

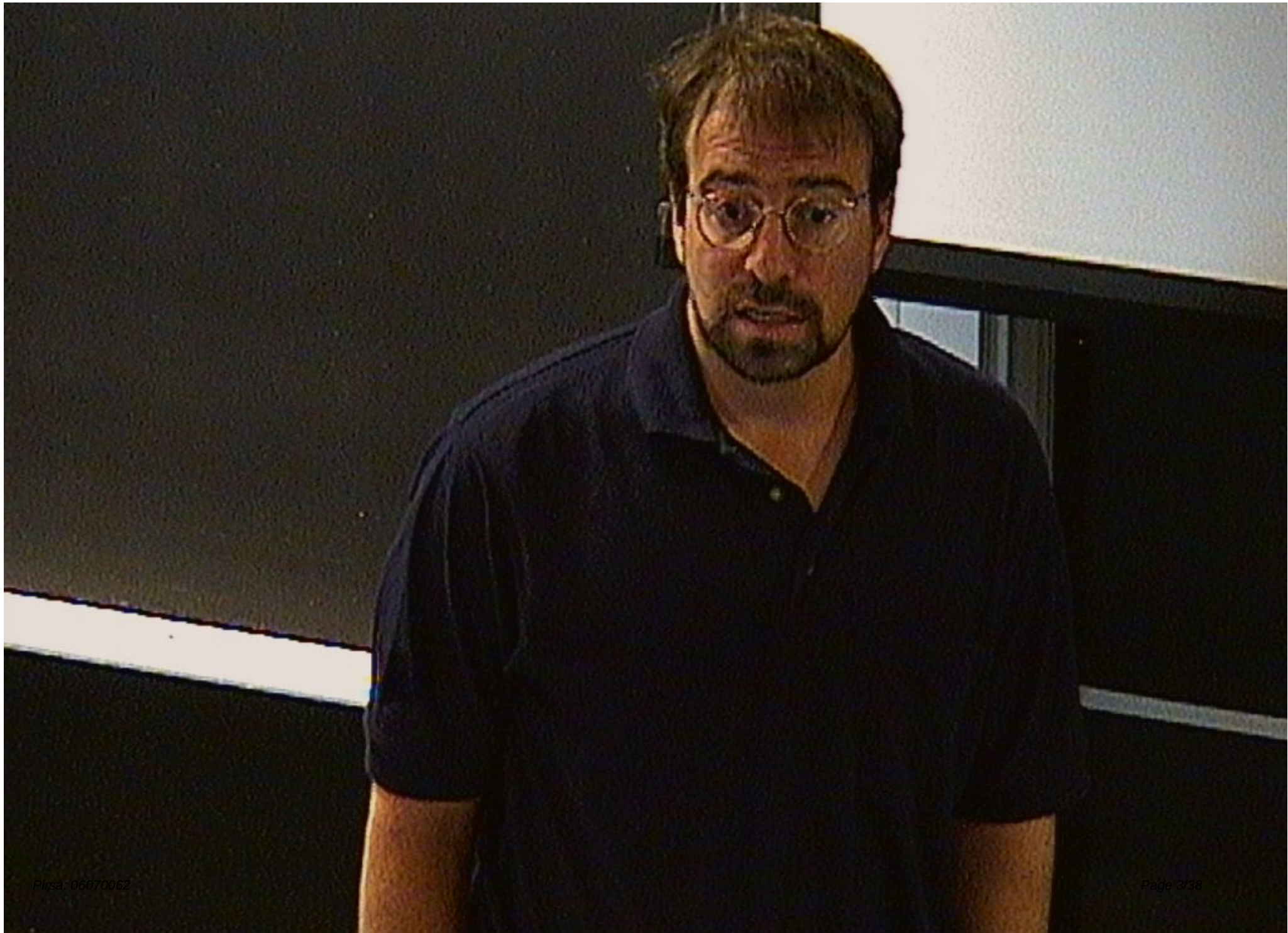
Title: Introduction and Math Primer: Differentiation

