Title: Causality in quantum theory and beyond: towards a theory of quantum gravity

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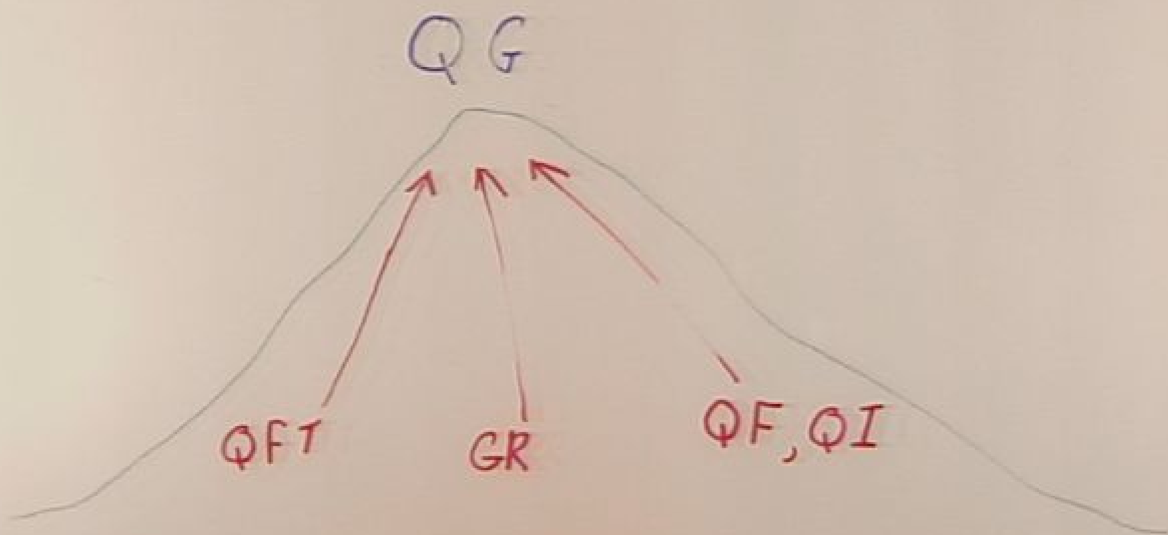
Abstract: The way we combine operators in quantum theory depends on the causal relationship involved. For spacelike separated spacetime regions we use the tensor product. For immediately sequential regions of spacetime we use the direct product. In the latter case we lose information Â– that is we cannot go from the direct product of two operators to the two original operators. This is a kind of compression. We will see that such compression is associated with causal adjacency. We will situate this in the context of a much broader framework potentially suitable for developing a theory of quantum gravity.


QG
QFT
GR
QF, QI

GR

δx^μ

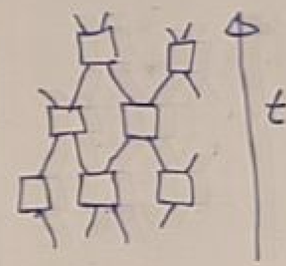
Causal structure (time like or space like) not fixed in advance

Must solve Einstein's field eqns.

$\Rightarrow$ $g_{\mu\nu}$

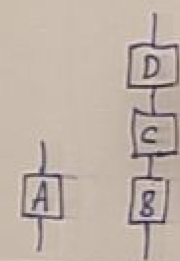
$\Rightarrow$ causal structure.

