Title: Math Primer: Differentiation Continued

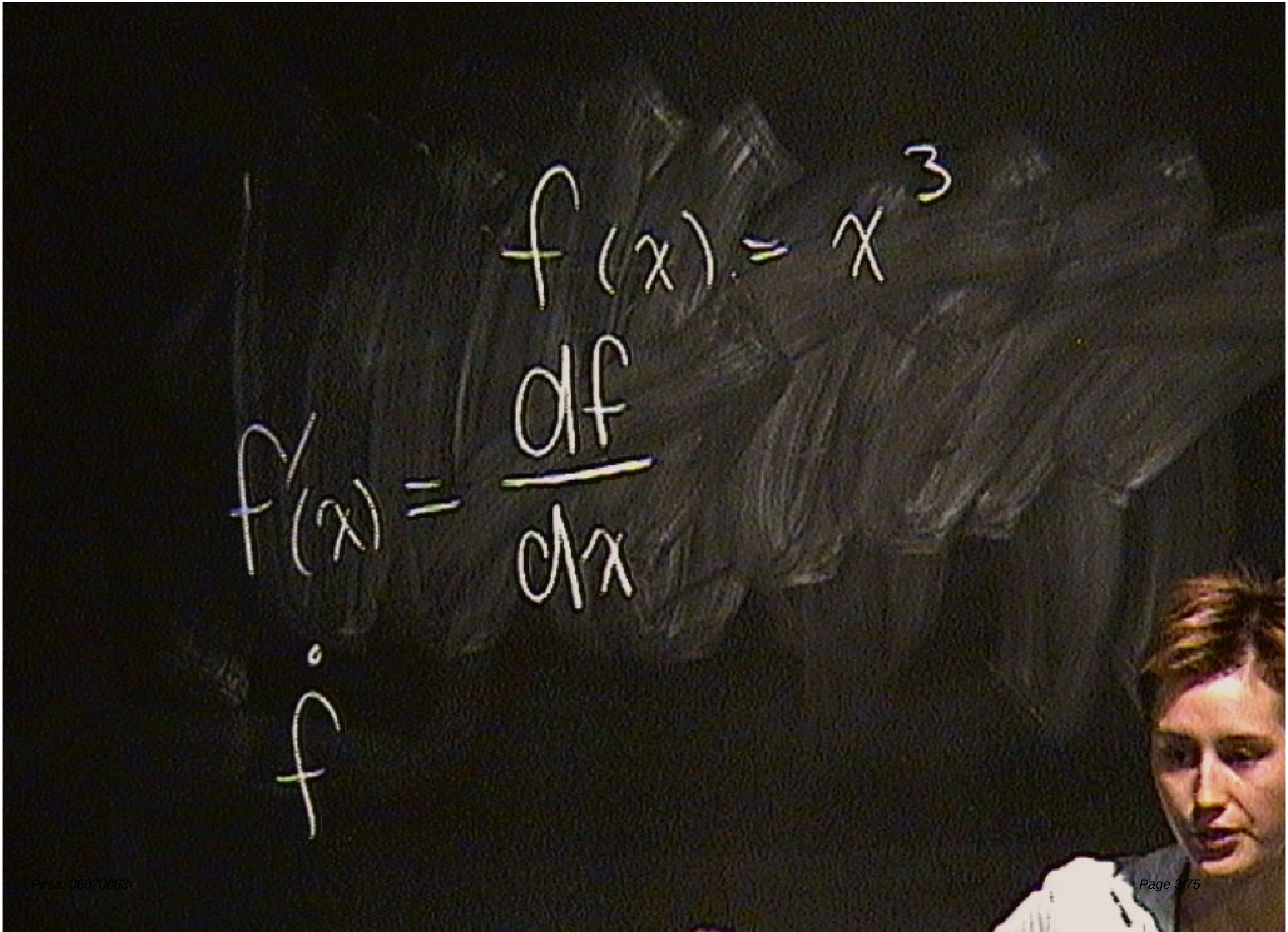
Date: Jul 24, 2006 10:40 AM

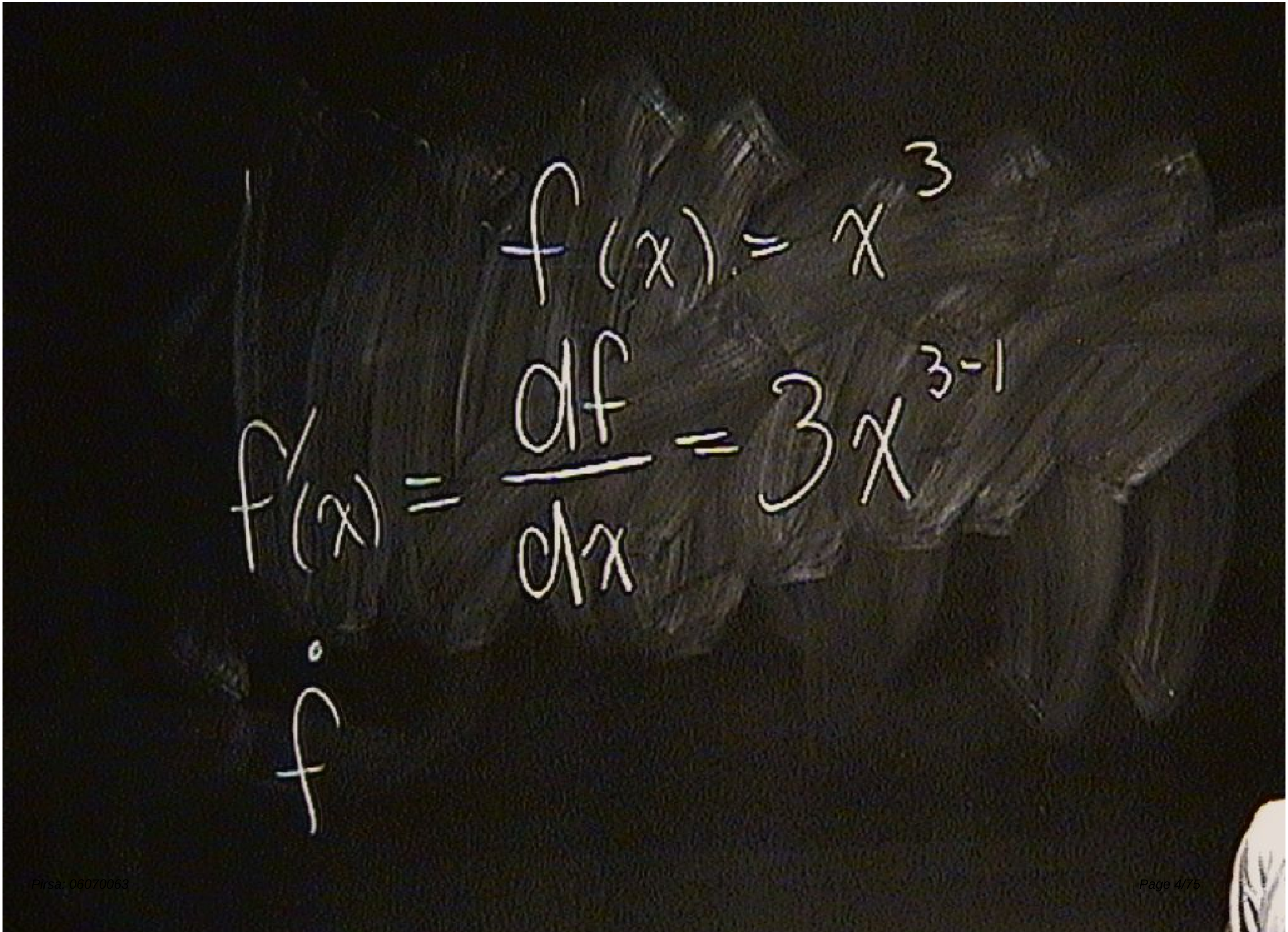
URL: http://pirsa.org/06070063

Abstract:

Power Rule

If n is a real number and $f(x) = x^n$, *then* $\frac{d}{dx}x^n = nx^{n-1}$

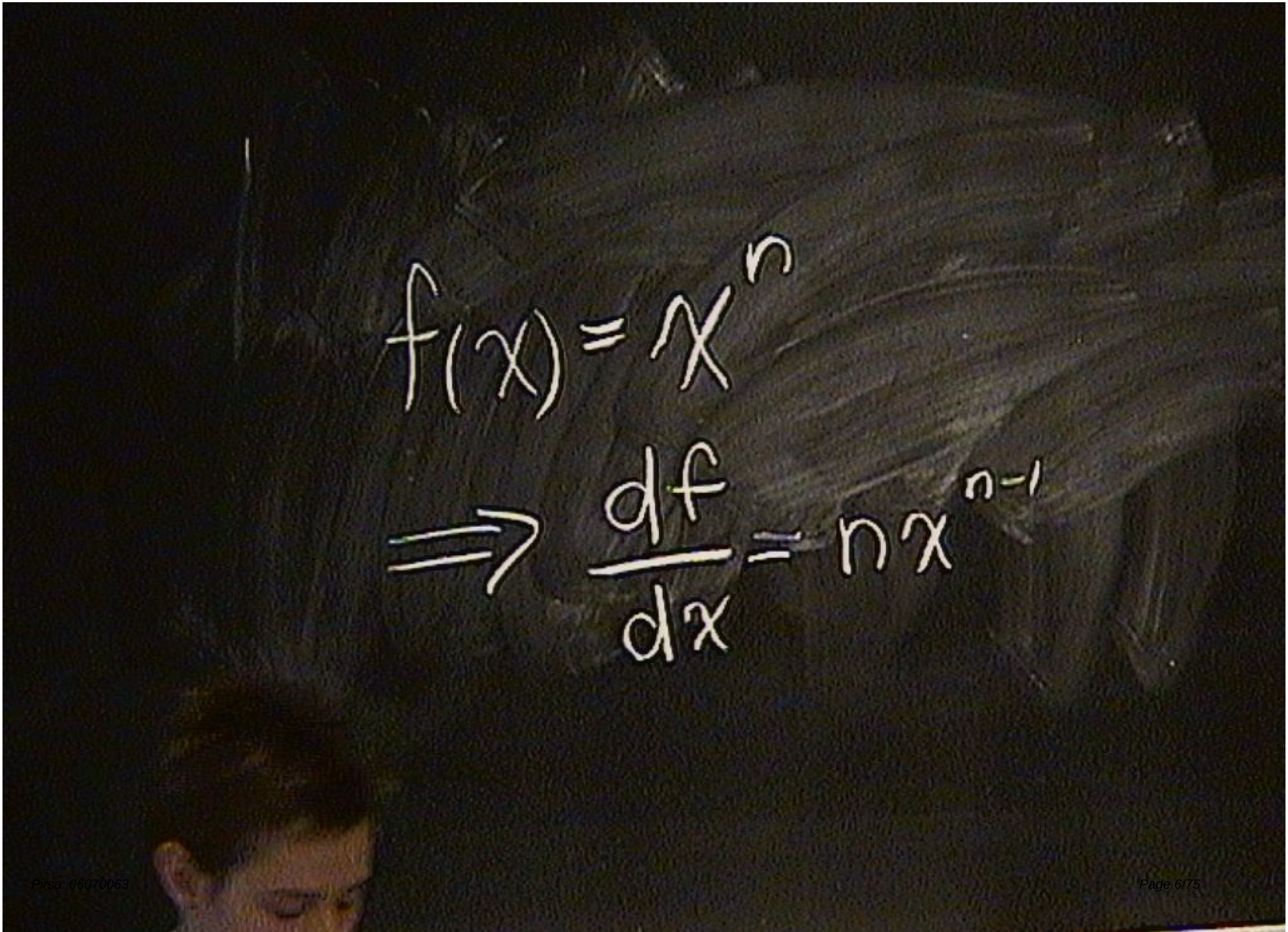

$f(x) = x^3$
$f'(x) = \frac{df}{dx}$
$\dot{f}$


$f(x) = x^3$
$f'(x) = \frac{df}{dx} = 3x^{3-1}$
$\dot{f}$

$$f(x) = x^3$$

$$f'(x) = \frac{df}{dx} = 3x^{3-1} = 3x^2$$

$$\dot{f}$$


$f(x) = x^n$
$\Rightarrow \frac{df}{dx} = nx^{n-1}$

Problem Set (2) – Power Rule

Differentiate the following with respect to the given variable:

1) $f(x) = x^7$

2) $g(t) = t^{\frac{5}{4}}$

3) $r(\theta) = \dfrac{1}{\theta}$

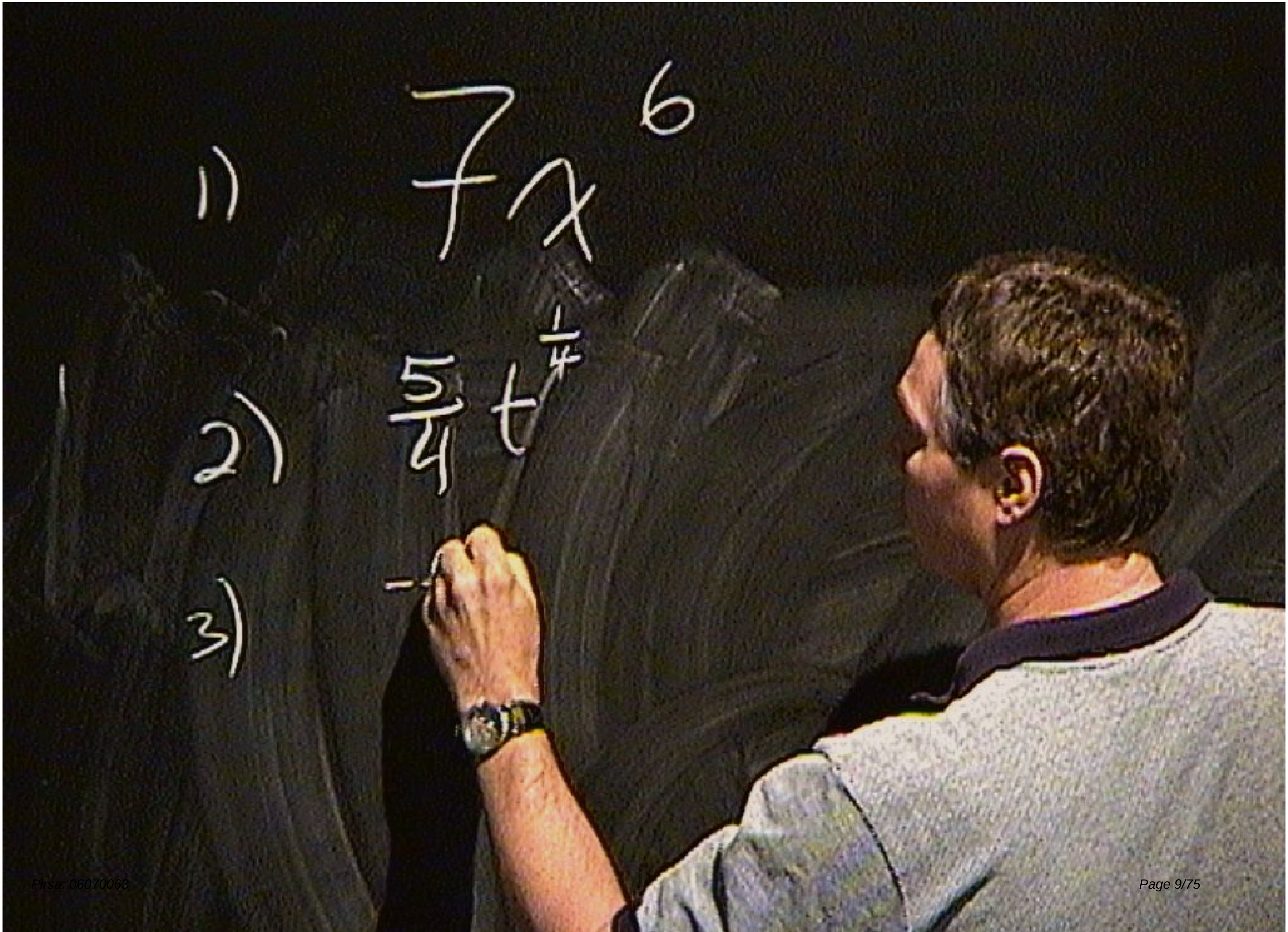
4) $f(y) = y^{1/2}$

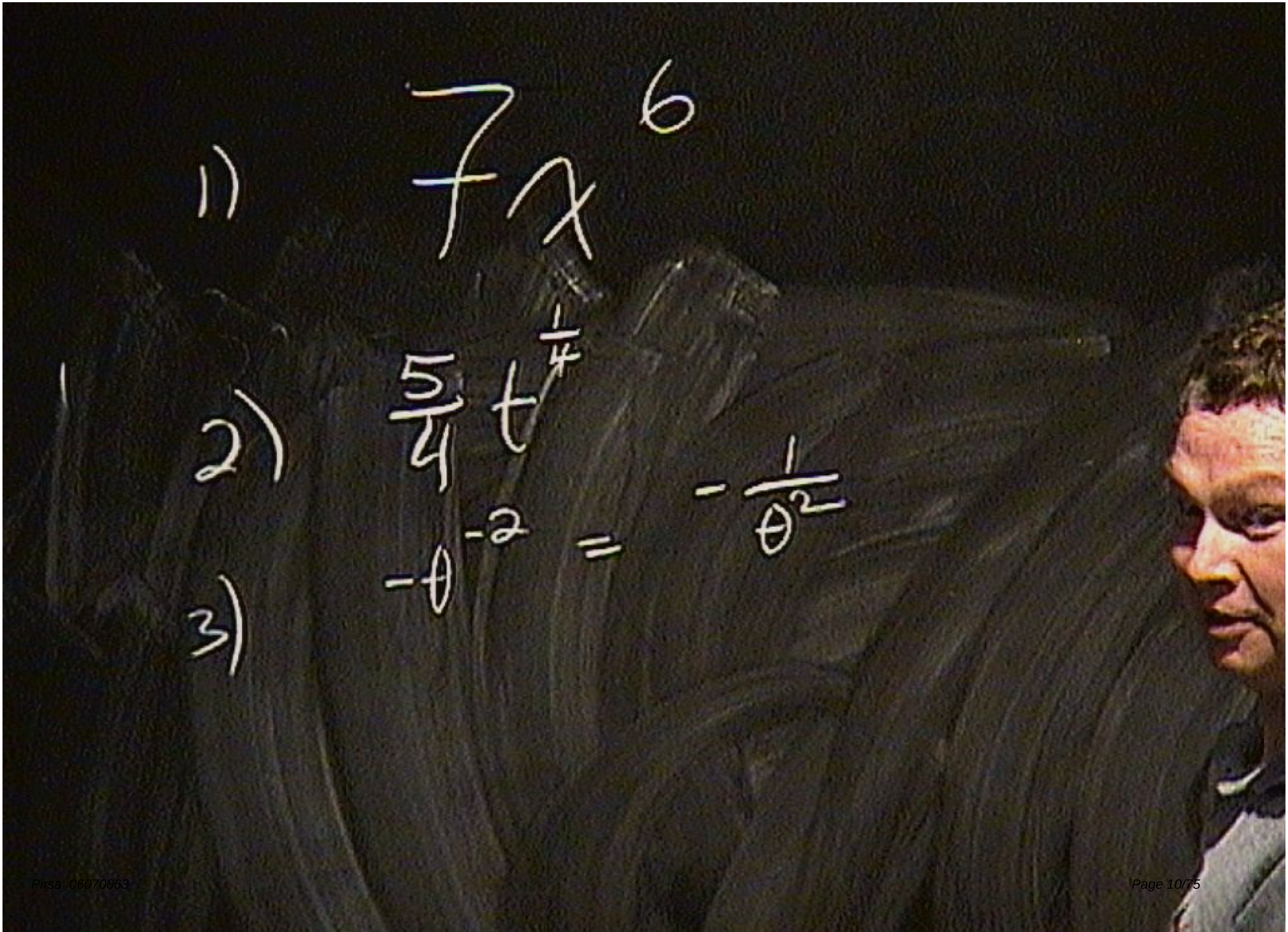
5) $h(x) = x$

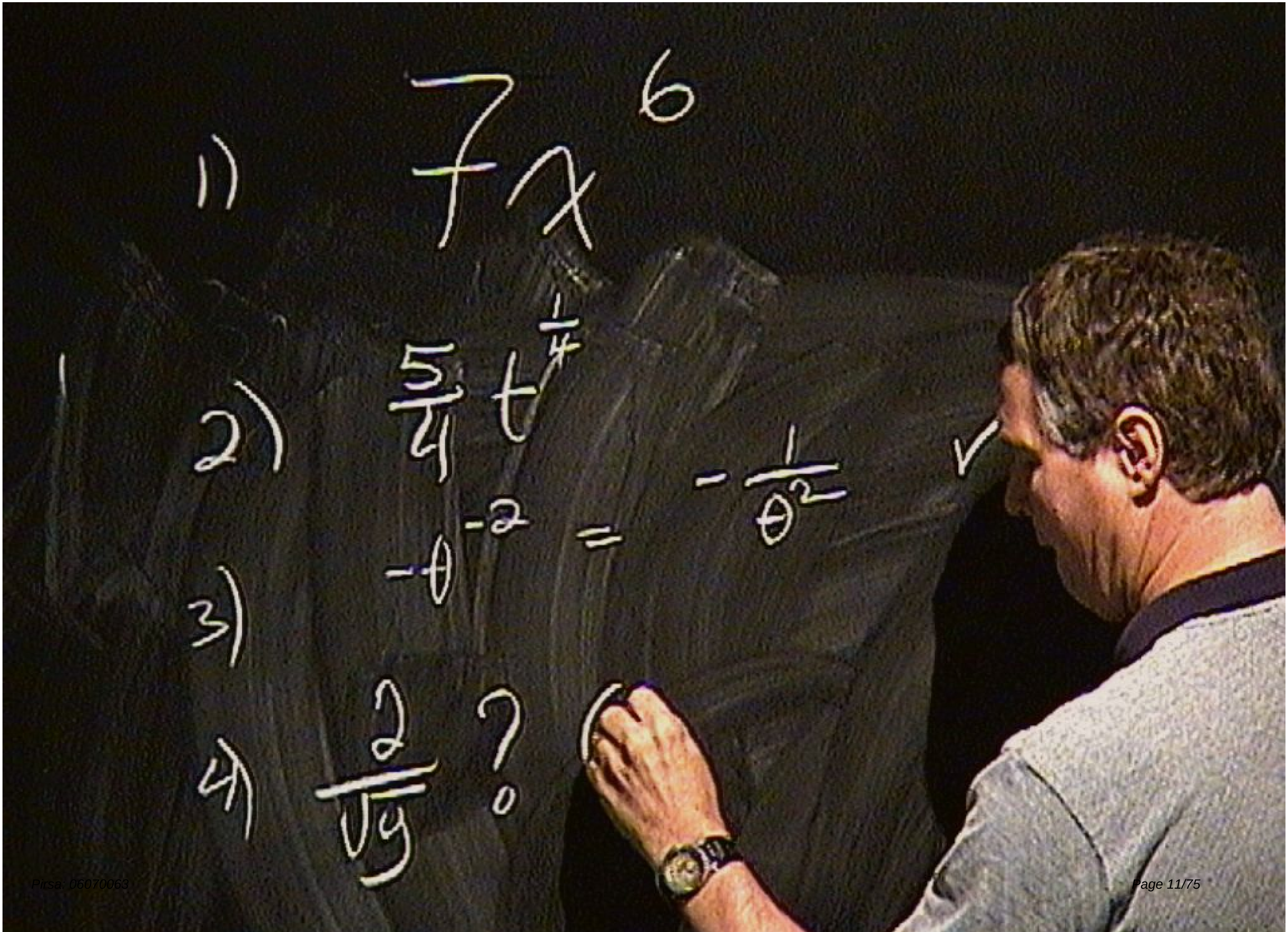
2)

Hint: $\frac{1}{x} = x^{-1}$

$$\sqrt{x} = x^{\frac{1}{2}}$$


1) 7x6
2) 5/4 t1/4
3)


1) $7x^6$
2) $\frac{5}{4}t^{\frac{1}{4}}$
3) $-\theta^{-2} = -\frac{1}{\theta^2}$


1) 7x 6
2) 5/4 t 1/4
3) -θ -2 = -1/θ2
4) 2/√5 ? (

1) $7x^6$

2) $\frac{5}{4} t^{\frac{1}{4}}$

3) $-\theta^{-2} = -\frac{1}{\theta^2}$ ✓

4) $\frac{2}{\sqrt{y}}$? , $\frac{1}{2\sqrt{y}}$

1) $7x^6$

2) $\frac{5}{4}t^{\frac{1}{4}}$

3) $-\theta^{-2} = -\frac{1}{\theta^2}$ ✓

4) $\frac{2}{\sqrt{y}}$? , $\frac{1}{2\sqrt{y}}$ $= \frac{1}{2}y^{-\frac{1}{2}}$

5) 1

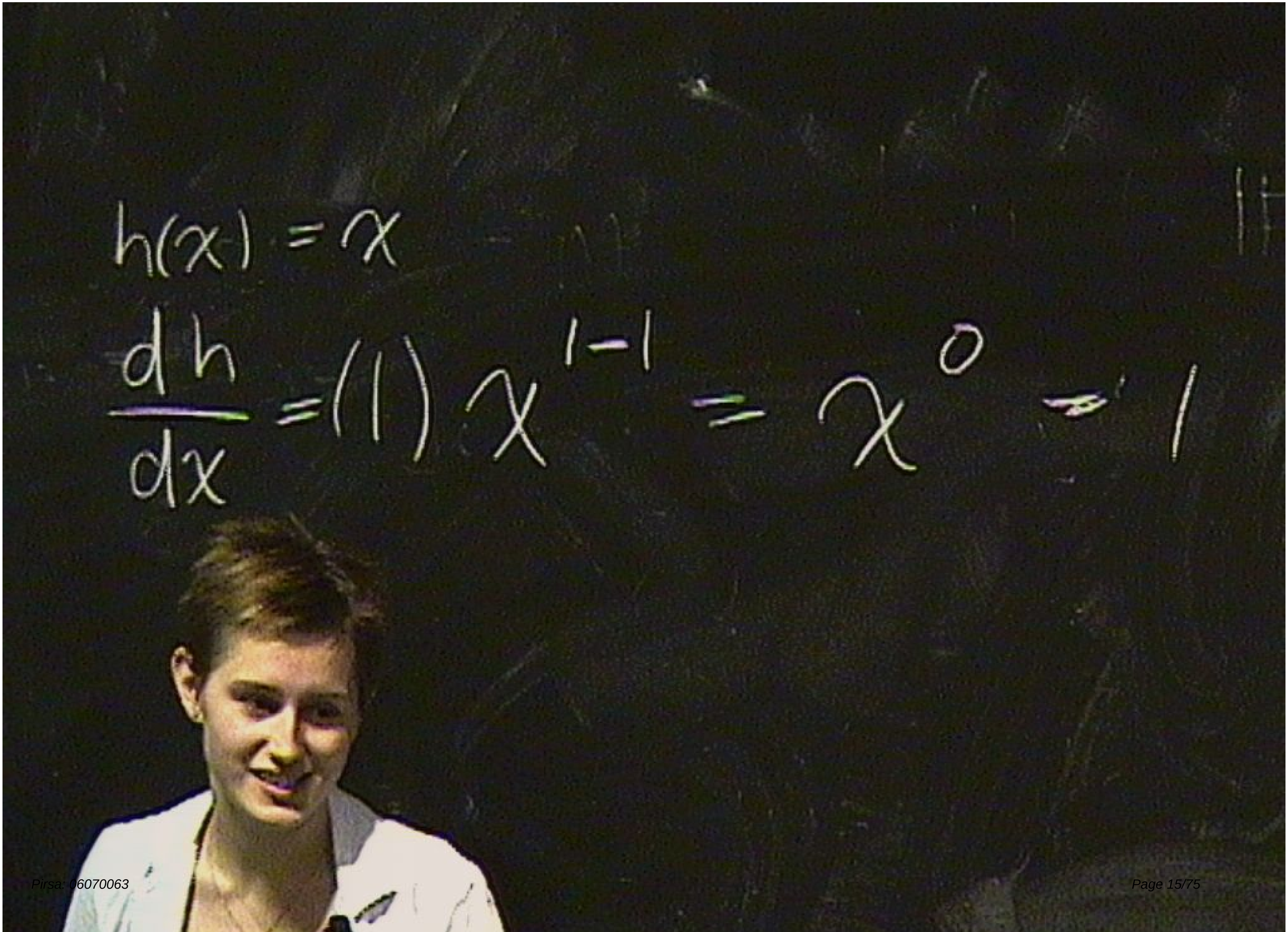
1) $7x^6$

2) $\frac{5}{4}t^{\frac{1}{4}}$

3) $-\theta^{-2} = -\frac{1}{\theta^2}$ ✓

4) $\cancel{\frac{2}{\sqrt{y}}}$? ☹ , $\frac{1}{2\sqrt{y}}$ ☺ $= \frac{1}{2}y^{-\frac{1}{2}}$

5) 1


$h(x) = x$
$\frac{dh}{dx} = (1)\, x^{1-1} = x^0 = 1$

Power Rule

If n is a real number and $f(x) = x^n$, *then* $\frac{d}{dx}x^n = nx^{n-1}$

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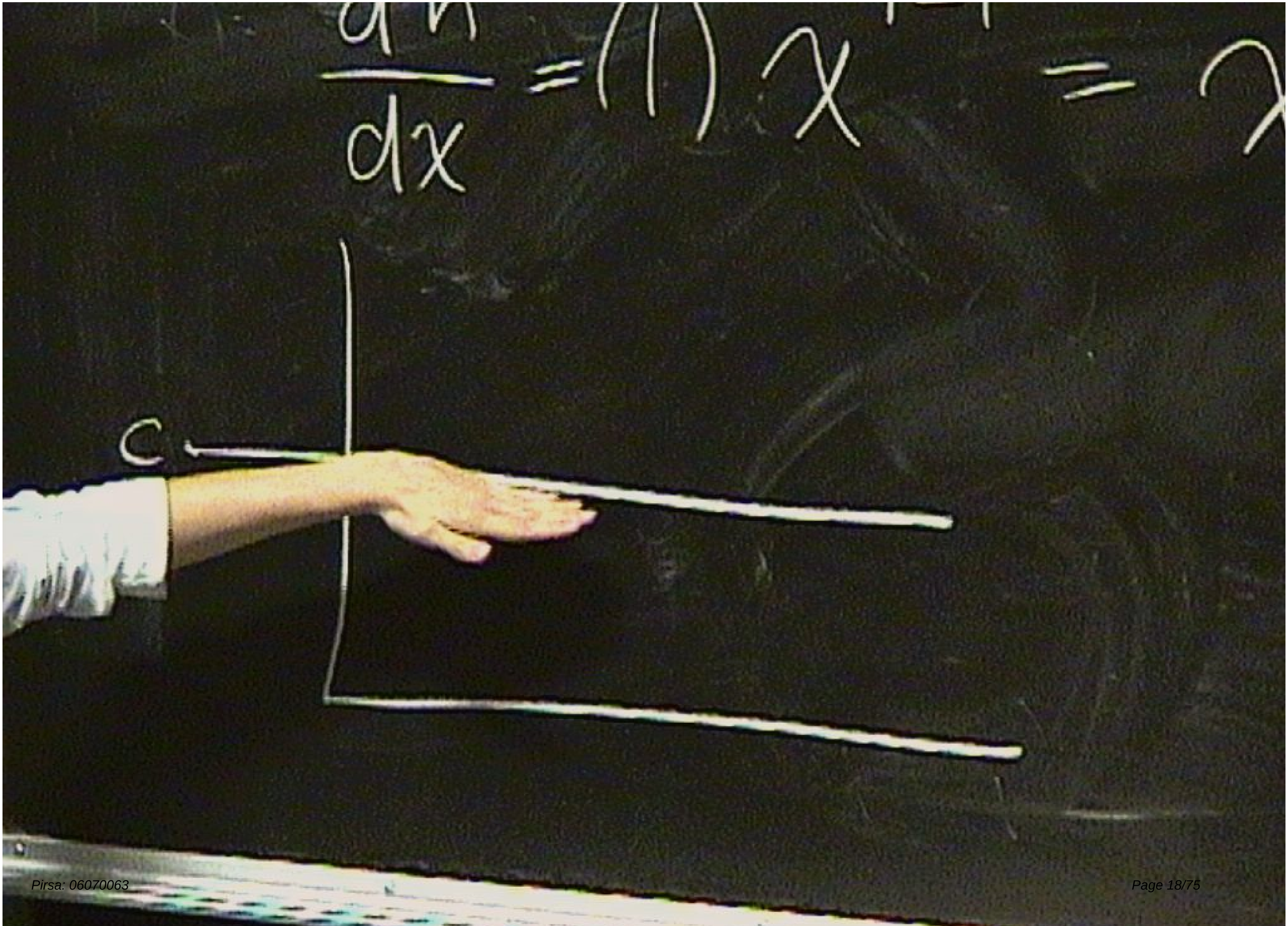
Constant Rule

