Title: Enrichment Presentation of Special Relativity Continued

Date: Jul 06, 2006 10:25 AM

URL: http://pirsa.org/06070009

Abstract:

- Let us denote the spacetime ‘distance’ or interval between A and B by s

- In frame S, this interval is made up from just time and so s=2L

  (Using ‘light time’ where we measure time by the distance light travels in the time under consideration.)

- In frame S’, the time t’ between A and B is given by

$$t' = \frac{\text{distance}}{\text{speed}} = \frac{2\sqrt{L^2 + \left(\frac{d}{2}\right)^2}}{c}$$

- Plugging L=s/2 into the above equation and squaring both sides yields

$$t'^2 = \frac{s^2 + d^2}{c^2}$$

$$\boxed{\therefore s^2 = (ct')^2 - d^2}$$



- This is the metric equation for special relativity

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# The metric of space and time

- Imagine that we would like to calculate the distance between points A and B on the map below.

- Use the Pythagorean theorem $d^2 = \Delta x^2 + \Delta y^2$ (nelgecting the earth's curvature)

This is called a *metric equation*

*metric* = technical term for distance

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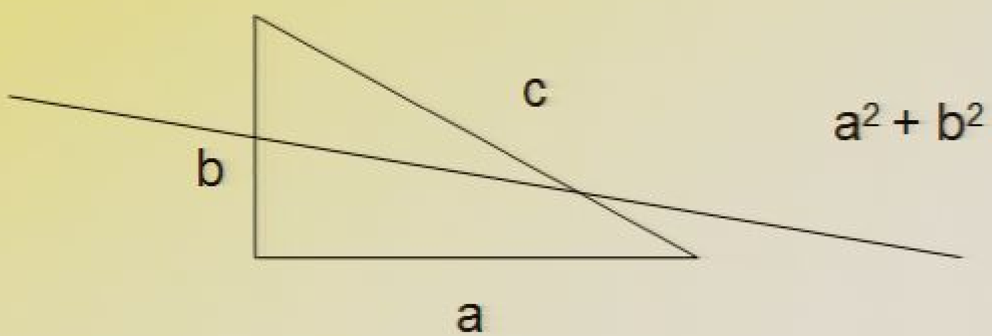
# What does the metric equation mean? Why the minus sign?

- Might have expected $s^2 = x^2 + (ct)^2$ but this does not work as the right-hand side varies for different observers. A metric is the same for all observers.

- The minus sign means that space and time are connected to each other in an unexpected manner.

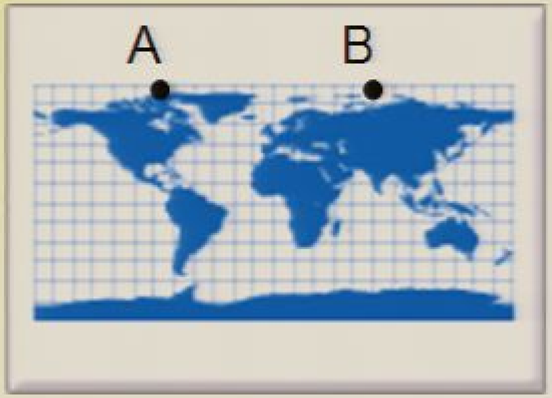
- Spacetime is *not* Pythagorean or Euclidean.




$a^2 + b^2 \neq c^2$

# What does the metric equation mean? cont.

- What are the implications of this?
- Consider the following example.
  Look at the Platte Carre map of the world:



