Title: Math Primer: Complex Numbers

Date: Jul 24, 2006 02:00 PM

URL: http://pirsa.org/06070066

Abstract:

Complex numbers

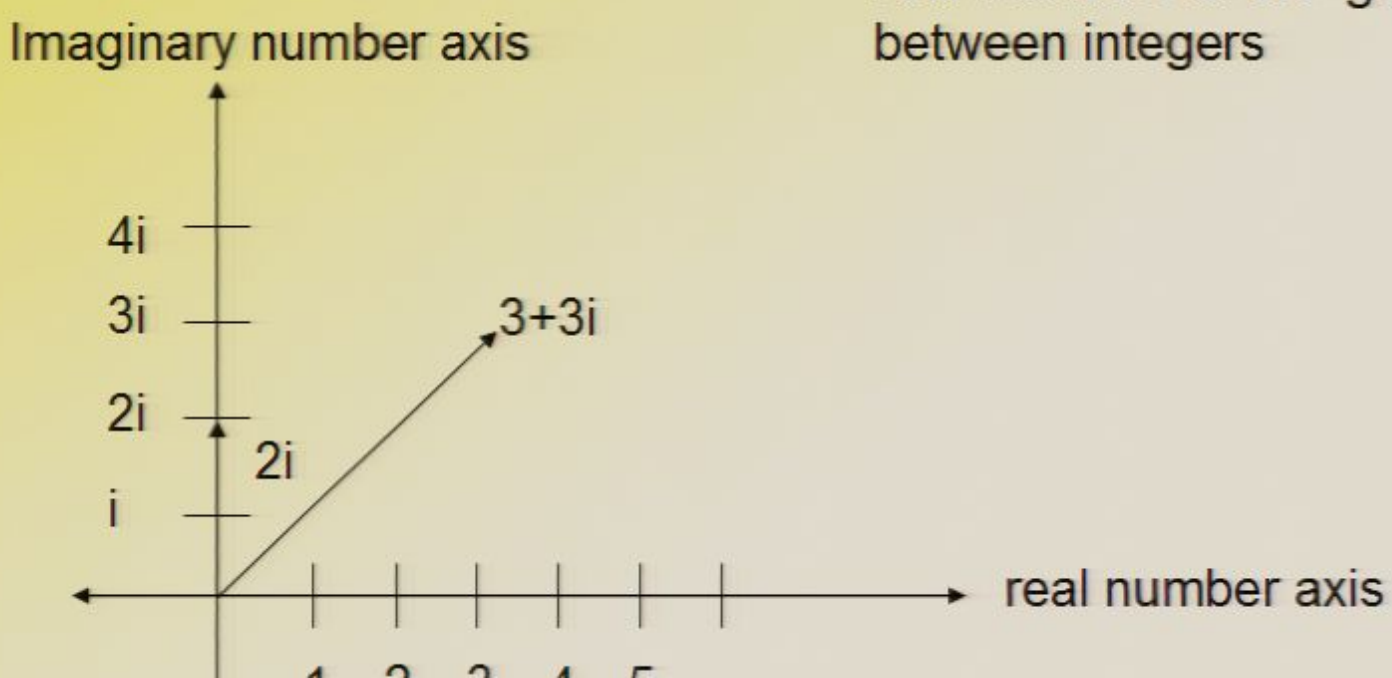
- integers -2,-1,0,1,2,3 …
- fractions 4/5, 1/10
- Real numbers: -4.53456, 3.14159
- Can we go any further? YES!

Complex numbers

- Another dimension of numbers. *Imaginary numbers*

- Incredibly useful in physics.

real numbers in the "gaps" between integers



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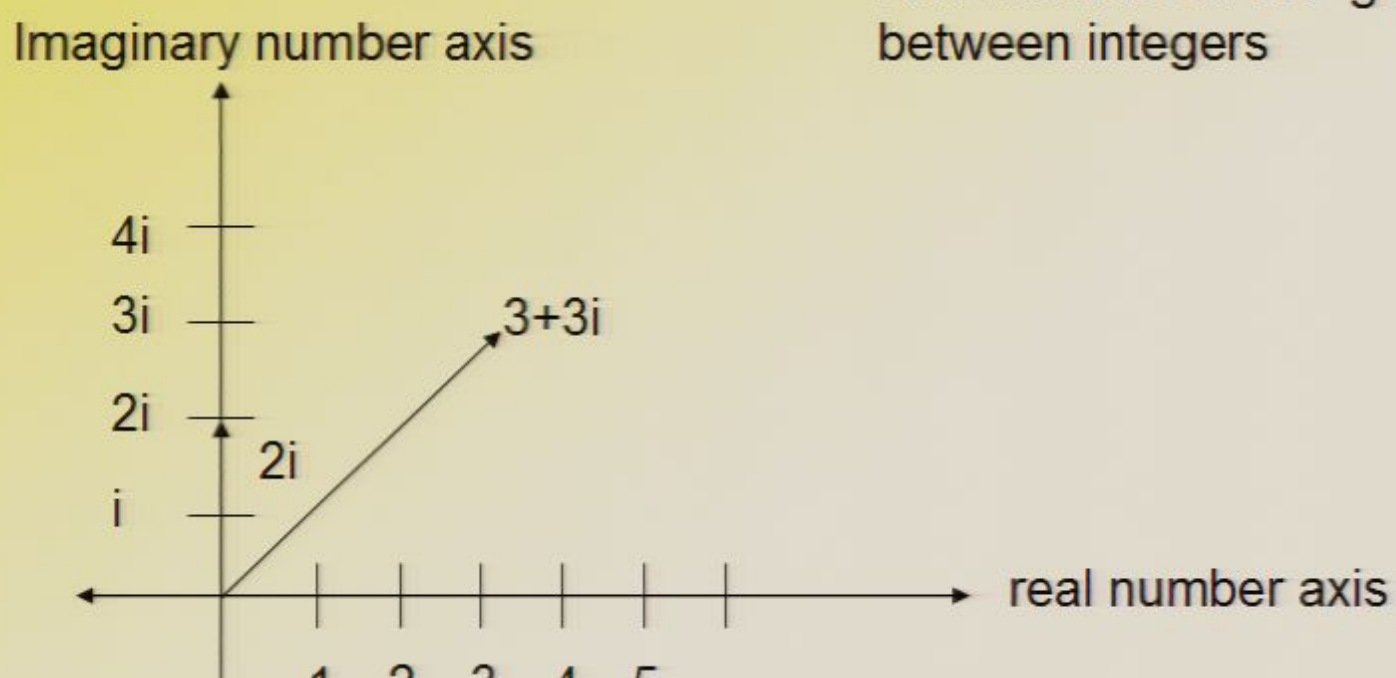

- What the heck is i?
- ANSWER: $i = \sqrt{-1}$ ← Definition of i
- But you can't take the square root of a negative number.
- If you're careful, you can. i is not a normal number. Imaginary dimension.

