Title: GPS and Relativity Continued

Date: Jul 15, 2006 11:00 AM

URL: http://pirsa.org/06070035

Abstract:

GPS and relativity

Physica Phantastica tour

- Ottawa
- Manatoulin Island
- Sudbury
- Waterloo
- Toronto
- Dryden
- Fort Francis
- Rainy River

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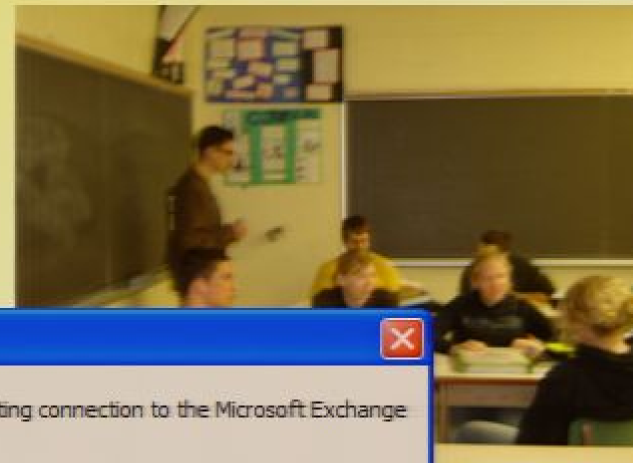

Microsoft Outlook

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OK

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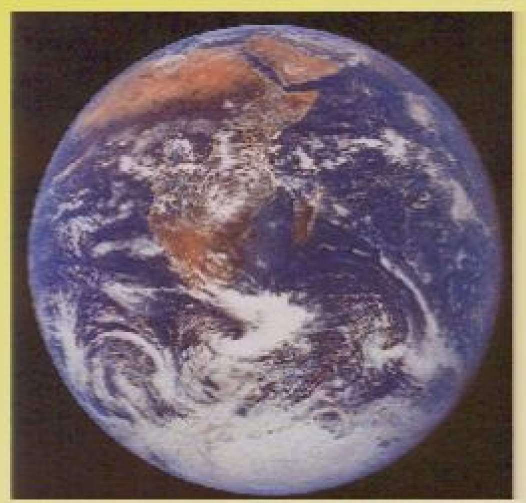
Global positioning system (GPS)



