Title: Enrichment Presentation - What does special relativity mean? Part 1 - Continued

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Abstract:

# Twin postulates of special relativity

- 1. **Relativity Postulate:** The laws of physics are the same in all inertial reference frames.
- 2. **Speed of light postulate:** Light travels at the constant speed of c ($3 \times 10^8$ $ms^{-1}$) through empty space relative to all inertial observers.
- *What is an inertial reference frame?*
  No acceleration.
  Newton's first law holds.
- Consider the relativity postulate:
  Physics should be universal, democratic

  In 1905, it was both revolutionary and pre-dated.

- Galileo in the 17th century: The laws of *mechanics* are the same in all inertial reference frames.

- Revolutionary aspect of the relativity postulate was that Einstein broadened ‘mechanics’ to ‘physics’.

- Part of Einstein’s genius was to boldly believe that that the laws of electromagnetism (Maxwell’s equations) applied to all inertial observers, even though he knew that this had radical implications. (Eg. he knew that this implied that the speed of light was independent of an observer’s velocity relative to the light’s source).

# Speed of light postulate

- Crudely speaking, 'Everyone measures light to travel at c'.
- Counter-intuitive


- What speed will you see the snowball moving towards you at?
  a) 45 $kmh^{-1}$
  b) 15 $kmh^{-1}$
  c) 2 $kmh^{-1}$
  d) 30 $kmh^{-1}$

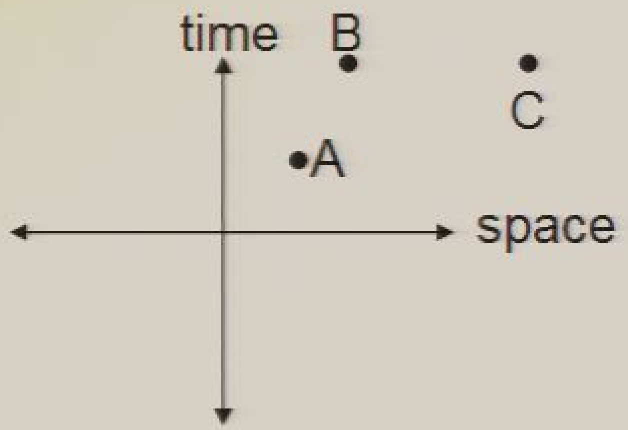
# Spacetime diagrams

- Consider the following unrelated happenings or *events*.
  An apple falling from a tree, (A)
  A bird taking flight from a tree, (B)
  A car coming to a stop at a set of traffic lights, (C)

  They all occur at a particular location and time

- Natural to plot or represent them graphically on a graph with space and time axes.

- By convention, we put time on the vertical axis.



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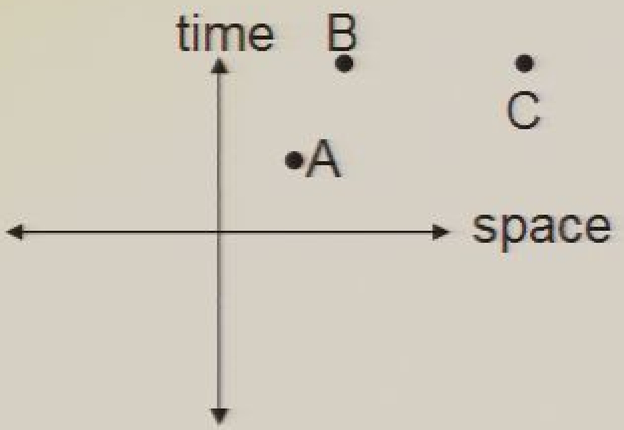
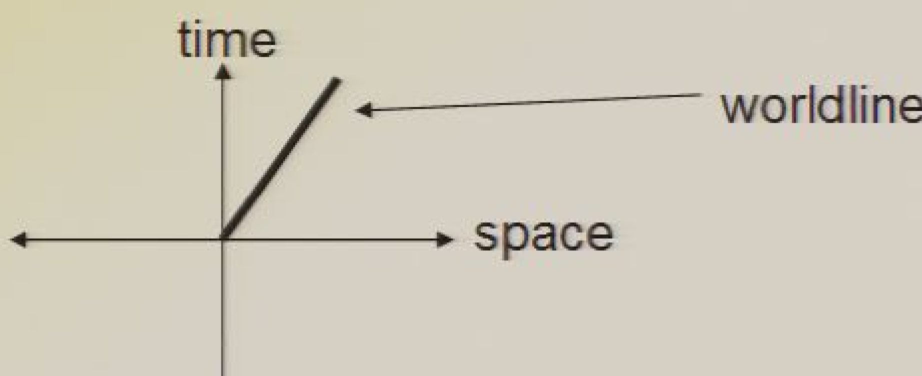


- Such a graph is called a *spacetime* diagram. Incredibly useful tool for understanding special relativity.

*"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."* — Hermann Minkowski

What do a series of events look like on a spacetime diagram? Eg. a car driving at $60kmh^{-1}$ in one direction on a highway.



# Activity

Draw spacetime diagrams that show the worldlines for:

i) A car that drives for one hour at 100 $kmh^{-1}$ in one direction, turns around, and then drives for an hour in the opposite direction

ii) A Grade 12 physics student standing still waiting for a bus

iii) An Olympic athlete running at a constant velocity in a 100 metres race.

iv) A car driving around in circles (n.b. this requires a 3D spacetime diagram).

# The metric of space and time

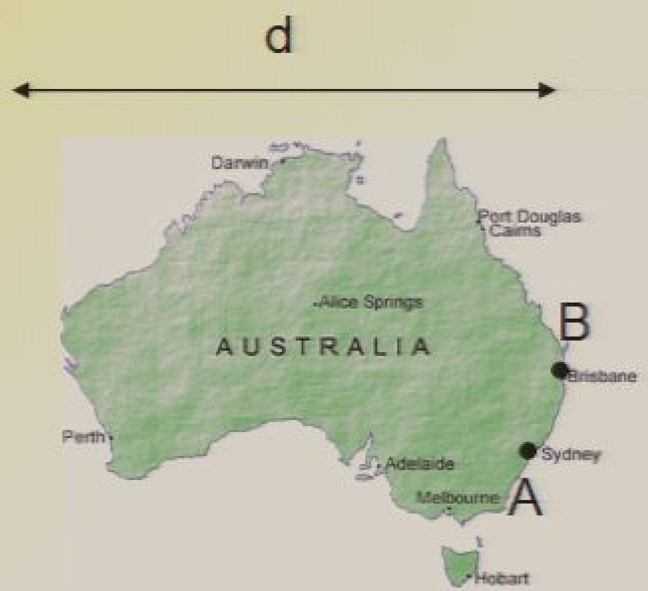
- Imagine that we would like to calculate the distance between points A and B on the map below.
- Use the Pythagorean theorem $d^2 = \Delta x^2 + \Delta y^2$ (nelgecting the earth's curvature)

This is called a *metric equation*

*metric* = technical term for distance

# What is the metric equation for spacetime

- As the two axes are x and t is it just $d^2 = \Delta x^2 + (c\Delta t)^2$?
- Or perhaps, according to Einstein, there is something different about space and time.
- To held decide, consider a “light clock”

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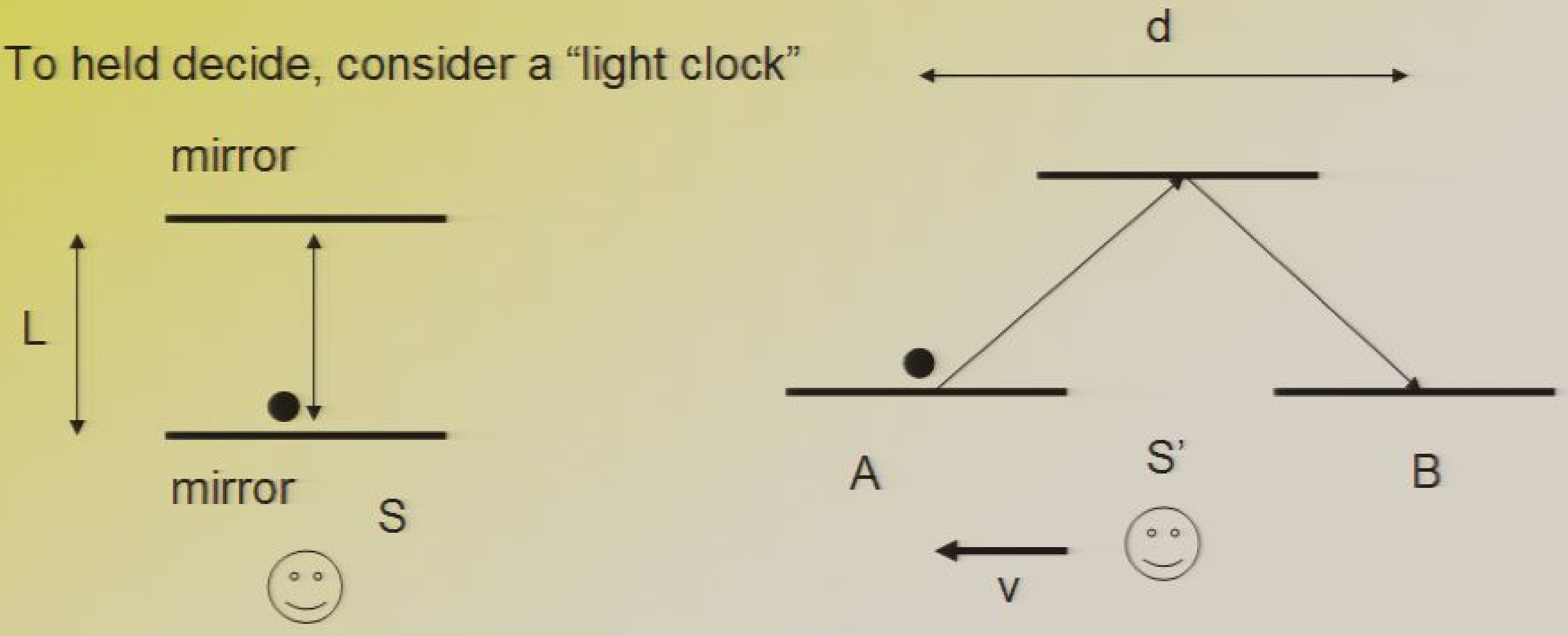


- A and B are the events corresponding to a photon hitting the bottom mirror on two successive occasions.

- Let us denote the spacetime ‘distance’ or interval between A and B by s

- In frame S, this interval is made up from just time and so s=2L

  (Using ‘light time’ where we measure time by the distance light travels in the time under consideration.)

- In frame S’, the time t’ between A and B is given by

$$t' = \frac{\text{distance}}{\text{speed}} = \frac{2\sqrt{L^2 + \left(\frac{d}{2}\right)^2}}{c}$$

- Plugging L=s/2 into the above equation and squaring both sides yields

$$t'^2 = \frac{s^2 + d^2}{c^2}$$

$$\therefore s^2 = (ct')^2 - d^2$$



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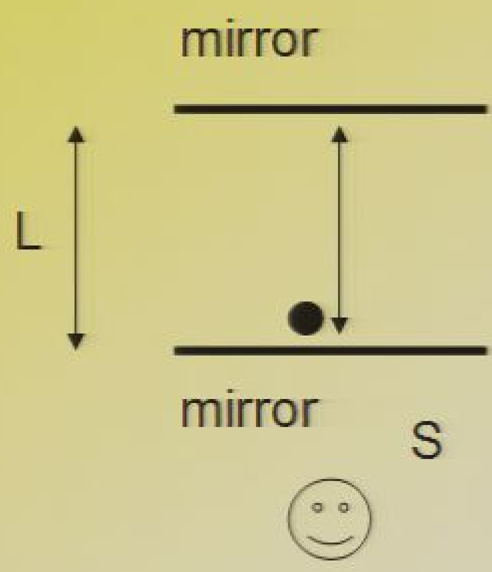


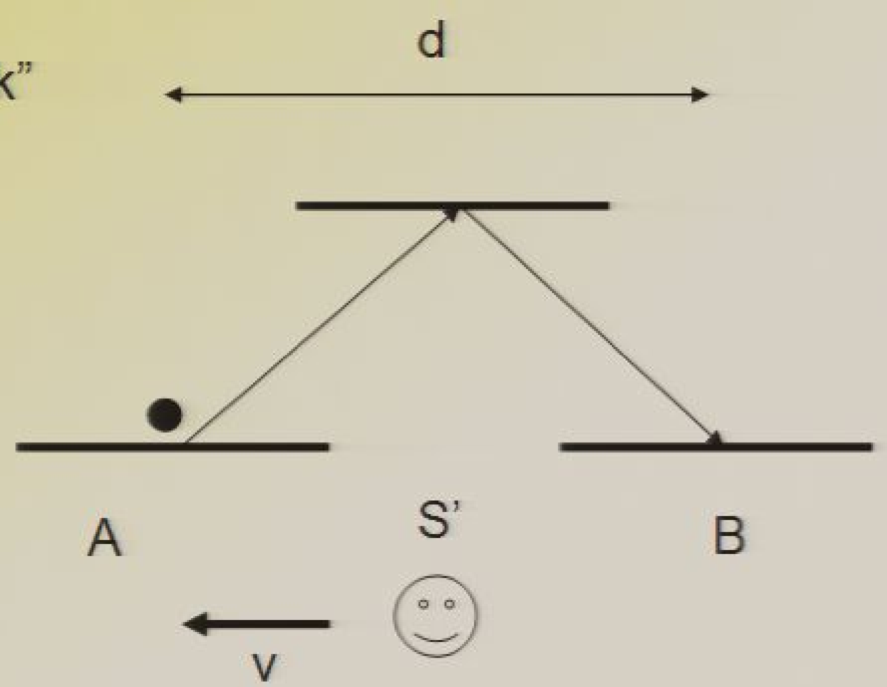


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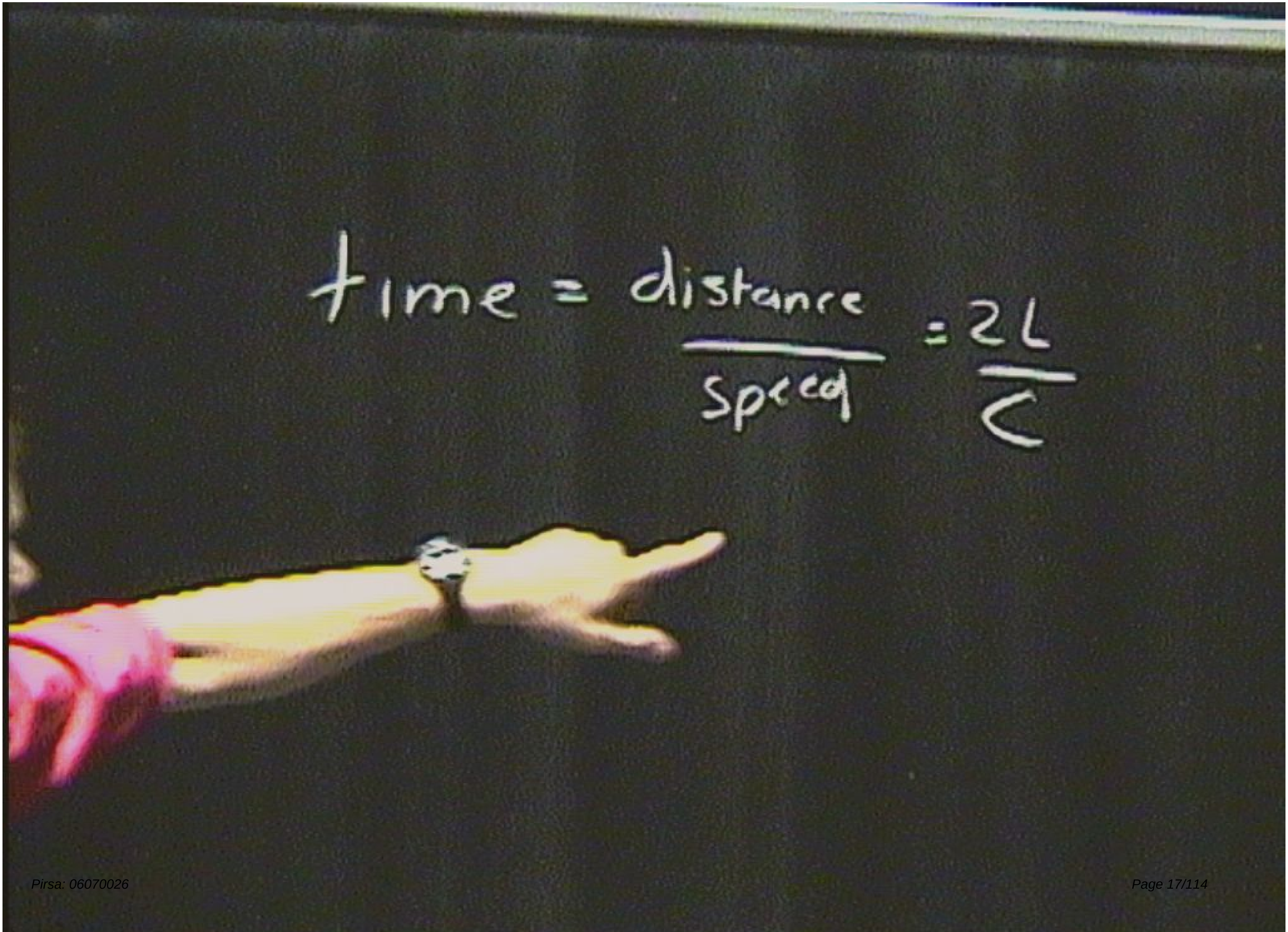
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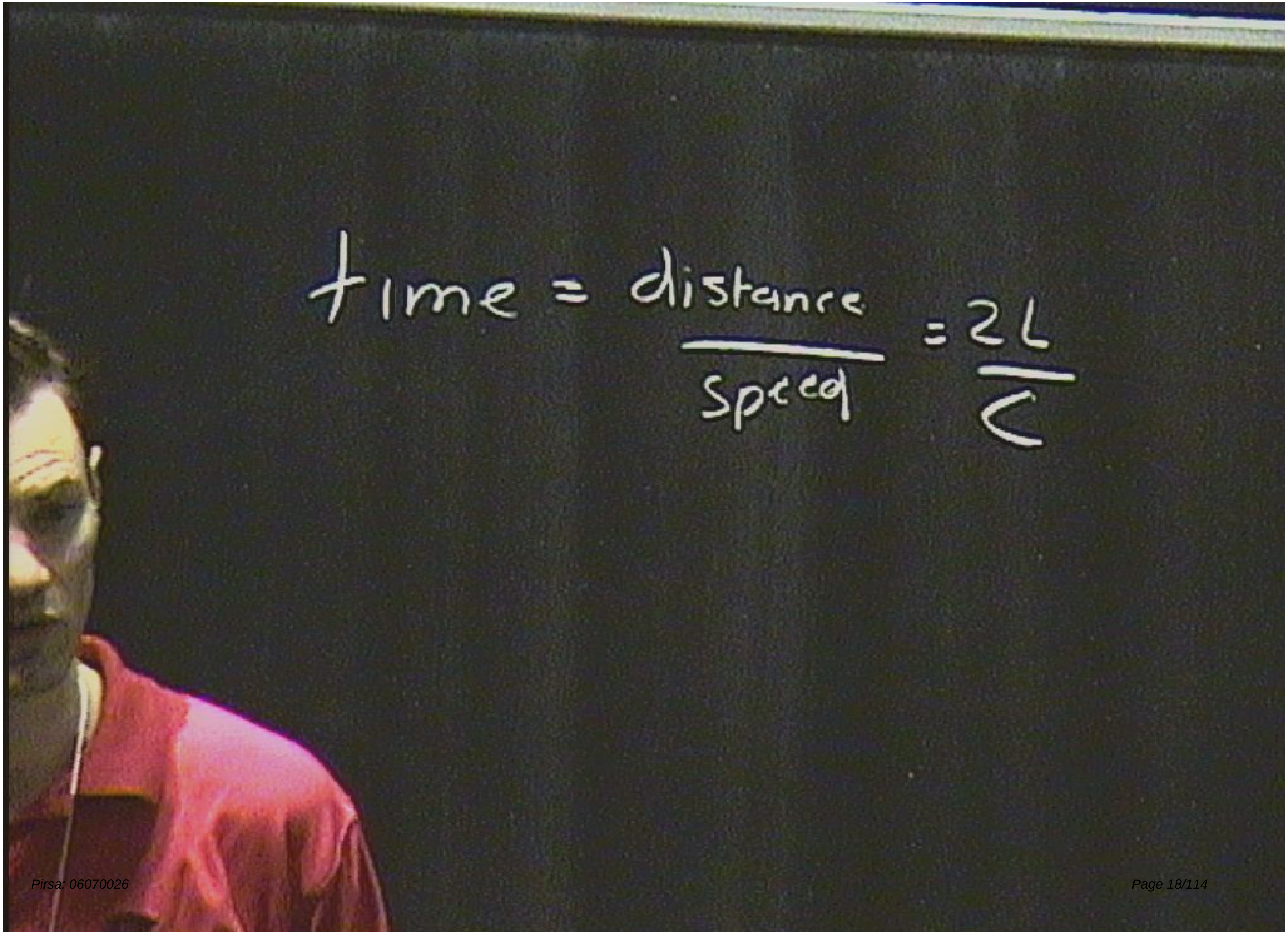
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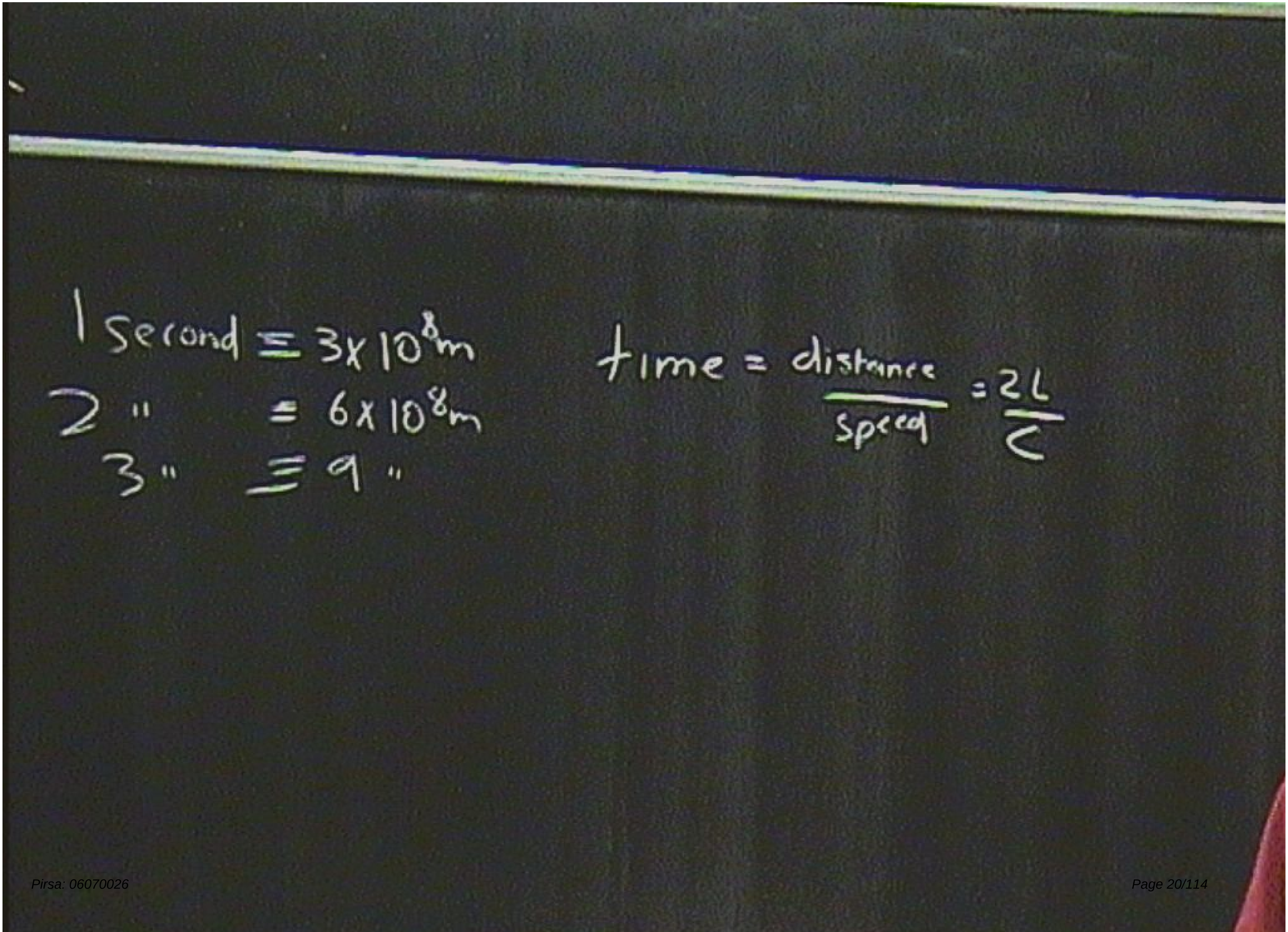
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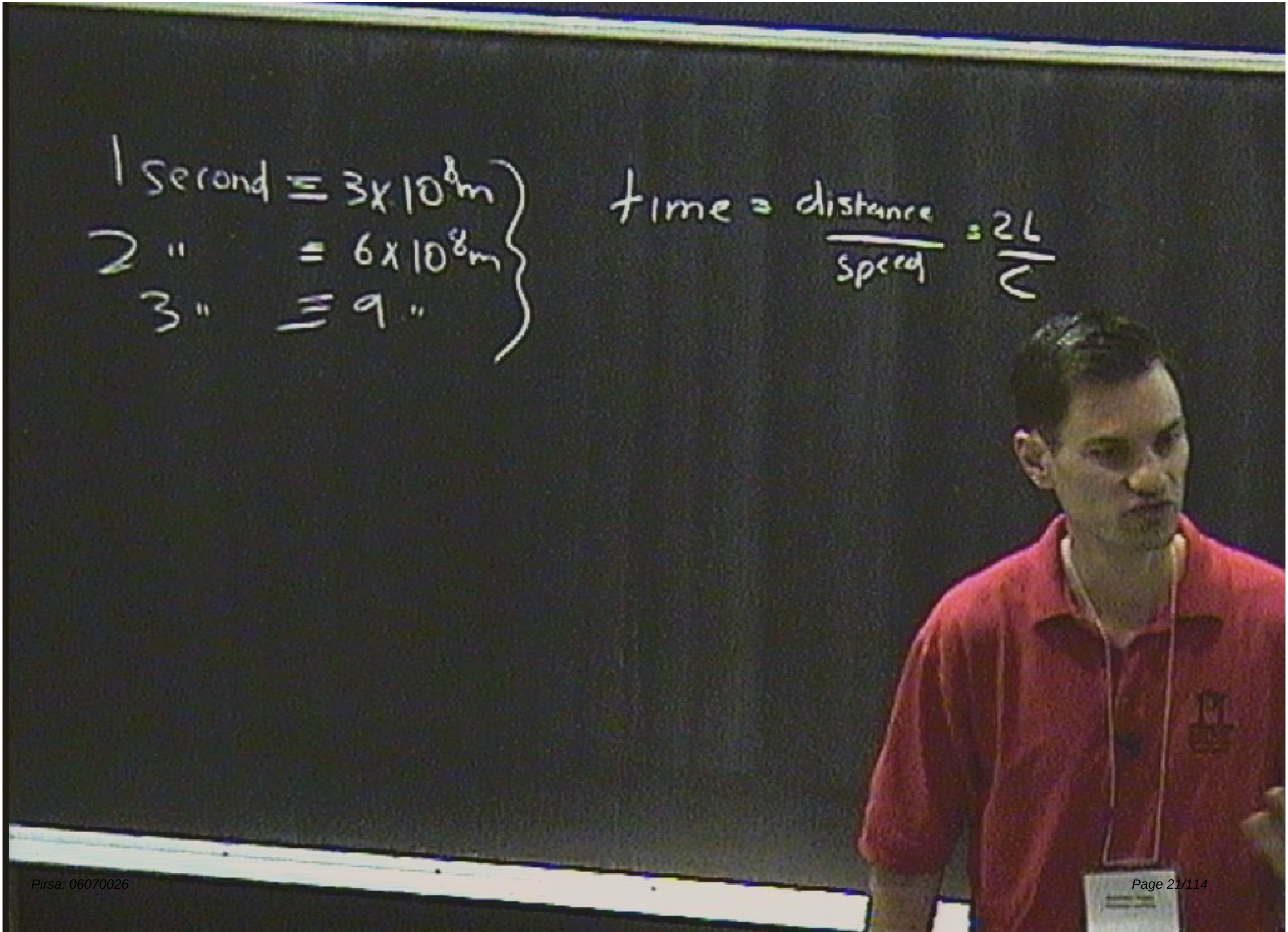
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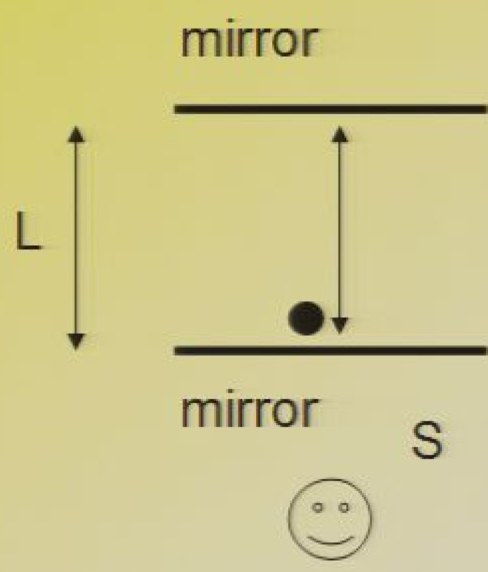


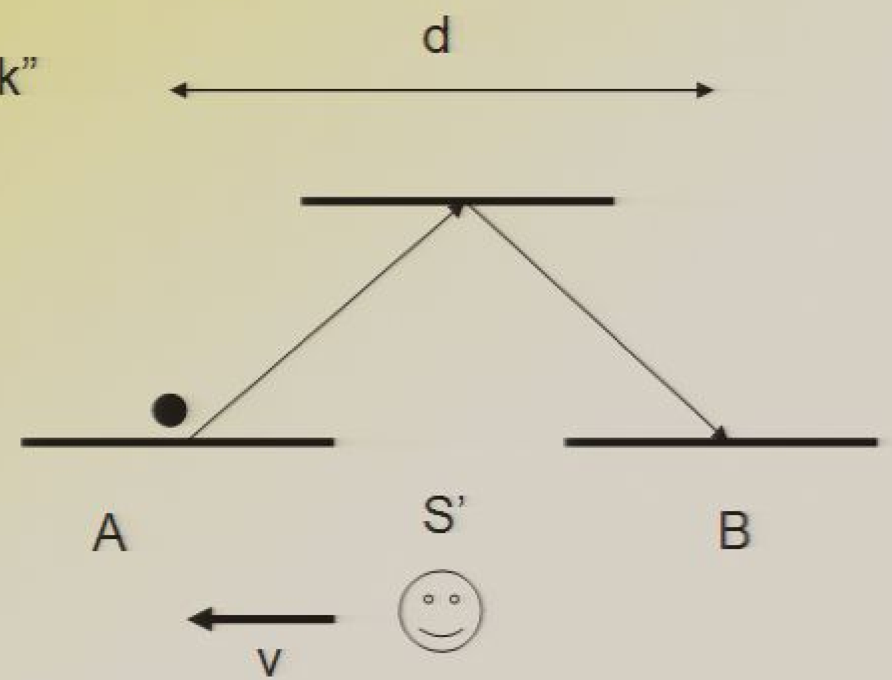


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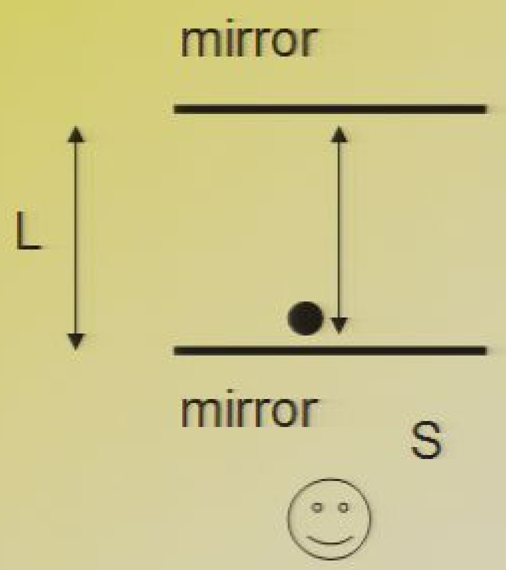


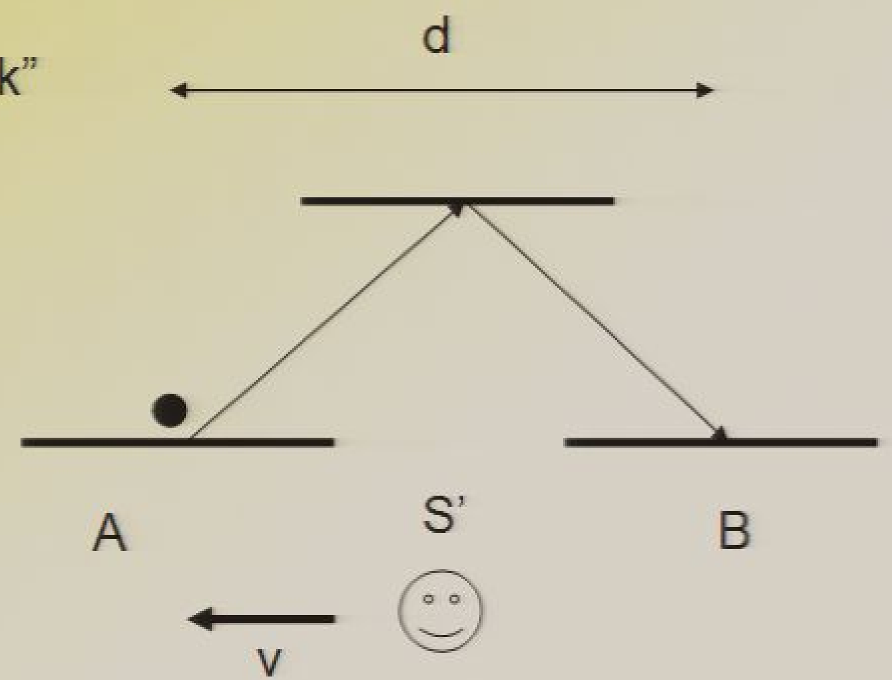


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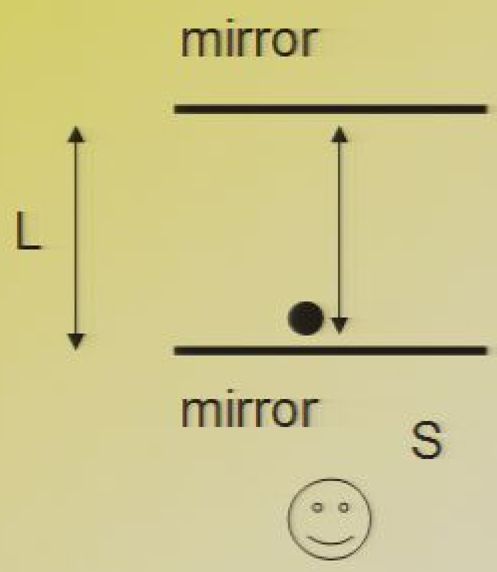


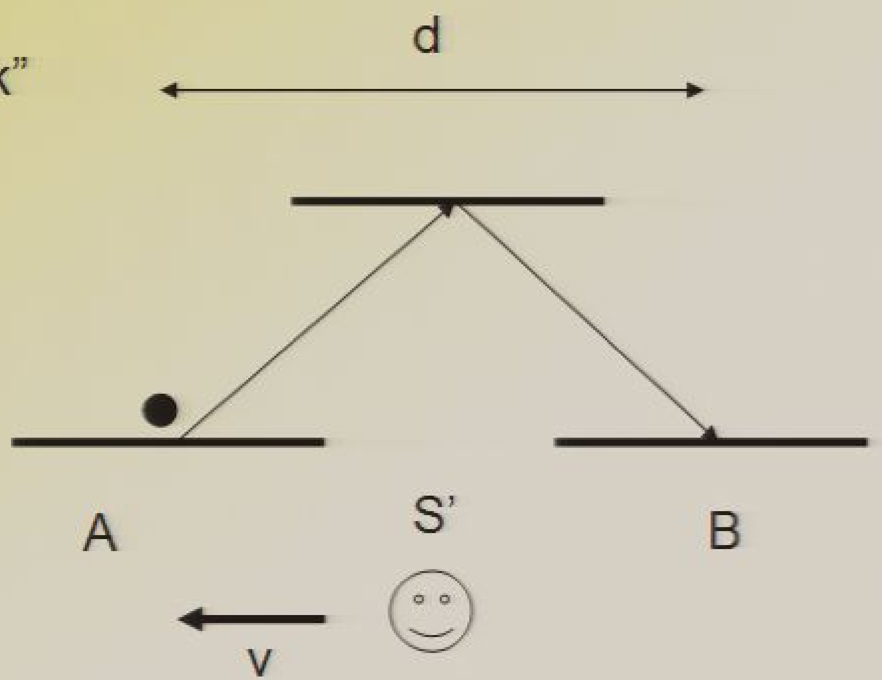


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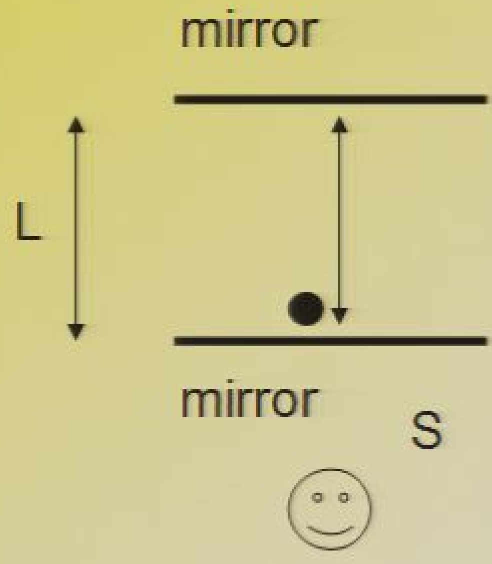


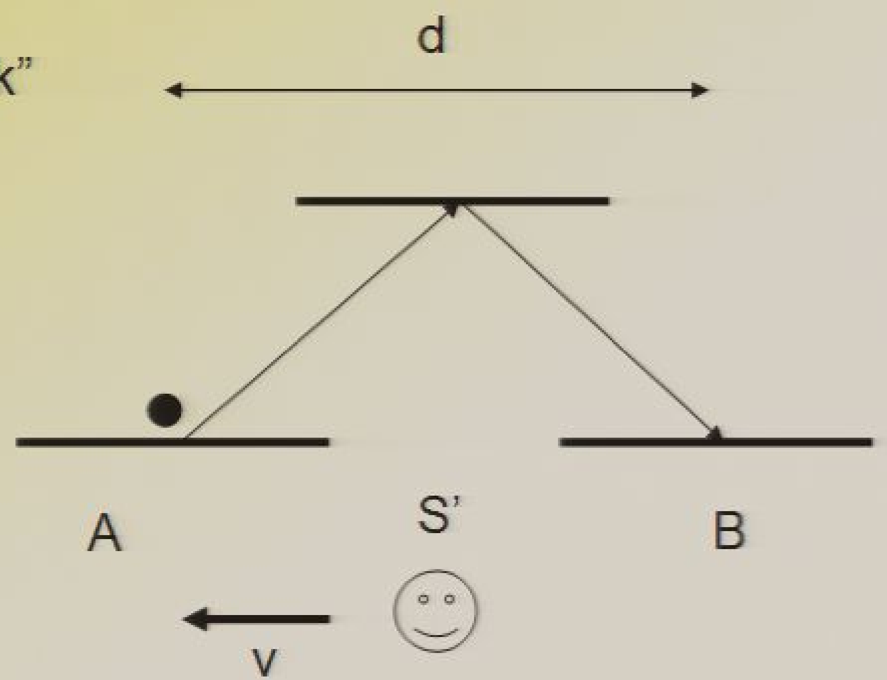


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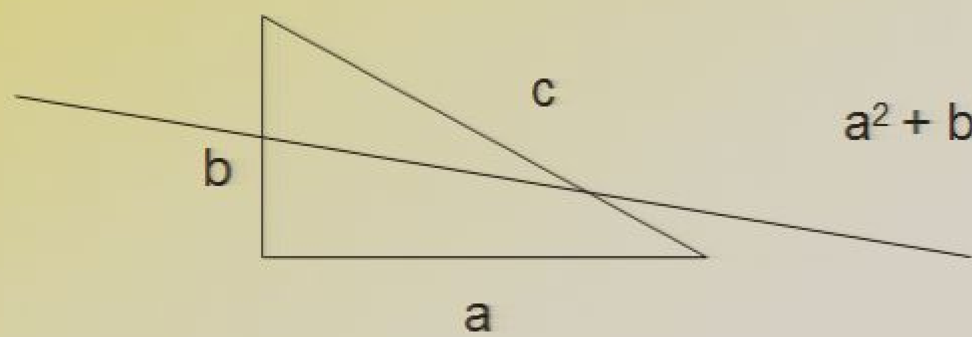
# What does the metric equation mean? Why the minus sign?

