Title: Enrichment presentation on Special Relativity continued

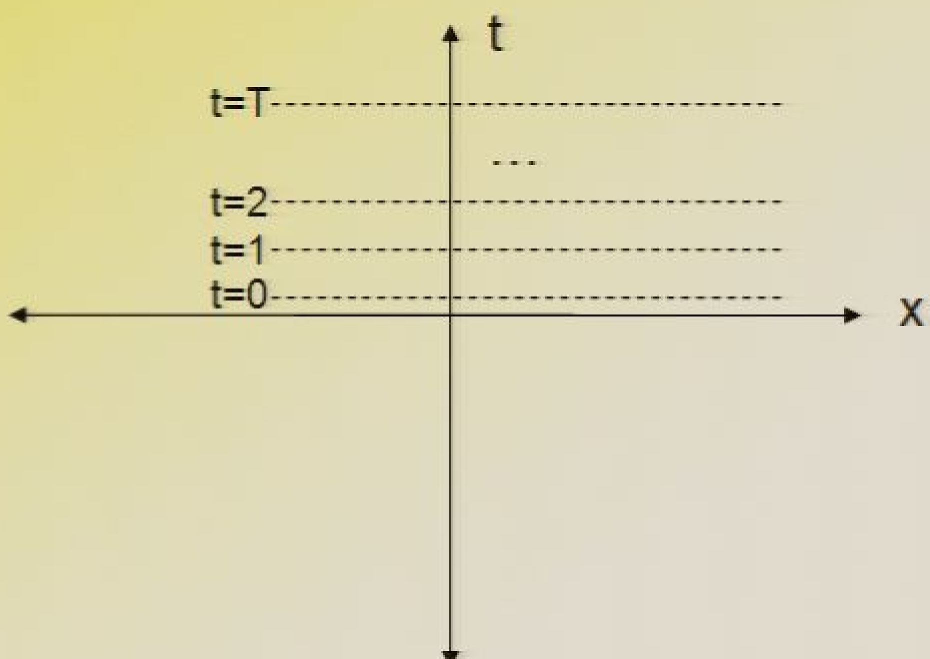
Date: Jul 07, 2006 11:00 AM

URL: http://pirsa.org/06070012

Abstract:

- If we did not know that Δt_s=T, then we could determine it from the graph through the following procedure:

- Start at A and draw a line of constant time (t=0). Continue to draw lines of constant time (t=1,2,3…) — these are parallel to the x axis — until we draw one that intersects with B. The time for this line is T.



- To determine the time interval between A and B as measured in Earth's reference frame E. Let us follow an analogous procedure.

- 1. The x' axis corresponds to the line t'=0 and so it is the first line.

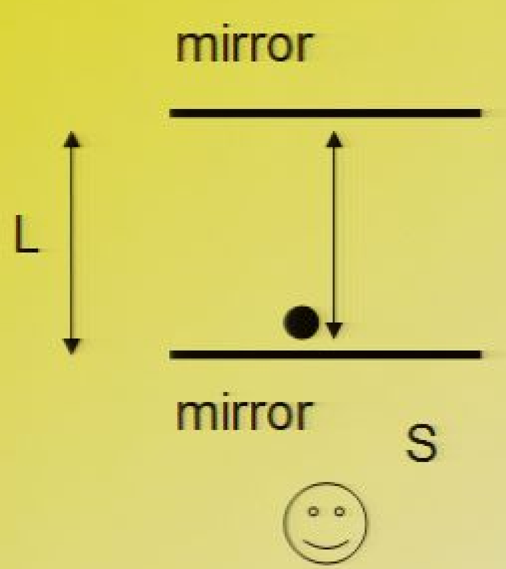
- 2. Draw another line parallel to the first that corresponds to t'=1. i.e. one unit of time upwards.

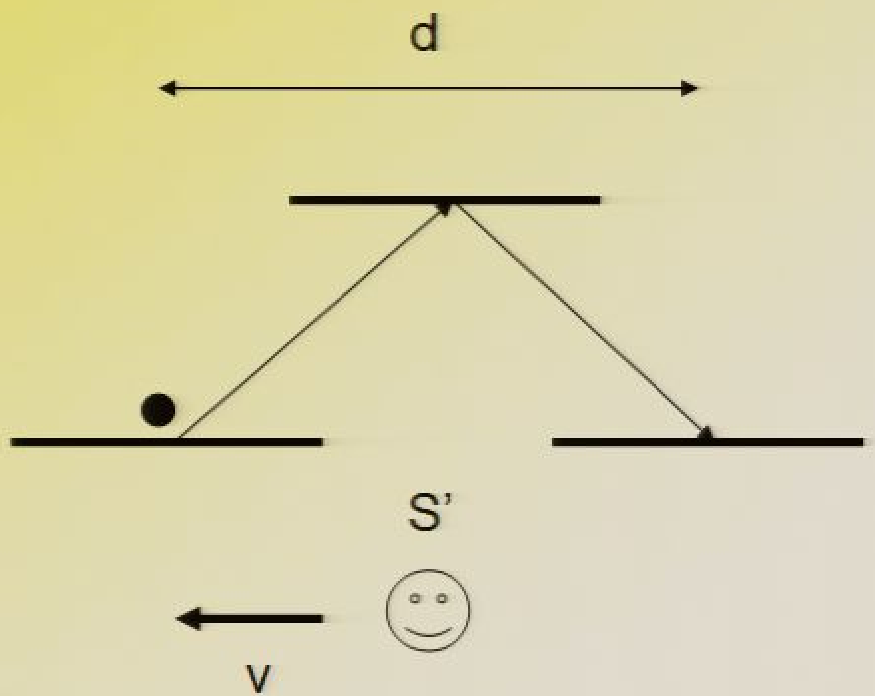
- 3. Continue this procedure until you draw a line that intersects B.

Core concepts of special relativity, Part 2

Time dilation

- Light clock





- A and B are the events corresponding to a photon hitting the bottom mirror on two successive occasions.

$$\Delta t = 2L / c$$

$$\Delta t' = \frac{\text{distance}}{\text{speed}} = \frac{2\sqrt{L^2 + \left(\frac{d}{2}\right)^2}}{c}$$

$$d = vt'$$

- If we did not know that Δt_S=T, then we could determine it from the graph through the following procedure:

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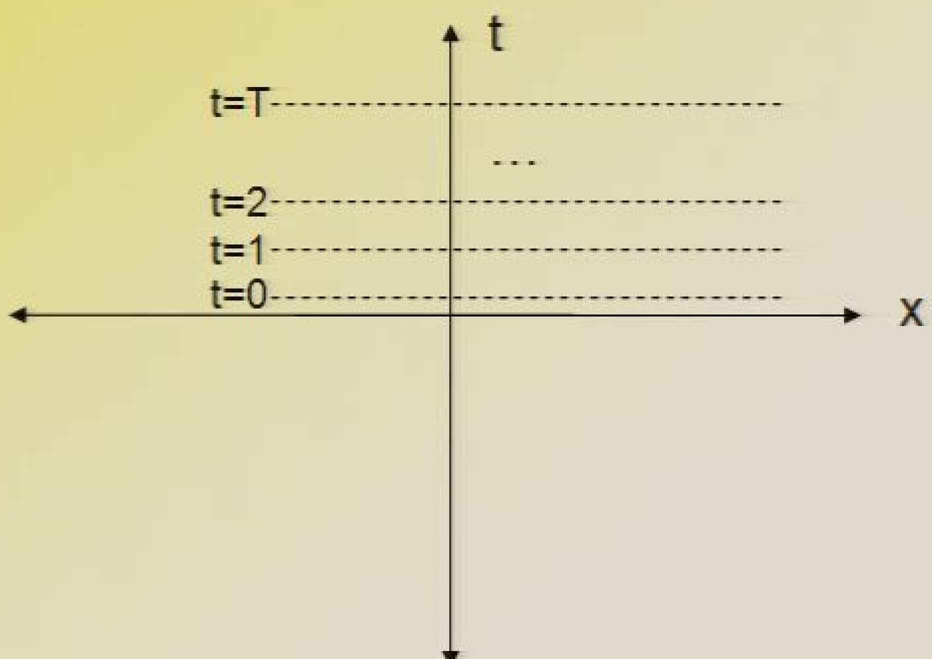
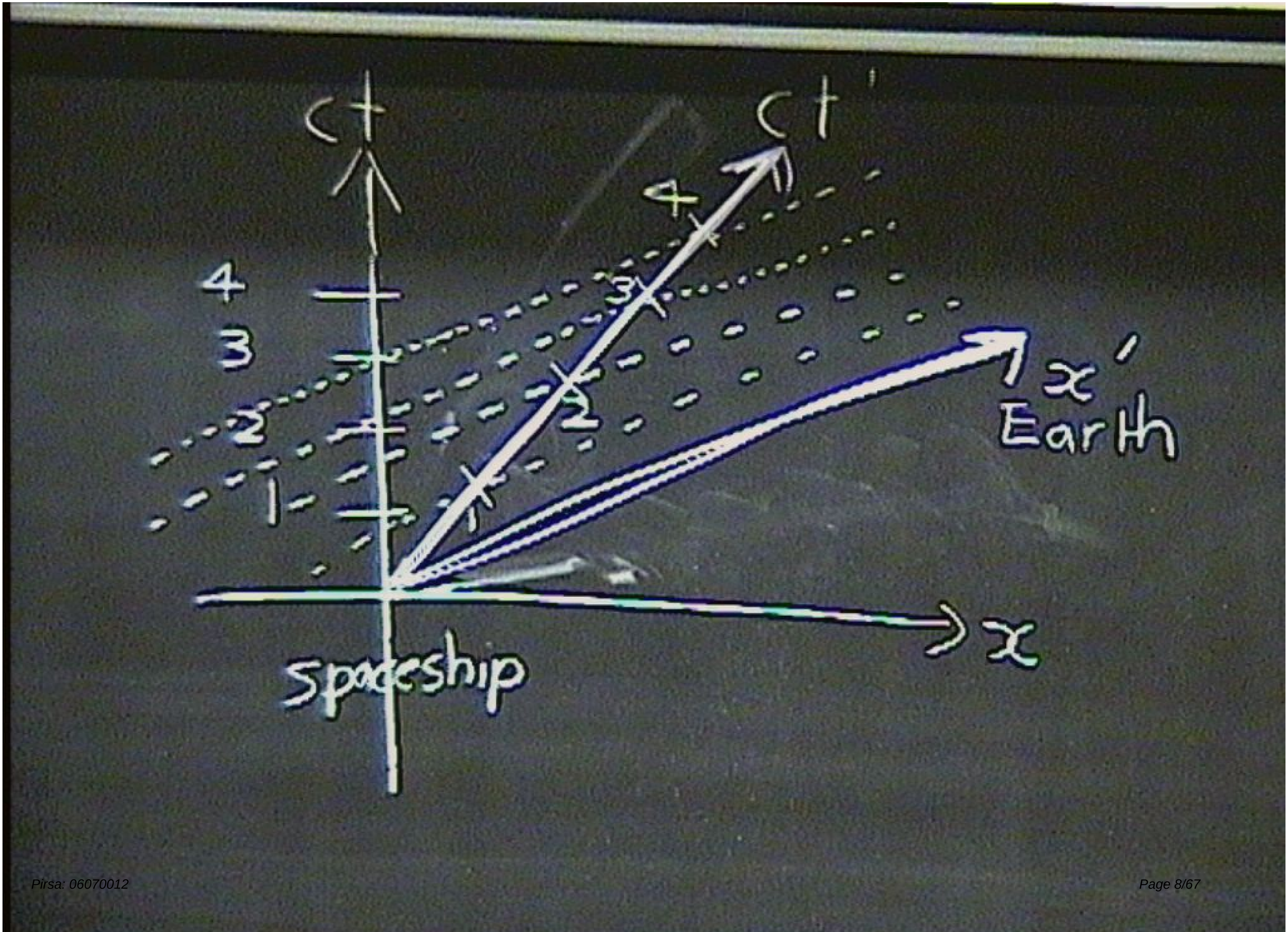
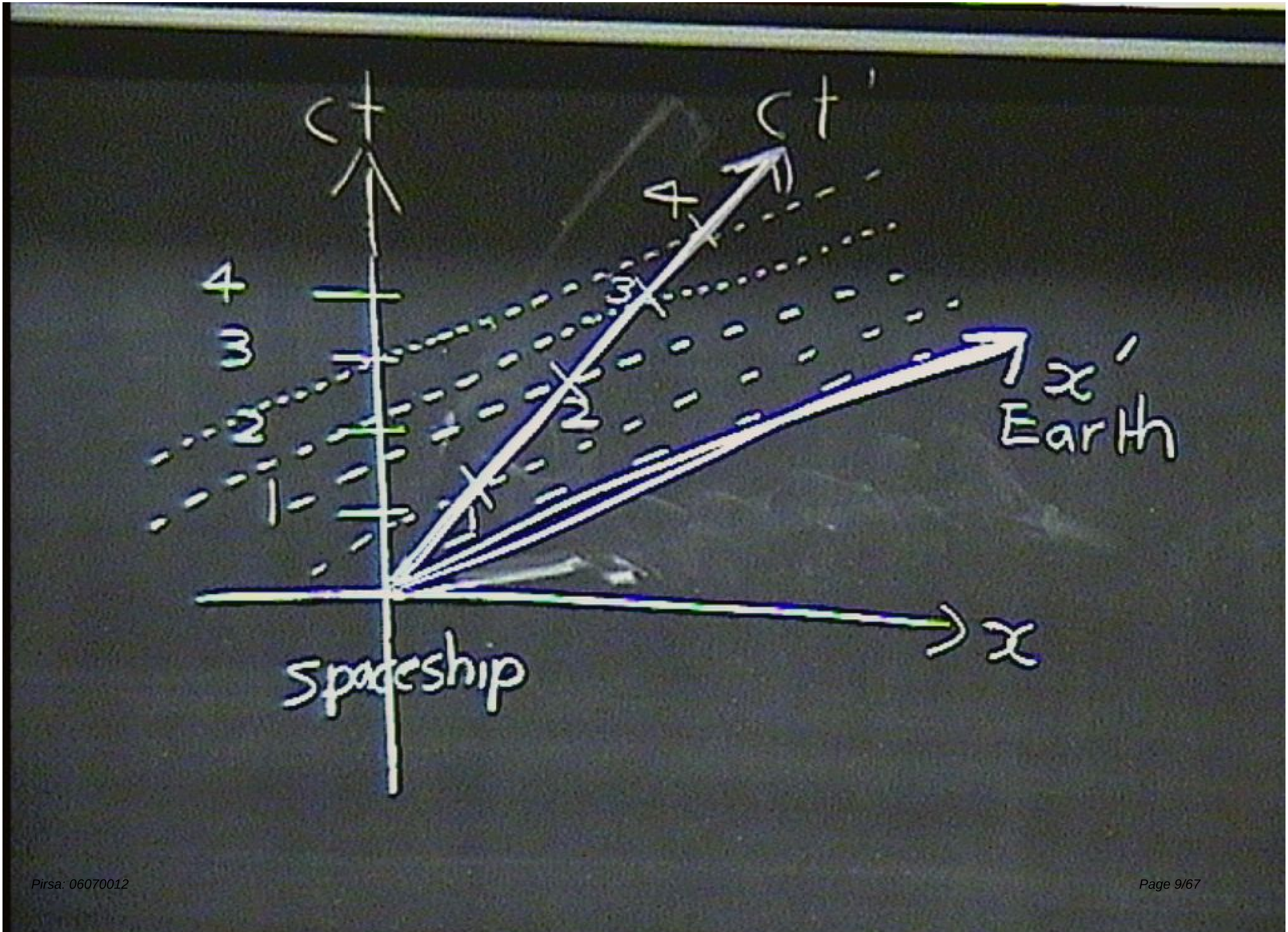
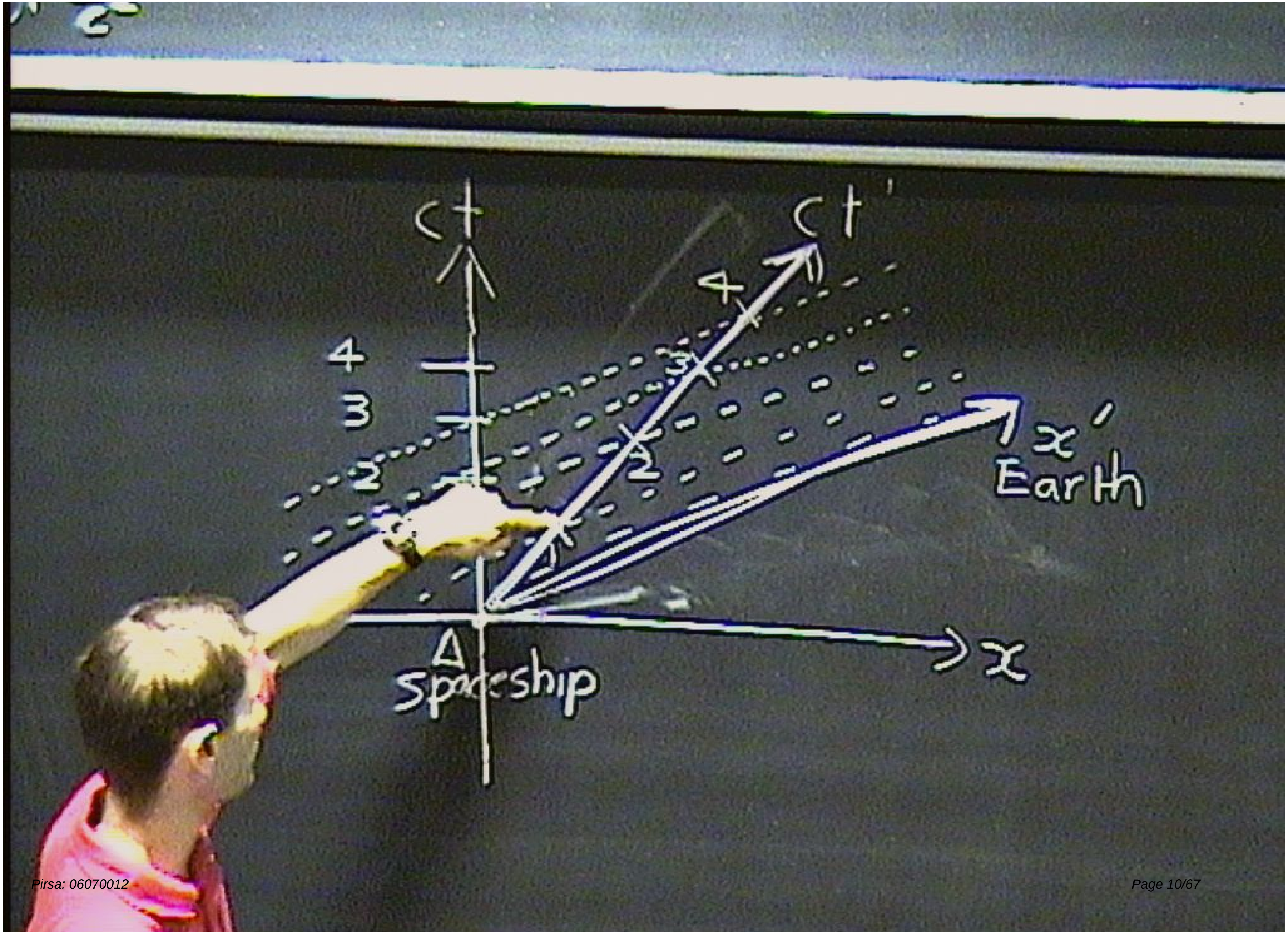


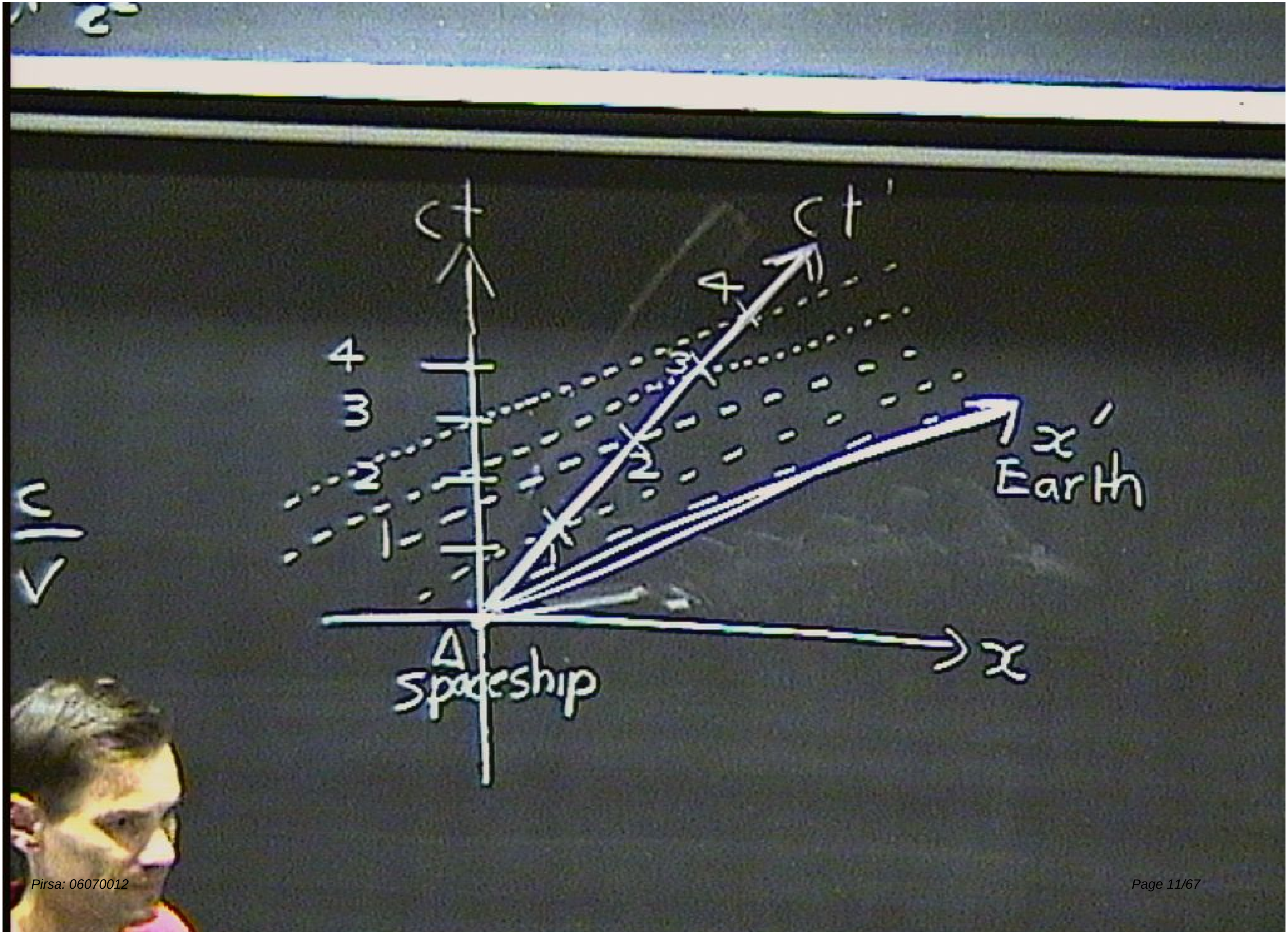
- To determine the time interval between A and B as measured in Earth's reference frame E. Let us follow an analogous procedure.
- 1. The x' axis corresponds to the line t'=0 and so it is the first line.
- 2. Draw another line parallel to the first that corresponds to t'=1. i.e. one unit of time upwards.
- 3. Continue this procedure until you draw a line that intersects B.