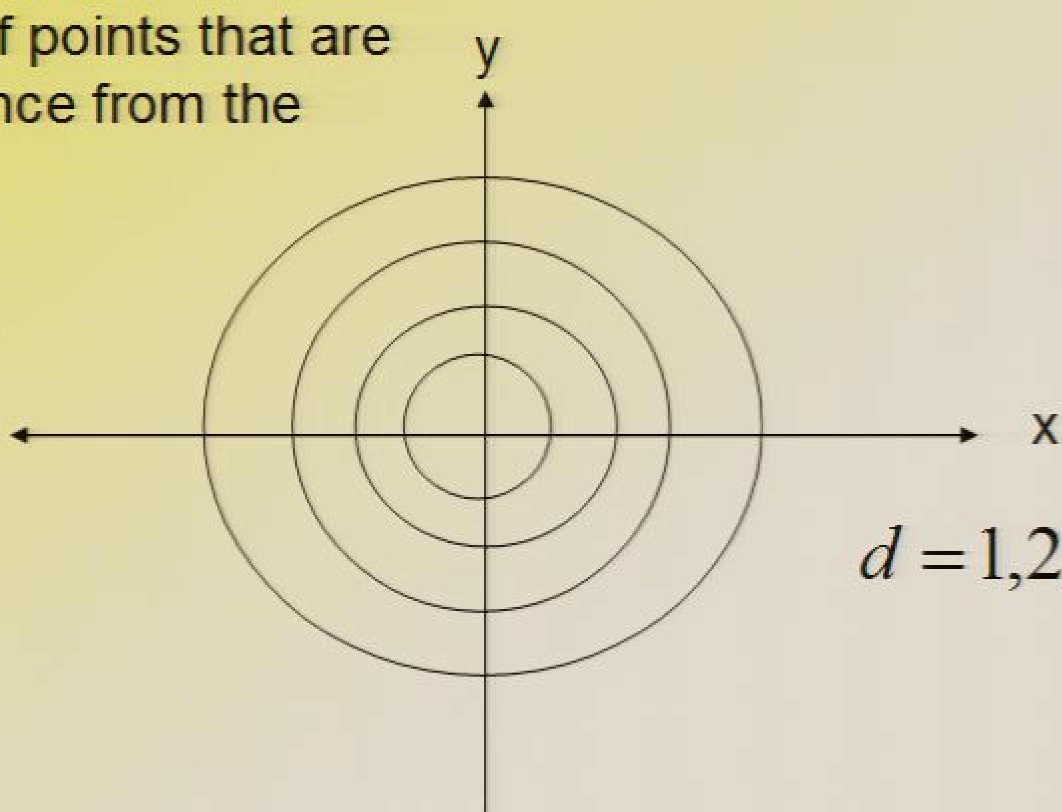

- We cannot interpret it naively. Eg. distance between A and B is actually zero as they are both at the North Pole
- Similarly, we cannot simply measure the spacetime distance or interval s between two events by simply measuring the distance between them on a spacetime diagram using a ruler.

- So, what is spacetime like then?

$$d = \sqrt{x^2 + y^2}$$

- For Euclidean x-y space, let us plot the locus of points that are the same distance from the origin.

- eg.