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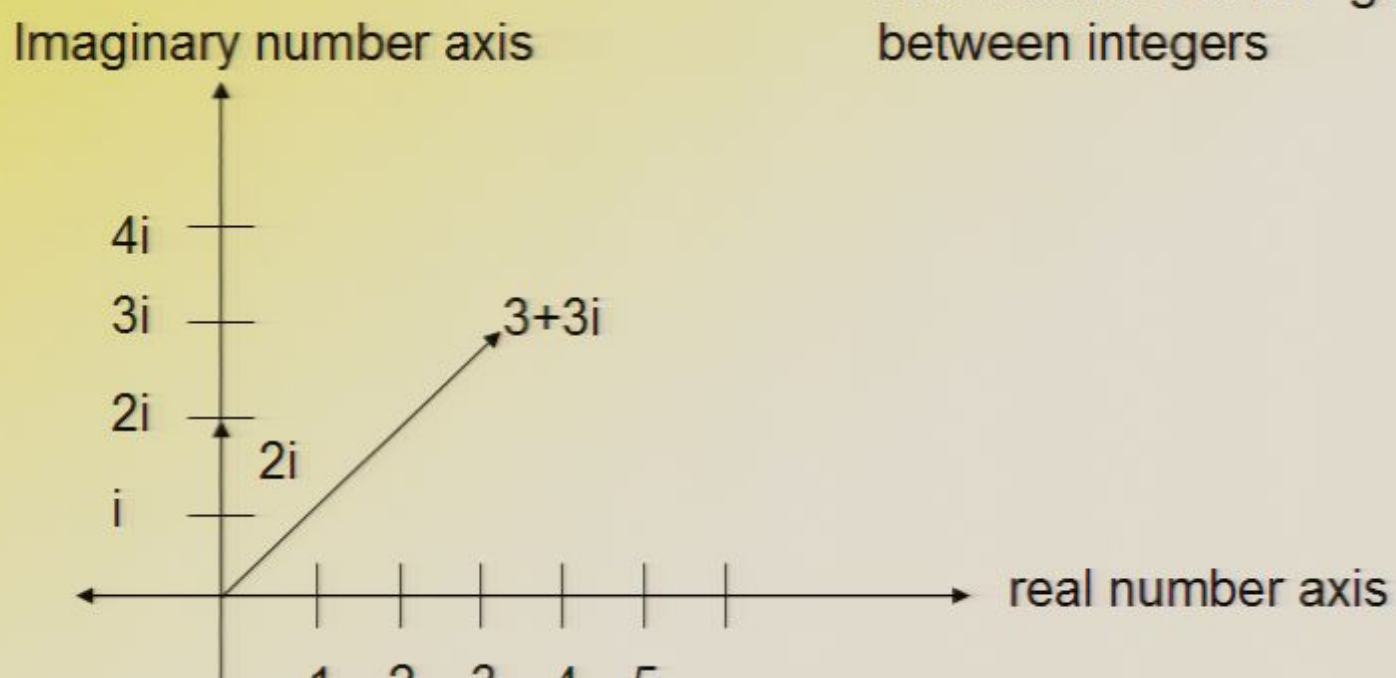

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Properties of i:

$$i^2 = i \times i = \sqrt{-1} \times \sqrt{-1} = -1$$

$$i \times -i = \sqrt{-1} \times (-\sqrt{-1}) = -(-1) = +1$$

numbers that are the sum of a real number and an imaginary numbers are called *complex numbers*.

z is the symbol we will use for complex numbers.

z = a + b i,
a is the *real component* and b is the *imaginary component*.
Eg. for z=3+4i, a=3 and b=4.

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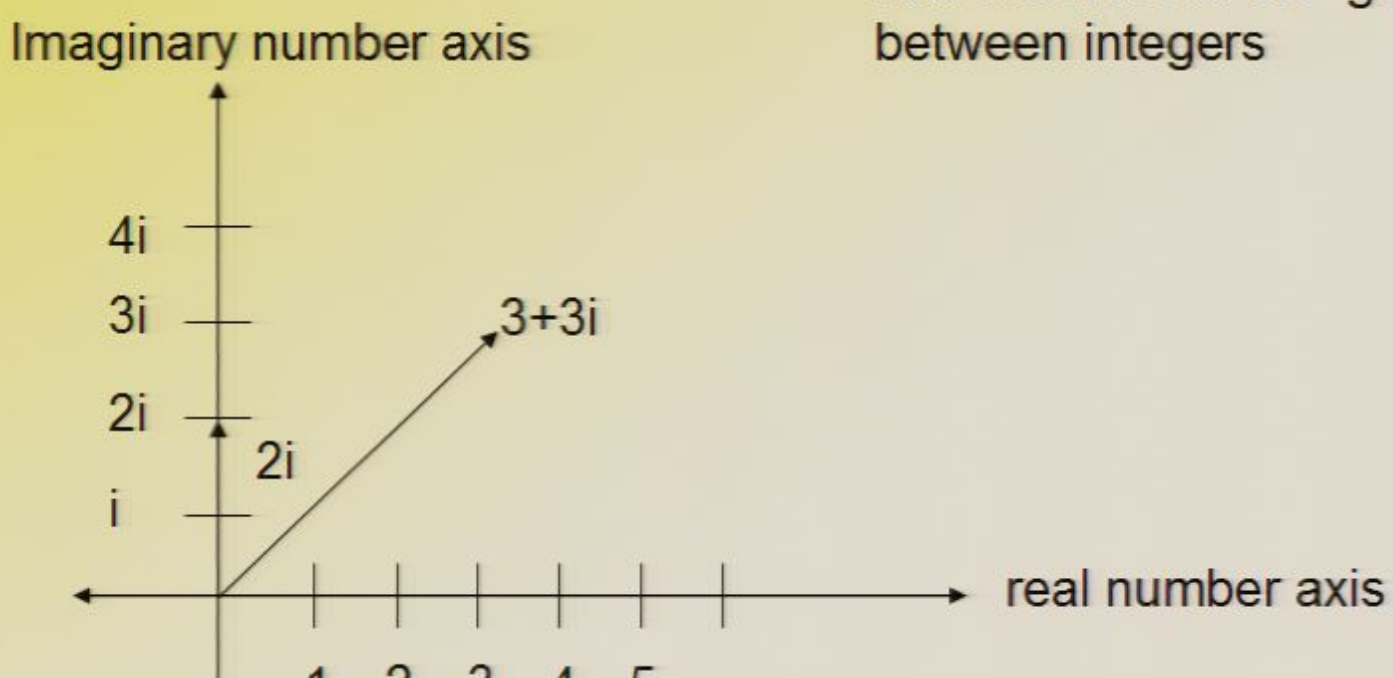