If f(x) is a constant function (i.e. f(x) = c), then $\frac{dc}{dx} = 0$



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Constant Multiple Rule

If $g(x) = cf(x)$, *then* $\frac{dg}{dx} = \frac{d}{dx}[cf(x)] = c\frac{df}{dx}$

$$g(x) = cf(x)$$

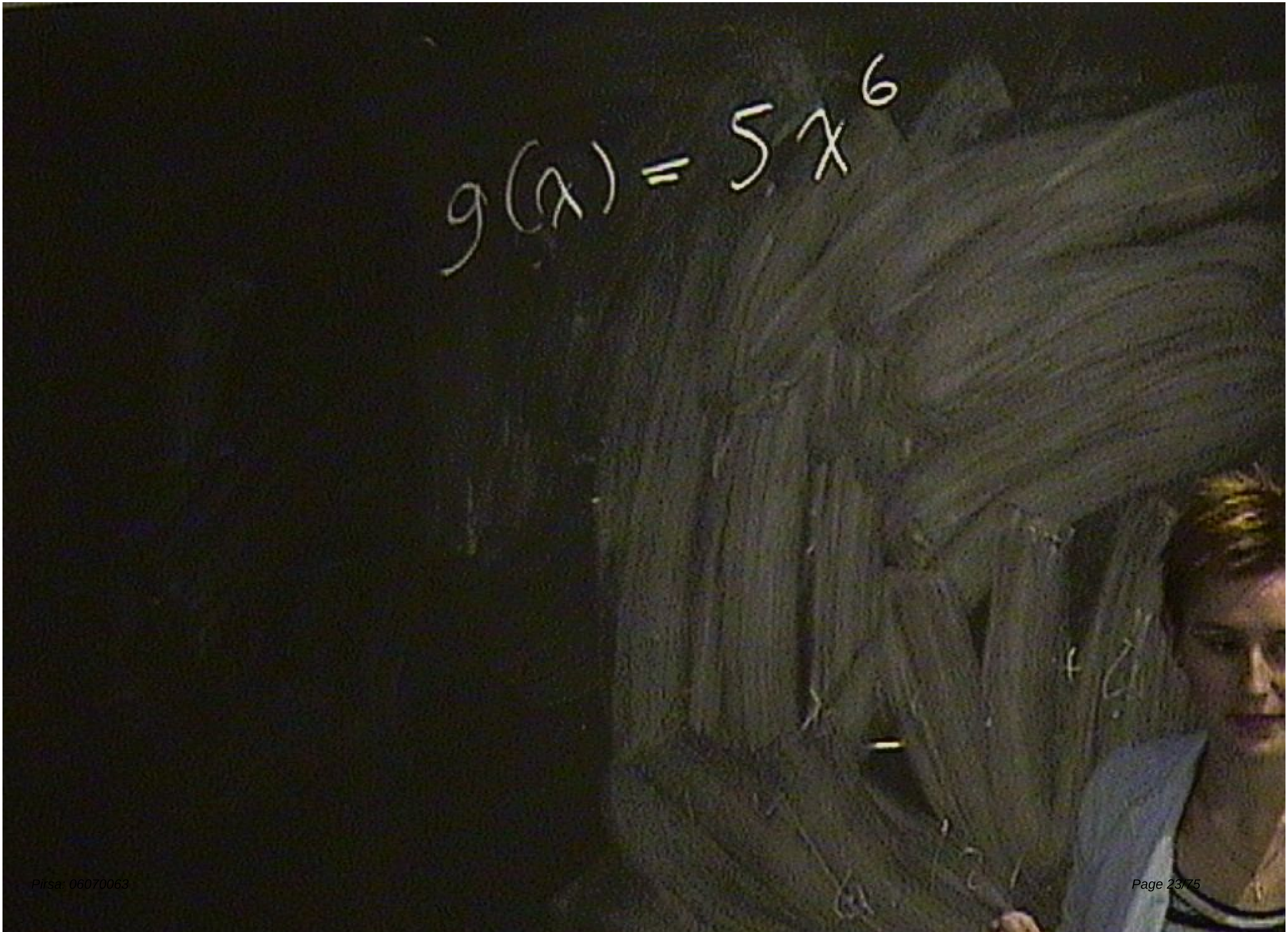
$$g' = cf'$$

$$\frac{dg}{dx} =$$

$$g(x) = cf(x)$$

$$g' = cf'$$

$$\frac{dg}{dx} = c\frac{df}{dx}$$


g(x) = 5x^6

$$g(x) = 5x^6$$

$$\frac{dg}{dx} = 5\,\frac{d}{dx}\left(x^6\right)$$

$$= 5 \times 6x^5$$

$$= 30x^5$$

Power Rule

If n is a real number and $f(x) = x^n$, *then* $\frac{d}{dx}x^n = nx^{n-1}$

Constant Rule

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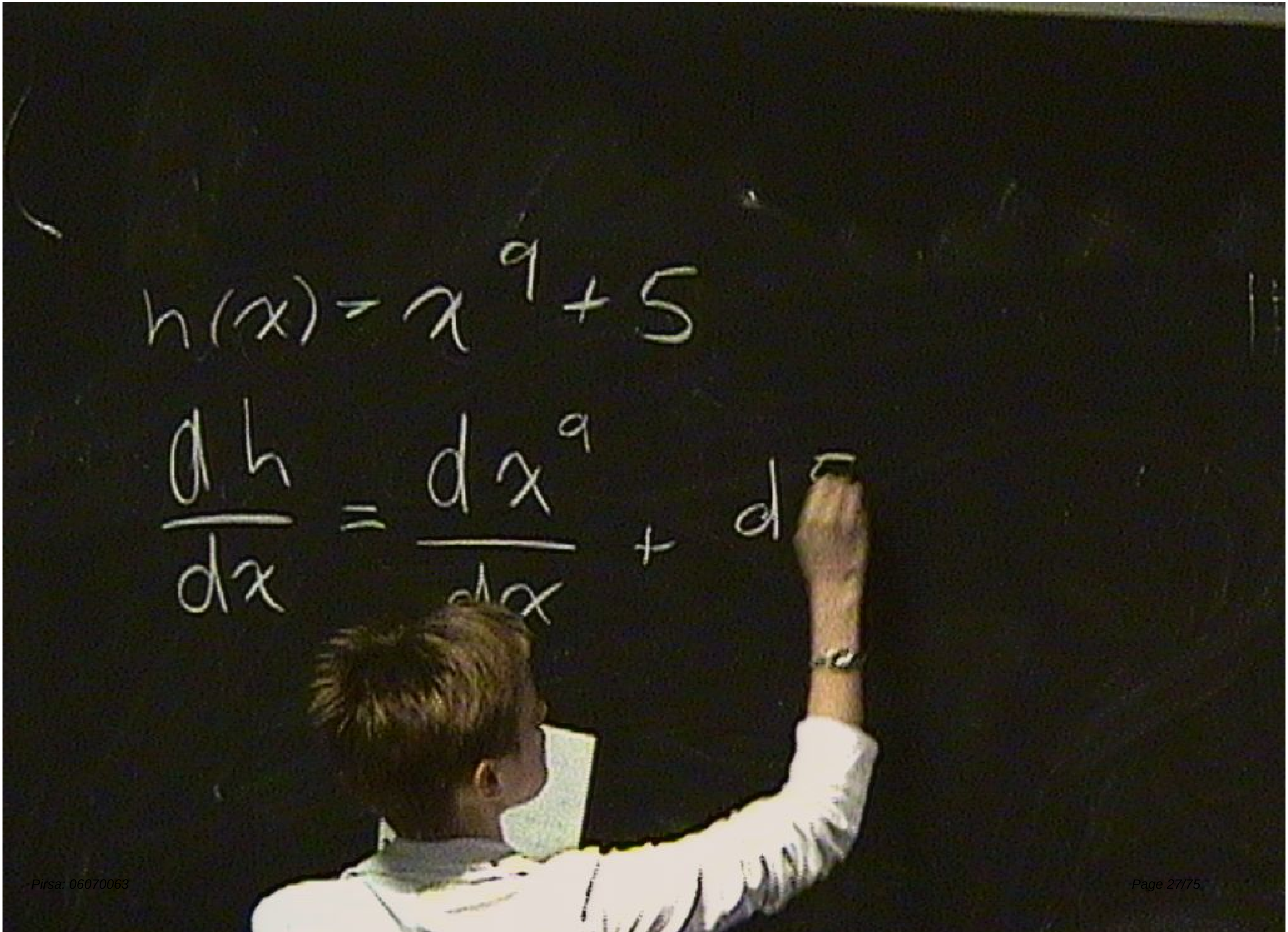
Constant Multiple Rule

If g(x) = cf(x), then $\frac{dg}{dx} = \frac{d}{dx}[cf(x)] = c\frac{df}{dx}$

Sum Rule

If h(x) = f(x) + g(x), then $\frac{dh}{dx} = \frac{d}{dx}[f(x)+g(x)] = \frac{df}{dx} + \frac{dg}{dx}$

$$h(x) = x^9 + 5$$


$h(x) = x^9 + 5$
$\frac{dh}{dx} = \frac{dx^9}{dx} + d$

$$h(x) = x^9 + 5$$

$$\frac{dh}{dx} = \frac{dx^9}{dx} + \frac{d5}{dx}$$

$$= 9x^8 + 0$$

Problem Set (3) – Polynomials

Differentiate the following with respect to the given variable:

1) $f(x) = x^2 + 7$

2) $\Phi(p) = 9p^5$

3) $r(t) = (t-2)^2$

4) $y(x) = 3x(x^2+1)^2 + 7$

Power Rule

If n is a real number and $f(x) = x^n$, *then* $\frac{d}{dx}x^n = nx^{n-1}$

Constant Rule

If f(x) is a constant function (i.e. f(x) = c), then $\frac{dc}{dx} = 0$

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Sum Rule

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4) $y(x) = 3x(x^2+1)^2 + 7$

1) $2x$

2) $45\rho^4$

3)

