

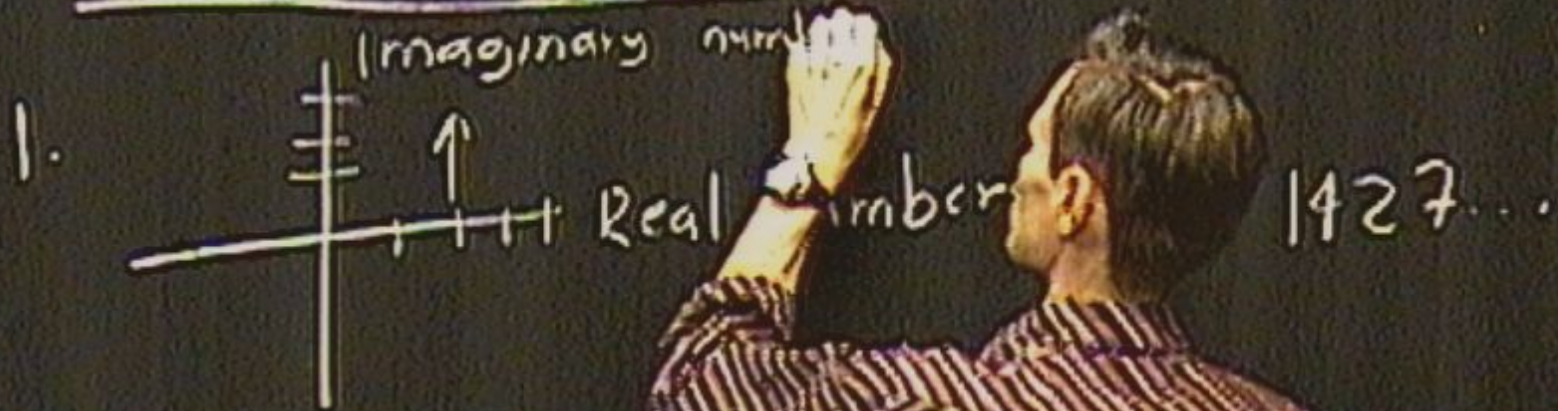
Title: Math Primer: Matrices

Date: Jul 25, 2006 09:00 AM

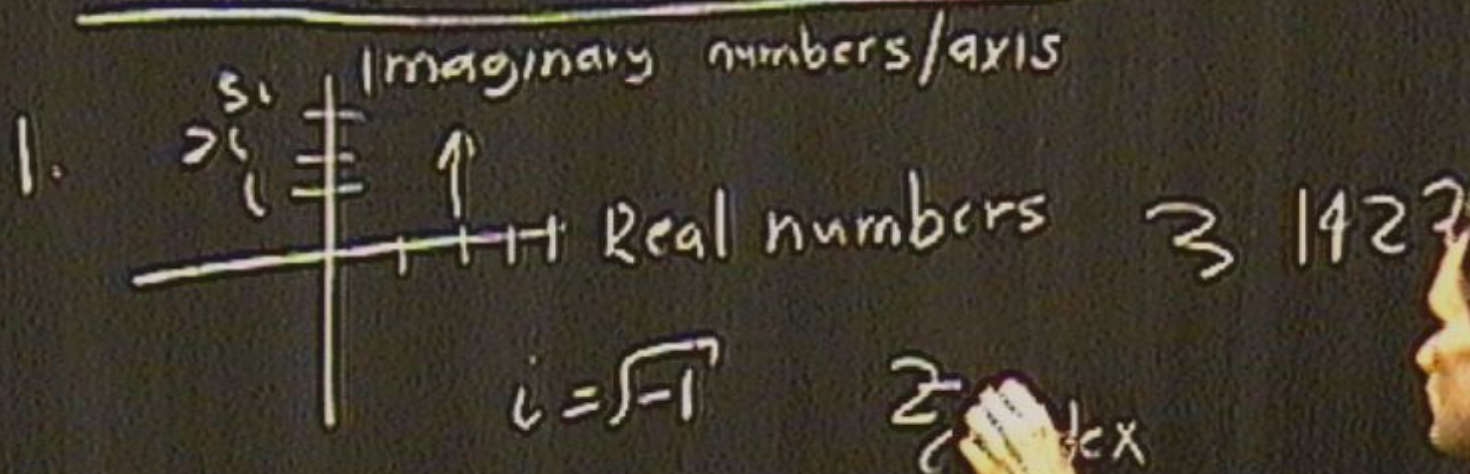
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Abstract:

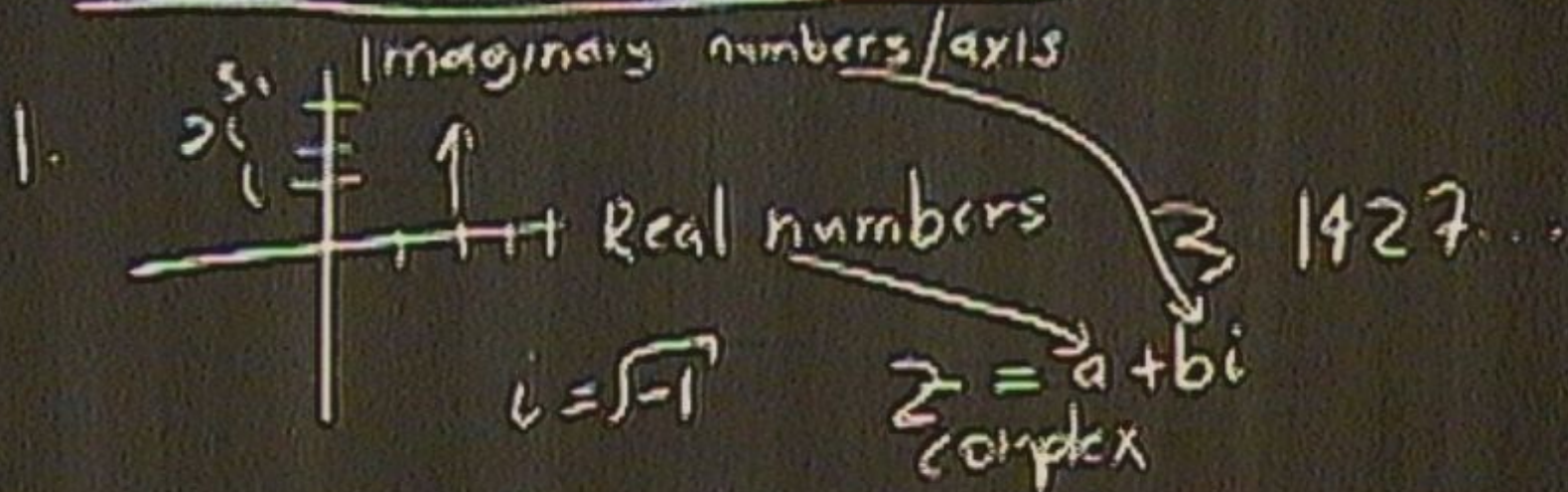
COMPLEX NUMBERS



COMPLEX NUMBERS



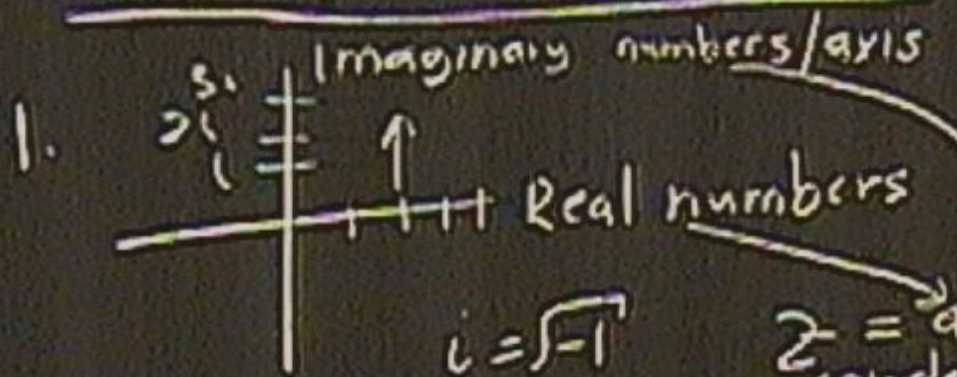
COMPLEX NUMBERS



2. $z = a + bi$

3

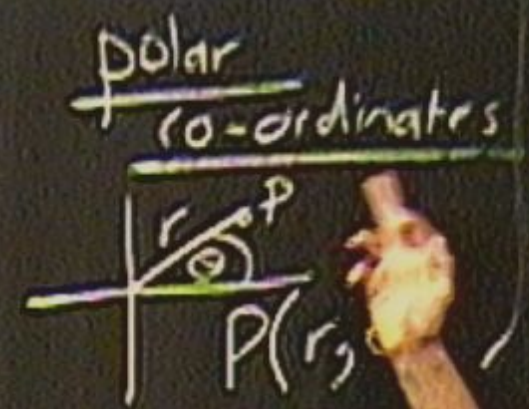
COMPLEX NUMBERS



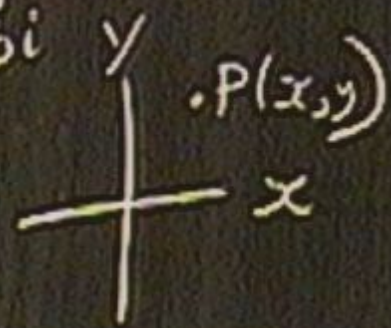
2. $z = a + bi$

$z = a + bi$
complex

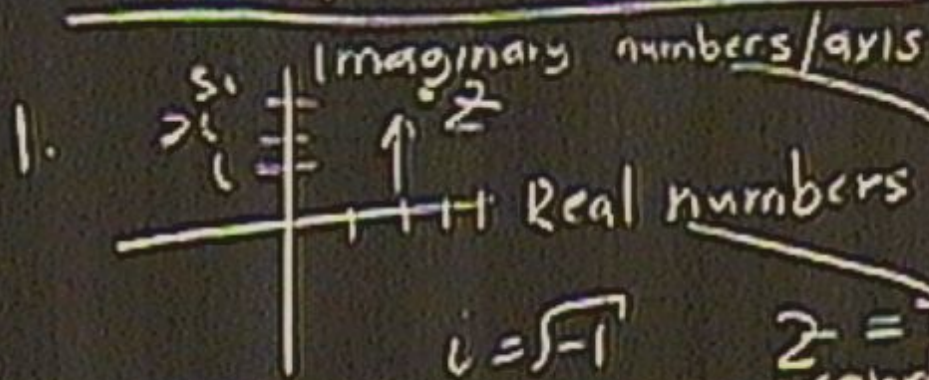
3



1427...



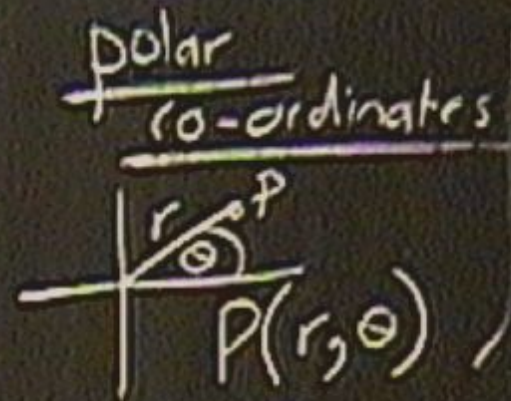
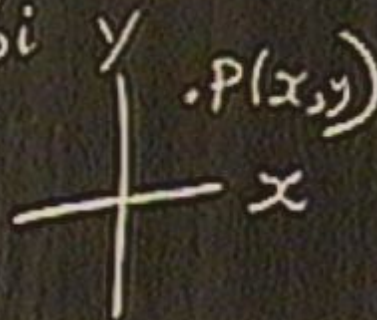
COMPLEX NUMBERS



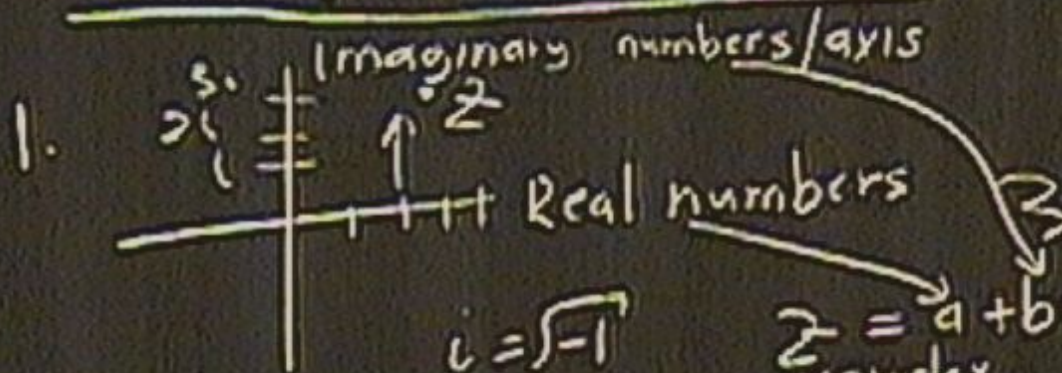
2. $z = a + bi$

$z = a + bi$
complex

3



COMPLEX NUMBERS

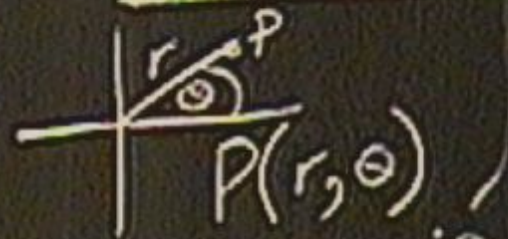


2. $z = a + bi$

$z = a + bi$
complex

③

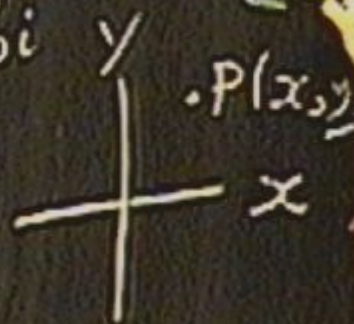
Polar co-ordinates



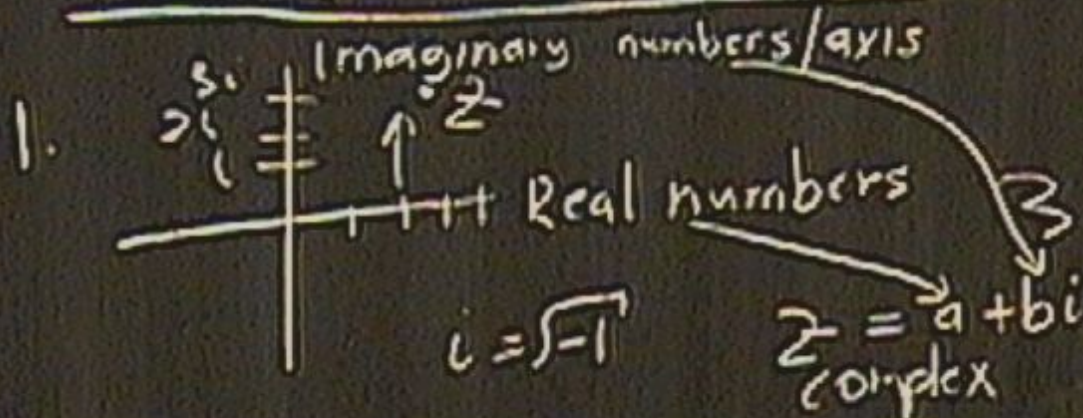
$z = r e^{i\theta}$

1427

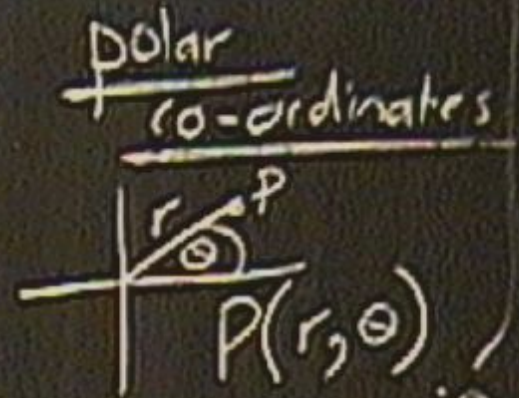
$z = 1$



COMPLEX NUMBERS



2. $z = a + bi$



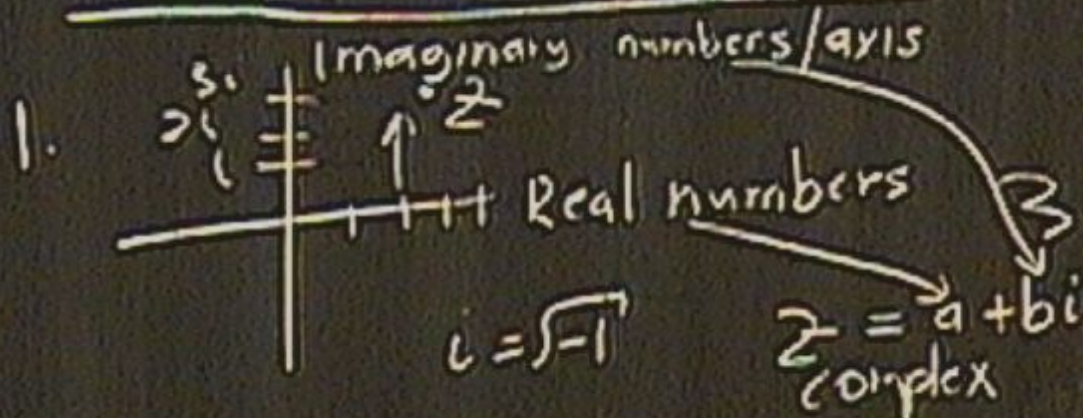
$$z = r e^{i\theta}$$

1427

$$u = |u| e^{i\theta}$$

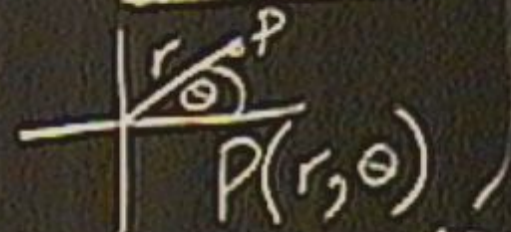
(x, y)