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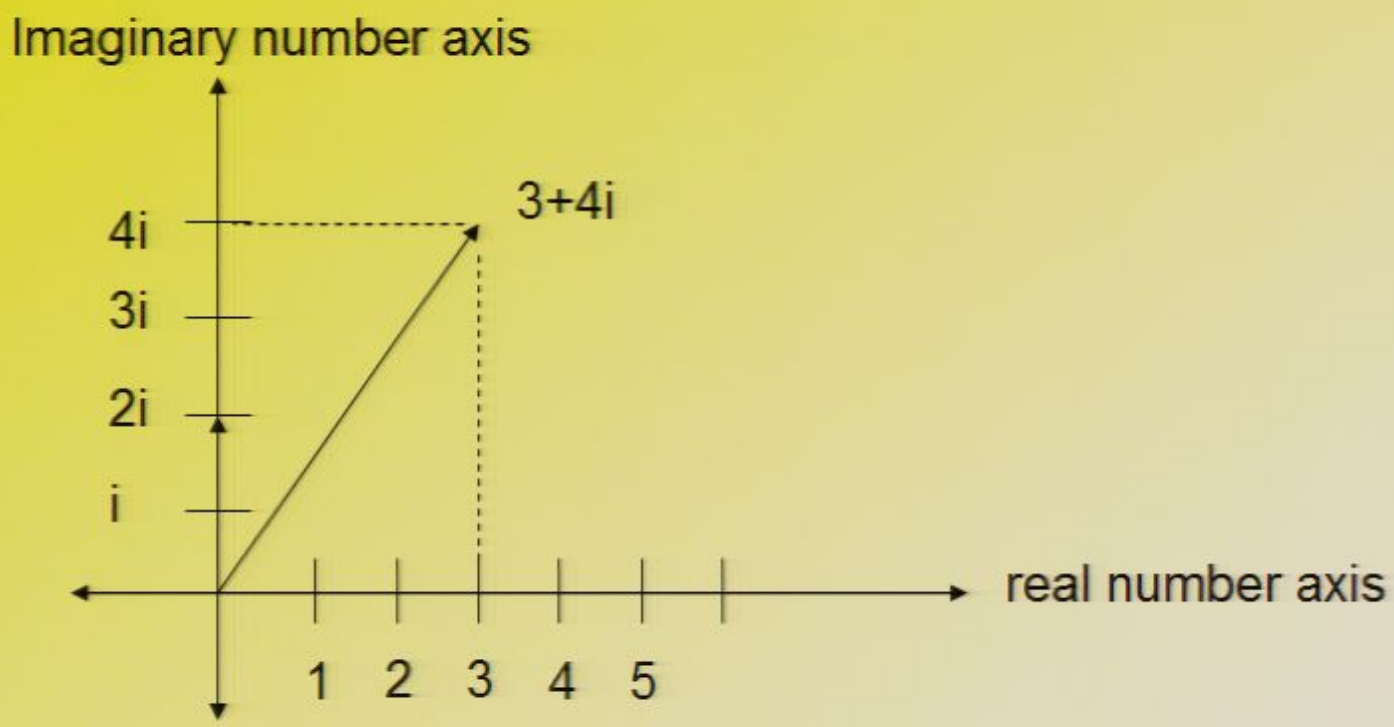

Imaginary number axis
3+4i
4i
3i
2i
i
real number axis
1
2
3
4
5

Student exercises

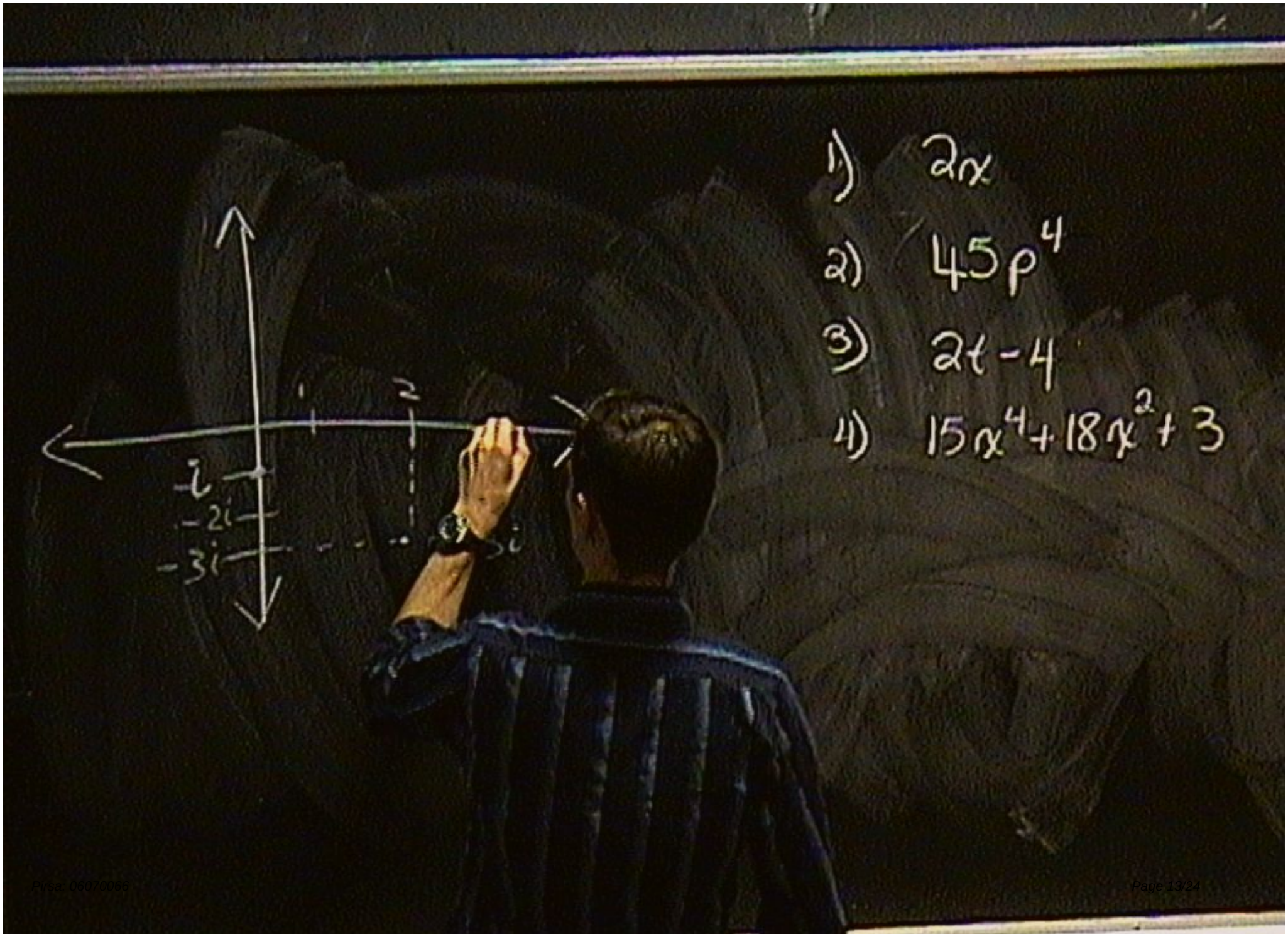
- 1. Plot 2-3i on a graph with real and imaginary axes similar to the one on the previous slide. Also plot 3.14 + 2.3i and -i