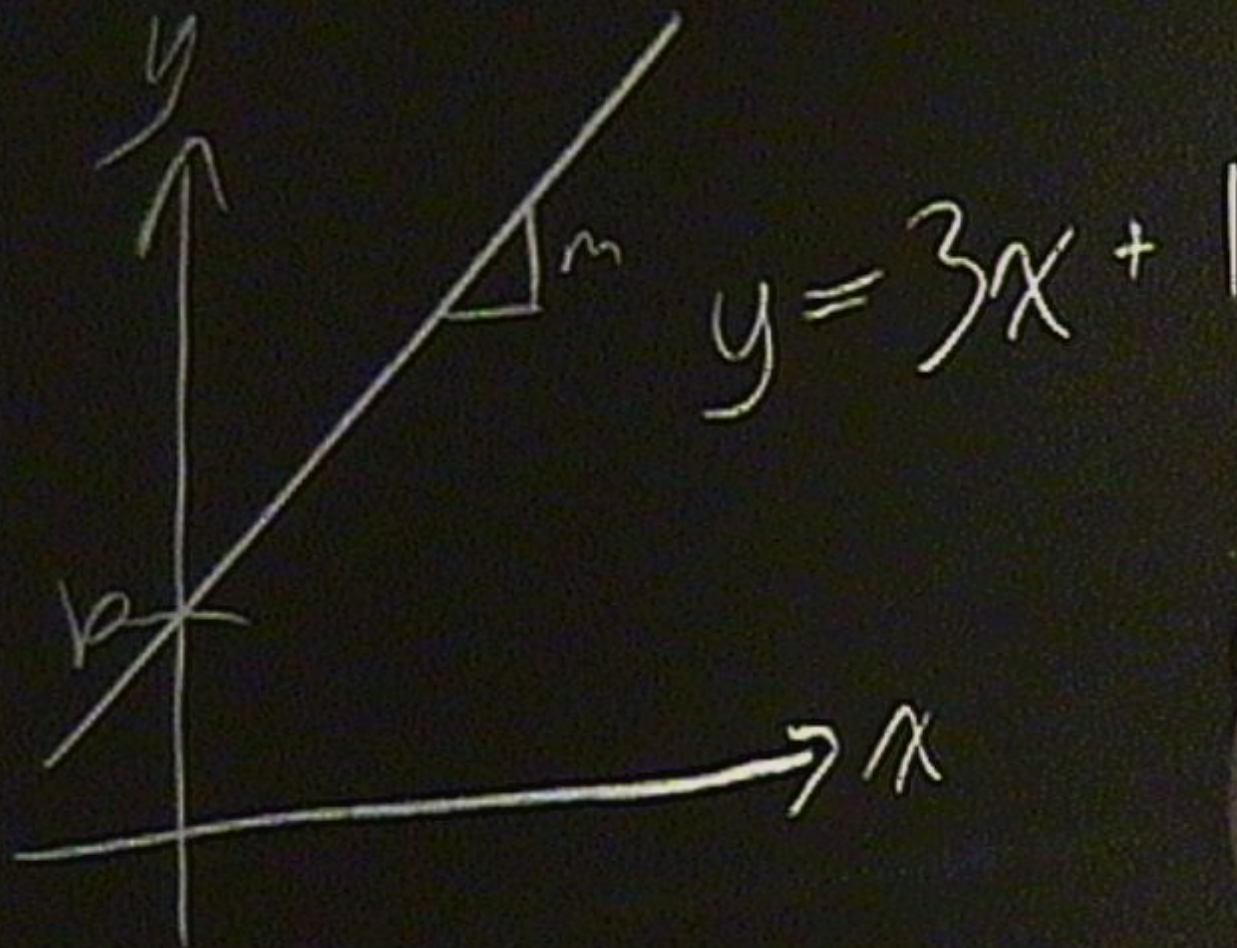
Date: Jul 24, 2006 09:00 AM

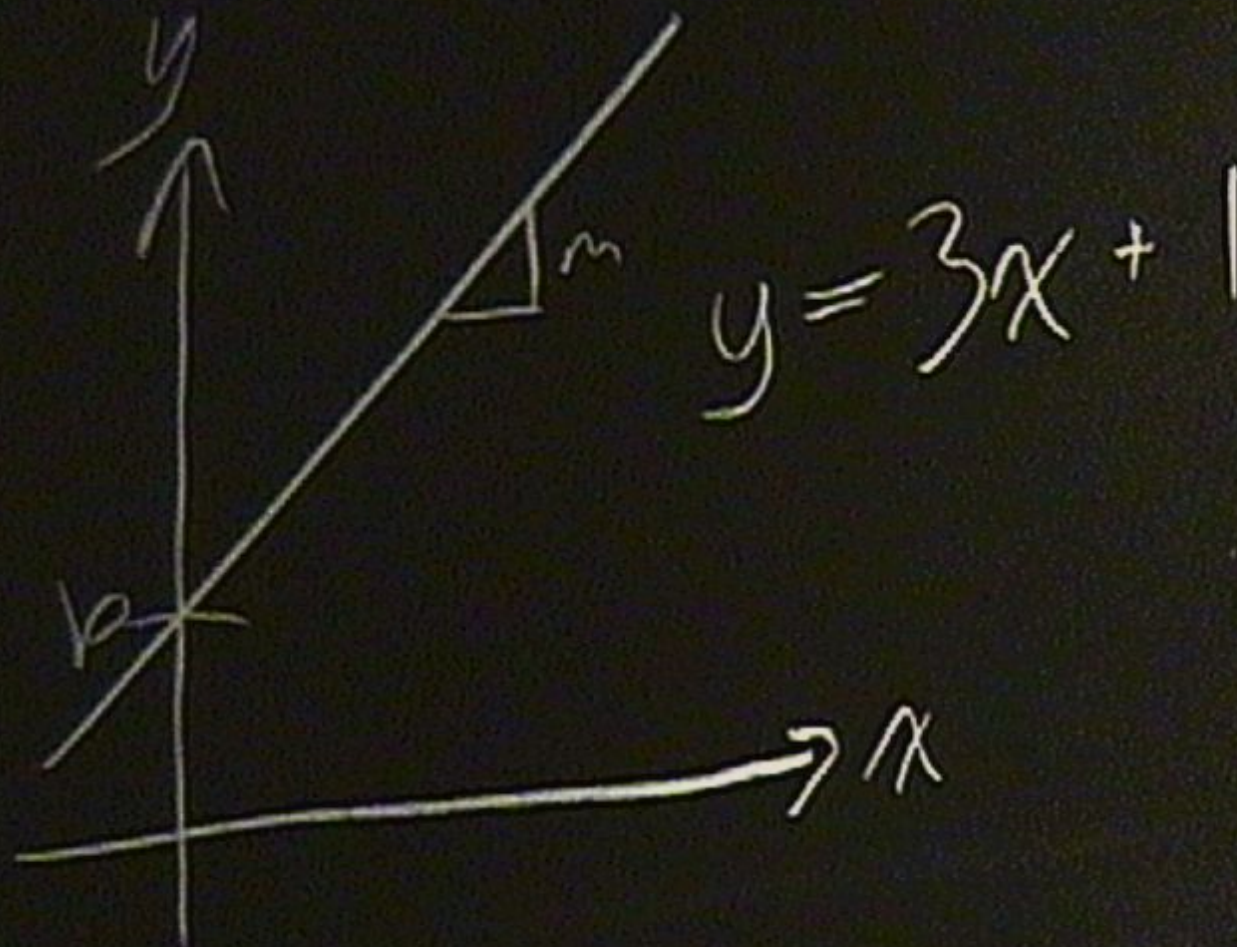
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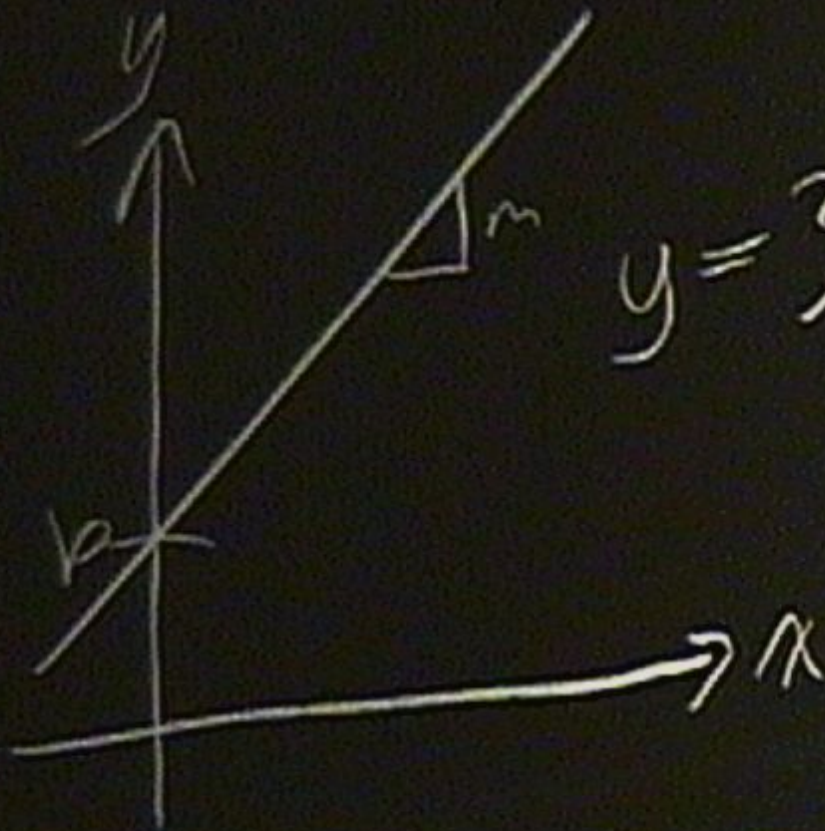
Abstract:





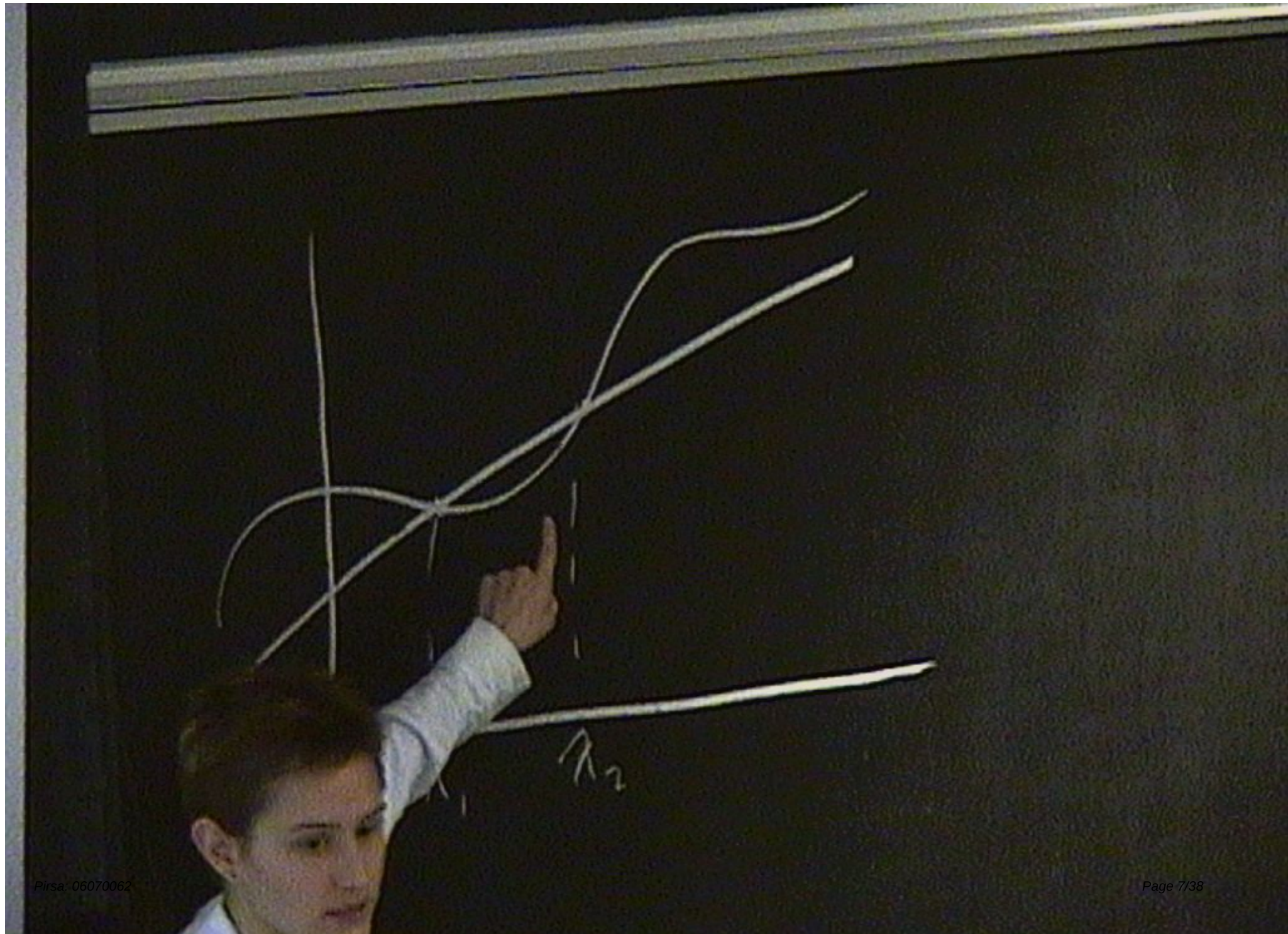


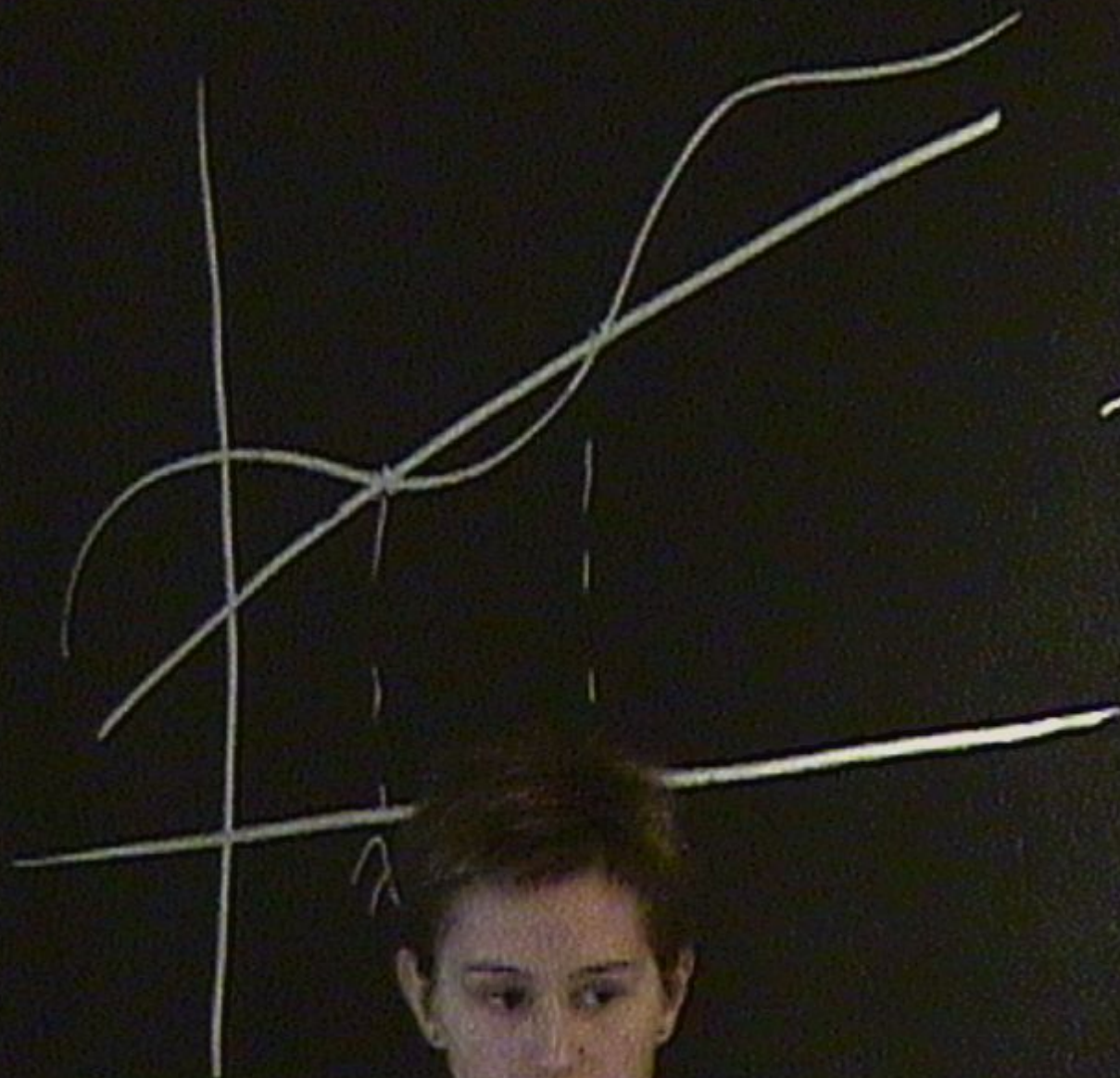




