Title: Entanglement and the Foundations of Statistical Mechanics

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Abstract:

ENTANGLEMENT AND THE FOUNDATIONS OF STATISTICAL MECHANICS

- SANDU POPESCU, TONY SHORT, ANDREAS WINTER

- YAKIR AHARONOV, NOAH LINDEN

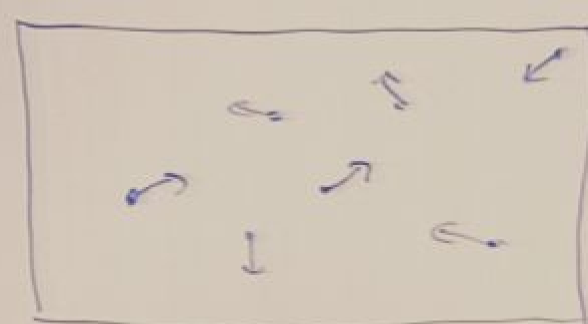
LARG NUMBER OF PARTICLES

$\Downarrow$

LACK OF KNOWLEDGE

$\Downarrow$

USE OF AVERAGES



GREAT PARADOX

SUBJECTIVE LACK OF KNOWLEDGE

HAS

OBJECTIVE PHYSICAL IMPLICATIONS !!

MACRO STATE (DETERMINED BY MACROSCOPIC PARAMETERS $P, V, E \ldots$)

$\Downarrow$

NUMBER OF (MICRO) STATES COMPATIBLE WITH THE MACRO STATE N

$\Downarrow$

$S = \ln N$ ENTROPY

$\Downarrow$

$$\frac{dS}{dE} = \frac{1}{T}$$

ENSEMBLE AVERAGE

$$\boxed{\text{POSTULATE OF EQUAL A-PRIORY PROBABILITIES}}$$

GIBBS ENSEMBLE

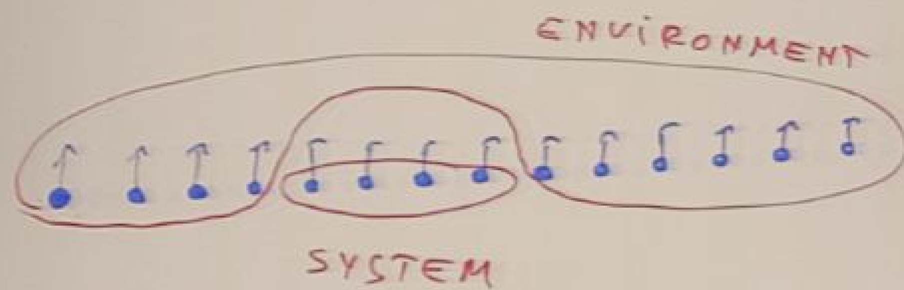
TIME AVERAGE

ERGODICITY

QUANTUM MECHANICS:

OBJECTIVE LACK OF KNOWLEDGE

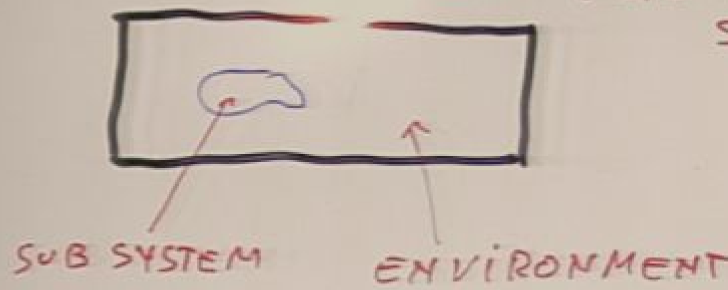
YAKIR AHARONOV



$$\boxed{\begin{array}{l} |\psi(0)\rangle \longrightarrow |\psi(t)\rangle \\ S(|\psi(t)\rangle) = 0 \\ S^{(t)}_{SYSTEM} \neq 0 \end{array}}$$

ENTANGLEMENT

UNIVERSE (BIG ISOLATED SYSTEM)



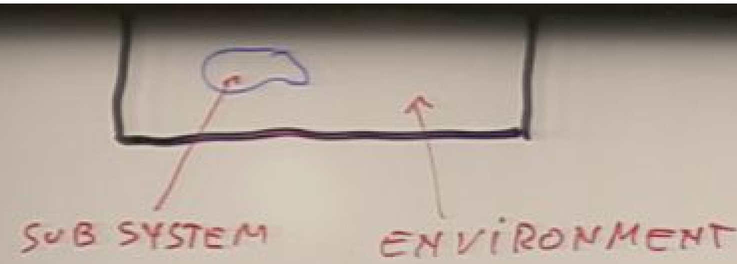
$$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$$

R = RESTRICTION

POSTULATE OF EQUAL A-PRIORY PROB.

$$\rho_U = \frac{\mathbb{I}_R}{\dim R}$$ (MICRO-CANONICAL DISTRIBUTION)

$$\Omega_S \equiv \mathrm{Tr}_E\, \rho_U = \mathrm{Tr}_E \frac{\mathbb{I}_R}{\dim R}$$



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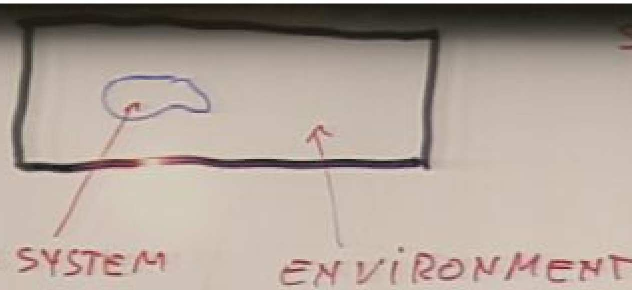

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$|\psi\rangle_U$

$\rho_S = \mathrm{Tr}_E \, |\psi\rangle_U \langle\psi|_U$

$$\boxed{\rho_S \approx \Omega_S \quad \text{FOR ALMOST ALL } |\psi\rangle_U}$$

$|\psi\rangle_U$

$\rho_S = \mathrm{Tr}_E \, |\psi\rangle_U \langle\psi|_U$

$$\boxed{\rho_S \simeq \Omega_S \quad \text{FOR ALMOST ALL } |\psi\rangle_U}$$

MAIN RESULTS 1.