COMPLEX NUMBERS



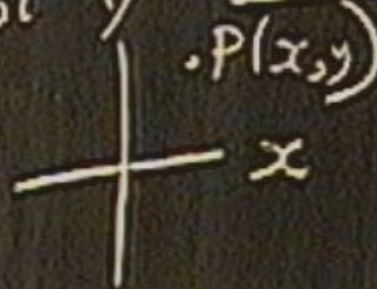
2. $z = a + bi$

Polar co-ordinates

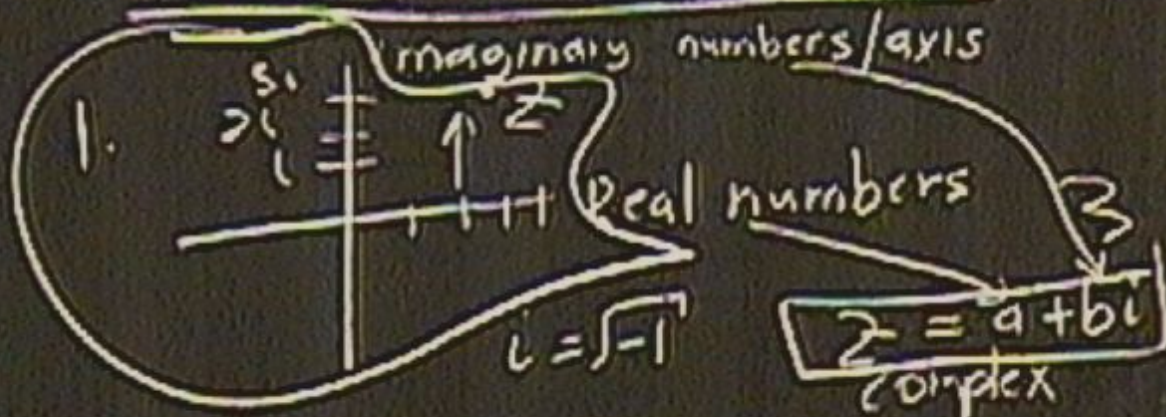


$$z = r e^{i\theta}$$

1427 $|z| = |u| e^{i\theta}$

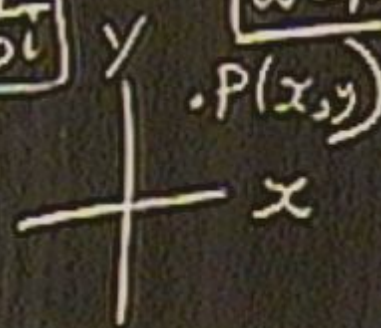


COMPLEX NUMBERS

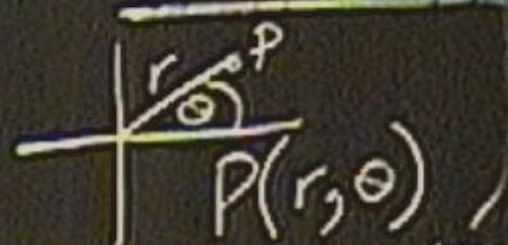


2. $z = a + bi$

3.



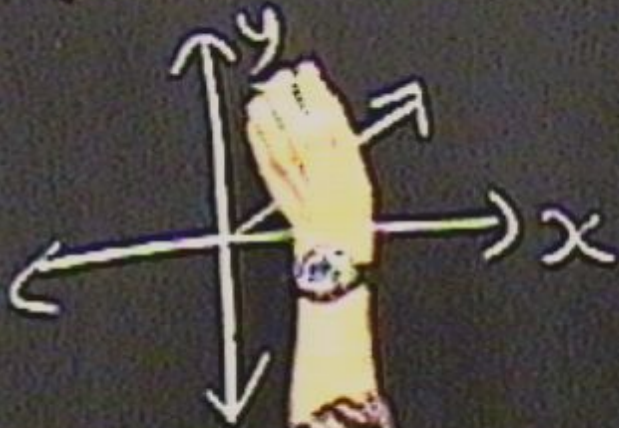
Polar co-ordinates



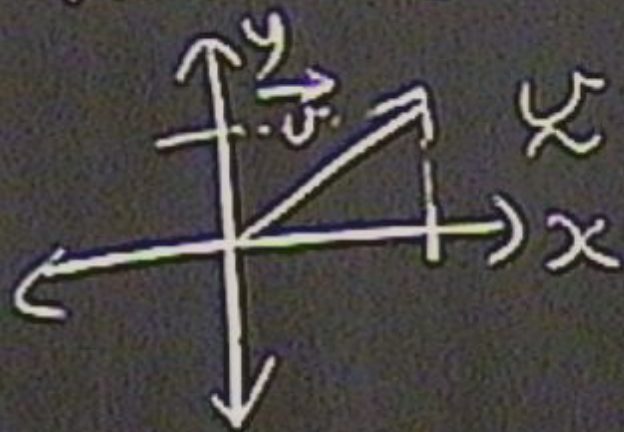
$z = r e^{i\theta}$

1427 $u = |u| e^{i\theta}$ *

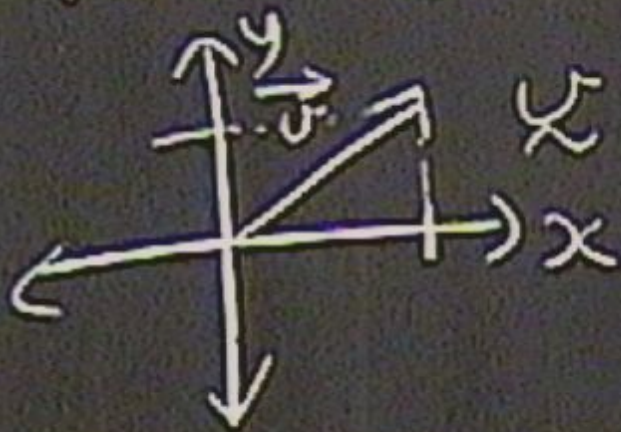
MATRICES



MATRICES

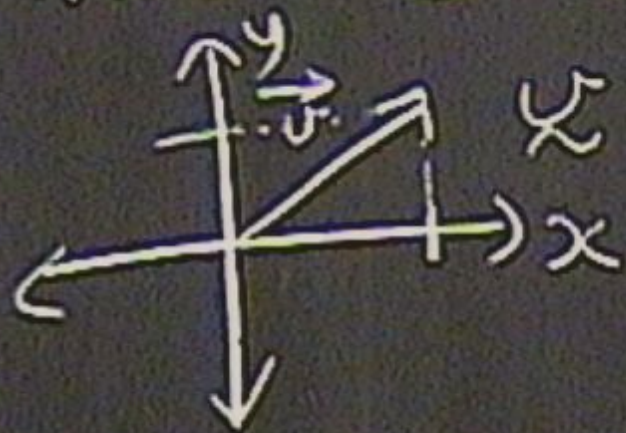


MATRICES



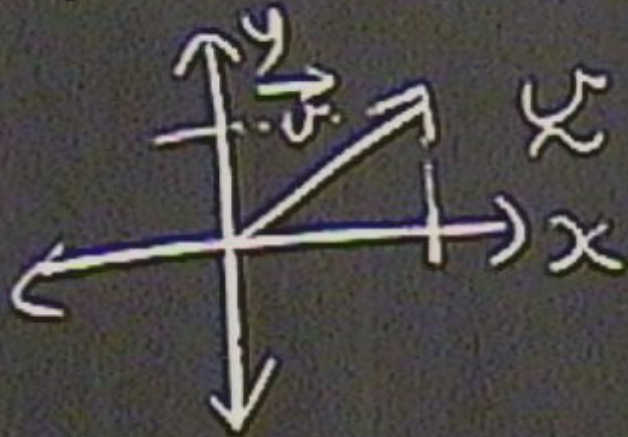
$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \left[\right.$$

