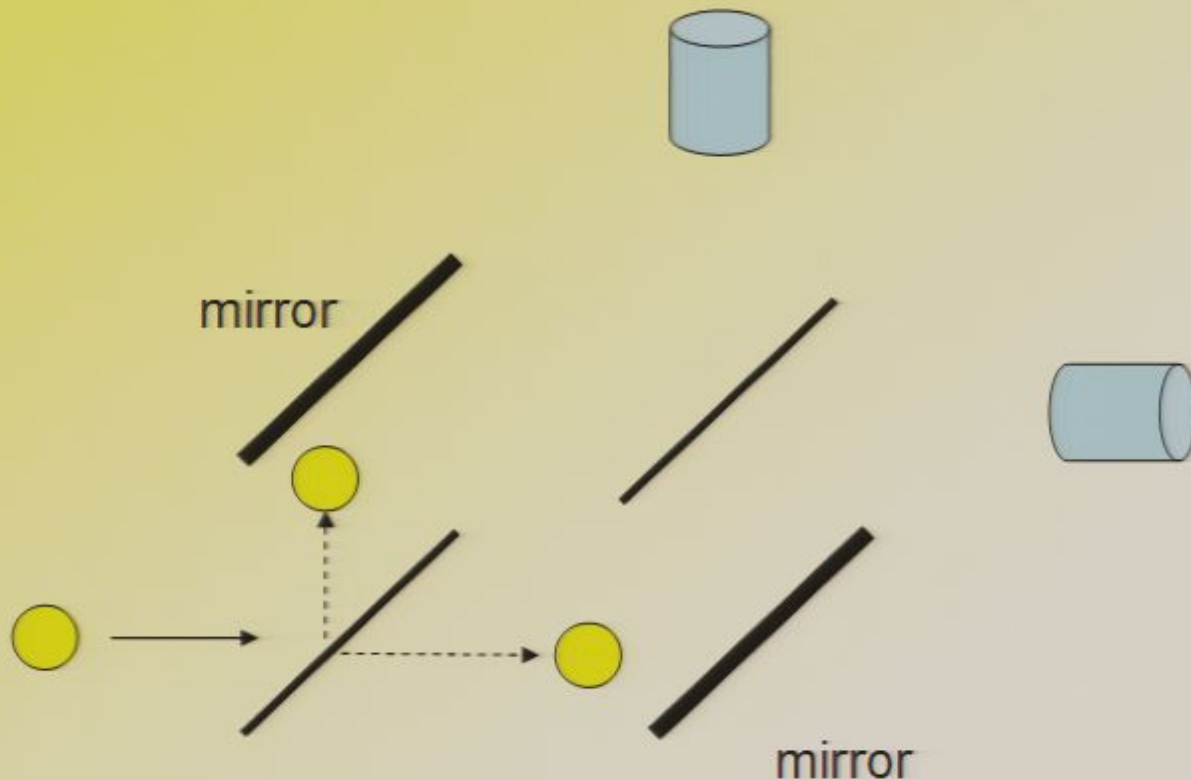
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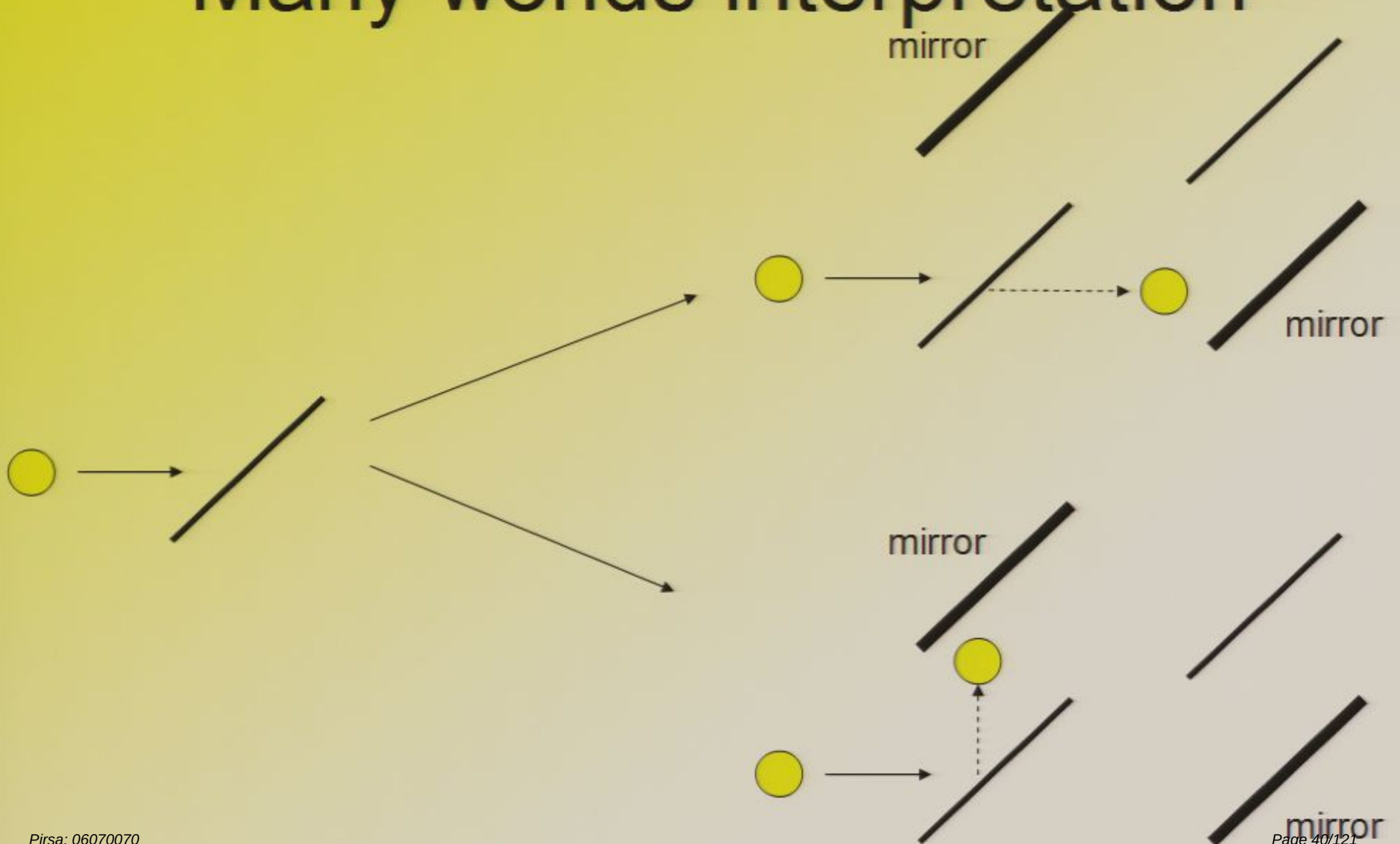
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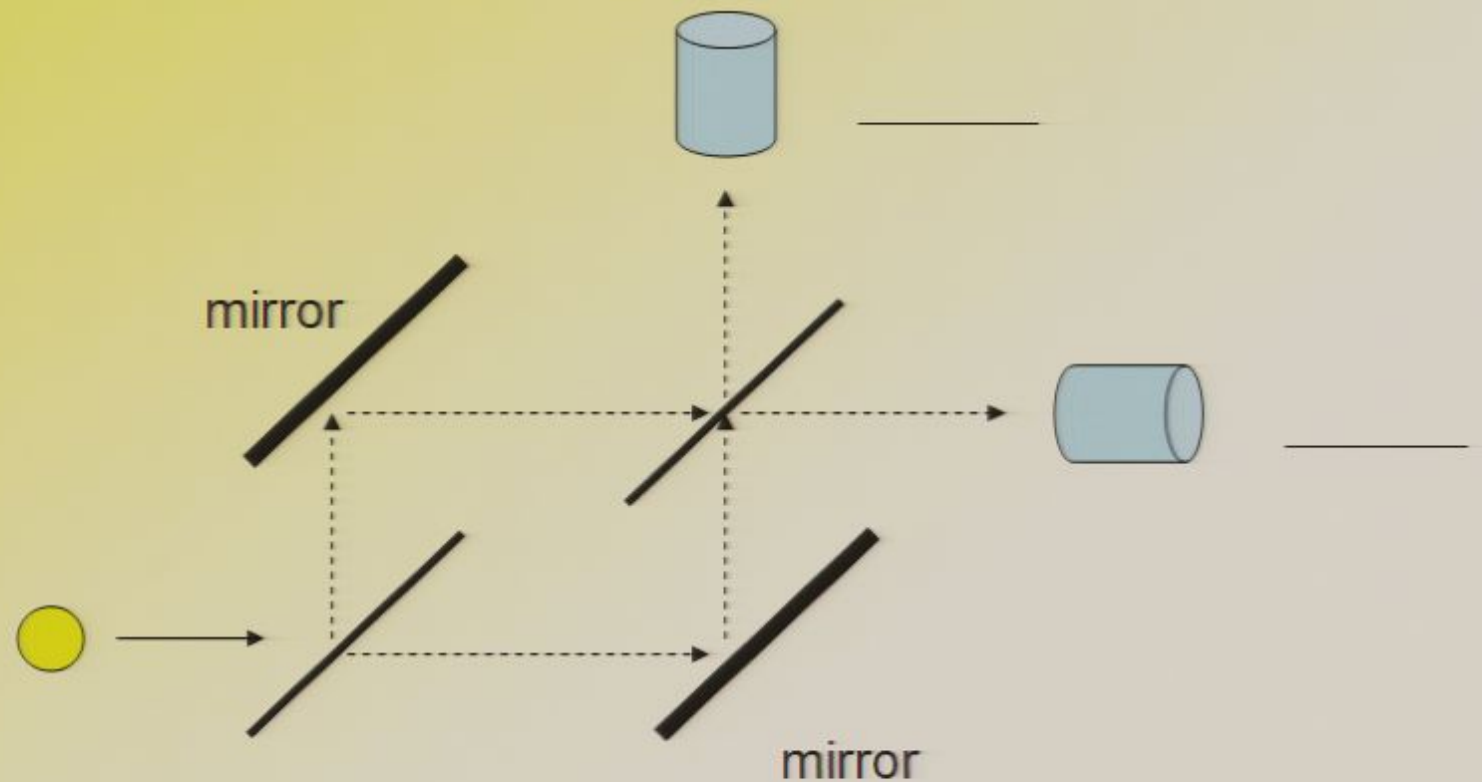
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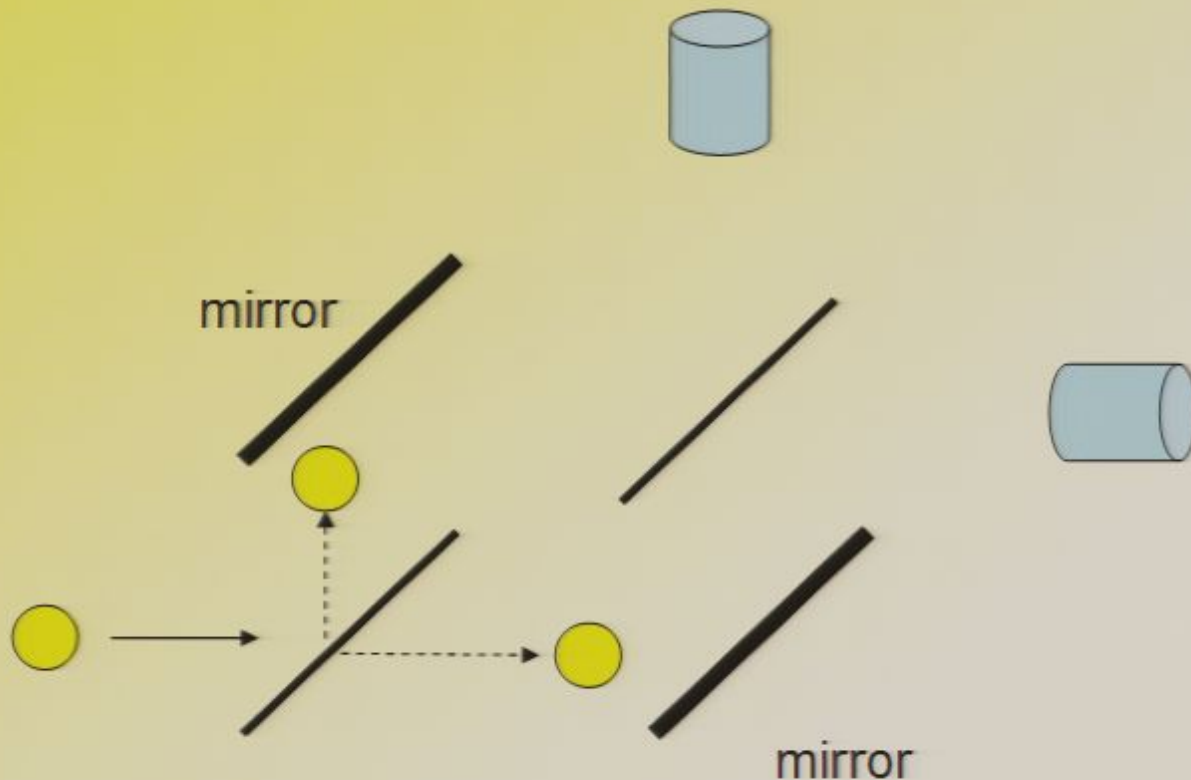