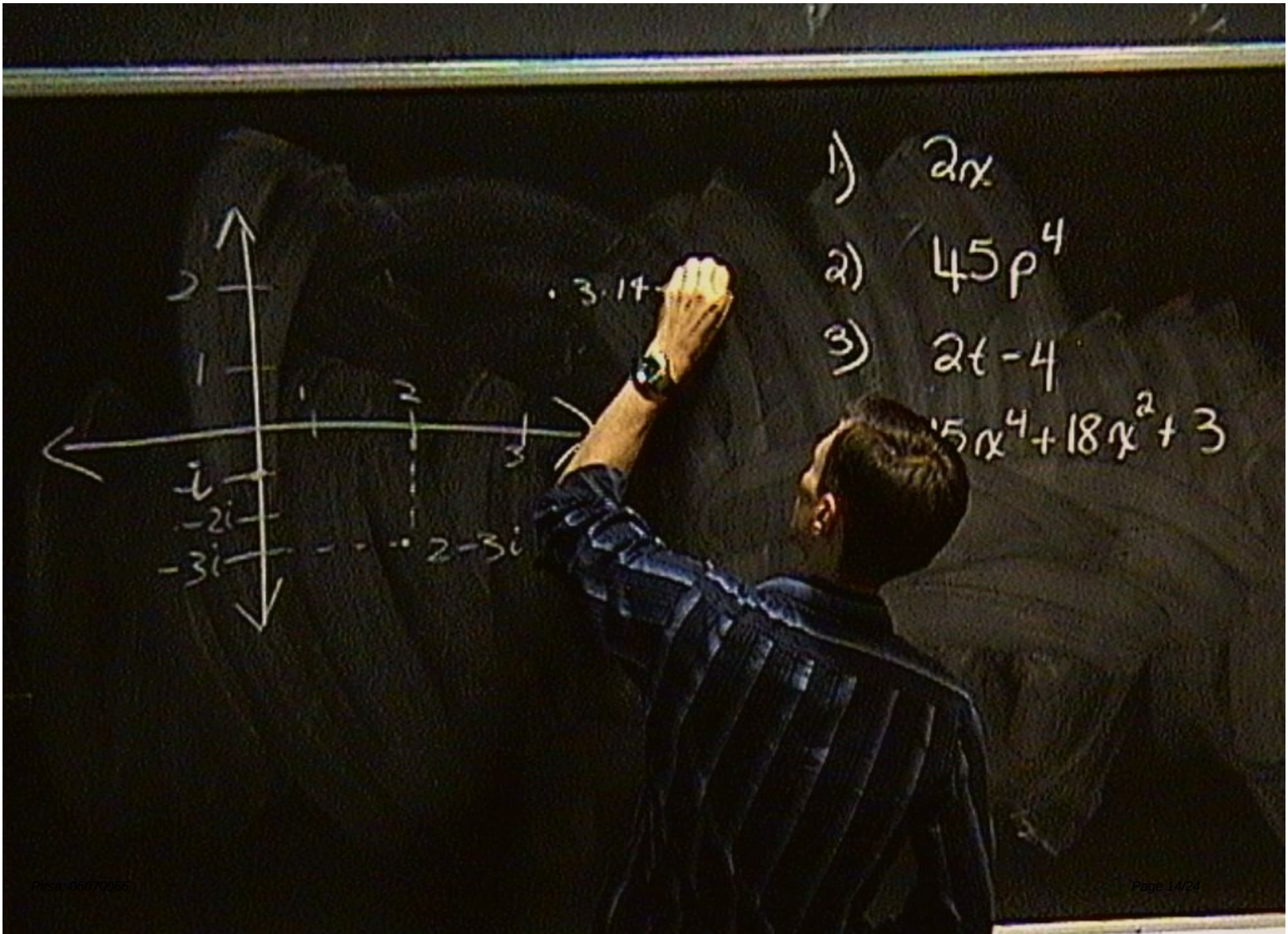
- 2. Calculate – *i x i*

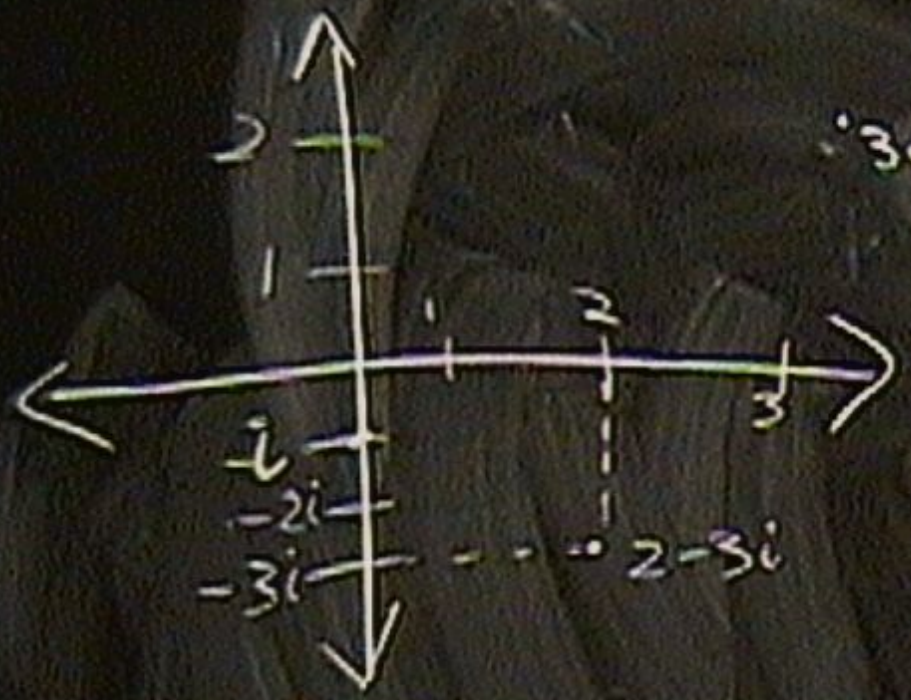
- 3. Calculate $\sqrt{-1} \times (-i)^2$

- **Extra question:**
  Calculate $(1+i)^n$
  (Hint: Read up on phasors in the math primer.)

  What is the result for n=10?


