

Title: Rotational analysis of a vibrational transition in the  $^{199}\text{Hg}_2$  molecule: a first step in an experimental realization of a spin-1/2 particle version of the EPR experiment Authors: Edward S. Fry and Xinmei Qu

Date: Jul 20, 2006 11:45 AM

URL: <http://pirsa.org/06070052>

Abstract: An experimental realization of our spin-1/2 particle version of the Einstein-Podolsky-Rosen (EPR) experiment will be briefly reviewed. In the proposed experiment, two  $^{199}\text{Hg}$  atoms in the ground  $1S_0$  electronic state, each with nuclear spin  $I=1/2$ , are generated in an entangled state with total nuclear spin zero. Such a state can be obtained by dissociation of a  $^{199}\text{Hg}_2$  molecule (dimer) using a spectroscopically selective stimulated Raman process. From symmetry considerations, the nuclear spin singlet state is guaranteed if the initial  $^{199}\text{Hg}_2$  molecule is in a rotational state with an even quantum number. Consequently, a thorough investigation and analysis of the rotational structure of the  $^{199}\text{Hg}_2$  molecule is required; results of this analysis will be presented.

Einstein together with colleagues  
Podolsky and Rosen:

Quantum Mechanics  
is  
"incomplete"

**Crux of the problem:**

**Classical** mechanics gives deterministic predictions

**Quantum** mechanics gives statistical predictions  
or probabilities

## John Bell

Considered an EPR type experiment

Assumed:

1. Locality
2. “Completion” QM
3. Positive Probabilities

**LOCALITY:** Two spatially separated systems can affect each other only after a time delay greater than the time it takes light to travel from one system to the other.

## John Bell Proved:

1. The statistical predictions of any local theory that “completes” quantum mechanics in the sense of Einstein must satisfy an inequality.
2. The statistical predictions of quantum mechanics can violate that inequality.

## John Bell Proved:

1. The statistical predictions of any local theory that “completes” quantum mechanics in the sense of Einstein must satisfy an inequality.
2. The statistical predictions of quantum mechanics can violate that inequality.

**A definitive laboratory  
experiment is possible**

## Initial experiments:

1972- Berkeley

violated Bell inequality  
and agreed with QM

1974- Harvard

satisfied Bell inequality  
and disagreed with QM

1976- TAMU

violated Bell inequality  
and agreed with QM

1982- Paris

violated Bell inequality  
and agreed with QM

•  
•  
•

These initial experiments had loopholes; they required additional assumptions in order to make an experiment feasible.

**Some more recent experiments (also at least one  
loophole):**

**Texas  
& M  
Physics  
A&M**

**P  
e  
r  
i  
m  
e  
t  
e  
r  
I  
n  
s  
t  
i  
t  
u  
t  
e  
2  
0  
0  
0**

Some more recent experiments (also at least one  
loophole):

**Maryland then Rochester (1986-88) -**

**Two photon down conversion to test Bell inequality**



Some more recent experiments (also at least one loophole):

**Maryland then Rochester (1986-88) -**

**Two photon down conversion to test Bell inequality**

**Paris (1997) -**

**Entanglement of atoms in high Q microwave cavity**

Some more recent experiments (also at least one loop-hole):

**Maryland then Rochester (1986-88) -**

**Two photon down conversion to test Bell inequality**

**Paris (1997) -**

**Entanglement of atoms in high Q microwave cavity**

**Geneva (1997) -**

**Tested Bell inequality with entangled photons and a detector separation of 10.9 km**

Some more recent experiments (also at least one loophole):

**Maryland then Rochester (1986-88) -**

Two photon down conversion to test Bell inequality

**Paris (1997) -**

Entanglement of atoms in high Q microwave cavity

**Geneva (1997) -**

Tested Bell inequality with entangled photons and a detector separation of 10.9 km

**Innsbruck (1998) -**

Tested Bell inequality with entangled photons under strict Einstein locality conditions

Some more recent experiments (also at least one loophole):

**Maryland then Rochester (1986-88) -**

Two photon down conversion to test Bell inequality

**Paris (1997) -**

Entanglement of atoms in high Q microwave cavity

**Geneva (1997) -**

Tested Bell inequality with entangled photons and a detector separation of 10.9 km

**Innsbruck (1998) -**

Tested Bell inequality with entangled photons under strict Einstein locality conditions

**Boulder (2001) -**

Tested Bell inequality with atoms (massive particles) and high (98%) efficiency detection

Some more recent experiments (also at least one loophole):

**Maryland then Rochester (1986-88) -**

Two photon down conversion to test Bell inequality

**Paris (1997) -**

Entanglement of atoms in high Q microwave cavity

**Geneva (1997) -**

Tested Bell inequality with entangled photons and a detector separation of 10.9 km

**Innsbruck (1998) -**

Tested Bell inequality with entangled photons under strict Einstein locality conditions

**Boulder (2001) -**

Tested Bell inequality with atoms (massive particles) and high (98%) efficiency detection

**Austria (2003) -**

Tested Bell inequality with space and spin components of a single neutron

**Note:**

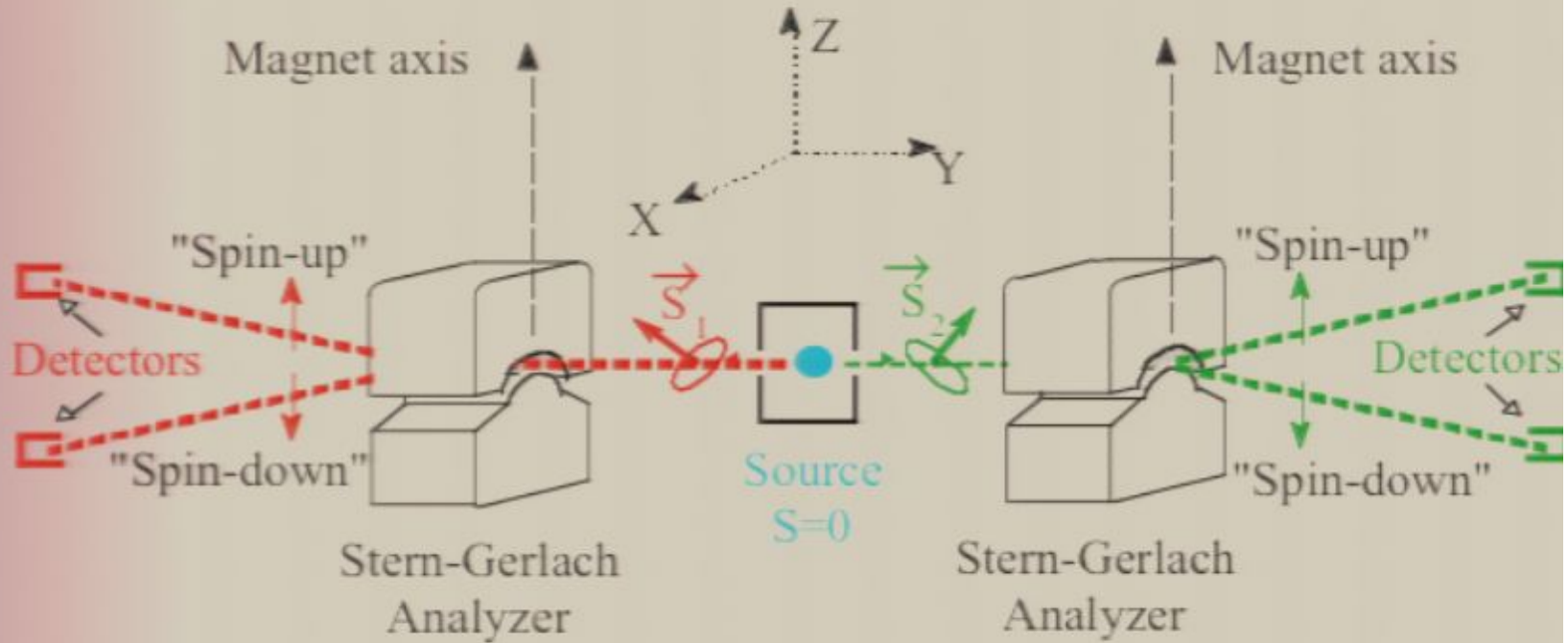
**Results of Bell inequality experiments require any hidden variable theory to be non-local (in order to explain the data).**

## Note:

Results of Bell inequality experiments require any hidden variable theory to be non-local (in order to explain the data).

But, results of Bell inequality experiments do NOT require quantum mechanics to be non-local.

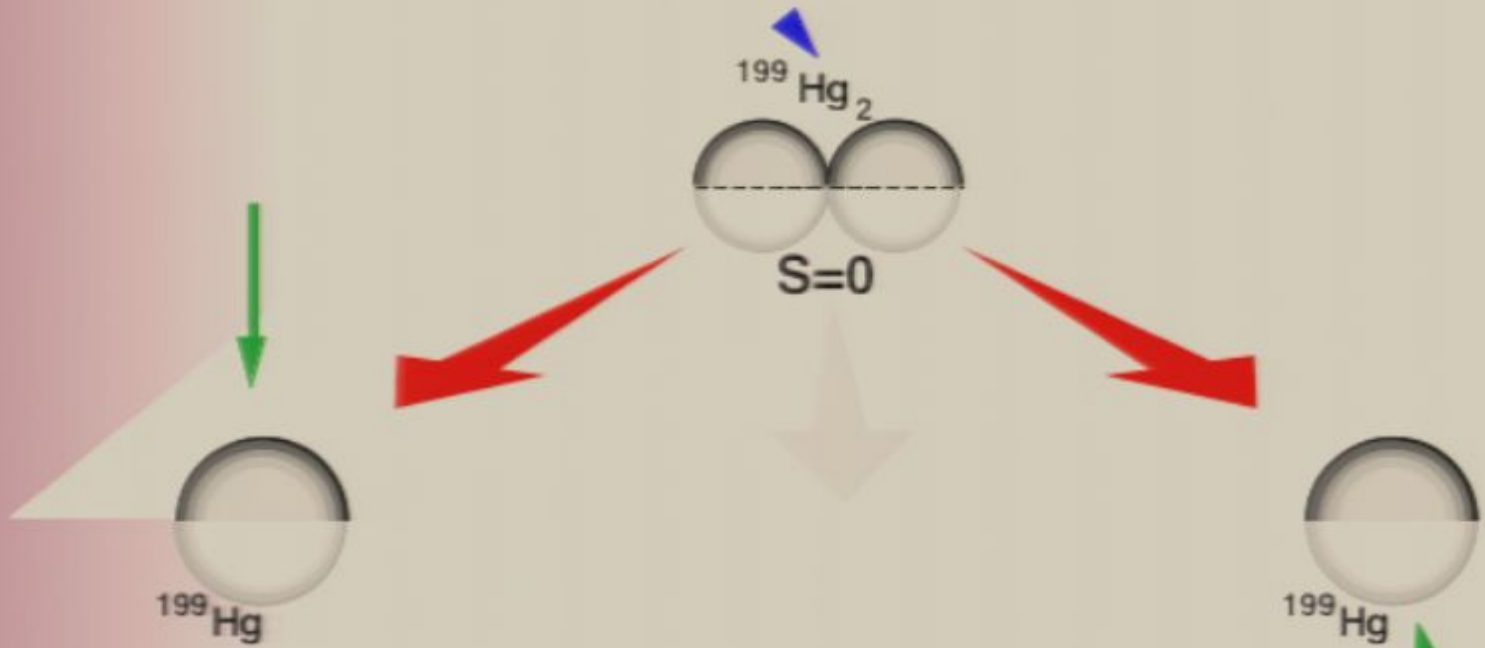
# Bohm's version of EPR



$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\}$$



# An experimental realization of Bohm's classic version of the Einstein-Podolsky-Rosen gedankenexperiment



Measurement of Correlations  
between  
Components of Nuclear Spin

# Features

When testing a fundamental concept, it is vital to do the study over as wide a range of the parameters as possible.

Our experiment with  $^{199}\text{Hg}$  dimers dramatically extends the parameter range over which Bell inequalities are tested:

# Features

When testing a fundamental concept, it is vital to do the study over as wide a range of the parameters as possible.

Our experiment with  $^{199}\text{Hg}$  dimers dramatically extends the parameter range over which Bell inequalities are tested:

## 1) Efficient detectors:

Detection efficiency of  $\approx 100\%$  closes the efficiency loophole.

# Features

When testing a fundamental concept, it is vital to do the study over as wide a range of the parameters as possible.

Our experiment with  $^{199}\text{Hg}$  dimers dramatically extends the parameter range over which Bell inequalities are tested:

1) Efficient detectors:

Detection efficiency of  $\approx 100\%$  closes the efficiency loophole.

2) Einstein locality:

Locality can be strictly enforced.

3) Spin one-half fermions rather than bosons:

A  $^{199}\text{Hg}$  atom is a fermion.

The first test of a Bell inequality with entangled fermions. The particles obey completely different quantum statistics than in all previous bell inequality tests.

3) Spin one-half fermions rather than bosons:  
A  $^{199}\text{Hg}$  atom is a fermion.  
The first test of a Bell inequality with entangled fermions. The particles obey completely different quantum statistics than in all previous bell inequality tests.

4) Massive particles vs. massless photons:  
Nonrelativistic massive particles obey the non-relativistic Schrödinger equation; photons are very different.

3) Spin one-half fermions rather than bosons:  
 A  $^{199}\text{Hg}$  atom is a fermion.  
 The first test of a Bell inequality with entangled fermions. The particles obey completely different quantum statistics than in all previous bell inequality tests.

4) Massive particles vs. massless photons:  
 Nonrelativistic massive particles obey the non-relativistic Schrödinger equation; photons are very different.  
 A test of a Bell inequality with massive particles is in a regime very different from tests with photons.

5) Inside the light cone rather than on it:



5) Inside the light cone rather than on it:

Massive particles must have a velocity less than  $c$  and trace out a world line inside the light cone.

5) Inside the light cone rather than on it:

Massive particles must have a velocity less than  $c$  and trace out a world line inside the light cone.

Photons (velocity  $c$ ) must always be on the light cone.

5) Inside the light cone rather than on it:

Massive particles must have a velocity less than  $c$  and trace out a world line inside the light cone.

Photons (velocity  $c$ ) must always be on the light cone.

Since Einstein locality plays a crucial role in the Bell inequalities, experimental tests well inside the light cone are especially important in comparison to the photon tests done on the light cone.

5) Inside the light cone rather than on it:

Massive particles must have a velocity less than  $c$  and trace out a world line inside the light cone.

Photons (velocity  $c$ ) must always be on the light cone.

Since Einstein locality plays a crucial role in the Bell inequalities, experimental tests well inside the light cone are especially important in comparison to the photon tests done on the light cone.

Since photons travel with the velocity of light in any reference frame, they cannot be strictly localized.

6) Entangled state exists for milliseconds:

In photon experiments, the entangled state typically exists a few nanoseconds before annihilation at the detectors.

6) Entangled state exists for milliseconds:

In photon experiments, the entangled state typically exists a few nanoseconds before annihilation at the detectors. Even in the 1997 experiment of Gisin, et al. (Geneva) it only existed for  $30 \mu\text{s}$ .

6) Entangled state exists for milliseconds:

In photon experiments, the entangled state typically exists a few nanoseconds before annihilation at the detectors.

Even in the 1997 experiment of Gisin, et al. (Geneva) it only existed for  $30 \mu s$ .

The  $^{199}\text{Hg}$  atoms travel slow compared to  $c$ ; the two atom entangled state must continue to exist at large spatial separations for milliseconds.

6) Entangled state exists for milliseconds:

In photon experiments, the entangled state typically exists a few nanoseconds before annihilation at the detectors.

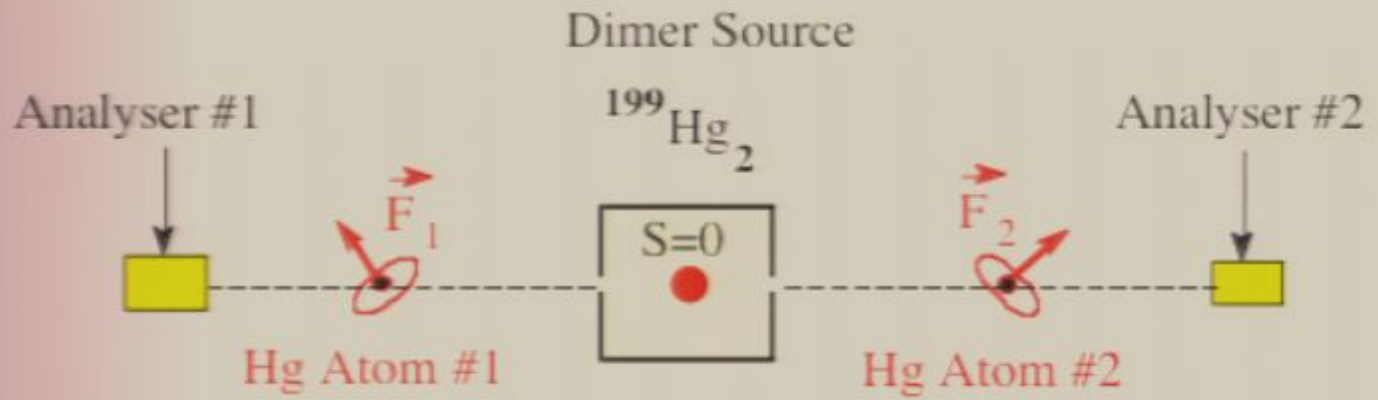
Even in the 1997 experiment of Gisin, et al. (Geneva) it only existed for  $30 \mu s$ .