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Student Activity

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i.e. that

$$|(1,1) \text{ element of matrix}|^2 = \frac{1}{2}$$

and

$$|(2,1) \text{ element of matrix}|^2 = \frac{1}{2}$$

2. Consider the matrix
$$\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} e^{i\pi/4} \end{bmatrix}.$$

If it represents the state of affairs of a photon, what is the probability of finding the photon in paths 1 and 2?

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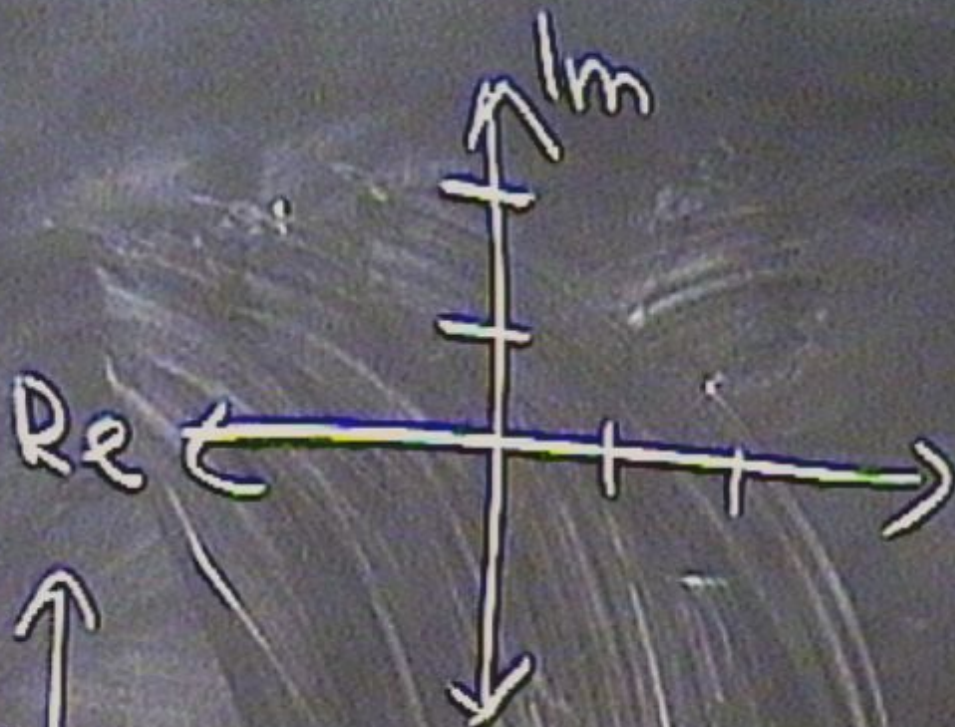
$$\left| -\frac{\hbar}{2m} \frac{\partial^2 \psi_{Am}}{\partial x^2} + V(x) \psi_{Am} = E \psi_{Am} \right. *$$