QT

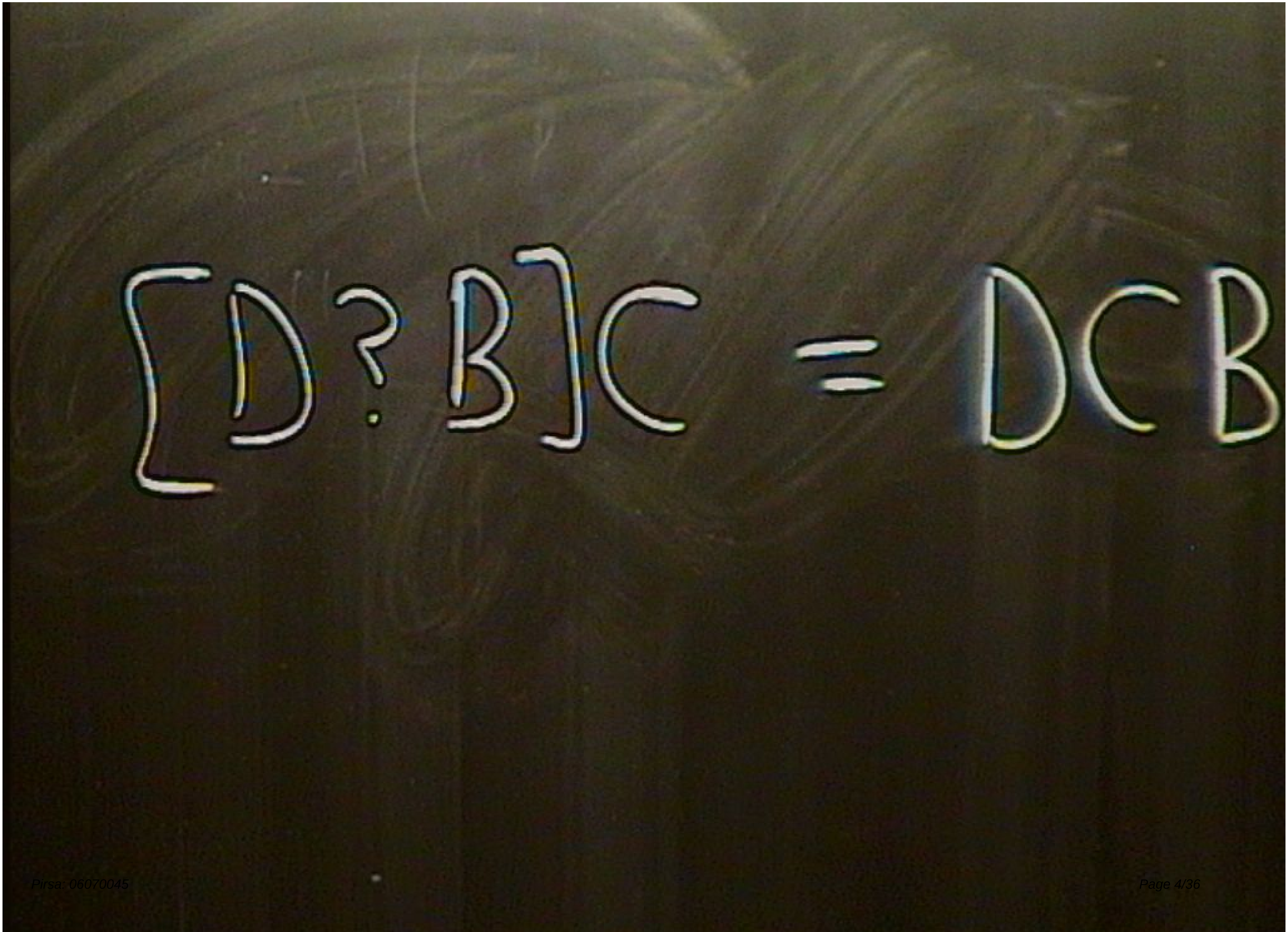


have background time t

$U(t)|\psi(0)\rangle$



	causal structure	compression
$\hat{A}\otimes\hat{B}$	space-like	none
$\hat{C}\hat{B}$	time-like	yes
$[D?B]$	time-like (with gap)	none


[D?B]C = DCB

Combine radical elements.

GR

deterministic

non-fixed causal structure

QT

fixed causal structure

probabilistic.

The causaloid framework

probabilistic

non-fixed causal structure

QT $\Longrightarrow$

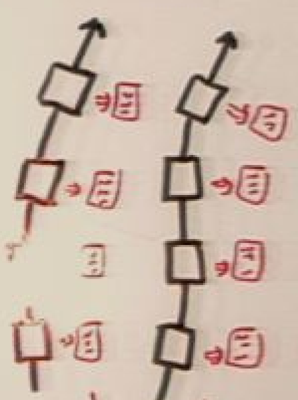
GR? $\Longrightarrow$

$\Longrightarrow$ QG?

An assertion

Assertion: A physical theory, whatever else it does, must correlate recorded data.

Ex. probes



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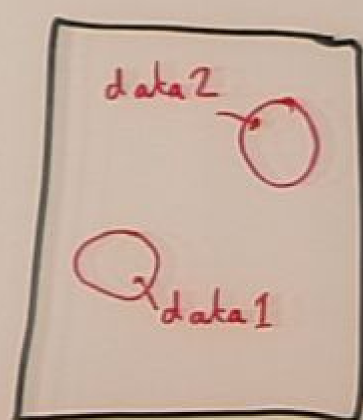

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prob(data1 | data2)

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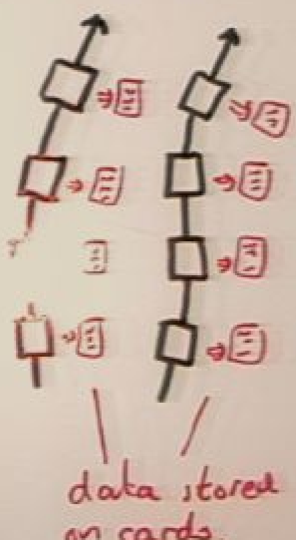

