Title: Peirce, James, and the Quantum Bayesians

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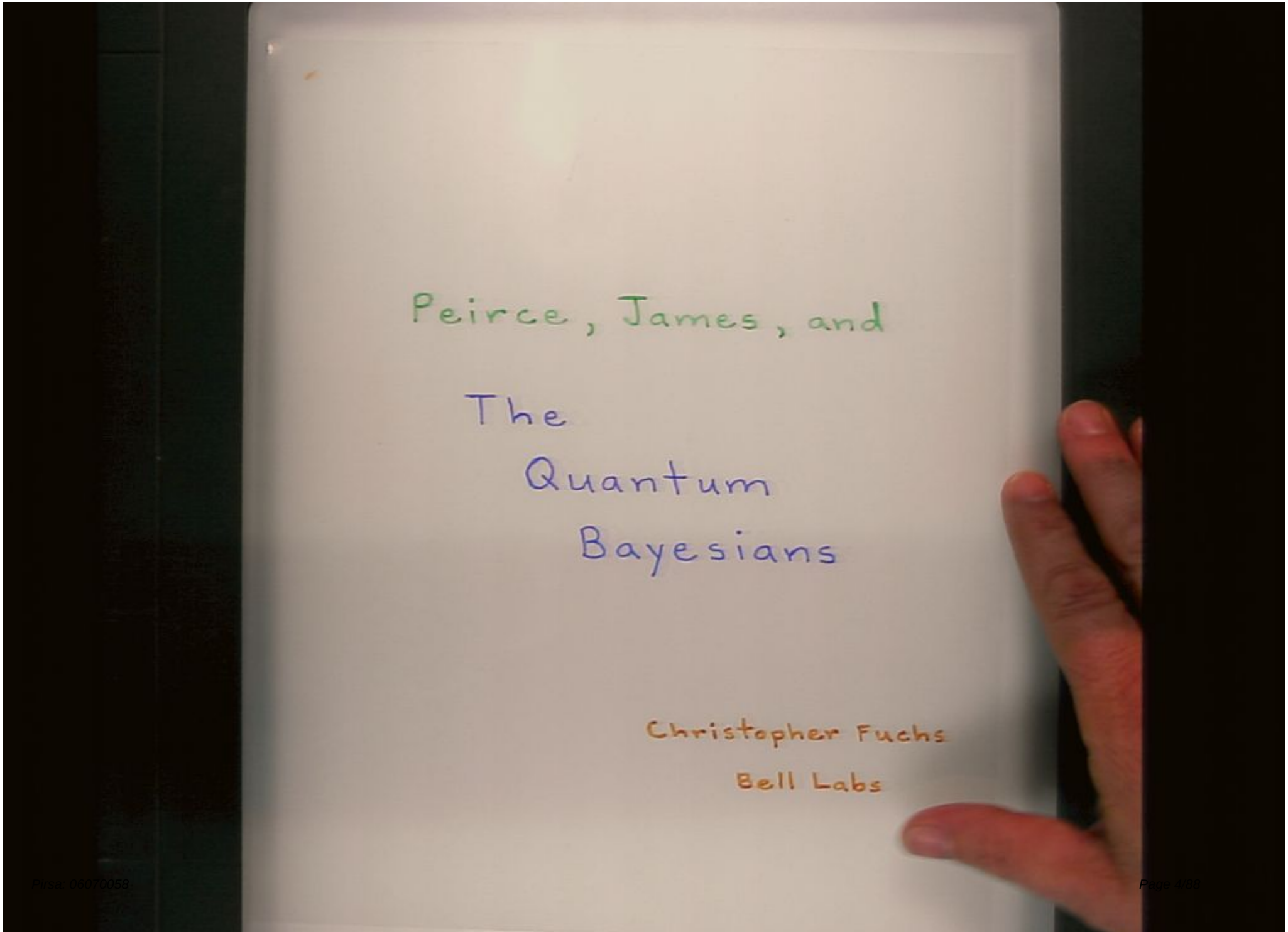
Abstract:

The hypothesis that there is an external world, not dependent on human minds, made of something.) is so obviously useful and so strongly confirmed by experience down through the ages that we can say without exaggerating that it is better confirmed than any other empirical hypothesis.

— Martin Gardner


Peirce, James, and
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Christopher Fuchs
Bell Labs

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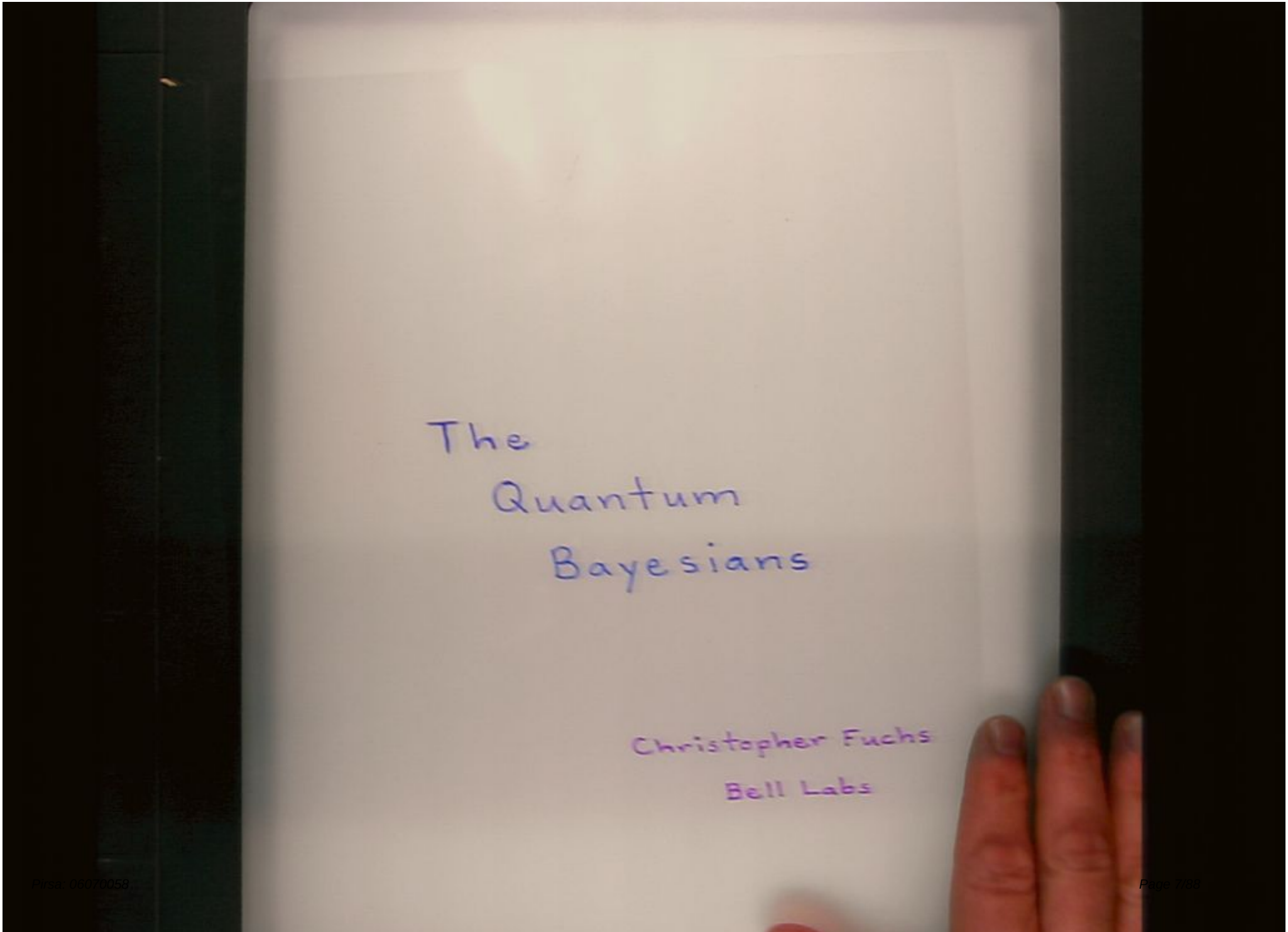
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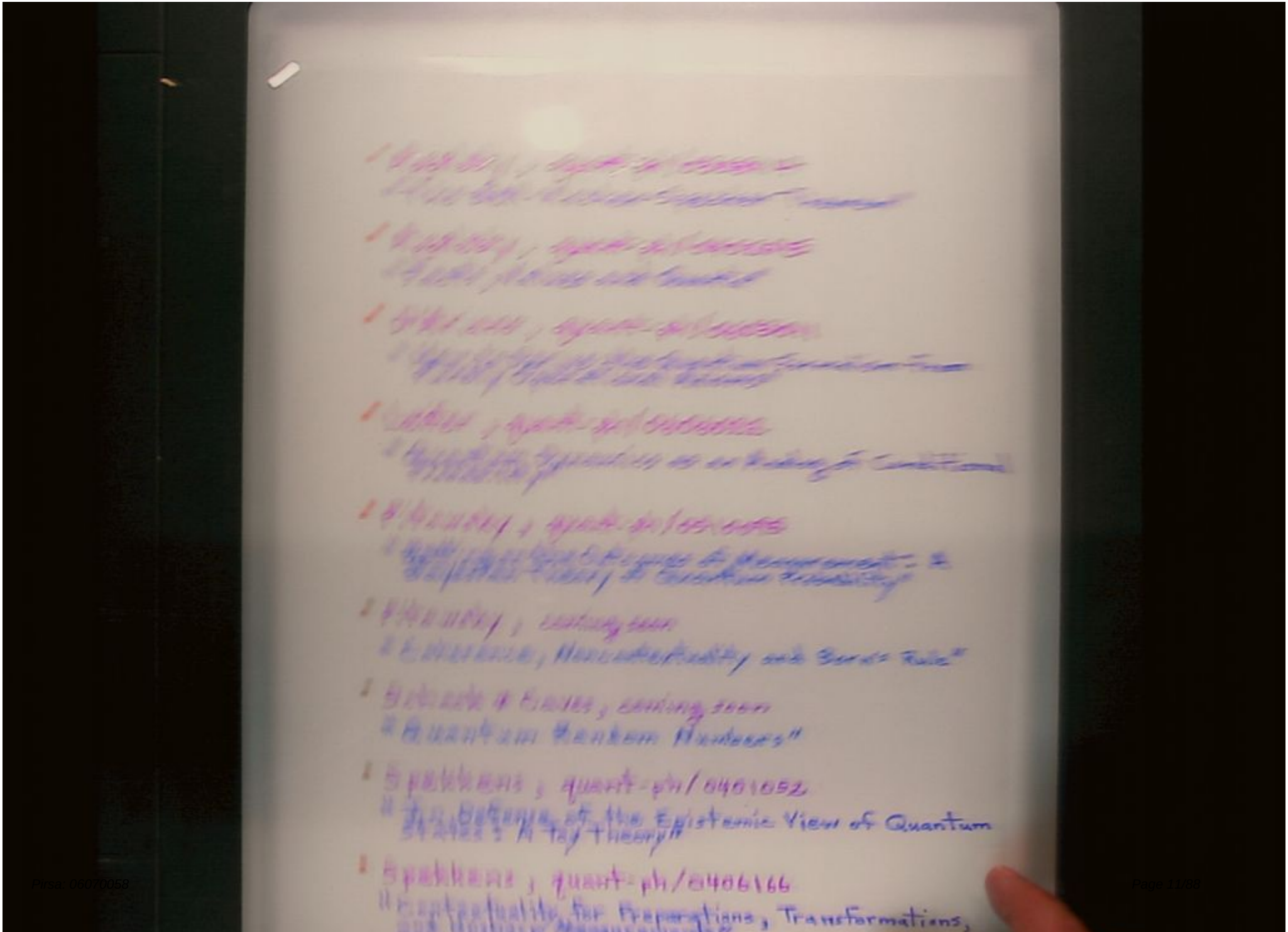