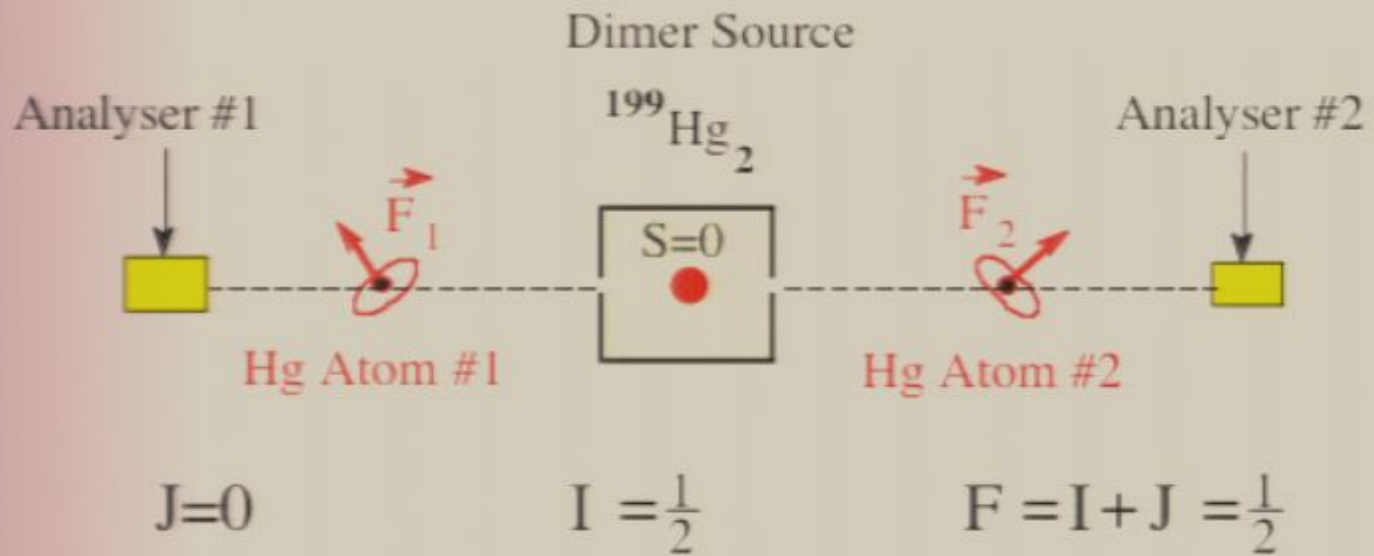
The  $^{199}\text{Hg}$  atoms travel slow compared to  $c$ ; the two atom entangled state must continue to exist at large spatial separations for milliseconds.

7) Spatially separated and independent storage of the two components of the entangled state:

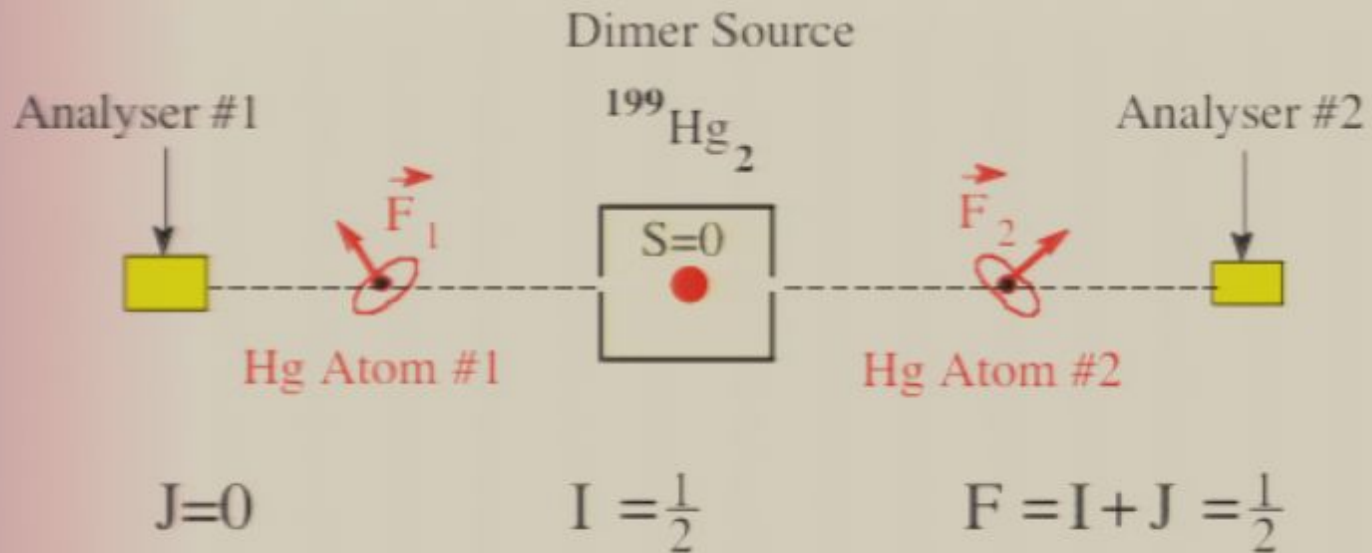
Frozen neon matrices offer the capability to store the two components of the entangled state in separate and movable locations for relatively long periods of time.







$$|\Psi\rangle \equiv \frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right\}$$



$$|\Psi\rangle \equiv \frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right\}$$

Analyzers determine component of  $F$  in a specific direction.

Measure correlations between different components of  $F$ .

## Hg Isotopes (natural abundance)

$^{196}\text{Hg}$	0.15%	I=0
$^{198}\text{Hg}$	10.1%	I=0
$^{199}\text{Hg}$	16.84%	I=1/2
$^{200}\text{Hg}$	23.1%	I=0
$^{201}\text{Hg}$	13.22%	I=3/2
$^{202}\text{Hg}$	29.65%	I=0
$^{204}\text{Hg}$	6.8%	I=0

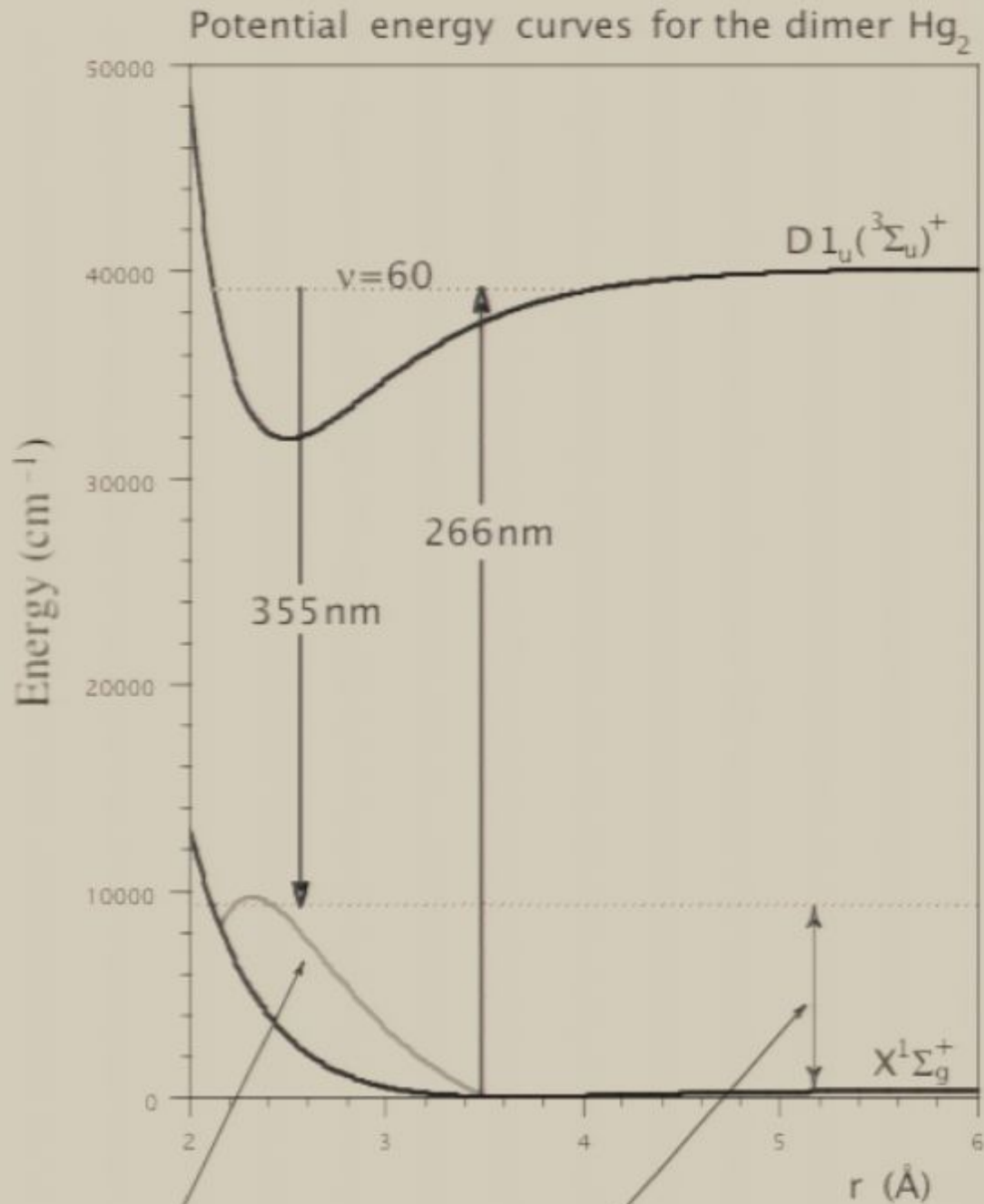
In a mercury dimer source, we have



with 2.84% abundance.

# DIMER

# DISSOCIATION



Mulliken Diff.Pot.

$1.17\text{eV} \Rightarrow v=753\text{m/sec}$

We require  $^{199}\text{Hg}_2$  in a nuclear spin singlet

We require  $^{199}\text{Hg}_2$  in a nuclear spin singlet

$^{199}\text{Hg}$  has total spin  $1/2 \Rightarrow$  fermion

Thus, the Pauli Principle  $\Rightarrow P\Psi = -\Psi$

where  $P$  is the particle exchange operator



We require  $^{199}\text{Hg}_2$  in a nuclear spin singlet

$^{199}\text{Hg}$  has total spin  $1/2 \Rightarrow$  fermion

Thus, the Pauli Principle  $\Rightarrow P\Psi = -\Psi$

where  $P$  is the particle exchange operator

$$P\Psi = (\sigma_{\text{el}}^i \Psi^{\text{el}}) \Psi^{\text{vib}} (C_2 \Psi^{\text{rot}}) (p_{\text{nuc}} \Psi^{\text{nuc}})$$

The  $^{199}\text{Hg}_2$  molecular ground state is  $X^1\Sigma_{\text{g}}^+$

We require  $^{199}\text{Hg}_2$  in a nuclear spin singlet

$^{199}\text{Hg}$  has total spin  $1/2 \Rightarrow$  fermion

Thus, the Pauli Principle  $\Rightarrow P\Psi = -\Psi$

where  $P$  is the particle exchange operator

$$P\Psi = (\sigma_{\text{el}} i_{\text{el}} \Psi^{\text{el}}) \Psi^{\text{vib}} (C_2 \Psi^{\text{rot}}) (p_{\text{nuc}} \Psi^{\text{nuc}})$$

The  $^{199}\text{Hg}_2$  molecular ground state is  $X^1\Sigma_g^+$

$g \Rightarrow$  reflection symmetry,  $\sigma_{\text{el}}$ , of the  $e^-$  wave function

$+ \Rightarrow$  inversion symmetry,  $i_{\text{el}}$ , of the  $e^-$  wave function

$$\sigma_{\text{el}} \Psi^{\text{el}} = +\Psi^{\text{el}}; \quad i_{\text{el}} \Psi^{\text{el}} = +\Psi^{\text{el}}; \quad C_2 \Psi^{\text{rot}} = (-1)^J \Psi^{\text{rot}};$$

We require  $^{199}\text{Hg}_2$  in a nuclear spin singlet

$^{199}\text{Hg}$  has total spin  $1/2 \Rightarrow$  fermion

Thus, the Pauli Principle  $\Rightarrow P\Psi = -\Psi$

where  $P$  is the particle exchange operator

$$P\Psi = (\sigma_{\text{el}} i_{\text{el}} \Psi^{\text{el}}) \Psi^{\text{vib}} (C_2 \Psi^{\text{rot}}) (p_{\text{nuc}} \Psi^{\text{nuc}})$$

The  $^{199}\text{Hg}_2$  molecular ground state is  $X^1\Sigma_g^+$

$g \Rightarrow$  reflection symmetry,  $\sigma_{\text{el}}$ , of the  $e^-$  wave function

$+ \Rightarrow$  inversion symmetry,  $i_{\text{el}}$ , of the  $e^-$  wave function

$$\sigma_{\text{el}} \Psi^{\text{el}} = +\Psi^{\text{el}}; \quad i_{\text{el}} \Psi^{\text{el}} = +\Psi^{\text{el}}; \quad C_2 \Psi^{\text{rot}} = (-1)^J \Psi^{\text{rot}};$$

$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

Nuclear spin singlet:

$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

Nuclear spin singlet:

$$\Psi^{\text{nuc}} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\} \Rightarrow p_{\text{nuc}} = -1$$

$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

Nuclear spin singlet:

$$\Psi^{\text{nuc}} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\} \Rightarrow p_{\text{nuc}} = -1$$

Thus,  $J$  must be an even integer.

In the nuclear spin singlet state, the rotational states must have even  $J$ :

$$J=0, 2, 4, \dots$$

$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

Nuclear spin singlet:

$$\Psi^{\text{nuc}} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\} \Rightarrow p_{\text{nuc}} = -1$$

Thus,  $J$  must be an even integer.

In the nuclear spin singlet state, the rotational states must have even  $J$ :

$$J=0, 2, 4, \dots$$


---

$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

### Nuclear spin singlet:

$$\Psi^{\text{nuc}} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\} \Rightarrow p_{\text{nuc}} = -1$$

Thus,  $J$  must be an even integer.

In the nuclear spin singlet state, the rotational states must have even  $J$ :

$$J=0, 2, 4, \dots$$

---

### Nuclear spin triplet:

$$\Psi^{\text{nuc}} = \begin{cases} |\uparrow\rangle_1 |\uparrow\rangle_2 \\ \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2 \} \\ |\downarrow\rangle_1 |\downarrow\rangle_2 \end{cases} \Rightarrow p_{\text{nuc}} = +1$$



$$P\Psi = (-1)^J p_{\text{nuc}} \Psi$$

### Nuclear spin singlet:

$$\Psi^{\text{nuc}} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\} \Rightarrow p_{\text{nuc}} = -1$$

Thus,  $J$  must be an even integer.