MATRICES



$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

MATRICES



$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

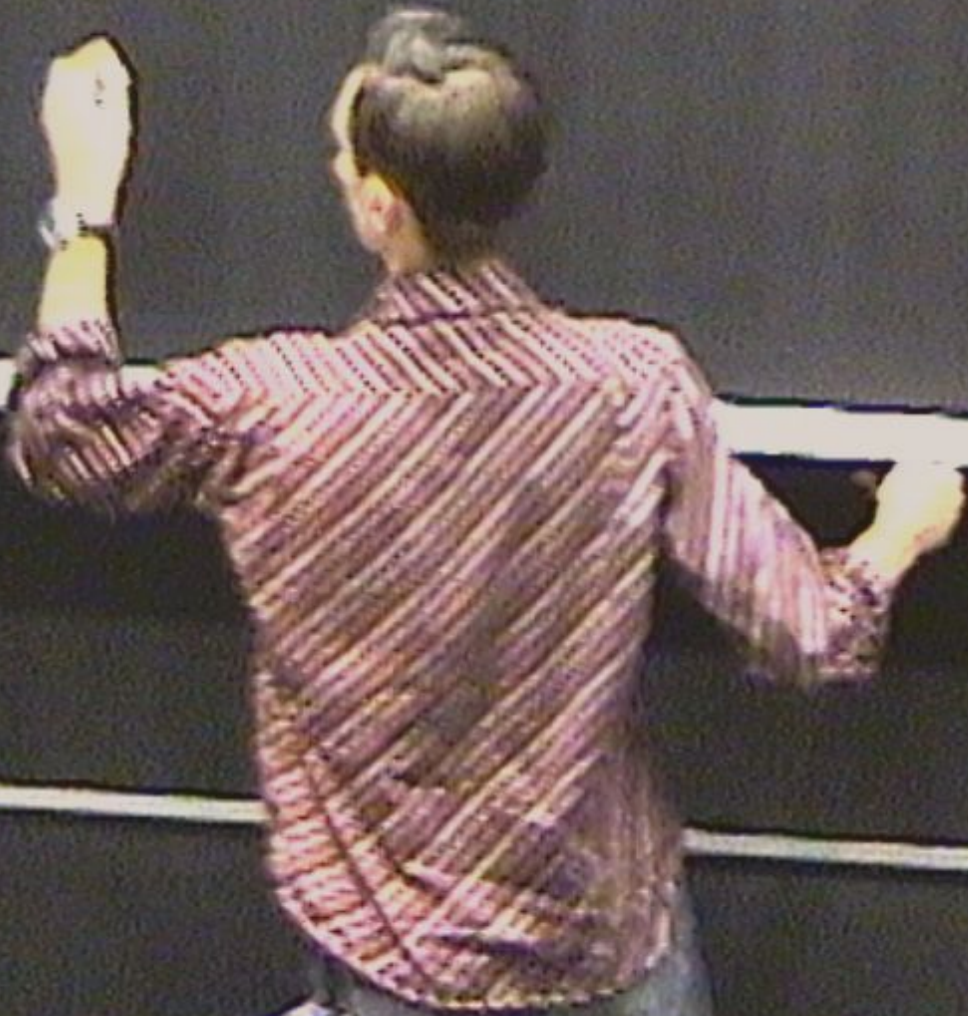
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$2 + 4 = 6$$

$$| \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} =$$

$$2 + 4 = 6$$

$$1 \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$



$$2 + 4 = 6$$

$$1 \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} =$$

$$2 + 4 = 6$$

$$1 \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

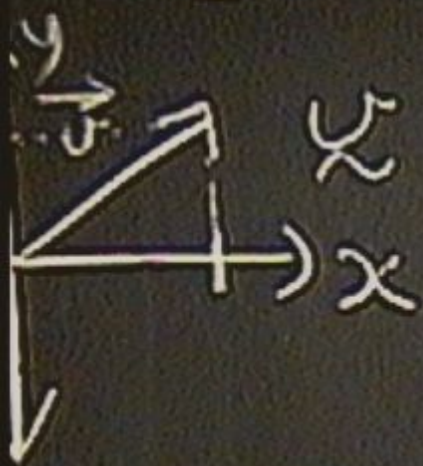
$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$$

$$2 + 4 = 6$$

$$1 \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

ICES



$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad 2 \times 2 \text{ matrix}$$

1. $\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 9 \\ 2 & 1 \end{bmatrix} =$

2. $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \end{bmatrix}$

$$1. \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix} =$$

$$2. \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 & 4 & i & e^{i\pi/4} \\ 7 & \pi & e & 3 & 9 \\ e^{i\pi/3} & \log_2 9 & 9 \cdot 2 & 1 & -17 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 & 5 & 6 \\ 8 & \pi^{-1} & \frac{i+1}{\sqrt{2}} & x & \end{bmatrix} =$$

$$\begin{bmatrix}
 1 & 2 & 4 & i & e^{i\frac{\pi}{4}} \\
 7 & \pi & e & 3 & 9 \\
 e^{i\frac{\pi}{3}} & \log_2 9 & 9.2 & 1 & -17
 \end{bmatrix}
 +
 \begin{bmatrix}
 1 & 3 & 4 & 5 & 6 \\
 8 & \pi^{-1} & \frac{i+1}{\sqrt{2}} & x & \psi \\
 2 & A & i & 243 & \text{☺}
 \end{bmatrix}
 =$$

$$\begin{bmatrix}
 1 & 2 & 4 & i & e^{i\pi/4} \\
 7 & \pi & e & 3 & 9 \\
 e^{i\pi/3} & \log_2 9 & 9 \cdot 2 & 1 & -17
 \end{bmatrix}
 +
 \begin{bmatrix}
 1 & 3 & 4 & 5 & 6 \\
 8 & \pi^{-1} & \frac{i+1}{\sqrt{2}} & x & \psi \\
 z & A & i & 243 & \text{☺}
 \end{bmatrix}
 =$$

\downarrow
 $9, 2$

$$\begin{bmatrix}
 1 & 2 & 4 & i & e^{i\pi/4} \\
 7 & \pi & e & 3 & 9 \\
 e^{i\pi/3} & \log_2 9 & 9.2 & 1 & -17
 \end{bmatrix}
 +
 \begin{bmatrix}
 1 & 3 & 4 & 5 & 6 \\
 8 & \pi^{-1} & \frac{i+1}{\sqrt{2}} & x & \psi \\
 2 & A & i & 2^{43} & \text{☺}
 \end{bmatrix}
 =$$

\downarrow
 9.2

$$9.2 = 9 \times 2$$

$$\begin{bmatrix}
 1 & 2 & 4 & i & e^{i\pi/4} \\
 7 & \pi & e & 3 & 9 \\
 e^{i\pi/3} & \log_2 9 & 9 \cdot 2 & 1 & -17
 \end{bmatrix}
 +
 \begin{bmatrix}
 1 & 3 & 4 & 5 & 6 \\
 8 & \pi^{-1} & \frac{i+1}{\sqrt{2}} & x & \psi \\
 2 & A & i & 243 & \text{☺}
 \end{bmatrix}
 =$$