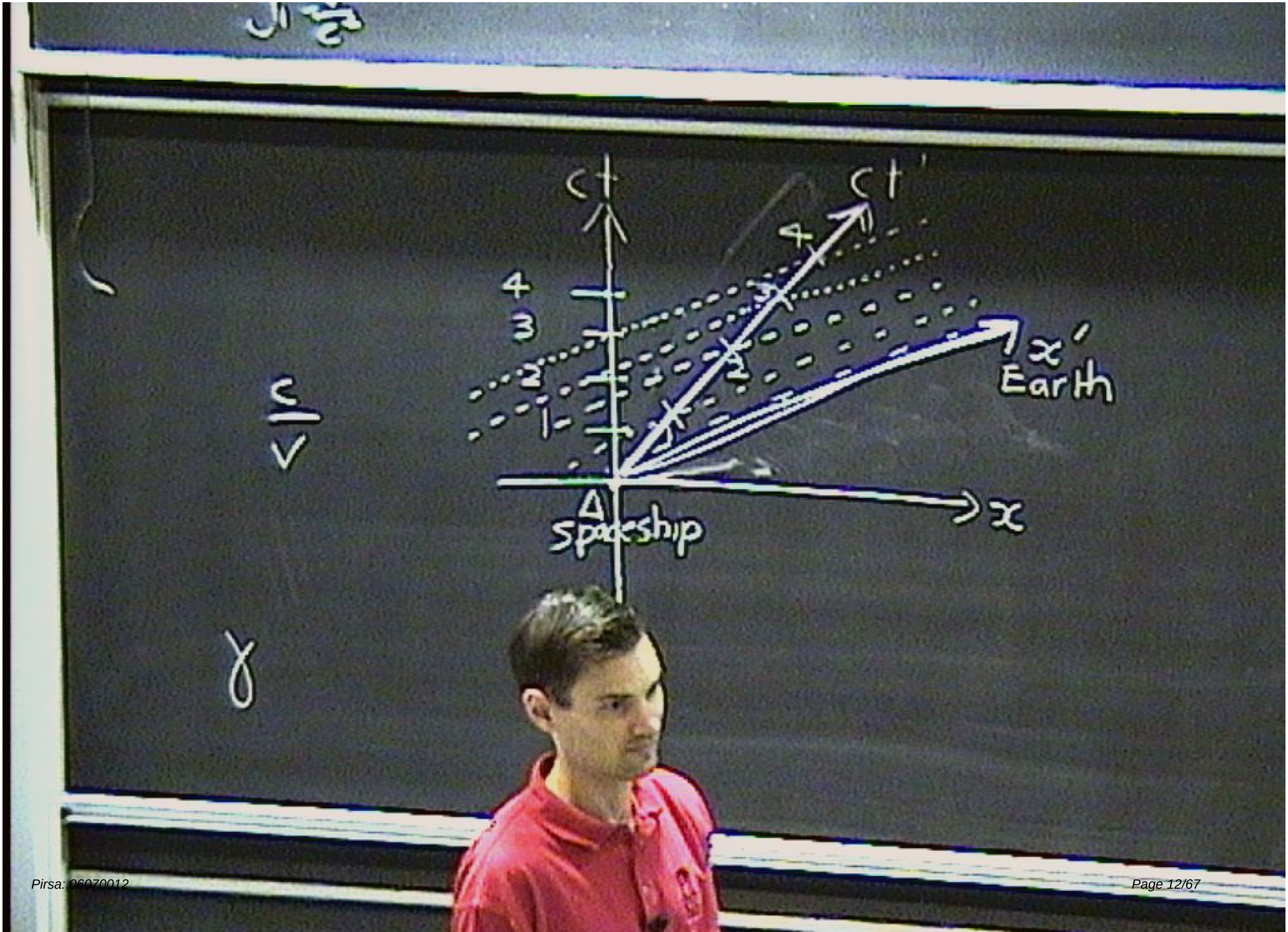

ct
ct'
x'
Earth
x
Spaceship
1
2
3
4



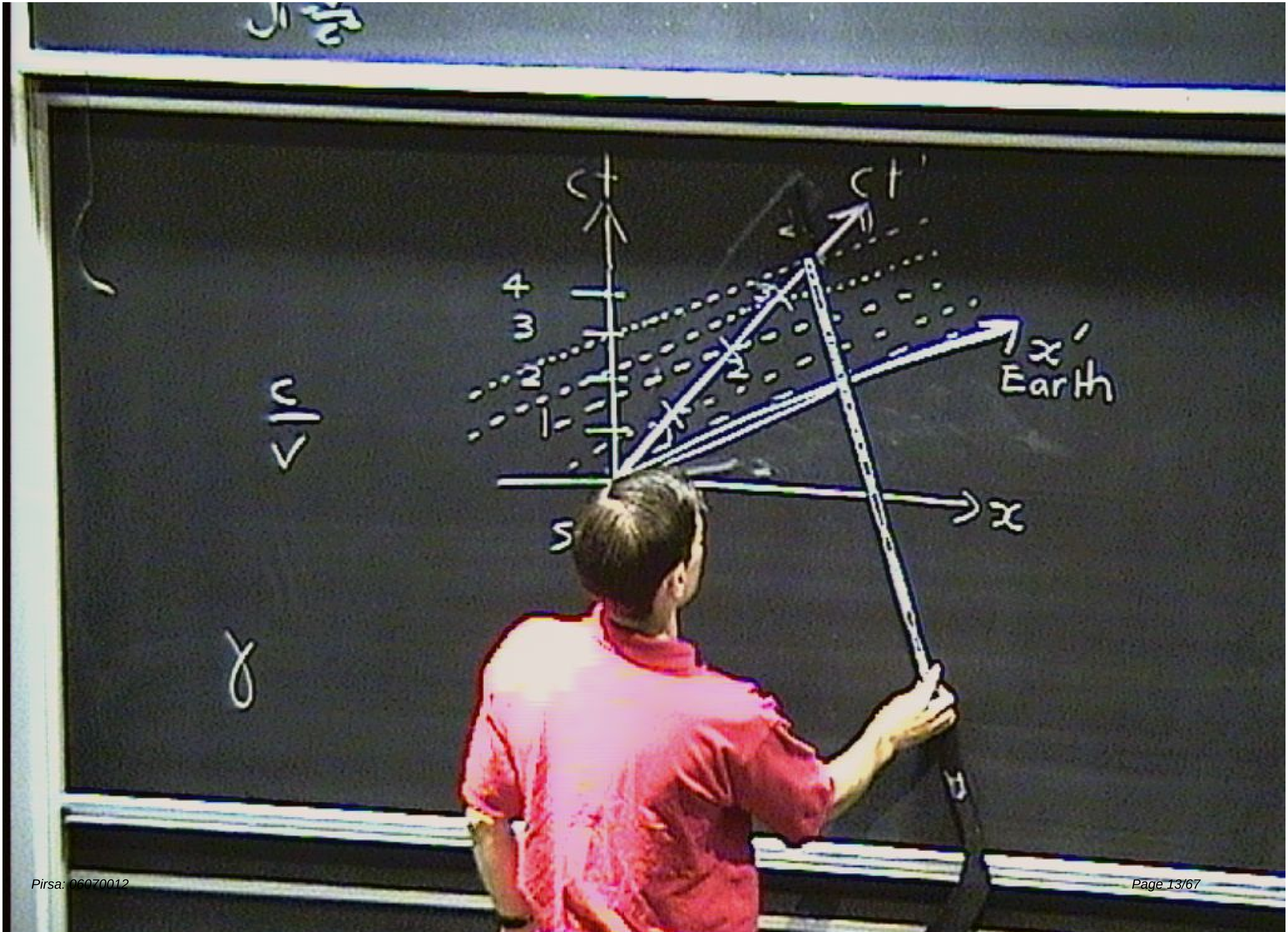
ct
ct'
4
3
2
1
4
3
2
1
x'
Earth
x
Spaceship


ct
ct'
4
3
2
4
3
2
x'
Earth
x
A
Spaceship

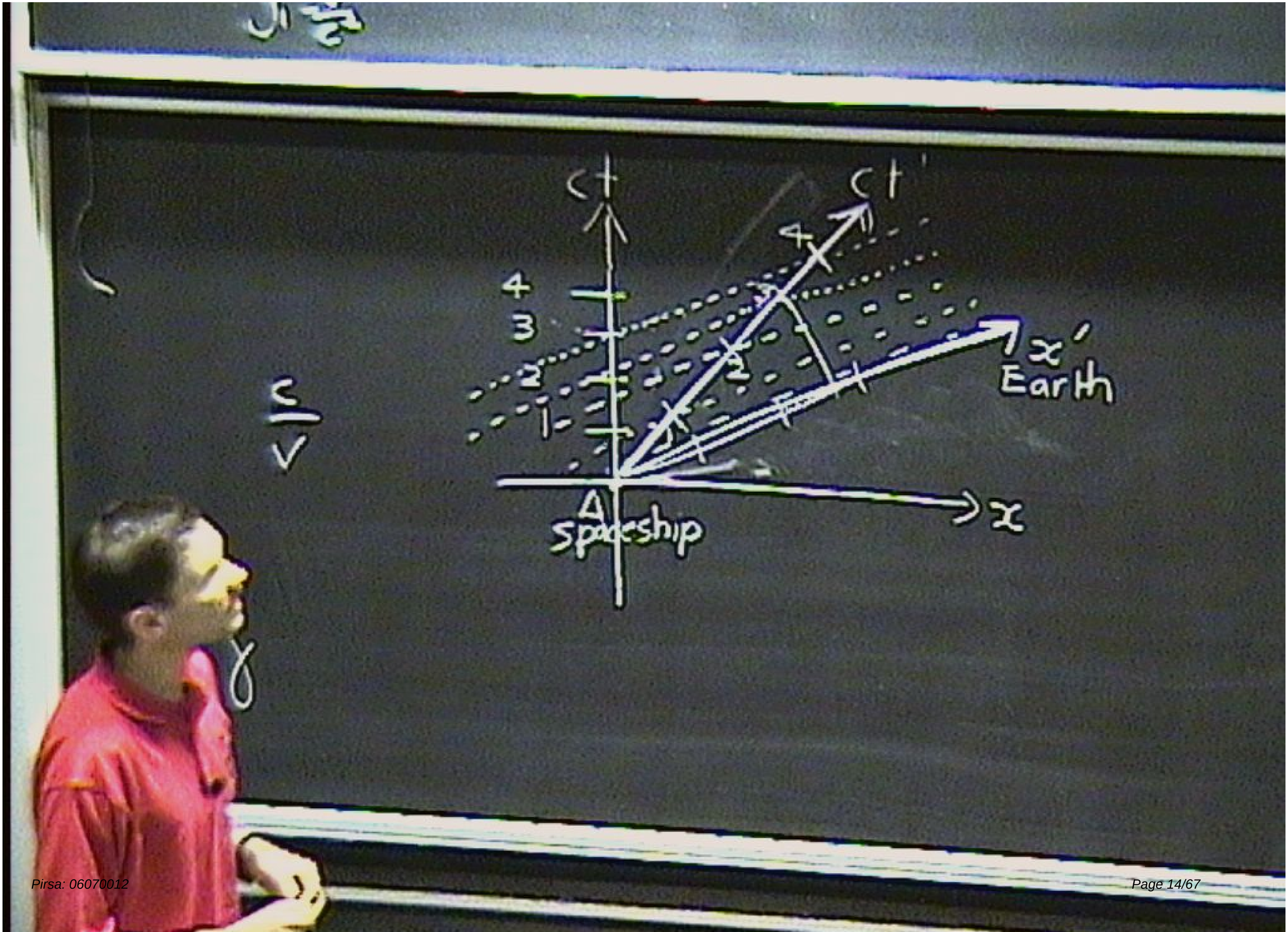

ct
ct'
4
3
2
1
x'
Earth
x
Spaceship



ct
ct'
4
3
2
1
4
2
x'
Earth
x
A
Spaceship
c
v
γ



ct
ct'
4
3
2
1
x'
Earth
x
S
c
v
γ



ct
ct'
4
3
2
1
x'
Earth
x
A
Spaceship
c
v
γ

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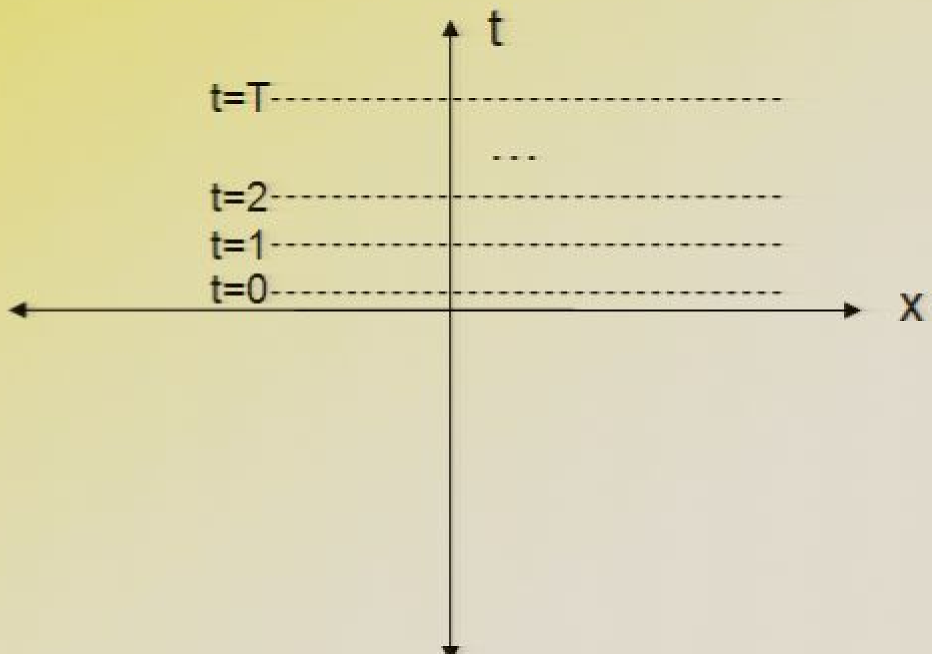
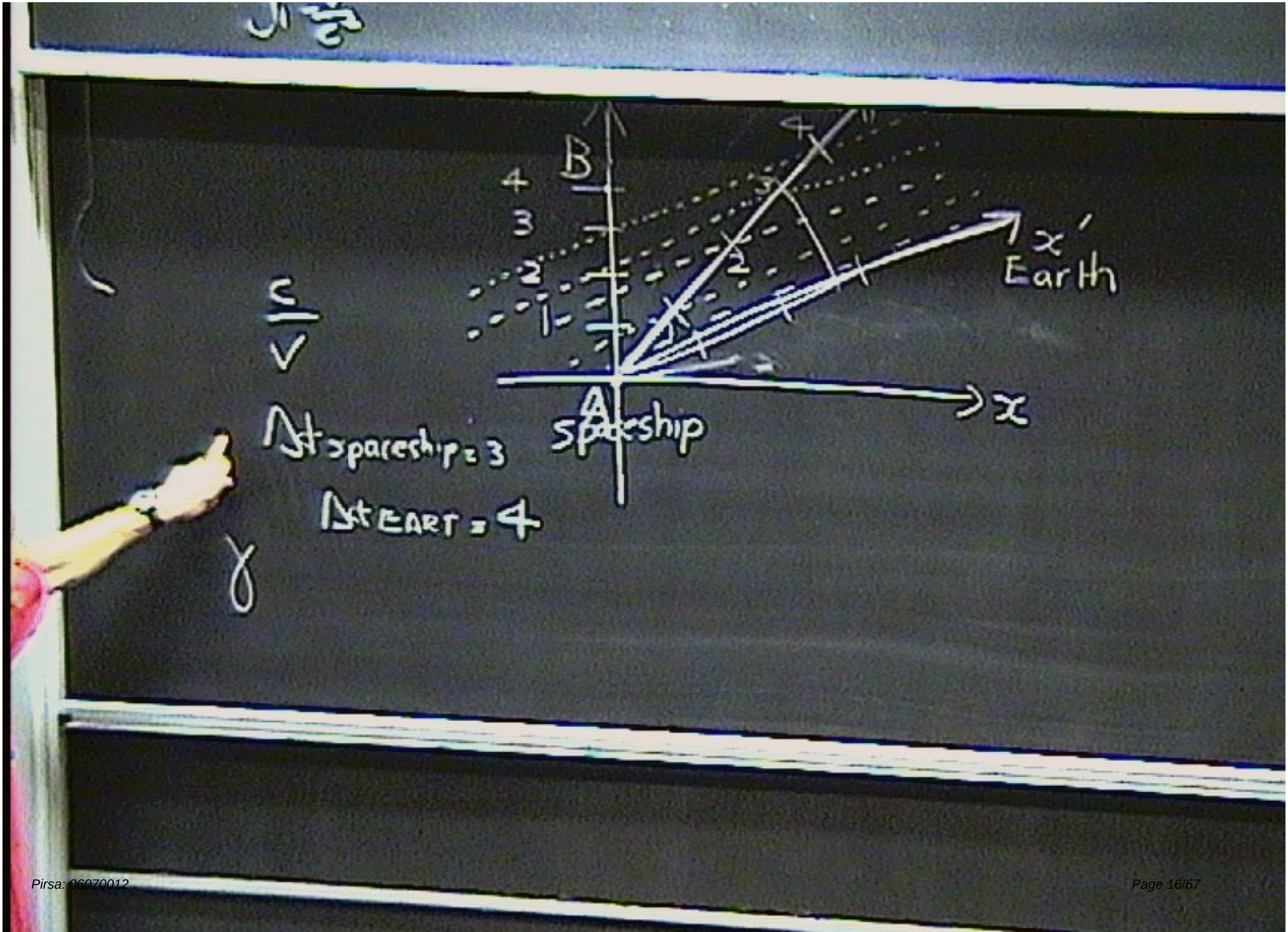
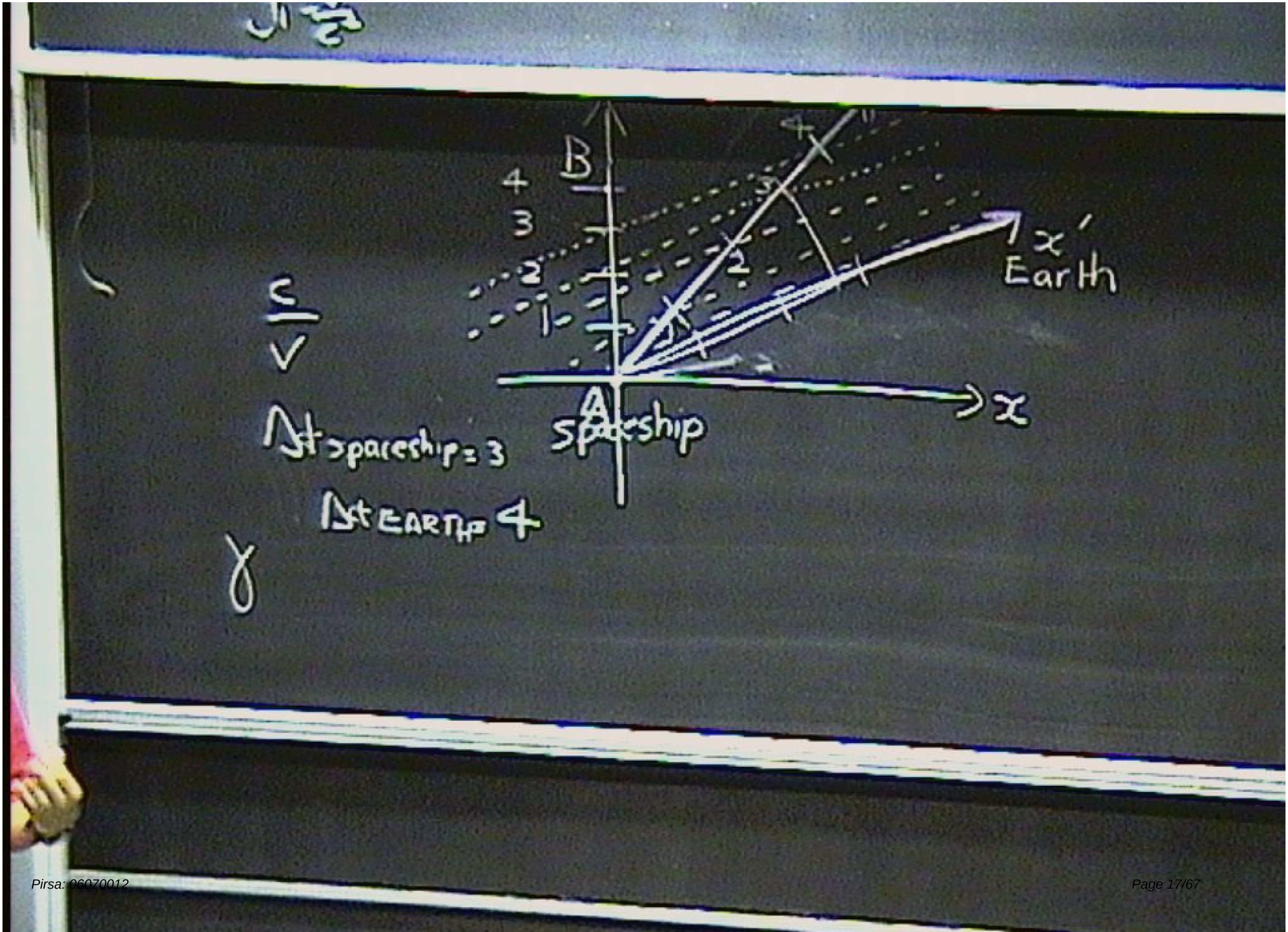



B
4
3
2
1
A
spaceship
x'
Earth
x
c
v
Δt spaceship = 3
Δt EART = 4
γ


B
4
3
2
1
x'
Earth
x
A
Spaceship
c
v
Δt spaceship = 3
Δt Earth = 4
γ

- If we did not know that Δt_S=T, then we could determine it from the graph through the following procedure:

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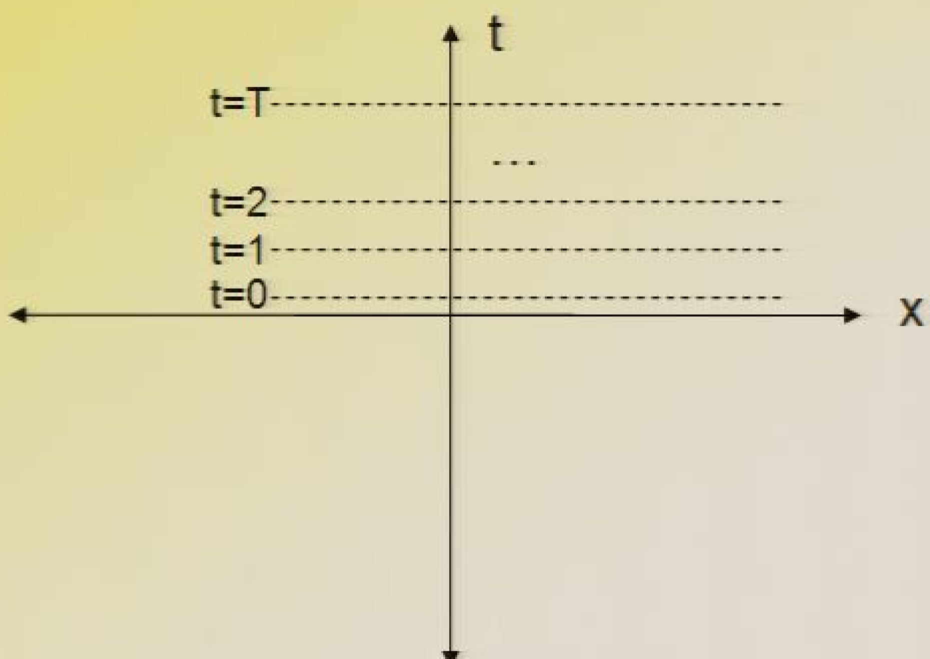
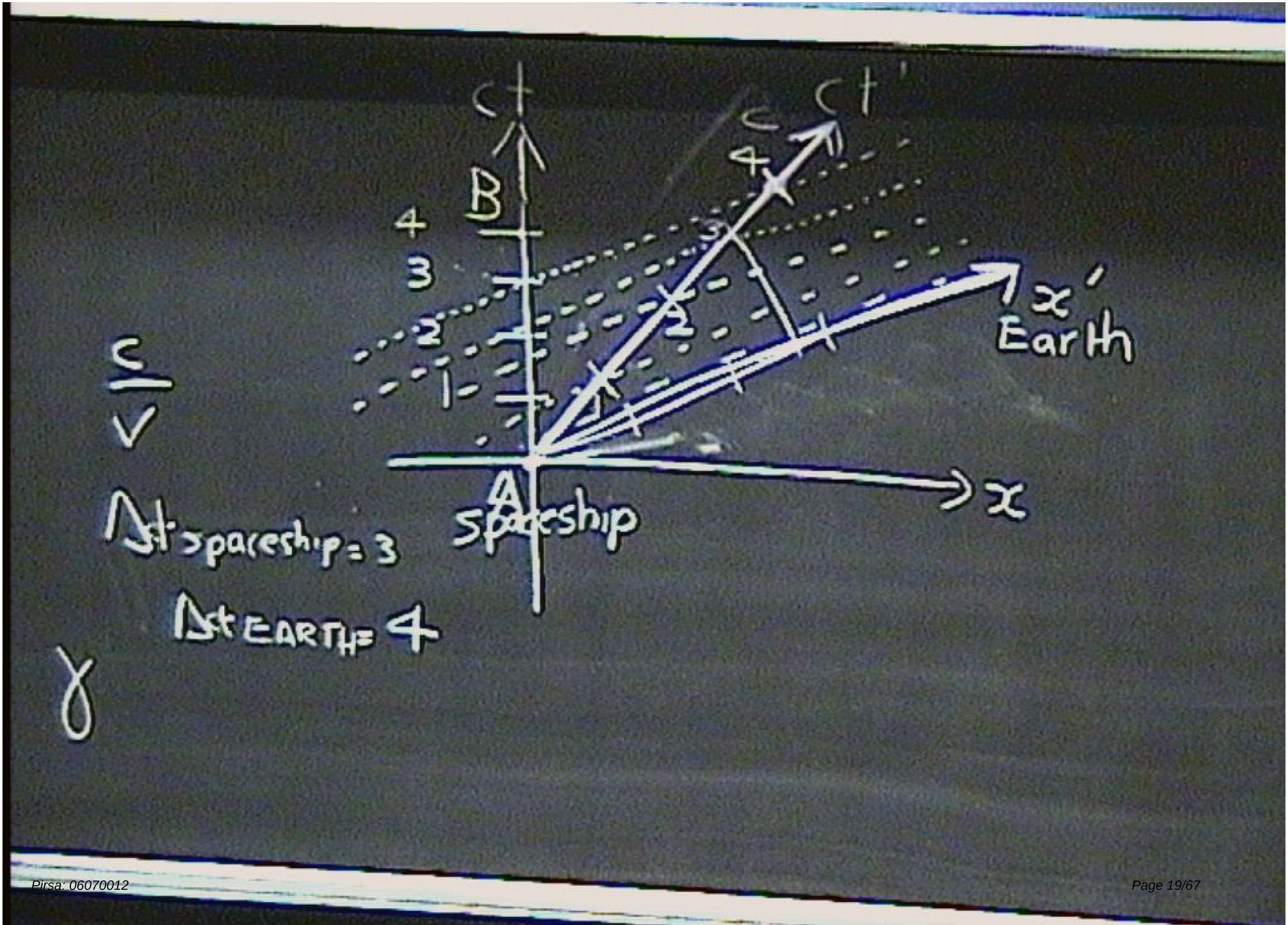
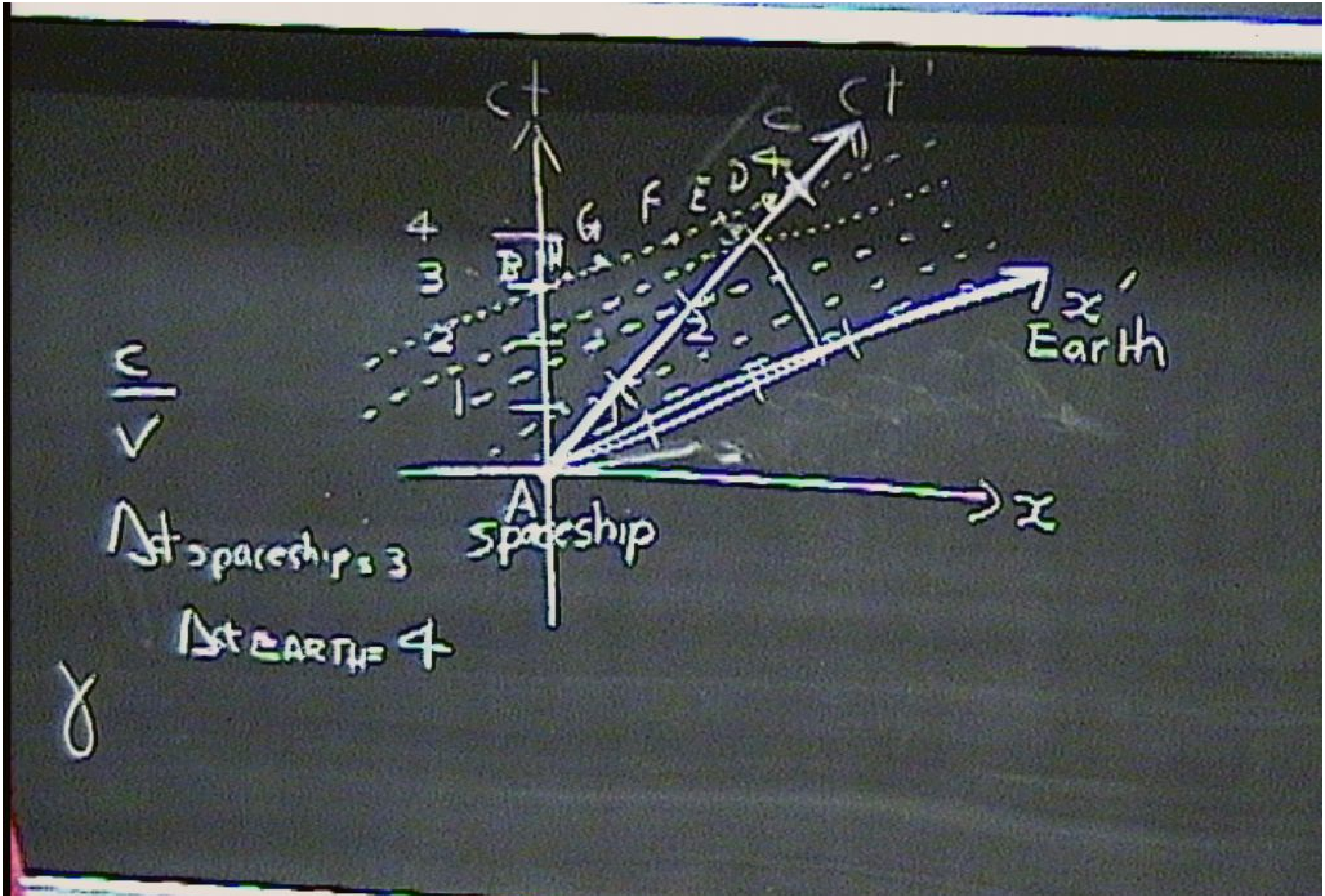



ct
ct'
B
4
3
2
1
x'
Earth
x
A
Spaceship
c
v
Δt spaceship = 3
Δt EARTH = 4
γ


ct
ct'
c
G F E D 4
4
3
B
2
1
x'
Earth
A
Spaceship
x
c
v
Δt spaceship = 3
Δt EARTH = 4
γ

- Question: How are the units of time marked out on the t' axis? Are they the same distance apart as for the t axis? No.