- Might have expected $s^2 = x^2 + (ct)^2$ but this does not work as the right-hand side varies for different observers. A metric is the same for all observers.

- The minus sign means that space and time are connected to each other in an unexpected manner.

- Spacetime is *not* Pythagorean or Euclidean.

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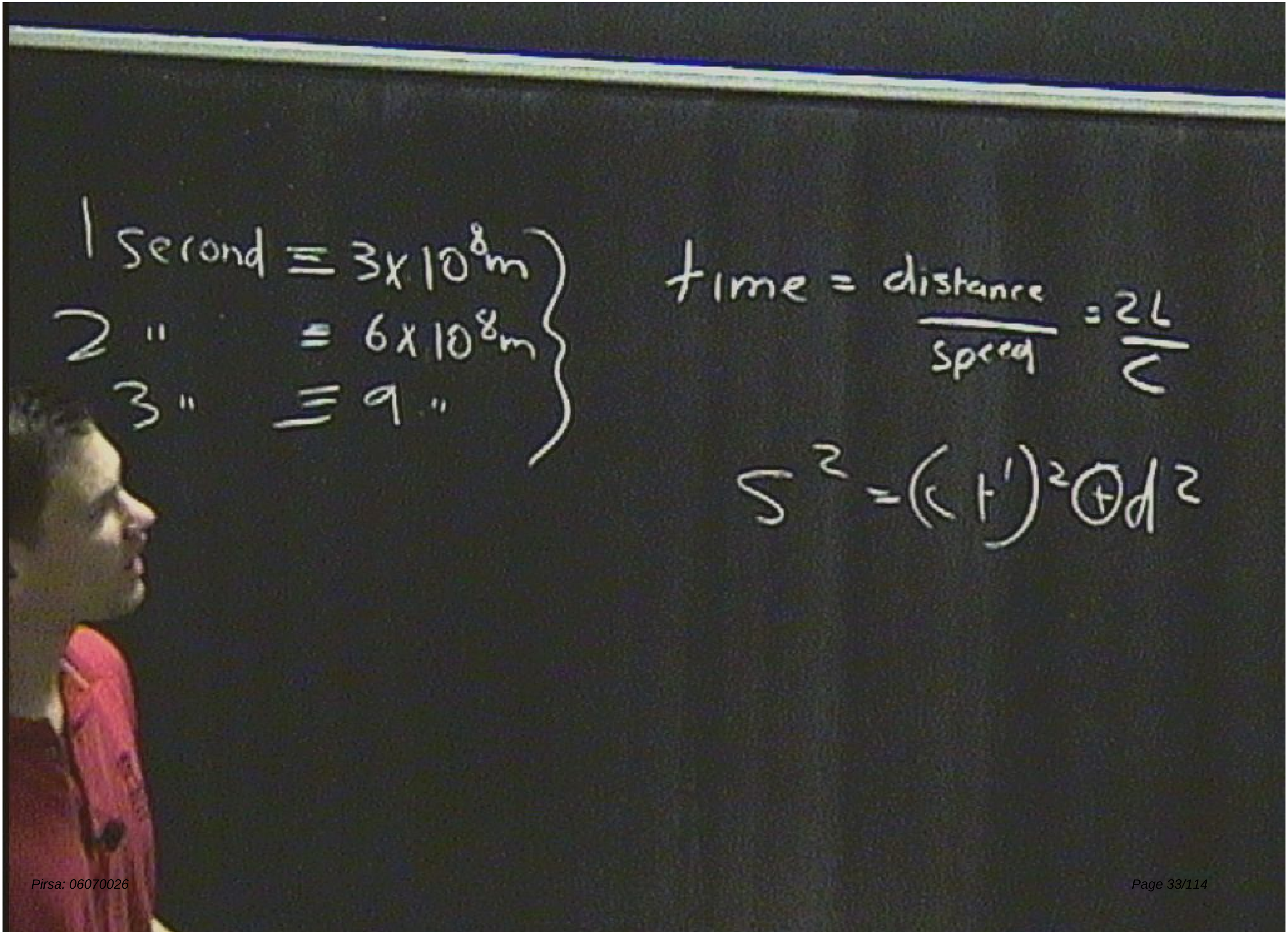
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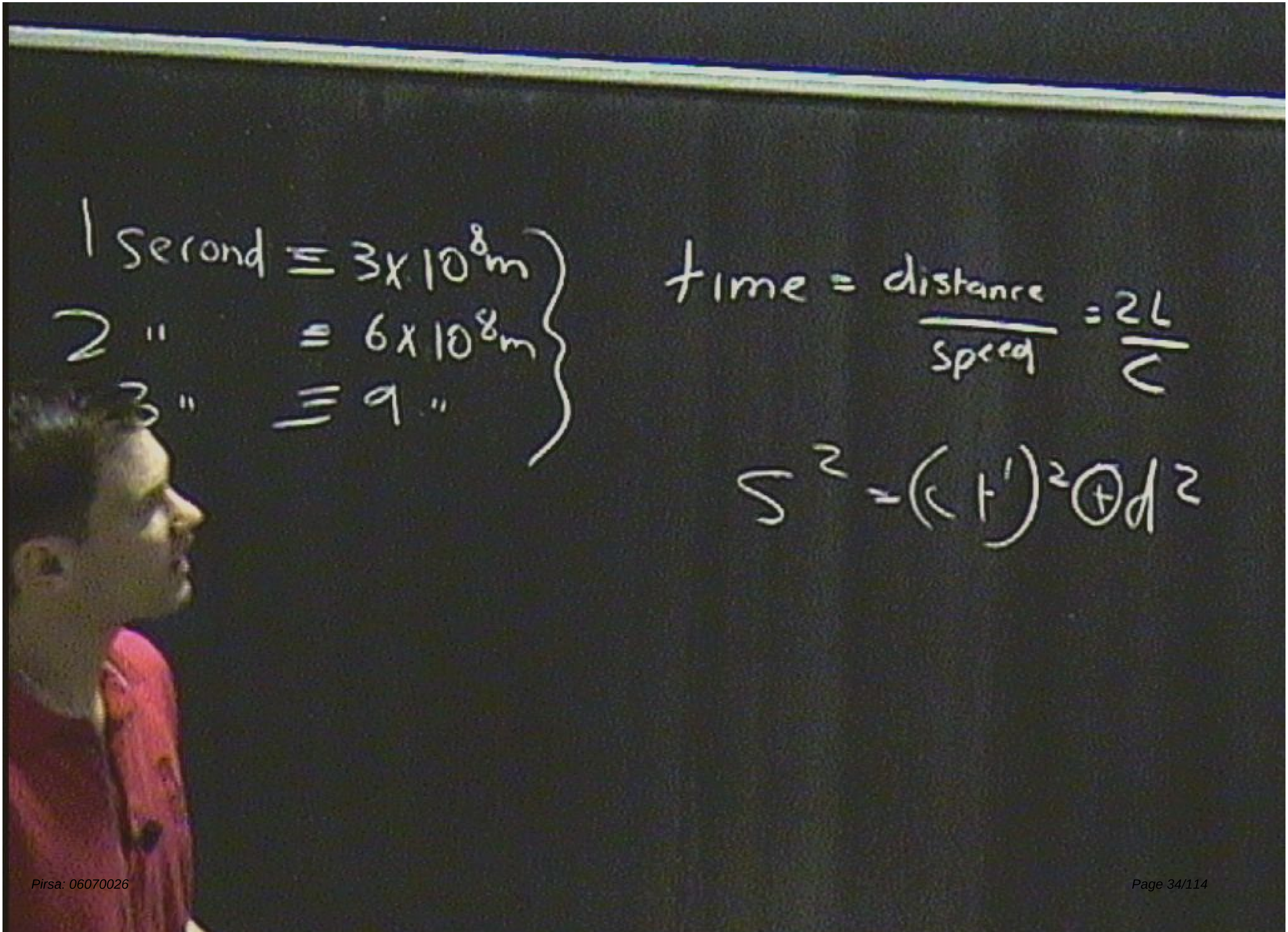
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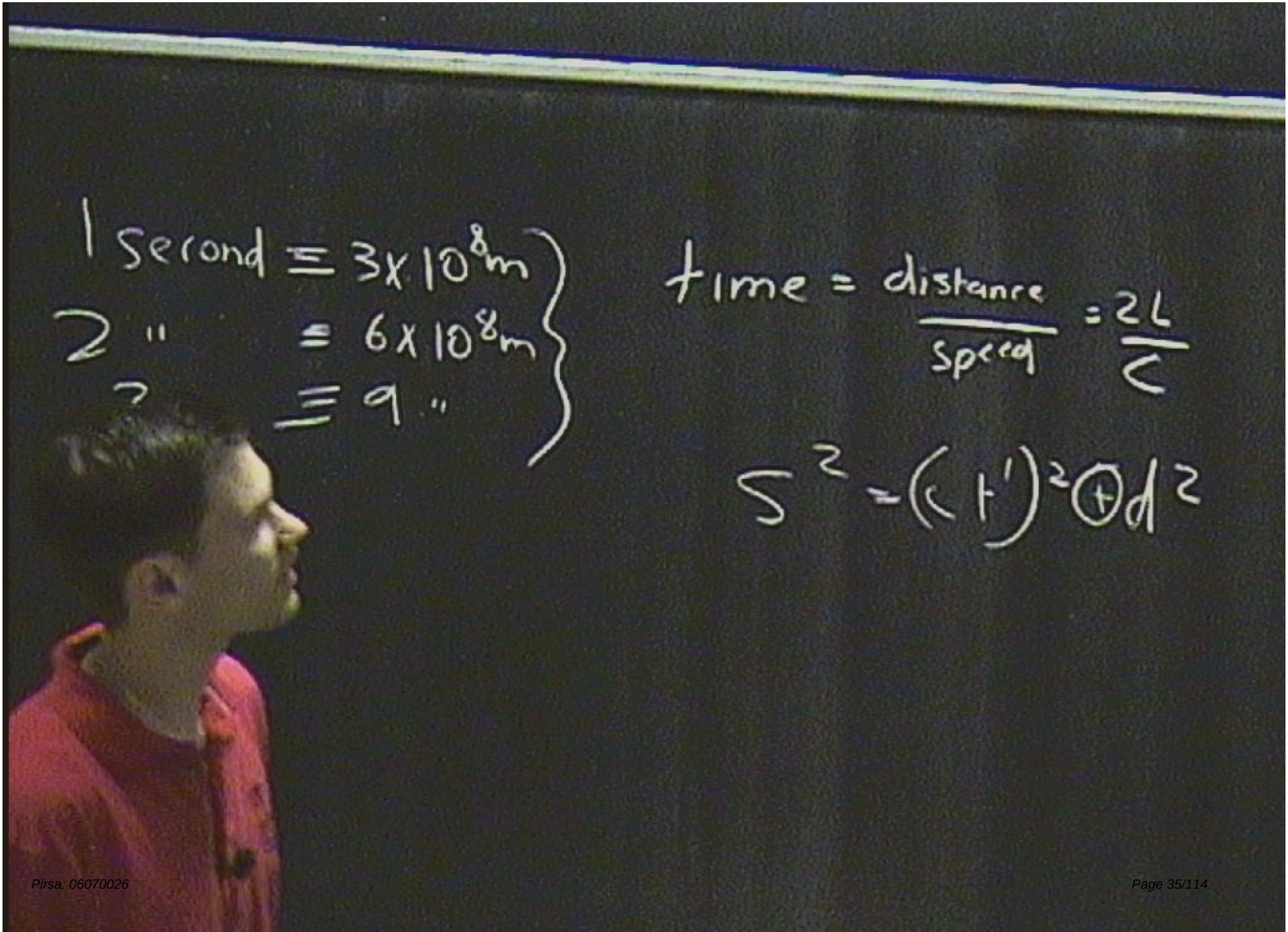
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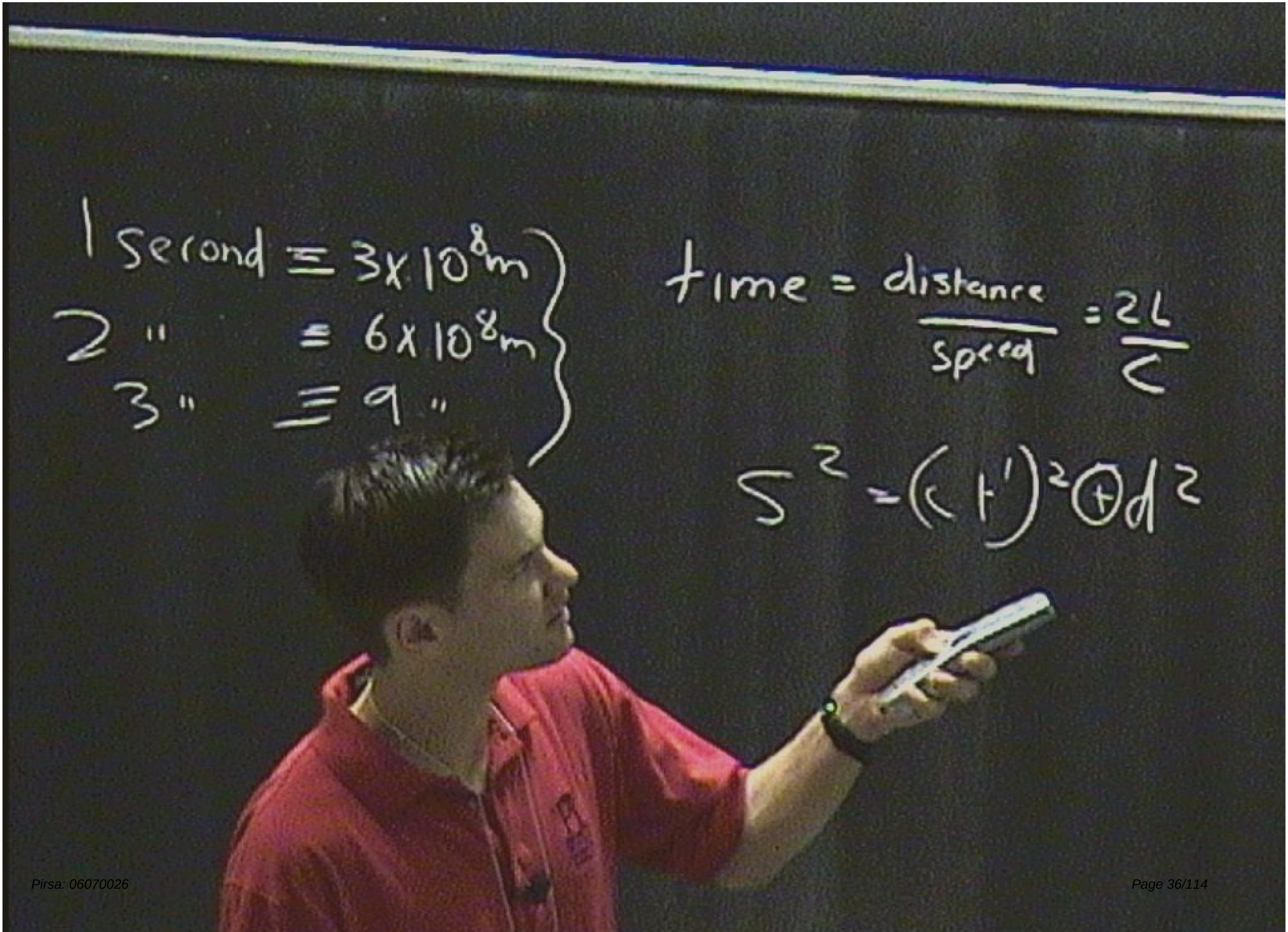
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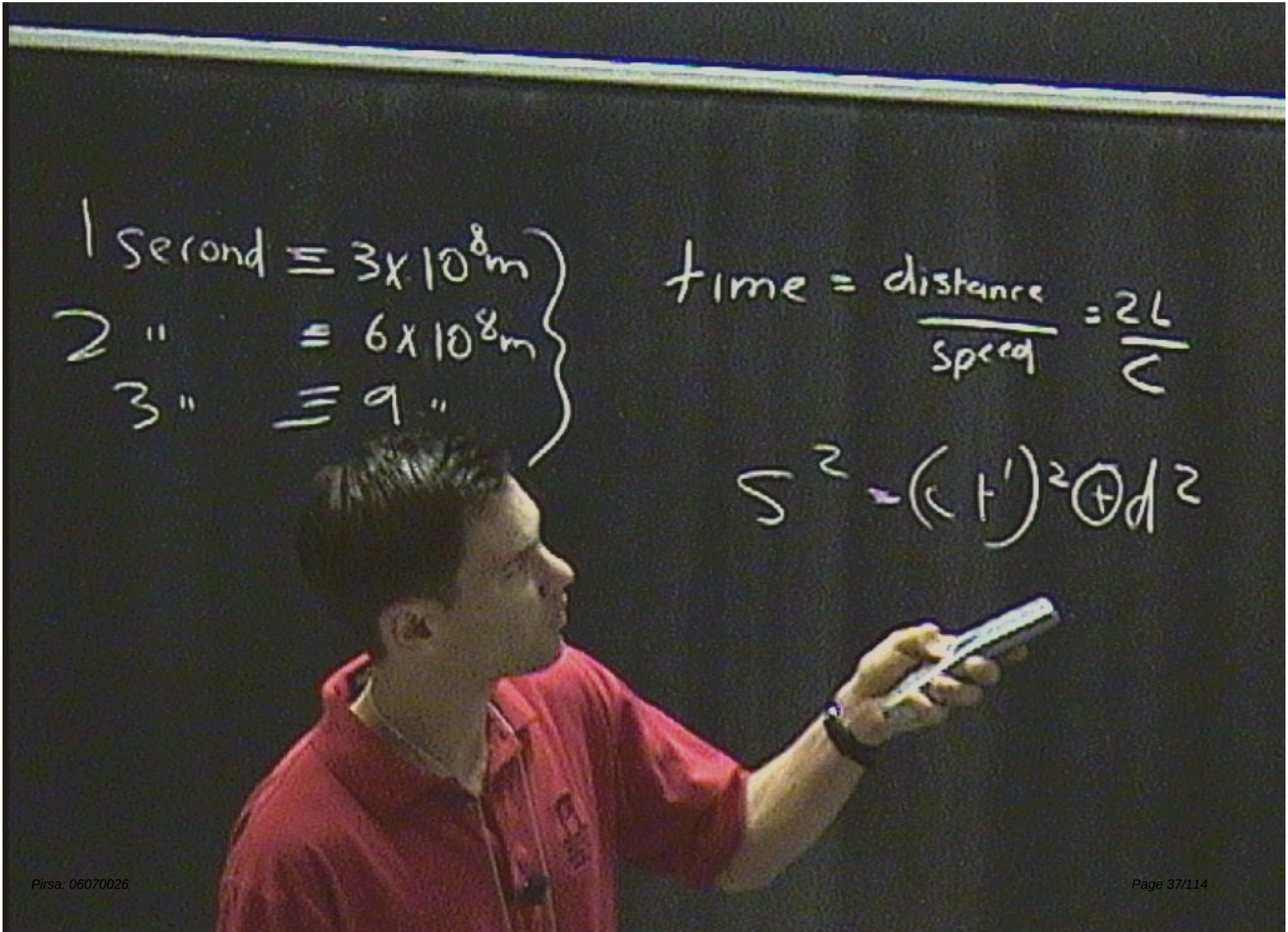

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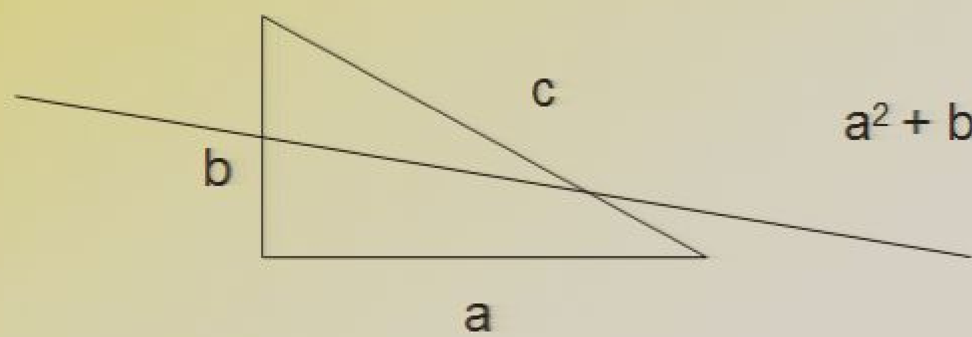


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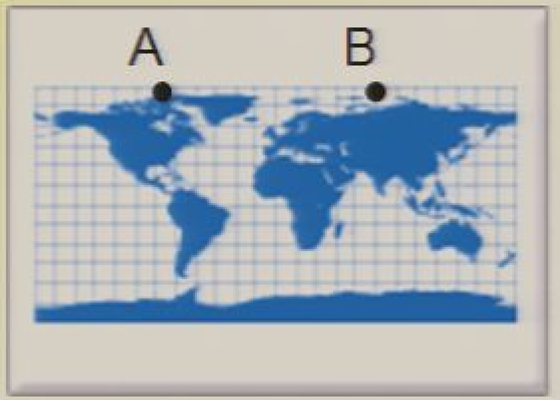
# What does the metric equation mean? cont.

- What are the implications of this?
- Consider the following example.
  Look at the Platte Carre map of the world:



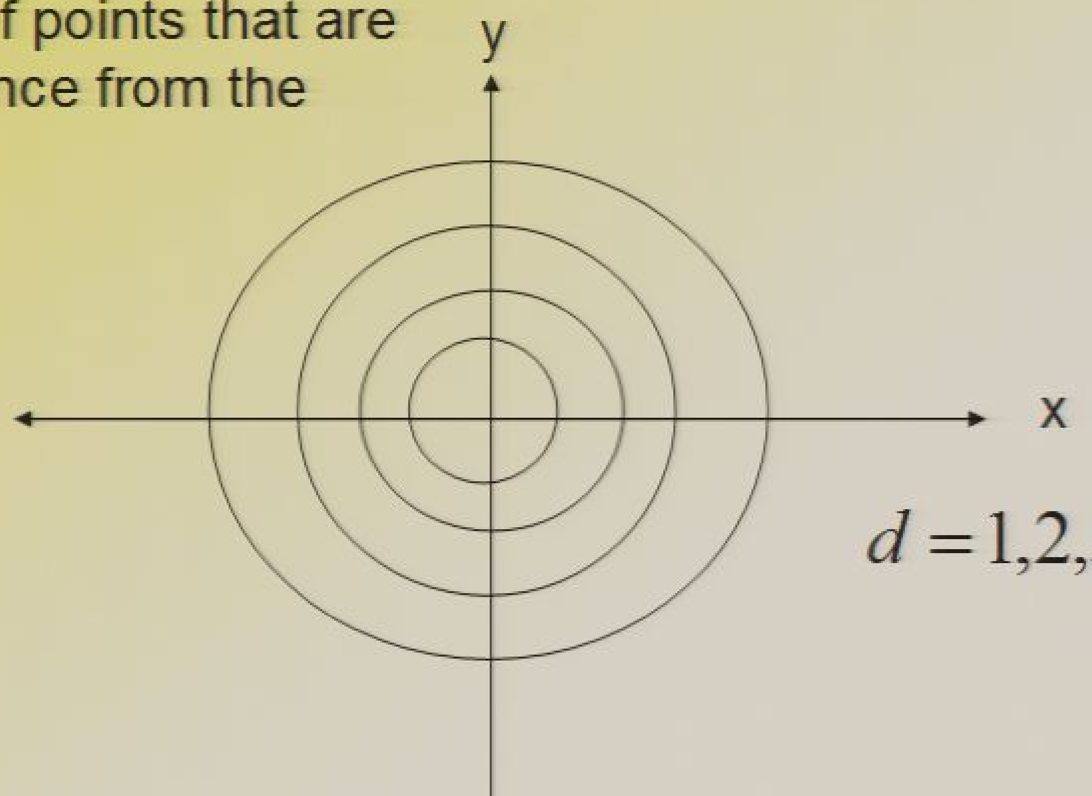

- We cannot interpret it naively. Eg. distance between A and B is actually zero as they are both at the North Pole
- Similarly, we cannot simply measure the spacetime distance or interval s between two events by simply measuring the distance between them on a spacetime diagram using a ruler.

- So, what is spacetime like then?

$$d = \sqrt{x^2 + y^2}$$

- For Euclidean x-y space, let us plot the locus of points that are the same distance from the origin.

- eg.

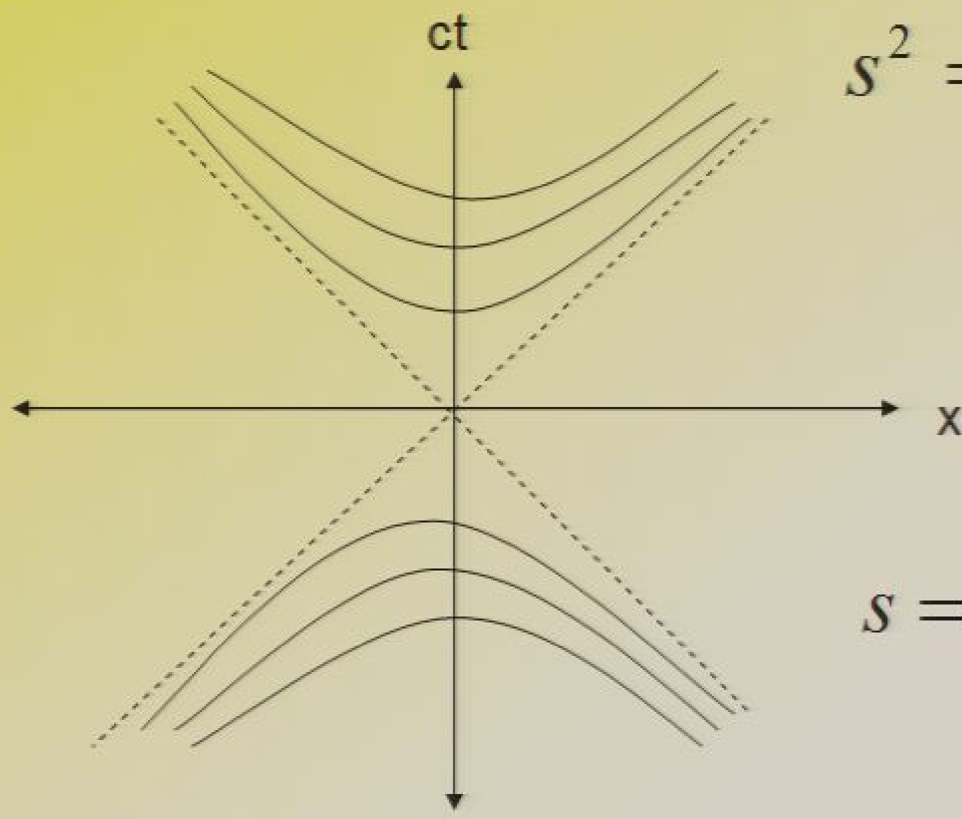



$$d = 1,2,3,4$$

- Let us do the analogous thing for spacetime

hyperbola

$$s^2 = ct^2 - x^2$$

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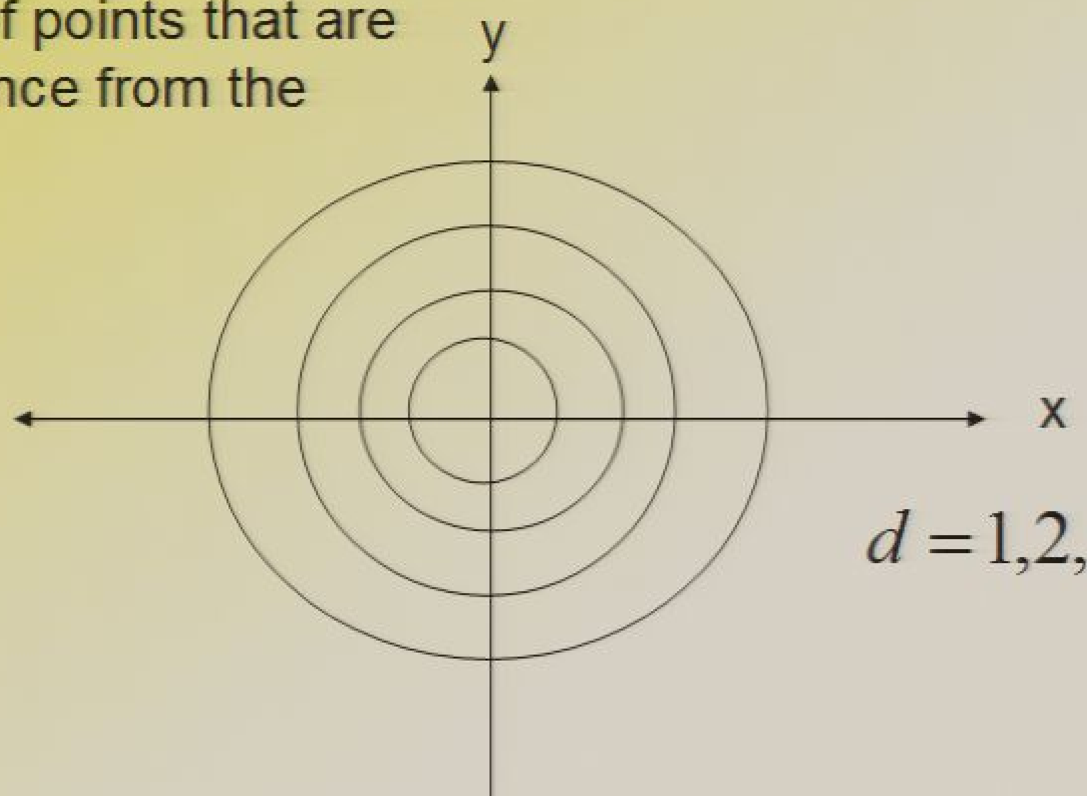


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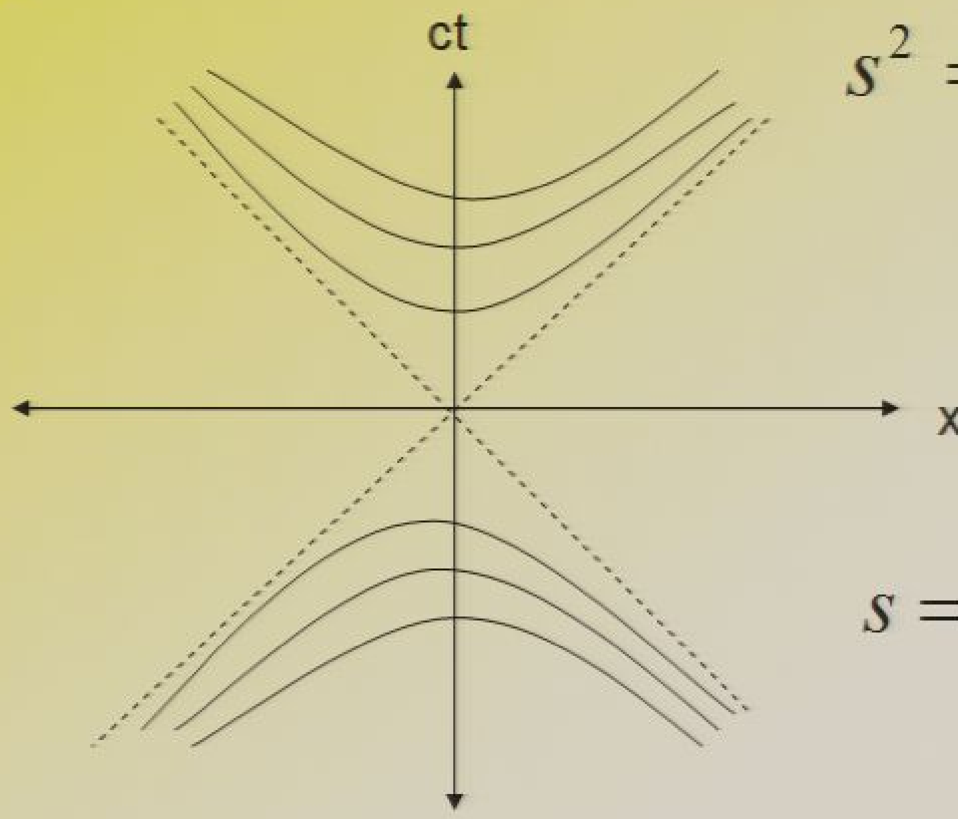


