Title: Unfinished Work: A Bequest

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Abstract: Here are some topics in physics and philosophy on which my work is incomplete. I invite my friends in this assembly, and their colleagues and students, to continue the work and inform me about their progress.

1. There is a well known theorem of Wigner that a necessary condition for a quantity M of a physical system O to be measured without distortion (i.e., if O is in an eigenstate u of M just prior to the measurement then it remains in u immediately afterwards) is the commutation of M with any additive conserved quantity. This theorem has been generalized by Stein and Shimony by relaxing the condition Â“without distortionÂ”, but the natural full generalization has neither been proven nor refuted by a counter-example.

2. In two-particle interferometry, using an ensemble of pairs of particles in a pure entangled state ?, the fringe visibility V12 of pairs counted in coincidence may be defined analogously to the fringe visibility V1 of single particles, and a complementarity relation has been derived: V12 + V122 = 1. Generalizations of this complementarity relation to n-tuples (n ?2) of entangled particle are desired.

3. Relations have been explored among various reasonable definitions of Â“degree of entanglementÂ”, but further systematization of these relations is desirable.

4. BellÂ’s Theorem shows that certain quantum mechanical predictions cannot be derived in any Â“local realisticÂ” theory, and experiments have overwhelmingly favored quantum mechanics in situations of theoretical conflict. Is it plausible to maintain Â“peaceful coexistenceÂ” between the nonlocality of quantum mechanics and the locality of relativity theory by citing the impossibility of using the former to send superluminal messages? But if this strategy fails, what is the proper adjudication of the conflict between these fundamental physical theories?

5. Stochastic modification of quantum dynamics (proposed by Ghirardi-Rimini-Weber, Gisin, Pearle, Wigner, Penrose, KÃ¡rolyhÃ¡zy, and others) has been proposed as a promising program for solving the quantum mechanical measurement problem. But theoretical refinements of the proposed modifications are desirable as well as definitive experimental tests. Two promising areas of relevant experimental research are quantum gravity and variants of the Â“quantum telegraphÂ” (an ensemble of atoms undergoing transitions between the ground state and a metastable state when exposed to appropriate laser beams).

6. Corinaldesi conjectured that the boson statistics of integral spin particles and the fermion statistics of half-integral spin particles are consequences of their dynamics rather than of their kinematics (the latter usually accepted because of PauliÂ’s spin and statistics theorem), so that obedience to the Pauli Exclusion Principle in a freshly formed

ensemble of electrons would become increasingly strict as the ensemble ages. To test CorinaldesiÂ’s conjecture it was proposed to form fresh ensembles of electrons by allowing a high velocity beam of Ne+ ions in a linear accelerator to be intersected by electrons from an electron gun, thereby neutralizing a subset of the ions. Transitions of electrons to the doubly occupied 1S shell in the complete Ne atoms will be monitored by suitable x-ray detectors at varying distances from the region of intersection, in order to scrutinize the conjectured diminution with time of violations of the Exclusion Principle. Refinements of this design and actual performance of the experiment are requested.

7. A test of the speculative conjecture that wave packet reduction
is a psycho-physical phenomenon, occurring only when a conscious
observer reads a measuring device, was performed in 1977 by three of my undergraduate students. They used a slow gamma emitter (about one emission per minute) monitored by a detector connected to registration devices in two separated rooms. There was a short time delay between the two registrations. The observer in the first room read his registration device in randomly chosen intervals of time and the observer in...

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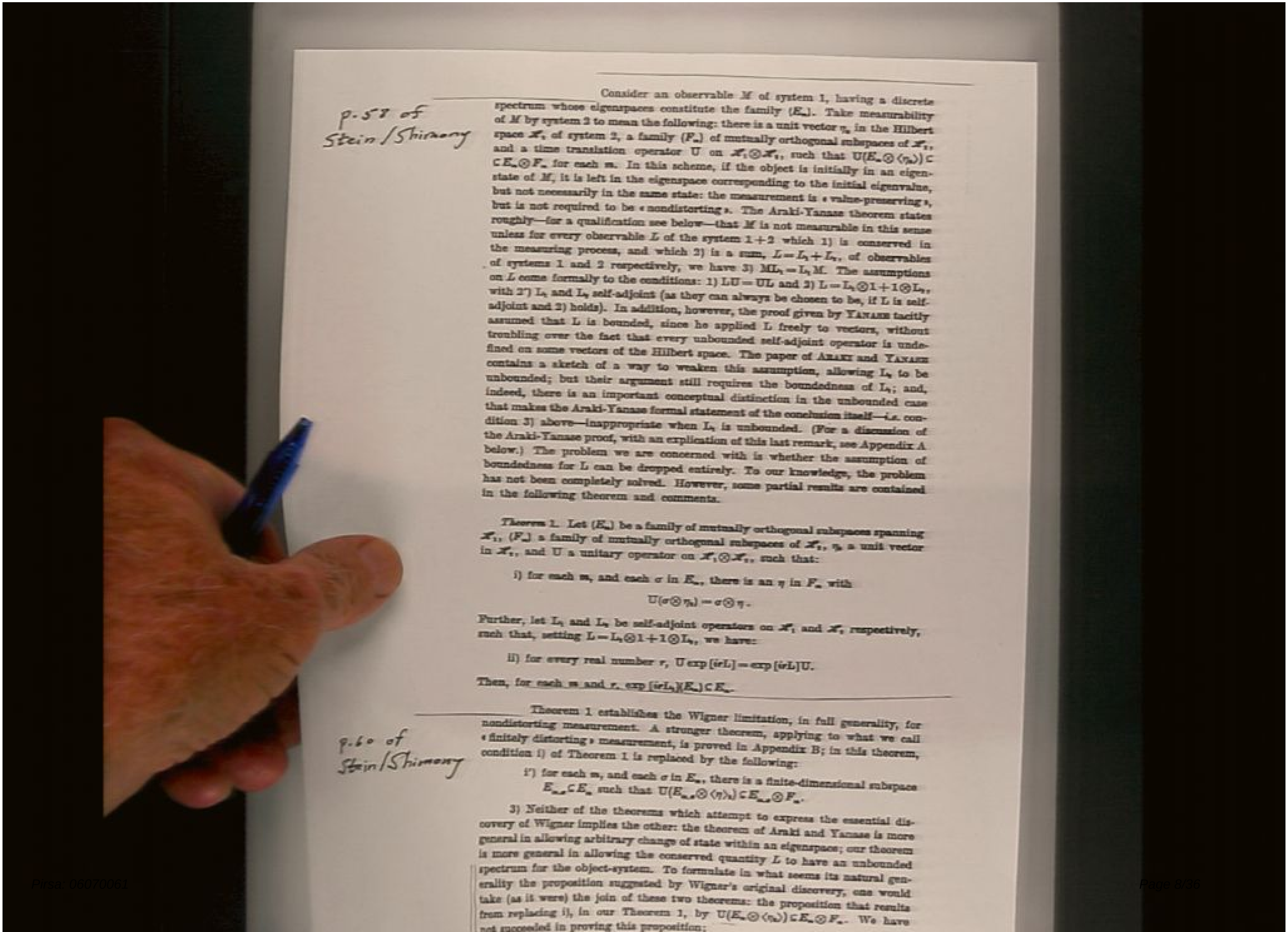

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Theorem 1. Let (E_m) be a family of mutually orthogonal subspaces spanning $\mathscr{H}_1$, (F_m) a family of mutually orthogonal subspaces of $\mathscr{H}_2$, η_0 a unit vector in $\mathscr{H}_2$, and U a unitary operator on $\mathscr{H}_1 \otimes \mathscr{H}_2$, such that:

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ii) for every real number r, $U \exp[irL] = \exp[irL] U$.

Then, for each m and r, $\exp[irL_1](E_m) \subset E_m$.

Theorem 1 establishes the Wigner limitation, in full generality, for nondistorting measurement. A stronger theorem, applying to what we call « finitely distorting » measurement, is proved in Appendix B; in this theorem, condition i) of Theorem 1 is replaced by the following:

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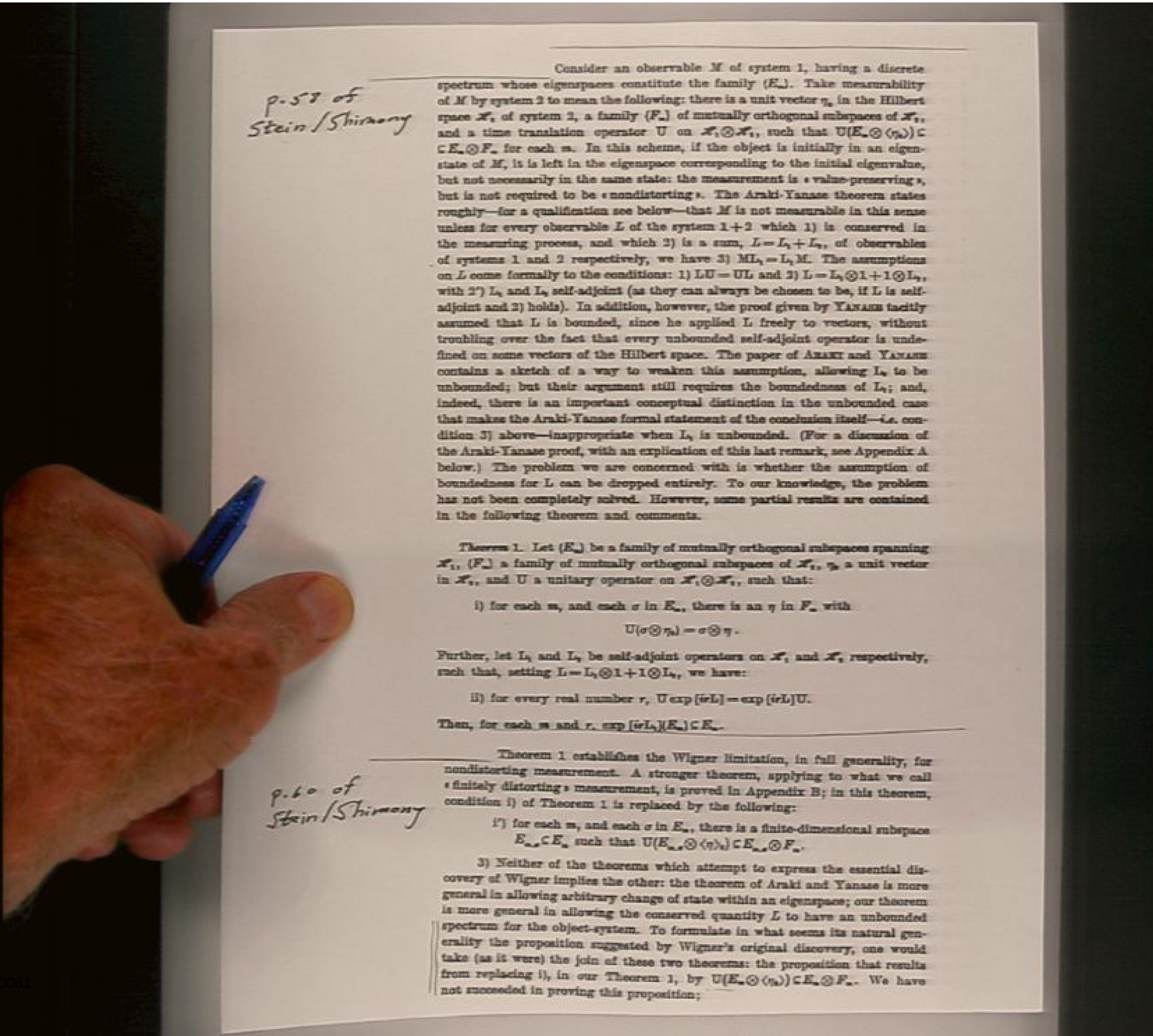
3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other: the theorem of Araki and Yanase is more general in allowing arbitrary change of state within an eigenspace; our theorem is more general in allowing the conserved quantity L to have an unbounded spectrum for the object-system. To formulate in what seems its natural generality the proposition suggested by Wigner's original discovery, one would take (as it were) the join of these two theorems: the proposition that results from replacing i), in our Theorem 1, by $U(E_m \otimes \langle \eta_0 \rangle) \subset E_m \otimes F_m$. We have not succeeded in proving this proposition:



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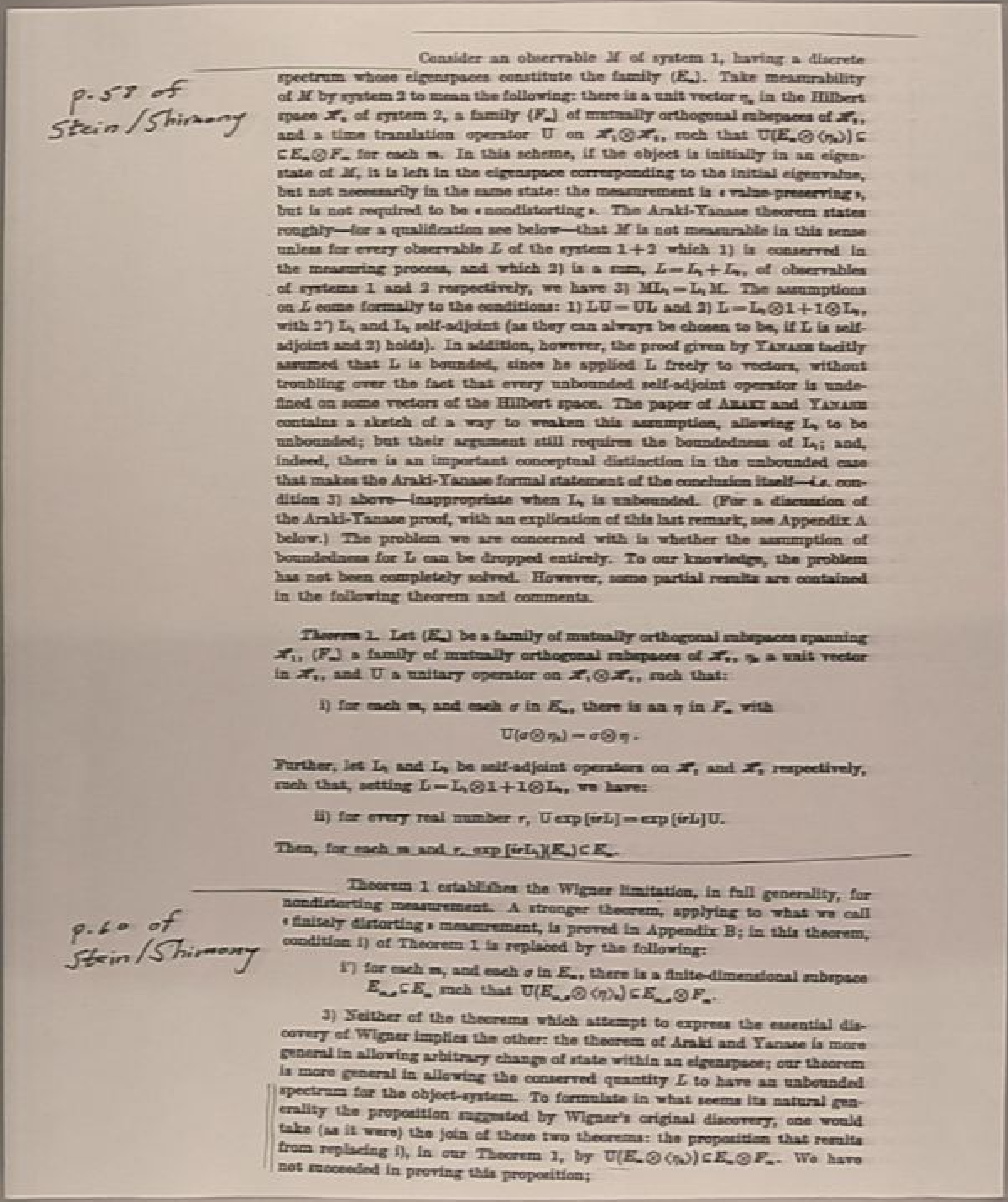