- To see this, consider again the hyperbolae corresponding to s=1,2,3…

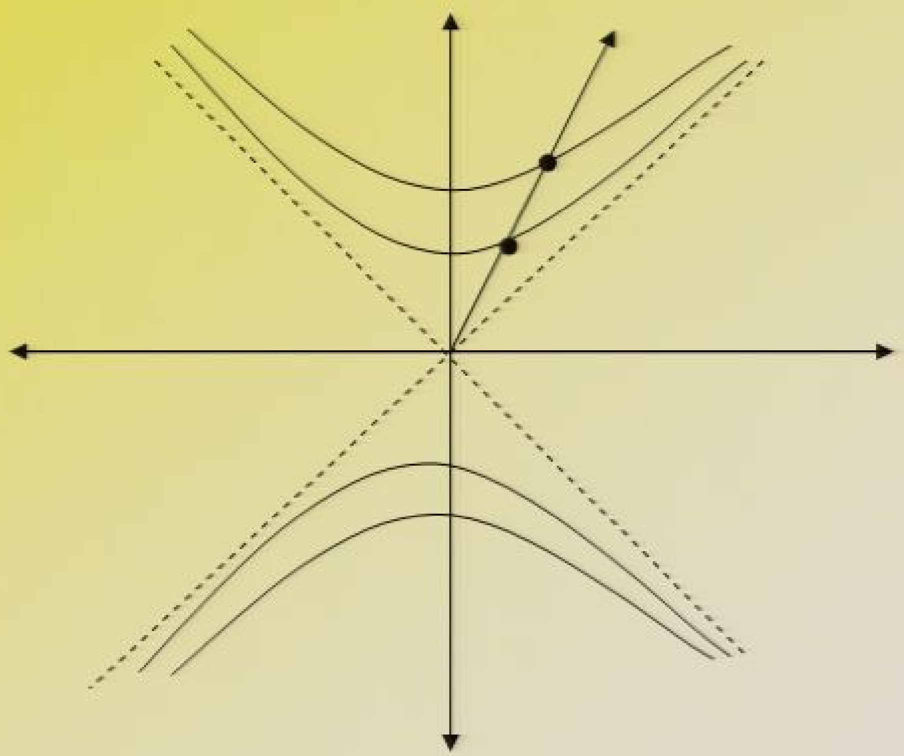
- Consider point C where the curve s intersects the t' axis. At C', x'=0 and so we have t'=s=1. i.e. C corresponds to t'=1.

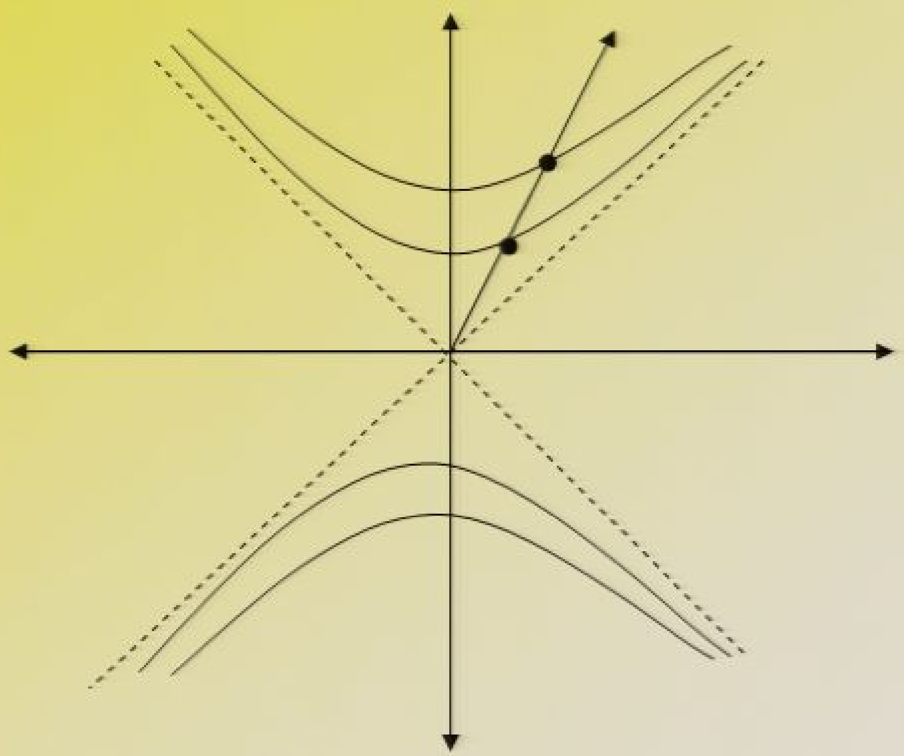
- Where the curve s=2, intersects denotes t'=2 etc.

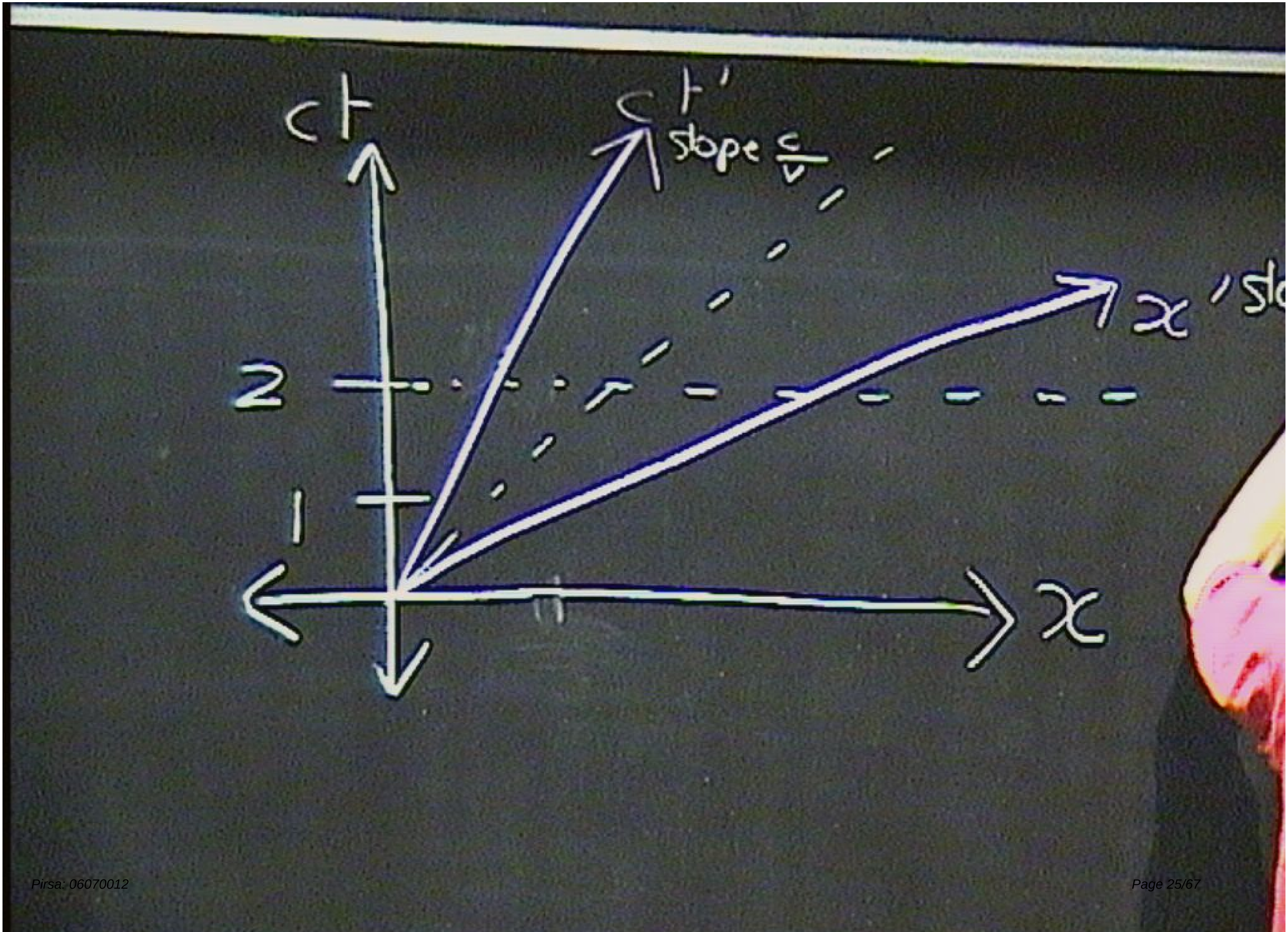
- Counterintuitively, the distance between time marks on the t' axis is *greater* than for the t axis.

- This leads to $t_E > t_S$, i.e. time dilation.

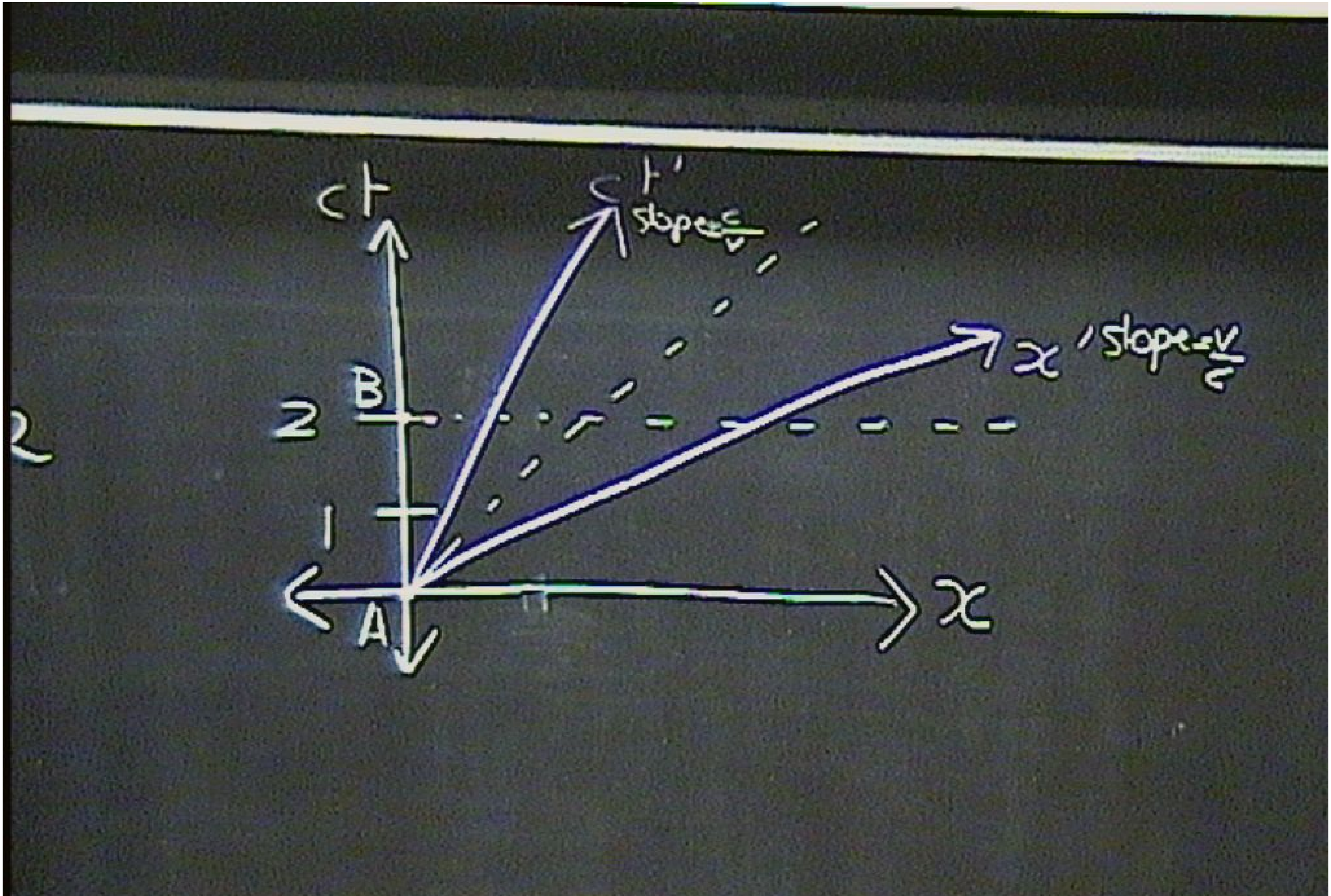
- Can calculate t_E without using the time dilation formula.