In the nuclear spin singlet state, the rotational states must have even  $J$ :

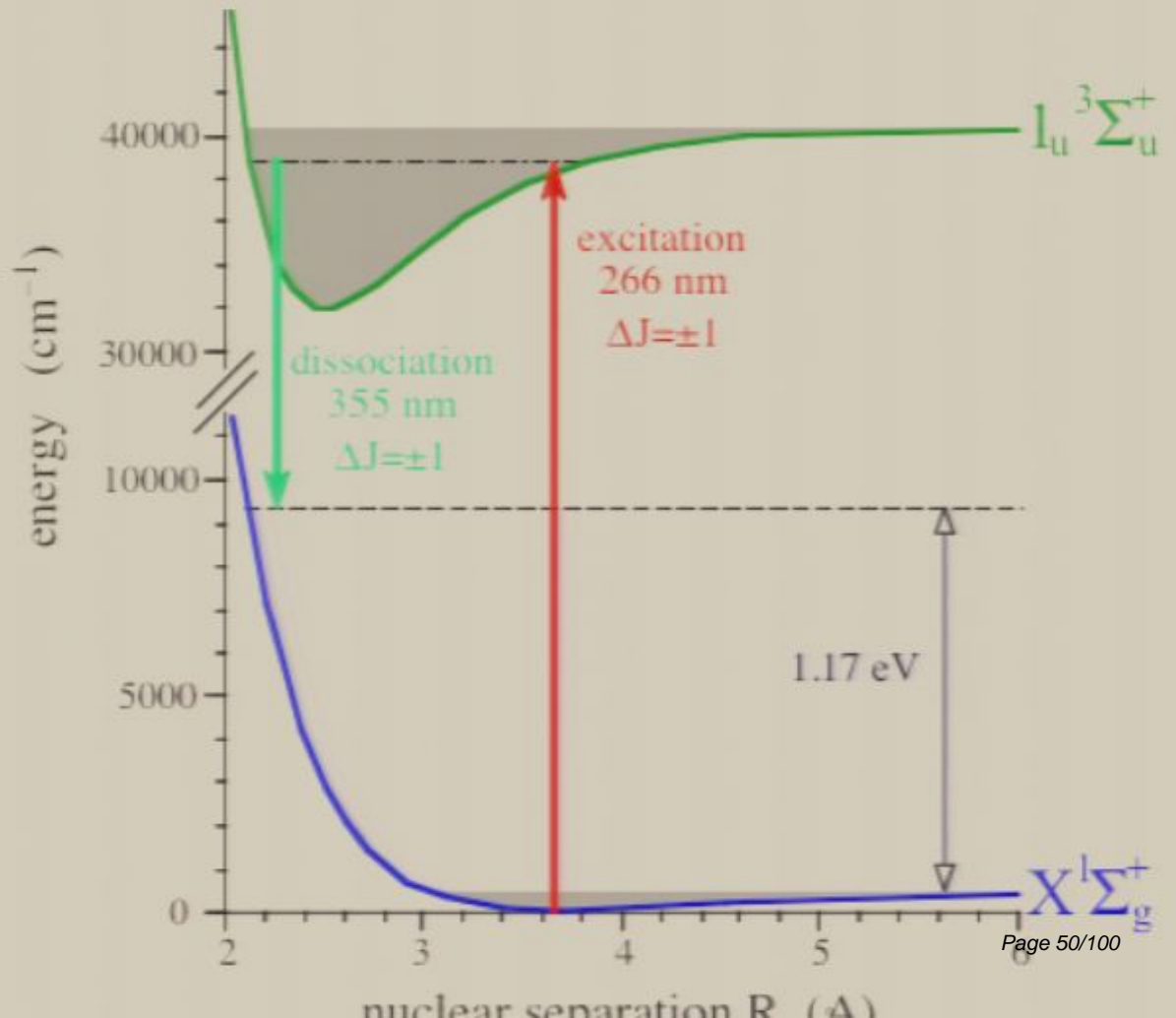
$$J=0, 2, 4, \dots$$

### Nuclear spin triplet:

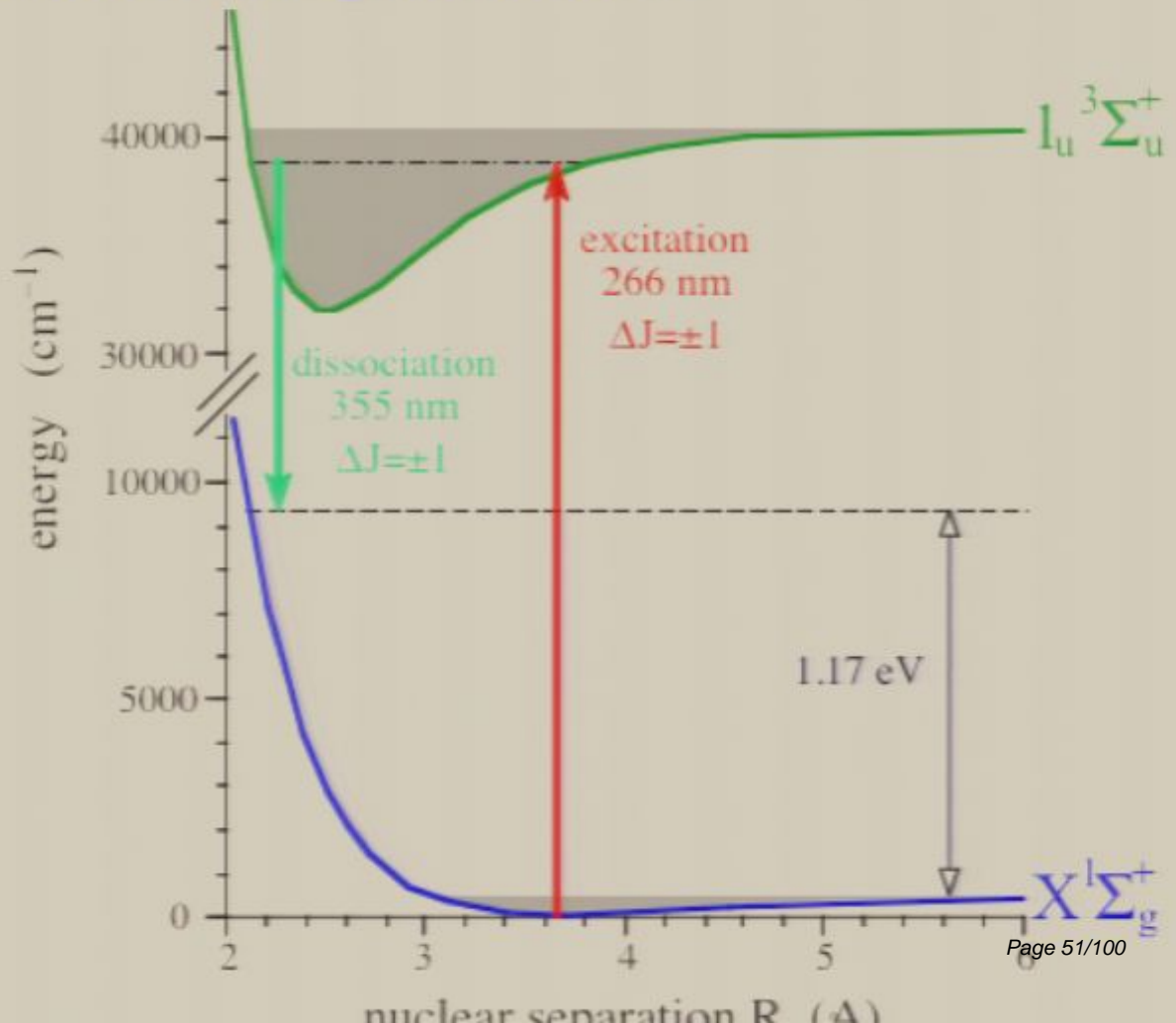
$$\Psi^{\text{nuc}} = \begin{cases} |\uparrow\rangle_1 |\uparrow\rangle_2 \\ \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2 \} \\ |\downarrow\rangle_1 |\downarrow\rangle_2 \end{cases} \Rightarrow p_{\text{nuc}} = +1$$

$$J=1, 3, 5$$

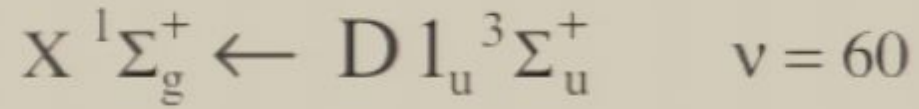
Since  $^{199}\text{Hg}_2$  is a homonuclear molecule, each leg of the Raman transition can only have  $\Delta J = \pm 1$ ;  $\Delta J$  cannot be 0.



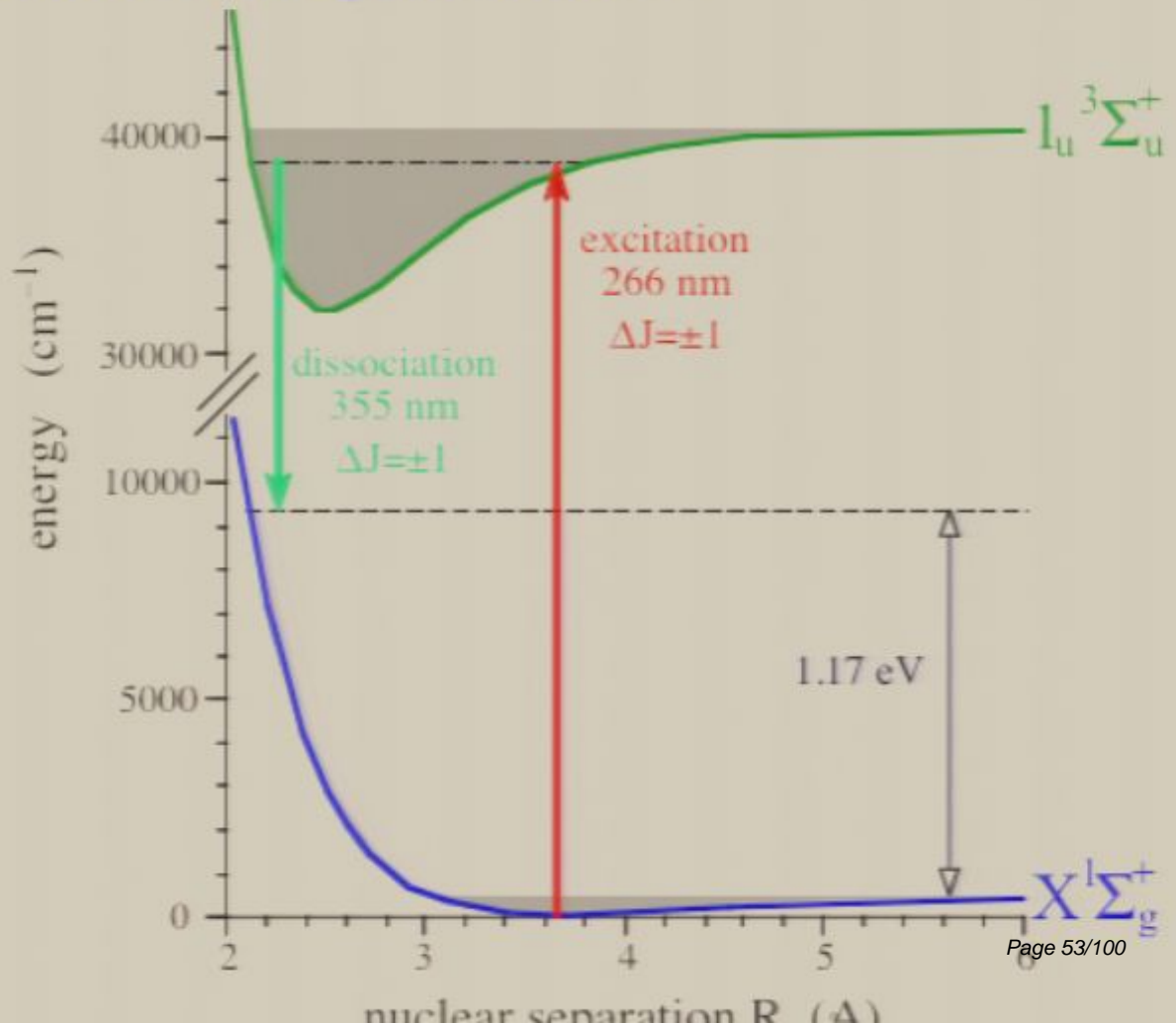
Since  $^{199}\text{Hg}_2$  is a homonuclear molecule, each leg of the Raman transition can only have  $\Delta J = \pm 1$ ;  $\Delta J$  cannot be 0. For the overall Raman transition,  $\Delta J = 0, \pm 2$ ; thus if the molecule starts in an even  $J$ , it ends up in an even  $J$  dissociating state in the ground state.



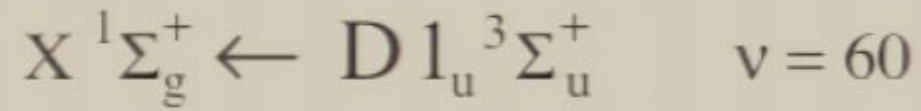
## Simulated Dimer Spectra



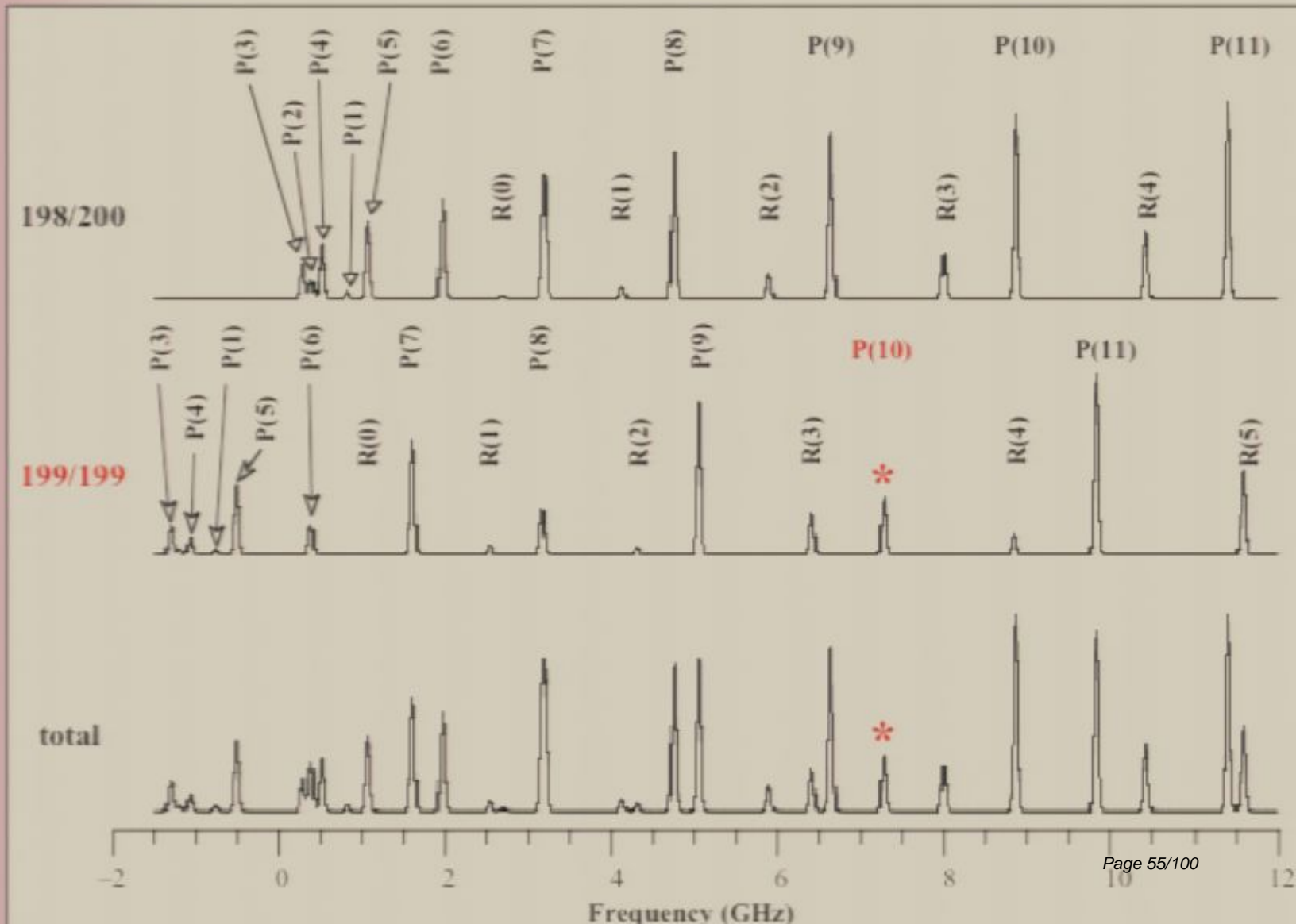
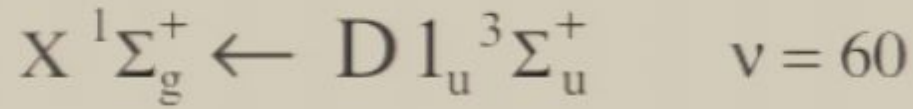
Since  $^{199}\text{Hg}_2$  is a homonuclear molecule, each leg of the Raman transition can only have  $\Delta J = \pm 1$ ;  $\Delta J$  cannot be 0. For the overall Raman transition,  $\Delta J = 0, \pm 2$ ; thus if the molecule starts in an even  $J$ , it ends up in an even  $J$  dissociating state in the ground state.