1) $2x$

2) $45p^4$

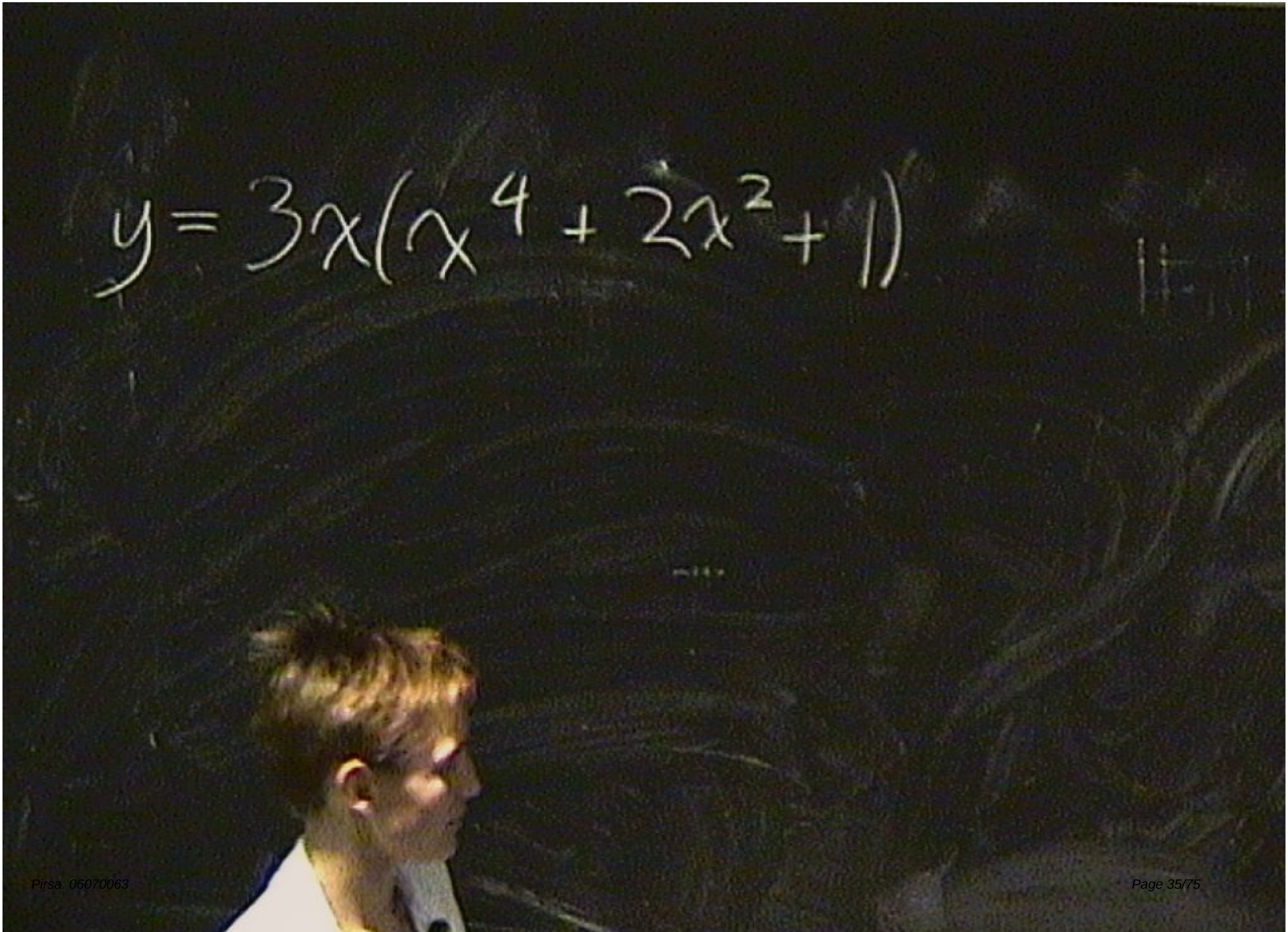
3) $2t-4$

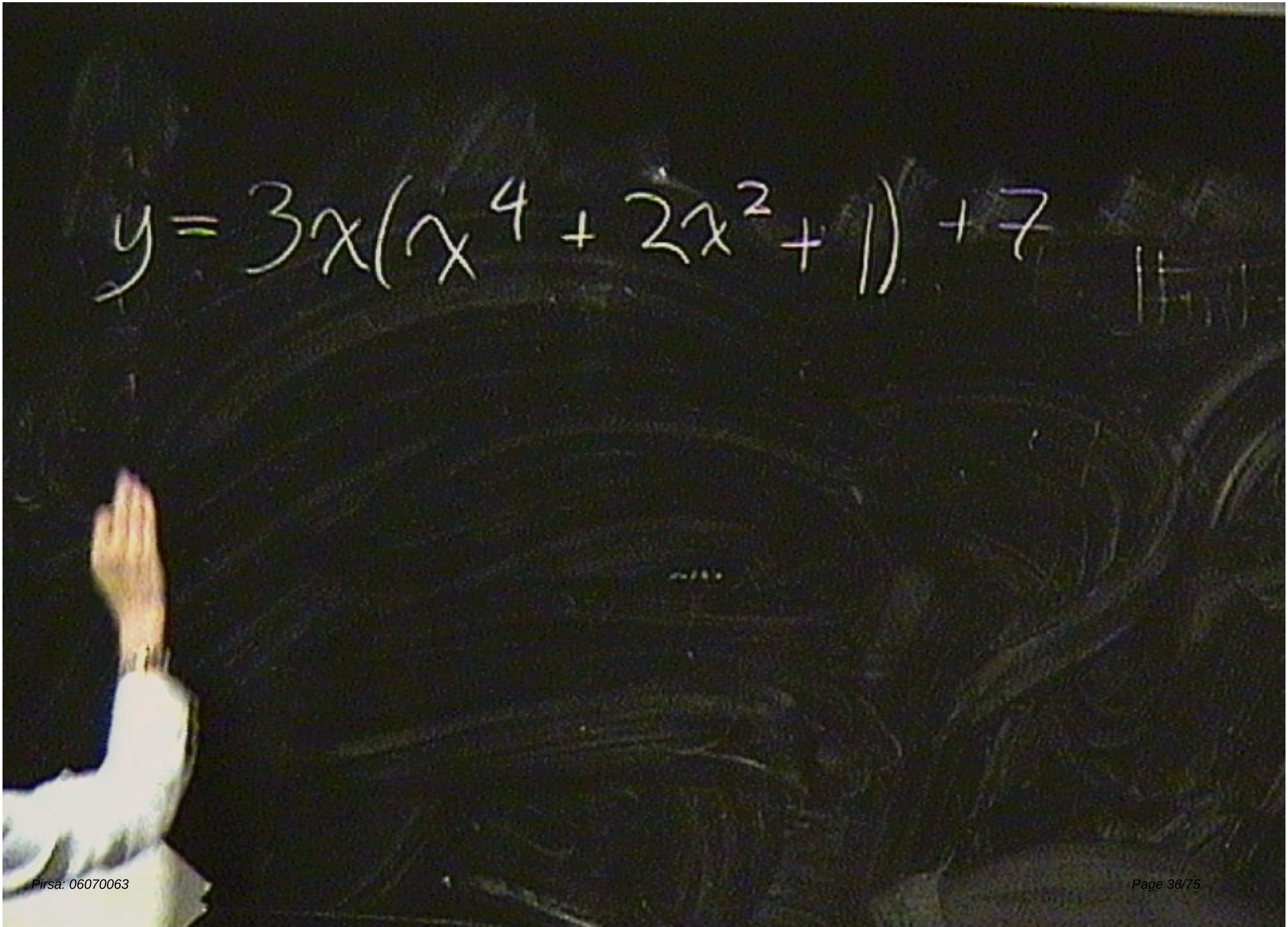
1) $2x$

2) $45p^4$

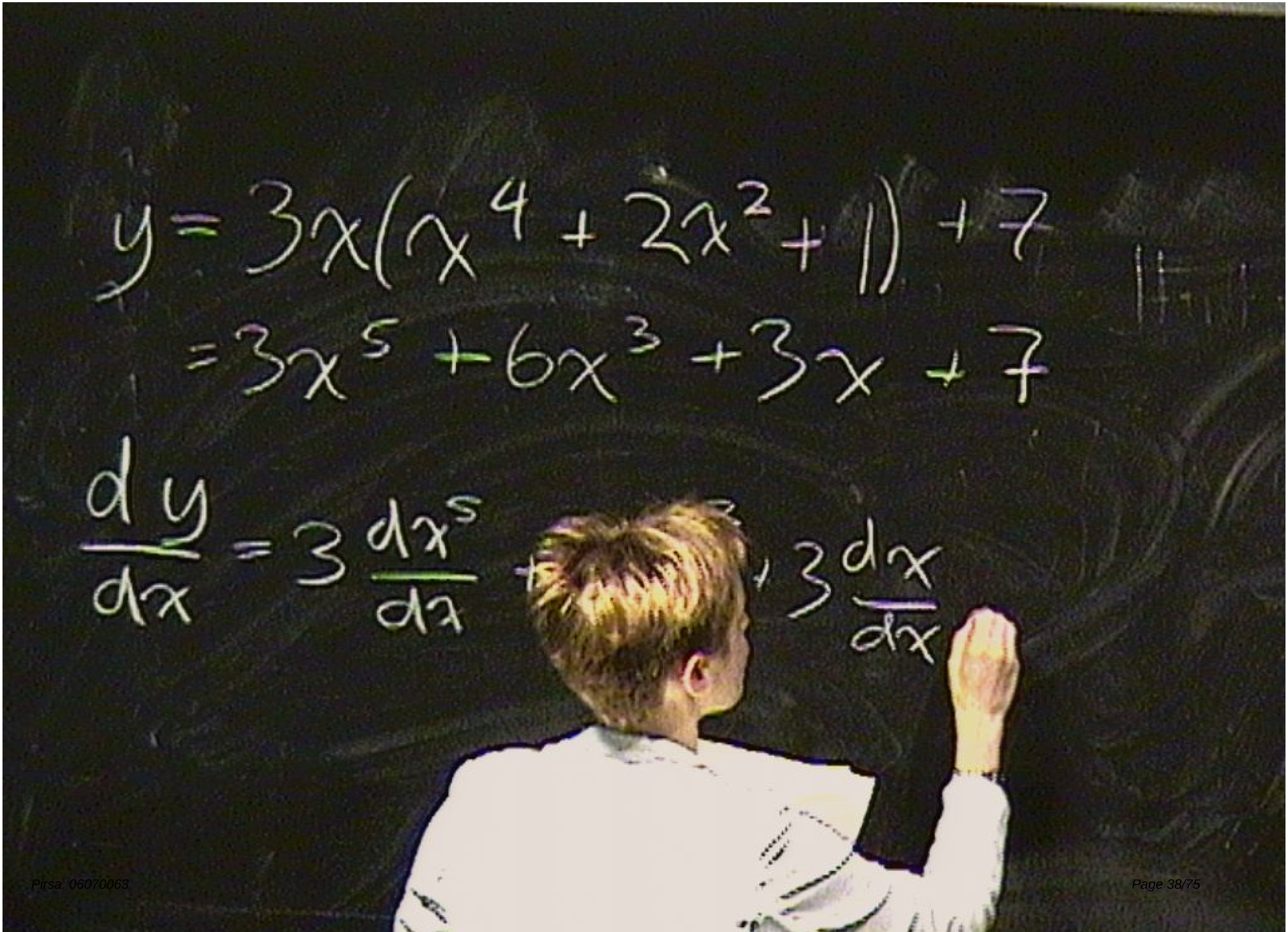
3) $2t-4$

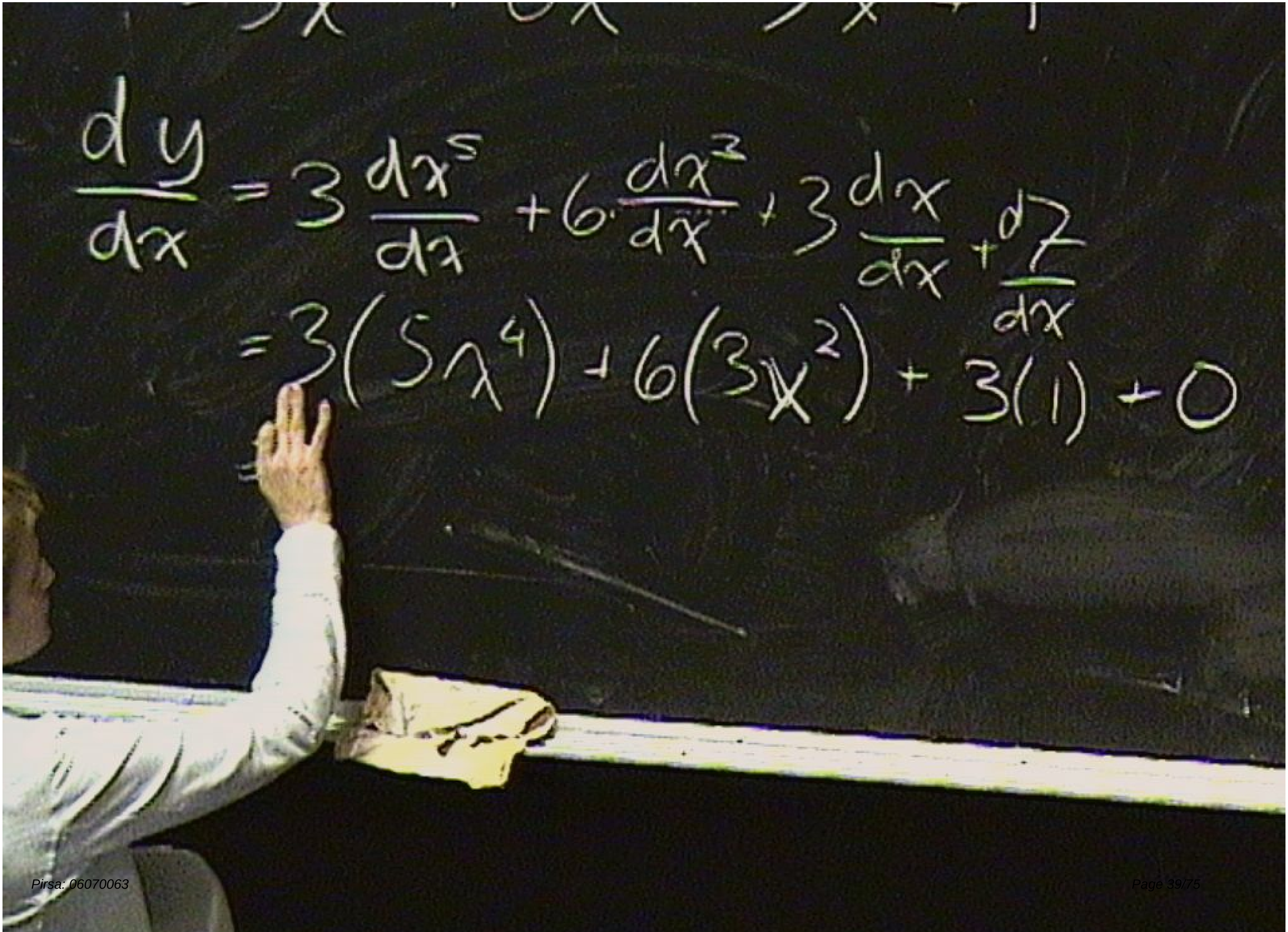
4) $15x^4+18x^2+3$


$y = 3x(x^4 + 2x^2 + 1)$


$y = 3x(x^4 + 2x^2 + 1) + 7$

$$y = 3x(x^4 + 2x^2 + 1) + 7$$
$$= 3x^5 + 6x^3 + 3x + 7$$


$y = 3x(x^4 + 2x^2 + 1) + 7$
$= 3x^5 + 6x^3 + 3x + 7$
$\frac{dy}{dx} = 3\frac{dx^5}{dx}$
$3\frac{dx}{dx}$


$\frac{dy}{dx} = 3\frac{dx^5}{dx} + 6\cdot\frac{dx^3}{dx} + 3\frac{dx}{dx} + \frac{d7}{dx}$
$= 3(5x^4) + 6(3x^2) + 3(1) + 0$
$=$

$$\frac{dy}{dx} = 3\frac{dx^5}{dx} + 6\frac{dx^3}{dx} + 3\frac{dx}{dx} + \frac{d7}{dx}$$

$$= 3(5x^4) + 6(3x^2) + 3(1) + 0$$

$$= 15x^4 + 18x^2 + 3$$

Constant Rule

If f(x) is a constant function (i.e. f(x) = c), then $\dfrac{dc}{dx} = 0$

Constant Multiple Rule

If g(x) = cf(x), then $\dfrac{dg}{dx} = \dfrac{d}{dx}\left[cf(x)\right] = c\dfrac{df}{dx}$

Sum Rule

If h(x) = f(x) + g(x), then $\dfrac{dh}{dx} = \dfrac{d}{dx}\left[f(x) + g(x)\right] = \dfrac{df}{dx} + \dfrac{dg}{dx}$