- 24 satellites
- 20,000 kms up
- radio waves

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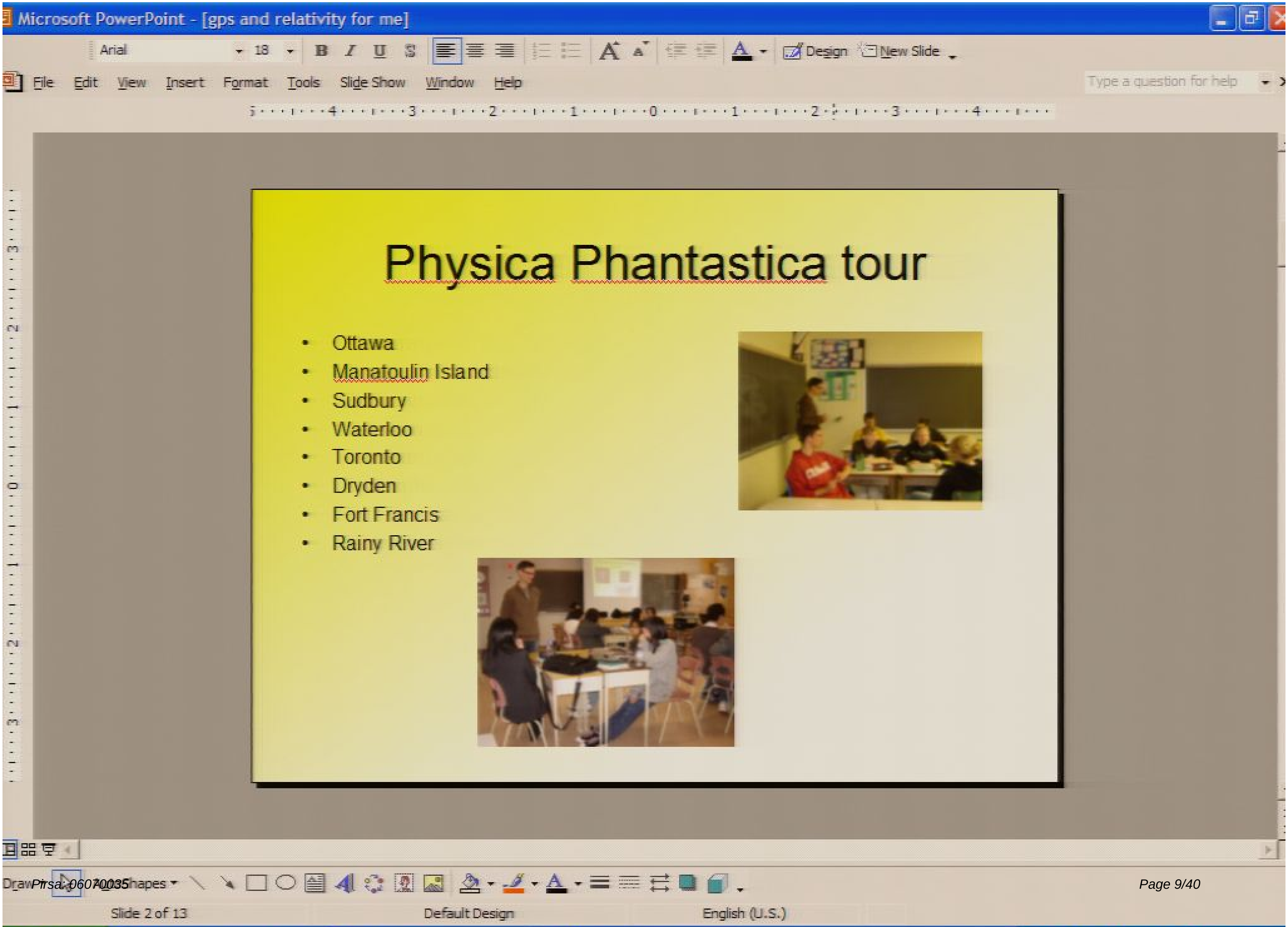
Microsoft PowerPoint - [gps and relativity for me]
Physica Phantastica tour
Ottawa
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Slide 2 of 13
Default Design
English (U.S.)

GPS and relativity

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Global positioning system (GPS)

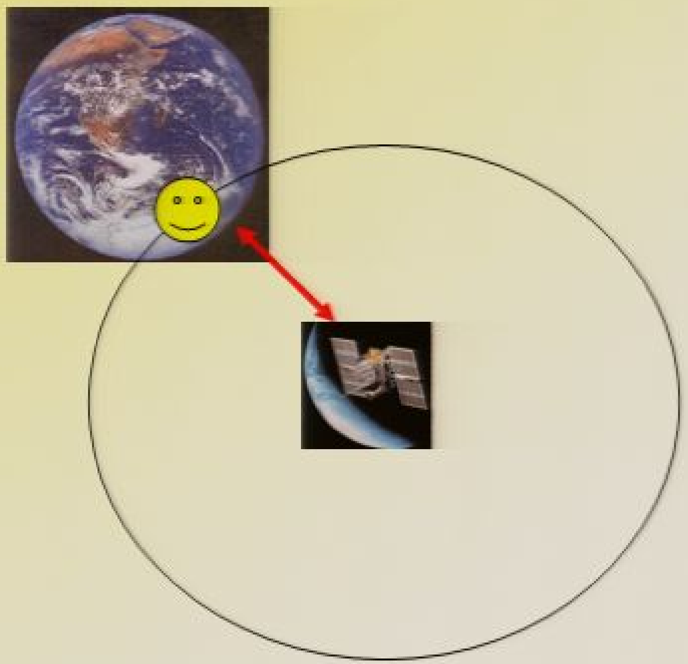
- 24 satellites
- 20,000 kms up
- radio waves