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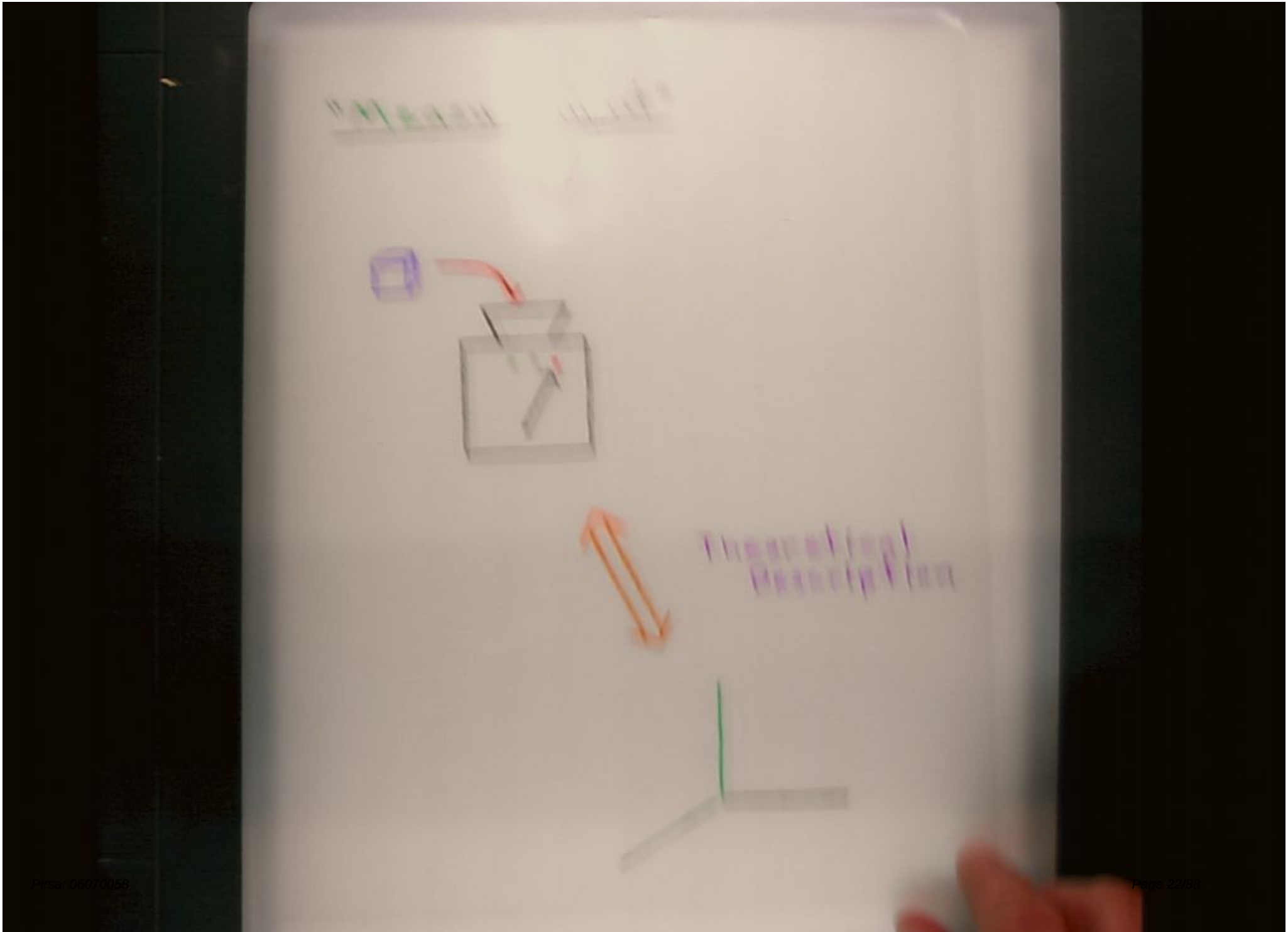
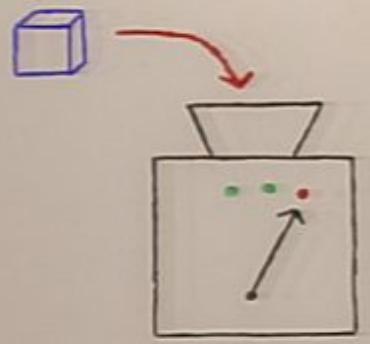
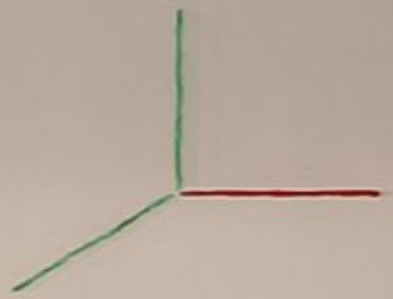
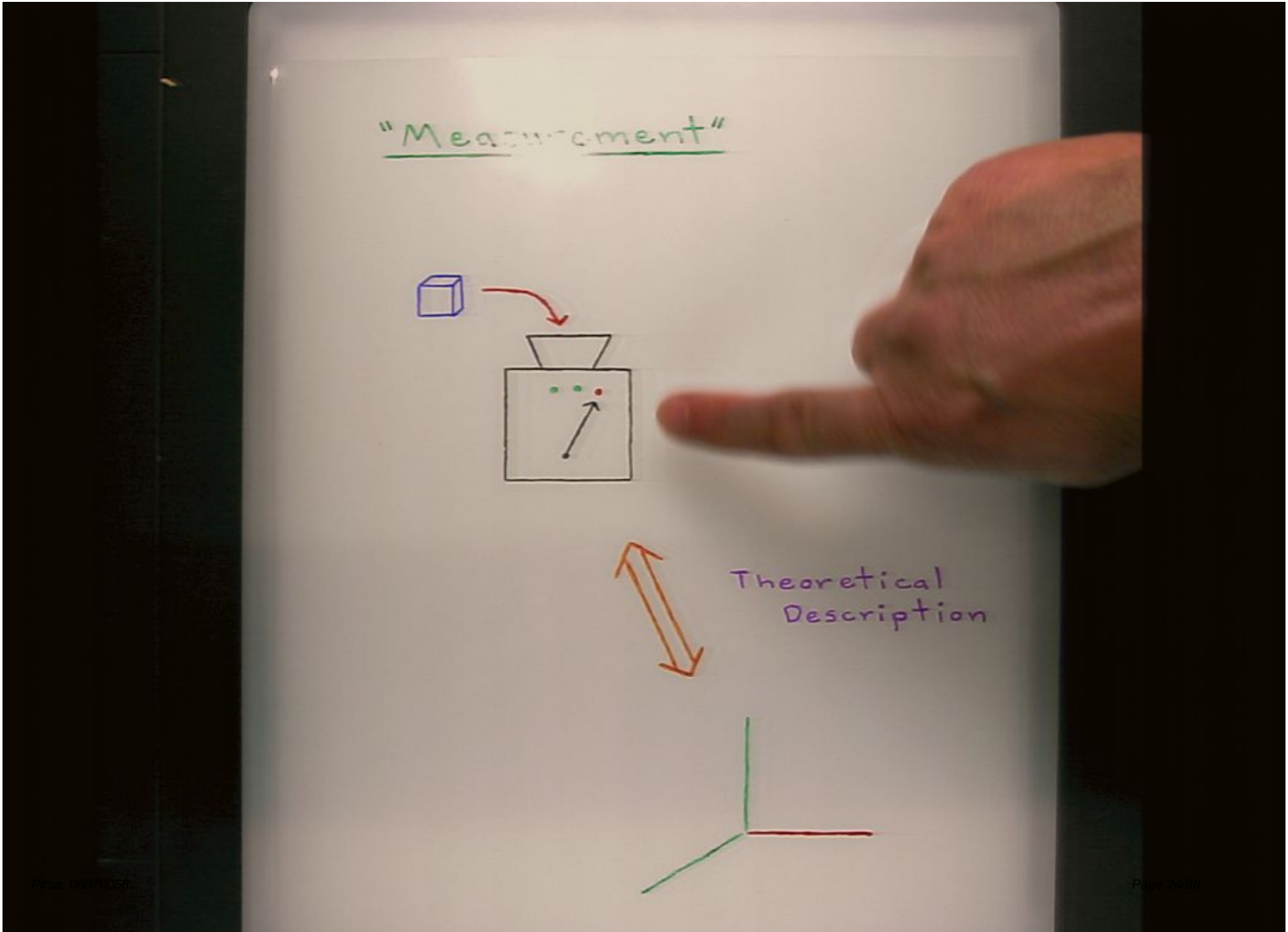


"Measurement"



Theoretical Description

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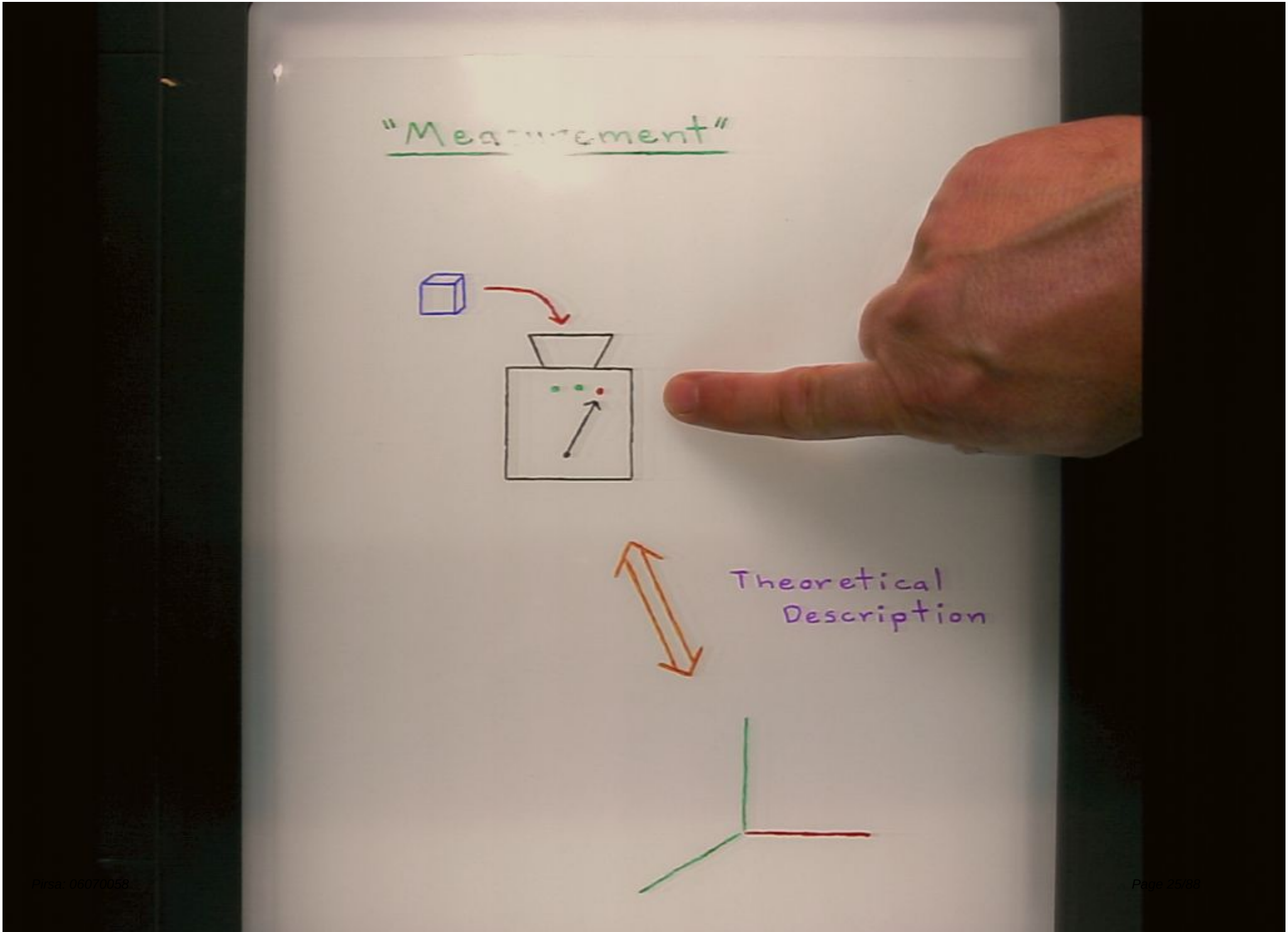
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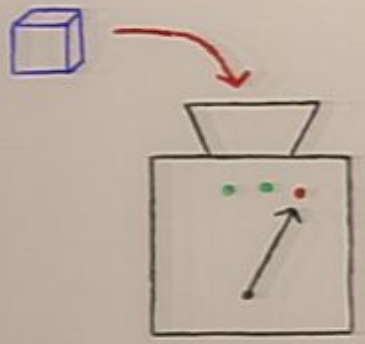

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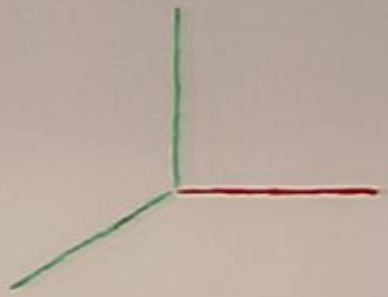

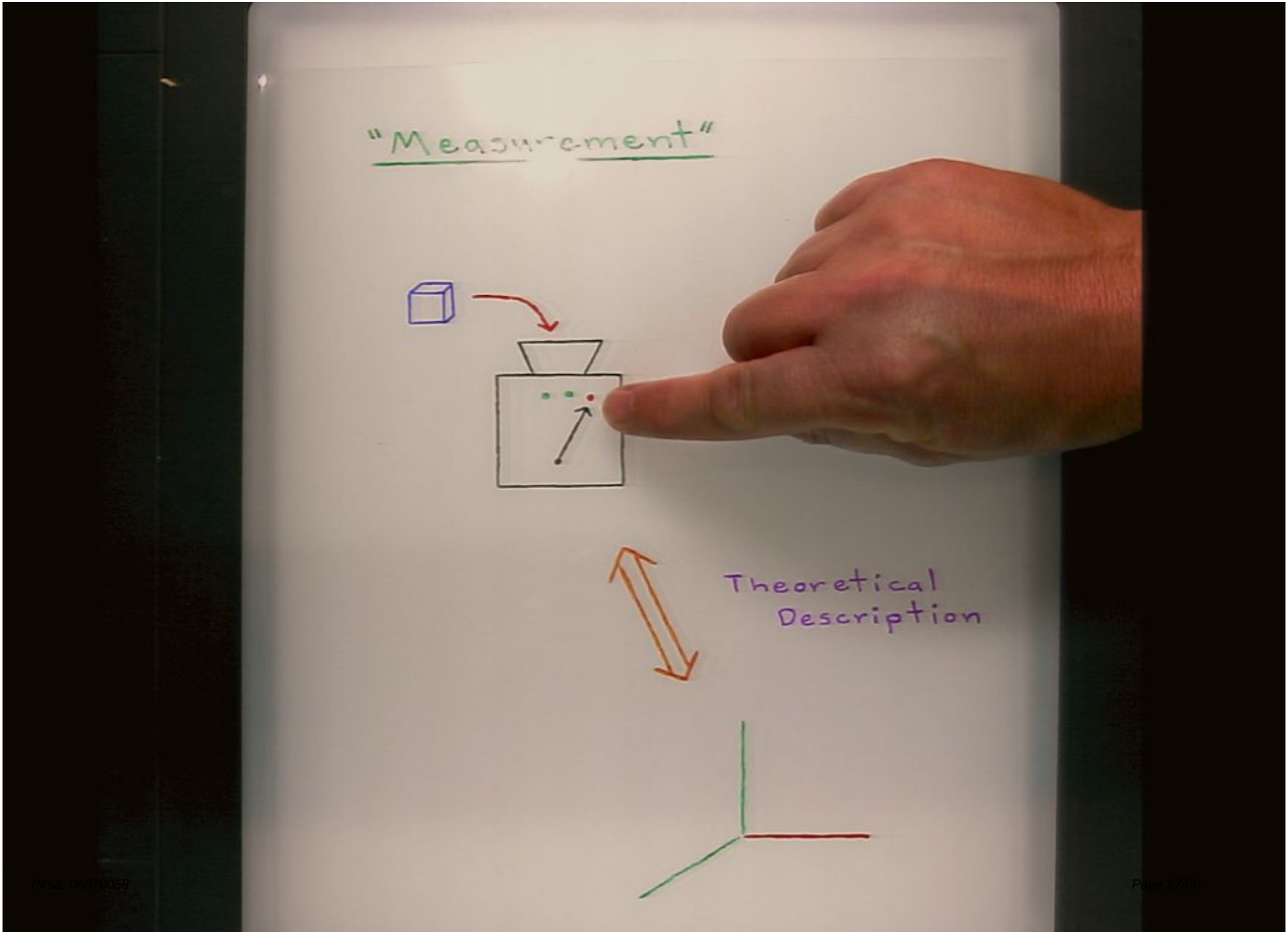
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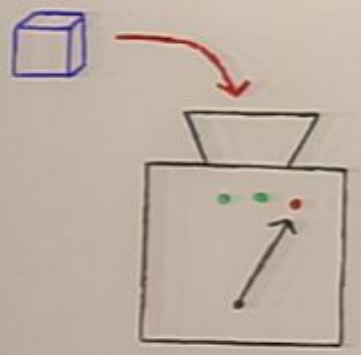
"Measurement"

Does it reveal a pre-existing, but unknown, value?

or

Does it in some sense go toward creating the very value?

"Measurement"



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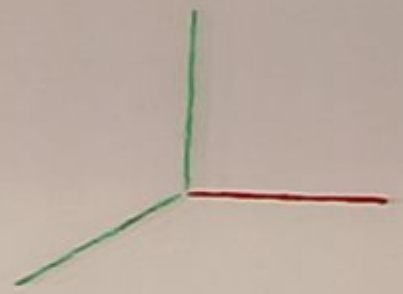

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EPR Criterion of Reality

"If, without in any way disturbing a system [one can gather the information required to] predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Motivated by EPR

Consider two spatially separated qutrits in a maximally entangled state:

$$|EPR\rangle = \sum_{i=1}^{3} |i\rangle|i\rangle$$

Assume locality.

Now measure the left one any way you like. Say with A or B, two nondegenerate noncommuting observables.

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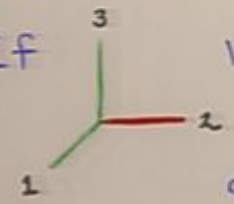
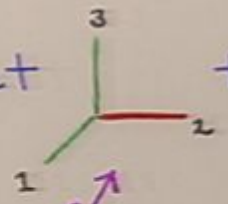
So measurement is simple revelation after all?