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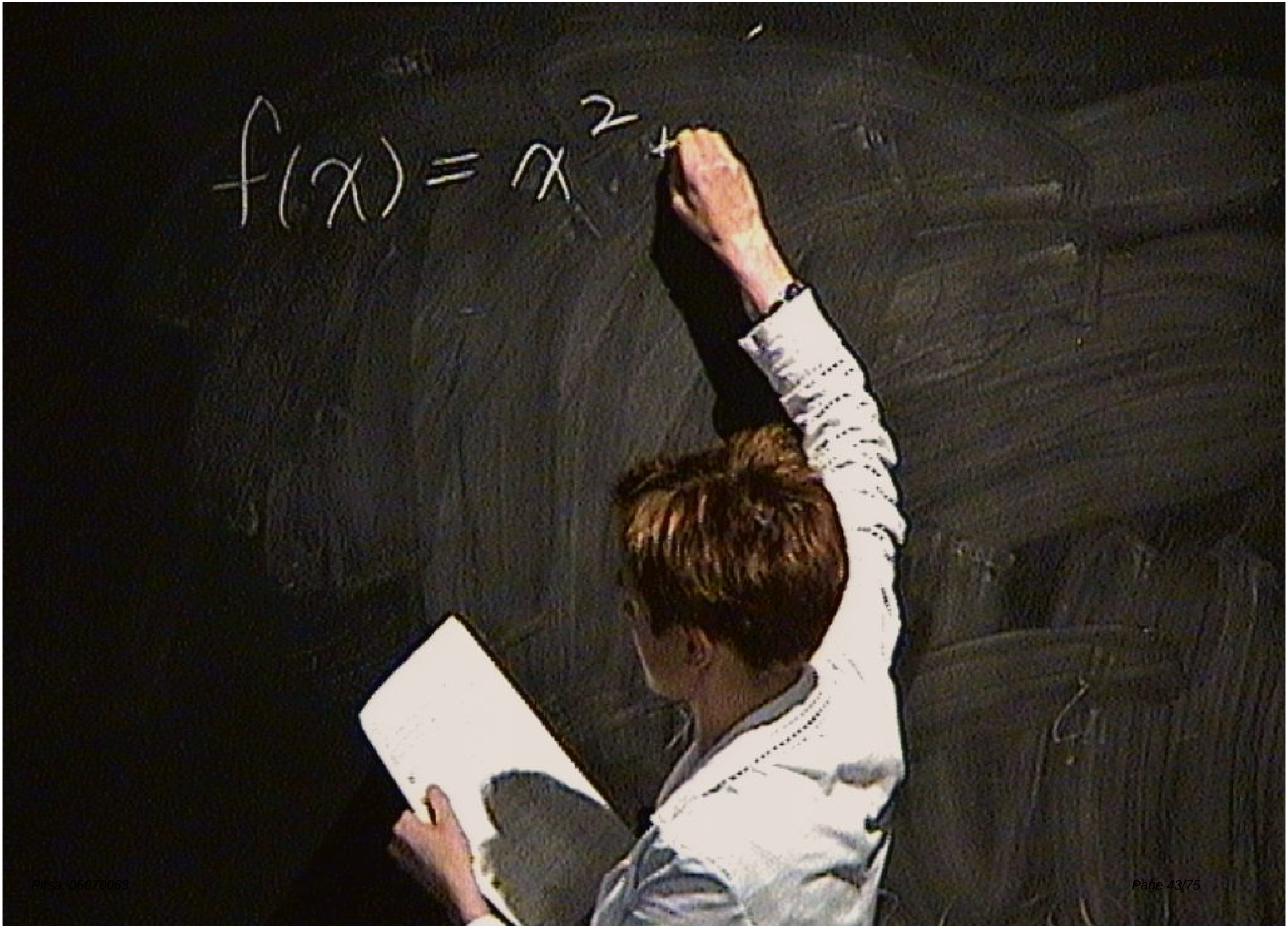
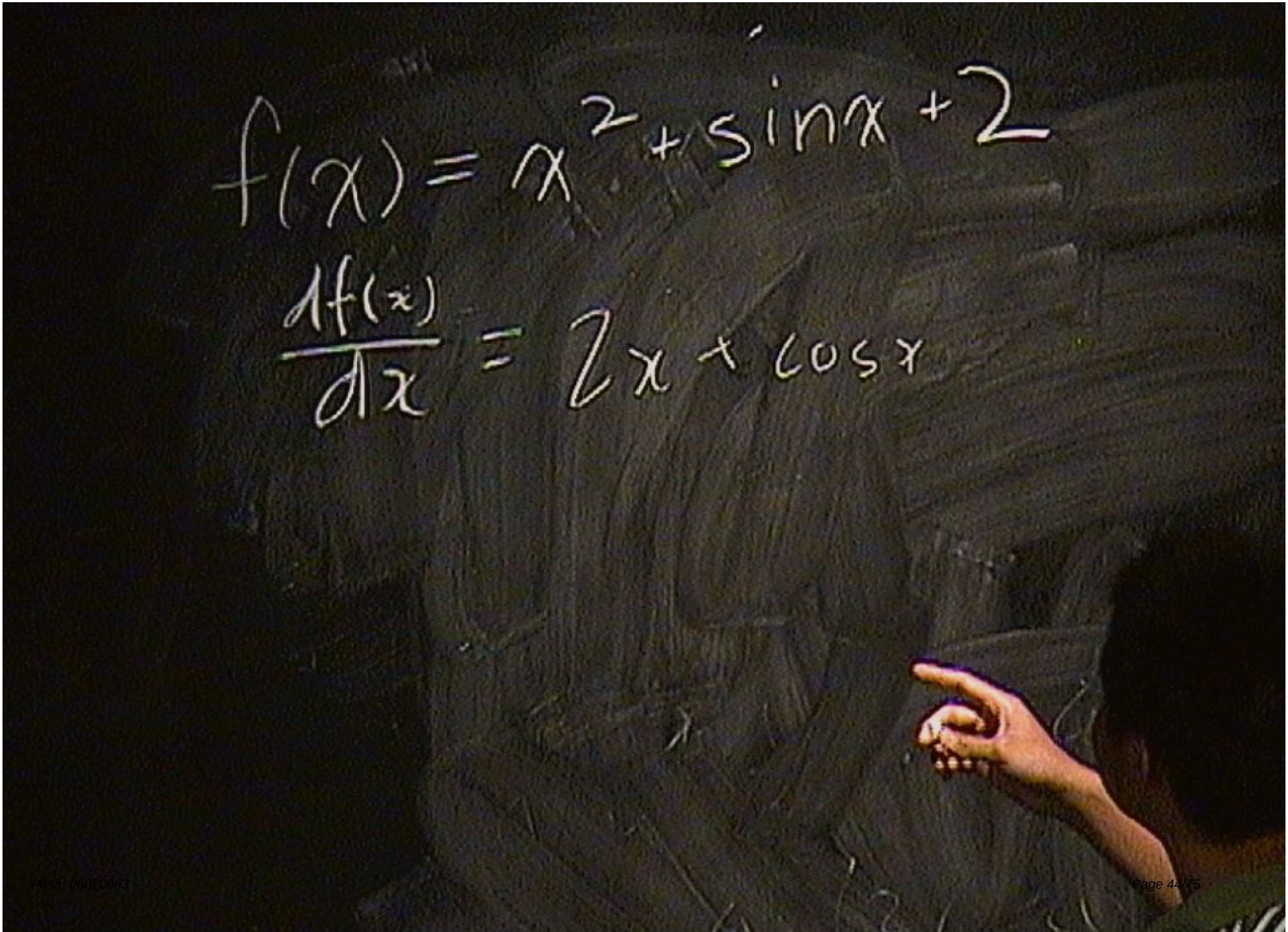
Exp/Trig/Log

$$\frac{d}{dx}e^x = e^x \quad \frac{d}{dx}\ln|x| = \frac{1}{x}$$

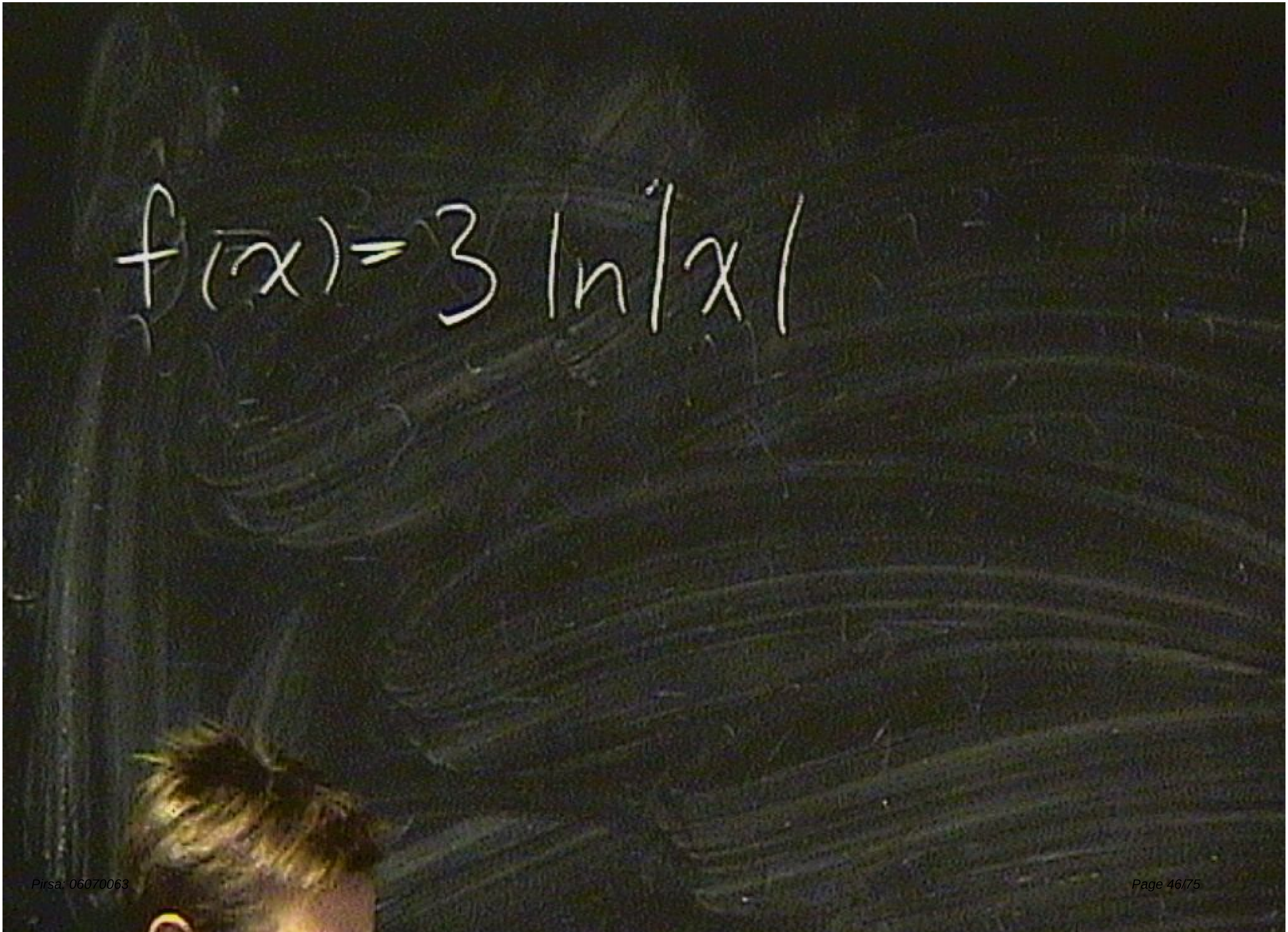
$$\frac{d}{dx}\sin x = \cos x \quad \frac{d}{dx}\cos x = -\sin x$$

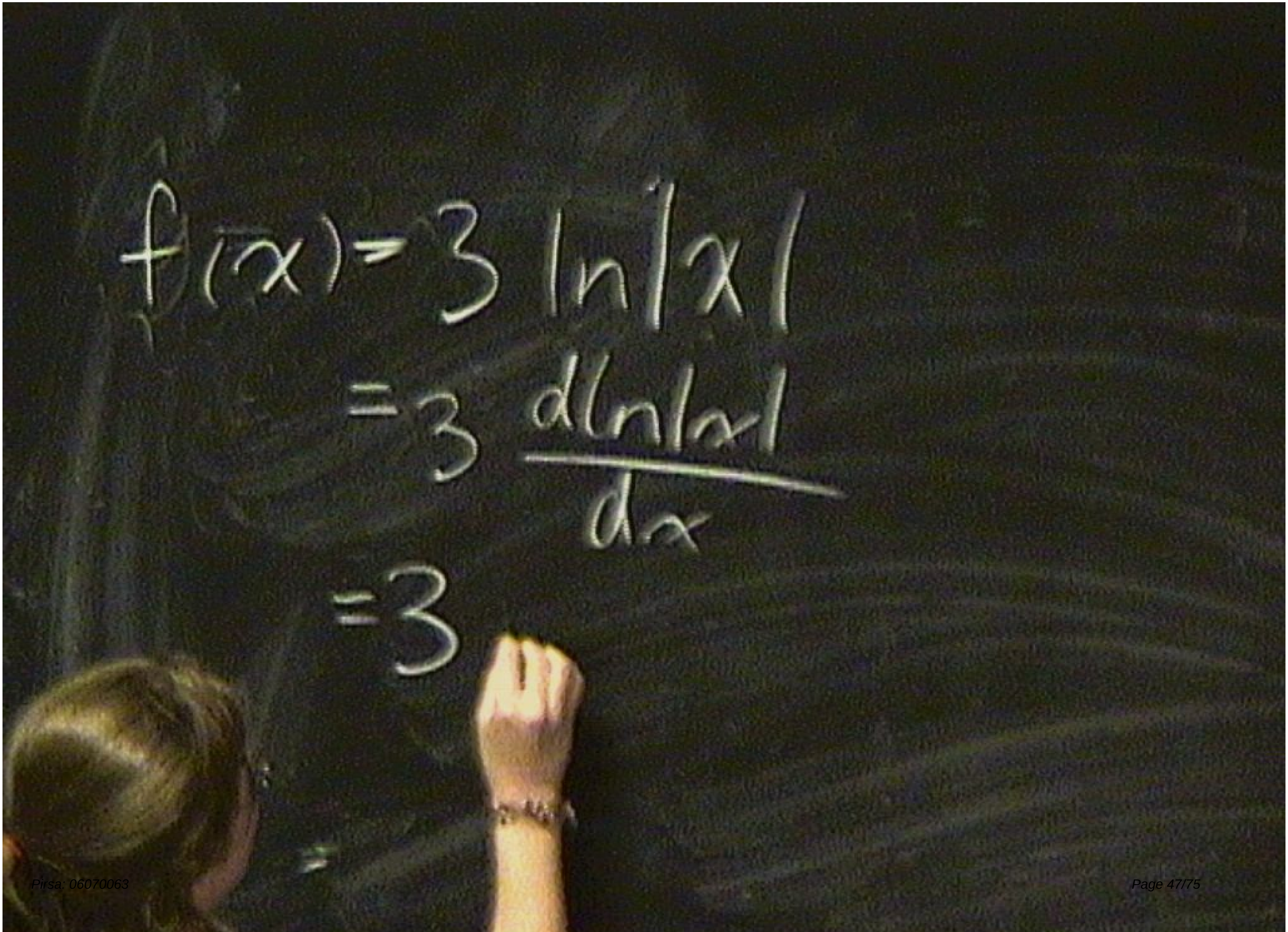

$f(x) = x^2 +$


$f(x) = x^2 + \sin x + 2$
$\frac{df(x)}{dx} = 2x + \cos x$

$$f(x) = x^2 + \sin x + 2$$

$$\frac{df(x)}{dx} = 2x + \cos x$$


$f(x) = 3 \ln|x|$


f'(x) = 3 ln|x|
= 3 d(ln|x|)/dx
= 3
=

$$f(x) = 4e^{x}$$

$$f(x) = 4e^x$$

$$f'(x) = 4e^x$$

If f(x) is a constant function (i.e. f(x) = c), then $\frac{dc}{dx} = 0$

Constant Multiple Rule

If g(x) = cf(x), then $\frac{dg}{dx} = \frac{d}{dx}[cf(x)] = c\frac{df}{dx}$

Sum Rule

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$$\frac{d}{dx}e^x = e^x \quad \frac{d}{dx}\ln|x| = \frac{1}{x}$$

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Chain Rule

If h(x) = (f∘g)(x) = f(g(x)), then $\frac{dh}{dx} = \left(\frac{df}{dg}\right)\left(\frac{dg}{dx}\right)$

Constant Multiple Rule

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Sum Rule

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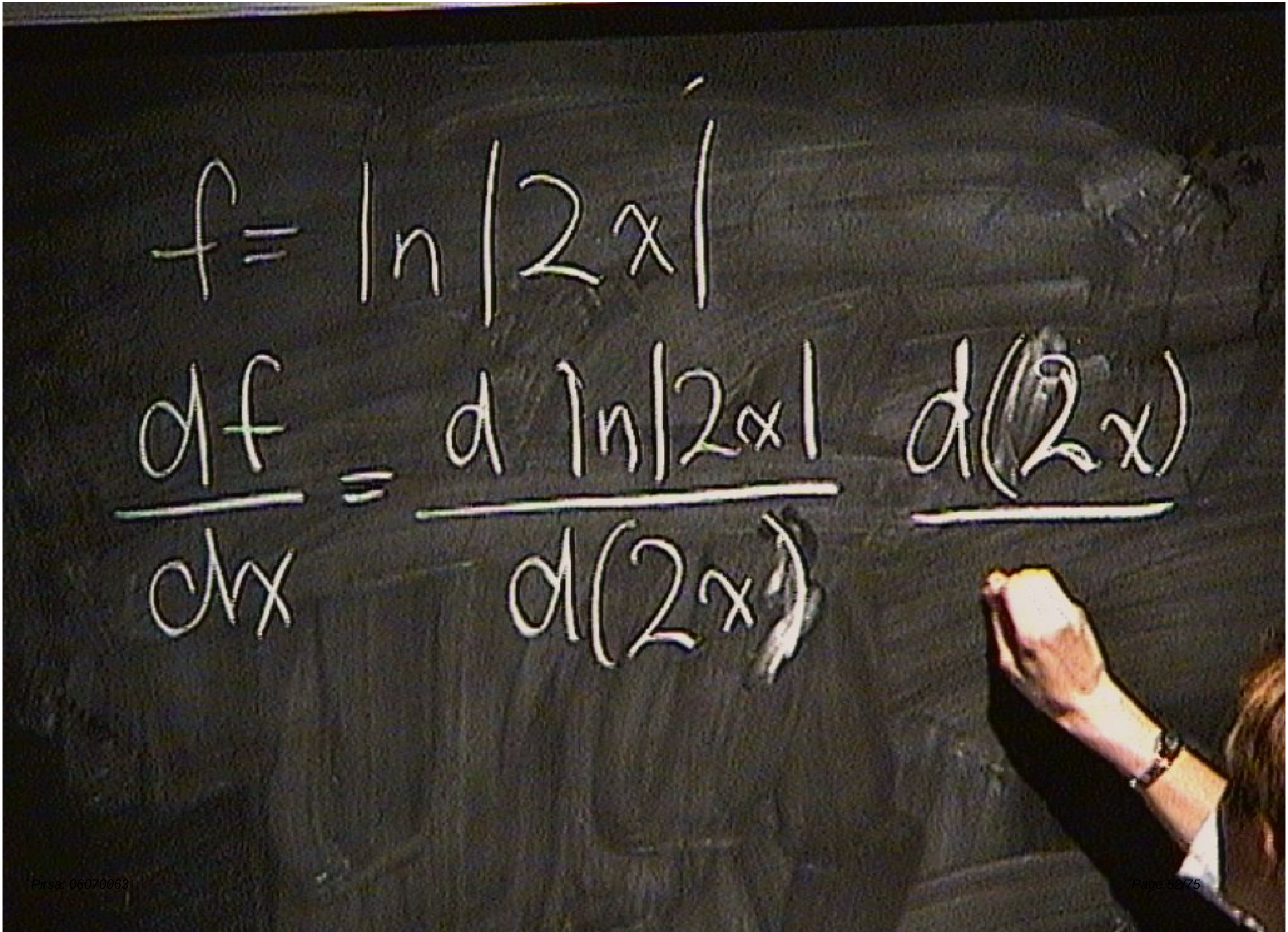