$$\left| -\frac{\hbar}{2m} \frac{\partial^2 \psi_{Am}}{\partial x^2} + V(x) \psi_{Am} = E \psi_{Am} \right. *$$

$$\dots + a_{1n}x_n = b_1$$

$$\dots + a_{qn}x_n = b_2$$

$$\dots + a_{nn}x_n = b_n$$



time ↑

$$\left(\frac{1}{2} \right)^2 \checkmark$$

$$\frac{1}{2} i$$

$$\sqrt{2}$$

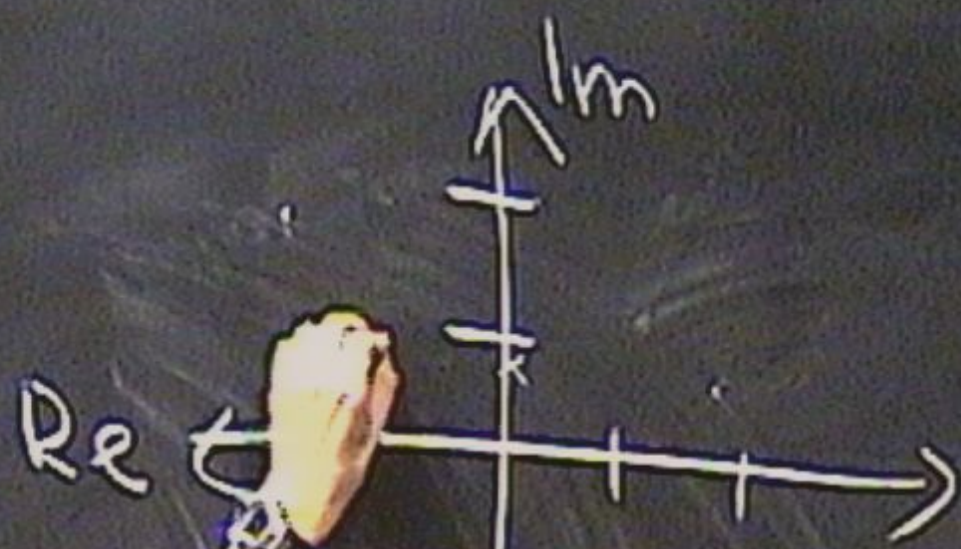
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a_{n1}
$\vdots$
a_{nn}

$$\left(\frac{1}{\sqrt{2}} \right)^2 = \bar{z}$$



time ↑

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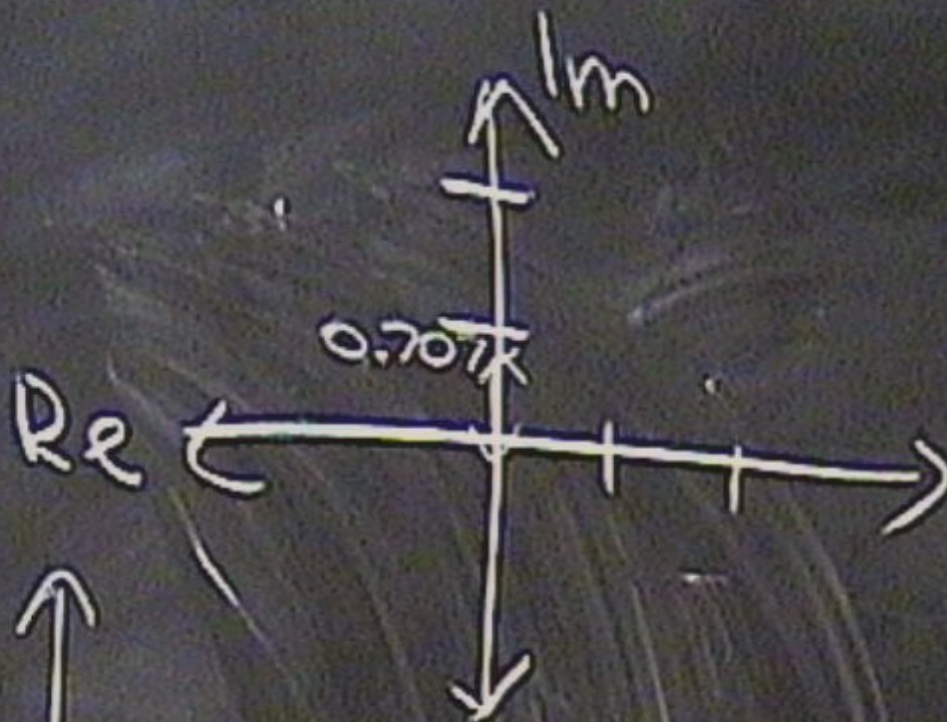
$$\dots + a_{nn}x_n = b_n$$

(n) a_{n1}

(n) a_{nn}

time ↑

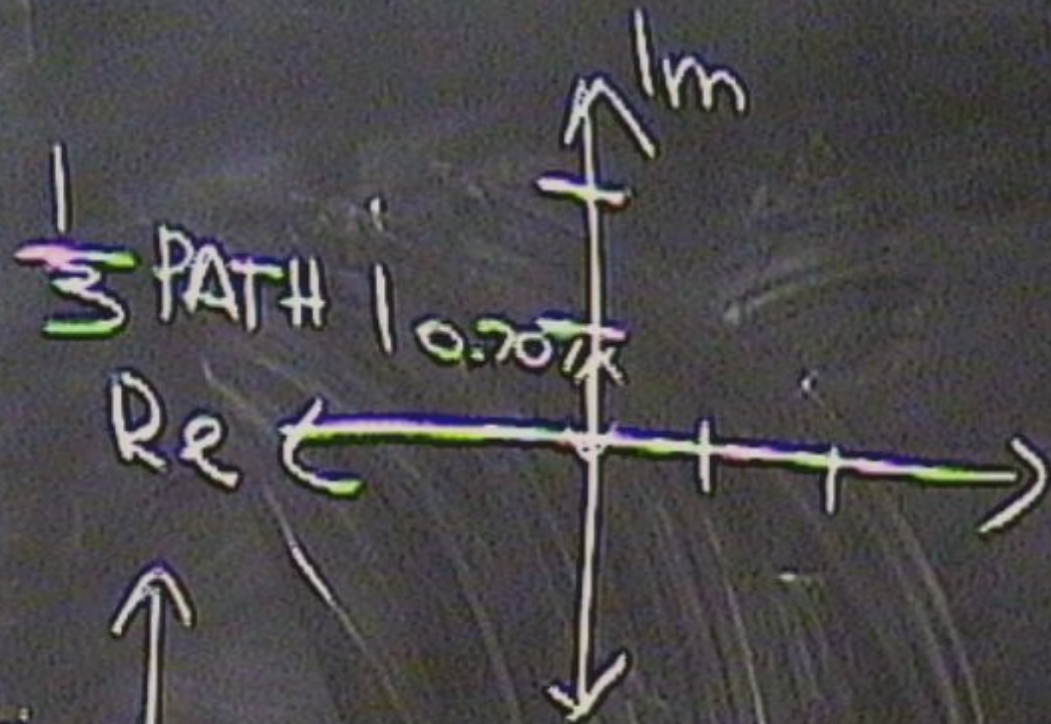
$$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \checkmark$$



