Title: Quantum in Gravity?

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Abstract: I will report my efforts to describe elementary Quantum behaviours, specifically single-particle interference and two-particle entanglement, in an accelerating frame.

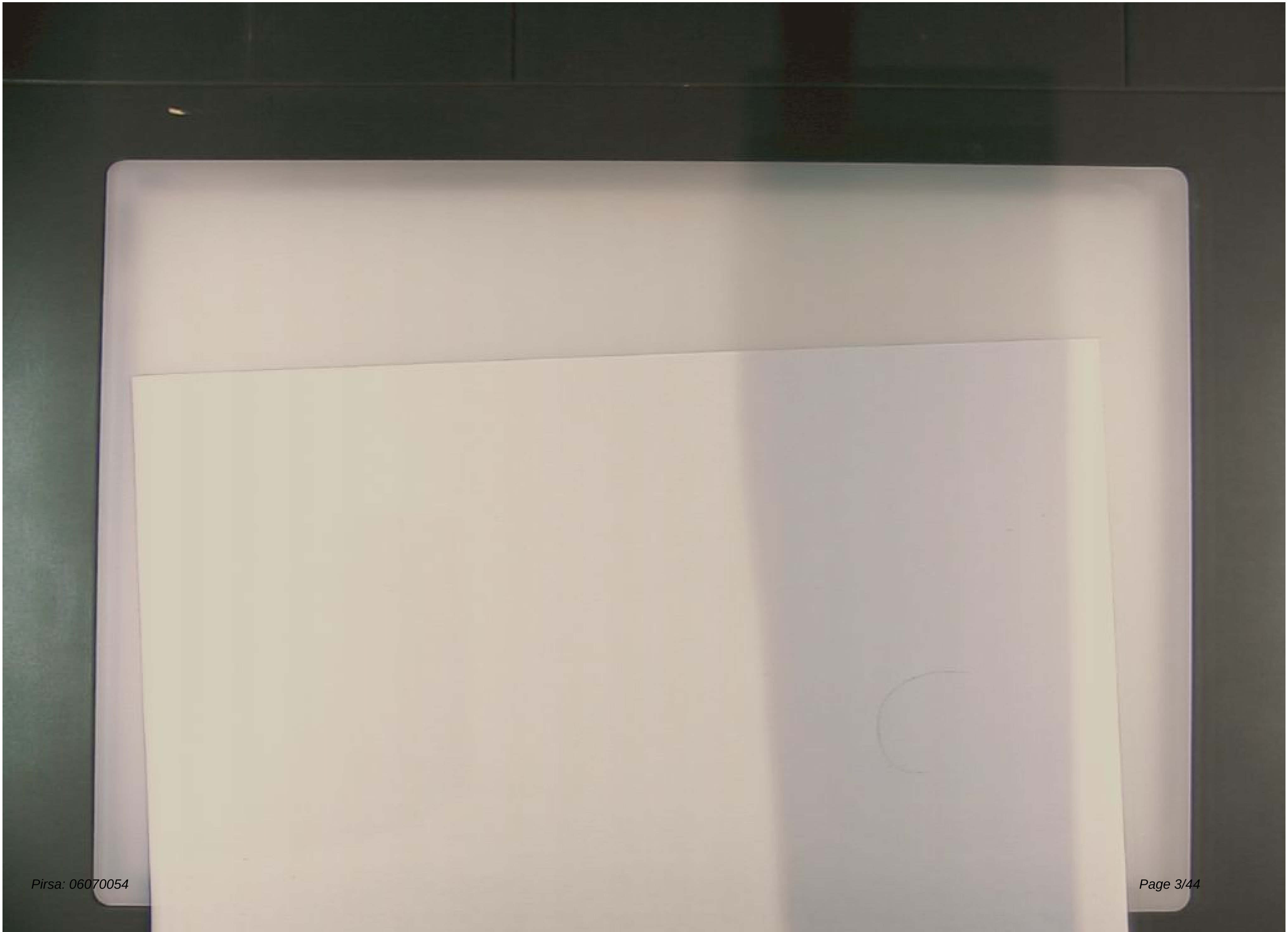
Quantum Mechanics in a Non-Inertial Frame

About Abner

Why this topic?

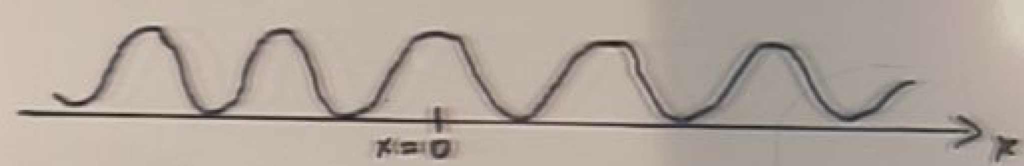
What quantum mechanics?

What non-inertial frame?

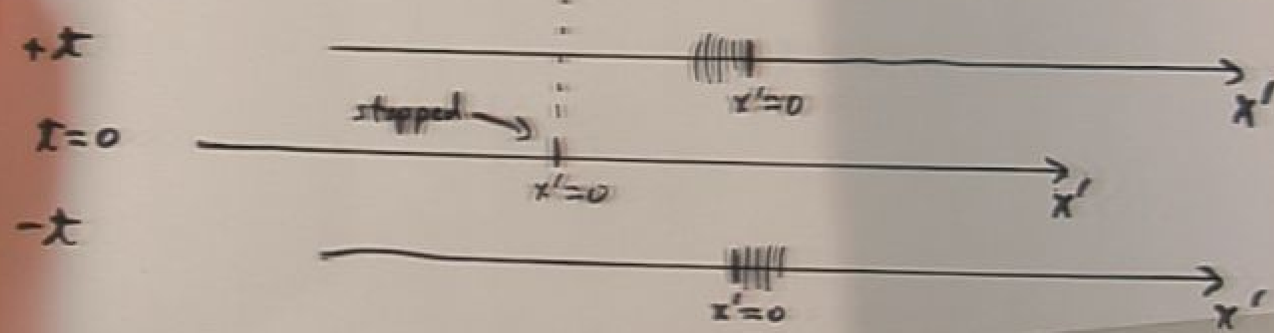


The Fringes and the Frames

Rest frame of Young's Fringes (Inertial, unprimed)



Non-inertial frame (primed)



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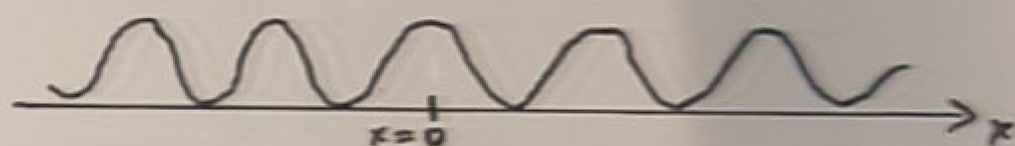

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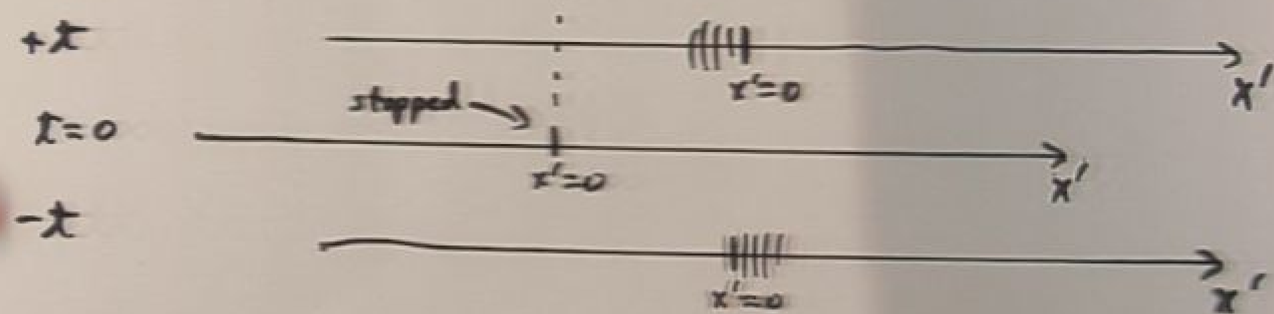