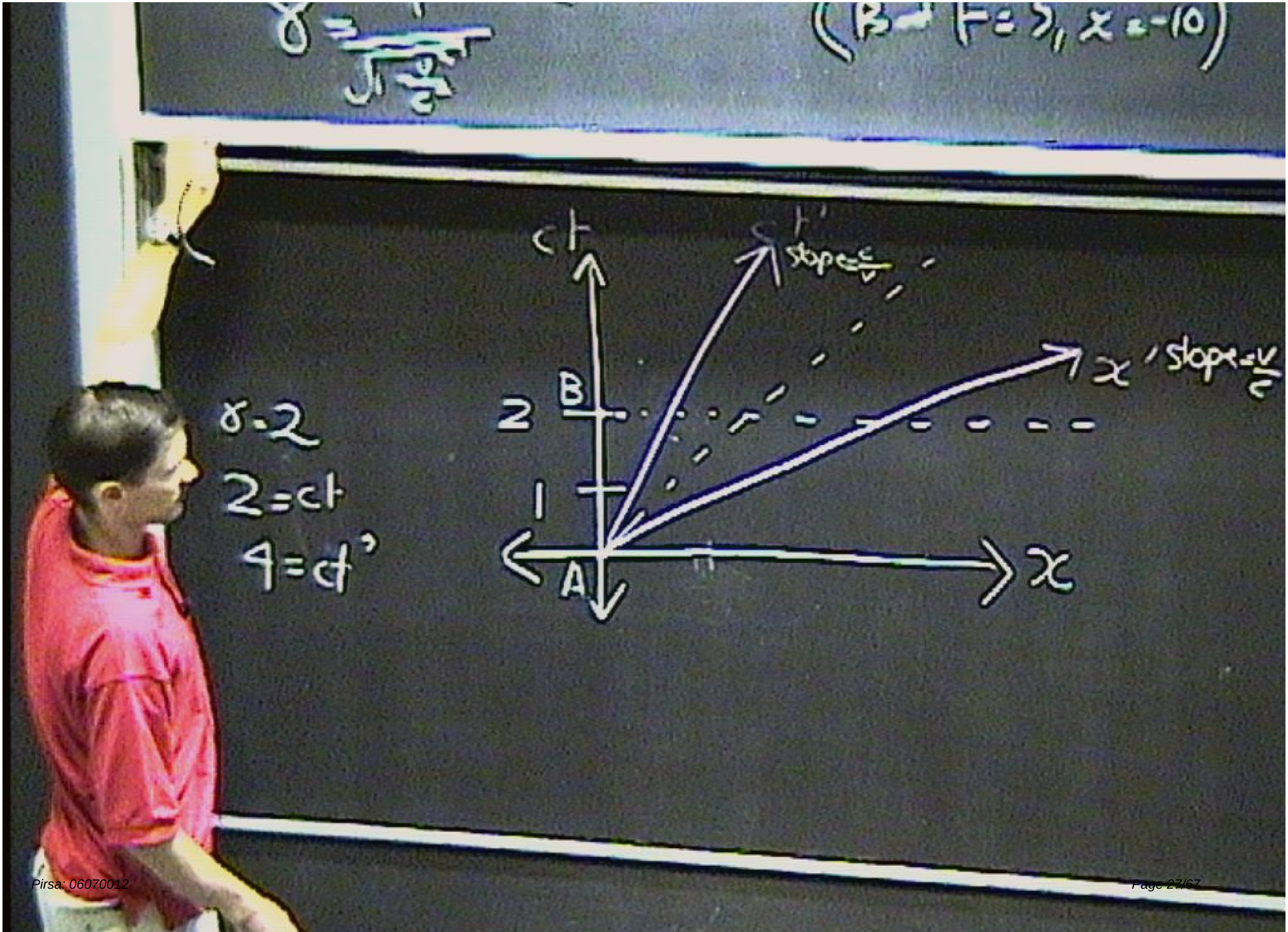


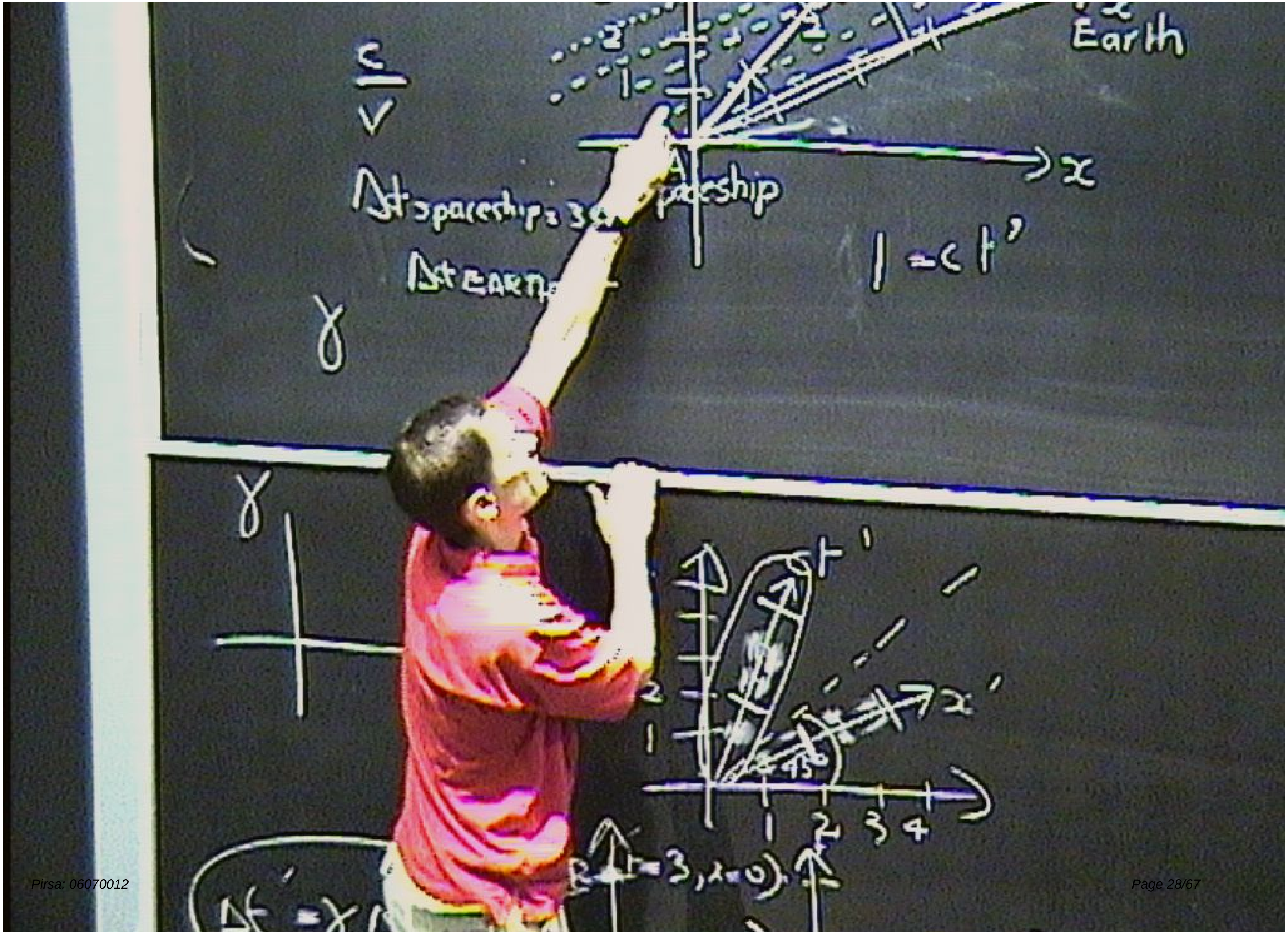


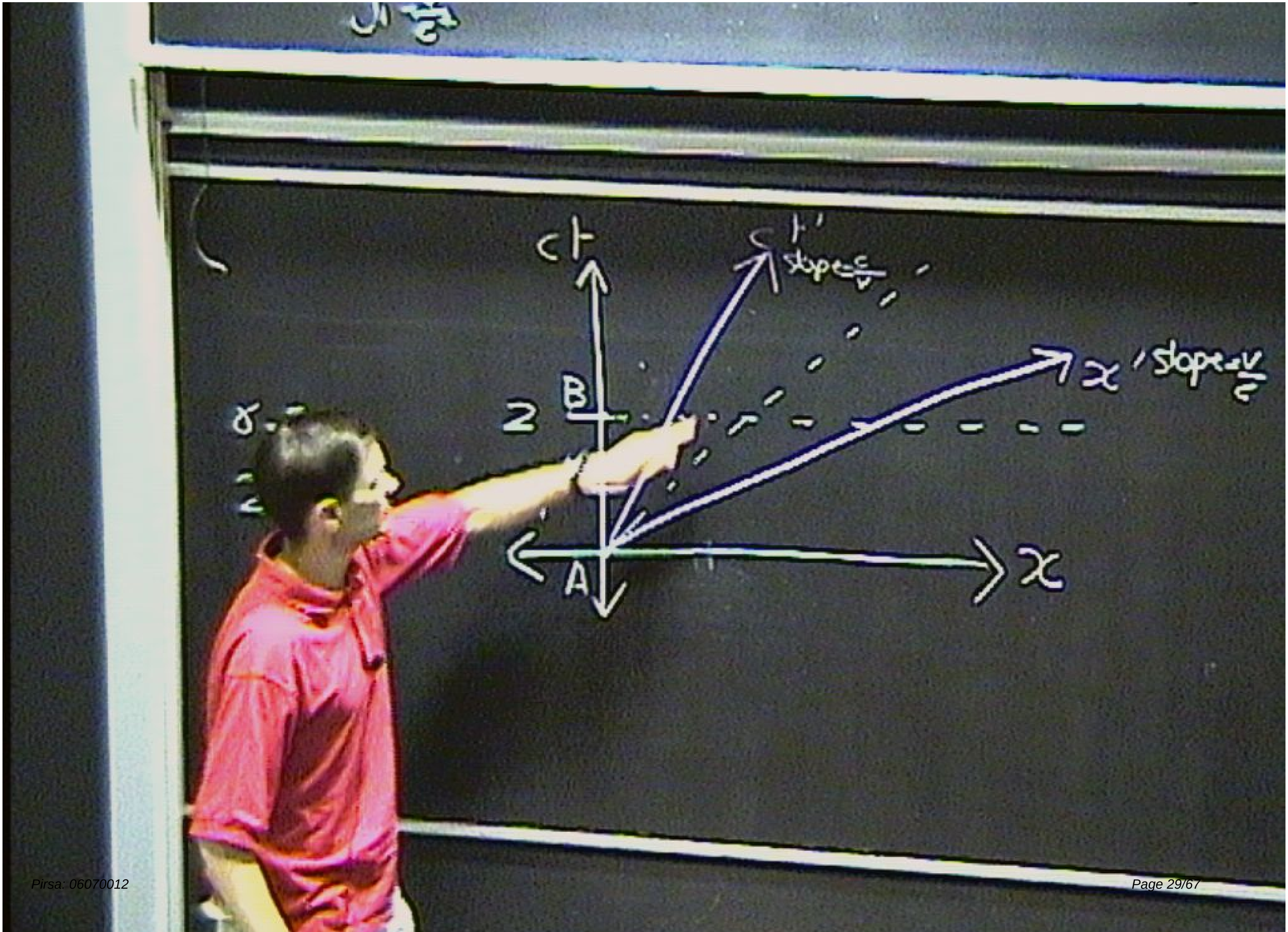


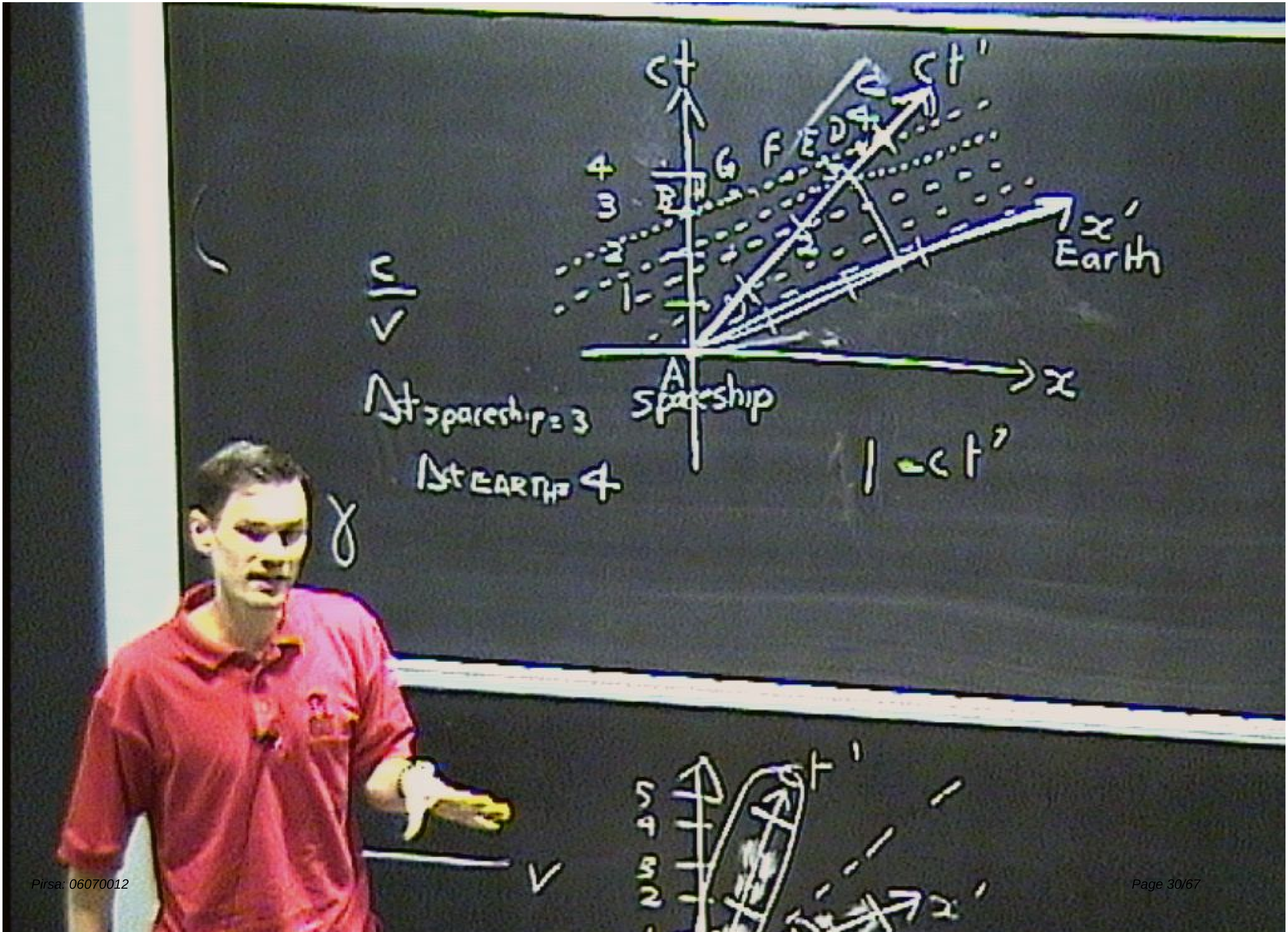
ct
ct'
slope c/v
x' slo
2
1
x

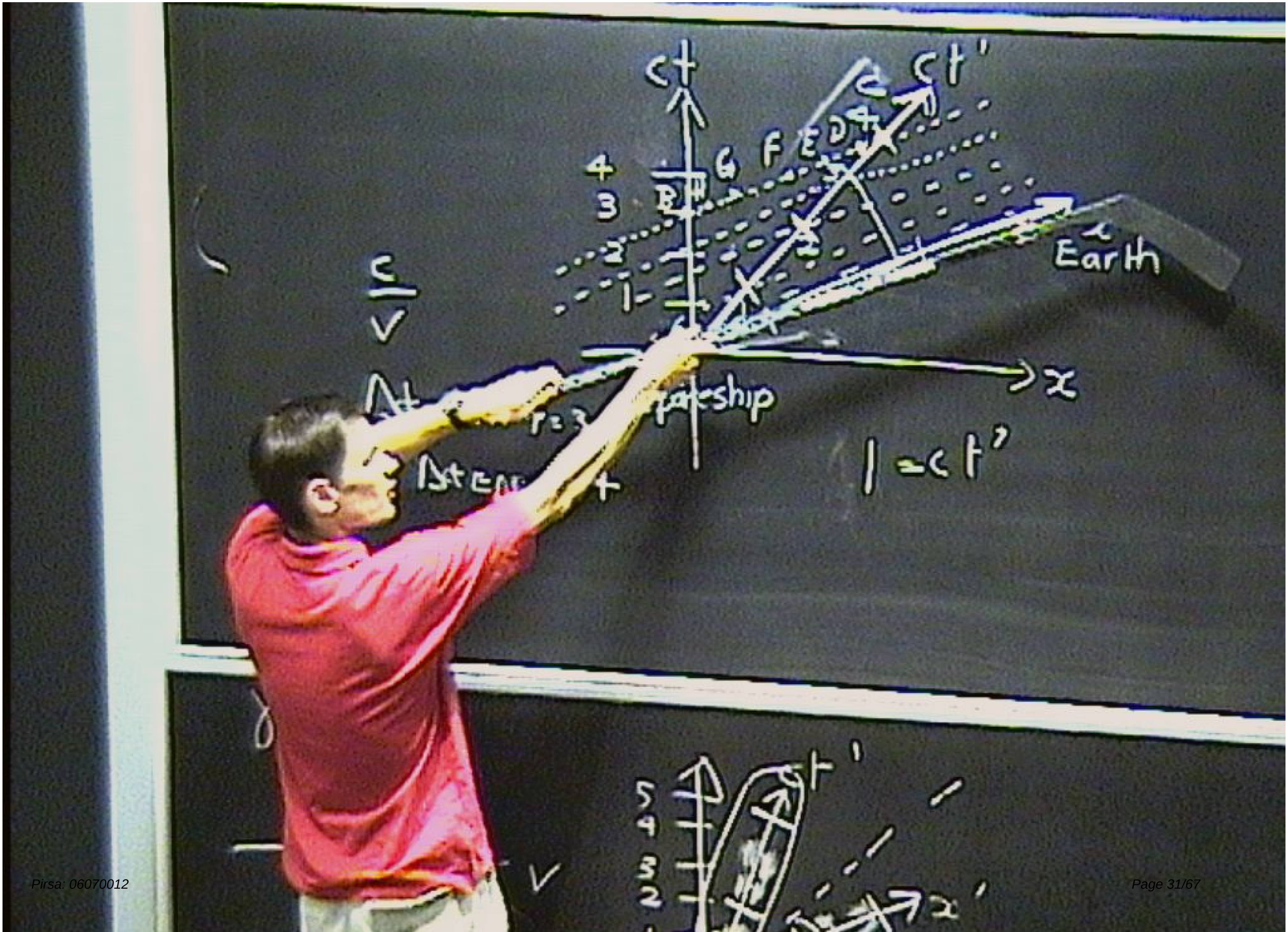

ct
ct'
slope = c/v
x'
slope = v/c
B
2
1
A
x

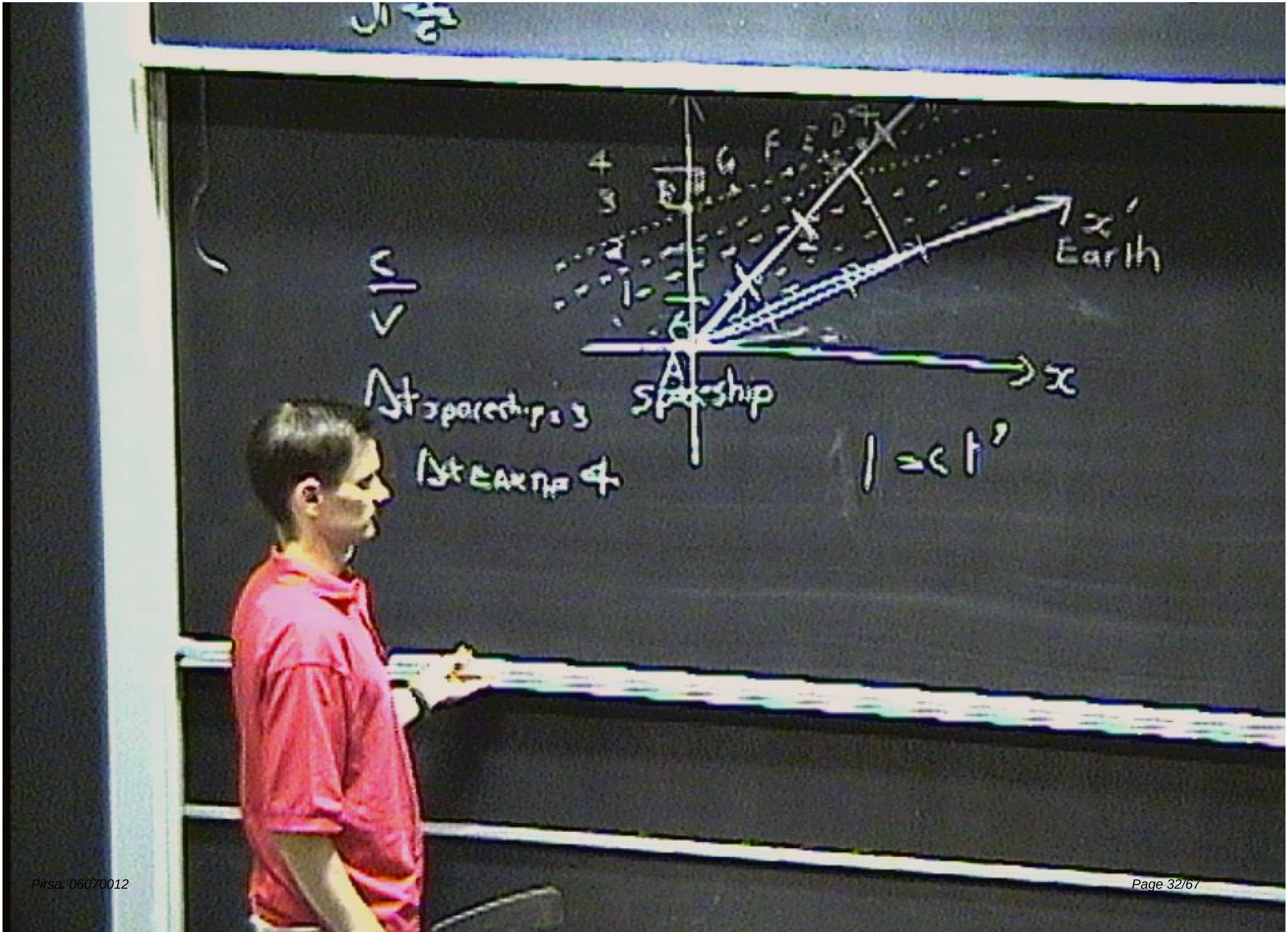

γ=2
2=ct
4=ct'
ct
ct'
slope=c/v
x'
slope=v/c
x
A
B
1
2

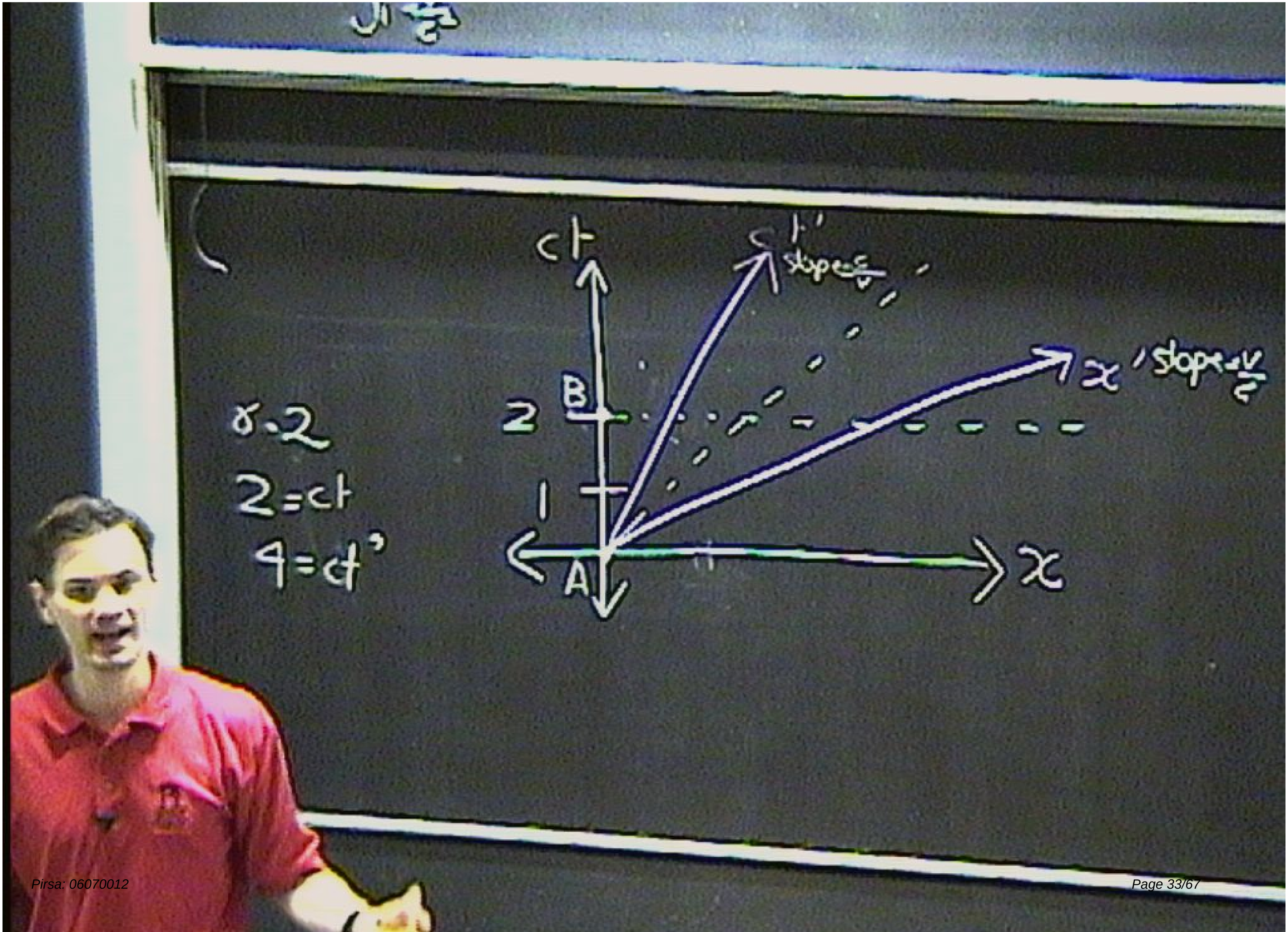

Earth
c
v
x
spaceship
1 = ct'
γ
γ
t'
x'
1
2
1 2 3 4

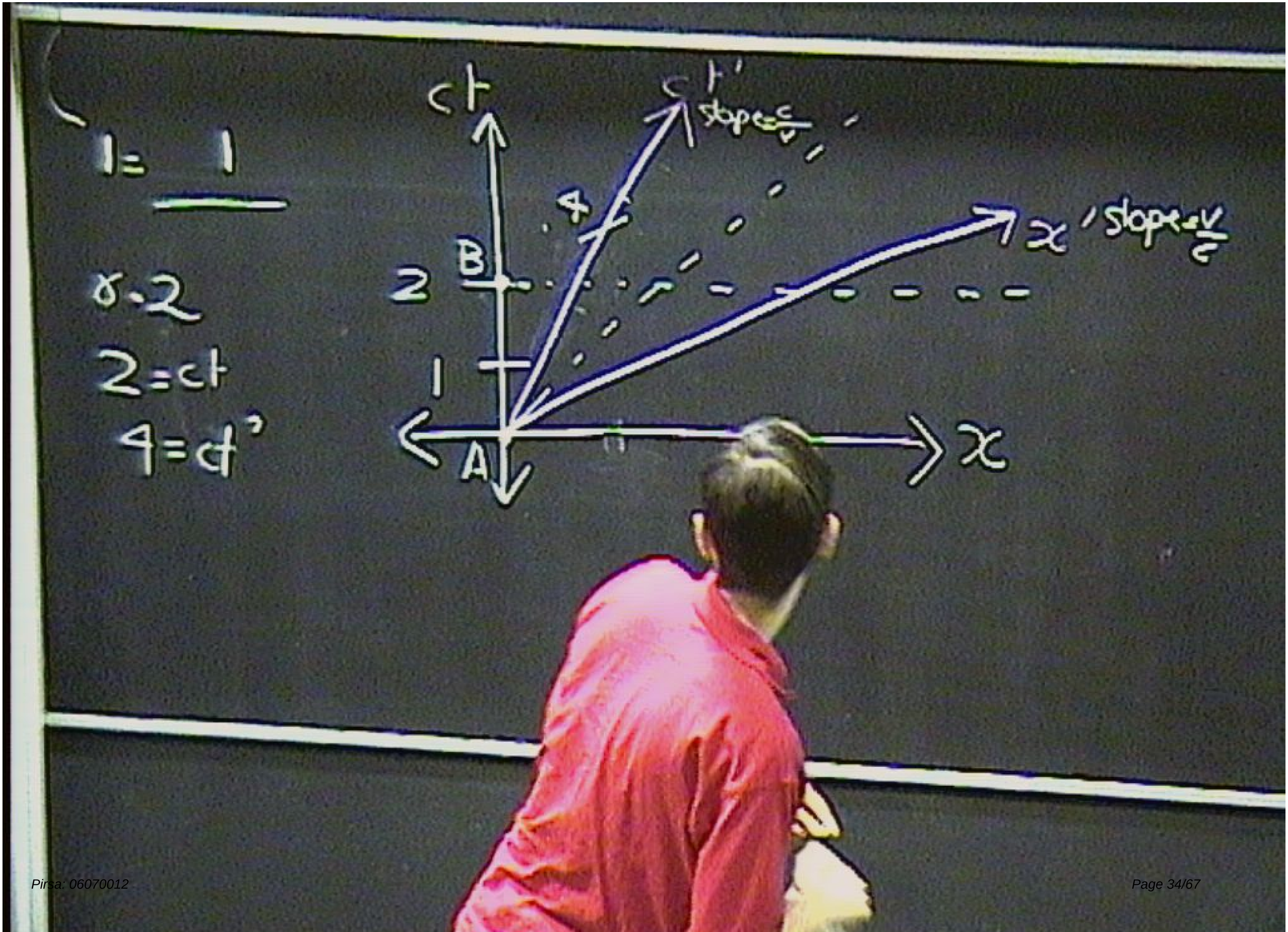

ct
ct'
slope=c/v
x' slope=v/c
B
2
A
x

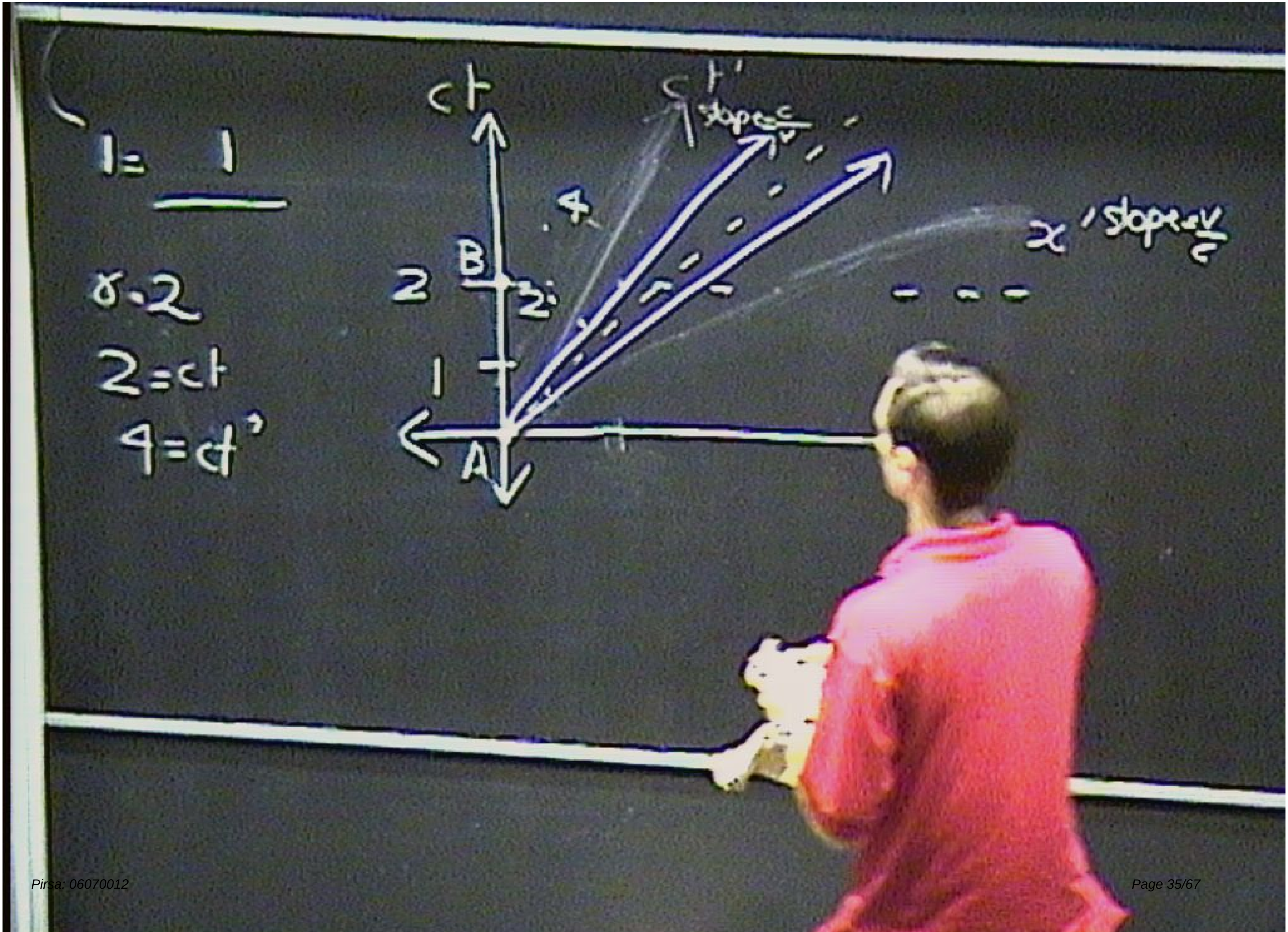

ct
ct'
x'
Earth
x
Spaceship
c
v
Δt spaceship = 3
Δt Earth = 4
| = ct'

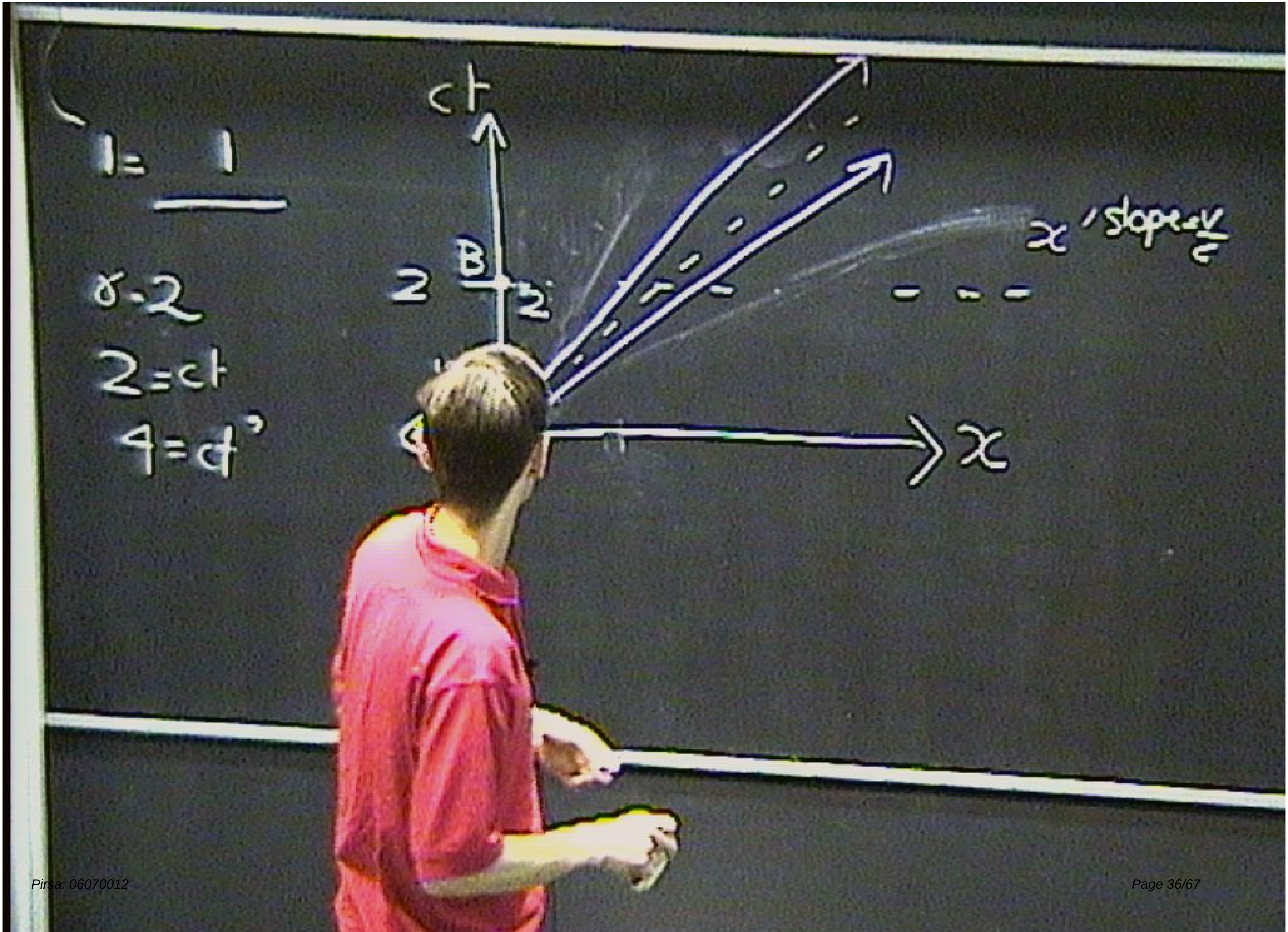

ct
ct'
G F E D
4
3
2
1
Earth
x
Spaceship
c
v
Δt
$1 = ct'$
x'
5
4
3
2

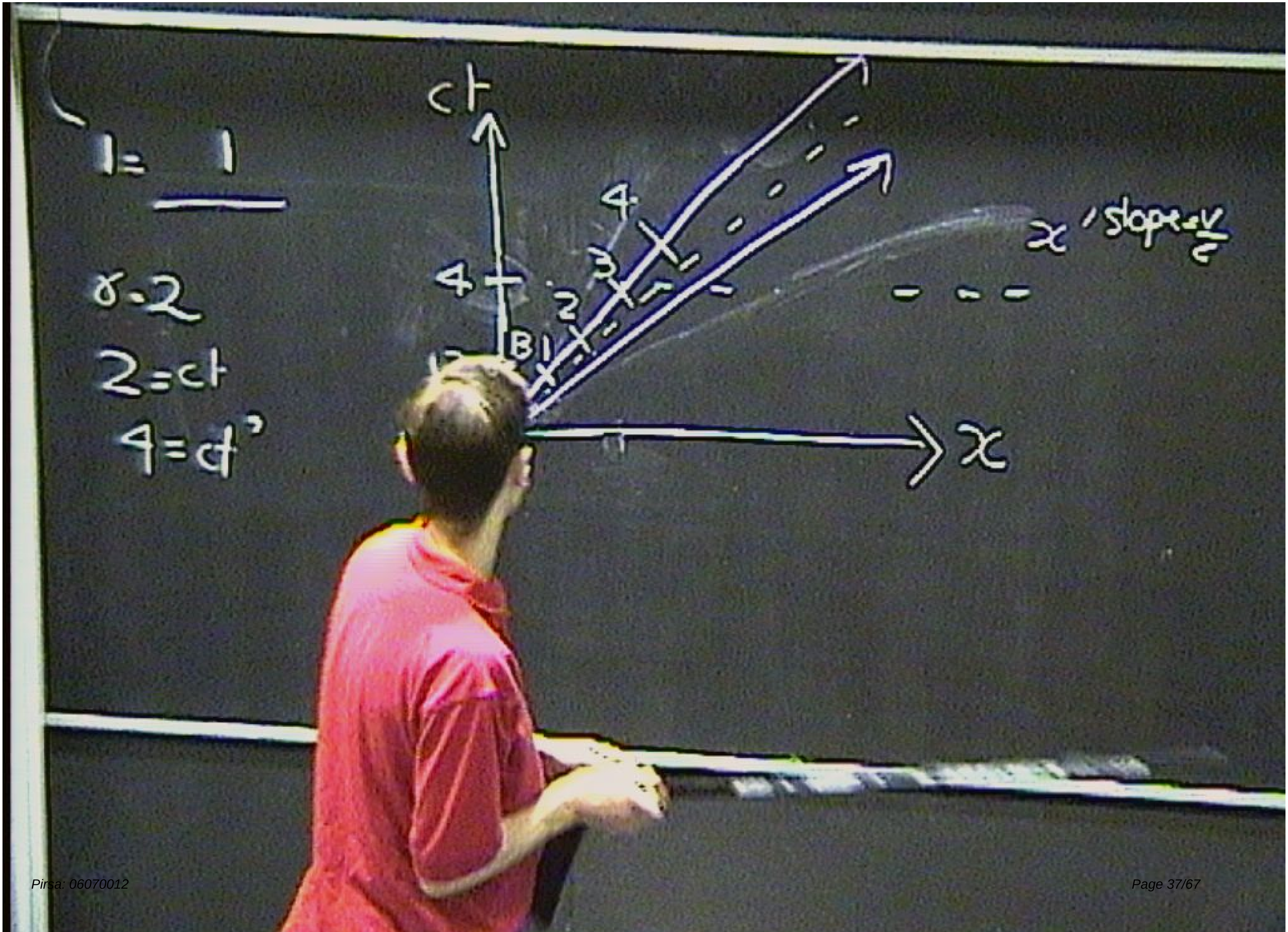

x'
Earth
x
Spaceship
l = c t'