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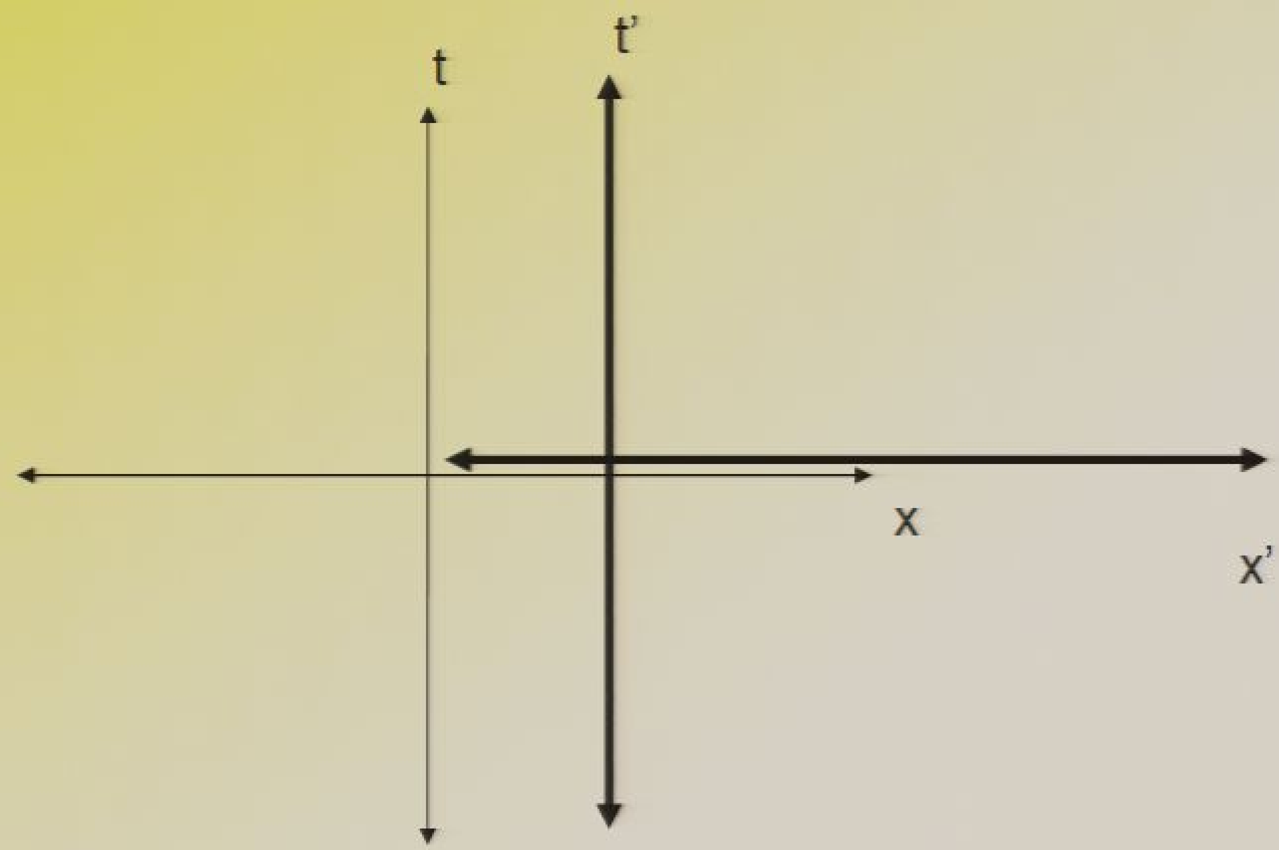
- Spacetime is based on *hyperbolae* not circles.
- Called *Lorentz space*

# Latest news on the nature of spacetime

- Today, scientists have even more unusual ideas about space and time
- **String theory:** Space only defined down to the Planck scale. There is a smallest possible distance, the Planck length, which is $10^{-35}$ metres. This limitation stops the theory from making nonsensical predictions when it combines general relativity with quantum theory.
- Extra dimensions: String theory says that there are nine dimensions of space!
- Loop quantum gravity. Space is also fundamentally grainy (discrete or quantized)
- The smallest possible area is $10^{-66}$ metres squared and the smallest possible volume $10^{-99}$ metres cubed.
- For more information on string theory, talk to a string theorist or loop quantum gravity researcher over lunch during the “chat-with-a-

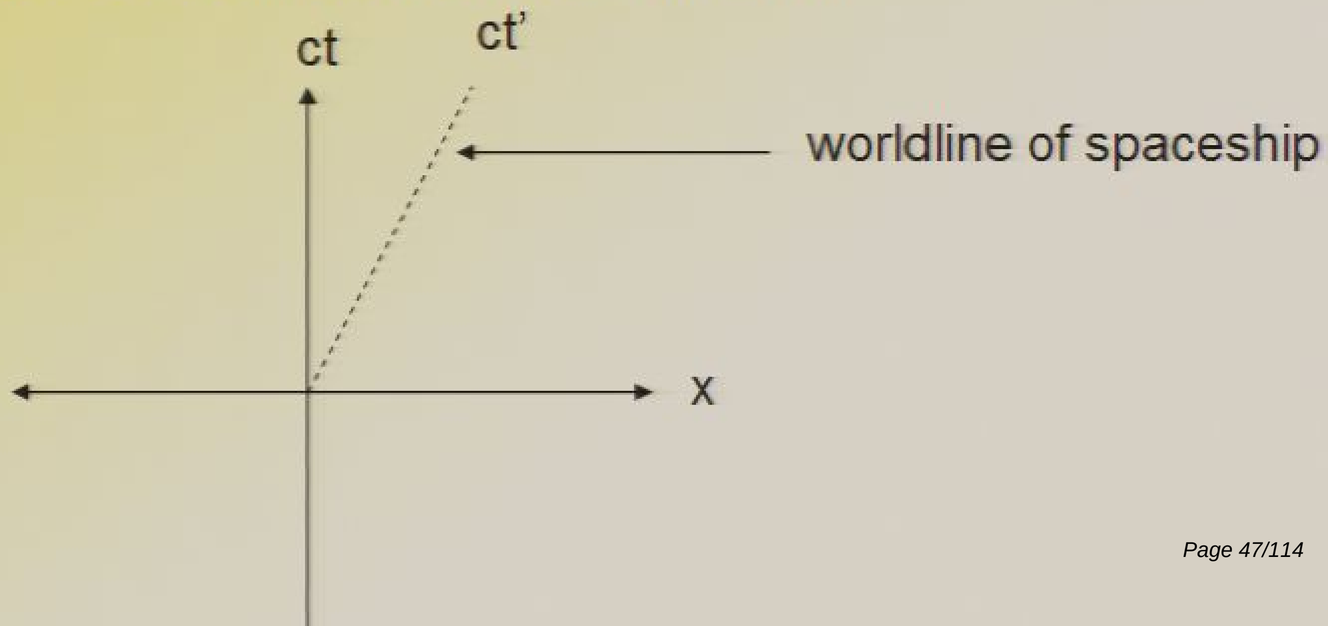
# Transforming between different frames of reference

- Galilean transformations



# Transforming between different frames of reference

- Imagine a spaceship moving at the constant velocity 0.5c past Earth
- What does its worldline look like?
- From the spaceship's perspective, it remains stationary and so all the points on its worldline have position x'=0. This defines the ct' axis just as the line x=0 defines the t axis for Earth.



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- For more information on string theory, talk to a string theorist or loop quantum gravity researcher over lunch during the “chat-with-a-

$$\boxed{y^2 - x^2 = a^2}$$

$$\frac{\text{distance}}{\text{speed}} = \frac{2L}{c}$$

HYPERBOLA

$$= (ct')^2 \oplus d^2 \qquad y^2 - x^2 = 1$$

$$y^2 - x^2 = a^2$$

$$\frac{\text{distance}}{\text{speed}} = \frac{2L}{c}$$

HYPERBOLA

$$= (ct')^2 + d^2$$

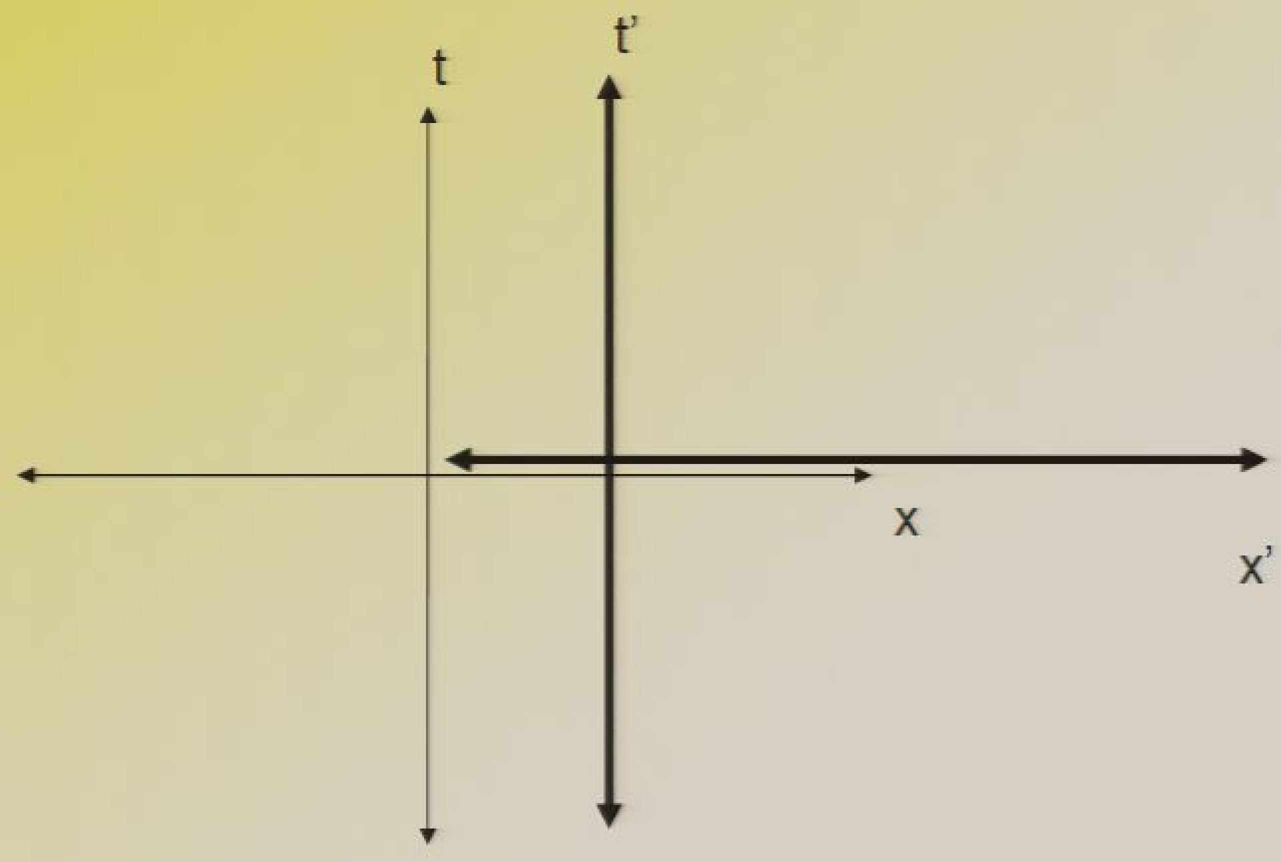
$$y^2 - x^2 = 1$$

# Latest news on the nature of spacetime

- Today, scientists have even more unusual ideas about space and time
- **String theory:** Space only defined down to the Planck scale. There is a smallest possible distance, the Planck length, which is $10^{-35}$ metres. This limitation stops the theory from making nonsensical predictions when it combines general relativity with quantum theory.
- Extra dimensions: String theory says that there are nine dimensions of space!
- Loop quantum gravity. Space is also fundamentally grainy (discrete or quantized)
- The smallest possible area is $10^{-66}$ metres squared and the smallest possible volume $10^{-99}$ metres cubed.
- For more information on string theory, talk to a string theorist or loop quantum gravity researcher over lunch during the "chat-with-a-

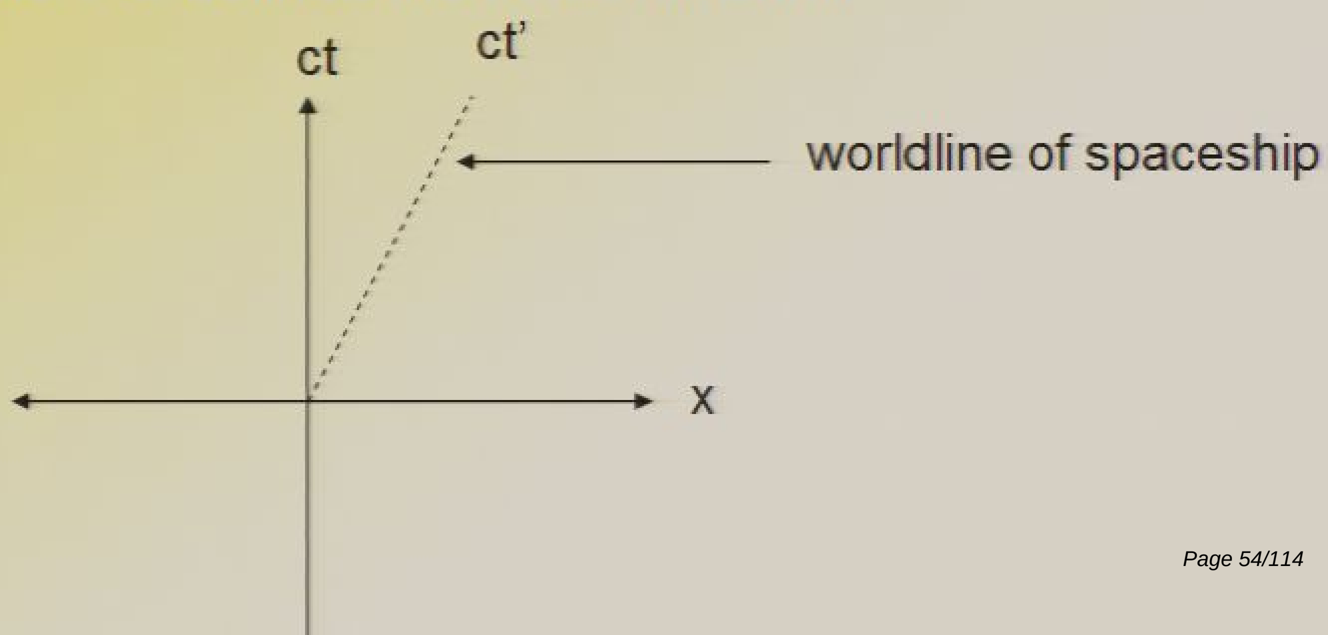
# Transforming between different frames of reference

- Galilean transformations



# Transforming between different frames of reference

- Imagine a spaceship moving at the constant velocity 0.5c past Earth
- What does its worldline look like?
- From the spaceship's perspective, it remains stationary and so all the points on its worldline have position $x'=0$. This defines the $ct'$ axis just as the line $x=0$ defines the t axis for Earth.



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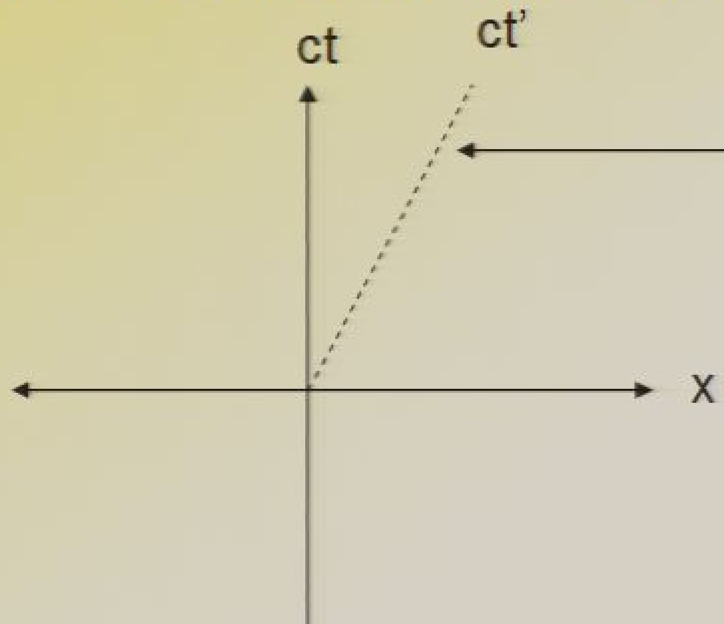
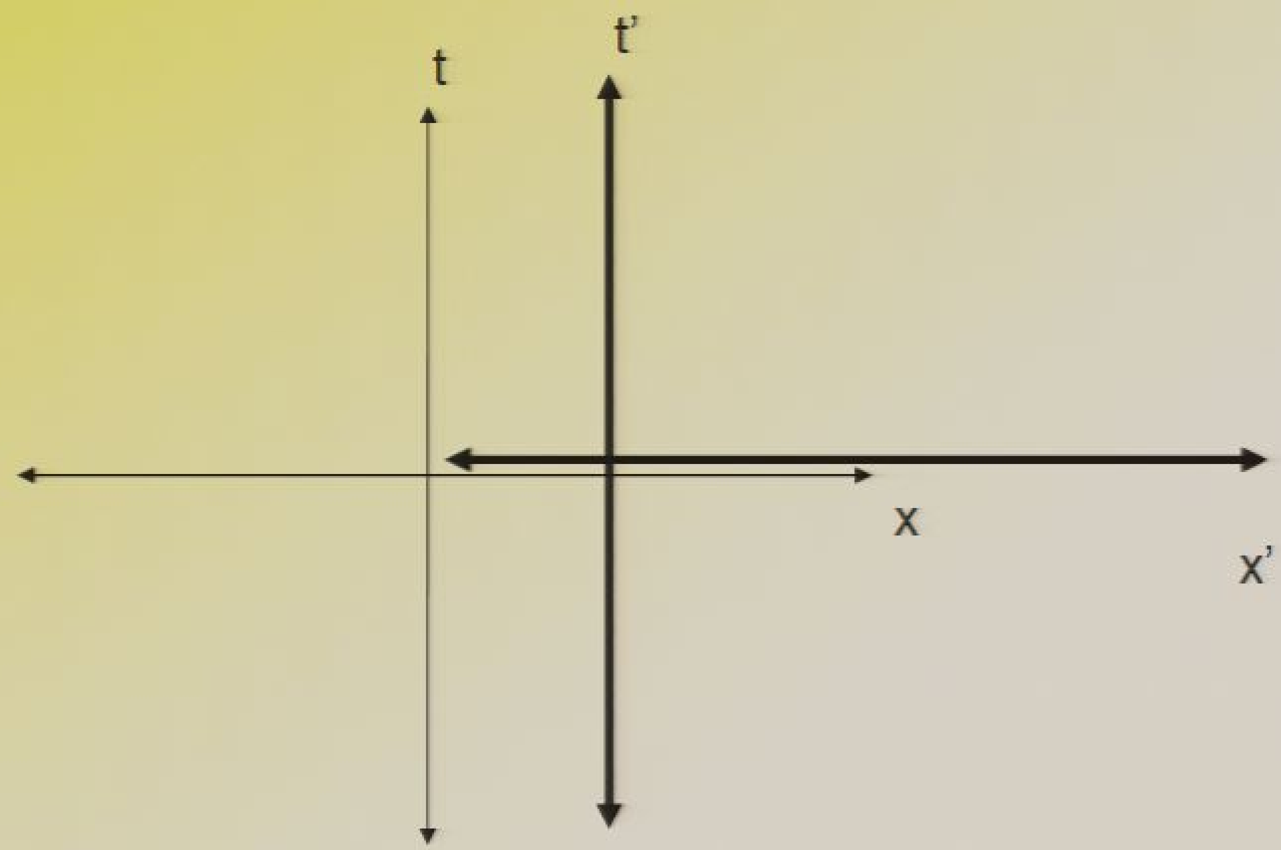


# Transforming between different frames of reference

- Galilean transformations



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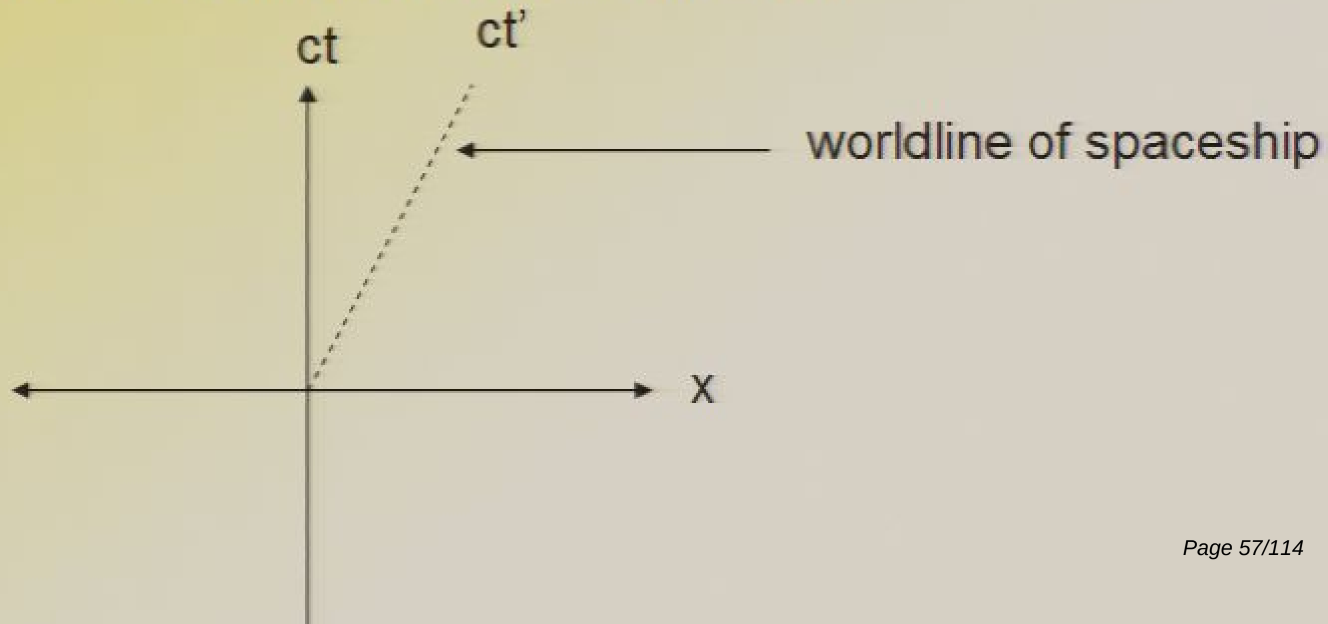
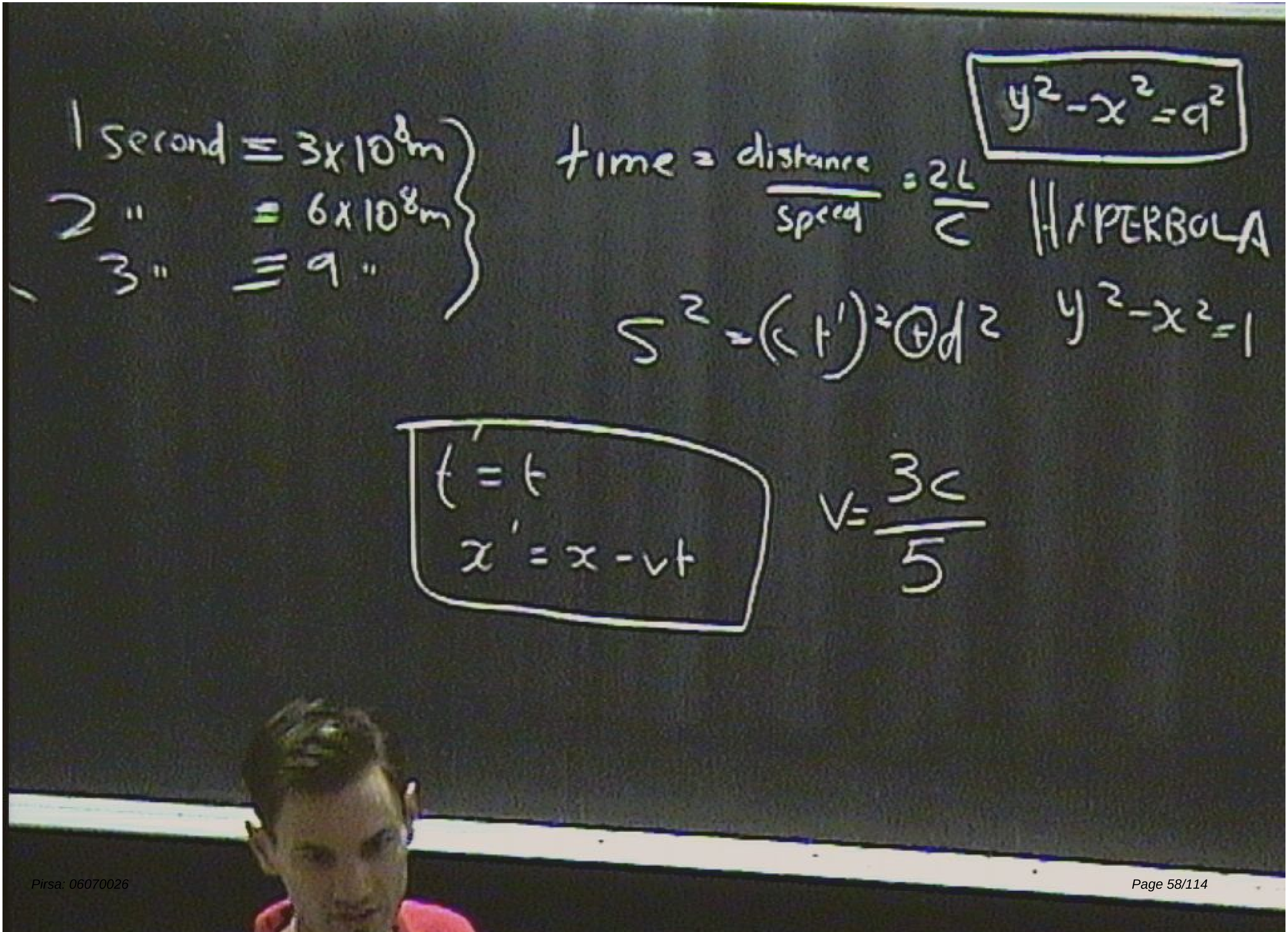
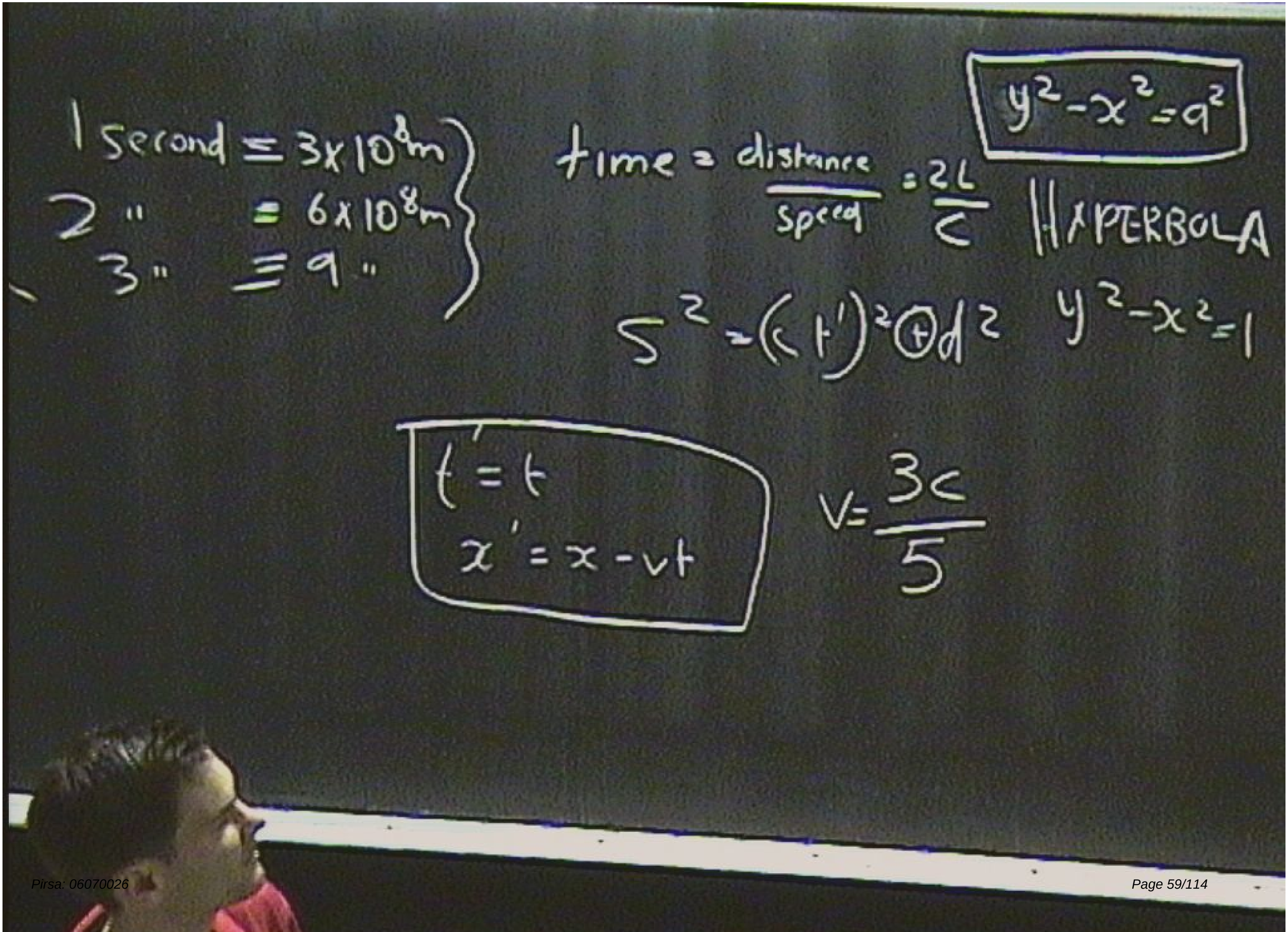