\downarrow
 $9 \cdot 2$

$$9 \cdot 2 = 9 \times 2$$

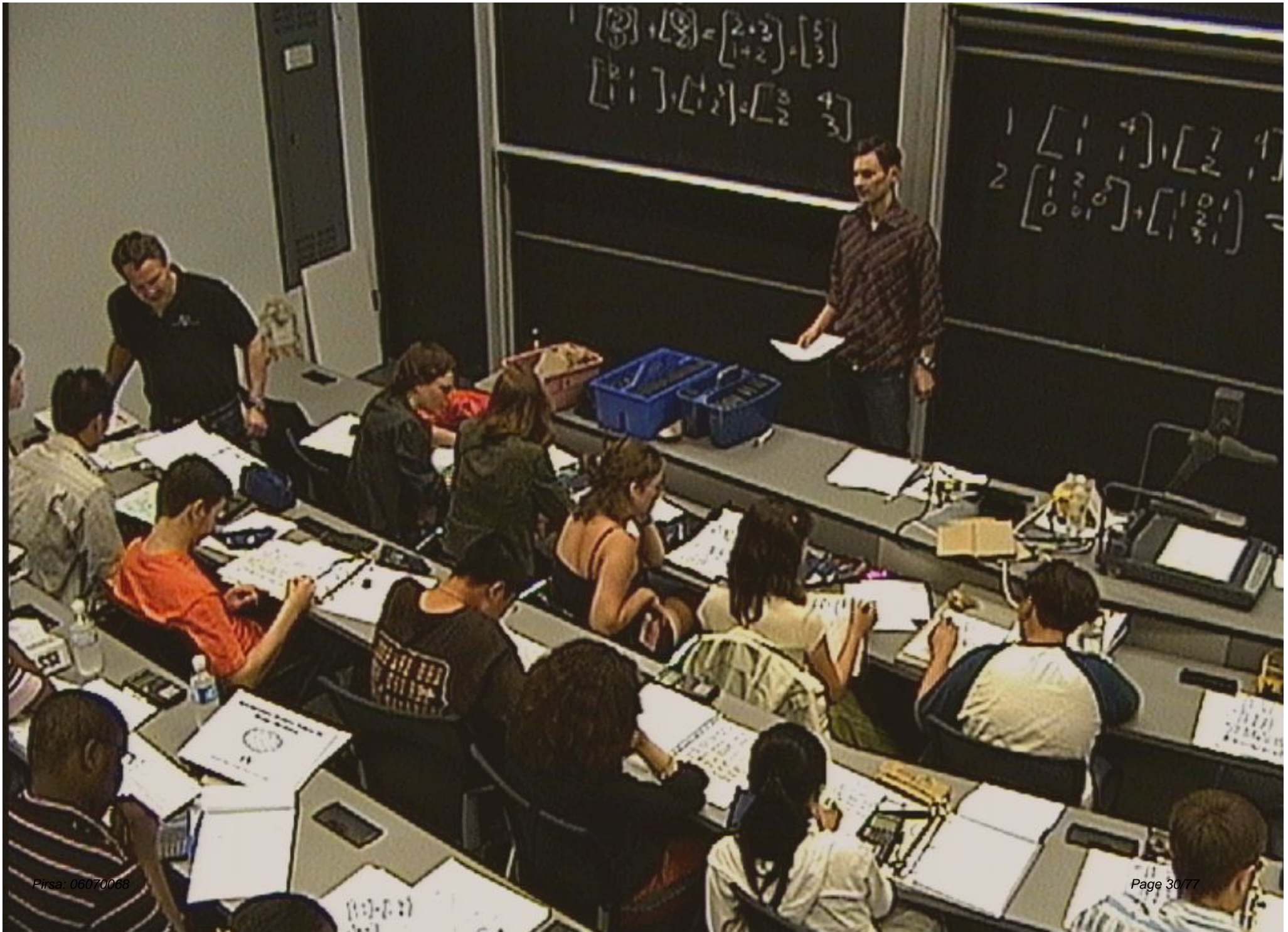
$$\begin{bmatrix} 1 & 2 & 4 & i & e^{i\frac{\pi}{4}} \\ 7 & \pi & e & 3 & 9 \\ e^{i\frac{\pi}{3}} & \log_2 9 & 9 \cdot 2 & 1 & -17 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 & 5 & 6 \\ 8 & \pi^{-1} & \frac{1+i}{\sqrt{2}} & x & \psi \\ 2 & A & i & 243 & \text{☺} \end{bmatrix} =$$

↓
9,2

$$9 \cdot 2 = 9 \times 2$$

→ 2.43
2.43
2,43

2 point 4 3



$$1. \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 9 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 13 \\ 3 & 2 \end{bmatrix}$$

$$2. \begin{bmatrix} 1 & 2 & 0^3 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix}$$

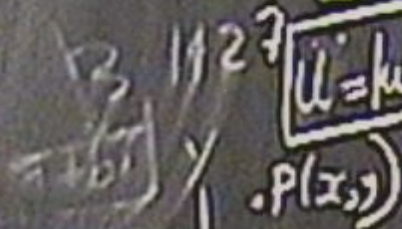
$$\left[\begin{array}{c} 2 \quad 5 \quad 8 \quad 5+i \quad 6re \\ \text{COORDINATES} \\ \text{CIRCLE} \end{array} \right]$$

Polar
Co-ordinates



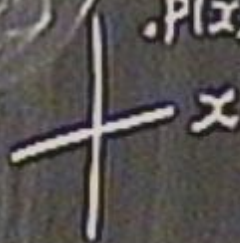
$$z = re^{i\theta}$$

$$u = kule^{i\theta} *$$



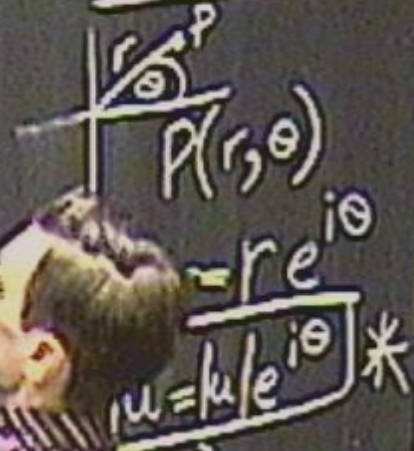
$$2 \quad z = a + bi$$

③



$$\left[\begin{array}{c|c|c|c} 2 & 5 & 8 & 5+i \\ \hline 15 & \pi + \frac{1}{\pi} & e + \frac{i}{\sqrt{2}} & 3+x \\ \hline & & & 6ie^{\frac{\pi}{4}} \end{array} \right]$$

polar
co-ordinates



2 $z = a + bi$

(3)

2	5	8	5+i	6ie ^{iπ/4}
15	π+π	e+ii/3x	9+y	
z+e ^{iπ/3}	A+log ₂ 9	9-2+i	3+ix	⊙ -17

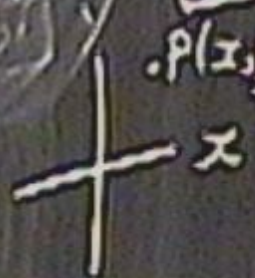
polar
co-ordinates



$$z = r e^{i\theta}$$

$$u = k u e^{i\theta} *$$

③



$$z = a + bi$$

2	5	8	5+i	6re ^{iπ}
15	$\pi + \frac{1}{\pi}$	$e^{i\frac{1}{\sqrt{2}}}$	3.6	9+ $\frac{1}{\sqrt{2}}$
$z + e^{\frac{1}{\sqrt{2}}}$	$\Delta + \log_2 9$	9.2+i	3.93	⊙ -17

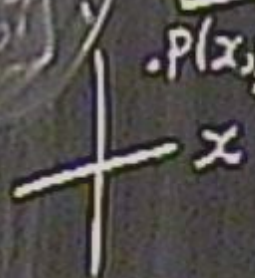
Polar co-ordinates



$$z = r e^{i\theta}$$

$$w = k u e^{i\theta} *$$

③



$$z = a + bi$$

9,2

$$9 \cdot 2 = 9 \times 2$$

2.43

2	5	8	5+i	6+e ^π
15	π+π	e ^{+iπ/√2}	3.68	9+ψ
z + e ^{π/3}	A + log ₂ 9	9.2+i	3.83	ψ-17

polar
(co-ordinates)



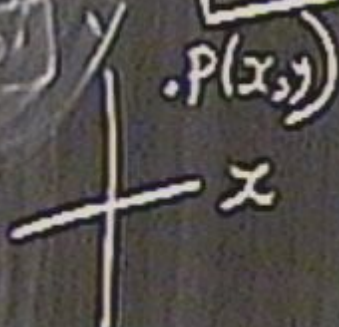
$$z = r e^{i\theta}$$

$$u = |u| e^{i\theta} *$$

$$\psi = \Psi$$

$$z = a + bi$$

③



$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} =$$



$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} \Rightarrow$$

$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix}$$

1ST COLUMN

$$\begin{bmatrix} a+e+f \\ - \end{bmatrix}$$

1ST ROW

$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix}$$

$$\begin{matrix} \text{1ST COLUMN} & \text{2ND COLUMN} \\ \text{1ST row} & \end{matrix} \begin{bmatrix} a \cdot e + b \cdot g & \text{---} \\ \text{---} & \text{---} \end{bmatrix}$$



$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix}$$

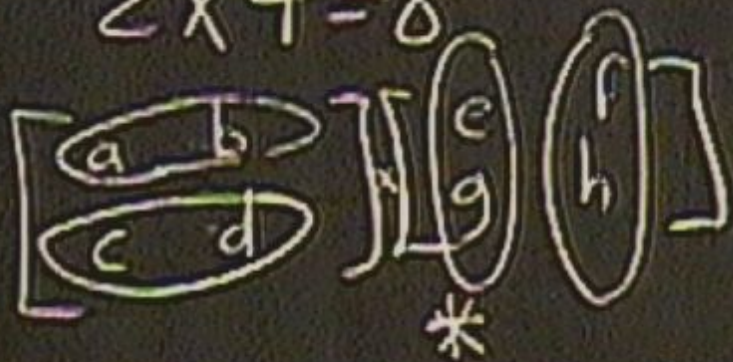
$$\begin{matrix} \text{1st column} & \text{2nd column} \\ \text{1st row} & \text{2nd row} \end{matrix} \begin{bmatrix} a \cdot e + b \cdot f & = \\ - & = \end{bmatrix}$$