How does it work?

time how long it takes signal to travel from satellite to receiver

speed of radio waves = 300,000 km per second

distance = speed x time



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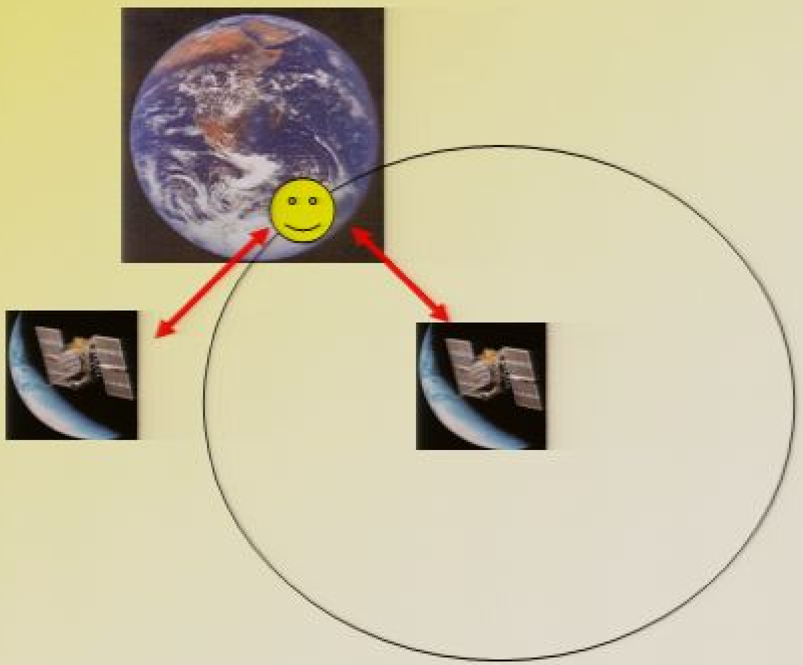