1 second ≡ 3x10^8 m
2 " = 6x10^8 m
3 " ≡ 9 "
time = distance / speed = 2L / c
y^2 - x^2 = a^2
HYPERBOLA
s^2 = (c t')^2 + d^2
y^2 - x^2 = 1
t' = t
x' = x - vt
V = 3c / 5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t' = t
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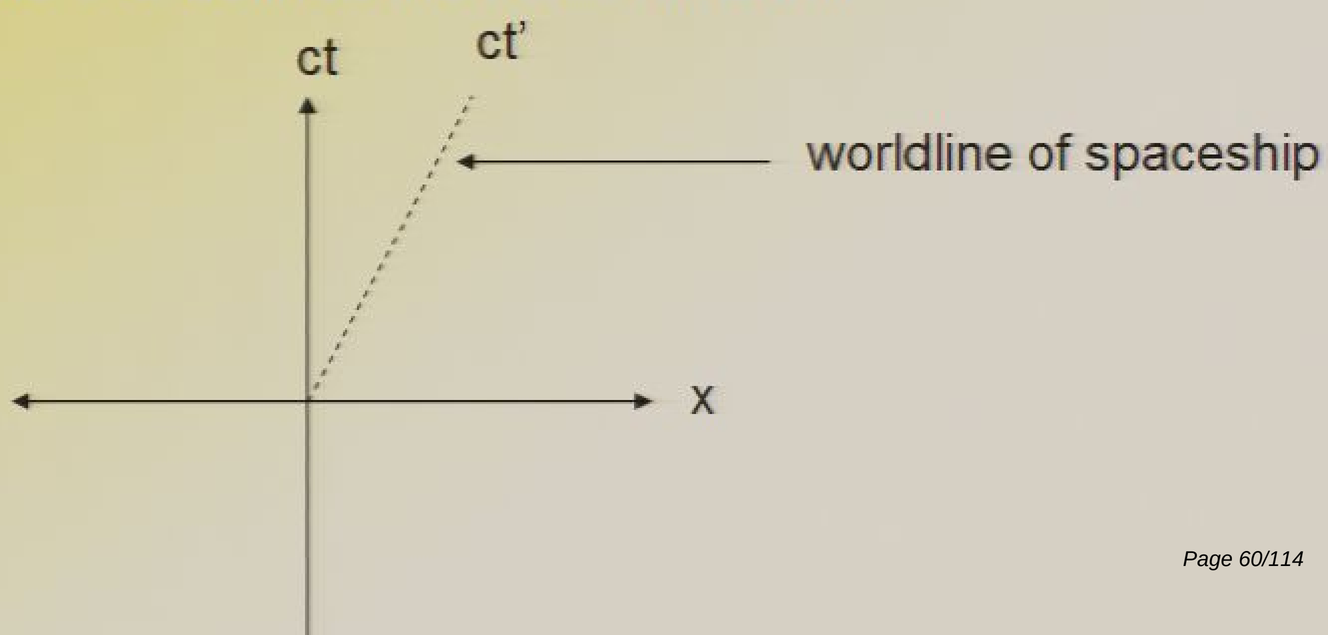
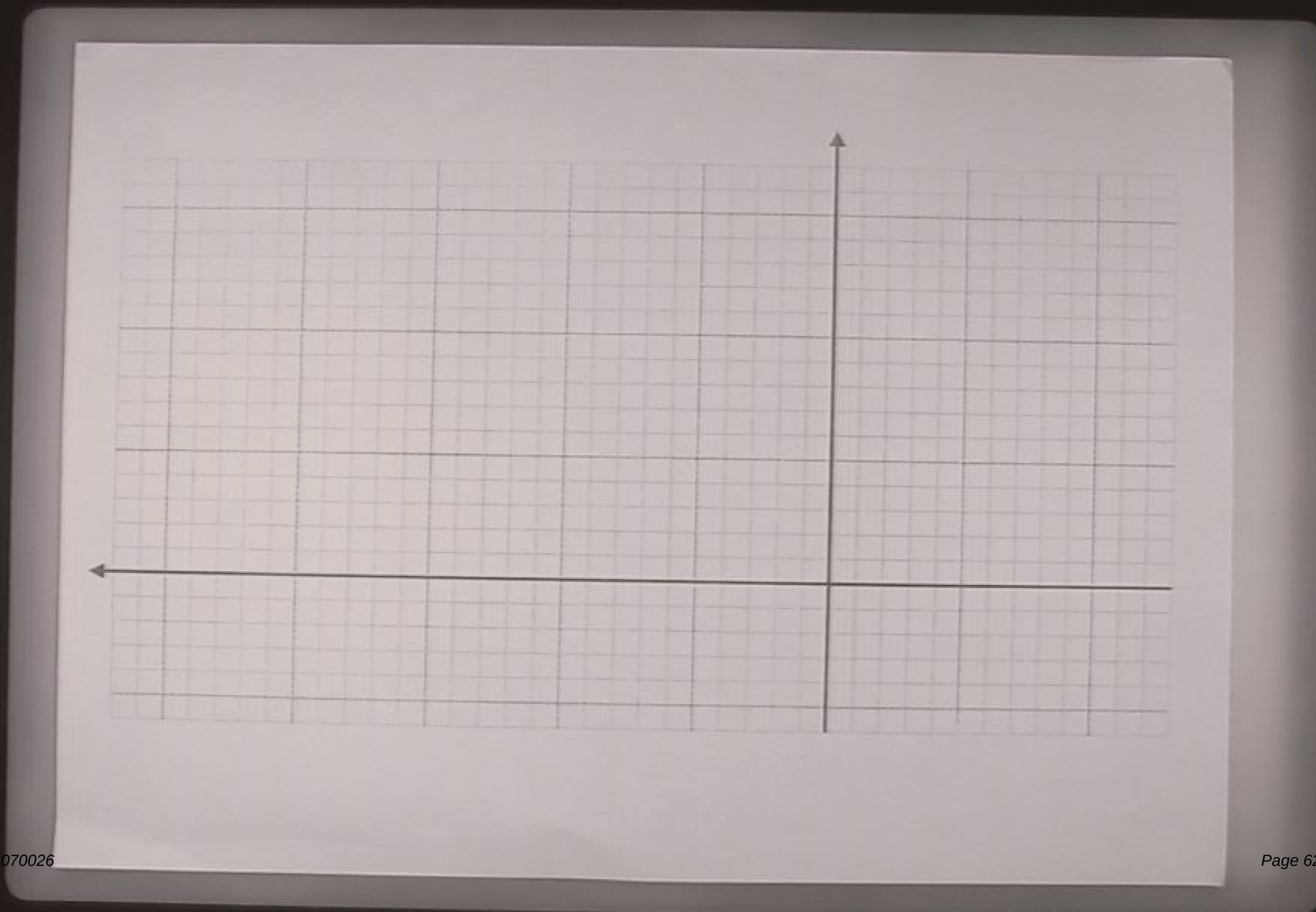
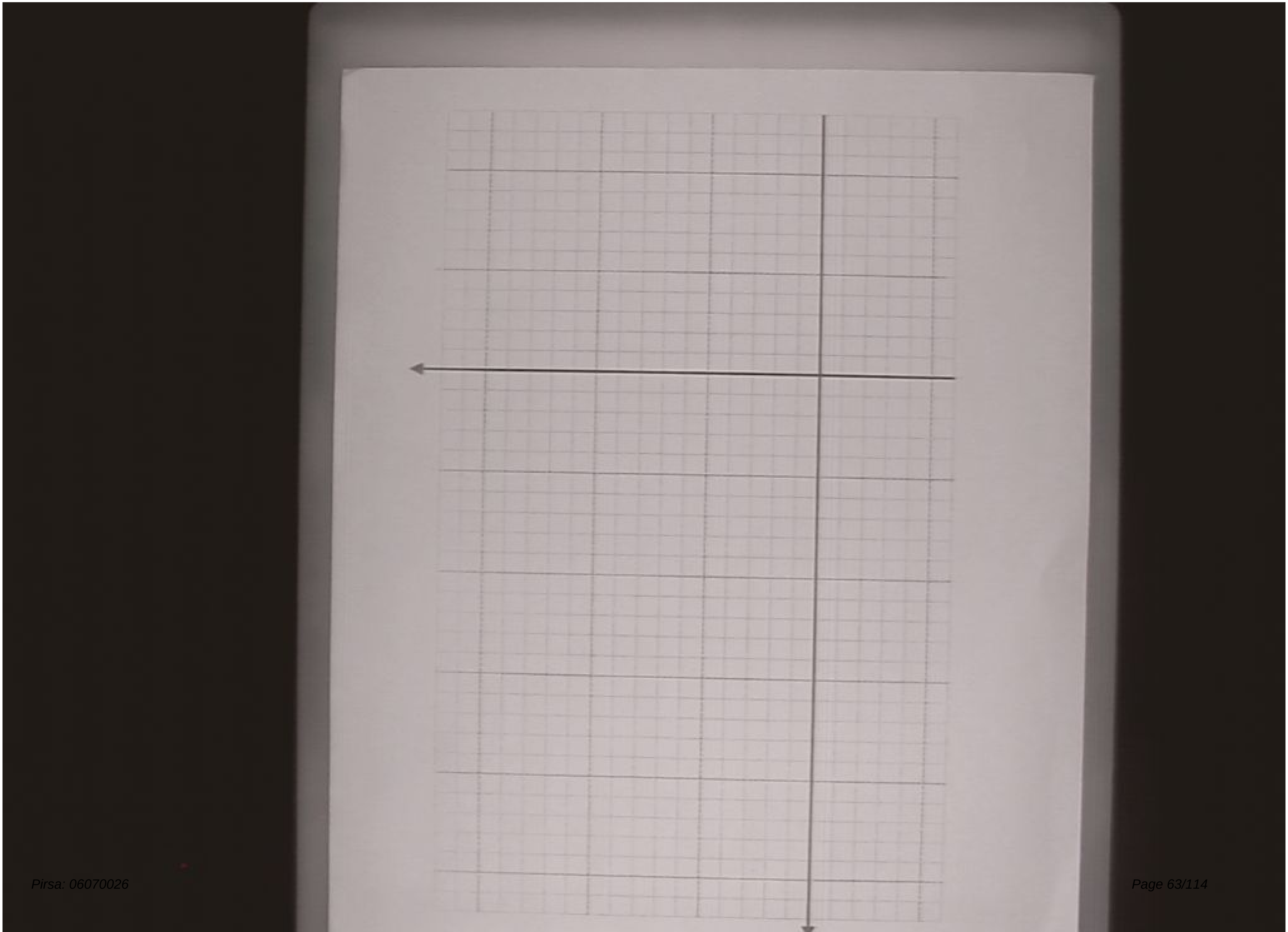
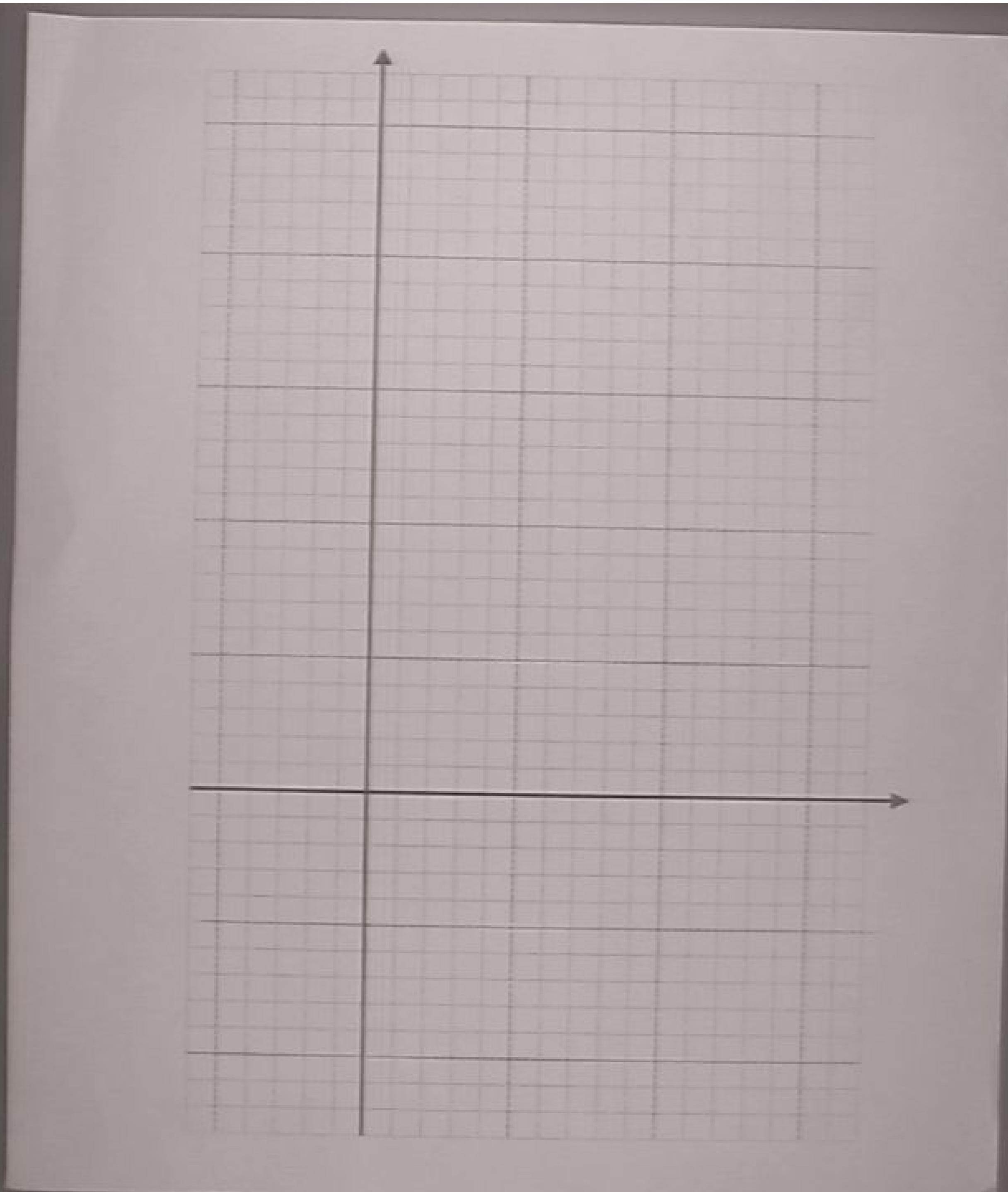
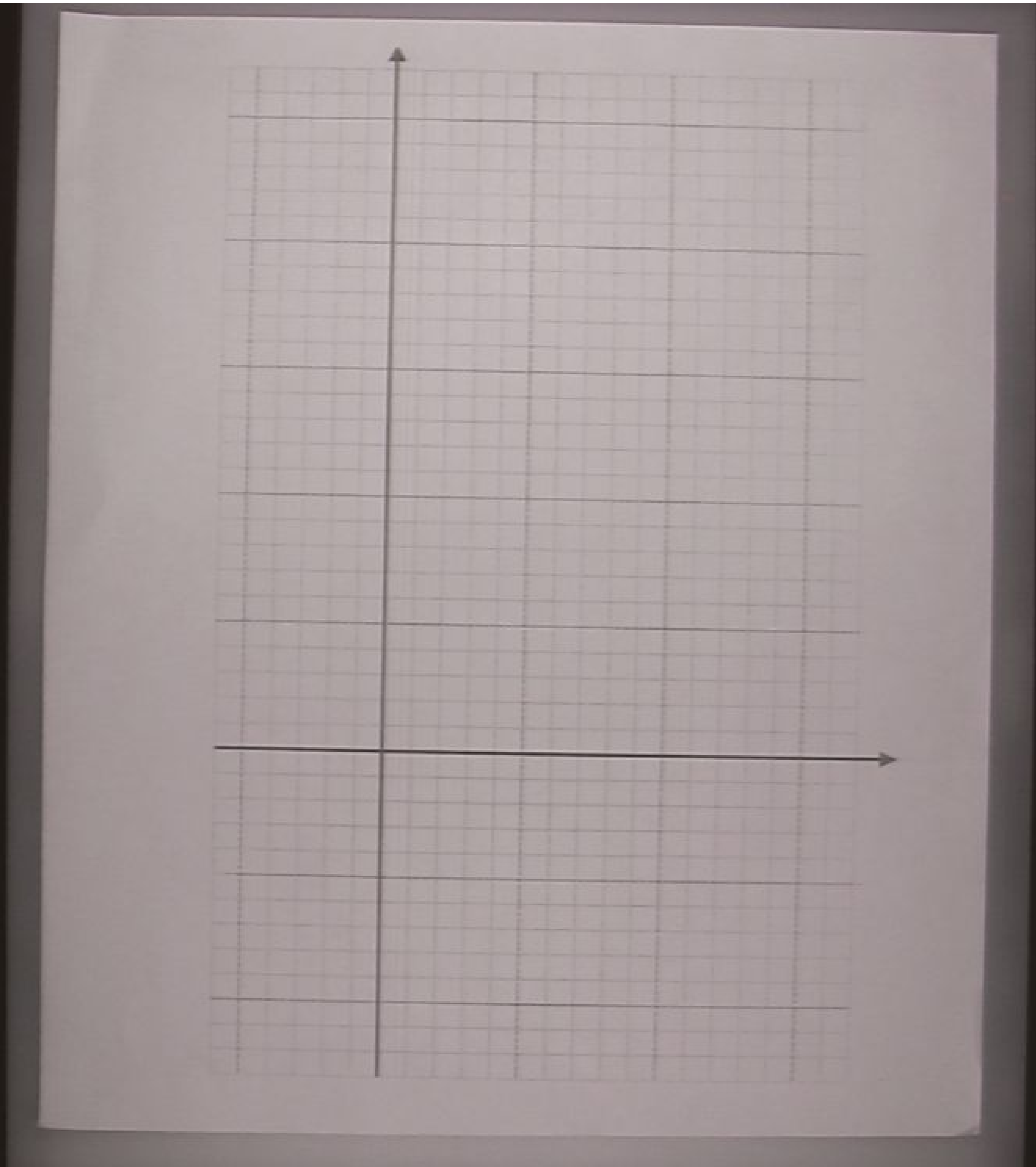
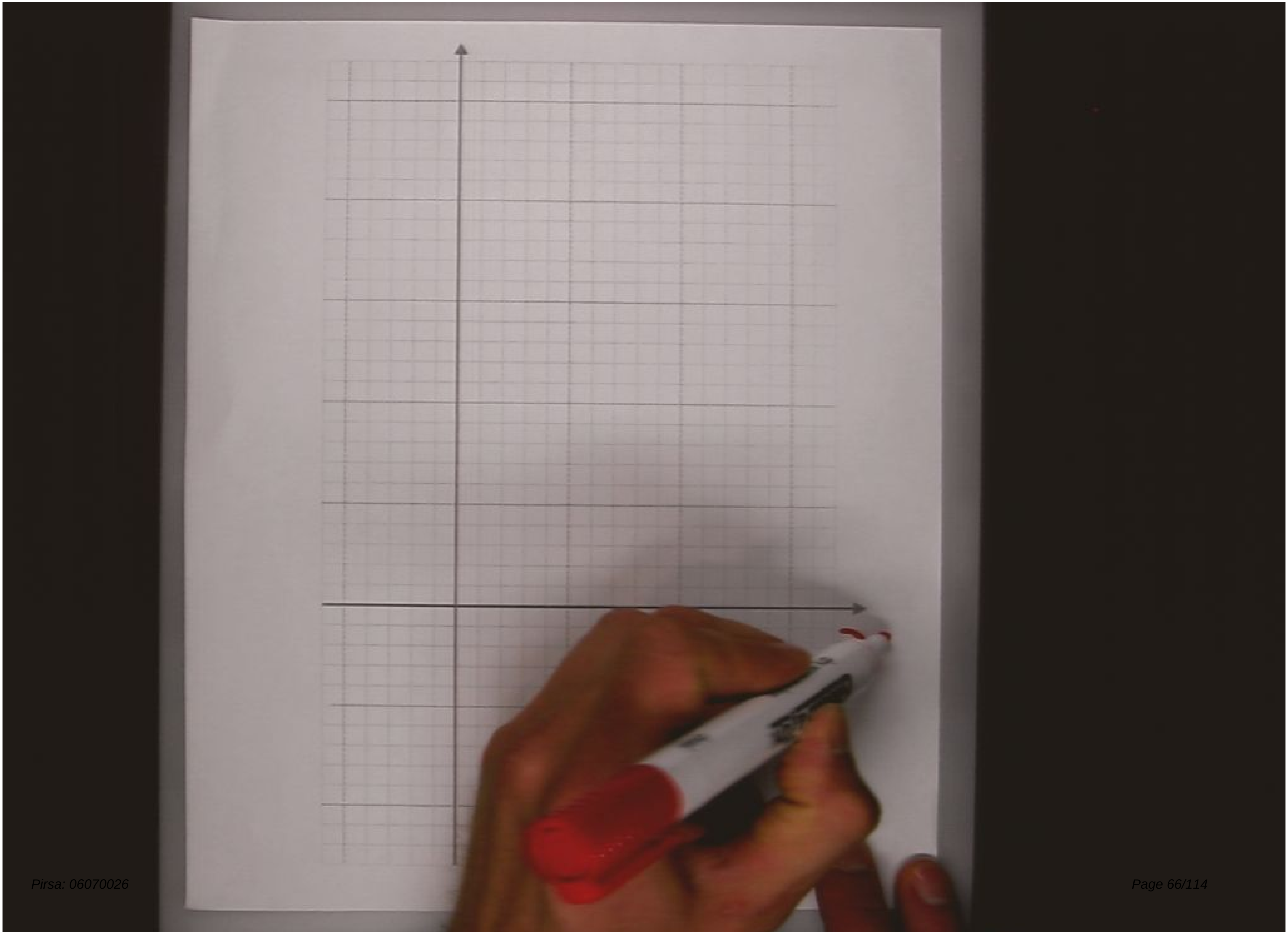