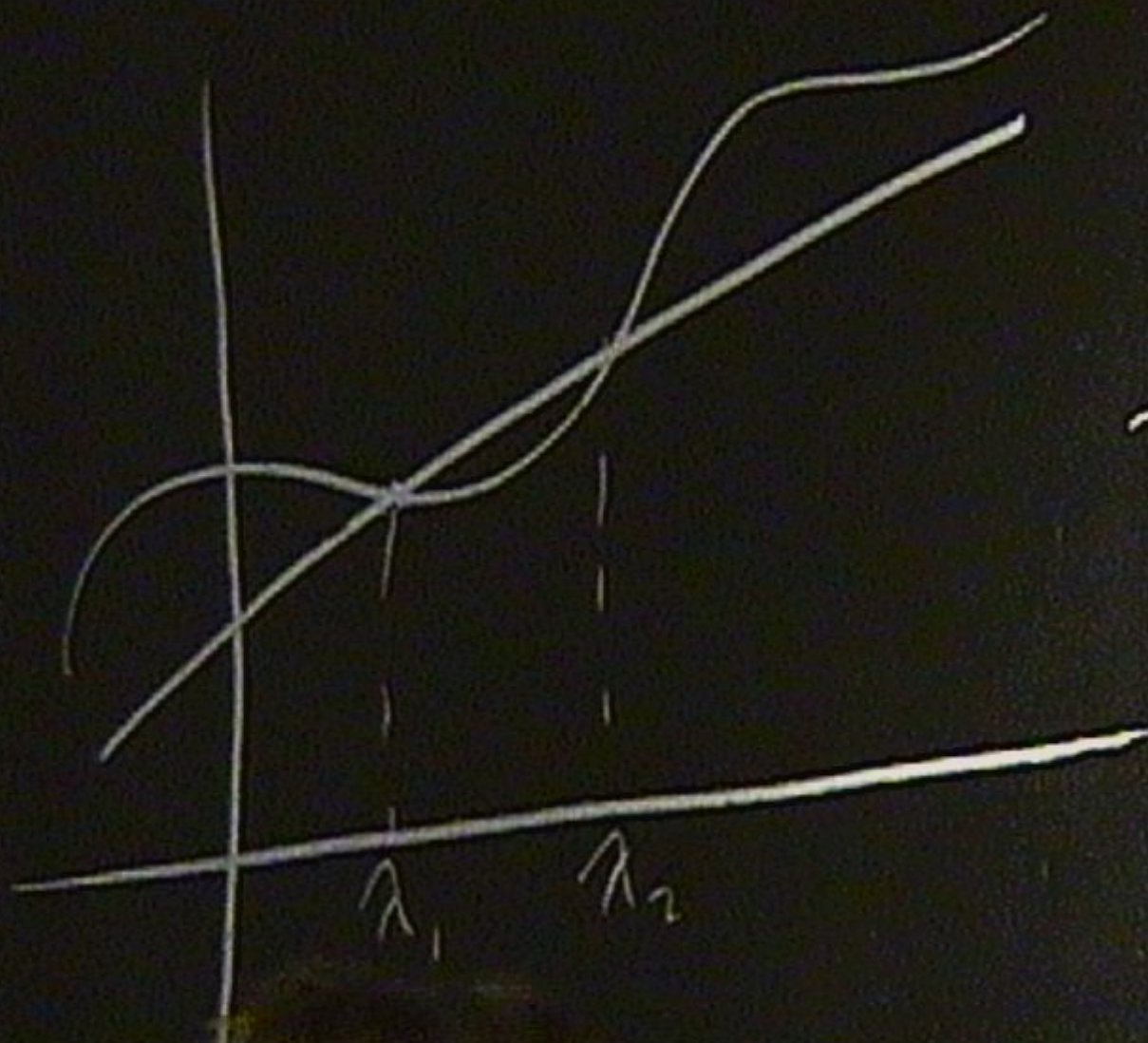
$$y = 3x + 1$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