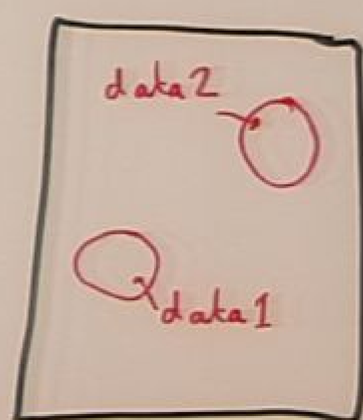

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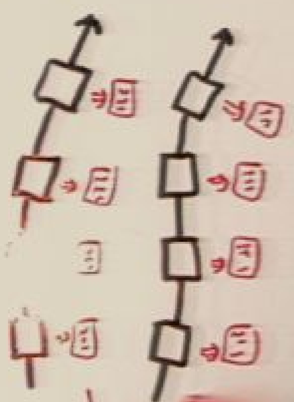

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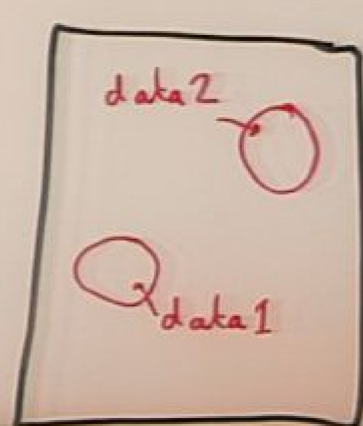

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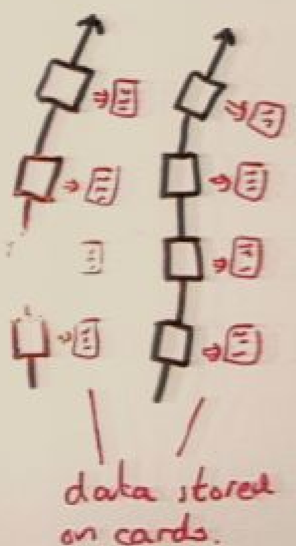

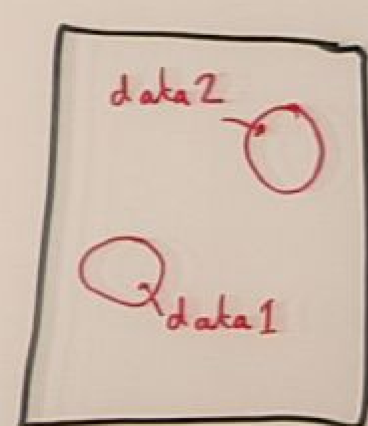

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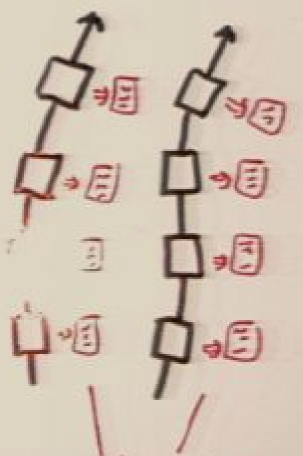

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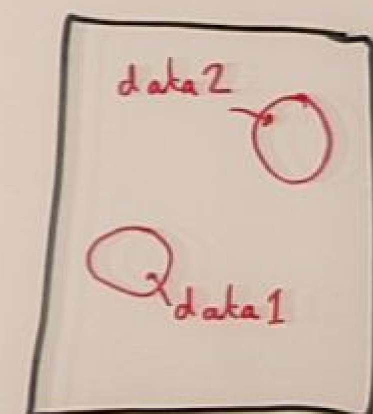

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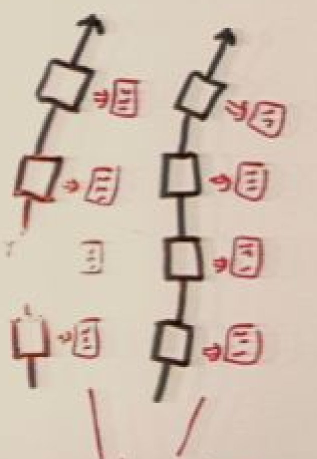

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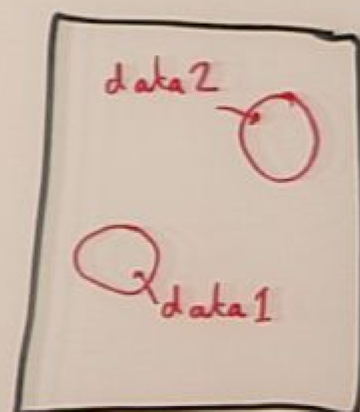

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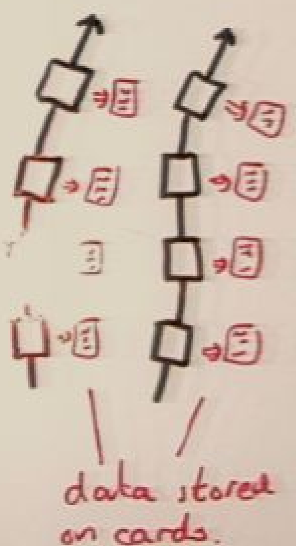