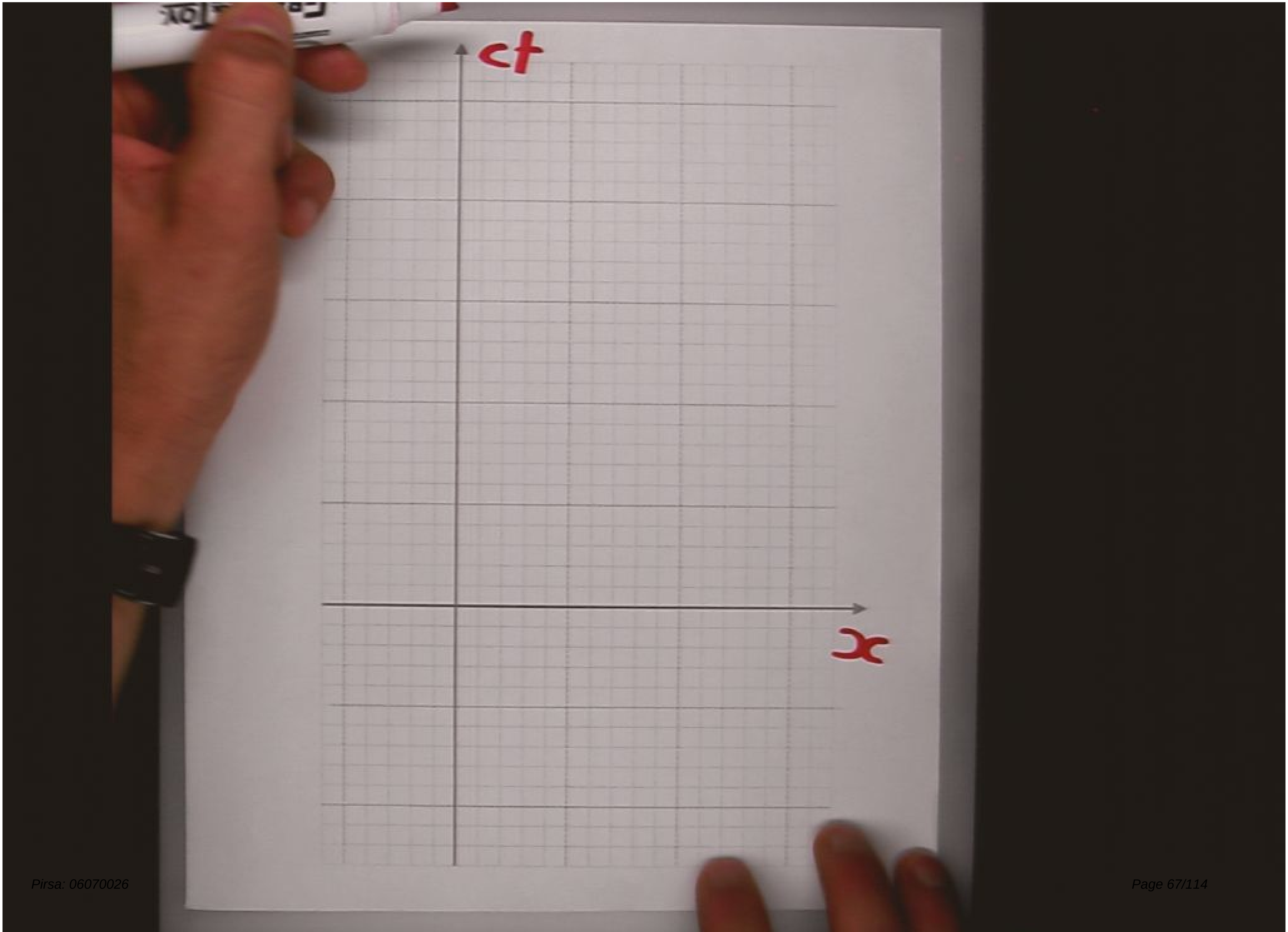




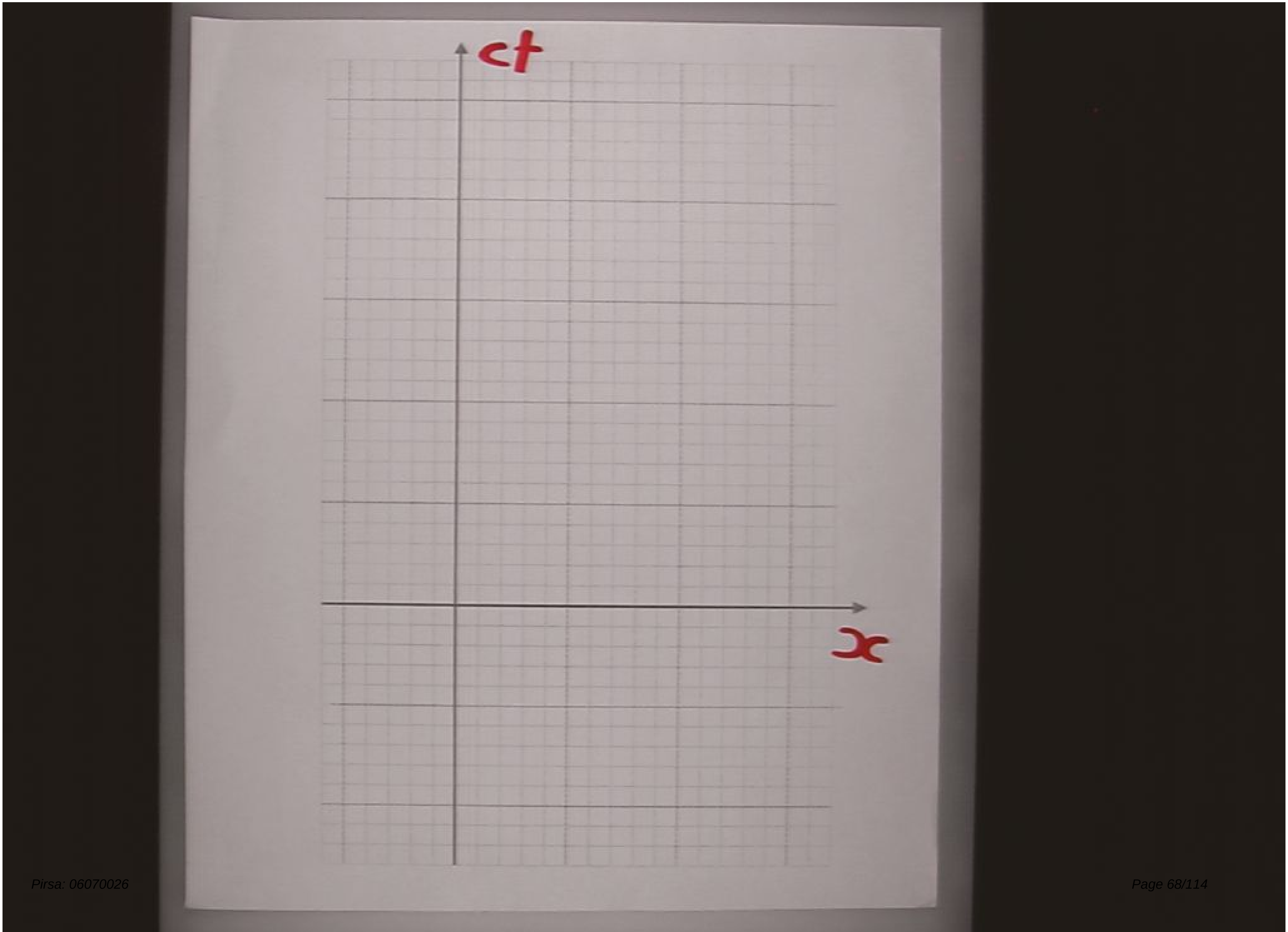


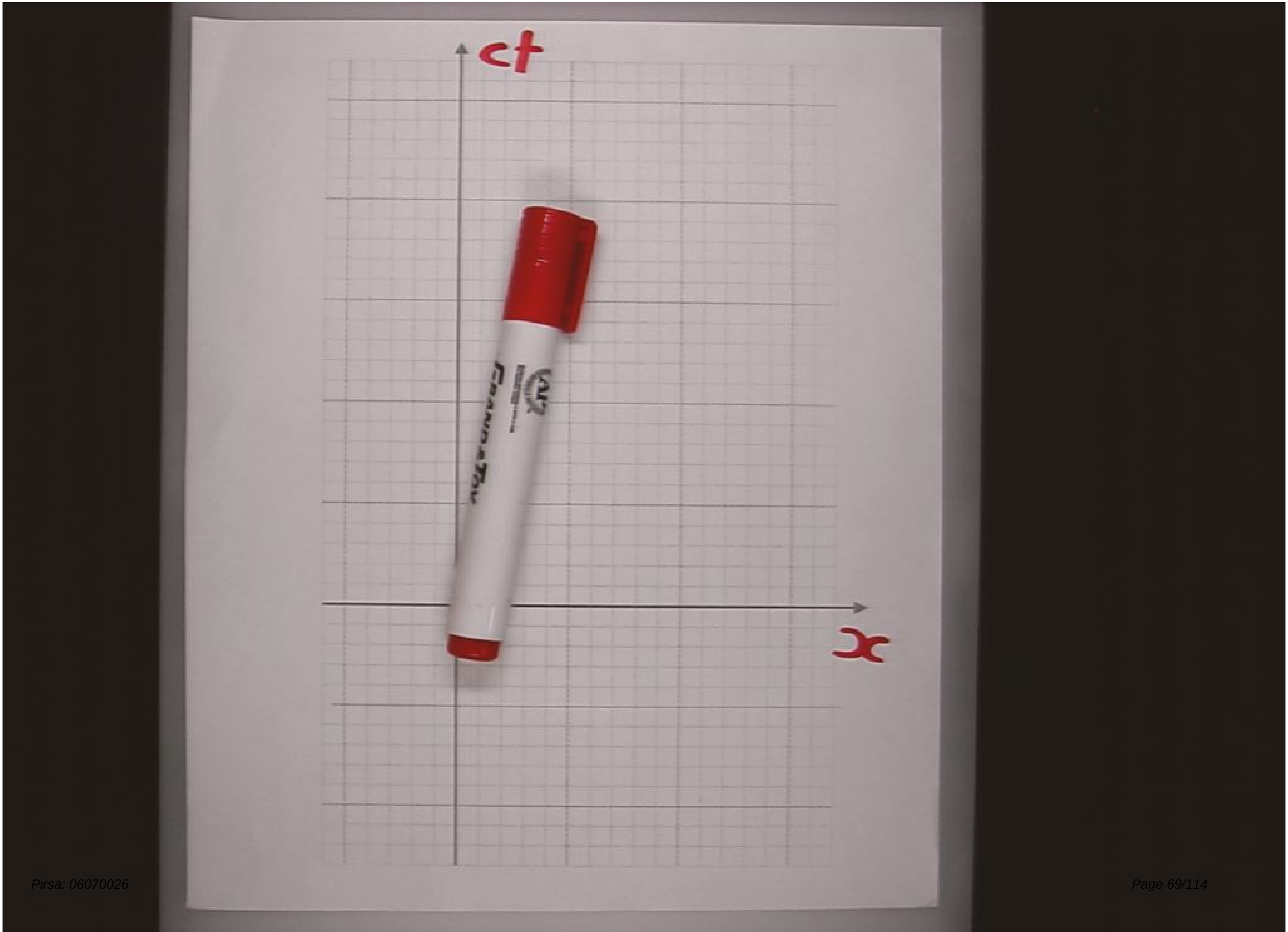


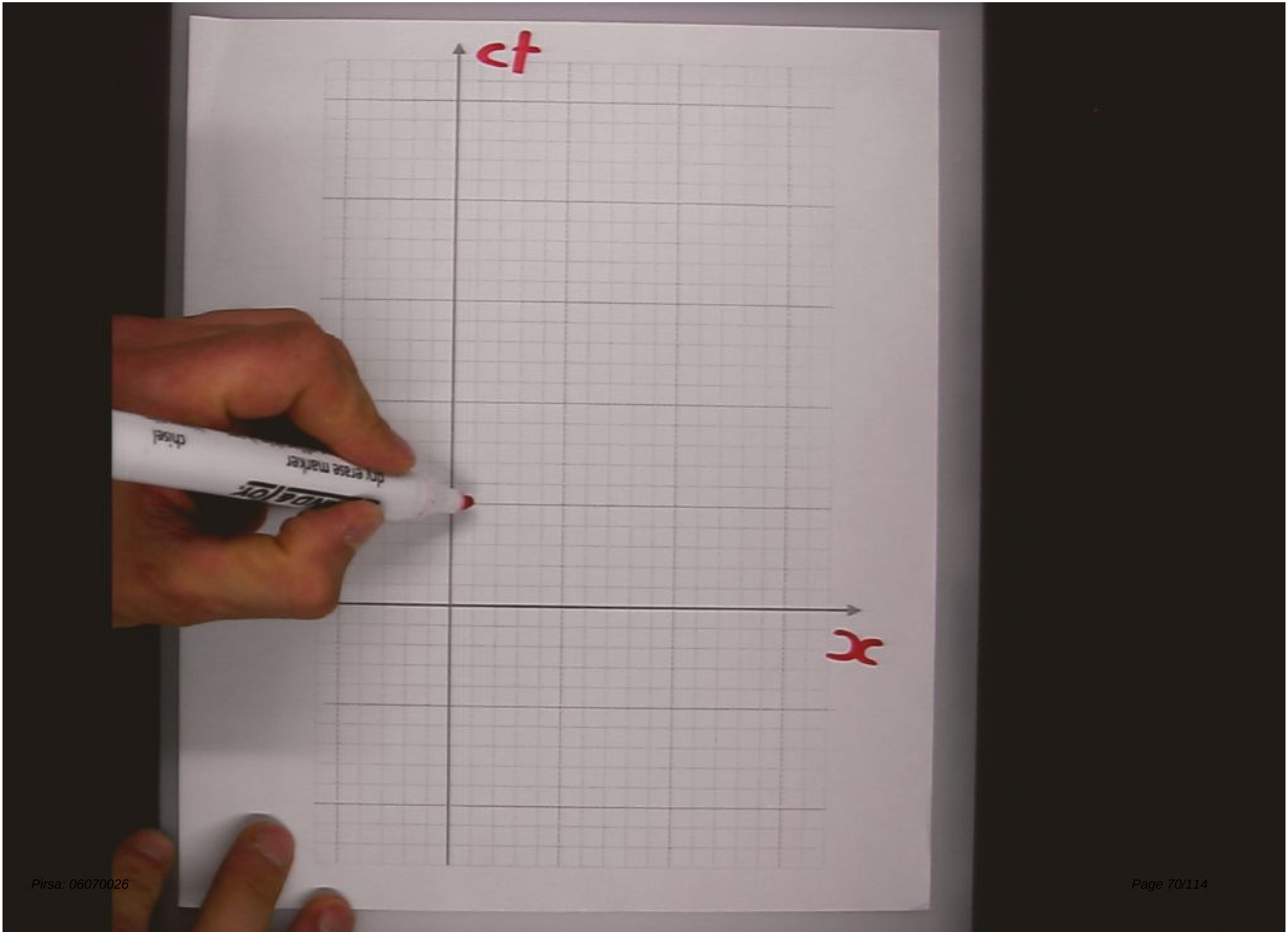


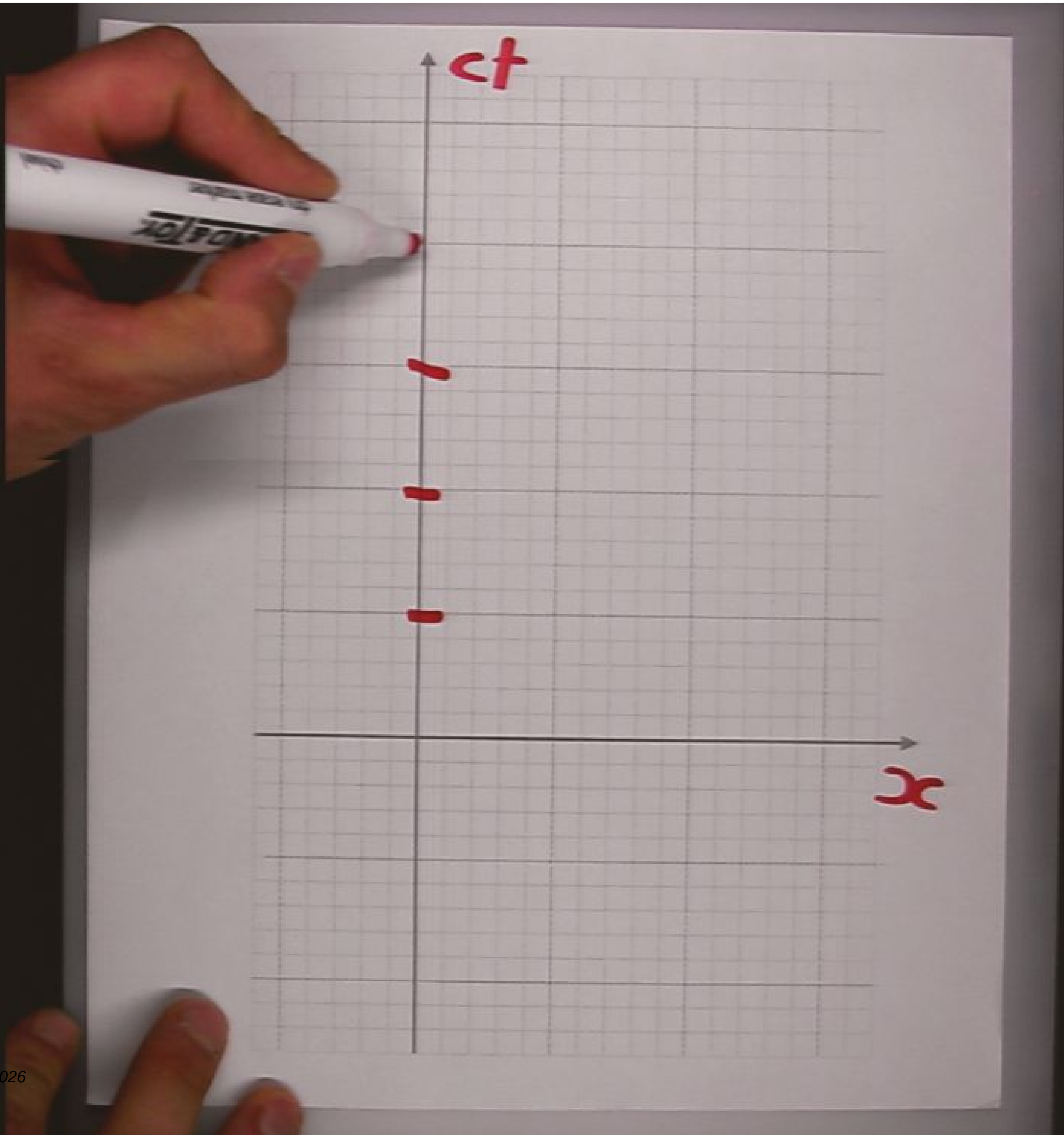


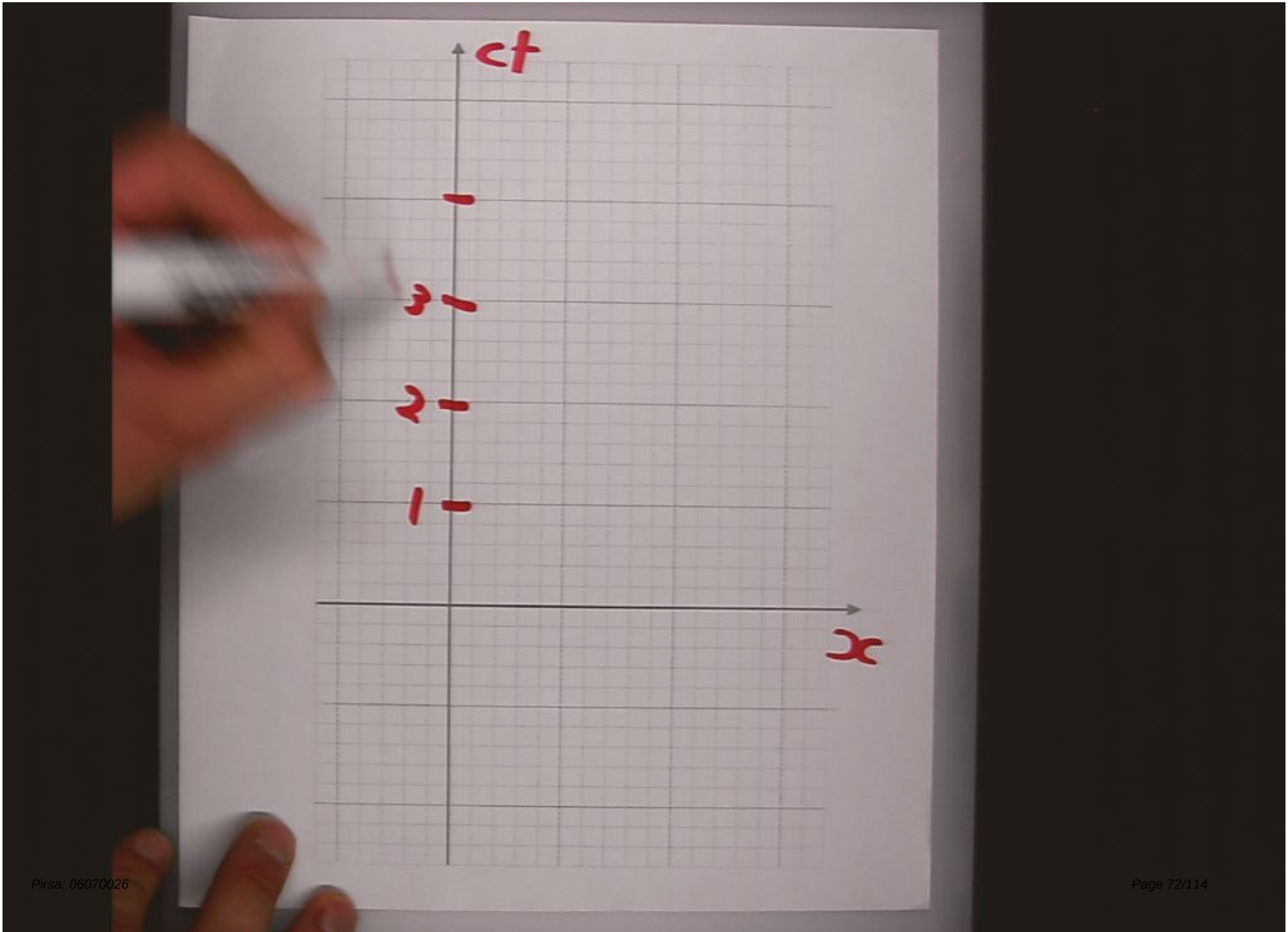
ct
x

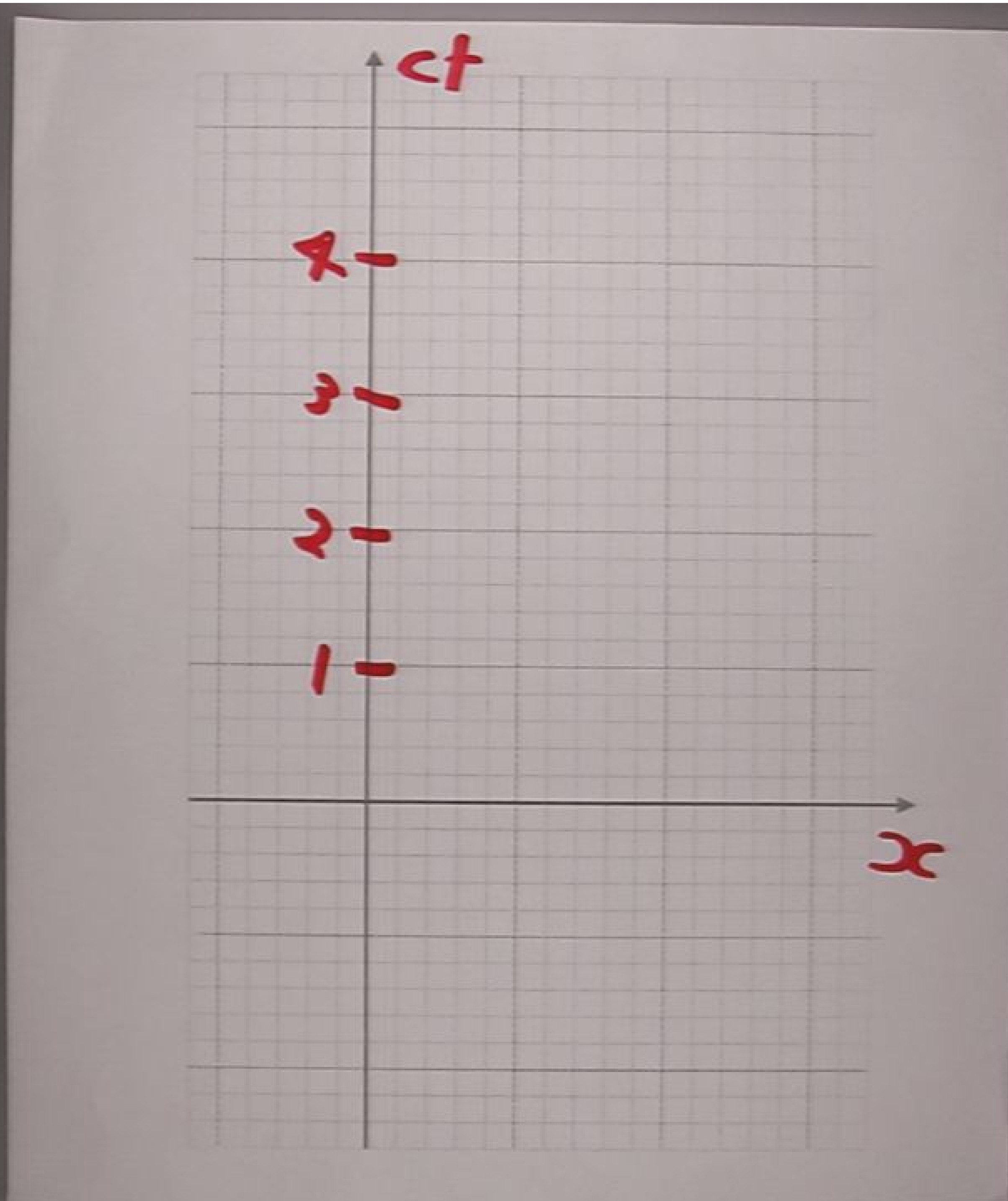

ct
x

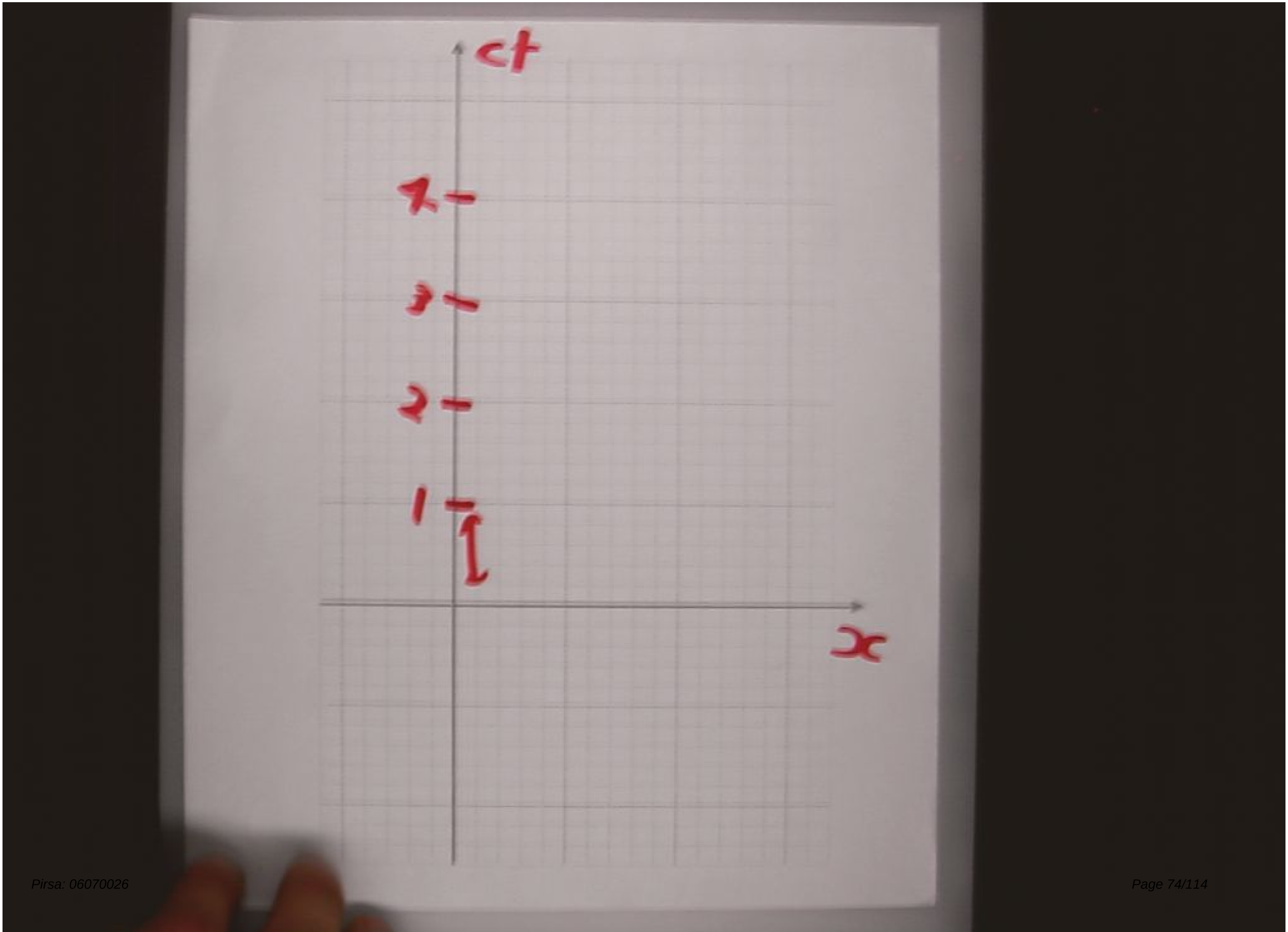

ct
x


ct
x

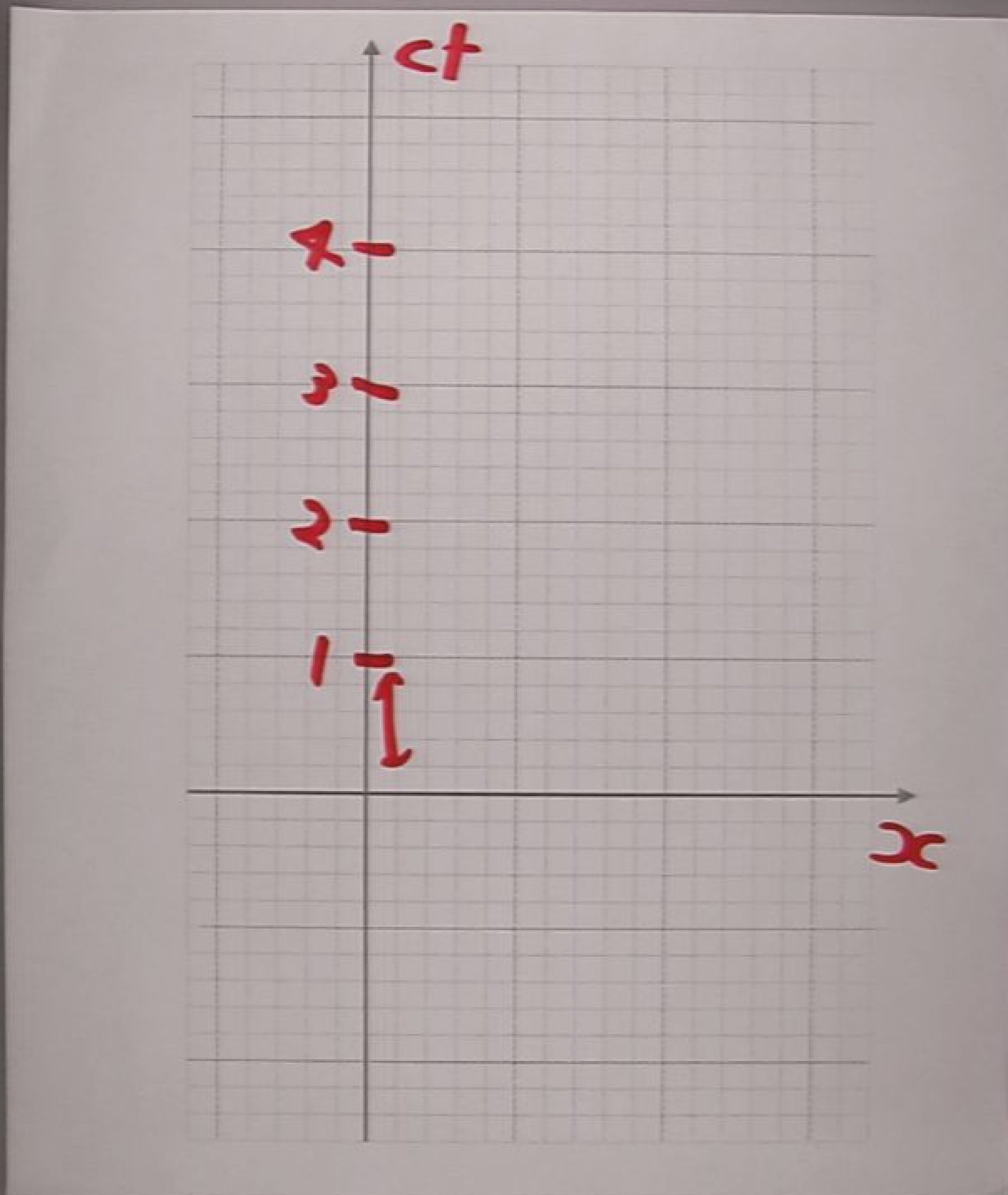

ct
x

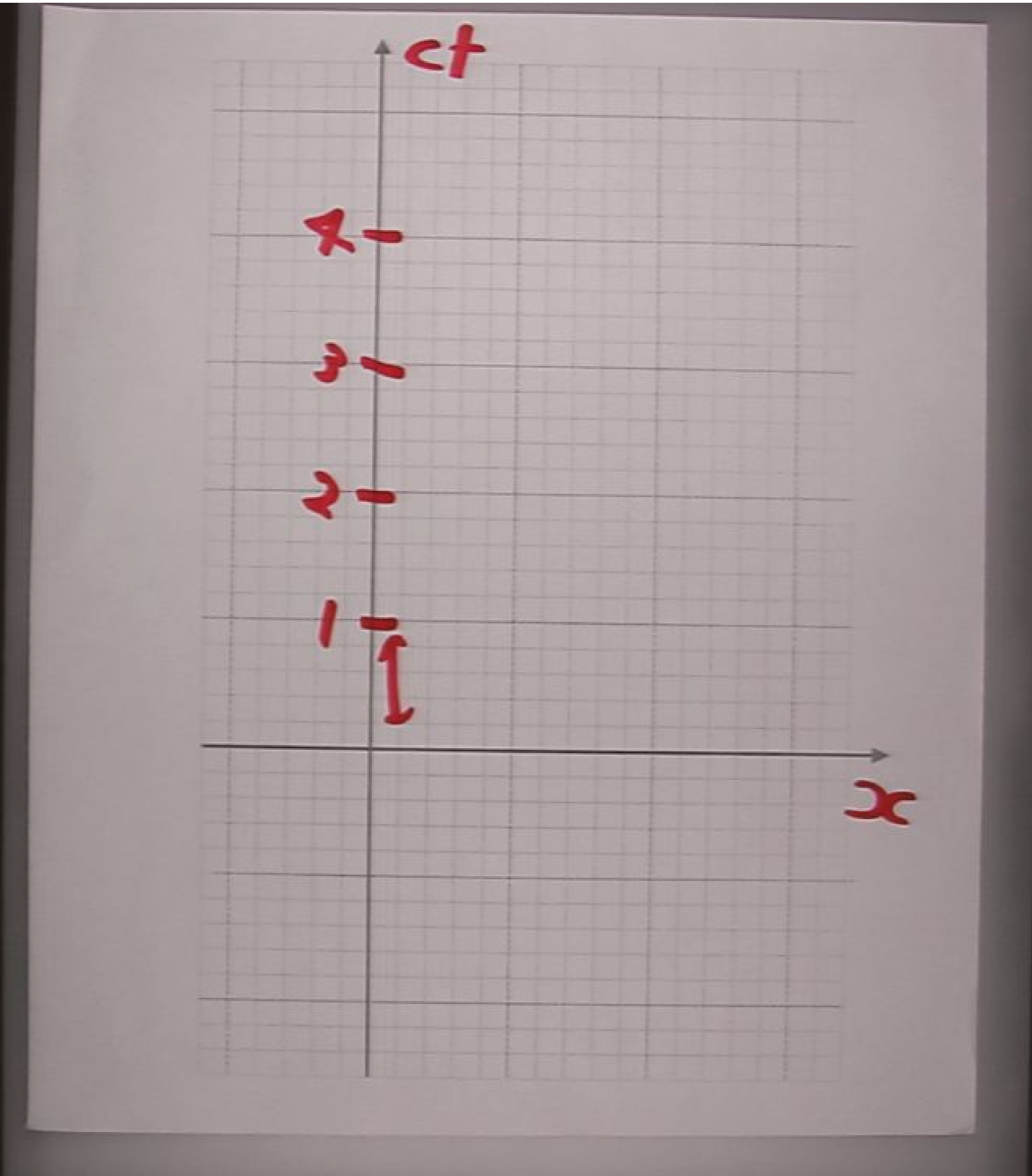

ct
3
2
1
x

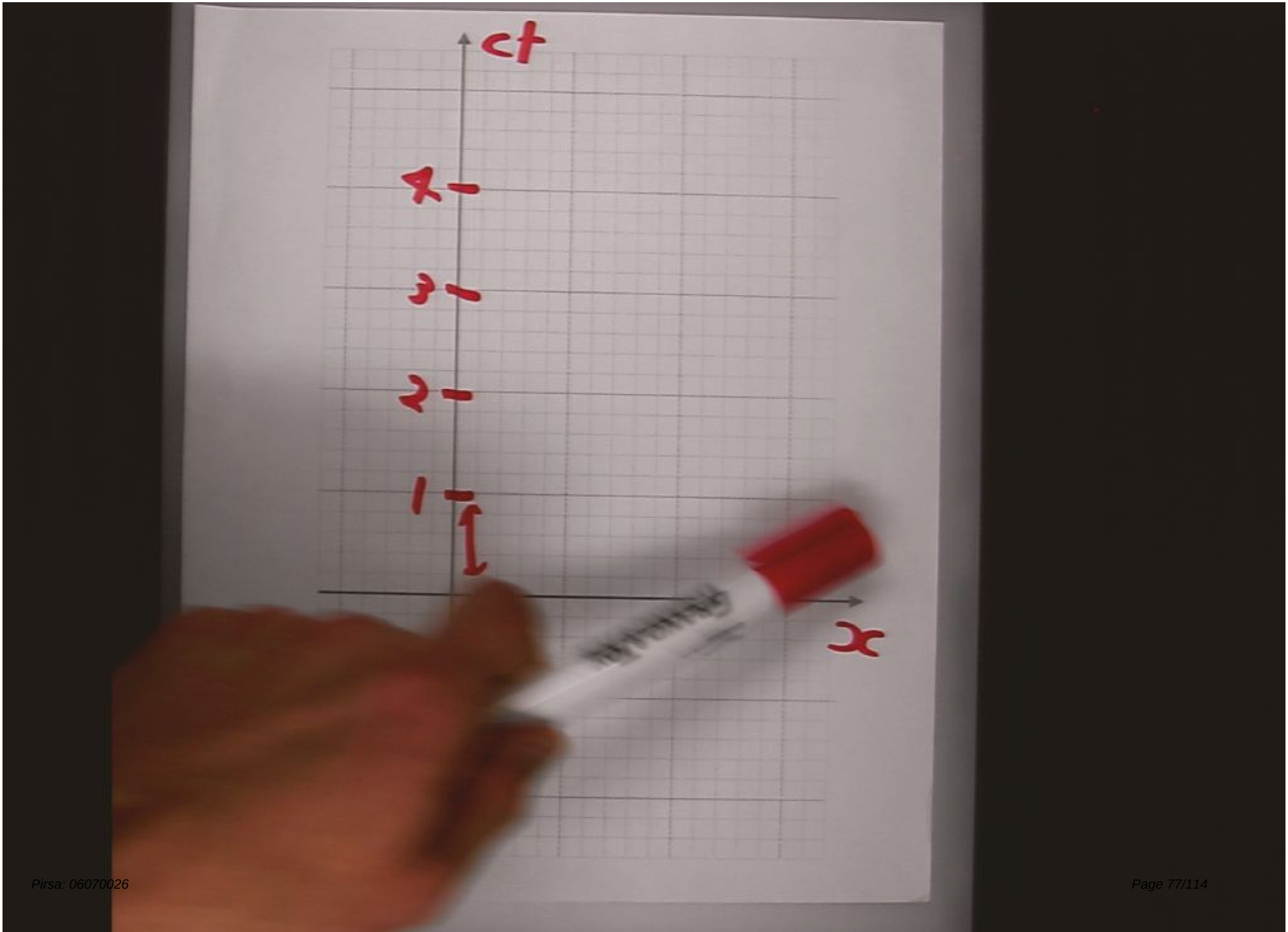

ct
4
3
2
1
x

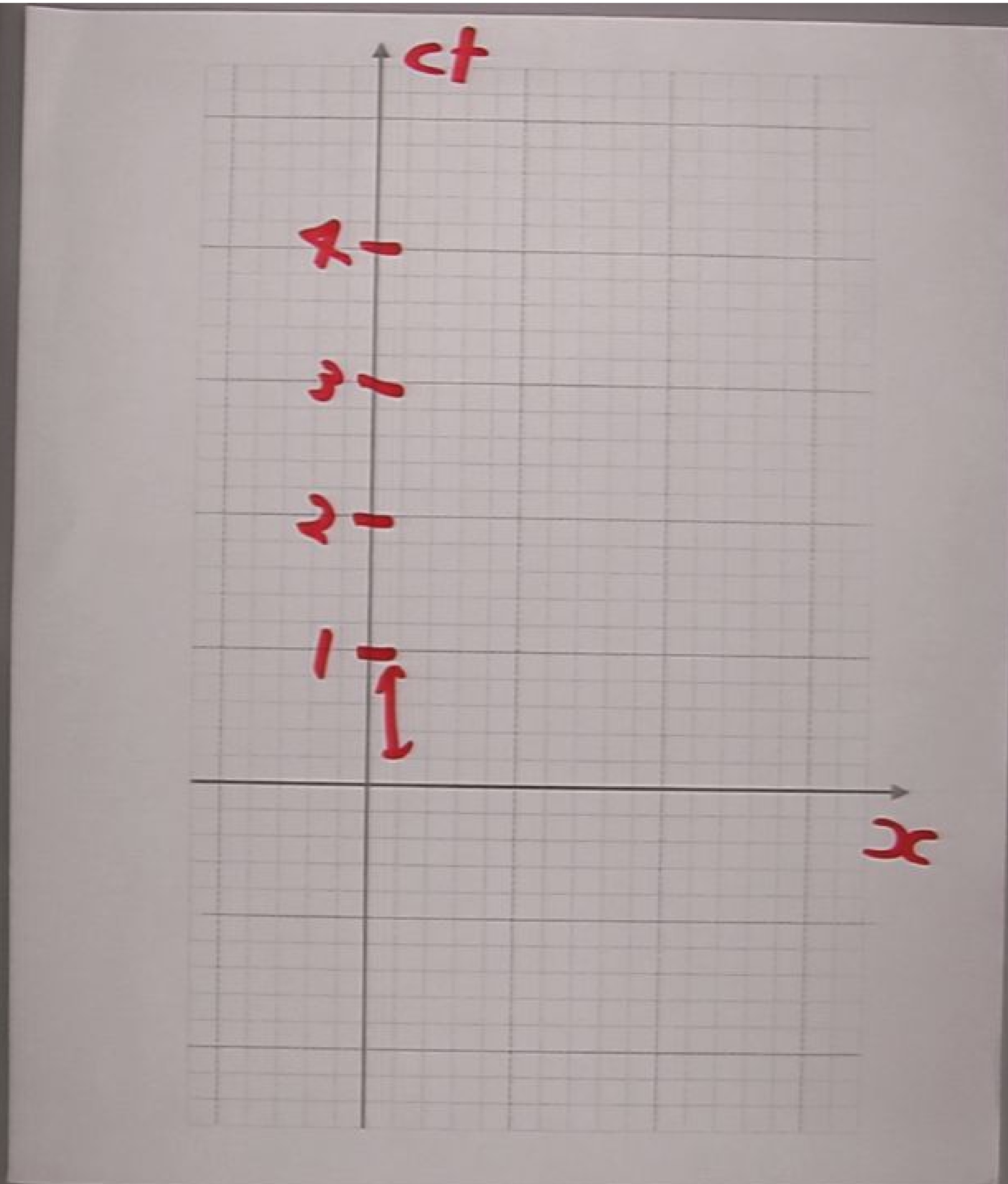


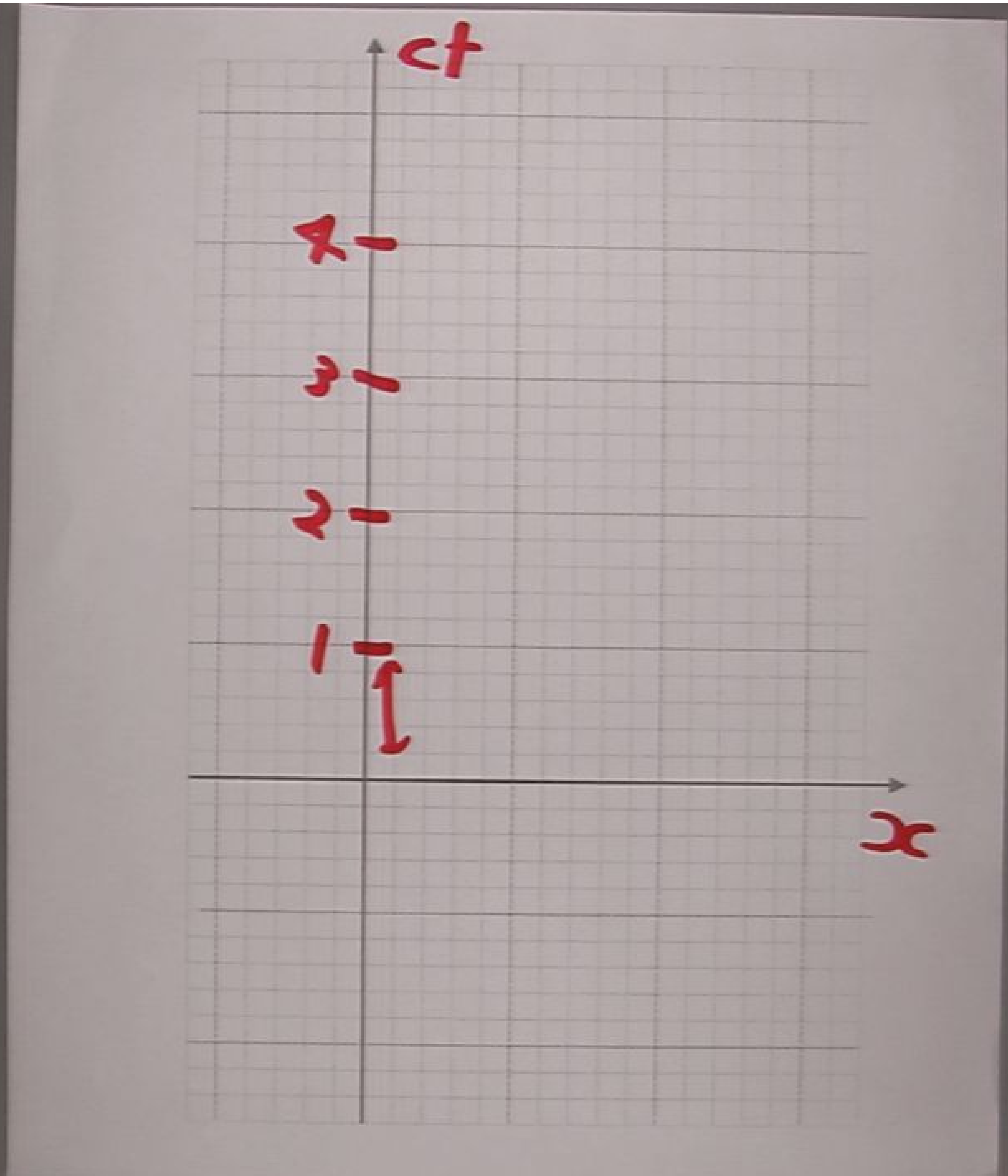
ct
4
3
2
1
x

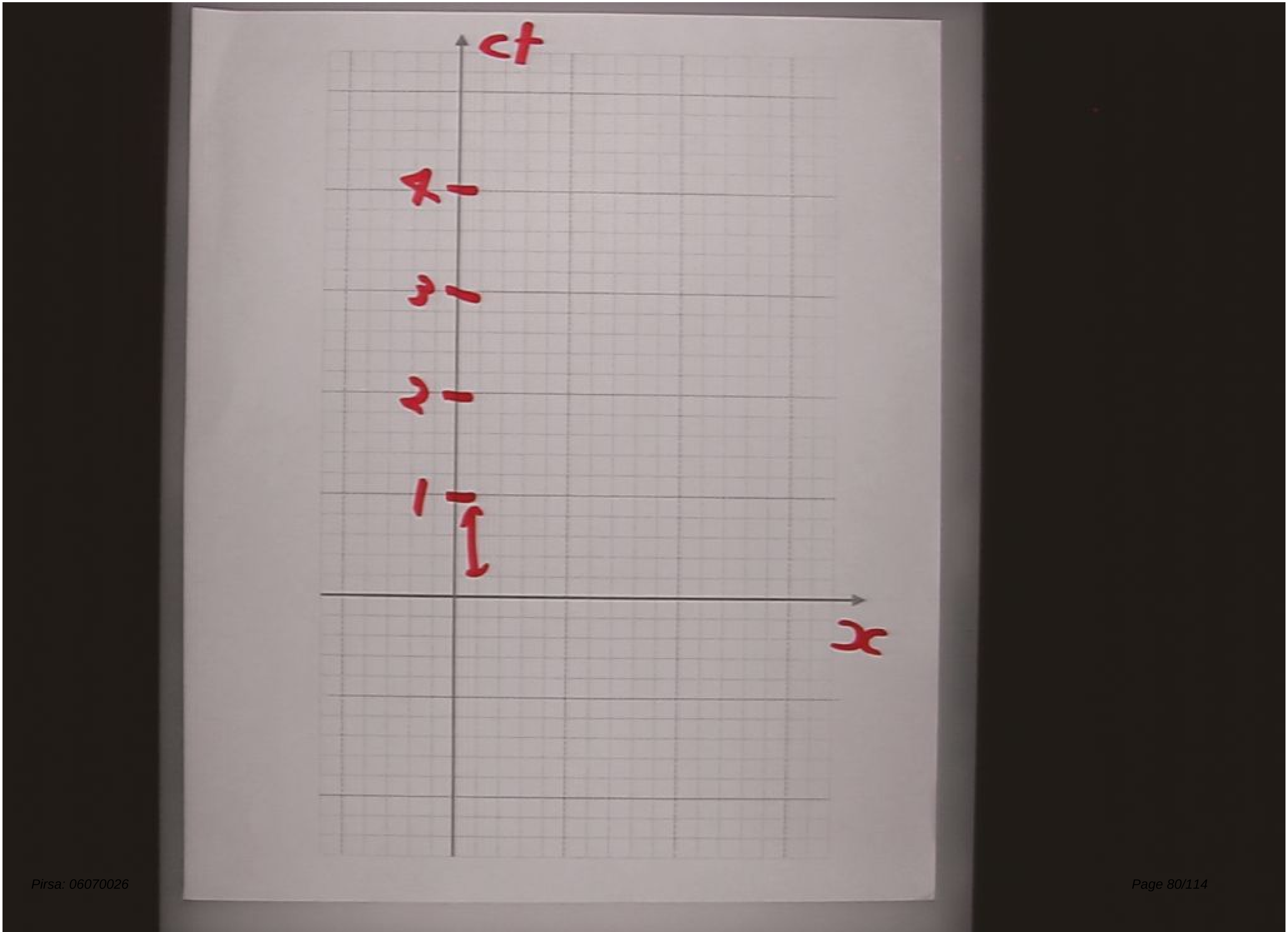

ct
4
3
2
1
x

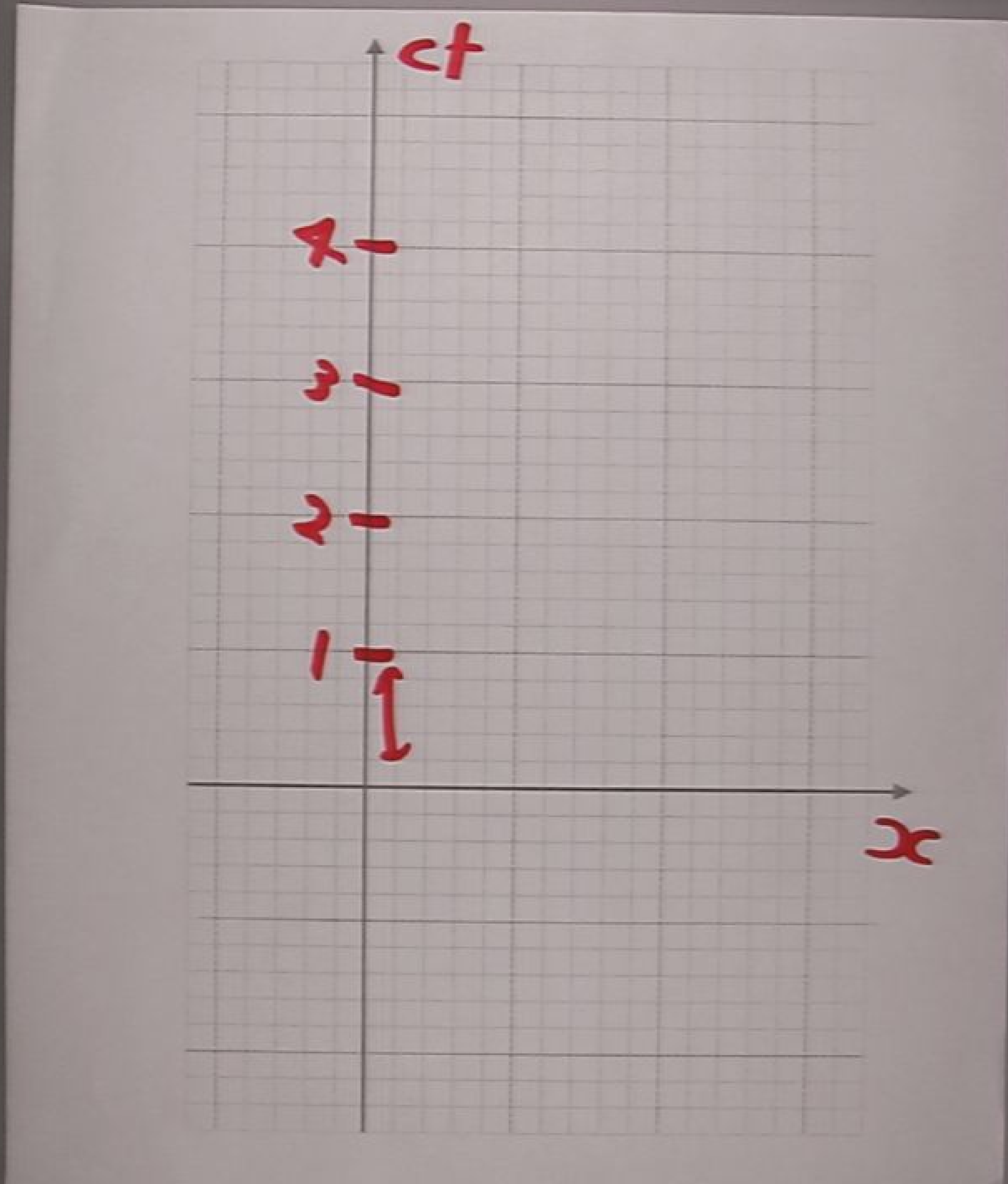

ct
4
3
2
1
x

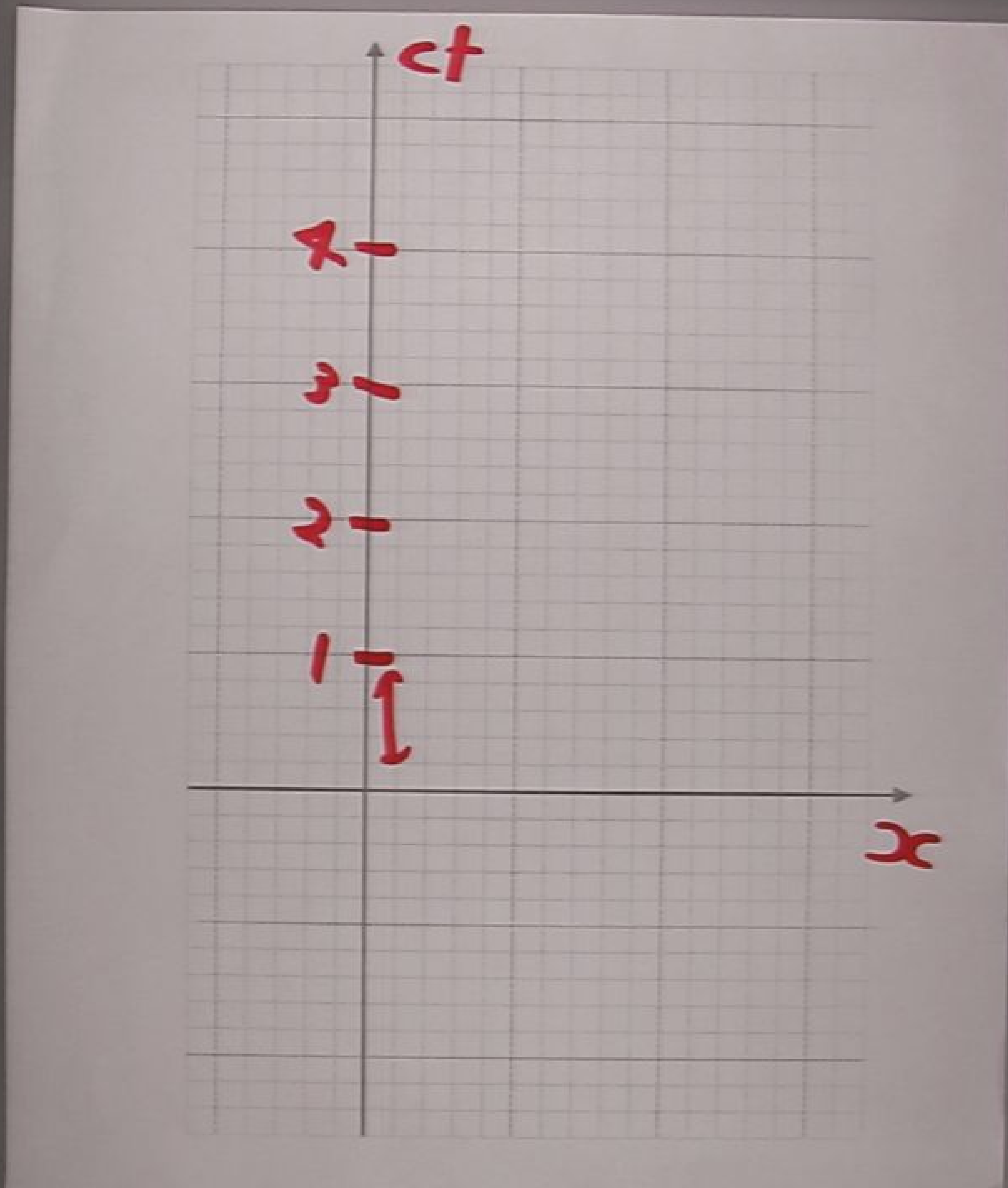

ct
4
3
2
1
x

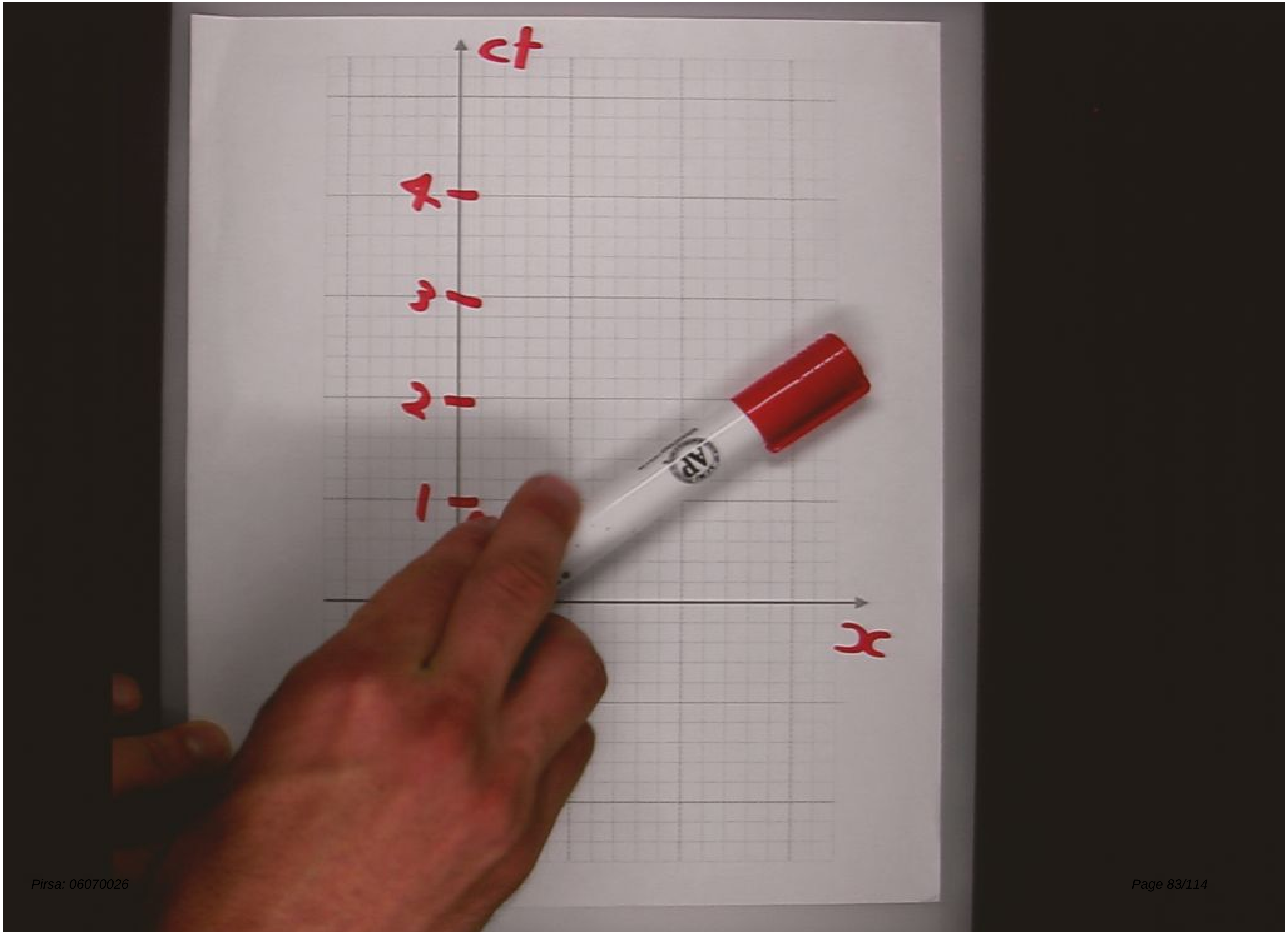

ct
4
3
2
1
x

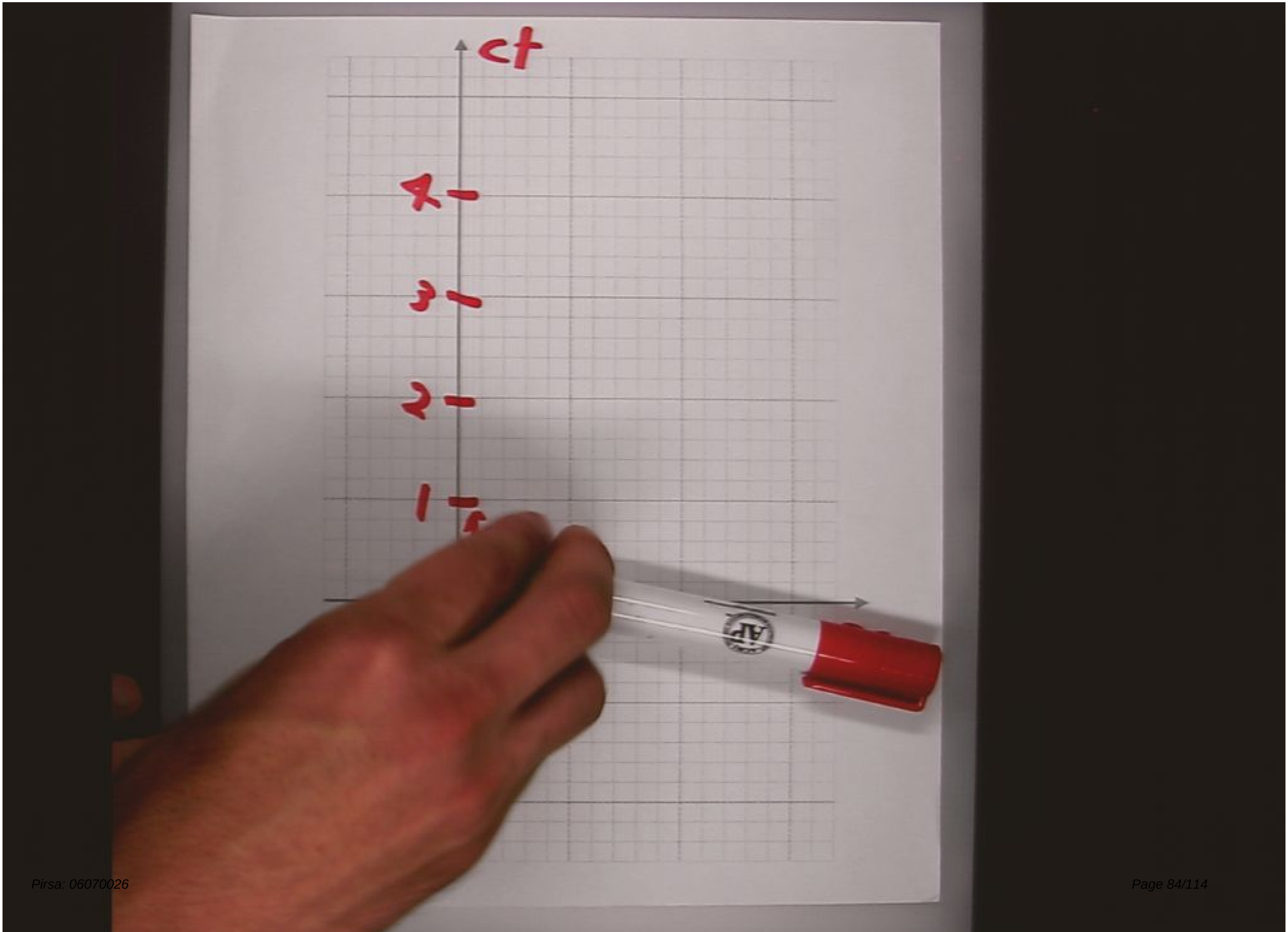