$$1 : 1 = \frac{1}{\sqrt{2}}$$

$$\dots + a_{1n} x_n = b_1$$

$$\dots + a_{2n} x_n = b_2$$



$$\dots + a_{nn} x_n = b_n$$

(i) a_{n1}

(ii) $\dot{a}_{n1}$

$$\left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} \checkmark$$

$$\left| \frac{i}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

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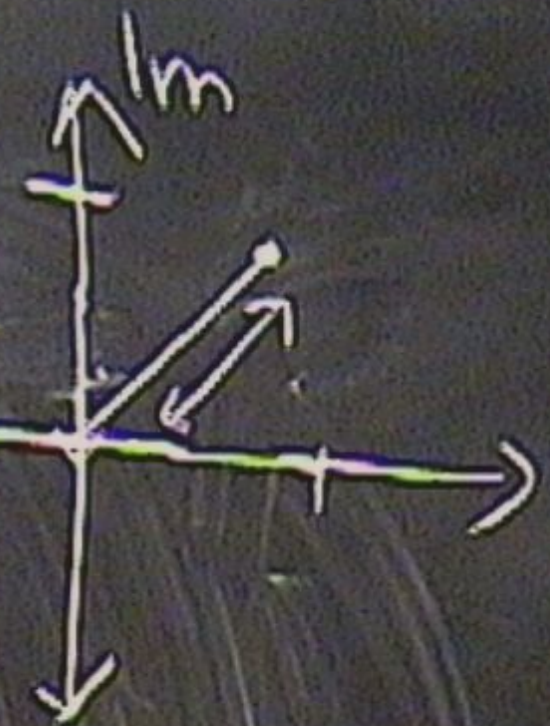
$$\dots + a_{nn}x_n = b_n$$

$$\begin{array}{l} a_{n1} \\ \vdots \\ a_{nn} \end{array}$$

$$\left(\frac{1}{\sqrt{2}} \right)$$

1/3 PATH 1...

Re

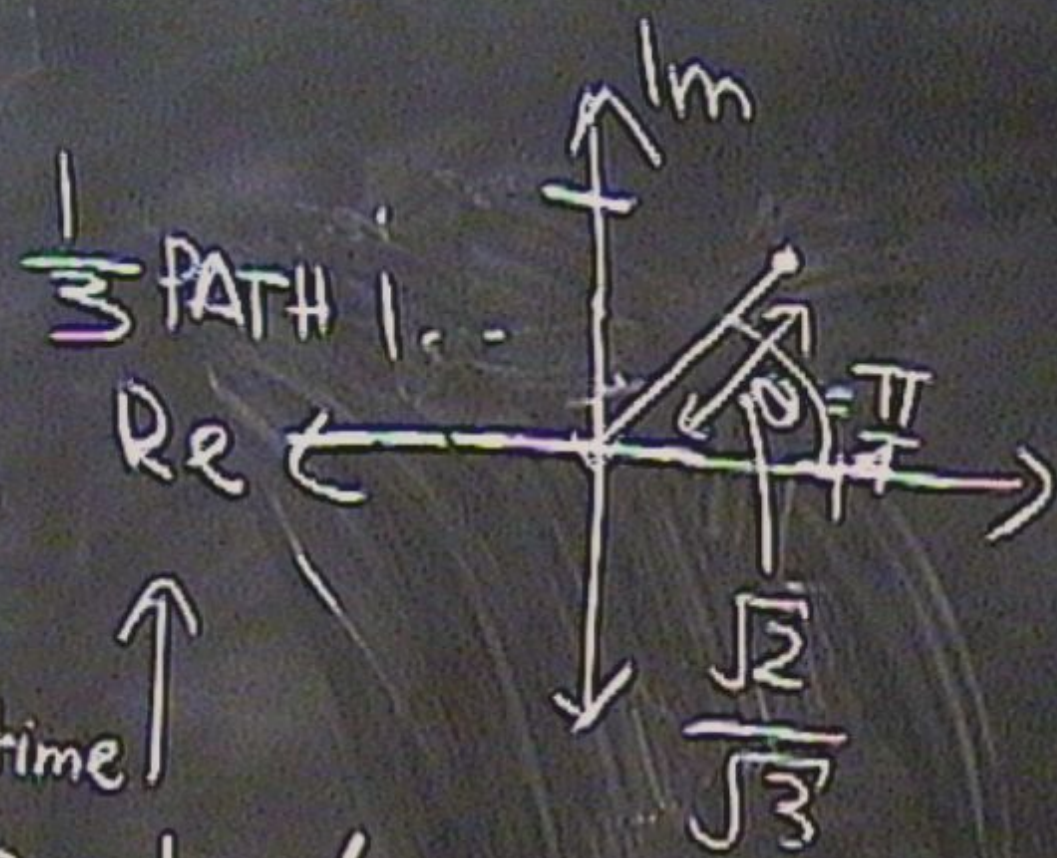


$$\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}$$

$$\dots + a_{1n} x_n = b_1$$

$$\dots + a_{2n} x_n = b_2$$

$$\dots + a_{mn} x_n = b_n$$



time ↑

$$1) z = \frac{1}{2} \sqrt{\frac{i}{2}} = \frac{1}{\sqrt{2}}$$

$$\dots + a_{1n} x_n = b_1$$

$$\dots + a_{2n} x_n = b_2$$

$$\dots + a_{nn} x_n = b_n$$

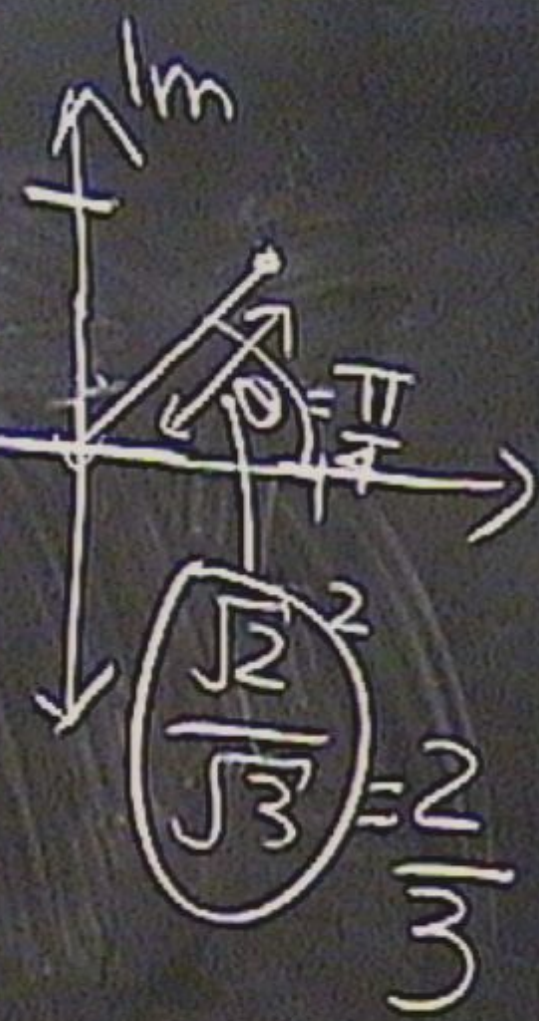
$$\begin{array}{l} \text{(i)} \quad a_{n1} \\ \text{(ii)} \quad \dot{a}_{n1} \end{array}$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \checkmark$$

$$\left| \frac{i}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

$\frac{1}{3}$ PATH 1. -

Re $\leftarrow$



time $\uparrow$

$$\frac{\sqrt{2}}{\sqrt{3}}$$

$\frac{1}{3}$ PATH i...