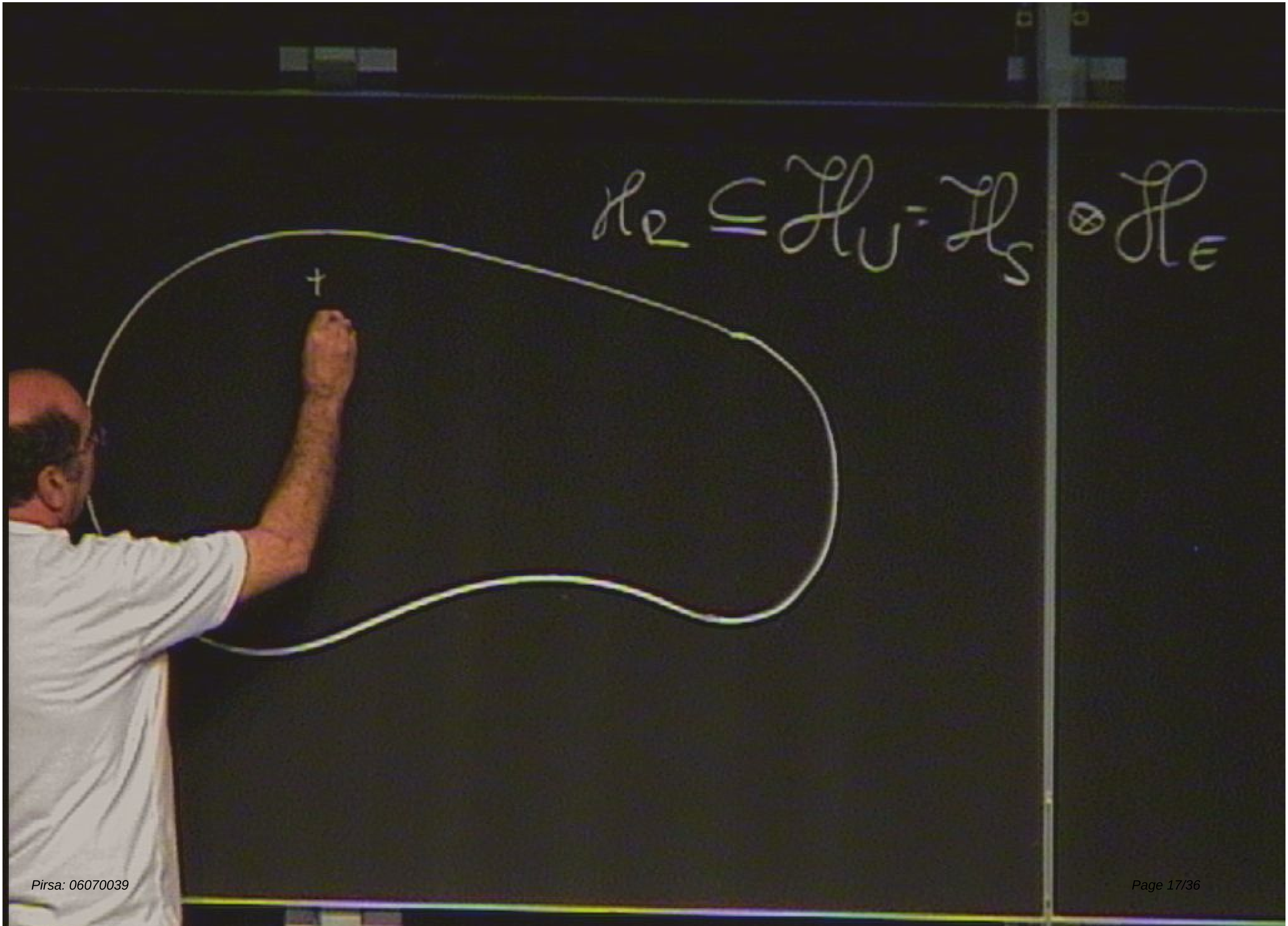
$$\langle \| \rho_S - \Omega_S \|_1 \rangle \leq \sqrt{\frac{d_S}{d_E^{eff}}}$$

$d_E \quad 2^{n-k}$

$d_S = 2^k$

MAIN RESULTS 2.