$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ f \\ g \end{bmatrix}$$

$$\begin{matrix} \text{1ST COLUMN} & \text{2ND COLUMN} \\ \text{1ST ROW} & \end{matrix} \begin{bmatrix} ae+bf & af+bg \\ - & - \end{bmatrix}$$

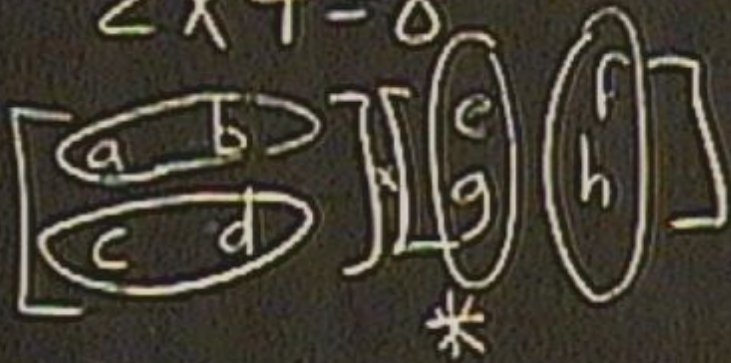
$$2 \times 4 = 8$$



1st column 2nd column

$$\begin{matrix} \text{1st row} \\ \text{2nd row} \end{matrix} \begin{bmatrix} a & e + b & a & f + b \\ c & e + d & c & h \end{bmatrix}$$

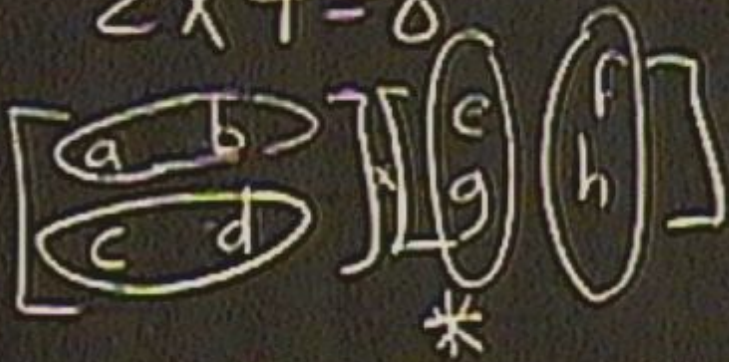
$$2 \times 4 = 8$$



$$\begin{matrix} \text{1ST COLUMN} & \text{2ND COLUMN} \\ \text{1ST ROW} & \end{matrix} \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \\ g & h \end{bmatrix}$$



$$2 \times 4 = 8$$

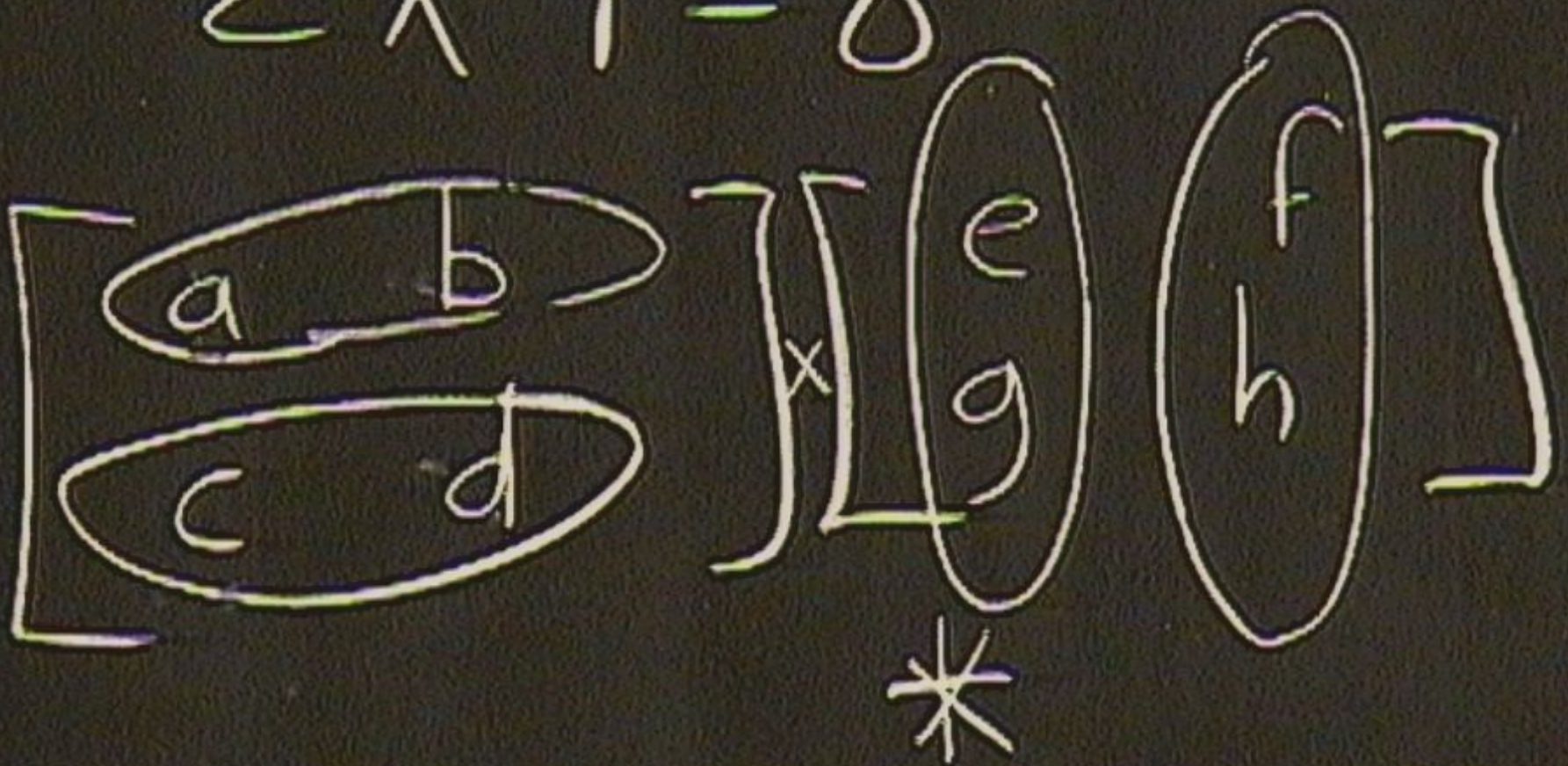


1st column 2nd column

$$\begin{matrix} \text{1st row} \\ \text{2nd row} \end{matrix} \begin{bmatrix} a & c & f \\ b & g & h \\ c & d & f \\ d & f & h \end{bmatrix}$$



$$2 \times 4 = 8$$



1ST COLUMN

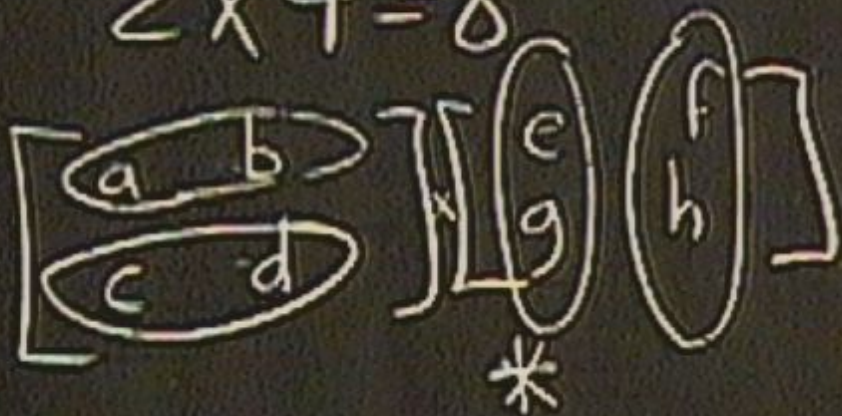
2ND COLUMN

1ST ROW

$$\begin{bmatrix} ax + e + bg \\ ce + dg \end{bmatrix}$$

$$\begin{bmatrix} af + bh \\ cf + dh \end{bmatrix}$$

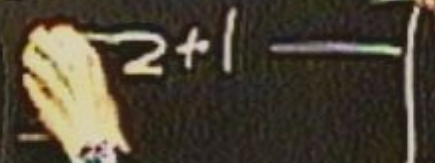
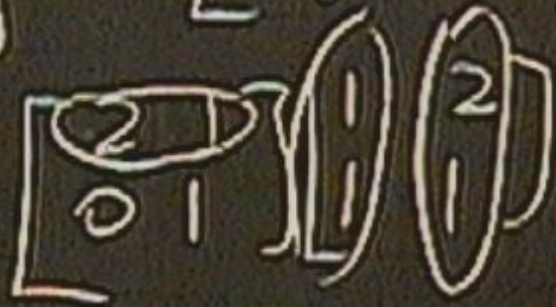
$$2 \times 4 = 8$$