Re ←

time ↑

$$= \frac{1}{2} \sqrt{\frac{i}{2}}$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$\frac{1}{3}$ PATH i ...
 $\text{Re} \leftarrow$

time $\uparrow$

$$z = \frac{1}{2} \sqrt{\frac{i}{2}}$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{\sqrt{3}} \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4} \right)$$

$\frac{1}{3}$ PATH i.
Re $\leftarrow$

time $\uparrow$

$$\left| \frac{i}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}}$$

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$$\frac{i}{\sqrt{2}}$$

$\frac{1}{3}$ PATH i. -
Re $\leftarrow$

time $\uparrow$

$$\frac{1}{\sqrt{3}} e^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \right) = \frac{1}{\sqrt{3}} (1 + i)$$

Re $\leftarrow$

time $\uparrow$

$$\frac{1}{\sqrt{2}} i =$$

$$\sqrt{3} e^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{3}}$$

Re $\leftarrow$

time $\uparrow$

$$\frac{1}{\sqrt{2}} \frac{i}{\sqrt{2}} =$$

$$\left(\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} \right)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{3}}$$

$$u_{\Delta} = a + b$$

$$a = \frac{1}{\sqrt{3}}$$

$$b =$$

p2 |

$$u = a + bi$$

$$a = \frac{1}{\sqrt{3}}$$

$$b = \frac{1}{\sqrt{3}}$$

$\Rightarrow$

$$+ i \sin \theta$$

$$\left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) =$$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = b$$

p2

$$u = a + bi$$

$$a = \frac{1}{\sqrt{3}}$$

$$b = 1$$

$$\frac{1}{\sqrt{3}}$$

+ i sin θ

$$\left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = |u| = \sqrt{a^2 + b^2}$$

$$= \sqrt{\frac{1}{3} + 1}$$

$$= \sqrt{\frac{4}{3}}$$

$$\frac{1}{\sqrt{3}} + \frac{i}{\sqrt{3}} = b$$

$$u = a + bi$$

$$a = \frac{1}{\sqrt{3}}$$

$$b = \frac{1}{\sqrt{3}}$$

+ i sin θ

$$\left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = |u| = \sqrt{a^2 + b^2}$$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = b \quad \boxed{\frac{2}{3}} \quad = \sqrt{\frac{1}{3} + \frac{1}{3}}$$

$$= \sqrt{\frac{2}{3}}$$

$$u = a + bi$$

$$a = \frac{1}{\sqrt{3}}$$

$$b = \frac{1}{\sqrt{3}}$$

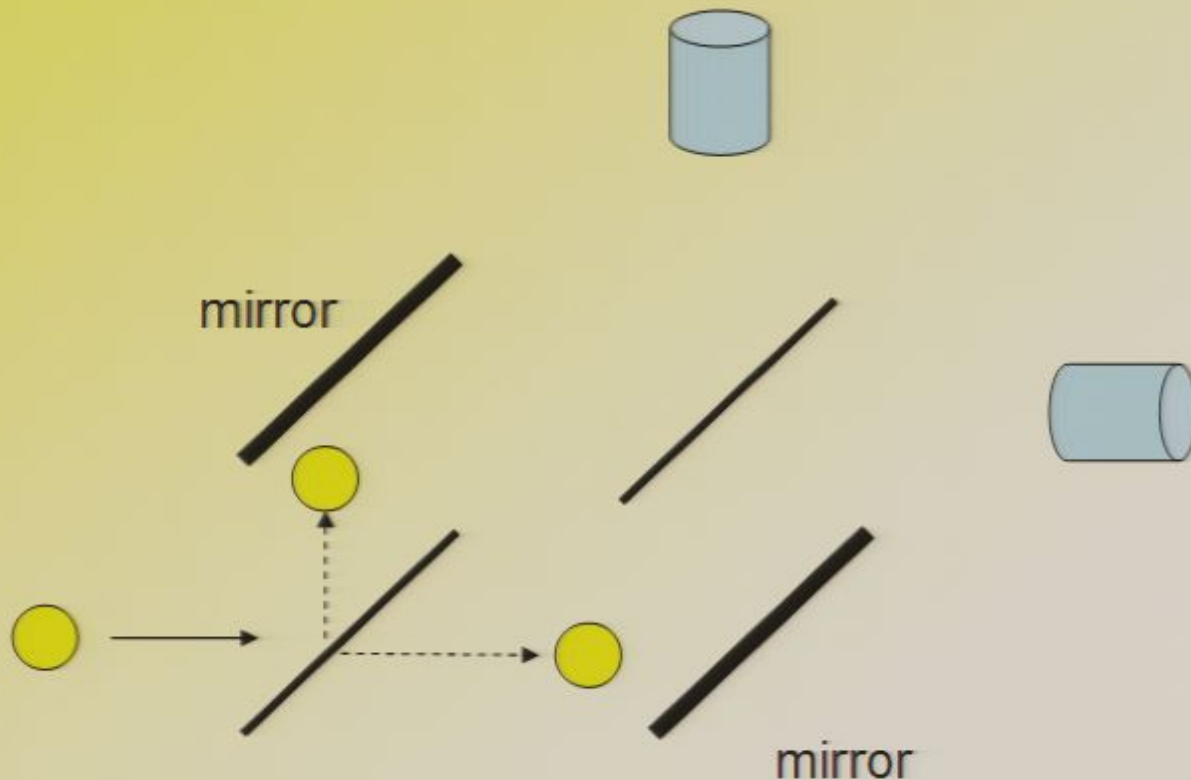
+ i sin θ

$$\left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = |u| = \sqrt{a^2 + b^2}$$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = b \quad \boxed{\frac{2}{3}} \quad = \sqrt{\frac{1}{3} + \frac{1}{3}}$$

Why?

- Each individual photon takes *both* paths.
- Two possible paths to reach each detector
- Overlap between the two leads to constructive or destructive interference (helps to think of wave interference)



- In Newtonian physics, these are only two possible states of affairs. Surely, each photon must either take the first path or the second one. These appear to be the only two options.

But, this is quantum physics. As we saw earlier, each photon can take both paths.

How do we represent this mathematically? Roughly speaking,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- 3. State the condition that the $(1,1)$ and $(2,1)$ elements must, jointly, satisfy due to the fact that the probability of finding the photon somewhere is always one.

$$\left| -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{\text{amy}}}{\partial x^2} + V(x) \psi_{\text{amy}} = E \psi_{\text{amy}} \right|$$

$$| (1,1) |^2 + | (1,1) |^2 = 1$$

$$\left| -\frac{\hbar}{2m} \frac{\partial^2 \psi_{\text{any}}}{\partial x^2} + V(x) \psi_{\text{any}} = E \psi_{\text{any}} \right|$$

$$|(1,1)|^2 + |(2,1)|^2 = 1$$

“Combination”/twin-possibility state

More accurately,

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ i \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Where did the factors of $\frac{1}{\sqrt{2}}$ and i come from?

- When a photon passes through a half-silvered mirror then there is a 50% probability of us *measuring it* to be reflected and a 50% probability of us *measuring it* to be transmitted.
- Therefore, we need some mathematical way to represent these probabilities.

$$\text{Probability}(\text{path 1}) = |(1,1) \text{ element of matrix}|^2$$

$$\text{Probability}(\text{path 2}) = |(2,1) \text{ element of matrix}|^2$$

What about the 'i'?

- Does not affect the probability as we take the magnitude.
- Affects how the two paths combine with each other when they overlap.
- i.e. constructive or destructive interference due to phase differences.
- We will see this in detail a little later ...