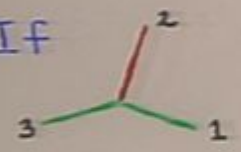
If 

here,

can predict 

there.

element of reality

If 

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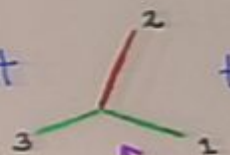
can predict 

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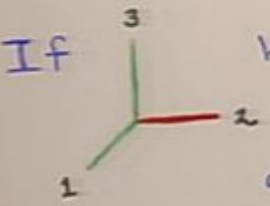
If 

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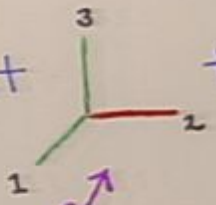
can predict 

there.

element of reality

If 

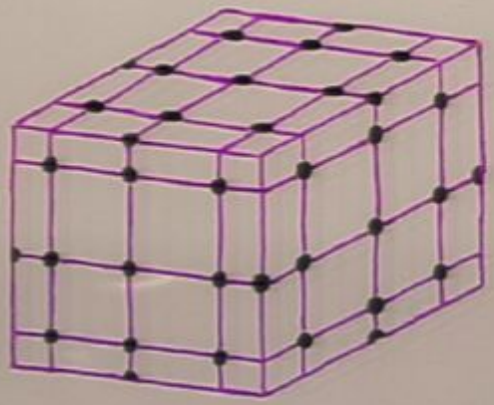
here,

can predict 

there.

element of reality

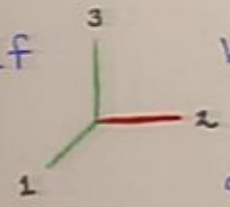
Cannot be colored:



33 rays, Peres

(when completed into full triads, consists of 40 triads made from 57 rays)

So me ement is simple revelati... after all?

If 

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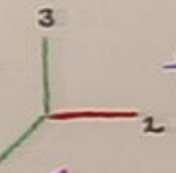
can predict 

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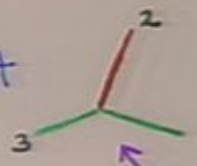
can predict 

there.

element of reality

Quantum measurements are generative:

Their outcomes do not pre-exist before the measurement interaction; they arise from the very process.

"For reasons of language this paper was written by Podolsky after much discussion. Still it did not come out as well as I had originally wanted; rather the essential thing was, so to speak, smothered by the formalism."

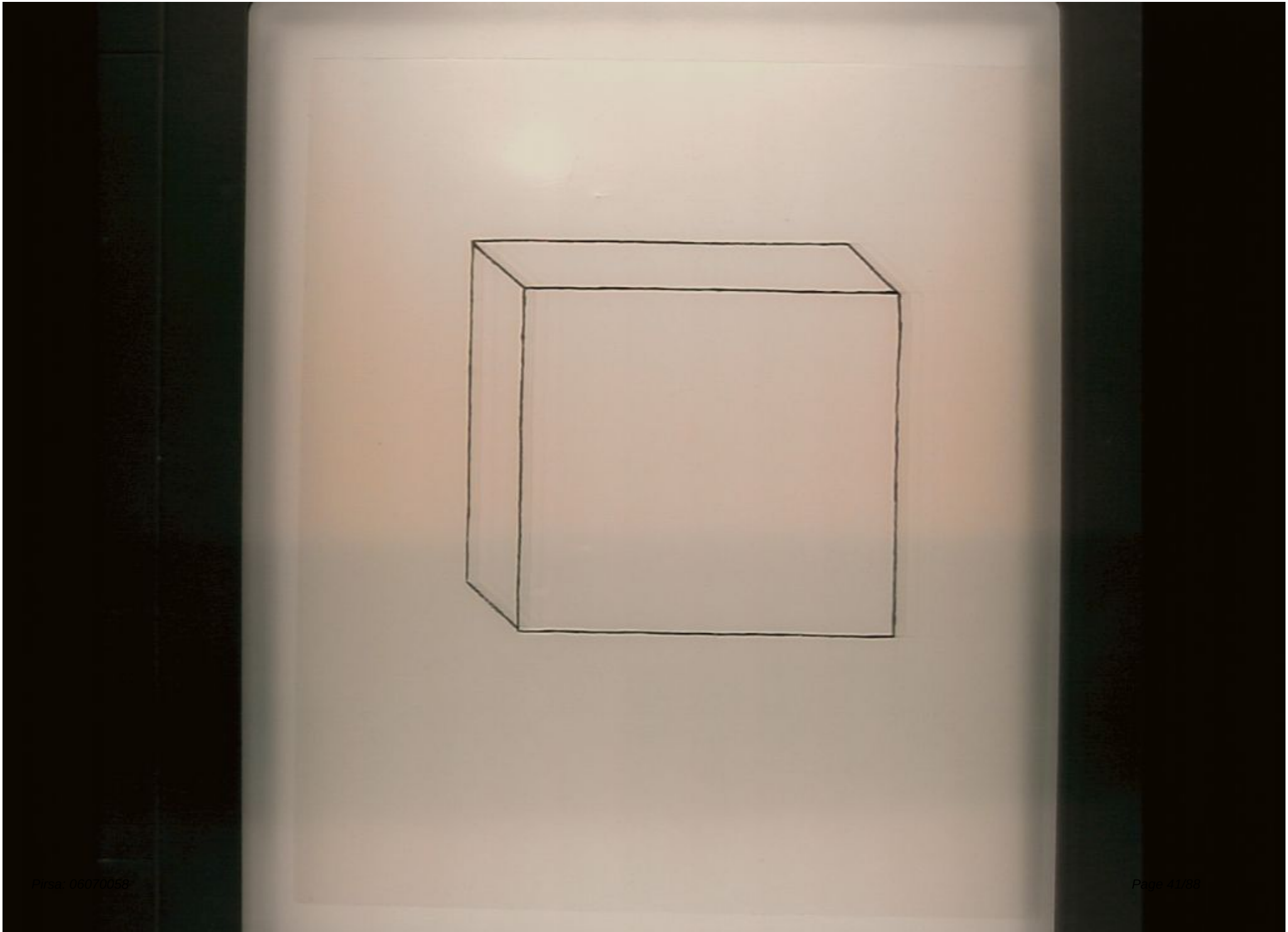
— Einstein to Schrödinger
19 June 1935

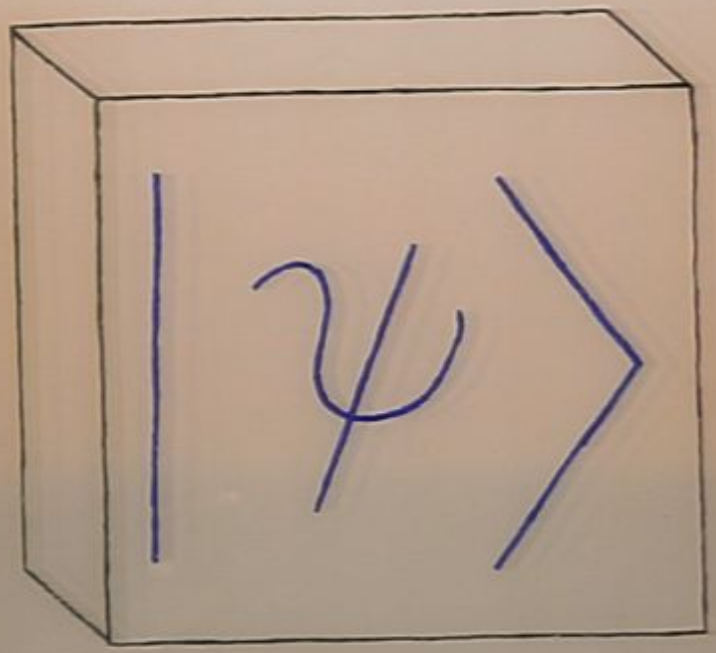
Granted: "The individual system (before the measurement) has no definite value of q (or p). The value of the measurement only arises in cooporation with the unique probability which is given to it in view of the ψ-function only through the act of measurement itself."

Consider spatially separated systems S_1 and S_2 initially attributed with an entangled quantum state ψ_{12}.

"Now it appears to me that one may speak of the real factual situation at S_2. ... [O]n one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with S_1. ... According to the type of measurement which I make of S_1, I get, however, a very different ψ_2 for $[S_2]$. ... For the same real situation of S_2 it is possible therefore to find, according to one's choice, different types of ψ-function.

If now [physicist B] accepts this consideration as valid, then [he] will have to give up his position that the ψ-function constitutes a complete description of a real factual situation. For in this case it would be impossible that two different types of ψ-functions could be coordinated with the identical factual situation of S_2."




|ψ⟩

On
the
Subject
of
Ducks

Quantum No Cloning

Want a device that

$$|\psi_i\rangle|s\rangle \longrightarrow |\psi_i\rangle|\psi_i\rangle$$

regardless of $i \in \{0,1\}$.

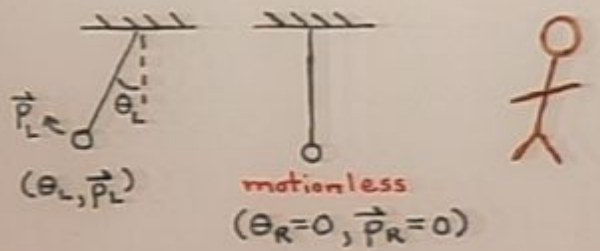
Cannot build if $0 < |\langle\psi_0|\psi_1\rangle| < 1$.

Because on total Hilbert space

initial inner product $= \langle\psi_0|\psi_1\rangle$

final inner product $= \langle\psi_0|\psi_1\rangle\langle\psi_0|\psi_1\rangle$

Belief Cloning?



$P_i(\theta_L, \vec{p}_L)$

$i \in \{0,1\}$

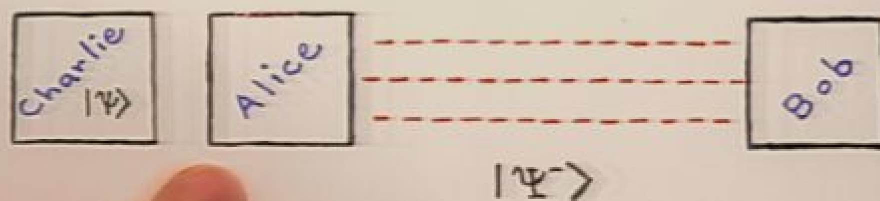
Is there any device I can build that will cause the first observer to describe the pendula according to

$$P_i(\theta_L, \vec{p}_L) \times P_i(\theta_R, \vec{p}_R) \; ?$$

Not if $0 < \int \sqrt{P_0(\theta_L, \vec{p}_L) P_1(\theta_L, \vec{p}_L)}\, d\theta_L d\vec{p}_L < 1$.

Because Liousville mechanics is phase-space volume preserving.

Teleportation



$|\psi\rangle|\Psi^-\rangle$ … $\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)$

$= \frac{\alpha}{\sqrt{2}}$ … $\rangle) + \frac{\beta}{\sqrt{2}}(|1\rangle|0\rangle|1\rangle - |1\rangle|1\rangle|0\rangle)$

$=$ …

… $|1\rangle)$

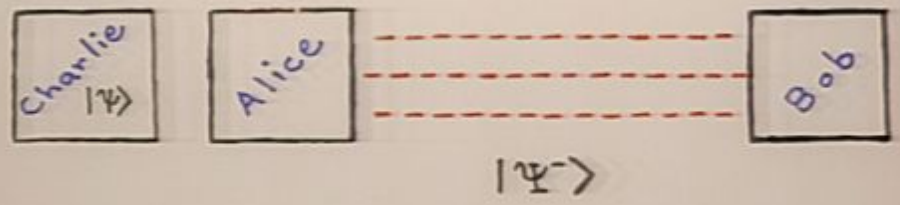
… $|0\rangle + \alpha|1\rangle)$

… $\Phi^+\rangle(-\beta|0\rangle + \alpha|1\rangle)$

… $+ \frac{1}{2}|\Psi^+\rangle(-\sigma_z|\psi\rangle)$

… $_x|\psi\rangle) + \frac{1}{2}|\Phi^+\rangle(i\sigma_y|\psi\rangle)$

Teleportation



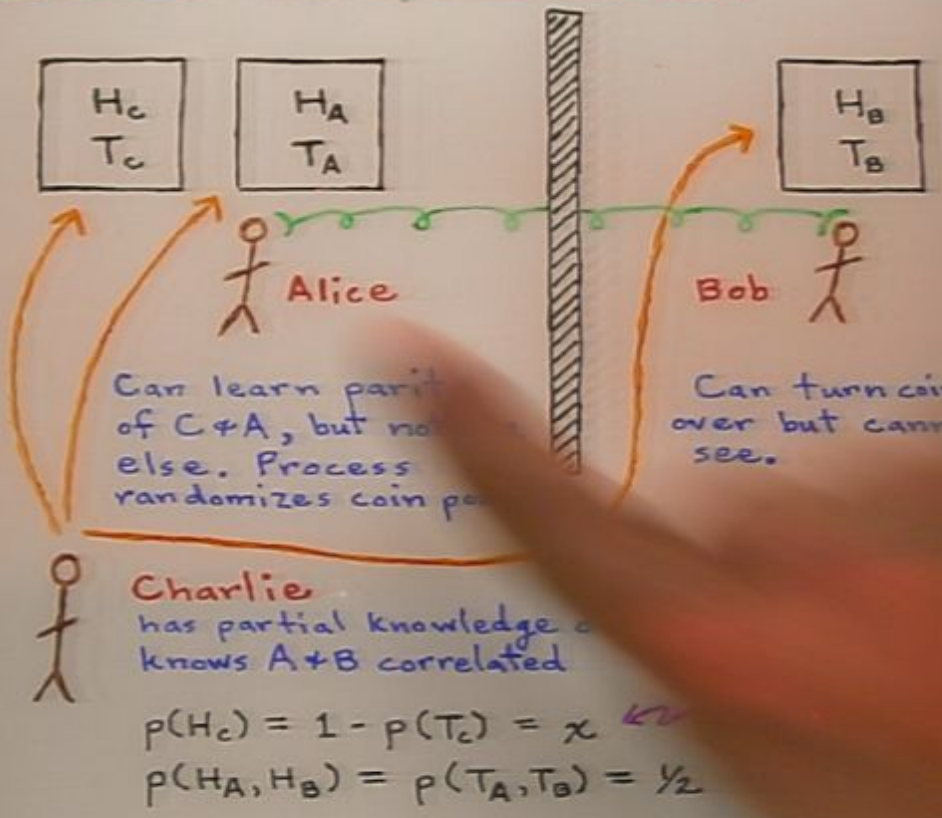
$$|\psi\rangle|\Psi^-\rangle = (\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)$$

$$= \frac{\alpha}{\sqrt{2}}(|0\rangle|0\rangle|1\rangle - |0\rangle|1\rangle|0\rangle) + \frac{\beta}{\sqrt{2}}(|1\rangle|0\rangle|1\rangle - |1\rangle|1\rangle|0\rangle)$$

$$= \frac{1}{2}|\Psi^-\rangle(-\alpha|0\rangle - \beta|1\rangle)$$
$$+ \frac{1}{2}|\Psi^+\rangle(-\alpha|0\rangle + \beta|1\rangle)$$
$$+ \frac{1}{2}|\Xi^-\rangle(\beta|0\rangle + \alpha|1\rangle)$$
$$+ \frac{1}{2}|\Xi^+\rangle(-\beta|0\rangle + \alpha|1\rangle)$$

$$= \frac{1}{2}|\Psi^-\rangle(-I|\psi\rangle) + \frac{1}{2}|\Psi^+\rangle(-\sigma_z|\psi\rangle)$$
$$+ \frac{1}{2}|\Xi^-\rangle(\sigma_x|\psi\rangle) + \frac{1}{2}|\Xi^+\rangle(i\sigma_y|\psi\rangle)$$

Belief Teleportation?