$$V\left(\|\rho_S - \Omega_S\|_1 \geq \varepsilon + \sqrt{\frac{d_S}{d_E^{eff}}} \right) \leq V_0\, e^{-C d_R \varepsilon^2}$$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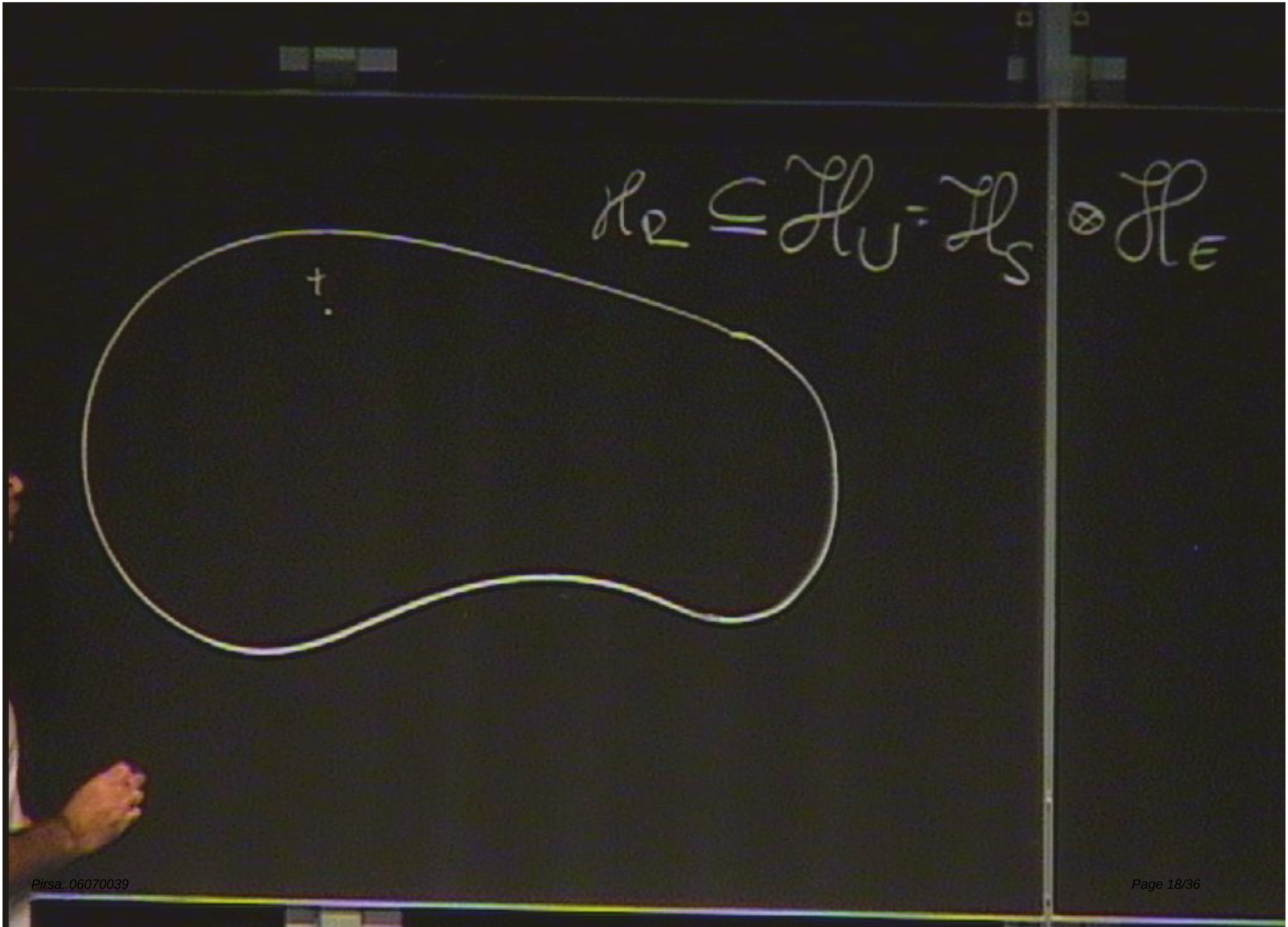