Uses

tractors

planes

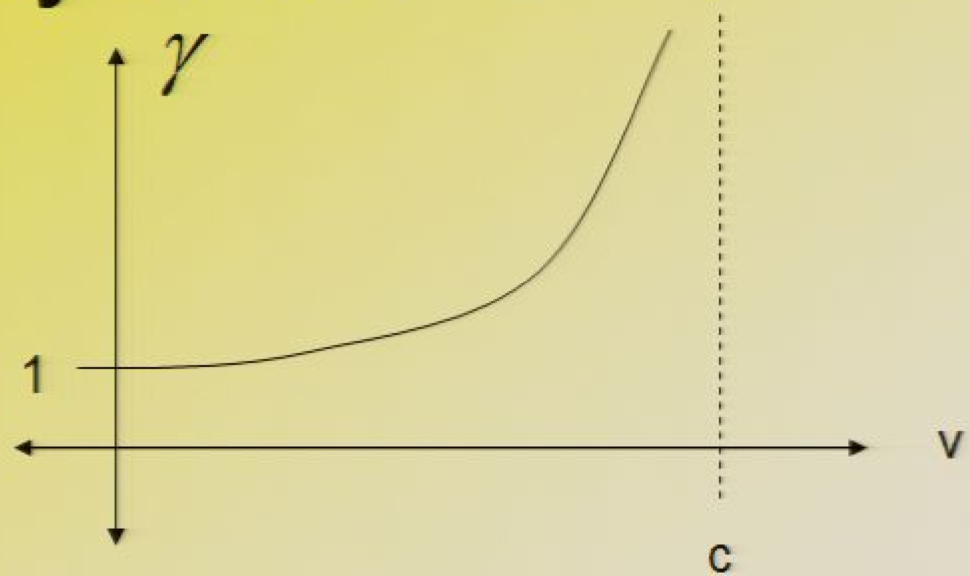
farmers

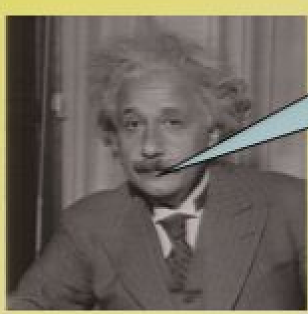
navigation

Relativity in the GPS

$$t = \gamma t_0$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$





If v=4km per second, what is γ (gamma)?

$$\gamma \approx 1 + \frac{v^2}{2c^2}$$

Calculate $\frac{v^2}{2c^2}$

Relativity in the GPS

$$t = \gamma t_0$$

$$\gamma = \frac{1}{\sqrt{\quad}}$$

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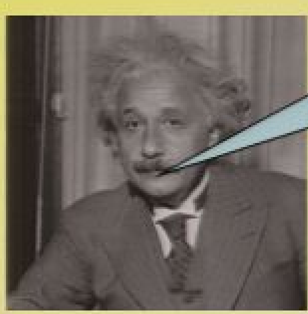
OK

(gamma)?



$$\gamma \approx 1 + \frac{v^2}{2c^2}$$

Calculate $\frac{v^2}{2c^2}$

Binomial expansion

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

But, the binomial expansion says that:

$(1+x)^n=1+nx+n(n-1)\,x^2/2 + n(n-1)(n-2)x^3/3! + \ldots$

Comparing $\quad (1+x)^n \; and \left(1-\frac{v^2}{c^2}\right)^{-1/2}$

the two are equivalent if $n=-1/2$ and $x=-v^2/c^2$.

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$$1 + nx$$

$$= 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right)$$

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Satellite clocks slow by around 10^{-10} seconds each second.

- corresponds to what distance error each second?
- a) 3 cm
- b) 3 m
- c) 3 mm

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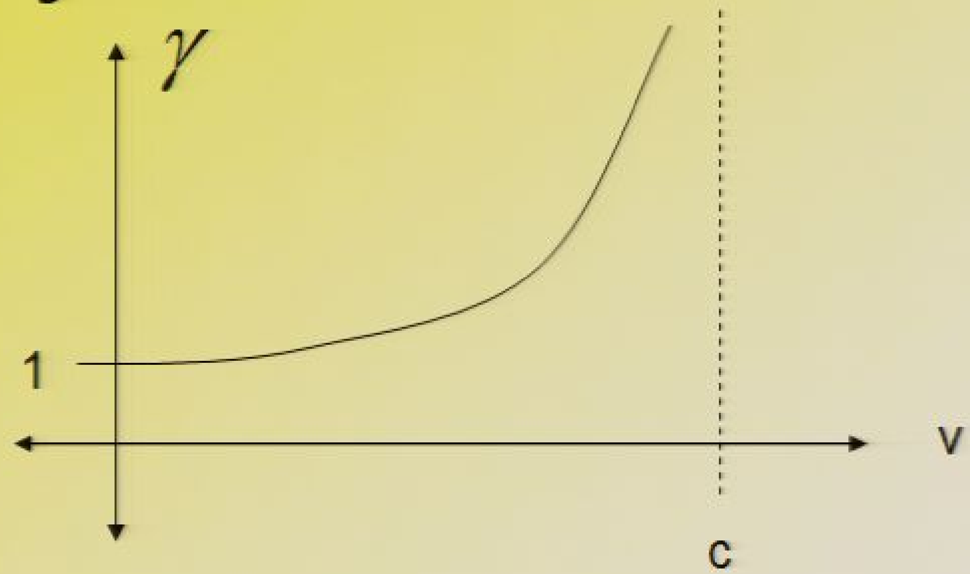