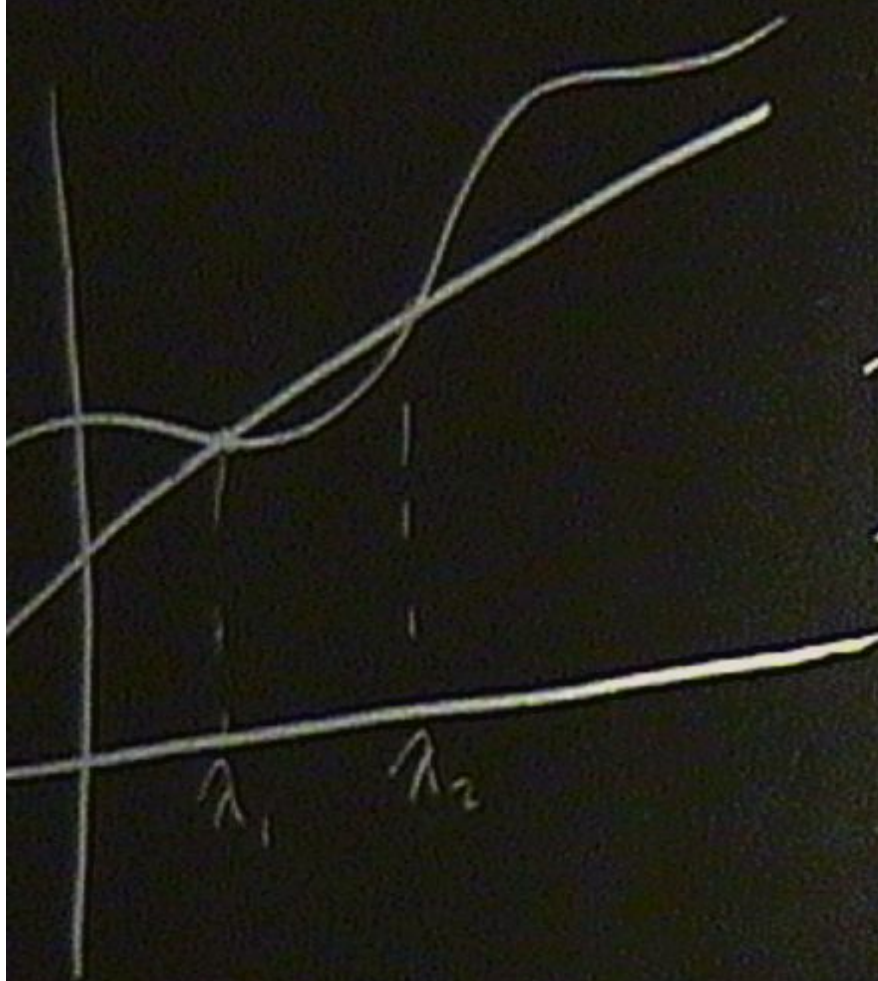




$$f(t) = t^2$$



$$f(t) = t^2$$



$$f(x) = x^2$$

$$x_1 = 3, x_2 = 5$$

$$\begin{aligned} m_s &= \frac{f(5) - f(3)}{5 - 3} \\ &= \frac{25 - 9}{2} \\ &= \frac{16}{2} = 8 \end{aligned}$$

Problem Set (1) – Slope

- 1) If $f(x) = 2x^3 + 5$, what is the slope of the secant between $x_1 = 1$ and $x_2 = 4$?

- 2) If $g(\theta) = \theta^2 - 3\theta$, what is the average rate of change of θ between $\theta_1 = 2$ and $\theta_2 = 7$?

- 3) If $r(t) = 8 - 2t^2$, what is the average speed between $t_1 = 0$ and $t_2 = 2$?

