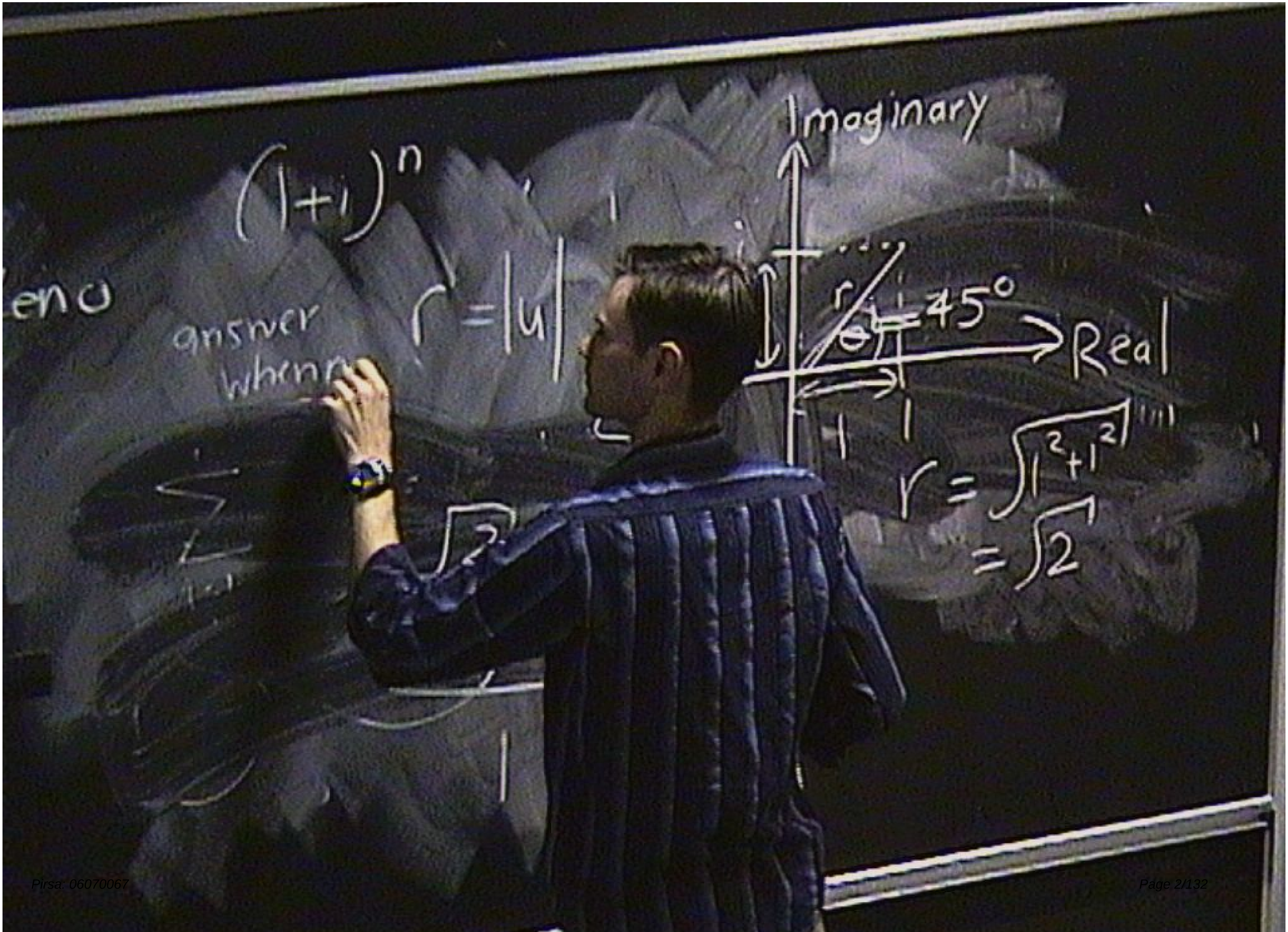
Title: Math Primer: Complex Numbers Continued

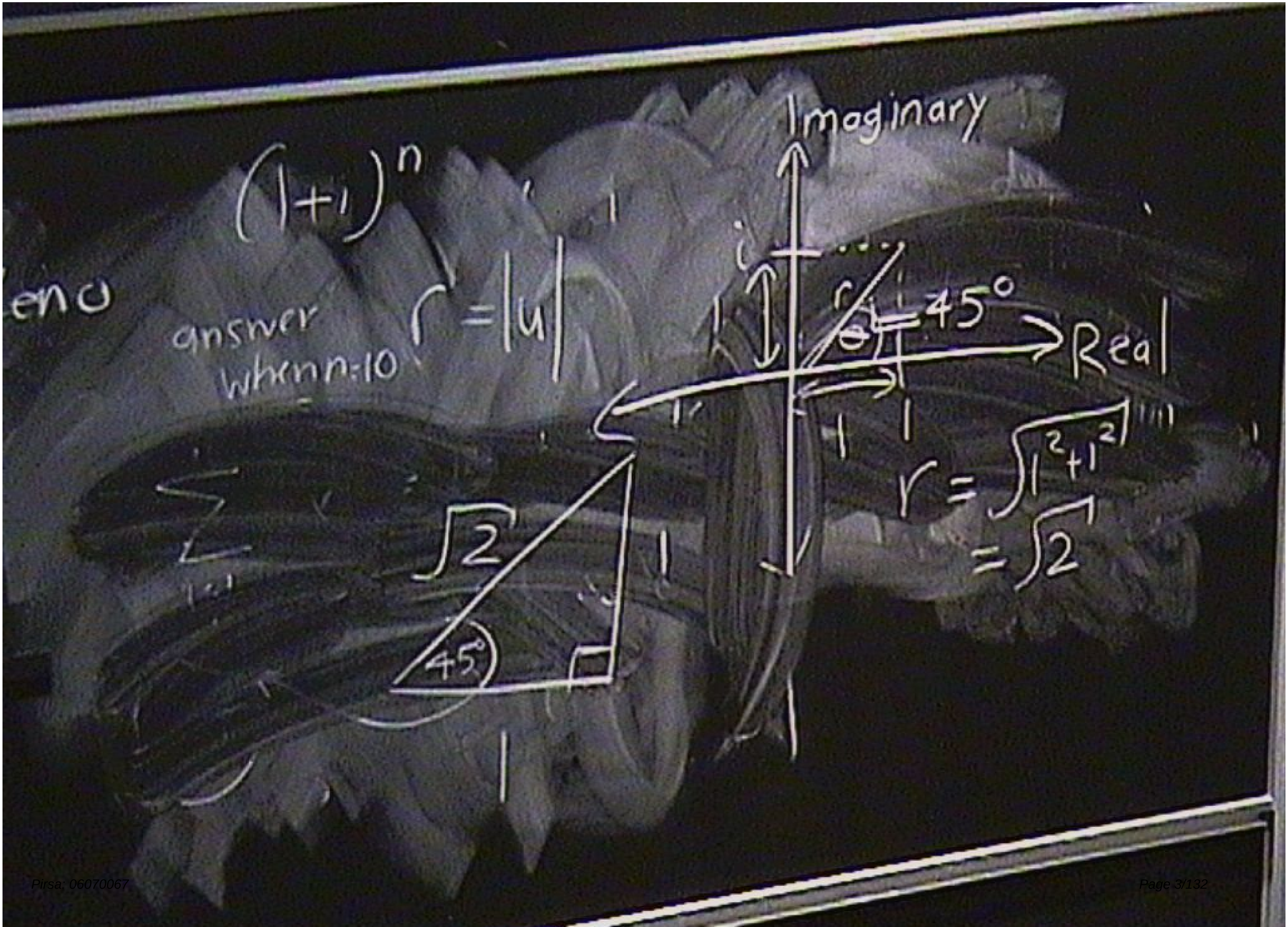
Date: Jul 24, 2006 03:45 PM

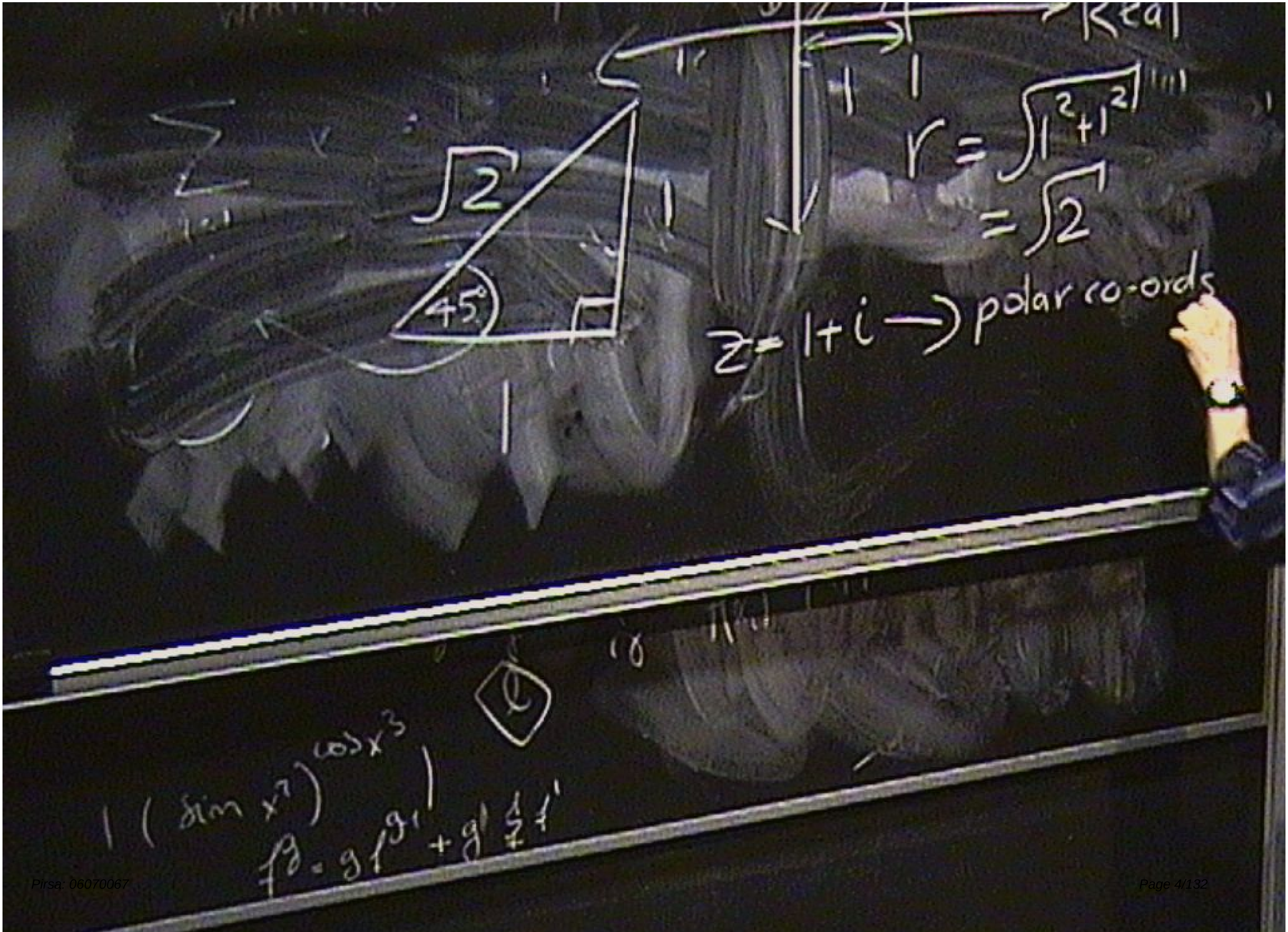
URL: http://pirsa.org/06070067

Abstract:

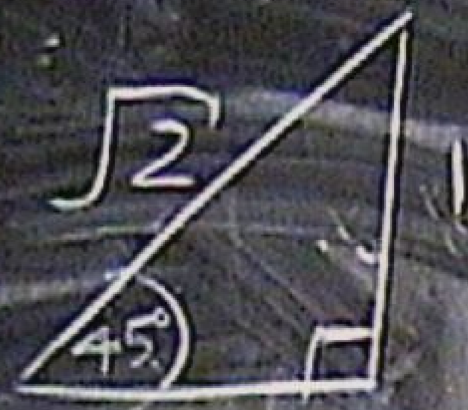


Imaginary
Real
$(1+i)^n$
$r=|u|$
answer
when n
45°
$r=\sqrt{1^2+1^2}$
$=\sqrt{2}$


(1+i)^n
Imaginary
Real
answer
when n=10
r = |u|
45°
r = √(1²+1²)
= √2
√2
45°
i
1


Real
r = √(1²+1²)
= √2
z = 1+i → polar co-ords
√2
45°
1
1
(sin x³)^cos x³
f' = g f^(g-1) + g' ln f

Real

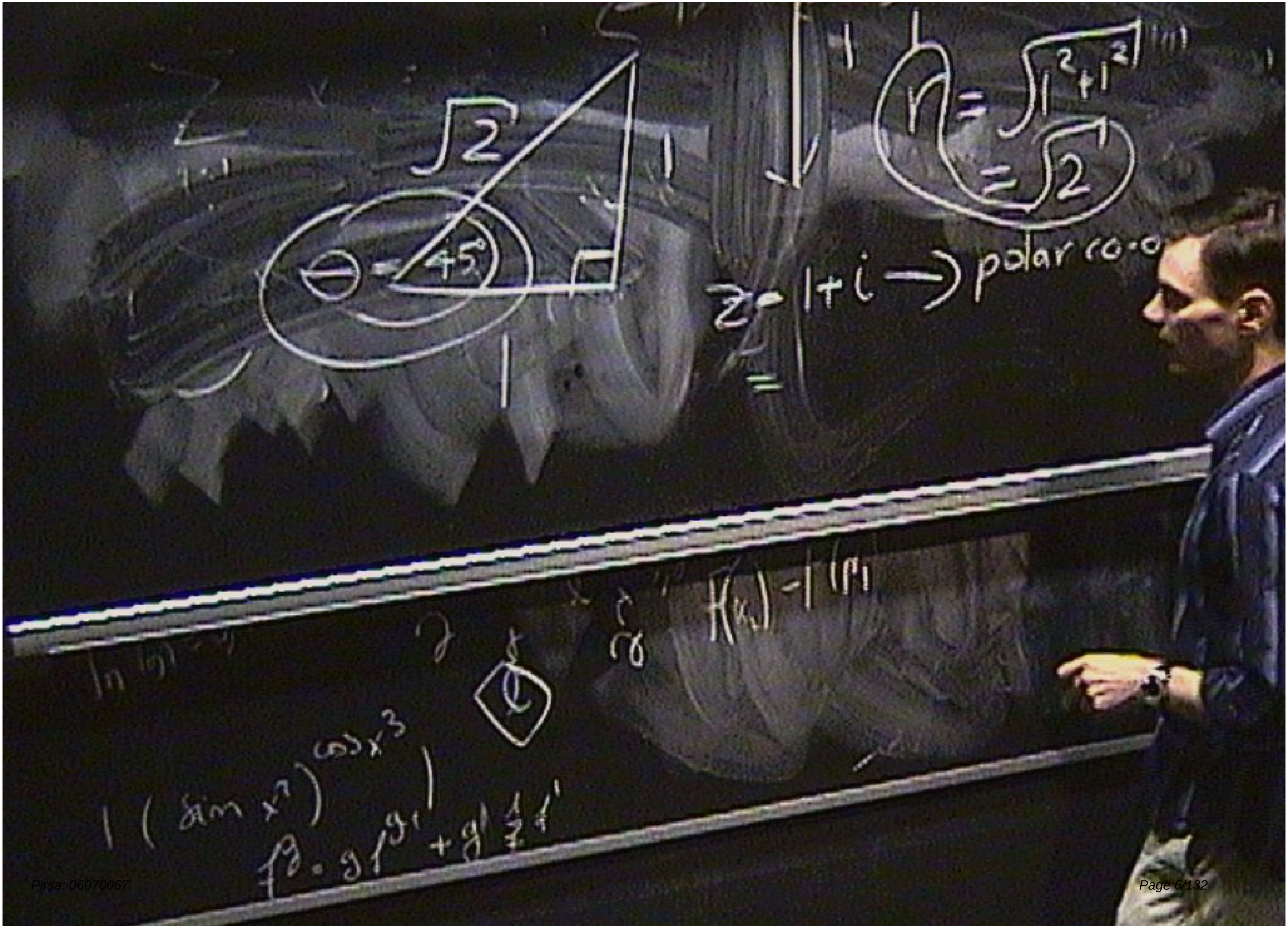


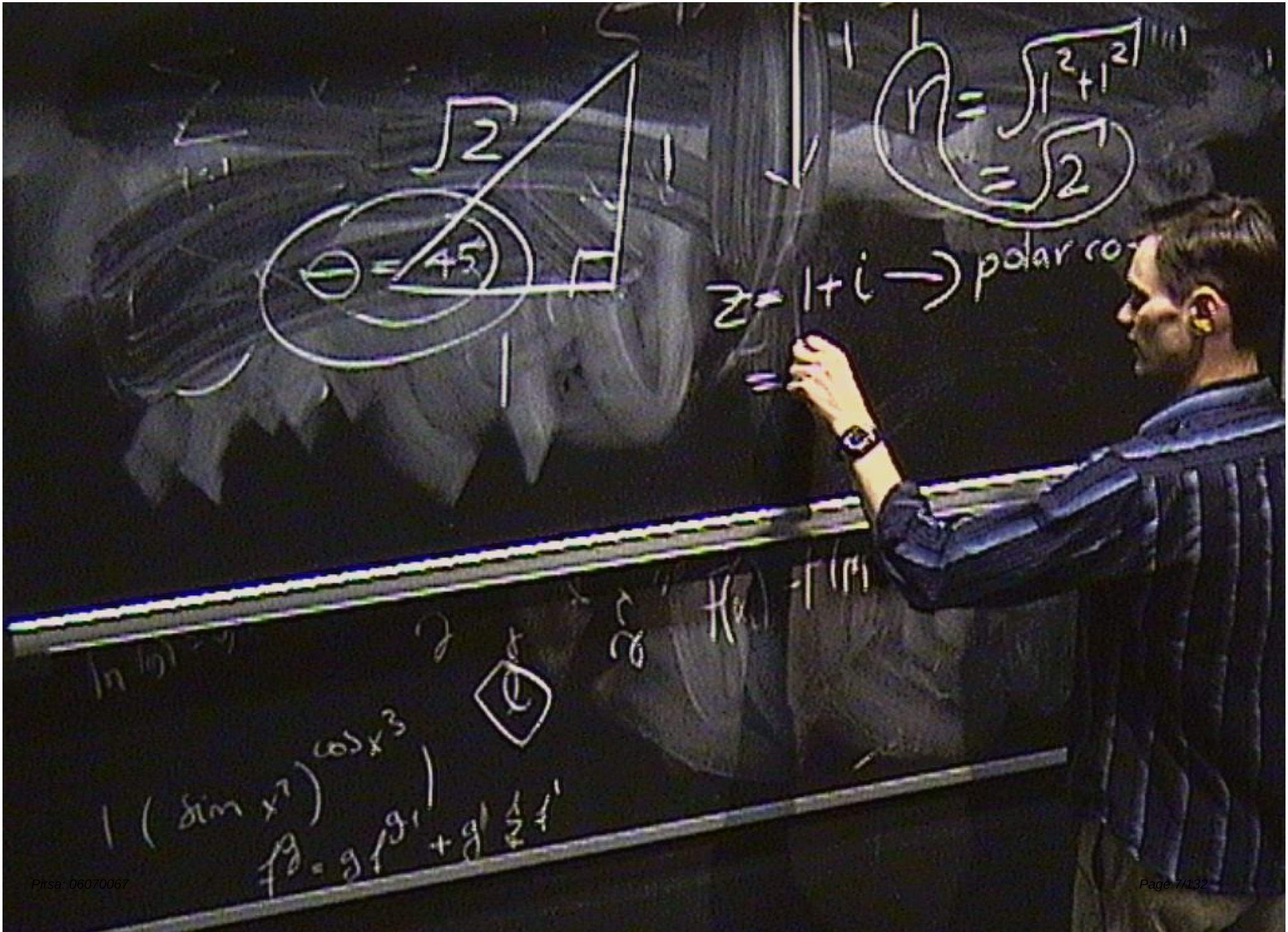
$$r = \sqrt{1^2+1^2}$$
$$= \sqrt{2}$$

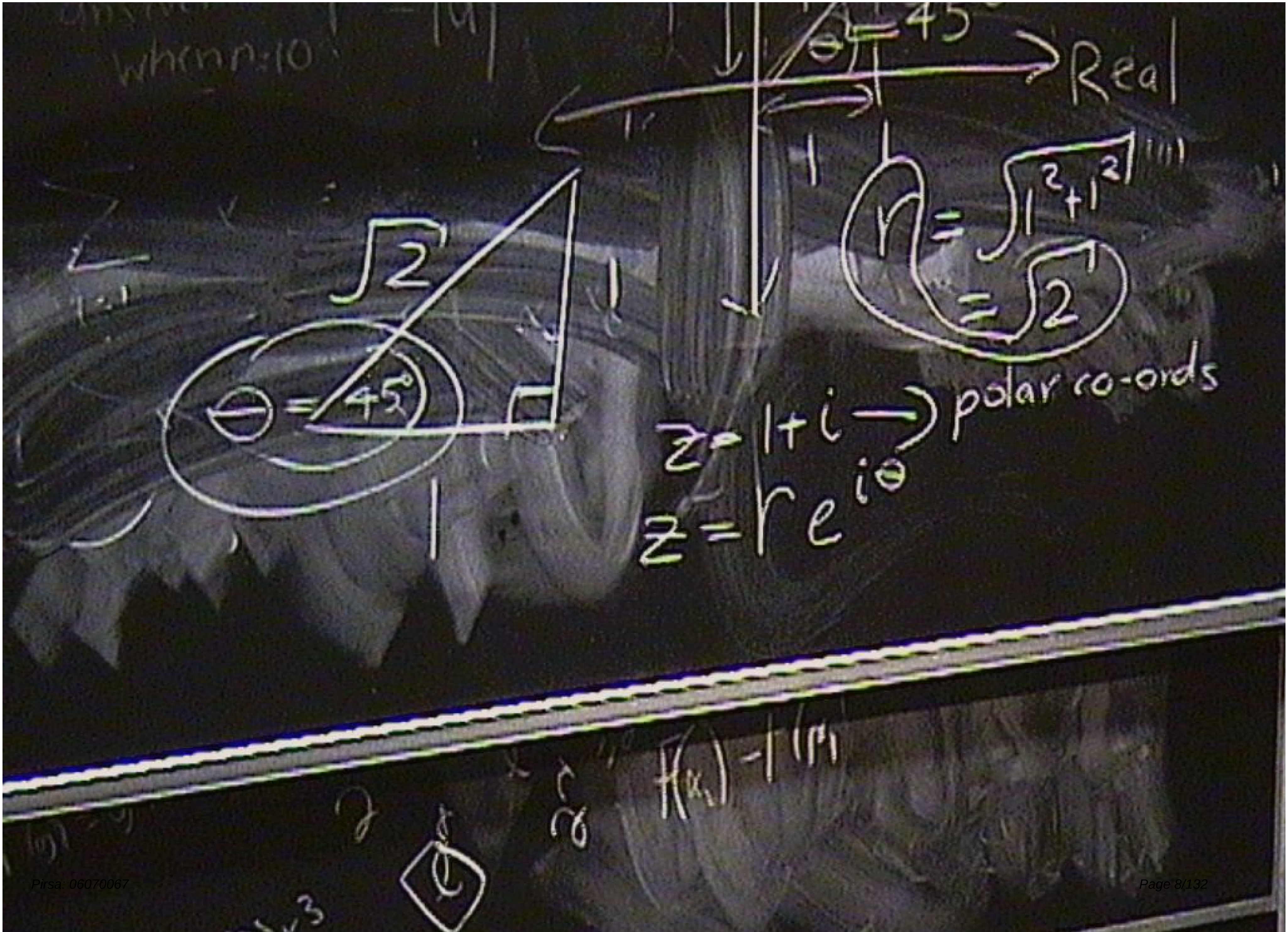
$z = 1+i \rightarrow$ polar co-ords

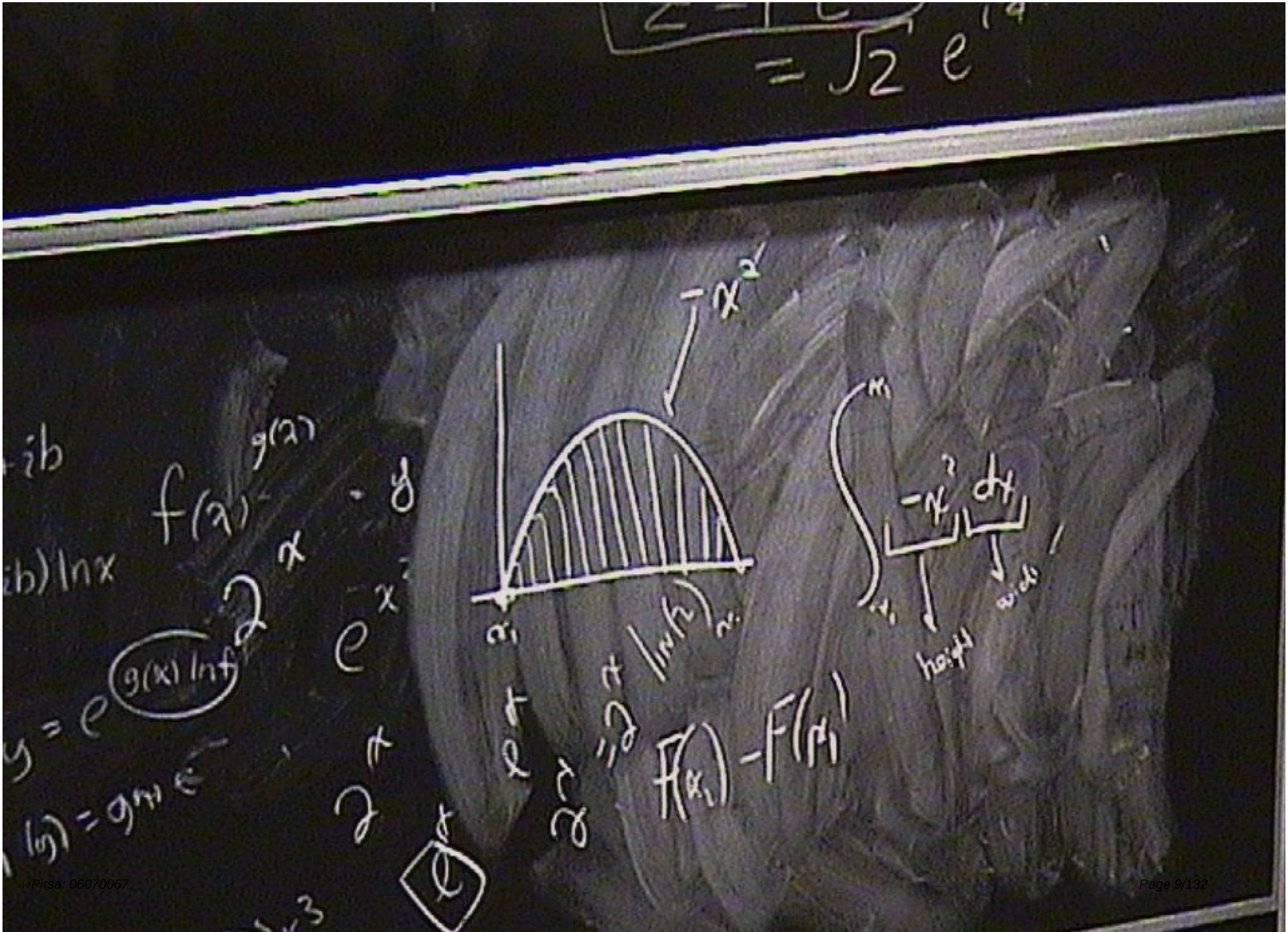
$(\sin x^3)^{\cos x^3}$

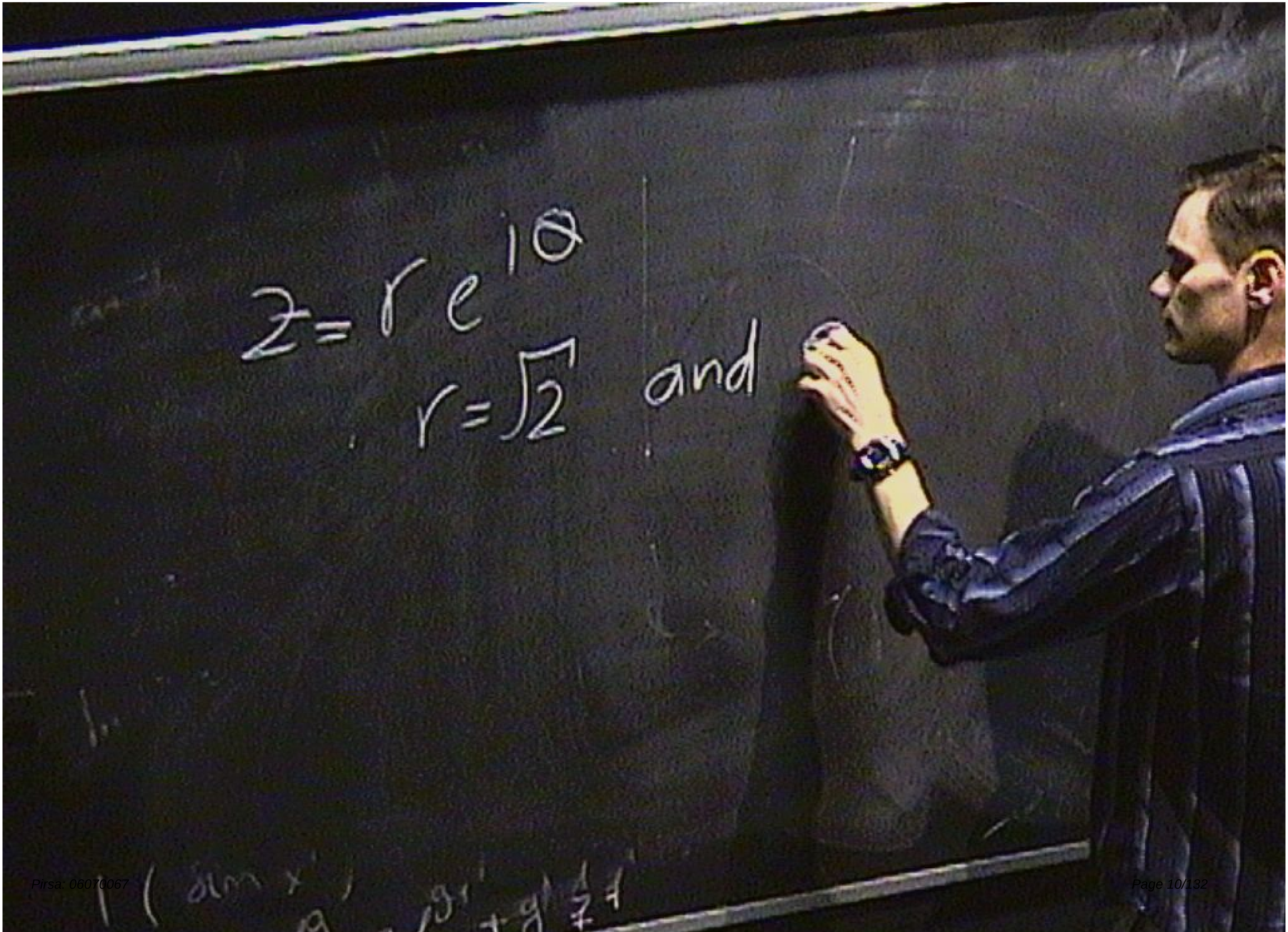
$f^g = g f^{g-1} + g' \ldots f'$

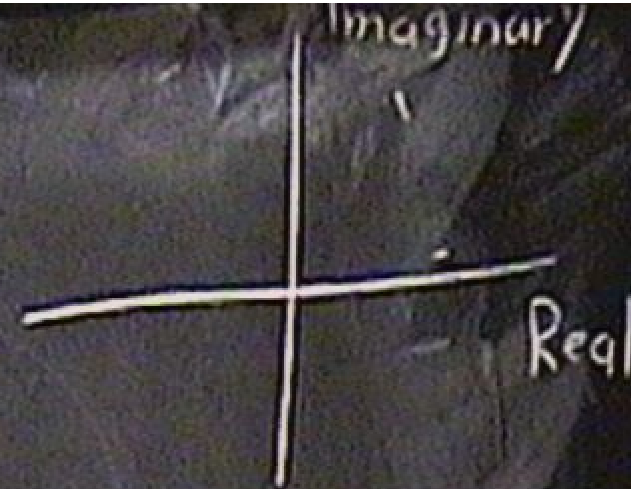

√2
θ = 45°
r = √(1²+1²) = √2
z = 1+i → polar co-o


√2
θ = 45
r = √(1²+1²)
= √2
z = 1+i → polar co


Real
√2
1
θ = 45°
r = √(1² + 1²) = √2
z = 1 + i → polar co-ords
z = re^{iθ}


= √2 e
e^{-x^2}
f(x)
g(x)
(ib) ln x
y = e^{g(x) ln f}
F(x_1) − F(x_1)
height


$z = re^{i\theta}$
$r = \sqrt{2}$ and



$$z = re^{i\theta}$$

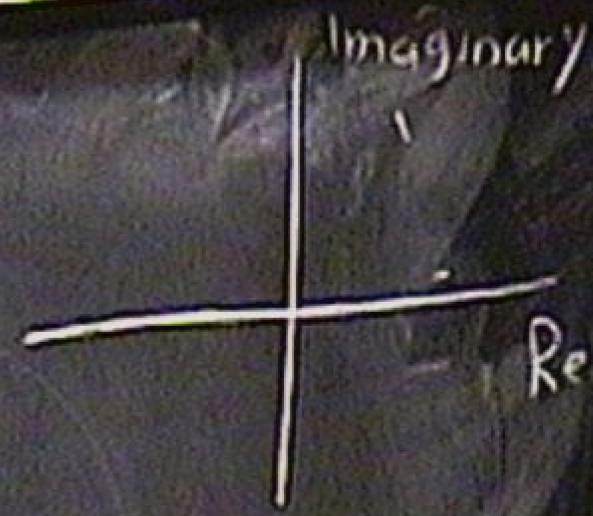
$$r = \sqrt{2} \quad \text{and} \quad \theta = \frac{\pi}{4}$$

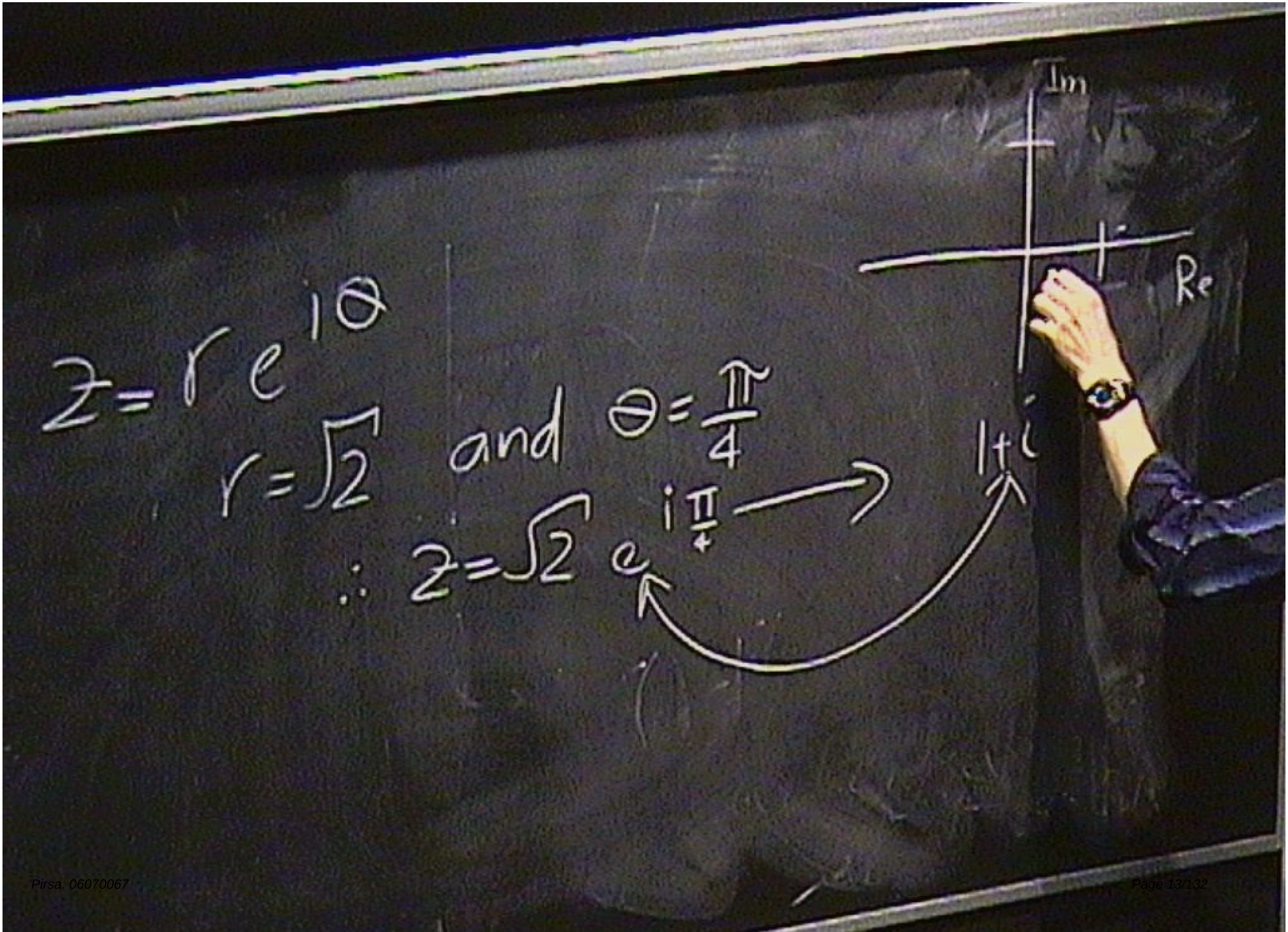
$$\therefore\ z = \sqrt{2}\, e^{i\frac{\pi}{4}} \longrightarrow 1+i$$

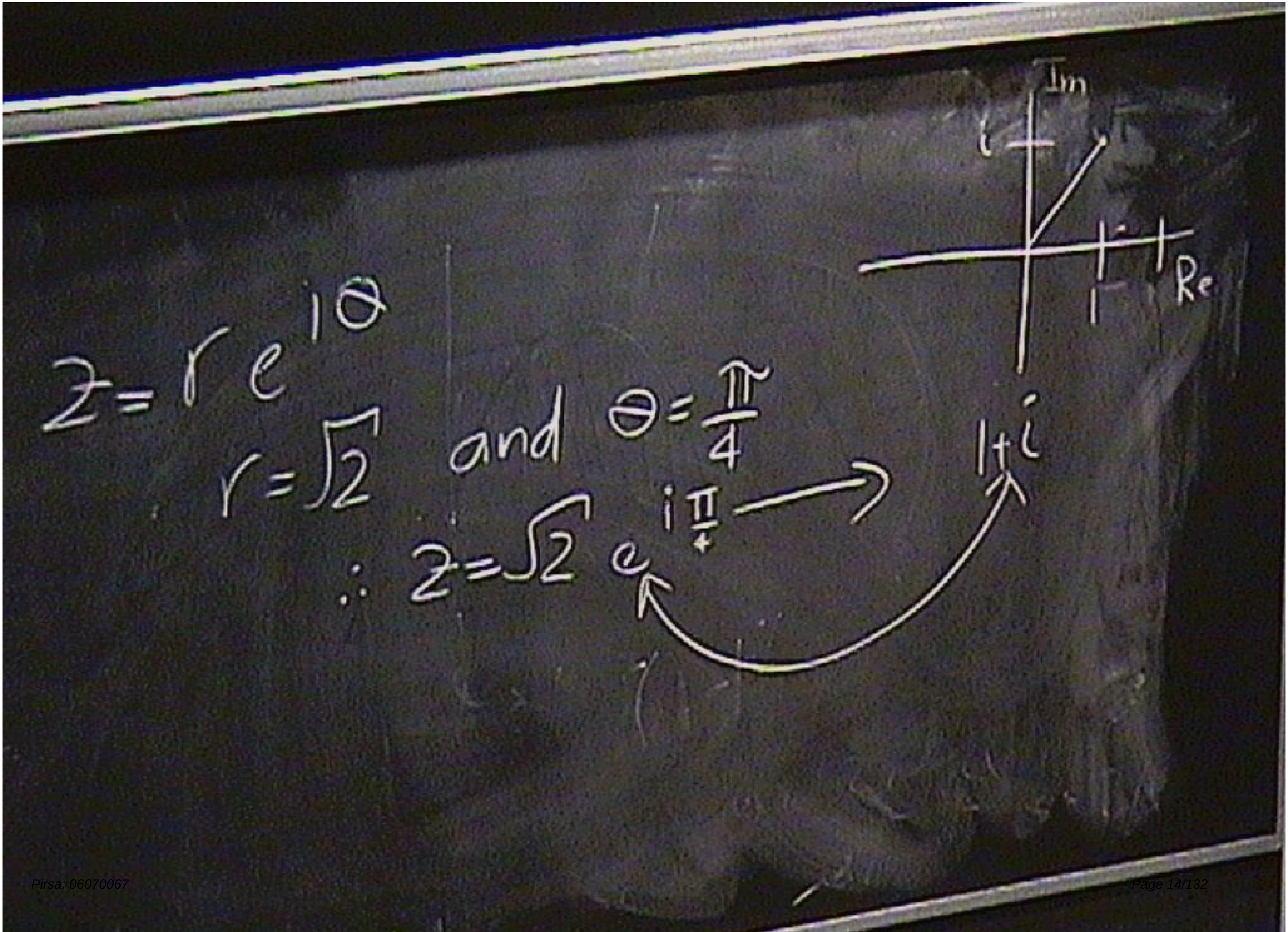
$$z = re^{i\theta}$$

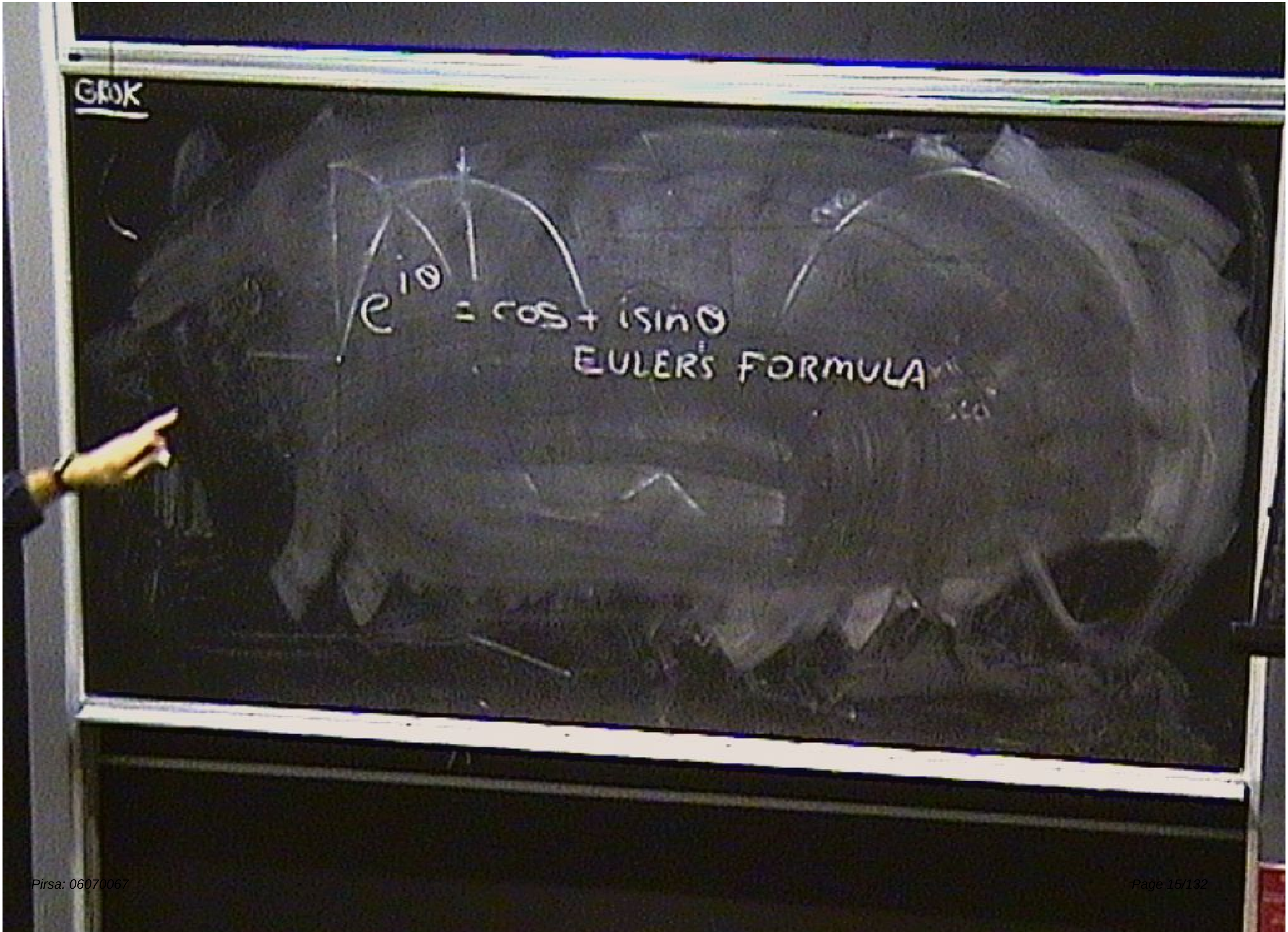
$r = \sqrt{2}$ and $\theta = \frac{\pi}{4}$