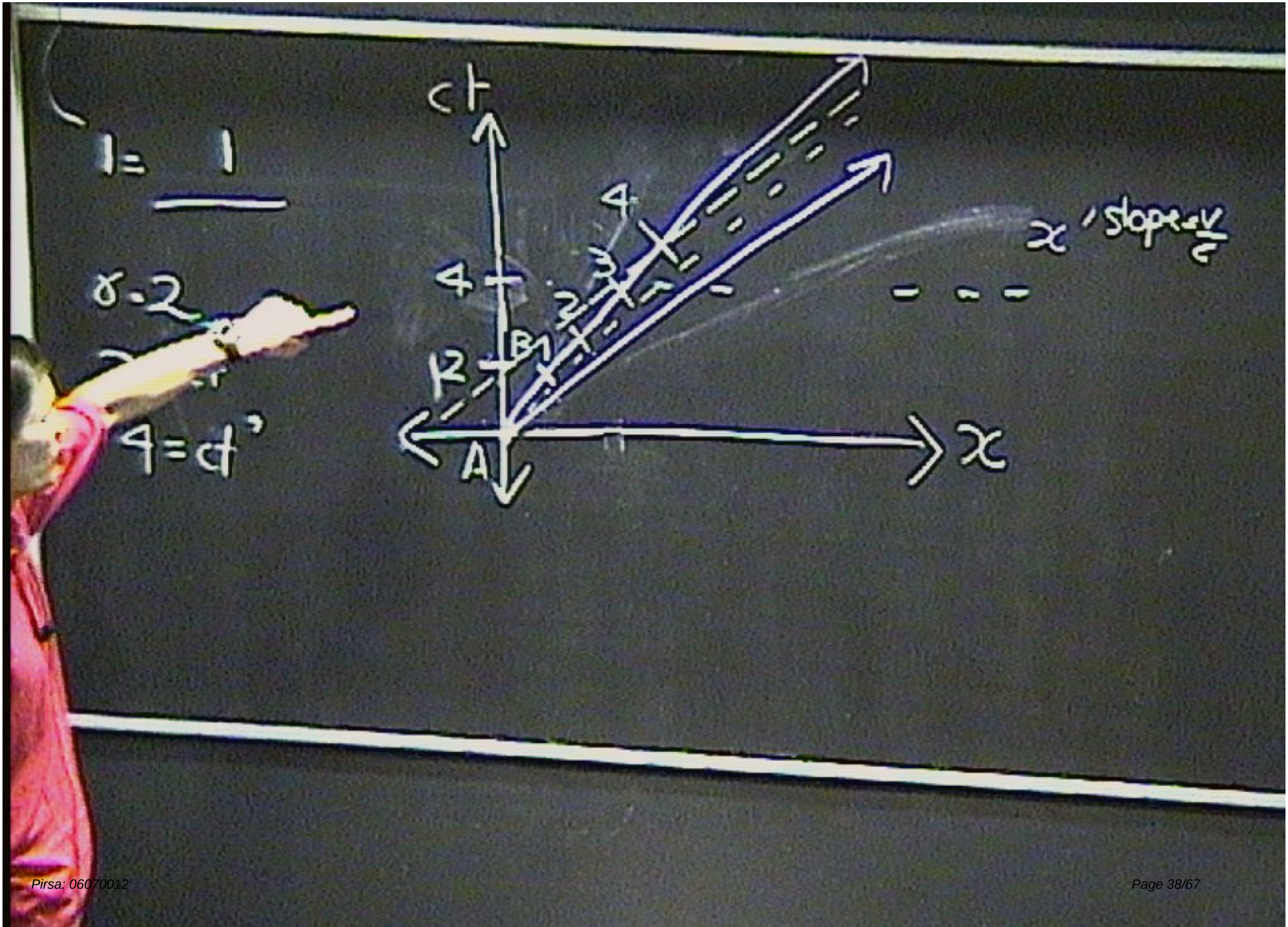

ct
ct'
slope = c/v
x' slope = v/c
B
2
1
A
x
γ = 2
2 = ct
4 = ct'


1= 1
γ=2
2=ct
4=ct'
ct
ct'
slope=c/v
x' slope=v/c
B
2
1
A
x
4

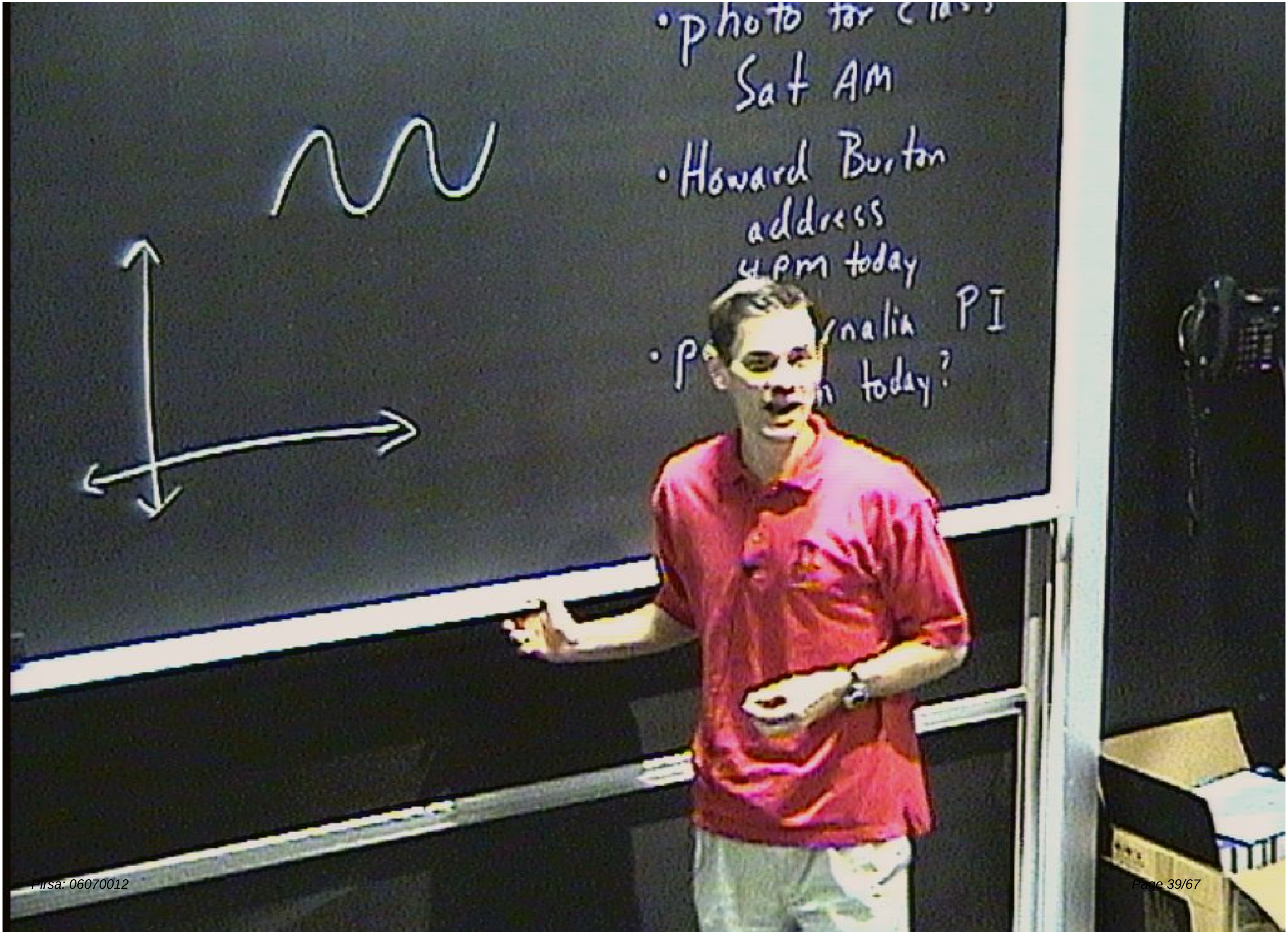

1= 1
γ=2
2=ct
4=ct'
ct
ct' slope=c/v
x' slope=v/c
B
A
2
1
4

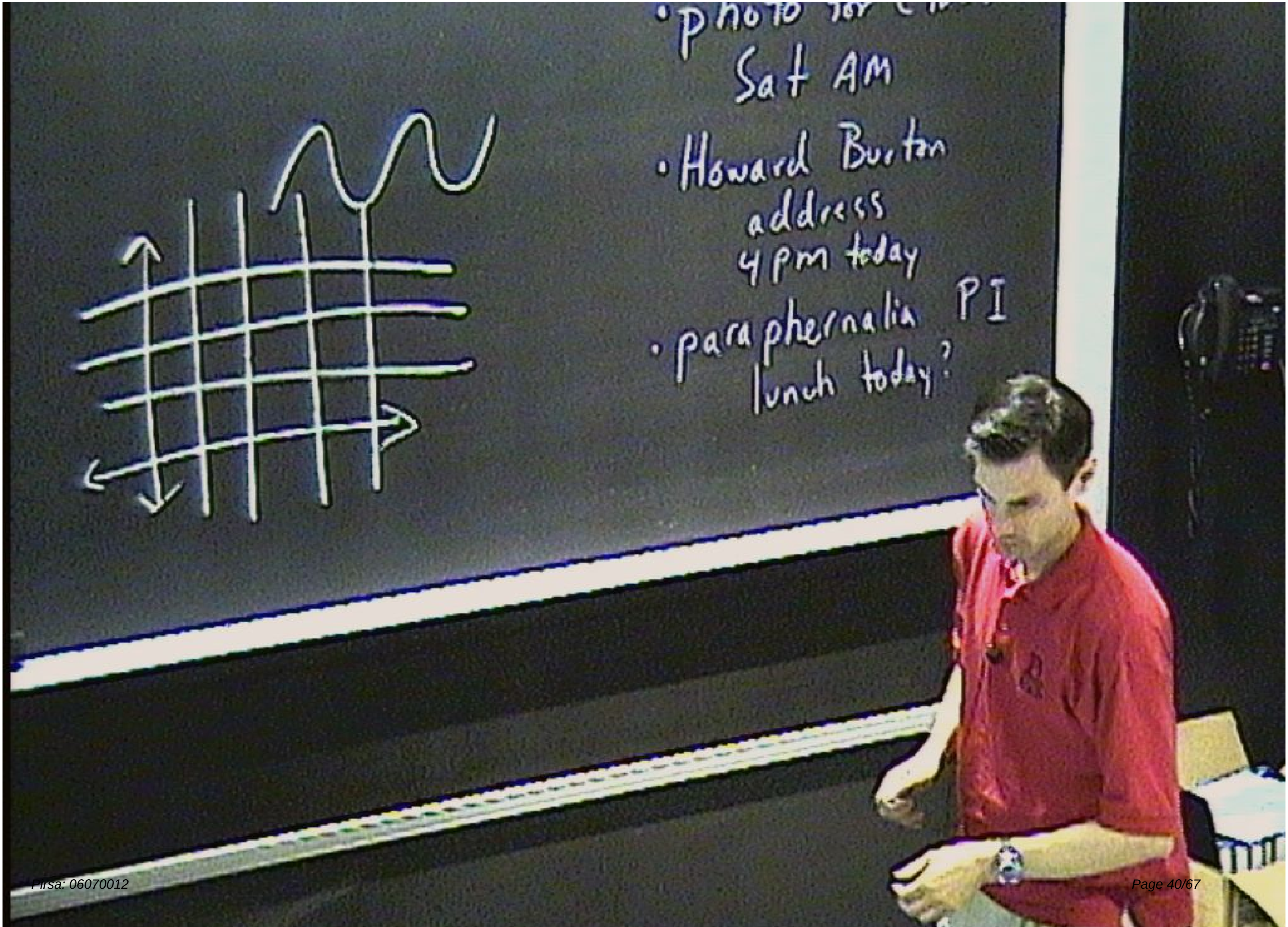

ct
x' slope = v/c
x

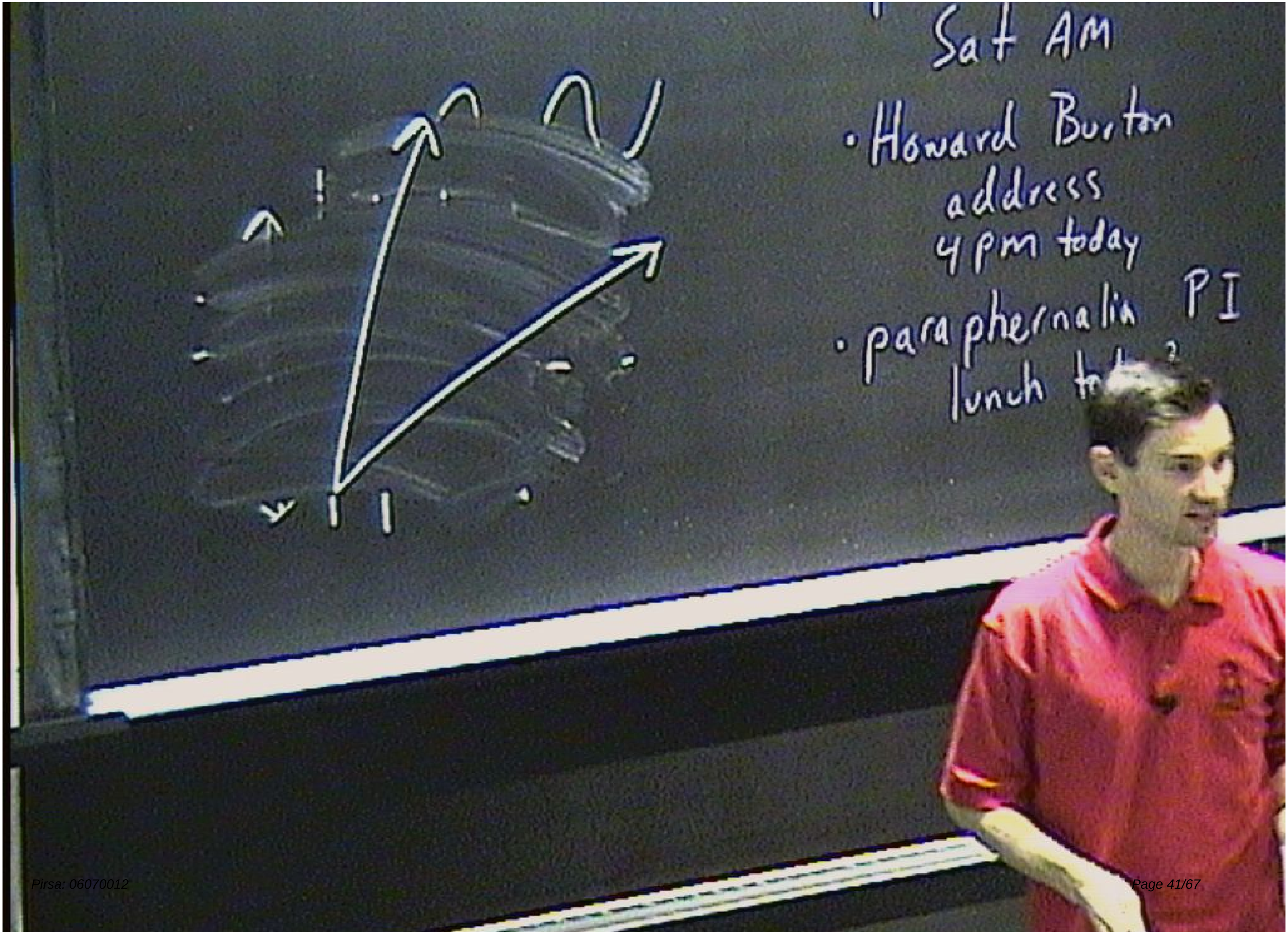

ct
x
x' slope = v/c
γ=2
2=ct
4=ct'

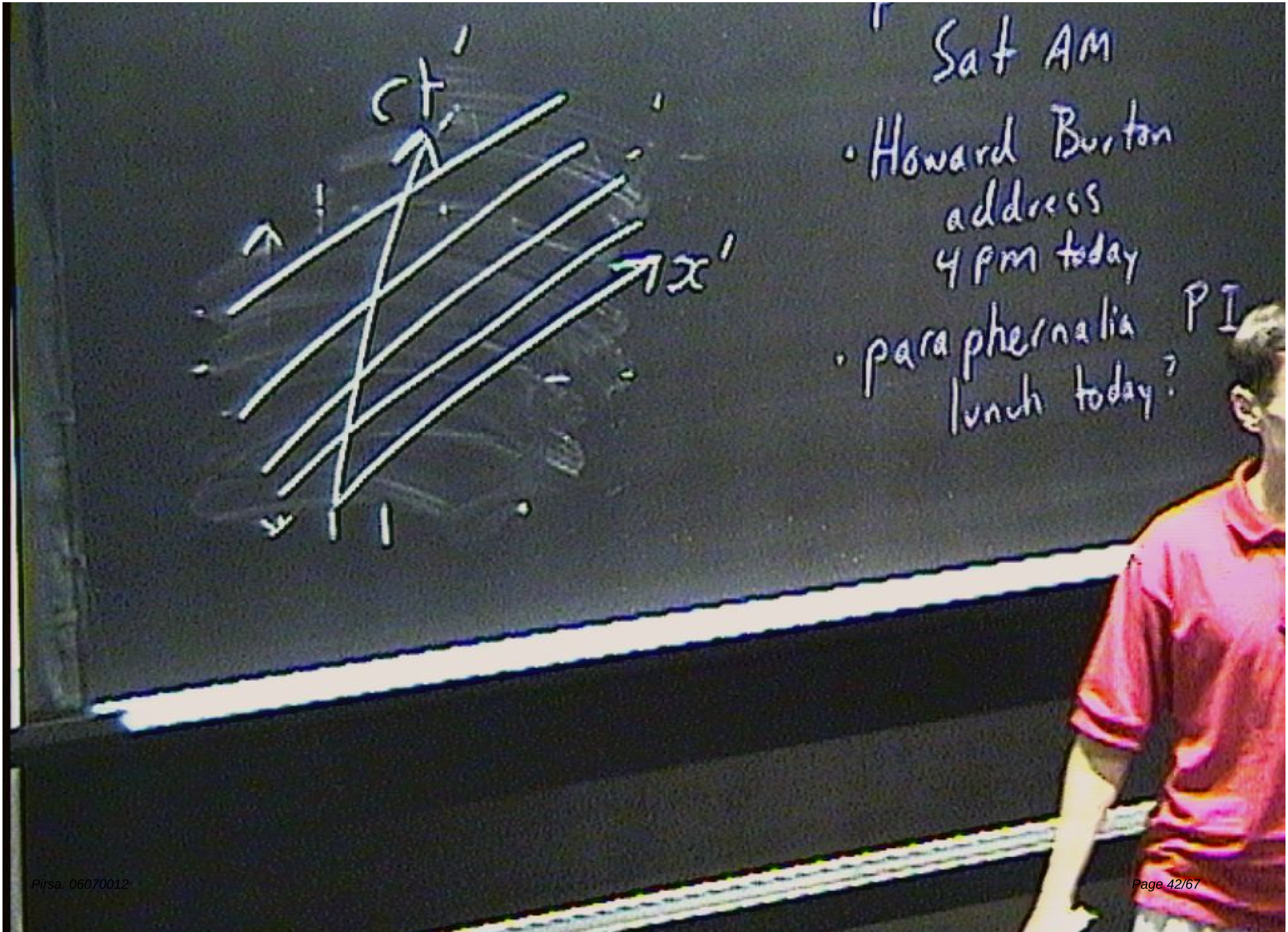


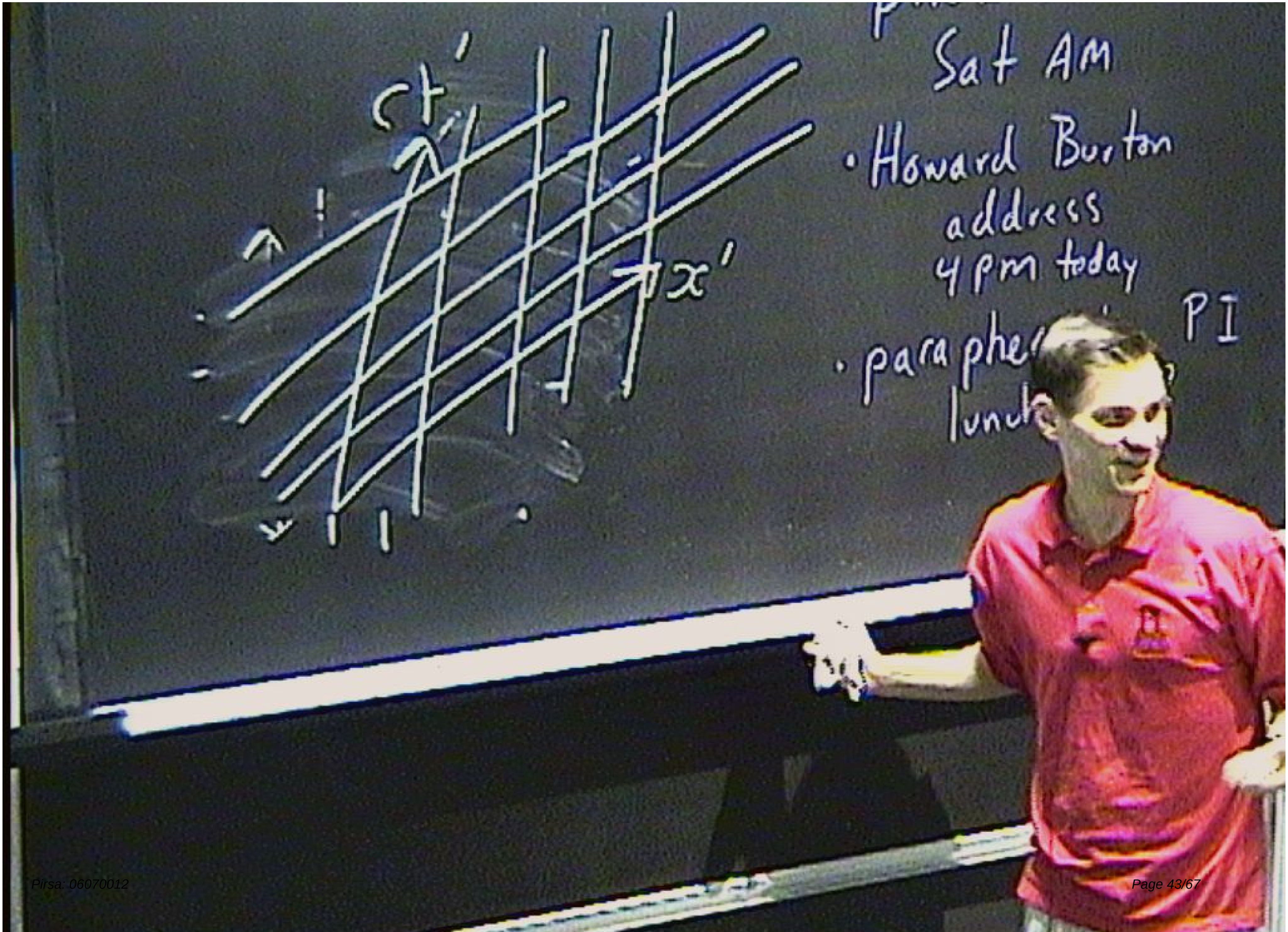
ct
x
A
B
x' slope = v/c
γ=2
4=ct'


• photo for
Sat AM
• Howard Burton
address
4pm today
PI
today?


Sat AM
• Howard Burton address
4 pm today
• paraphernalia PI
lunch today?


Sat AM
• Howard Burton address 4pm today
• paraphernalia PI
lunch


ct'
x'
Sat AM
· Howard Burton
address
4 pm today
· paraphernalia PI
lunch today?