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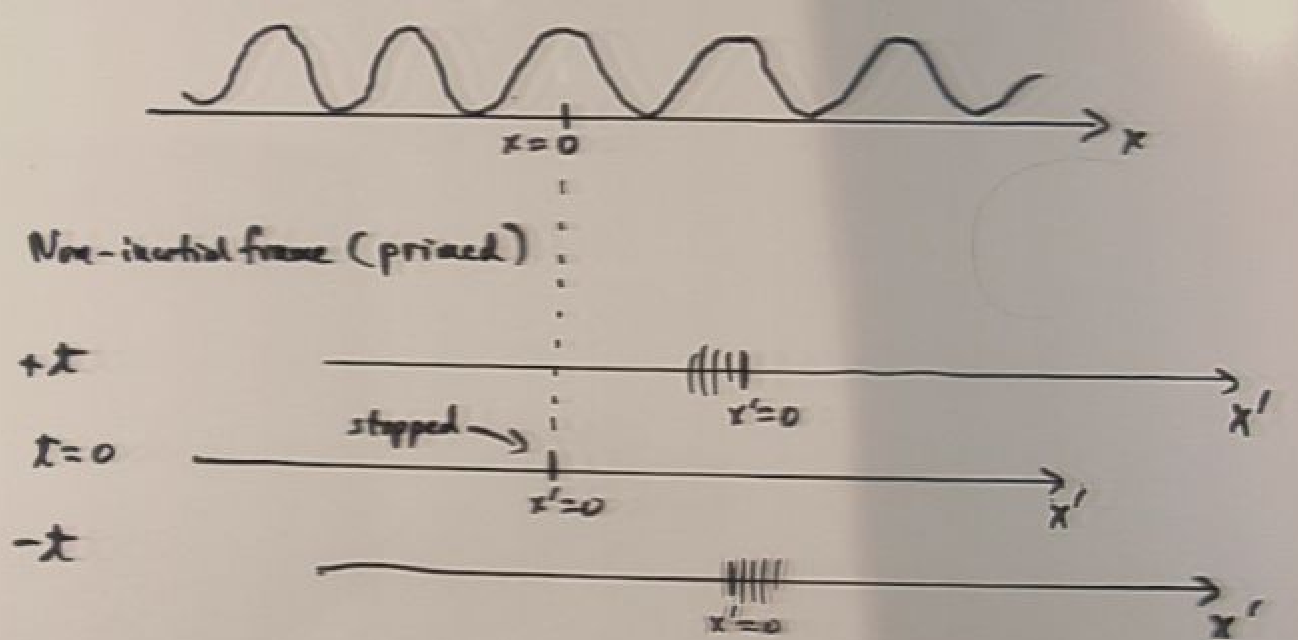

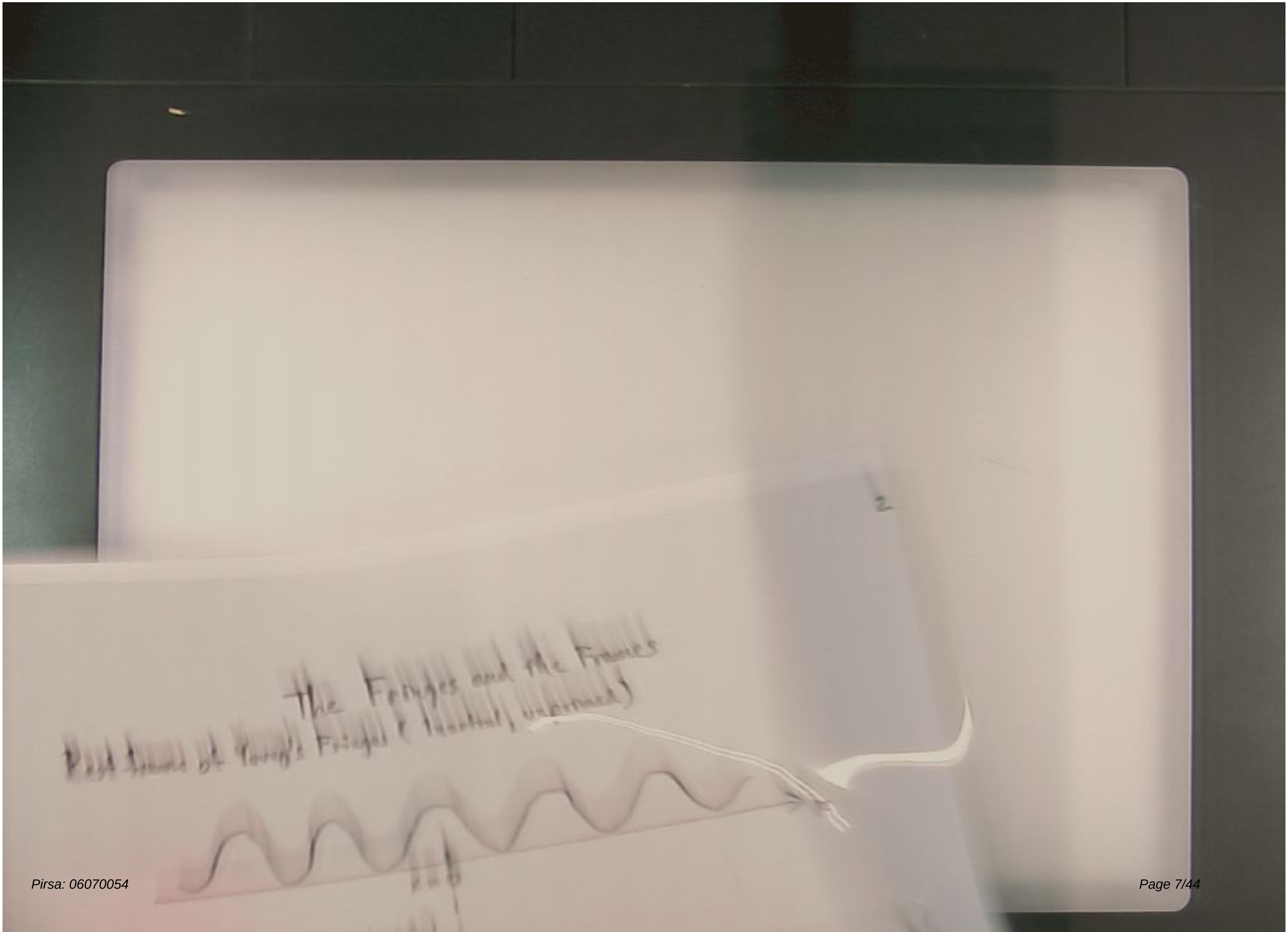
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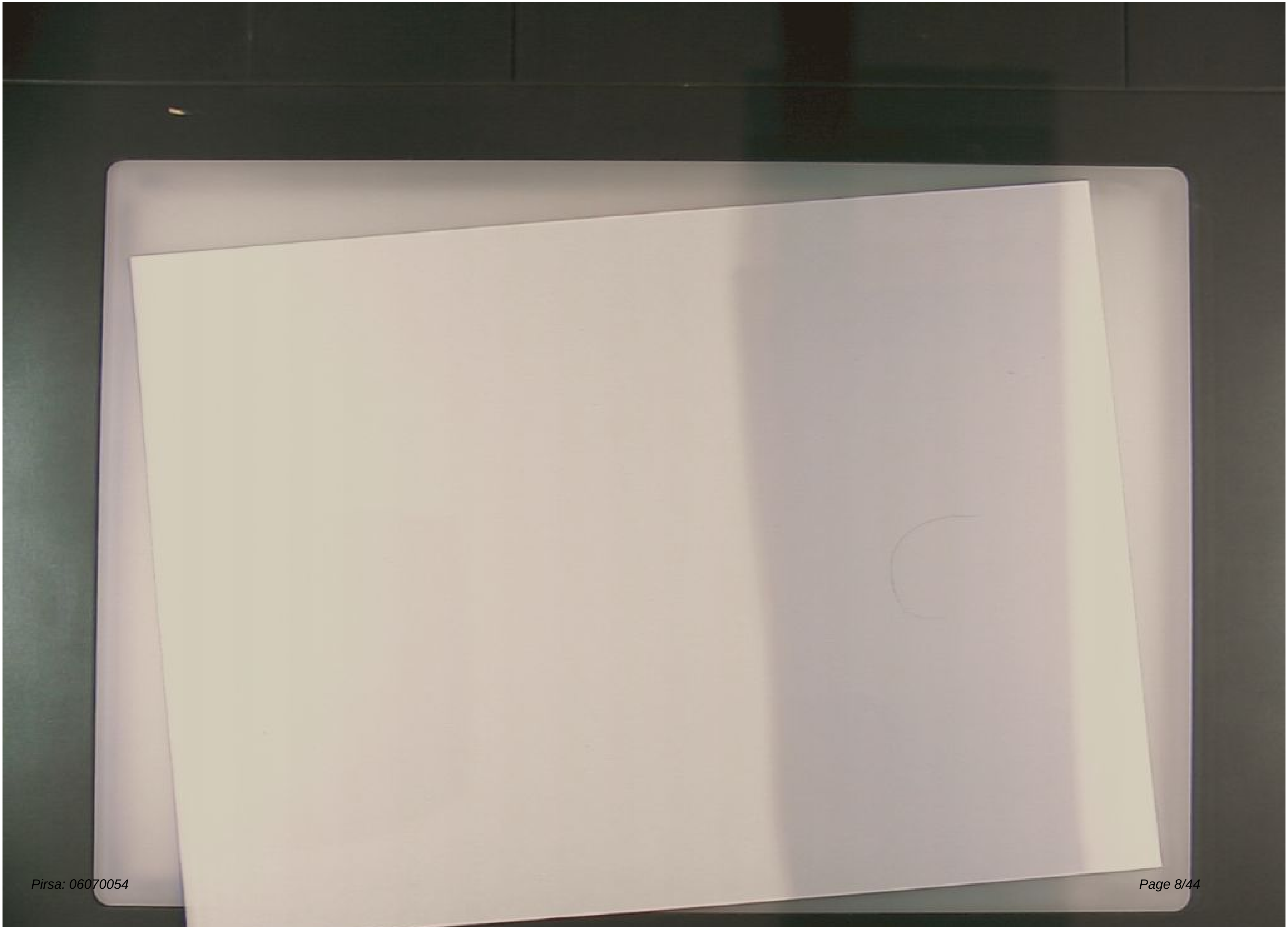