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}}$$

$$e^{i\theta} = \cos\theta$$

$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{\sqrt{3}}$$

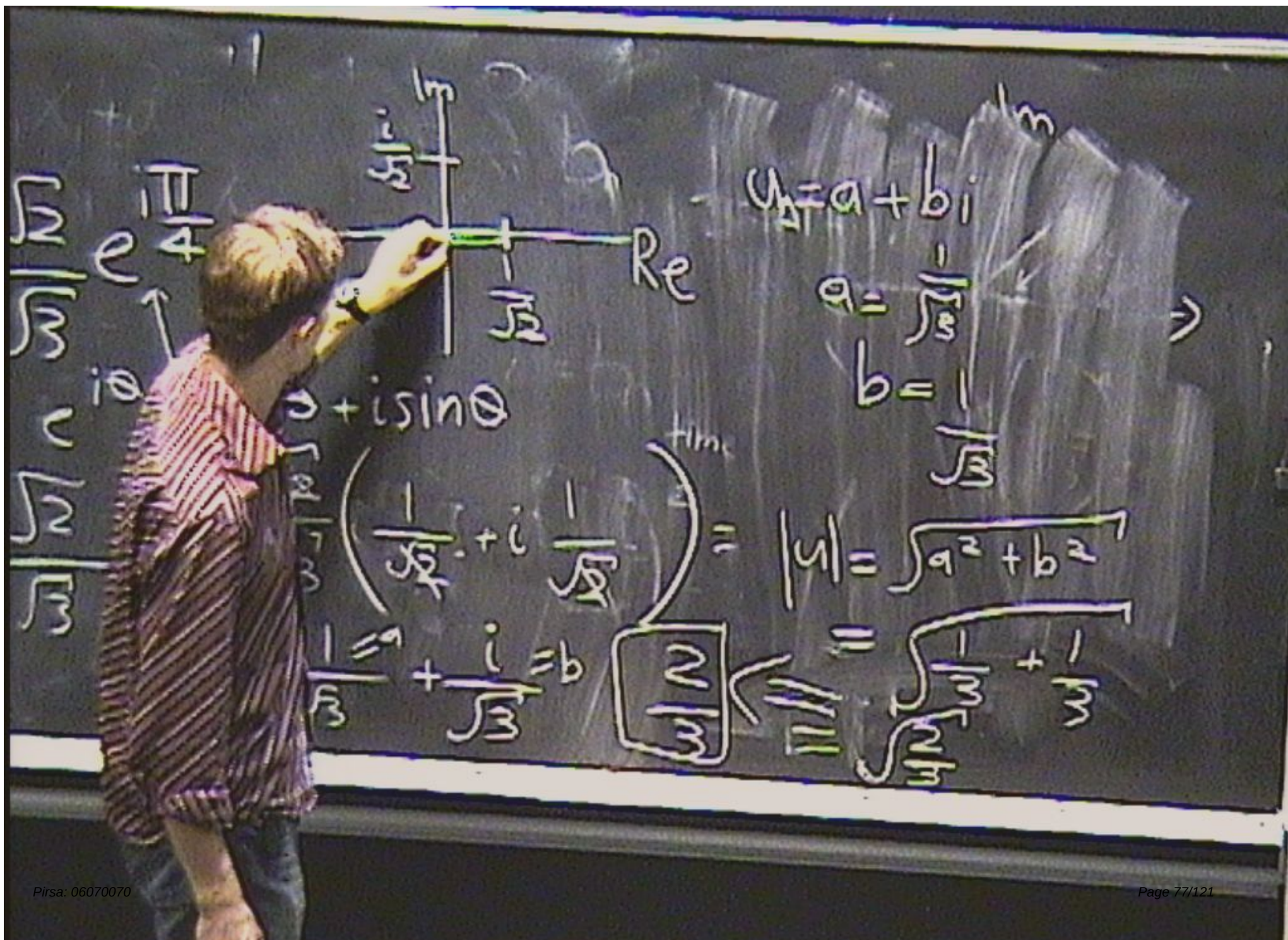
$$= \frac{1}{\sqrt{3}}$$

Im
 $\uparrow$

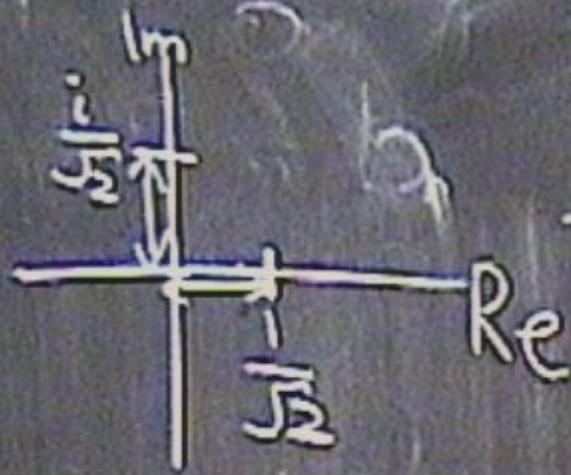
Re

$u = a + bi$
 $a = \frac{1}{\sqrt{3}}$
 $b = 1$

$|u| = \sqrt{a^2 + b^2}$
 $= \sqrt{\frac{1}{3} + 1}$
 $= \sqrt{\frac{4}{3}}$
 $= \frac{2}{\sqrt{3}}$



$$\frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}}$$



$$u_A = a + bi$$

$$a = \frac{1}{\sqrt{3}}$$

$$b = \frac{1}{\sqrt{3}}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\begin{aligned} \frac{\sqrt{2}}{\sqrt{3}} e^{i\frac{\pi}{4}} &= \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = |u| = \sqrt{a^2 + b^2} \\ &= \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{3}} = b \end{aligned}$$

$\left(\frac{2}{3} \right) \leftarrow \sqrt{\frac{1}{3} + \frac{1}{3}}$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

initial
matrix



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$



$$\begin{bmatrix} \end{bmatrix}$$

2nd $\frac{1}{2}$ silvered



half

$$|u| = \sqrt{\frac{1}{2} + \frac{1}{3}}$$

$$= \sqrt{\frac{5}{6}}$$

$$= \sqrt{\frac{2}{3}}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

initial
matrix

$$\longrightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \longrightarrow \begin{bmatrix} ? \\ ? \end{bmatrix}$$

2nd $\frac{1}{2}$ silvered



half-s

$$= |u| = \sqrt{b^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3}}$$

$$= \sqrt{\frac{2}{3}}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

initial
matrix



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ i\frac{1}{\sqrt{2}} \end{bmatrix}$$



$$\begin{bmatrix} ? \\ ? \end{bmatrix}$$

2nd $\frac{1}{2}$ silvered



half-silvered

$$= \sqrt{1 + \frac{1}{3}}$$

$$2a + b = 7$$

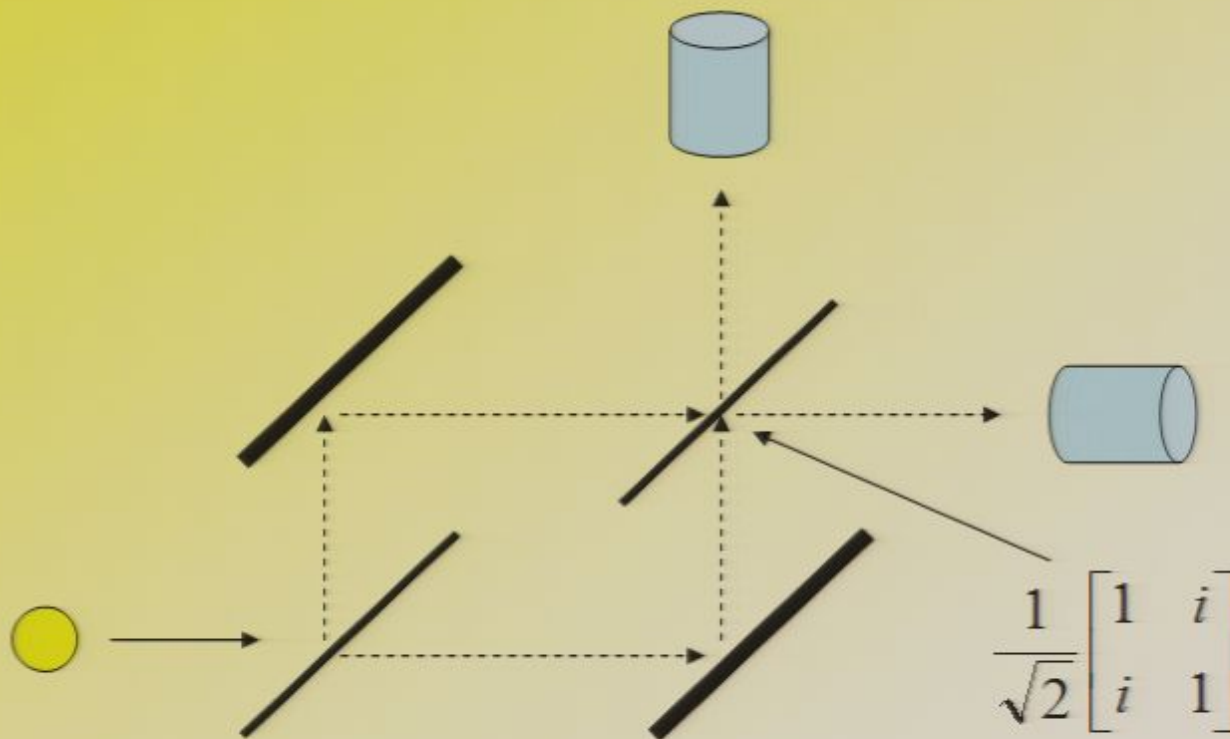
- What do we do with the second half-silvered mirror?
- Transforms from one 2x1 matrix to another.