1ST COLUMN 2ND COLUMN

1ST ROW

$$\begin{bmatrix} \underline{ae+bg} & \underline{af+bh} \\ \underline{ce+dg} & \underline{cf+dh} \end{bmatrix}$$



$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e \\ g \\ h \end{bmatrix} *$$

$$\begin{matrix} \text{1ST COLUMN} & \text{2ND COLUMN} \\ \text{1ST ROW} & \end{matrix} \begin{bmatrix} \underline{ae+bg} & \underline{af+bh} \\ \underline{ce+dg} & \underline{cf+dh} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+1 & 5 \end{bmatrix}$$

$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = *$$

$$\begin{matrix} \text{1ST COLUMN} & \text{2ND COLUMN} \\ \text{1ST ROW} & \end{matrix} \begin{bmatrix} \underline{ae+bf} & \underline{af+bh} \\ \underline{ce+dg} & \underline{cf+dh} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2+1 & 5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix}$$

11. $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

$$1. \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} =$$



$$1. \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2 \times 4 = 8$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c & d \\ e & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

if equal system

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c & d \\ e & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$1. \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 1 & 2 \end{bmatrix}$$

←←

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ \text{ } \end{bmatrix}$$

2×2 matrix 2×1 matrix

$$11. \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 1 & 2 \end{bmatrix}$$

←

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ | \\ | \end{bmatrix}$$

2 x 2 matrix 2 x 1 matrix

←

$$11. \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ -1 & 2 \end{bmatrix}$$

←

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

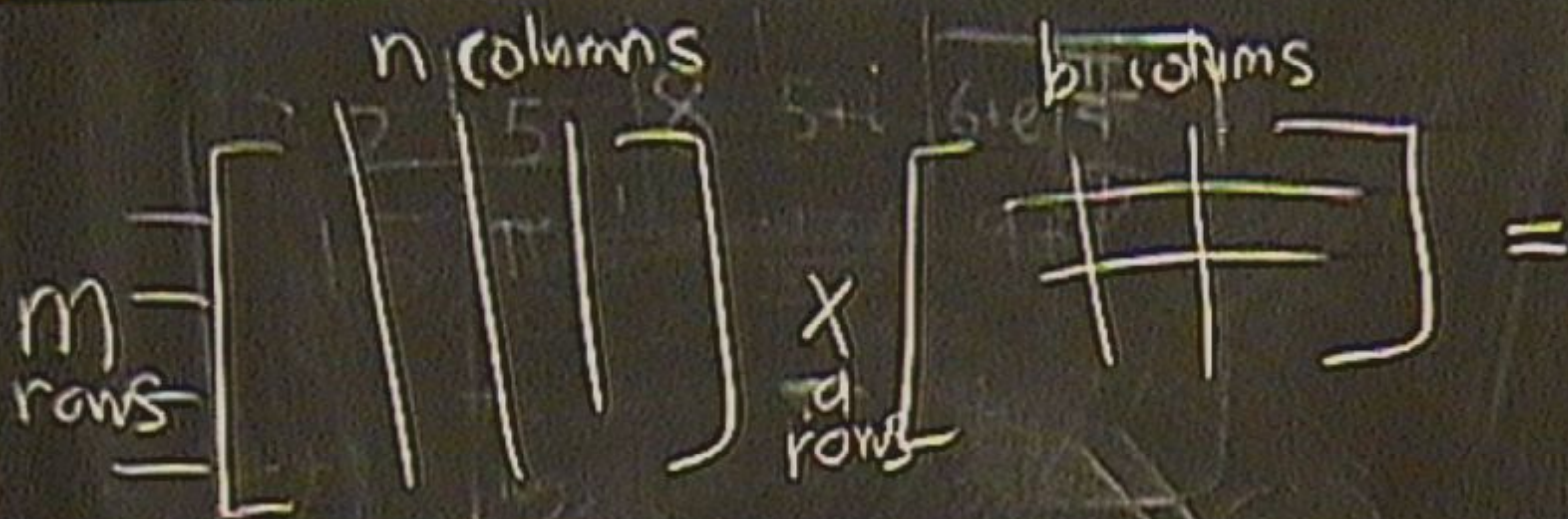
2 x 2 matrix 2 x 1 matrix

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$

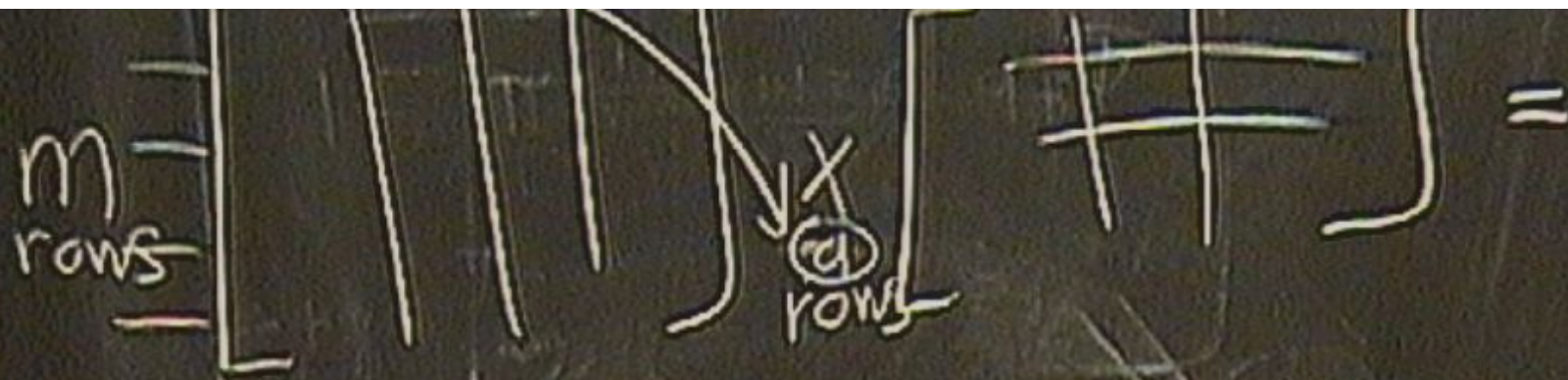
$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 1+3 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 11 & 11 \end{bmatrix} \times \begin{bmatrix} 5+i & 6+ie \\ 9+2j & 4 \end{bmatrix}$$





+



no. of columns in first =
no. of columns in 2



m
rows

x
rows



no. of columns in first =
no. of rows in 2nd



MATRICES

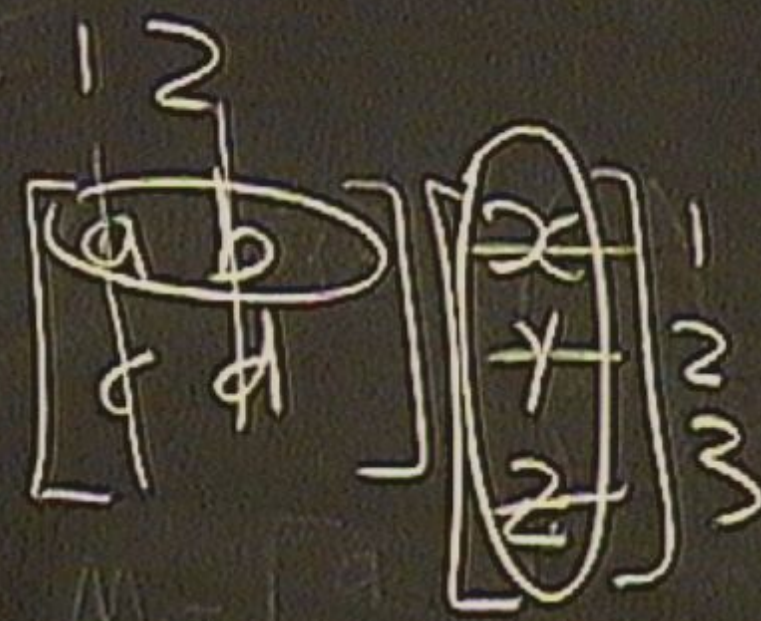


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

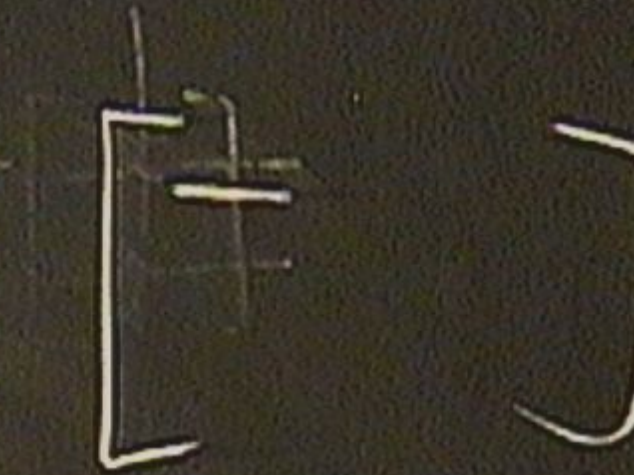
$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