3

Van Name Motion

Starting from rest, particle of mass M_0 experiences a constant force

① $x(t) = (1+t^2)^{1/2} - 1$

② $\beta(t) = \dfrac{t}{(1+t^2)^{1/2}}$

③ $a(t) = \dfrac{a_0}{(1+t^2)^{3/2}}$

④ $\beta(x) = \dfrac{(2x+x^2)^{1/2}}{1+x}$

where $\begin{cases} x \equiv \dfrac{F x_{conv}}{M_0 c^2} \\ t \equiv \dfrac{F t_{conv}}{M_0 c} \\ \beta \equiv \dfrac{v_{conv}}{c} \\ a_0 \equiv \dfrac{F}{M_0} \end{cases}$

derivation

$(F\,dt = dp)_{conv} \rightarrow dt = d(\gamma\beta) \Rightarrow t = \gamma\beta = \dfrac{\beta}{(1-\beta^2)^{\frac{1}{2}}} \Rightarrow$ ② $\Rightarrow \begin{cases} ① \\ ③ \\ ④ \end{cases}$

alternately for ④

$(F\,dx = dE)_{conv} \rightarrow dx = d\gamma \Rightarrow x = \gamma - 1 = \dfrac{1}{(1-\beta^2)^{\frac{1}{2}}} - 1 \Rightarrow$ ④

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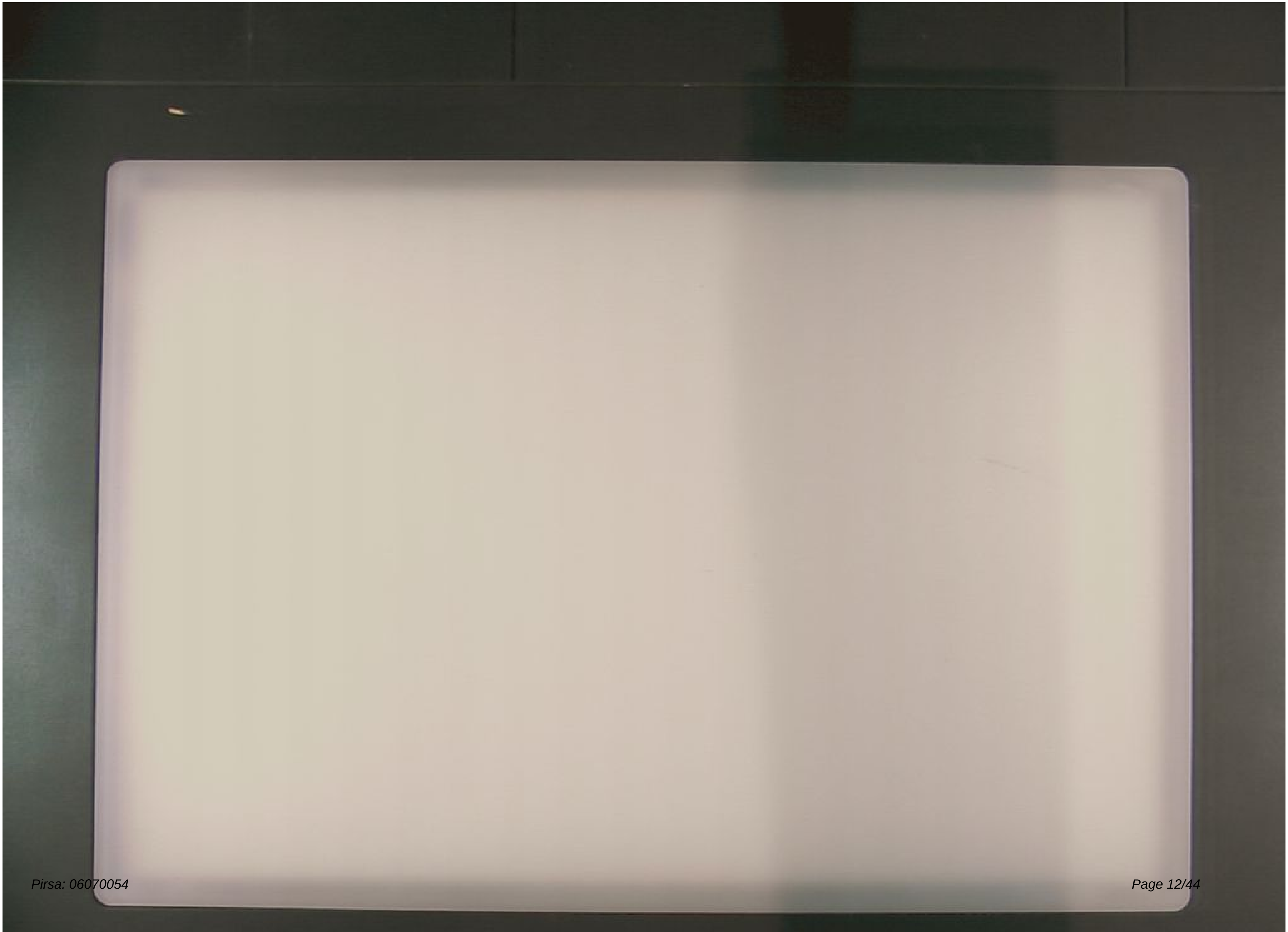