1) 2x
2) 45p^4
3) 2t-4
4) 15x^4+18x^2+3
1
2
-i
-2i
-3i
3i


1) 2x
2) 45p^4
3) 2t-4
5x^4+18x^2+3
3.14
2-3i
-i
-2i
-3i



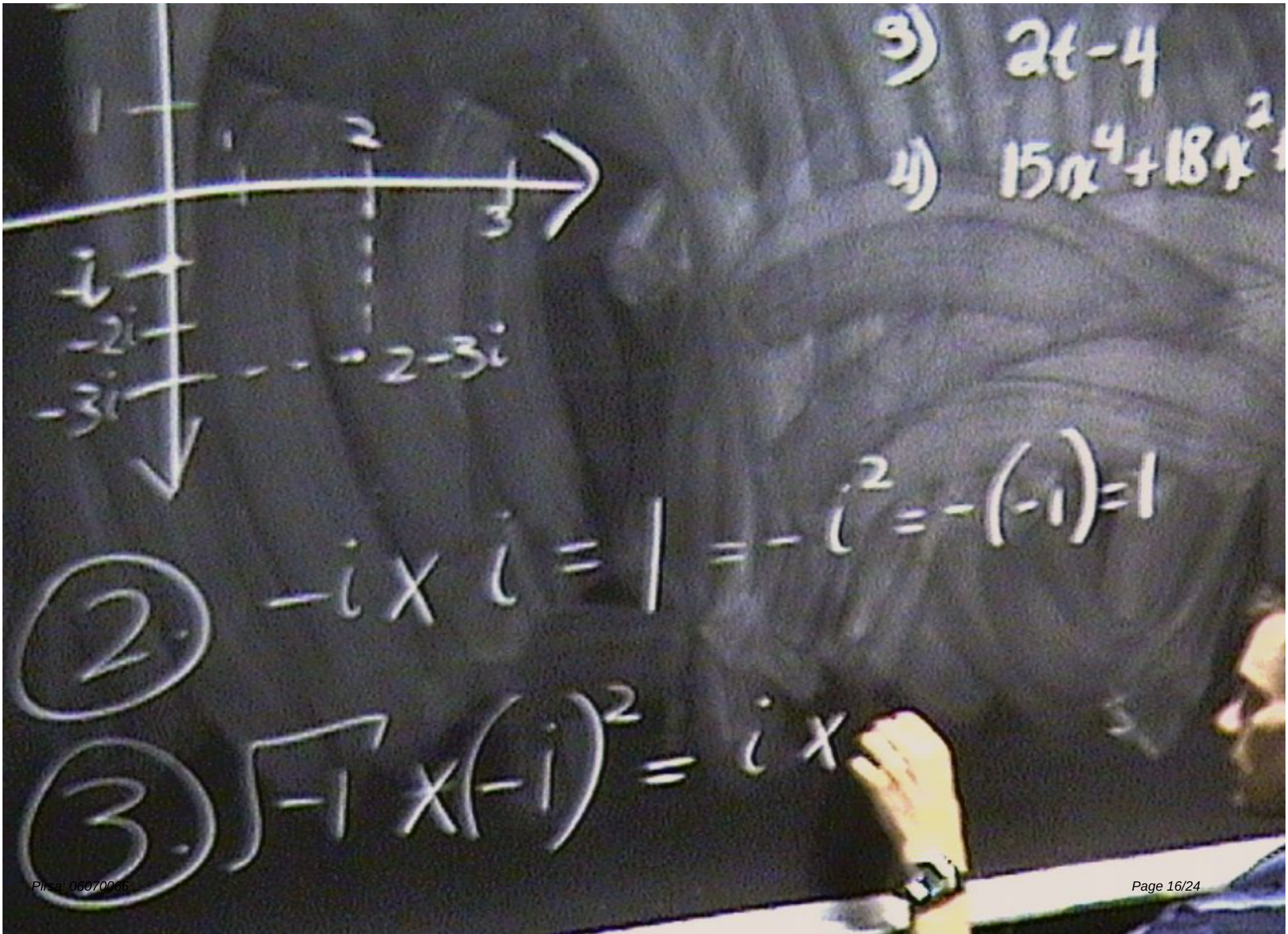
$3.14+2i$

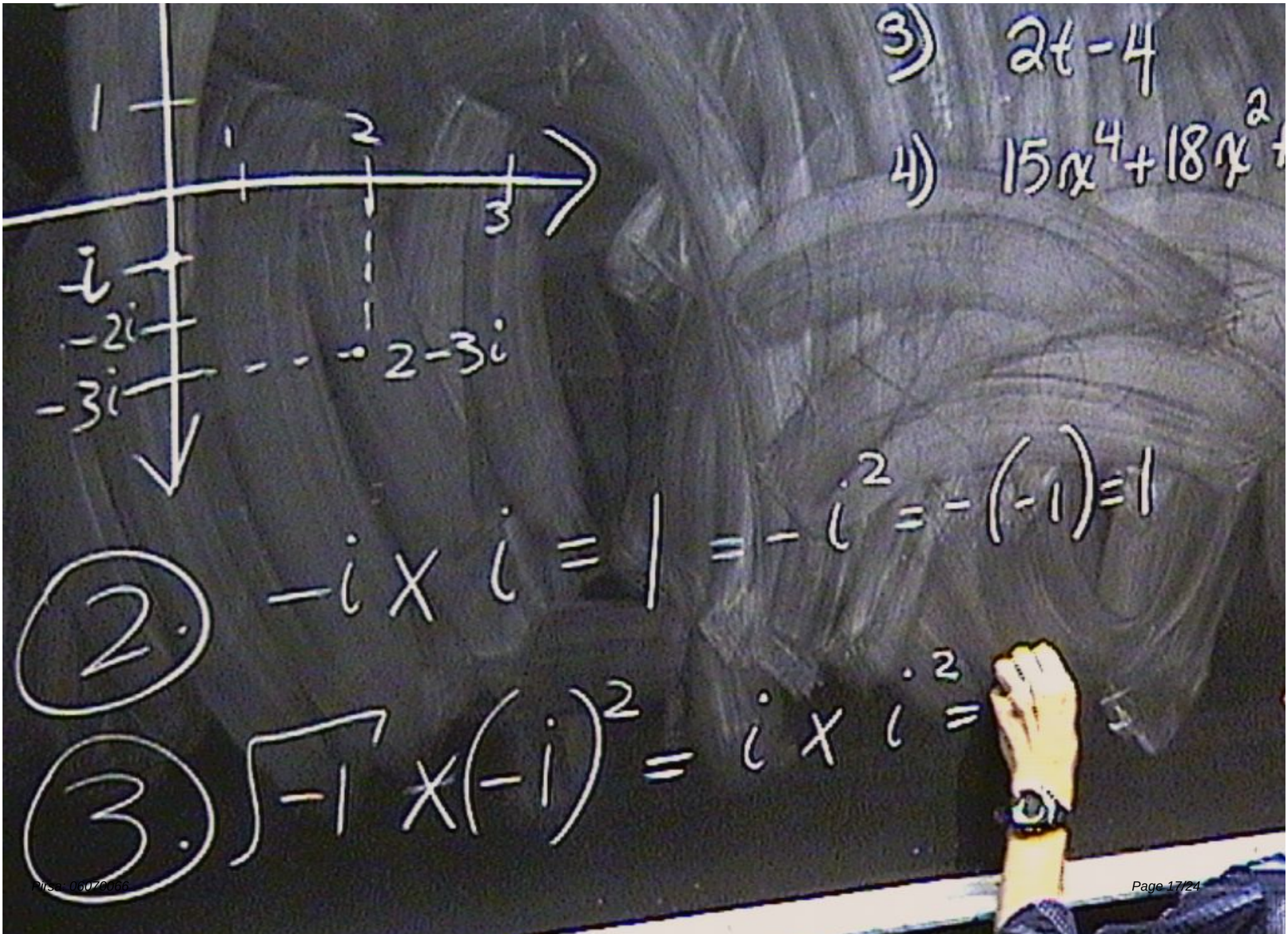
1) $2x$

2) $45p^4$

3) $2t-4$

4) $15x^4+18x^2+3$


3) 2t-4
4) 15x^4+18x^2
1
2
3
i
-2i
-3i
2-3i
2. -i x i = 1 = -i^2 = -(-1) = 1
3. √-1 x (-i)^2 = i x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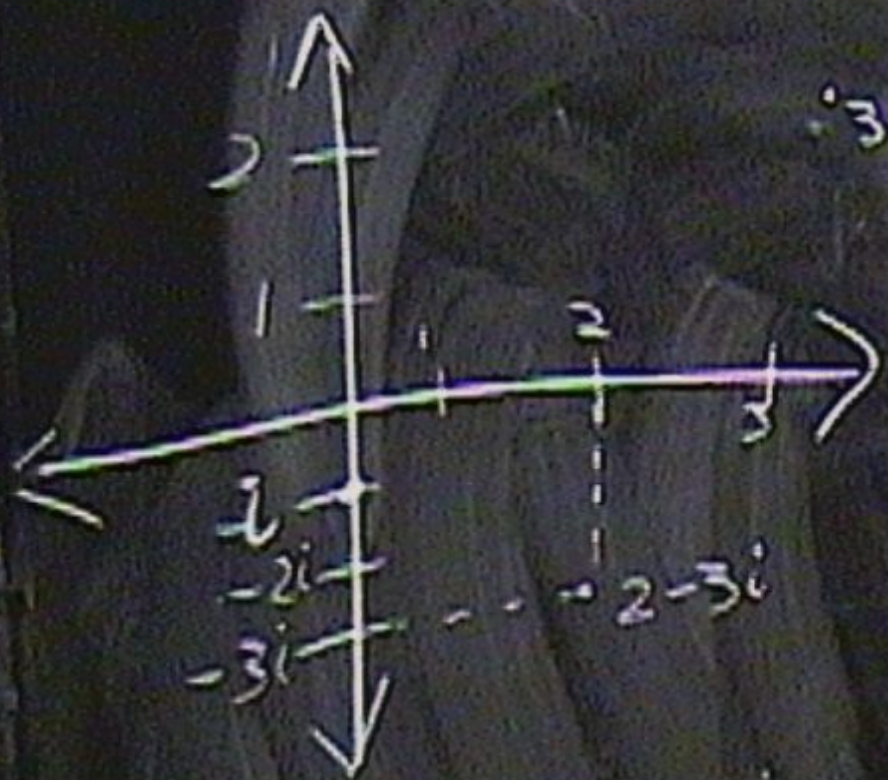
1.