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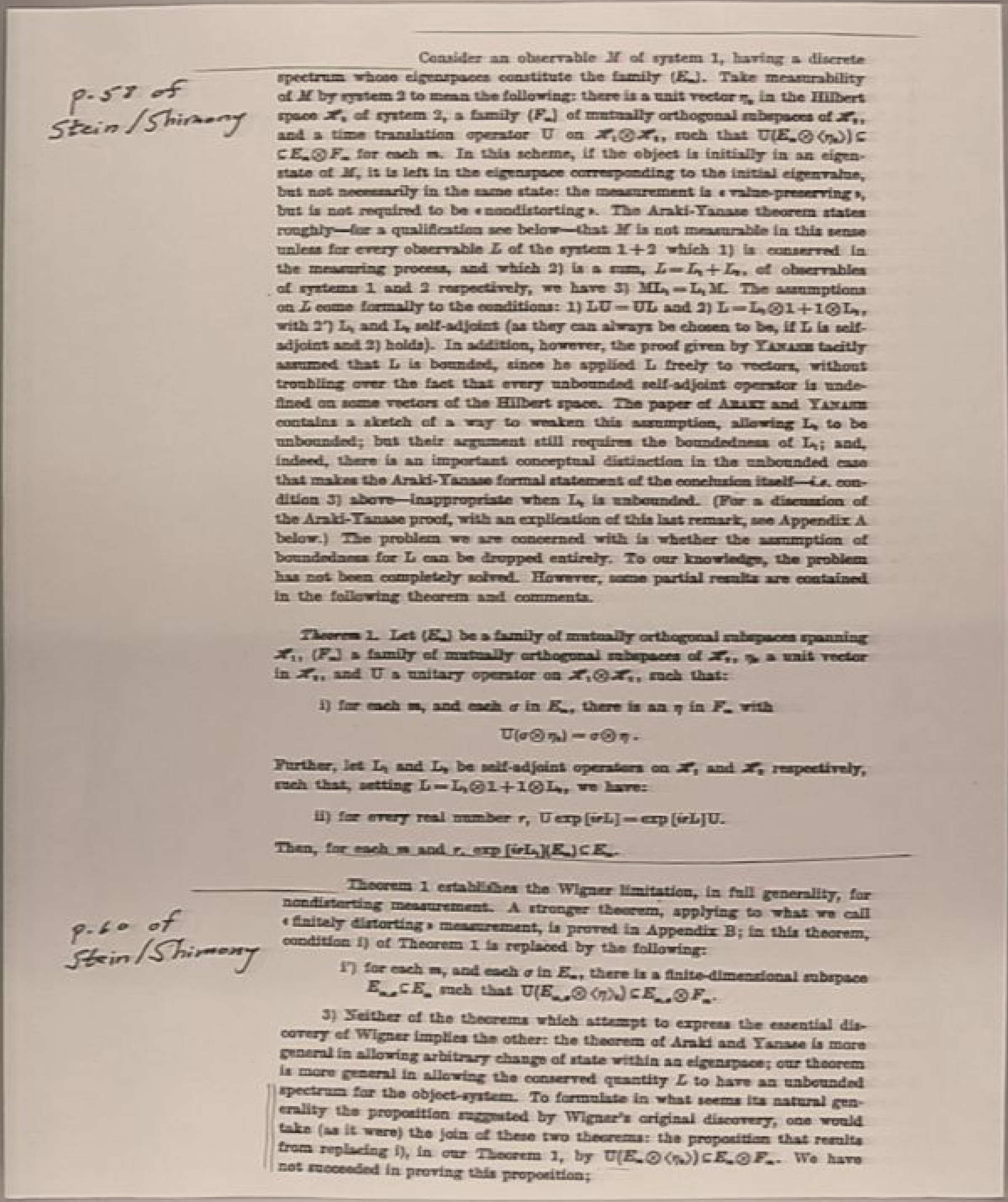

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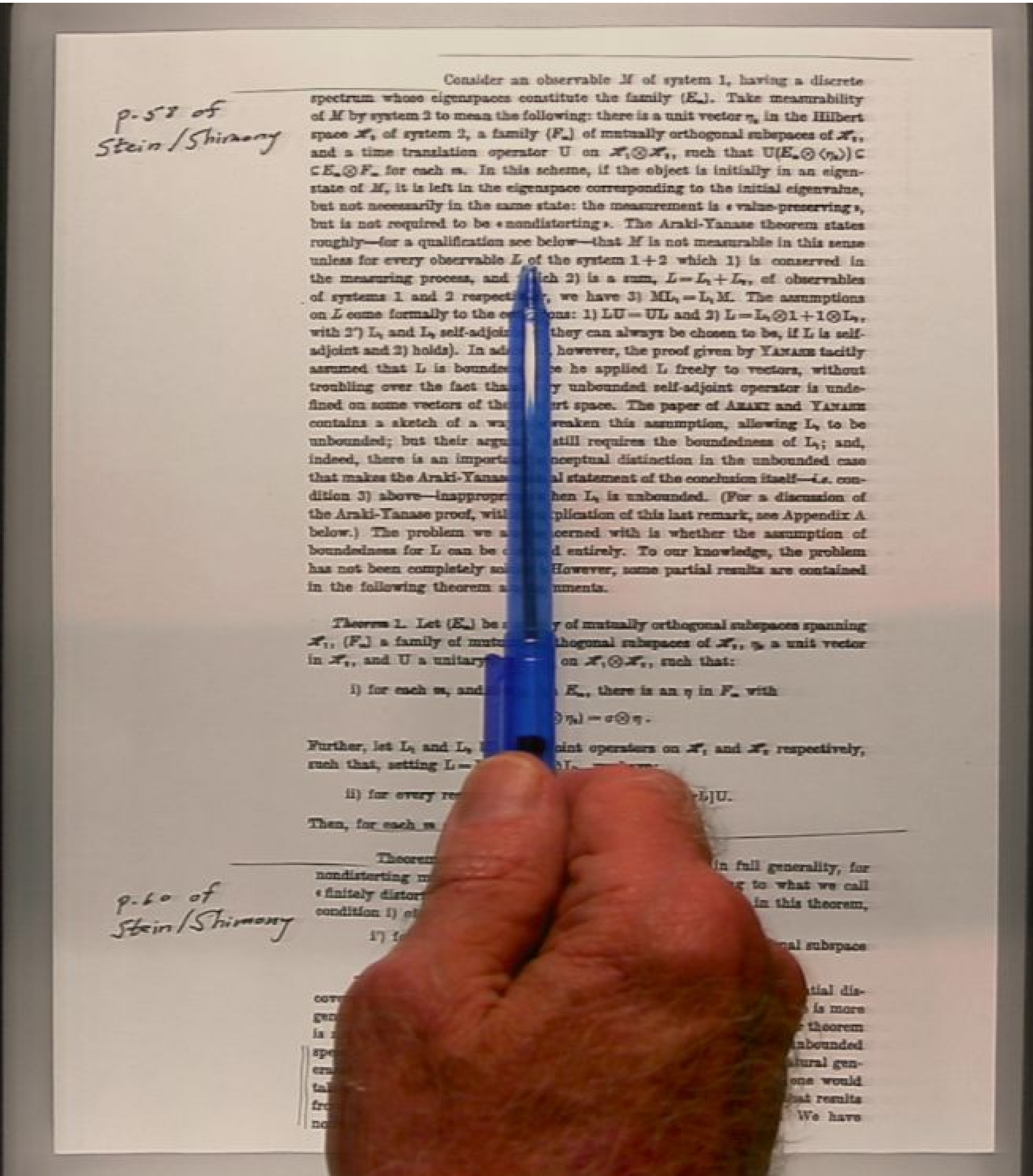

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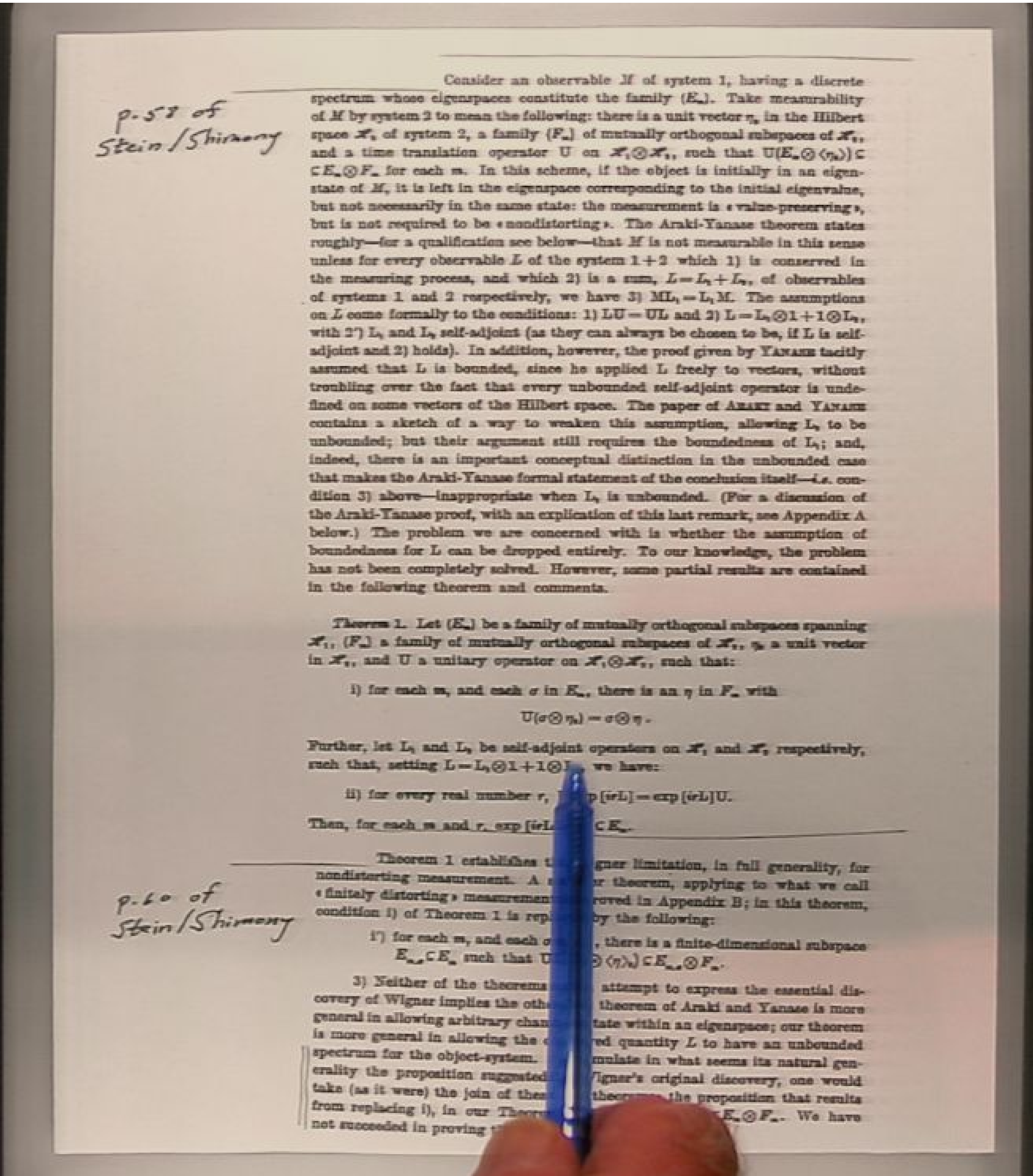

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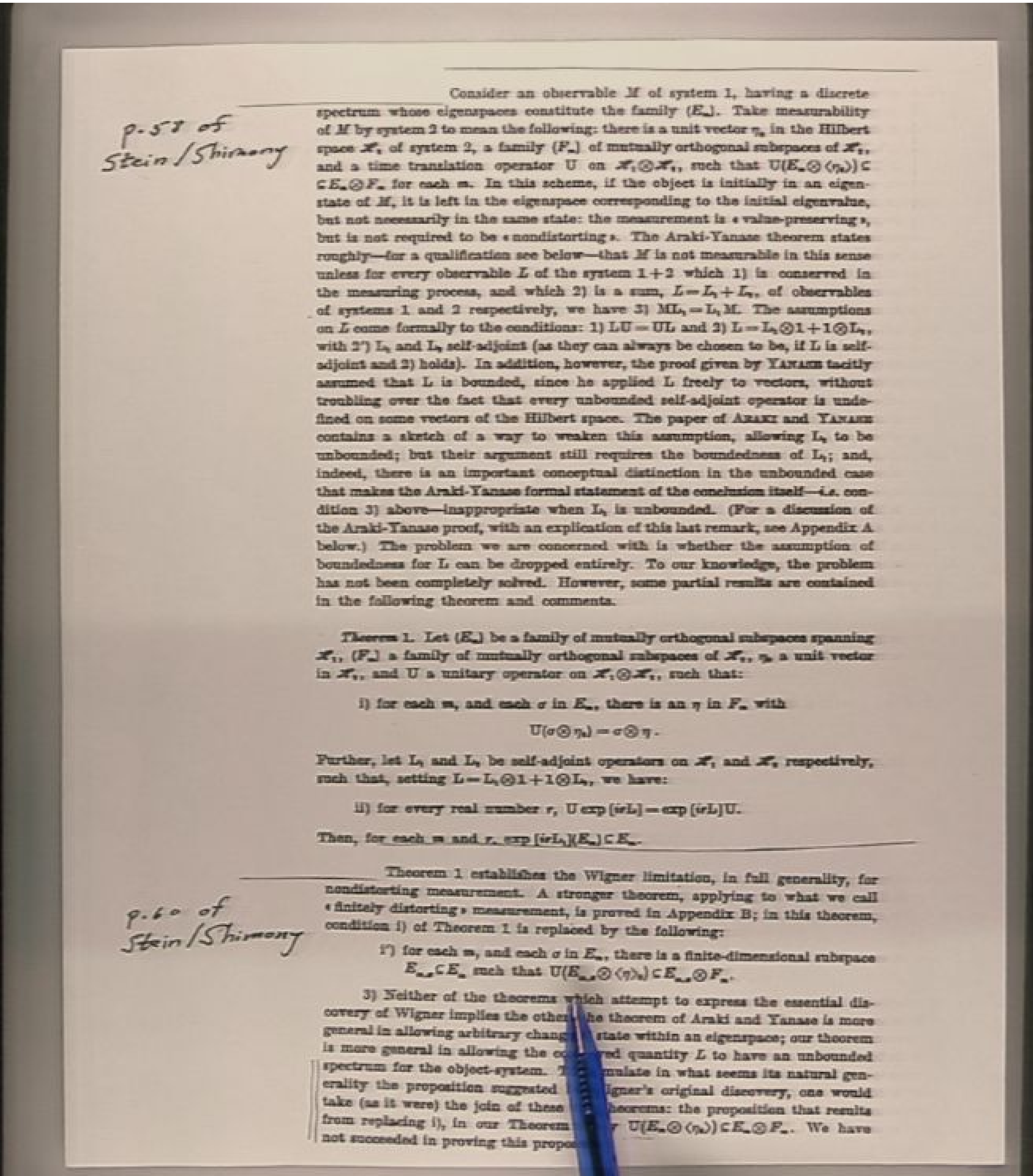