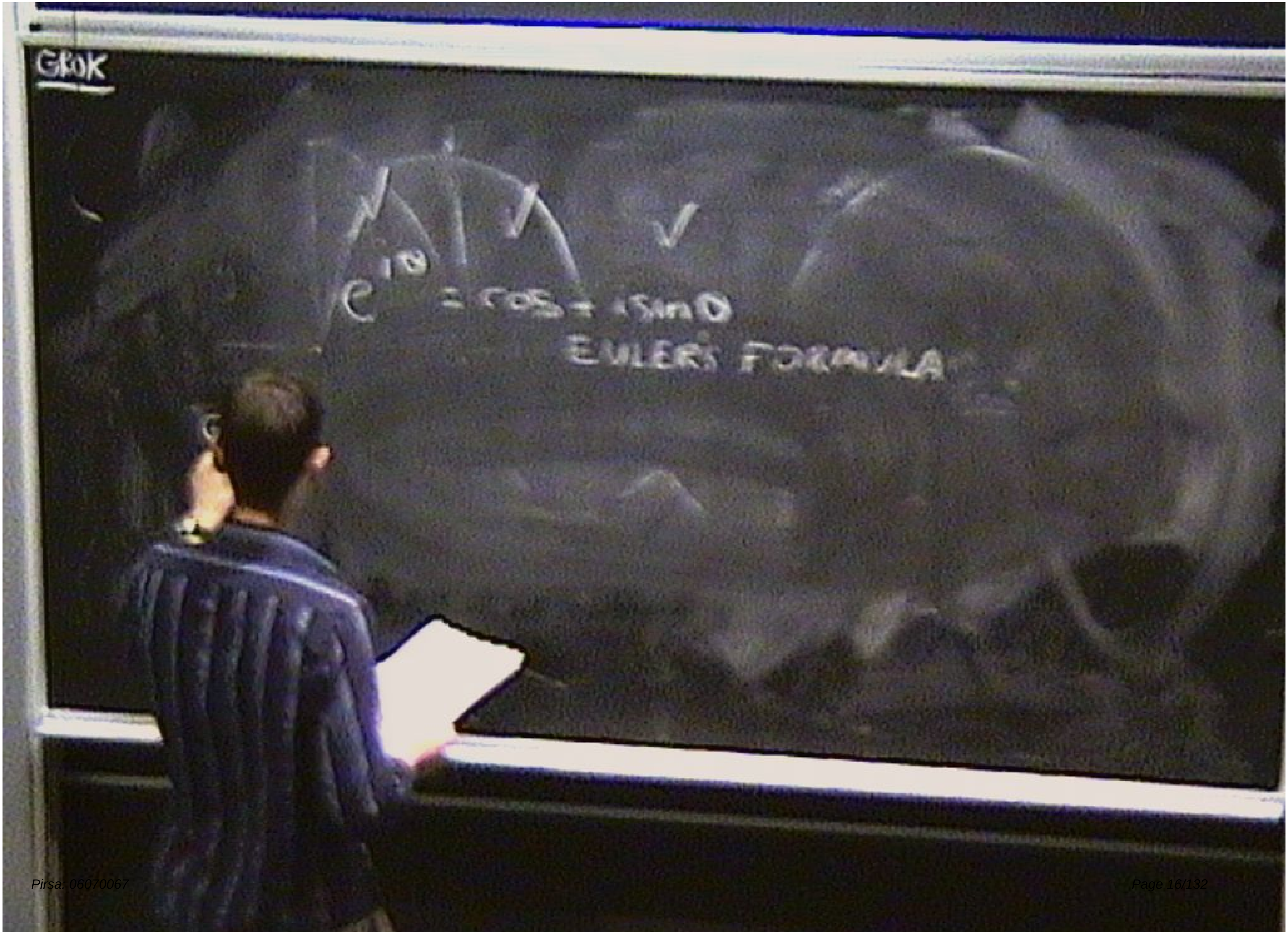
$$\therefore z = \sqrt{2}\, e^{i\frac{\pi}{4}} \longrightarrow 1+i$$

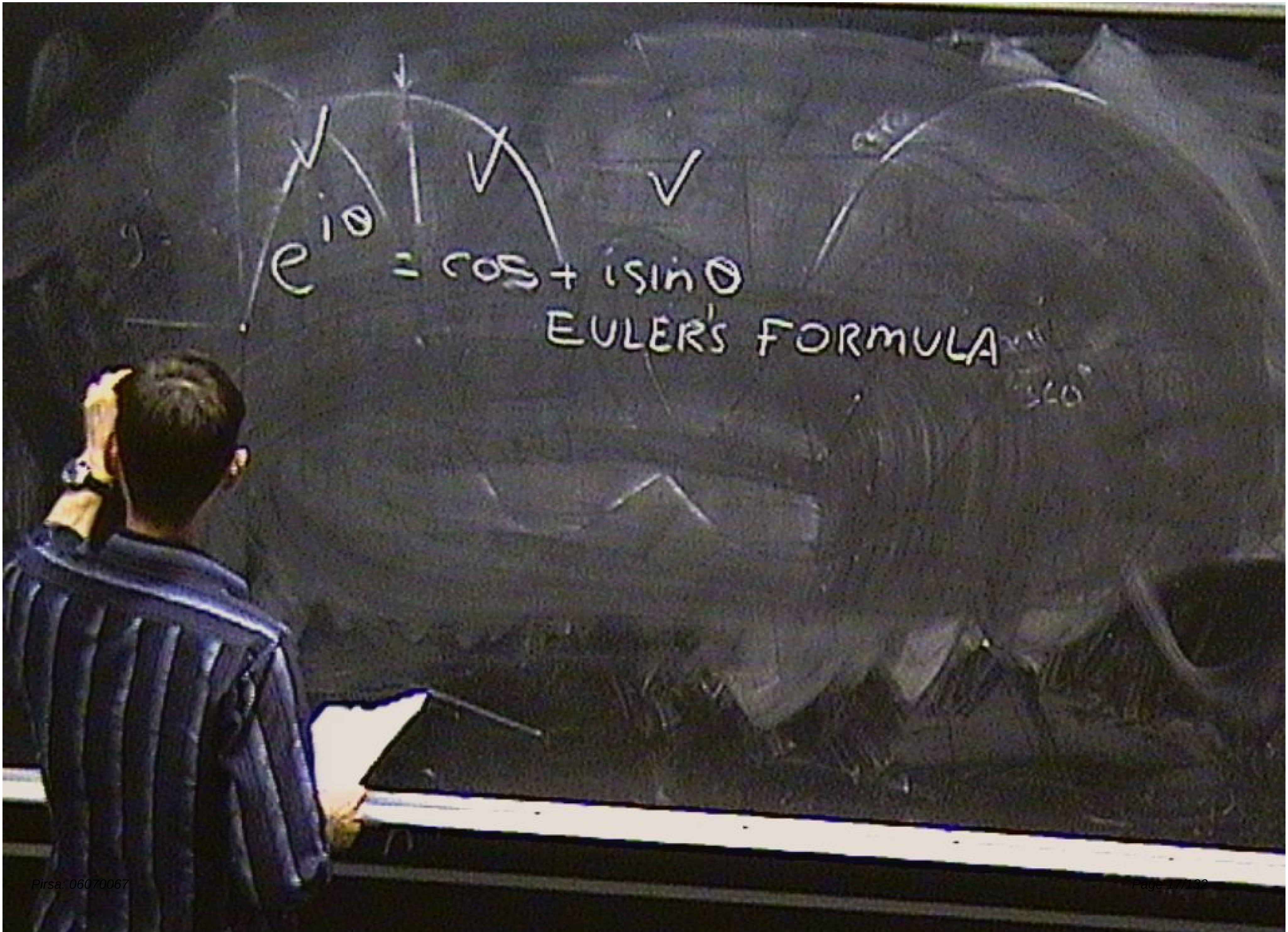


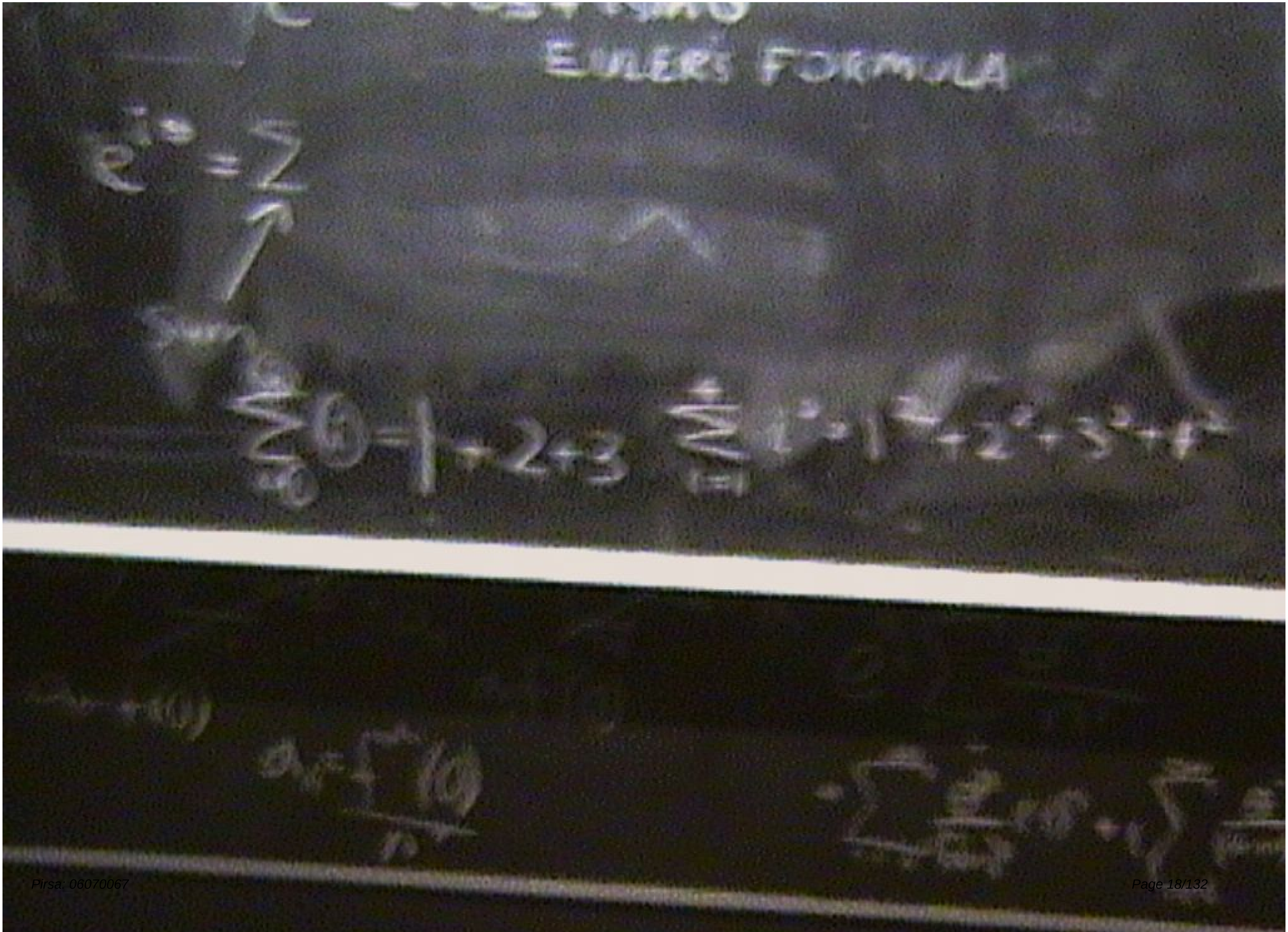

Im
Re
z = re^{iθ}
r = √2 and θ = π/4
∴ z = √2 e^{iπ/4}
1+i


Im
i
1
Re
z = re^{iθ}
r = √2 and θ = π/4
1+i
∴ z = √2 e^{iπ/4}


GROK
e iθ = cos + i sin θ
EULER'S FORMULA


GROK
e^{iθ} = cos θ + i sin θ
EULER'S FORMULA


e^{iθ} = cos + i sin θ
EULER'S FORMULA


EULER'S FORMULA

EULER'S FORMULA

$e^{i\theta} = \sum$

Sum

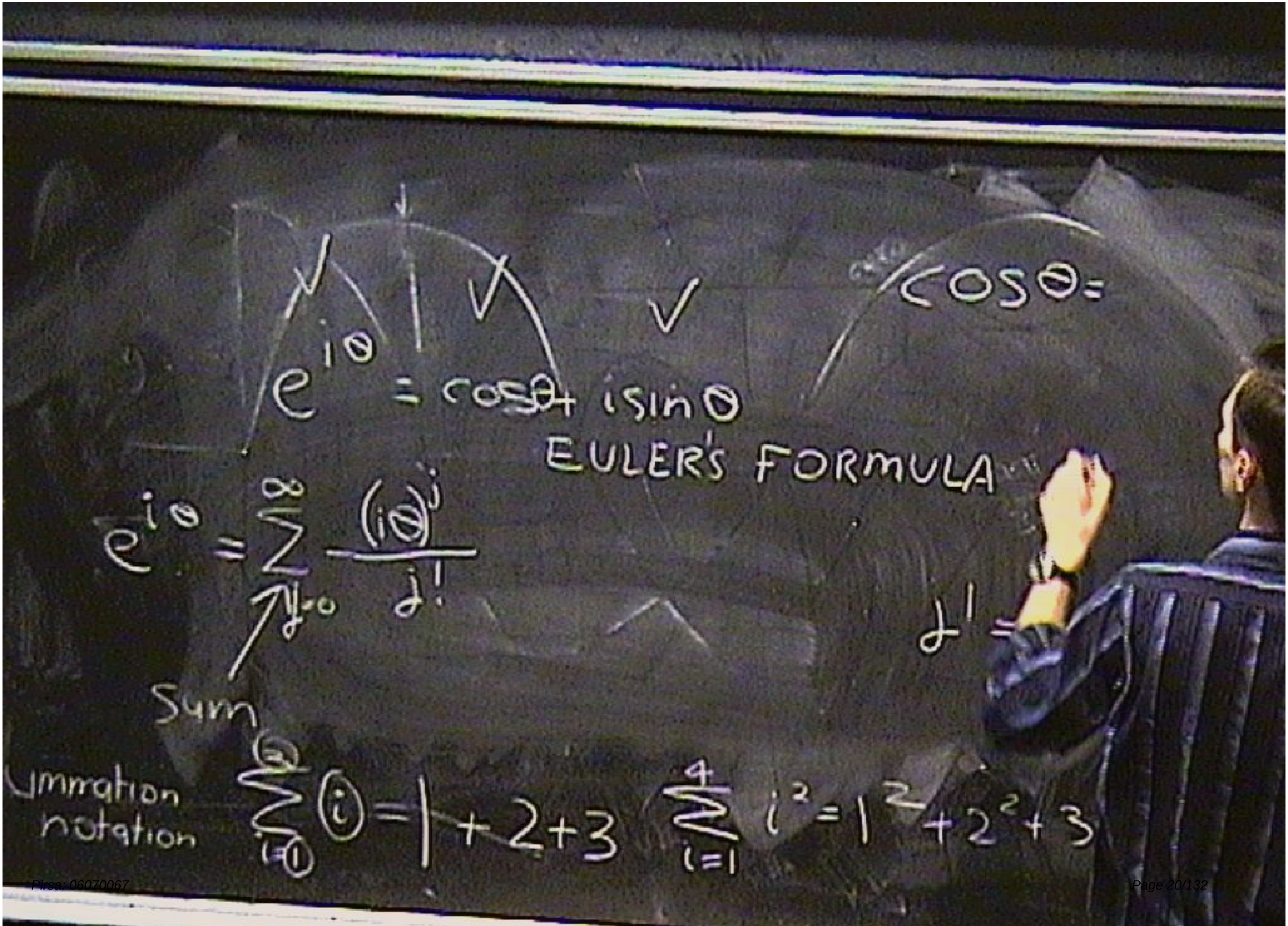
$\sum_{i=1}^{3} i = 1 + 2 + 3 \qquad \sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$

$a_0 = f(0) \qquad a_1 = f'(0)$

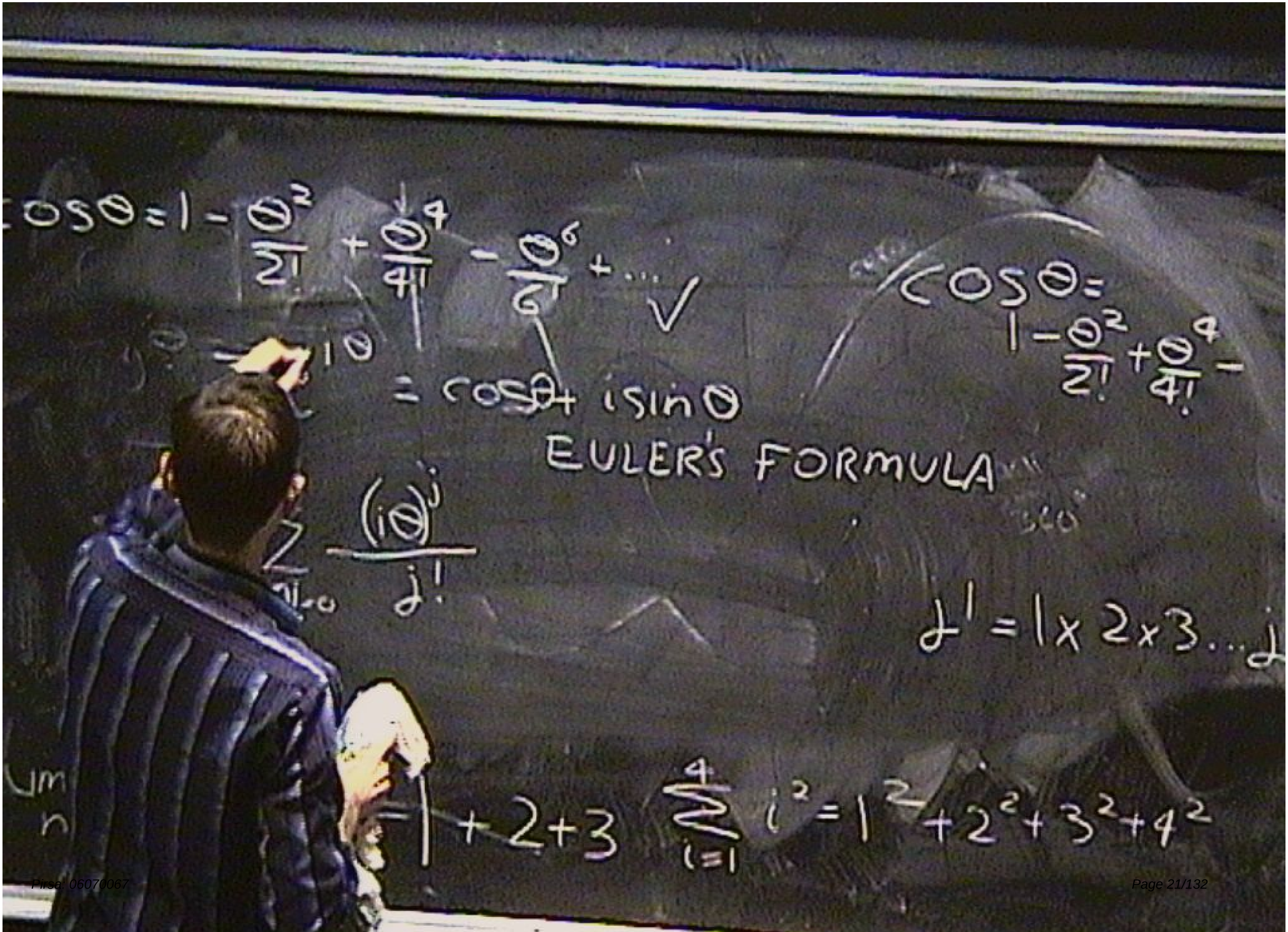
$a_n = \frac{f^{(n)}(0)}{n!}$

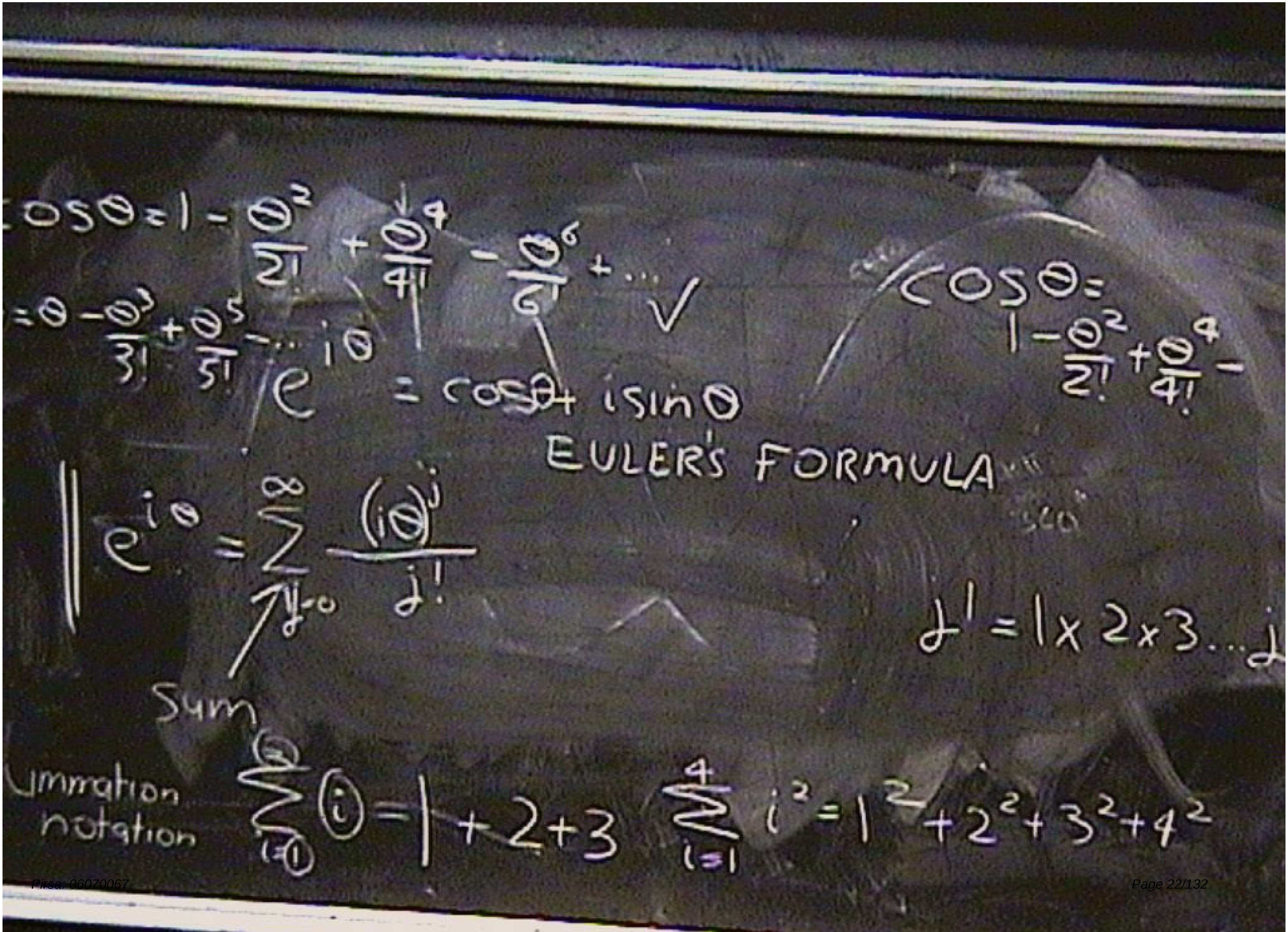
$e^{i\theta} = \sum_{n=0} \frac{\theta^n i^n}{n!}$

$= \sum_{n=0}^{\infty} \frac{\theta^{2n}}{(2n)!}(-1)^n + i \sum_{n=0}^{\infty} \frac{\theta}{(2n+1)!}$

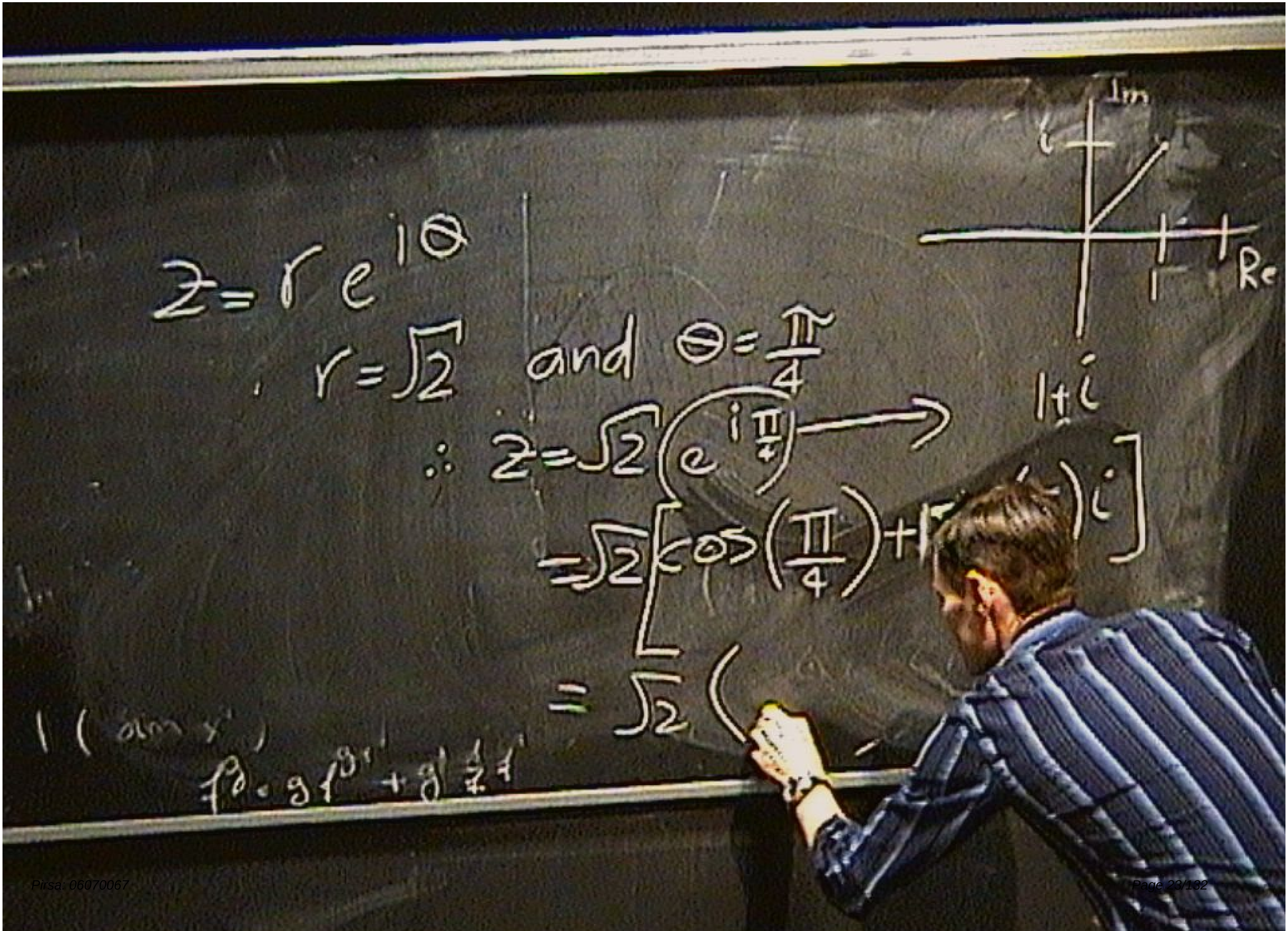


cosθ=
e^iθ = cosθ + isinθ
EULER'S FORMULA
e^iθ = Σ (iθ)^j / j!
j!=
Sum
ummation
notation
Σ i = 1 + 2 + 3
Σ i² = 1² + 2² + 3

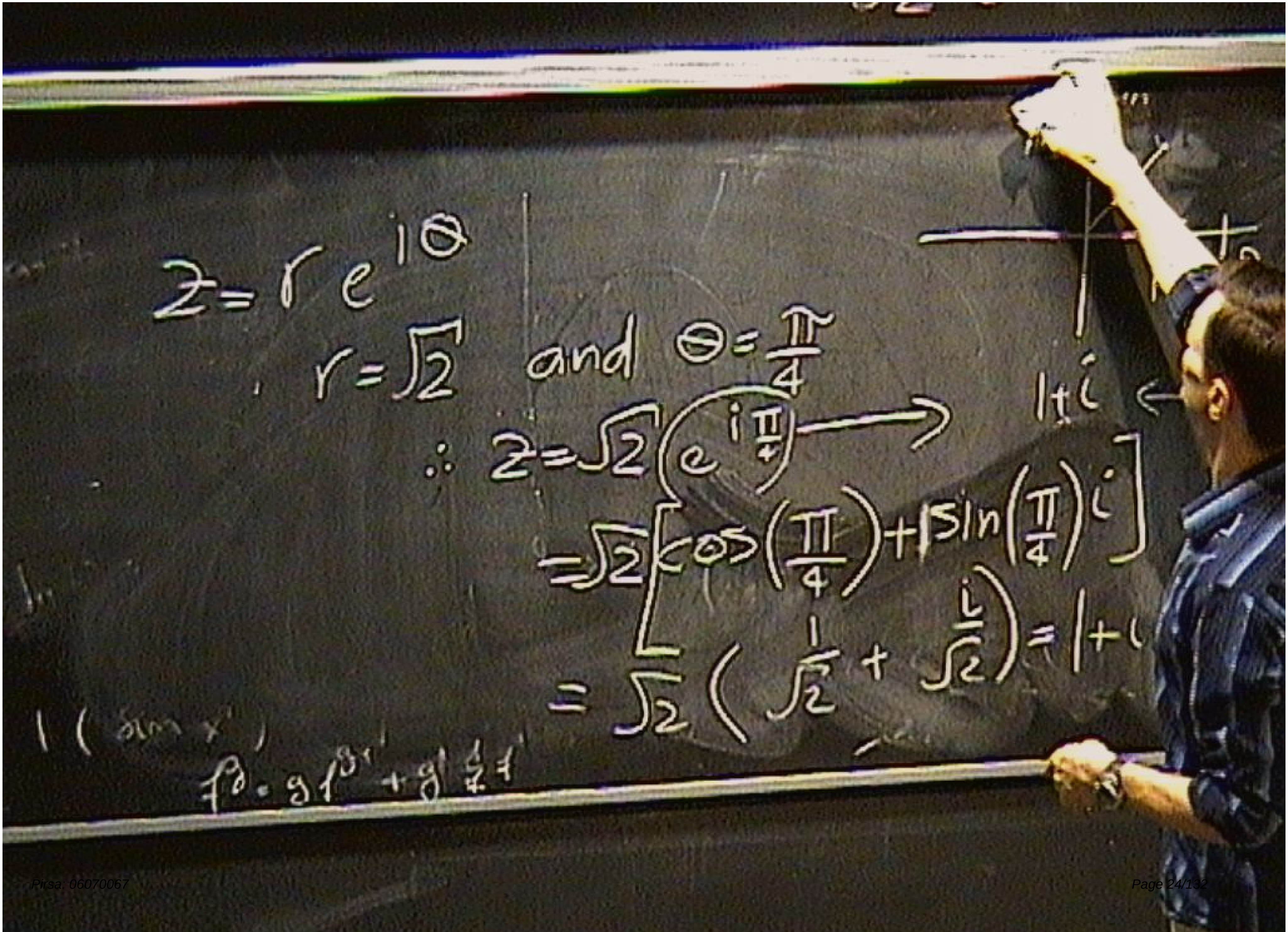

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$
$= \cos\theta + i\sin\theta$
EULER'S FORMULA
$\sum_{j=0} \frac{(i\theta)^j}{j!}$
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} -$
$j! = 1 \times 2 \times 3 \ldots j$
$+2+3$
$\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$

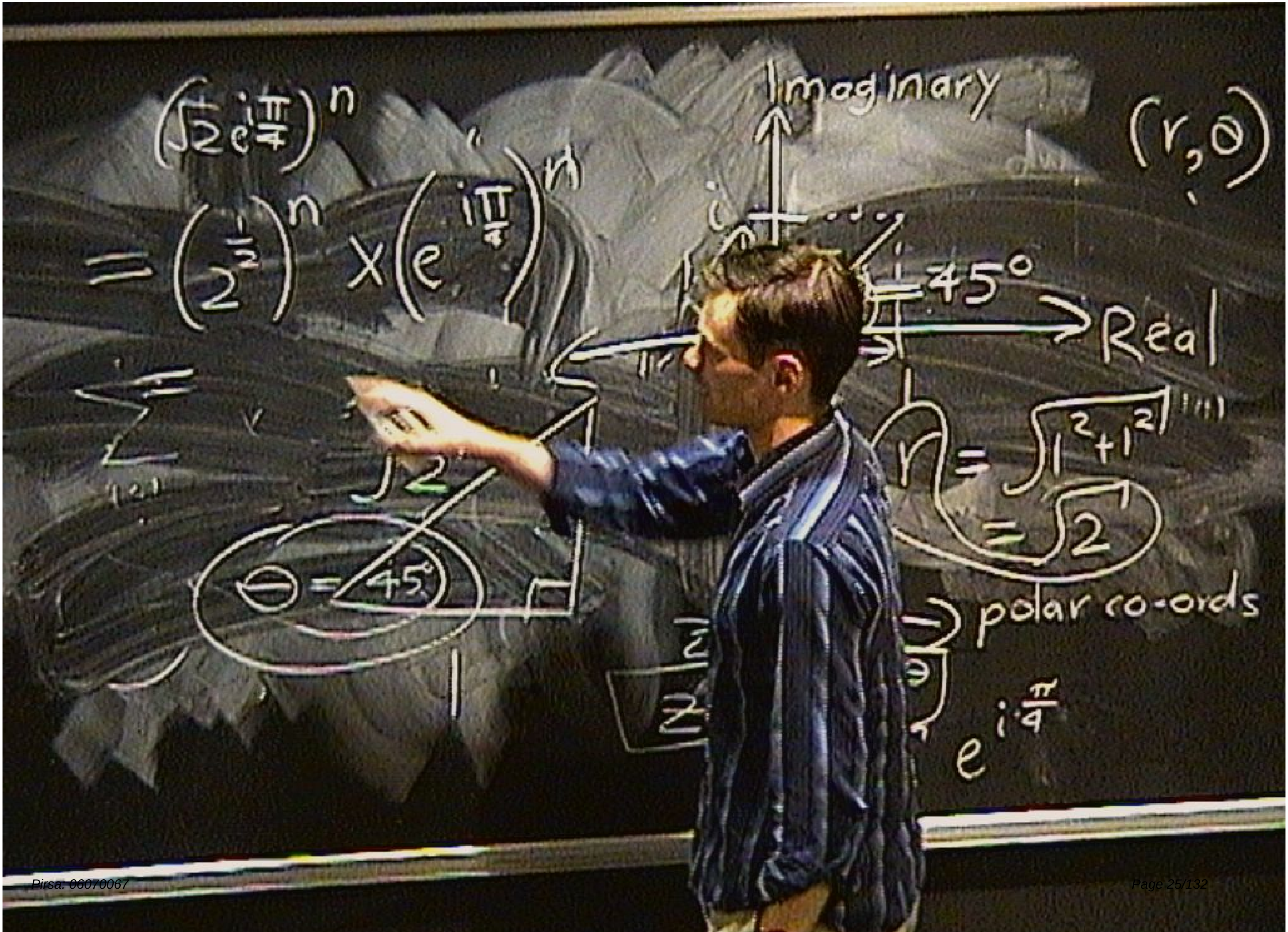


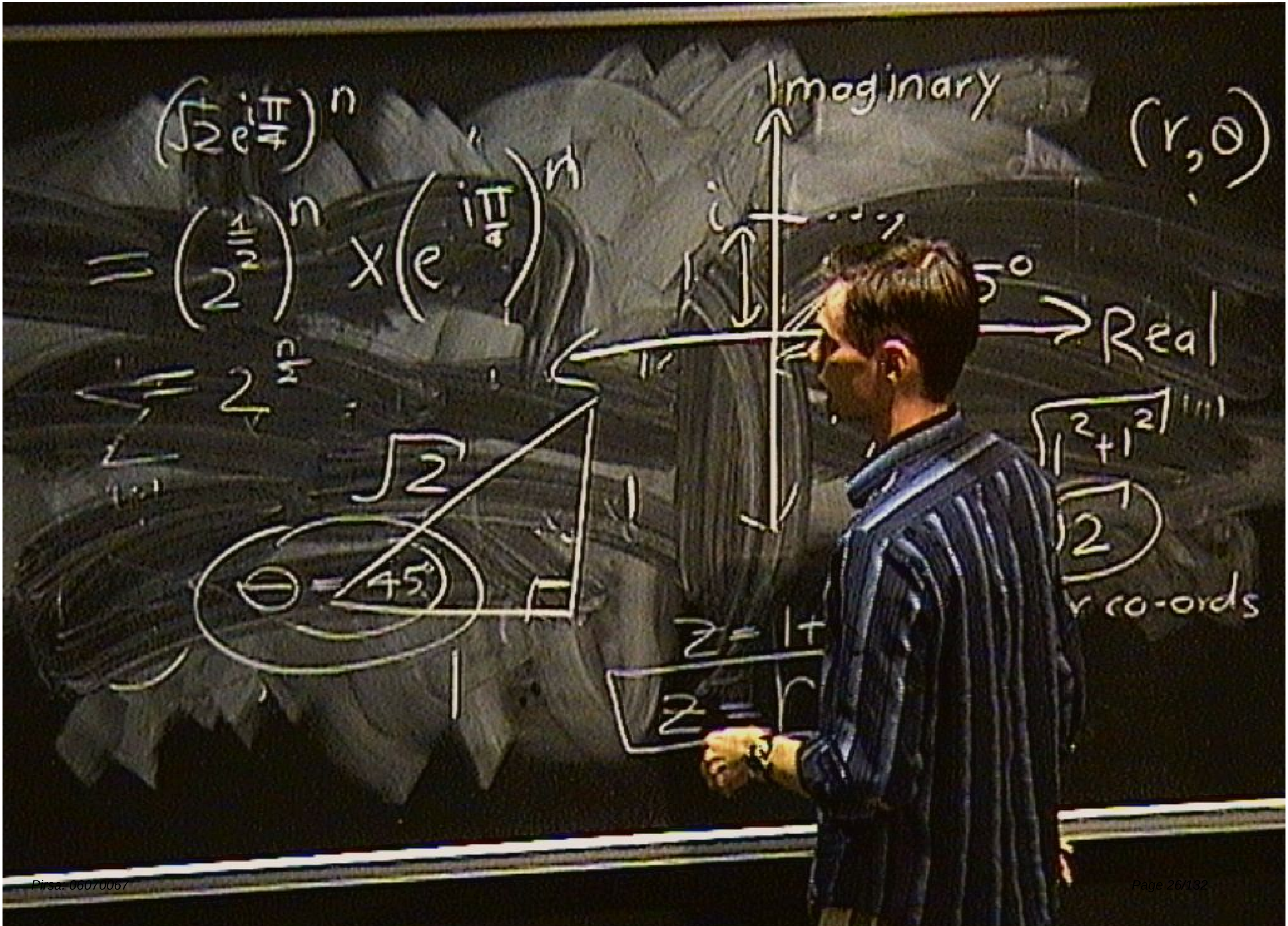
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$
$= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$
$e^{i\theta} = \cos\theta + i\sin\theta$
EULER'S FORMULA
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} -$
$e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!}$
$j! = 1 \times 2 \times 3 \dots j$
Sum
ummation notation
$\sum_{i=1}^{3} i = 1 + 2 + 3$
$\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$

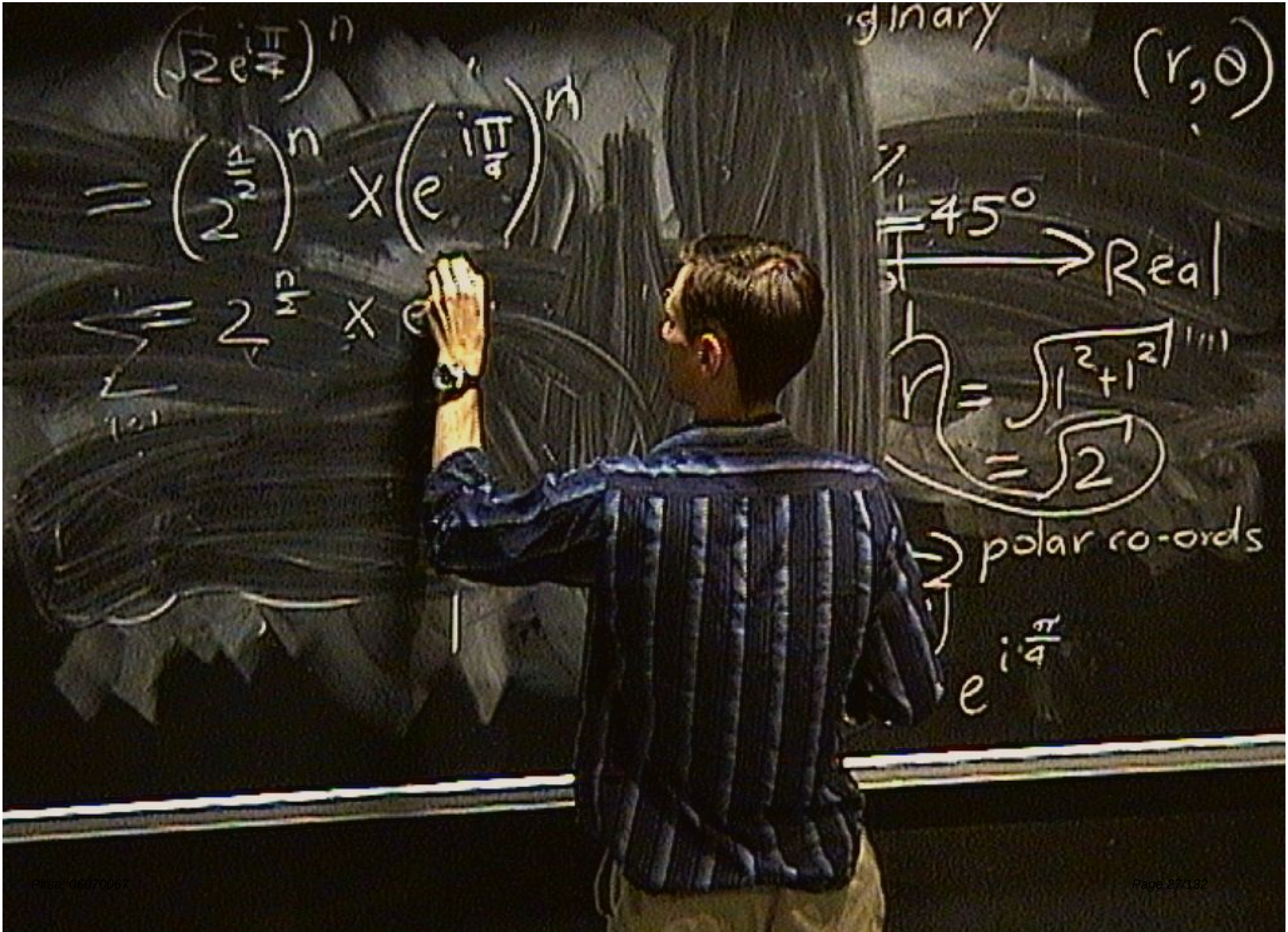


z = re^{iθ}
∴ r = √2 and θ = π/4
∴ z = √2 (e^{iπ/4}
1+i
= √2 [cos(π/4) + i
= √2 (
Im
Re