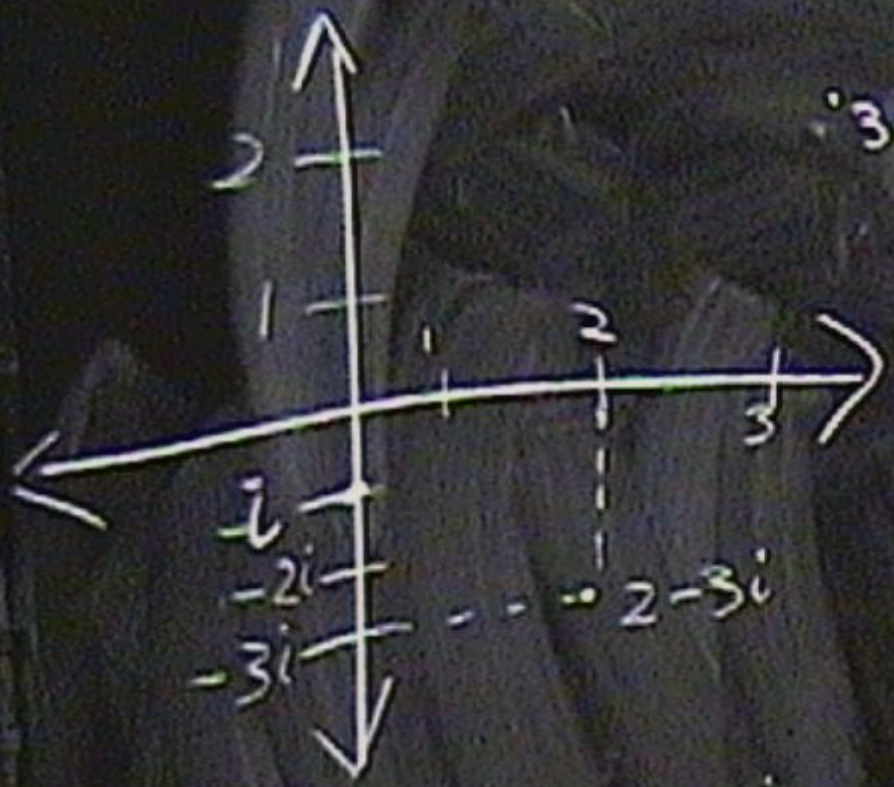
$3.14+2i$

4.

2. $-i \times i = 1 = -i^2 = -(-1) = 1$

3. $\sqrt{-1} \times (-i)^2 = i \times i^2 = i \times -1 = -i$

1.