Eg.

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

- What sort of matrix does this?
- a) 2 x 2 matrix
- b) 3 x 3 matrix
- c) 1 x 2 matrix
- d) 2 x 1 matrix

Extra questions: i) Prove this. ii) Prove that none of the other three options work?
iii) Is there any type of matrix that transforms one 2x1 matrix into another?



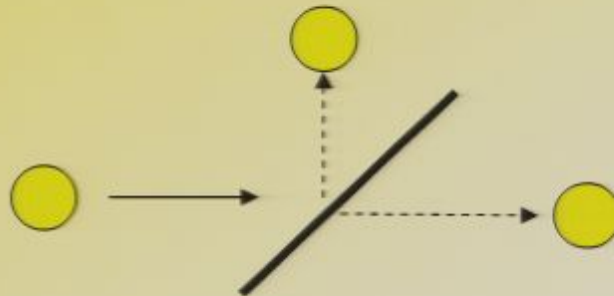
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

Does the 2x2 matrix make sense?

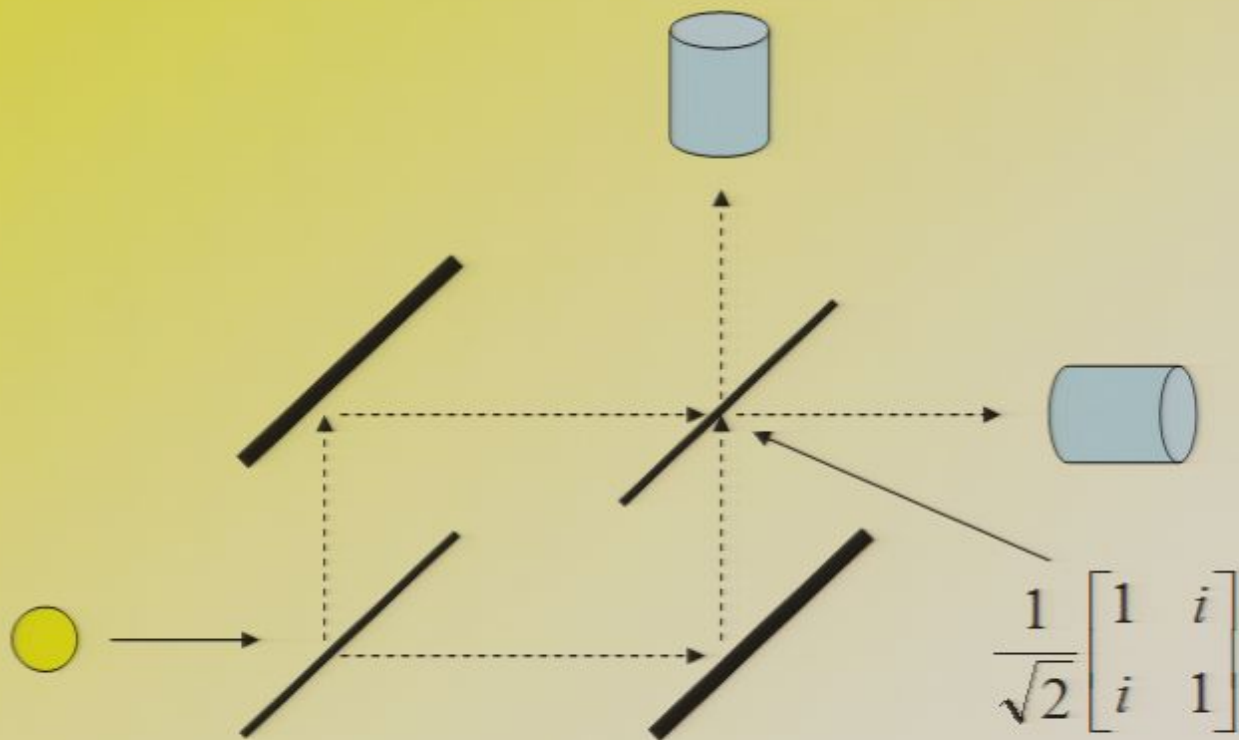
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

This is precisely the combination state of affairs we encountered before.



$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$


Another “state” in which the photon is taking both paths.



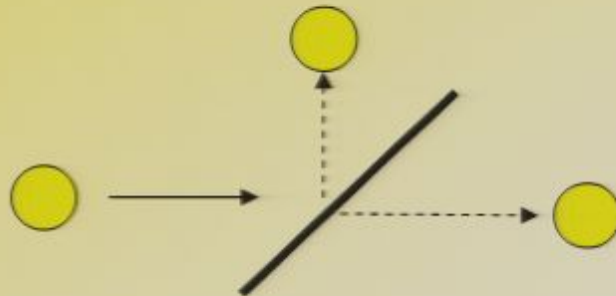
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

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Does the 2x2 matrix make sense?

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

This is precisely the combination state of affairs we encountered before.



$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$


Another “state” in which the photon is taking both paths.

What do we mean by the 'state'?

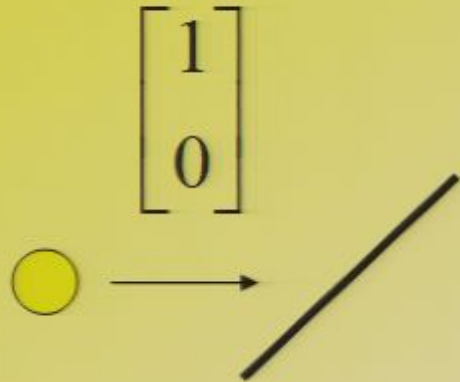
- Mathematical summary of what the photon is doing, its state of affairs.
- Consider a six-sided die that we roll once.
- $1/6$ probability of it landing 1,2,3,4,5 or 6.
- The following mathematical probability distribution mathematically summarizes this fact:

Result	1	2	3	4	5	6
Pr(result)	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

- The state is an analogous mathematical object for the photon.

- What is the difference between $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ i \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ and $\begin{bmatrix} \frac{i}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$?
- The probabilities for the photons to be measured in both paths are the same.
- As we shall soon see, they differ when we look at how different possibilities combine and overlap with each other.

- If we input one photon at the left of the experiment, i.e. then the initial state is

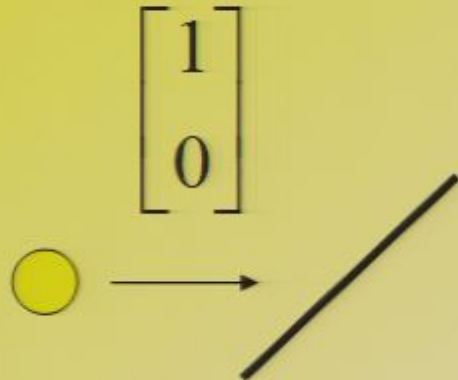


- STUDENT ACTIVITY:
What is the state after the photon has passed through the half-silvered mirror?

- What is the difference between $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ i \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ and $\begin{bmatrix} \frac{i}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$?
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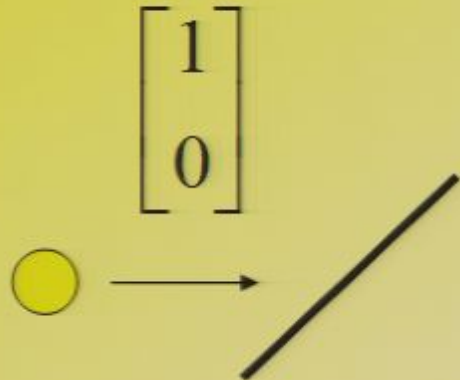
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- STUDENT ACTIVITY:
What is the state after the photon has passed through the half-silvered mirror?

- What is the state after the photon has passed through the second half-silvered mirror?
- Check that the probabilities that this state gives for finding the photon in paths 1 and 2 are in agreement with the quantum probabilities for this experiment we saw earlier.