$p(H_C) = 1 - p(T_C) = x$

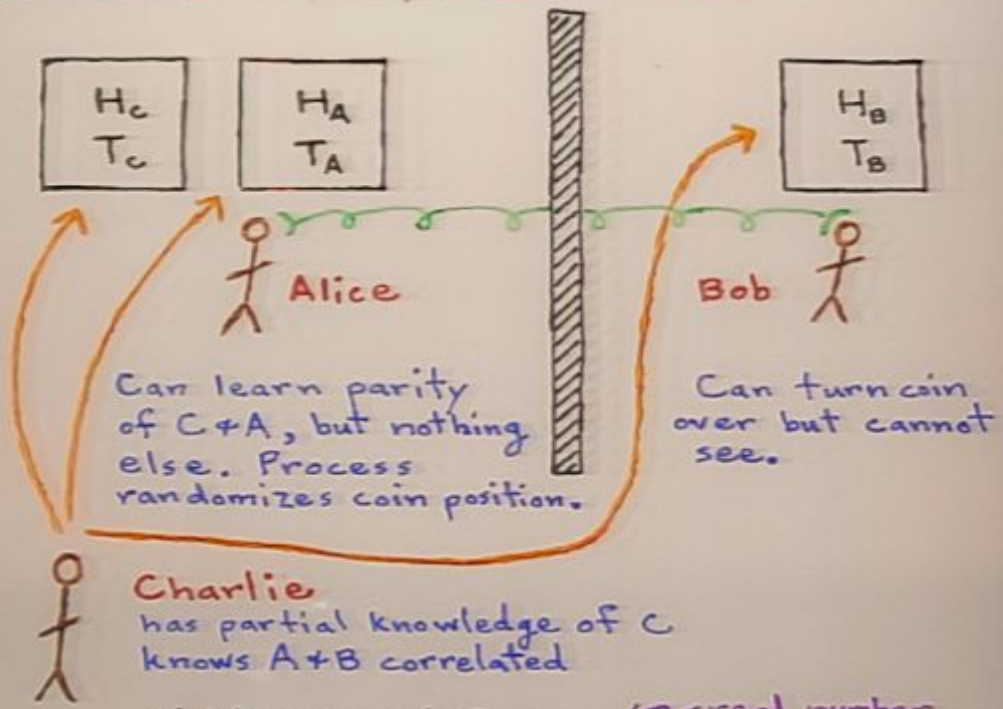
$p(H_A, H_B) = p(T_A, T_B) = 1/2$

Teleportation:

1) Alice checks parity, announces result Bob

2) Bob turns coin over if parity odd; otherwise does nothing

Belief Teleportation?



$p(H_C) = 1 - p(T_C) = x$ ← real number

$p(H_A, H_B) = p(T_A, T_B) = 1/2$

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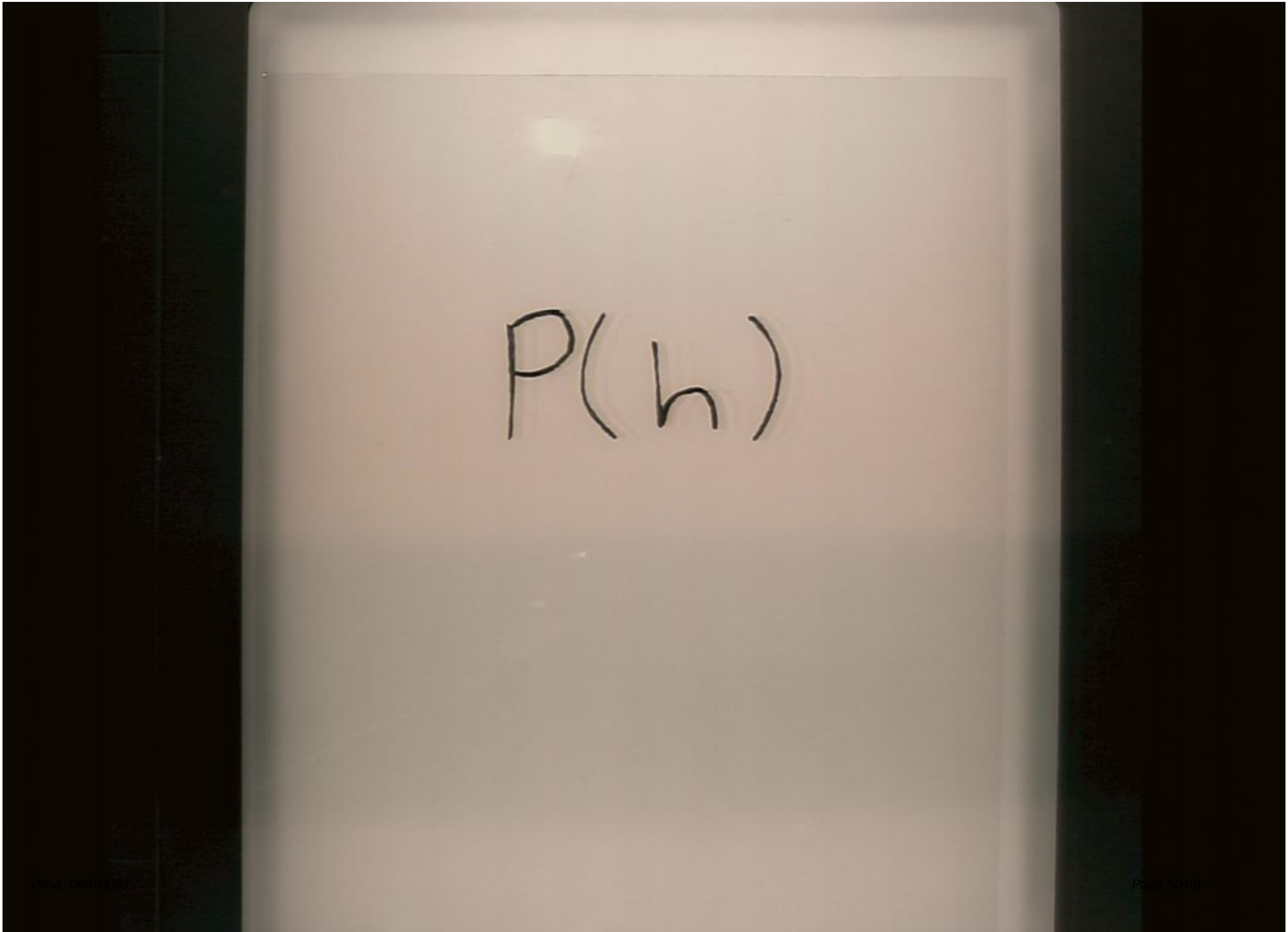
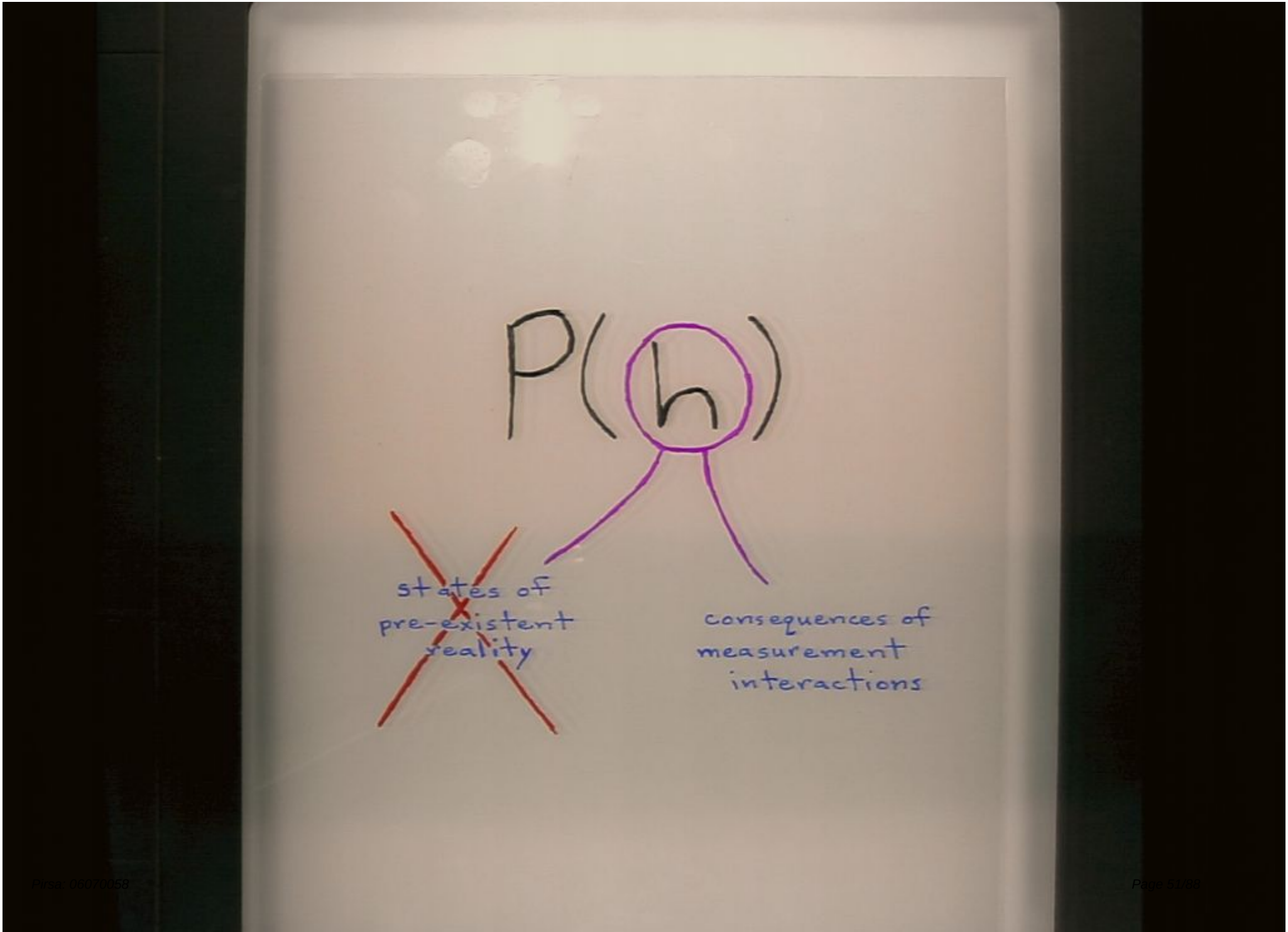

P(h)


P(h)
states of pre-existent reality
consequences of measurement interactions

They went to sea in a Sieve, they did;
In a Sieve they went to sea:
In spite of all their friends could say,
On a winter's morn, on a stormy day,
In a Sieve they went to sea!
And when the Sieve turned round and round,
And every one cried, "You'll all be drowned!"
They called aloud, "Our Sieve ain't big;
But we don't care a button! we don't care a fig!
In a Sieve we'll go to sea."

— Edward Lear, from *The Jumblies*

The Pauli'an Idea, 1

[Einstein and I] often discussed these questions, and I invariably profited very greatly even when I could not agree with Einstein's views. "Physics is after all the description of reality," he said to me, continuing, with a sarcastic glance in my direction, "or should I perhaps say physics is the description of what one merely imagines?" This question clearly shows Einstein's concern that the objective character of physics might be lost through a theory of the type of quantum mechanics, in that as a consequence of its wider conception of objectivity of an explanation of nature the difference between physical reality and dream or hallucination become blurred.

The objectivity of physics is however fully ensured in quantum mechanics in the following sense. Although in principle, according to the theory, it is in general only the statistics of series of experiments that is determined by laws, the observer is unable, even in the unpredictable single case, to influence the result of his observation—as for example the response of a counter at a particular instant of time. Further, personal qualities of the of the observer do not come into the theory in any way—the observation can be made by objective registering apparatus, the results of which are objectively available for anyone's inspection. Just as in the theory of relativity a group of mathematical transformations connects all possible coordinate systems, so in quantum mechanics a group of mathematical transformations connects the possible experimental arrangements.

Einstein however advocated a narrower form of the reality concept ...

— Wolfgang Pauli
"Albert Einstein and the Development of Physics," 1958

The Pauli'an Idea, 1

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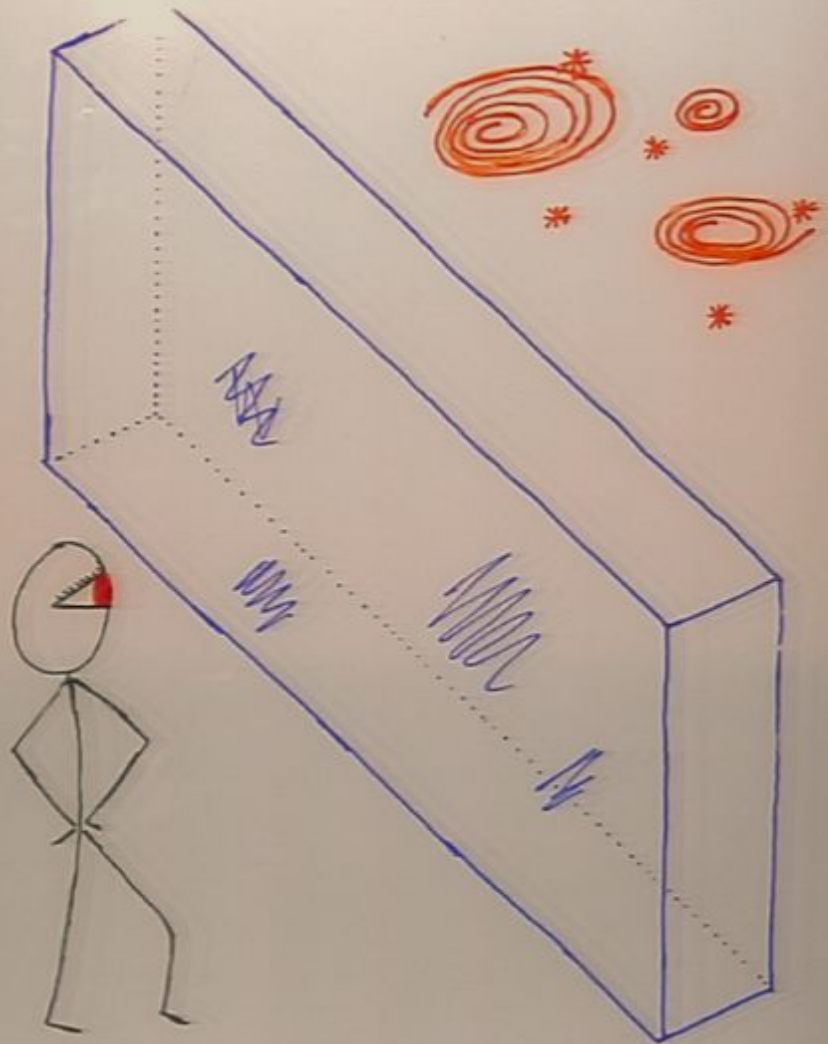
The Pauli'an Idea, 2