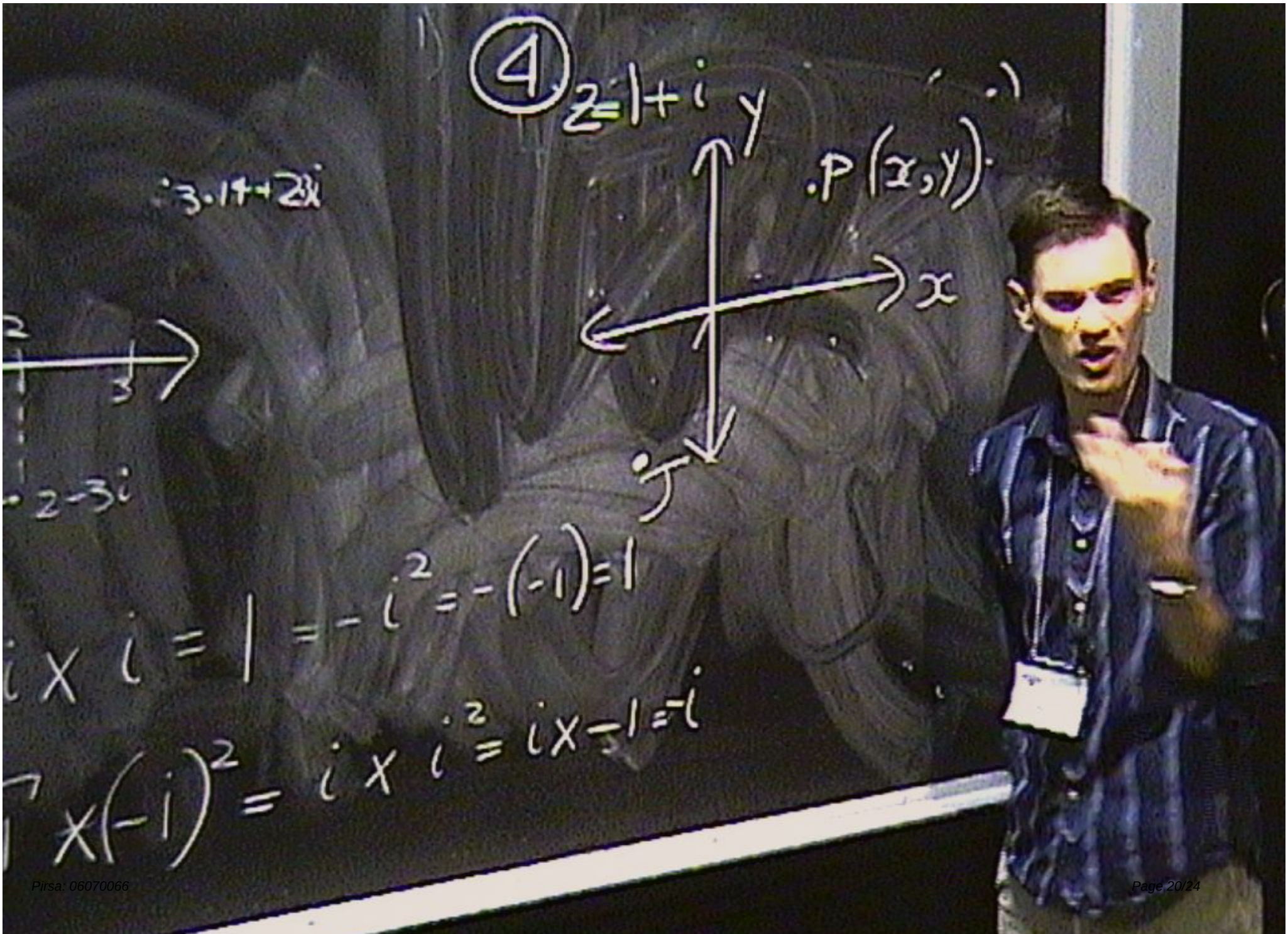


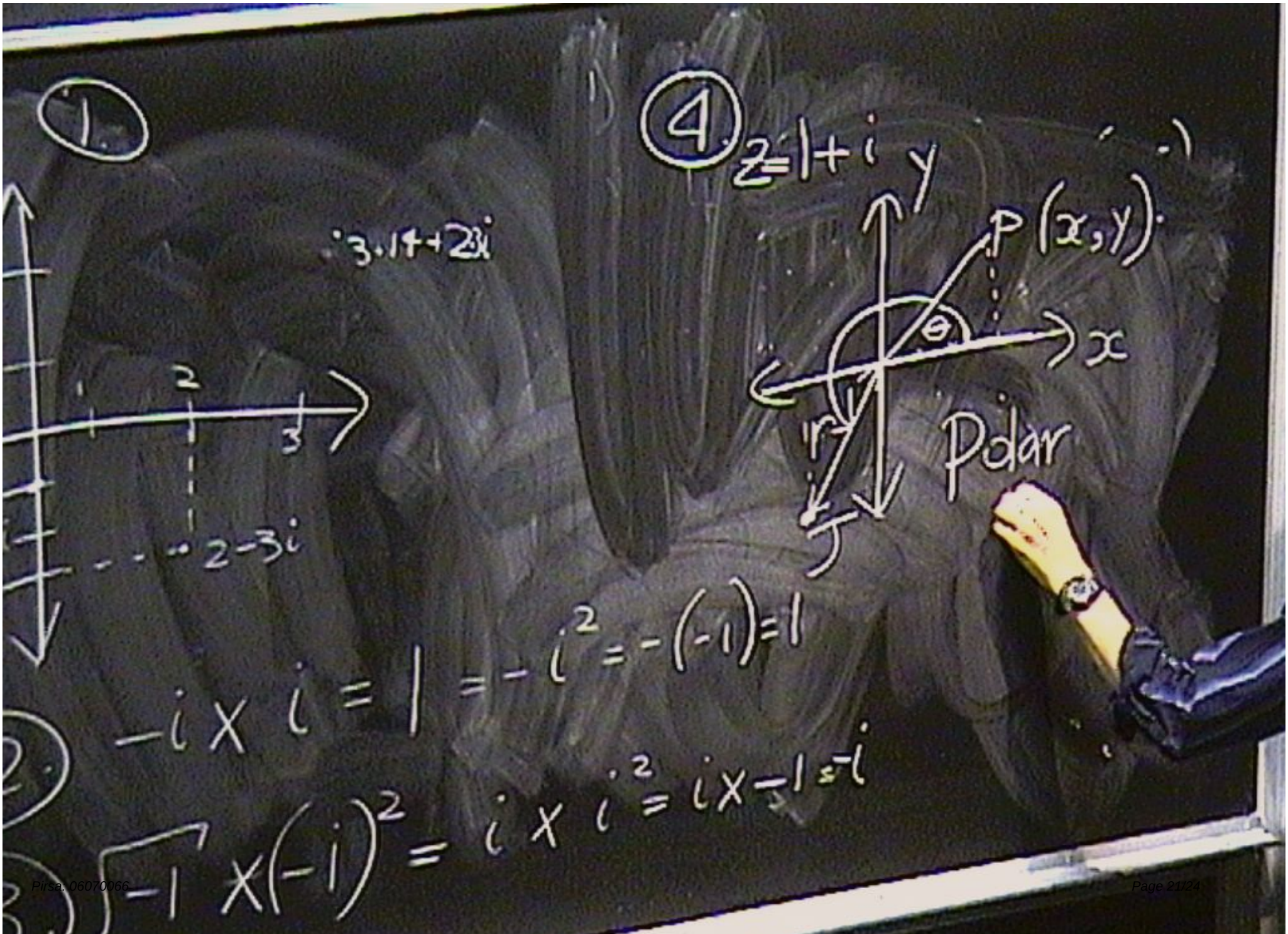
3.14+2i

4. 1+i

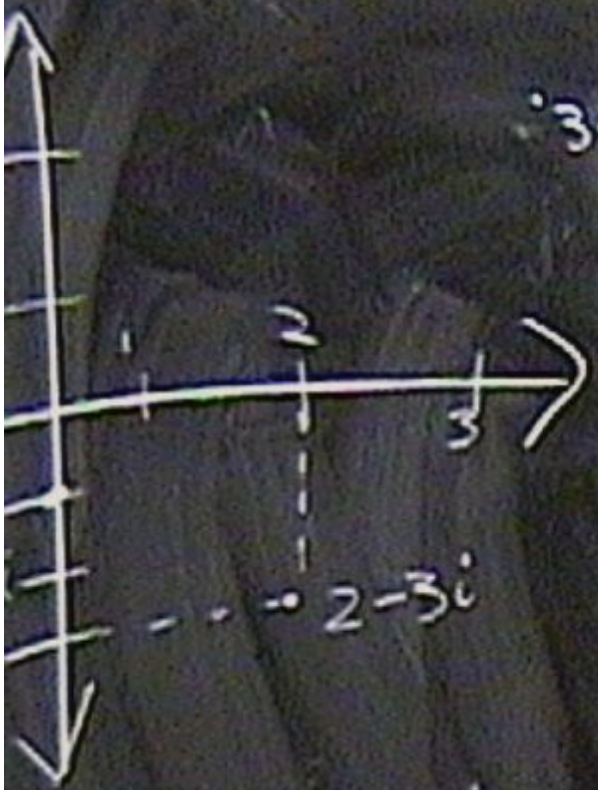
2. $-i \times i = 1 = -i^2 = -(-1) = 1$

3. $\sqrt{-1} \times (-i)^2 = i \times i^2 = i \times -1 = -i$


4
z=1+i
y
P(x,y)
x
3.14+2i
3
2-3i
i x i = 1 = -i² = -(-1) = 1
(-i)² = i x i² = i x -1 = -i


1.
3.14+2i
2
3
2-3i
4
z=1+i
y
P (x, y)
θ
x
r
Polar
-i x i = 1 = - i^2 = -(-1) = 1
2.
3.
√-1 x (-i)^2 = i x i^2 = i x -1 = -i

1.



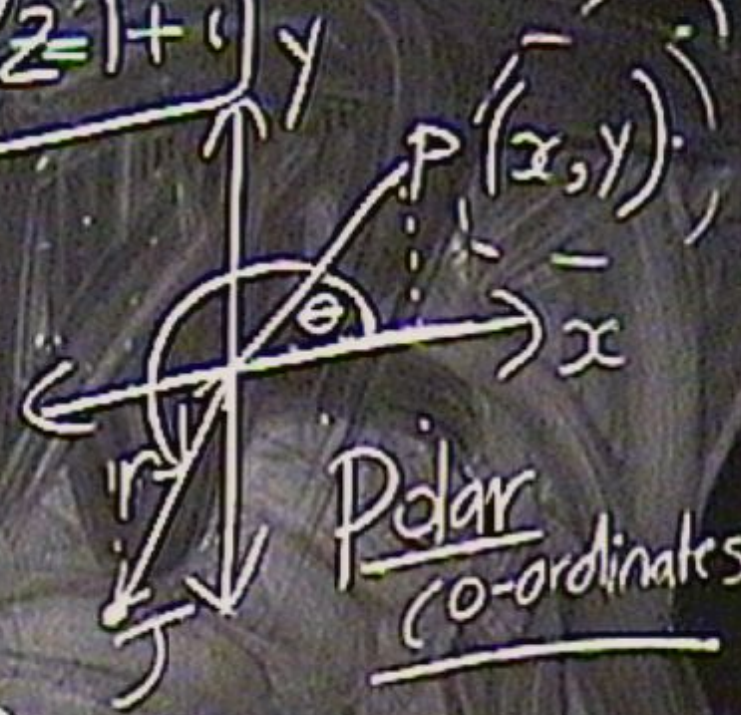
$3.14+2.3i$

(x, y)

$\downarrow$

(r, θ)