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$$f(x) = 2x^2 + 4$$

$$f(2)$$

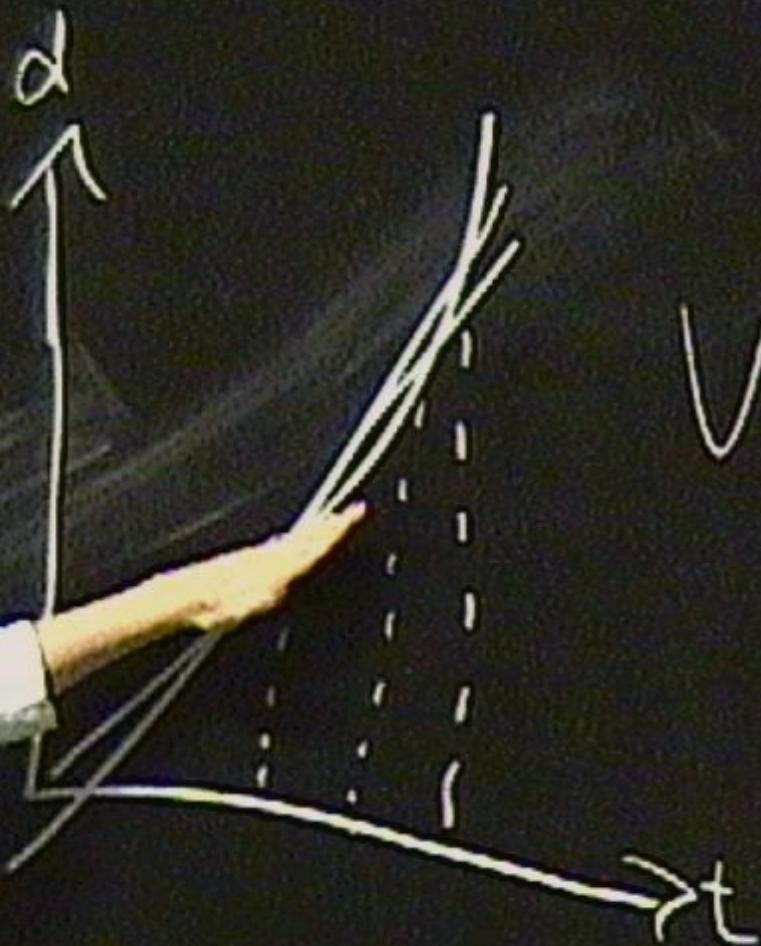
$$m_s = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$v = \frac{\Delta d}{\Delta t}$$

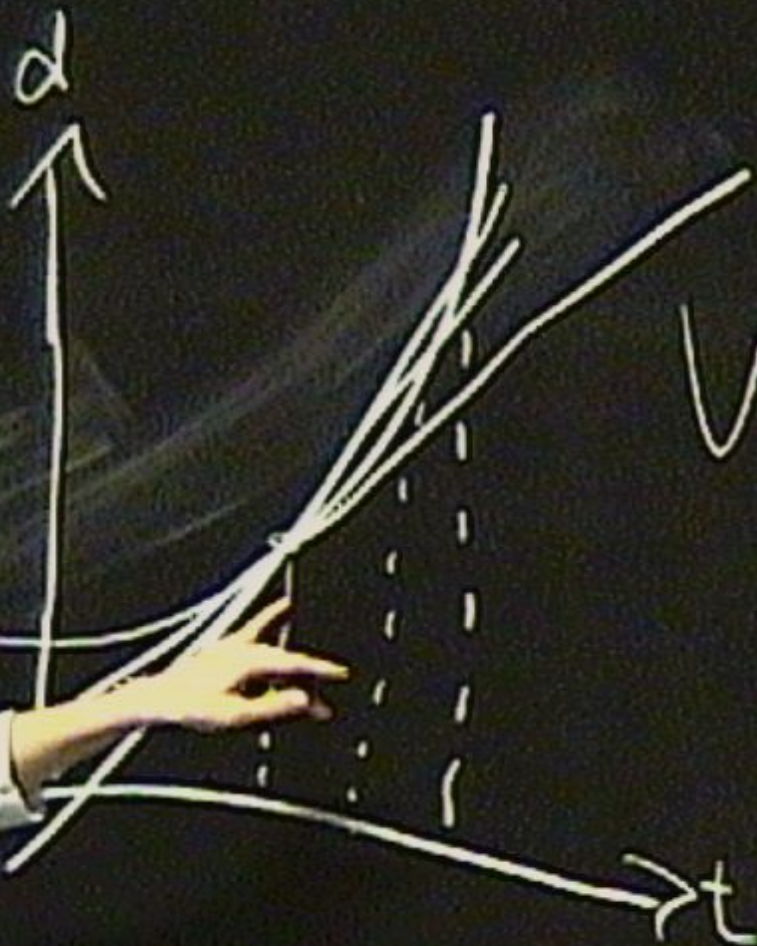
$$v = \frac{\Delta d}{\Delta t}$$



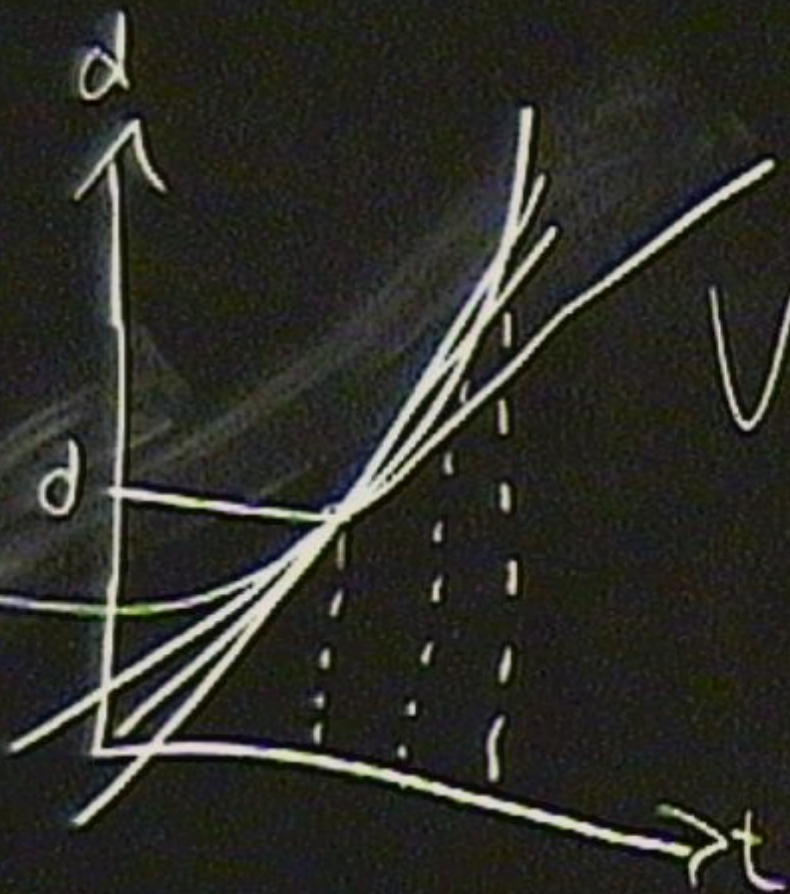
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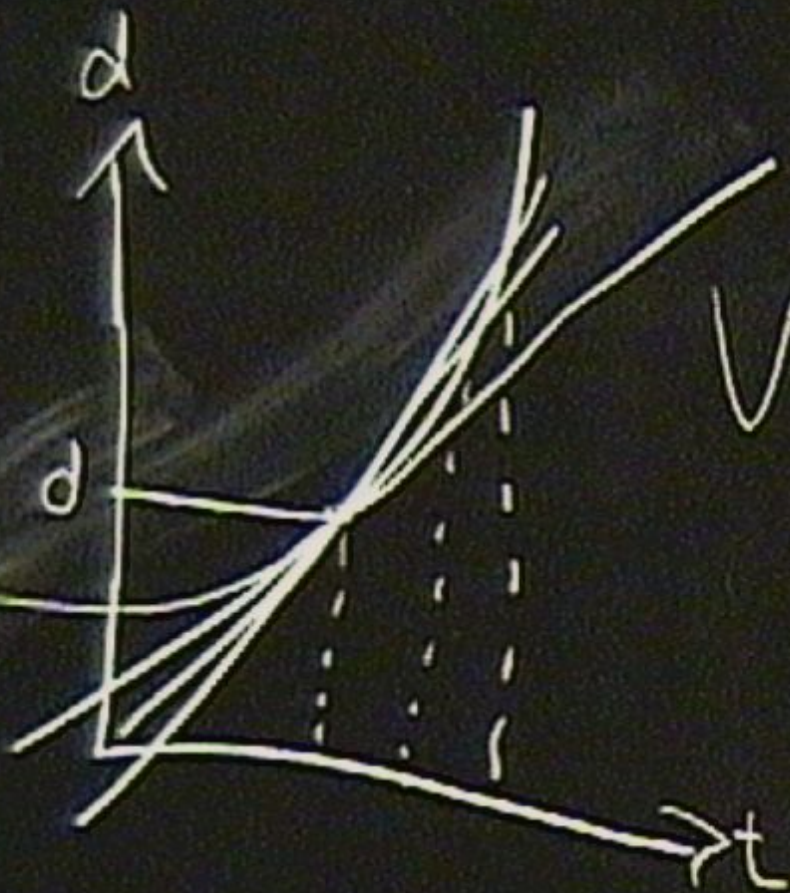
$$v = \frac{\Delta d}{\Delta t}$$