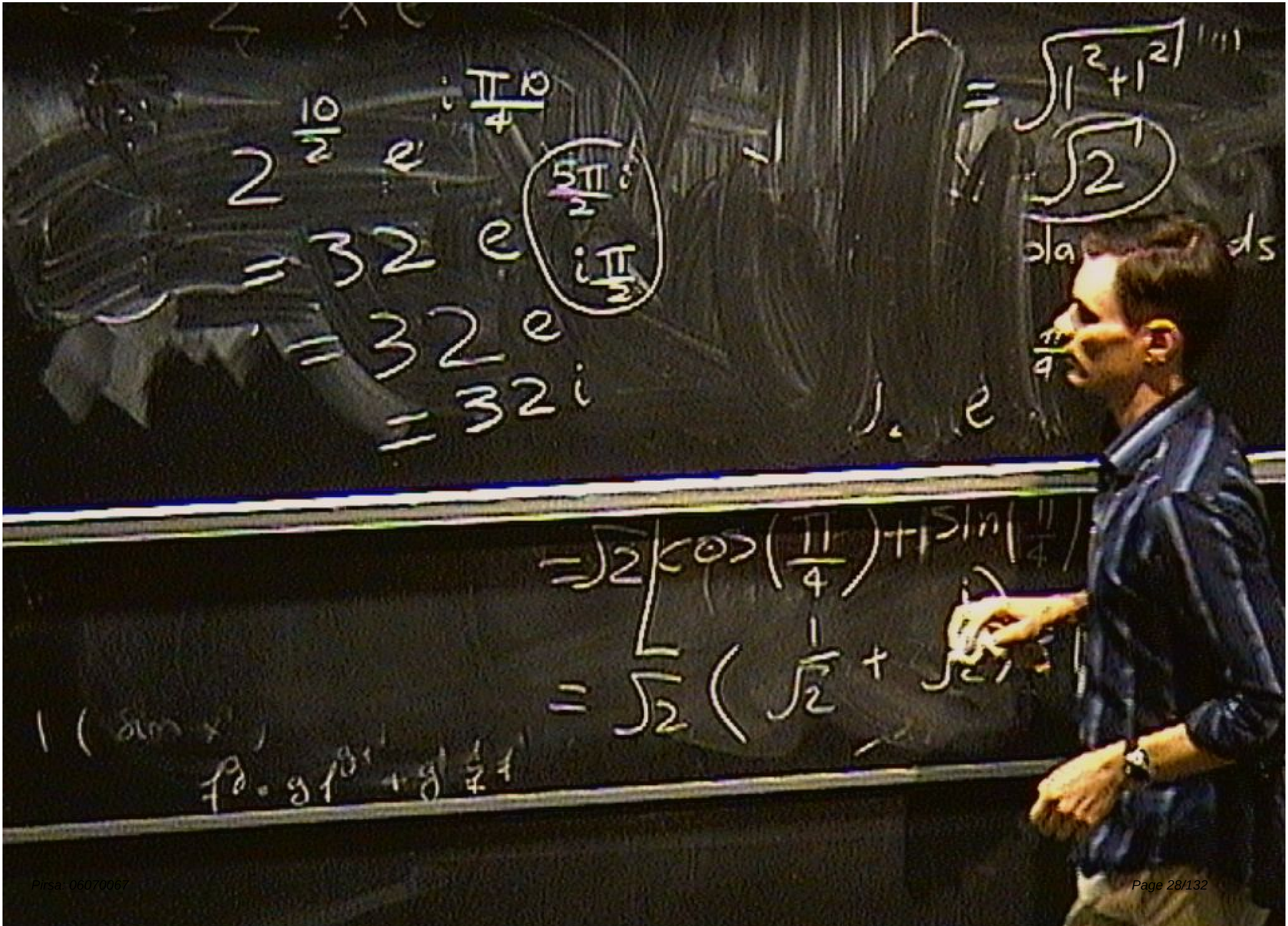

$z = r e^{i\theta}$
$r = \sqrt{2}$ and $\theta = \frac{\pi}{4}$
$\therefore z = \sqrt{2}\left(e^{i\frac{\pi}{4}}\right) \longrightarrow 1+i$
$= \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$
$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = 1+i$

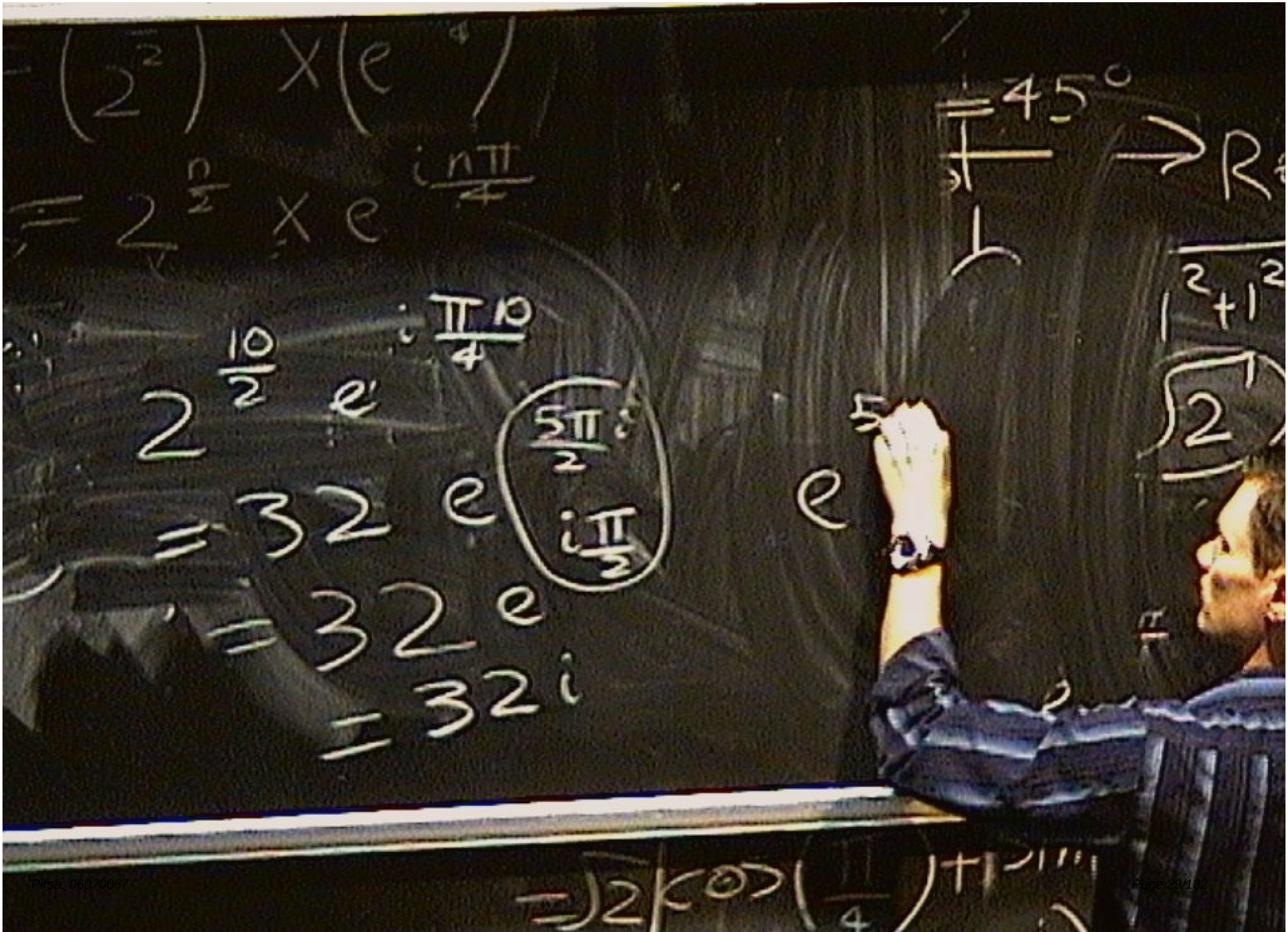

Imaginary
Real
(r, θ)
45°
θ = 45°
r = √(1²+1²) = √2
polar co-ords
e^{i·π/4}

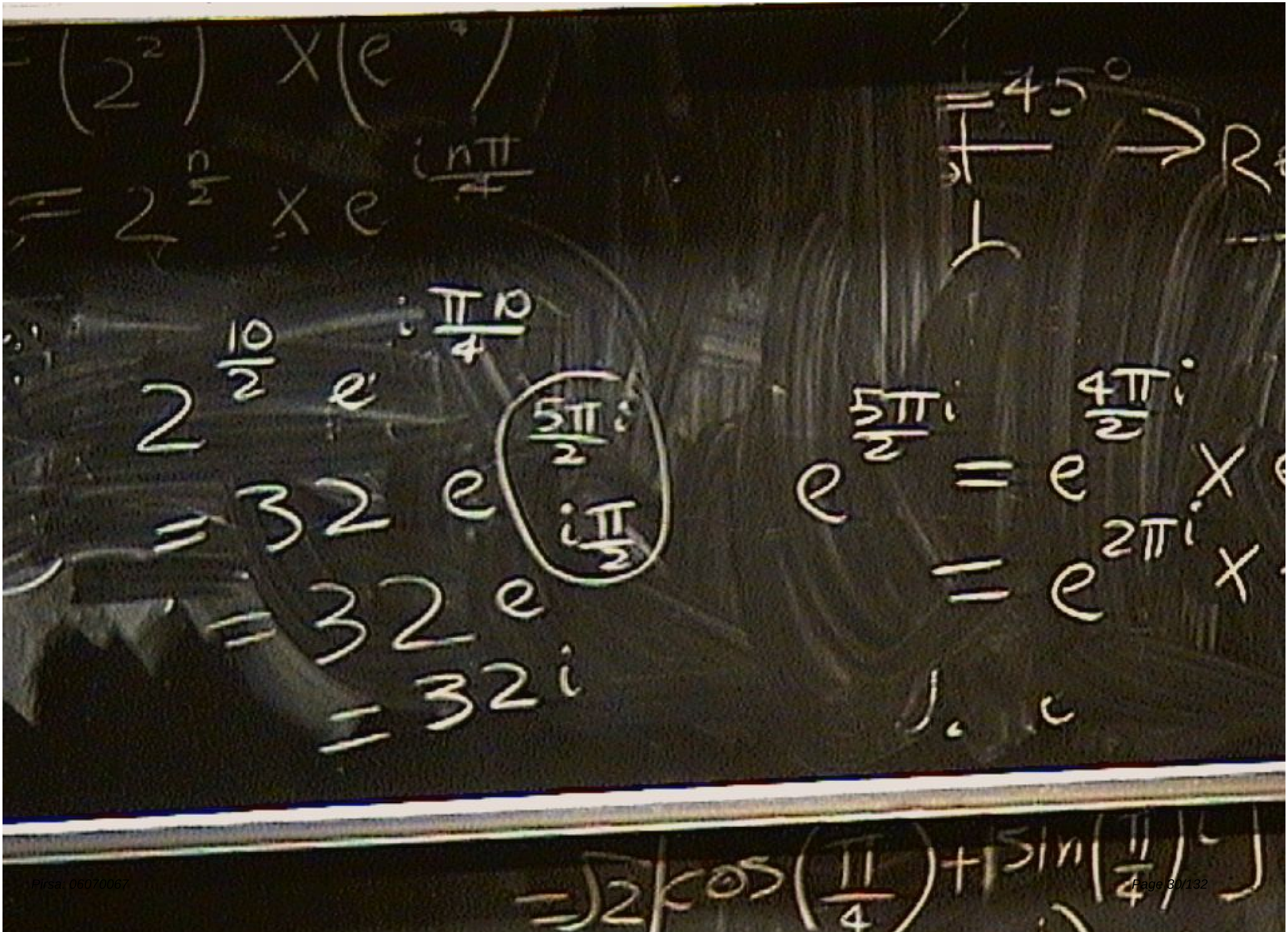

Imaginary
Real
(r,θ)
co-ords

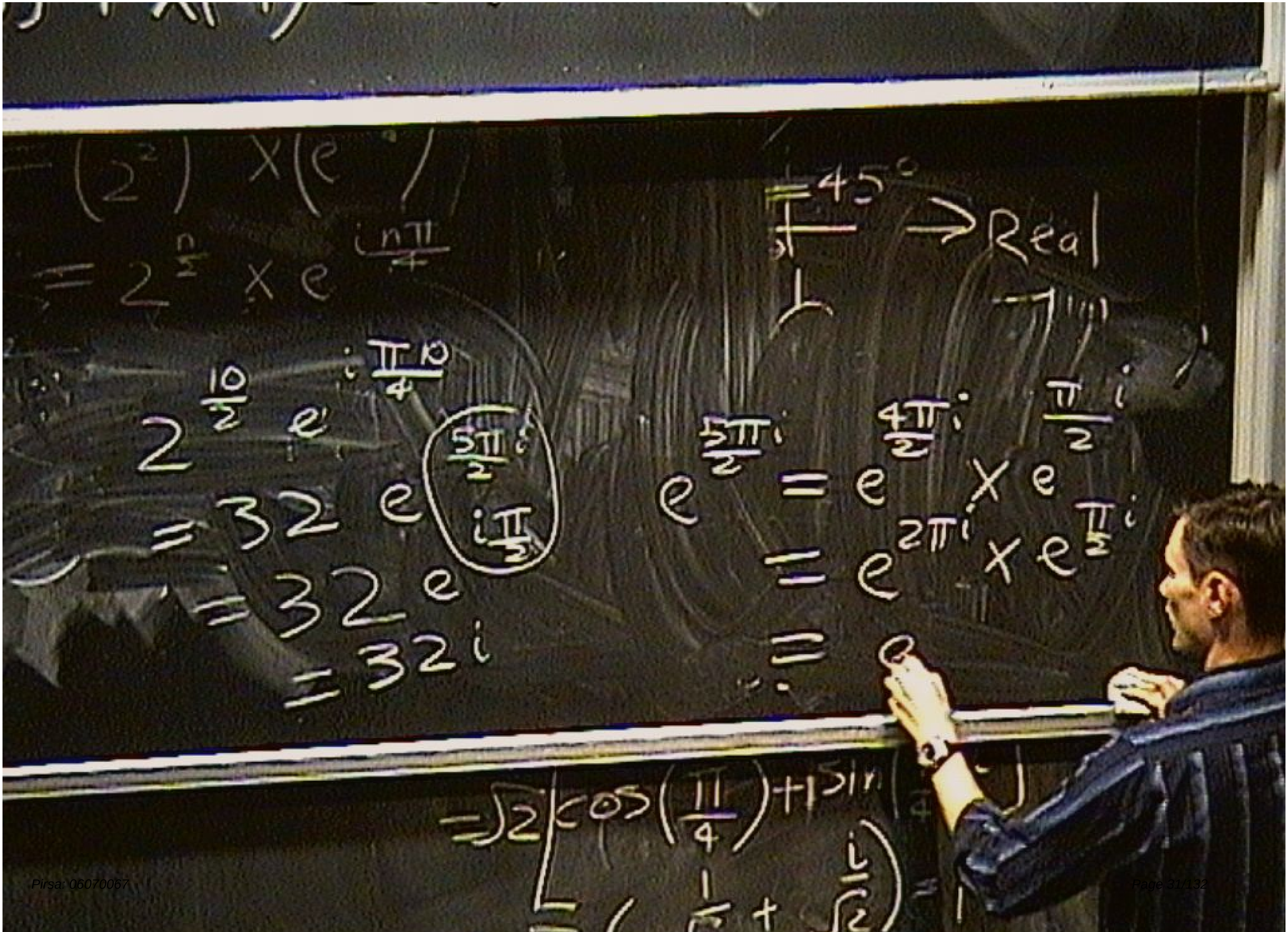


(r,θ)
45°
Real
polar co-ords


$2^{\frac{10}{2}} e^{i\frac{\pi}{4}10}$
$= 32 e^{\frac{5\pi}{2} i}$
$= 32 e^{i\frac{\pi}{2}}$
$= 32i$
$= \sqrt{1^2+1^2}$
$\sqrt{2}$
$= \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$
$= \sqrt{2}\left(\frac{1}{\sqrt{2}} +\right.$




$= 2^{\frac{n}{2}} \times e^{\frac{i n \pi}{4}}$
$45°$
$\Rightarrow R$
$2^{\frac{10}{2}} e^{i \frac{\pi 10}{4}}$
$= 32 e^{\frac{5\pi i}{2}}$
$e^{i\frac{\pi}{2}}$
$= 32 e$
$= 32i$
$e^{\frac{5\pi i}{2}} = e^{\frac{4\pi i}{2}} \times$
$= e^{2\pi i} \times$



45°
Real
=32i
=√2 cos

$= (2^2) \times (e \ldots)$

$= 2^{\frac{n}{2}} \times e^{i\frac{n\pi}{4}}$

$45^\circ \Rightarrow$ Real

$2^{\frac{10}{2}} e^{i\frac{\pi}{4}10}$

$= 32 e^{\frac{5\pi}{2}i}$

$= 32 e^{i\frac{\pi}{2}}$

$= 32i$

$\cos\left(\frac{5\pi}{2}\right) + \sin\left(\frac{5\pi}{2}\right) i$

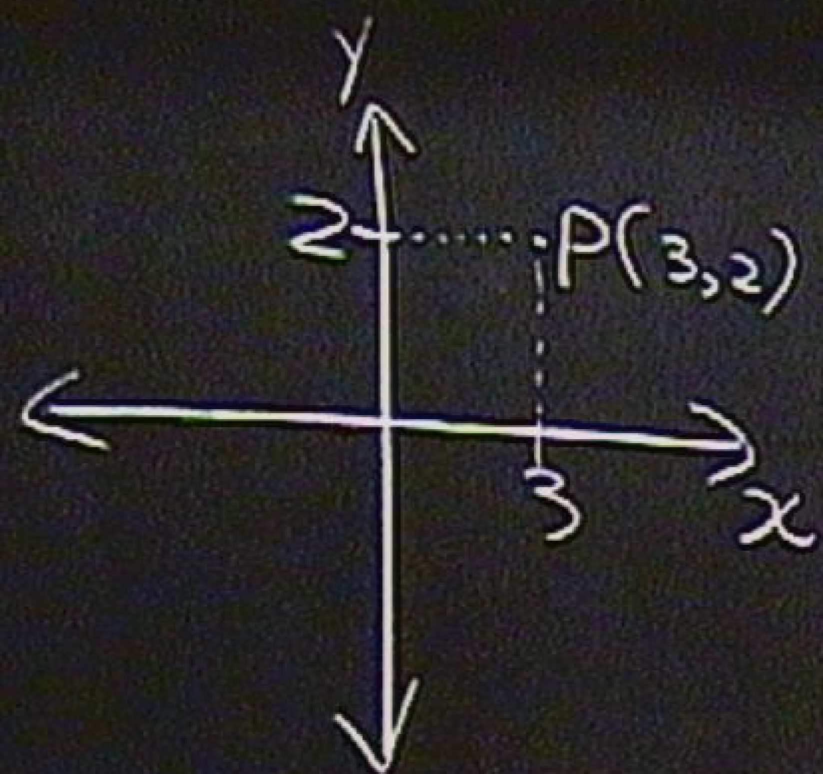
$e^{\frac{5\pi}{2}i} = e^{\frac{4\pi}{2}i} \times e^{\frac{\pi}{2}i}$

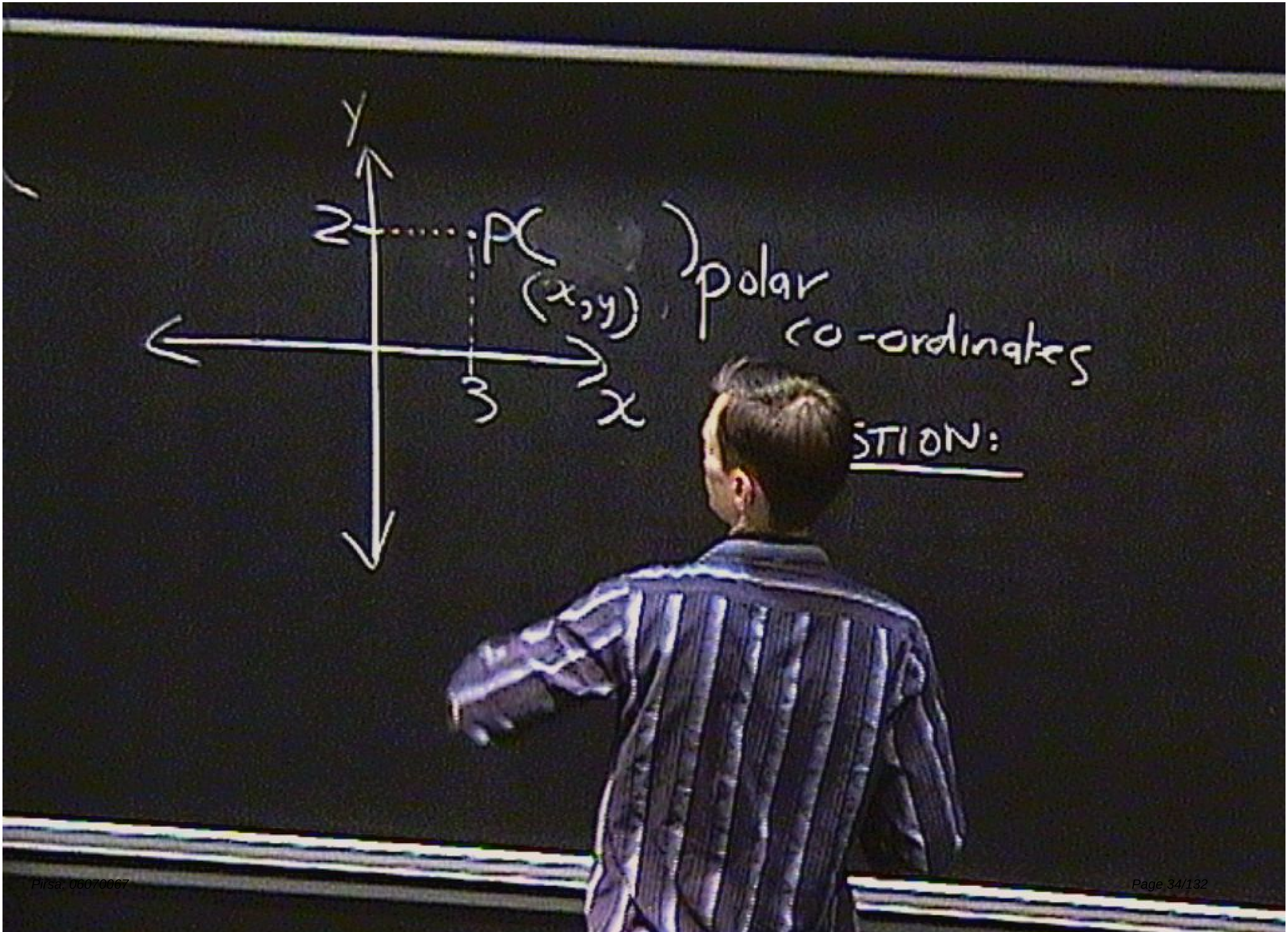
$= e^{2\pi i} \times e^{\frac{\pi}{2}i}$

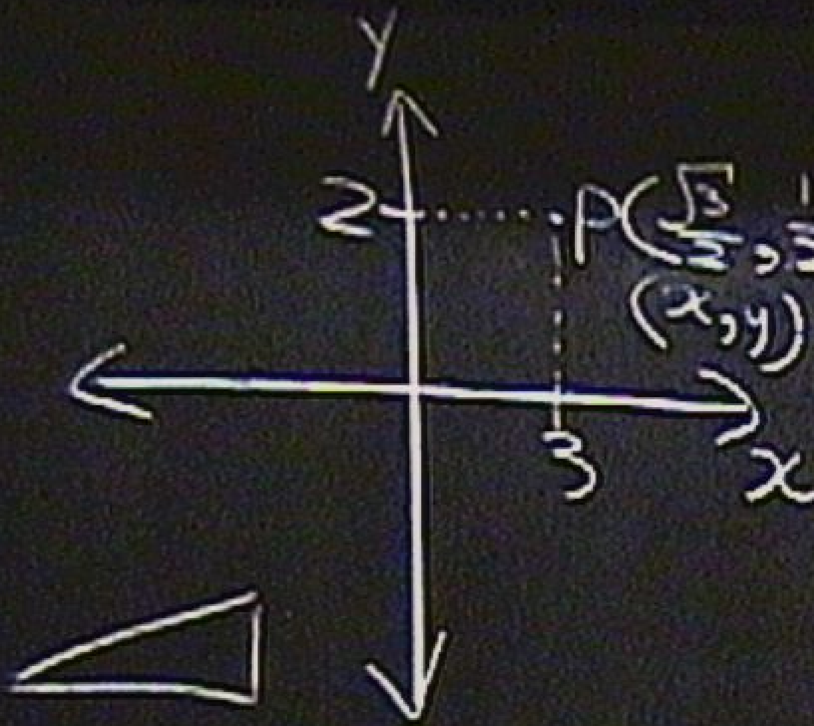
$= i$

$= \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$

$= \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = 1 + i$


y
2
P(3,2)
3
x

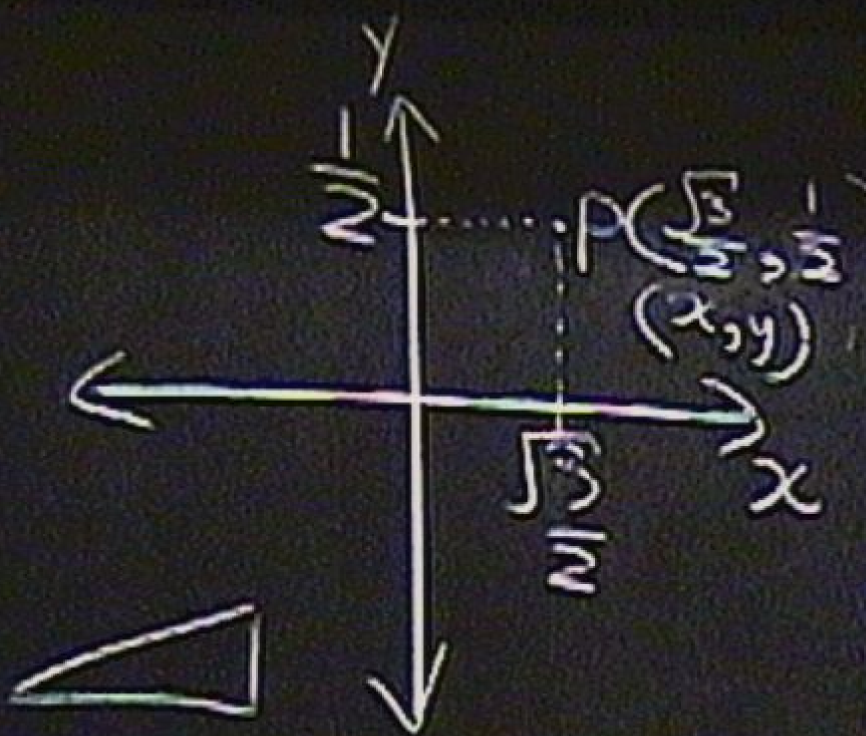

y
2
P(
)
polar
(x,y)
co-ordinates
3
x
STION:



polar co-ordinates

QUESTION:

represent point P in polar co-ordinates.



polar co-ordinates

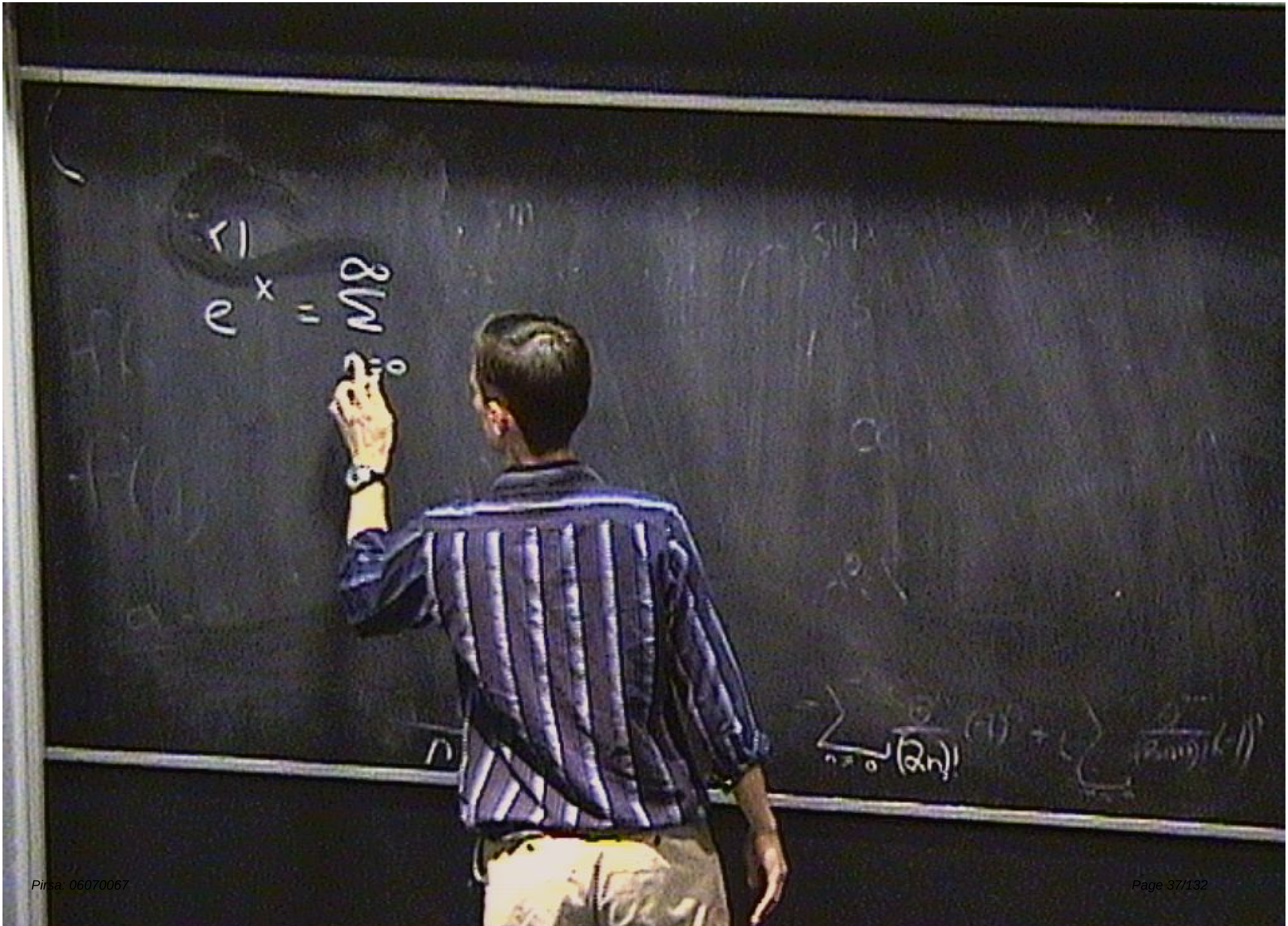
QUESTION:

represent point P in polar co-ordinates

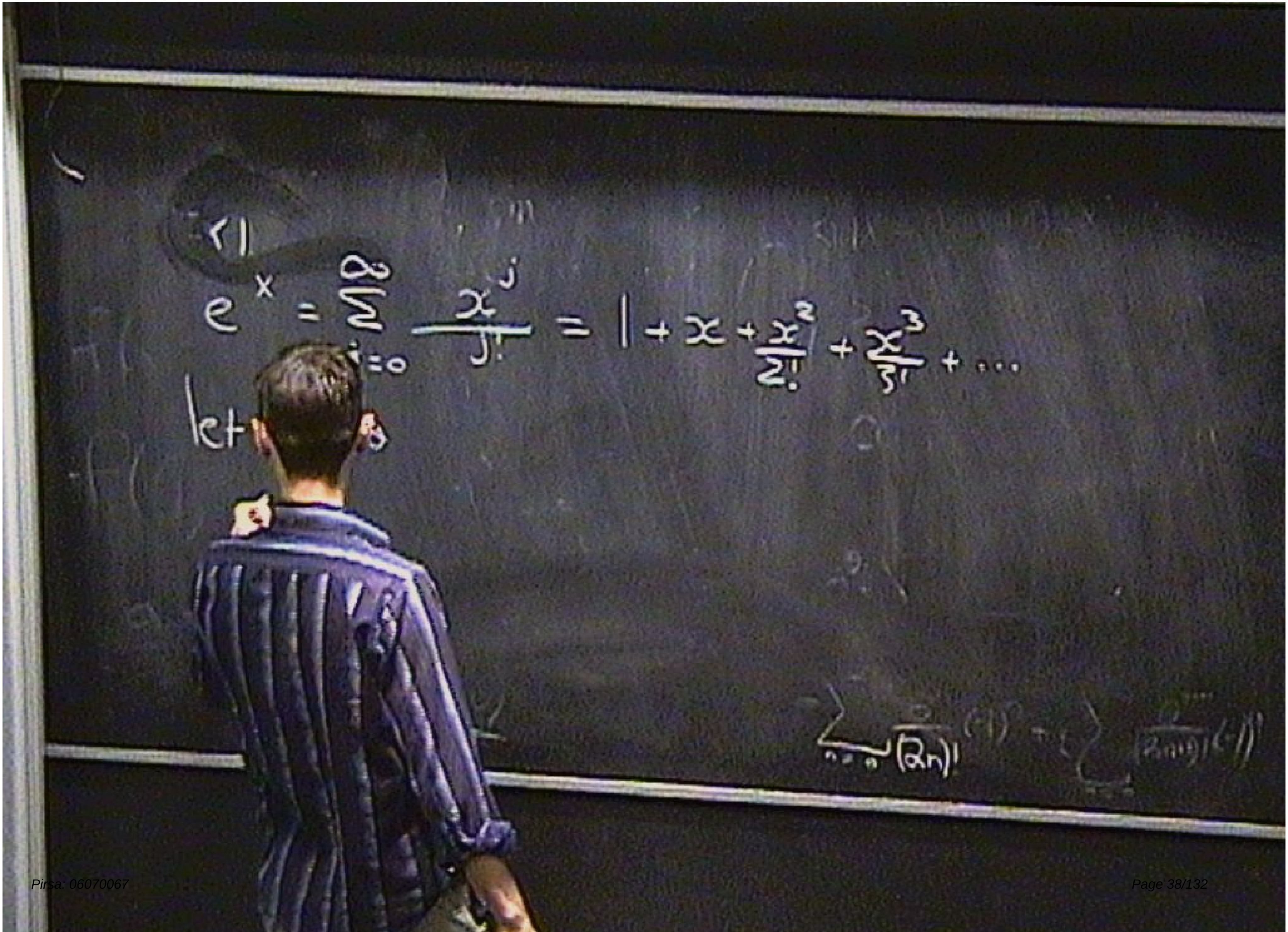
* **EXTRA:** prove that

$e^{i\theta} = \cos\theta + i\sin\theta$

using power series



<1
e^x = ∑^∞


<1
$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
let

ak

$$*\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{j=0,2,4\dots}^{\infty} \frac{\quad}{j!}$$

$$*\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

EULER'S FORMULA

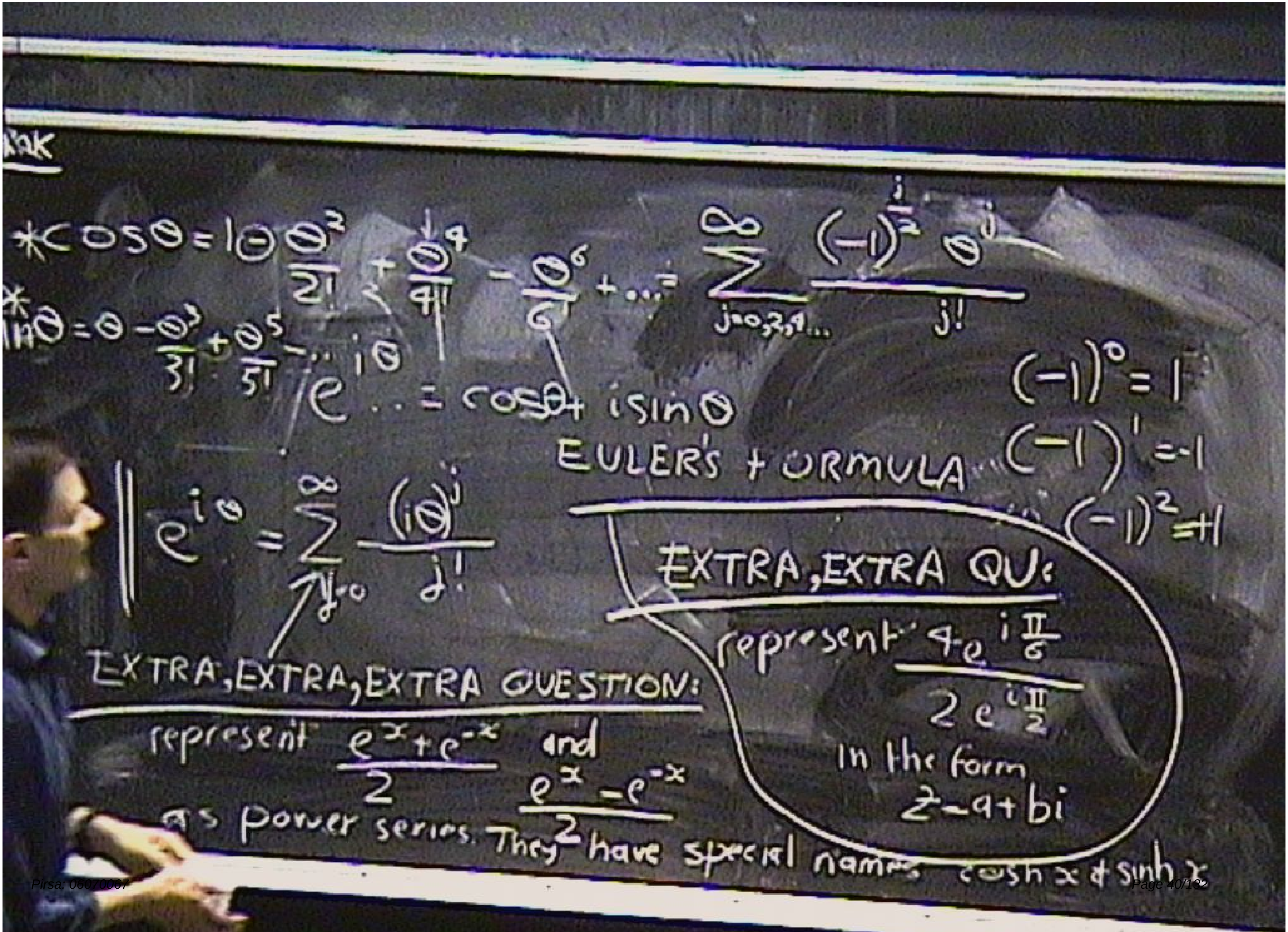
$$\left| e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!} \right.$$

EXTRA, EXTRA QU...