$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$
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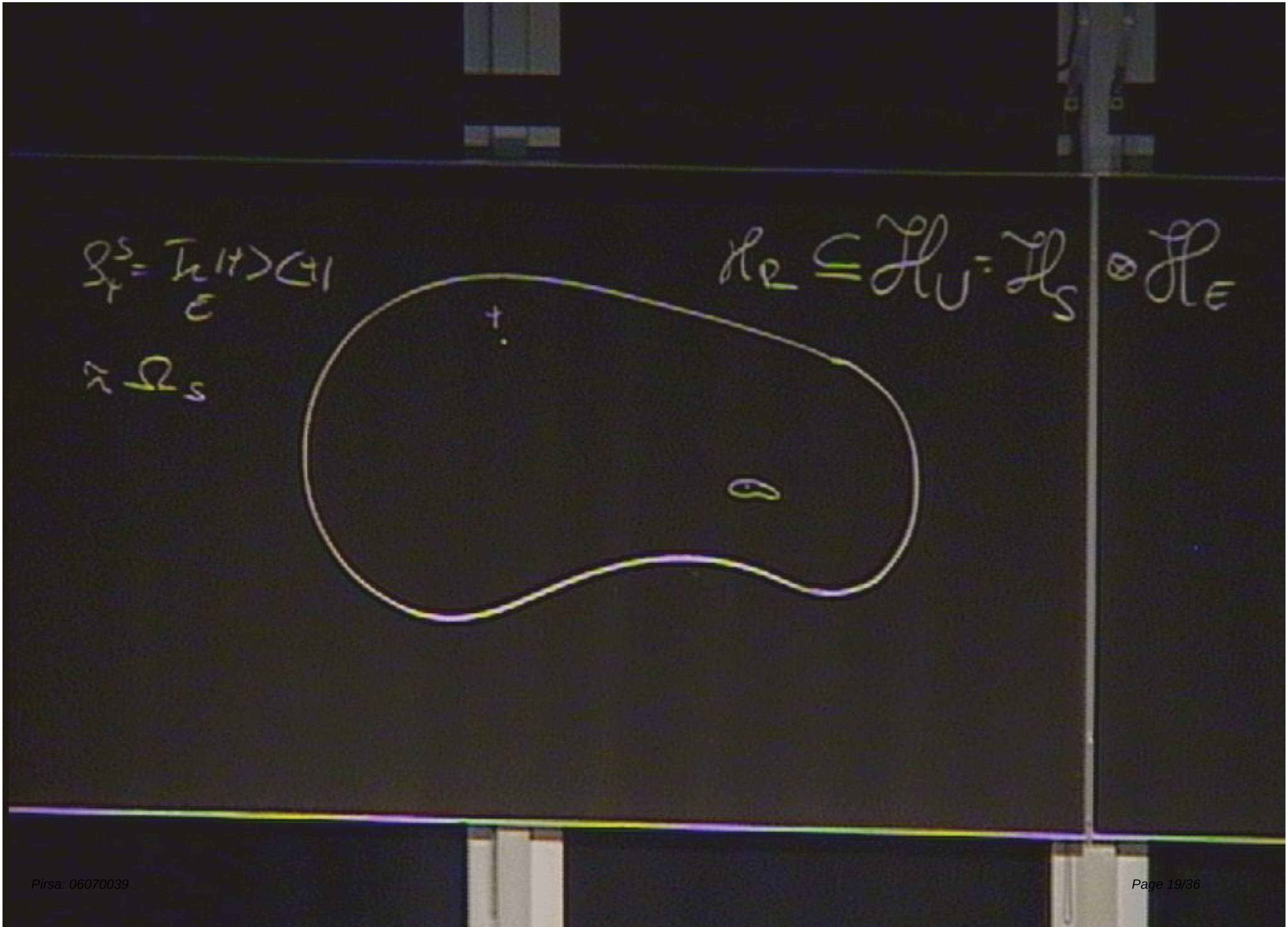
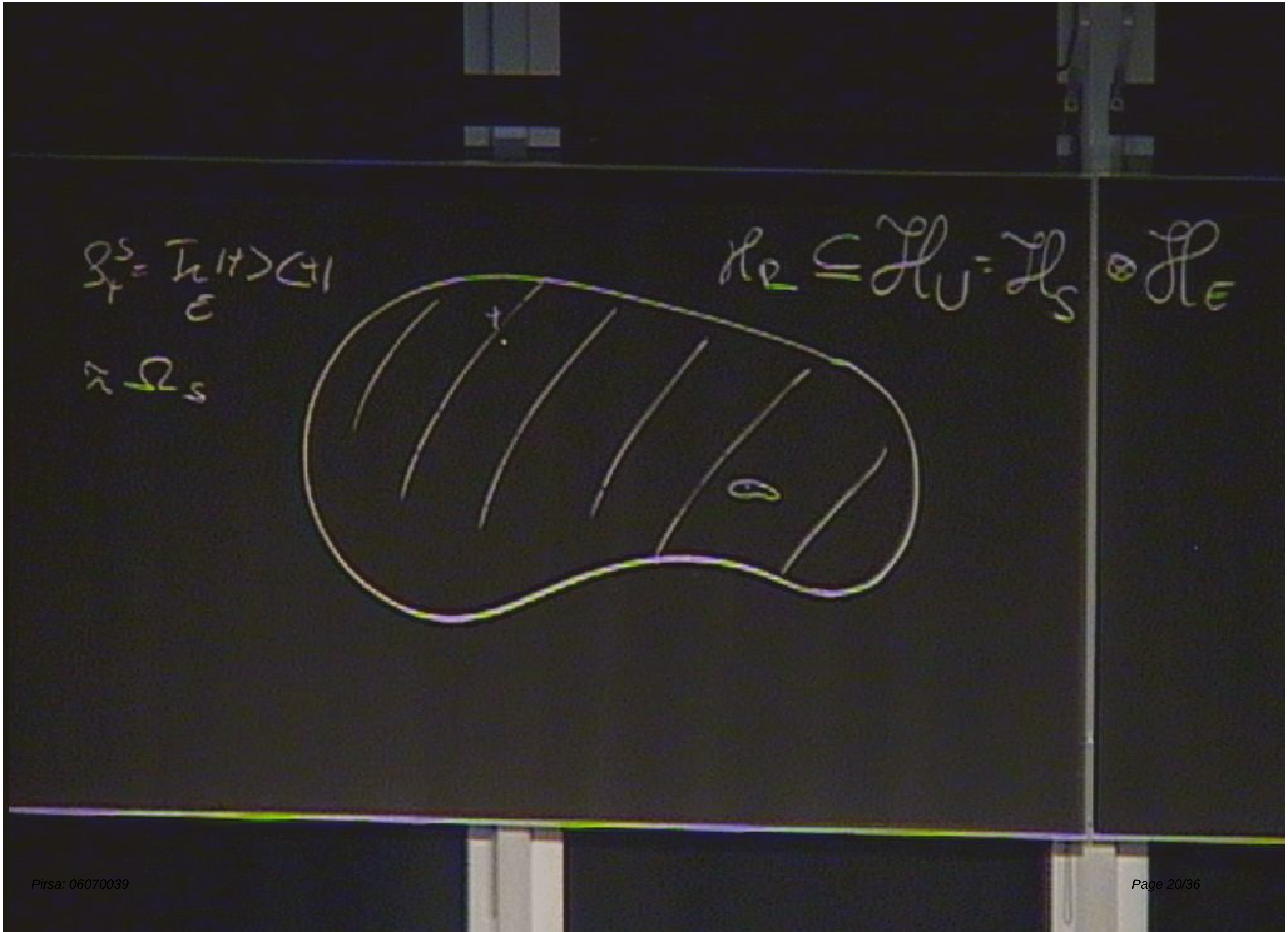