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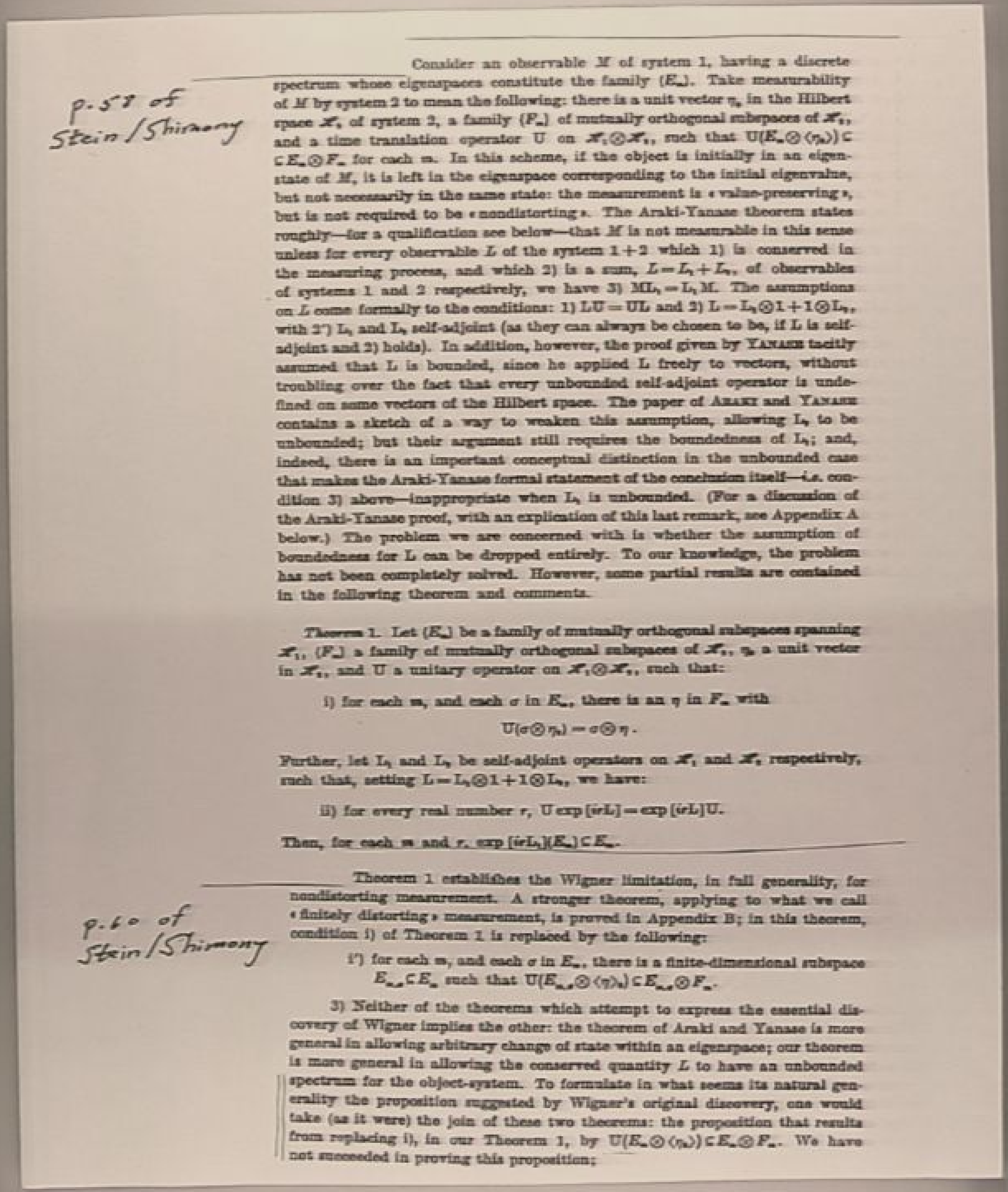

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2. A complementarity in two-particle interferometry, and a quest for generalization to n-particle interferometry. (G. Jaeger, A. Shimony, L. Vaidman, Phys. Rev. A 51, 54-67, 1995).

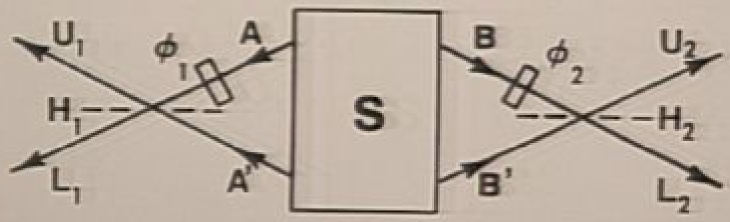

FIG. 2. Schematic two-particle four-beam inteferometer, using beam splitters H_1 and H_2.

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The single-particle fringe visibilities V_1 and V_2 can be determined by inspection from Eqs. (9), (77), and (78), together with $\alpha \ge \beta$. Clearly,

$$[P(U_1)]_{\max}=[P(U_2)]_{\max}=\alpha^2 , \tag{79a}$$

$$[P(U_1)]_{\min}=[P(U_2)]_{\min}=\beta^2 . \tag{79b}$$

Hence,

$$V_i=\frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}=\alpha^2-\beta^2 \quad (i=1,2) . \tag{80}$$

We note that $P(U_1)$ achieves its maximum and minimum when a has the respective values 1 and 0. When a is unity (hence b is zero), a photon in C will go with certainty into U_1 and a photon in C' will go with certainty into L_1. When a is zero (hence b is unity), the exit states are reversed. A similar statement can be made concerning photon 2, relating the vectors $|D\rangle$ and $|D'\rangle$ to exit in beams U_2 and L_2.

We turn now to the two-particle fringe visibility V_{12}. As pointed out in Ref. [10], one cannot capture the intuitive meaning of two-particle fringe visibility by using the attractive definition

$$V_{12}=\frac{[P(U_1,U_2)]_{\max}-[P(U_1,U_2)]_{\min}}{[P(U_1,U_2)]_{\max}+[P(U_1,U_2)]_{\min}} \quad (\text{No}) ; \tag{81}$$

this expression would yield a nonzero value even if $|\Theta\rangle$ is a product state, for in that case $P(U_1U_2)$ is the product of $P(U_1)$ and $P(U_2)$, and these vary respectively with T_1 and T_2. As in Ref. [10] we define a "corrected" joint probability $\tilde{P}(U_1,U_2)$ by subtracting the product $P(U_1)P(U_2)$ from $P(U_1,U_2)$ and adding a constant as a compensation against excessive subtraction:

$$\tilde{P}(U_1,U_2)=P(U_1,U_2)-P(U_1)P(U_2)+\tfrac{1}{4} . \tag{82a}$$

By Eqs. (72), (77), and (78),

$$\tilde{P}(U_1,U_2)=\alpha^2\beta^2\cos\mu\cos\nu+\tfrac{1}{2}\alpha\beta\sin\mu\sin\nu\cos\Phi+\tfrac{1}{4} , \tag{82b}$$

where

$$a=\cos\frac{\mu}{2},\quad b=\sin\frac{\mu}{2},\quad c=\cos\frac{\nu}{2},\quad d=\sin\frac{\nu}{2} . \tag{82c}$$

A rationale for the term $\frac{1}{4}$ in Eq. (82a) is the fact that $\frac{1}{4}$ is the least real number s such that $P(U_1,U_2)-P(U_1)P(U_2)+s$ is non-negative for all two-particle vectors of the form of Eq. (66a) and all unitary mappings T_1 and T_2 of the classes under consideration, as can be checked from Eqs. (72), (77), and (78). We now parallel Ref. [10] and define the two-particle fringe visibility V_{12} as

$$V_{12}=\frac{[\tilde{P}(U_1,U_2)]_{\max}-[\tilde{P}(U_1,U_2)]_{\min}}{[\tilde{P}(U_1,U_2)]_{\max}+[\tilde{P}(U_1,U_2)]_{\min}} . \tag{83}$$

To find the extrema of $\tilde{P}(U_1,U_2)$ we use Eq. (82b) and set partial derivatives to zero: first,

$$0=\frac{\partial\tilde{P}(U_1,U_2)}{\partial\Phi}=-\frac{1}{2}\alpha\beta\sin\mu\sin\nu\sin\Phi . \tag{84}$$

If $\alpha\beta\neq 0$, then Eq. (84) can be satisfied only if one of the two following conditions is satisfied: (i) $\sin\mu\sin\nu=0$, in which case

$$\tfrac{1}{4}-\alpha^2\beta^2\le\tilde{P}(U_1,U_2)\le\tfrac{1}{4}+\alpha^2\beta^2 , \tag{85}$$

or (ii) $\sin\Phi=0$, in which case

$$\tilde{P}(U_1,U_2)=\alpha^2\beta^2\cos\mu\cos\nu\pm\tfrac{1}{2}\alpha\beta\sin\mu\sin\nu+\tfrac{1}{4} , \tag{86a}$$

and

$$0=\frac{\partial\tilde{P}}{\partial\mu}=-\alpha^2\beta^2\sin\mu\cos\nu\pm\tfrac{1}{2}\alpha\beta\cos\mu\sin\nu , \tag{86b}$$

$$0=\frac{\partial\tilde{P}}{\partial\nu}=-\alpha^2\beta^2\cos\mu\sin\nu\pm\tfrac{1}{2}\alpha\beta\sin\mu\cos\nu . \tag{86c}$$

If $\alpha^2\beta^2\neq\frac{1}{2}\alpha\beta$, then Eqs. (86b) and (86c) imply $\cos\mu\sin\nu=\sin\mu\cos\nu=0$, which is possible only if one of two conditions is satisfied: (iia) $(\mu,\nu)=(m\pi,n\pi)$, with m,n integers, in which case Eq. (85) is again satisfied, or (iib) $(\mu,\nu)=(\pi/2,\pi/2)$, values are all modπ, in which case

$$\tilde{P}(U_1,U_2)=\tfrac{1}{4}\pm\tfrac{1}{2}\alpha\beta . \tag{87}$$

But

$$\tfrac{1}{2}\alpha\beta\ge\alpha^2\beta^2 , \tag{88}$$

and therefore a review of all the cases (i), (iia), and (iib) yields

$$[\tilde{P}(U_1,U_2)]_{\max}=\tfrac{1}{4}+\tfrac{1}{2}\alpha\beta , \tag{89a}$$

$$[\tilde{P}(U_1,U_2)]_{\min}=\tfrac{1}{4}-\tfrac{1}{2}\alpha\beta . \tag{89b}$$

Note that in the neglected case of $\beta=0$ these equations continue to hold, as does Eq. (80), because α was assumed $\ge\beta$. It follows that without exception

$$V_{12}=2\alpha\beta . \tag{90}$$

By Eqs. (80) and (90),

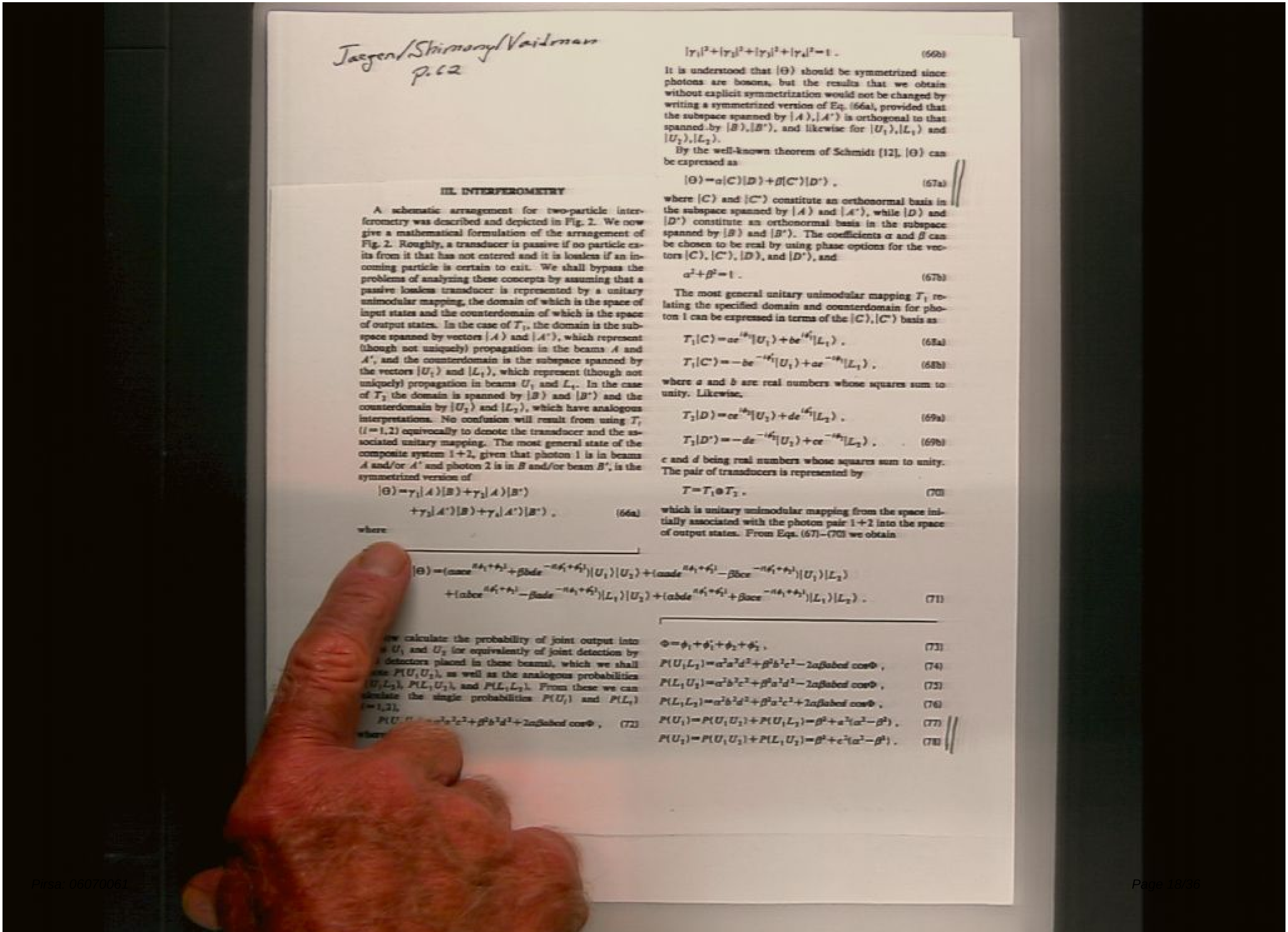
$$\begin{aligned} V_{12}^2+V_i^2&=4\alpha^2\beta^2+(\alpha^2-\beta^2)^2 \quad (i=1,2)\\ &=(\alpha^2+\beta^2)^2=1 , \end{aligned} \tag{91}$$

which is the expression for the complementarity of one-particle and two-particle visibilities promised in the Introduction (slightly generalized, since $i=1$ or 2).