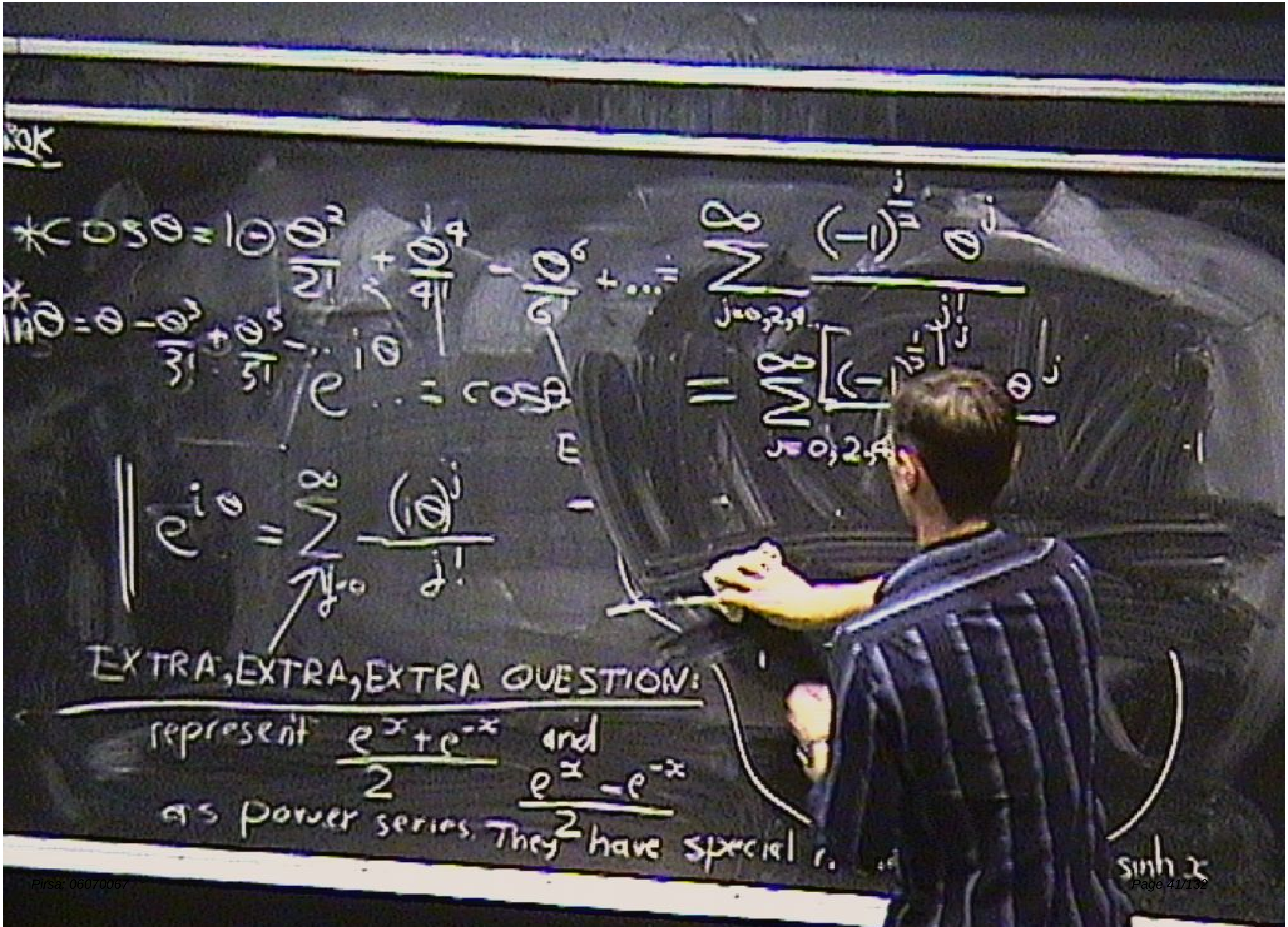
represent $\dfrac{4e^{i\frac{\pi}{6}}}{2e^{i\frac{\pi}{2}}}$ in the form $z = a+bi$

EXTRA, EXTRA, EXTRA QUESTION:

represent $\dfrac{e^x+e^{-x}}{2}$ and $\dfrac{e^x-e^{-x}}{2}$ as power series. They have special names $\cosh x$ & $\sinh x$



$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots = \sum_{j=0,2,4,\ldots}^{\infty} \frac{(-1)^{\frac{j}{2}}\theta^j}{j!}$
$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$
$e^{i\theta} \ldots = \cos\theta + i\sin\theta$
EULER'S FORMULA
$(-1)^0 = 1$
$(-1)^1 = -1$
$(-1)^2 = +1$
$e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!}$
EXTRA, EXTRA QU
represent $\frac{4e^{i\frac{\pi}{6}}}{2e^{i\frac{\pi}{2}}}$ in the form $z = a + bi$
EXTRA, EXTRA, EXTRA QUESTION:
represent $\frac{e^x + e^{-x}}{2}$ and $\frac{e^x - e^{-x}}{2}$ as power series. They have special names: cosh x & sinh x


$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots = \sum_{j=0,2,4\ldots}^{\infty} \frac{(-1)^{\frac{j}{2}} \theta^j}{j!}$
$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$
$e^{i\theta} \ldots = \cos\theta$
$e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!}$
EXTRA, EXTRA, EXTRA QUESTION:
represent $\frac{e^x + e^{-x}}{2}$ and $\frac{e^x - e^{-x}}{2}$
as power series. They have special

OK

$$*\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{j=0,2,4\dots}^{\infty} \frac{(-1)^{\frac{j}{2}}\,\theta^j}{j!}$$

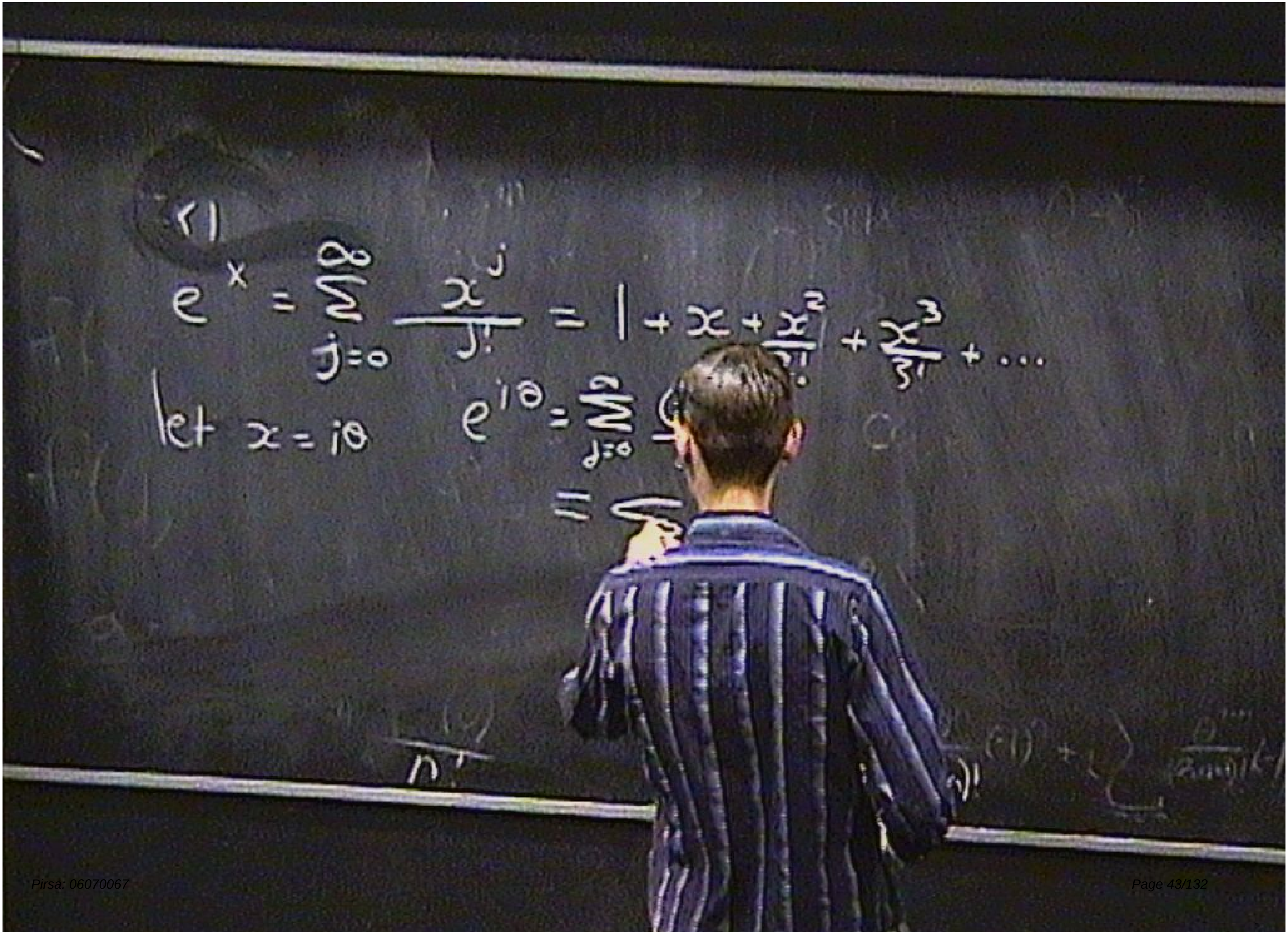
$$*\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

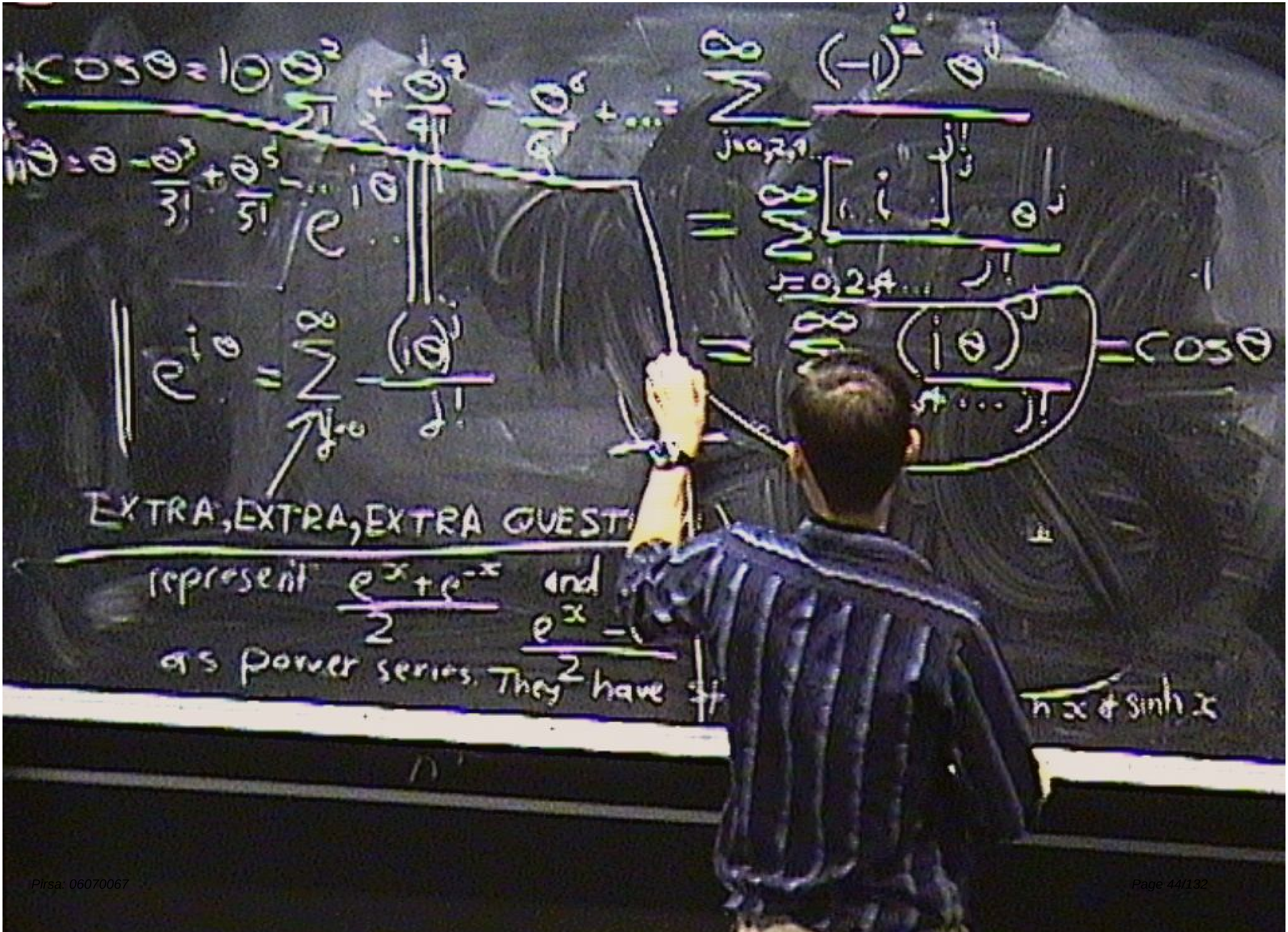
$$e^{i\theta} = \sum_{j=0,2,4\dots}^{\infty} \frac{[i]^j\,\theta^j}{j!}$$

$$e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!} = \sum_{j=0,2,4\dots}^{\infty} \frac{(i\theta)^j}{j!} =$$

EXTRA, EXTRA, EXTRA QUESTION:

represent $\frac{e^{x}+e^{-x}}{2}$ and $\frac{e^{x}-e^{-x}}{2}$ as power series. They have special names $\cosh x$ & $\sinh$


$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
let $x = i\theta$
$e^{i\theta} = \sum_{j=0}^{\infty}$


EXTRA, EXTRA, EXTRA
represent
and
as power series. They have

as power series. They have special names $\cosh x$ & $\sinh x$

(1)

$$e^{x} = \sum_{j=0}^{\infty} \frac{x^{j}}{j!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

let $x = i\theta$

$$e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!}$$

$$= \sum_{j=0,2,4\dots}^{\infty} \frac{(i\theta)^j}{j!} + \sum_{j=1,3,5\dots}^{\infty} \frac{(i\theta)^j}{j!}$$

as power series. They have special names $\cosh x$ & $\sinh x$

(1)

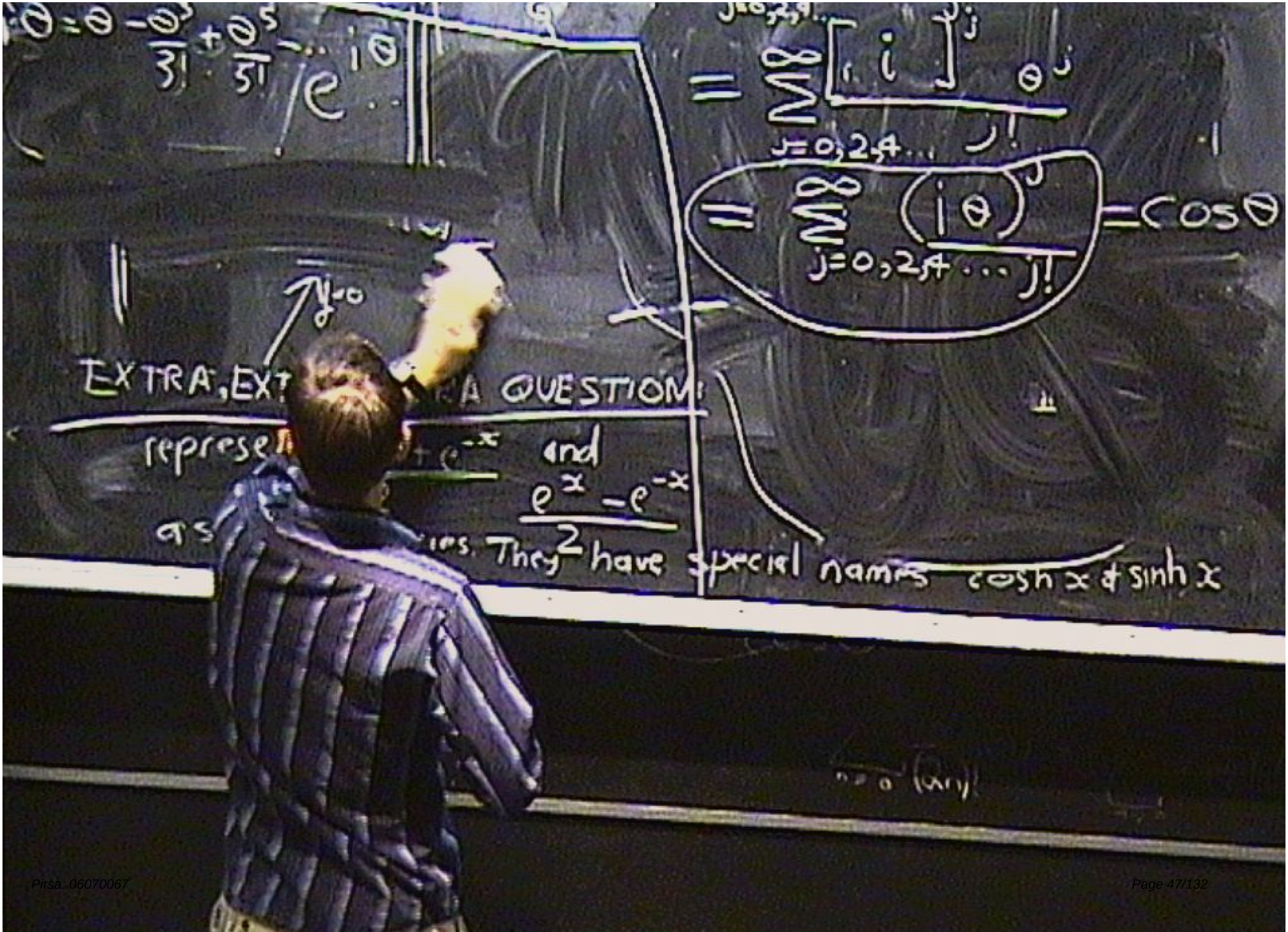
$$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

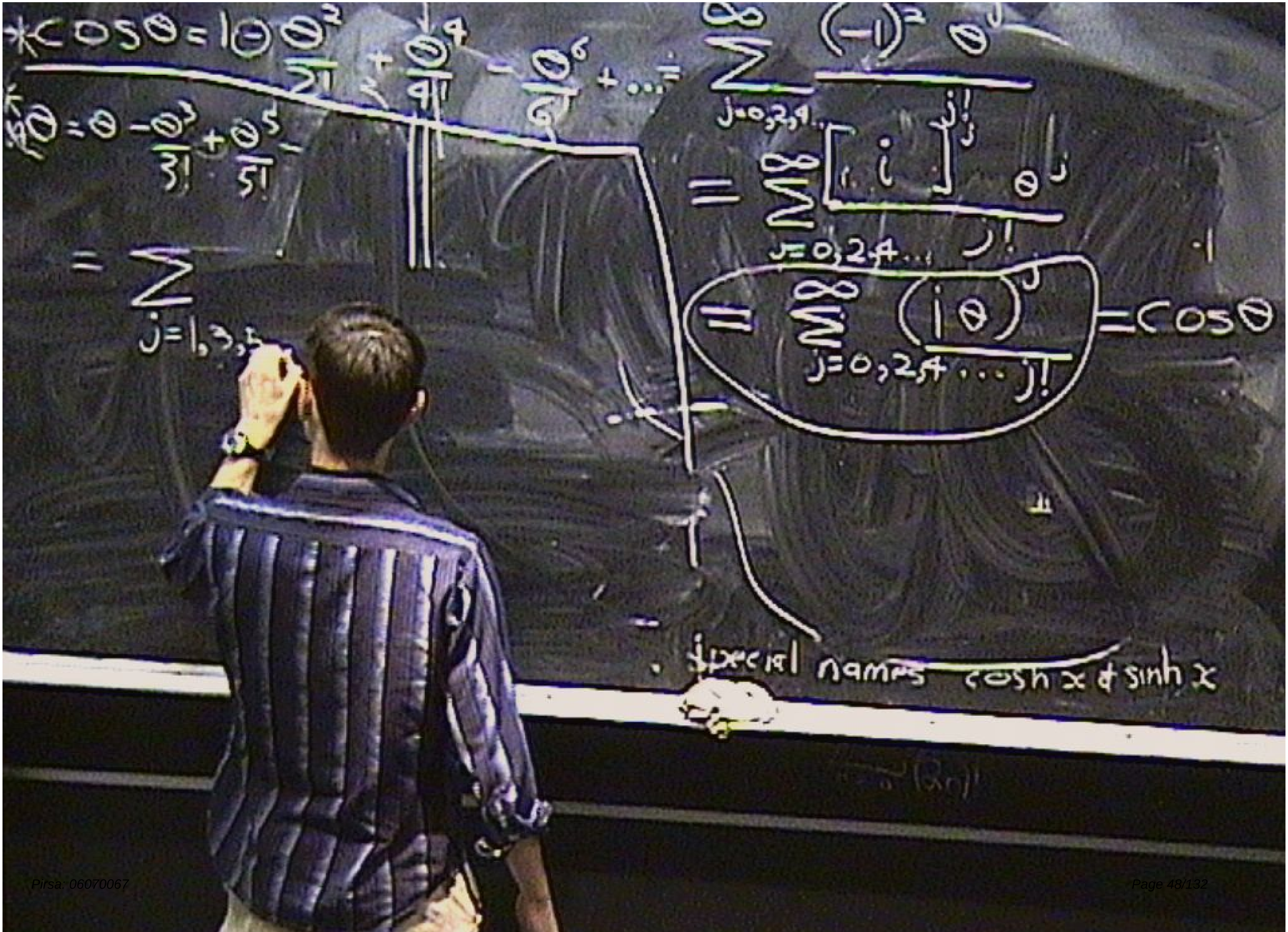
let $x = i\theta$

$$e^{i\theta} = \sum_{j=0}^{\infty} \frac{(i\theta)^j}{j!}$$

$$= \sum_{j=0,2,4\dots}^{\infty} \frac{(i\theta)^j}{j!} + \sum_{j=1,3,5\dots}^{\infty} \frac{(i\theta)^j}{j!}$$

$$= \cos\theta$$


EXTRA, EXT
QUESTION:
represe
and
They have special names cosh x & sinh x
= cos θ



$*\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{j=0,2,4\dots}^{\infty} \frac{(-1)^{j/2}\,\theta^j}{j!}$
$*\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$
$= \sum_{j=0,2,4\dots}^{\infty} \frac{[i^j]\,\theta^j}{j!}$
$= \sum_{j=0,2,4\dots}^{\infty} \frac{(i\theta)^j}{j!} = \cos\theta$
$= \sum_{j=1,3,5}$
special names cosh x & sinh x

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots = \sum_{j=0,2,4\dots}^{\infty} \frac{(-1)^{j/2}\,\theta^j}{j!}$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$= \sum_{j=1,3,5\dots}^{\infty} \frac{\dots}{j!}$$

$$= \sum_{j=0,2,4\dots}^{\infty} \frac{[\,i\,]^j\,\theta^j}{j!}$$

$$= \sum_{j=0,2,4\dots}^{\infty} \frac{(i\theta)^j}{j!} = \cos\theta$$

• special names $\cosh x$ & $\sinh x$

GROK

$$* \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots = \sum_{j=0,2,4\ldots}^{\infty} \frac{(-1)^{j/2}\,\theta^j}{j!}$$

$$= \sum_{j=0,2,4\ldots}^{\infty} \frac{[i]^j\,\theta^j}{j!}$$

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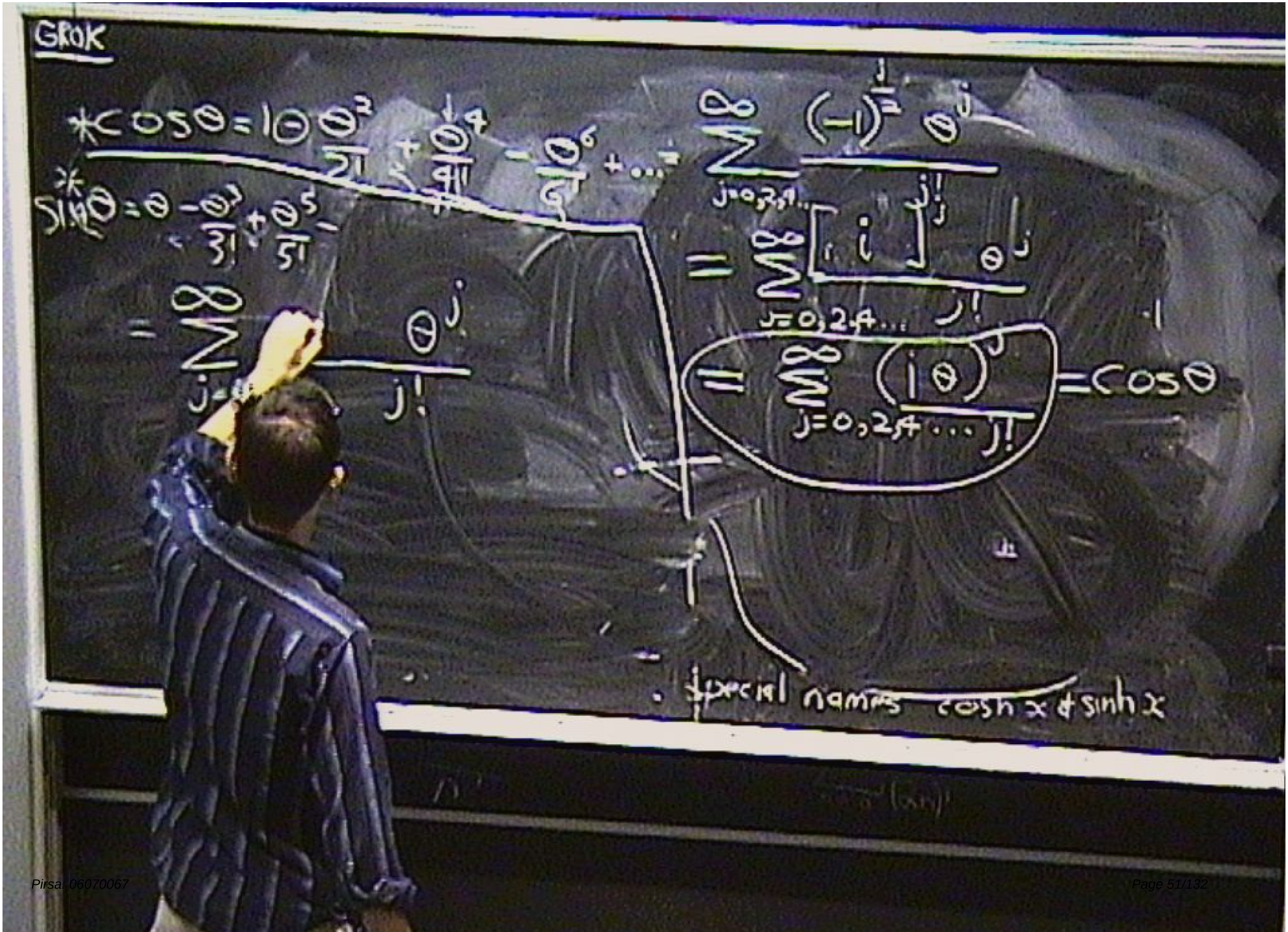
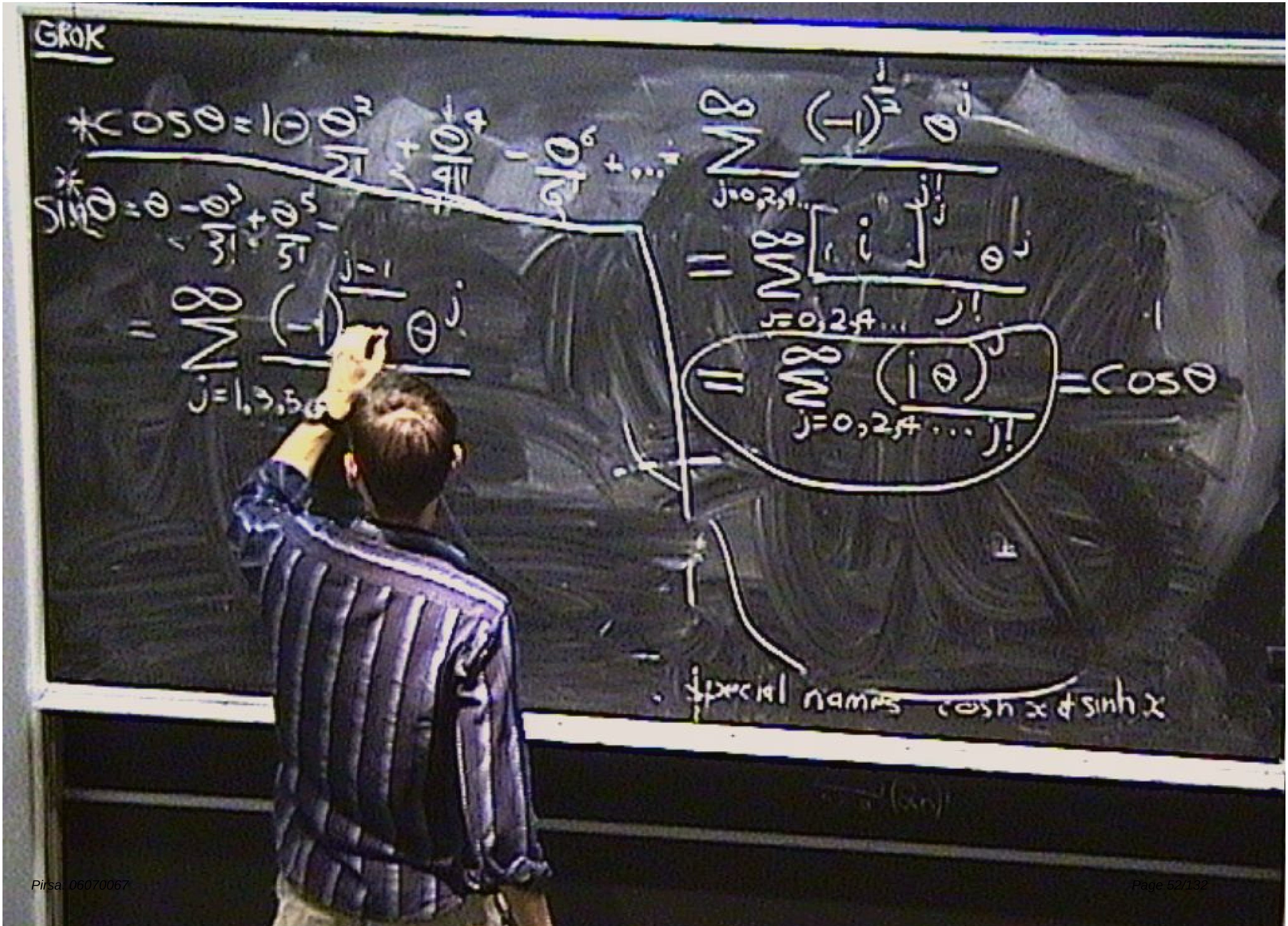
$$* \sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$$

$$= \sum_{j=1,3,5\ldots}^{\infty} \frac{\theta^j}{j!}$$

special names $\cosh x$ & $\sinh x$


GROK
* cos θ = 1 − θ²/2! + θ⁴/4! − θ⁶/6! + … = Σ_{j=0,2,4,…}^∞ (−1)^{j/2} θ^j / j!
* sin θ = θ − θ³/3! + θ⁵/5! − …
= Σ θ^j / j!
= Σ_{j=0,2,4…}^∞ [i]^j θ^j / j!
= Σ_{j=0,2,4…}^∞ (iθ)^j / j! = cos θ
special names cosh x & sinh x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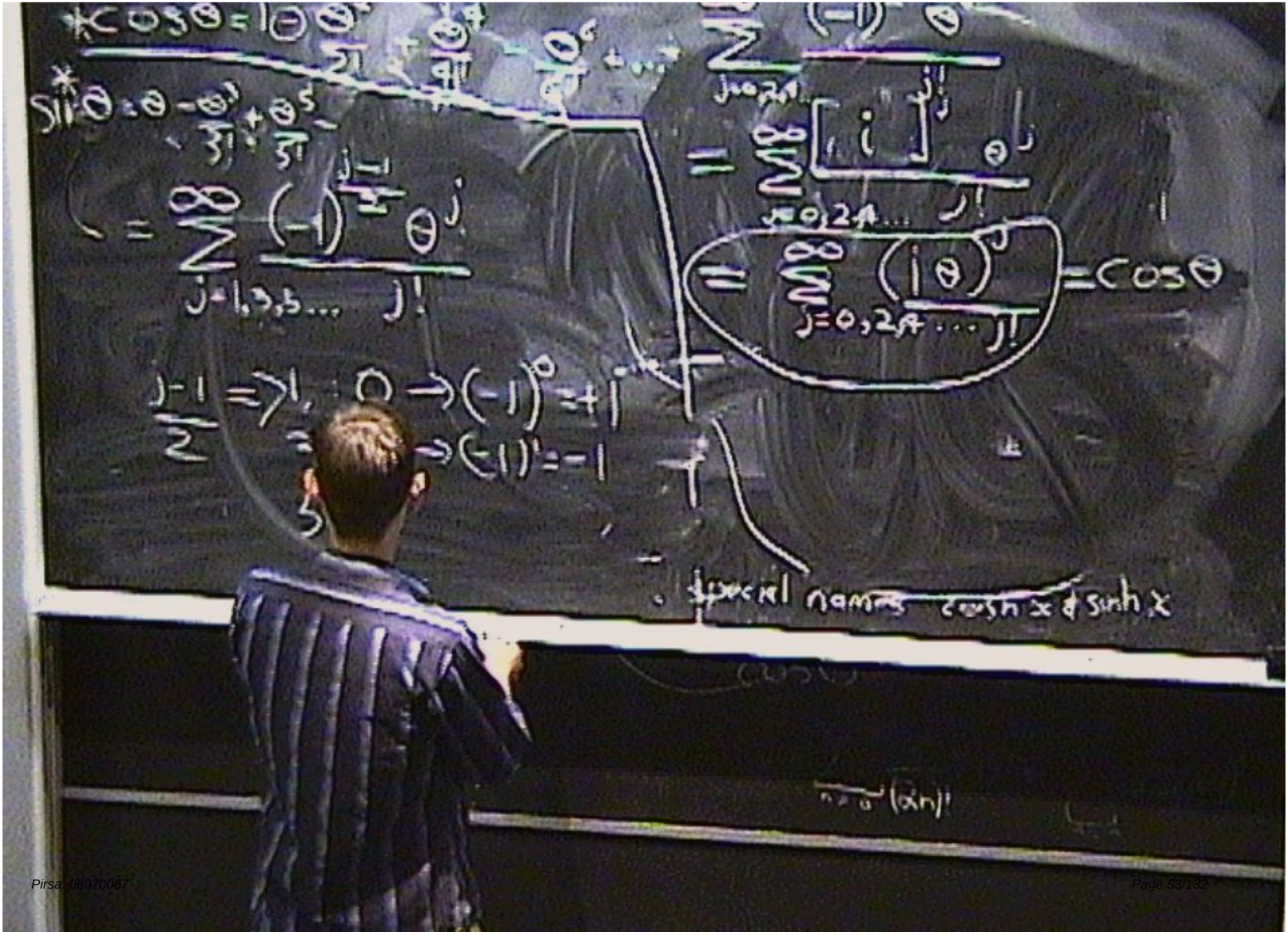
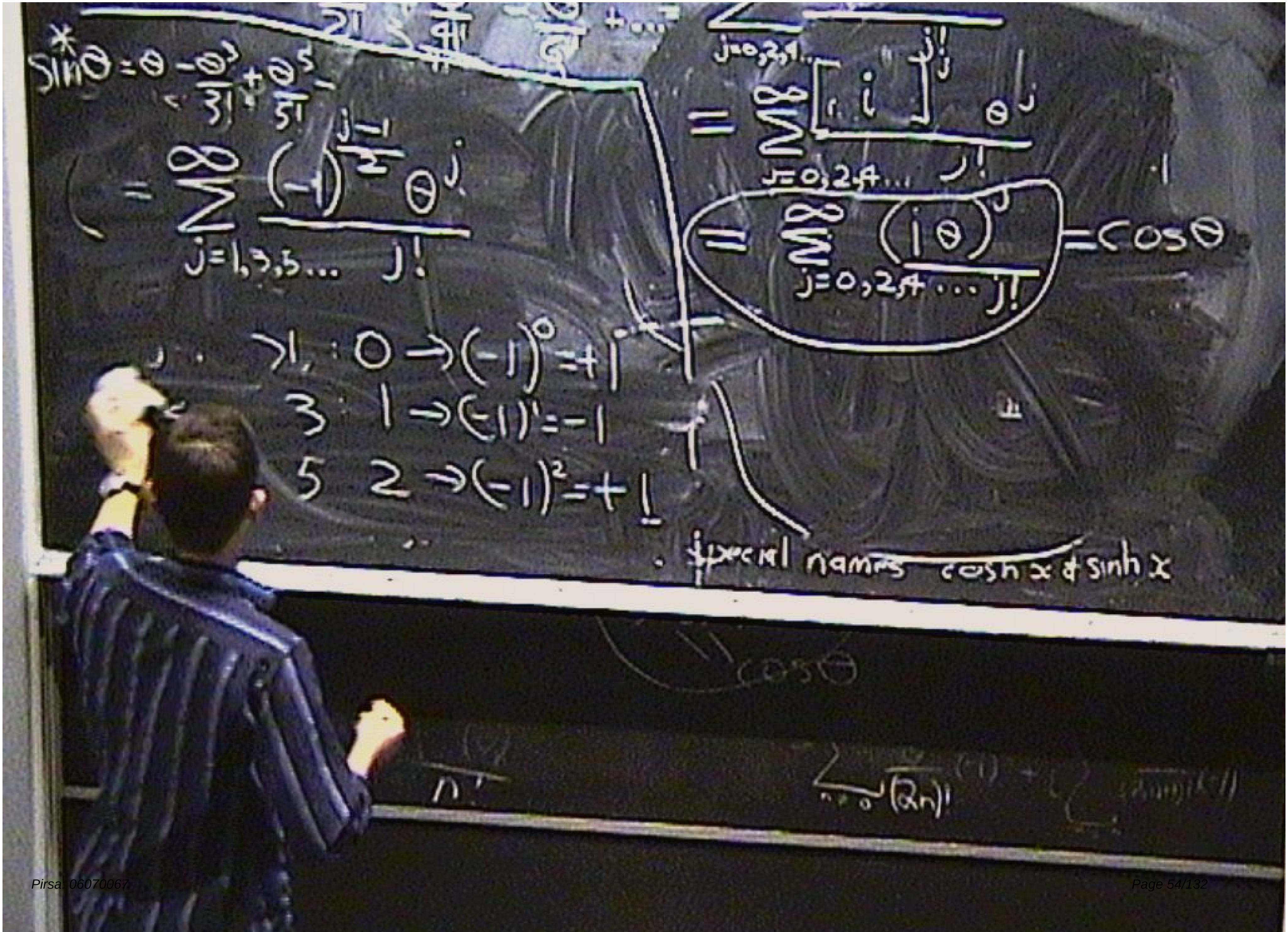
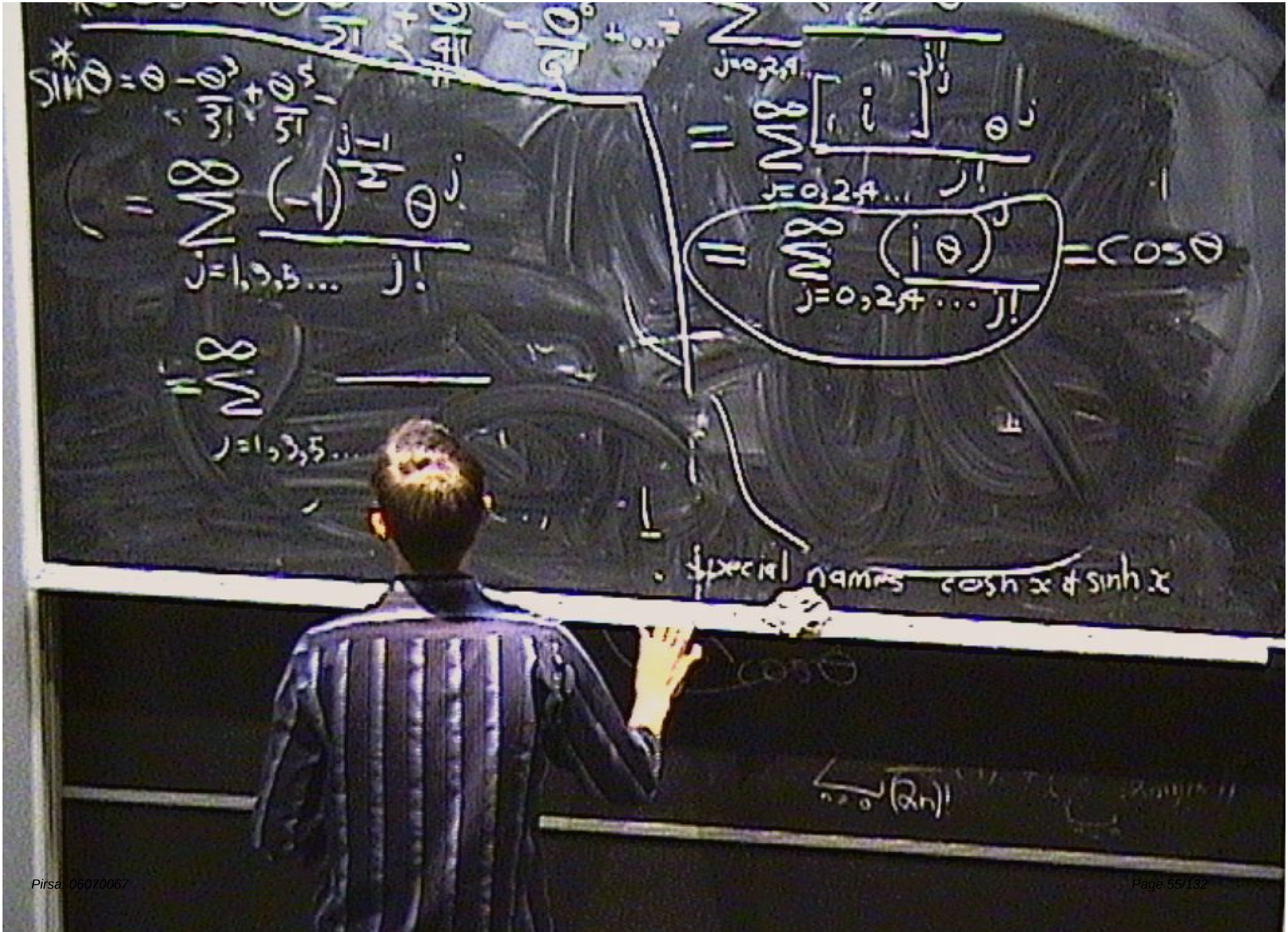

cos θ = 1 − θ²/2! + θ⁴/4! − θ⁶/6! + ...
sin θ = θ − θ³/3! + θ⁵/5!
j=1,3,5...
j=0,2,4...
= cos θ
special names cosh x & sinh x


sinθ = θ − θ³/3! + θ⁵/5! − …
= Σ_{j=1,3,5…}^∞ (−1)^{(j−1)/2} θ^j / j!
1 : 0 → (−1)⁰ = +1
3 : 1 → (−1)¹ = −1
5 2 → (−1)² = +1
= Σ_{j=0,2,4…}^∞ [i]^j θ^j / j!
= Σ_{j=0,2,4…}^∞ (iθ)^j / j! = cosθ
special names cosh x & sinh x