$f = \ln|2x|$
$\frac{df}{dx} = \frac{d \ln|2x|}{d(2x)} \, \frac{d(2x)}{}$

$$f = \ln|2x|$$

$$\frac{df}{dx} = \frac{d\,\ln|2x|}{d(2x)}\,\frac{d(2x)}{dx}$$

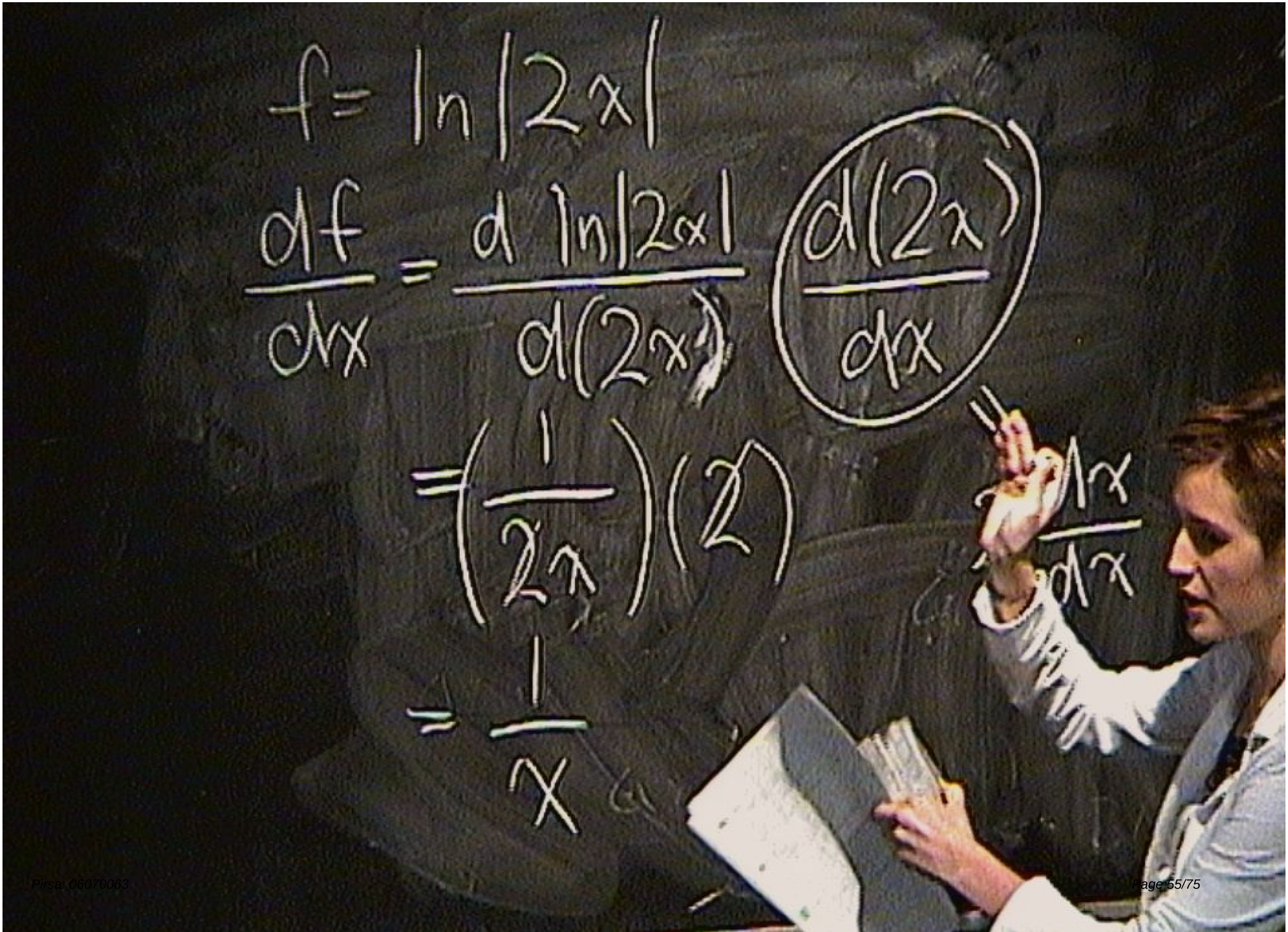
$$= \left(\frac{1}{2x}\right)(2)$$

$$f = \ln|2x|$$

$$\frac{df}{dx} = \frac{d\ln|2x|}{d(2x)}\,\frac{d(2x)}{dx}$$

$$= \left(\frac{1}{2x}\right)(2)$$

$$= \frac{1}{x}$$


$f = \ln|2x|$
$\frac{df}{dx} = \frac{d \ln|2x|}{d(2x)} \frac{d(2x)}{dx}$
$= \left(\frac{1}{2x}\right)(2)$
$= \frac{1}{x}$

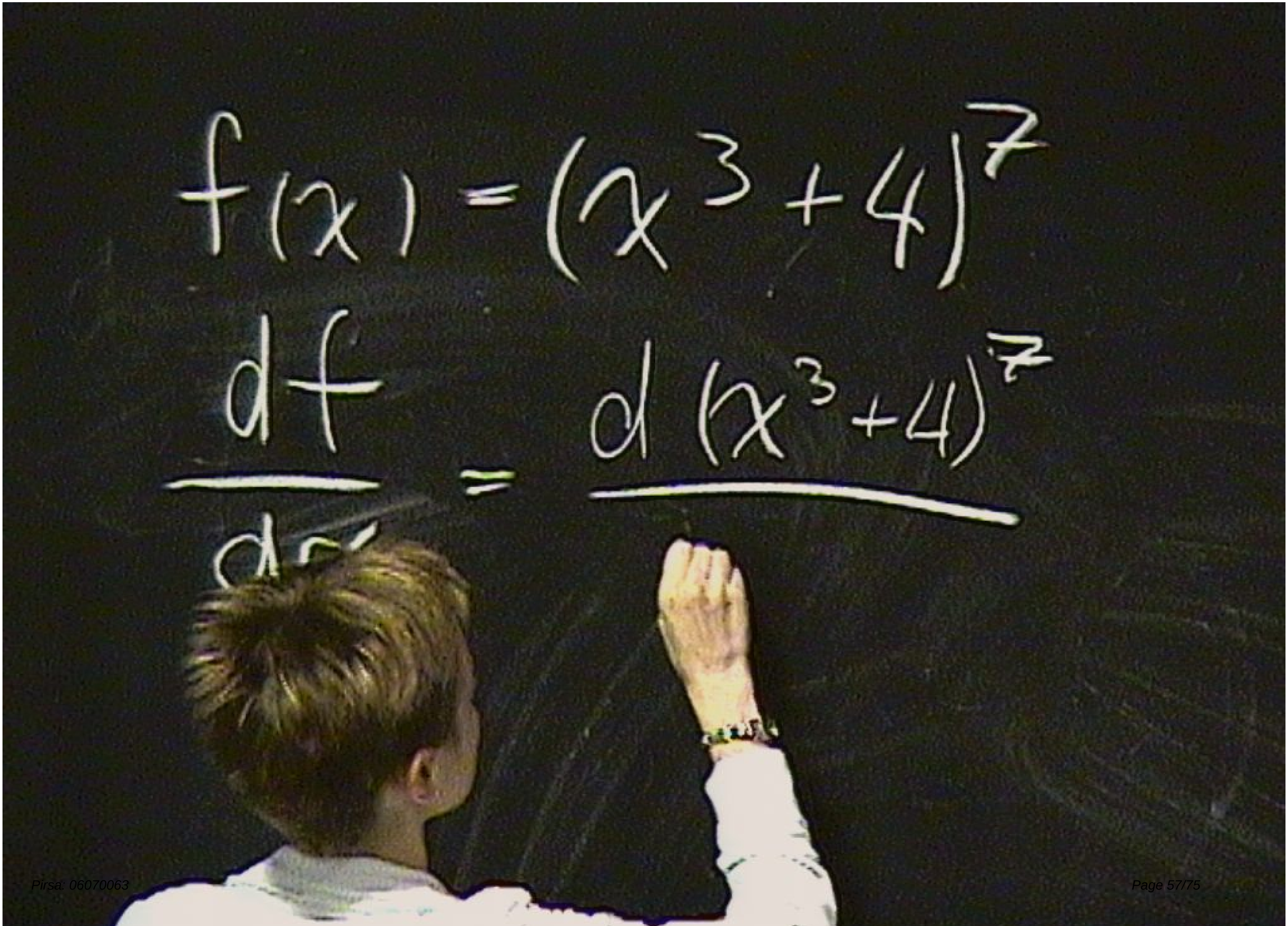
$$f = \ln|2x|$$

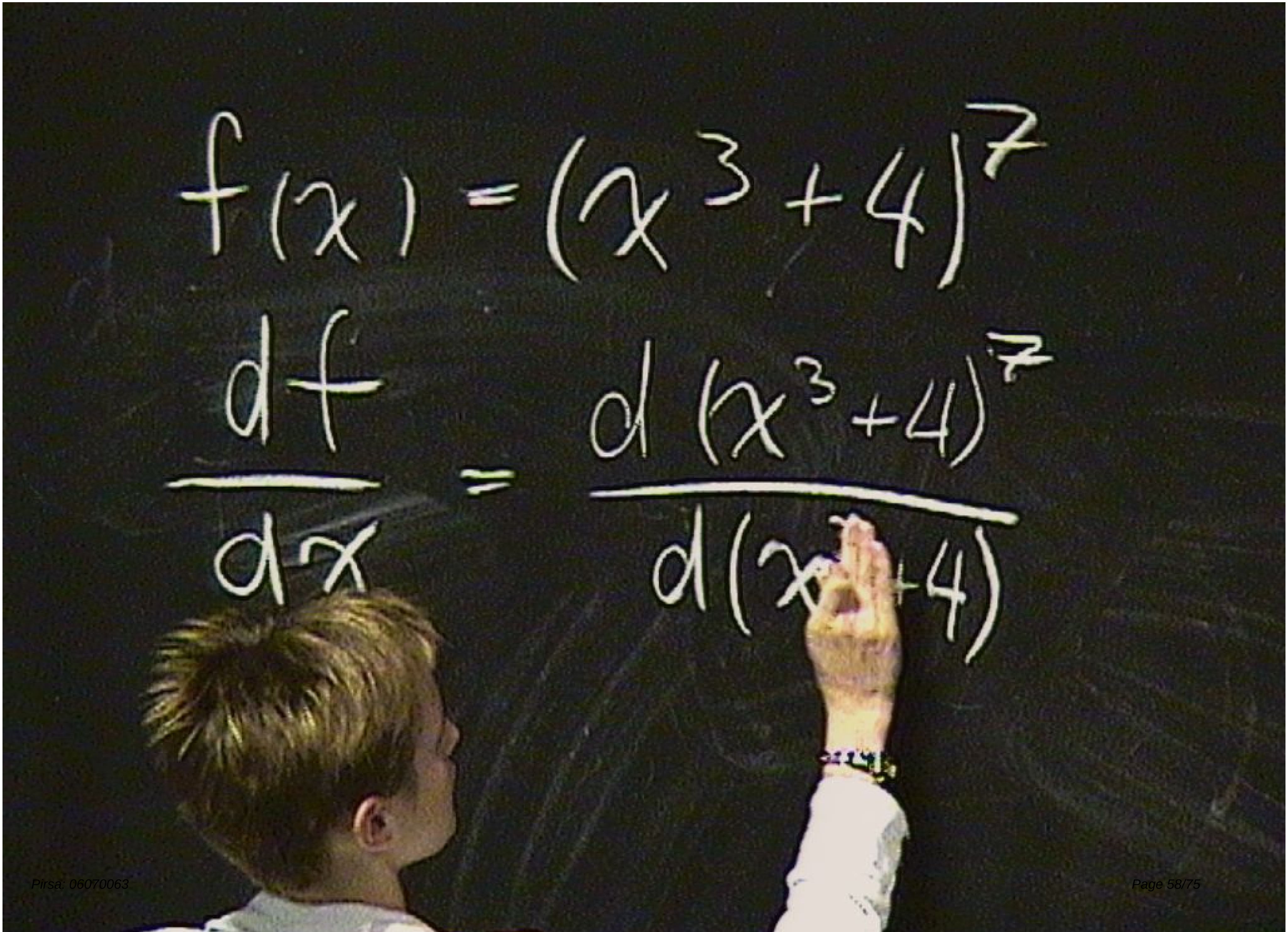
$$\frac{df}{dx} = \frac{d \ln|2x|}{d(2x)} \frac{d(2x)}{dx}$$