## Simulated Dimer Spectra



# Simulated Dimer Spectra



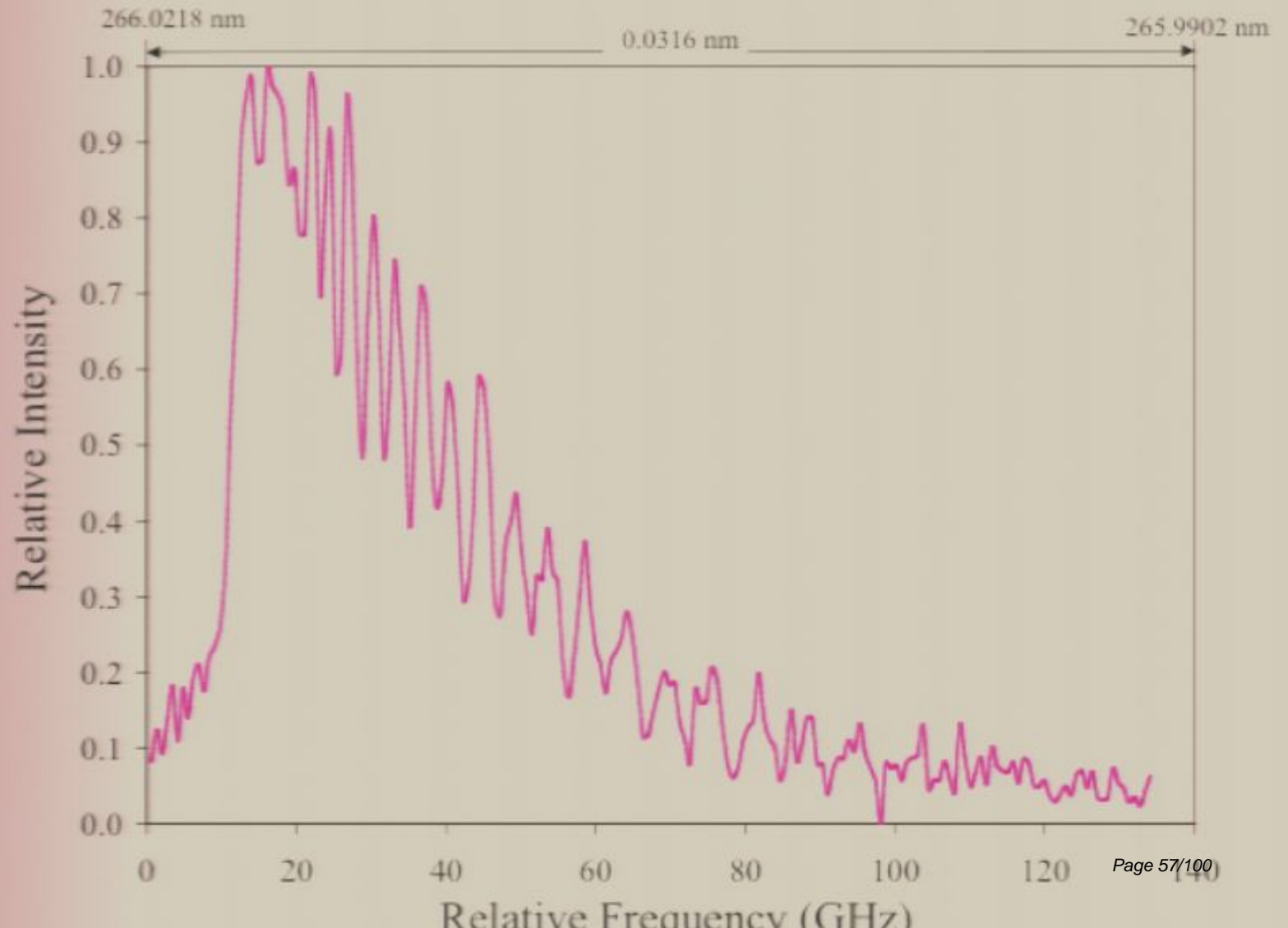
# Preliminary Dimer Spectra Data

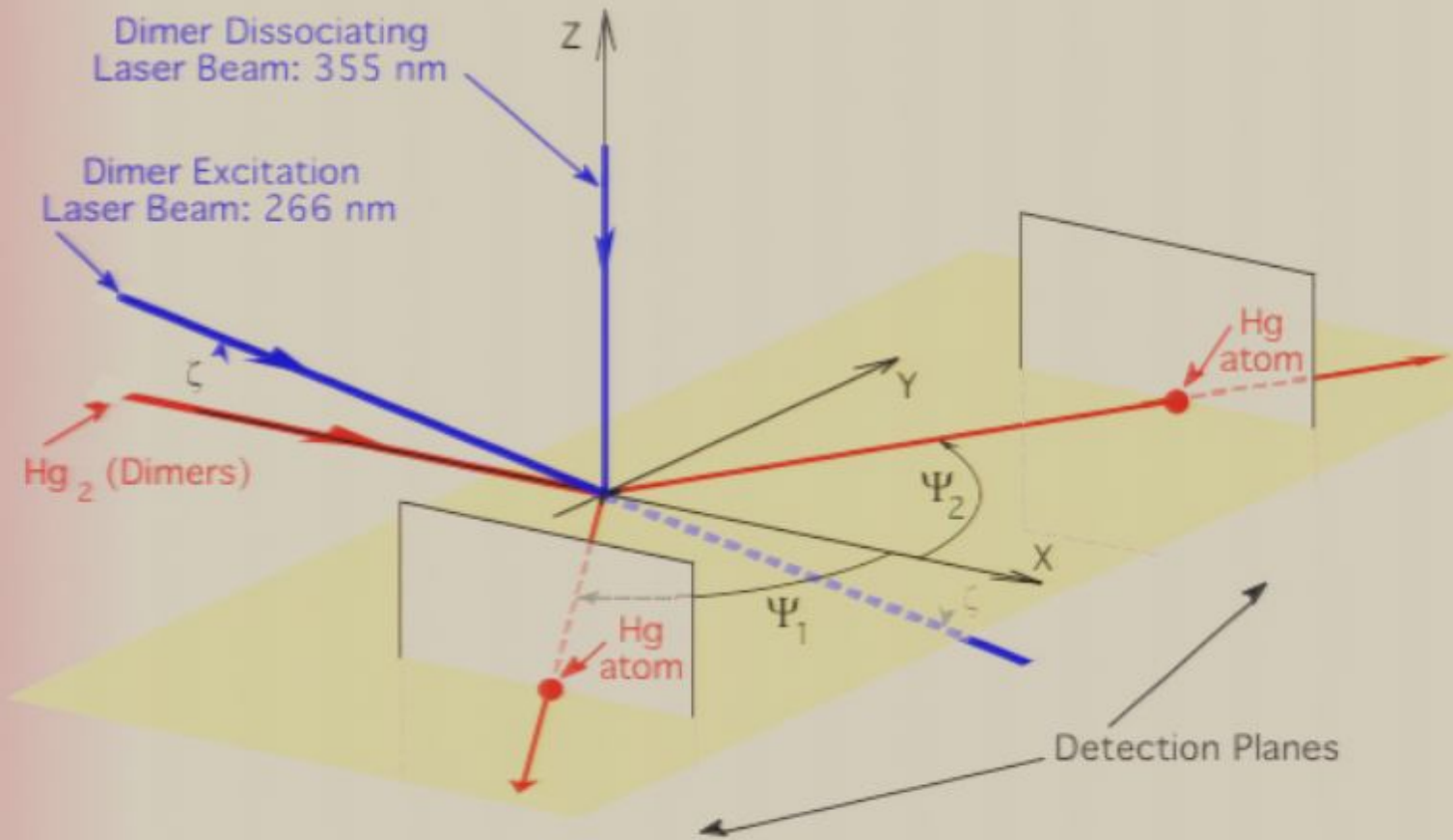
Mass 398 isotopomer  $X \ ^1\Sigma_g^+ \leftarrow D \ 1_u \ ^3\Sigma_u^+ \quad v = 60$



# Preliminary Dimer Spectra Data

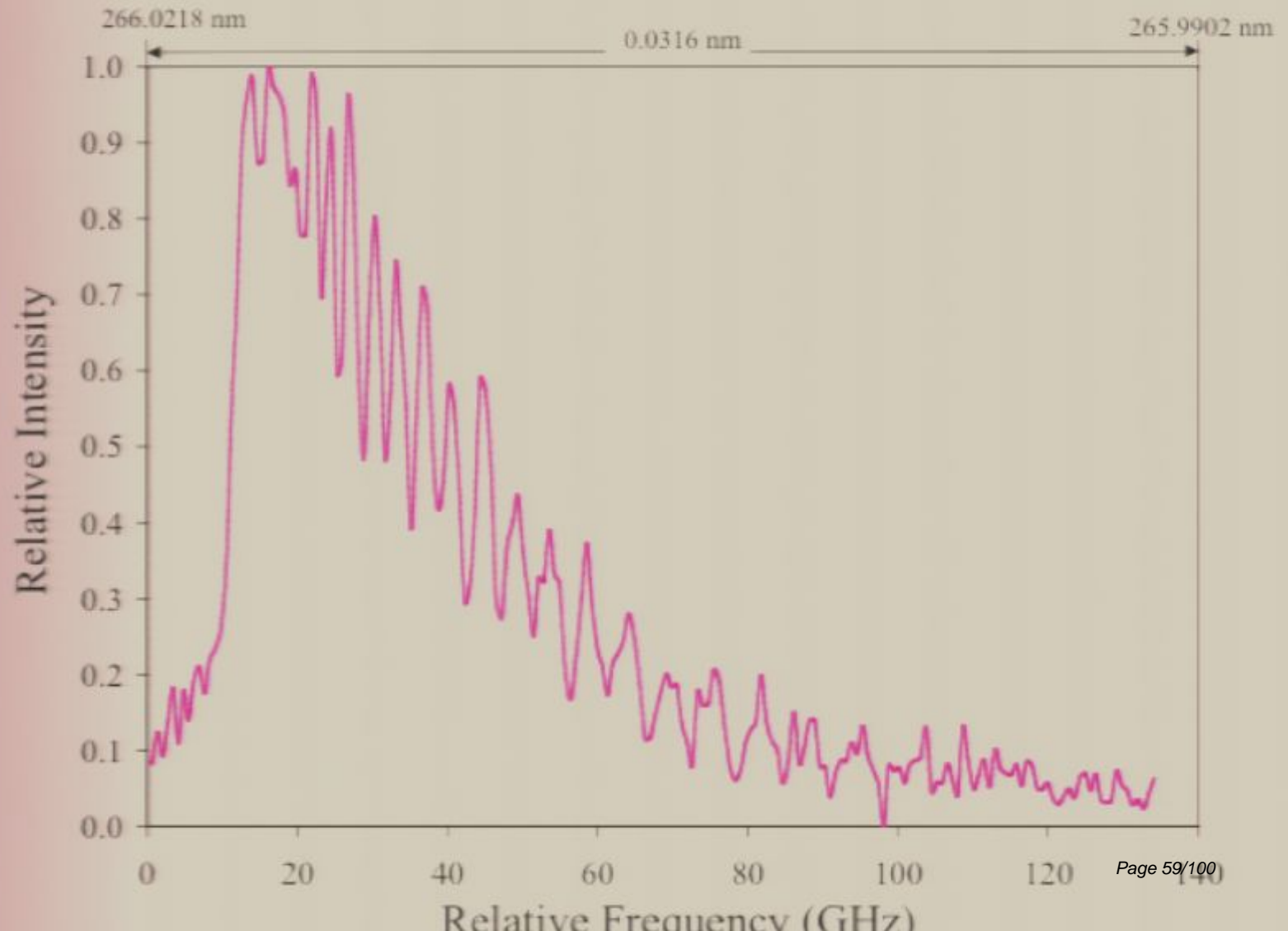
Mass 398 isotopomer  $X^1\Sigma_g^+ \leftarrow D1_u^3\Sigma_u^+ \quad v = 60$





# Preliminary Dimer Spectra Data

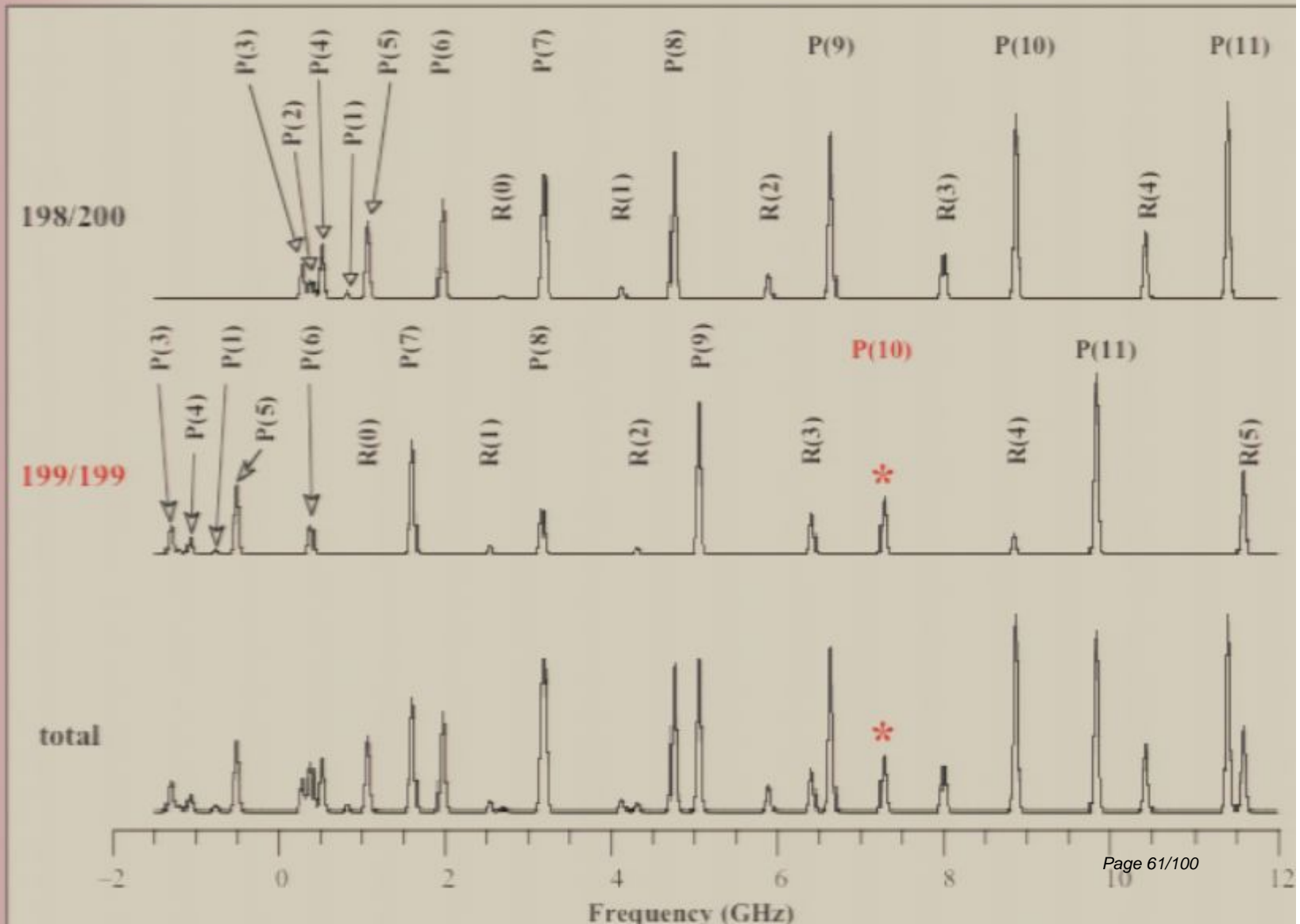
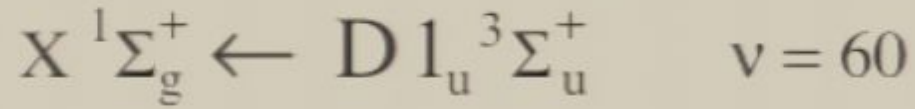
Mass 398 isotopomer  $X^1\Sigma_g^+ \leftarrow D1_u^3\Sigma_u^+ \quad v = 60$