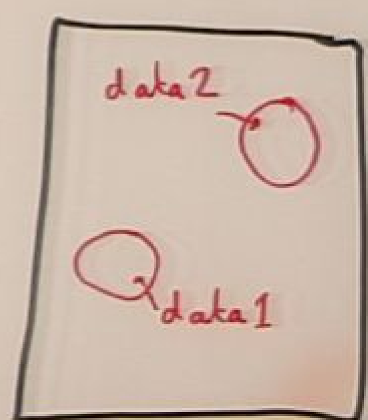

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$$\text{prob}(\text{data1} \mid \text{data2})$$

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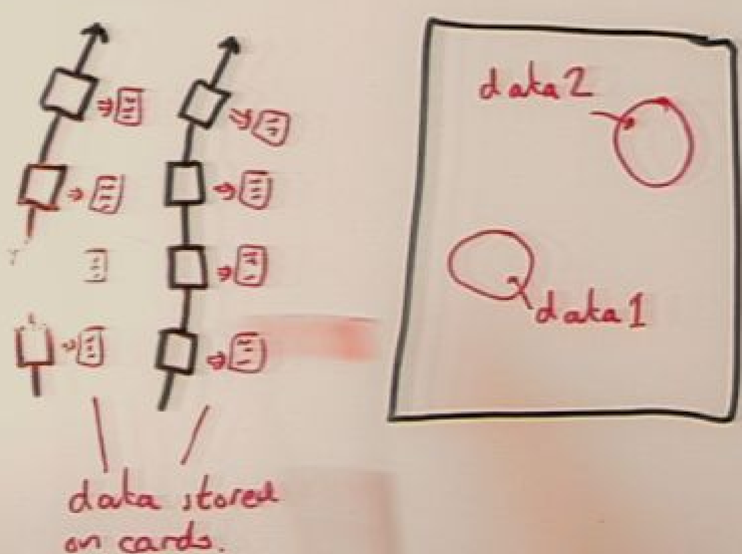

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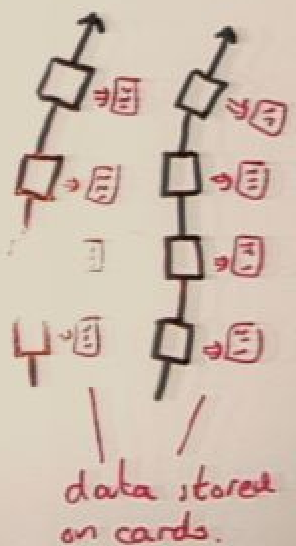

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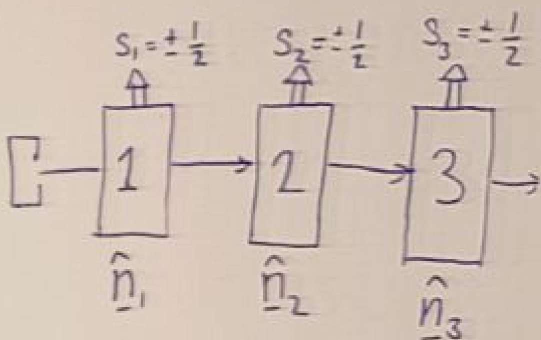
prob(data1 | data2)

Well defined Probabilities

Ex.



$\text{prob}\left(s_2 = +\frac{1}{2} \,\middle|\, \hat{\underline{n}}_2,\ s_1 = +\frac{1}{2},\ \hat{\underline{n}}_1\right)$ is well defined

$\text{prob}\left(s_3 = +\frac{1}{2} \,\middle|\, \hat{\underline{n}}_3,\ s_1 = +\frac{1}{2},\ \hat{\underline{n}}_1\right)$ is <u>not</u> well defined.

In general probabilities will not be well defined.

Usually, we restrict our attention to probabilities we know are well defined by referring to some known causal structure.

New approach

Write down directly an expression which tells us

(a) If prob(data2|data1) is well defined

(b) and, if well defined, what it is equal to.

Key ideal: compression due to physical theory

Example: spin half particle

$$\hat{\rho} = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \iff \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

$$a = p_{x+} - i p_{y+} - \frac{(1-i)}{2}(p_{z+} + p_{z-})$$

Can we $\underline{p}$ to calculate spin along arbitary dir$^{\underline{n}}$ $\underline{\hat{n}}$

$$\underset{\alpha}{\begin{pmatrix} \vdots \\ p_{\underline{\hat{n}}+} \\ \vdots \end{pmatrix}} \quad \xrightarrow{\text{linear compression}} \quad \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ℓ, α

Fiducial set (ℓ)

The physical theory (QT) compresses down to just four probabilities in this case.