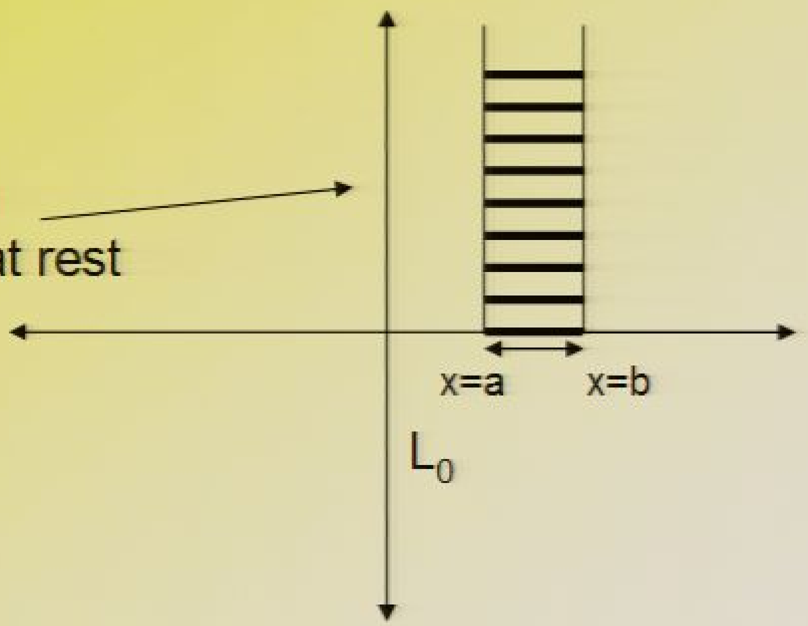

ct'
x'
Sat AM
· Howard Burton
address
4 pm today
PI

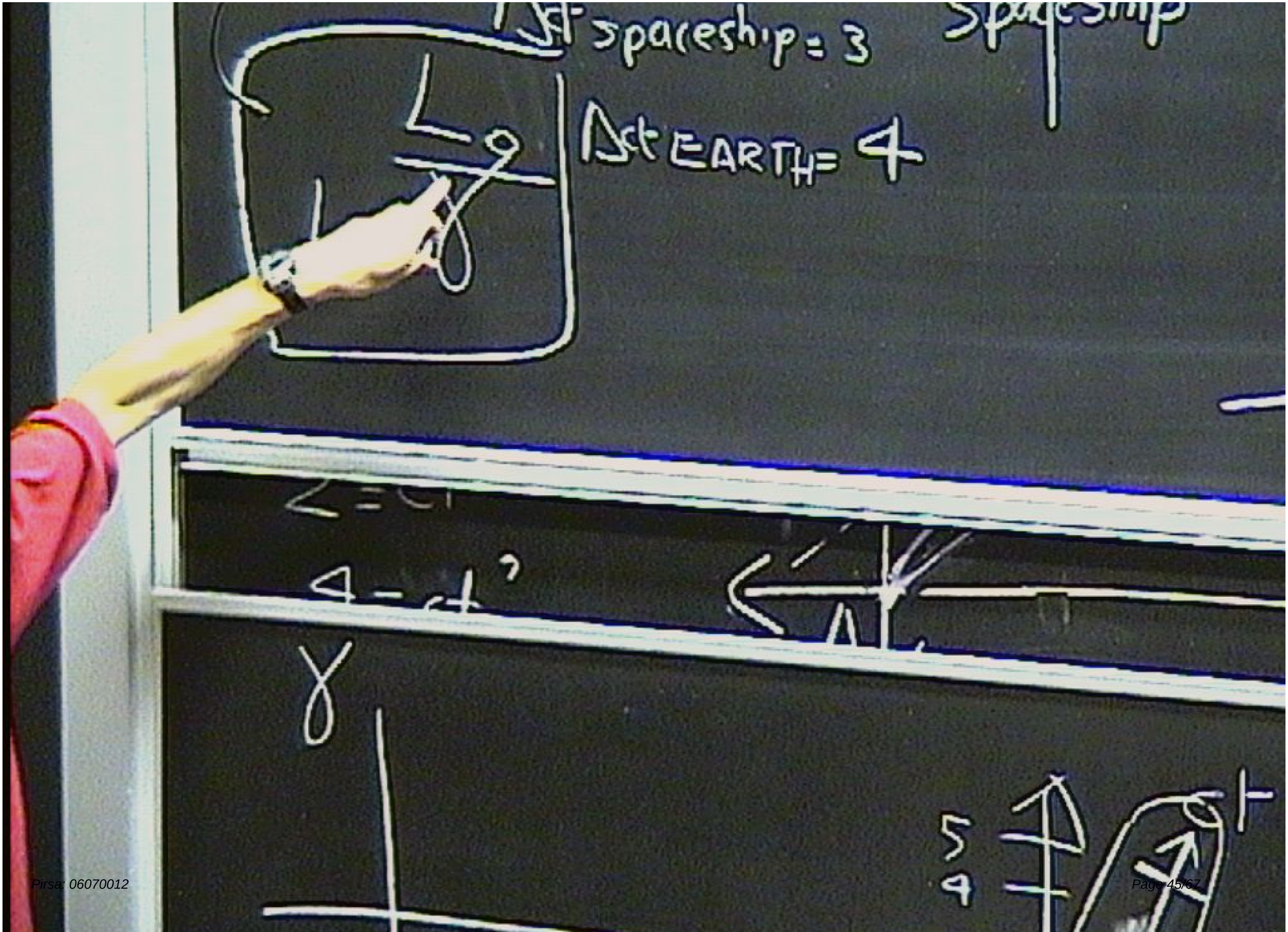
Length contraction

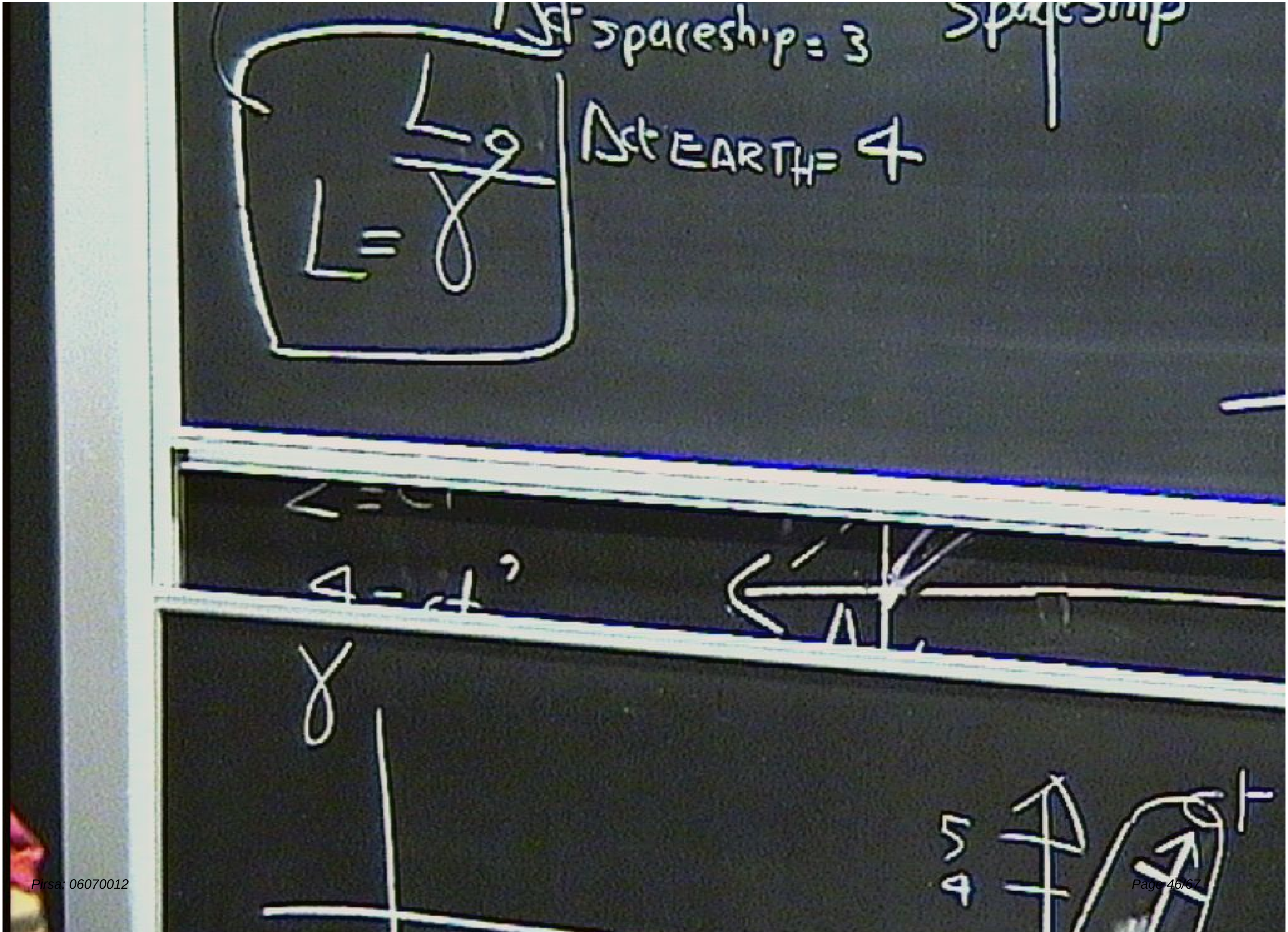
- Analogous process for length contraction

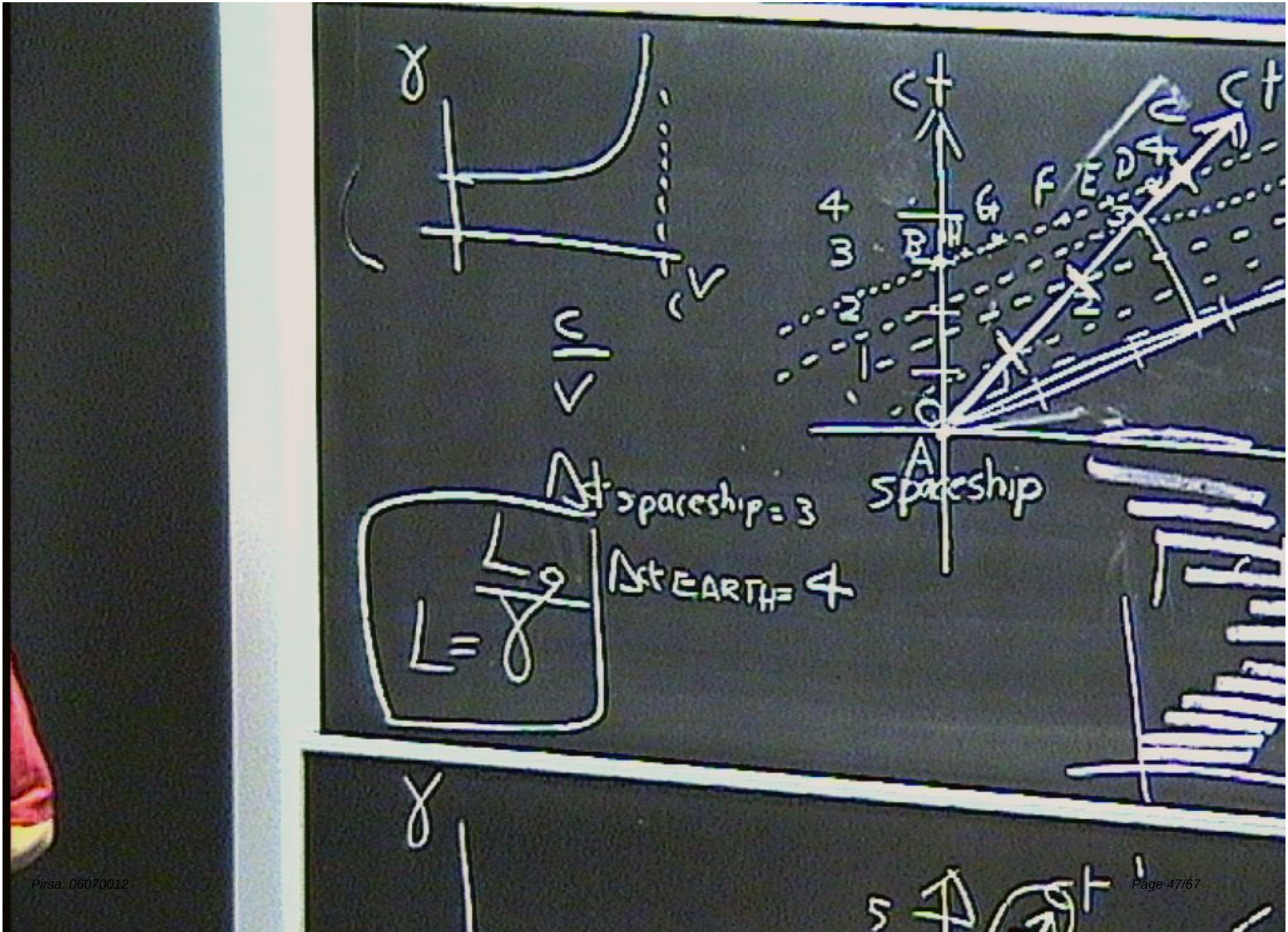
- Consider a stick lying on the ground at rest.
- What is its length in frame S' moving at speed v relative to the stick?

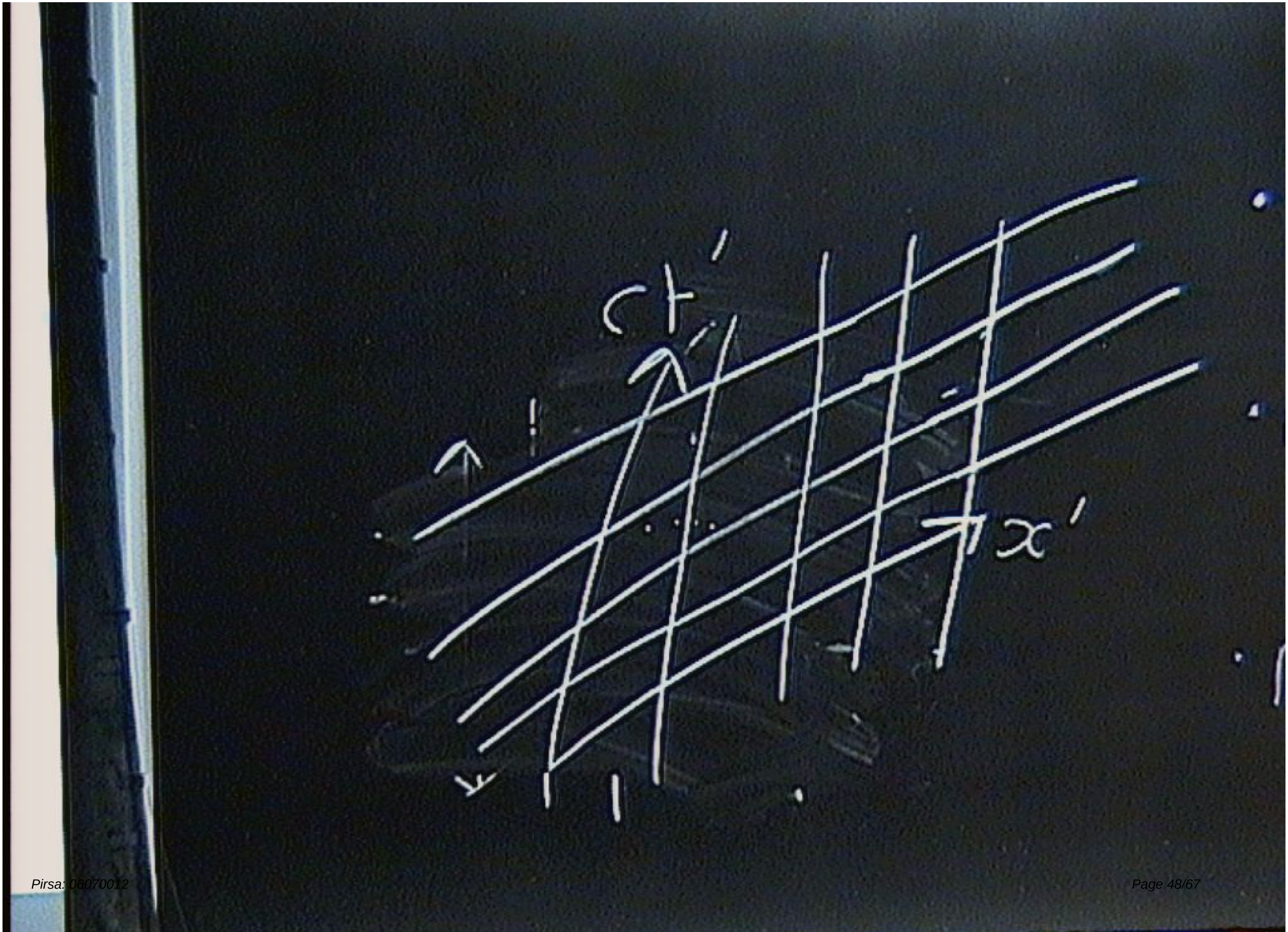
worldsheet for a
horizontal stick at rest




Δt spaceship = 3
Spaceship
Δt EARTH = 4


Δt spaceship = 3
Δt EARTH = 4
L = L₀/γ


γ
v
c
c/v
ct
ct'
A
B
G F E D
Spaceship
Δt spaceship = 3
Δt EARTH = 4
L = L0/γ


ct'
x'

- To determine its length in S graphically, consider the t axis which corresponds to the line x=0.

- Draw a line parallel to it that passes through the point x=a and then continue to draw parallel lines to the right of the first one corresponding to x=a+1,a+2,a+3 … until we draw one that passes through the point x=b. The length L_0 is given by b-a.

- Although this process is laborious and unnecessary, it helps us to understand how to measure L' graphically.

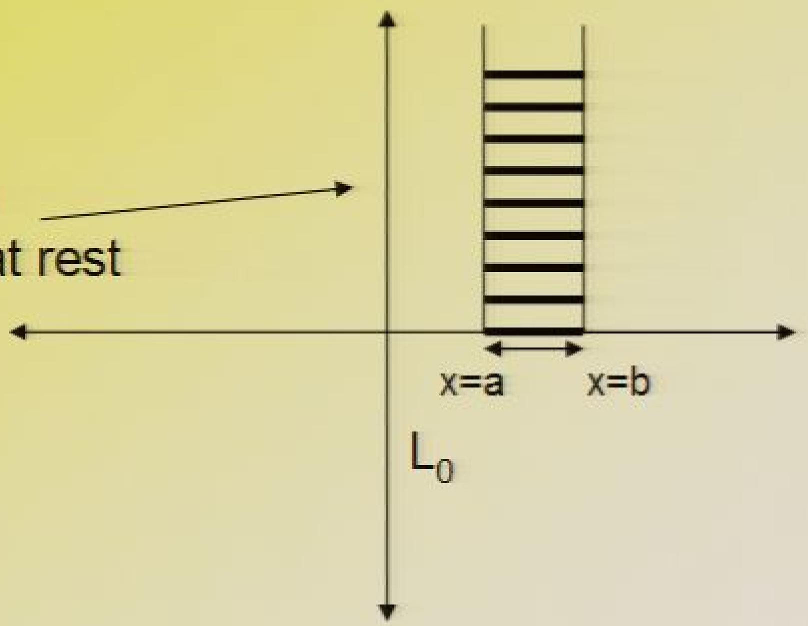
- Let us perform the analogous procedure in the frame S'

Length contraction

- Analogous process for length contraction

- Consider a stick lying on the ground at rest.
- What is its length in frame S' moving at speed v relative to the stick?

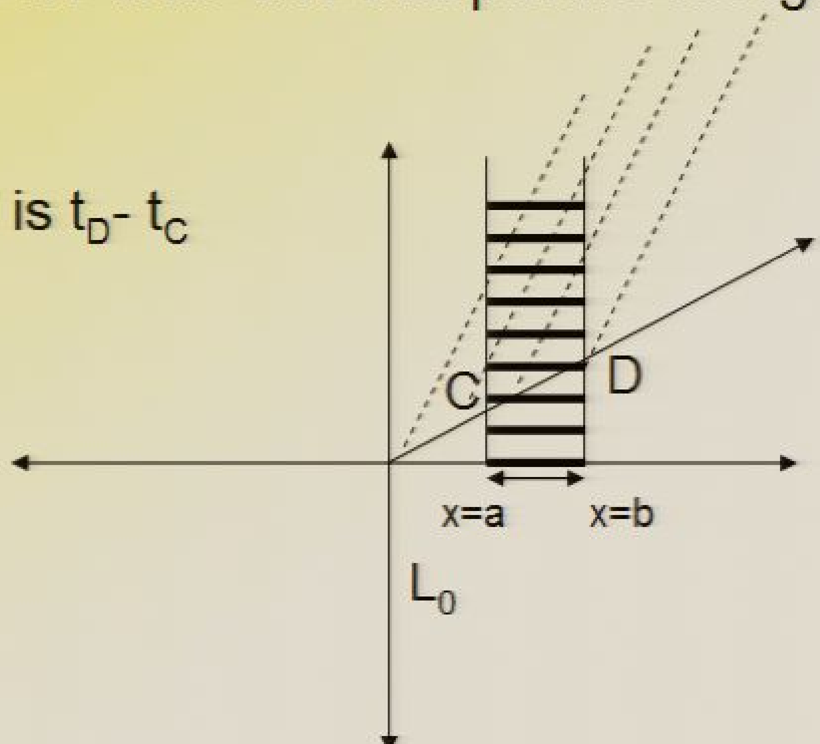
worldsheet for a horizontal stick at rest

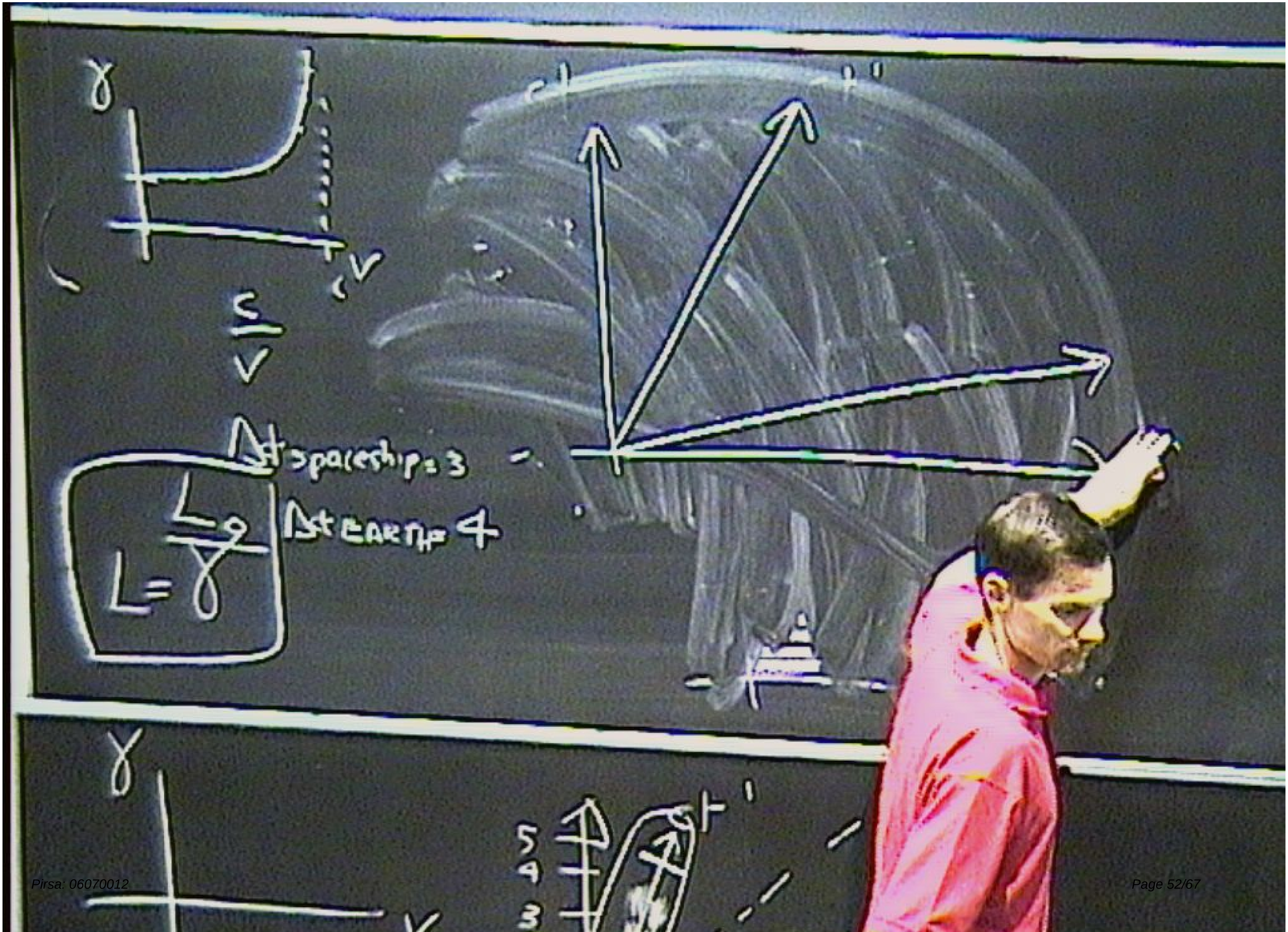


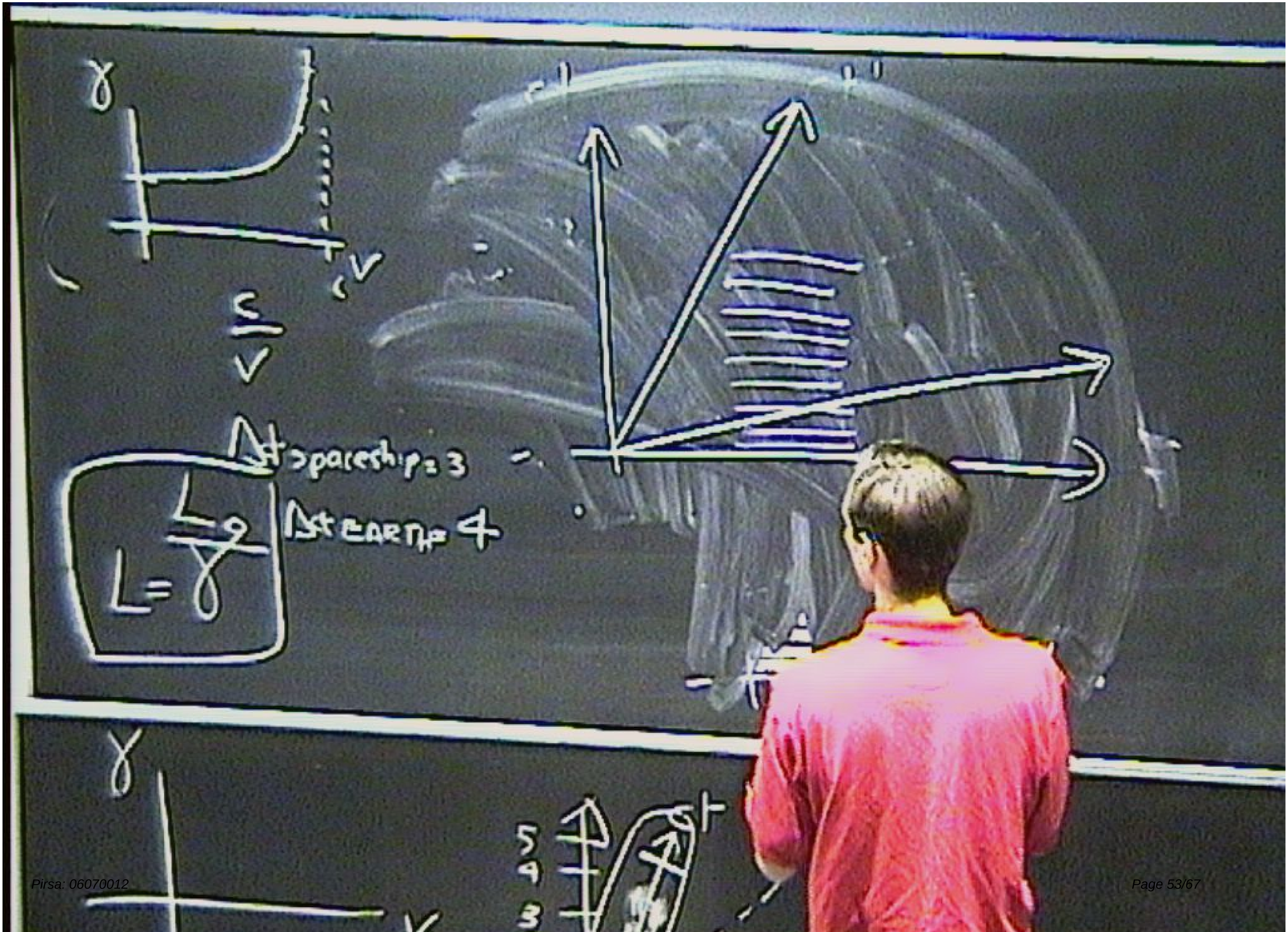
- First draw the axes for S'