# Preliminary Dimer Spectra Data

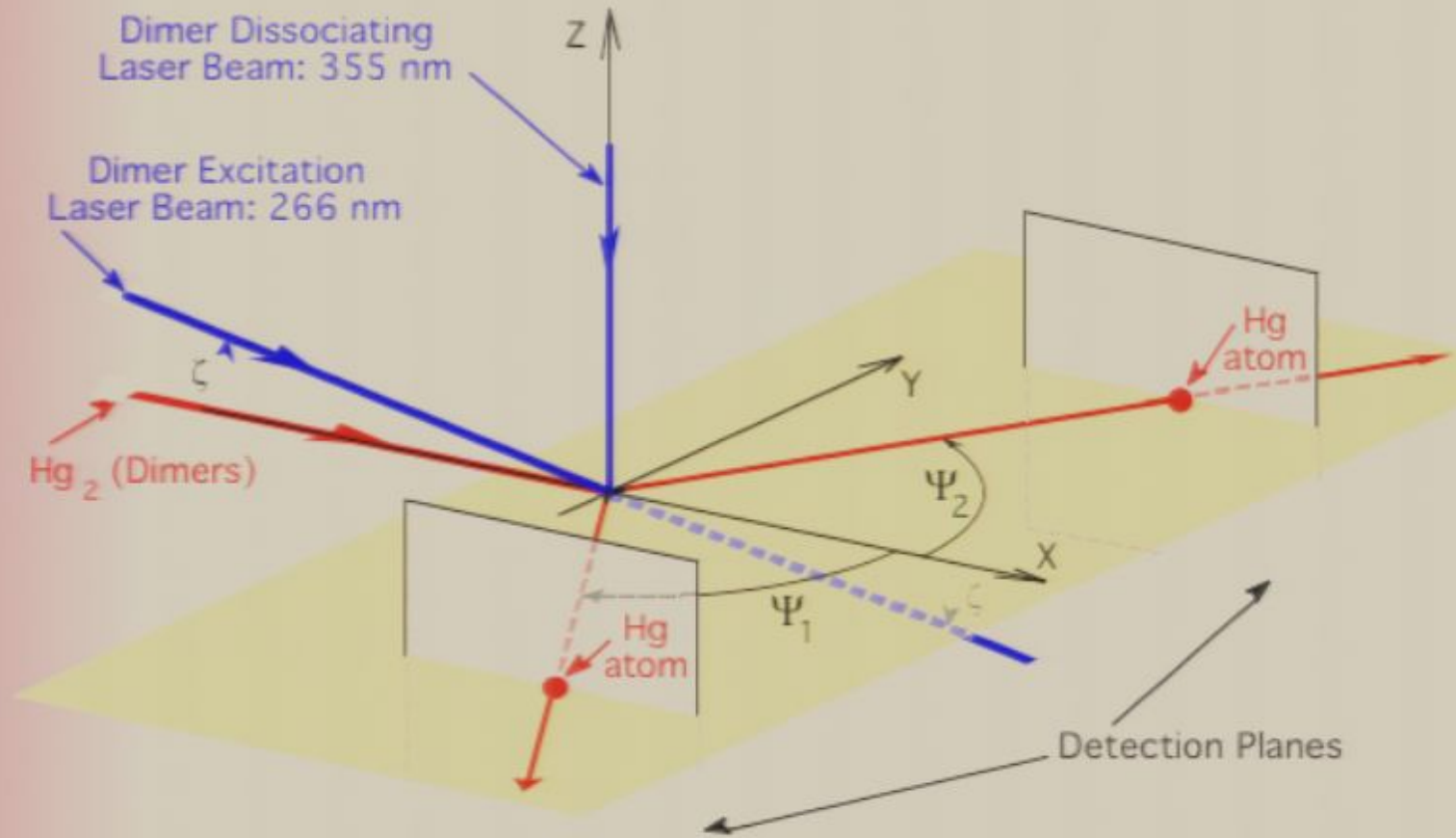
Mass 398 isotopomer  $X \ ^1\Sigma_g^+ \leftarrow D \ 1_u \ ^3\Sigma_u^+ \quad v = 60$

# Simulated Dimer Spectra



# Preliminary Dimer Spectra Data

Mass 398 isotopomer  $X \ ^1\Sigma_g^+ \leftarrow D \ 1_u \ ^3\Sigma_u^+ \quad v = 60$



# Dissociation Lasers



# Dissociation Lasers

## 263 – 269 nm System

### Alexandrite Laser

150 ns FWHM pulses

4 MHz linewidth

40 mJ fundamental

Developed high voltage ramp/Q-switch combination to cancel chirp. Narrowest linewidth of existing pulsed lasers?

# Dissociation Lasers

## 263 – 269 nm System

### Alexandrite Laser

150 ns FWHM pulses

4 MHz linewidth

40 mJ fundamental

Developed high voltage ramp/Q-switch combination to cancel chirp. Narrowest linewidth of existing pulsed lasers?

## 355 nm System

### Excimer pumped dye laser

15 ns FWHM pulses

6 GHz linewidth

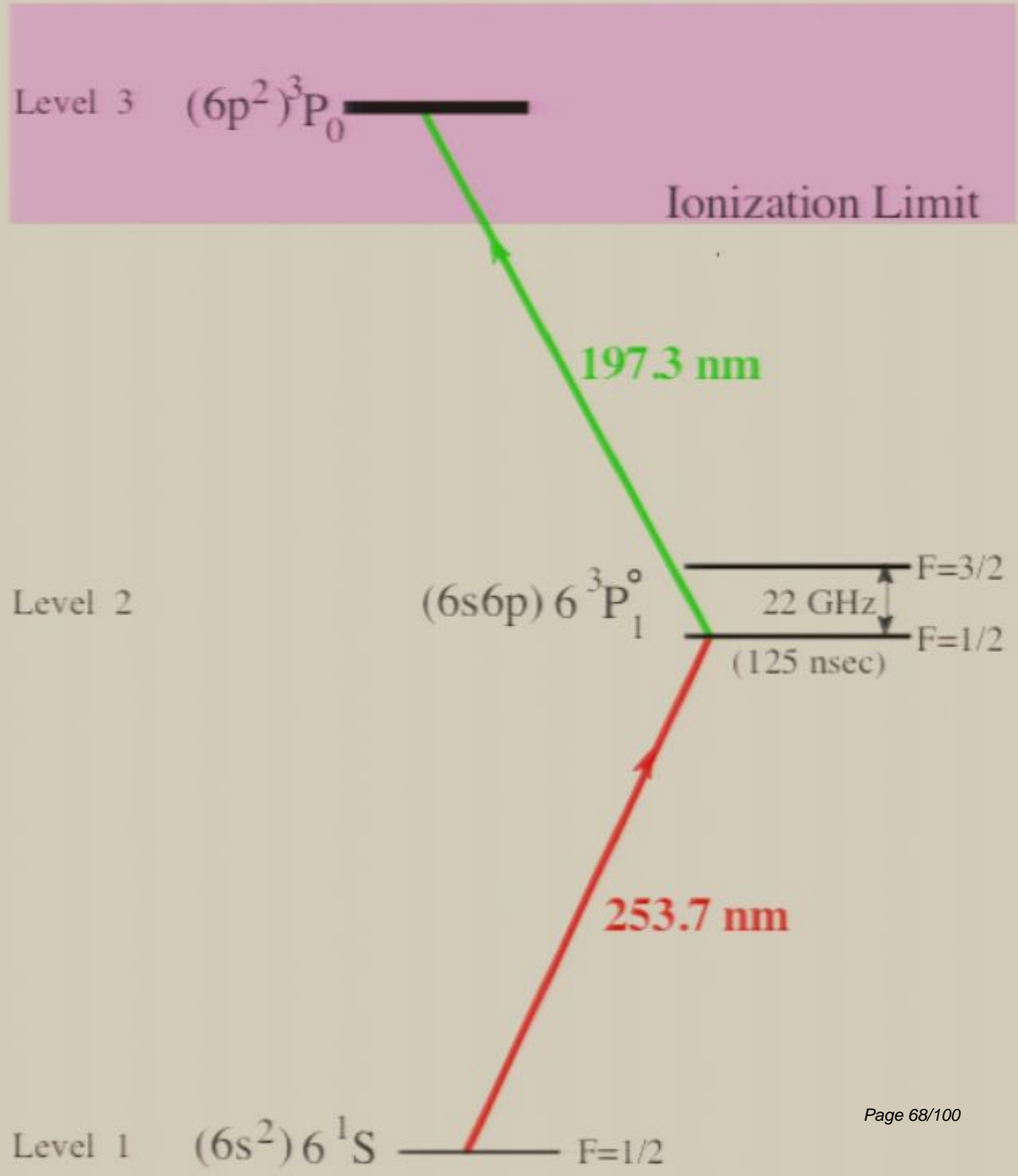
4 mJ

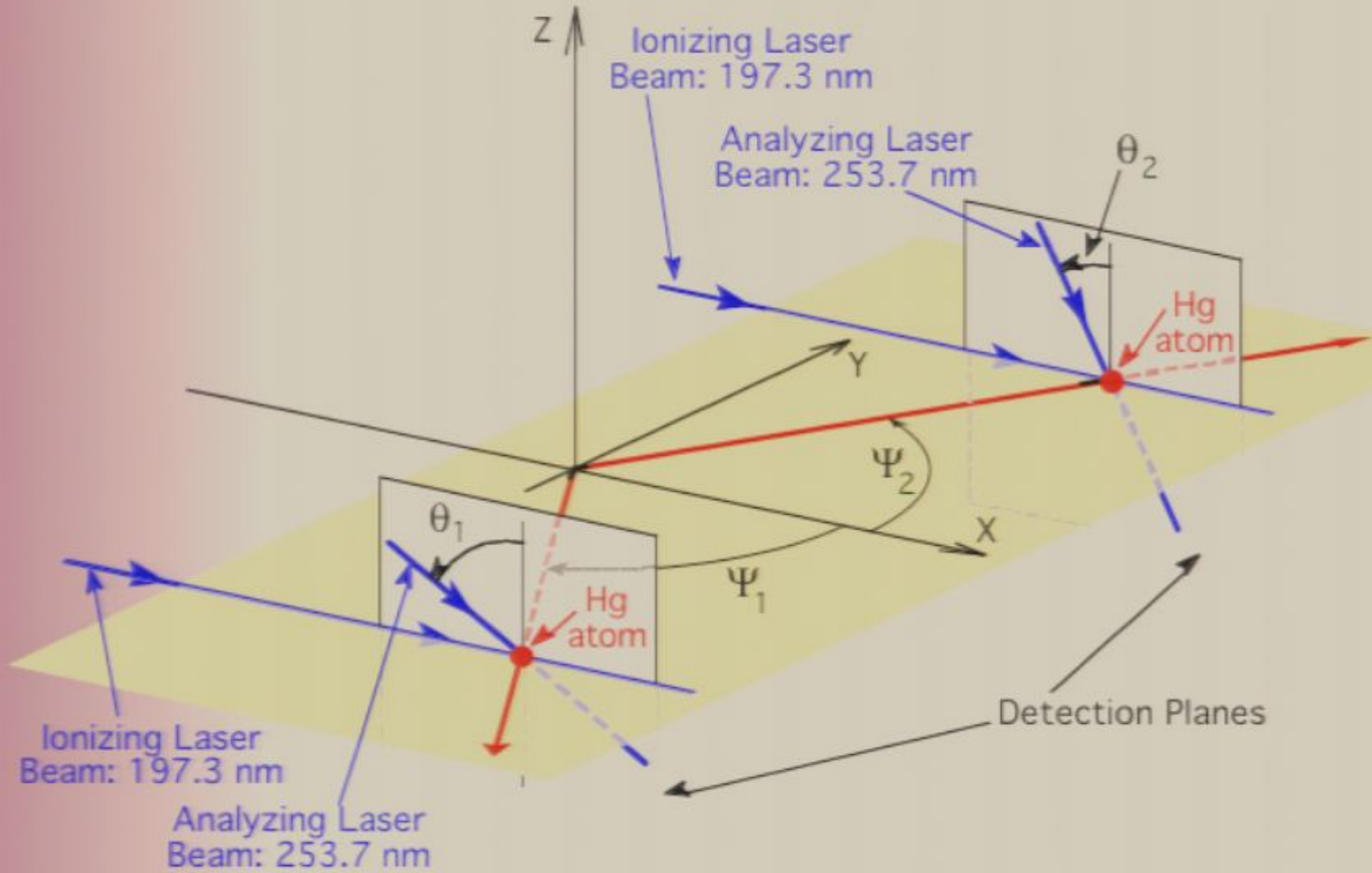
# MERCURY ATOM

# EXCITATION

&

# IONIZATION





## Auto-ionizing transition:

Linewidth  $(6p^2)^3P_0$ :

$$\Gamma = 9 \text{ cm}^{-1} = 270 \text{ GHz}$$

Oscillator Strength (calculated):

$$f = 0.362$$

Absorption Cross-section:

$$\sigma = 2.3 \times 10^{-14} \text{ cm}^2$$

Radiative lifetime:

$$\tau_r = 0.5 \text{ nanosec}$$

Non-radiative lifetime:

$$\tau_i = 3.7 \text{ picosec}$$

Pulse energy to saturate transition:

$$E \approx 100 \mu\text{J}$$

Level 3  $(6p^2)^3P_0$

Ionization Limit

197.3 nm

Level 2

$(6s6p) 6^3P_1^o$   $F=3/2$   $F=1/2$   
 22 GHz  
 (125 nsec)