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- STUDENT ACTIVITY:
What is the state after the photon has passed through the half-silvered mirror?

- What is the state after the photon has passed through the second half-silvered mirror?
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$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

initial
matrix

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}}$$



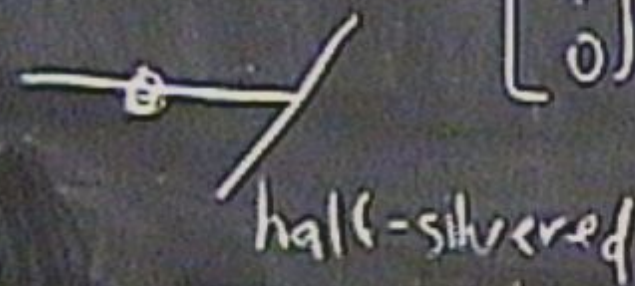
half-silvered

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{bmatrix}$$

initial matrix

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} / \sqrt{2}$$



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & i \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

- What is the state after the photon has passed through the second half-silvered mirror?
- Check that the probabilities that this state gives for finding the photon in paths 1 and 2 are in agreement with the quantum probabilities for this experiment we saw earlier.

No Signal

VGA-1

No Signal

VGA-1

No Signal

VGA-1

No Signal
VGA-1

No Signal
VGA-1

No Signal

VGA-1

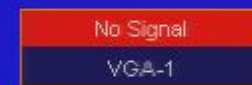
No Signal

VGA-1

No Signal
VGA-1

No Signal
VGA-1

No Signal
VGA-1





No Signal
VGA-1



$$(a+b)^0 =$$

$$(a+b)^0 =$$

$$(a+b)^1 =$$

$$(a+b)^2 =$$

$$(a+b)^3 =$$

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 =$$

$$(a+b)^{10}$$

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^{10}$$

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 =$$

$$(a+b)^{10}$$



$$(a+b)^0 = 1$$

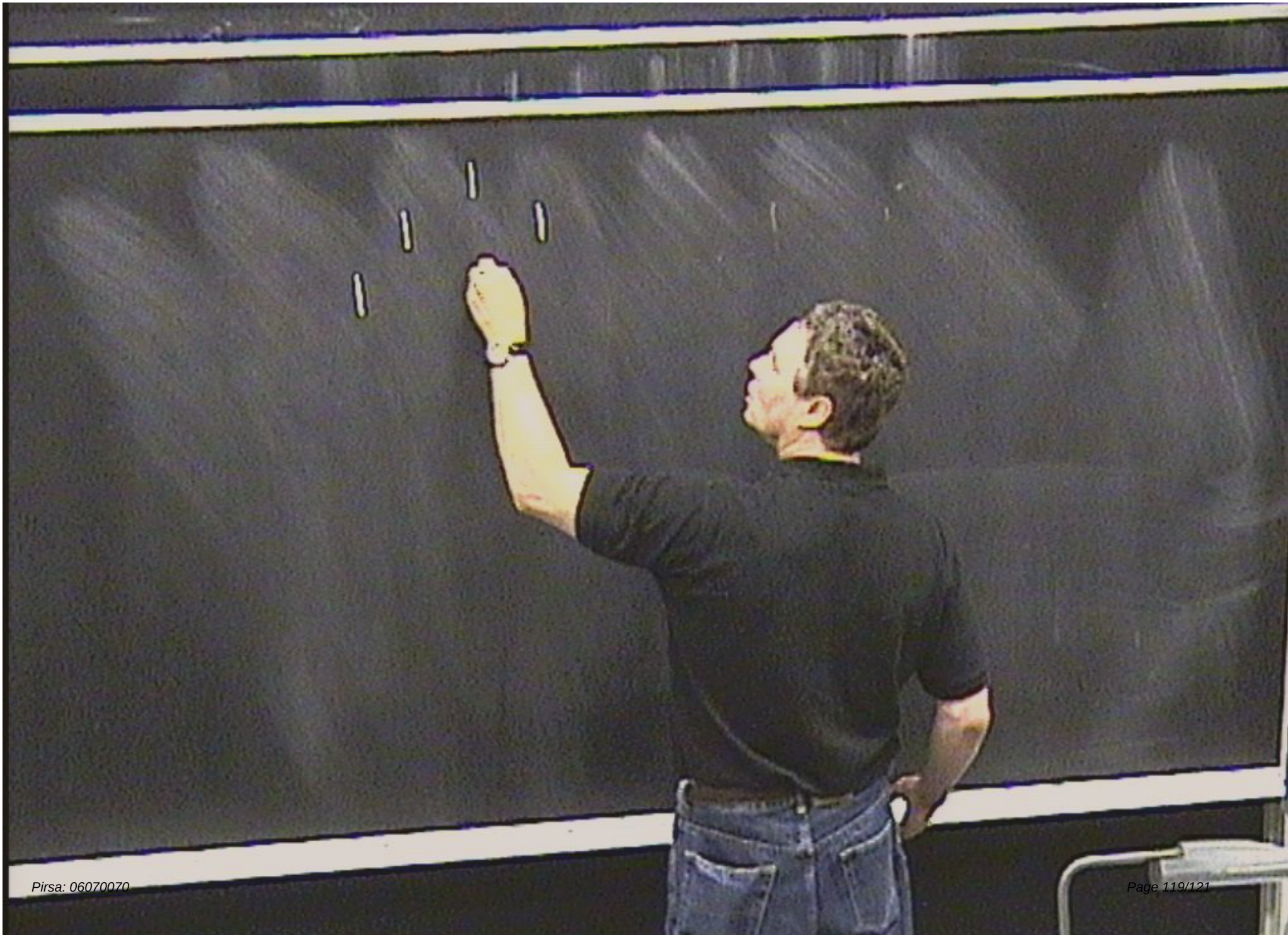
$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

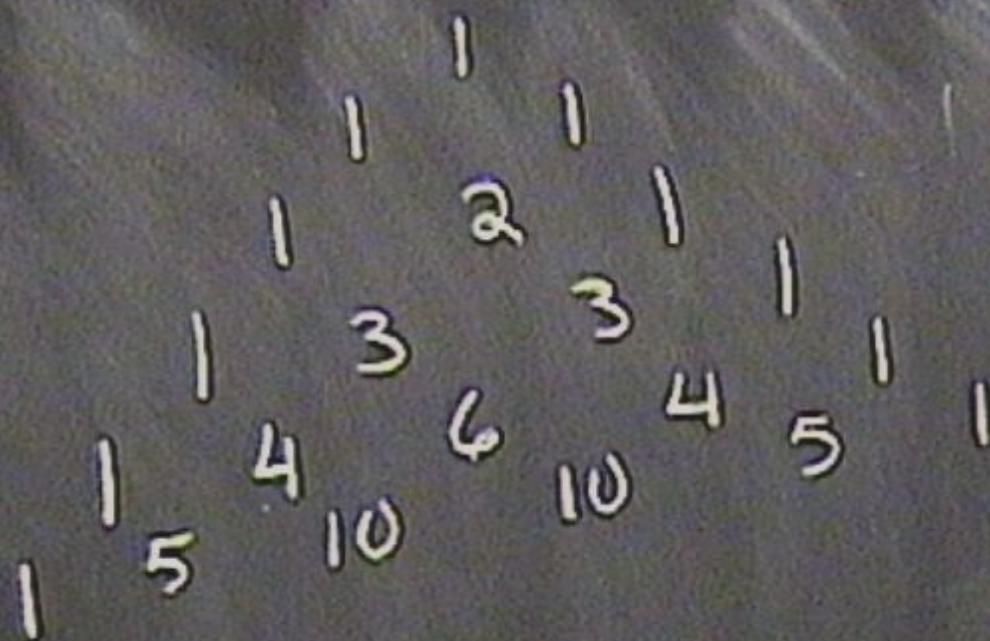
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 =$$

$$(a+b)^{10}$$



Pascal's Triangle



$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^{10} = \dots$$

$$(a+b)^{100} = \dots$$