Hyperbolic Motion

Time t as function of proper time τ of the moving point (clock)

$$t = \sinh \tau$$

derivation

$$d\tau = (1-\beta^2(t))^{\frac{1}{2}} dt \Rightarrow \tau = \int_0^t (1-\beta^2(t))^{\frac{1}{2}} dt = \sinh^{-1} t \Rightarrow t = \sinh$$

Position x as function of τ

$$x = \cosh \tau - 1$$

derivation

$$x = (1+t^2)^{\frac{1}{2}} - 1 = (1+\sinh^2\tau)^{\frac{1}{2}} - 1 = \cosh\tau - 1$$

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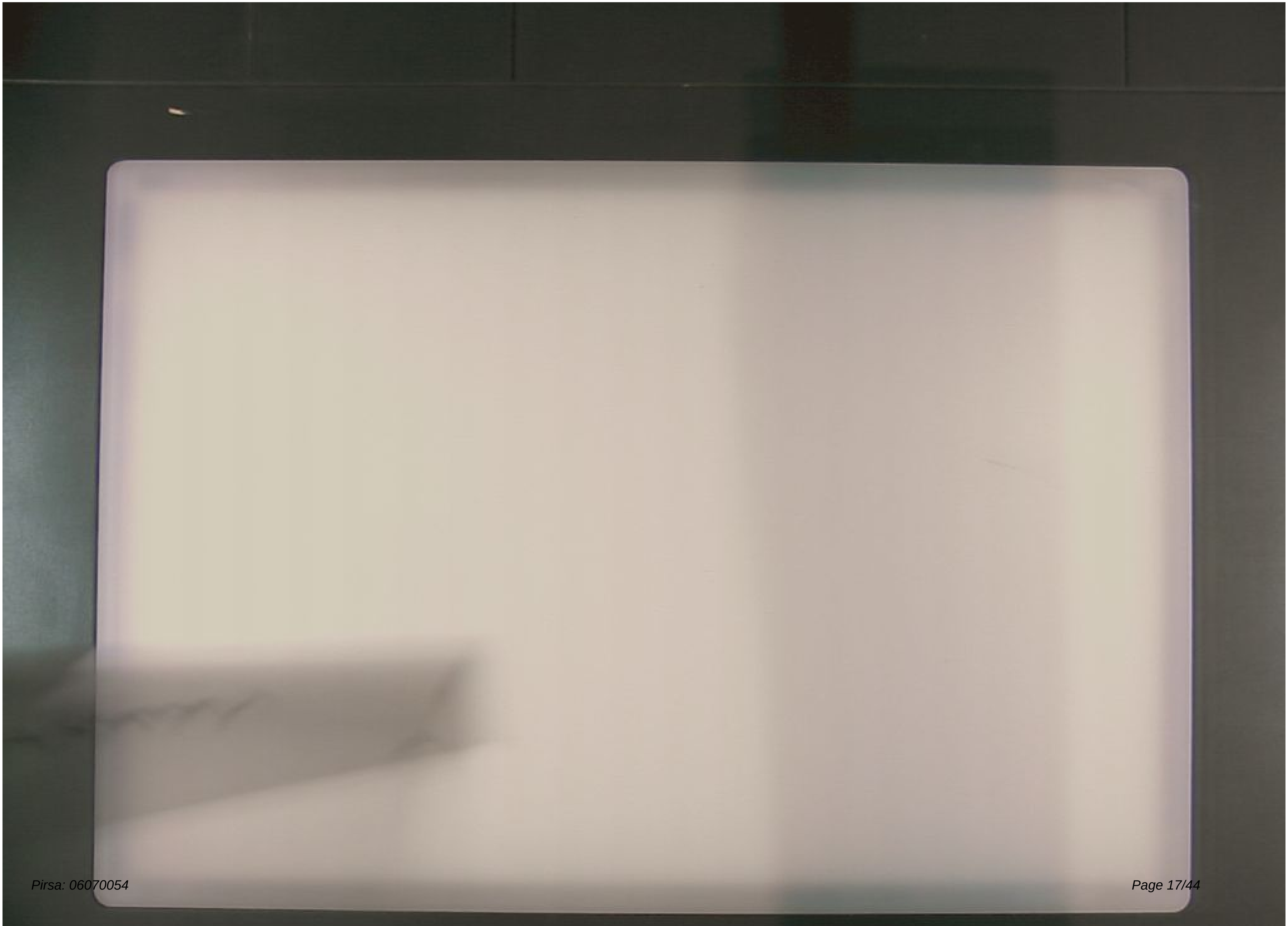