253.7 nm

Level 1  $(6s^2) 6^1S$   $F=1/2$

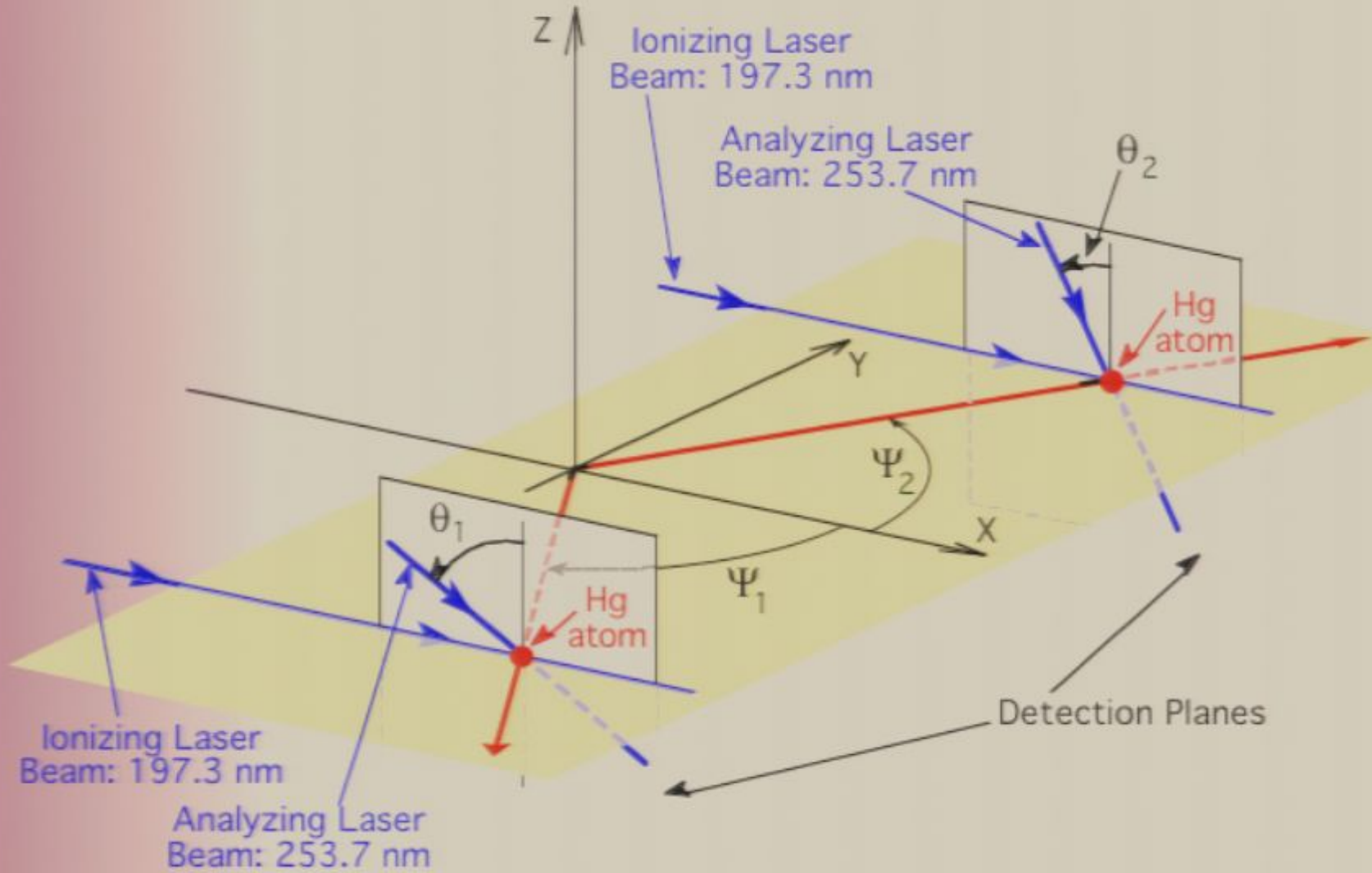
# MERCURY ATOM

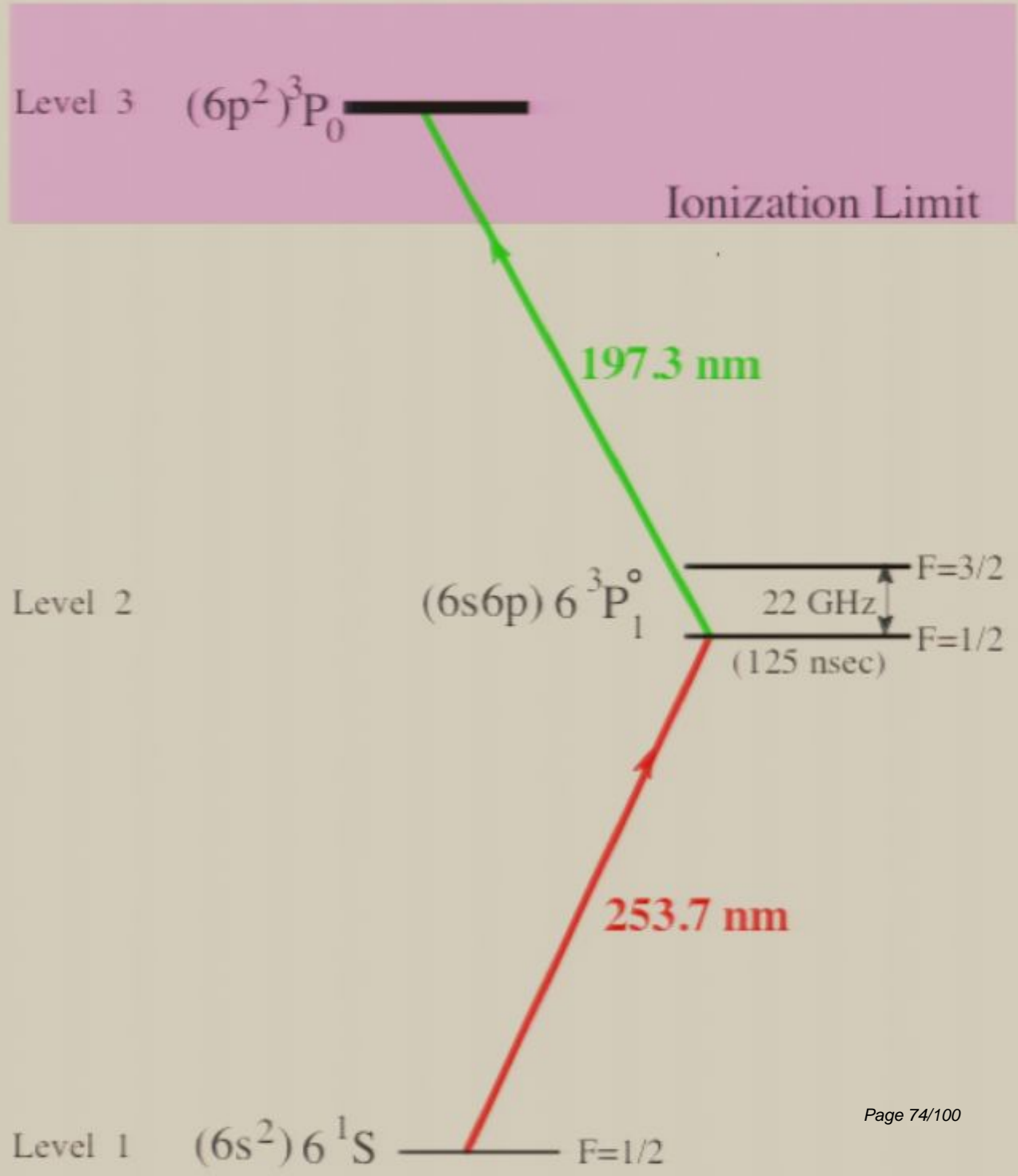
# EXCITATION

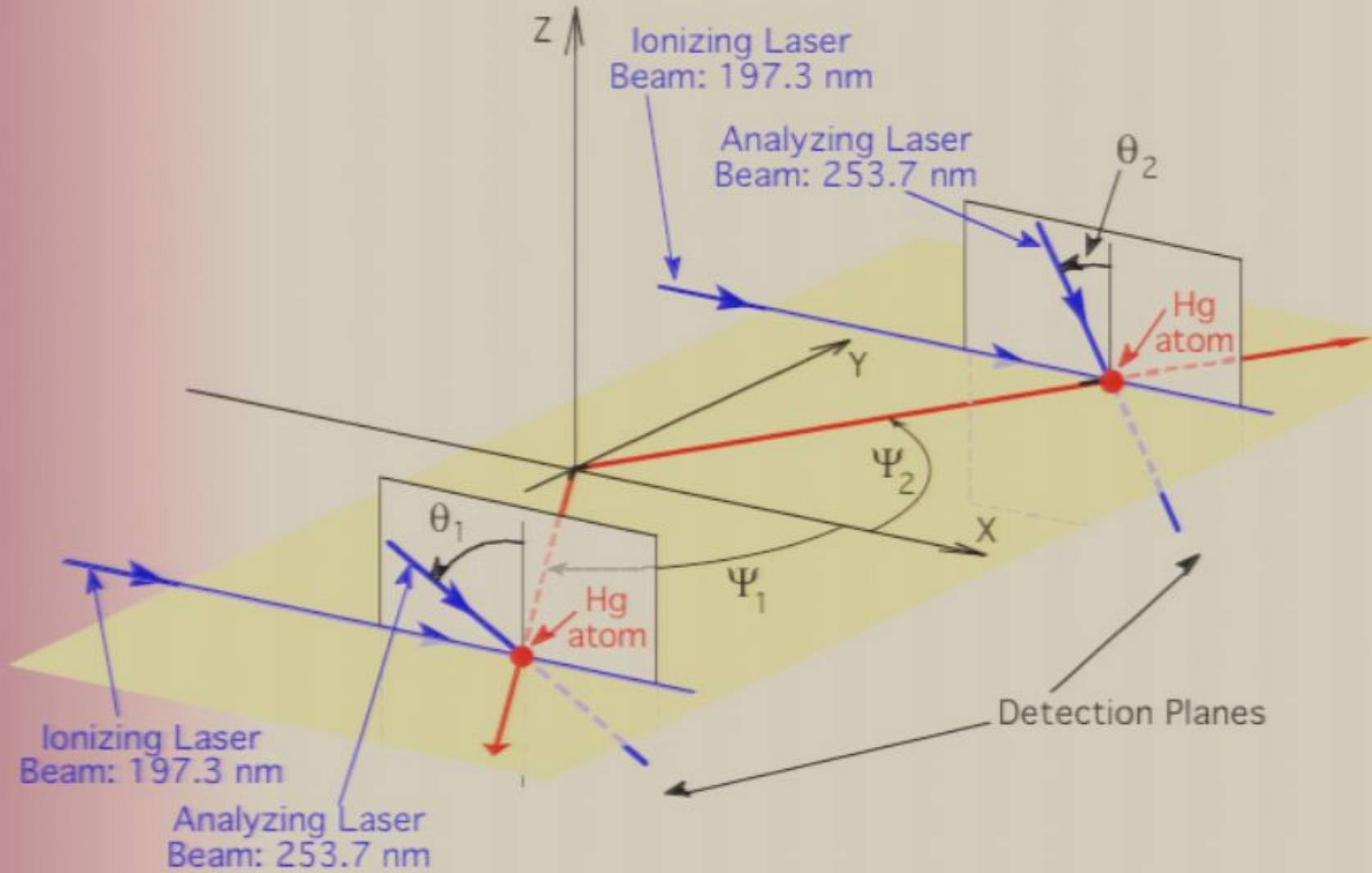
&

# IONIZATION









## Auto-ionizing transition:

Linewidth  $(6p^2)^3P_0$ :

$$\Gamma = 9 \text{ cm}^{-1} = 270 \text{ GHz}$$

Oscillator Strength (calculated):

$$f = 0.362$$

Absorption Cross-section:

$$\sigma = 2.3 \times 10^{-14} \text{ cm}^2$$

Radiative lifetime:

$$\tau_r = 0.5 \text{ nanosec}$$

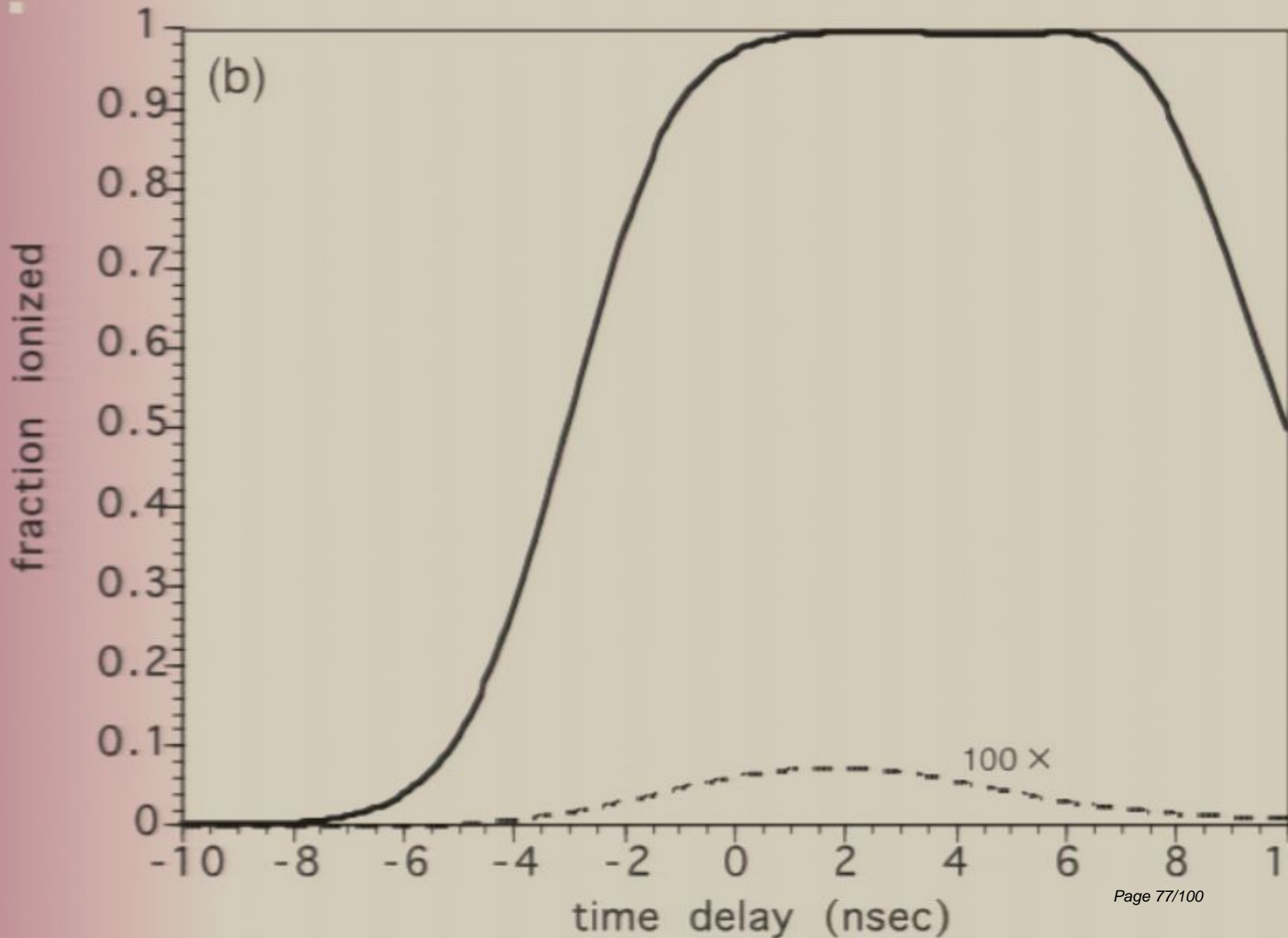
Non-radiative lifetime:

$$\tau_i = 3.7 \text{ picosec}$$

Pulse energy to saturate transition:

$$E \approx 100 \mu\text{J}$$

# Ionization Fraction



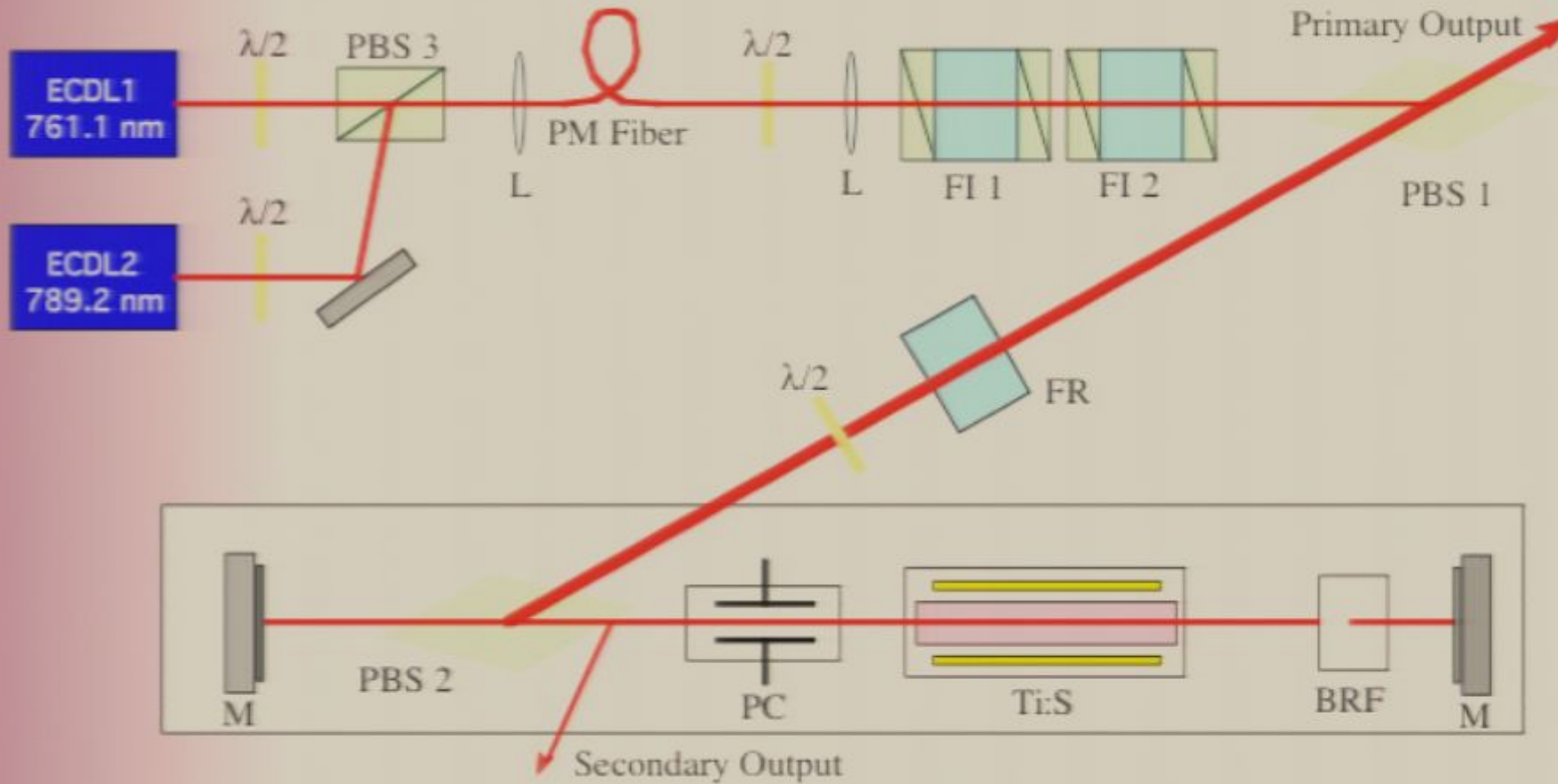
## Excitation/Ionization Lasers

**253.7 nm:** 3rd harmonic of 761.1 nm;

**197.3 nm:** 4th harmonic of 789.2 nm

761.1 nm and 789.2 nm are produced simultaneously by a flashlamp pumped Ti:Sapphire laser.