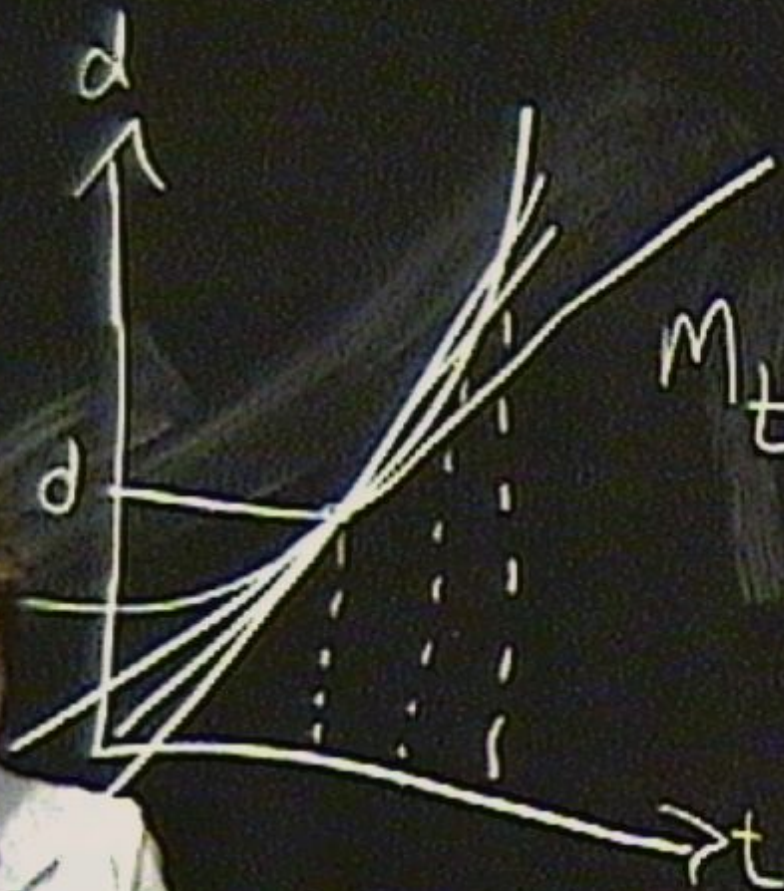
$$v = \frac{\Delta d}{\Delta t} = \frac{d}{0}$$