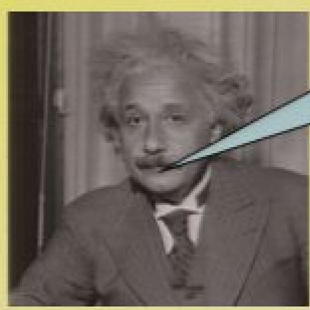

If v=4km per second, what is γ (gamma)?

$$\gamma \approx 1 + \frac{v^2}{2c^2}$$

Calculate $\frac{v^2}{2c^2}$ $\quad 8.89 \times 10^{-11}$

- As $x = -v^2/c^2 << 1$, we only need to consider the first two terms in the expansion as the remaining ones are very, very small.

- Eg. for v=4km/s, $v^2/c^2 = 1.8 \times 10^{-10} = x$
  and so $n(n-1)x^2/2 = v^4/(8c^4) \approx 10^{-20} << nx$ etc.

Satellite clocks slow by around 10^{-10} seconds each second.

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Errors accumulate!

- Imagine that you are the pilot of a Boeing 747 full of 300 passengers about to land at Toronto Pearson International airport in the middle of a snowstorm. You cannot see the runway.

  Like many commercial pilots, you are using GPS navigation to help guide you. Let's pretend that you think that Einstein's theory of relativity is "just a theory" and completely impractical and so, somehow, turned off the relativity corrections on your GPS unit a day ago.

STUDENT ACTIVITY:

How far away from the runway will the resulting error cause you to land (assuming that you are solely using GPS navigation) and that you have neglected the effects of special relativity for *one day*.

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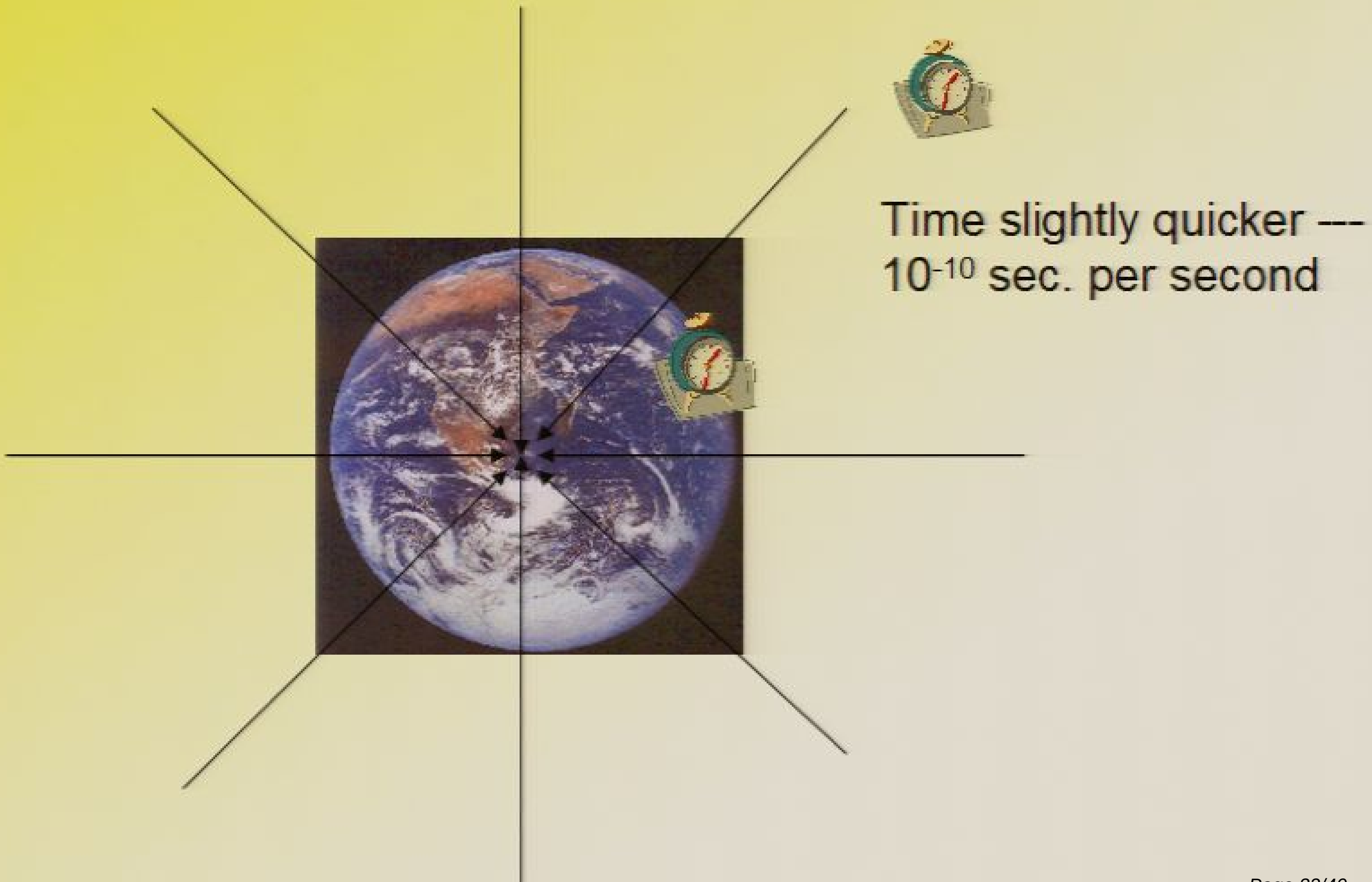
Answer