$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$
$= \sum_{j=1,3,5\ldots}^{\infty} \frac{(-1)^{\frac{j-1}{2}} \theta^j}{j!}$
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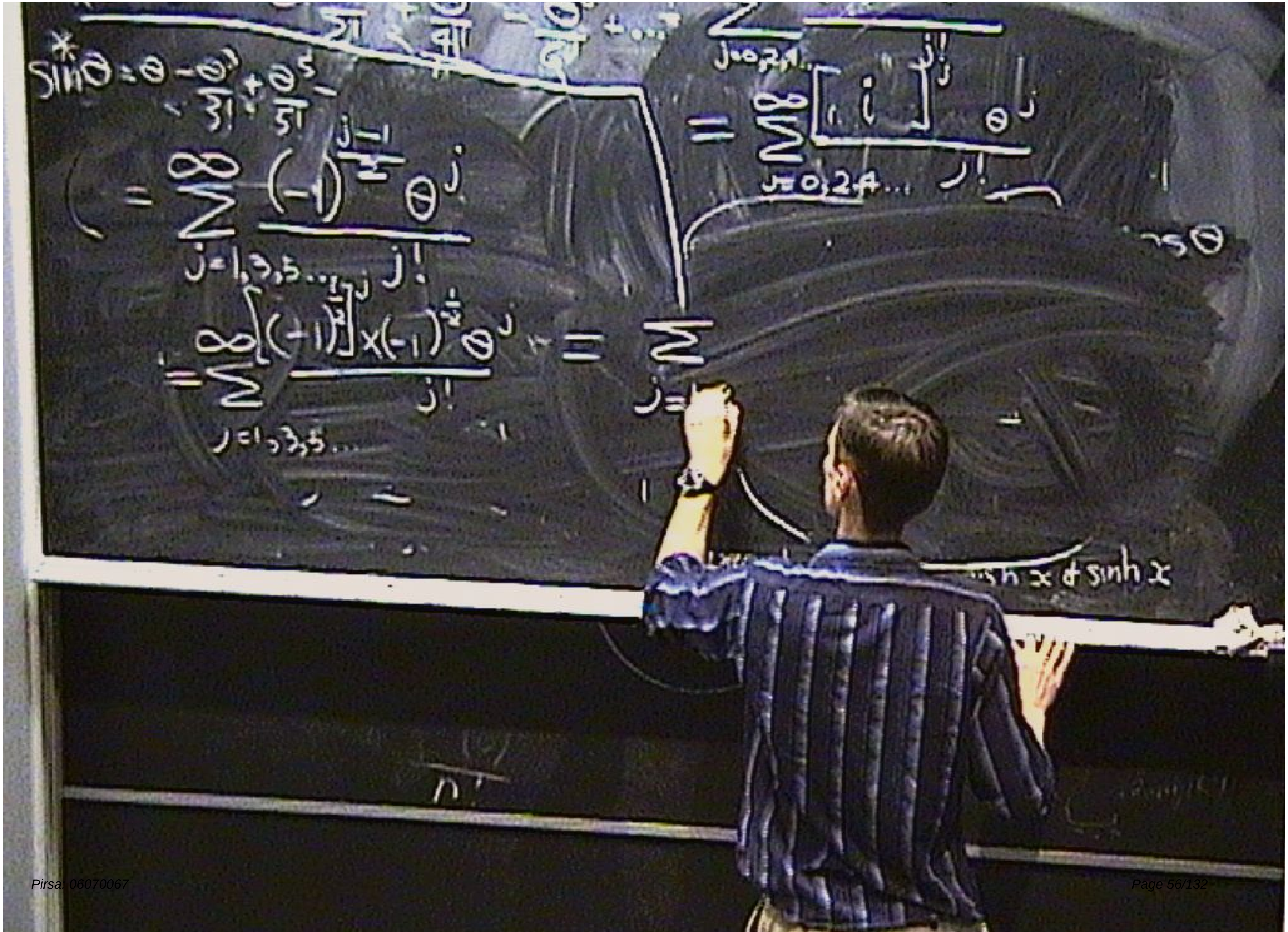
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= Σ_{j=0,2,4...}^∞ [i]^j θ^j / j!
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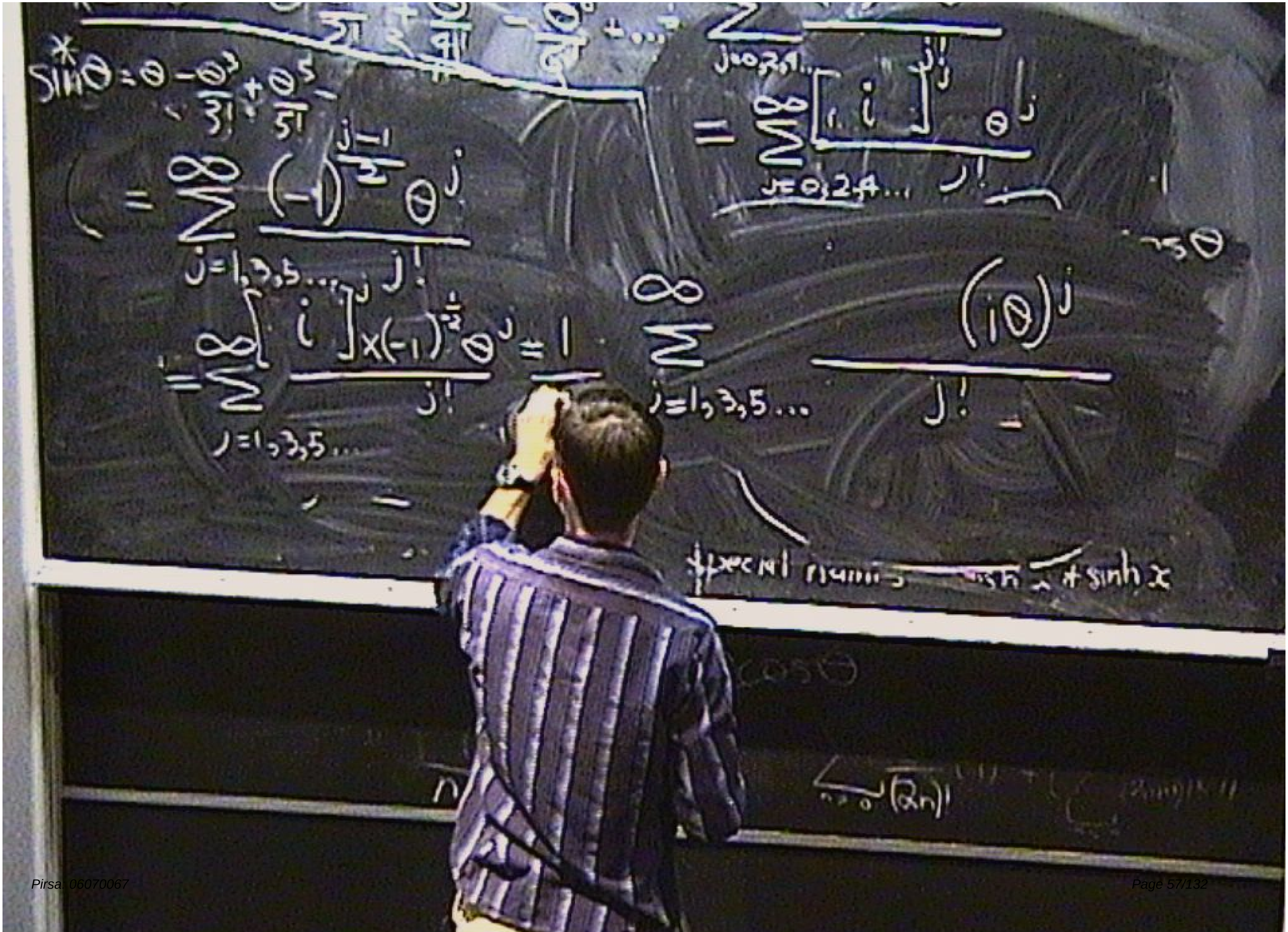
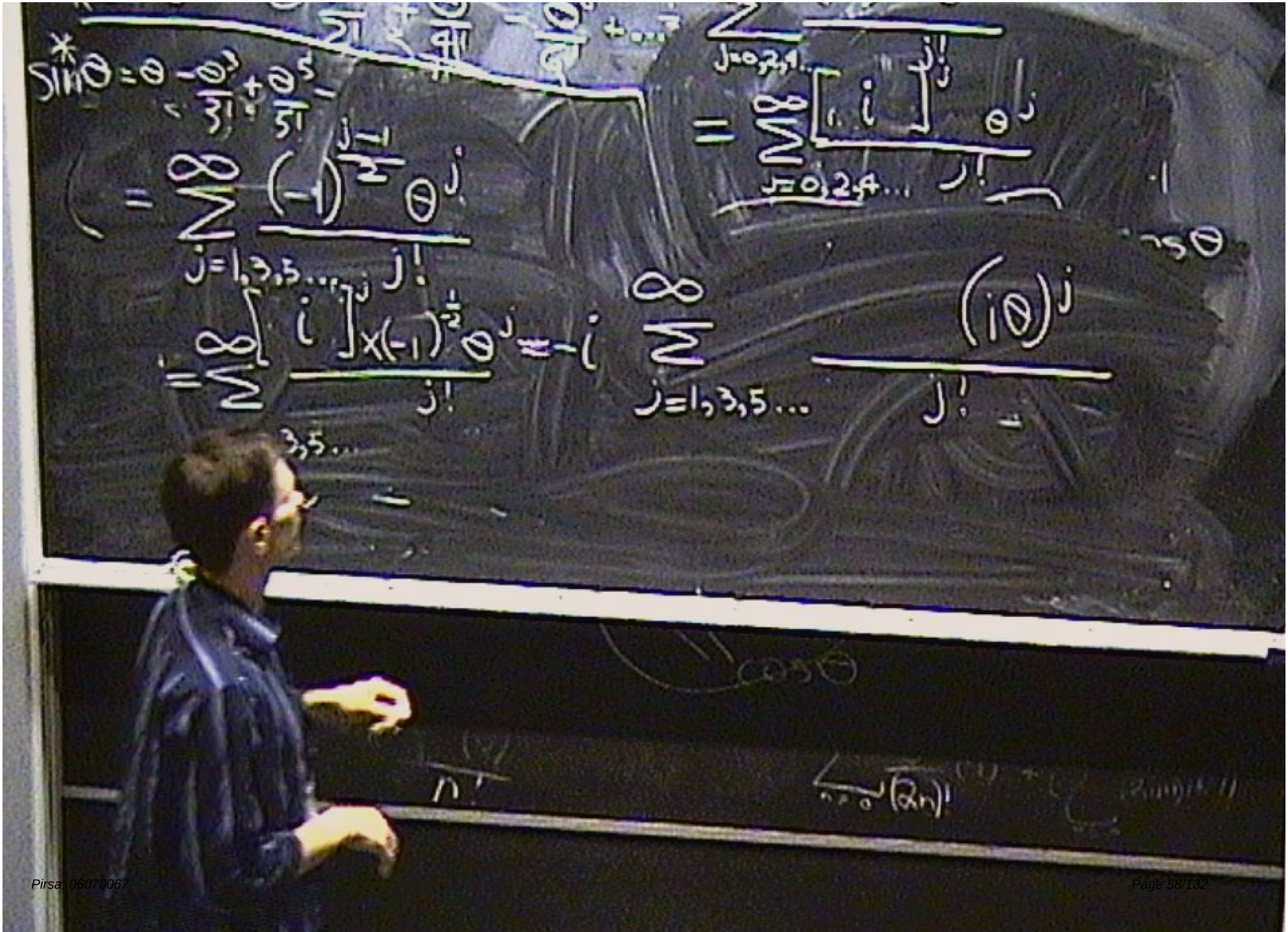
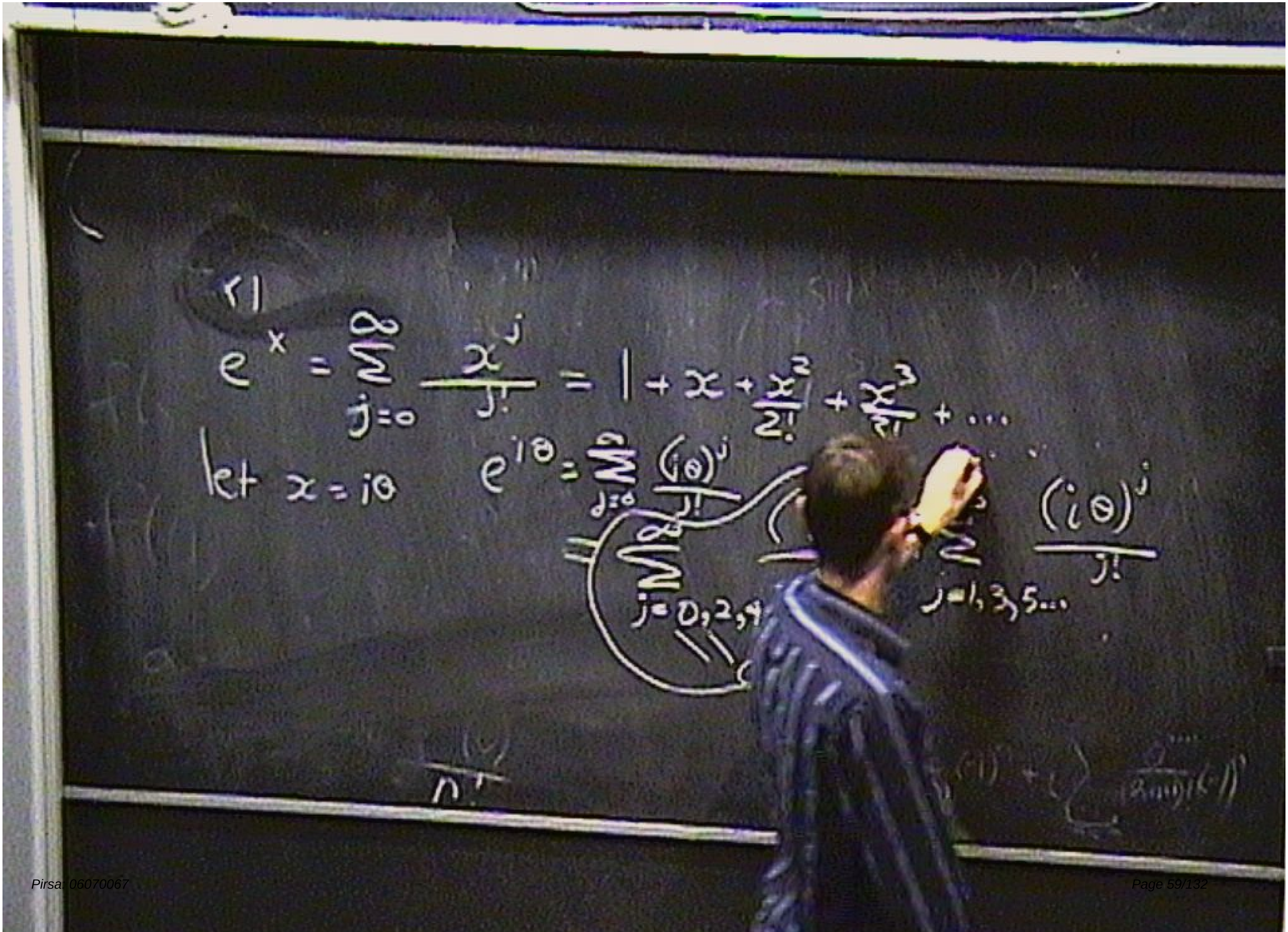



sinθ = θ − θ³/3! + θ⁵/5! −
= Σ_{j=1,3,5...}^∞ (−1)^{(j−1)/2} θ^j / j!
= Σ^∞ [i]^j x (−1)^{j/2} θ^j / j!
= −i Σ_{j=1,3,5...}^∞ (iθ)^j / j!
= Σ_{j=0,2,4...}^∞ [i]^j θ^j / j!
cosθ


$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
let $x = i\theta$
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$= \sum_{j=0,2,4\dots}$
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$$\sum_{j=0,2,4\dots}^{\infty} \frac{(i\theta)^j}{j!} = \cos\theta \qquad \sum_{j=1,3,5\dots}^{\infty} \frac{(i\theta)^j}{j!} = i\sin\theta$$

Adding together complex numbers.

- add the real and imaginary parts individually.
- eg. $u=a+bi$ and $v=ac+di$
- What is $u + v$?
- $u+v = (a+c) + (b+d)i$.
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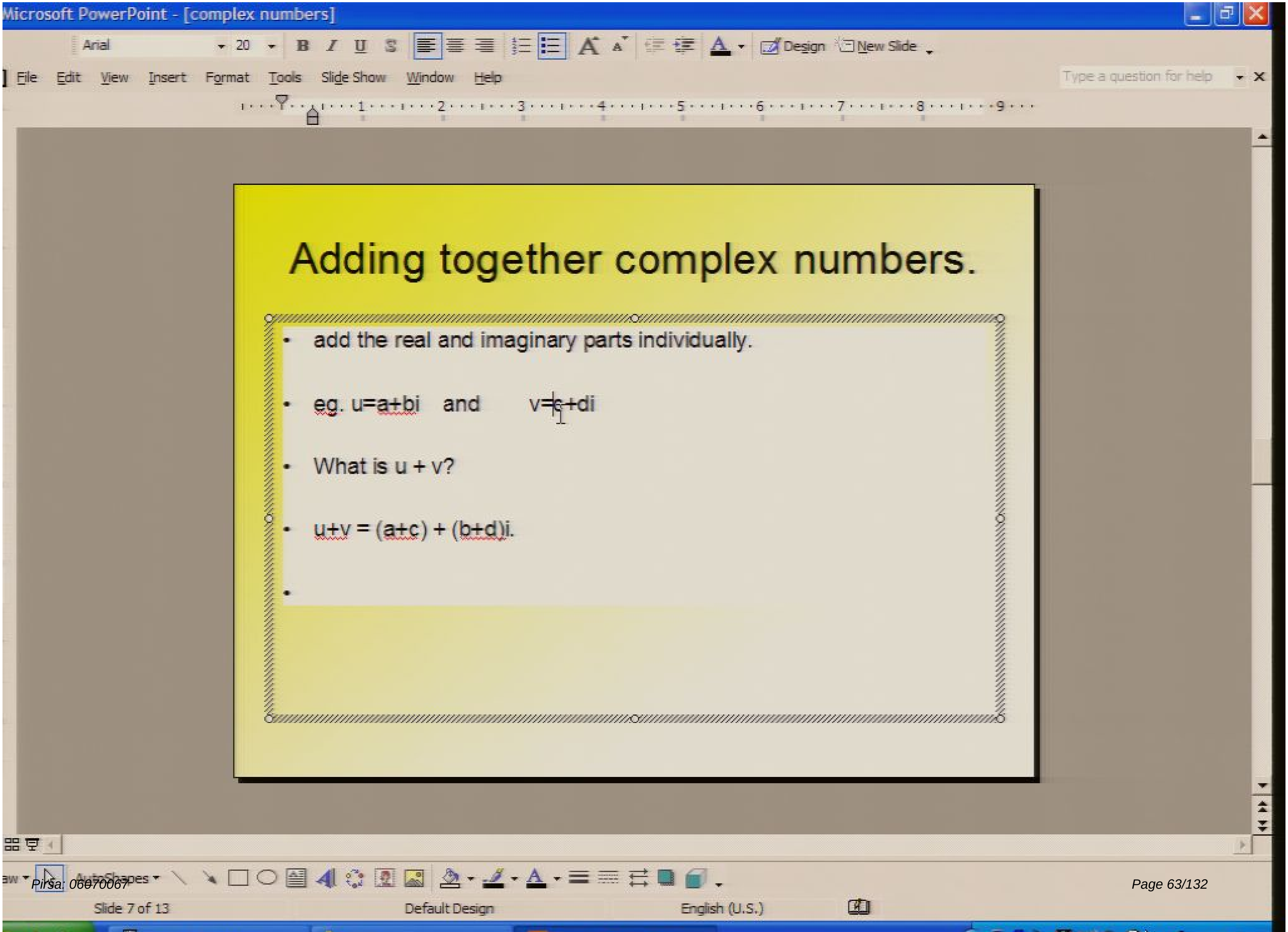
Microsoft PowerPoint - [complex numbers]
Adding together complex numbers.
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Slide 7 of 13
Default Design
English (U.S.)

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Complex numbers

- integers -2,-1,0,1,2,3 …
- fractions 4/5, 1/10
- Real numbers: -4.53456, 3.14159
- Can we go any further? YES!

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Multiplication

- What is u x v?
- Expand out the brackets and use the fact that i^2=-

- u=a+bi and v=ac+di

- $u \times v = (a+bi) \times (c+di) = ac + (ad + bc)i + bd \times i^2$

$$= (ac - bd) + (ad + bc)i$$

Note the minus sign.

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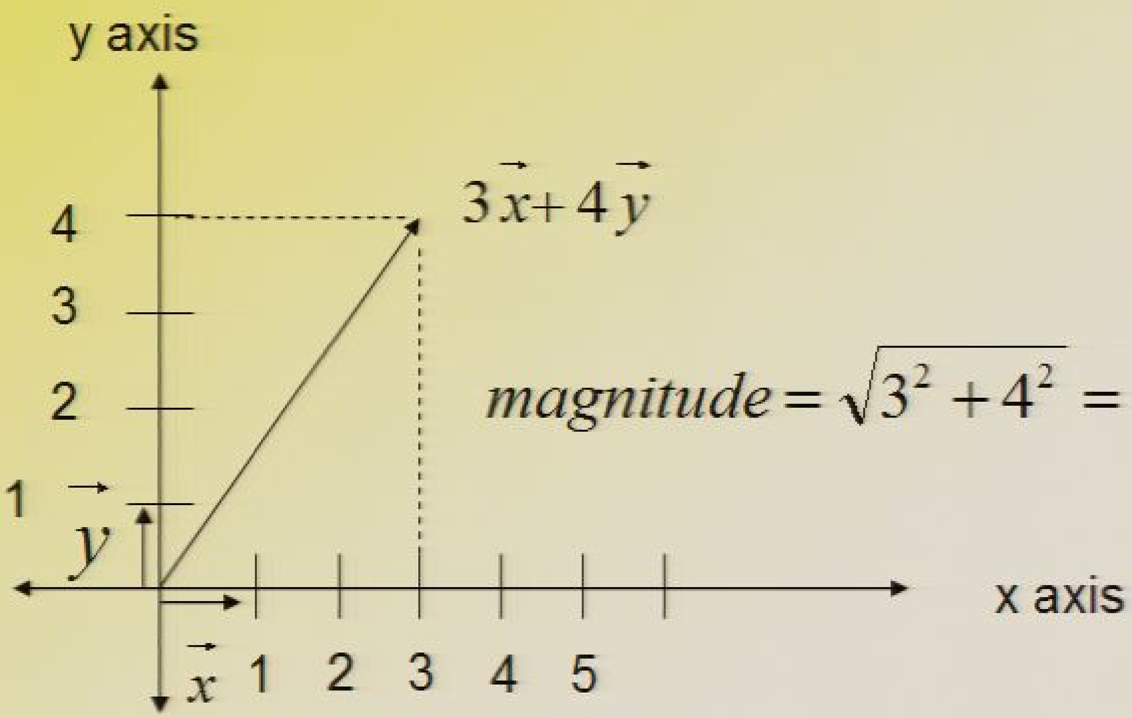
Student activity

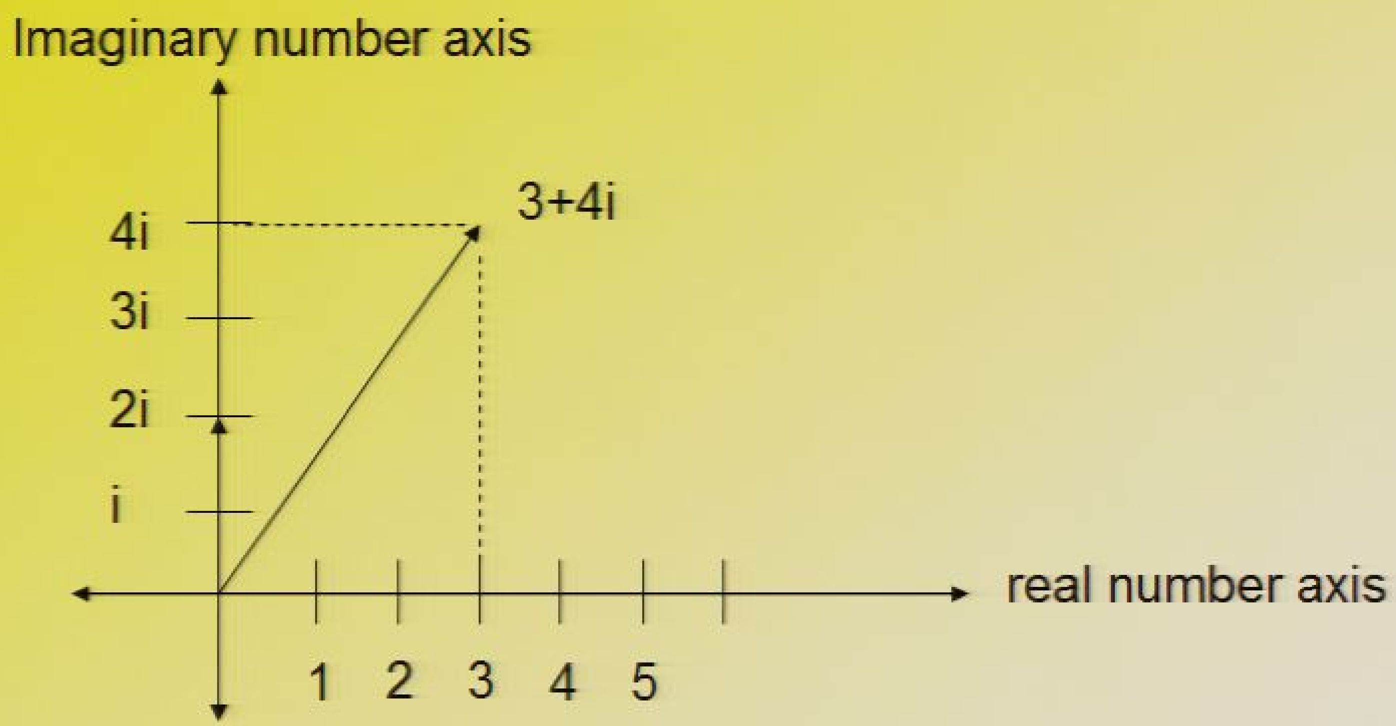
- 1. calculate u + v, where u=4 + 2i and v = 5 + 7i

Absolute value

- also known as magnitude.
- Compare with the magnitude of a vector.



$$magnitude = \sqrt{3^2+4^2} = \sqrt{25} = 5$$



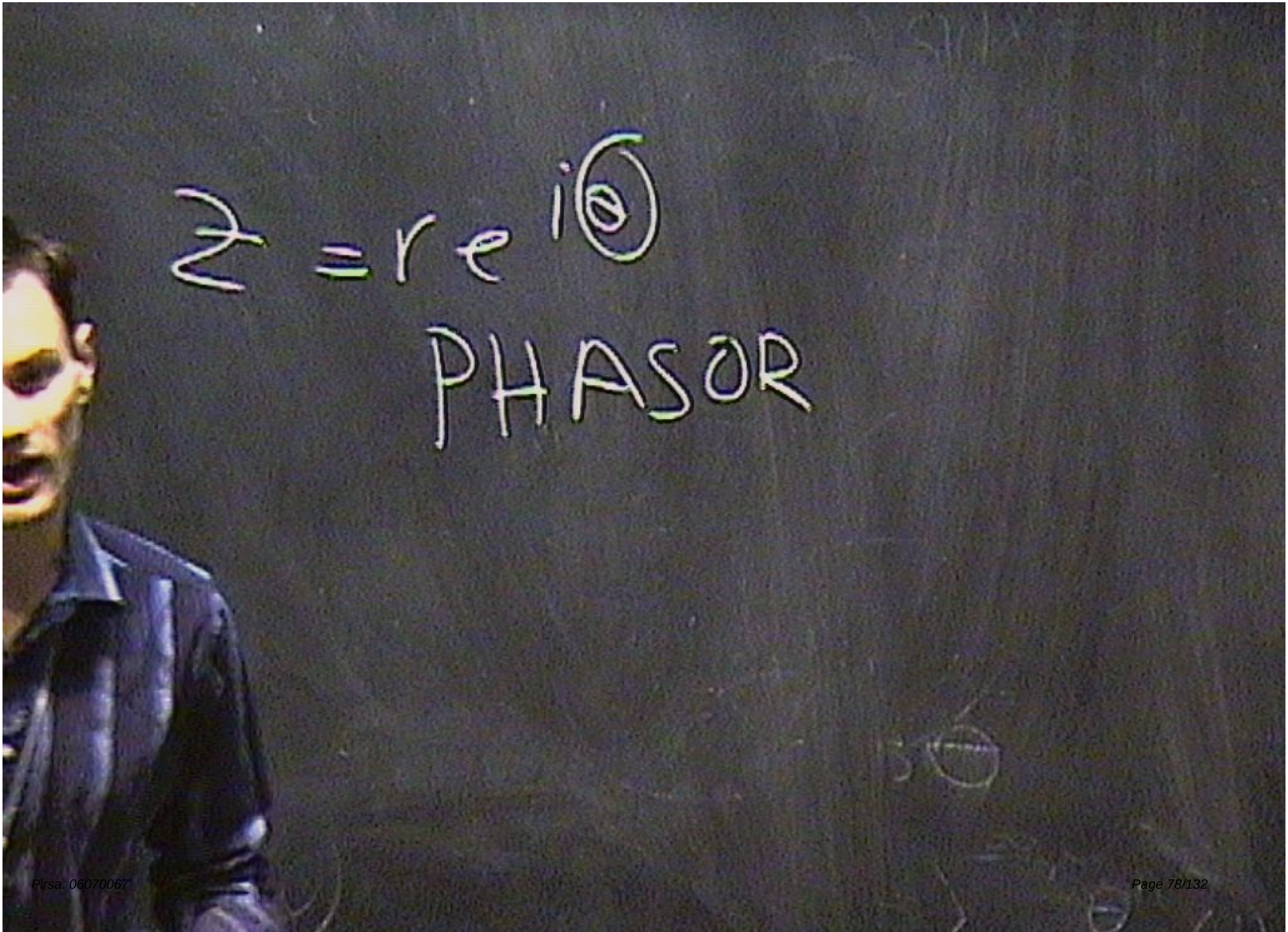
- Same thing for complex numbers.
- u = 3 + 4i
- $|u| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$$u = a + bi$$

$$|u| = \sqrt{a^2 + b^2}$$

Phasors

Phasors


$z = re^{i\theta}$
PHASOR

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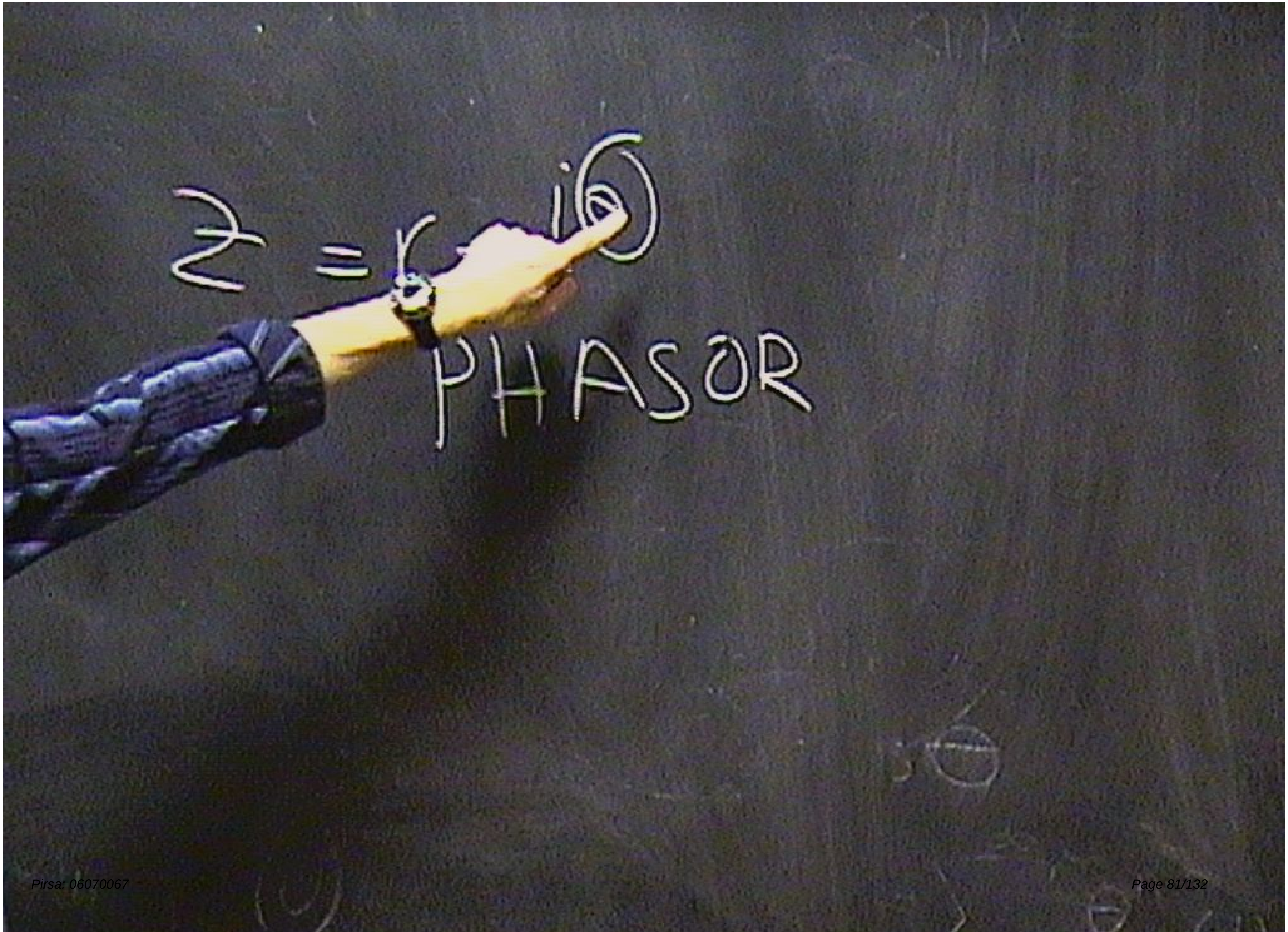
PHASOR


PHASOR

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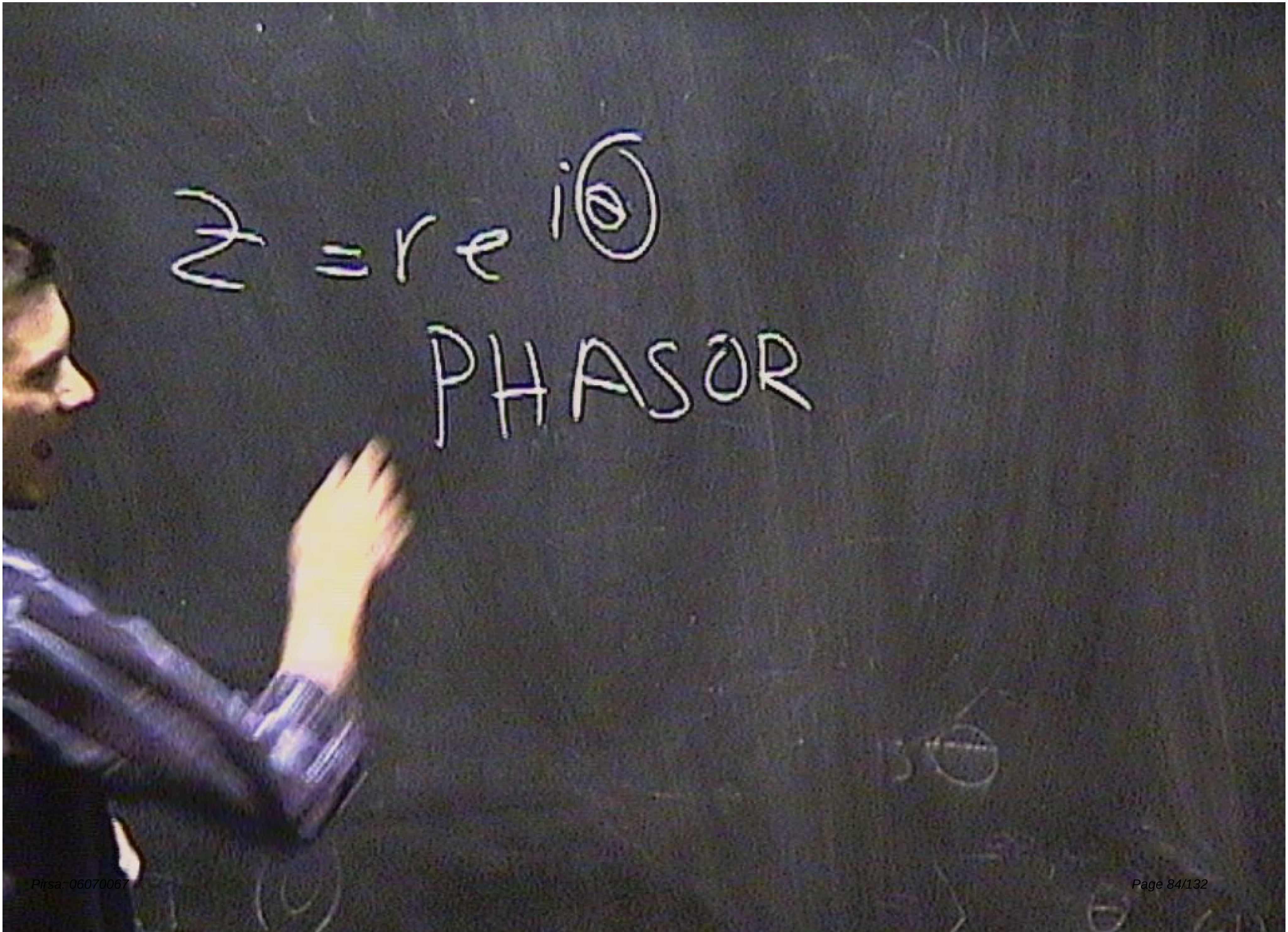
PHASOR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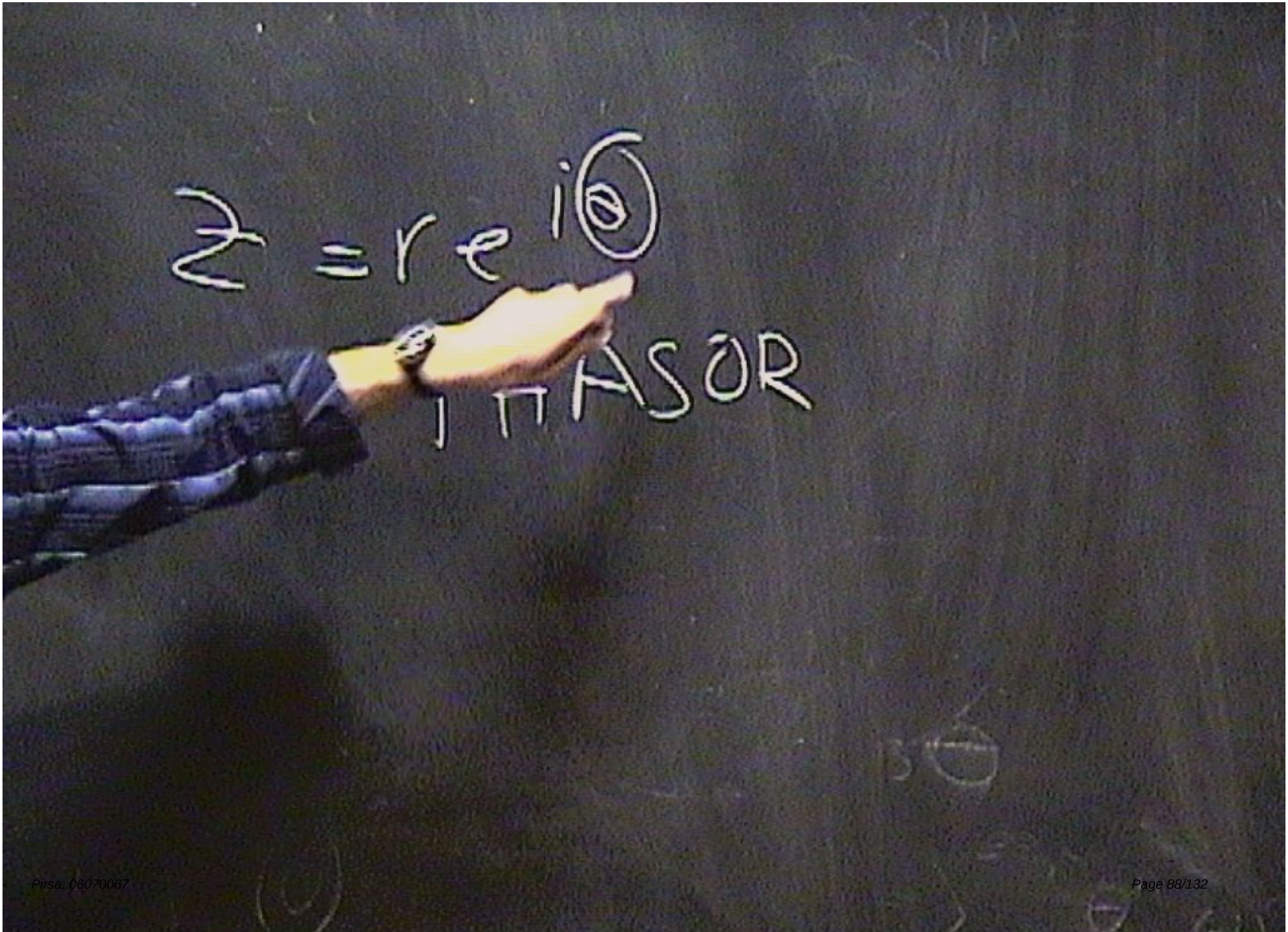

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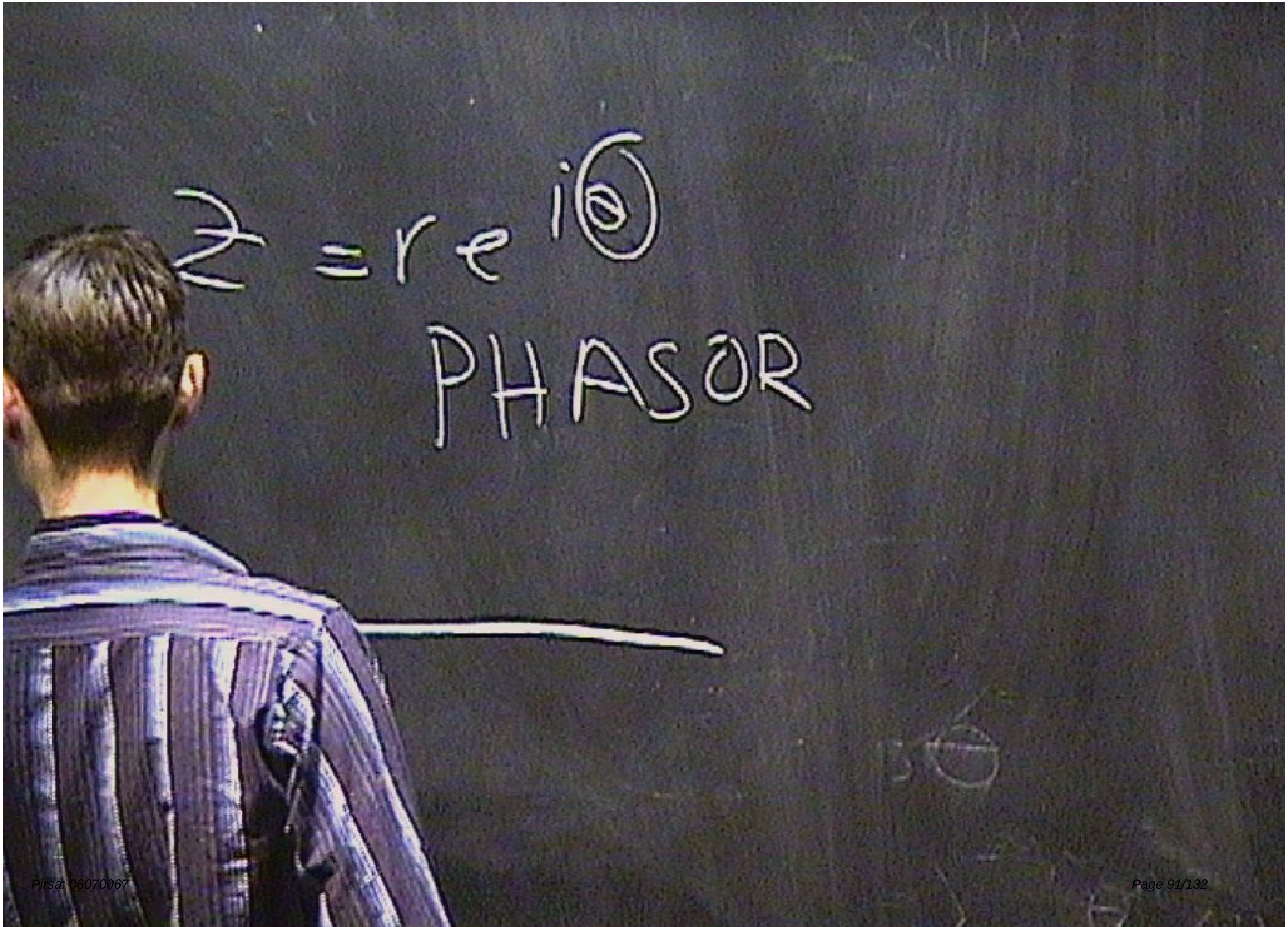
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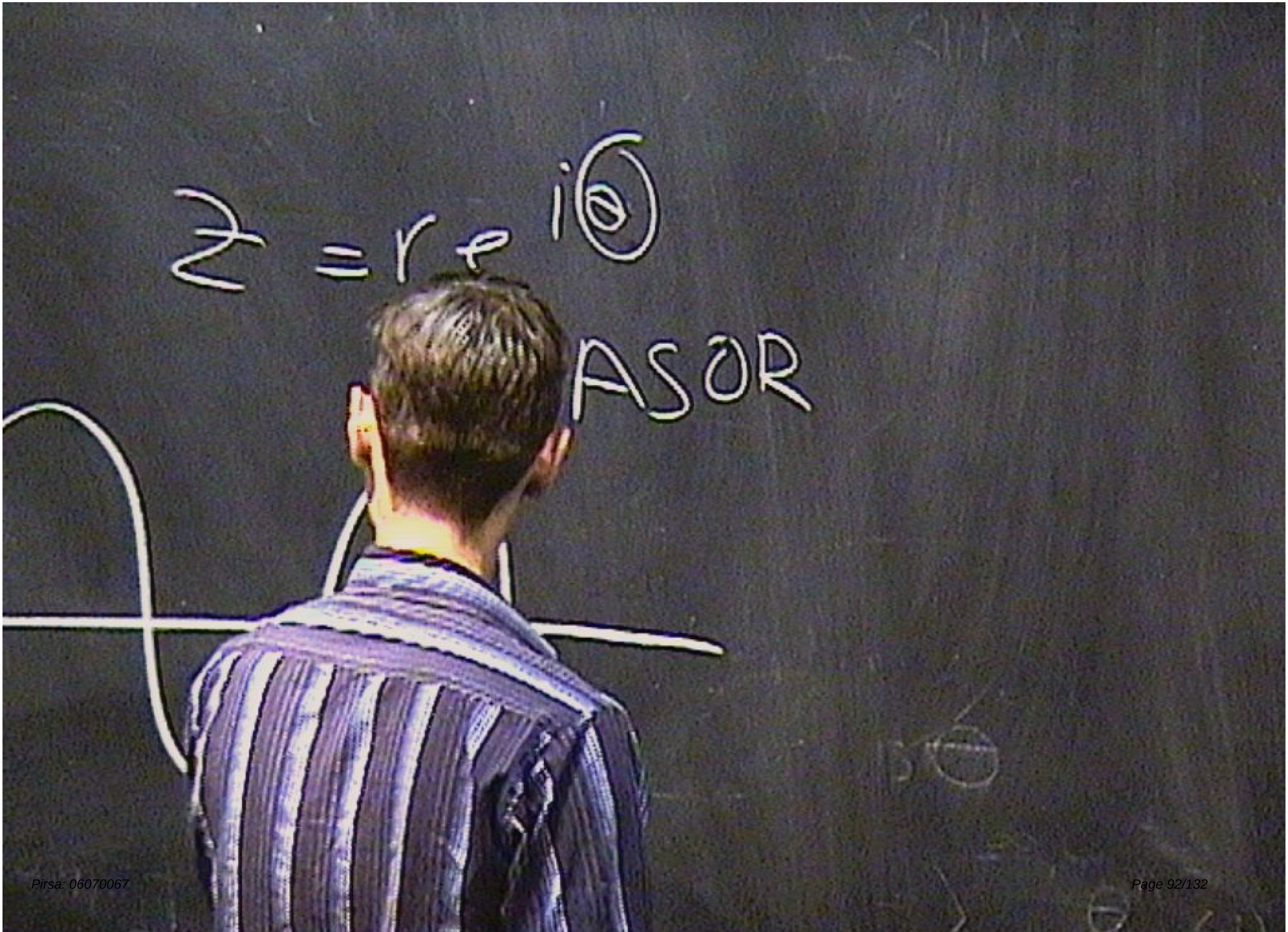
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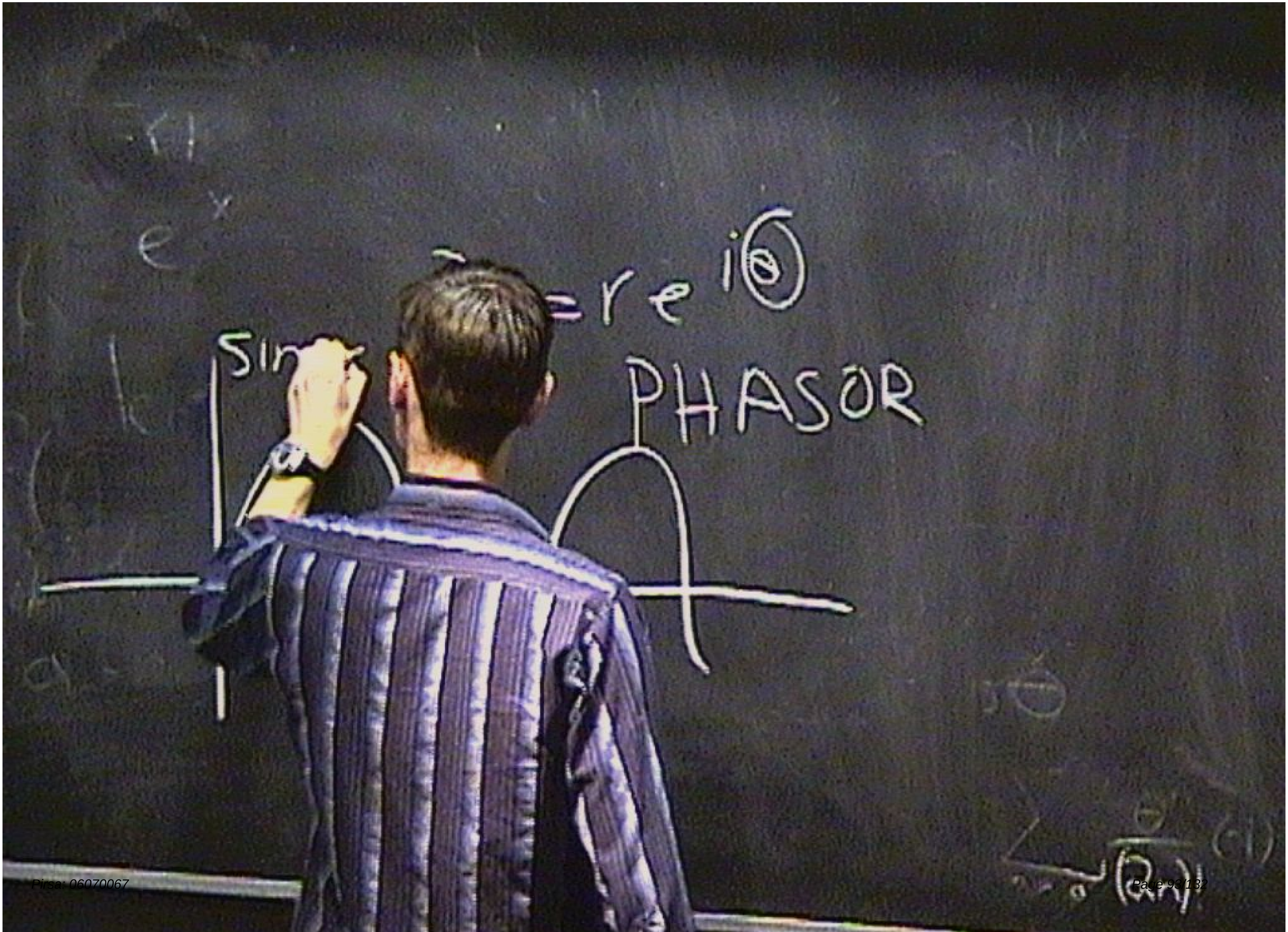
ASOR