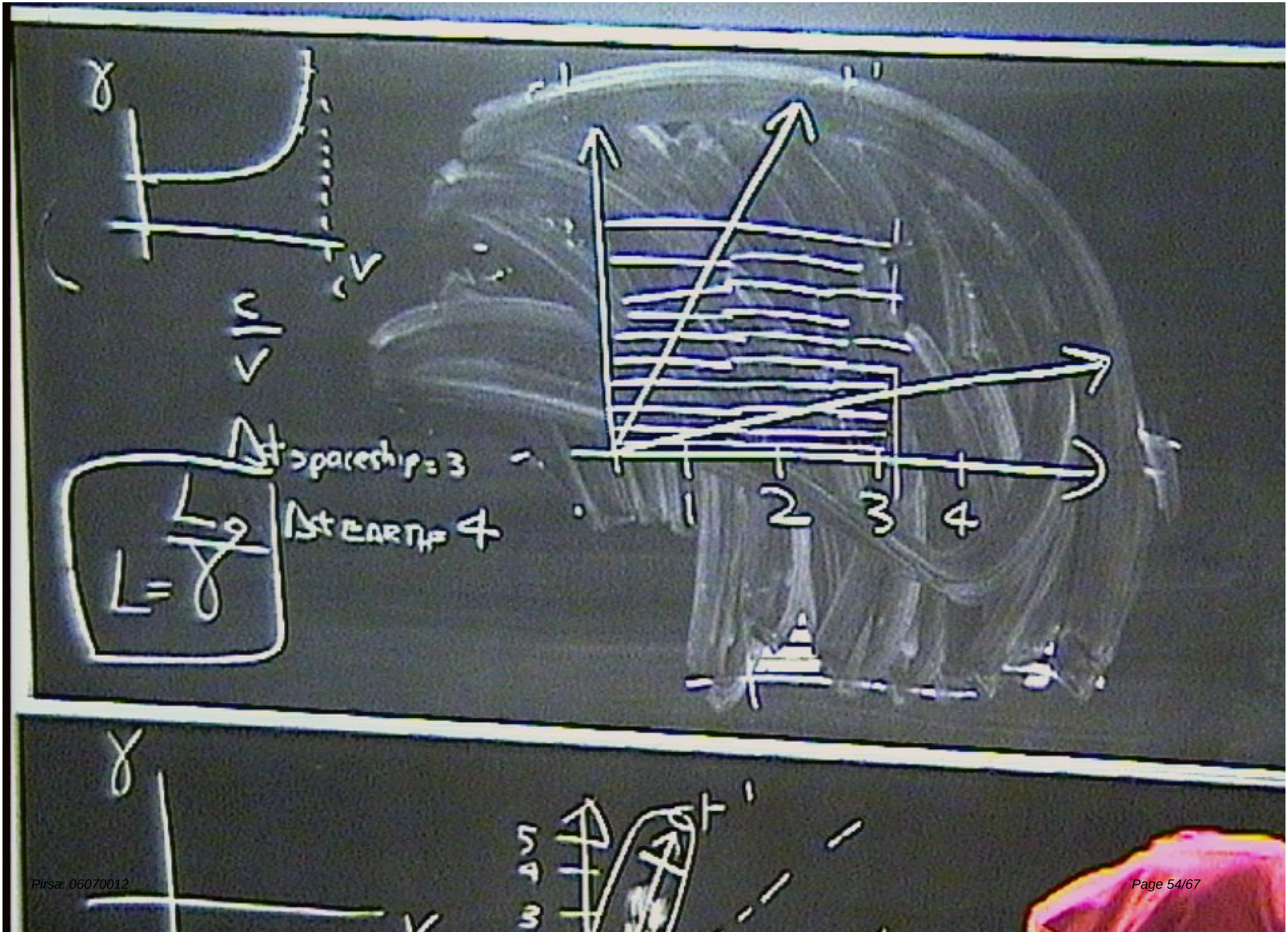
- Next, draw a line parallel with the t' axis that passes through point C and then draw lines parallel to it and to the right of it that correspond to $t'=t_C+1$, t_C+2, t_C+3… until we draw one that passes though point D with time t_D.

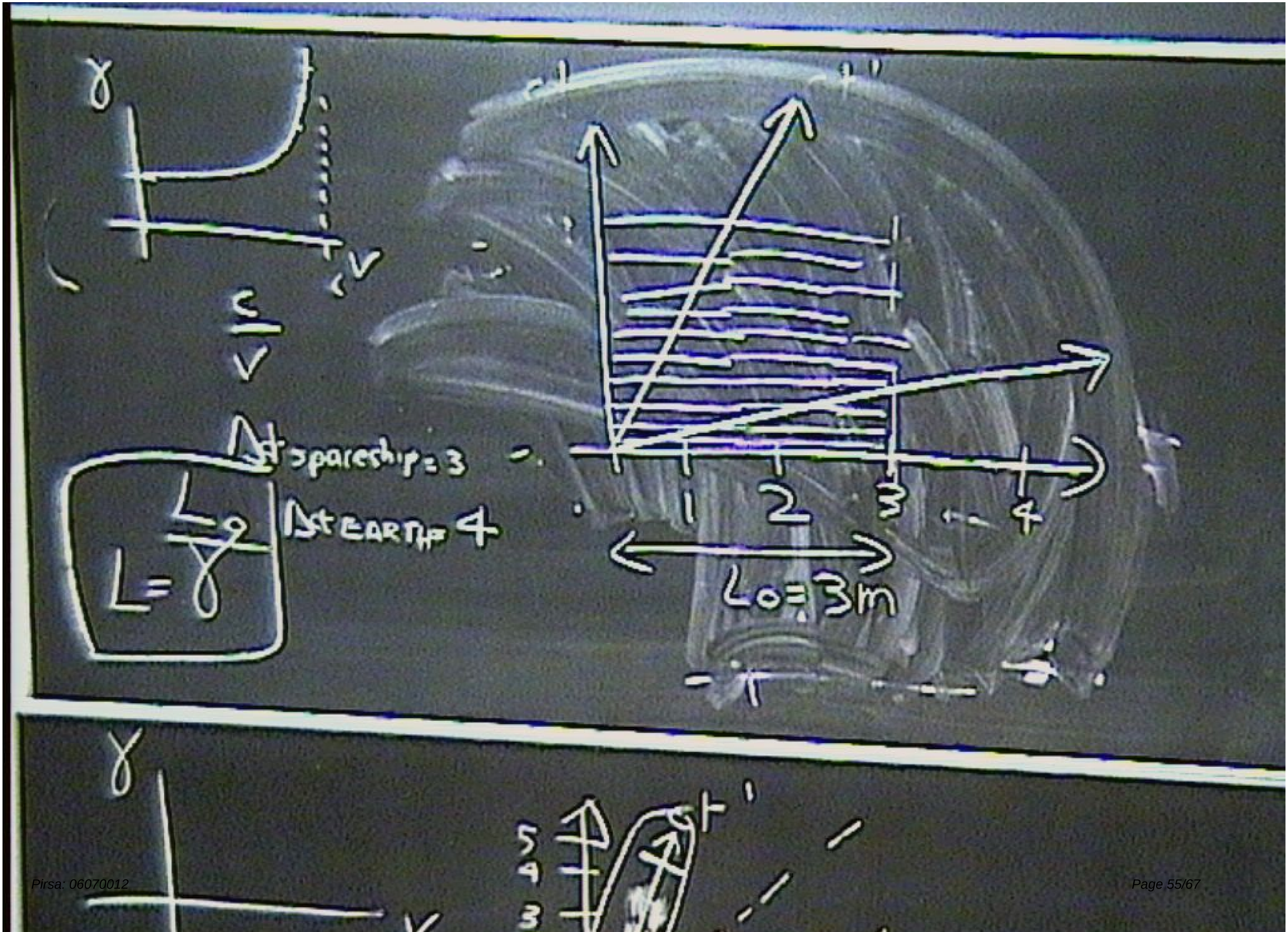
- The length of the stick in S' is $t_D - t_C$