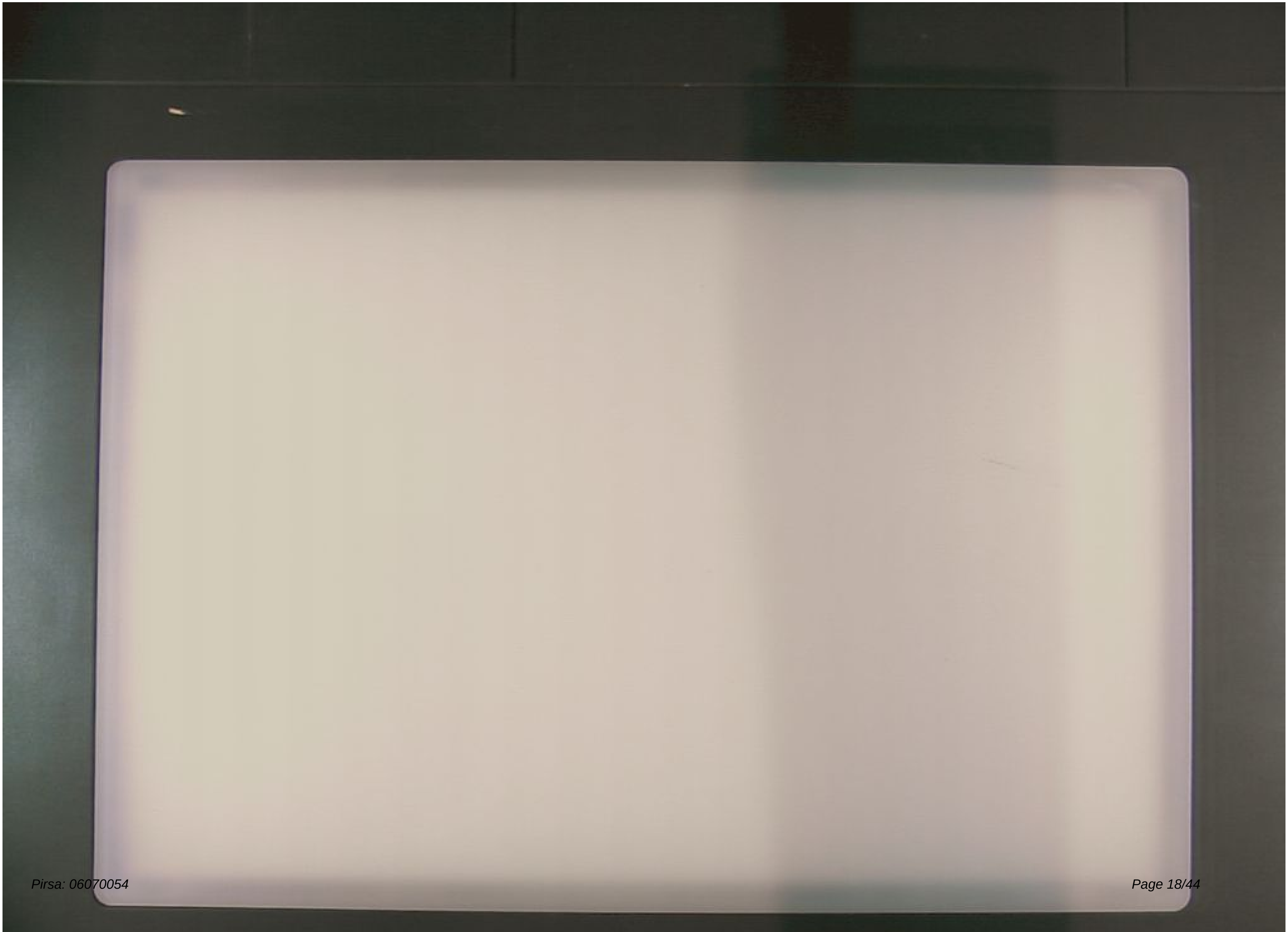

Hanging on x'-axis and Spending the Time τ

$$x = (1 + x')\cosh\tau - 1$$

$$t = (1 + x')\sinh\tau$$

$$x' = [(1+x)^2 - t^2]^{1/2} - 1$$

$$\tau = \tanh^{-1}\left(\frac{t}{1+x}\right)$$

⌜derivation?⌟

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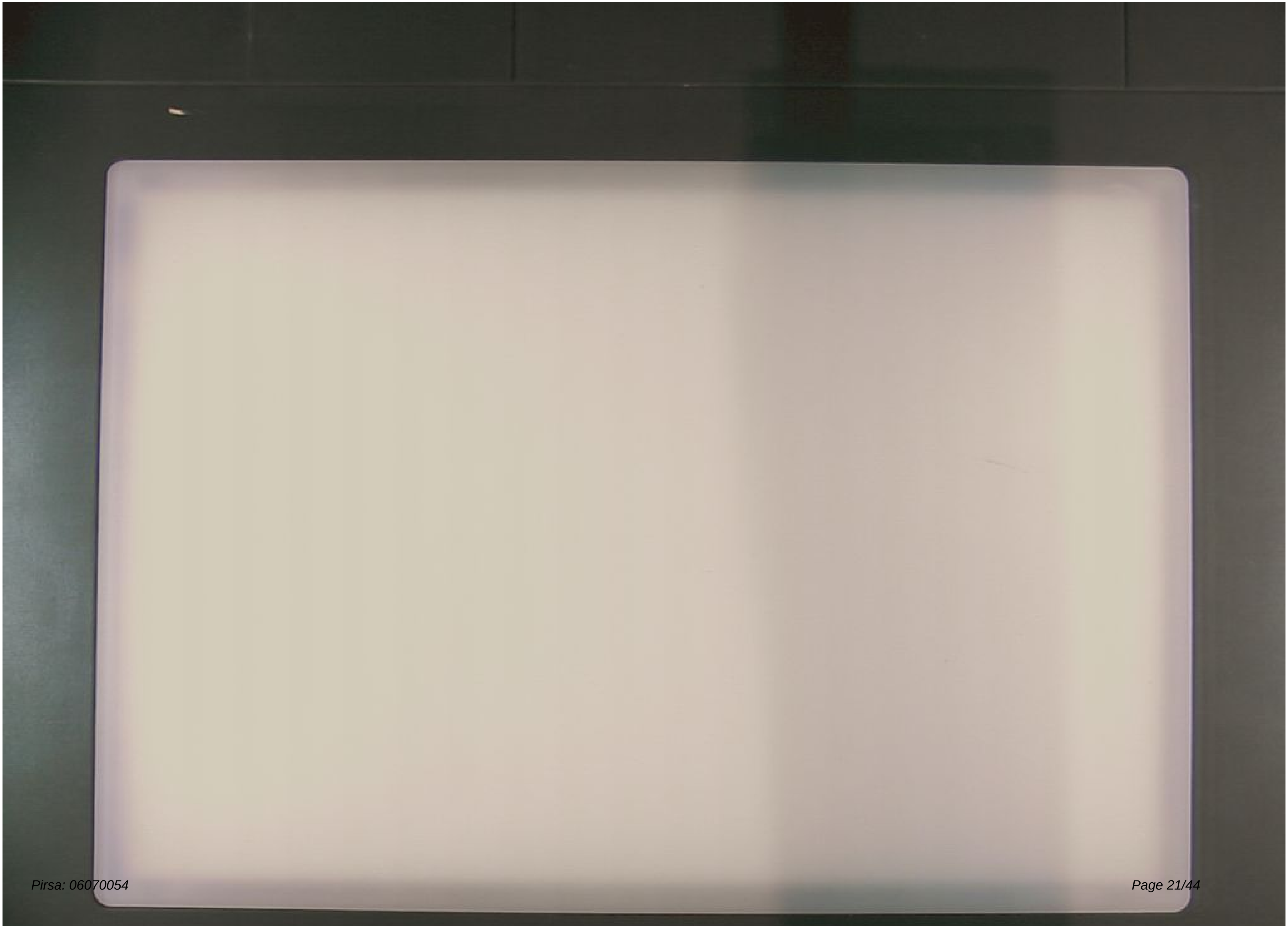

Three Disappointments

6

1. Unequal relative speeds of the frames

$$\left.\frac{dx}{dt}\right|_{x'=\text{constant}} = \frac{\bar{\lambda}}{1+x} \qquad\qquad \left.\frac{dx'}{d\tau}\right|_{x=\text{constant}} = -\underbrace{(1+x')}_{\text{ugh!}}\left(\frac{\bar{\lambda}}{1+x}\right)$$

2. Breakdown of speed addition:

$$\text{ugh!} \rightarrow \frac{\left(\frac{u'}{c}\right)}{(1+x')} = \frac{\left(\frac{u}{v}\right) - \left(\frac{\bar{\lambda}}{1+x}\right)}{1 - \left(\frac{u}{v}\right)\left(\frac{\bar{\lambda}}{1+x}\right)} \qquad \text{where} \begin{cases} u' \equiv \dfrac{dx'}{d\tau} \\ u \equiv \dfrac{dx}{dt} \end{cases}$$

3. Alternative calculations of momentum (and energy) inconsistent

momentum $\quad \gamma\left(\frac{u'}{c}\right)\left(\frac{u'}{c}\right. \neq \gamma\left(\frac{u}{c}\right)\gamma\left(\frac{\bar{\lambda}}{1+x}\right)\left[\frac{u}{c} - \left(\frac{\bar{\lambda}}{1+x}\right)\right]$

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$$\left.\frac{dx}{dt}\right|_{x'=\text{constant}} = \frac{\dot{x}}{1+x} \qquad \left.\frac{dx'}{d\tau}\right|_{x=\text{constant}} = -(1+x')\left(\frac{\dot{x}}{1+x}\right)$$

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ugh! $\rightarrow$

$$\frac{\left(\frac{\mu'}{c}\right)}{(1+x')} = \frac{\left(\frac{\mu}{v}\right) - \left(\frac{\dot{x}}{1+x}\right)}{1 - \left(\frac{\mu}{v}\right)\left(\frac{\dot{x}}{1+x}\right)} \quad \text{where} \begin{cases} \mu' \equiv \frac{dx'}{d\tau} \\ \mu \equiv \frac{dx}{dt} \end{cases}$$