In Ref. [10] a more restricted set of transducers was considered than the class permitted here. Each T_i was taken to consist of a symmetric beam splitter with reflectivity r and transmittivity t both equal to $1/\sqrt{2}$, together with a phase shifter in one beam incident upon the beam splitter. The small letters v_i $(i=1,2)$ and v_{12} denote the one-particle and two-particle visibilities under this restriction. It was shown that for a large class [13] of two-particle vectors $|\Theta\rangle$, the inequality

$$v_{12}^2+v_i^2\le 1 \tag{92}$$

This is the complementarity
in two-particle interferometry



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III. INTERFEROMETRY

A schematic arrangement for two-particle interferometry was described and depicted in Fig. 2. We now give a mathematical formulation of the arrangement of Fig. 2. Roughly, a transducer is passive if no particle exits from it that has not entered and it is lossless if an incoming particle is certain to exit. We shall bypass the problems of analyzing these concepts by assuming that a passive lossless transducer is represented by a unitary unimodular mapping, the domain of which is the space of input states and the counterdomain of which is the space of output states. In the case of T_1, the domain is the subspace spanned by vectors $|A\rangle$ and $|A'\rangle$, which represent (though not uniquely) propagation in the beams A and A', and the counterdomain is the subspace spanned by the vectors $|U_1\rangle$ and $|L_1\rangle$, which represent (though not uniquely) propagation in beams U_1 and L_1. In the case of T_2 the domain is spanned by $|B\rangle$ and $|B'\rangle$ and the counterdomain by $|U_2\rangle$ and $|L_2\rangle$, which have analogous interpretations. No confusion will result from using T_i $(i=1,2)$ equivocally to denote the transducer and the associated unitary mapping. The most general state of the composite system 1+2, given that photon 1 is in beams A and/or A' and photon 2 is in B and/or beam B', is the symmetrized version of

$$|\Theta\rangle=\gamma_1|A\rangle|B\rangle+\gamma_2|A\rangle|B'\rangle$$
$$+\gamma_3|A'\rangle|B\rangle+\gamma_4|A'\rangle|B'\rangle\,, \qquad (66a)$$

where

$$|\gamma_1|^2+|\gamma_2|^2+|\gamma_3|^2+|\gamma_4|^2=1\,. \qquad (66b)$$

It is understood that $|\Theta\rangle$ should be symmetrized since photons are bosons, but the results that we obtain without explicit symmetrization would not be changed by writing a symmetrized version of Eq. (66a), provided that the subspace spanned by $|A\rangle,|A'\rangle$ is orthogonal to that spanned by $|B\rangle,|B'\rangle$, and likewise for $|U_1\rangle,|L_1\rangle$ and $|U_2\rangle,|L_2\rangle$.

By the well-known theorem of Schmidt [12], $|\Theta\rangle$ can be expressed as

$$|\Theta\rangle=\alpha|C\rangle|D\rangle+\beta|C'\rangle|D'\rangle\,, \qquad (67a)$$

where $|C\rangle$ and $|C'\rangle$ constitute an orthonormal basis in the subspace spanned by $|A\rangle$ and $|A'\rangle$, while $|D\rangle$ and $|D'\rangle$ constitute an orthonormal basis in the subspace spanned by $|B\rangle$ and $|B'\rangle$. The coefficients α and β can be chosen to be real by using phase options for the vectors $|C\rangle$, $|C'\rangle$, $|D\rangle$, and $|D'\rangle$, and

$$\alpha^2+\beta^2=1\,. \qquad (67b)$$

The most general unitary unimodular mapping T_1 relating the specified domain and counterdomain for photon 1 can be expressed in terms of the $|C\rangle,|C'\rangle$ basis as

$$T_1|C\rangle=ae^{i\phi_1}|U_1\rangle+be^{i\phi_1'}|L_1\rangle\,, \qquad (68a)$$

$$T_1|C'\rangle=-be^{-i\phi_1'}|U_1\rangle+ae^{-i\phi_1}|L_1\rangle\,, \qquad (68b)$$

where a and b are real numbers whose squares sum to unity. Likewise,

$$T_2|D\rangle=ce^{i\phi_2}|U_2\rangle+de^{i\phi_2'}|L_2\rangle\,, \qquad (69a)$$

$$T_2|D'\rangle=-de^{-i\phi_2'}|U_2\rangle+ce^{-i\phi_2}|L_2\rangle\,, \qquad (69b)$$

c and d being real numbers whose squares sum to unity. The pair of transducers is represented by

$$T=T_1\otimes T_2\,, \qquad (70)$$

which is unitary unimodular mapping from the space initially associated with the photon pair 1+2 into the space of output states. From Eqs. (67)–(70) we obtain

$$|\Theta\rangle=(\alpha ace^{i(\phi_1+\phi_2)}+\beta bde^{-i(\phi_1'+\phi_2')})|U_1\rangle|U_2\rangle+(\alpha ade^{i(\phi_1+\phi_2')}-\beta bce^{-i(\phi_1'+\phi_2)})|U_1\rangle|L_2\rangle$$
$$+(\alpha bce^{i(\phi_1'+\phi_2)}-\beta ade^{-i(\phi_1+\phi_2')})|L_1\rangle|U_2\rangle+(\alpha bde^{i(\phi_1'+\phi_2')}+\beta ace^{-i(\phi_1+\phi_2)})|L_1\rangle|L_2\rangle\,. \qquad (71)$$

w calculate the probability of joint output into U_1 and U_2 (or equivalently of joint detection by detectors placed in these beams), which we shall te $P(U_1,U_2)$, as well as the analogous probabilities $U_1,L_2)$, $P(L_1,U_2)$, and $P(L_1,L_2)$. From these we can lculate the single probabilities $P(U_i)$ and $P(L_i)$ $=1,2)$.

$$P(U \ldots =\ldots a^2c^2+\beta^2b^2d^2+2\alpha\beta abcd\cos\Phi\,, \qquad (72)$$

wher

$$\Phi=\phi_1+\phi_1'+\phi_2+\phi_2'\,, \qquad (73)$$

$$P(U_1L_2)=\alpha^2a^2d^2+\beta^2b^2c^2-2\alpha\beta abcd\cos\Phi\,, \qquad (74)$$

$$P(L_1U_2)=\alpha^2b^2c^2+\beta^2a^2d^2-2\alpha\beta abcd\cos\Phi\,, \qquad (75)$$

$$P(L_1L_2)=\alpha^2b^2d^2+\beta^2a^2c^2+2\alpha\beta abcd\cos\Phi\,, \qquad (76)$$

$$P(U_1)=P(U_1U_2)+P(U_1L_2)=\beta^2+a^2(\alpha^2-\beta^2)\,, \qquad (77)$$

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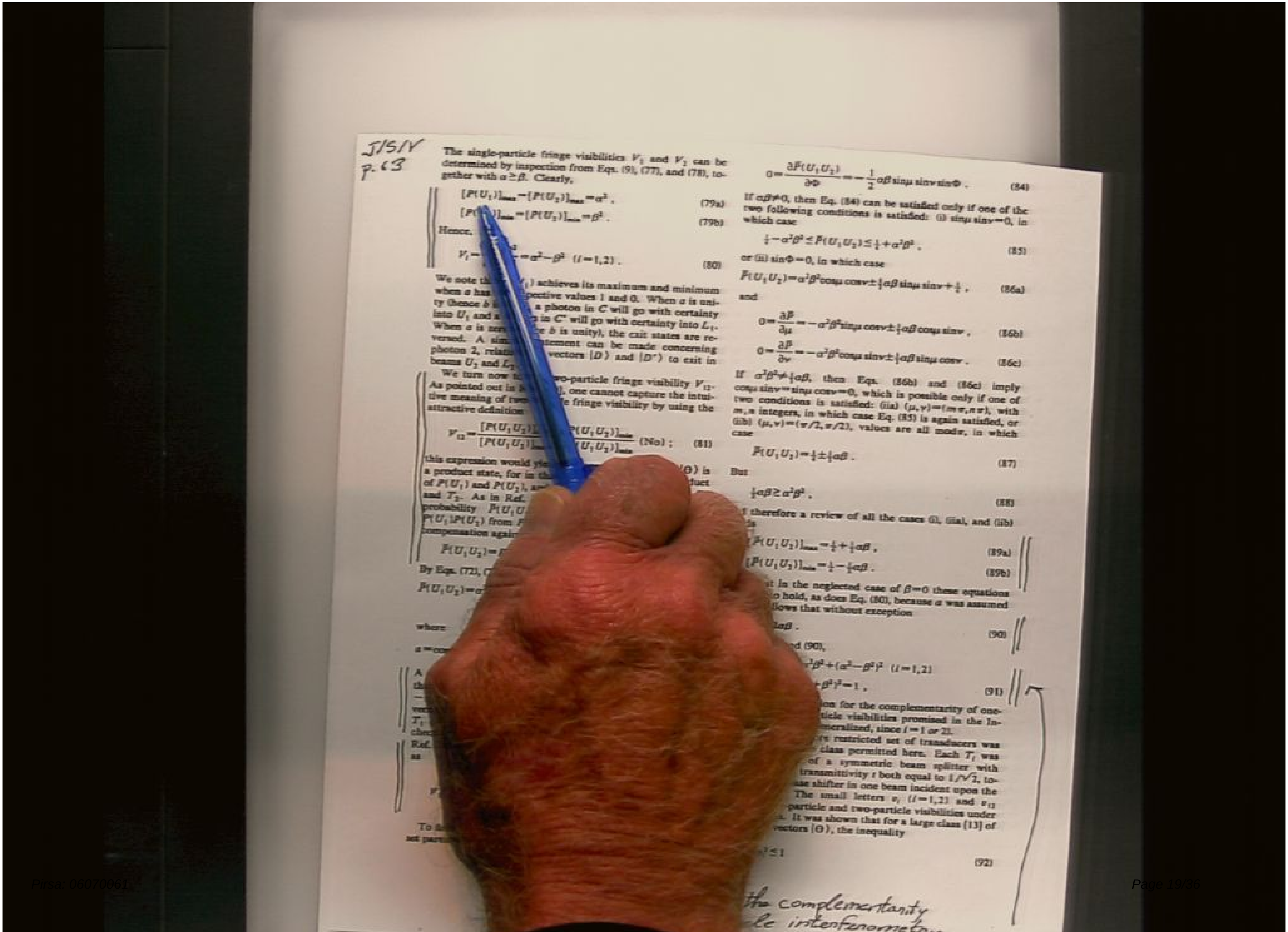

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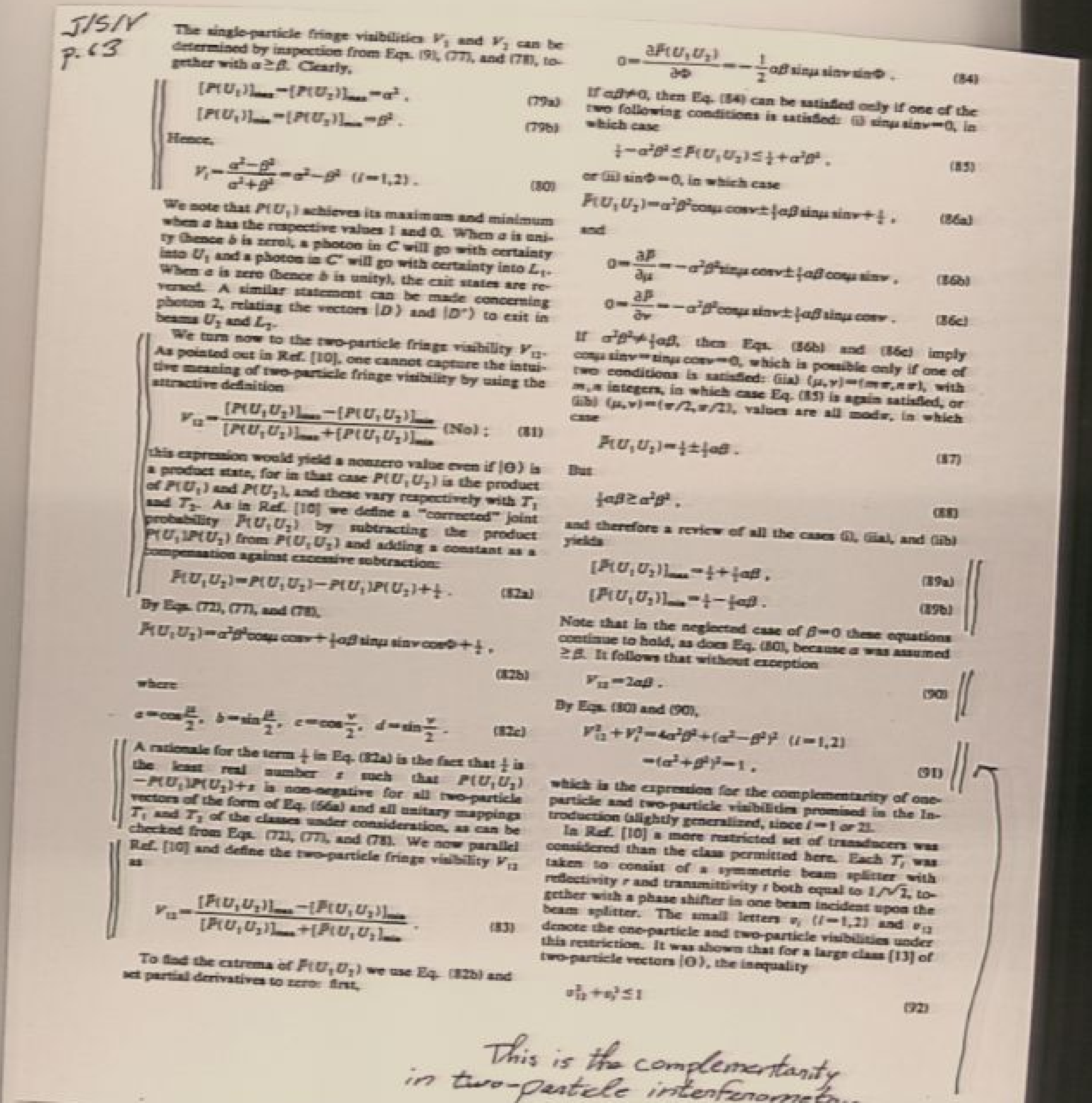