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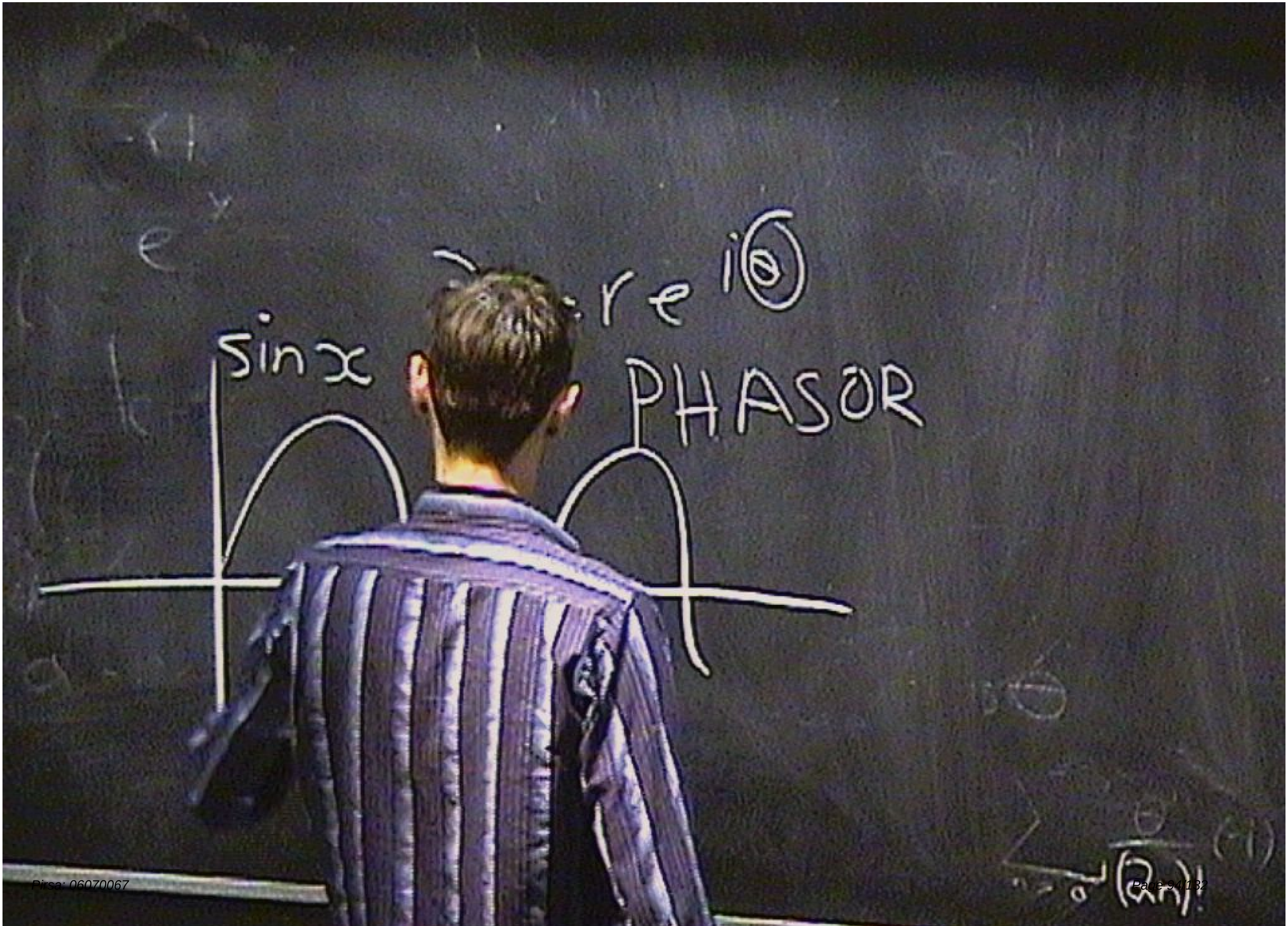
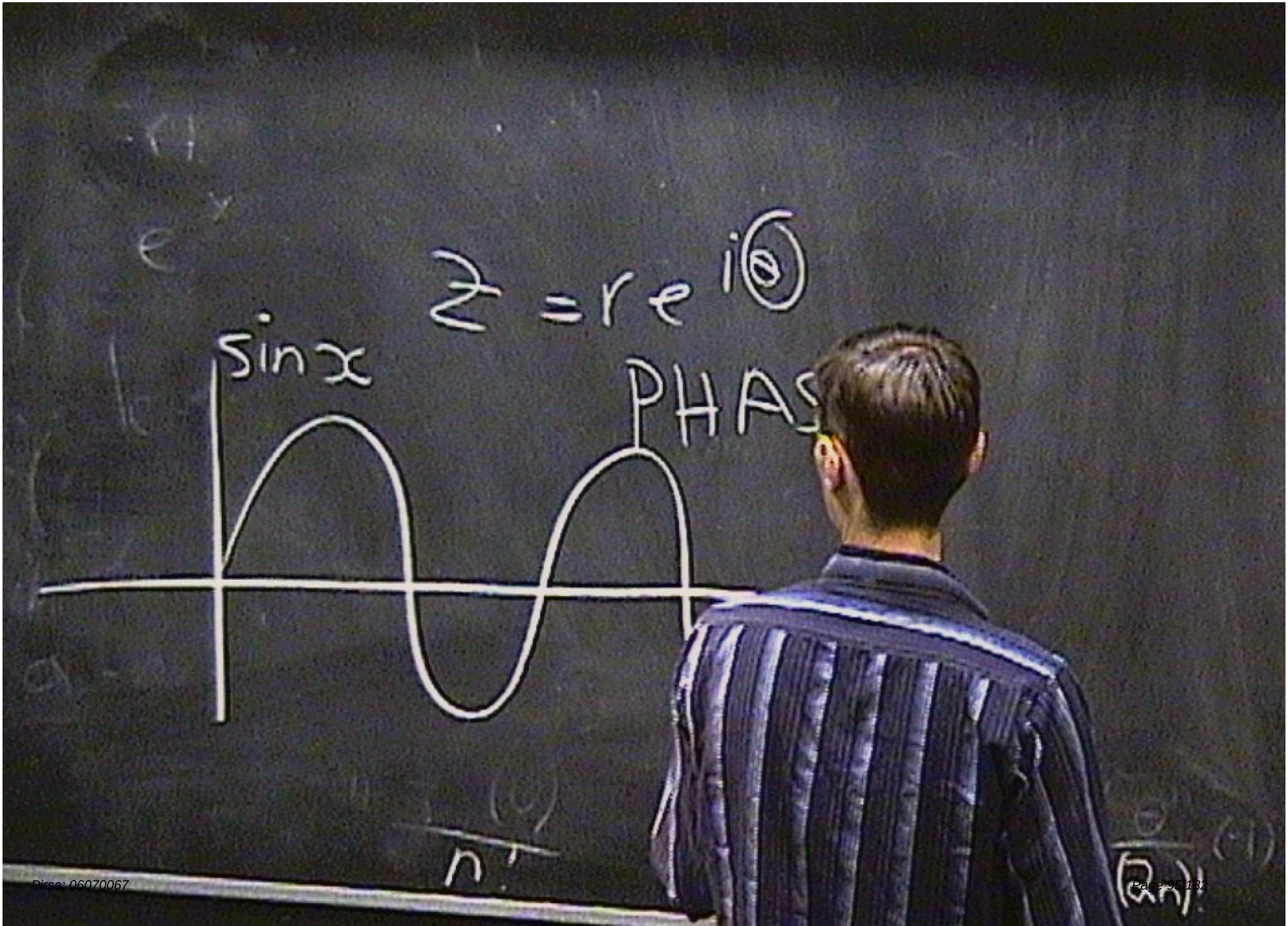
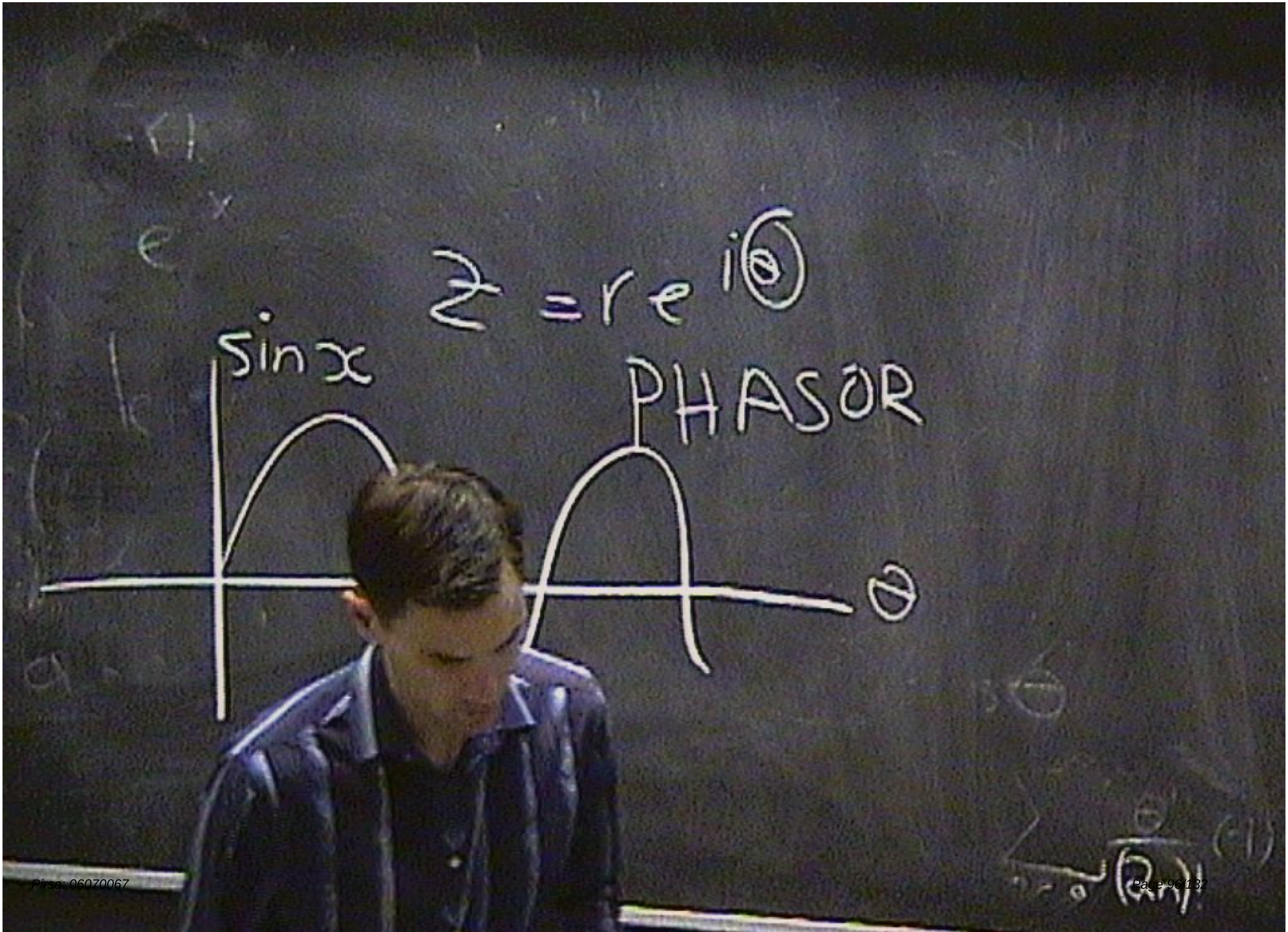
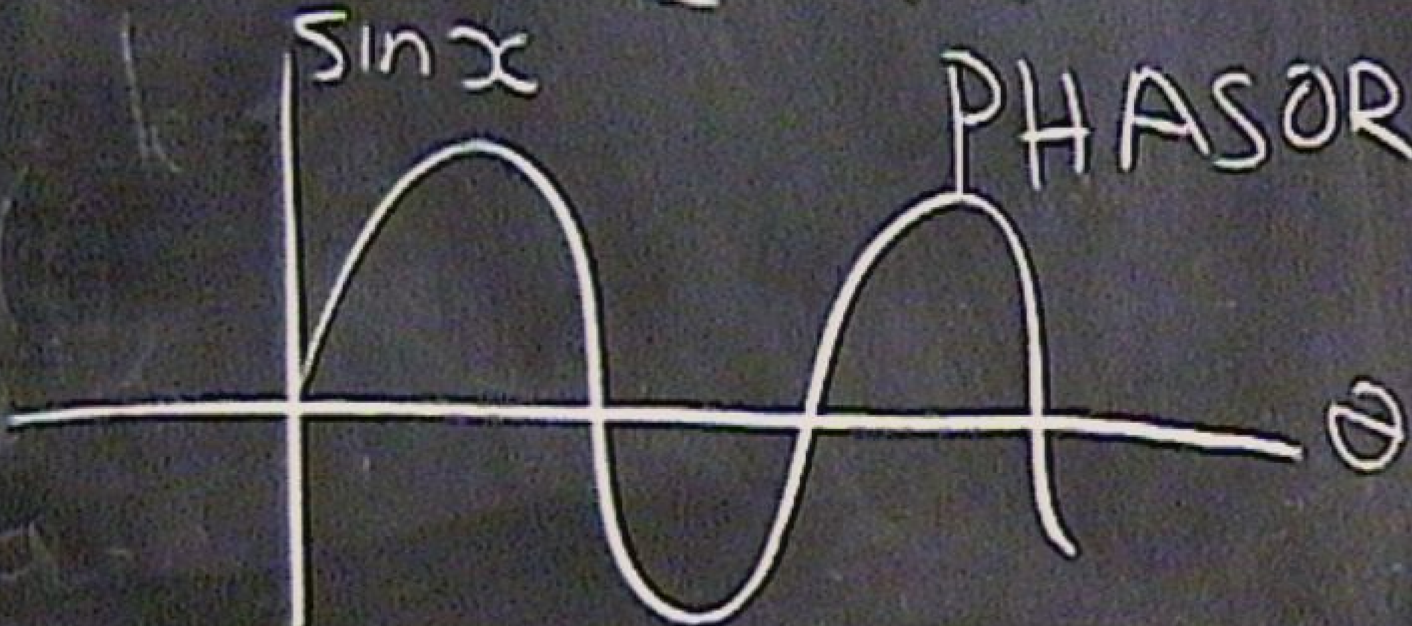

sin x
re iθ
PHASOR

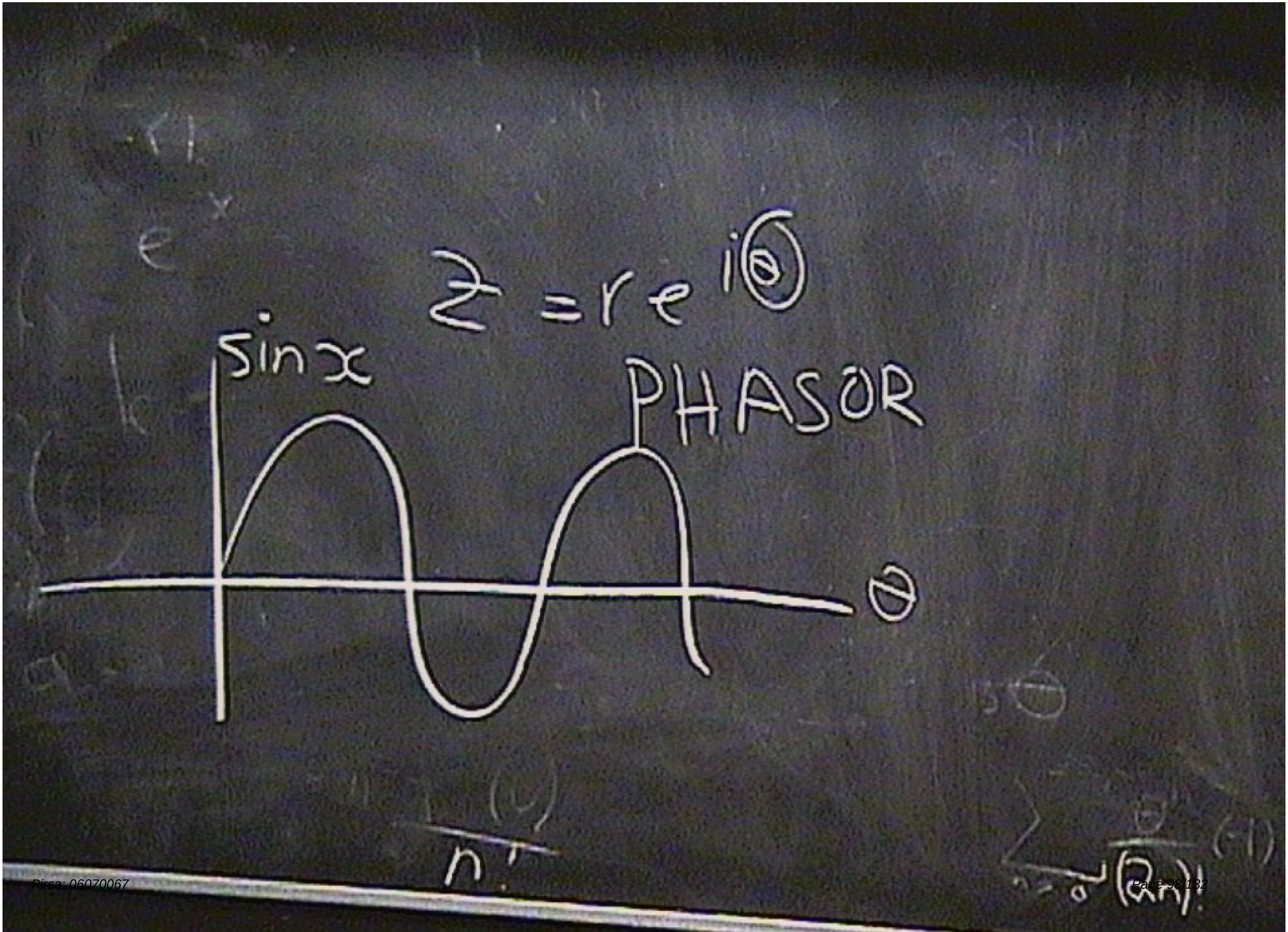


z = re^{iθ}
sinx
PHAS


z = re iθ
sin x
PHASOR
θ

$$z = re^{i\theta}$$

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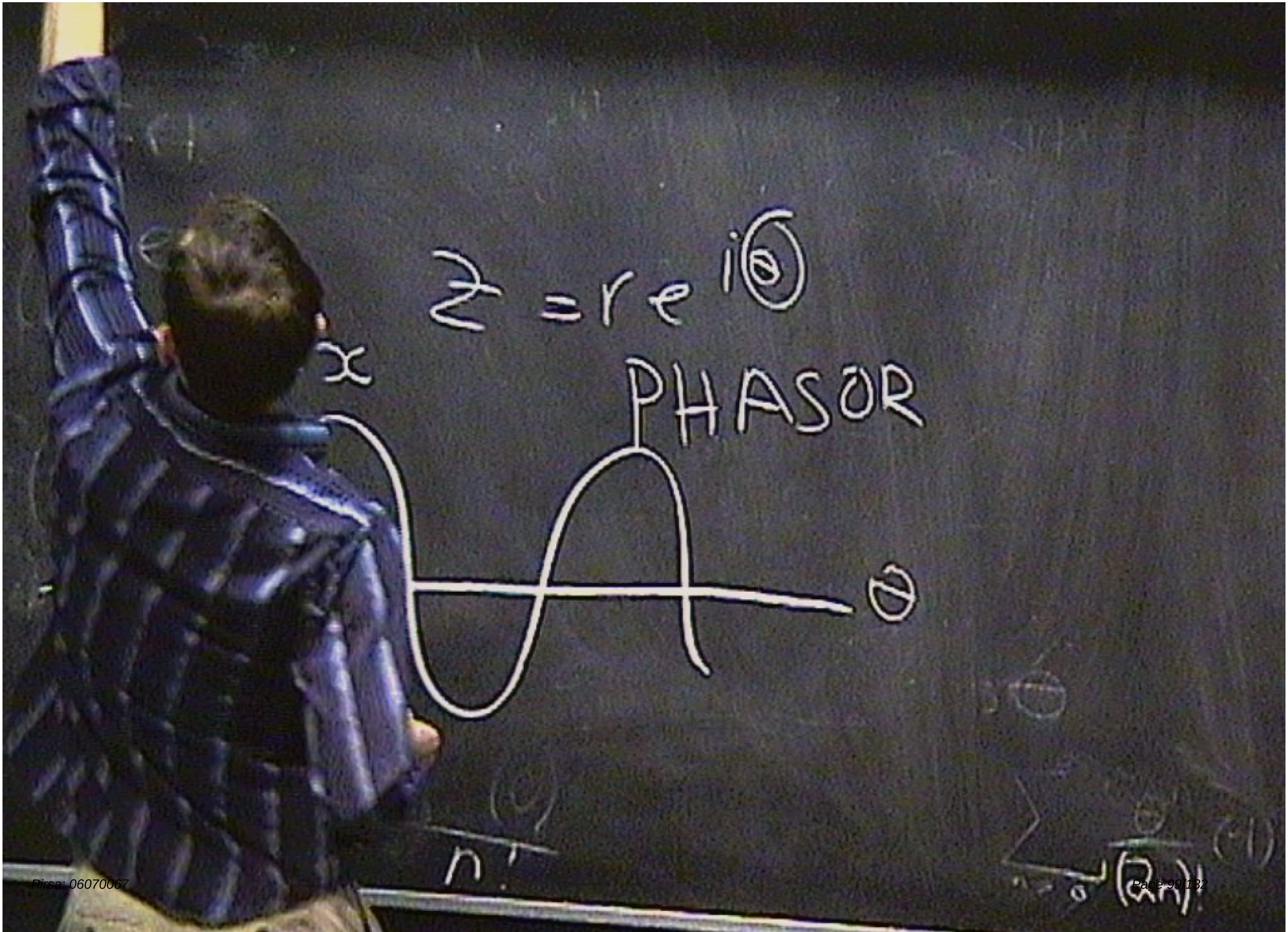
z = re iθ
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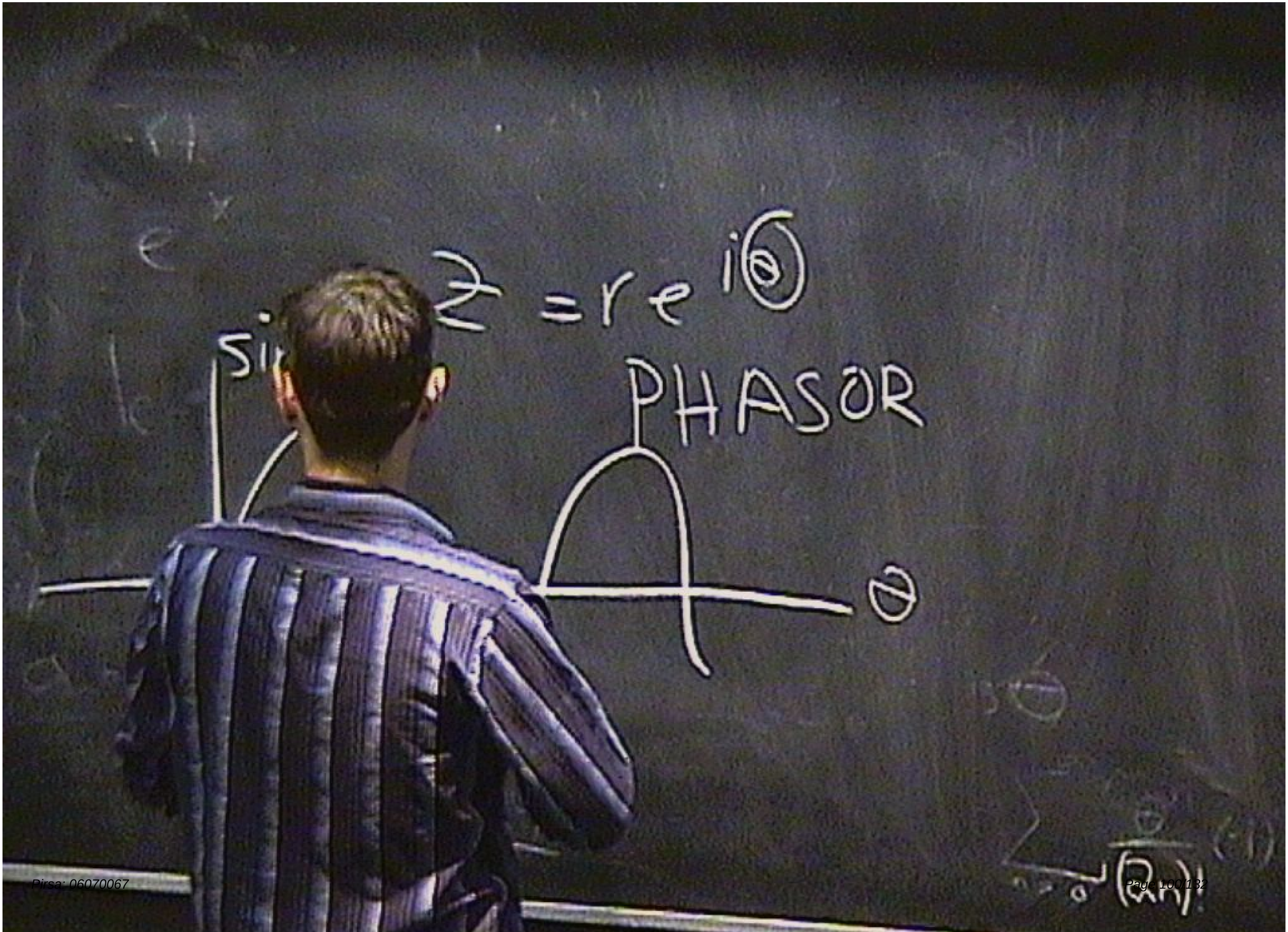
PHASOR
θ
n!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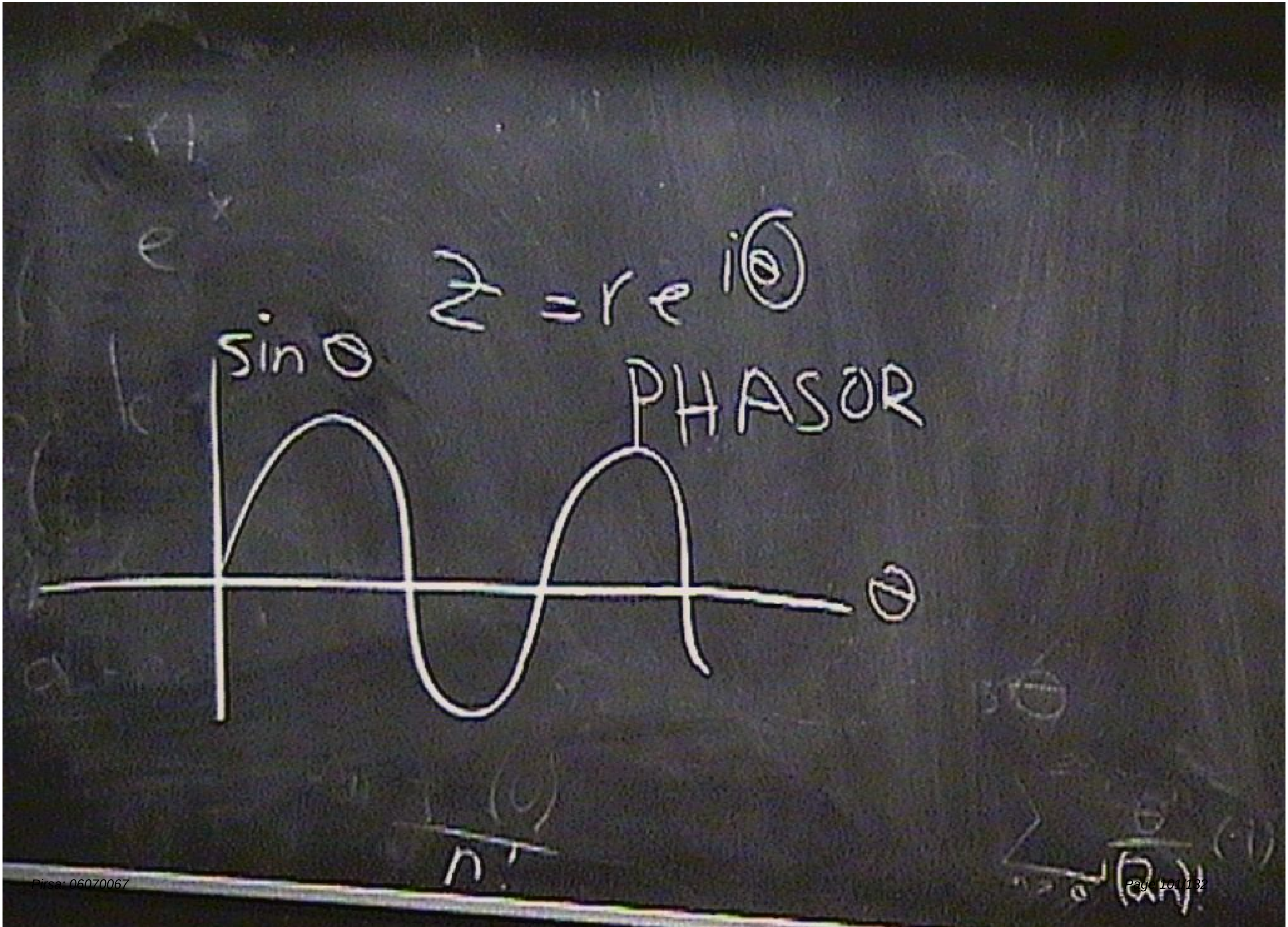
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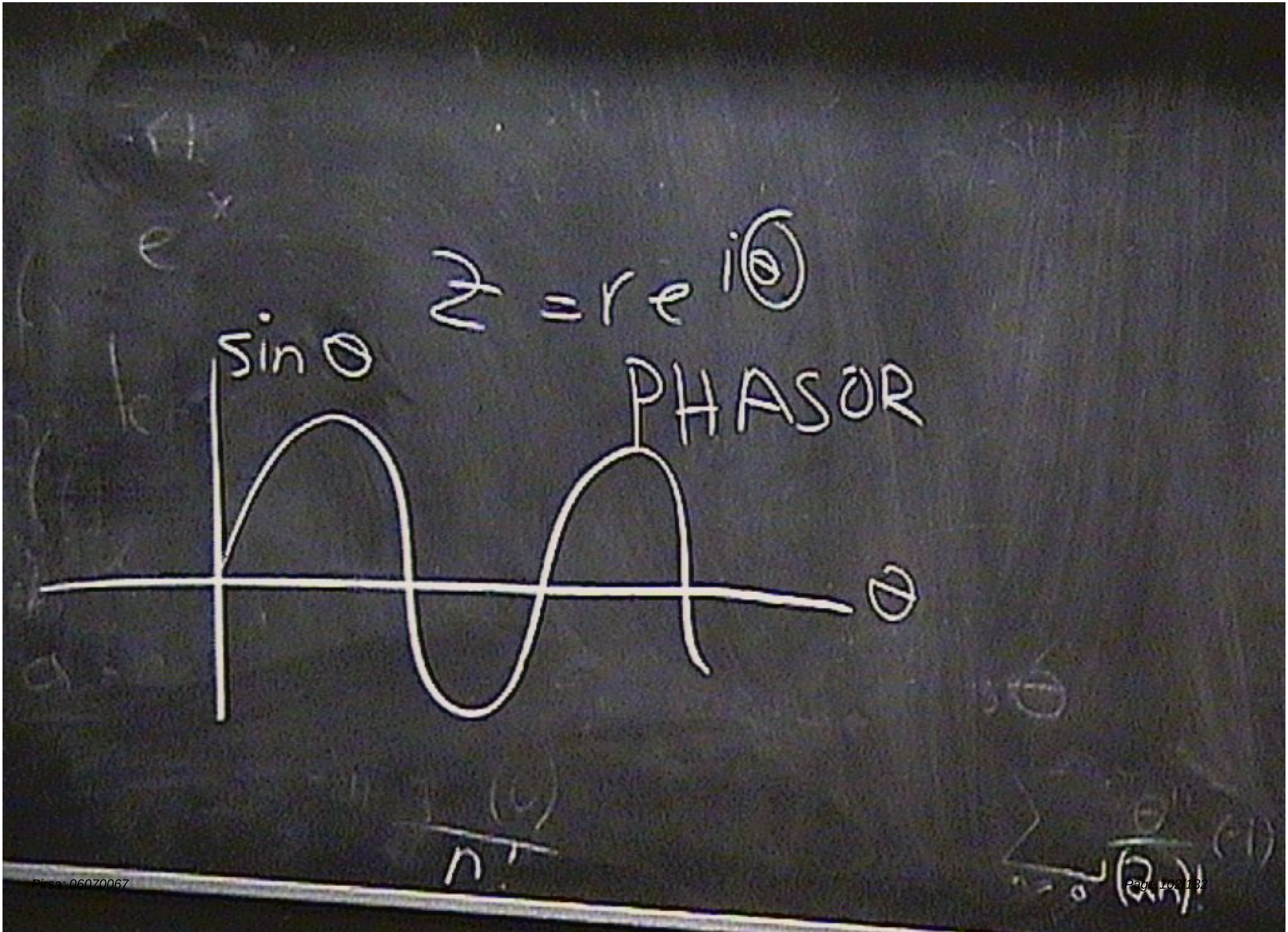
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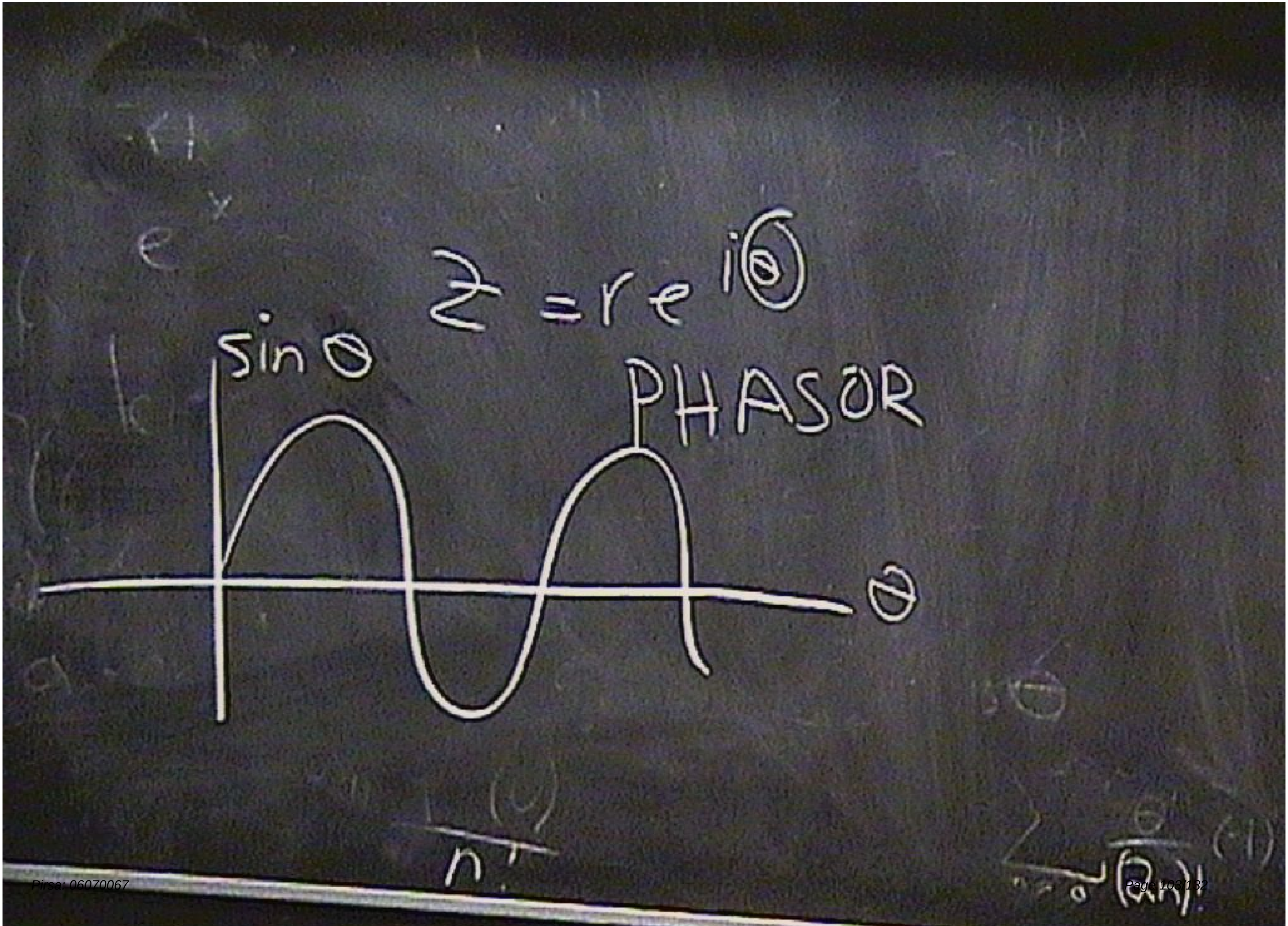