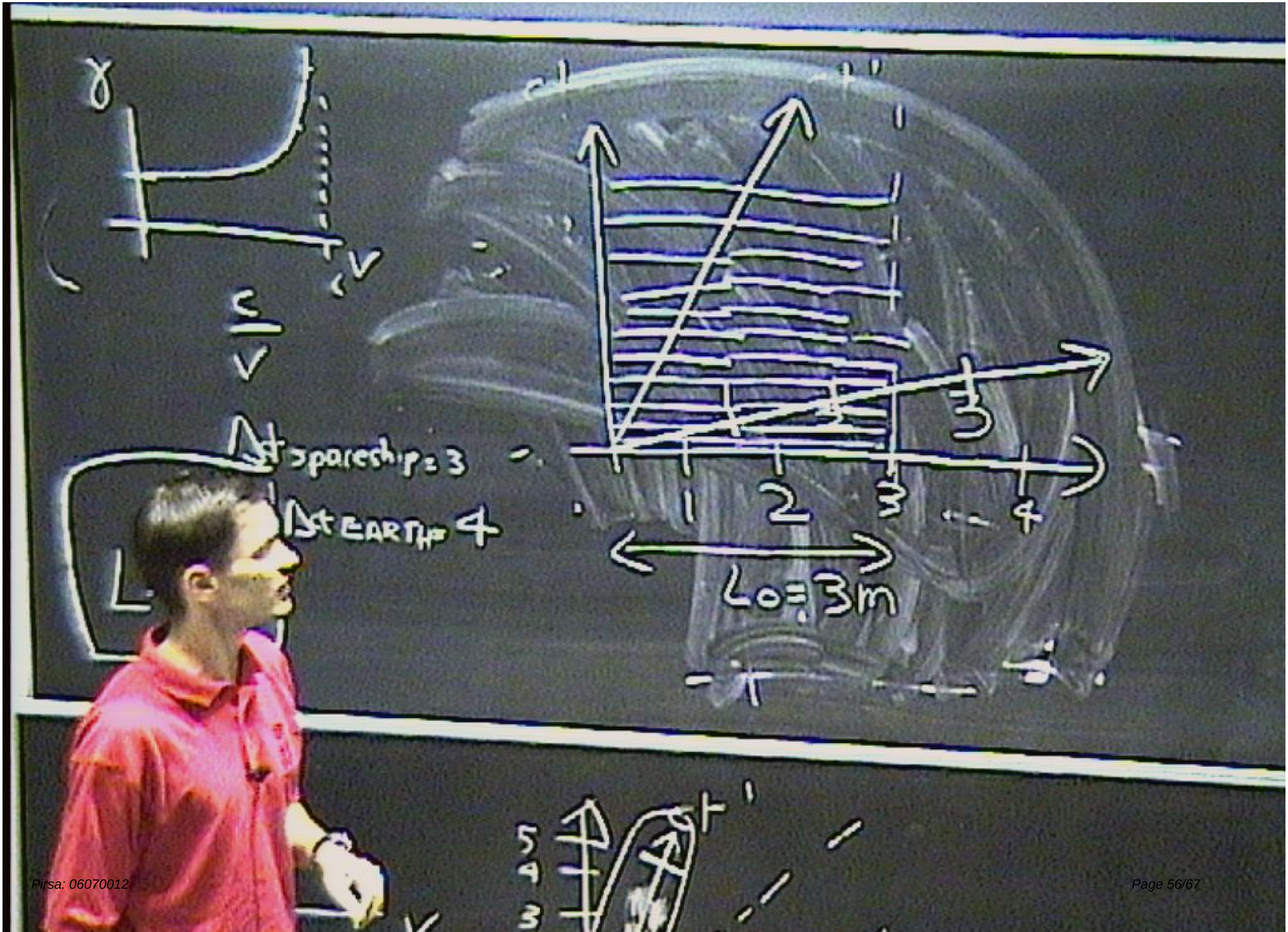


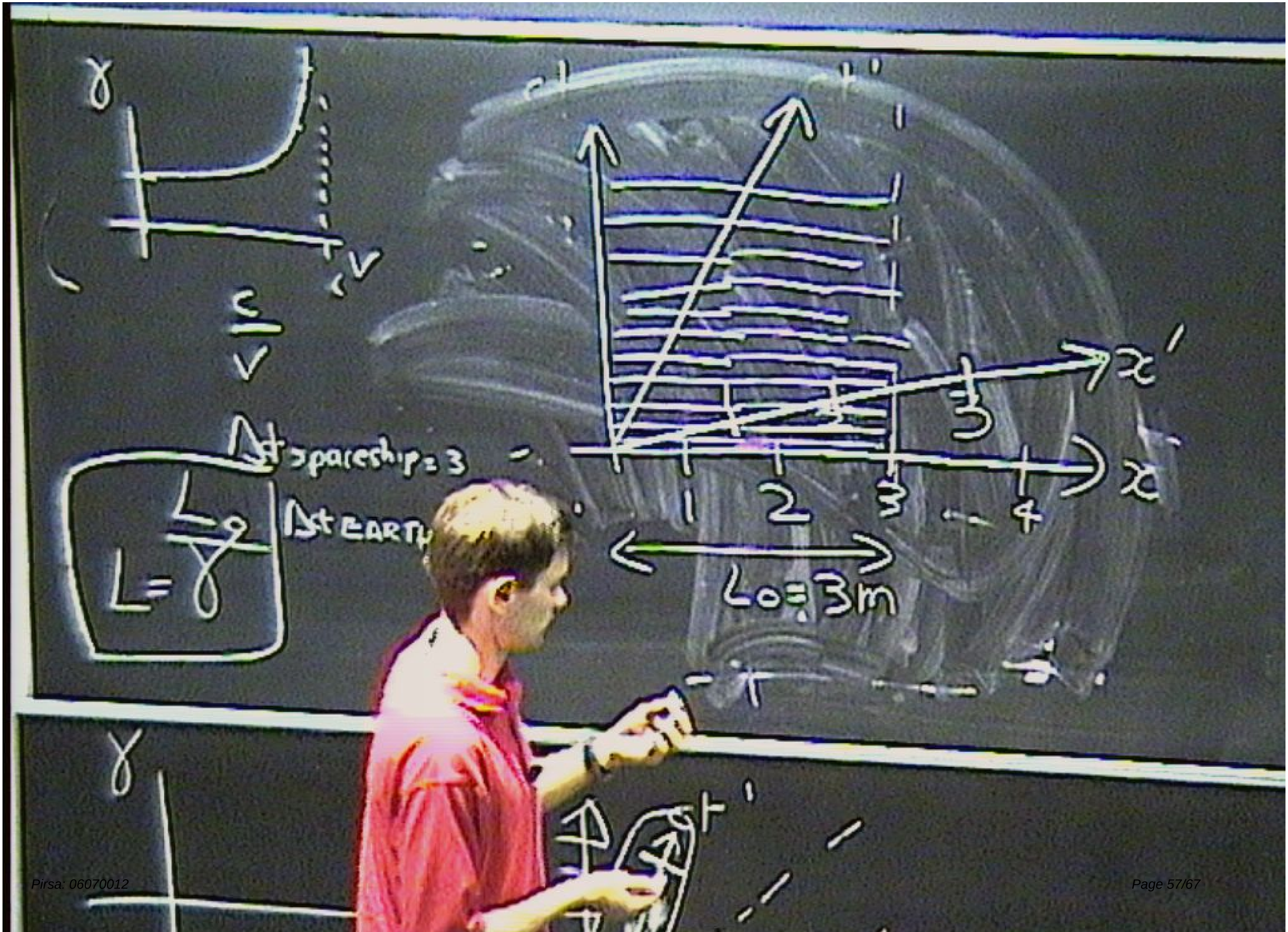

γ
c/V
Δt spaceship = 3
Δt earth = 4
L = Lo/γ
5
4
3

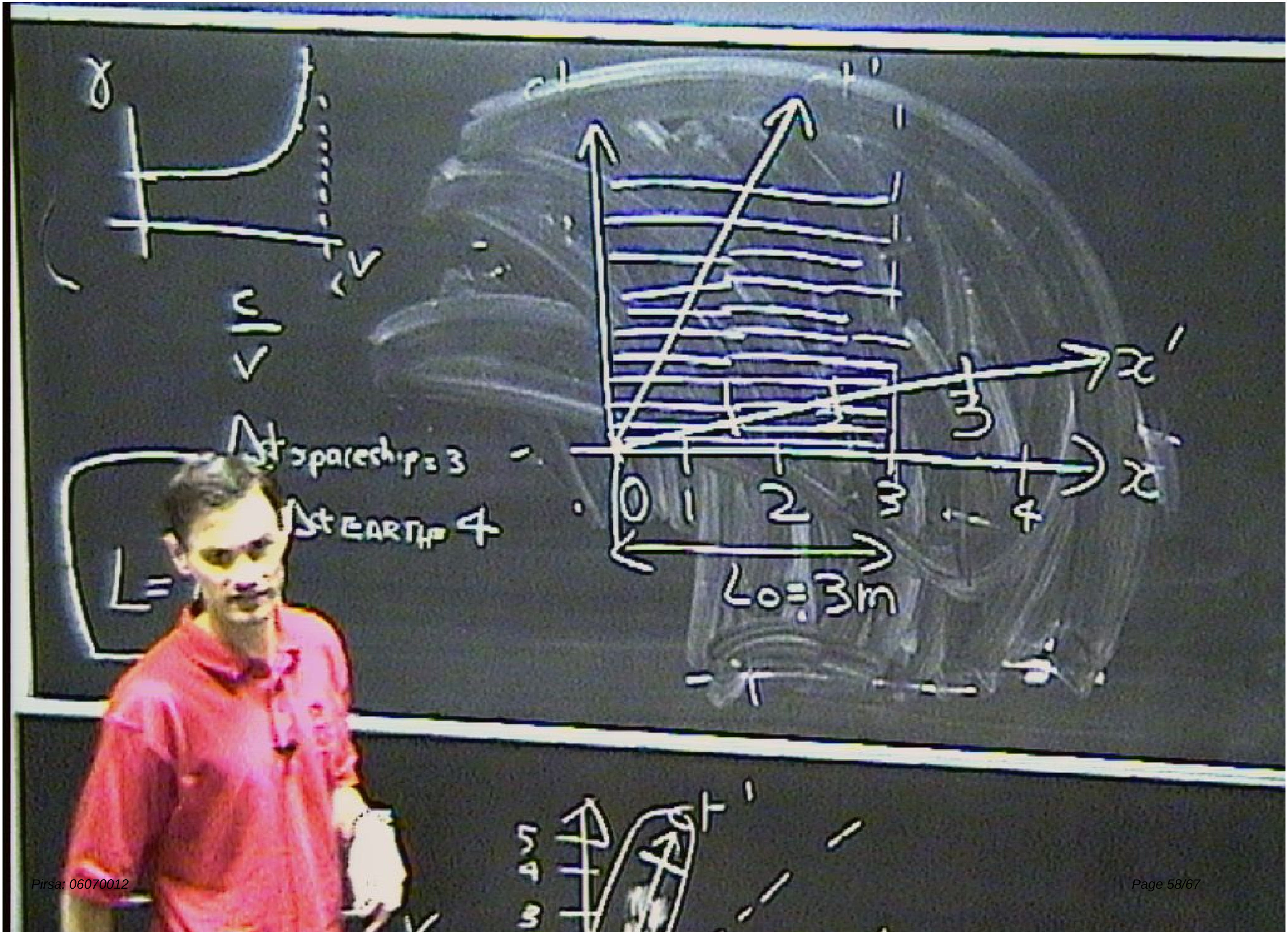

Δt spaceship = 3
Δt earth = 4
L = L₀/γ


γ
v
c
c/v
Δt spaceship = 3
Δt earth = 4
L = L0/γ
1
2
3
4
5
4
3

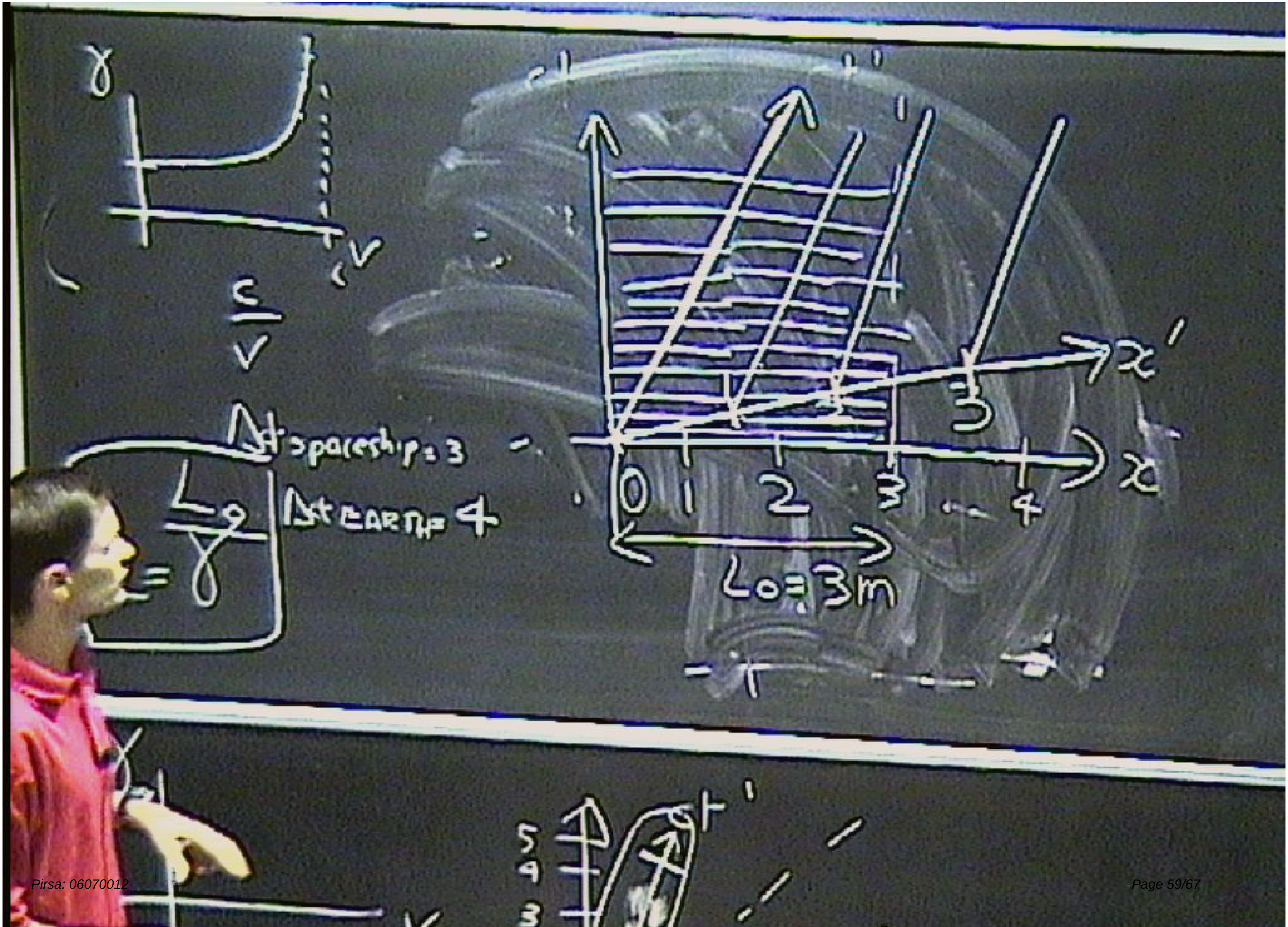

γ
v
c
c/v
Δt spaceship = 3
Δt earth = 4
L = L₀/γ
1
2
3
4
L₀ = 3m
5
4
3

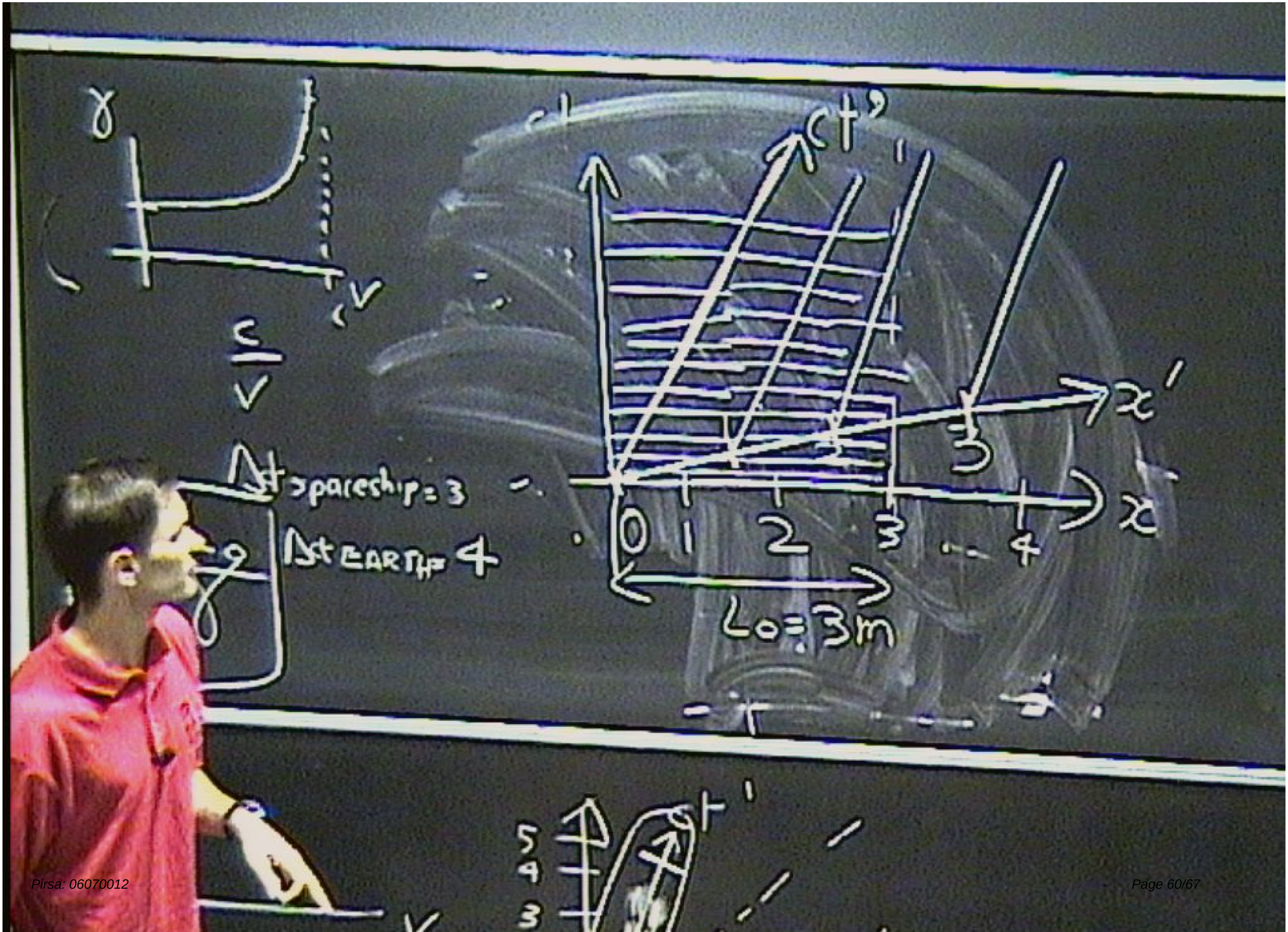

γ
v
c
c/v
Δt spaceship = 3
Δt EARTH = 4
1 2 3 4
3
L₀ = 3m
5
4
3

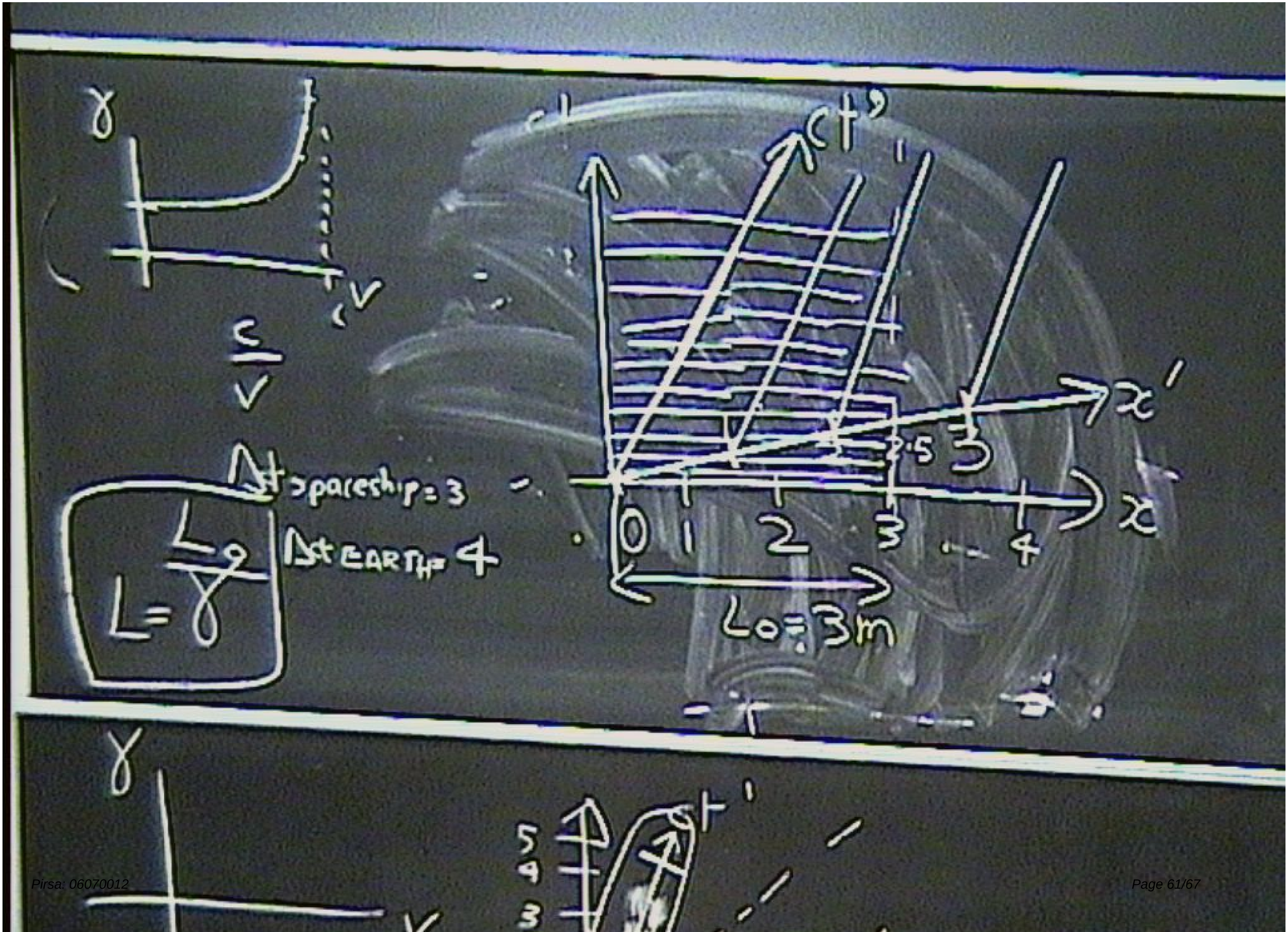

γ
v
c
c/v
Δt spaceship = 3
Δt earth
L = L₀/γ
ct
ct'
x'
x
1 2 3 4
3
L₀ = 3m

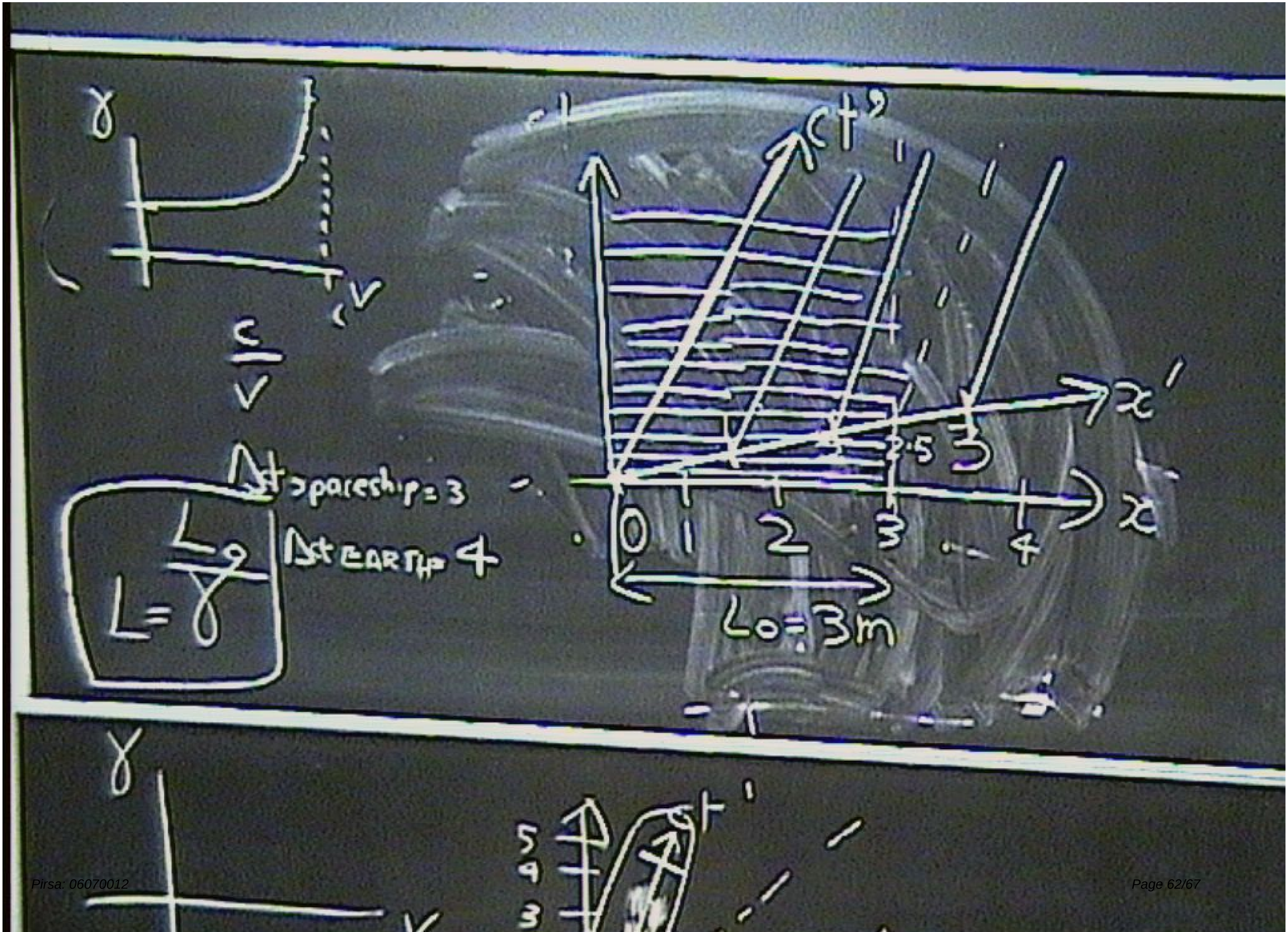


γ
v
c/v
Δt spaceship = 3
Δt earth = 4
L =
x'
x
0
1
2
3
4
L0 = 3m

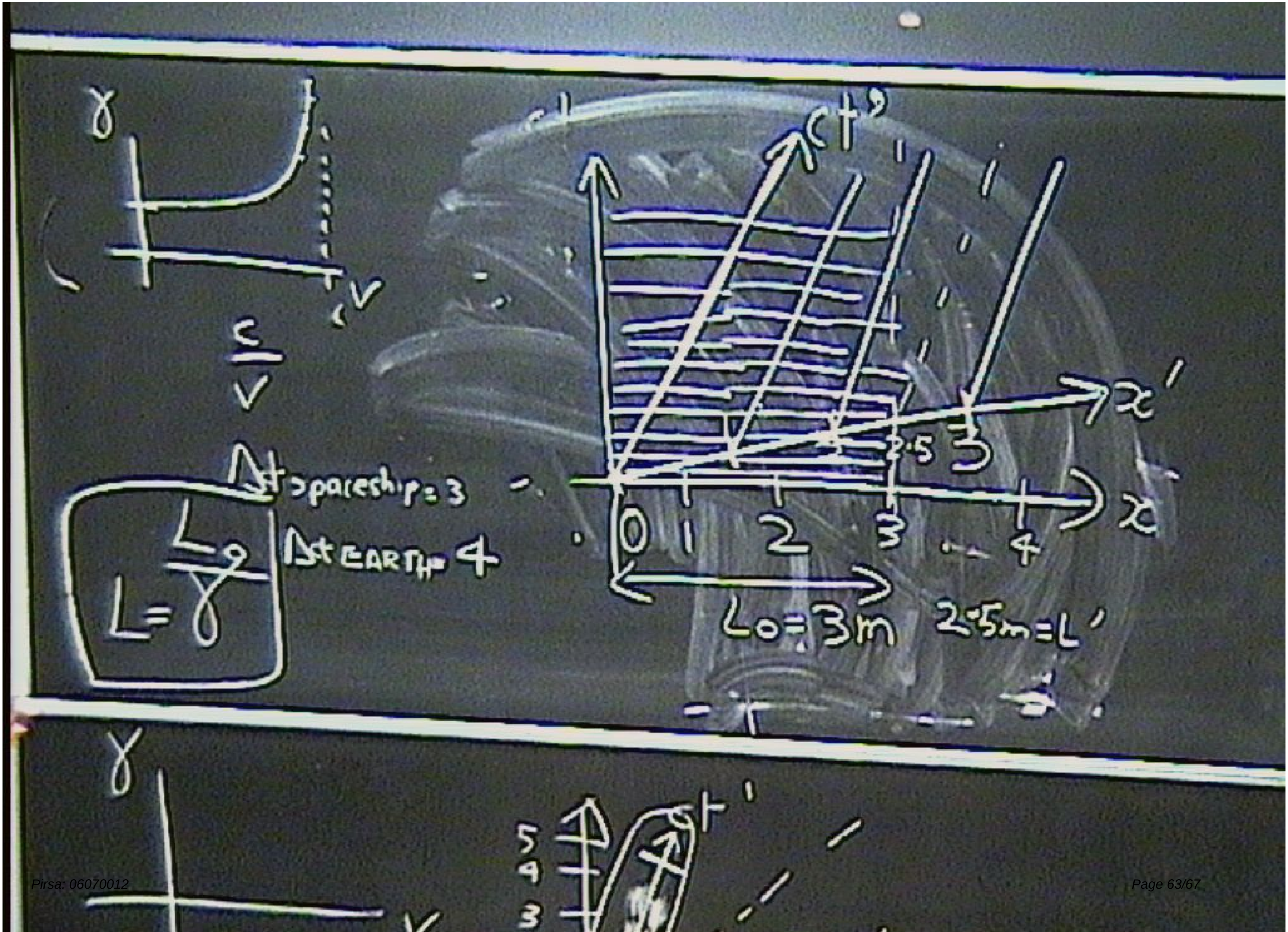

Δt spaceship = 3
Δt earth = 4
L₀ = 3m
x'
x
0 1 2 3 4

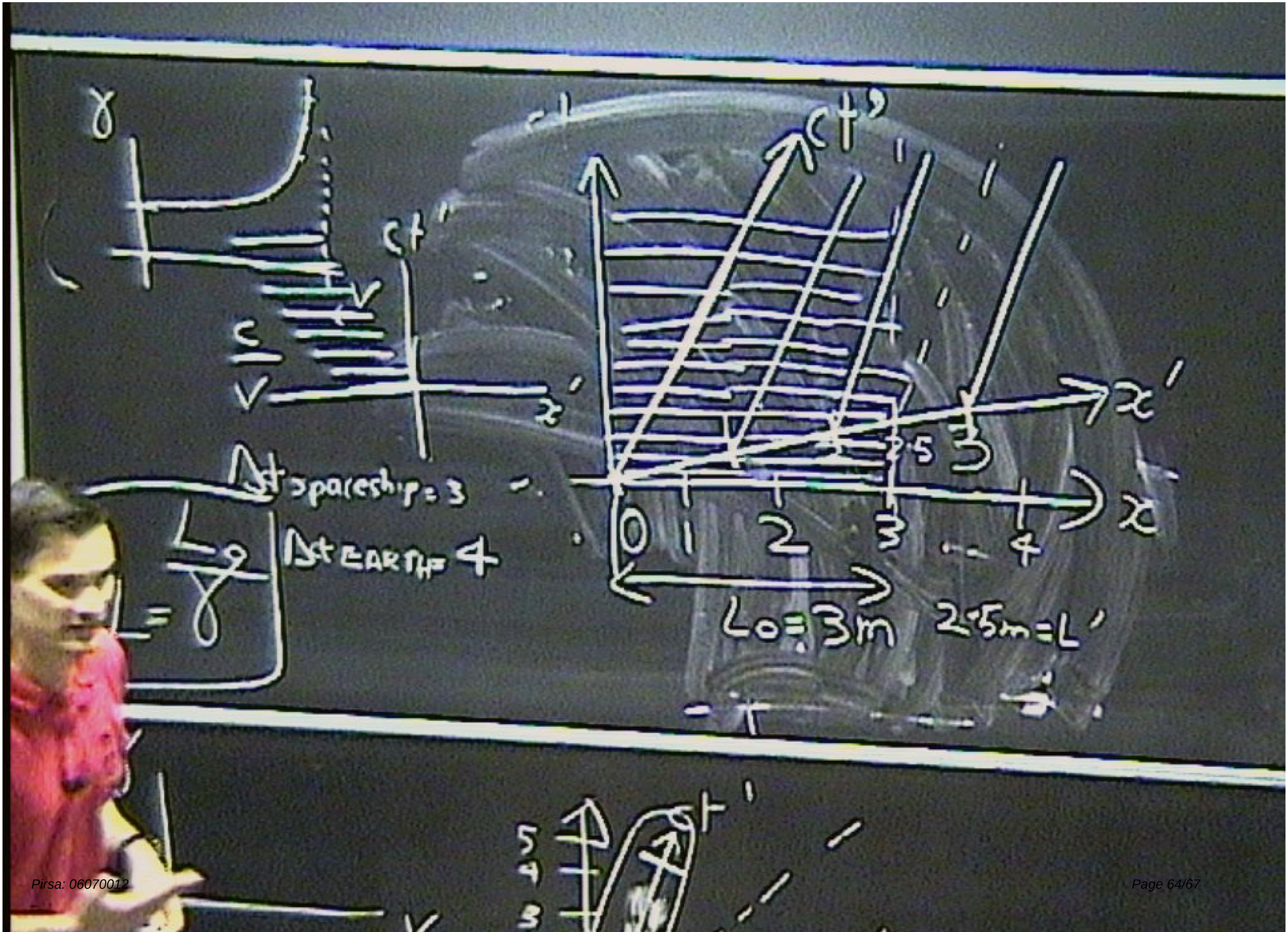

γ
v
c
c/v
Δt spaceship = 3
Δt Earth = 4
ct'
x'
x
0
1
2
3
4
L₀=3m

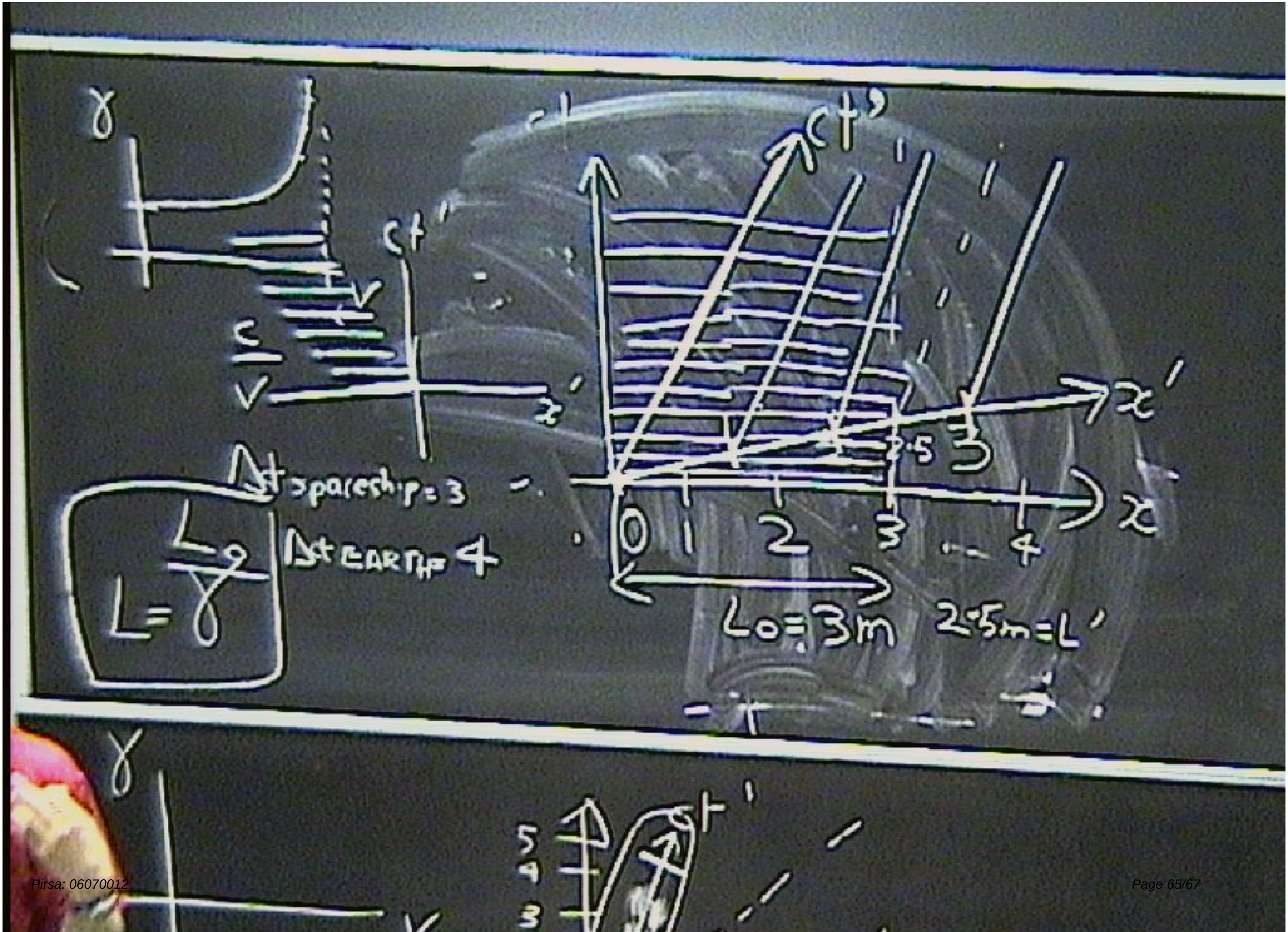

γ
v
c/v
Δt spaceship = 3
Δt EARTH = 4
L = L0/γ
ct'
x'
x
0
1
2
3
4
2.5
L0 = 3m

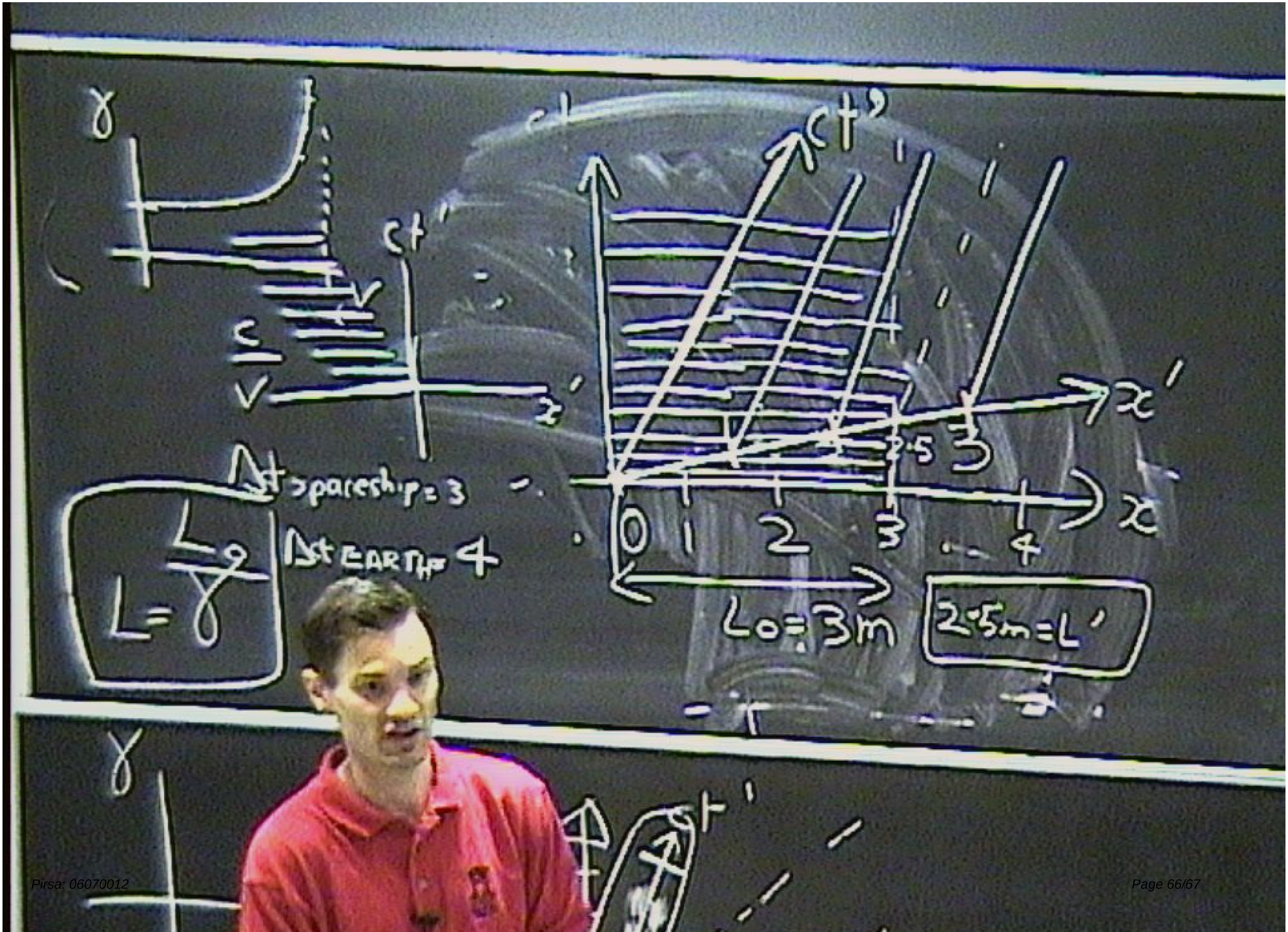


γ
v
c/v
Δt spaceship = 3
Δt earth = 4
L = L0/γ
ct
ct'
x'
x
0 1 2 3 4
2.5 3
L0 = 3m


γ
v
c
v
ct'
x'
x
2.5
3
0 1 2 3 4
$\Delta t_{spaceship} = 3$
$\Delta t_{EARTH} = 4$
$L = \frac{L_0}{\gamma}$
$L_0 = 3m$
$2.5m = L'$


γ
c
v
ct'
x'
Δt spaceship = 3
Δt earth = 4
L = L₀/γ
ct
x
0 1 2 3 4
2.5 3
L₀ = 3m
2.5m = L'


γ
ct'
c
v
x'
ct
Δt spaceship = 3
Δt EARTH = 4
$L = \frac{L_0}{\gamma}$
0 1 2 3 4
2.5 3
x'
x
Lo = 3m
2.5m = L'


γ
ct'
ct'
c
v
x'
x'
x
0
1
2
3
4
2.5
3
Δt spaceship = 3
Δt earth = 4
L = L0/γ
L0 = 3m
2.5m = L'

- First draw the axes for S’

- Next, draw a line parallel with the t’ axis that passes through point C and then draw lines parallel to it and to the right of it that correspond to $t'=t_C+1$, t_C+2, t_C+3… until we draw one that passes though point D with time t_D.

- The length of the stick in S’ is $t_D - t_C$