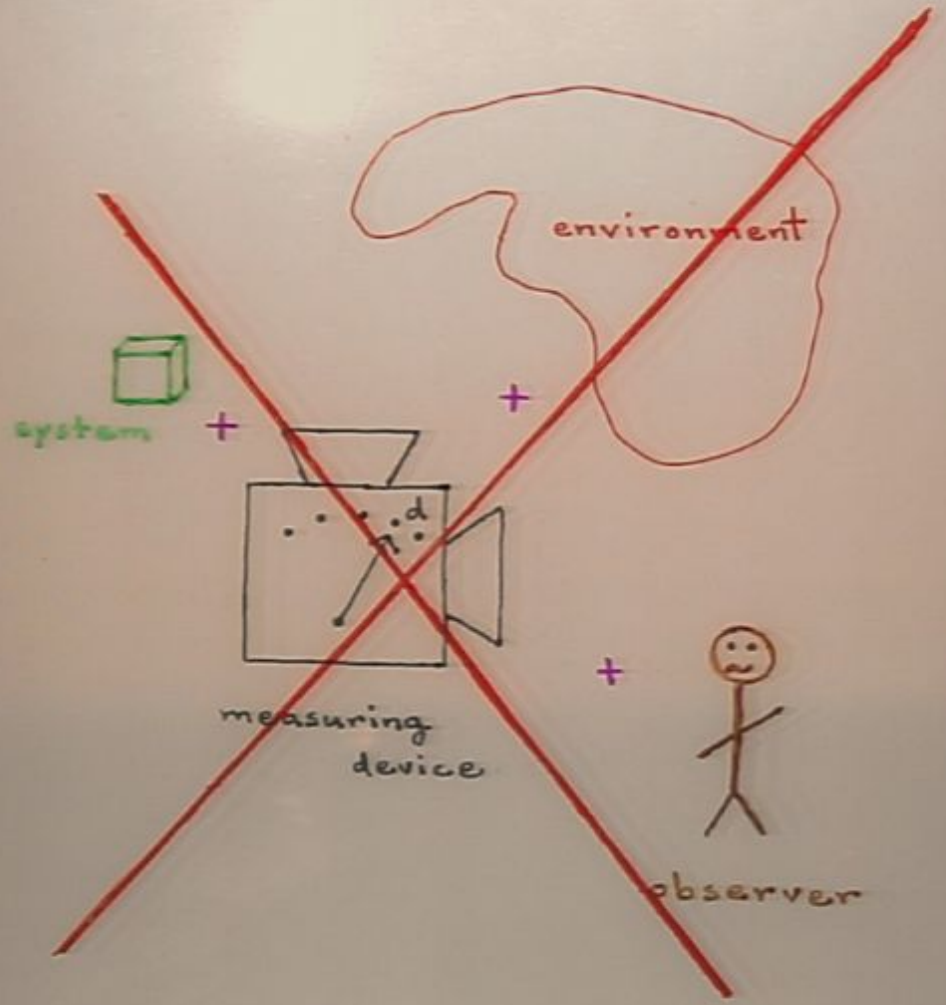
It seems to me appropriate to call the conceptual description of nature in classical physics, which Einstein so wishes to retain, "the ideal of the detached observer". To put it drastically, the observer has according to this ideal to disappear entirely in a discrete manner as hidden spectator, never as actor, nature being left alone in a predetermined course of events, independent of the way in which the phenomena are observed. "Like the moon has a definite position" Einstein said to me last winter, "whether or not we look at the moon, the same must also hold for the atomic objects, as there is no sharp distinction possible between these and macroscopic objects. Observation cannot *create* an element of reality like a position, there must be something contained in the complete description of physical reality which corresponds to the *possibility* of observing a position, already before the observation has been actually made." It is this kind of postulate which I call the ideal of the detached observer.

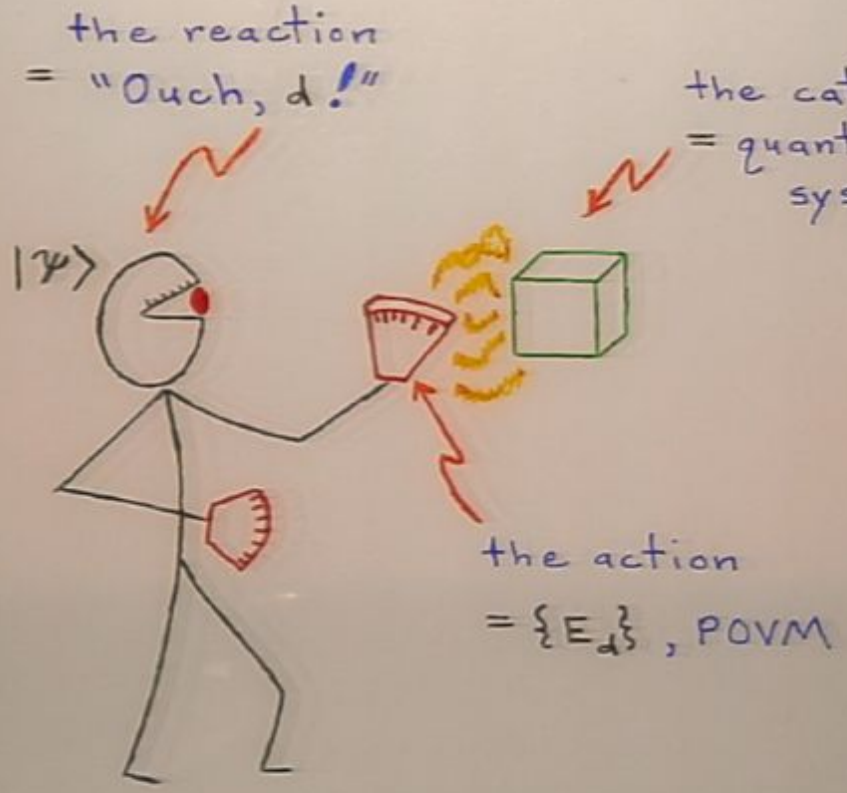
In quantum mechanics, on the contrary, an observation changes in general the "state" of the observed system in a way not contained in the mathematically formulated *laws*, which only apply to the time dependence of the state of a *closed* system. I think here on the passage to a new phenomenon by observation which is taken into account by the so called "reduction of the wave packets." As it is allowed to consider the instruments of observation as a kind of prolongation of the sense organs of the observer, I consider the impredictable change of the state by a single observation to be *an abandonment of the idea of the isolation (detachment) of the observer from the course of physical events outside himself.*

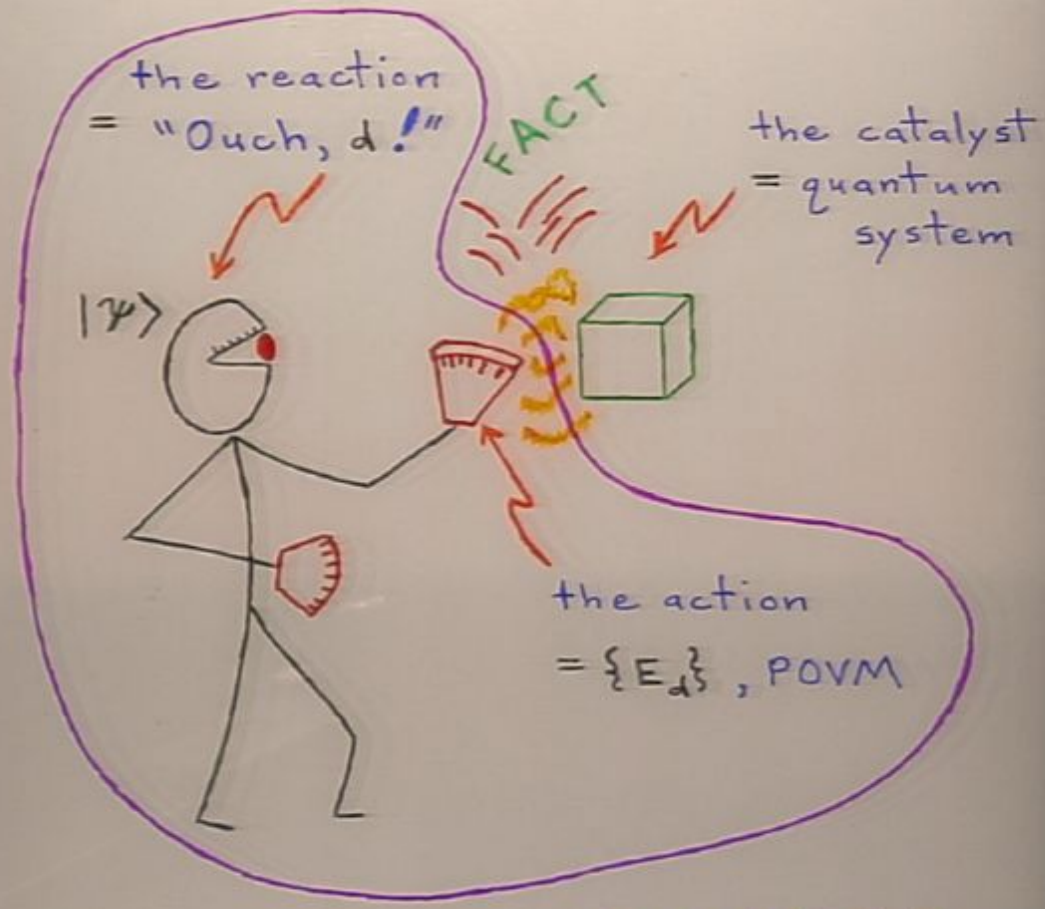
— Wolfgang Pauli
Letter to Bohr, 1955



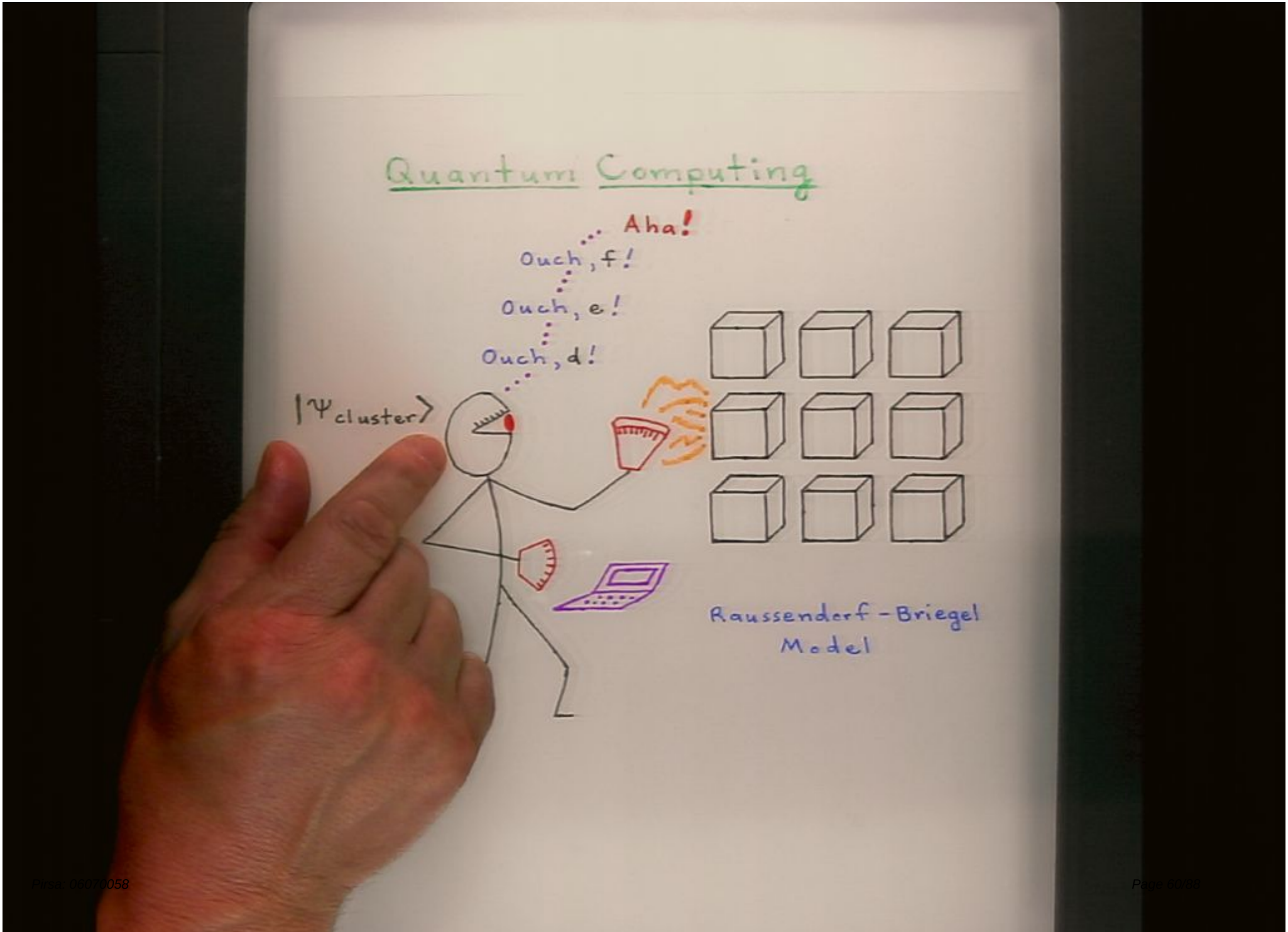
Courtesy of
John Wheeler

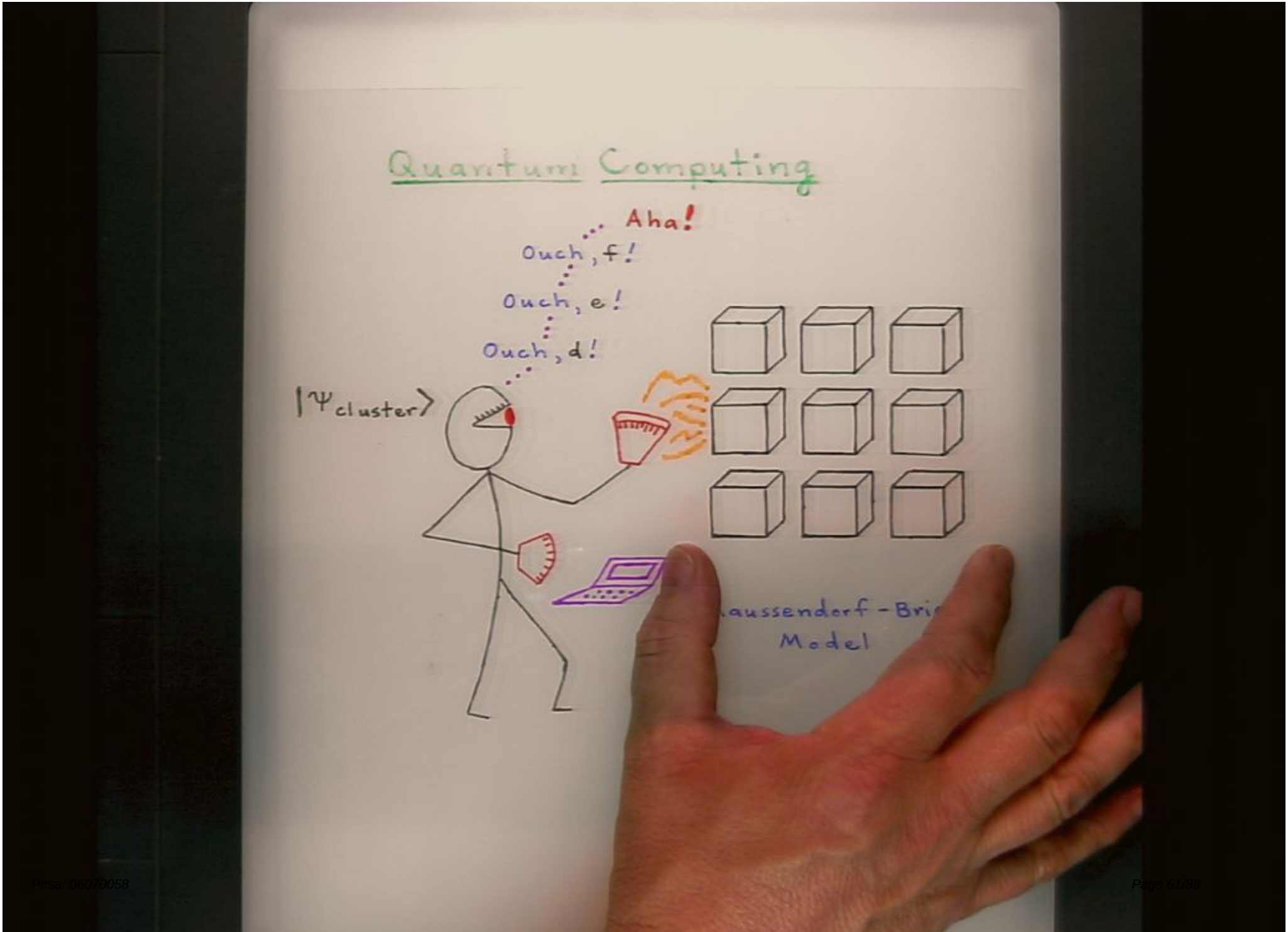

environment
system
+
+
+
+
measuring
device
observer


the reaction
= "Ouch, d!"
the catalyst
= quantum system
|ψ⟩
the action
= {E_d} , POVM



Except for "d",
the terms of quantum
mechanics are only
(directly) concerned with the inside
of the purple boundary.