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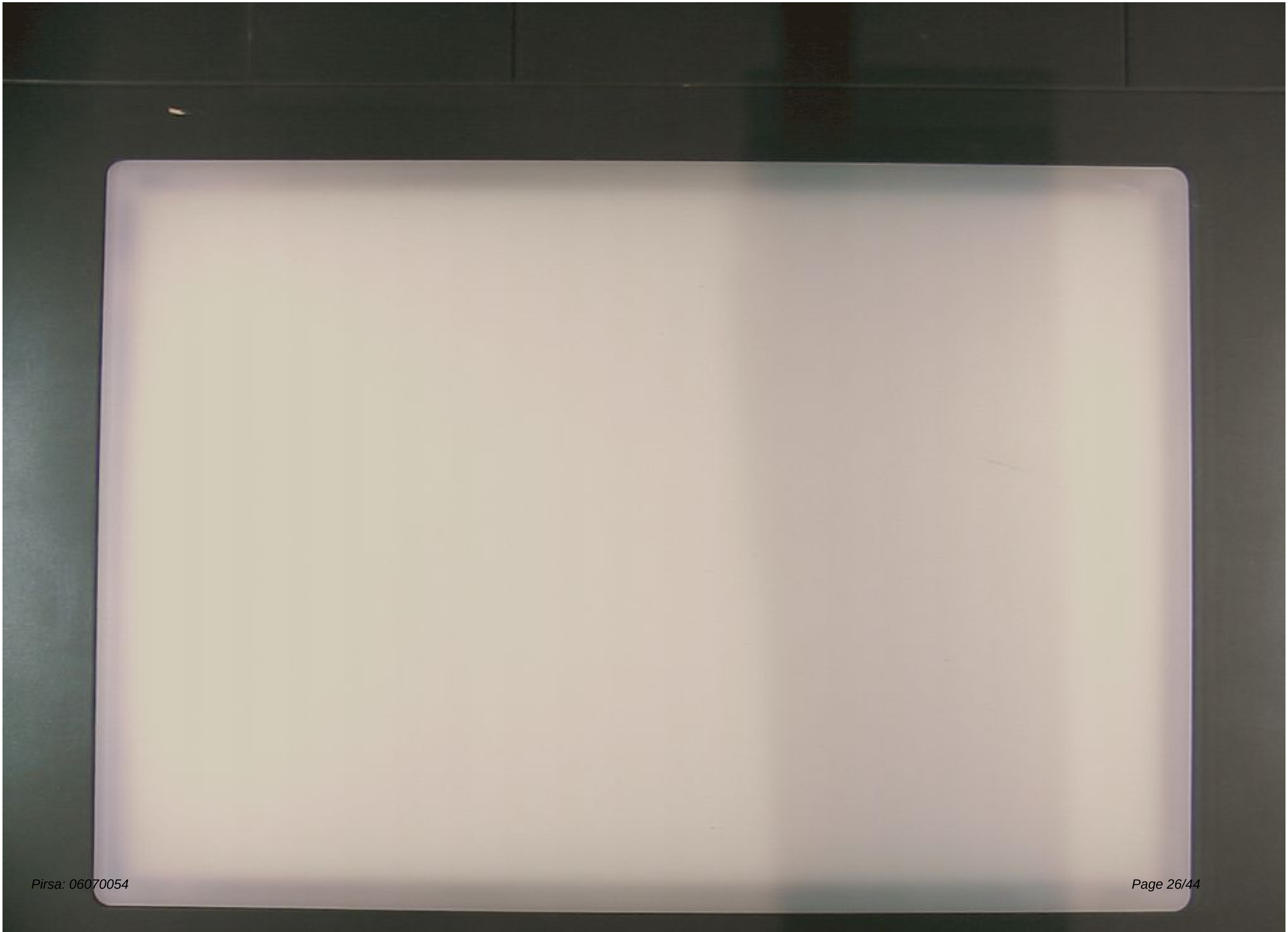

The Quick Fix

Prime-frame clock at x' keeps t' time, not τ time, where

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then

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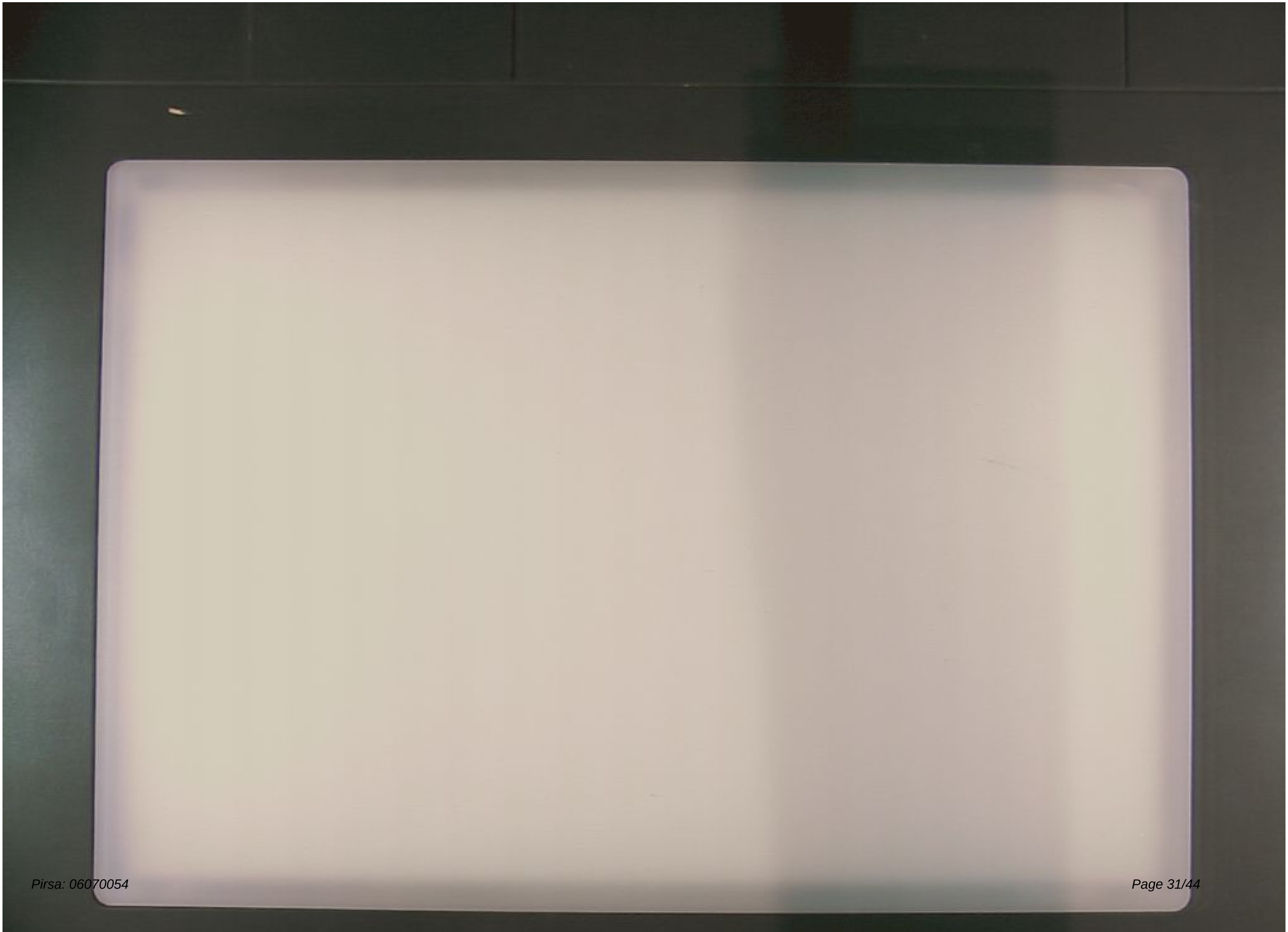

On to the Quantum Mechanics

Single-Particle Fringes

$$\psi = e^{i(p_R x - E_R t)/\hbar} + e^{i(p_L x - E_L t)/\hbar}$$

$$\Rightarrow \psi^* \psi = 2 + 2\cos\left\{\underbrace{(p_R - p_L)x/\hbar - (E_R - E_L)t/\hbar}_{\equiv\, \Phi}\right\}$$