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p. 63

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A rationale for the term $\frac{1}{4}$ in Eq. (82a) is the fact that $\frac{1}{4}$ is the least real number s such that $P(U_1,U_2)-P(U_1)P(U_2)+s$ is non-negative for all two-particle vectors of the form of Eq. (66a) and all unitary mappings T_1 and T_2 of the classes under consideration, as can be checked from Eqs. (72), (77), and (78). We now parallel Ref. [10] and define the two-particle fringe visibility V_{12} as

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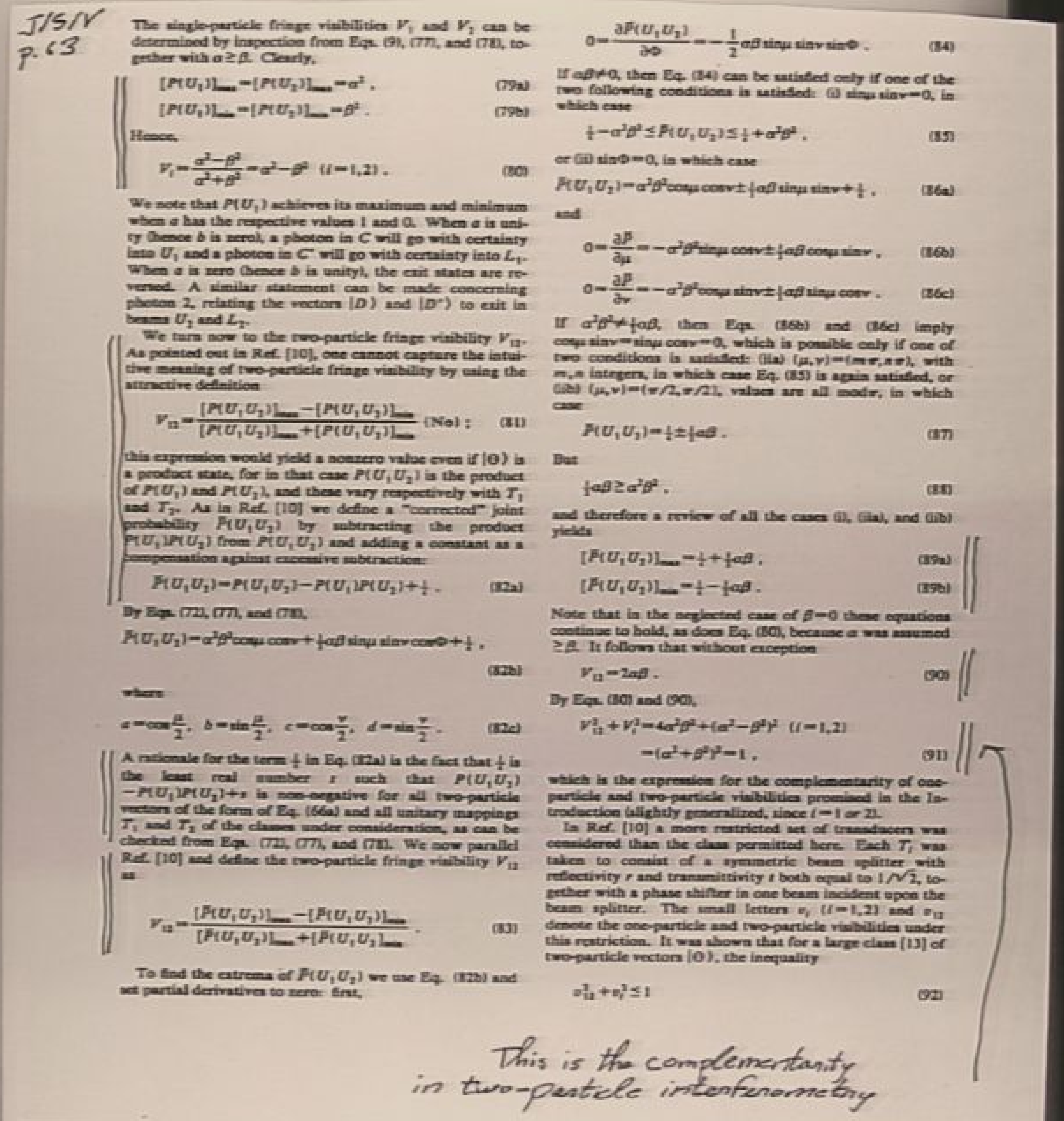

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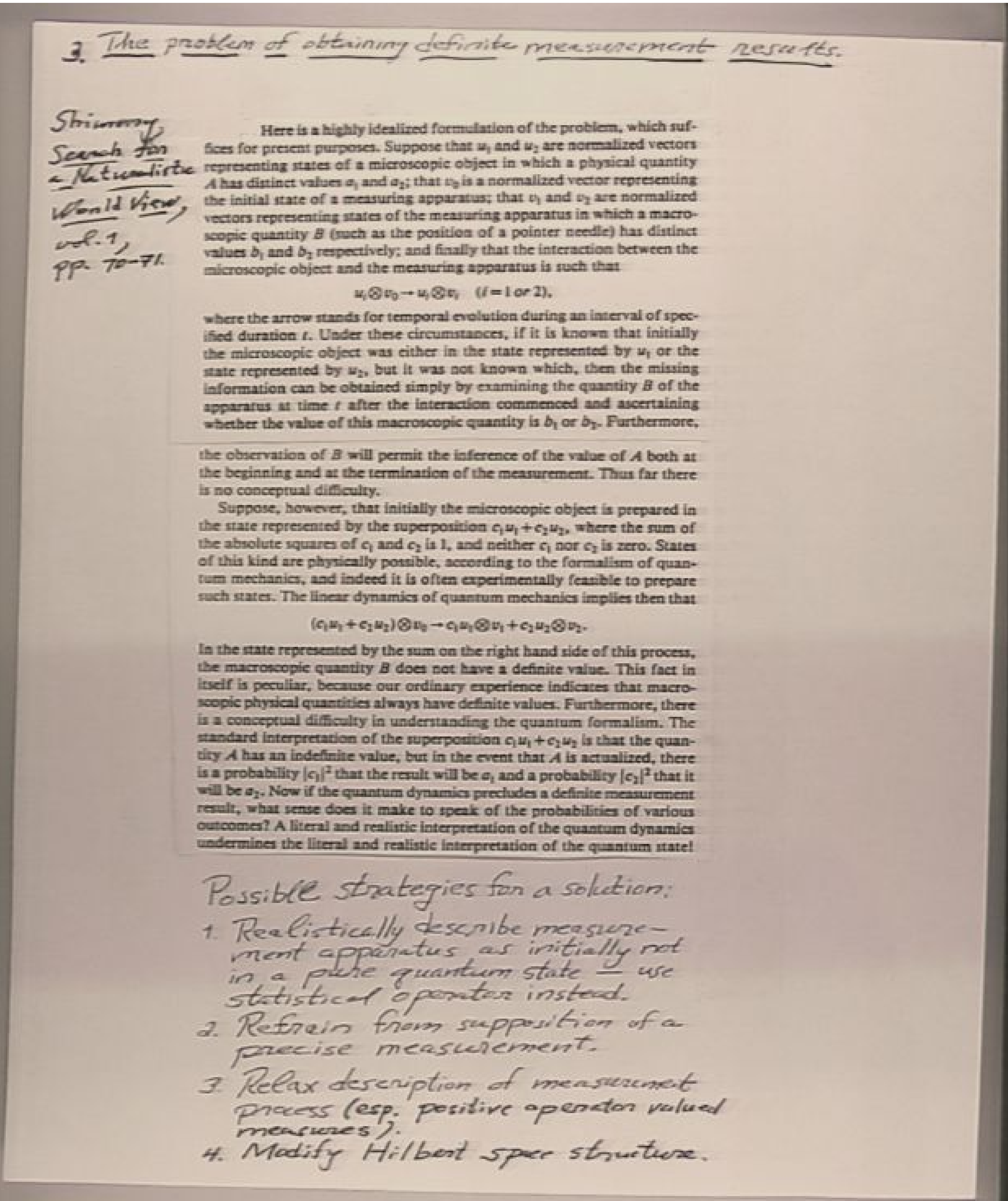


3. The problem of obtaining definite measurement results.

Shimony, Search for a Naturalistic World View, vol. 1, pp. 70-71.

Here is a highly idealized formulation of the problem, which suffices for present purposes. Suppose that u_1 and u_2 are normalized vectors representing states of a microscopic object in which a physical quantity A has distinct values a_1 and a_2; that v_0 is a normalized vector representing the initial state of a measuring apparatus; that v_1 and v_2 are normalized vectors representing states of the measuring apparatus in which a macroscopic quantity B (such as the position of a pointer needle) has distinct values b_1 and b_2 respectively; and finally that the interaction between the microscopic object and the measuring apparatus is such that

$$u_i \otimes v_0 \rightarrow u_i \otimes v_i \quad (i = 1 \text{ or } 2),$$

where the arrow stands for temporal evolution during an interval of specified duration t. Under these circumstances, if it is known that initially the microscopic object was either in the state represented by u_1 or the state represented by u_2, but it was not known which, then the missing information can be obtained simply by examining the quantity B of the apparatus at time t after the interaction commenced and ascertaining whether the value of this macroscopic quantity is b_1 or b_2. Furthermore, the observation of B will permit the inference of the value of A both at the beginning and at the termination of the measurement. Thus far there is no conceptual difficulty.

Suppose, however, that initially the microscopic object is prepared in the state represented by the superposition $c_1u_1 + c_2u_2$, where the sum of the absolute squares of c_1 and c_2 is 1, and neither c_1 nor c_2 is zero. States of this kind are physically possible, according to the formalism of quantum mechanics, and indeed it is often experimentally feasible to prepare such states. The linear dynamics of quantum mechanics implies then that

$$(c_1u_1 + c_2u_2) \otimes v_0 \rightarrow c_1u_1 \otimes v_1 + c_2u_2 \otimes v_2.$$

In the state represented by the sum on the right hand side of this process, the macroscopic quantity B does not have a definite value. This fact in itself is peculiar, because our ordinary experience indicates that macroscopic physical quantities always have definite values. Furthermore, there is a conceptual difficulty in understanding the quantum formalism. The standard interpretation of the superposition $c_1u_1 + c_2u_2$ is that the quantity A has an indefinite value, but in the event that A is actualized, there is a probability $|c_1|^2$ that the result will be a_1 and a probability $|c_2|^2$ that it will be a_2. Now if the quantum dynamics precludes a definite measurement result, what sense does it make to speak of the probabilities of various outcomes? A literal and realistic interpretation of the quantum dynamics undermines the literal and realistic interpretation of the quantum state!

Possible strategies for a solution:

1. Realistically describe measurement apparatus as initially not in a pure quantum state — use statistical operator instead.
2. Refrain from supposition of a precise measurement.
3. Relax description of measurement process (esp. positive operator valued measures).
4. Modify Hilbert space structure.

Shimony,
"Approximate
Measurement"
Part II,
Phys. Rev. D9
(1974),
2321.
Also Search
for =
Naturalistic
World View,
vol. II,
pp. 42-43.

one can give the present formulation of approximate measurement in two conditions:

(a) $\{E_m\}$ is a finite or denumerably infinite family of mutually orthogonal subspaces spanning $\mathcal{H}_1$. $\{E_m\}$ is a family of mutually orthogonal subspaces of $\mathcal{H}_2$, and U is a unitary operator on $\mathcal{H}_1 \otimes \mathcal{H}_2$;
(b) T is a statistical operator on $\mathcal{H}_2$ such that for every m and every $v \in E_m$, $U(P_v \otimes T)U^{-1}$ can be expressed in the form $\sum_{n,r} a_{nr} P_{\chi_{nr}}$, where $\chi_{nr} \in \mathcal{H}_1 \otimes E_n$, and the a_{nr} are non-negative real numbers summing to 1 such that

$$\sum_{\substack{r\\ n\neq m}} a_{nr} = \epsilon_m \ll 1.$$

The theorem of Sec. II implies that if these two conditions are satisfied and if the number of subspaces E_m is greater than one, then there exist initial states of the object for which the final statistical state of the object plus apparatus is not expressible as a mixture of eigenstates of the apparatus observable.

II. A THEOREM ON MEASUREMENT

It will be convenient for proving the theorem of this section to use the Dirac bra and ket notation, in which $\langle\phi|\phi\rangle = 1$ implies that $|\phi\rangle\langle\phi|$ is the projection operator P_ϕ.

The theorem is the following.

Hypotheses:
(i) *u_1, u_2 are normalized orthogonal vectors of $\mathcal{H}_1$, $\{E_n\}$ is a family of mutually orthogonal subspaces of $\mathcal{H}_2$, U is a unitary operator on $\mathcal{H}_1 \otimes \mathcal{H}_2$, and T is a statistical operator on $\mathcal{H}_2$;*
(ii) *there exist orthonormal sets $\{\xi^1_{nr}\}$, $\{\xi^2_{nr}\}$ such that*

$$\xi^j_{nr} \in \mathcal{H}_1 \otimes E_n \quad \textit{for } j = 1, 2,$$

and

$$U(P_{u_j} \otimes T)U^{-1} = \sum_{n,r} b^j_{nr} |\xi^j_{nr}\rangle\langle\xi^j_{nr}|;$$