Quantum Computing
Aha!
Ouch, f!
Ouch, e!
Ouch, d!
$|\Psi_{cluster}\rangle$
Raussendorf - Briegel Model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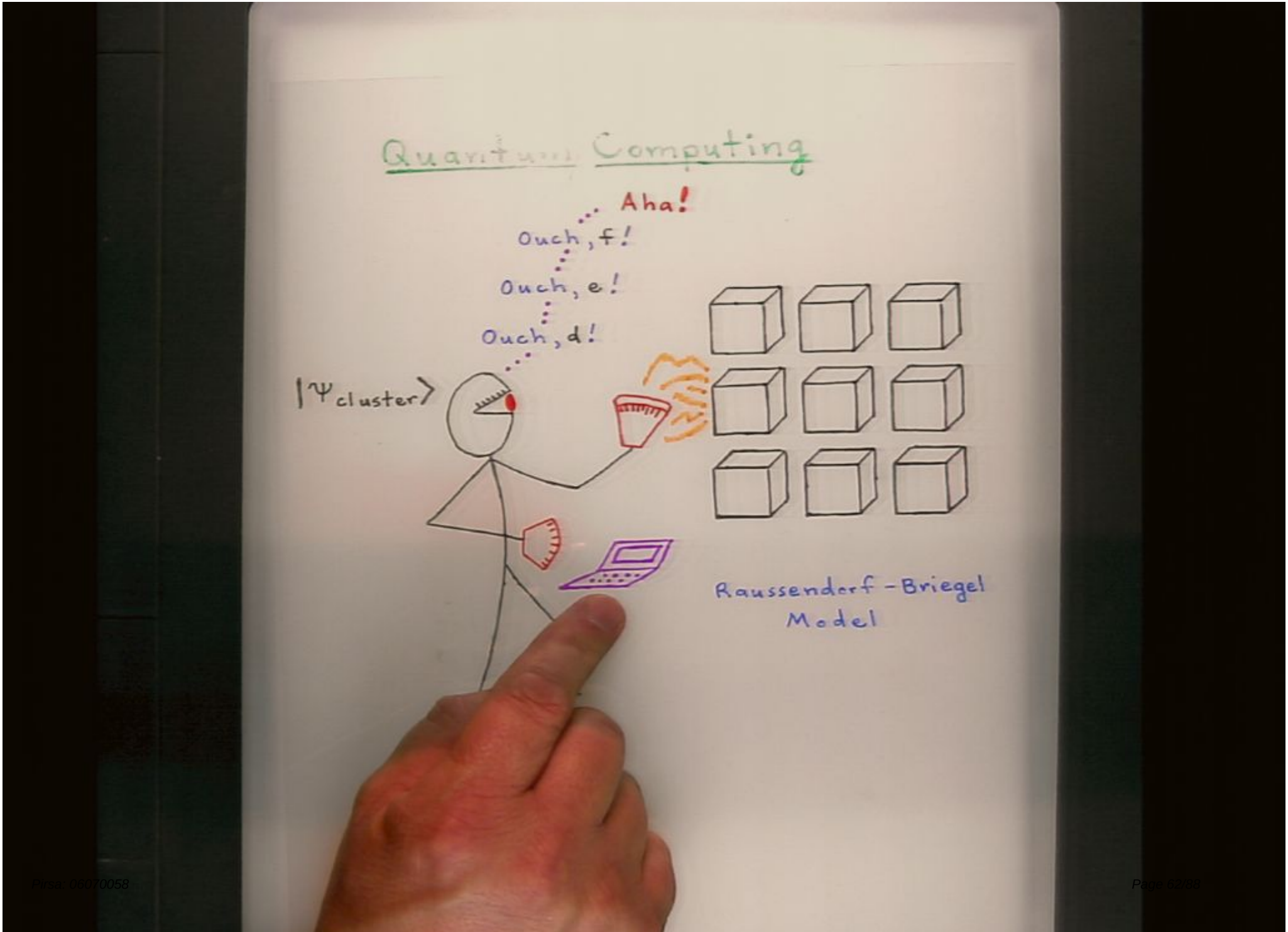
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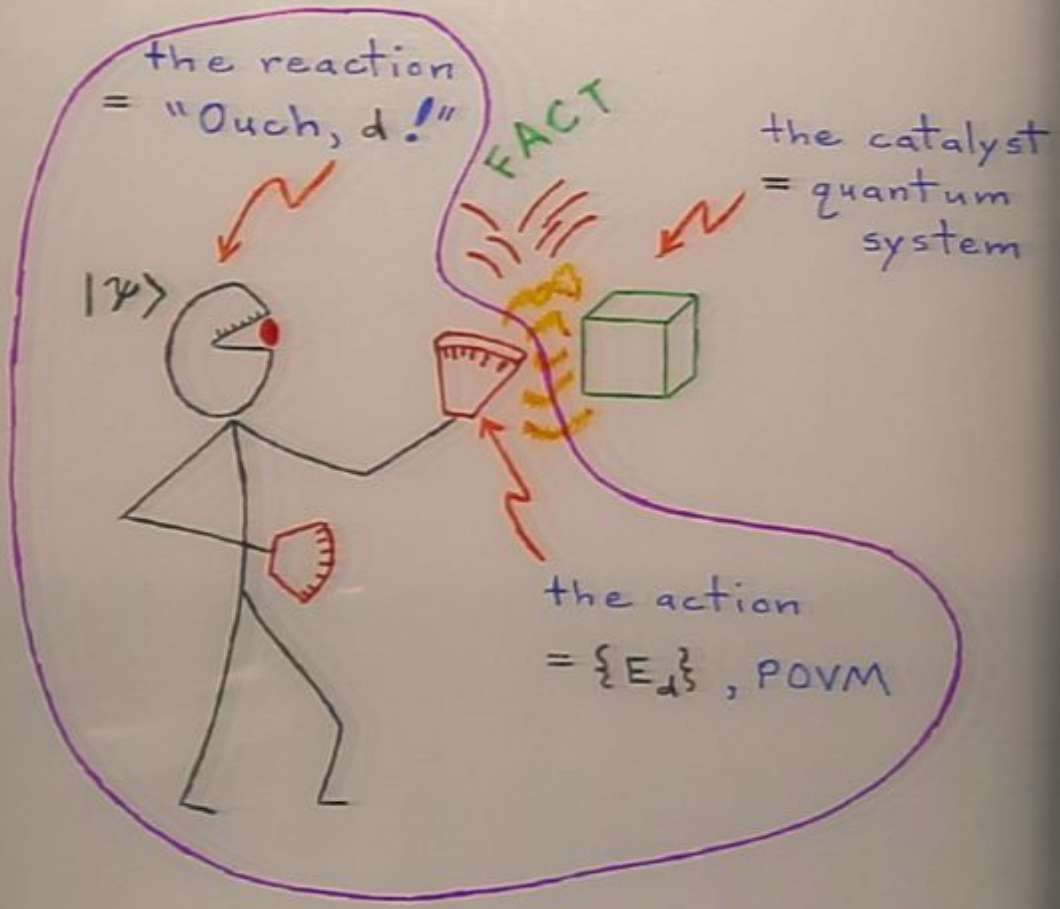

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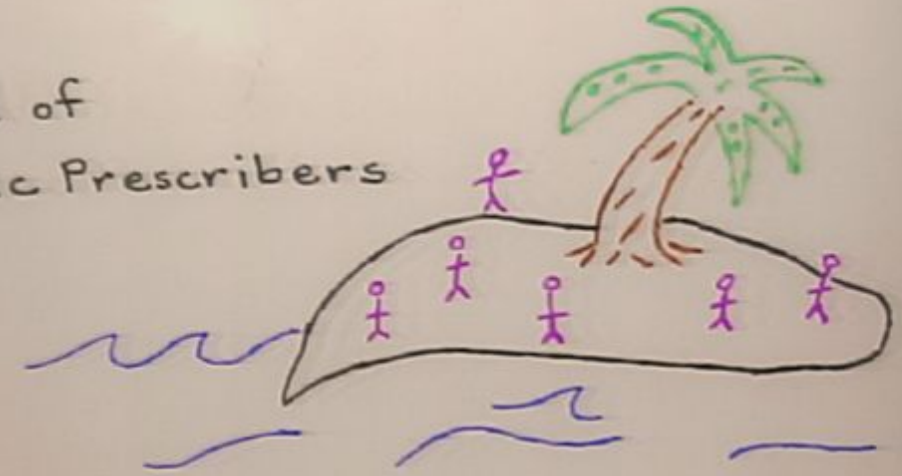
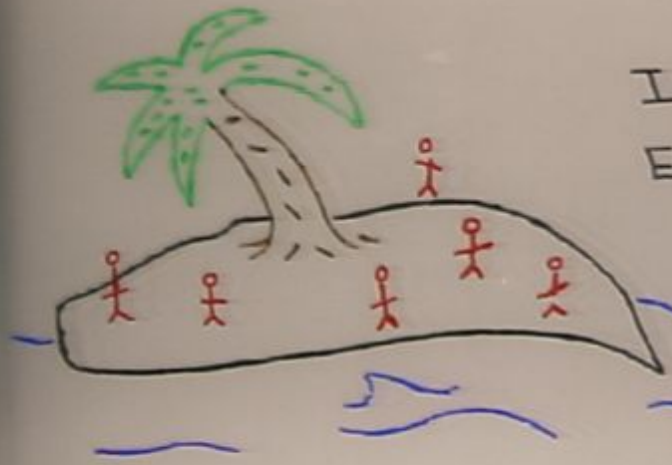
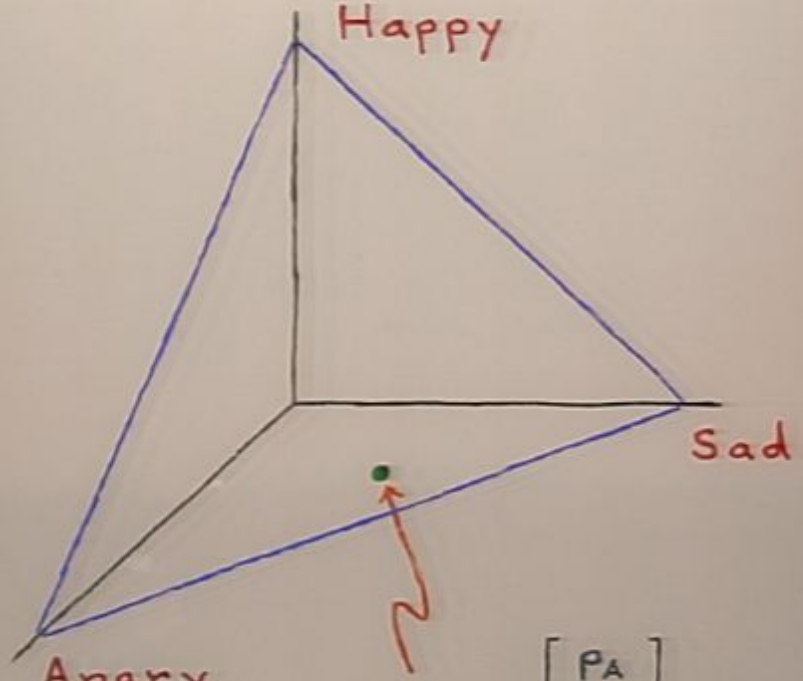
Island of
Prozac Prescribers





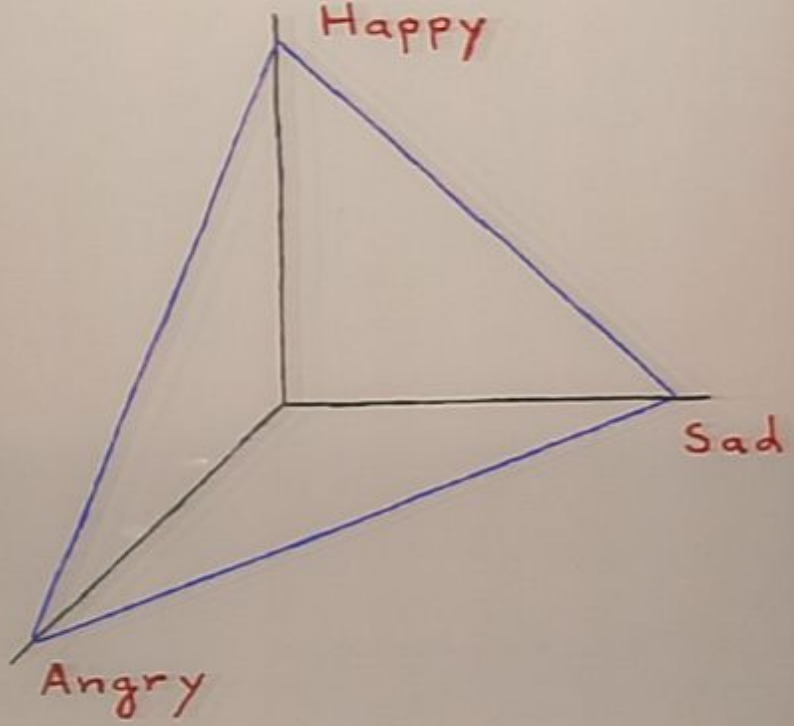
Island of
Emotional
Rollercoasters



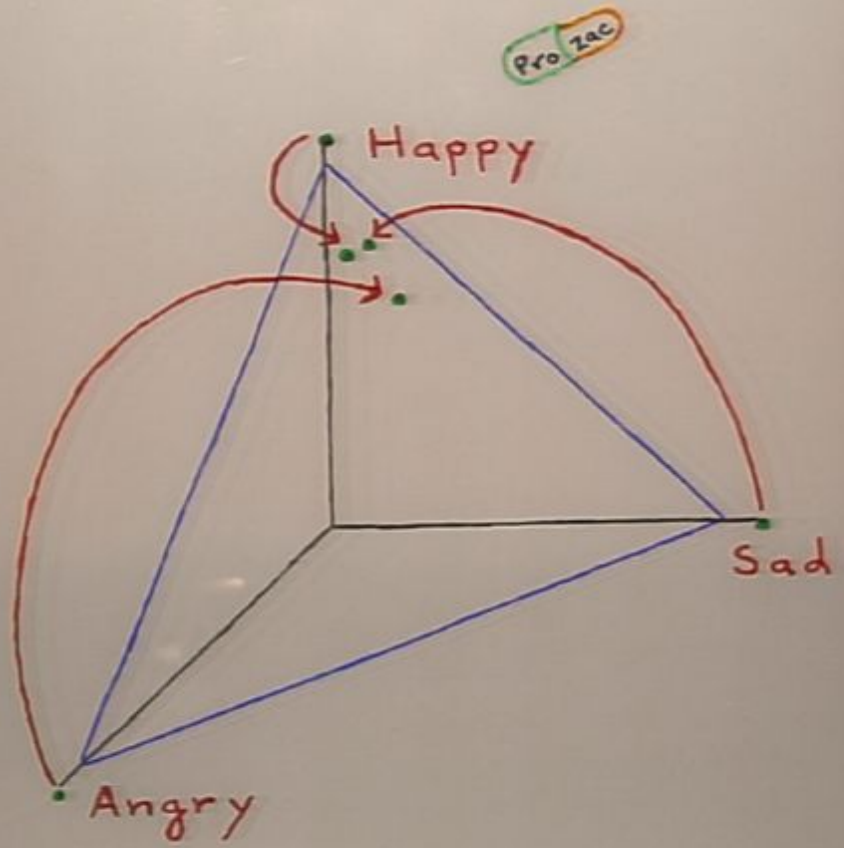
$$\vec{p} = \begin{bmatrix} p_A \\ p_S \\ p_H \end{bmatrix}$$