ct
4
3
2
1
x

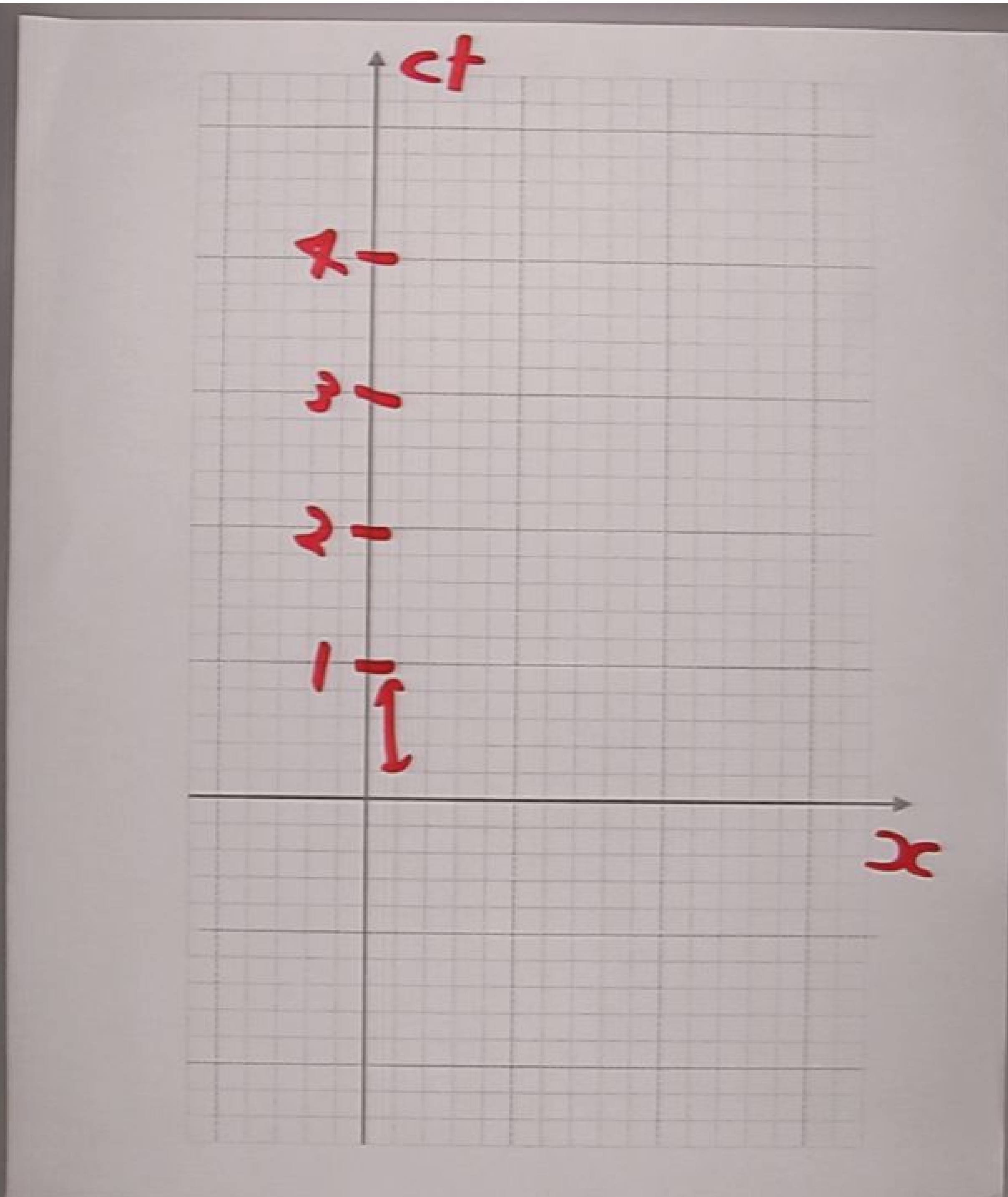

ct
4
3
2
1
x

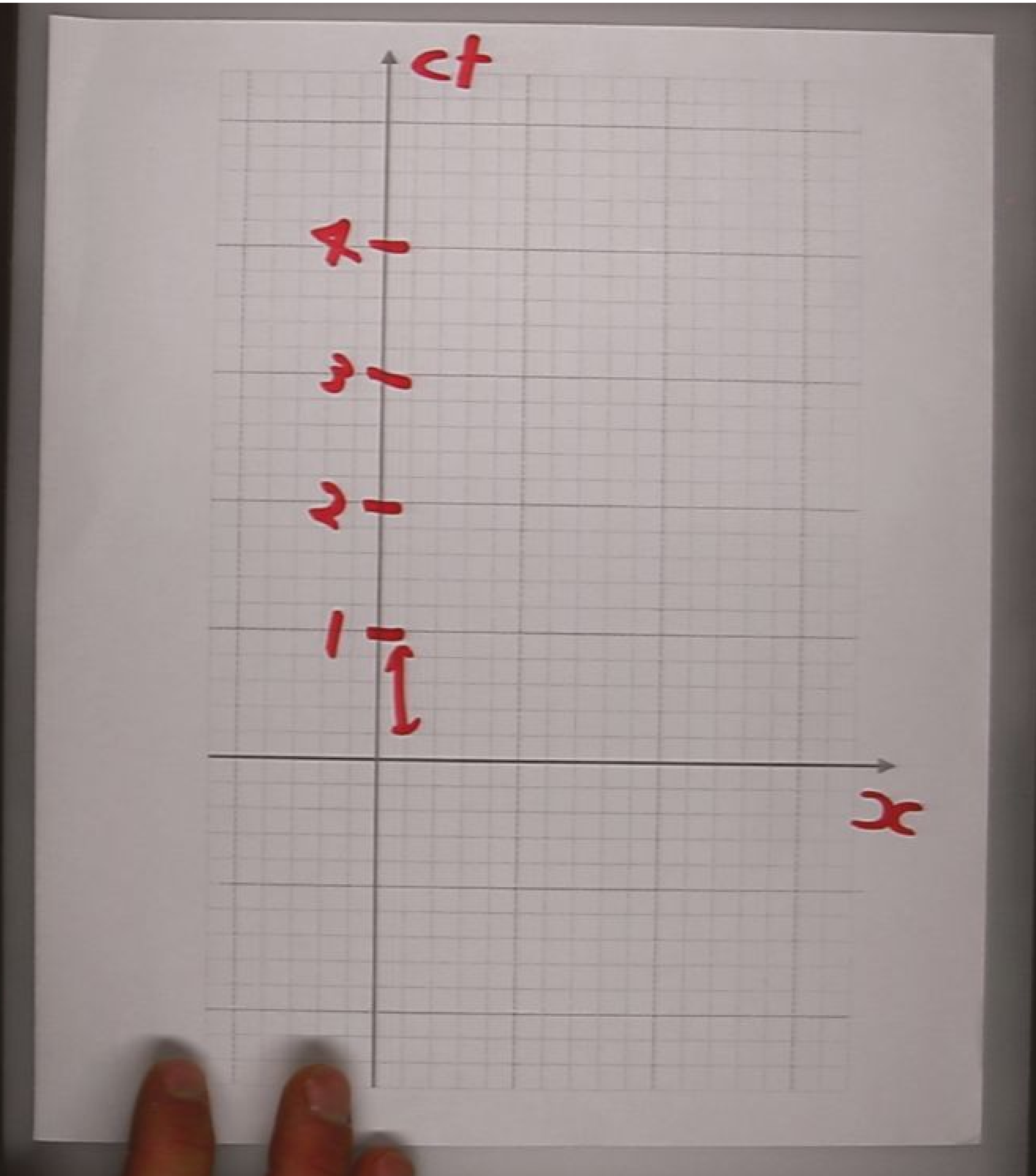

ct
4
3
2
1
x

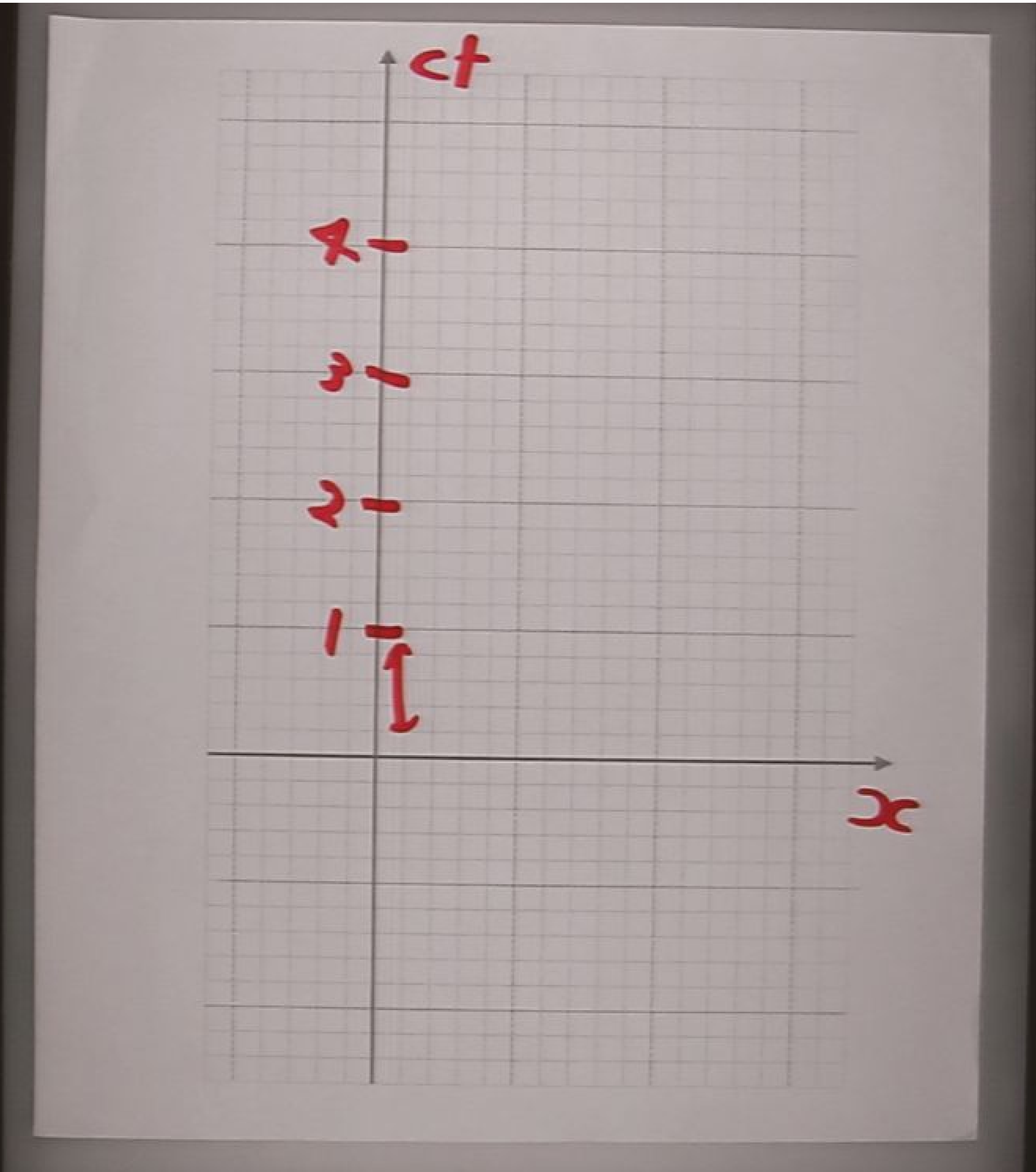

ct
4
3
2
1
x

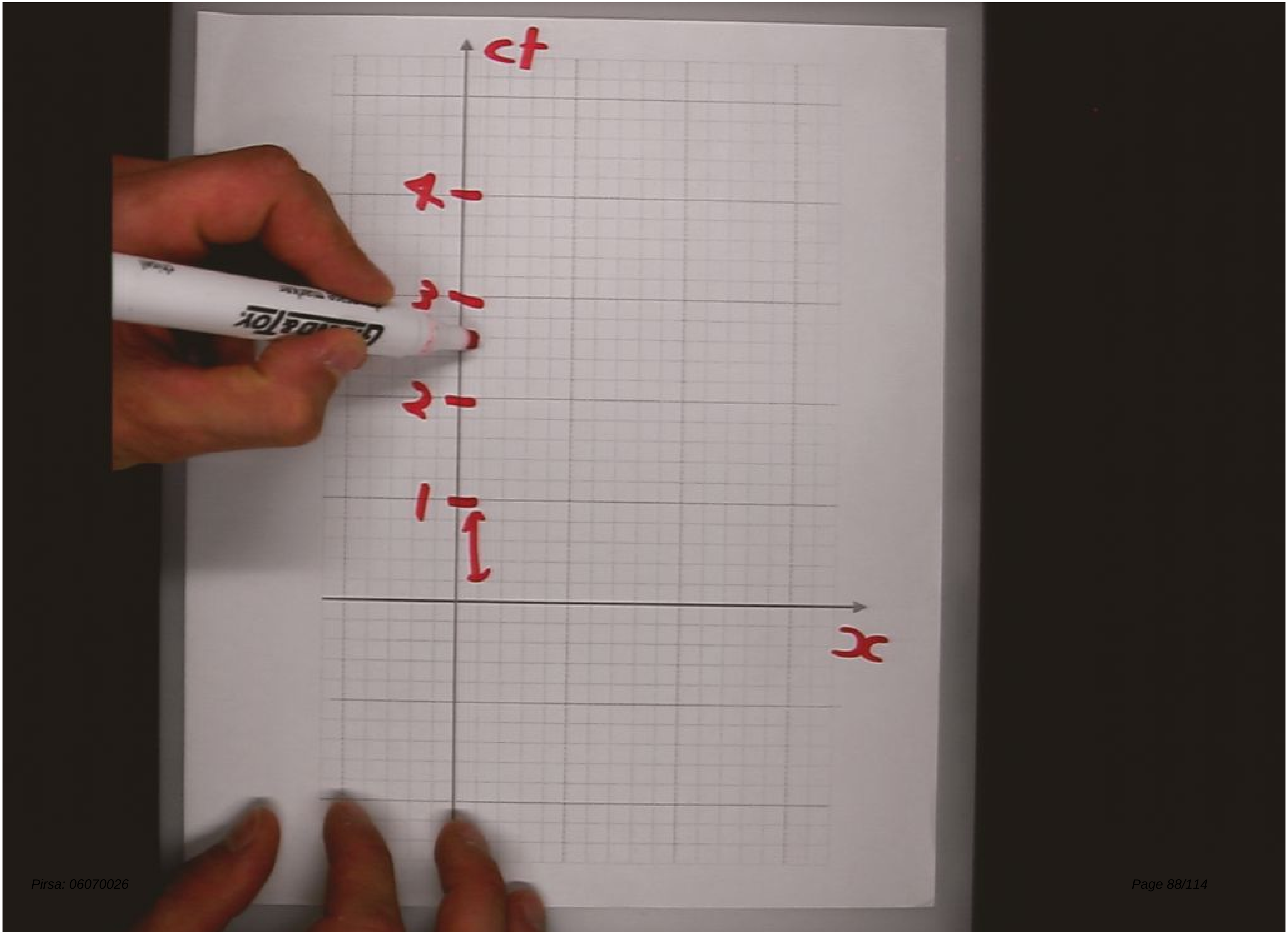

ct
4
3
2
1
x

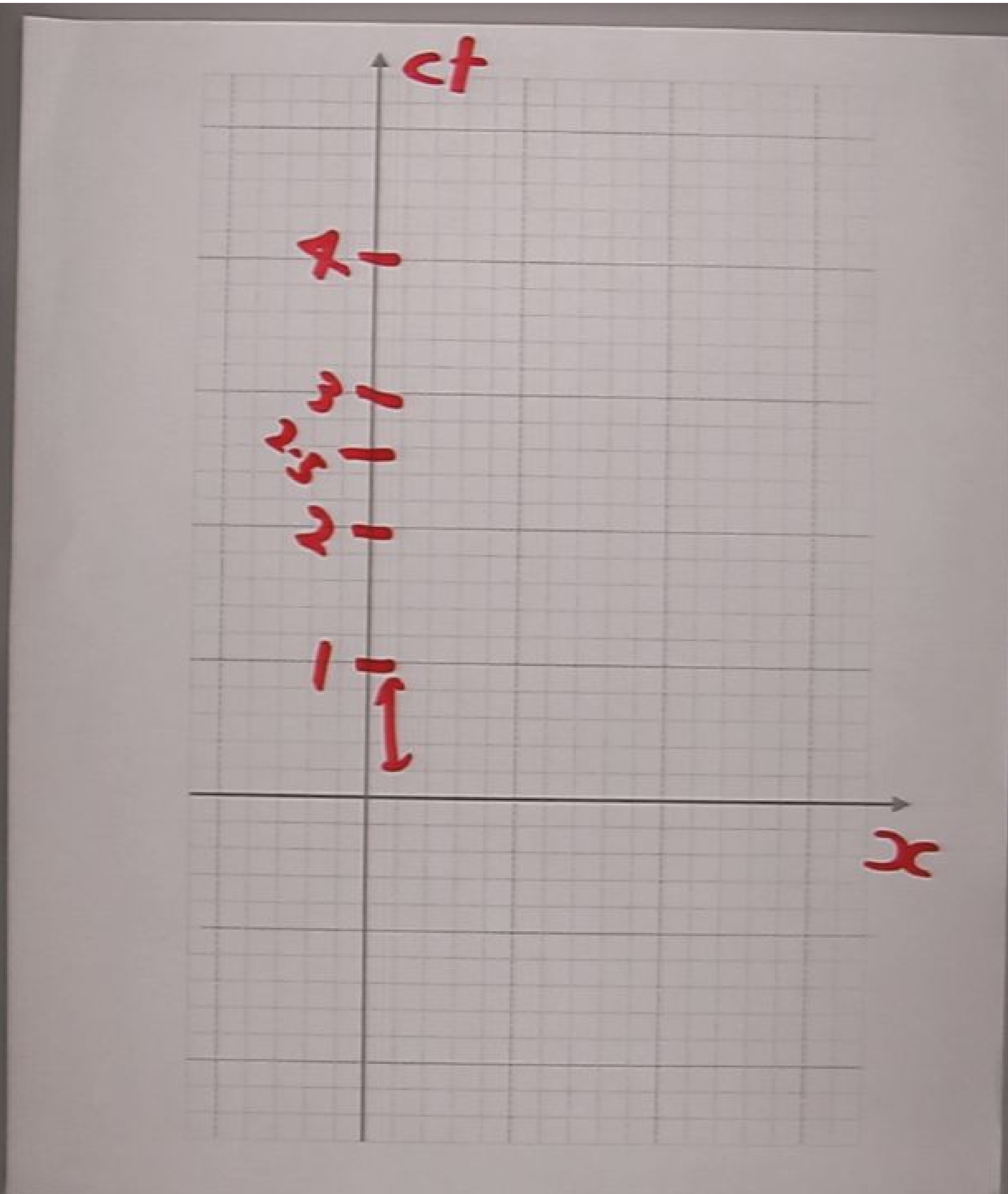

ct
4
3
2
1

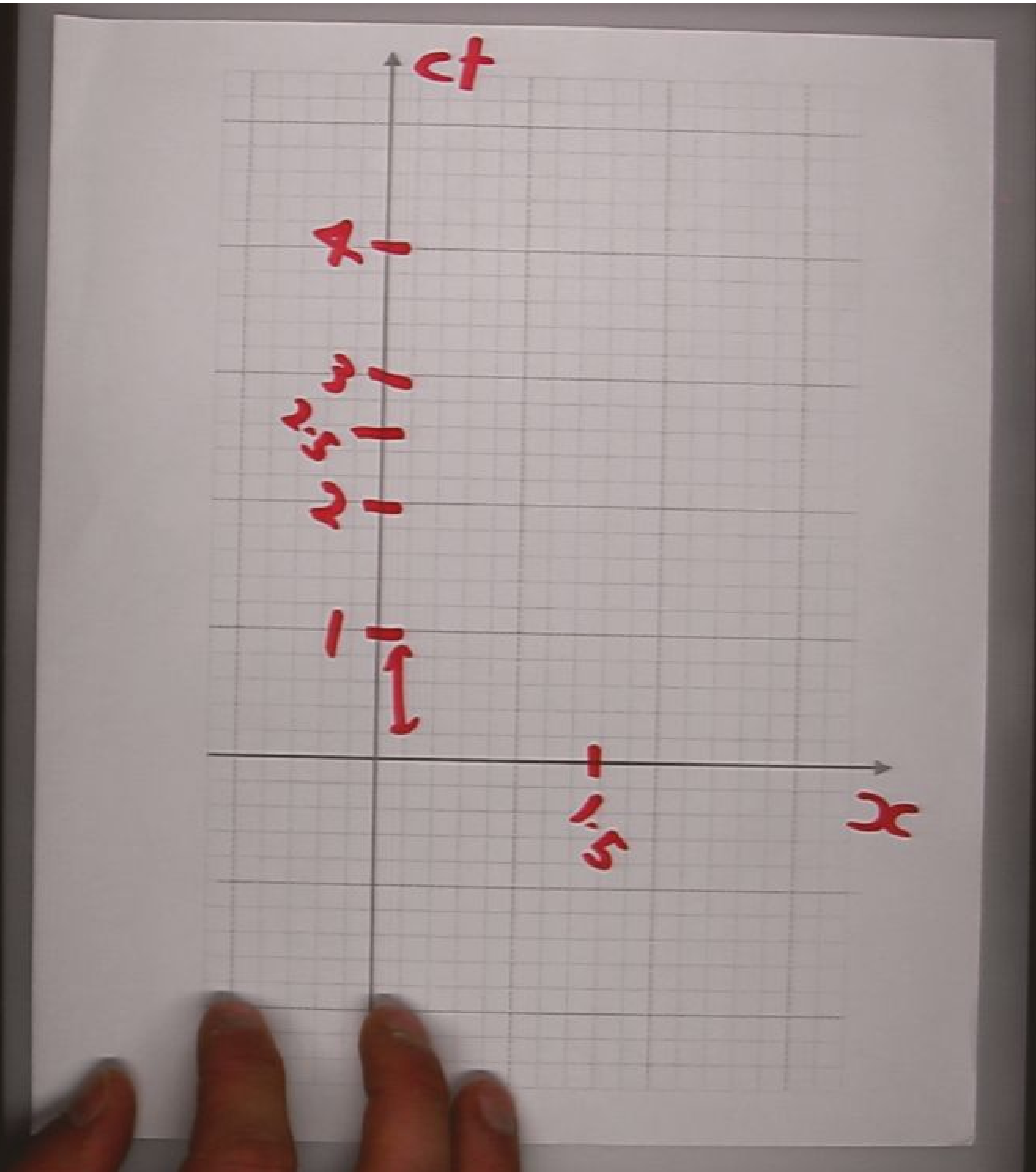

ct
4
3
2
1
x

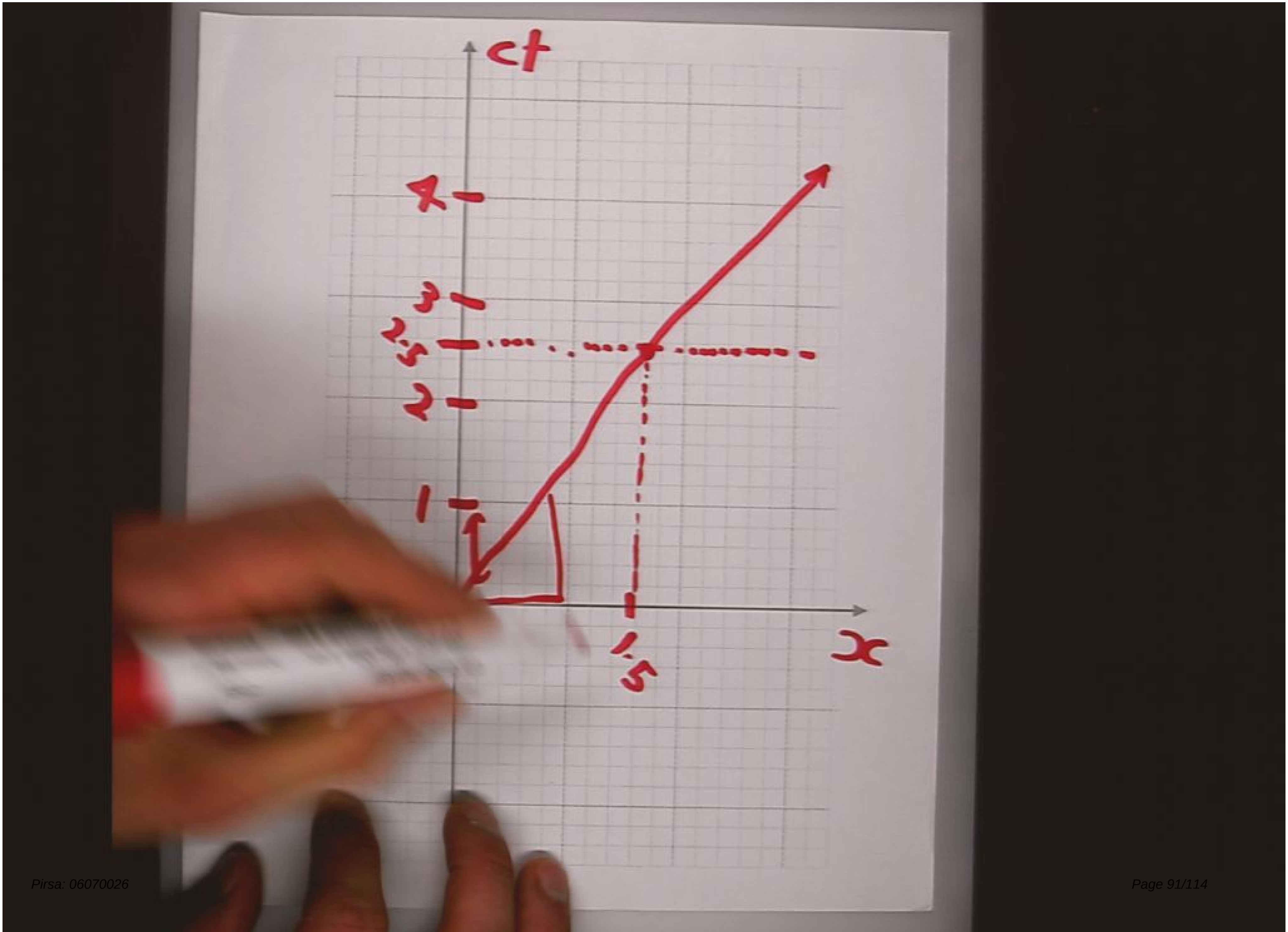

ct
4
3
2
1
x


ct
4
3
2
1
x

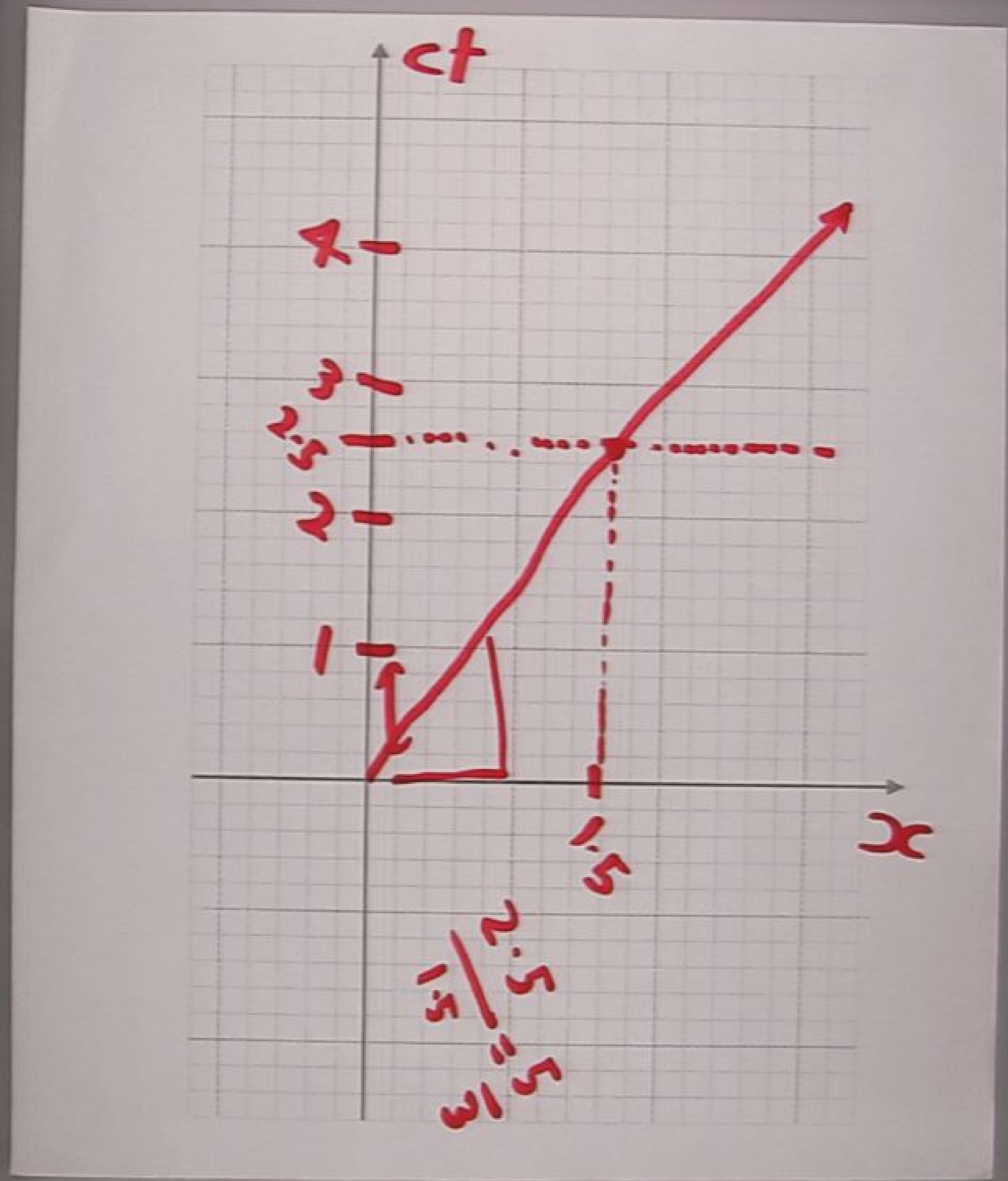

ct
4
3
2
1
x

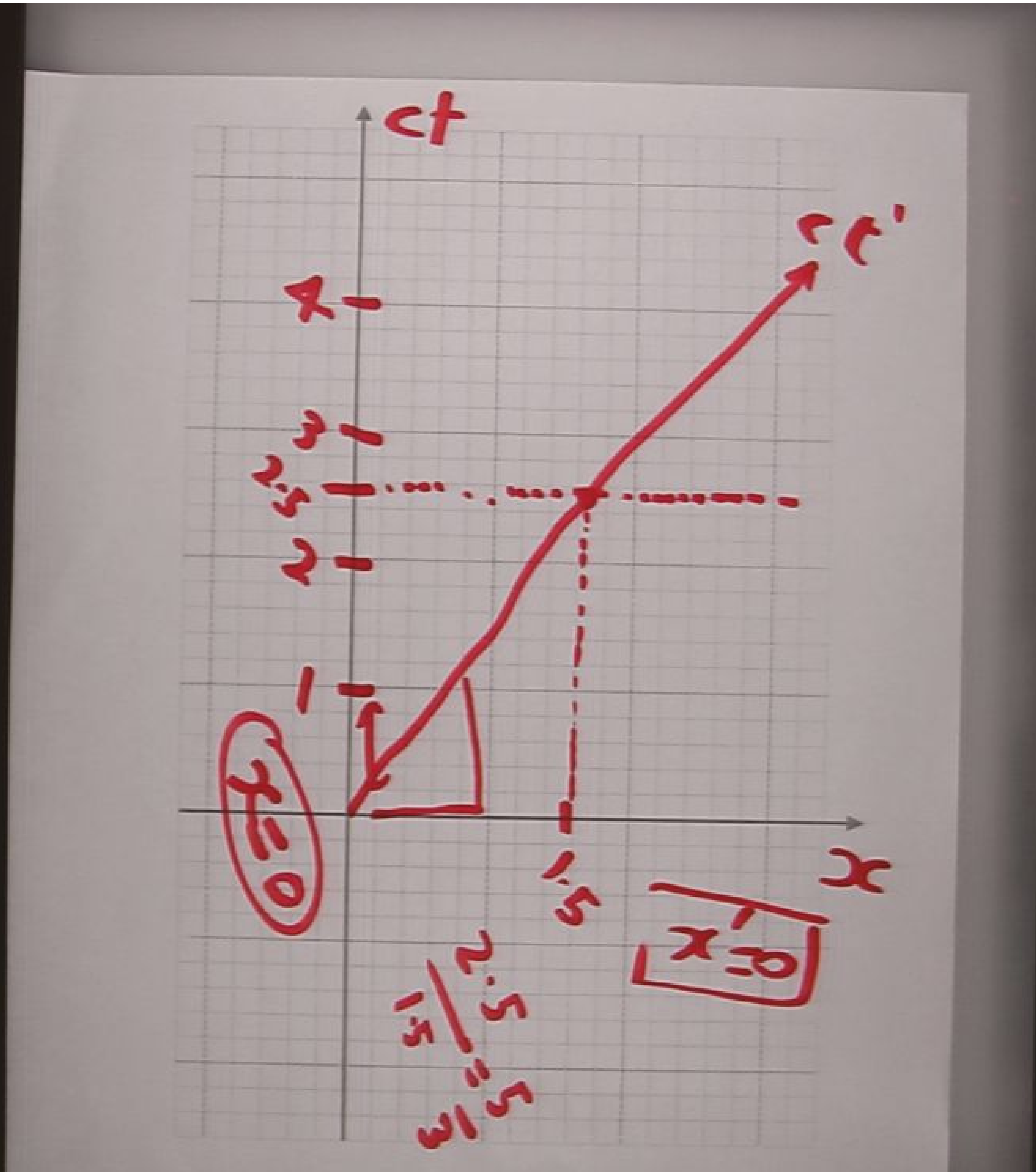

ct
4
3
2.5
2
1
x


ct
4
3
2.5
2
1
1.5
x



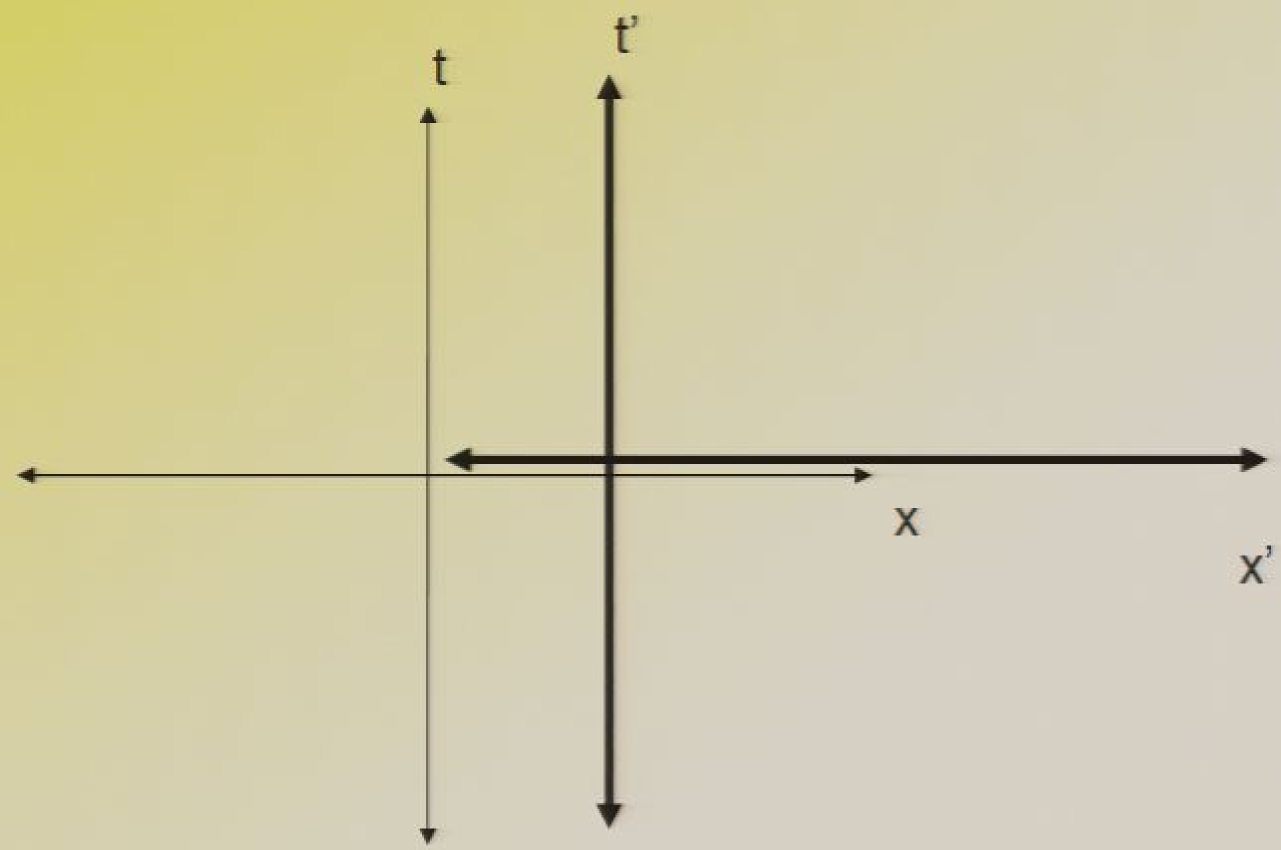
ct
4
3
2.5
2
1
1.5
x

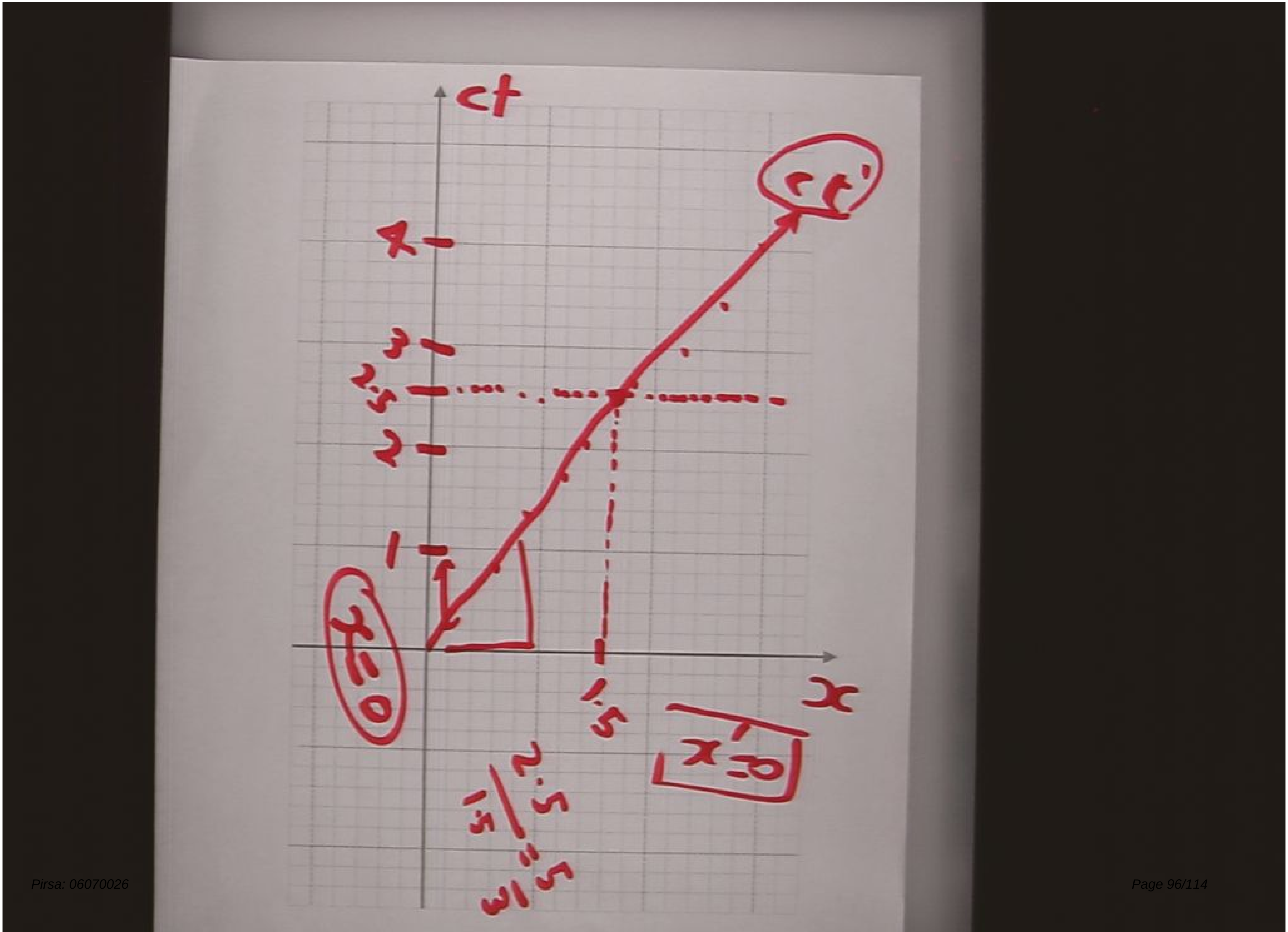

ct
4
3
2.5
2
1
x
1.5
2.5
1.5
= 5
3

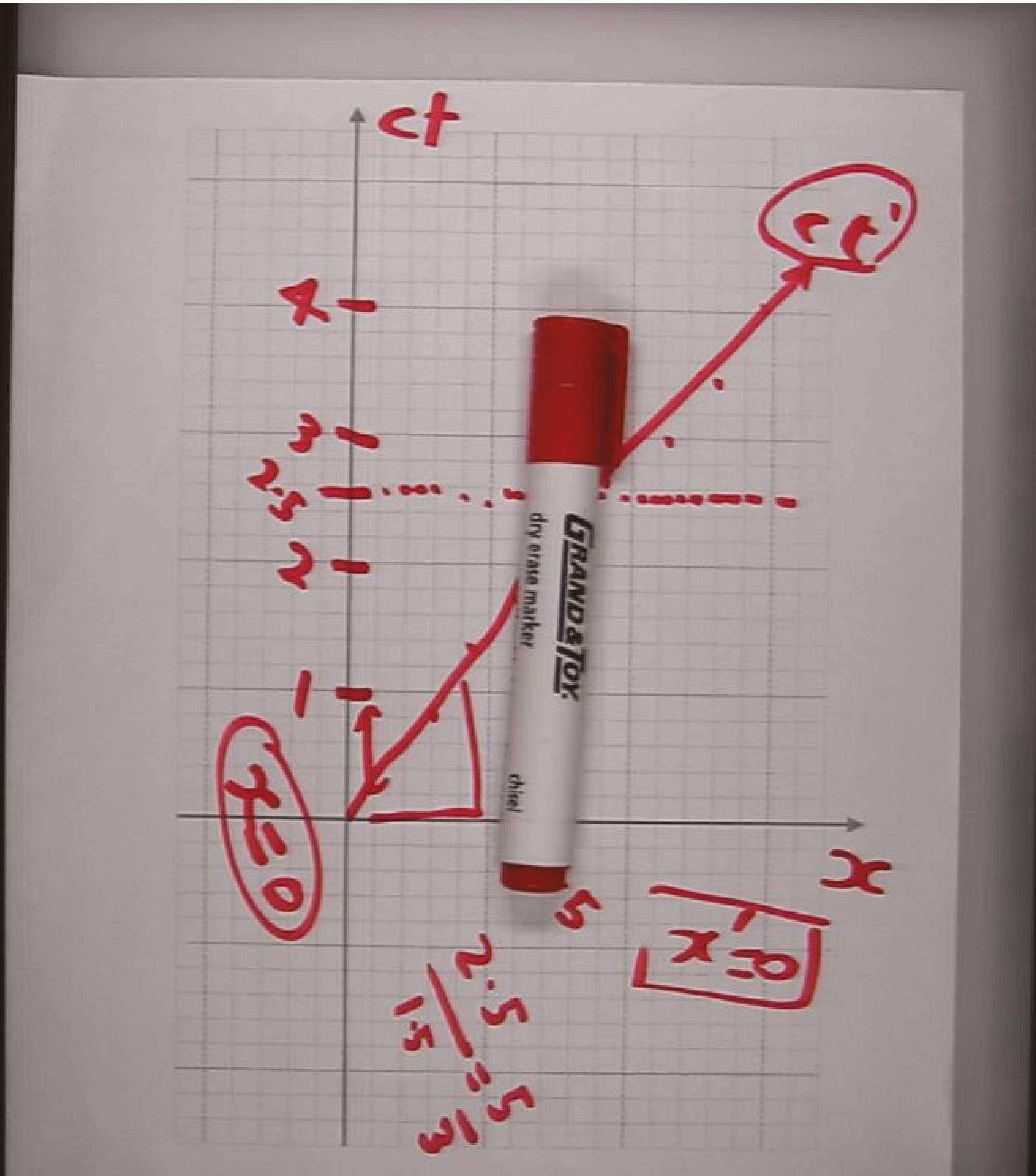

ct
ct'
4
3
2.5
2
1
x
1.5
x=0
x'=0
2.5
1.5
= 5
3

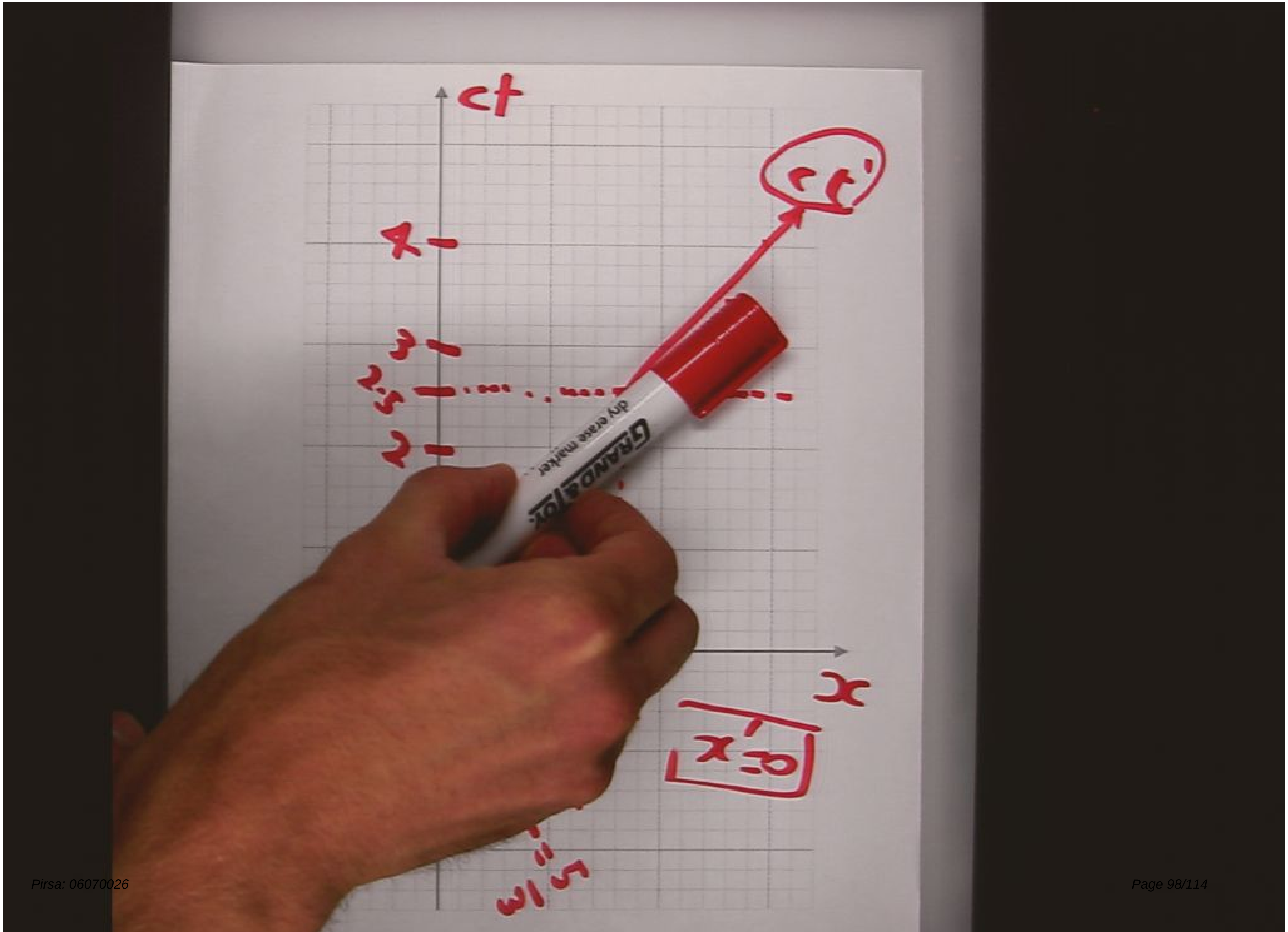
# Transforming between different frames of reference

- Galilean transformations