$$\frac{d(2x)}{dx} = 2\frac{dx}{dx}$$

$$= \left(\frac{1}{2x}\right)(2)$$

$$= \frac{1}{x}$$


$f(x) = (x^3+4)^7$
df
$=$
$d(x^3+4)^7$

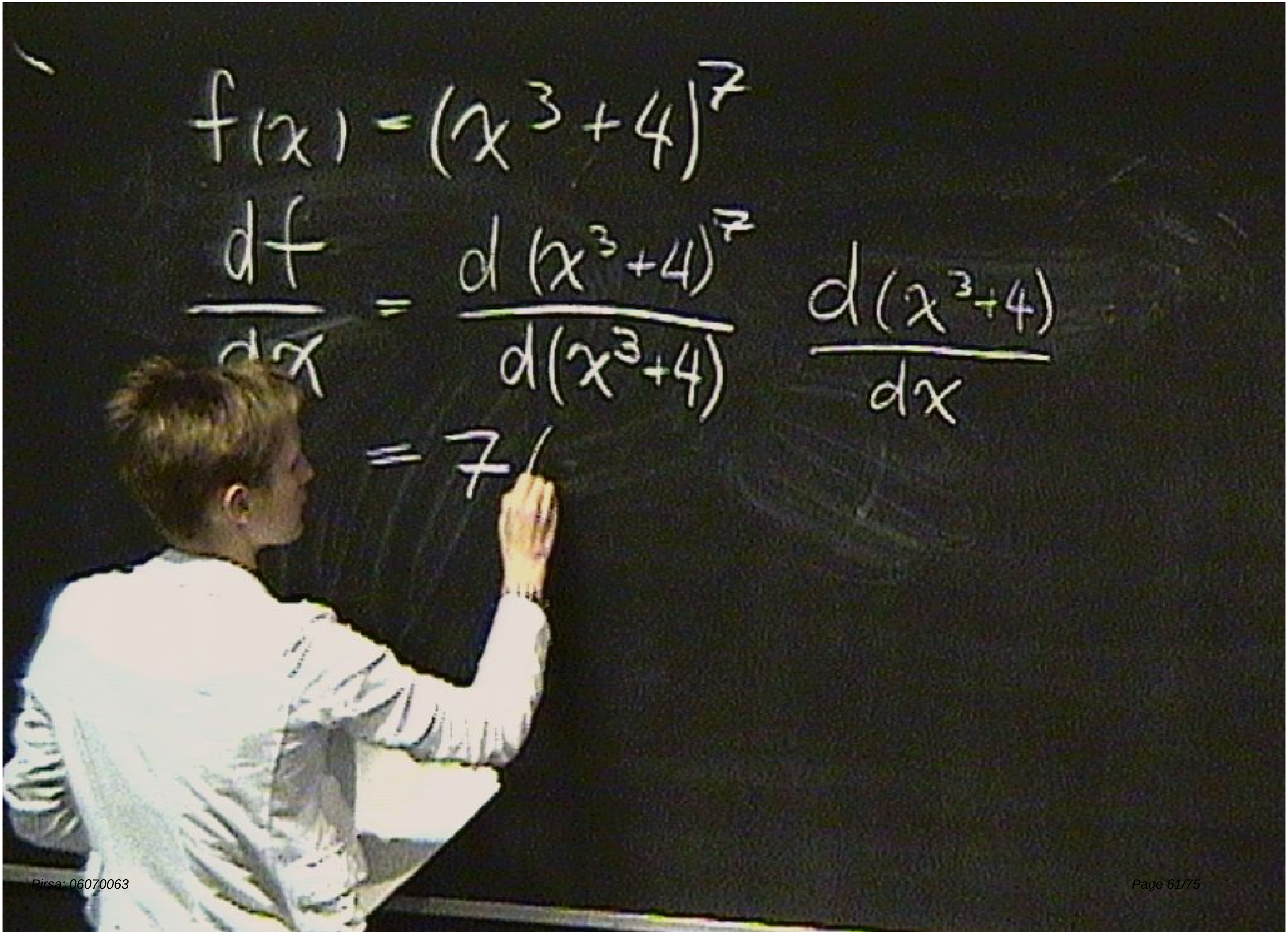

$f(x) = (x^3+4)^7$
$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)}$

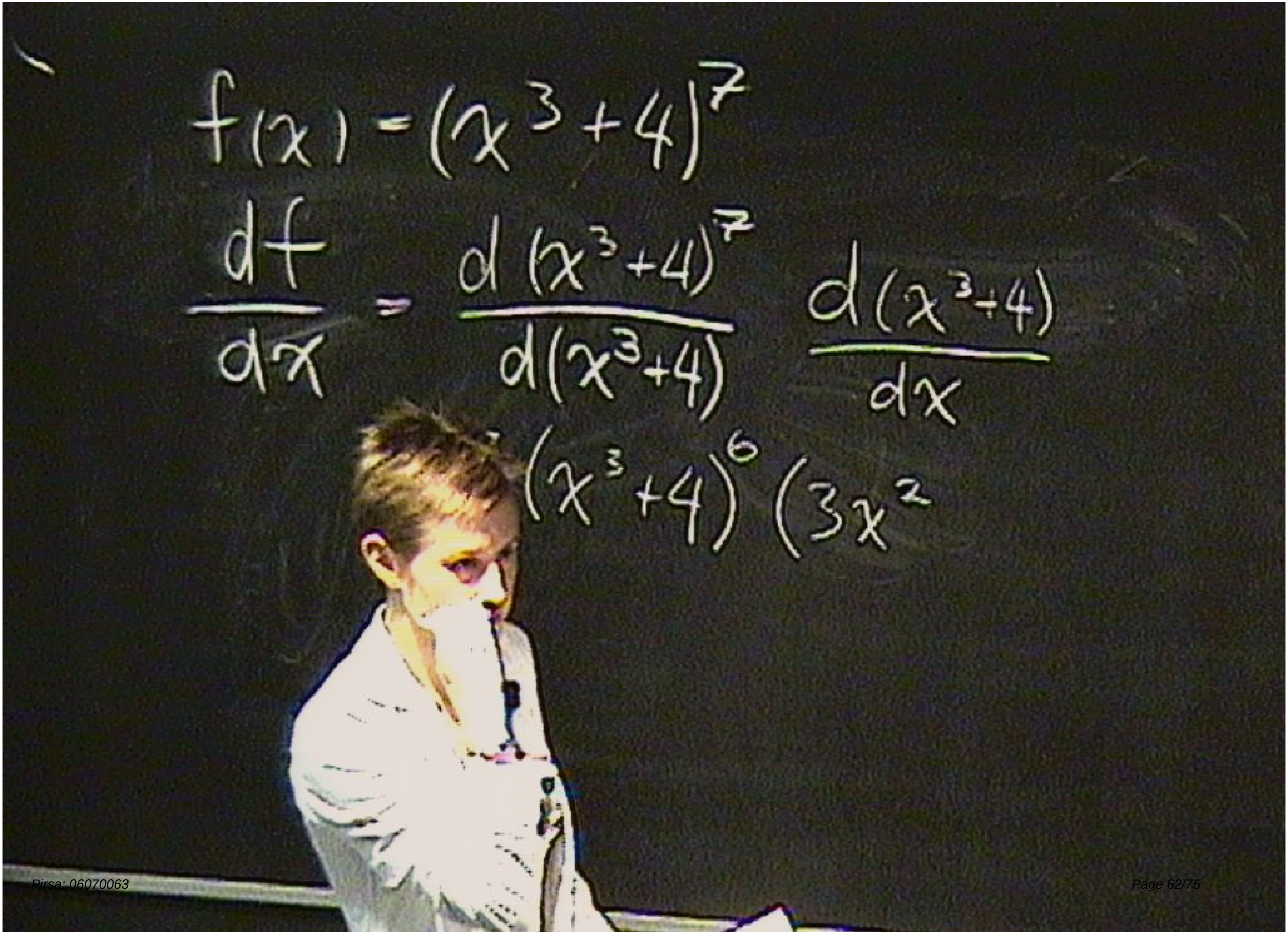
$$f(x) = (x^3+4)^7$$

$$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)}$$

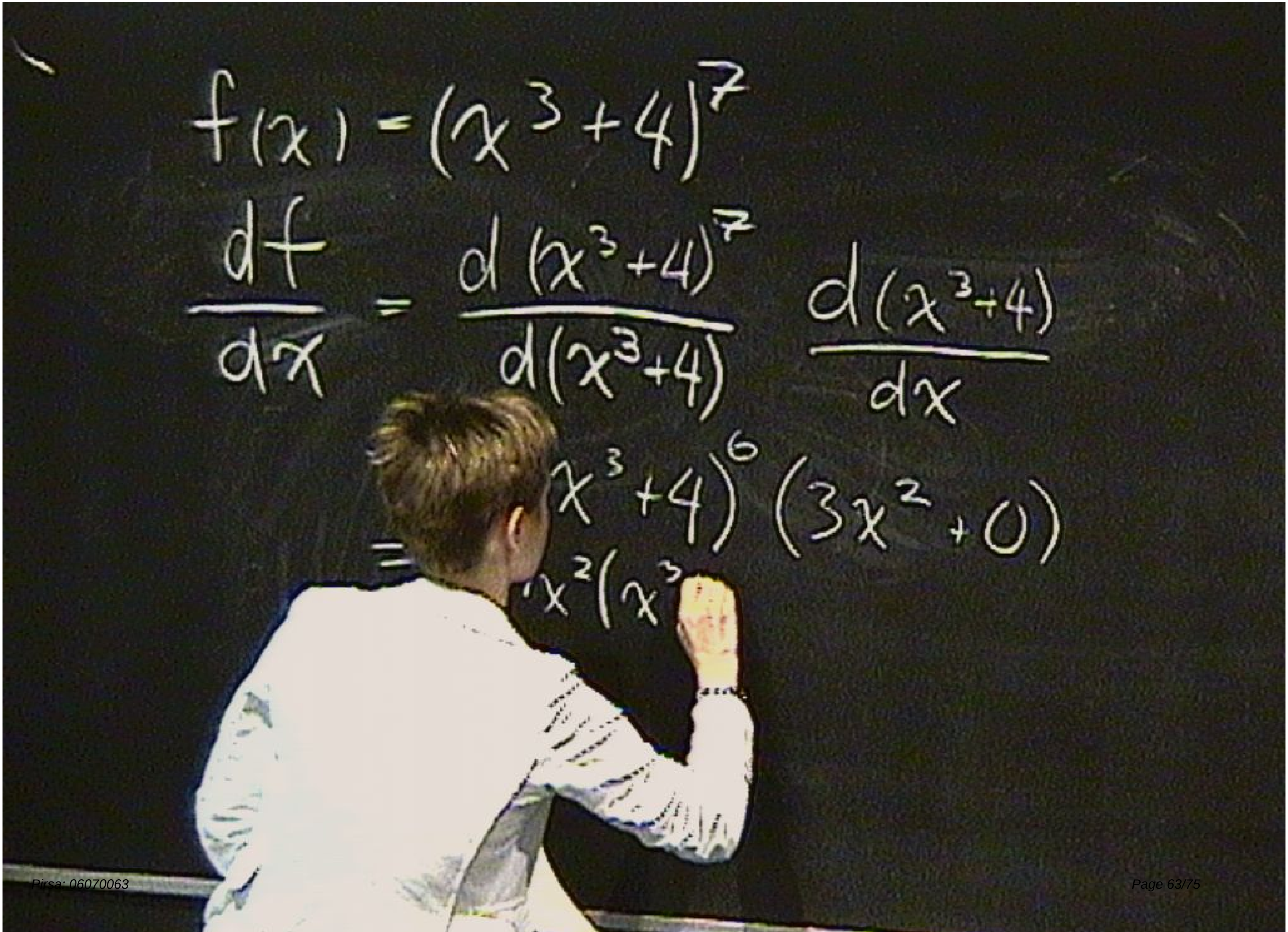
$$f(x) = (x^3+4)^7$$

$$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)} \quad \frac{d(x^3+4)}{dx}$$


$f(x) = (x^3+4)^7$
$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)} \frac{d(x^3+4)}{dx}$
$= 7($



$f(x) = (x^3+4)^7$
$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)} \frac{d(x^3+4)}{dx}$
$(x^3+4)^6 (3x^2$


$f(x) = (x^3+4)^7$
$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)} \frac{d(x^3+4)}{dx}$
$x^3+4)^6 (3x^2+0)$
$= x^2(x^3$

$$f(x) = (x^3+4)^7$$

$$\frac{df}{dx} = \frac{d(x^3+4)^7}{d(x^3+4)} \frac{d(x^3+4)}{dx}$$

$$= 7(x^3+4)^6 (3x^2+0)$$

$$= 21 \cdot x^2 (x^3+4)^6$$

Constant Multiple Rule

$$\text{If } g(x) = cf(x), \text{ then } \frac{dg}{dx} = \frac{d}{dx}[cf(x)] = c\frac{df}{dx}$$

Sum Rule

$$\text{If } h(x) = f(x) + g(x), \text{ then } \frac{dh}{dx} = \frac{d}{dx}[f(x) + g(x)] = \frac{df}{dx} + \frac{dg}{dx}$$

Exp/Trig/Log

$$\frac{d}{dx}e^x = e^x \quad \frac{d}{dx}\ln|x| = \frac{1}{x}$$

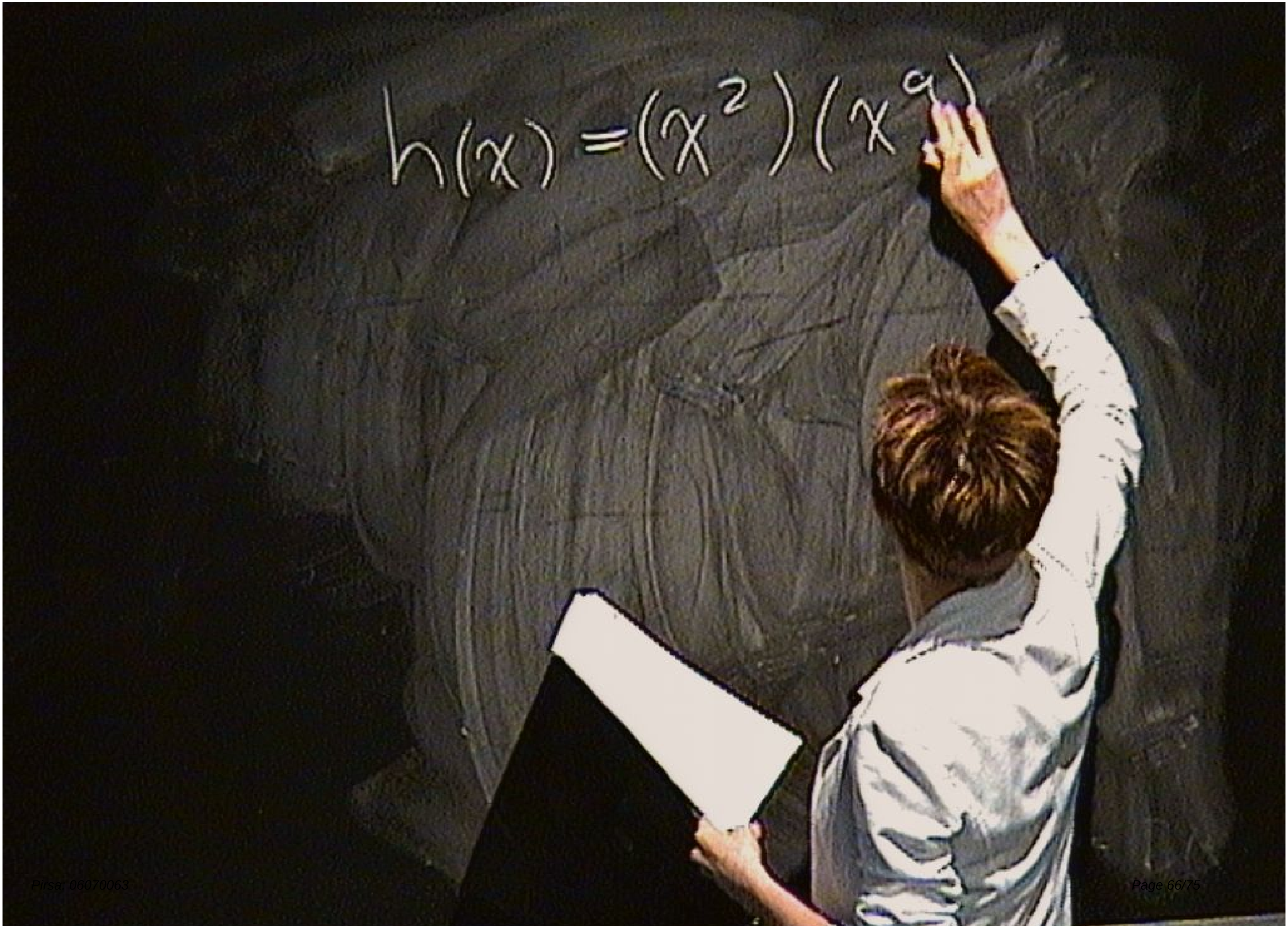
$$\frac{d}{dx}\sin x = \cos x \quad \frac{d}{dx}\cos x = -\sin x$$