$\rho_t^S = \mathrm{Tr}_E |\psi_t\rangle\langle\psi_t|$
$\approx \Omega_S$
$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$
t



ρ_t^S = Tr_E |t⟩⟨t|
≈ Ω_S
H_R ⊆ H_U = H_S ⊗ H_E
t

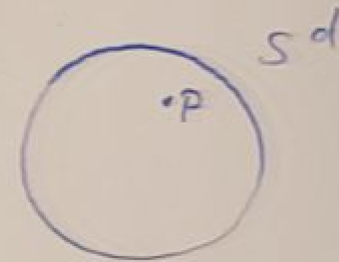
~~POSTULATE OF EQUAL A-PRIORY PROBABILITIES~~

PRINCIPLE OF APPARENTLY EQUAL A-PRIORY PROBABILITIES

$$\Omega_s \simeq e^{-}$$

$$\rho_s \simeq \Omega_s^R$$

MAIN TOOL: LEVY'S LEMMA



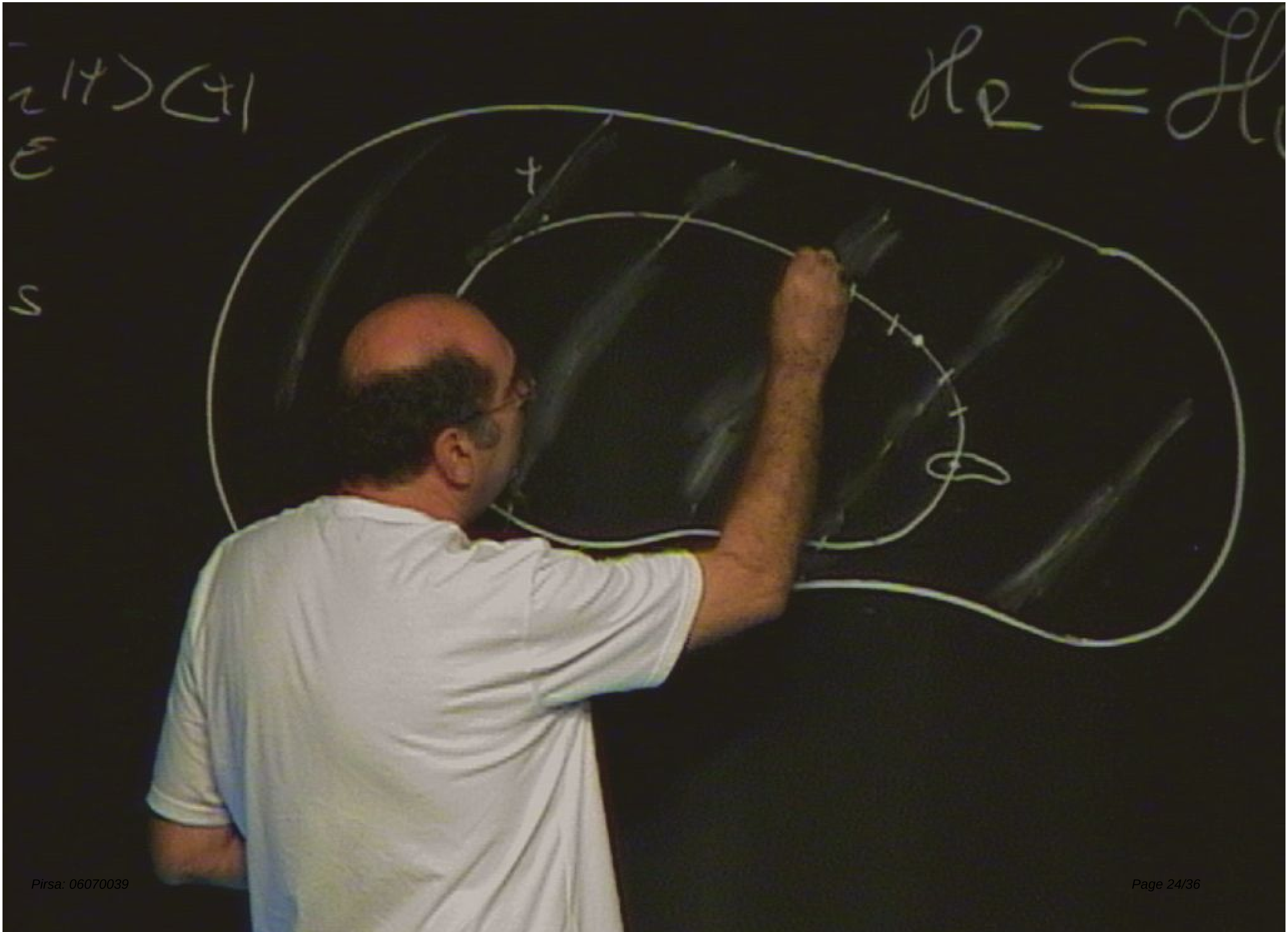
$$f : S^d \longrightarrow R$$

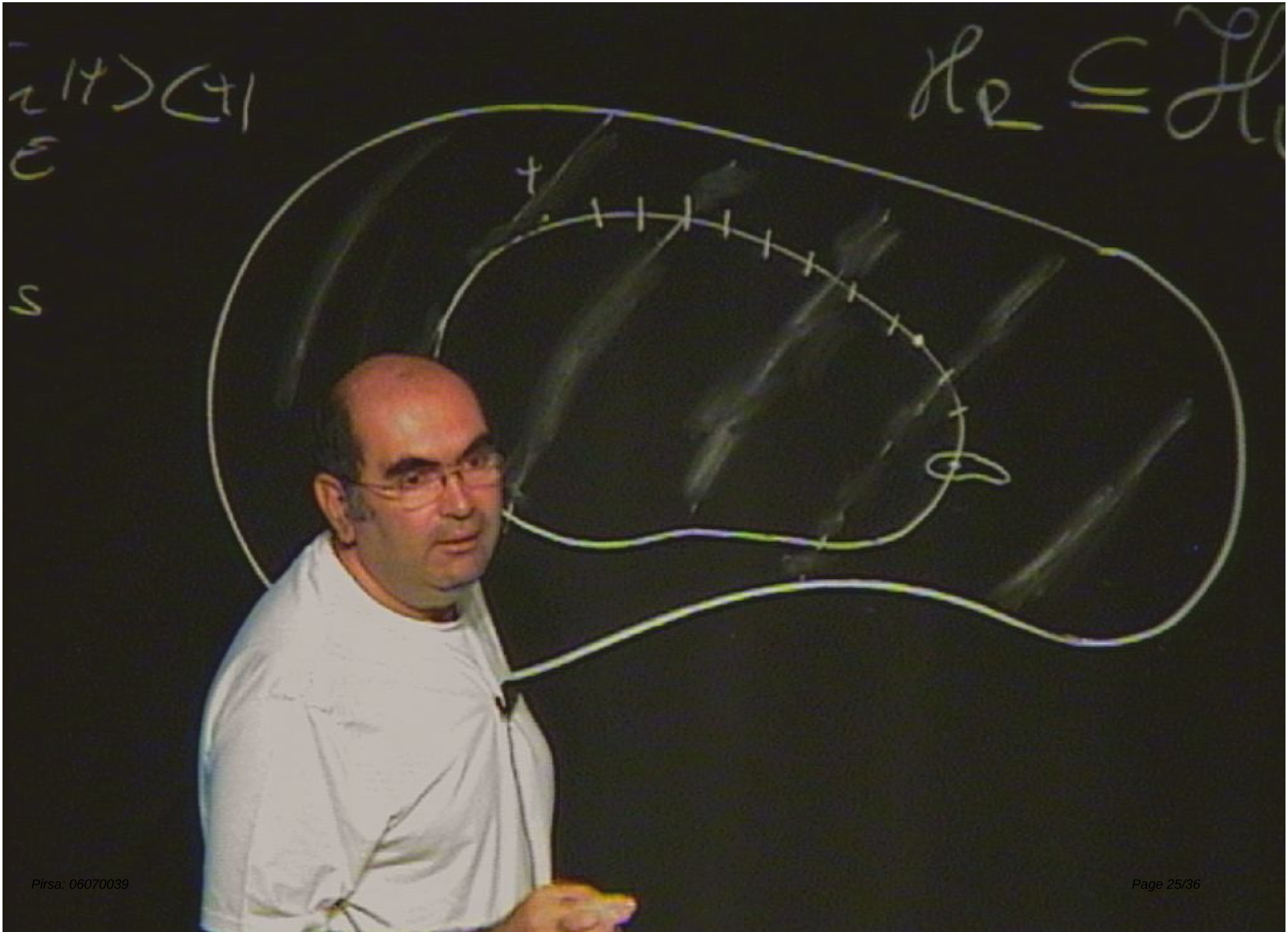
$$\text{Prob}\left[\,|f(p) - \langle f \rangle| \geq \epsilon\,\right] \leq 2\, e^{-\frac{C(d+1)\,\epsilon^2}{\eta^2}}$$