In QT

$$\text{prob} = tr(\hat{A}\,\hat{\rho})$$

$$= \underline{\Gamma} \cdot \underline{p}$$

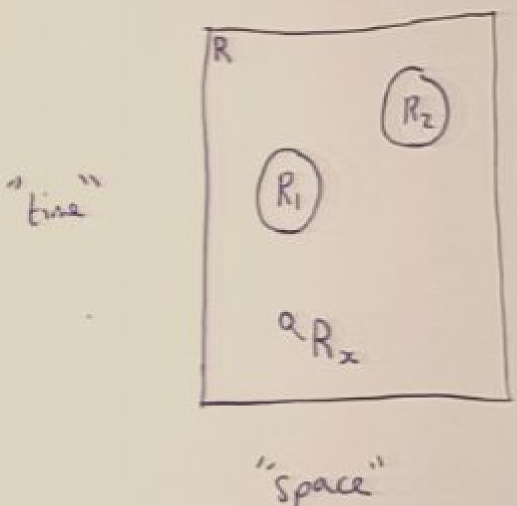
$$\hat{A} \Longleftrightarrow \underline{\Gamma}$$

$\hat{A} \otimes \hat{B}$

$\hat{A}\hat{B}$ $[\hat{A} ? \hat{B}]$ $\Longleftrightarrow$ $\underline{\Gamma}_A \otimes^{\wedge} \underline{\Gamma}_B$

The causaloid product.

Regions



R everything.

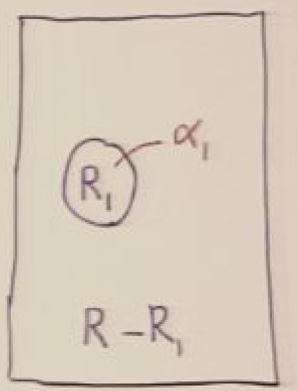
R_1, R_2 general regions.

R_x elementary region

$$R_1 = \bigcup_{x \in \mathcal{O}_1} R_x$$

Three levels of physical Compression

First level



$$\left.\begin{array}{ll} X_{R_1}^{\alpha_1} & \text{outcomes in } R_1 \\ F_{R_1}^{\alpha_1} & \text{choices of settings in } R_1 \end{array}\right\} \text{label with } \alpha_1$$

$$\left.\begin{array}{ll} X_{R-R_1} & \text{outcomes in } R-R_1 \\ F_{R-R_1} & \text{choices of settings in } R-R_1 \end{array}\right\} \text{Generalised preparation for } R_1$$

$$p_{\alpha_1} = \text{prob}\left(X_{R_1}^{\alpha_1} \cup X_{R-R_1} \mid F_{R_1}^{\alpha_1} \cup F_{R-R_1}\right)$$

physical compression.

$$\text{state in } R_1 \Longleftrightarrow \begin{pmatrix} \vdots \\ p_{\alpha_1} \\ \vdots \end{pmatrix} \qquad \underline{p}(R_1) = \begin{pmatrix} \vdots \\ p_{l_1} \\ \vdots \end{pmatrix} \qquad l_1 \in \Omega_1$$

Linear physical compression

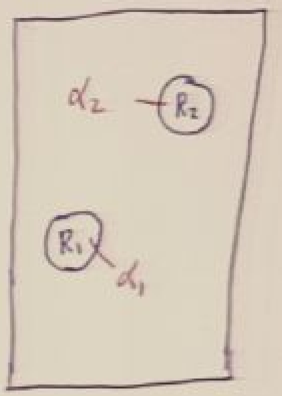
$$p_{\alpha_1} = \underline{r}_{\alpha_1} \cdot \underline{p}$$

Decompression encoded in $\underline{r}_{\alpha_1}$'s. Define

$$\Lambda_{\alpha_1}^{l_1} \equiv \underline{r}_{\alpha_1}\Big|_{l_1}$$

Encodes 1st level (de)compression.

Second level involves two or more regions (disjoint)