$p_A \geq 0$, $p_S \geq 0$, $p_H \geq 0$

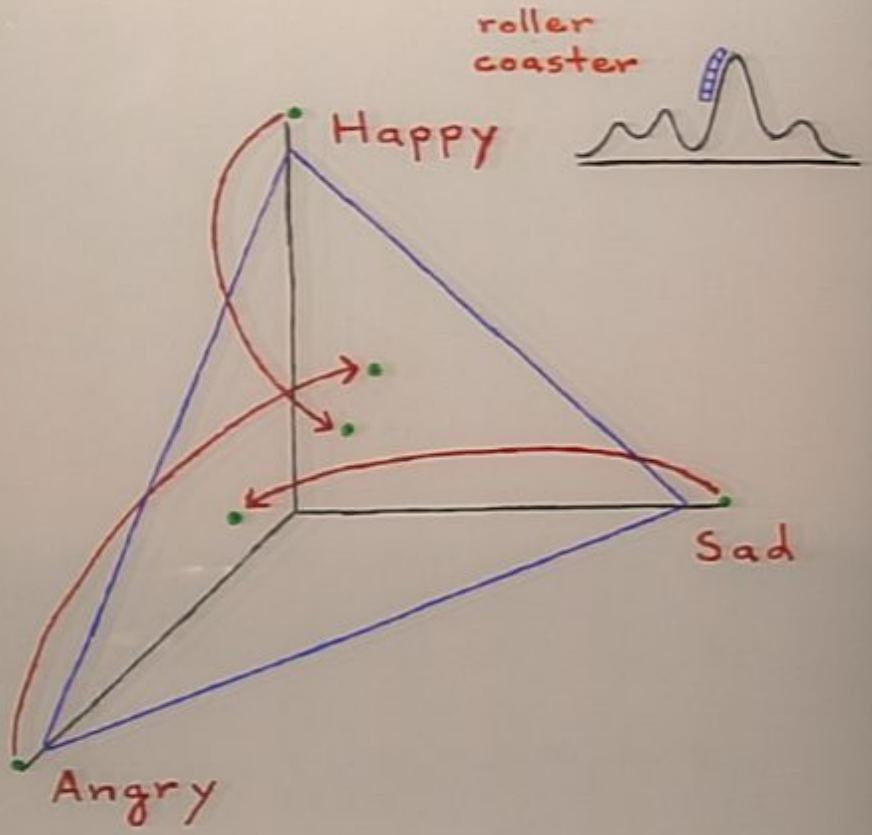
$p_A + p_S + p_H = 1$


Happy
Sad
Angry

Island of Prozac Prescribers



Island = Emotional Rollercoasters

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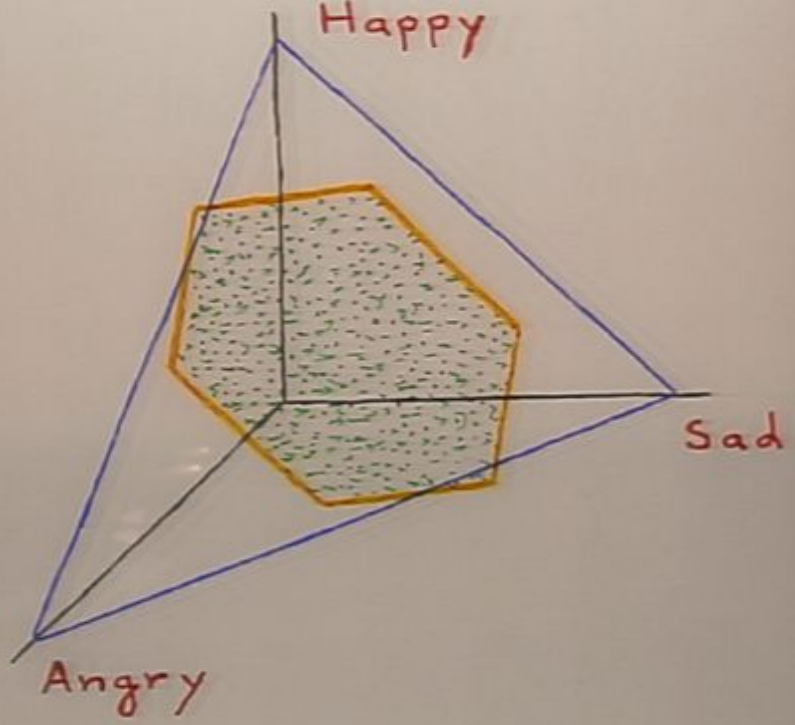
Happy
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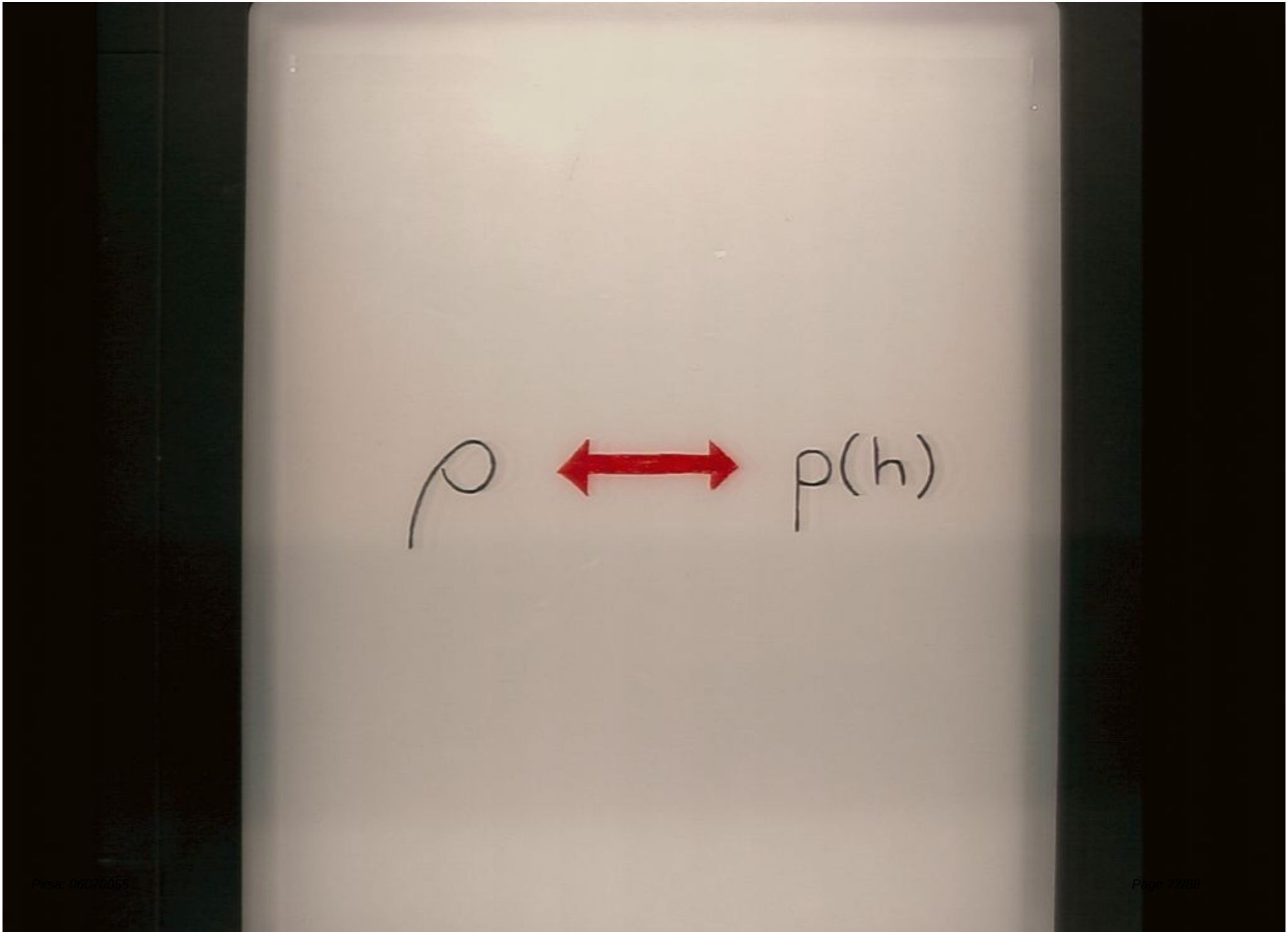
Angry

The Lesson

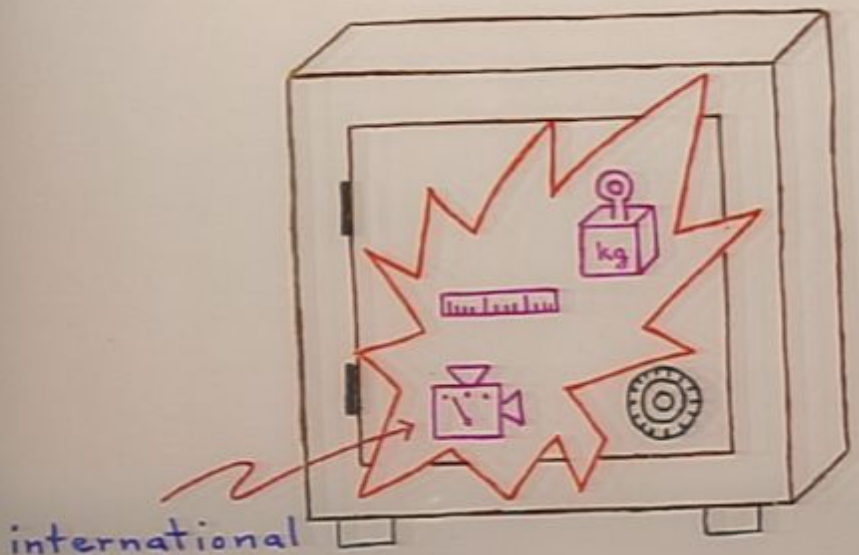
p(h) - state of belief

h - not even a state
of the external world

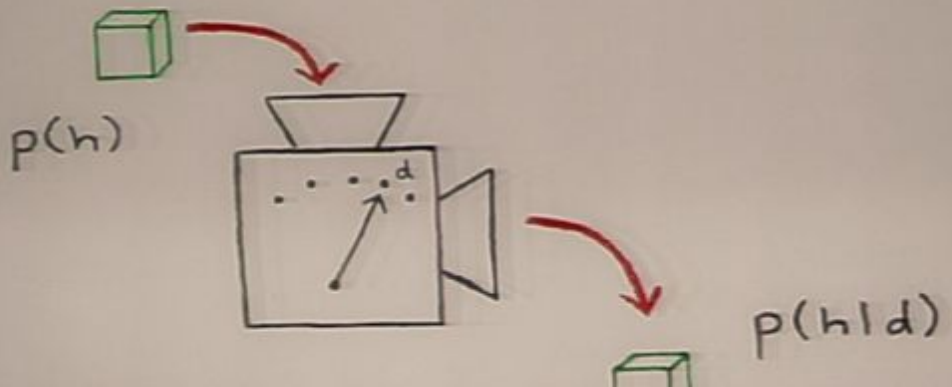
Yet, REGION gives direct
information about components
of the external world.

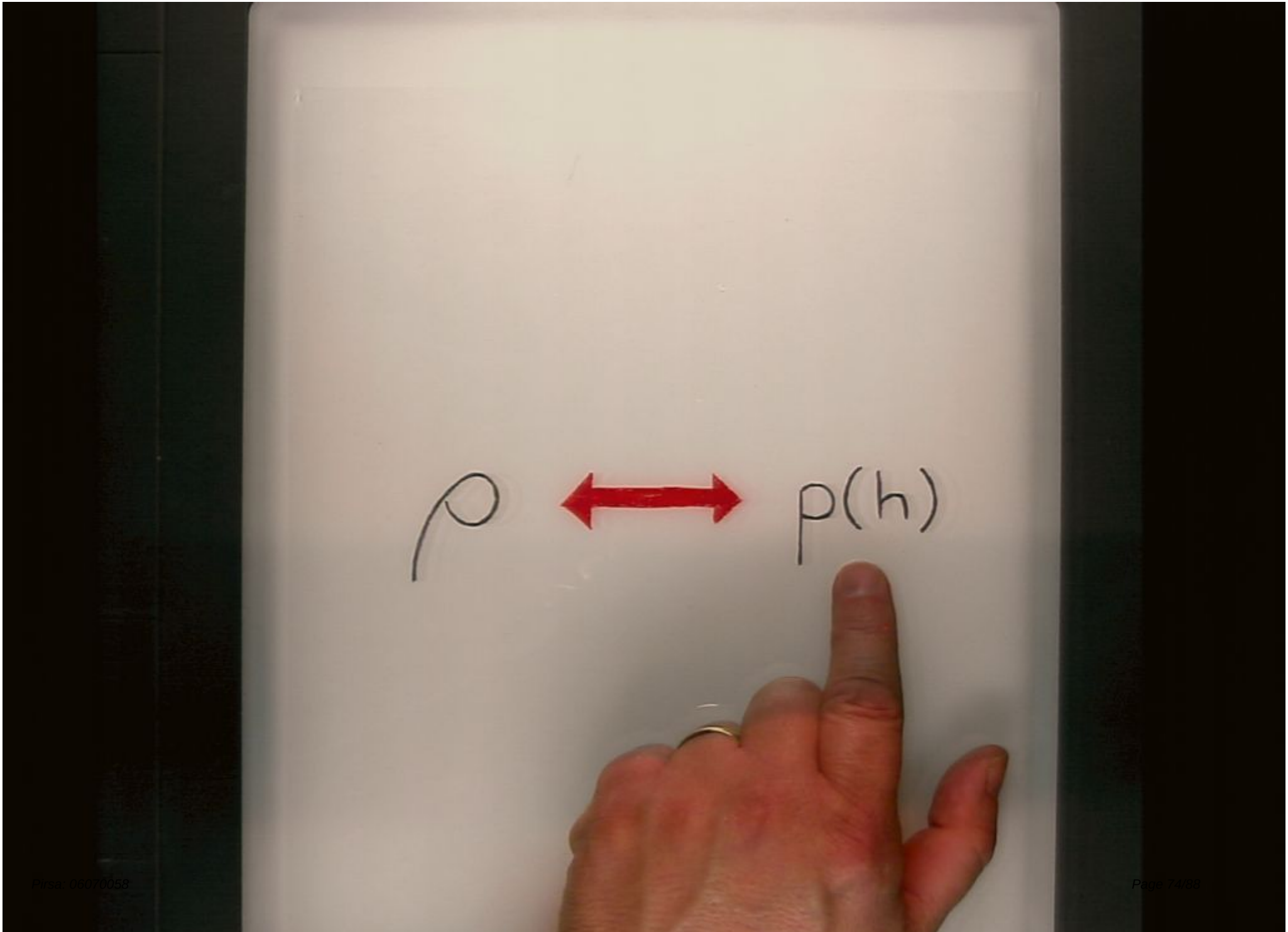

ρ
p(h)