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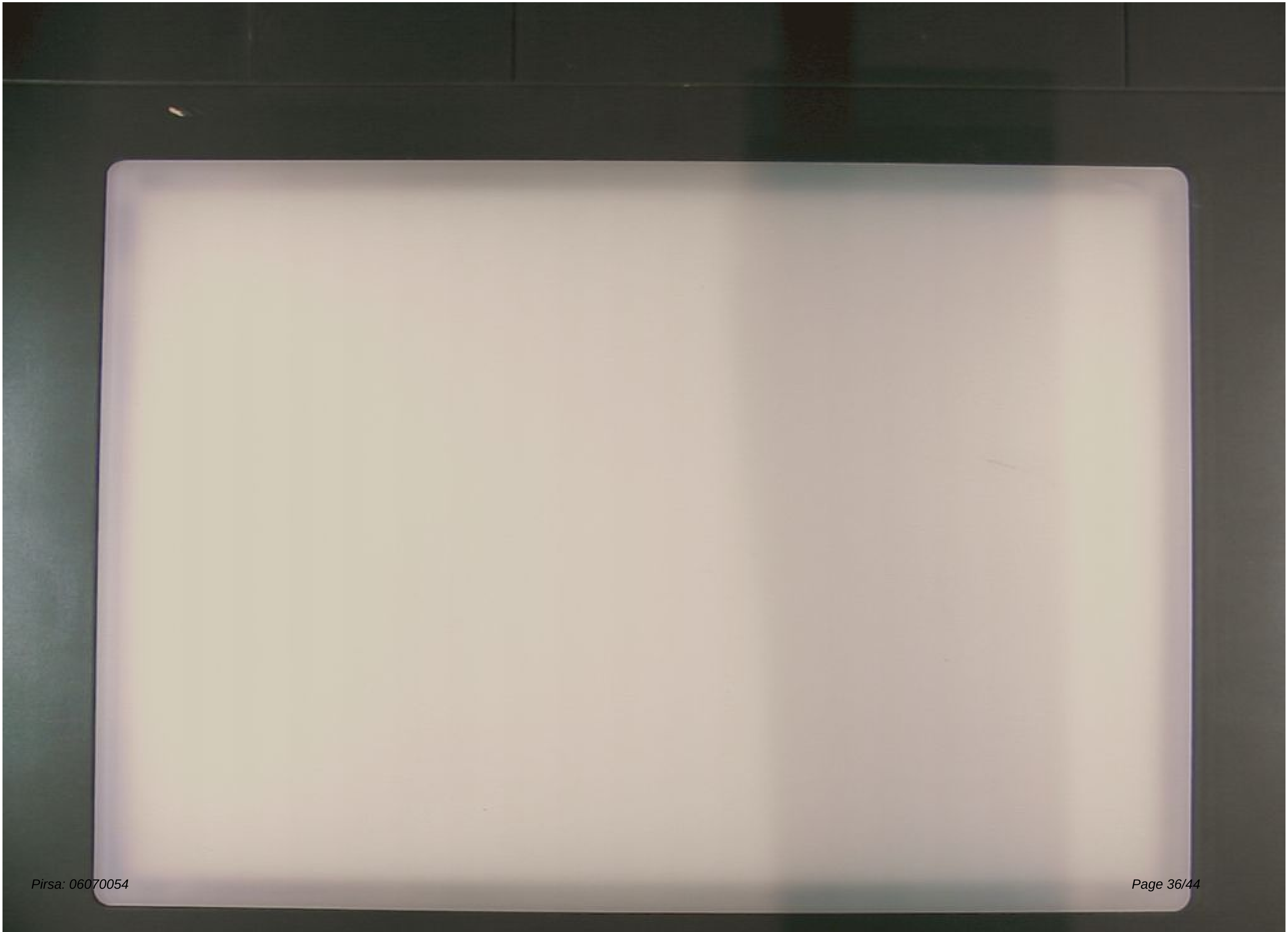

Φ in Three Frames

1. Rest frame of the fringes (particle mass m_0, speeds $\pm u$), inertial

$$\Phi = 2\gamma\left(\frac{u}{c}\right) m_0 u x / \hbar \Rightarrow \text{fringe width} \equiv W_0 = \left[1-\left(\frac{u}{c}\right)^2\right]^{\frac{1}{2}} \frac{\pi\hbar}{m_0 u}$$

2. Another inertial frame, moving right at speed v

$$\Phi' = \gamma\left(\frac{v}{c}\right)\gamma\left(\frac{u}{c}\right)\left[2m_0 u x'/\hbar + 2\left(\frac{u}{c}\right)\left(\frac{v}{c}\right) m_0 c^2 t'/\hbar\right]$$

$$\Rightarrow \begin{cases} \text{fringe width} \equiv W = \left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} W_0 \\ \text{fringe speed} = \left.\frac{dx}{dt}\right|_{\Phi = \text{const}} = \frac{E_R - E_L}{p_R - p_L} = \frac{-2\left(\frac{u}{c}\right)\left(\frac{v}{c}\right) m_0 c^2}{2 m_0 u} = -v \end{cases}$$

3. The non-inertial frame

$$\Phi = \text{same as 2., with } \frac{v}{c} \rightarrow \left(\frac{x}{1+x}\right)$$

$$\Rightarrow \begin{cases} \text{fringe width} = \left[1-\left(\frac{x}{1+x}\right)^2\right]^{1/2} W_0 \\ \text{fr[illegible]d} = -\left(\frac{x}{1+x}\right) \end{cases}$$

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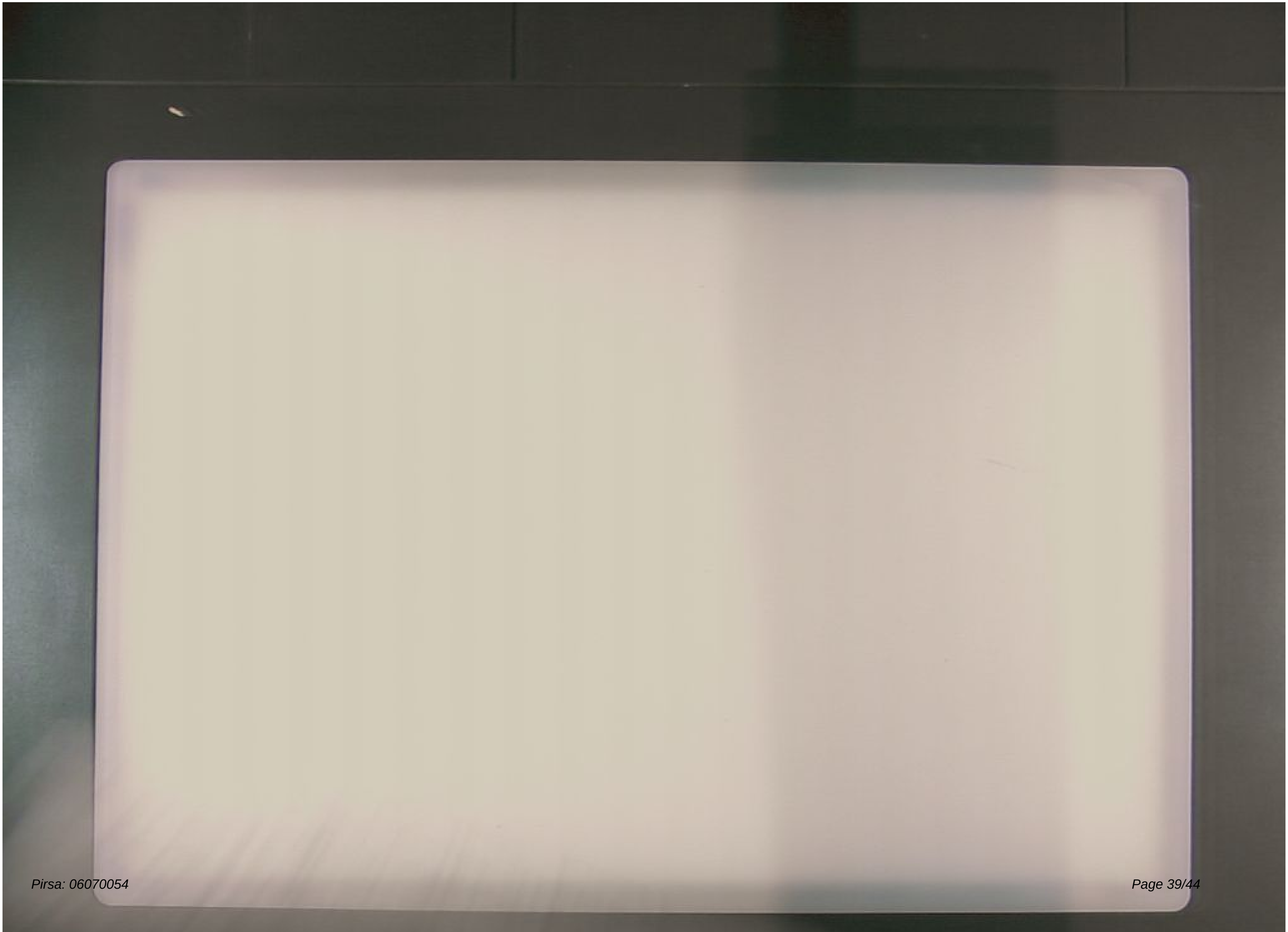