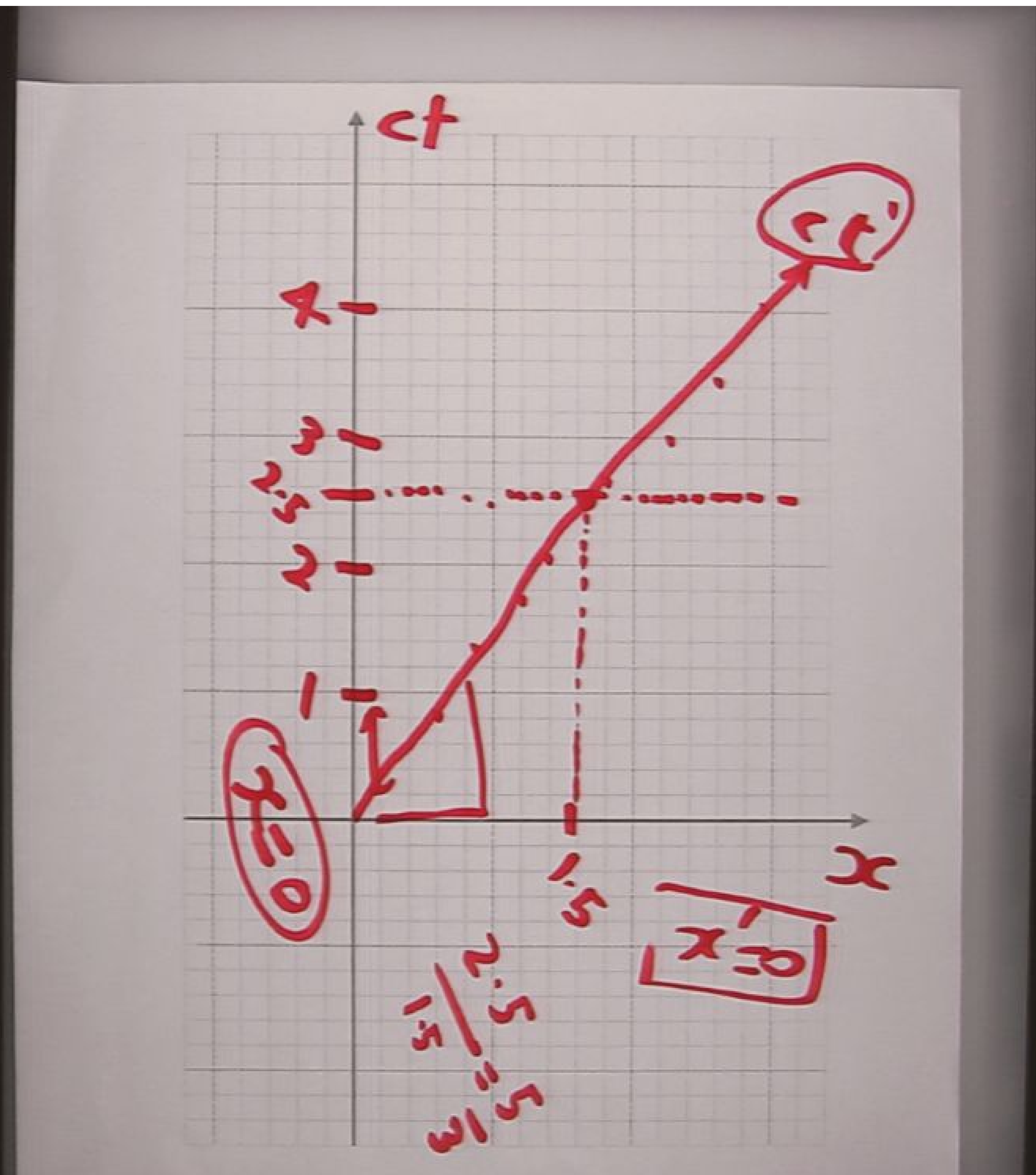


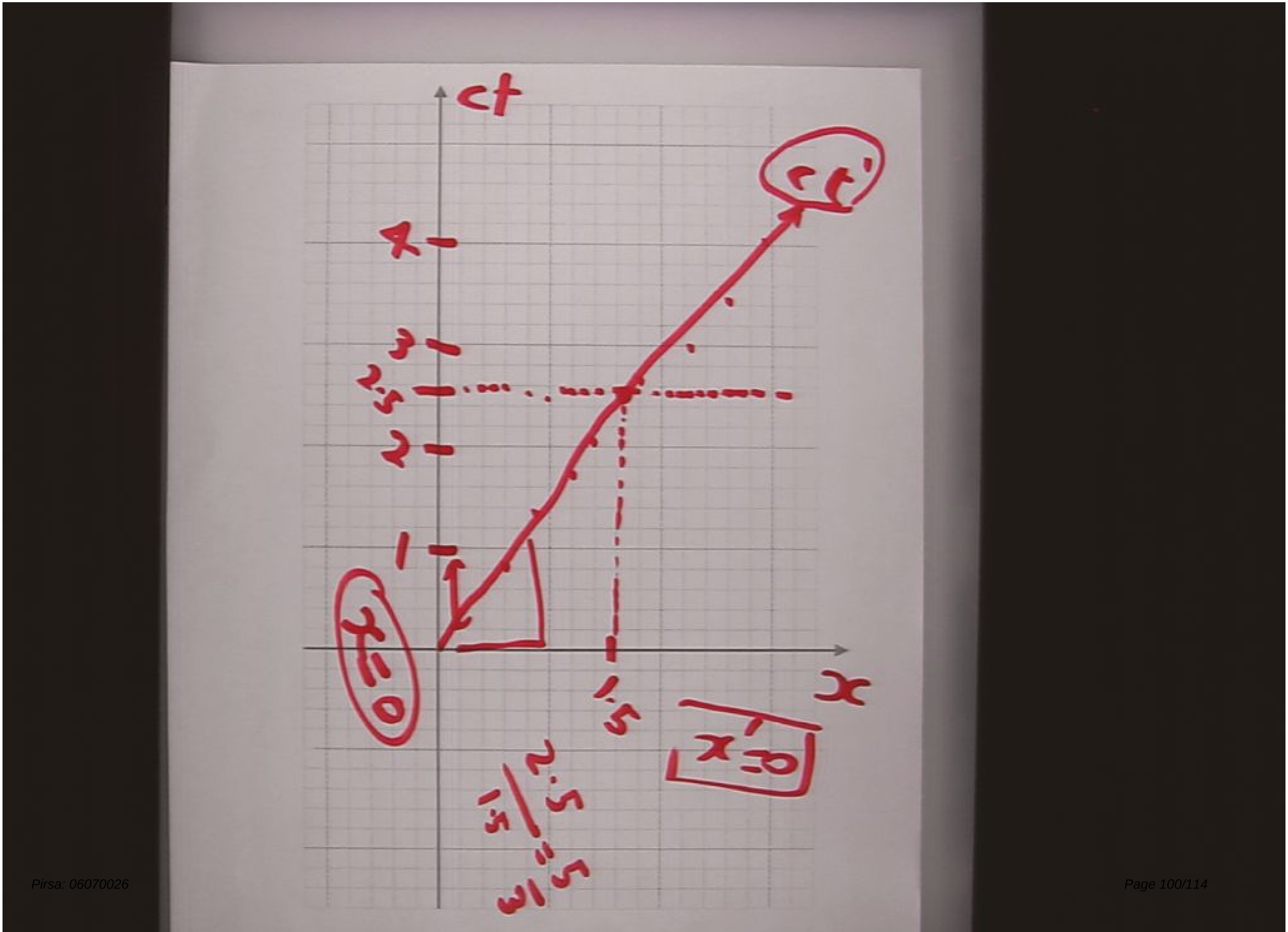

ct
ct'
4
3
2.5
2
1
x=0
x
1.5
x'=0
2.5
1.5
= 5
3


ct
ct'
4
3
2.5
2
1
x=0
x
x'=0
5
2.5
1.5
=5/3
GRAND&TOY
dry erase marker
chisel



ct
4
3
2.5
2
ct'
x
x'
dry erase marker


ct
ct'
4
3
2.5
2
1
x=0
1.5
x
x'=0
2.5
1.5
= 5
3

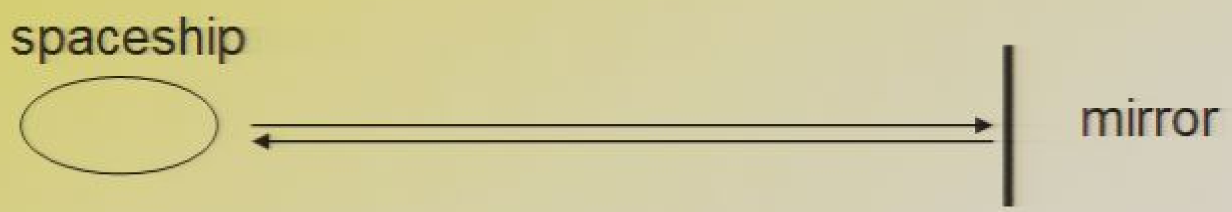

ct
ct'
4
3
2.5
2
1
x = 0
x
1.5
x' = 0
2.5 / 1.5 = 5/3

- What about the x' axis?

- In analogy with the ct' axis, it is defined by the equation t'=0.

- Let us determine it via the following procedure:

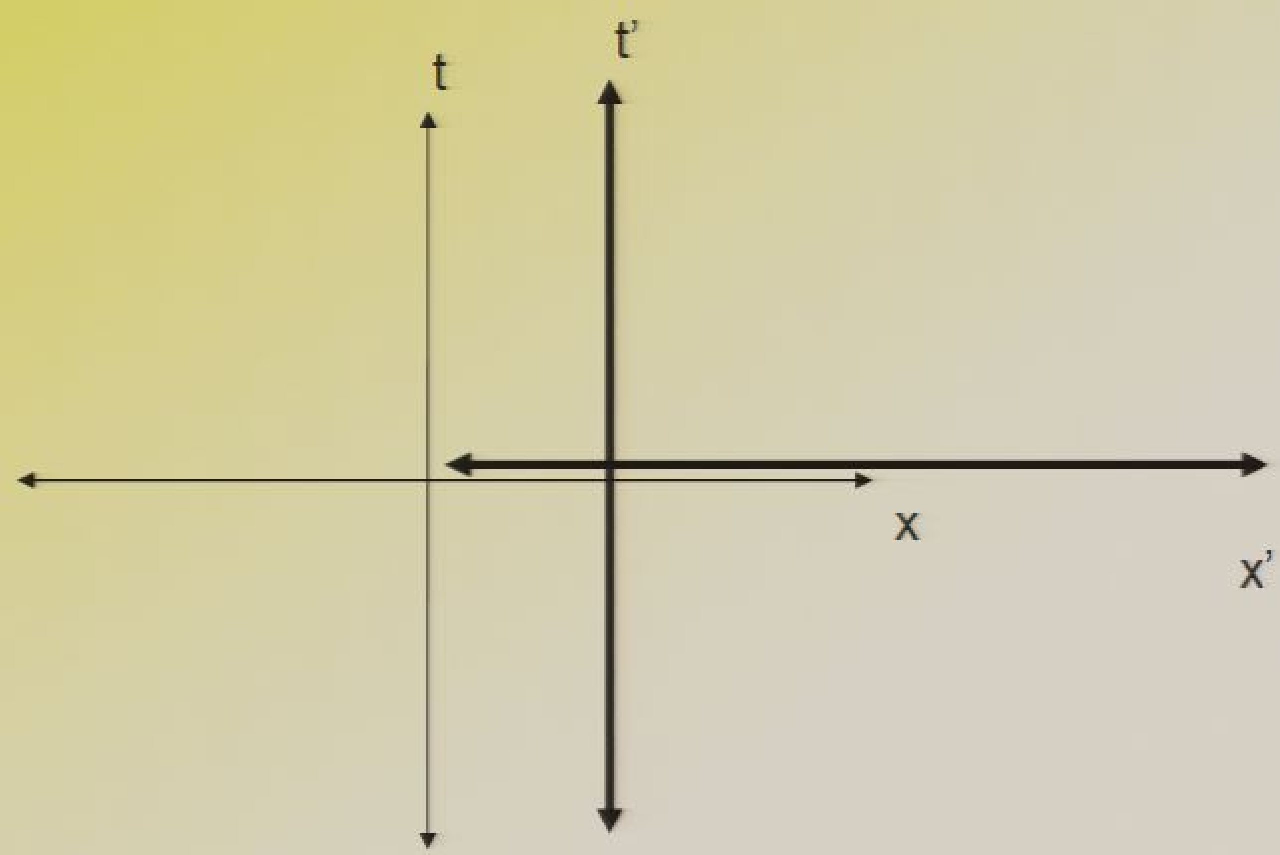
  Consider the spaceship's commander turning on a high-powered flashlight at time t'=-T. Assume that the light travels away from them in the +x direction. Once it reaches a certain point B, a mirror there reflects it back to the spaceship at time t'=T.