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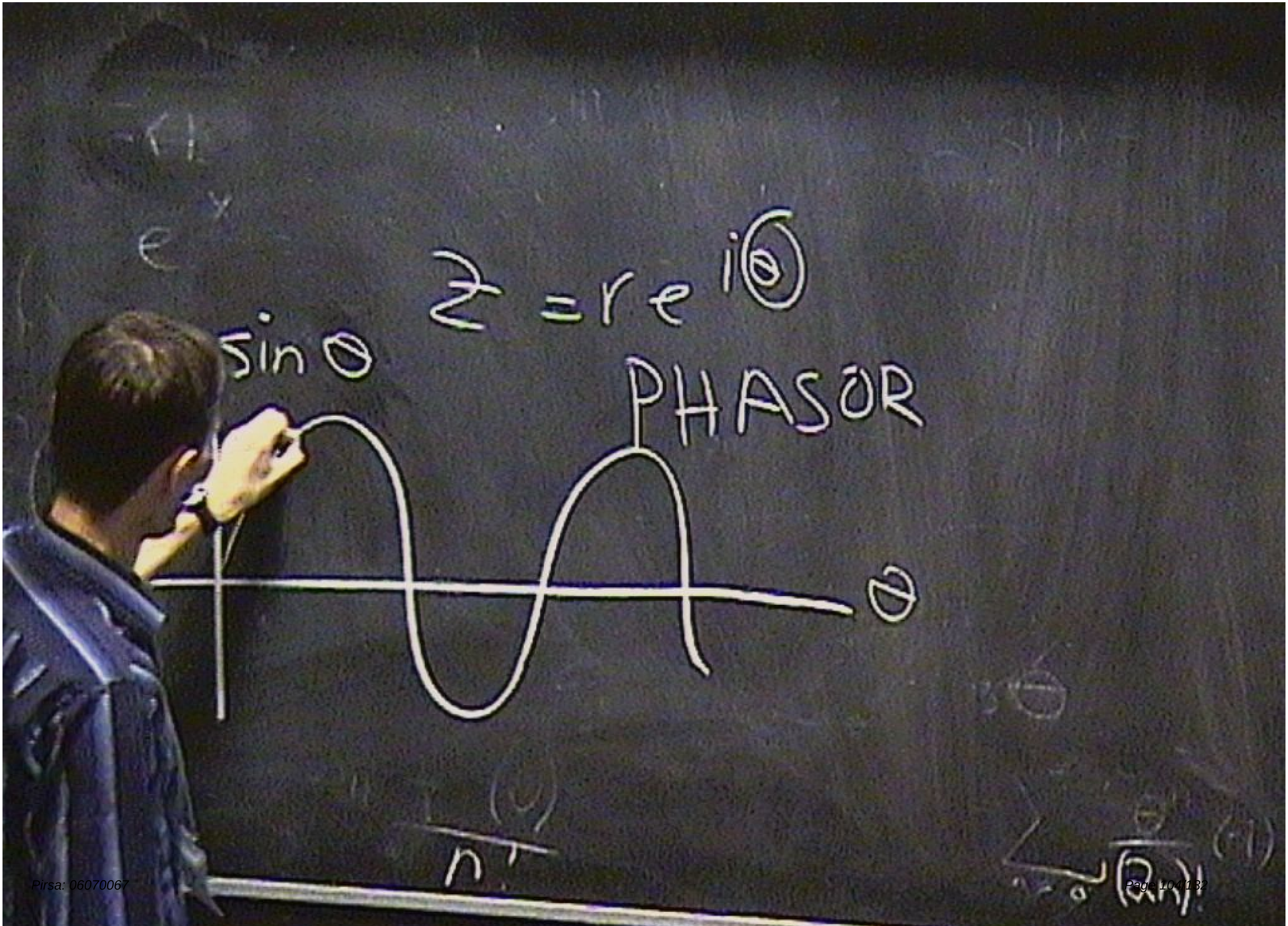
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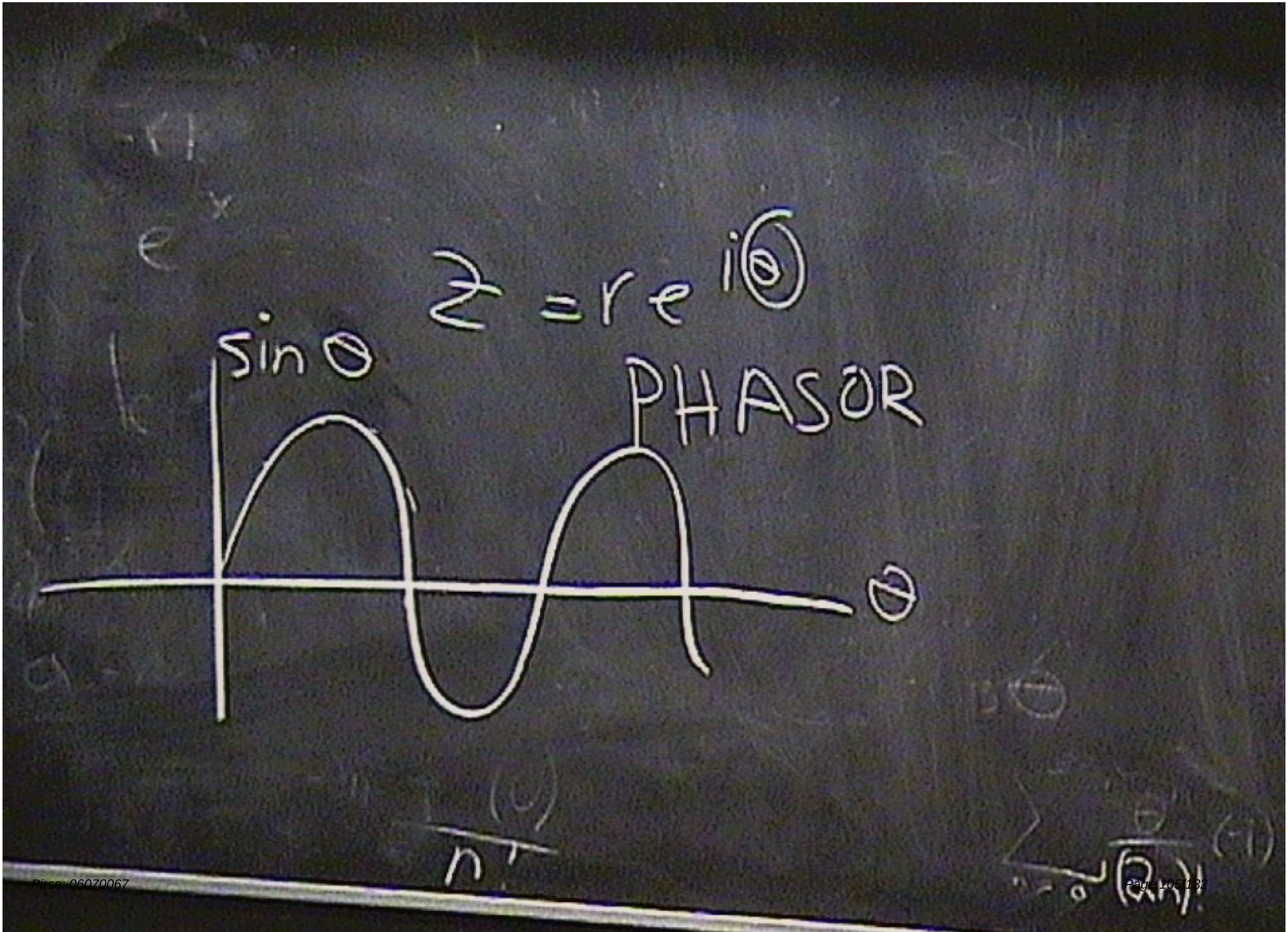
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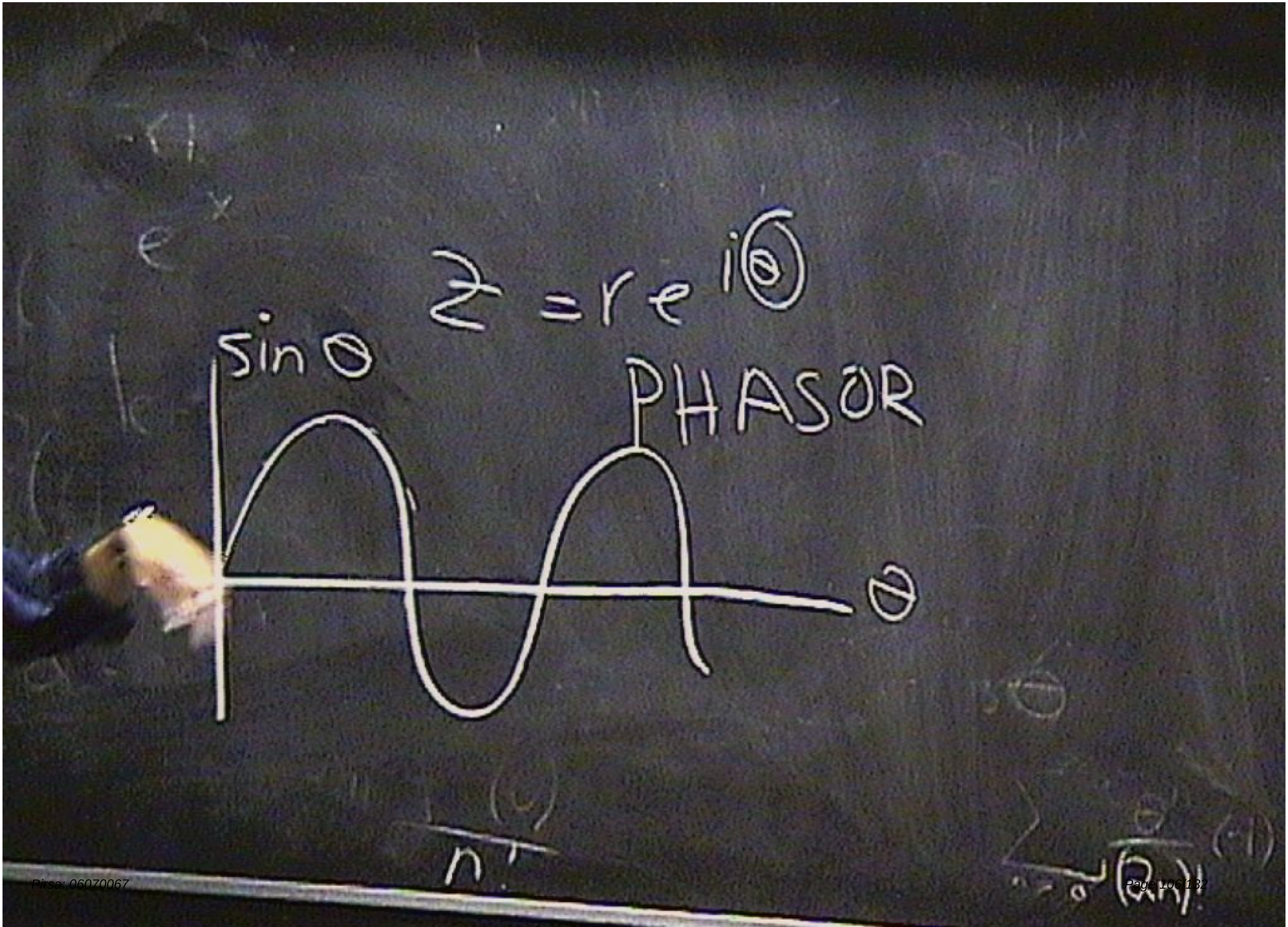
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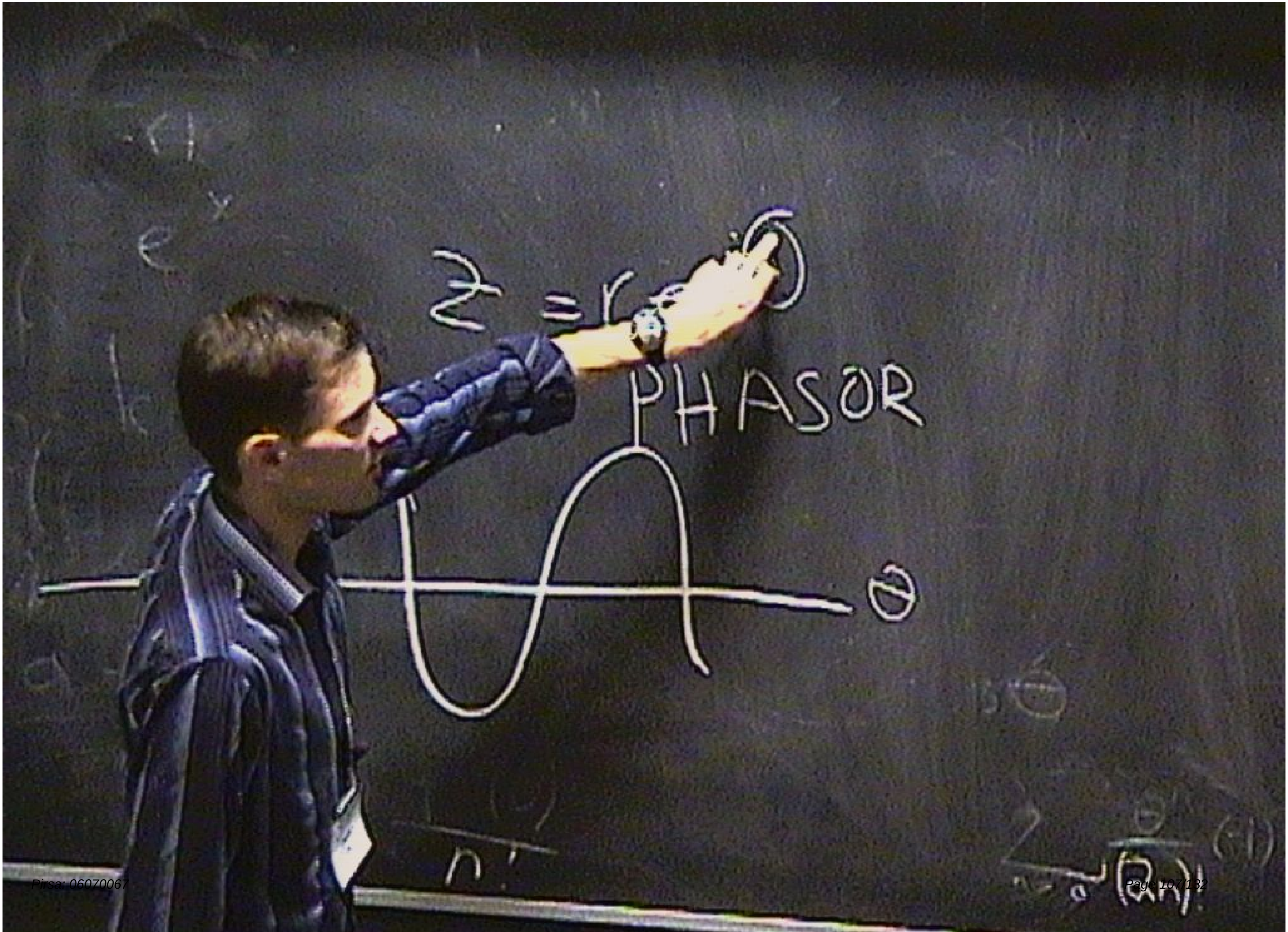
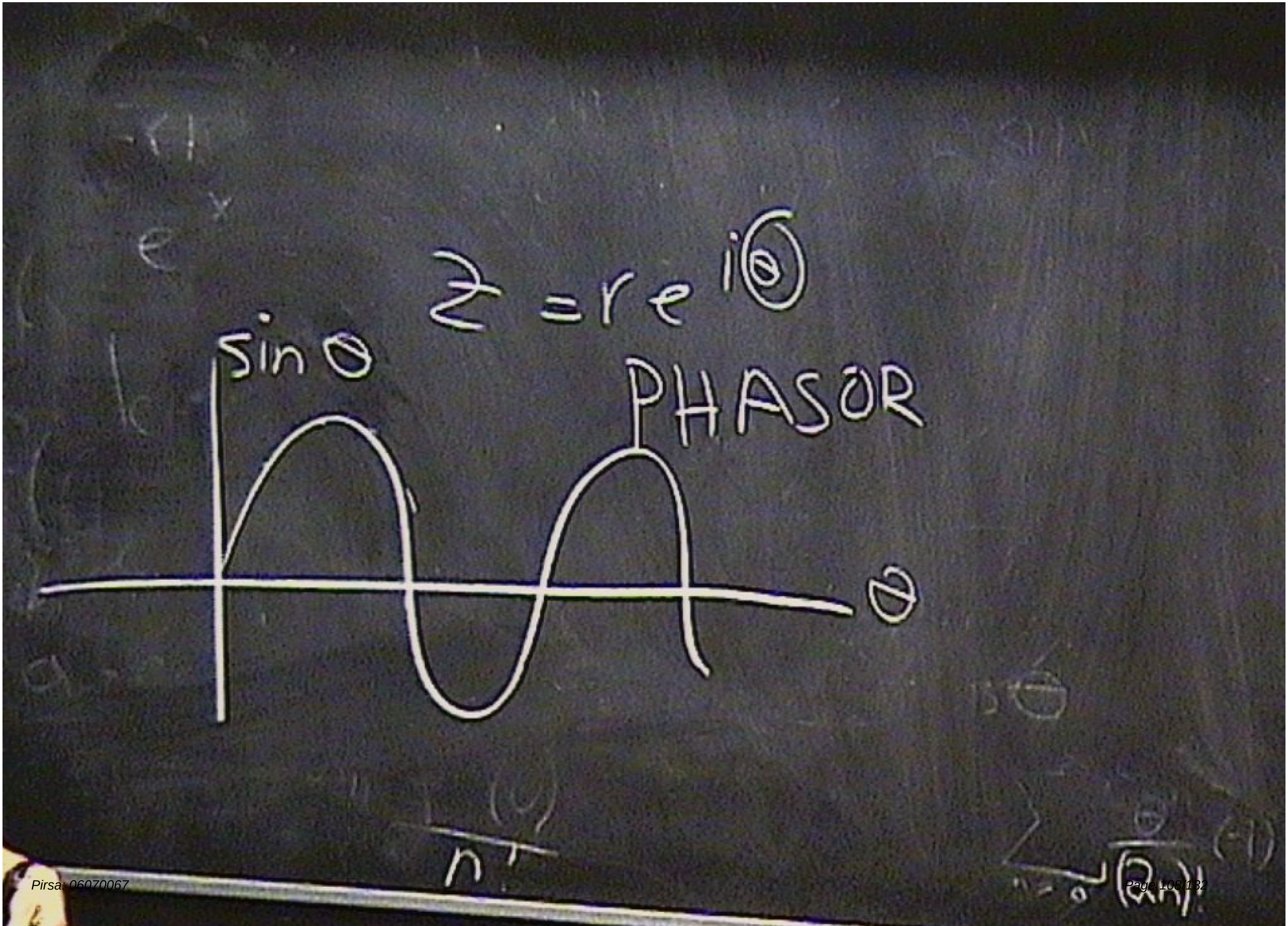

PHASOR
θ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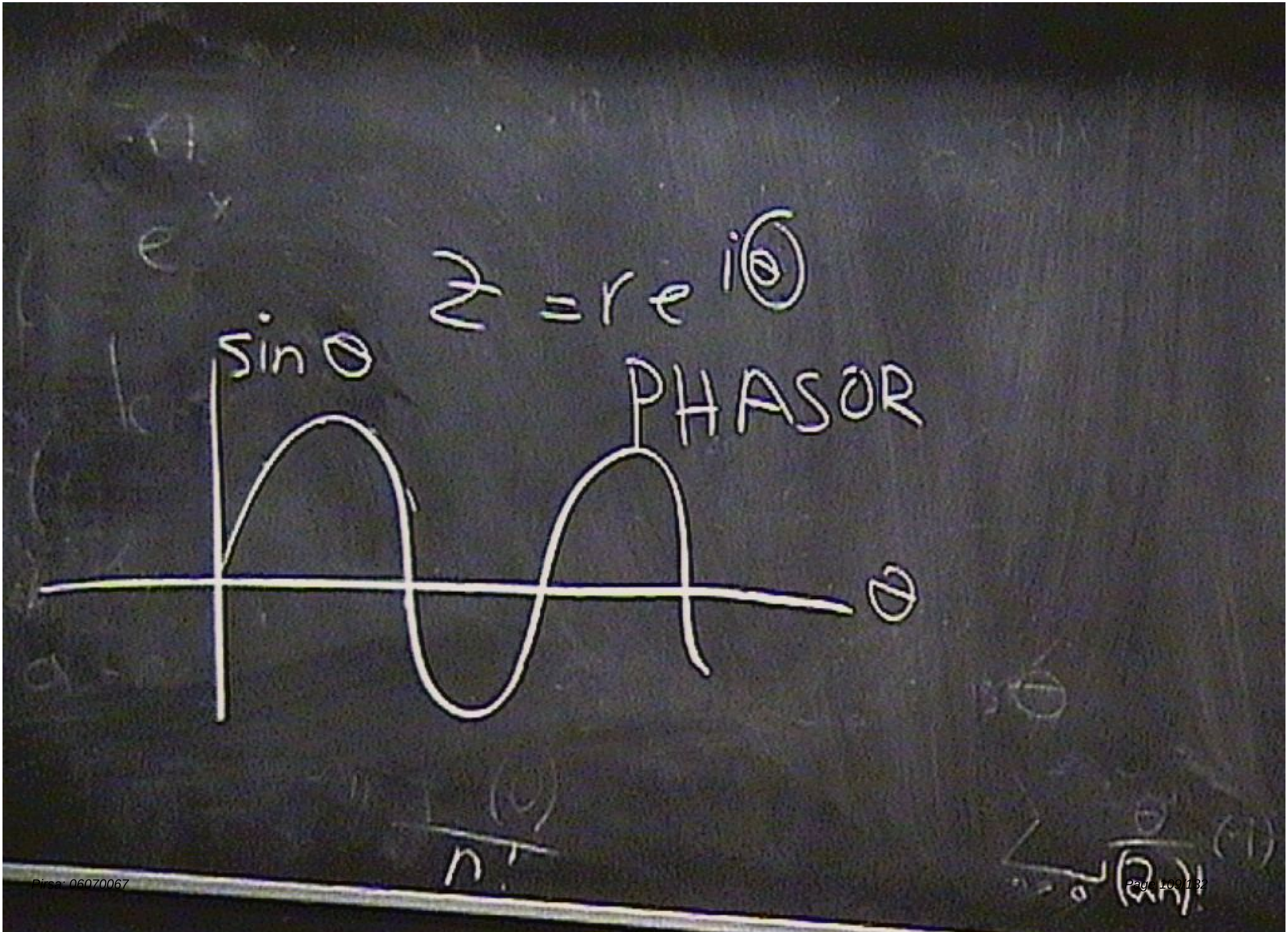
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PHASOR
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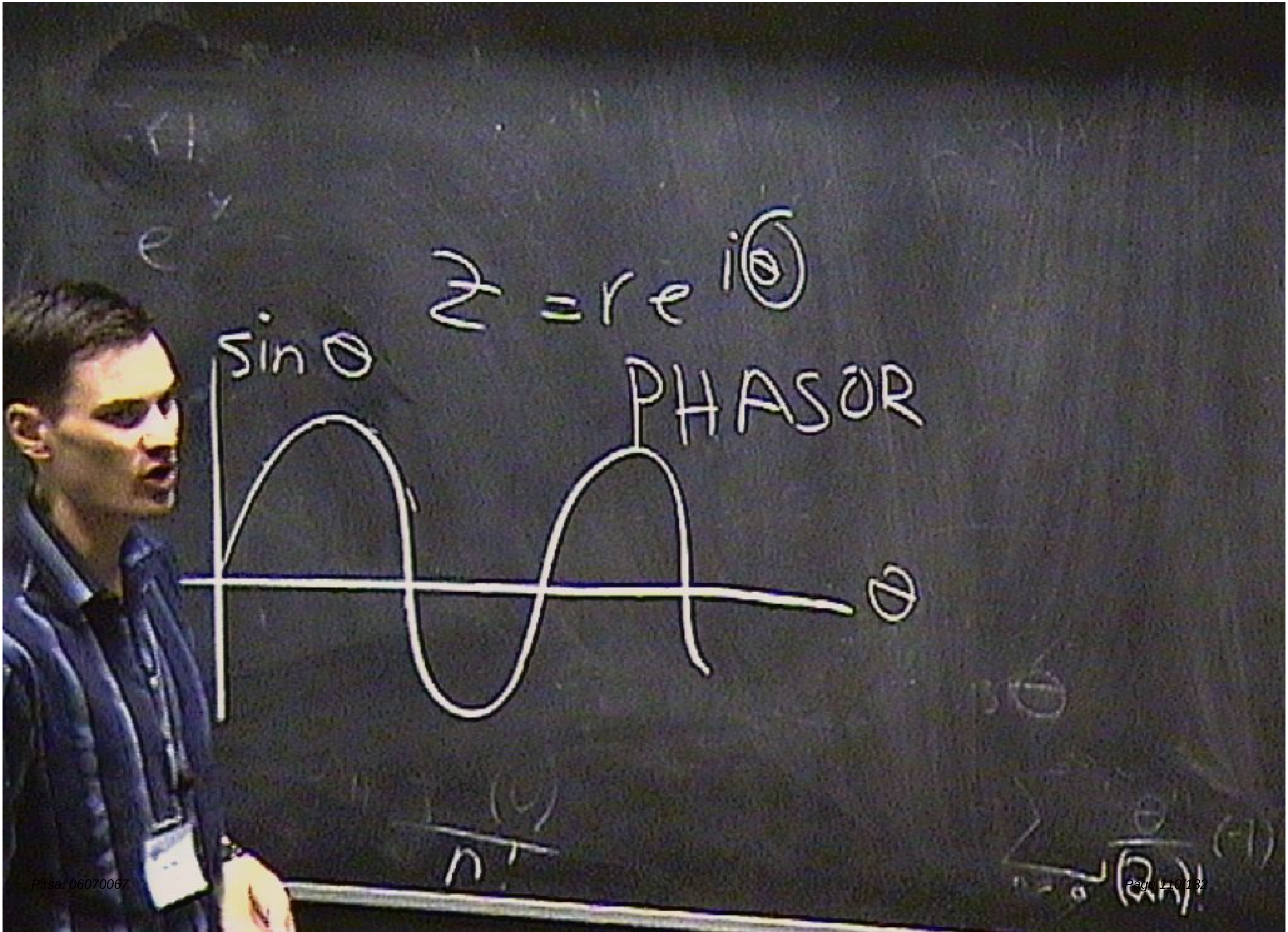
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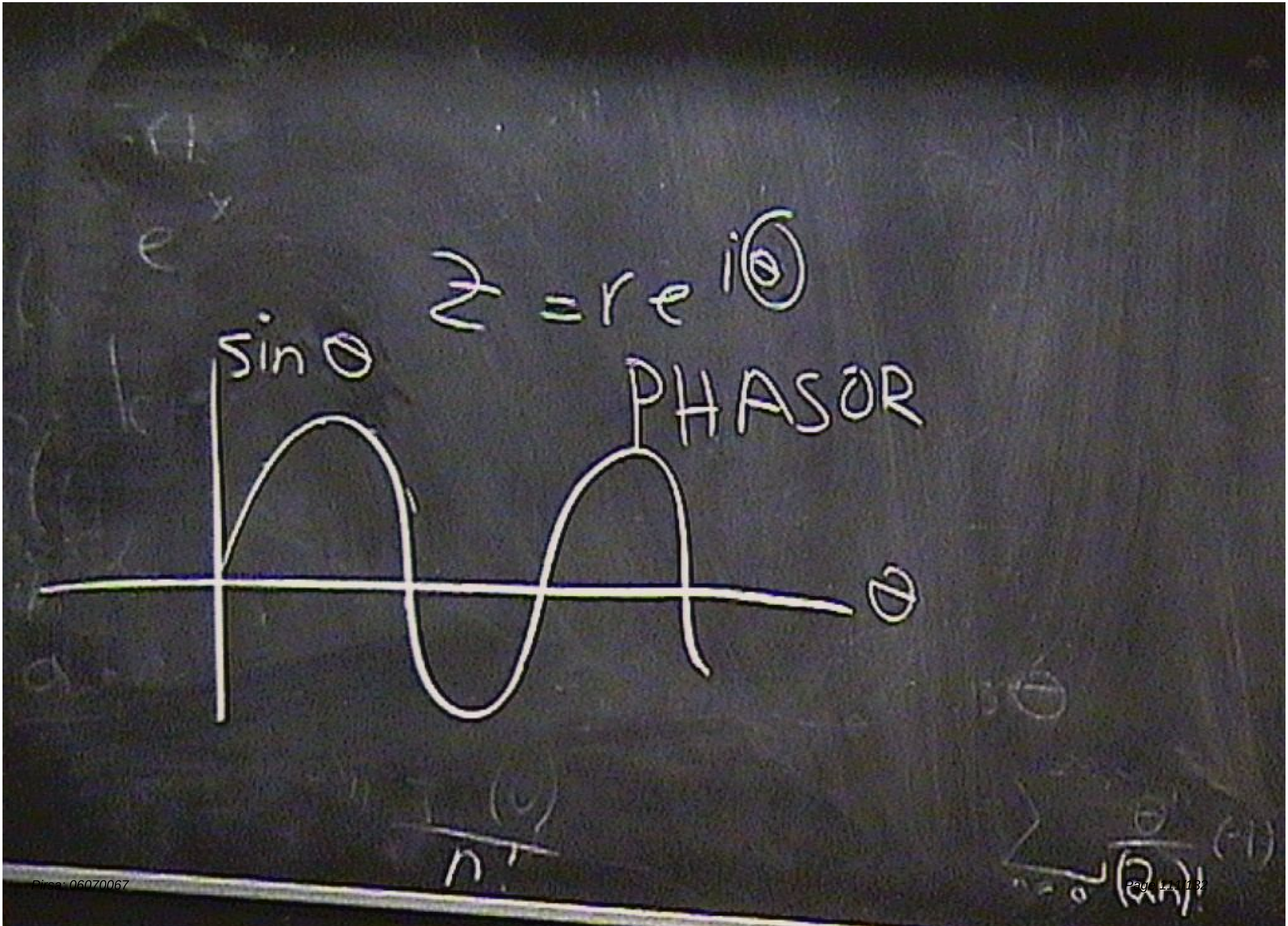
PHASOR
sin θ
θ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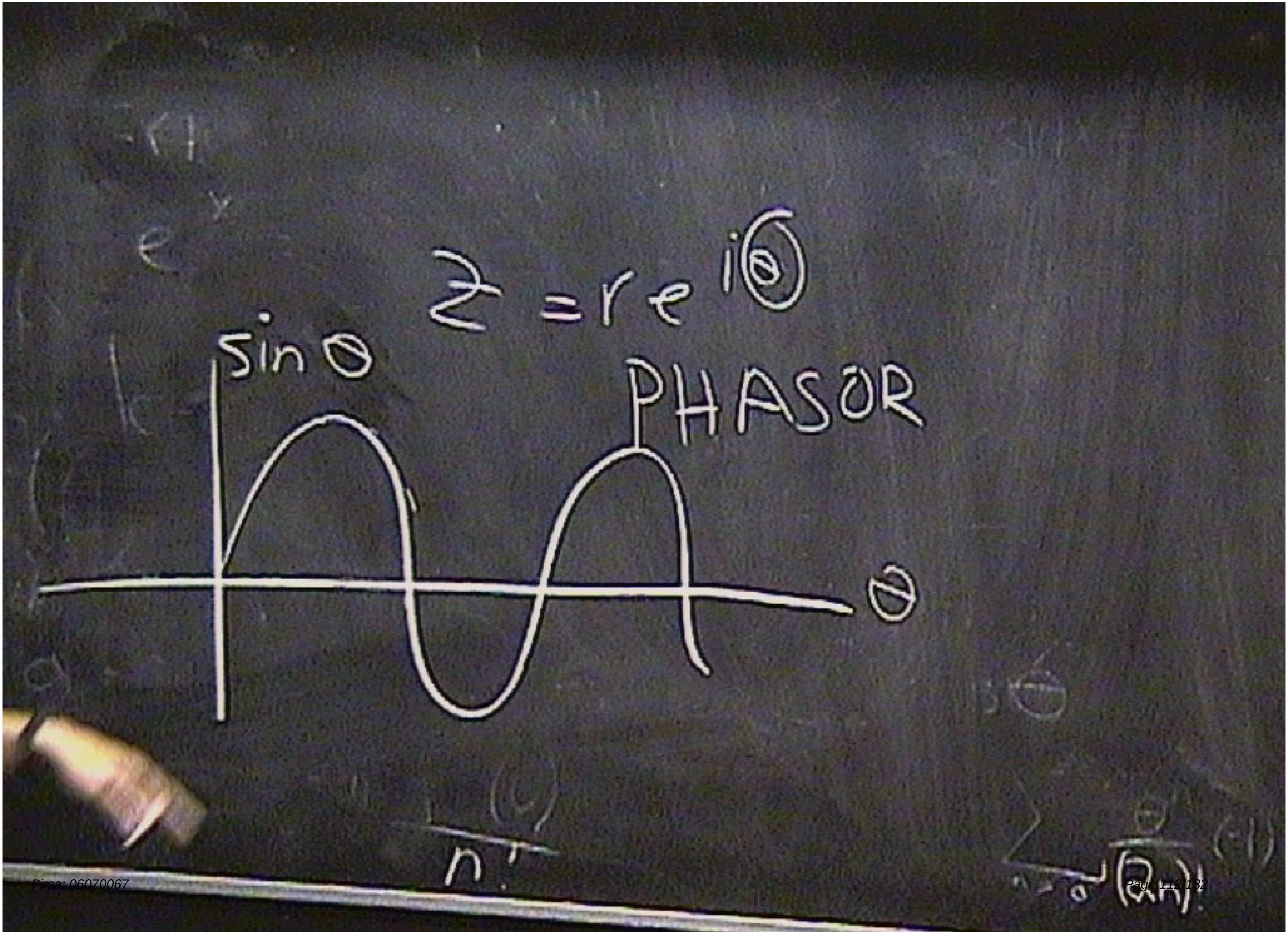
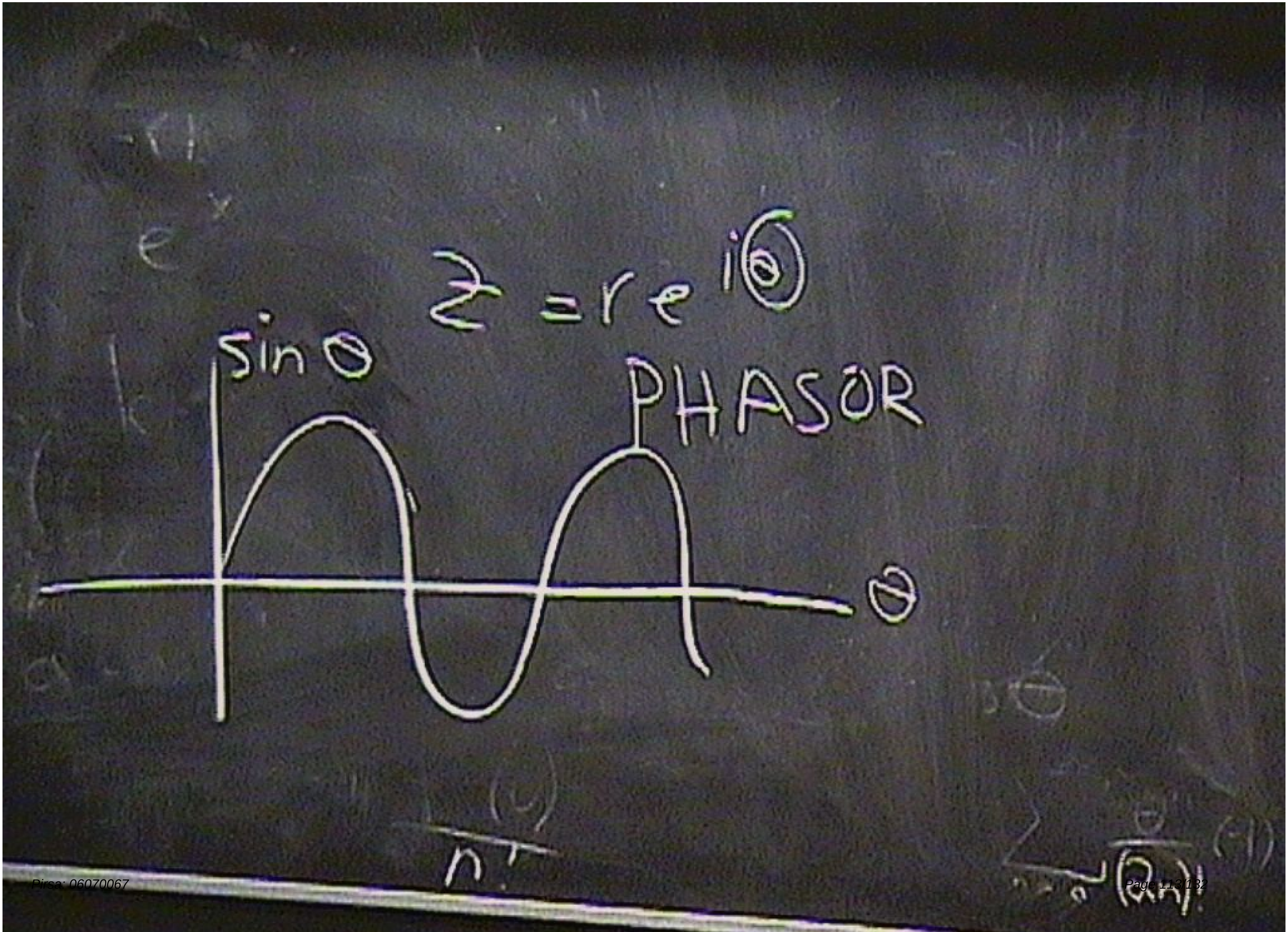
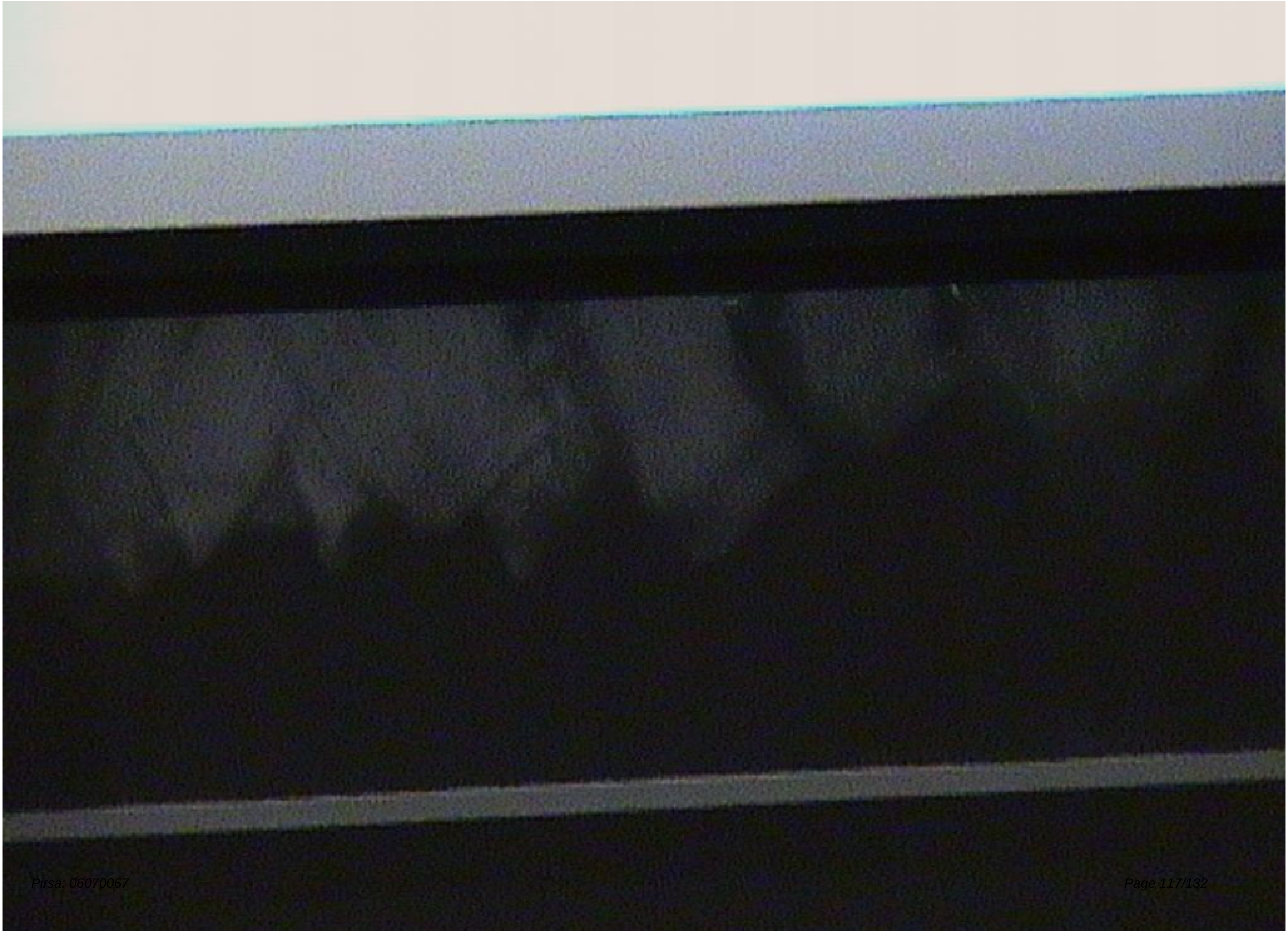
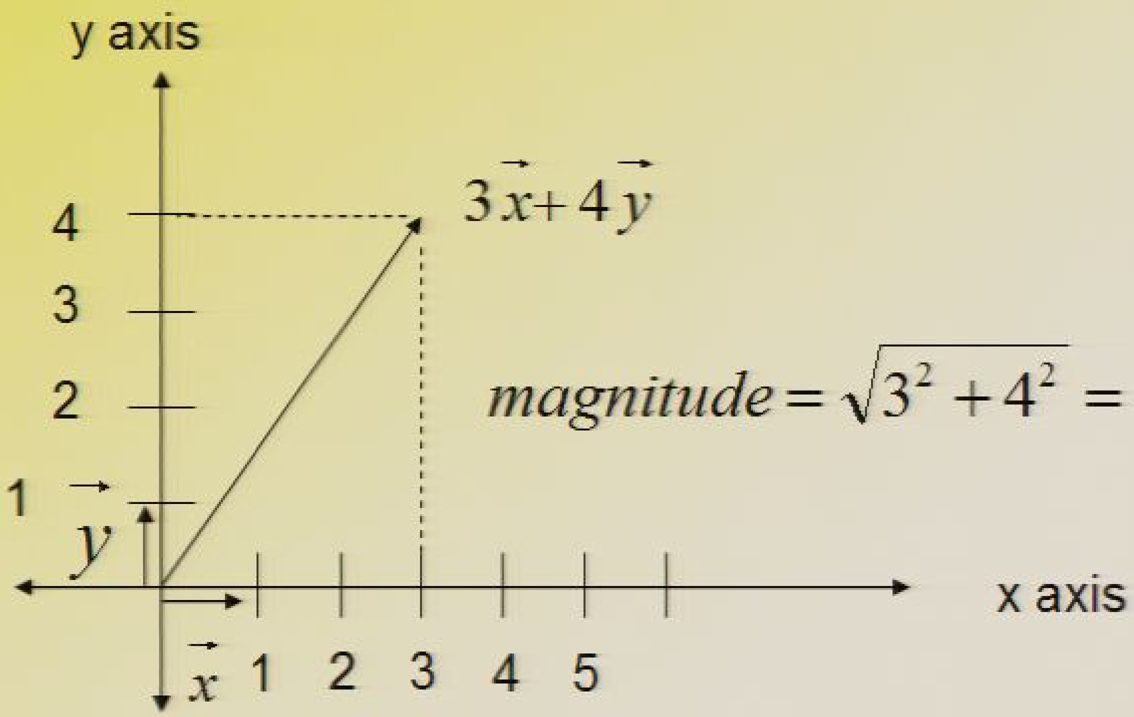

z = re iθ
sin θ
PHASOR
θ
n!
(2n)!


z = re^{iθ}
sin θ
PHASOR
θ
n!