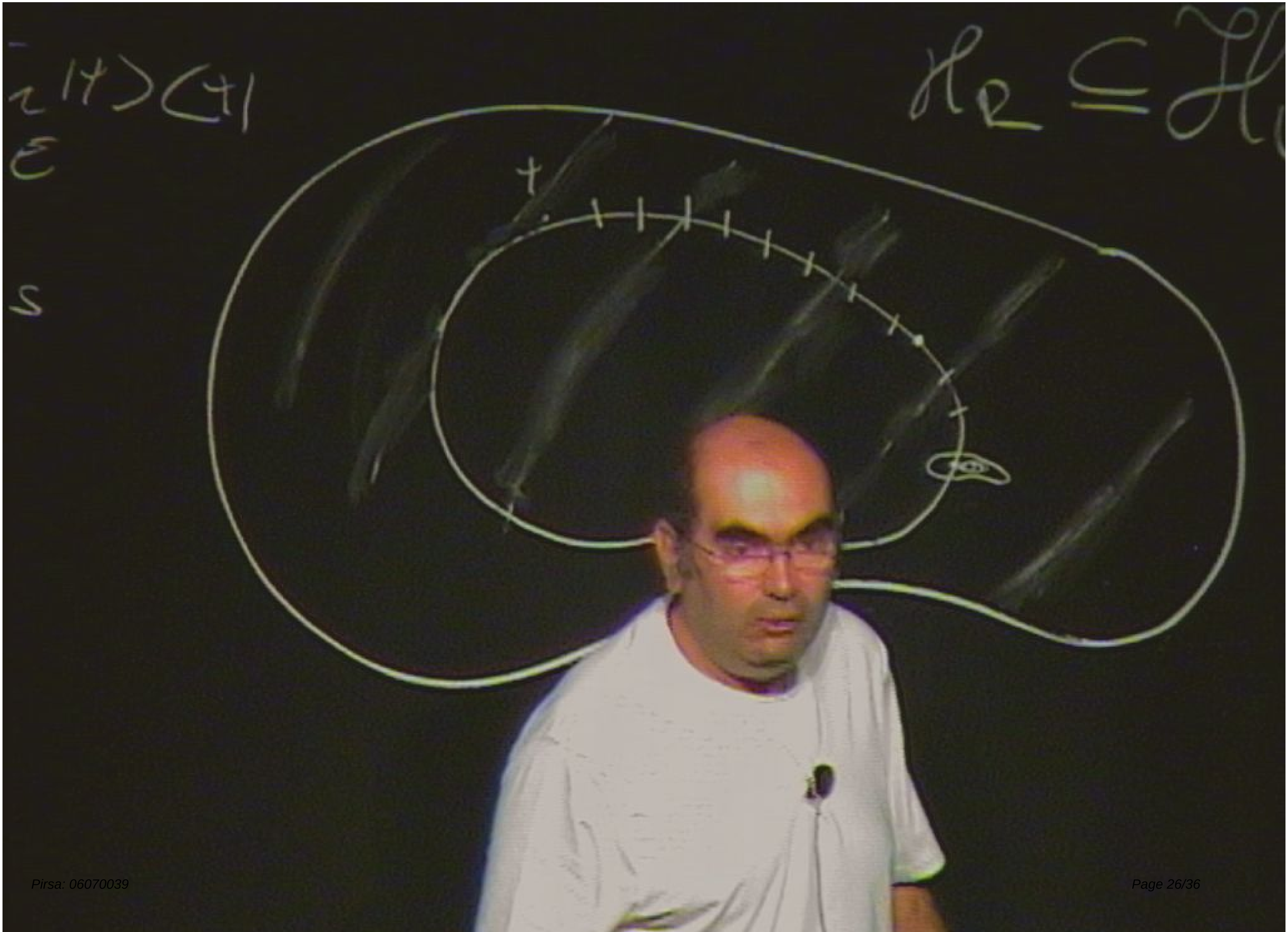
$$\eta = \max |\nabla f|$$

$$f(|\psi\rangle) = \|\rho_S(\psi) - \Omega_S\|_1$$

$$\|M\|_1 \equiv \mathrm{Tr}\sqrt{MM^{+}}$$




$H_R \subseteq \mathcal{H}$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