4. $\boxed{z = 1+i}$



Polar co-ordinates

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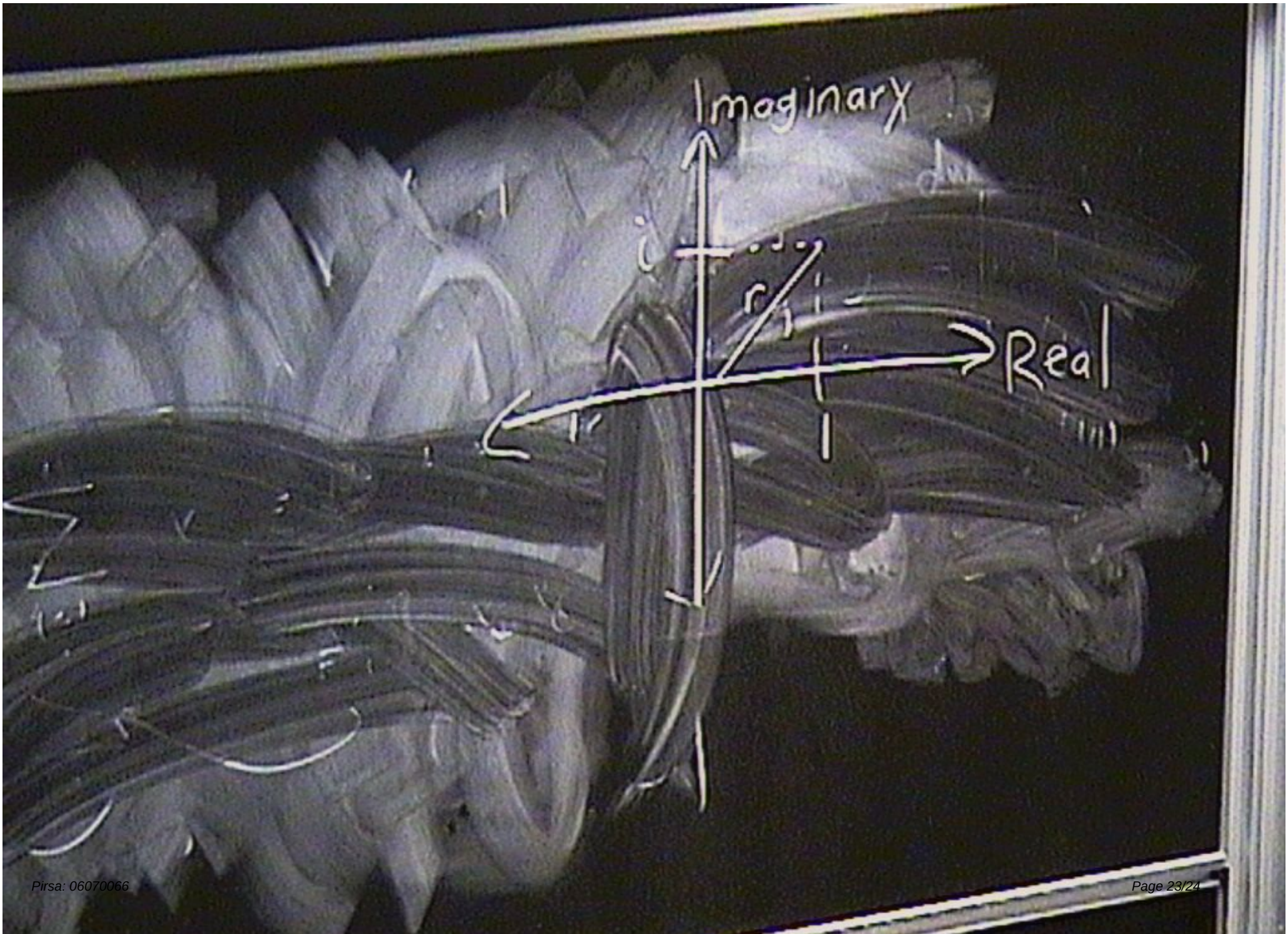
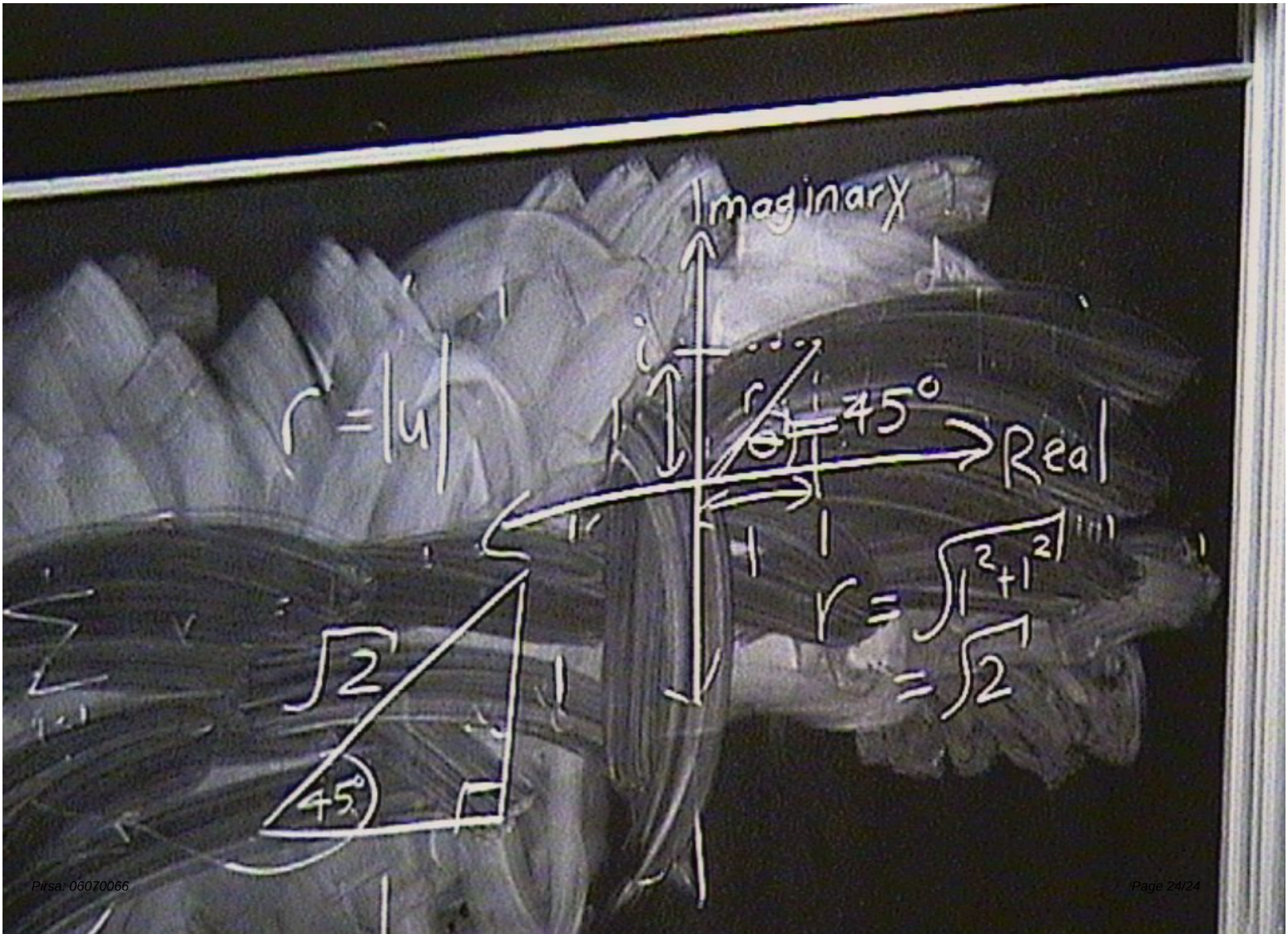

Imaginary
Real


Imaginary
Real
r = |u|
θ = 45°
r = √(1² + 1²)
= √2
√2
45°