# Dual frequency Ti:Sapphire Laser



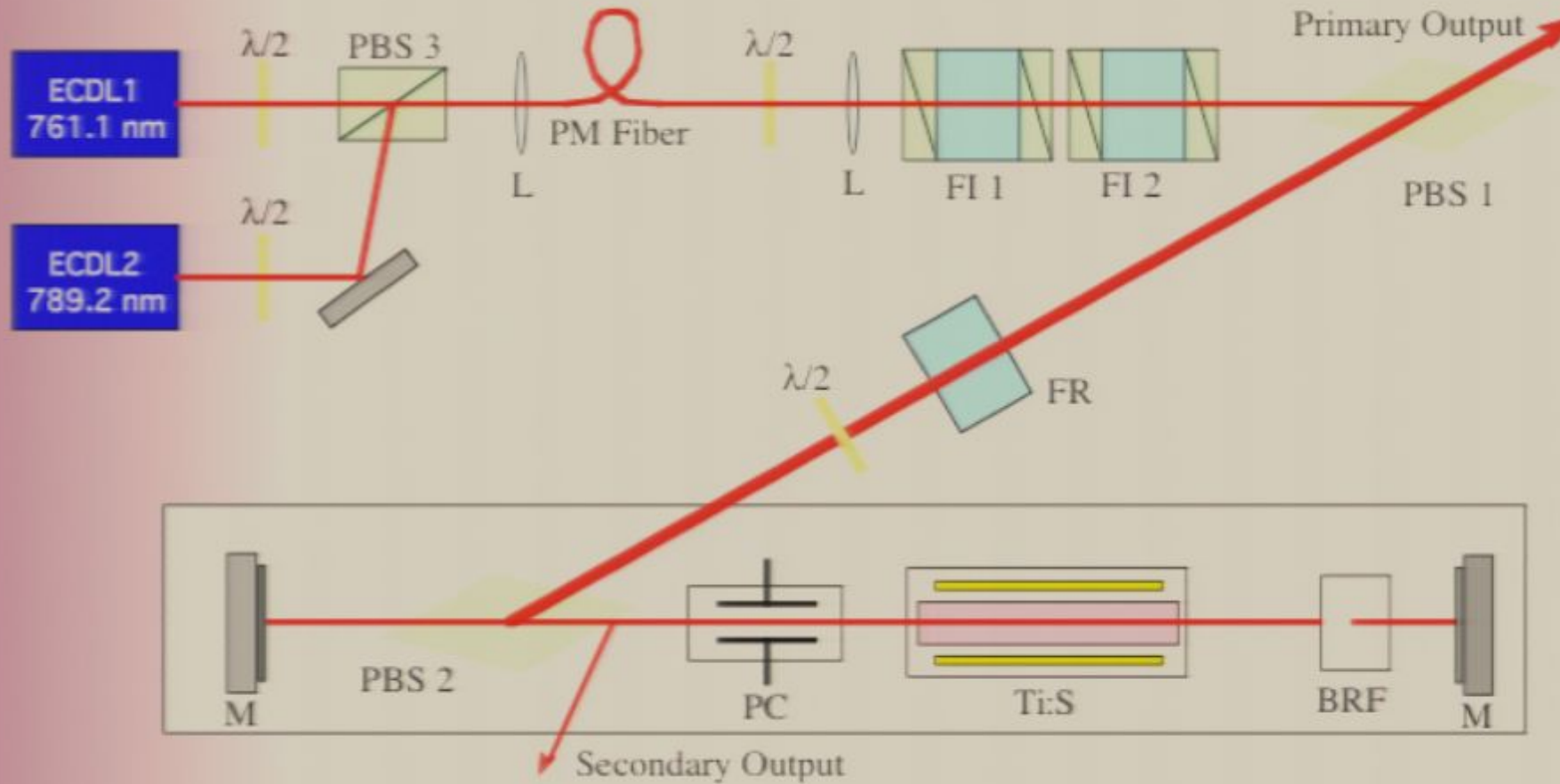
# MERCURY

# ATOM

# DETECTION



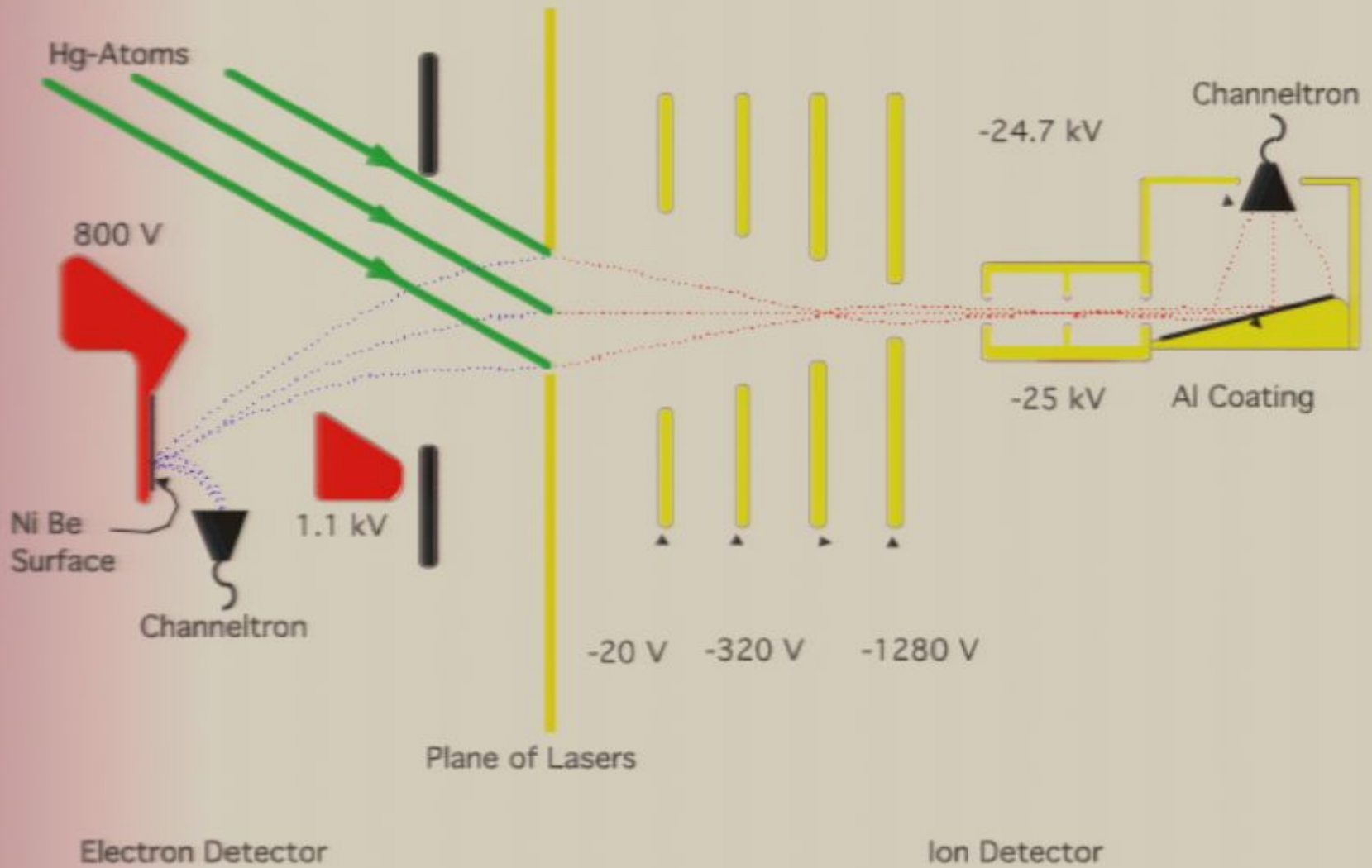
# Dual frequency Ti:Sapphire Laser



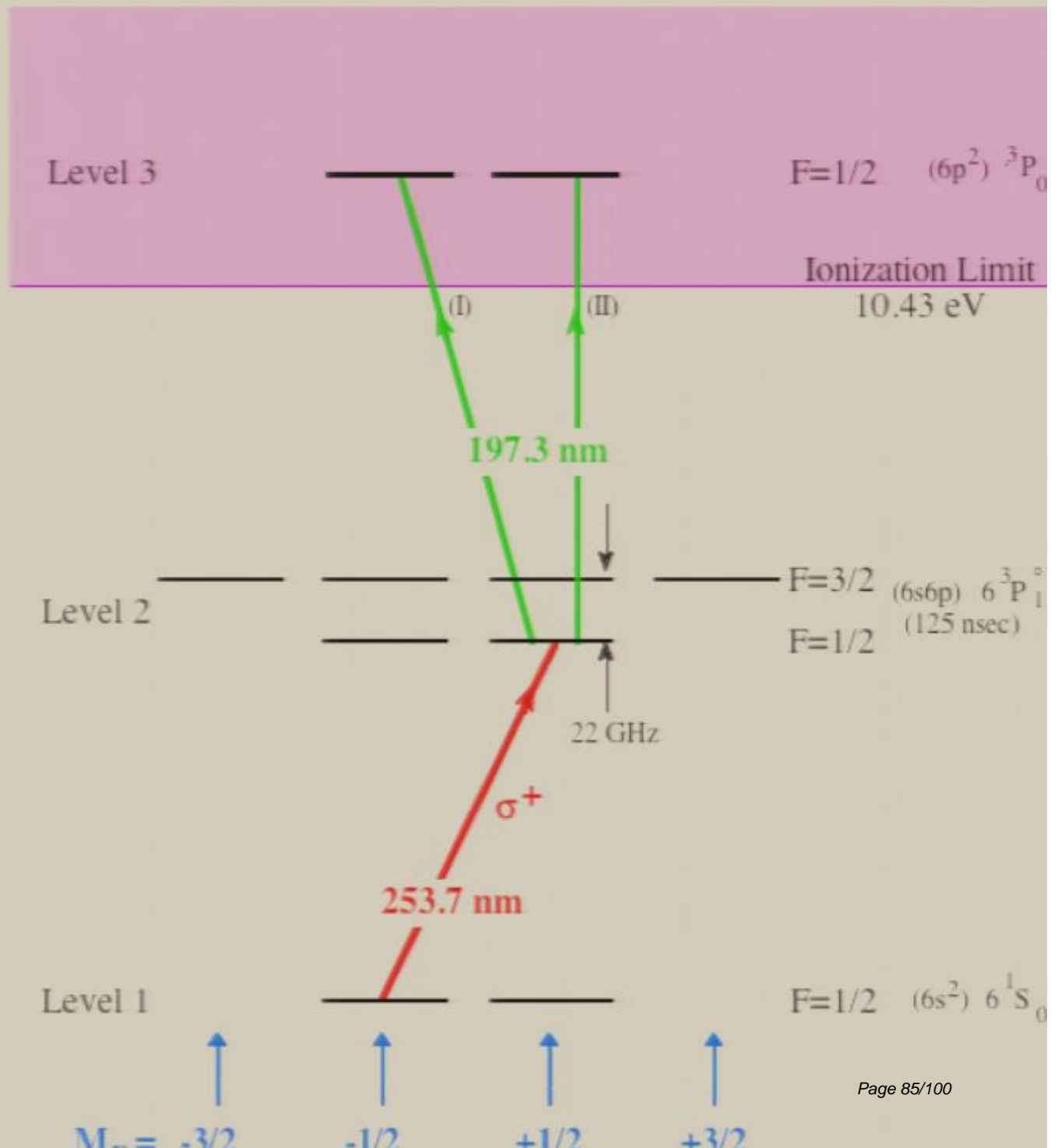
# MERCURY

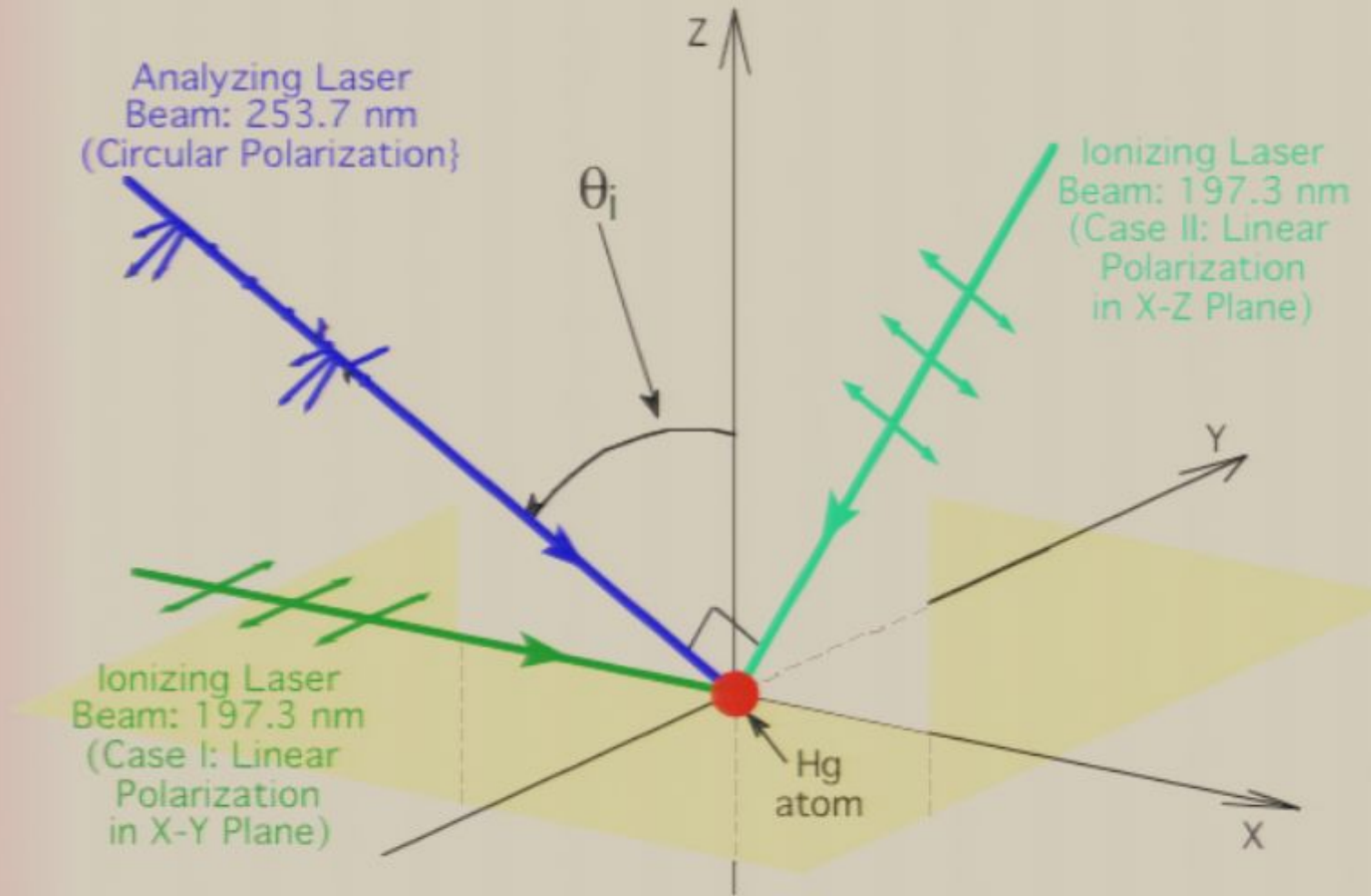
# ATOM

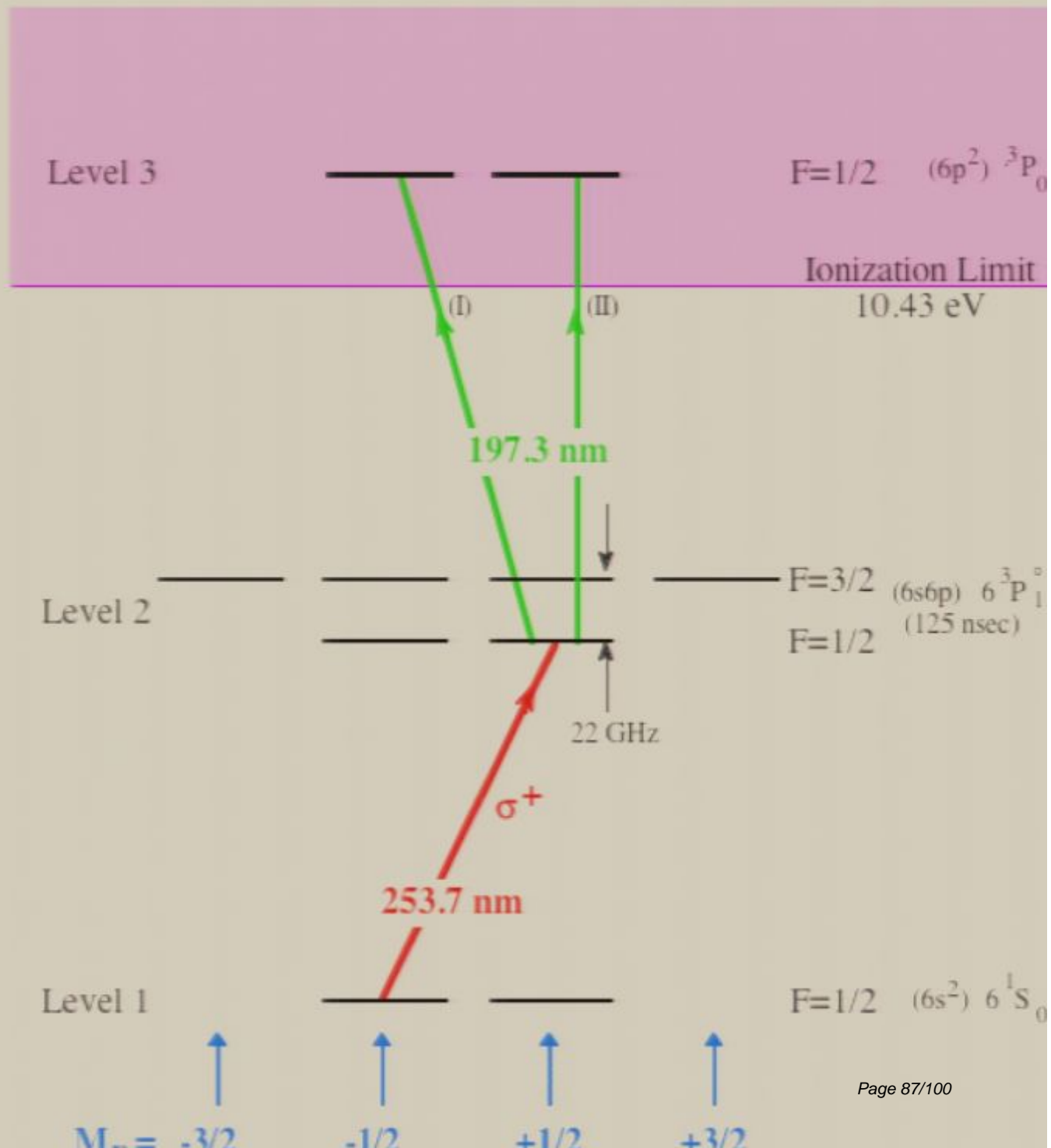
# DETECTION

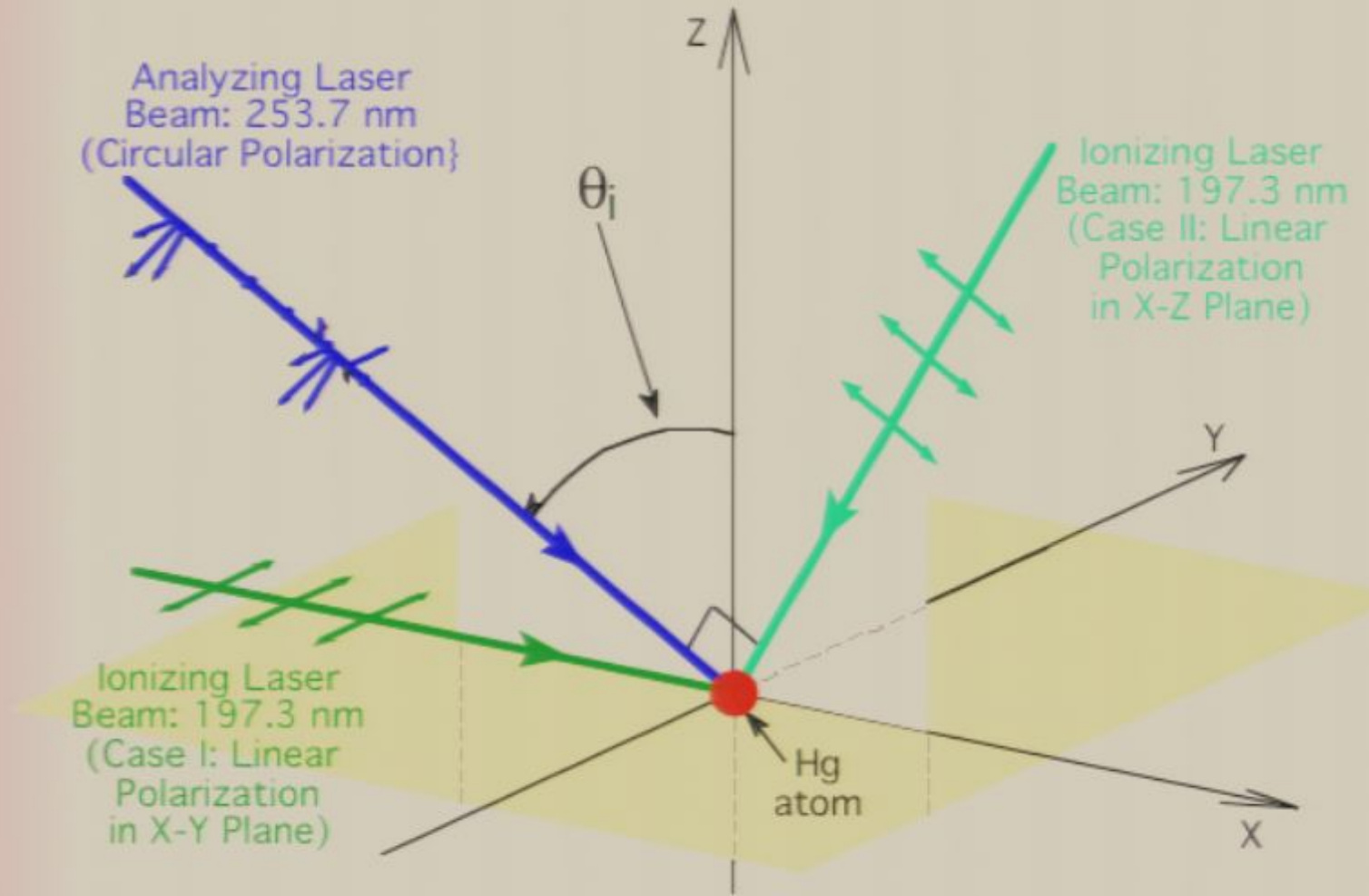


# NUCLEAR SPIN ANALYSIS











# QUANTUM MECHANICAL PREDICTIONS

One may choose various quantities for the correlation measurement.

Consider the case of measuring the component  $M_F = +1/2$  of both atoms in various directions.

### Quantum Predictions

$$R_{++}(\theta_1, \theta_2) = g\eta^2 N \left[ \frac{1}{4} - \frac{1}{4} \cos(\theta_1 - \theta_2) \right]$$

$$R_{1+}(\theta_1) = \frac{\eta N}{2}$$

The strong Bell Inequality is

$$\frac{R_{++}(\theta_1, \theta_2) - R_{++}(\theta_1, \theta'_2) + R_{++}(\theta'_1, \theta_2) + R_{++}(\theta'_1, \theta'_2)}{R_{1+}(\theta'_1) + R_{1+}(\theta_2)} \leq 1$$

Taking

$$\theta_1 = 45^\circ, \theta'_1 = 135^\circ, \theta_2 = 270^\circ, \theta'_2 = 0^\circ$$

The LHS is

$$1.207 \eta g$$

In order for the quantum mechanical predictions to violate the inequality, we must have

$$\eta g \geq \frac{1}{1.207} = 0.828$$

We expect

$$\eta \geq 0.98, g \approx 0.97$$

$$\eta g \geq 0.95$$

One may choose various quantities for the correlation measurement.

Consider the case of measuring the component  $M_F = +1/2$  of both atoms in various directions.

### Quantum Predictions

$$R_{++}(\theta_1, \theta_2) = g\eta^2 N \left[ \frac{1}{4} - \frac{1}{4} \cos(\theta_1 - \theta_2) \right]$$