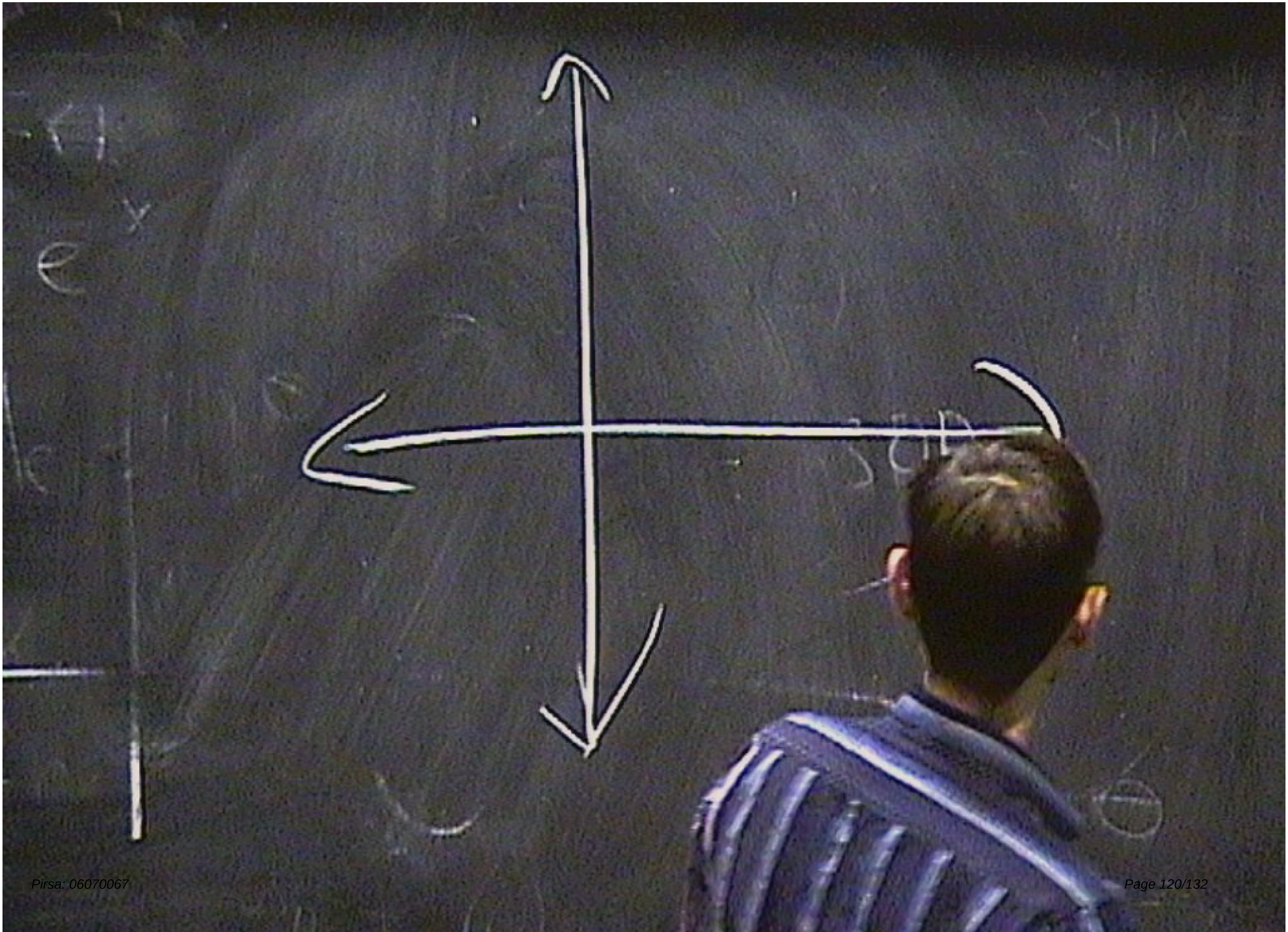


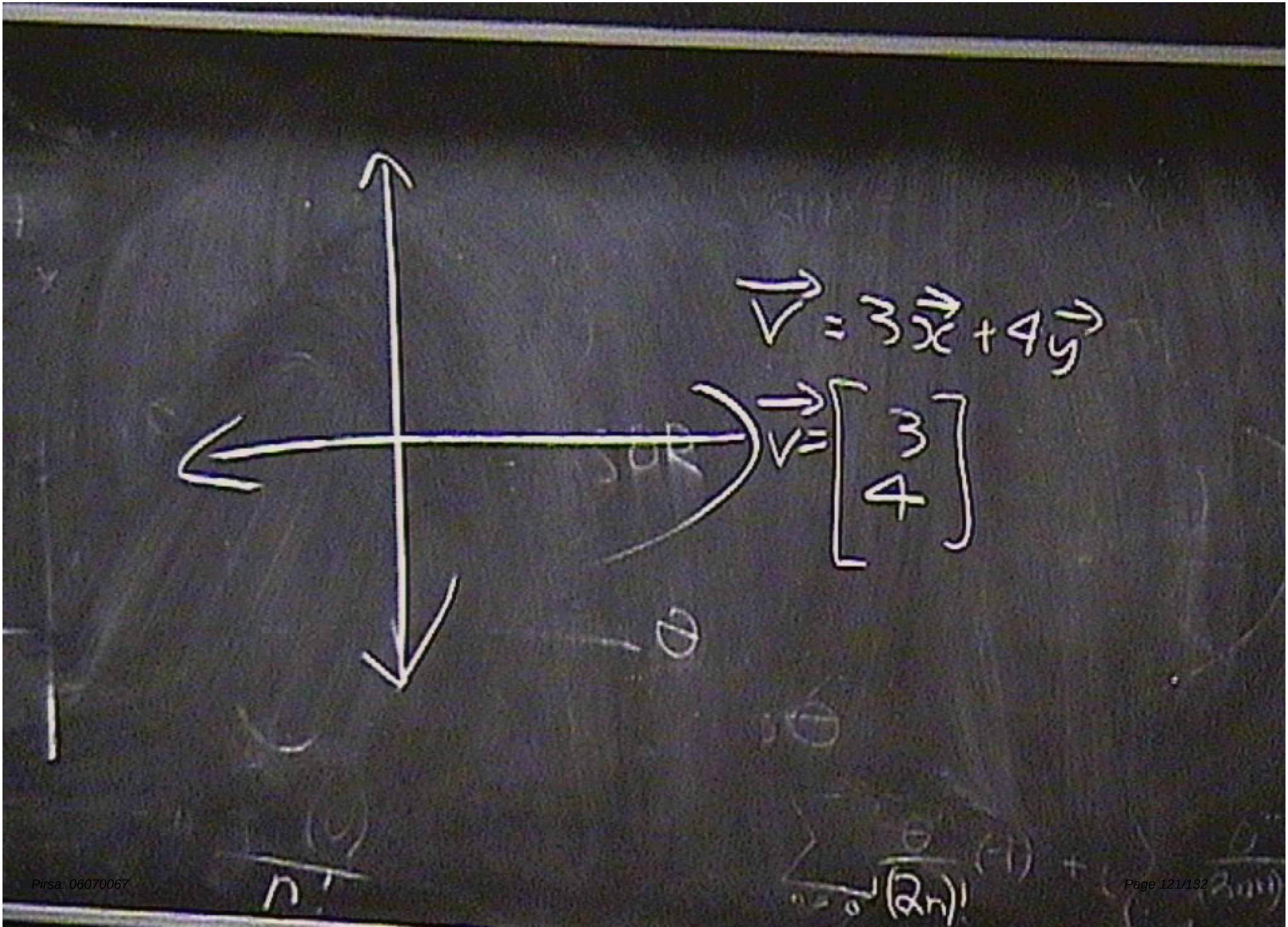
Absolute value

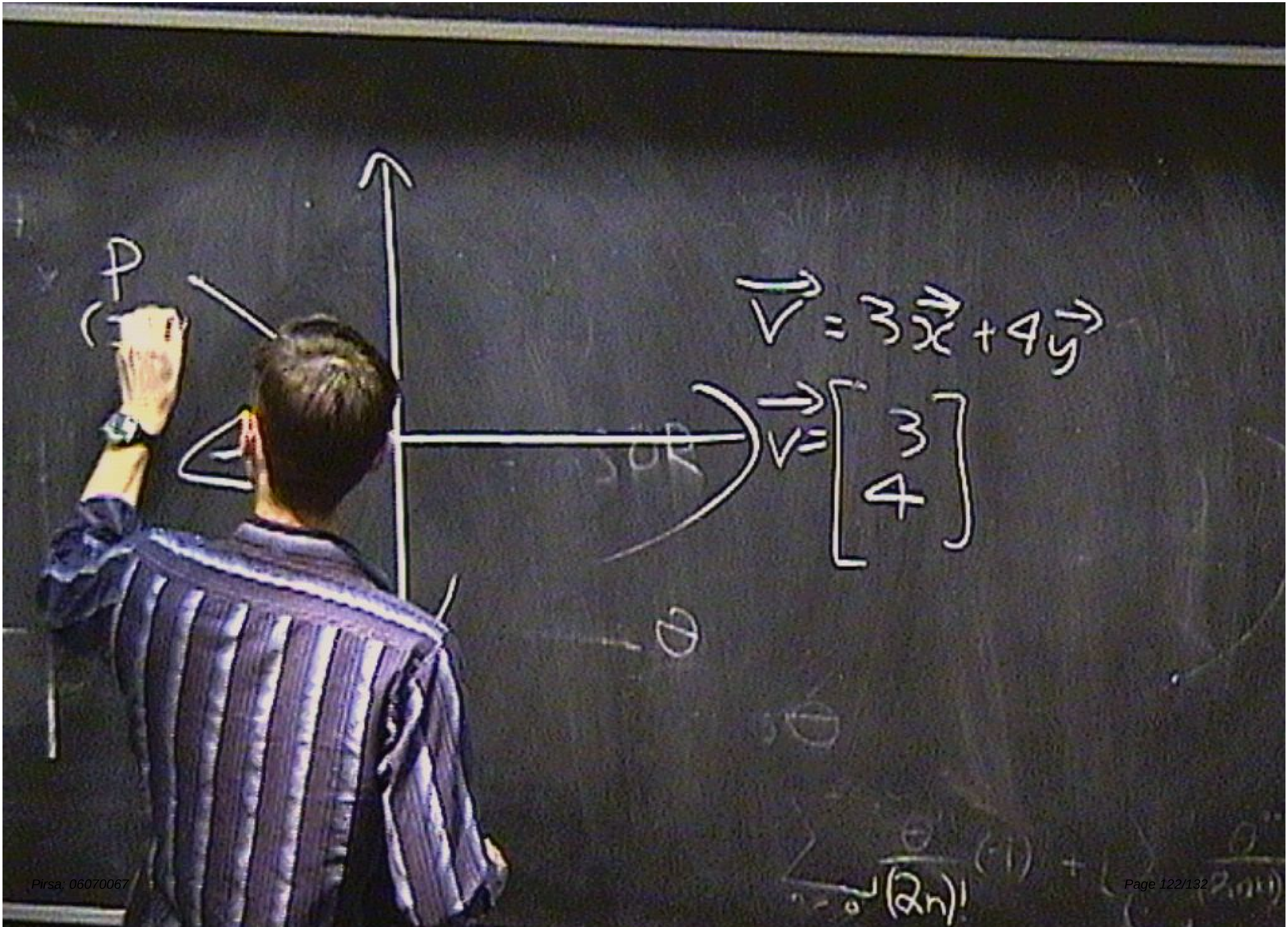
- also known as magnitude.
- Compare with the magnitude of a vector.

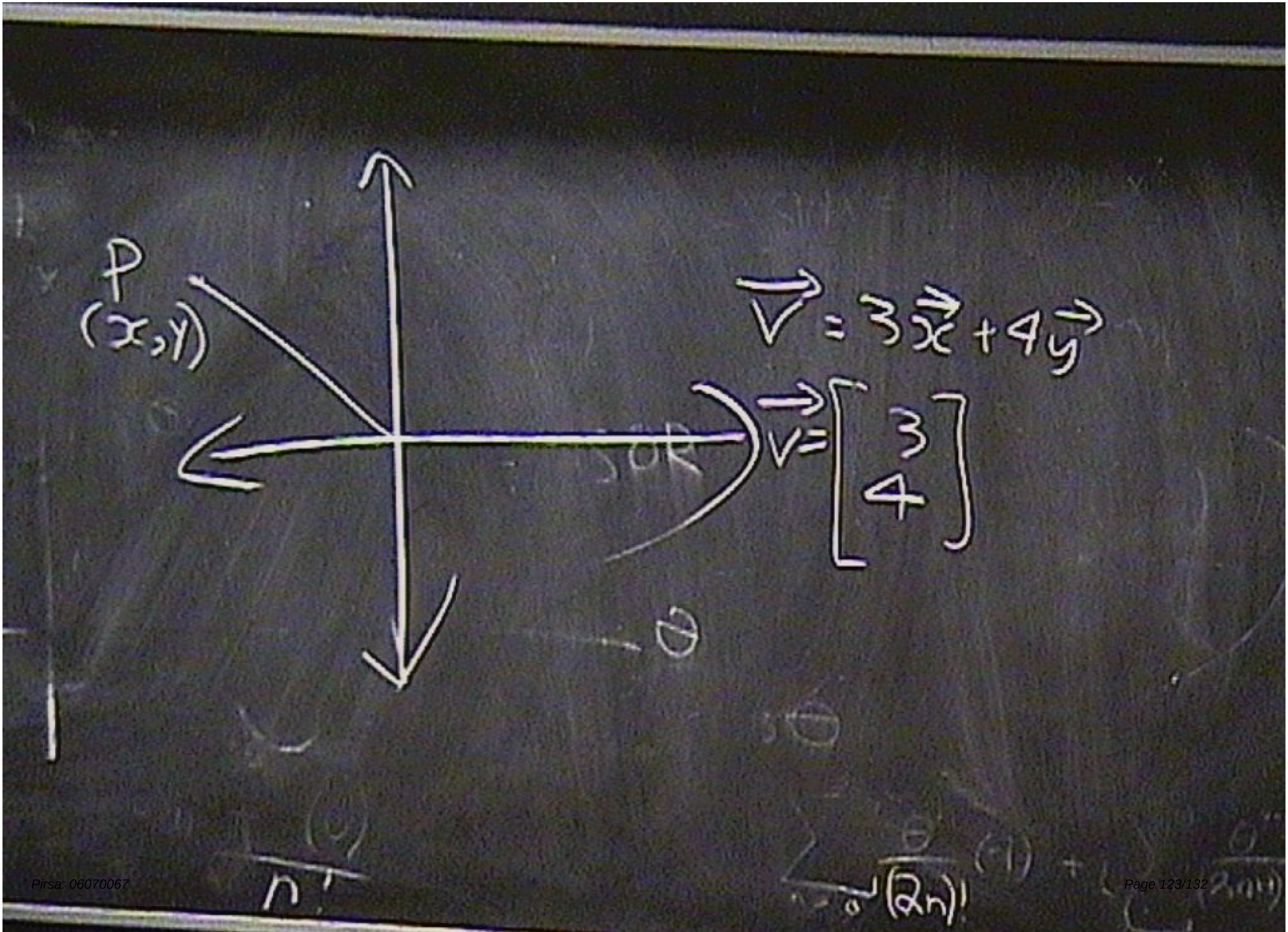


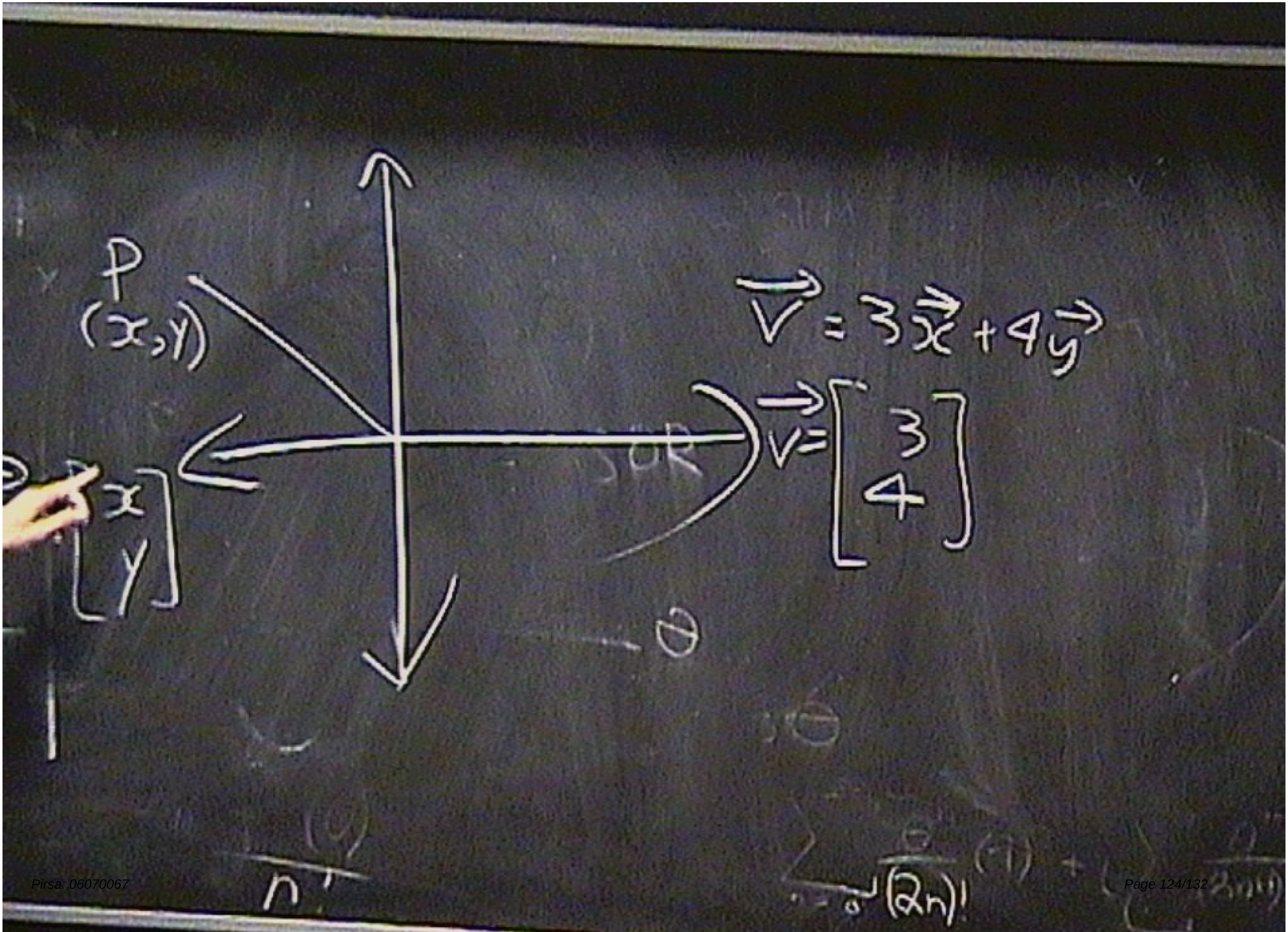
$$magnitude = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

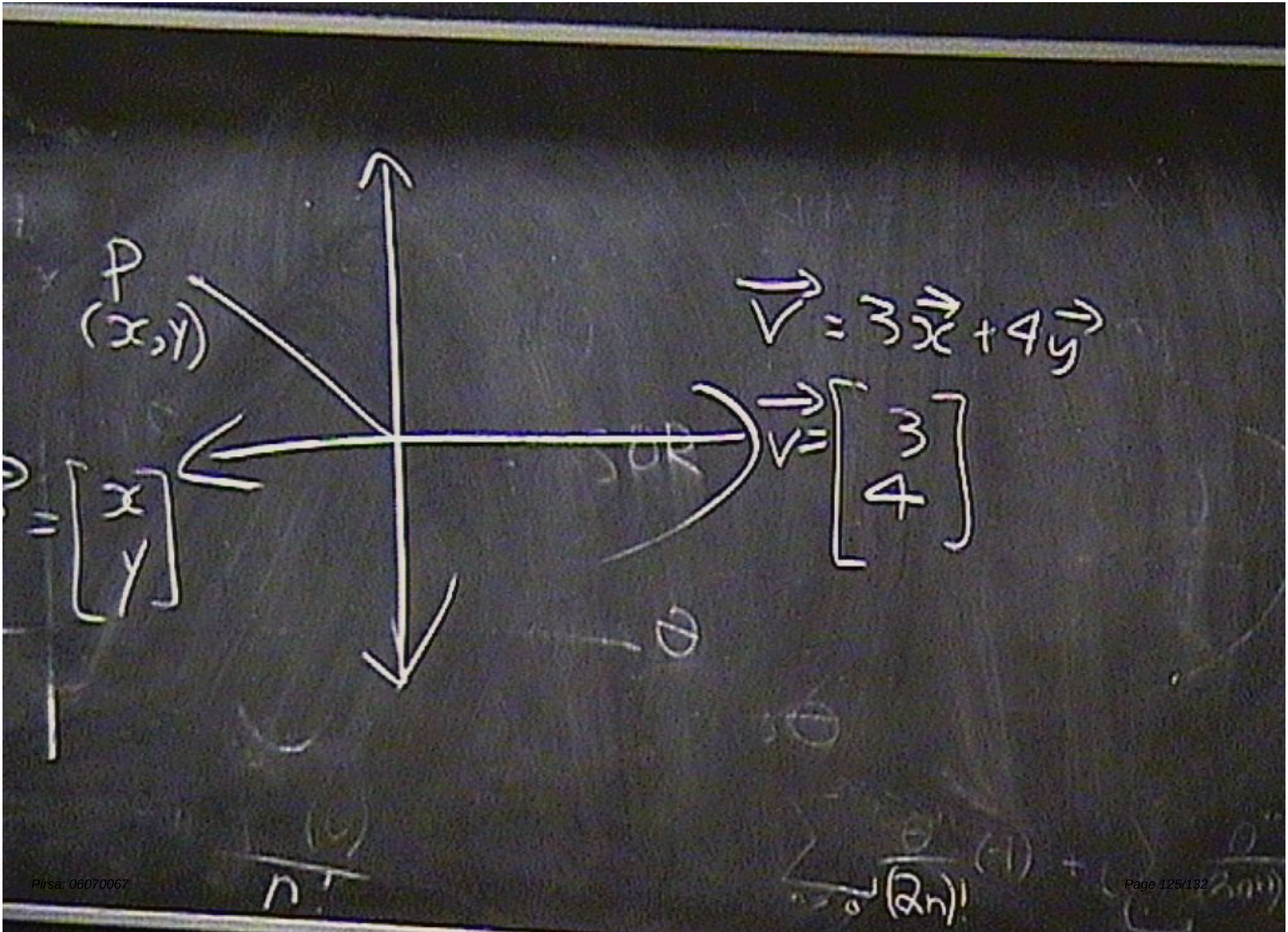



$\vec{v} = 3\vec{x} + 4\vec{y}$
$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
θ
n!
(2n)!

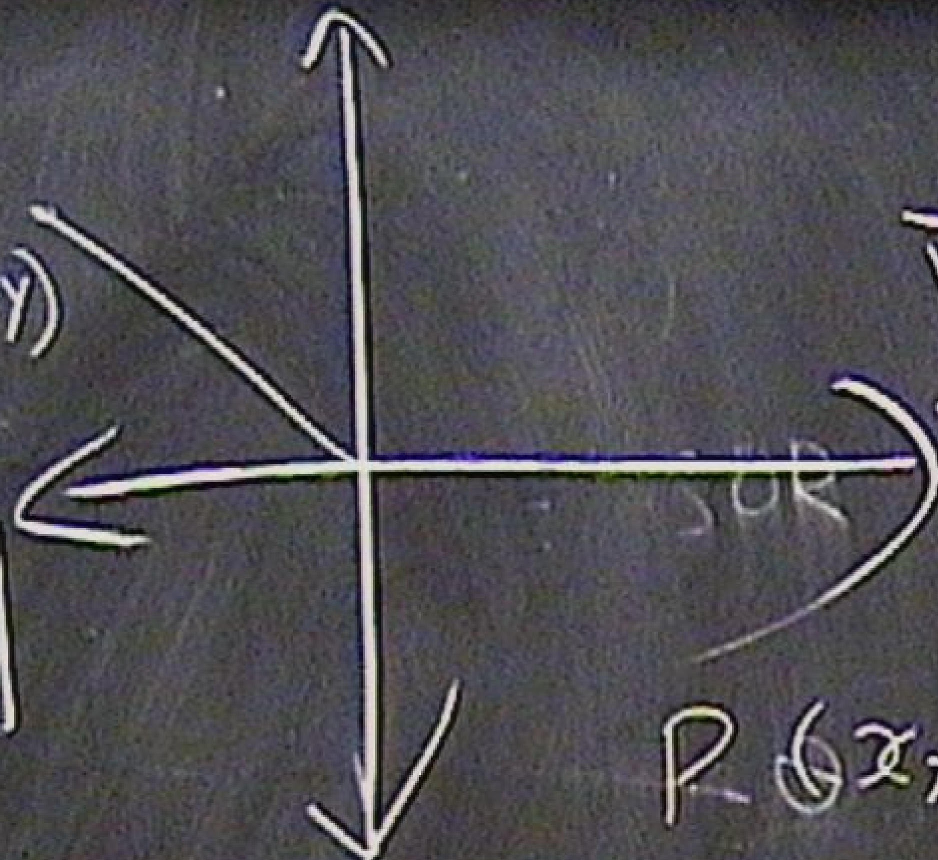

P
$\vec{V} = 3\vec{x} + 4\vec{y}$
$\vec{V} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
θ
$(2n)!$


P
(x,y)
$\vec{v} = 3\vec{x} + 4\vec{y}$
$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
θ
n!
(2n)!


P
(x, y)
$\vec{V} = 3\vec{x} + 4\vec{y}$
$\vec{V} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
$\begin{bmatrix} x \\ y \end{bmatrix}$


P
(x,y)
$\vec{V} = 3\vec{x} + 4\vec{y}$
$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
$= \begin{bmatrix} x \\ y \end{bmatrix}$
θ
n!
(2n)!

P
(x, y)



$\vec{V} = 3\vec{x} + 4\vec{y}$

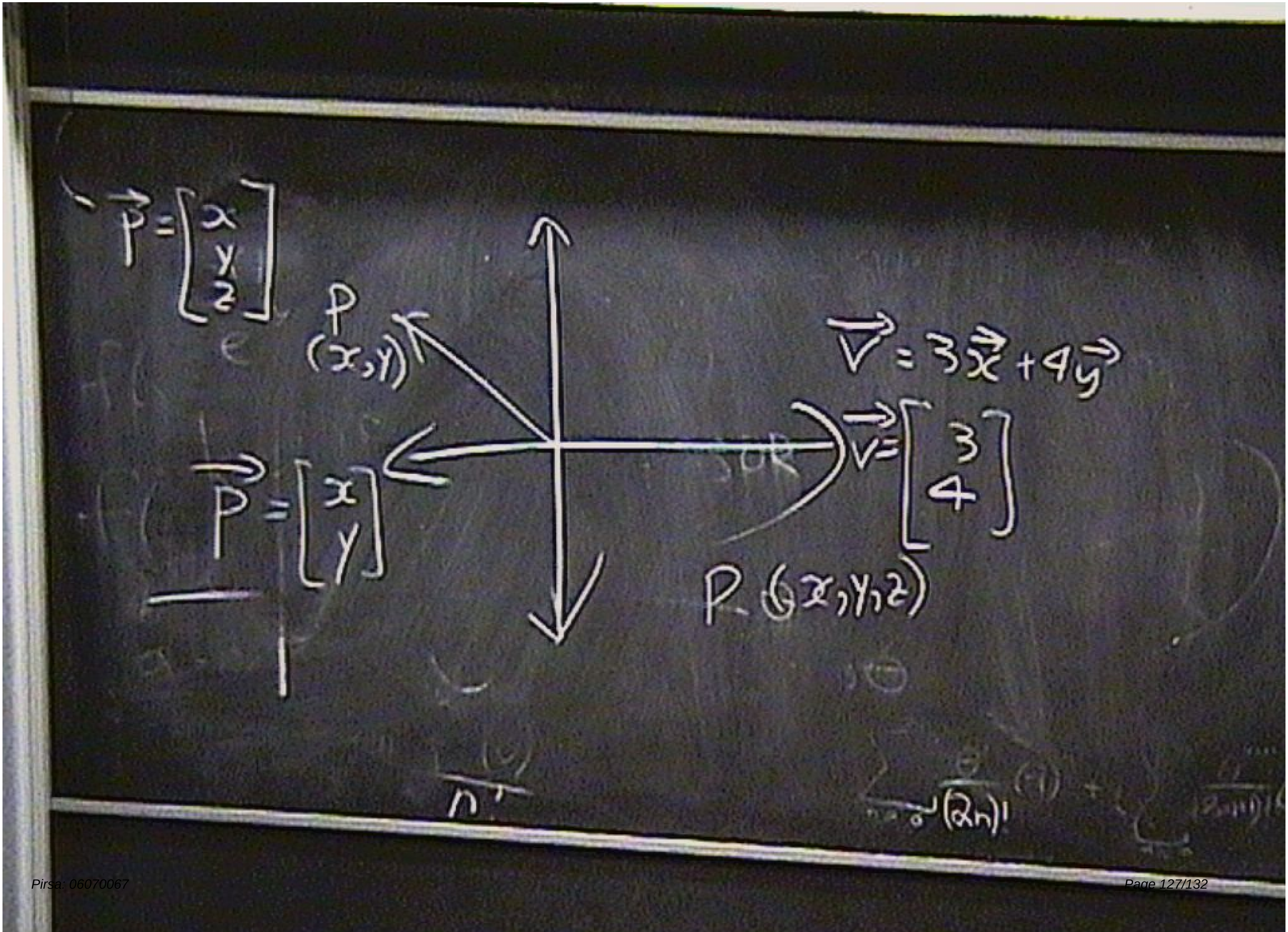
$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

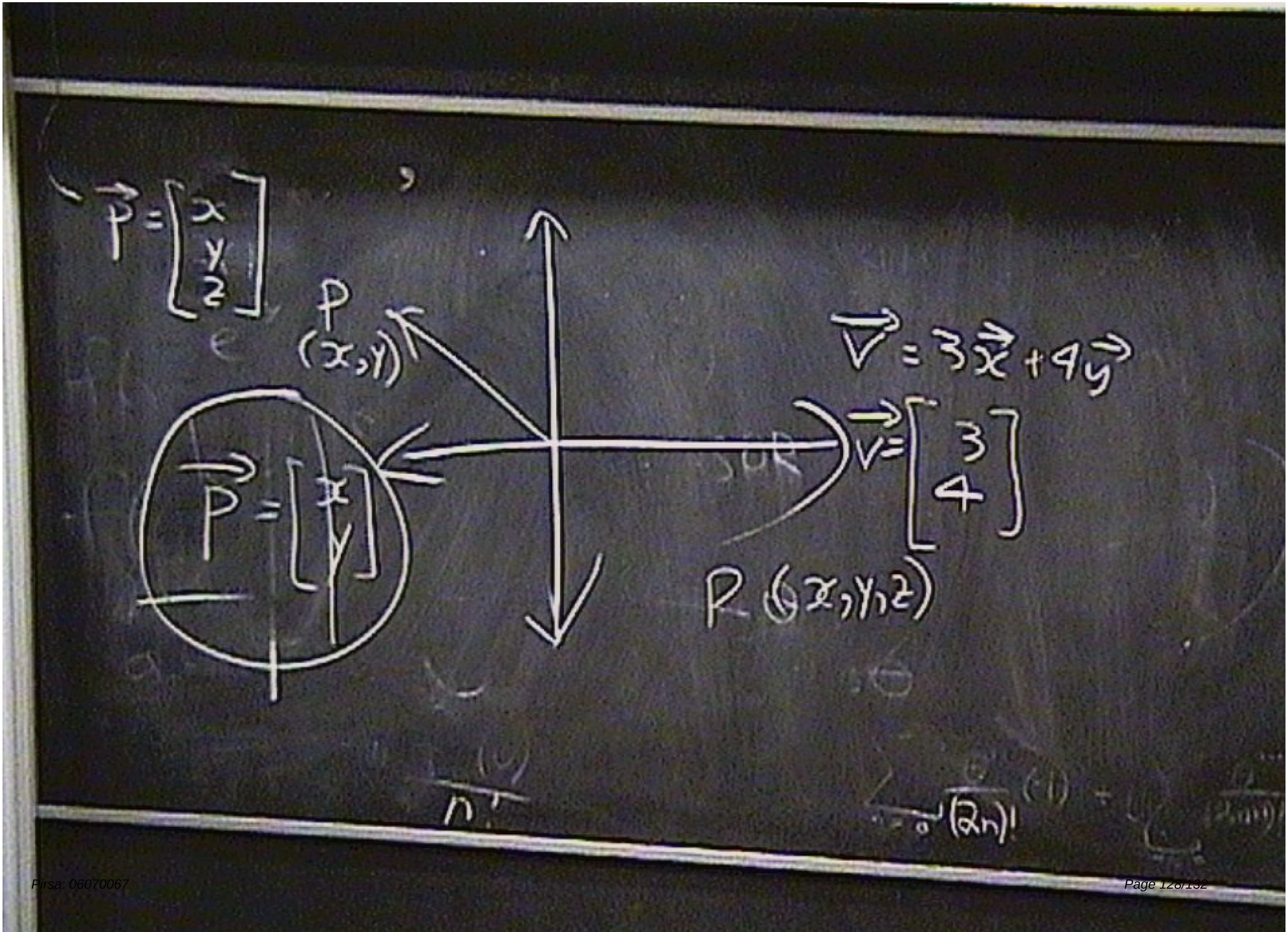
$= \begin{bmatrix} x \\ y \end{bmatrix}$

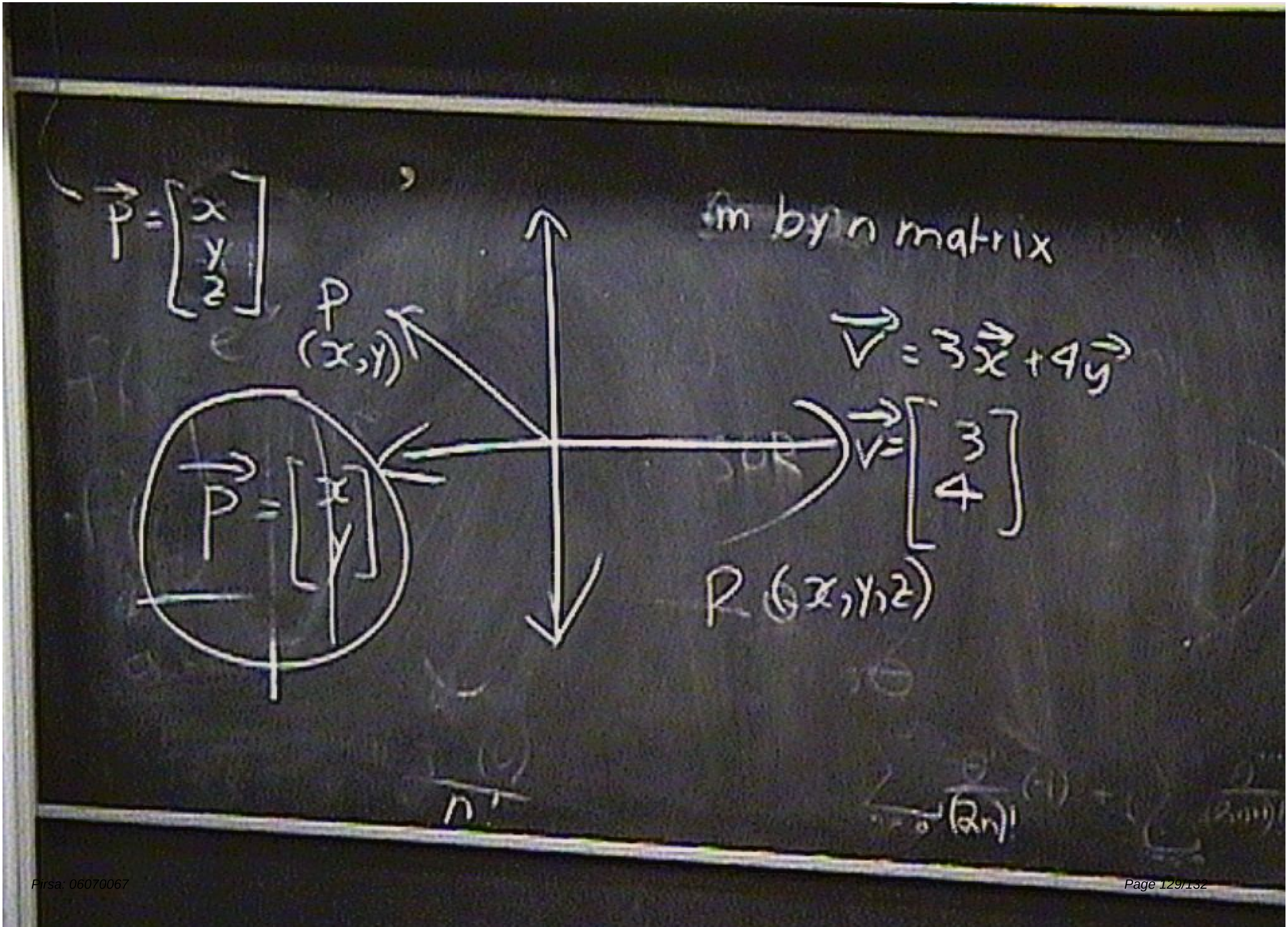
$P(x, y, z)$

$n!$

$(2n)!$


$\vec{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
P
(x, y)
$\vec{v} = 3\vec{x} + 4\vec{y}$
$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
$\vec{p} = \begin{bmatrix} x \\ y \end{bmatrix}$
P (x, y, z)


P = [x y z]
P (x,y)
V = 3x + 4y
V = [3 4]
P = [x y]
P (x,y,z)
n!
(2n)!



P = [x y z]
m by n matrix
P
(x,y)
V = 3x + 4y
v = [3 4]
P = [x y]
R (x,y,z)
n!
(2n)!

$\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

P

(x, y)

m by n matrix

2 by 1

2x1 $\vec{V} = 3\vec{x} + 4\vec{y}$

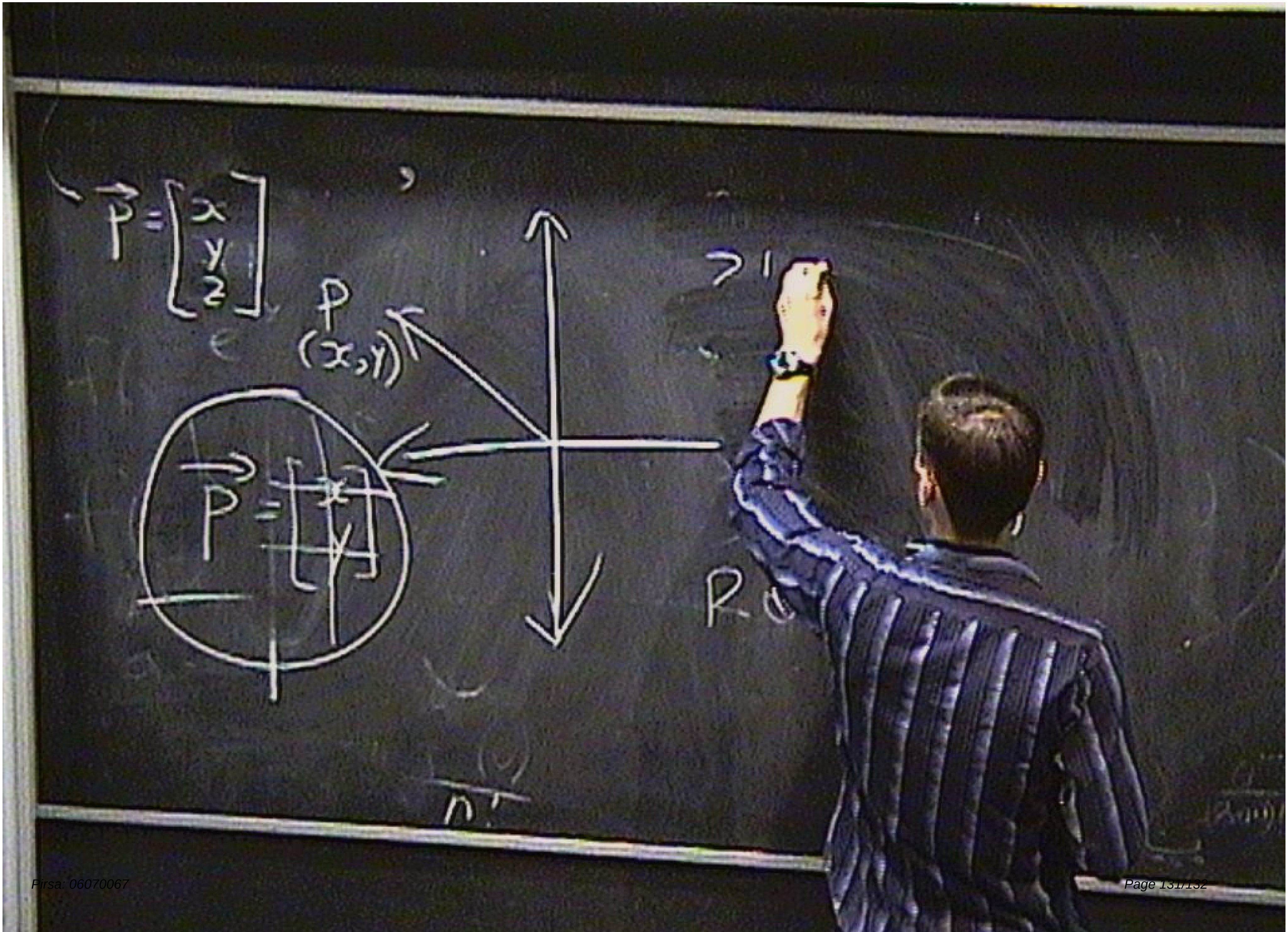
$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\vec{P} = \begin{bmatrix} x \\ y \end{bmatrix}$

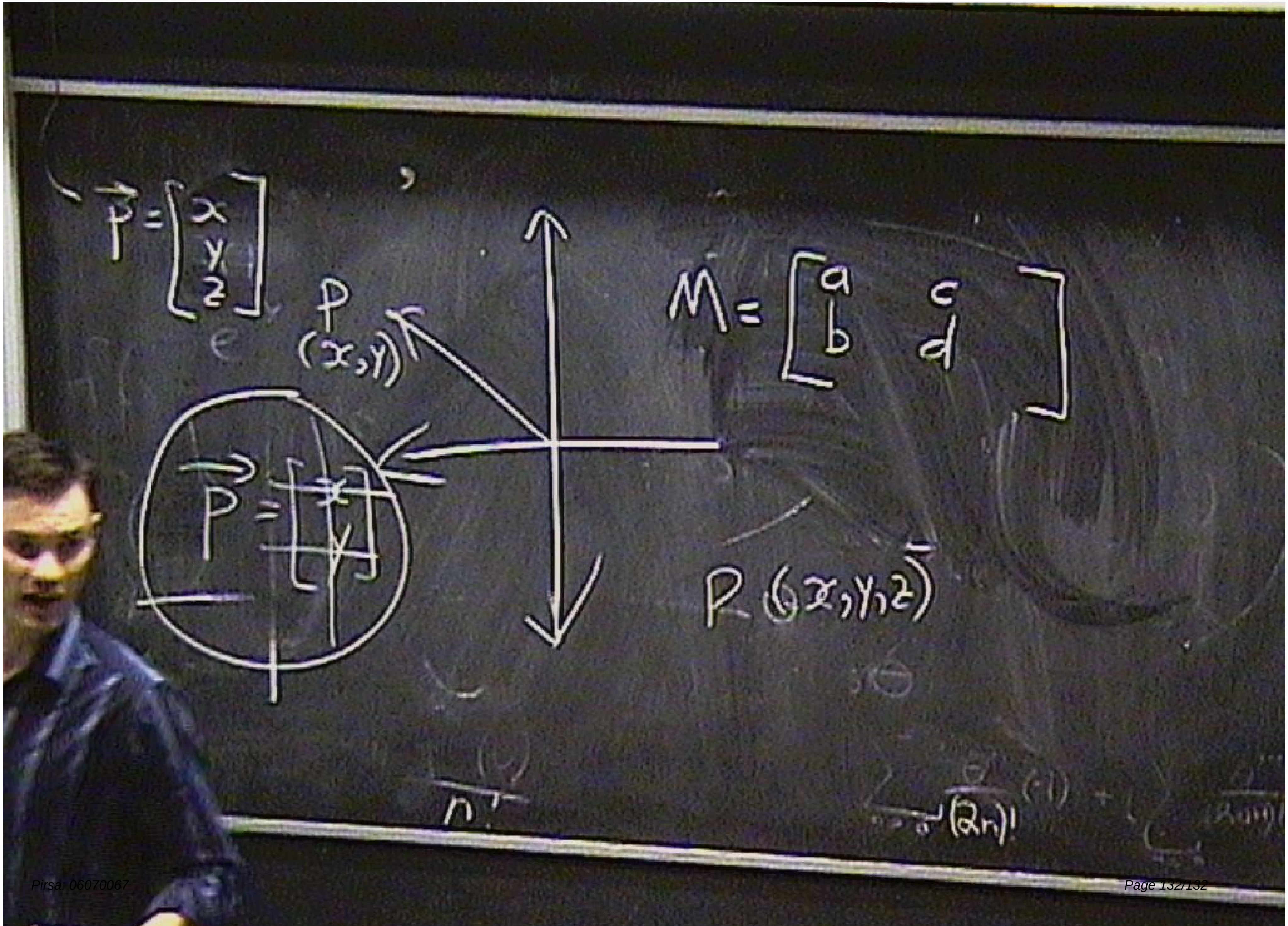
P (x, y, z)

n!

(2n)!



$\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
P
(x, y)
$\vec{P} = \begin{bmatrix} x \\ y \end{bmatrix}$
R


P = [x y z]
P (x,y)
P = [x y]
M = [a c b d]
P (x,y,z)