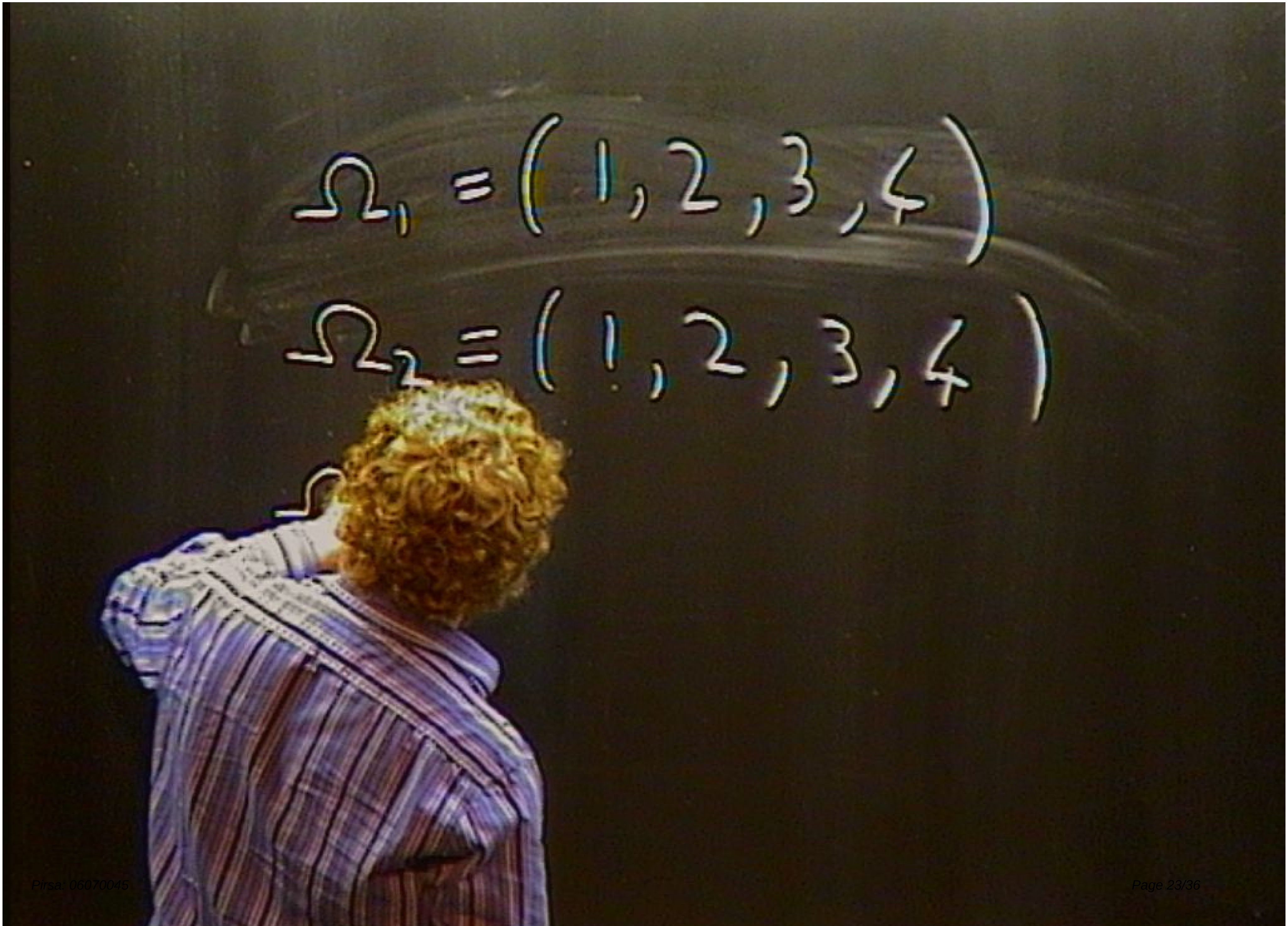
$$p_{\alpha_1,\alpha_2} = \text{prob}\left(X_{R_1}^{\alpha_1} \cup X_{R_2}^{\alpha_2} \cup X_{R-R_1-R_2} \,\middle|\, F_{R_1}^{\alpha_1} \cup F_{R_2}^{\alpha_2} \cup F_{R-R_1-R_2} \right)$$

linear physical compression

$$\text{state } R_1 \cup R_2 \iff \begin{pmatrix} \vdots \\ p_{\alpha_1\alpha_2} \\ \vdots \end{pmatrix} \iff \mathbf{p}(R_1 \cup R_2) = \begin{pmatrix} \vdots \\ p_{k_1k_2} \\ \vdots \end{pmatrix} \quad k_1k_2 \in \Omega_{12}$$

Important theorem

$$\Omega_{12} \subseteq \Omega_1 \times \Omega_2$$


$\Omega_1 = (1,2,3,4)$
$\Omega_2 = (1,2,3,4)$

$$\Omega_1 = (1, 2, 3, 4)$$

$$\Omega_2 = (1, 2, 3, 4)$$

$$\Omega_1 \times \Omega_2 = (11, 12, \ldots 21, 22$$

$$\Omega_{12} = (11, 21, 31, 41)$$

If

$$\Omega_{12} \subset \Omega_1 \times \Omega_2$$

have (nontrivial) 2nd level physical compression.