- As light travels at c relative to all observers, someone on Earth (E) always sees light as moving at c. This means that the worldlines for light are always at a 45 degree angle relative to the x axis.

# Transforming between different frames of reference

- Galilean transformations



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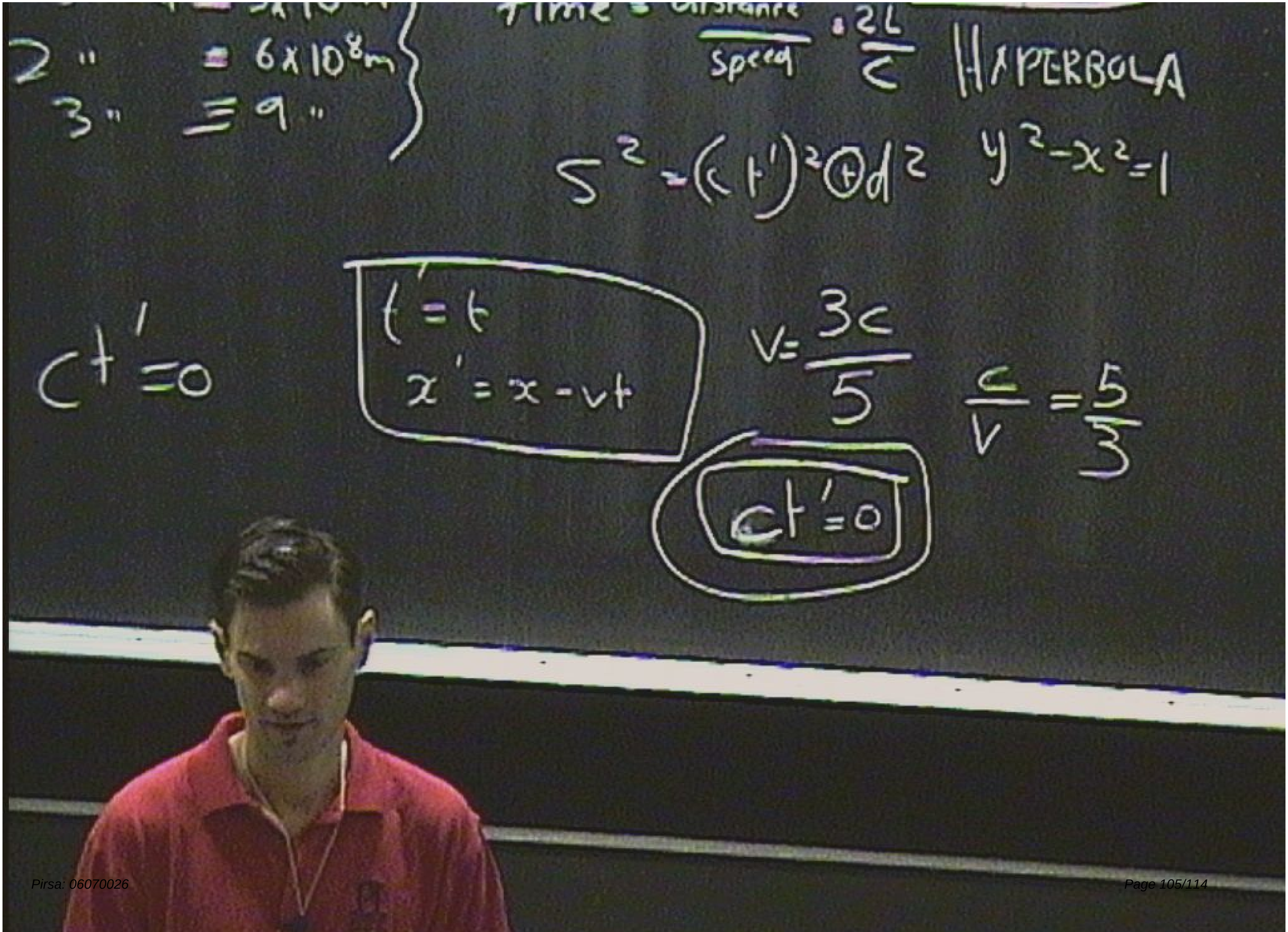
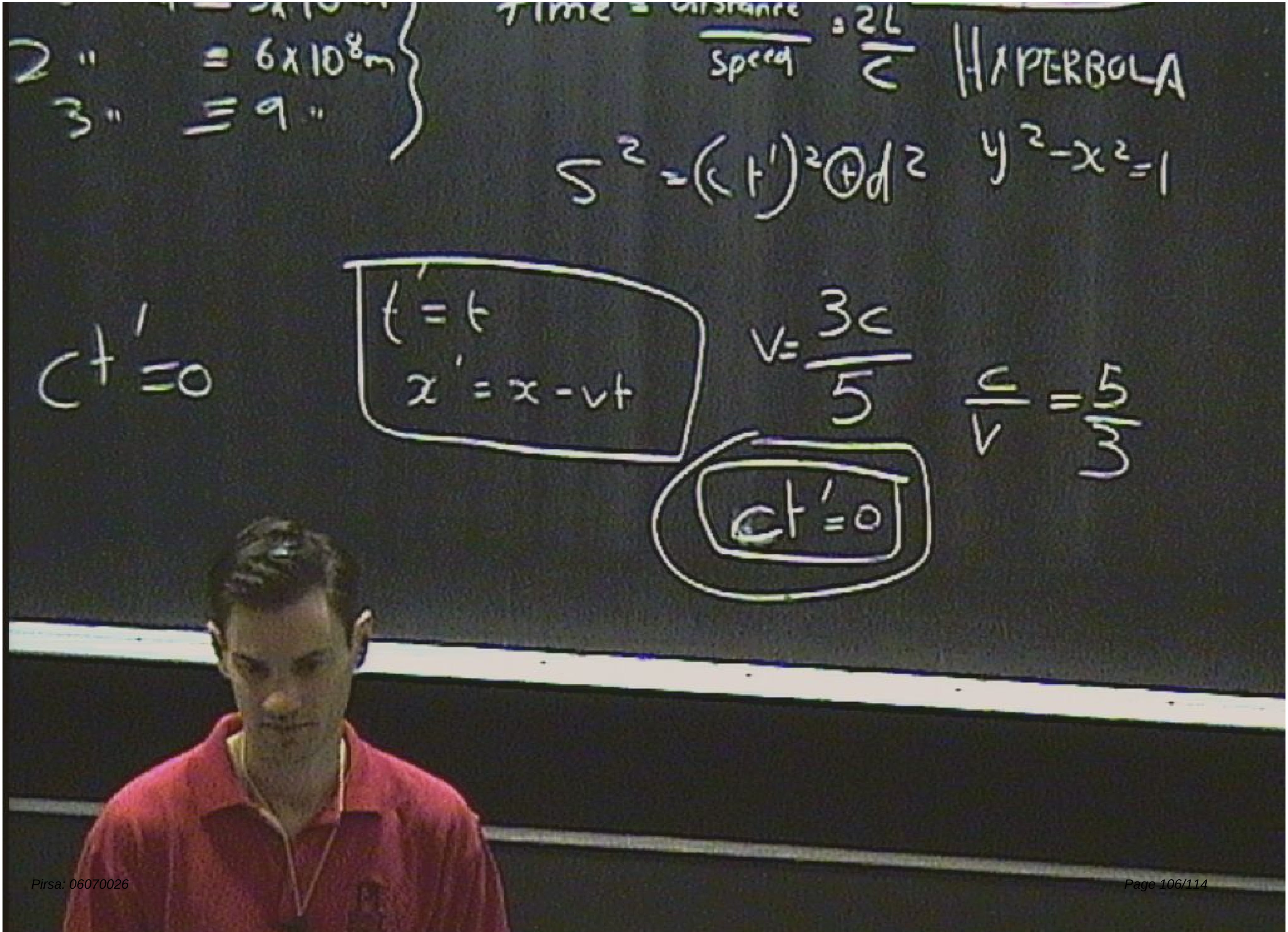
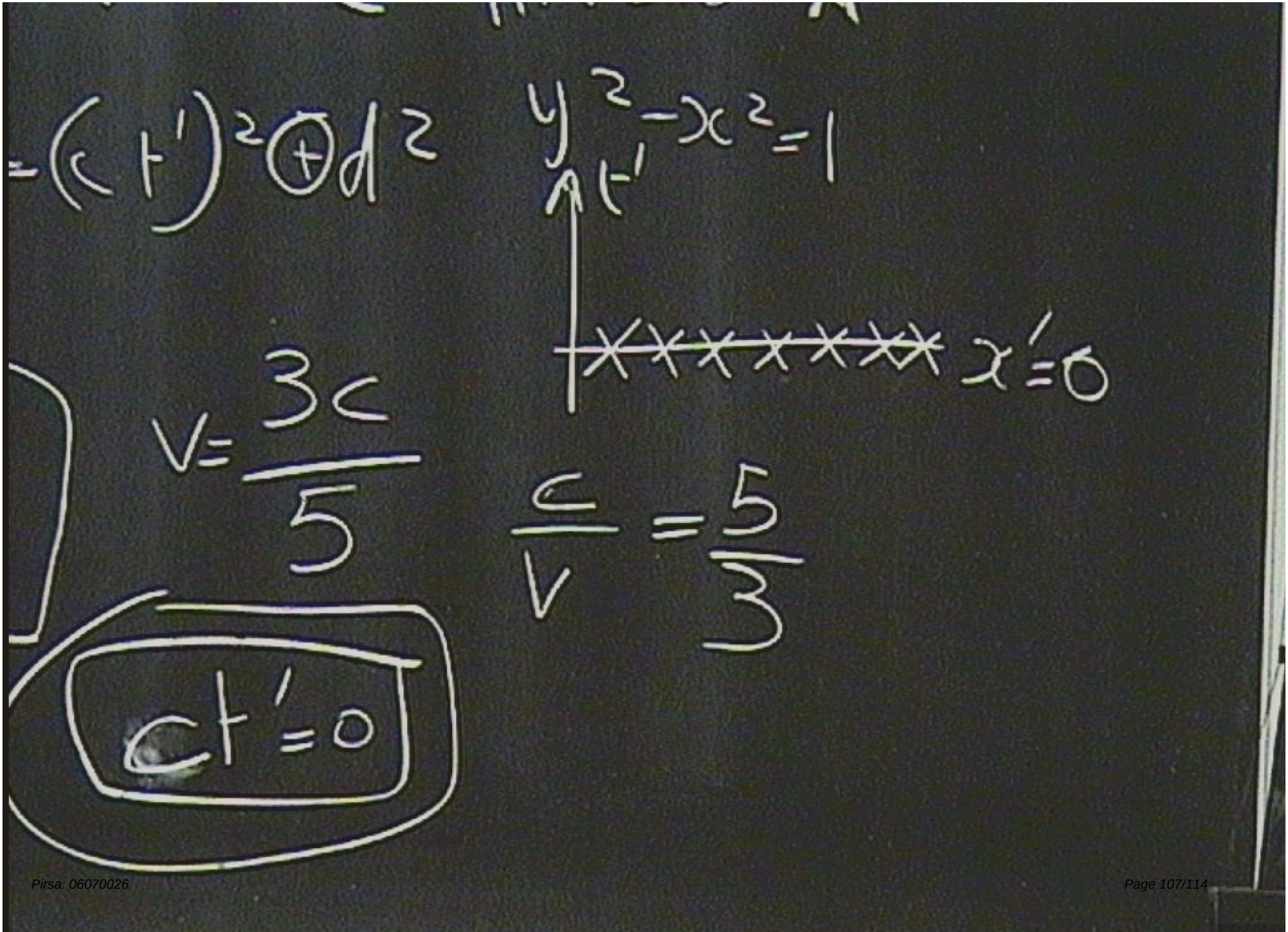
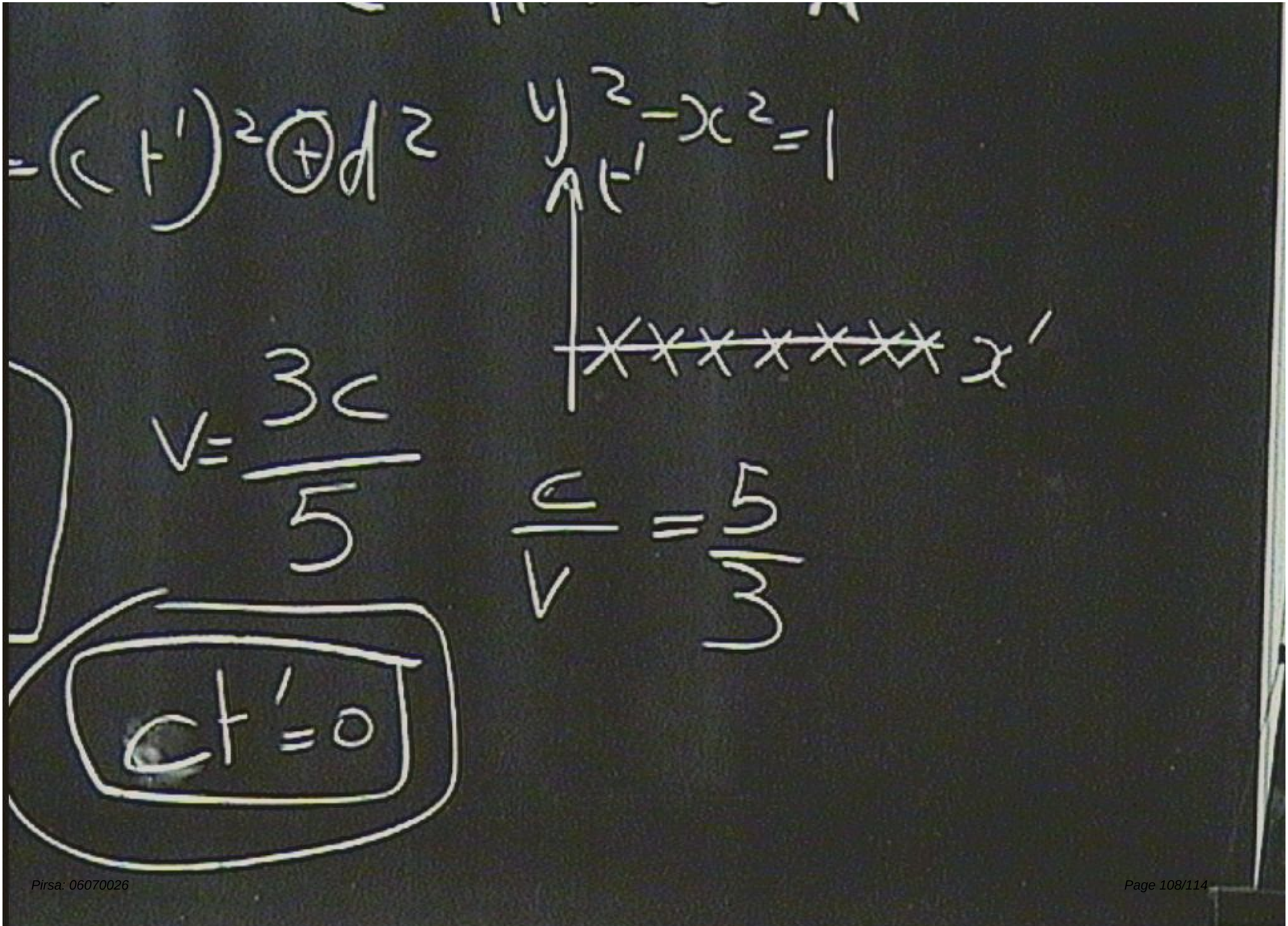
- As light travels at c relative to all observers, someone on Earth (E) always sees light as moving at c. This means that the worldlines for light are always at a 45 degree angle relative to the x axis.


2 " = 6x10^8 m
3 " = 9 "
speed
2L/c
HYPERBOLA
s^2 = (ct')^2 + d^2
y^2 - x^2 = 1
ct' = 0
t' = t
x' = x - vt
v = 3c/5
c/v = 5/3
ct' = 0



2 " = 6x10⁸m
3 " = 9 "
speed
HYPERBOLA
y² - x² = 1
t' = t
x' = x - vt
ct' = 0
v = 3c/5
c/v = 5/3
ct' = 0


(ct')² ⊕ d²
y² − x² = 1
t'
x' = 0
V = 3c/5
c/v = 5/3
ct' = 0


(ct')² ⊕ d²
y² − x² = 1
t'
x'
V = 3c/5
c/v = 5/3
ct' = 0

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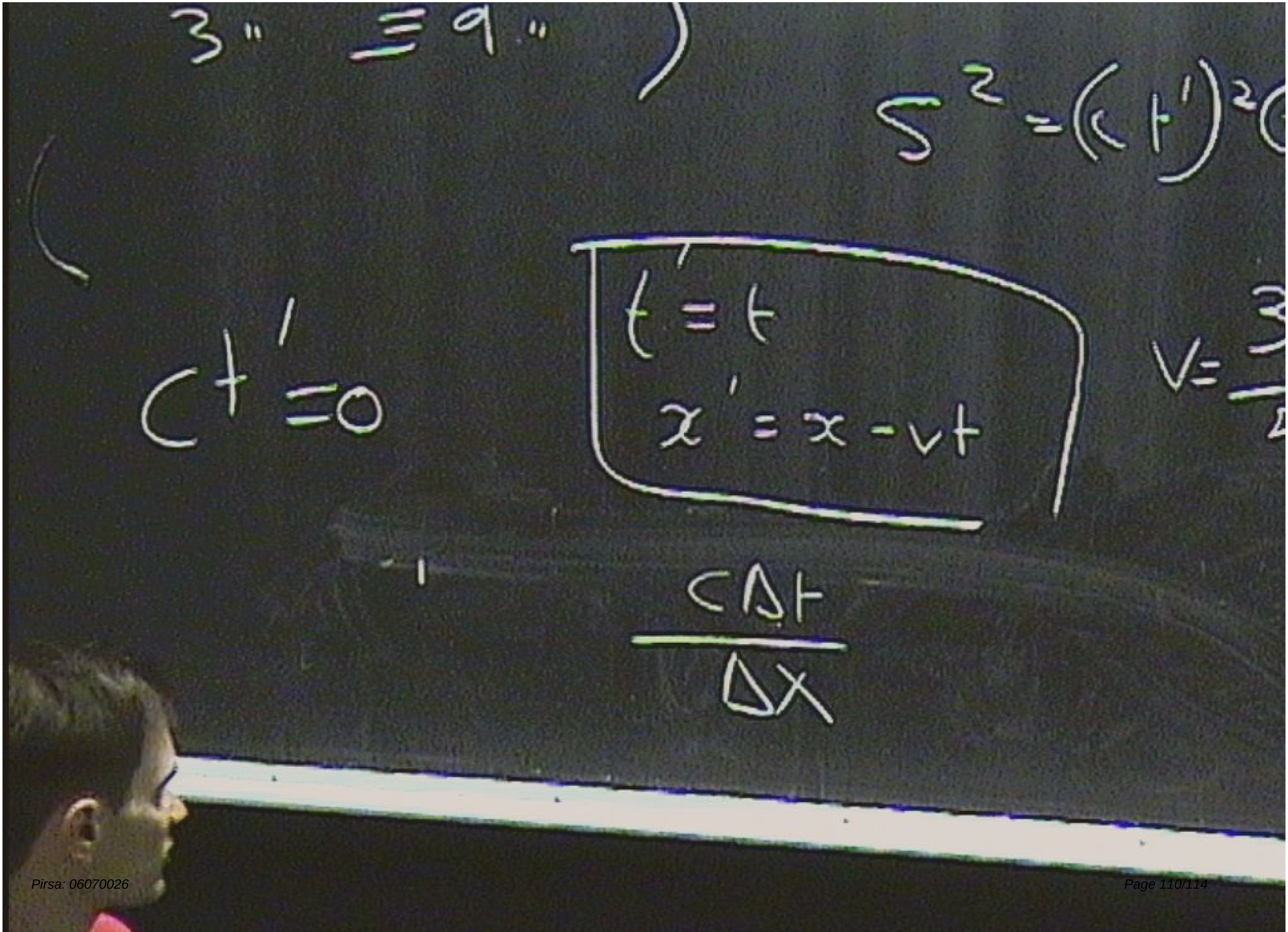
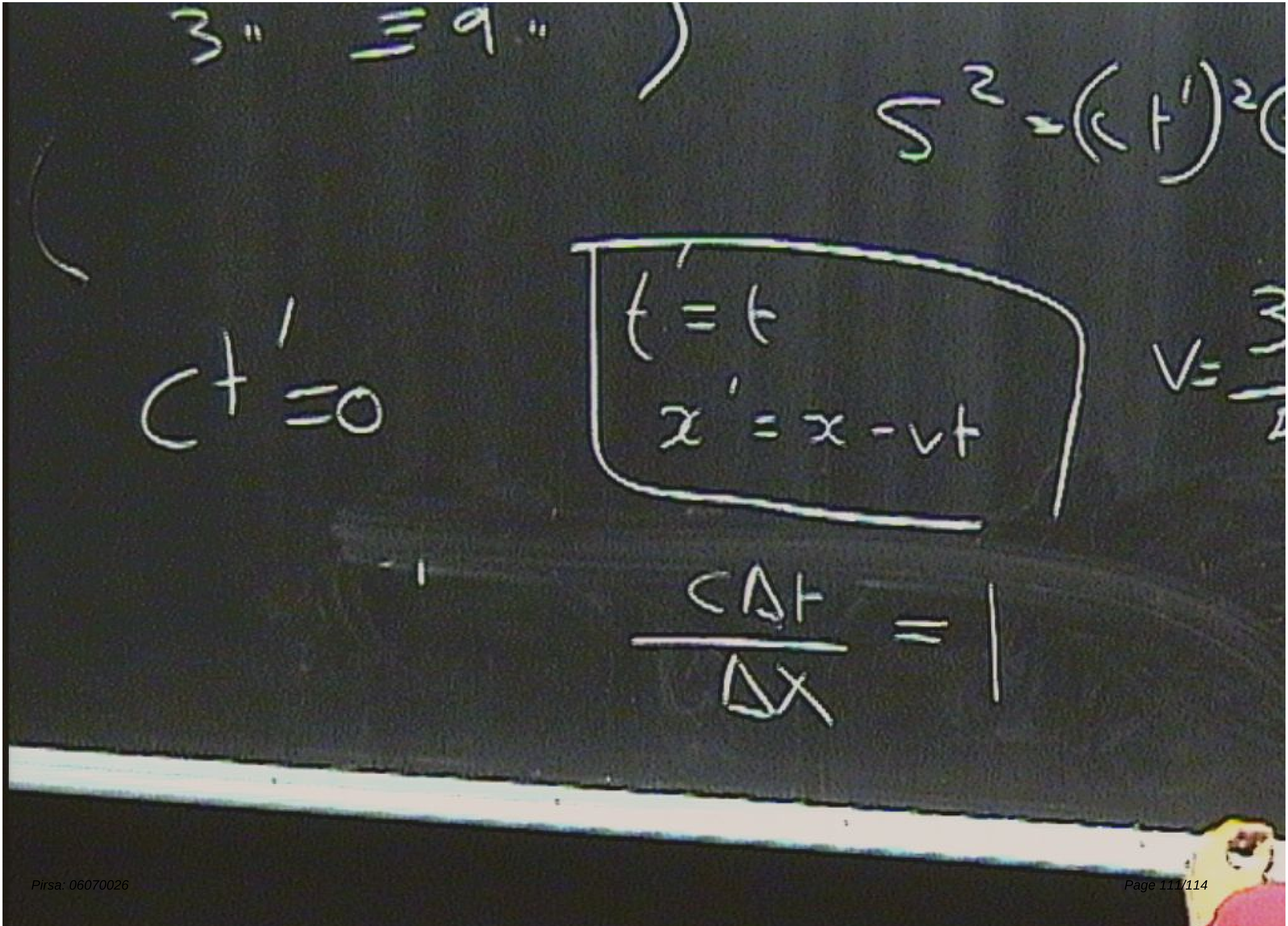
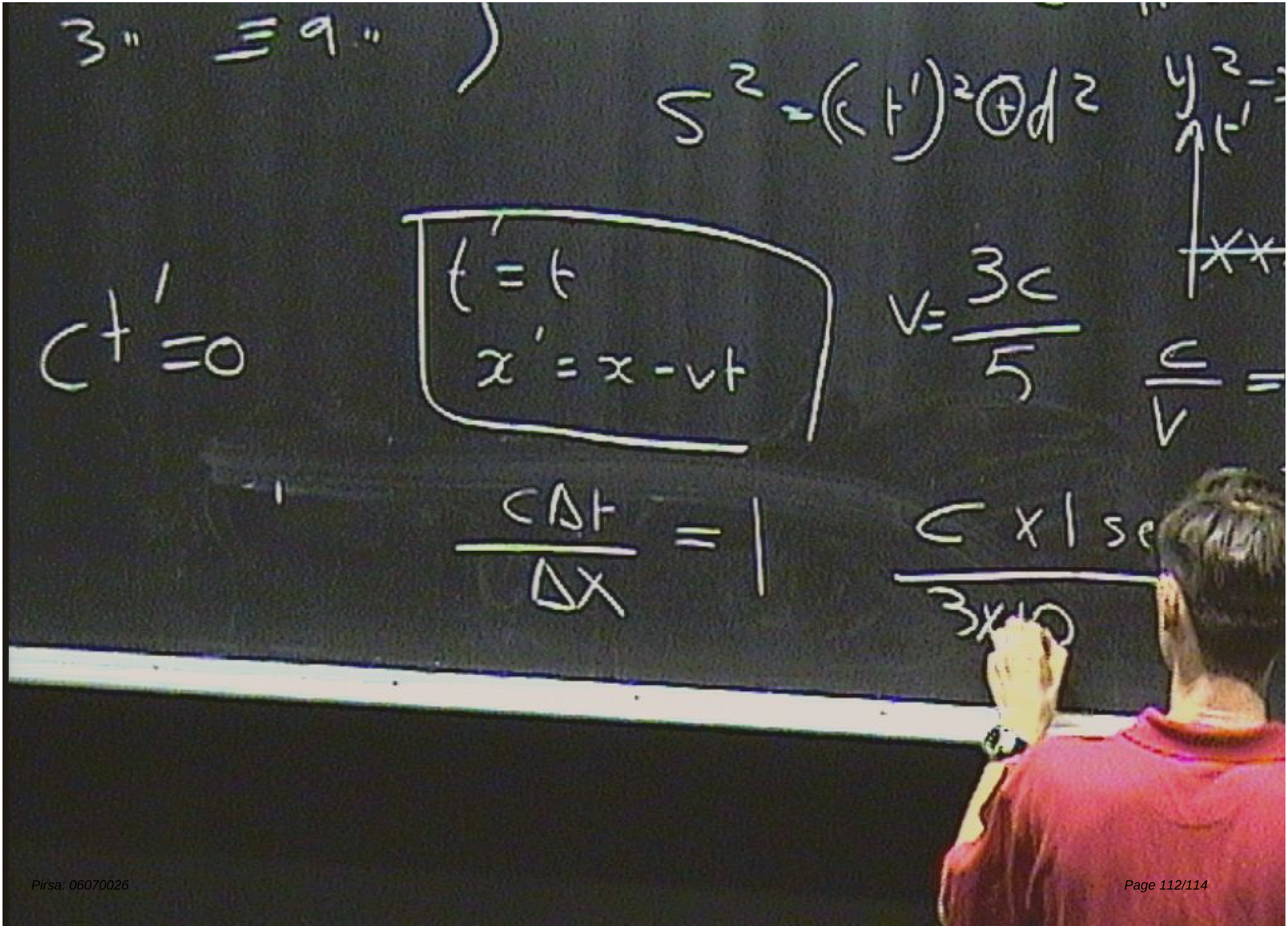
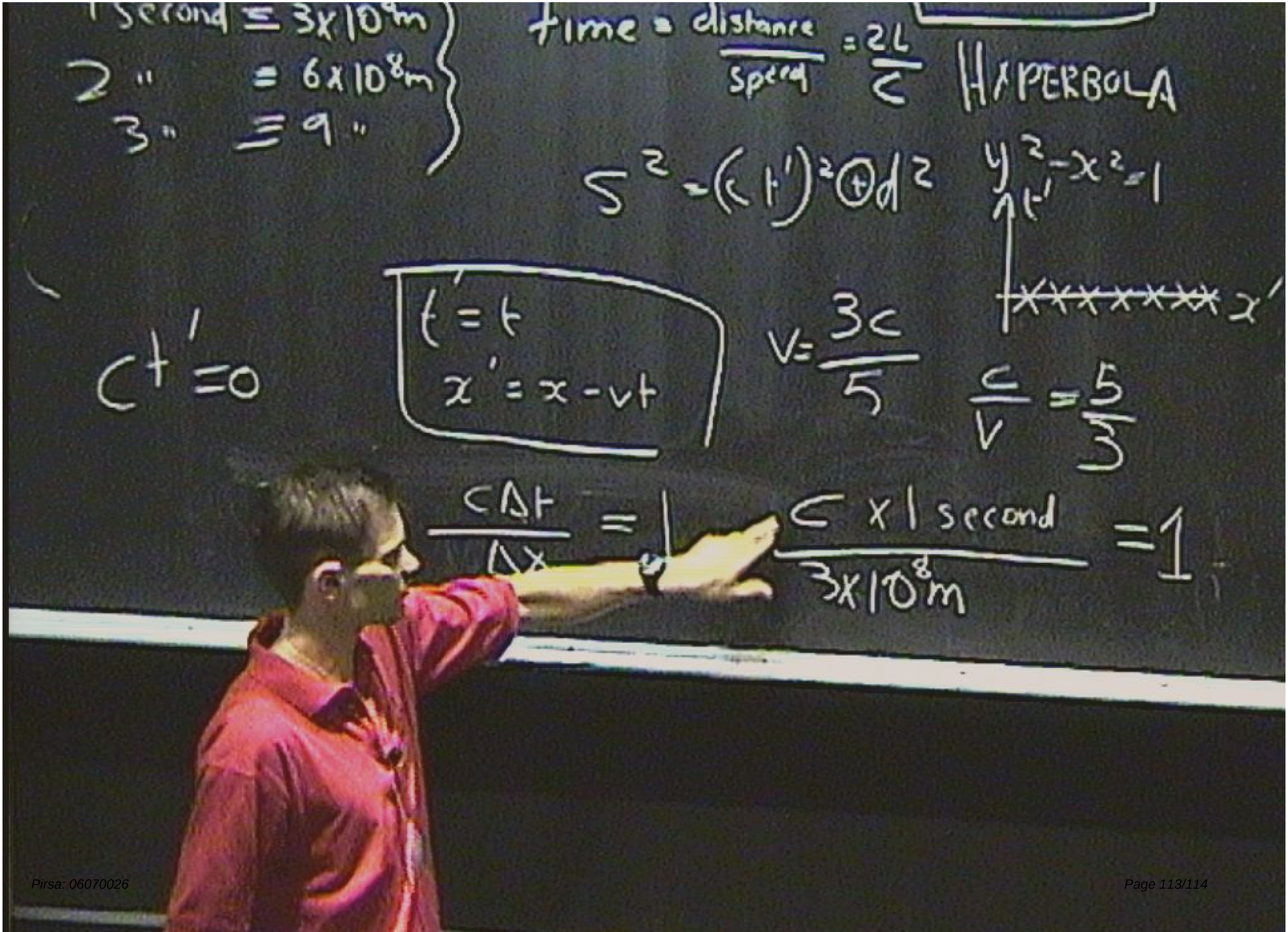
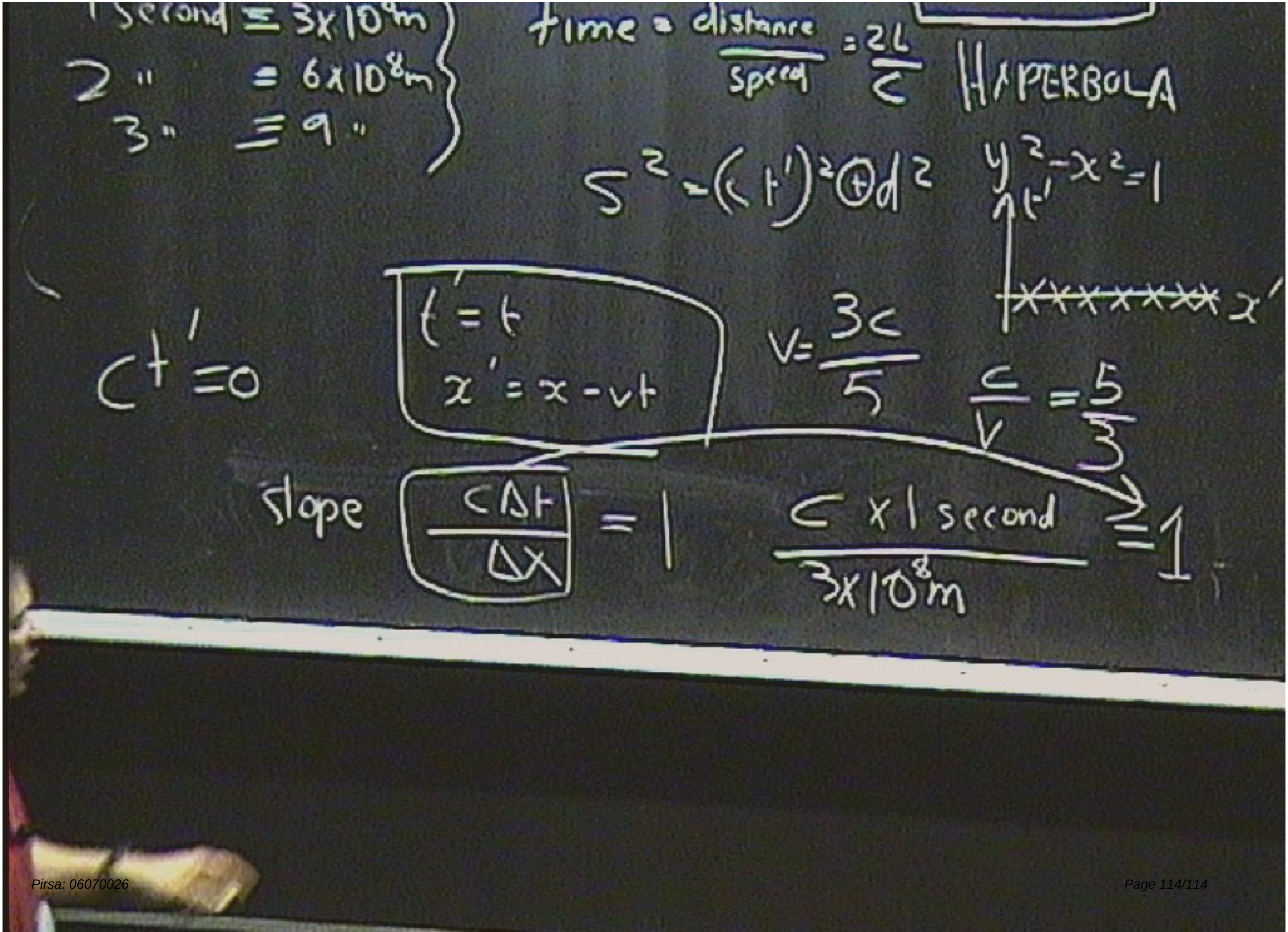

3" ≡ 9"
s² = (ct')²
ct' = 0
t' = t
x' = x - vt
V=
cΔt
Δx


3" ≡ 9"
S² = (ct')²
ct' = 0
t' = t
x' = x - vt
cΔt / Δx = 1
V =



3" ≡ 9"
ct' = 0
t' = t
x' = x - vt
V = 3c / 5
c / V =
cΔt / Δx = 1
c x 1 se
3x10


1 second ≡ 3x10^8 m
2 " = 6x10^8 m
3 " ≡ 9 "
time = distance / speed = 2L / c
HYPERBOLA
s^2 = (ct')^2 + d^2
y^2 - x^2 = 1
t'
x'
t' = t
x' = x - vt
ct' = 0
V = 3c / 5
c / v = 5 / 3
cΔt / Δx = 1
c x 1 second / 3x10^8 m = 1


1 second ≡ 3x10^8 m
2 " = 6x10^8 m
3 " ≡ 9 "
time = distance / speed = 2L / c
HYPERBOLA
s^2 = (ct')^2 + d^2
y^2 - x^2 = 1
ct'
x'
ct' = 0
t' = t
x' = x - vt
V = 3c / 5
c / v = 5 / 3
slope cΔt / Δx = 1
c x 1 second / 3x10^8 m = 1