$$R_{1+}(\theta_1) = \frac{\eta N}{2}$$

The strong Bell Inequality is

$$\frac{R_{++}(\theta_1, \theta_2) - R_{++}(\theta_1, \theta'_2) + R_{++}(\theta'_1, \theta_2) + R_{++}(\theta'_1, \theta'_2)}{R_{1+}(\theta'_1) + R_{1+}(\theta_2)} \leq 1$$

Taking

$$\theta_1 = 45^\circ, \theta'_1 = 135^\circ, \theta_2 = 270^\circ, \theta'_2 = 0^\circ$$

The LHS is

$$1.207 \eta g$$

In order for the quantum mechanical predictions to violate the inequality, we must have

$$\eta g \geq \frac{1}{1.207} = 0.828$$

We expect

$$\eta \geq 0.98, g \approx 0.97$$

$$\eta g \geq 0.95$$

One may choose various quantities for the correlation measurement.

Consider the case of measuring the component  $M_F = +1/2$  of both atoms in various directions.

### Quantum Predictions

$$R_{++}(\theta_1, \theta_2) = g\eta^2 N \left[ \frac{1}{4} - \frac{1}{4} \cos(\theta_1 - \theta_2) \right]$$

$$R_{1+}(\theta_1) = \frac{\eta N}{2}$$

The strong Bell Inequality is

$$\frac{R_{++}(\theta_1, \theta_2) - R_{++}(\theta_1, \theta'_2) + R_{++}(\theta'_1, \theta_2) + R_{++}(\theta'_1, \theta'_2)}{R_{1+}(\theta'_1) + R_{1+}(\theta_2)} \leq 1$$

Taking

$$\theta_1 = 45^\circ, \theta'_1 = 135^\circ, \theta_2 = 270^\circ, \theta'_2 = 0^\circ$$

The LHS is

$$1.207 \eta g$$

In order for the quantum mechanical predictions to violate the inequality, we must have

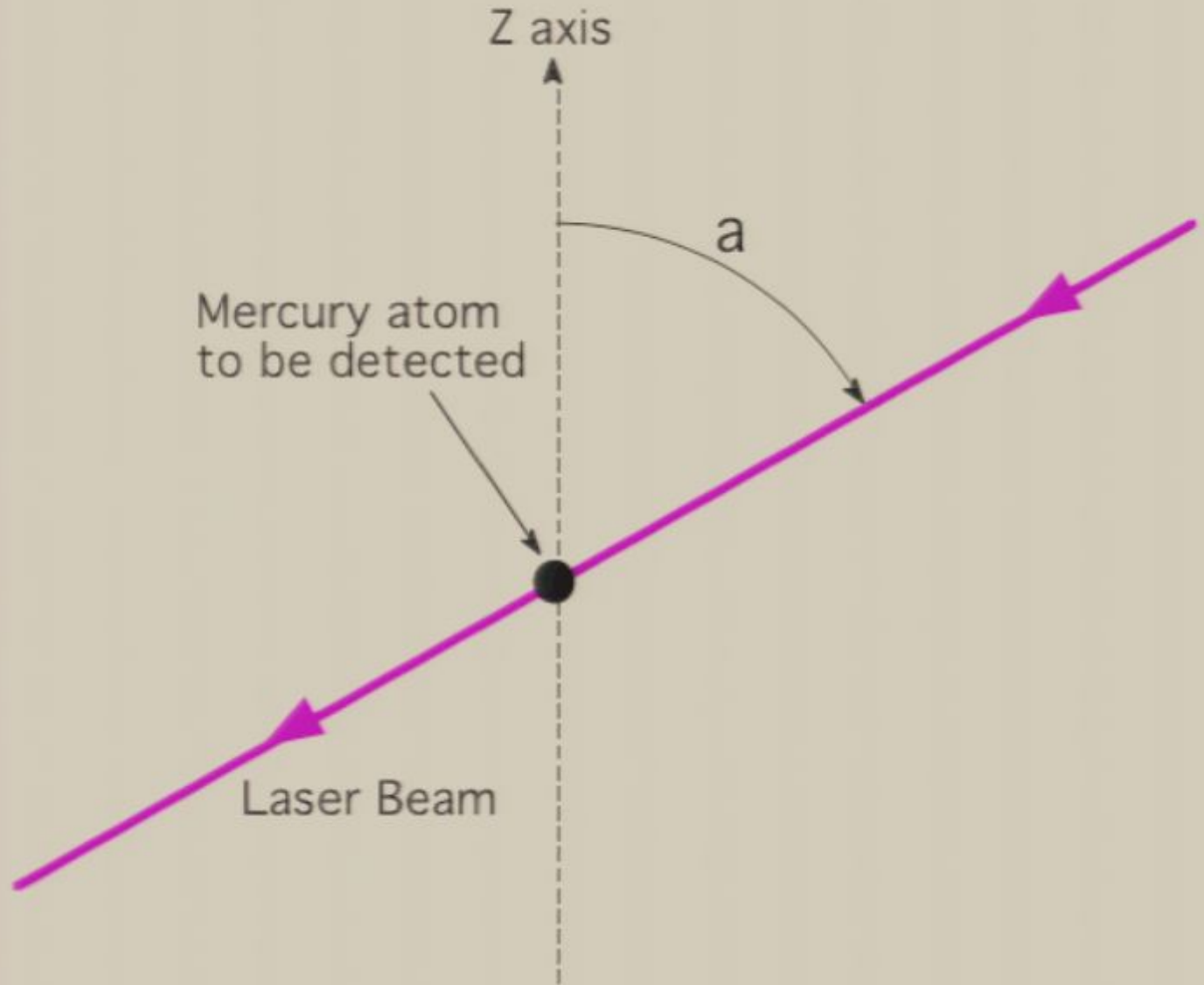
$$\eta g \geq \frac{1}{1.207} = 0.828$$

We expect

$$\eta \geq 0.98, g \approx 0.97$$

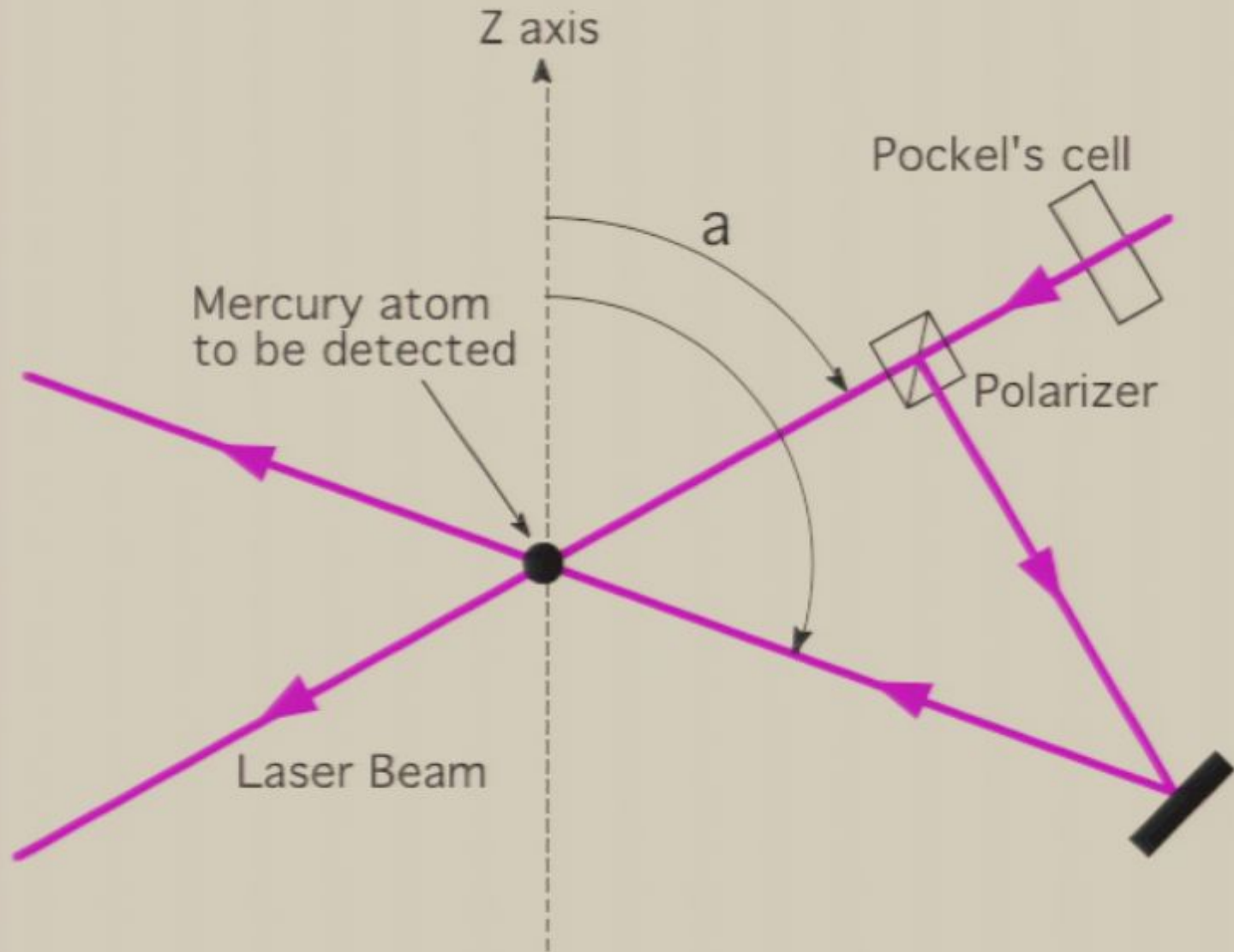
$$\eta g \geq 0.95$$

# Enforcing locality





# Enforcing locality



Texas  
Physics  
&  
M

THE END

Honoring  
Abner Shimony  
Perimeter  
Institute  
200