Have

$$p_{\alpha_1 \alpha_2} = \underline{\Gamma}_{\alpha_1 \alpha_2} \cdot \underline{p}$$

Define

$$\Lambda^{k_1 k_2}_{l_1 l_2} \equiv \underline{\Gamma}_{l_1 l_2}\Big|_{k_1 k_2} \qquad \begin{array}{l} k_1 k_2 \in \Omega_{12} \\ l_1 l_2 \in \Omega_1 \times \Omega_2 \end{array}$$

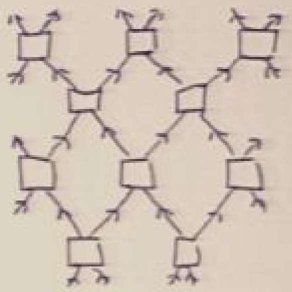
Third level physical compression

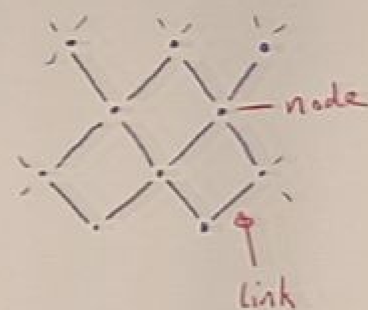
Have

$$\left(\begin{array}{ll} \Lambda_{\alpha_x}^{l_x} & \forall\, x \\ \Lambda_{l_x l_{x'}}^{k_x k_{x'}} & \forall\, x, x' \\ \Lambda_{l_x l_{x'} l_{x''}}^{k_x k_{x'} k_{x''}} & \forall\, x, x', x'' \\ \vdots & \end{array} \right) \xrightarrow{\text{compress}} \left[\begin{array}{l} \Lambda(j) \\ \text{plus} \\ \text{RULES} \end{array} \right] = \underline{\Lambda}$$

The causaloid

Example. QT of interacting qubits





Causaloid specified by

$\Lambda^{l_x}_{\alpha_x}$ for each node

$\Lambda^{k_x k_{x'}}_{l_x l_{x'}}$ for each link

some RULES

$= \underline{\Lambda} \Rightarrow$ All predictions of QT.

Making Predictions

$$p \equiv \text{prob}\left(X_2^{\alpha_2} \,\middle|\, F_2^{\alpha_2}, X_1^{\alpha_1}, F_1^{\alpha_1}\right) = \frac{\underline{\Gamma}_{\alpha_1 \alpha_2} \cdot \underline{P}}{\sum_{\beta_2} \underline{\Gamma}_{\alpha_1 \beta_2} \cdot \underline{P}}$$

consistent with $F_2^{\alpha_2}$

Well defined iff

$$\underline{\Gamma}_{\alpha_1 \alpha_2} \quad // \quad \sum_{\beta_2} \underline{\Gamma}_{\alpha_1 \beta_2}$$

then equal to p where

$$\underline{\Gamma}_{\alpha_1 \alpha_2} = p \sum_{\beta_2} \underline{\Gamma}_{\alpha_1 \beta_2}$$

The Causaloid Formalism

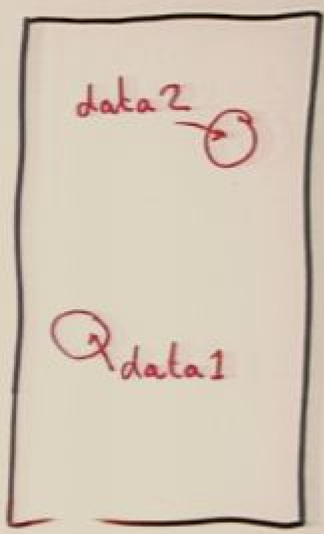
The causaloid (theory dependent

Some basic equations (theory independent)

can predict all probabilities like

$$\text{prob}(X_{R_1} \mid X_{R_2}, F_{R_1}, F_{R_2})$$

How do we usually go about correlating data?



Three approaches:

1) Evolving state (e.g. in Q.T. $|\psi(t)\rangle$)

2) Local laws (e.g. in GR $G_{\mu\nu} = 8\pi T_{\mu\nu}$)

3) Histories (e.g. Feynman path integrals)

All problematic for QG.

All approaches here require consideration of other data besides data being considered.

The problem of time in QG.

Different distributions of matter

$\Rightarrow$ different causal structure.

Quantum superposition of different matter distributions

$\Rightarrow$ no "matter-of-fact" for causal structure

But

Q.T. has state $\psi(t)$ evolving in time.