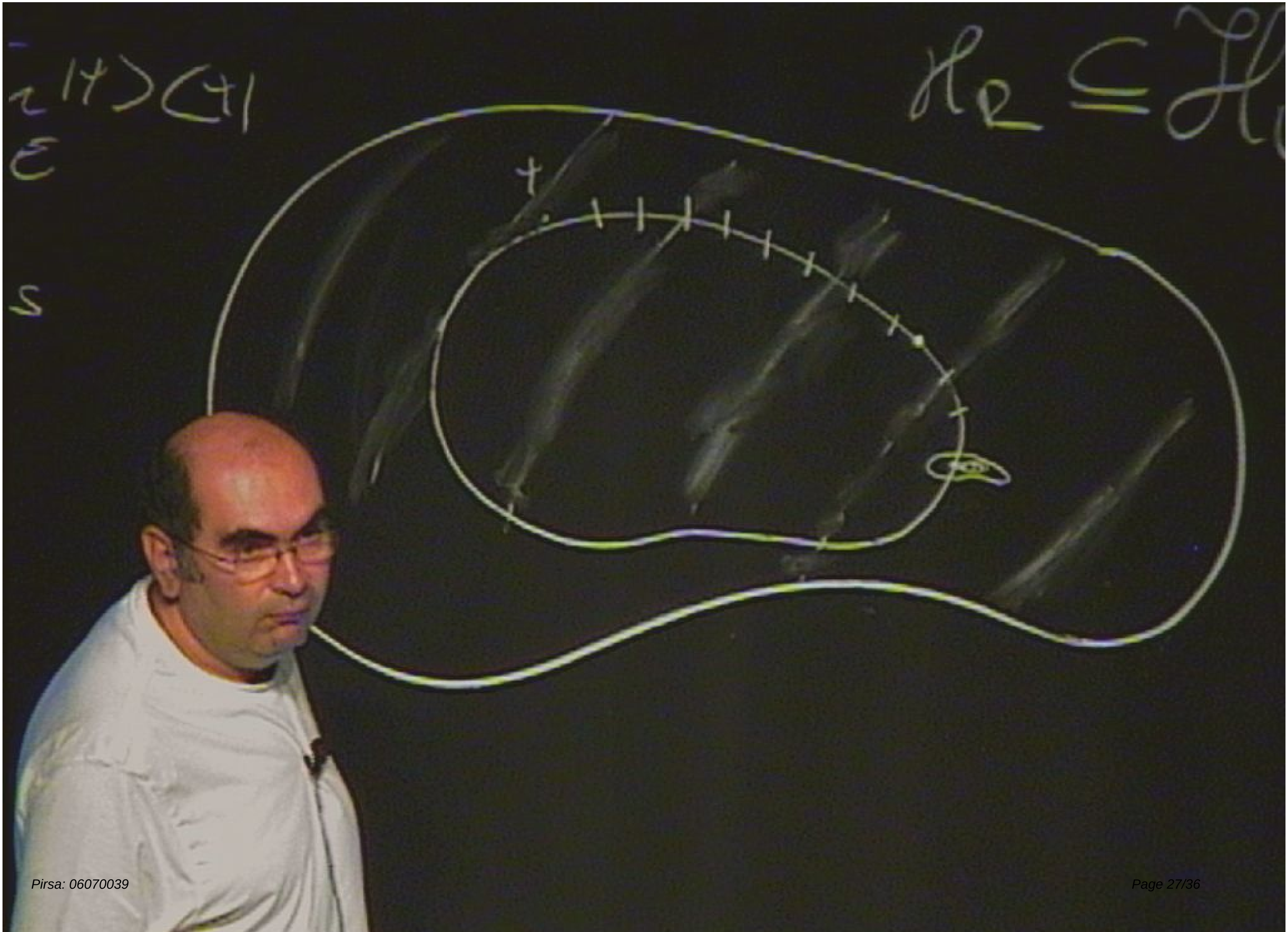
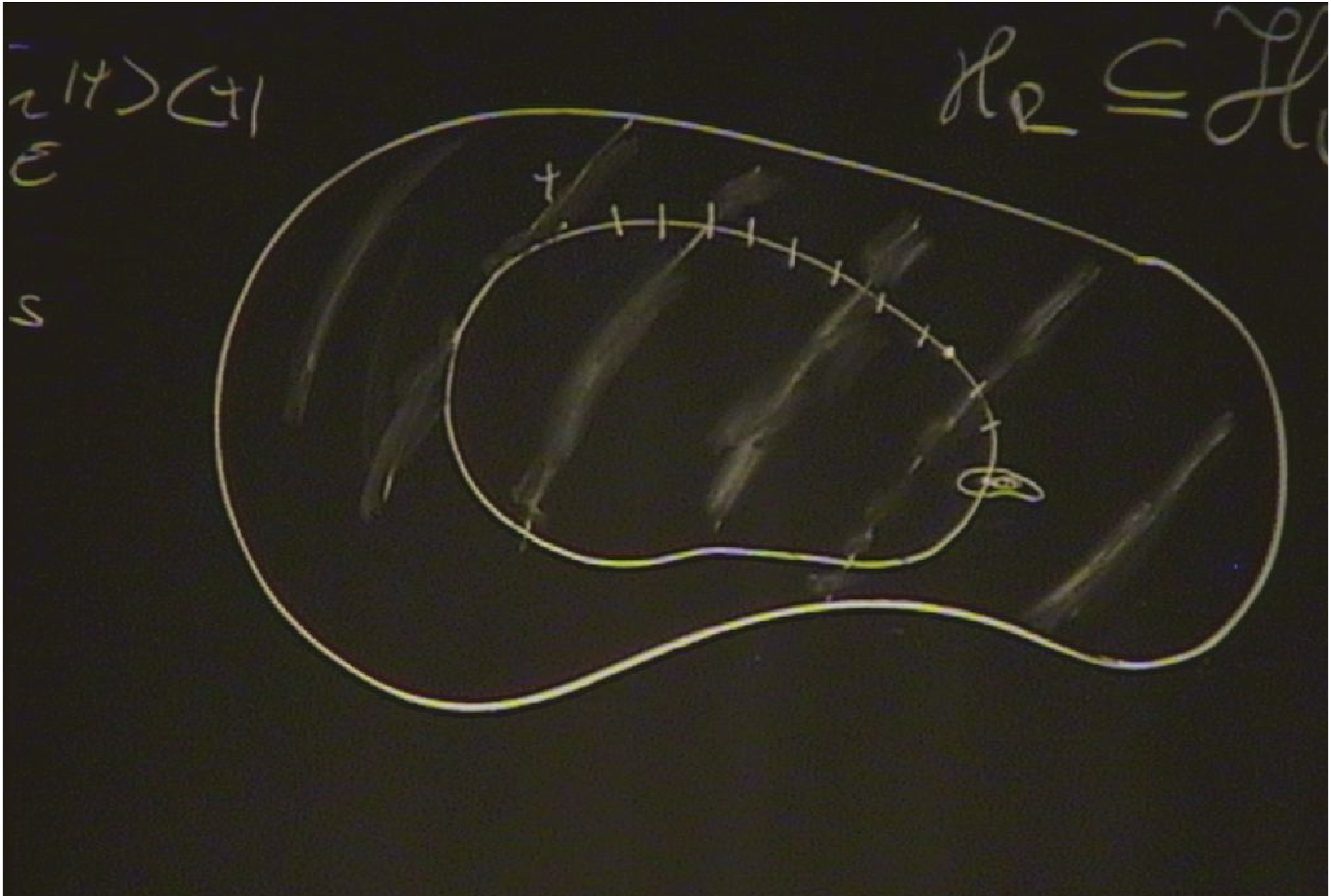
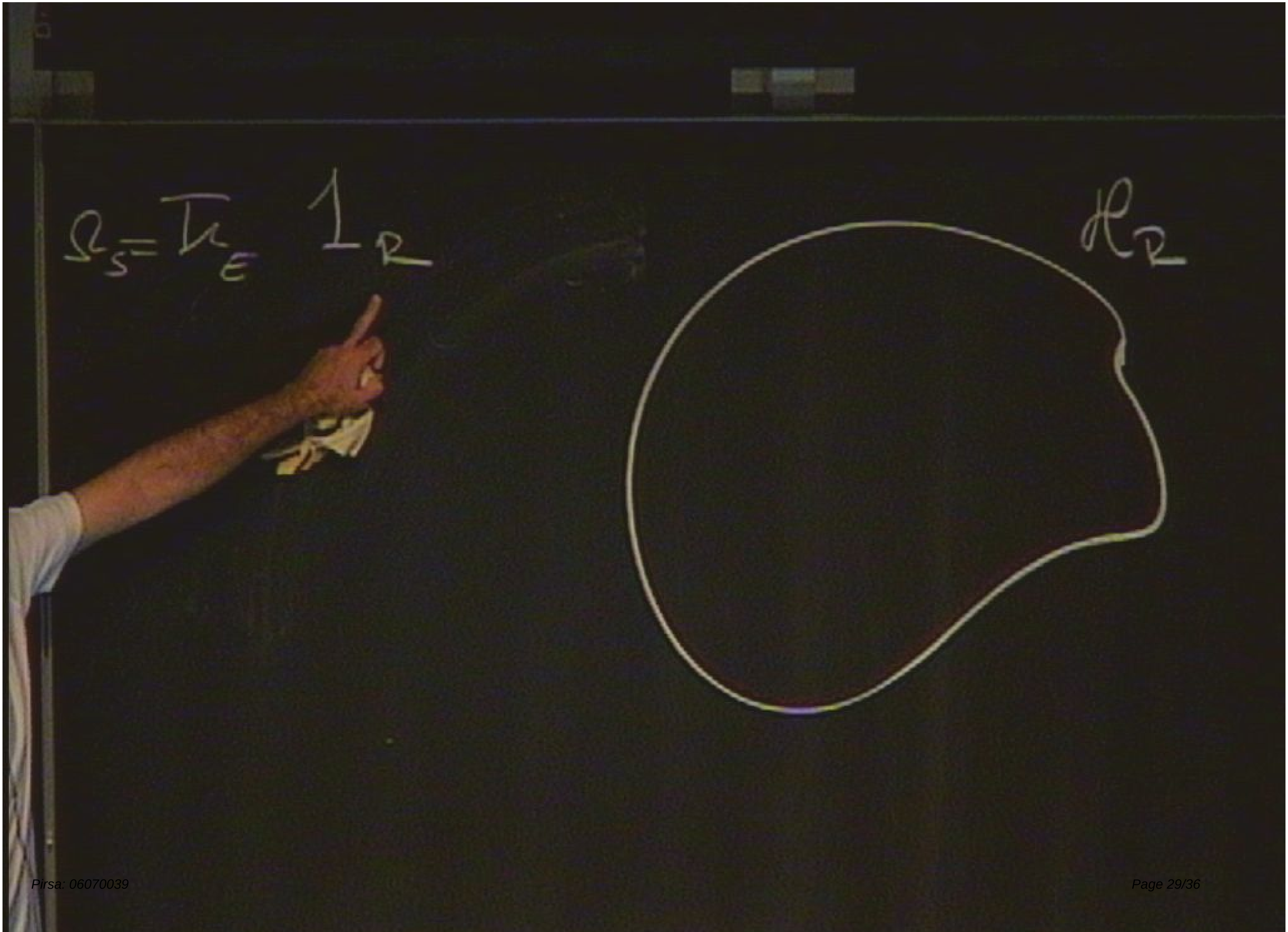
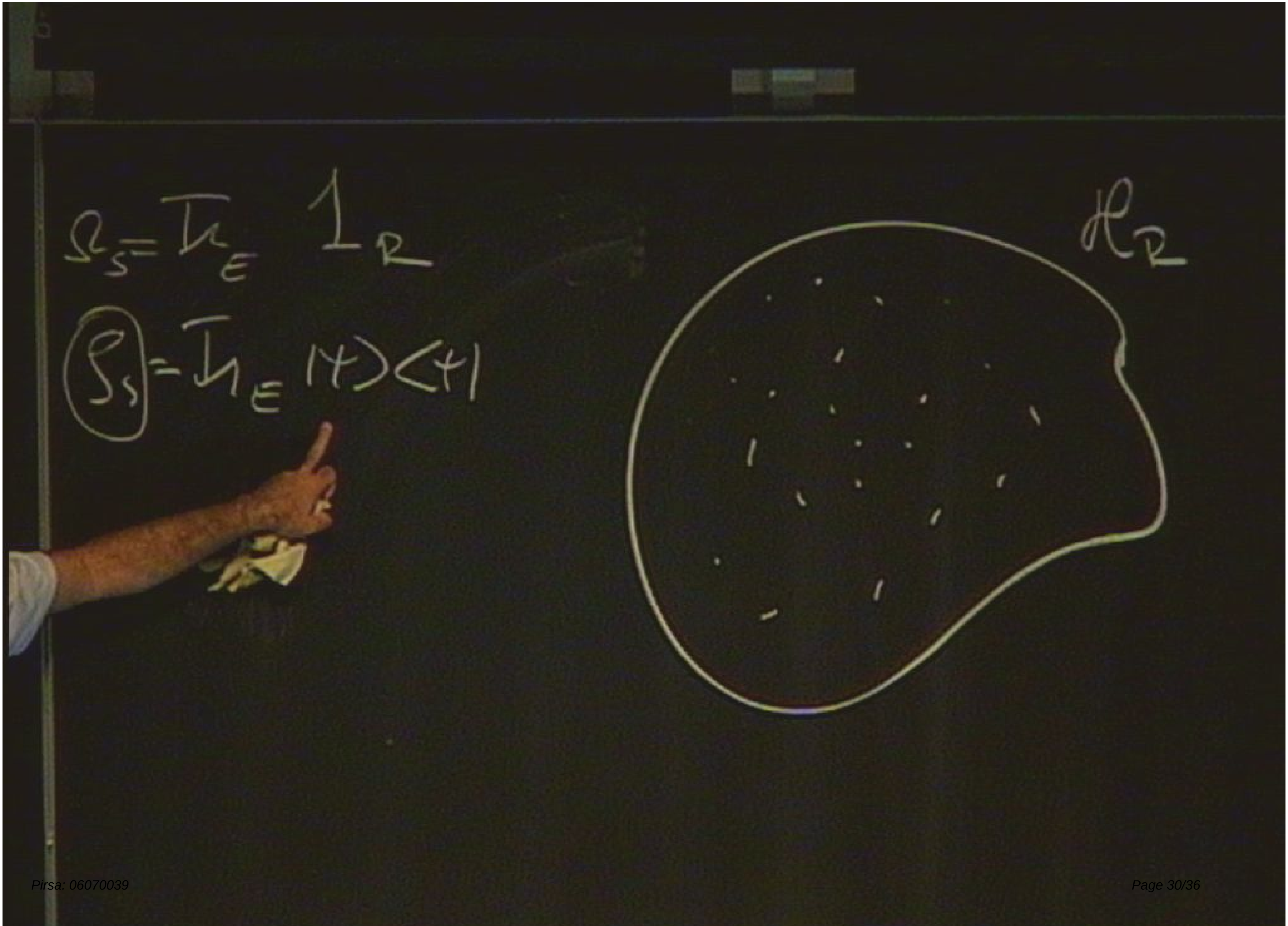
ε
s
H_R ⊆ H




$|t\rangle\langle t|$
E
S
t
$\mathcal{H}_R \subseteq \mathcal{H}$


$\Omega_S = \mathrm{Tr}_E \, 1_R$
$\mathcal{H}_R$


$\rho_S = \mathrm{Tr}_E \, 1_R$
$\rho_S = \mathrm{Tr}_E \, |\psi\rangle\langle\psi|$
$\mathcal{H}_R$

$$\rho_S = \mathrm{Tr}_E \, \mathbb{1}_R$$

$$\rho_S = \mathrm{Tr}_E \, |\psi\rangle\langle\psi|$$