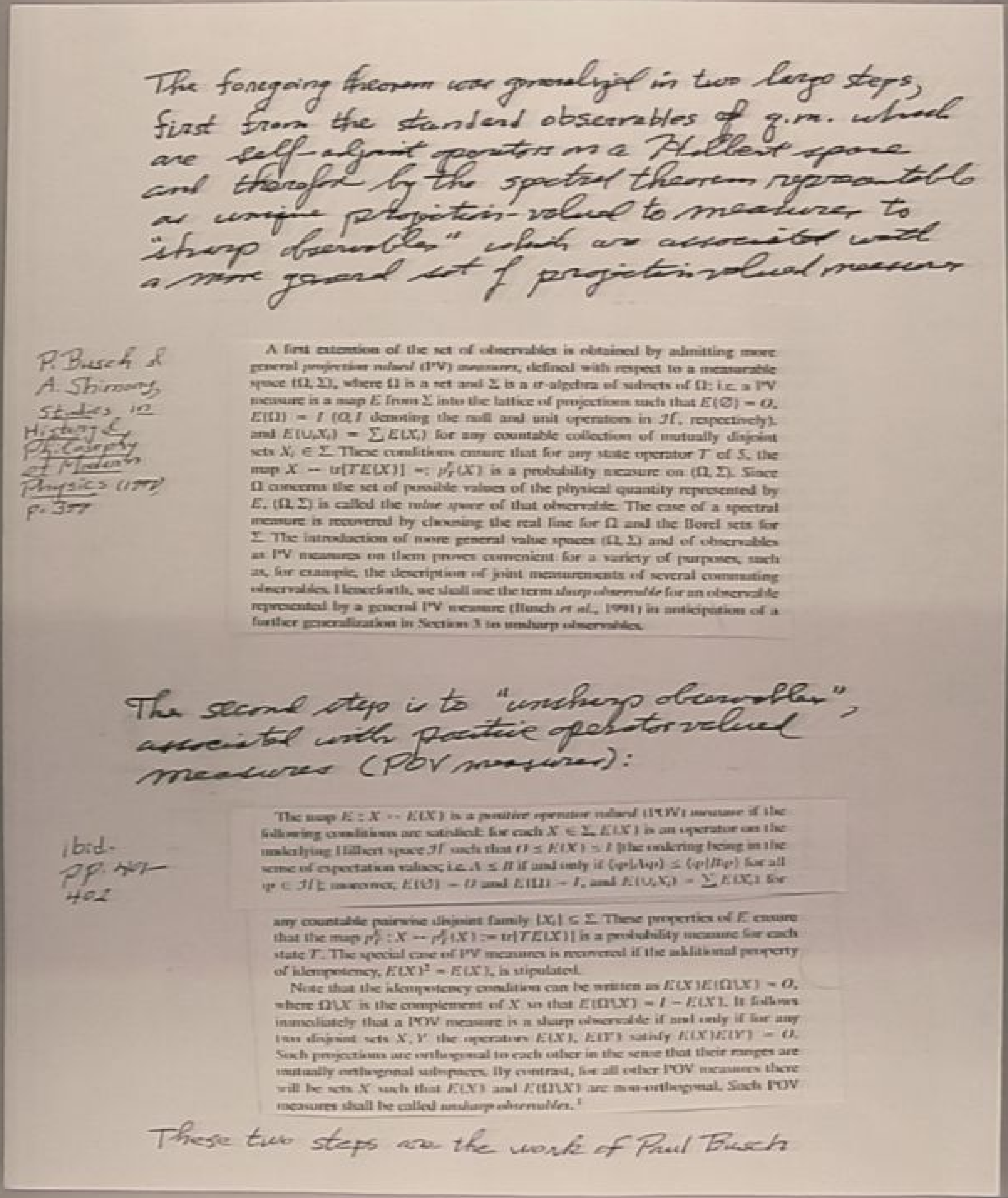
and for some value of n,

$$\sum_r b^1_{nr} \neq \sum_r b^2_{nr}.$$

Conclusion: If u is defined as $g_1u_1 + g_2u_2$, with both g_1 and g_2 nonzero, then there exists no orthonormal set $\{\psi_{nr}\}$ with $\psi_{nr} \in \mathcal{H}_1 \otimes E_n$ and no coefficients $\{b_{nr}\}$ such that $\sum_{n,r} b_{nr} = 1$ and

$$U(P_u \otimes T)U^{-1} = \sum_{n,r} b_{nr} |\psi_{nr}\rangle\langle\psi_{nr}|.$$

This theorem shows that strategies 1 and 2 for obtaining definite measurement results will not succeed.



The foregoing theorem was generalized in two large steps, first from the standard observables of q.m. which are self-adjoint operators on a Hilbert space and therefore by the spectral theorem representable as unique projection-valued measures to "sharp observables" which are associated with a more general set of projection valued measures

P. Busch & A. Shimony, Studies in History & Philosophy of Modern Physics (1997) p. 397

A first extension of the set of observables is obtained by admitting more general *projection valued* (PV) *measures*, defined with respect to a measurable space (Ω, Σ), where Ω is a set and Σ is a σ-algebra of subsets of Ω; i.e. a PV measure is a map E from Σ into the lattice of projections such that $E(\emptyset) = O$, $E(\Omega) = I$ (O, I denoting the null and unit operators in $\mathcal{H}$, respectively), and $E(\cup_i X_i) = \sum_i E(X_i)$ for any countable collection of mutually disjoint sets $X_i \in \Sigma$. These conditions ensure that for any state operator T of S, the map $X \to \mathrm{tr}[TE(X)] =: p_T^E(X)$ is a probability measure on (Ω, Σ). Since Ω concerns the set of possible values of the physical quantity represented by E, (Ω, Σ) is called the *value space* of that observable. The case of a spectral measure is recovered by choosing the real line for Ω and the Borel sets for Σ. The introduction of more general value spaces (Ω, Σ) and of observables as PV measures on them proves convenient for a variety of purposes, such as, for example, the description of joint measurements of several commuting observables. Henceforth, we shall use the term *sharp observable* for an observable represented by a general PV measure (Busch *et al.*, 1991) in anticipation of a further generalization in Section 3 to unsharp observables.

The second steps is to "unsharp observables", associated with positive operator valued measures (POV measures):

ibid. PP. 401–402

The map $E: X \to E(X)$ is a *positive operator valued* (POV) *measure* if the following conditions are satisfied: for each $X \in \Sigma$, $E(X)$ is an operator on the underlying Hilbert space $\mathcal{H}$ such that $O \leq E(X) \leq I$ [the ordering being in the sense of expectation values; i.e. $A \leq B$ if and only if $\langle\varphi|A\varphi\rangle \leq \langle\varphi|B\varphi\rangle$ for all $\varphi \in \mathcal{H}$]; moreover, $E(\emptyset) = O$ and $E(\Omega) = I$, and $E(\cup_i X_i) = \sum_i E(X_i)$ for any countable pairwise disjoint family $\{X_i\} \subseteq \Sigma$. These properties of E ensure that the map $p_T^E: X \to p_T^E(X) := \mathrm{tr}[TE(X)]$ is a probability measure for each state T. The special case of PV measures is recovered if the additional property of idempotency, $E(X)^2 = E(X)$, is stipulated.

Note that the idempotency condition can be written as $E(X)E(\Omega\backslash X) = O$, where $\Omega\backslash X$ is the complement of X so that $E(\Omega\backslash X) = I - E(X)$. It follows immediately that a POV measure is a sharp observable if and only if for any two disjoint sets X, Y the operators $E(X)$, $E(Y)$ satisfy $E(X)E(Y) = O$. Such projections are orthogonal to each other in the sense that their ranges are mutually orthogonal subspaces. By contrast, for all other POV measures there will be sets X such that $E(X)$ and $E(\Omega\backslash X)$ are non-orthogonal. Such POV measures shall be called *unsharp observables*.[1]

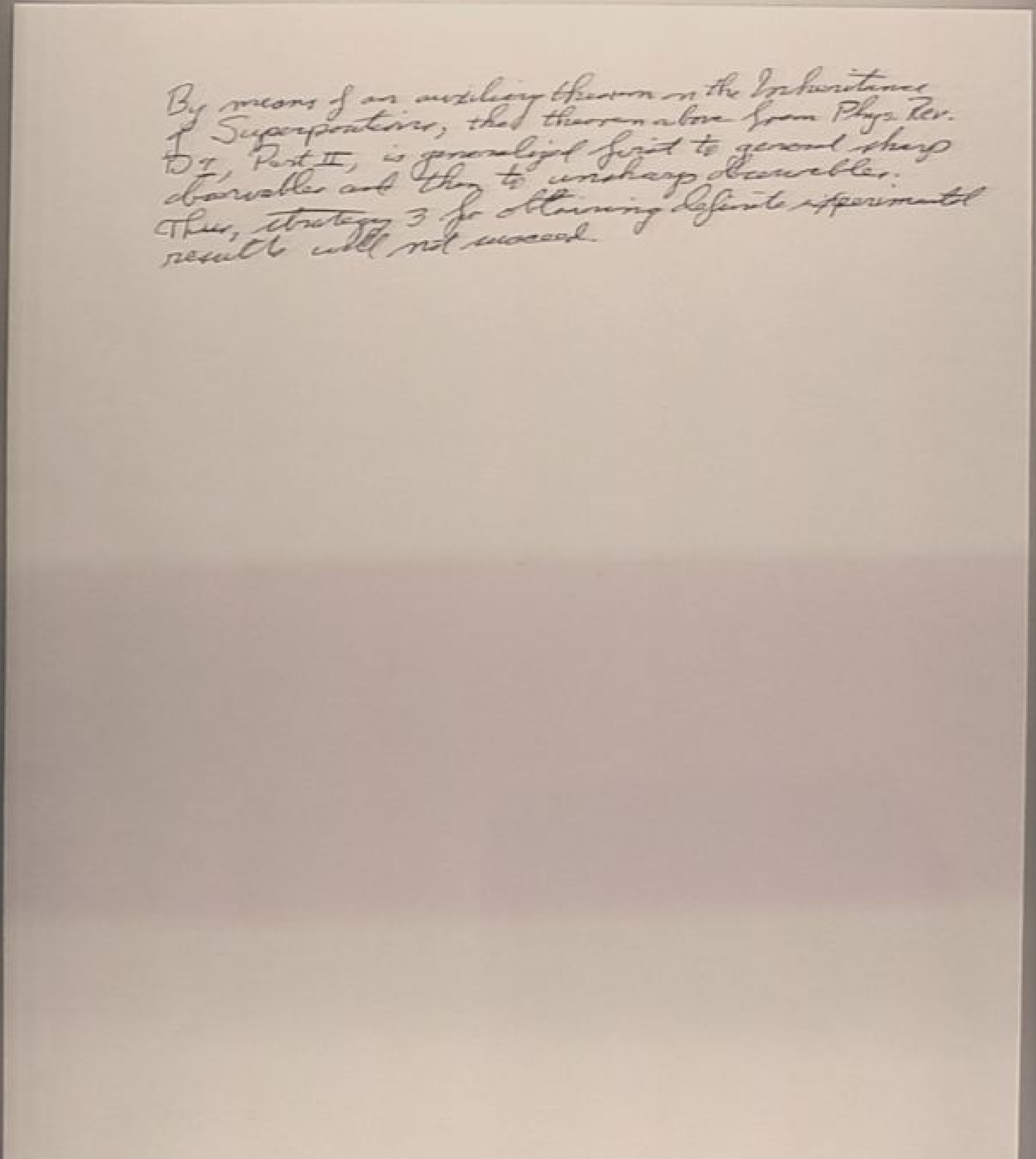
These two steps are the work of Paul Busch

A rough operational definition of "projection-valued measure": a rule for partitioning an arbitrary quantum state into projections onto eigenspaces of some physical quantity of a system and then labeling the projected pieces with the eigenvalues associated with the eigenspaces. (Must be refined in case of continuous spectra.)

A "position-operator valued measure" is also a rule for partitioning an arbitrary quantum state and for labeling the pieces, but in general labeling with less information than eigenvalues — less than the whole truth but not a distortion of the truth.

By means of an auxiliary theorem on the Inheritance of Superpositions, that theorem above from Phys. Rev. D9, Part II, is generalized first to general sharp observables and then to unsharp observables.

Thus, strategy 3 for obtaining definite experimental results will not succeed



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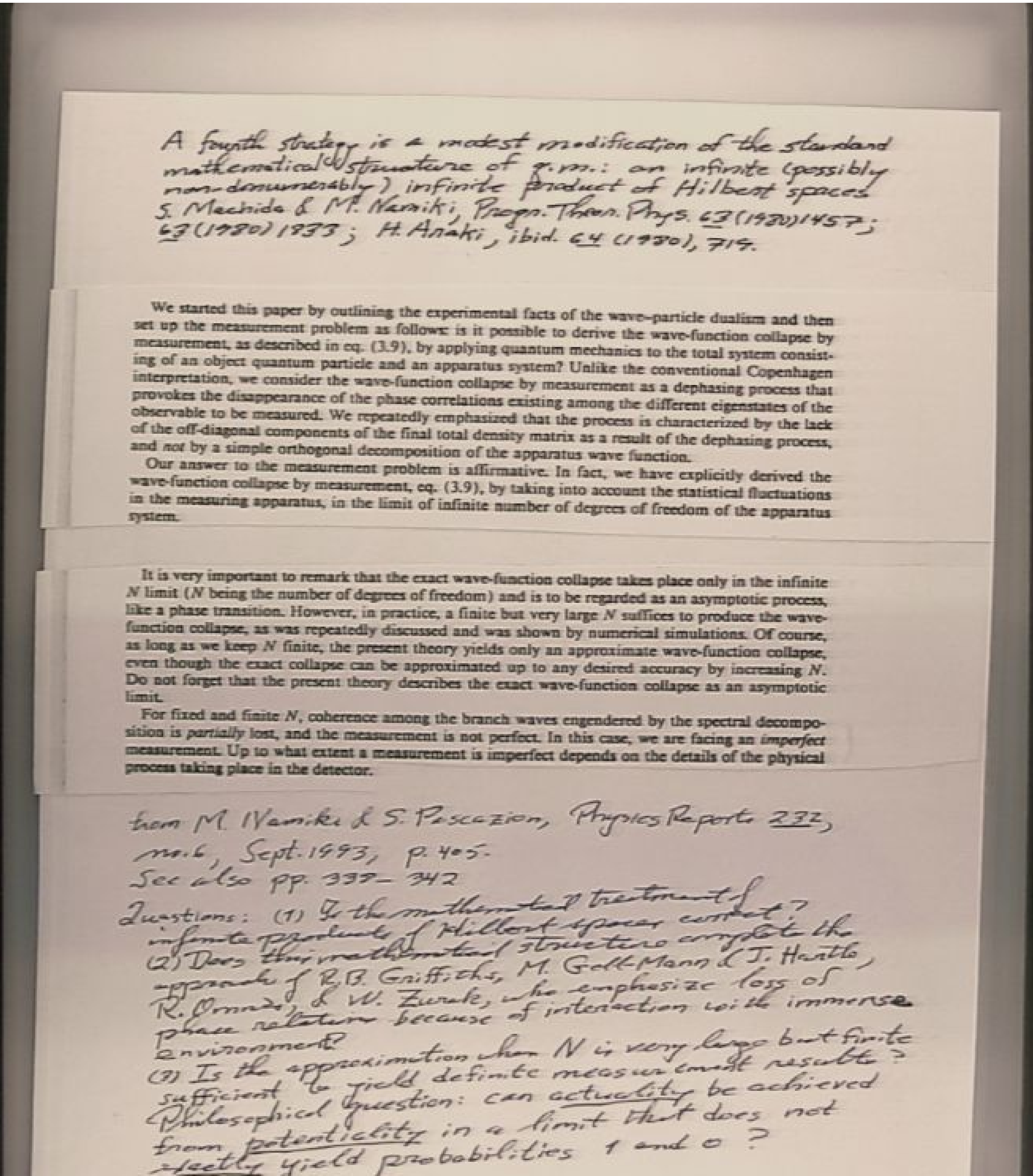


A fourth strategy is a modest modification of the standard mathematical structure of q.m.: an infinite (possibly non-denumerably) infinite product of Hilbert spaces
S. Machida & M. Namiki, Progr. Theor. Phys. 63 (1980) 1457; 63 (1980) 1833; H. Araki, ibid. 64 (1980), 719.

We started this paper by outlining the experimental facts of the wave-particle dualism and then set up the measurement problem as follows: is it possible to derive the wave-function collapse by measurement, as described in eq. (3.9), by applying quantum mechanics to the total system consisting of an object quantum particle and an apparatus system? Unlike the conventional Copenhagen interpretation, we consider the wave-function collapse by measurement as a dephasing process that provokes the disappearance of the phase correlations existing among the different eigenstates of the observable to be measured. We repeatedly emphasized that the process is characterized by the lack of the off-diagonal components of the final total density matrix as a result of the dephasing process, and *not* by a simple orthogonal decomposition of the apparatus wave function.