Conclusions

The causaloid formalism

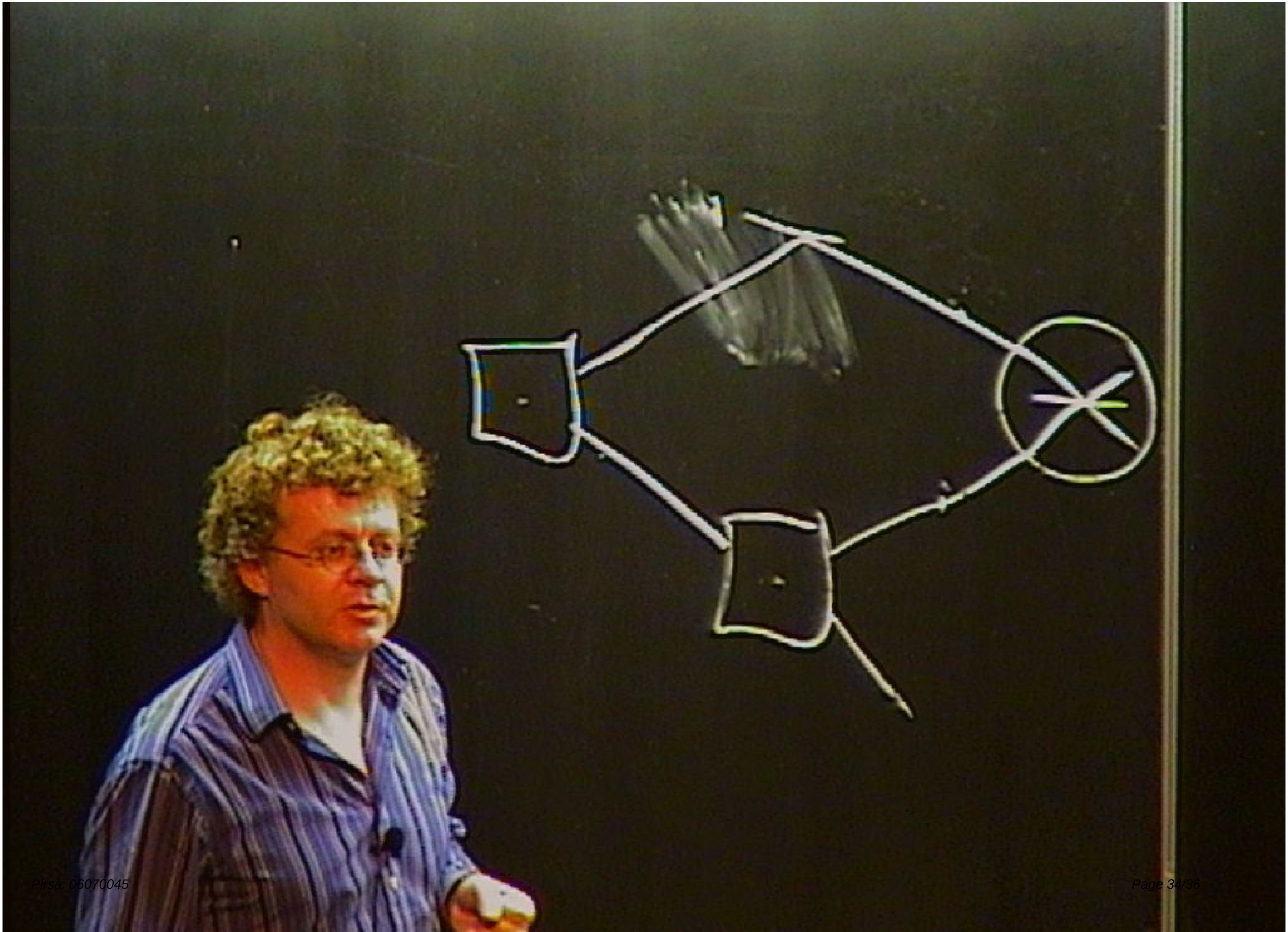
(1) Treats all pairs of elementary regions on an equal footing — it provides a unification of

$$\hat{A} \otimes \hat{B}, \; \hat{A}\hat{B}, \; [\hat{A}?\hat{B}] \text{ in Q.T.}$$

(2) We can formulate QT in this formalism.

(3) We can hope to formulate GR

(4) Provides a mathematical setting for QG



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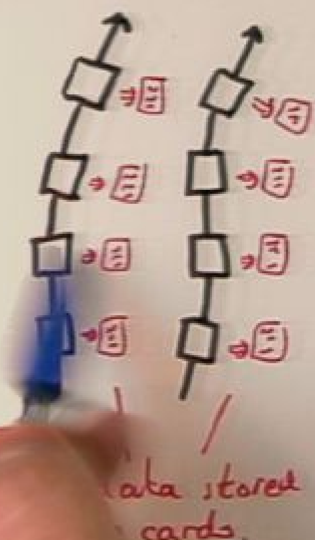

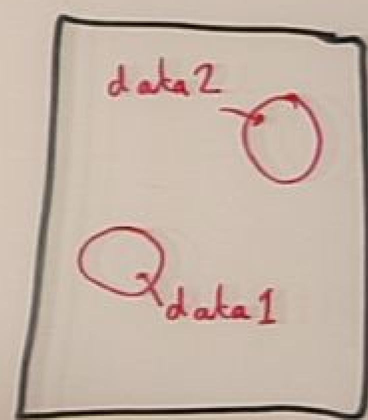

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