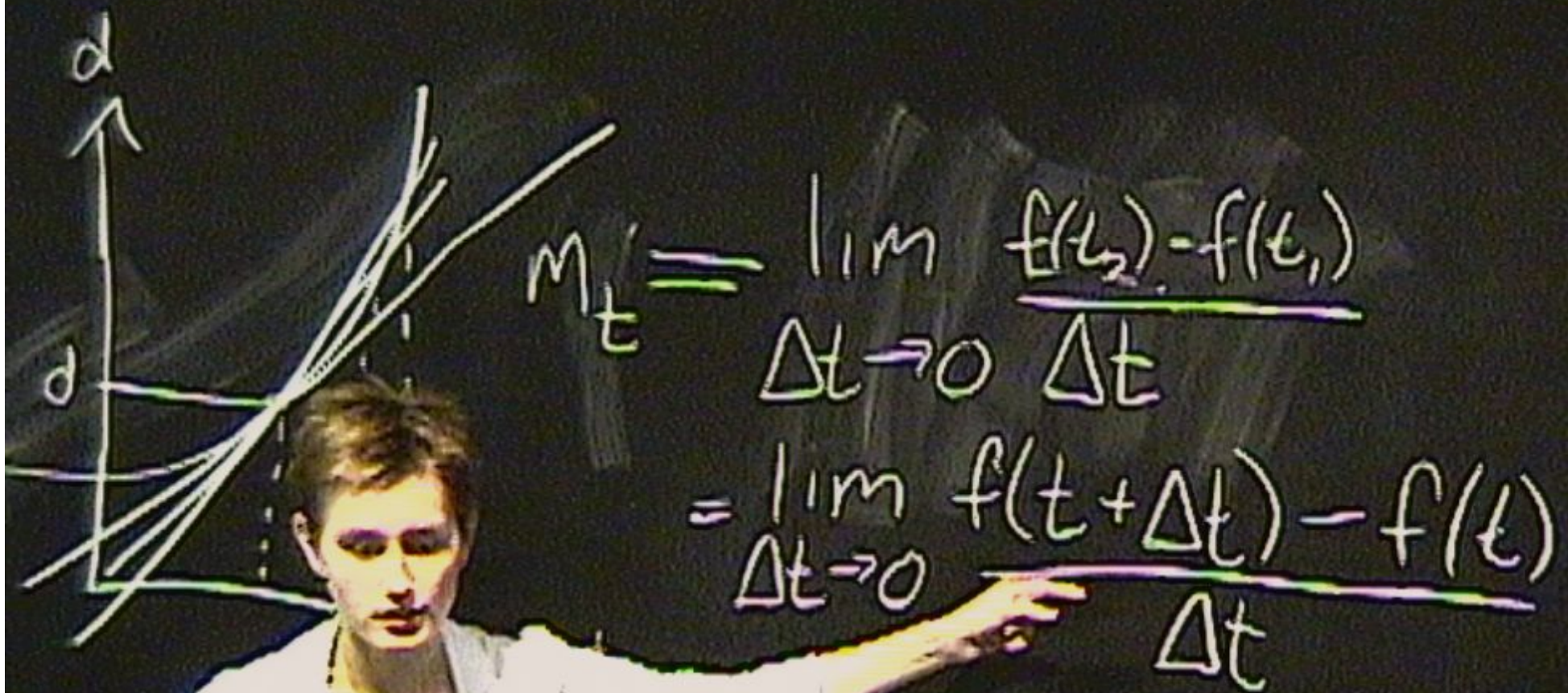
$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta d}{\Delta t}$$



$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta d}{\Delta t}$$



$$m_t = \lim_{\Delta t \rightarrow 0} \frac{\Delta d}{\Delta t}$$

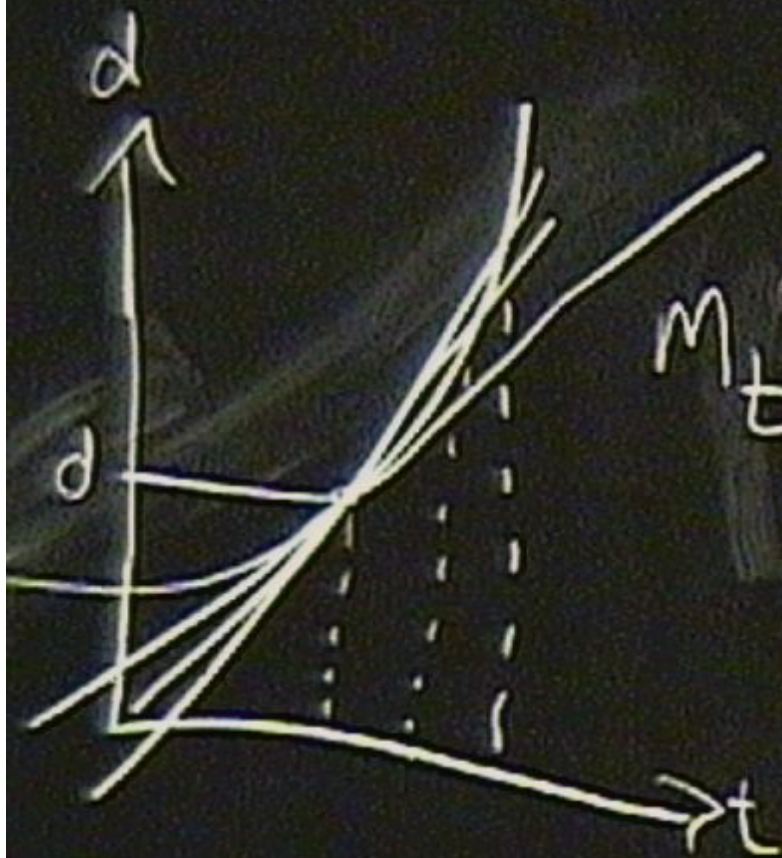


The image shows a chalkboard with a graph on the left and mathematical equations on the right. A person is standing in front of the board, pointing at the equations.

The graph on the left shows a coordinate system with a vertical axis labeled d and a horizontal axis labeled t . A curve is drawn, and several tangent lines are shown at different points on the curve, illustrating the concept of a derivative.

The equations on the right are:

$$m_t = \lim_{\Delta t \rightarrow 0} \frac{f(t_2) - f(t_1)}{\Delta t}$$
$$= \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$



$$m_t = \lim_{\Delta t \rightarrow 0} \frac{f(t_2) - f(t_1)}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

$$m_t = \lim_{\Delta t \rightarrow 0} \frac{f(t_2) - f(t_1)}{\Delta t}$$

$$\frac{df(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

$$m_t = \lim_{\Delta t \rightarrow 0} \frac{f(t_2) - f(t_1)}{\Delta t}$$

$$\frac{df(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

$$= \frac{\Delta y}{\Delta t}$$

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$$\frac{df(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

$$= \frac{\Delta y}{\Delta t}$$

$$f(x) = x^2$$

$$\frac{df}{dx}$$

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0}$$

$$= \lim_{\Delta x \rightarrow 0}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

x

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0}$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0}$$

$$(x + \Delta x)^2 - x^2$$

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0}$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} (x + \Delta x)^2 - x^2$$

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0}$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0}$$

$$\frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \cancel{(\Delta x)^2} - x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \cancel{\Delta x})}{\Delta x}$$

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

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$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \cancel{\Delta x})}{\Delta x} =$$

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \cancel{(\Delta x)^2} - x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = 2x$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \rightarrow 0} 2x + \Delta x$$

$$\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x \\ &= 2x\end{aligned}$$