There are 24 hours in each day, each of which has 3600 seconds.

Therefore, the error is:

3cm x 24 hours x 3600 seconds = 2.5 km per day!

Gravity slows down time



50,000 ns DAY FAST GR

11,000 ns " SLOW SR

39,000 ns FAST TOTAL

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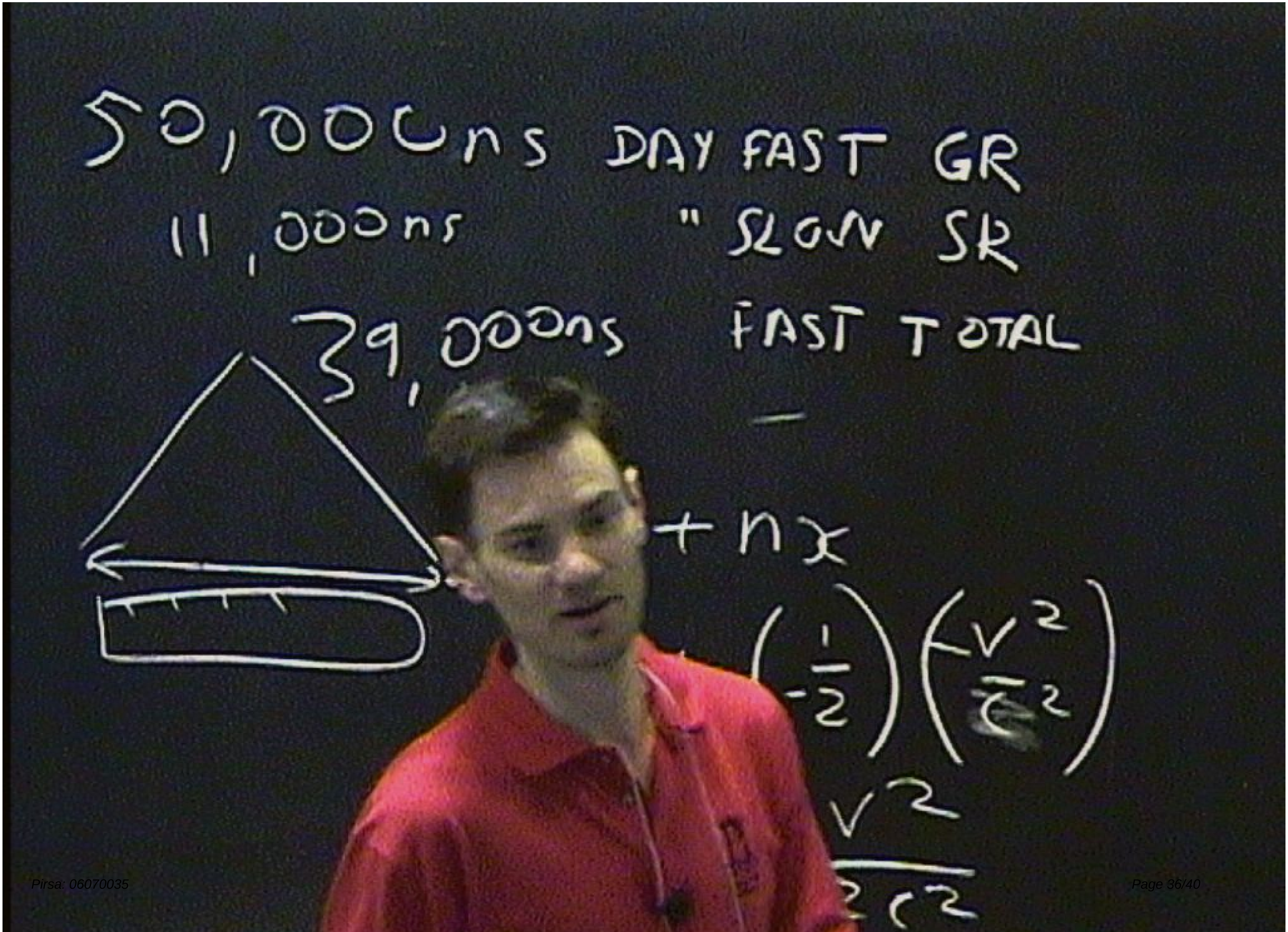
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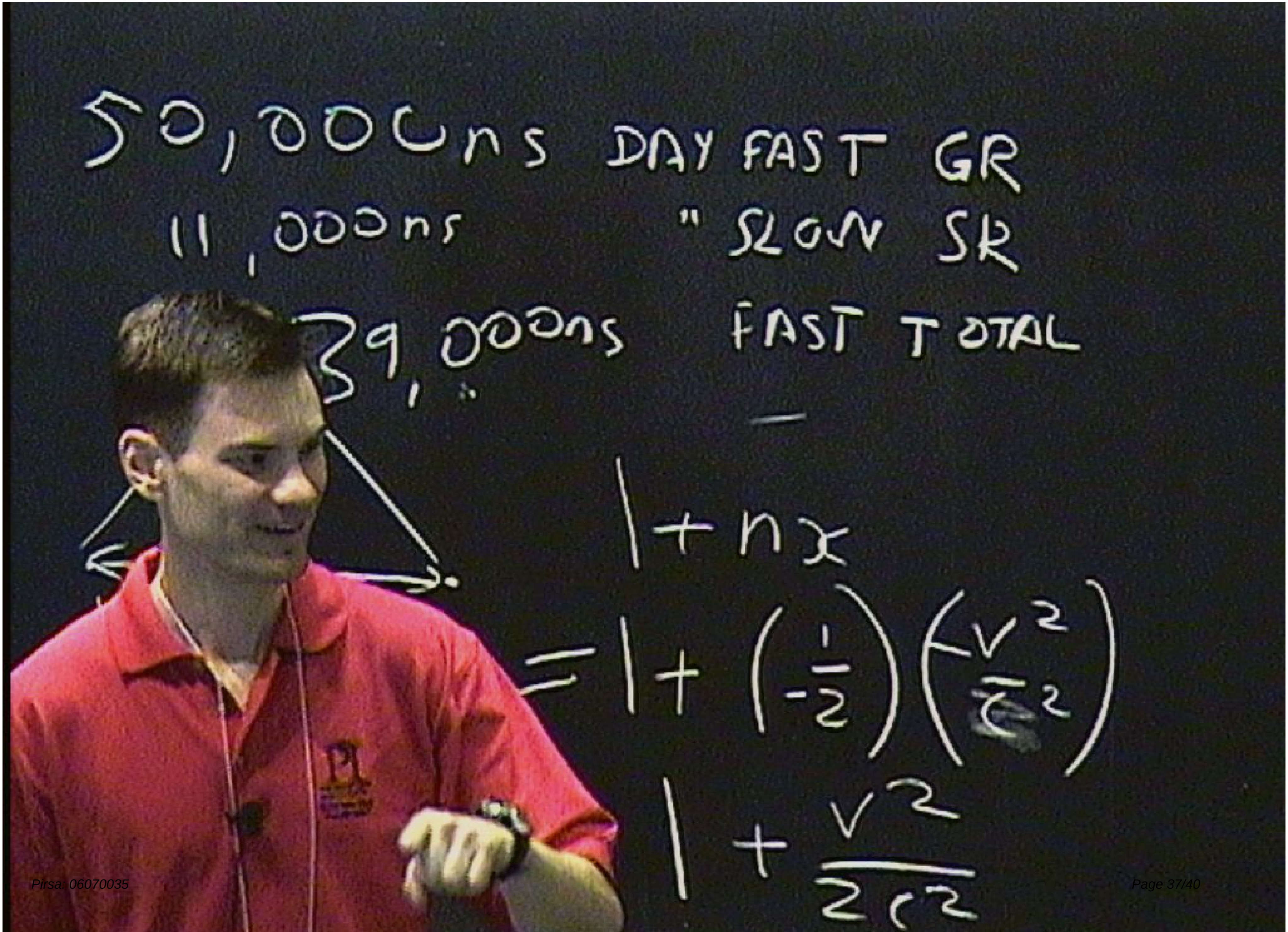