2 x 2 matrix

MATRICES



$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



2 x 2 matrix

MATRICES 10

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$A \cdot X = \text{Matrix}$$

MATRICES 10

Transpose:

$$A^T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

MATRICES

Transpose:

A^T

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

MATRICES

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

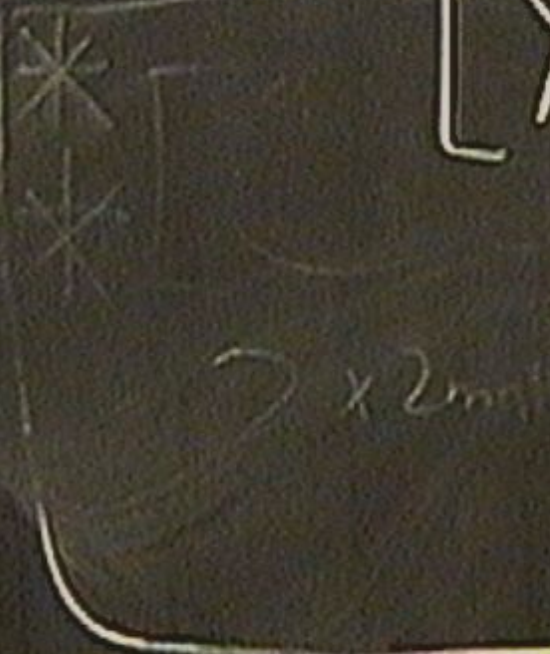
Transpose:

A^T

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

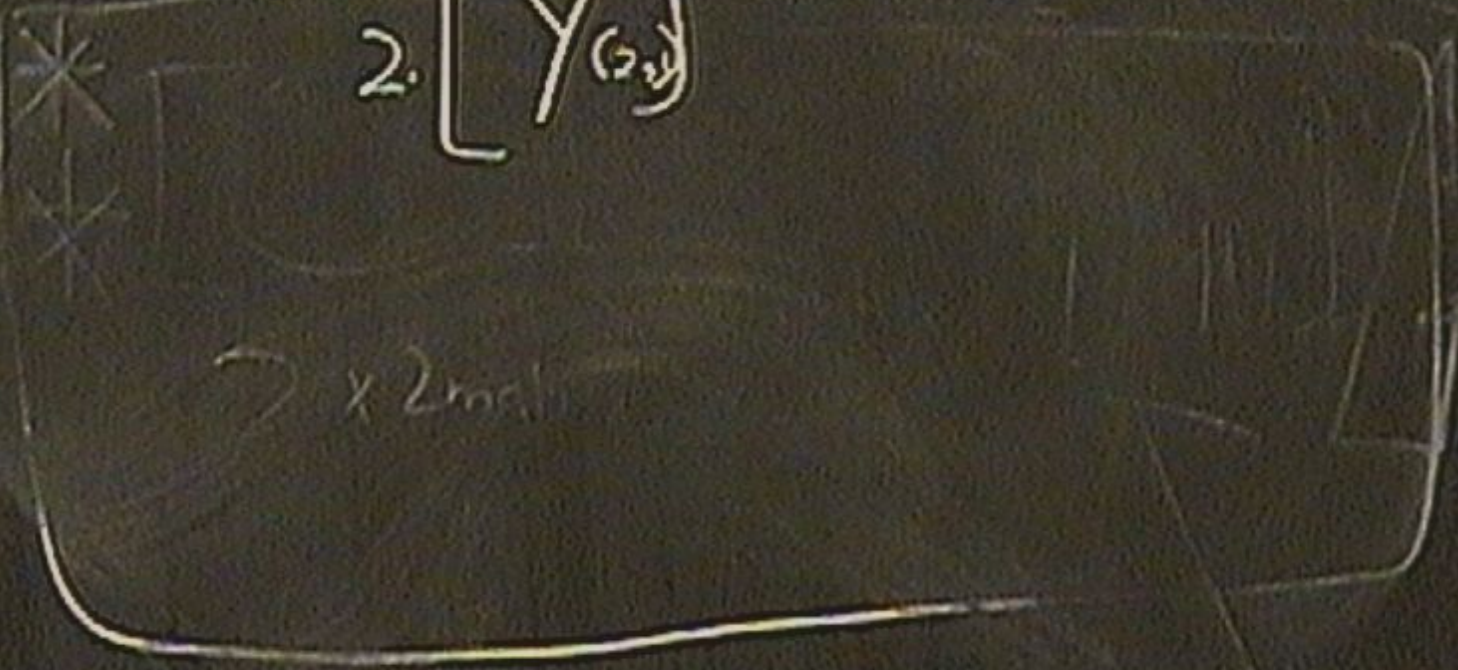
11.
$$B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ -1 & 2 \end{bmatrix}$$



11.

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 10 \\ -1 & 2 \end{bmatrix}$$

$$B = \begin{matrix} 1. \\ 2. \end{matrix} \begin{bmatrix} x_{(1,1)} \\ y_{(2,2)} \end{bmatrix}$$



11.

$$B = \begin{matrix} 1. \\ 2. \end{matrix} \begin{bmatrix} x_{(1,1)} \\ y_{(2,1)} \end{bmatrix}$$

$$C = \begin{matrix} 1. & 2. \\ 2. \end{matrix} \begin{bmatrix} a_{(1,1)} & b_{(1,2)} \\ c_{(2,1)} & d_{(2,2)} \end{bmatrix}$$

2 x 2 mat

11.

$$B = \begin{matrix} 1. \\ 2. \end{matrix} \begin{bmatrix} x_{(1,1)} \\ y_{(2,1)} \end{bmatrix}$$

$$C = \begin{matrix} 1. & 2. \\ 2. \end{matrix} \begin{bmatrix} a_{(1,1)} & b_{(1,2)} \\ c_{(2,1)} & d_{(2,2)} \end{bmatrix}$$

$$C^T = \begin{bmatrix} a \\ \end{bmatrix}$$

(1,2) → (2,1)

$$B = \begin{matrix} 1. \\ 2. \end{matrix} \begin{bmatrix} x_{(1,1)} \\ y_{(2,2)} \end{bmatrix}$$

$x_{(1,1)}$	$x_{(1,2)}$
$y_{(2,1)}$	$y_{(2,2)}$

$$C = \begin{matrix} 1. & 2. \\ 1. & 2. \end{matrix} \begin{bmatrix} a_{(1,1)} & b_{(1,2)} \\ c_{(2,1)} & d_{(2,2)} \end{bmatrix}$$

$$C^T = \begin{bmatrix} a \\ b \end{bmatrix}$$

$(1,2) \rightarrow (2,1)$

$$B = \begin{matrix} 1. \\ 2. \end{matrix} \begin{bmatrix} x_{(1,1)} \\ y_{(2,2)} \end{bmatrix}$$

$$1. \begin{matrix} (1,1) \rightarrow (1,1) \\ (2,1) \end{matrix}$$

$$1. \begin{matrix} (1,2) \rightarrow (2,1) \end{matrix}$$

$$2. \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$(1,1)$	$(1,2)$
$(2,1)$	$(2,2)$

$x_{2 \times 2}$

$$B = \begin{matrix} 1. & \begin{bmatrix} x_{(1,1)} \\ y_{(2,1)} \end{bmatrix} \\ 2. & \end{matrix}$$

$$\begin{matrix} (1,1) \rightarrow (1,1) \\ (2,1) \rightarrow (1,2) \end{matrix}$$

$$\begin{matrix} (1,2) \rightarrow (2,1) \\ (2,2) \rightarrow (2,2) \end{matrix}$$

$$B = \begin{bmatrix} x_{(1,1)} \\ y_{(2,1)} \end{bmatrix}$$

1. $(1,1) \rightarrow (1,1)$
2. $(2,1) \rightarrow (1,2)$

$$B^T = \begin{bmatrix} x & y \end{bmatrix}$$

row matrix

$$B = \begin{bmatrix} x_{(1,1)} \\ y_{(2,1)} \end{bmatrix}$$

1. $(1,1) \rightarrow (1,1)$
 2. $(2,1) \rightarrow (1,2)$

$$B^T = [x \quad y]$$

$$2 \times 1 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$1 \times 2 [x \quad y]$$

$$2 \times 2 \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

row matrix

1 x 2 matrix