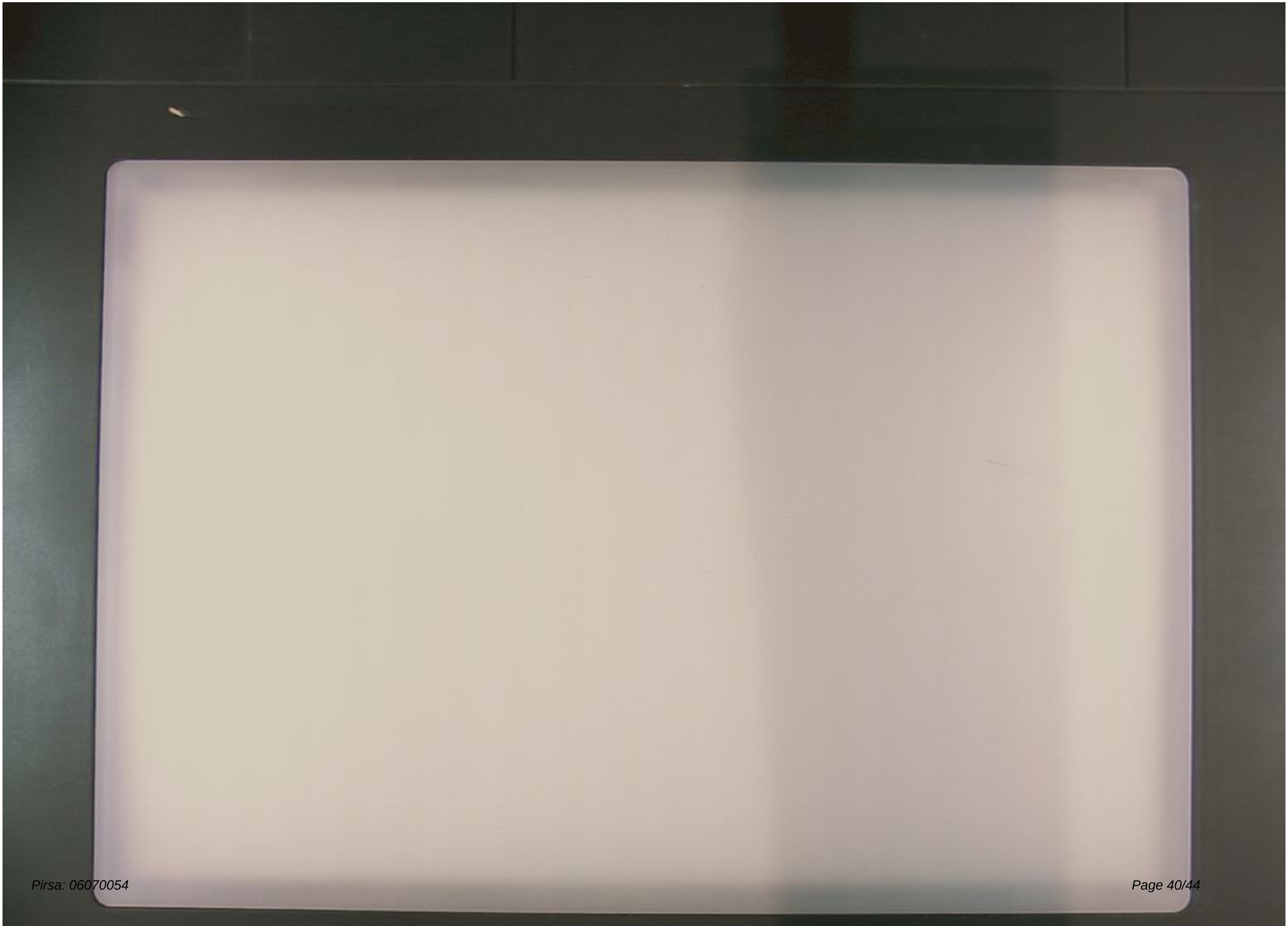

Two-Particle Fringes (Conditional)

$$\psi = e^{i\phi_R^1} e^{i\phi_L^2} + e^{i\phi_L^1} e^{i\phi_R^2}$$

$$\Rightarrow \quad \psi^*\psi = 2 + 2\,[\underbrace{(\phi_R^1 - \phi_L^1) - (\phi_R^2 - \phi_L^2)}_{\equiv\, \Phi}]$$

For mass m_0 and speed $\pm u$ in the inertial frame }, Φ in the non-inertial frame is

$$\Phi' = \Big\{ \gamma\left(\tfrac{t_1}{1+X_1}\right)\gamma\left(\tfrac{u}{c}\right)\left[2 m_0 u\, x_1'/\hbar + 2\left(\tfrac{t_1}{1+X_1}\right)\left(\tfrac{u}{c}\right) m_0 c^2\, t_1'/\hbar\right]$$
$$- \gamma\left(\tfrac{t_2}{1+X_2}\right)\gamma\left(\tfrac{u}{c}\right)\left[2 m_0 u\, x_2'/\hbar + 2\left(\tfrac{t_2}{1+X_2}\right)\left(\tfrac{u}{c}\right) m_0 c^2\, t_2'/\hbar\right] \Big\}$$

Since

$$\rho(x_2, t_2) \sim \int_{\text{one spatial cycle}} \psi^*\psi\, dx_1' = \int_{\text{one temp cycle}} \psi^*\psi\, dt_1' = \underline{\text{constant}},$$

$$\rho(x_1, t_1 \mid x_2, t_2) = \rho(x_1, t_1; x_2, t_2)/\rho(x_2, t_2) \sim \psi^*\psi \quad \text{itself!}$$

$\Longrightarrow$ conditional fringes have correct <u>width</u>, <u>speed</u>, and <u>space-time location</u>.

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For $\left.\begin{array}{l}\text{mass } m_0 \text{ and speed } \pm u \\ \text{in the inertial frame}\end{array}\right\}$, Φ in the non-inertial frame is

$$\Phi' = \Big\{ \gamma\left(\frac{t_1}{1+x_1}\right) \gamma\left(\frac{u}{c}\right) \left[2 m_0 u x_1'/\hbar + 2\left(\frac{t_1}{1+x_1}\right)\left(\frac{u}{c}\right) m_0 c^2 t_1'/\hbar \right]$$
$$- \gamma\left(\frac{t_2}{1+x_2}\right) \gamma\left(\frac{u}{c}\right) \left[2 m_0 u x_2'/\hbar + 2\left(\frac{t_2}{1+x_2}\right)\left(\frac{u}{c}\right) m_0 c^2 t_2'/\hbar \right] \Big\}$$

Since

$$\rho(x_2, t_2) \sim \int_{\text{one spatial cycle}} \psi^* \psi \, dx_1' = \int_{\text{one temp cycle}} \psi^* \psi \, dt_1' = \underline{\text{constant}},$$

$$\rho(x_1, t_1 | x_2, t_2) = \rho(x_1, t_1; x_2, t_2) / \rho(x_2, t_2) \sim \psi^* \psi \quad \text{itself!}$$

$\Rightarrow$ conditional fringes have correct <u>width</u>, <u>speed</u>, and <u>space-time location</u>.