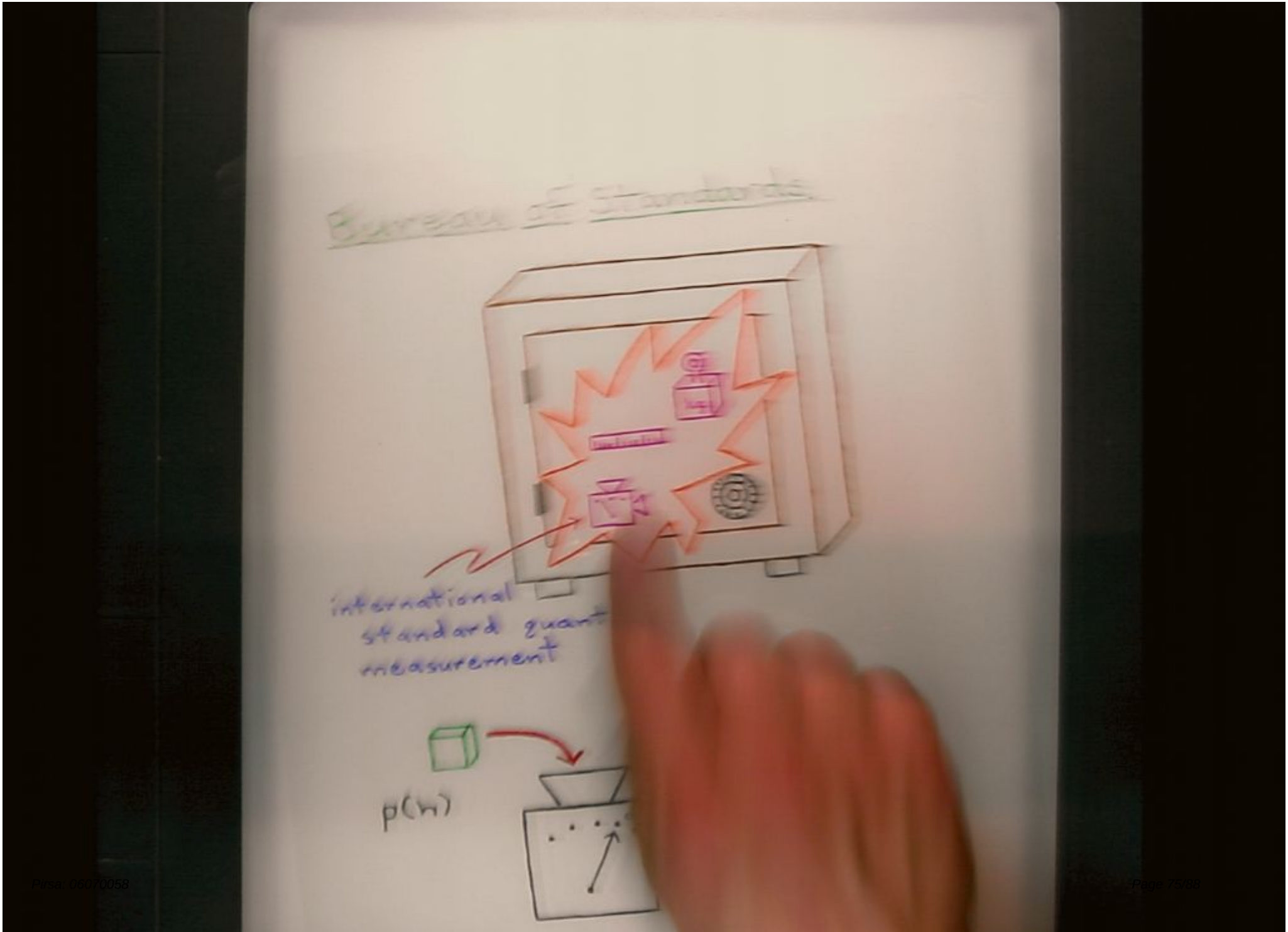
Bureau of Standards



international standard quantum measurement




ρ
p(h)


Bureau of Standards
international
standard quant
measurement
p(n)

Standard Measurements	Generalized Measurements
$\{\Pi_i\}$	$\{E_b\}$
$\langle\psi\|\Pi_i\|\psi\rangle \geq 0, \forall\|\psi\rangle$	$\langle\psi\|E_b\|\psi\rangle \geq 0, \forall\|\psi\rangle$
$\sum_i \Pi_i = I$	$\sum_b E_b = I$
$p(i) = tr\,\rho\Pi_i$	$p(b) = tr\,\rho E_b$
$\Pi_i \Pi_j = \delta_{ij}\Pi_i$	

Standard Measurements	Generalized Measurements
$\{\Pi_i\}$	$\{E_b\}$
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$\sum_i \Pi_i = \mathbb{I}$	$\sum_b E_b = \mathbb{I}$
$p(i) = \mathrm{tr}\,\rho\Pi_i$	$p(b) = \mathrm{tr}\,\rho E_b$
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Informational Completeness

quantum states

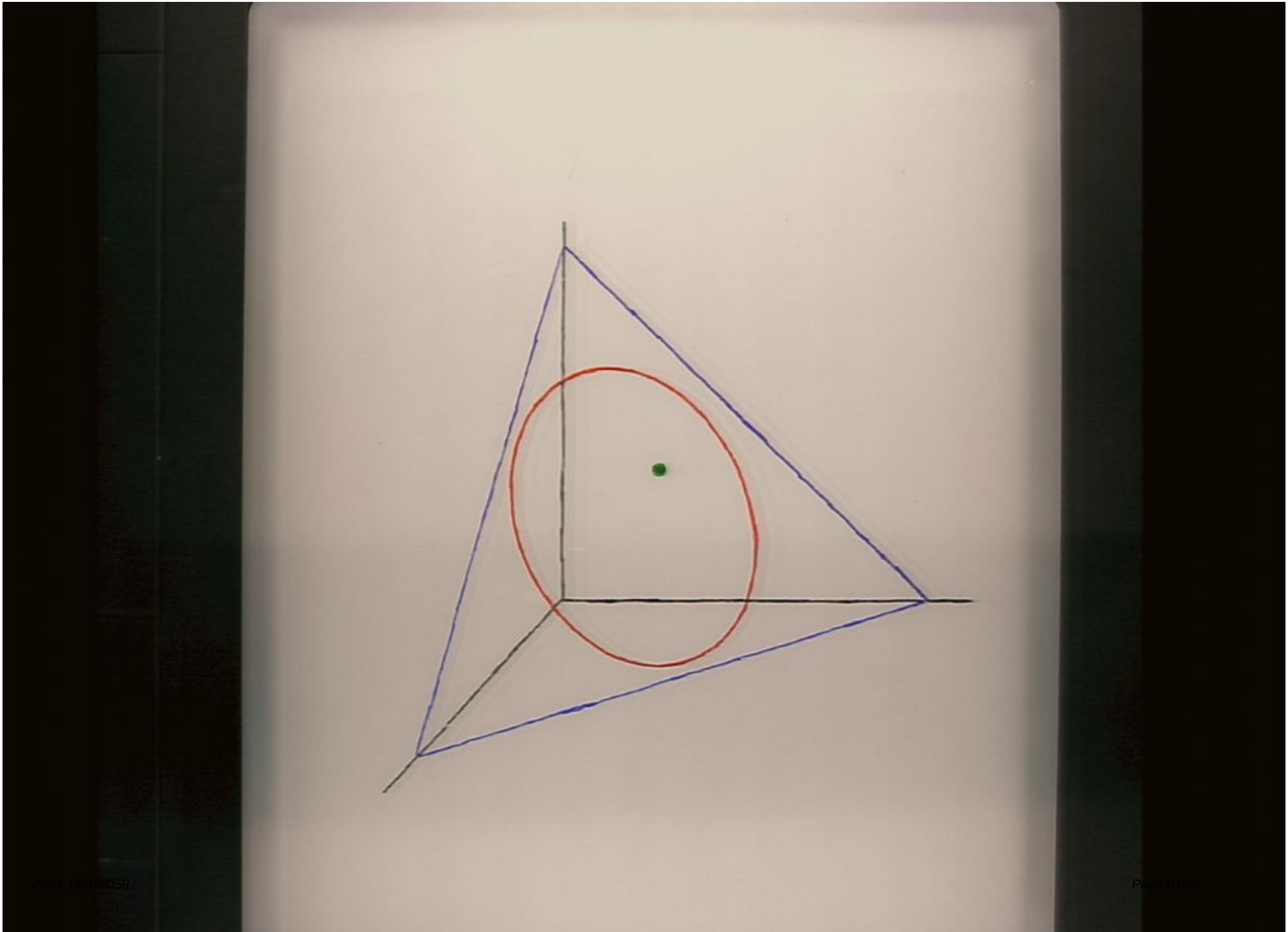
$\rho \in \mathcal{L}(\mathcal{H}_0)$ — D^2-dimensional vector space

Choose POVM $\{E_h\}$, $h = 1, \ldots, D^2$, with E_h all linearly independent. (Can be done.)

D^2 numbers $p(h) = \mathrm{tr}\,\rho E_h$ determine ρ.

↑ projection of ρ onto E_h

Any such $\{E_h\}$ can be the standard quantum measurement.



A Very Fundamental Mmt ?

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$ satisfying

$$\mathrm{tr}\,\Pi_i \Pi_j = \frac{1}{d+1} \quad , \quad i \neq j$$

exist.

Can prove:

1) the Π_i linearly independent

2) $\sum_i \frac{1}{d} \Pi_i = I$

So good for Bureau of Standards.

Also

$$p(i) = \frac{1}{d}\,\mathrm{tr}\,\rho\Pi_i$$

$$\boxed{\rho = \sum_i [(d+1)p(i) - \tfrac{1}{d}]\Pi_i}$$

Pure States in SIC Language

Conditions

$$\rho^+ = \rho \quad , \quad \mathrm{tr}\,\rho^2 = \mathrm{tr}\,\rho^3 = 1$$

translate to

$$\sum_i p(i)^2 = \frac{2}{d(d+1)}$$

and

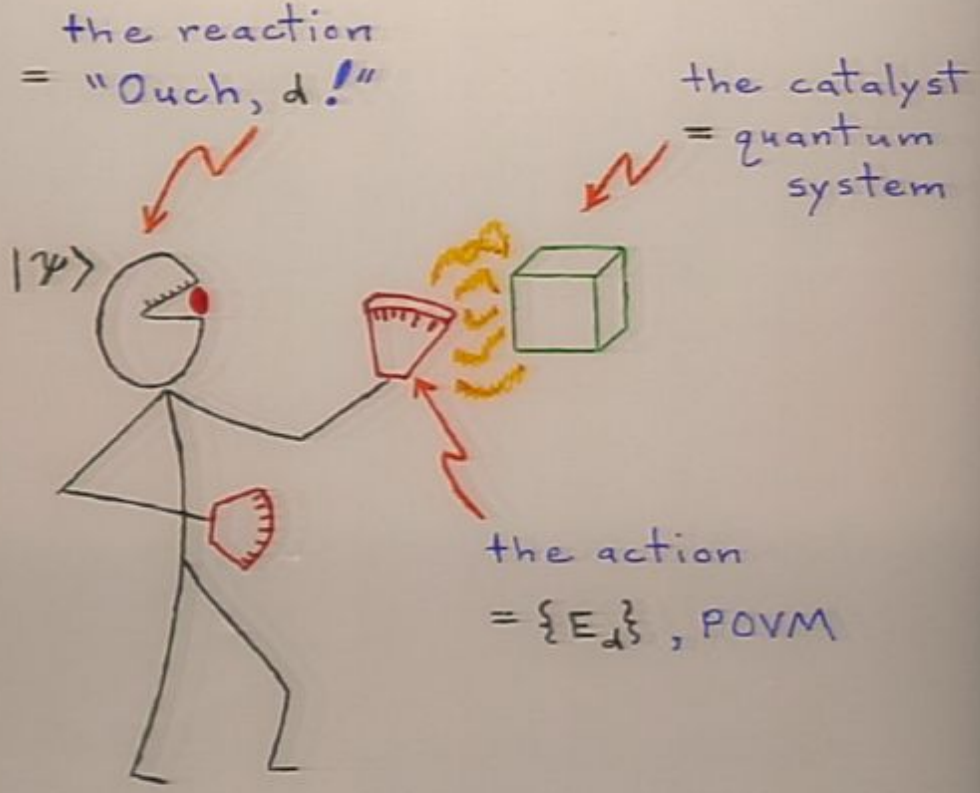
$$\sum_{jkl} c_{jkl}\, p(j)\, p(k)\, p(l) = \frac{d+7}{(d+1)^3}$$

where

$$c_{jkl} = \mathrm{Re}\ \mathrm{tr}\,\Pi_j \Pi_k \Pi_l$$

Could these be independently motivatable physical constants?

What island do we live on?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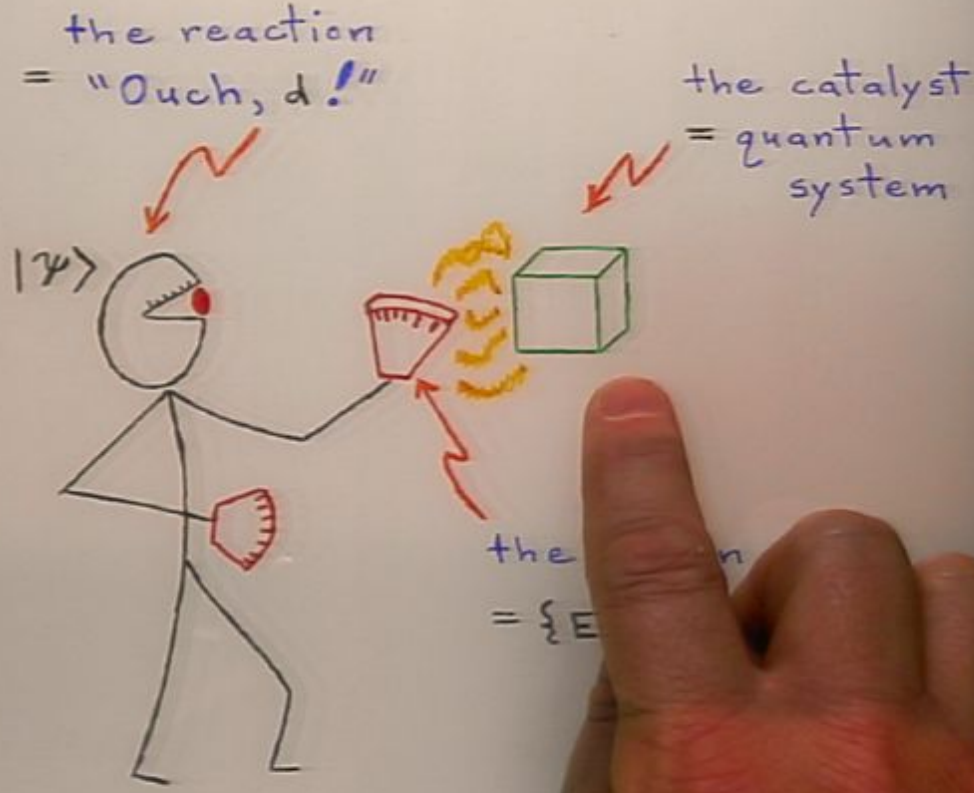
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R. W. S[illegible]ns, quant-ph/0401052

"In Defense of the Epistemic View of Quantum States: A Toy Theory"

- noncommutivity of measurements
- interference
- monogomy of "entanglement"
- no cloning
- no broadcasting
- teleportation
- superdense coding
- impossibility of universal state inverter

⋮

"The diversity and quality of these analogies provides compelling evidence for the view that quantum states are states of knowledge rather than states of reality, and that maximal knowledge is incomplete knowledge."