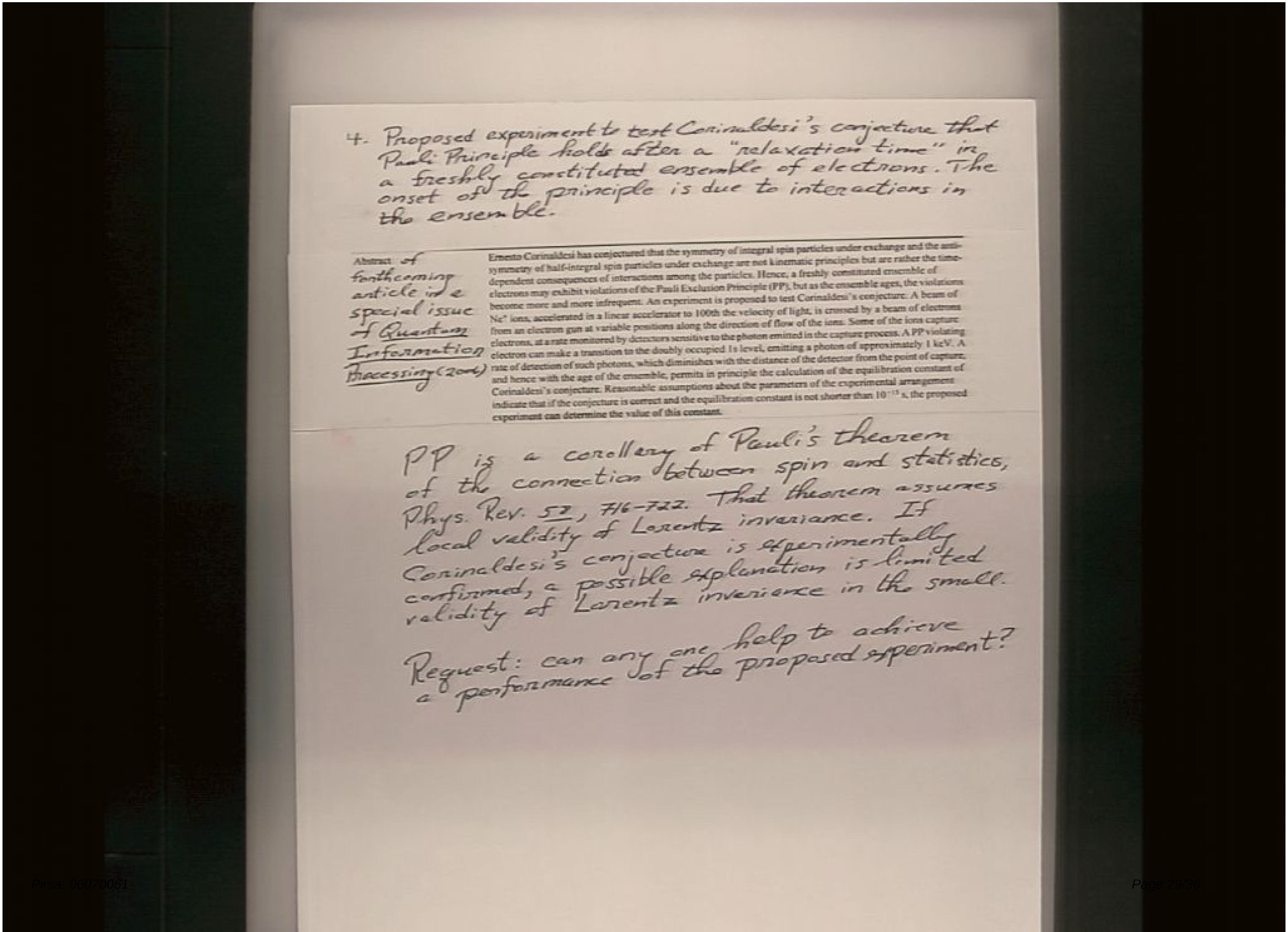
Our answer to the measurement problem is affirmative. In fact, we have explicitly derived the wave-function collapse by measurement, eq. (3.9), by taking into account the statistical fluctuations in the measuring apparatus, in the limit of infinite number of degrees of freedom of the apparatus system.

It is very important to remark that the exact wave-function collapse takes place only in the infinite N limit (N being the number of degrees of freedom) and is to be regarded as an asymptotic process, like a phase transition. However, in practice, a finite but very large N suffices to produce the wave-function collapse, as was repeatedly discussed and was shown by numerical simulations. Of course, as long as we keep N finite, the present theory yields only an approximate wave-function collapse, even though the exact collapse can be approximated up to any desired accuracy by increasing N. Do not forget that the present theory describes the exact wave-function collapse as an asymptotic limit.

For fixed and finite N, coherence among the branch waves engendered by the spectral decomposition is *partially* lost, and the measurement is not perfect. In this case, we are facing an *imperfect* measurement. Up to what extent a measurement is imperfect depends on the details of the physical process taking place in the detector.

from M. Namiki & S. Pascazio, Physics Reports 232, no. 6, Sept. 1993, p. 405.
See also pp. 339 – 342

Questions: (1) Is the mathematical treatment of infinite products of Hilbert spaces correct?
(2) Does this mathematical structure complete the approach of R.B. Griffiths, M. Gell-Mann & J. Hartle, R. Omnès, & W. Zurek, who emphasize loss of phase relations because of interaction with immense environment?
(3) Is the approximation when N is very large but finite sufficient to yield definite measurement results?
Philosophical question: can actuality be achieved from potentiality in a limit that does not exactly yield probabilities 1 and 0?



4. Proposed experiment to test Corinaldesi's conjecture that Pauli Principle holds after a "relaxation time" in a freshly constituted ensemble of electrons. The onset of the principle is due to interactions in the ensemble.

Abstract: of forthcoming article in a special issue of Quantum Information Processing (2006)

Ernesto Corinaldesi has conjectured that the symmetry of integral spin particles under exchange and the anti-
symmetry of half-integral spin particles under exchange are not kinematic principles but are rather the time-
dependent consequences of interactions among the particles. Hence, a freshly constituted ensemble of
electrons may exhibit violations of the Pauli Exclusion Principle (PP), but as the ensemble ages, the violations
become more and more infrequent. An experiment is proposed to test Corinaldesi's conjecture. A beam of
Ne⁺ ions, accelerated in a linear accelerator to 100th the velocity of light, is crossed by a beam of electrons
from an electron gun at variable positions along the direction of flow of the ions. Some of the ions capture
electrons, at a rate monitored by detectors sensitive to the photon emitted in the capture process. A PP violating
electron can make a transition to the doubly occupied 1s level, emitting a photon of approximately 1 keV. A
rate of detection of such photons, which diminishes with the distance of the detector from the point of capture,
and hence with the age of the ensemble, permits in principle the calculation of the equilibration constant of
Corinaldesi's conjecture. Reasonable assumptions about the parameters of the experimental arrangement
indicate that if the conjecture is correct and the equilibration constant is not shorter than 10^{-13} s, the proposed
experiment can determine the value of this constant.

PP is a corollary of Pauli's theorem of the connection between spin and statistics, Phys. Rev. 57, 716-722. That theorem assumes local validity of Lorentz invariance. If Corinaldesi's conjecture is experimentally confirmed, a possible explanation is limited validity of Lorentz invariance in the small.

Request: can any one help to achieve a performance of the proposed experiment?

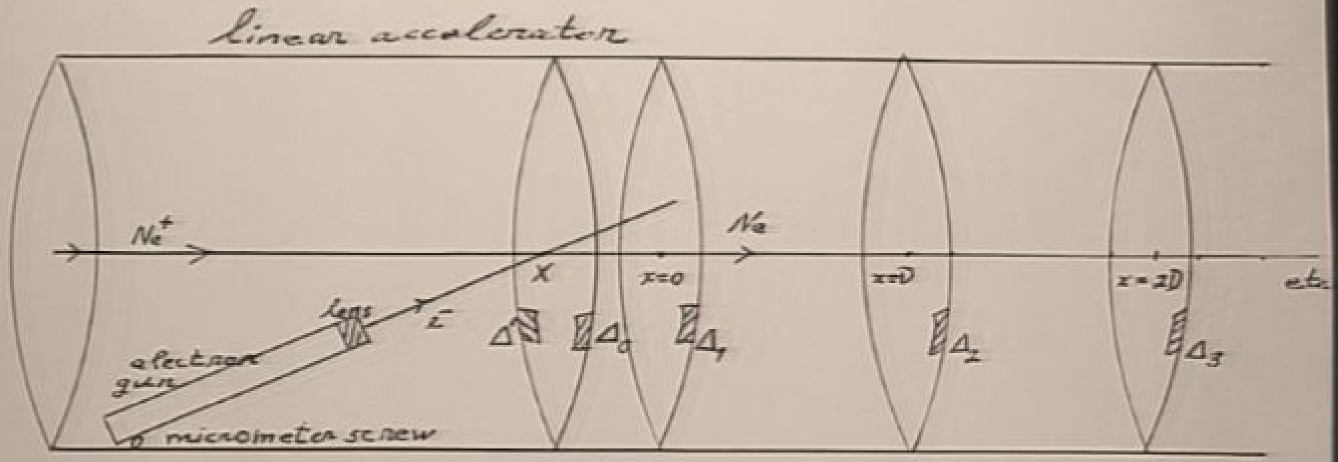


Fig. 1 Micrometer screw moves X in steps of 10^{-4} cm.
Lens moves X in steps of 10^{-6} cm.
Detectors Δ_0 and Δ' at X, both movable.
Detectors $\Delta_1, ---, \Delta_{10}$ fixed, separated by 80 cm.
Diameter of each detector: 2.5 cm.
Diagram not drawn to scale.

5. How should one treat the tension between the nonlocality exhibited in careful experimental tests of Bell's Inequality and the locality implicit in the special theory of relativity?

A much-favored proposal: the nonlocality in the Bell tests (which is implied by q.m.) cannot be used to send a superluminal signal: theorems of P. Eberhard, Nuovo Cimento 46B (1978), 392; G.C. Ghirardi, A. Rimini, & T. Weber, Lett. Nuov. Cim. 27 (1980), 293; D. Page, Phys. Lett. 91A (1982), 57.

Bell's rejection of this anthropocentric variety of peaceful coexistence:

> The obvious definition of 'local causality' does not work in quantum mechanics, and this cannot be attributed to the 'incompleteness' of that theory[11].
>
> Experimenters have looked to see if the relevant predictions of quantum mechanics are in fact true[1,2,9,12]. The consensus is that quantum mechanics works excellently, with no sign of an error [redacted]. It is often said then that experiment has decided against the locality inequality. Strictly speaking that is not so. The actual experiments depart too far from the ideal[13], and only after the various deficiencies are 'corrected' by theoretical extrapolation do the actual experiments become critical. There is a school of thought[14] which stresses this fact, and advocates the idea that better experiments may contradict quantum mechanics and vindicate locality. I do not myself entertain that hope. I am too impressed by the quantitative success of quantum mechanics, for the experiments already done, to hope that it will fail for more nearly ideal ones.
>
> Do we then have to fall back on 'no signalling faster than light' as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have lost the idea that correlations can be explained, or at least this idea awaits reformulation. More importantly, the 'no signalling...' notion rests on concepts which are desperately vague, or vaguely applicable. The assertion that 'we cannot signal faster than light' immediately provokes the question:
>
> Who do we think we are?
>
> *We* who can make 'measurements', *we* who can manipulate 'external fields', *we* who can 'signal' at all, even if not faster than light? Do *we* include chemists, or only physicists, plants, or only animals, pocket

J.S. Bell,
Speakable & Unspeakable in Q.M.,
2nd ed.
(Cambridge U. Press, 2004),
p. 245.

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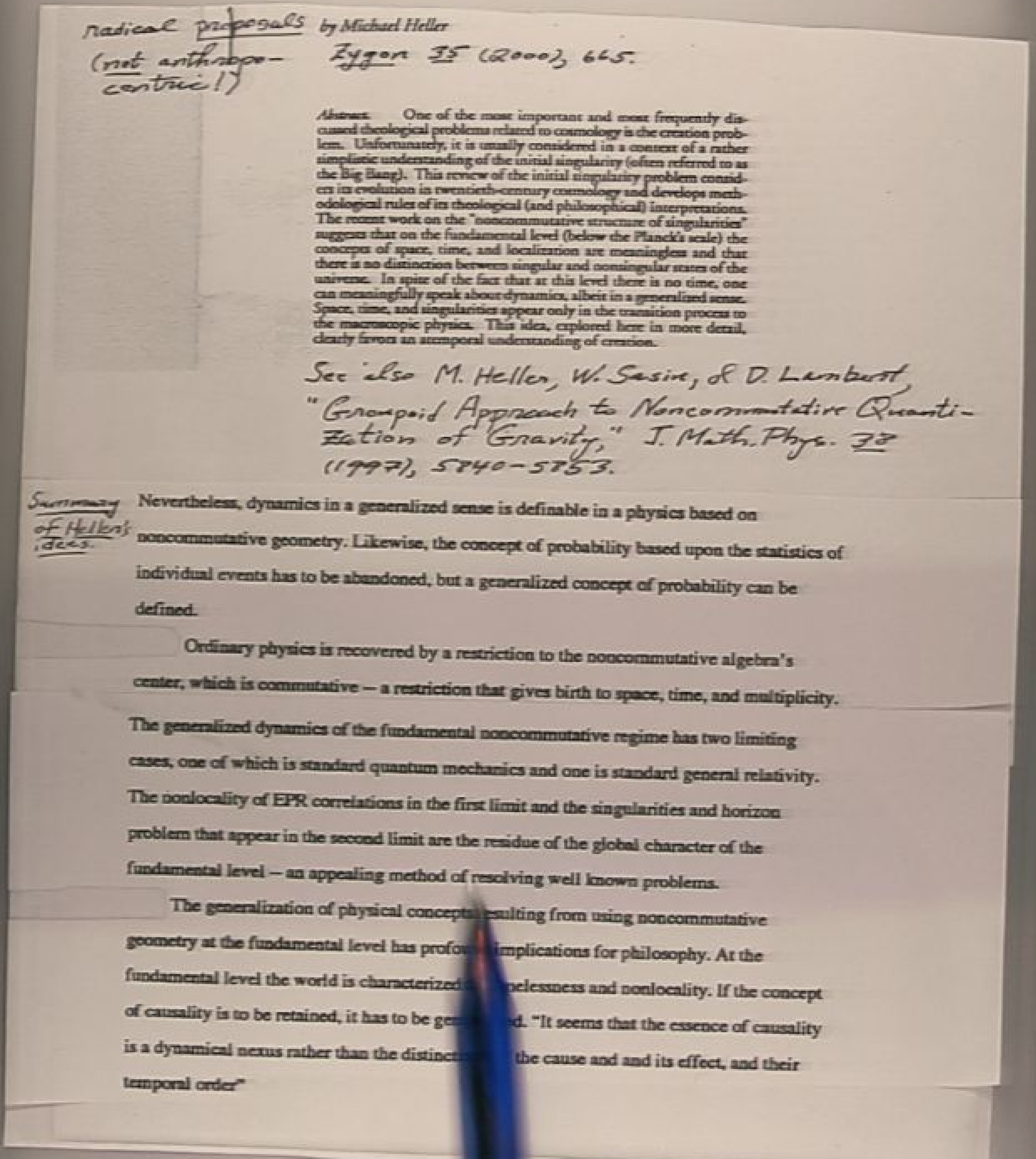


radical proposals
(not anthropo-
centric!)

by Michael Heller

Zygon 35 (2000), 665.