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1+nx
=1+(1/-2)(-v2/c2)
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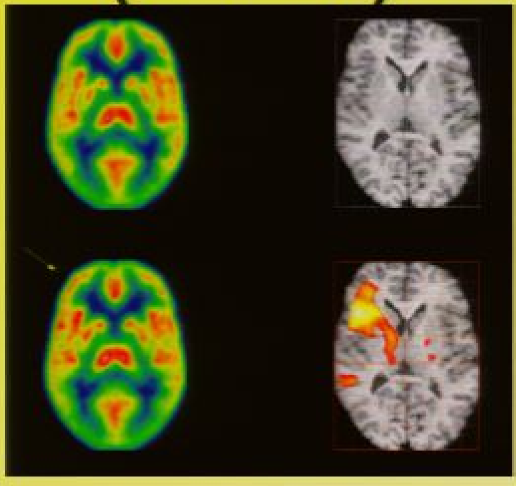
Positron Emission Tomography (PET): Einstein in the hospital



cancer detection, brain research

quantum physics & special relativity

antimatter!

$E=mc^2$

$m_{electron} = m_{positron} = 9.11 \times 10^{-31}$ kg

$\therefore$

Microsoft PowerPoint - [gps and relativity for me]

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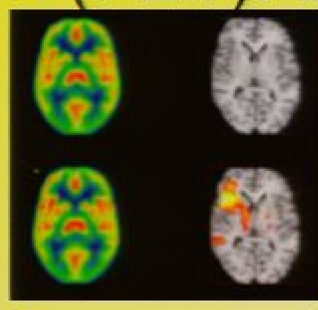

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$E=mc^2$

$m_{electron} = m_{positron} = 9.11\text{x}10^{-31}$ kg

$\therefore \quad E = 1.6 \times 10^{-13}$ J

Slide 13 of 13 Default Design English (U.S.)