Chain Rule

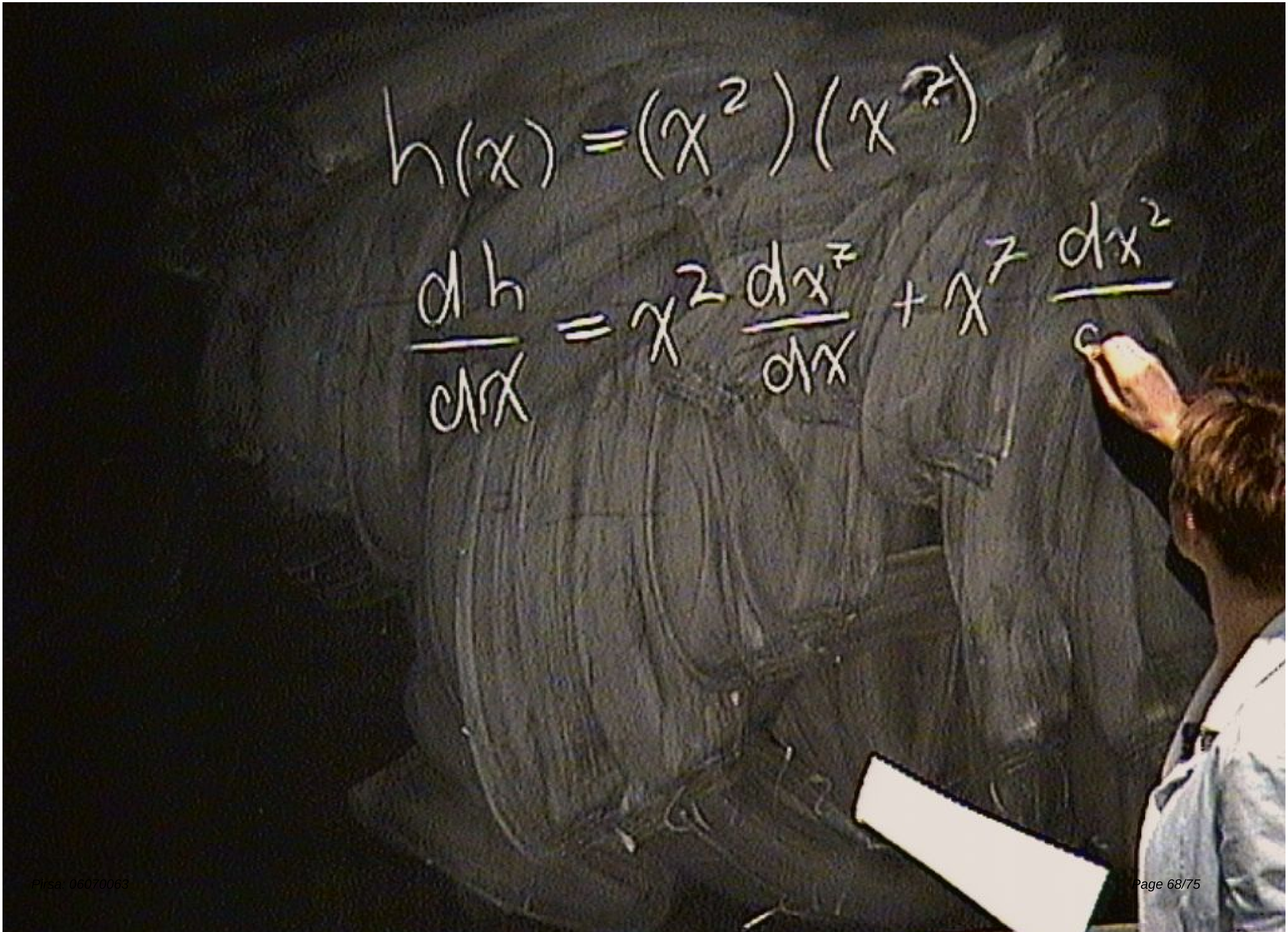
$$\text{If } h(x) = (f \circ g)(x) = f(g(x)), \text{ then } \frac{dh}{dx} = \left(\frac{df}{dg}\right)\left(\frac{dg}{dx}\right)$$

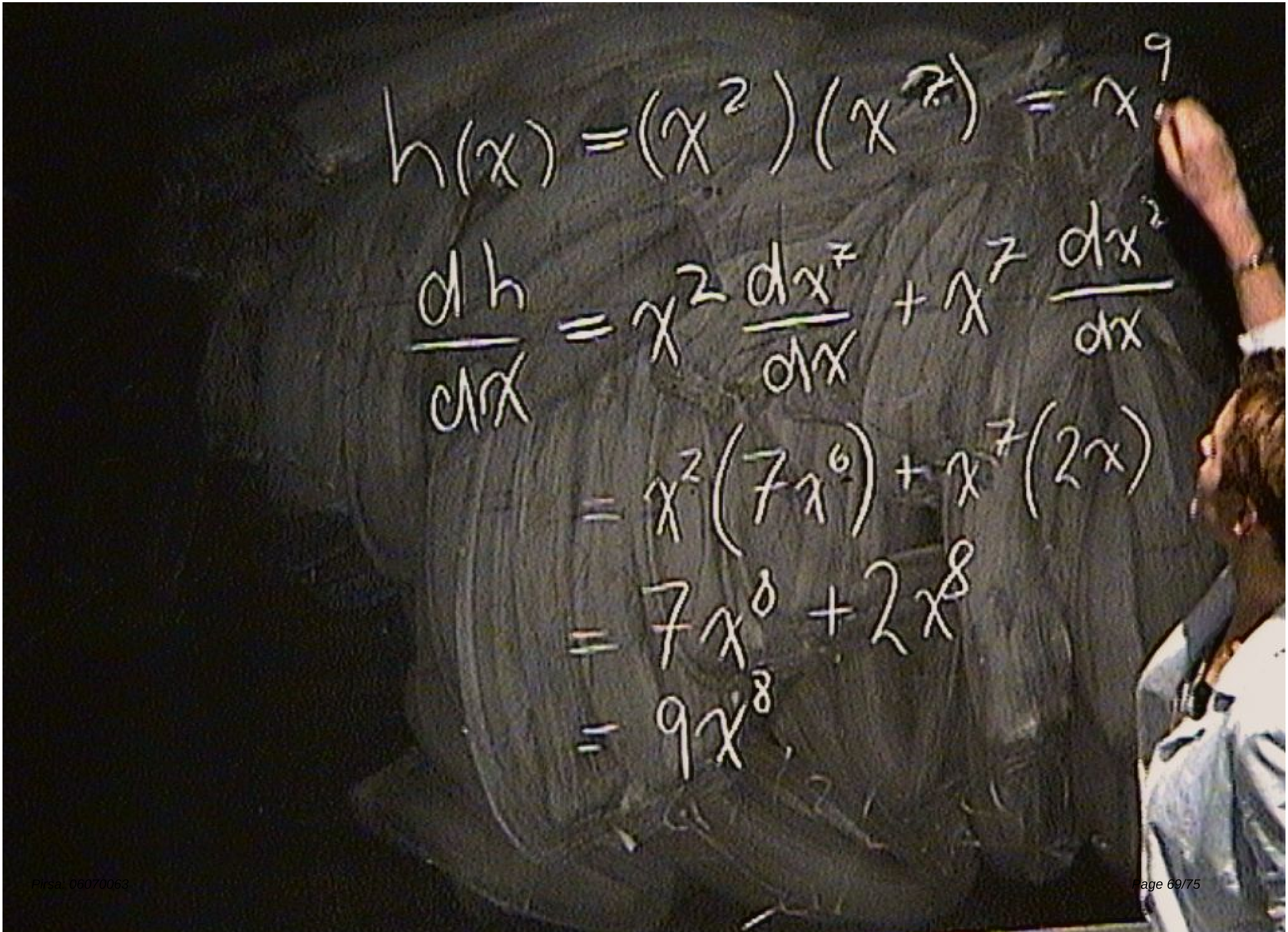
Product Rule

$$\text{If } h(x) = f(x)\, g(x), \text{ then } \frac{dh}{dx} = \frac{d}{dx}[f(x)g(x)] = g(x)\frac{df}{dx} + f(x)\frac{dg}{dx}$$


$h(x)=(x^2)(x^9)$

$$h(x) = (x^2)(x^{\pi})$$


h(x) = (x^2)(x^7)
dh/dx = x^2 dx^7/dx + x^7 dx^2/d


$h(x) = (x^2)(x^7) = x^9$
$\frac{dh}{dx} = x^2 \frac{dx^7}{dx} + x^7 \frac{dx^2}{dx}$
$= x^2(7x^6) + x^7(2x)$
$= 7x^8 + 2x^8$
$= 9x^8$

$$h(x) = (x^2)(x^7) = x^9$$

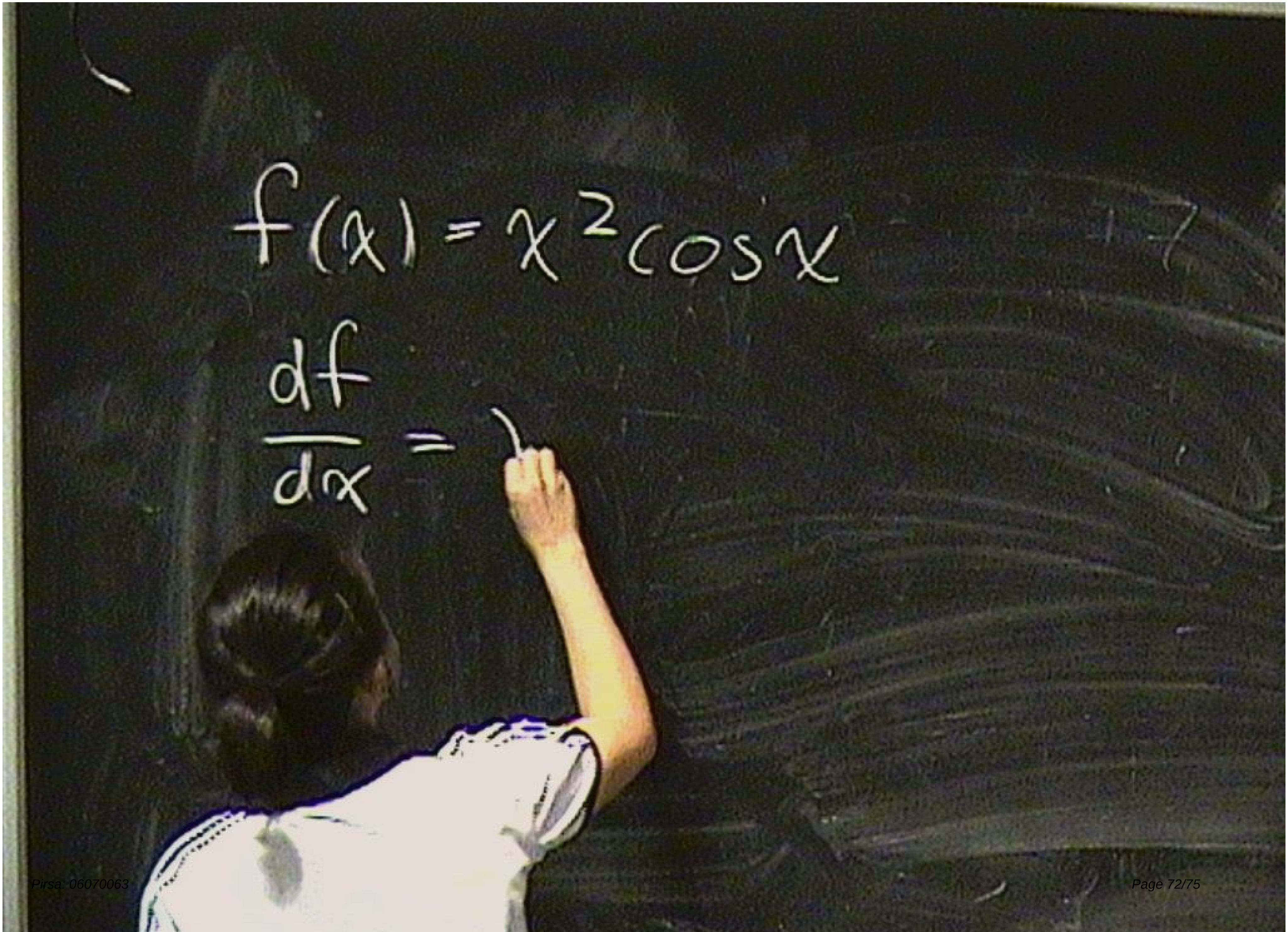
$$\frac{dh}{dx} = x^2 \frac{dx^7}{dx} + x^7 \frac{dx^2}{dx}$$

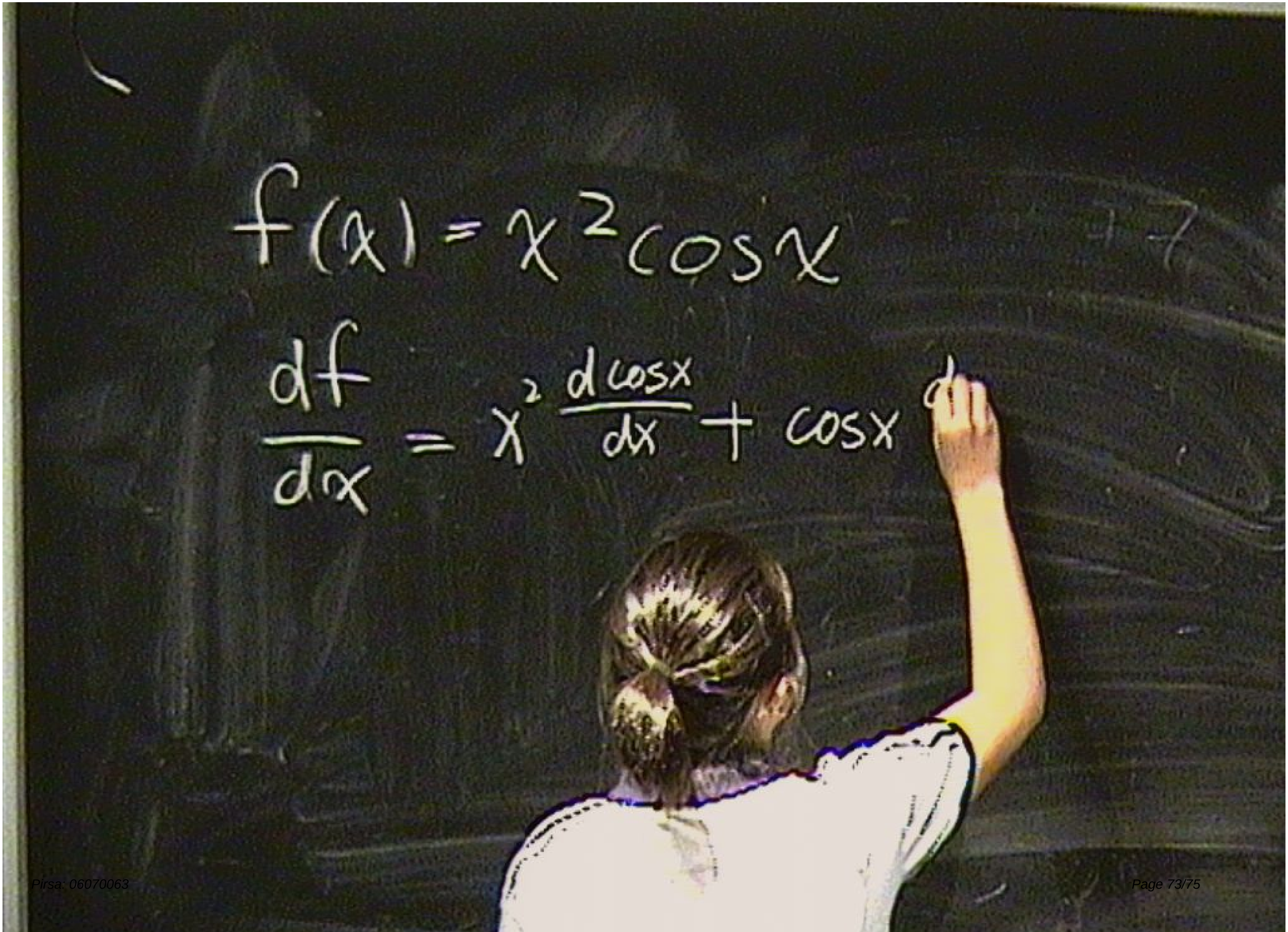
$$= x^2(7x^6) + x^7(2x)$$

$$= 7x^8 + 2x^8$$

$$= 9x^8$$

$$f(x) = x^2 \cos x$$


$f(x) = x^2 \cos x$
$\frac{df}{dx} =$


$f(x) = x^2 \cos x$
$\frac{df}{dx} = x^2 \frac{d\cos x}{dx} + \cos x \, d$

$$f(x) = x^2 \cos x$$

$$\frac{df}{dx} = x^2 \frac{d\cos x}{dx} + \cos x \frac{dx^2}{dx}$$

$$= -x^2 \sin x + 2x \cos x$$

Problem Set (4) – Complete Derivative Rules

Differentiate the following with respect to the given variable:

1) $h(x) = e^{2x+3}$

2) $r(t) = t \ln|t|$

3) $f(\theta) = \theta^3 \sin\theta + \dfrac{\cos\theta}{\theta}$