Abstract. One of the most important and most frequently discussed theological problems related to cosmology is the creation problem. Unfortunately, it is usually considered in a context of a rather simplistic understanding of the initial singularity (often referred to as the Big Bang). This review of the initial singularity problem considers its evolution in twentieth-century cosmology and develops methodological rules of its theological (and philosophical) interpretations. The recent work on the "noncommutative structure of singularities" suggests that on the fundamental level (below the Planck's scale) the concepts of space, time, and localization are meaningless and that there is no distinction between singular and nonsingular states of the universe. In spite of the fact that at this level there is no time, one can meaningfully speak about dynamics, albeit in a generalized sense. Space, time, and singularities appear only in the transition process to the macroscopic physics. This idea, explored here in more detail, clearly favors an atemporal understanding of creation.

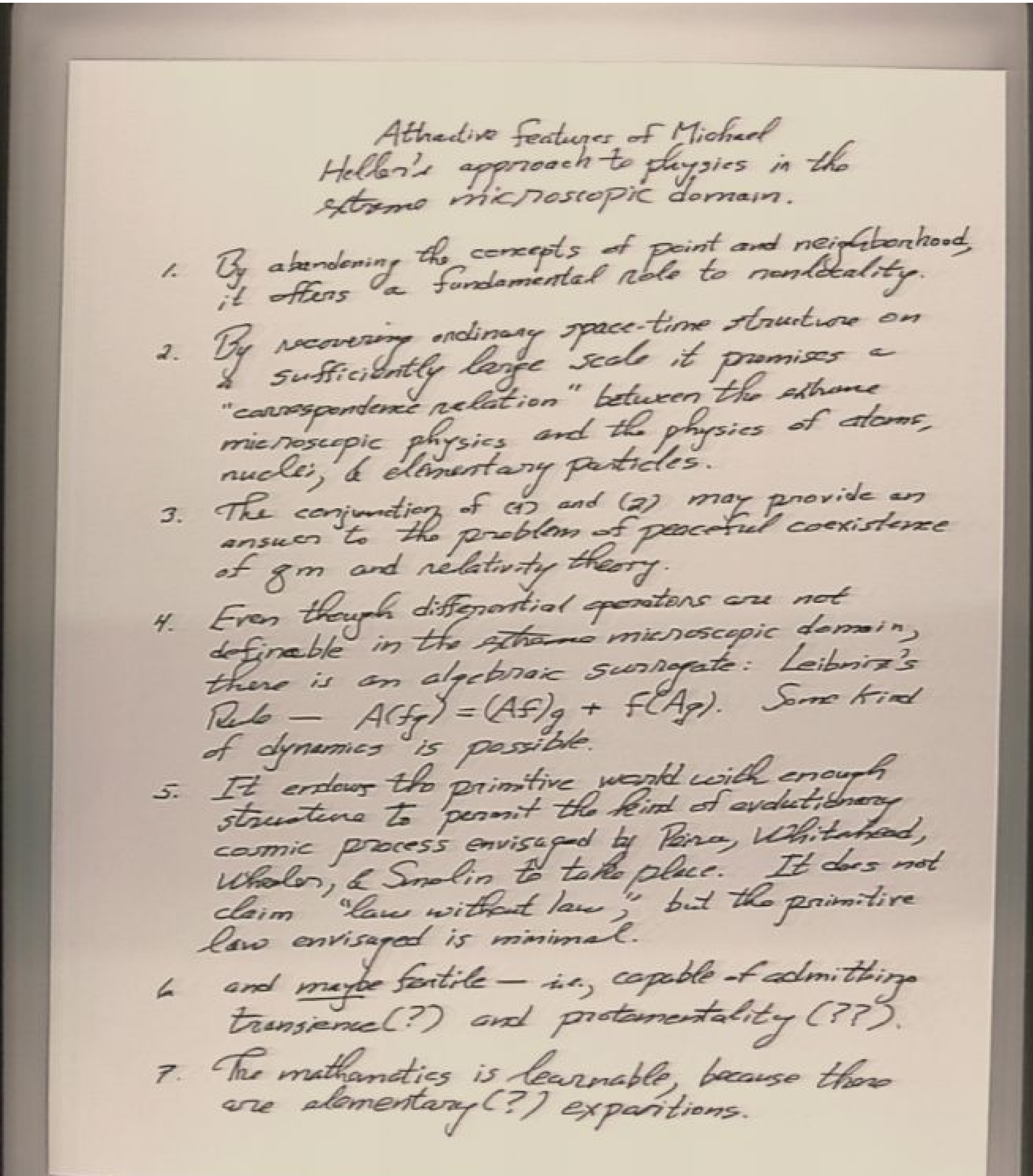
See also M. Heller, W. Sasin, & D. Lambert,
"Groupoid Approach to Noncommutative Quanti-
zation of Gravity," J. Math. Phys. 38
(1997), 5840–5853.

Summary of Heller's ideas.

Nevertheless, dynamics in a generalized sense is definable in a physics based on noncommutative geometry. Likewise, the concept of probability based upon the statistics of individual events has to be abandoned, but a generalized concept of probability can be defined.

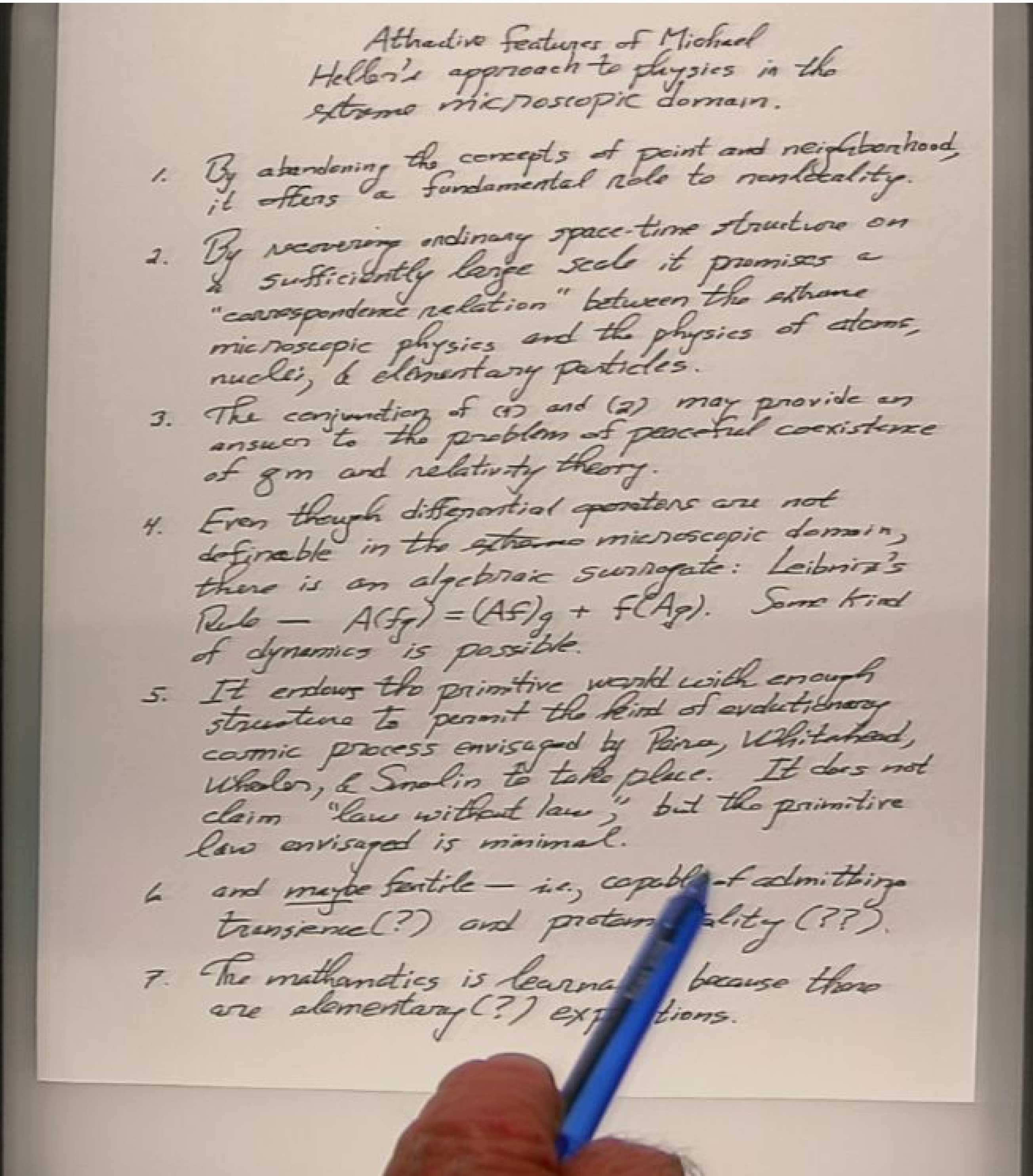
Ordinary physics is recovered by a restriction to the noncommutative algebra's center, which is commutative – a restriction that gives birth to space, time, and multiplicity. The generalized dynamics of the fundamental noncommutative regime has two limiting cases, one of which is standard quantum mechanics and one is standard general relativity. The nonlocality of EPR correlations in the first limit and the singularities and horizon problem that appear in the second limit are the residue of the global character of the fundamental level – an appealing method of resolving well known problems.

The generalization of physical concept[illegible]esulting from using noncommutative geometry at the fundamental level has profo[illegible] implications for philosophy. At the fundamental level the world is characterized [illegible]elessness and nonlocality. If the concept of causality is to be retained, it has to be ge[illegible]d. "It seems that the essence of causality is a dynamical nexus rather than the distinc[illegible] the cause and and its effect, and their temporal order"



Attractive features of Michael
Heller's approach to physics in the
extreme microscopic domain.

1. By abandoning the concepts of point and neighborhood, it offers a fundamental role to nonlocality.
2. By recovering ordinary space-time structure on a sufficiently large scale it promises a "correspondence relation" between the extreme microscopic physics and the physics of atoms, nuclei, & elementary particles.
3. The conjunction of (1) and (2) may provide an answer to the problem of peaceful coexistence of qm and relativity theory.
4. Even though differential operators are not definable in the extreme microscopic domain, there is an algebraic surrogate: Leibniz's Rule — $A(fg) = (Af)g + f(Ag)$. Some kind of dynamics is possible.
5. It endows the primitive world with enough structure to permit the kind of evolutionary cosmic process envisaged by Peirce, Whitehead, Wheeler, & Smolin to take place. It does not claim "law without law," but the primitive law envisaged is minimal.
6. and *maybe* fertile — i.e., capable of admitting transience(?) and protomentality(??).
7. The mathematics is learnable, because there are elementary(?) expositions.



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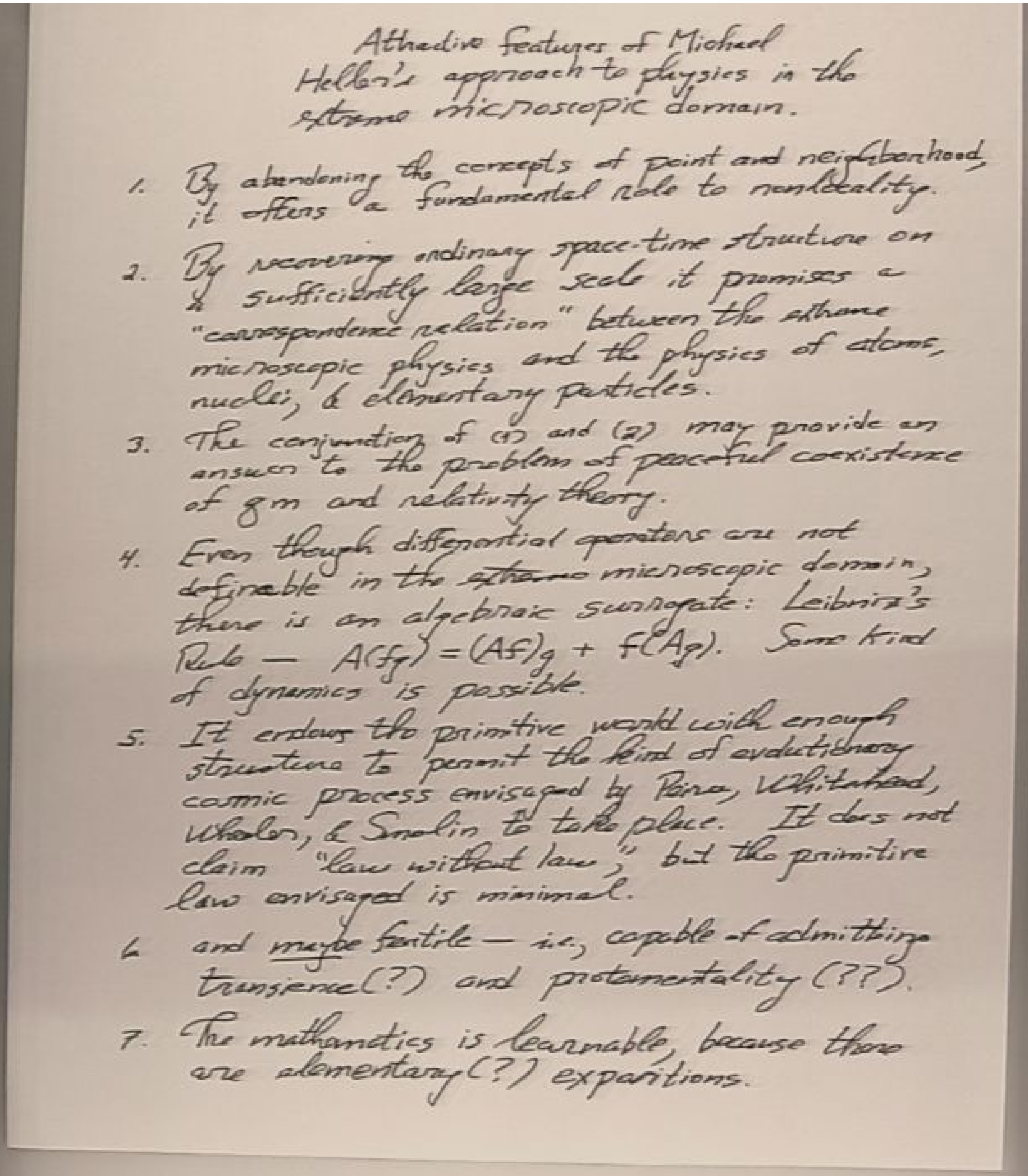

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