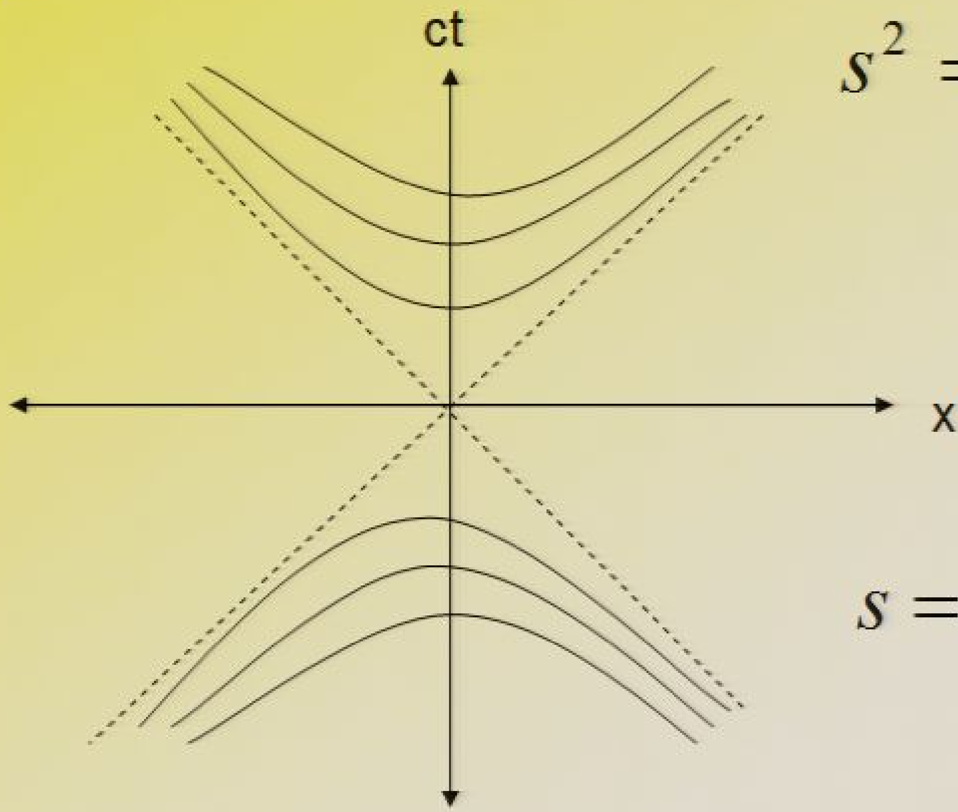



$$d = 1,2,3,4$$

- Let us do the analogous thing for spacetime

hyperbola

$$s^2 = ct^2 - x^2$$

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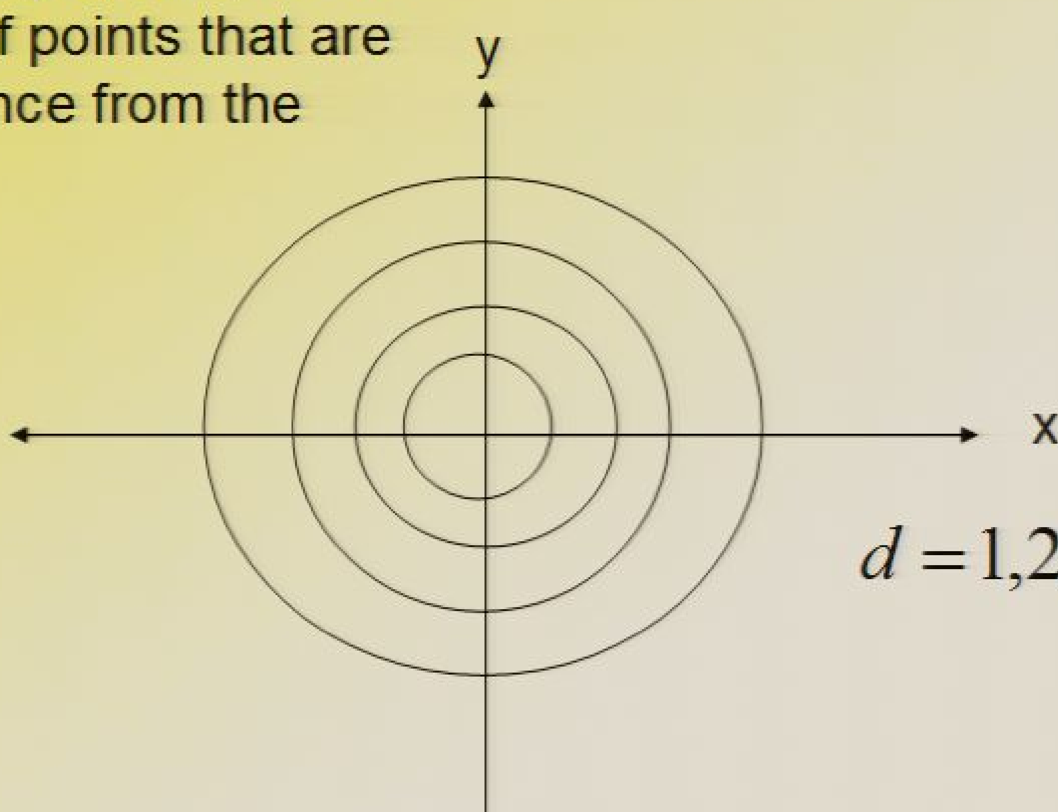


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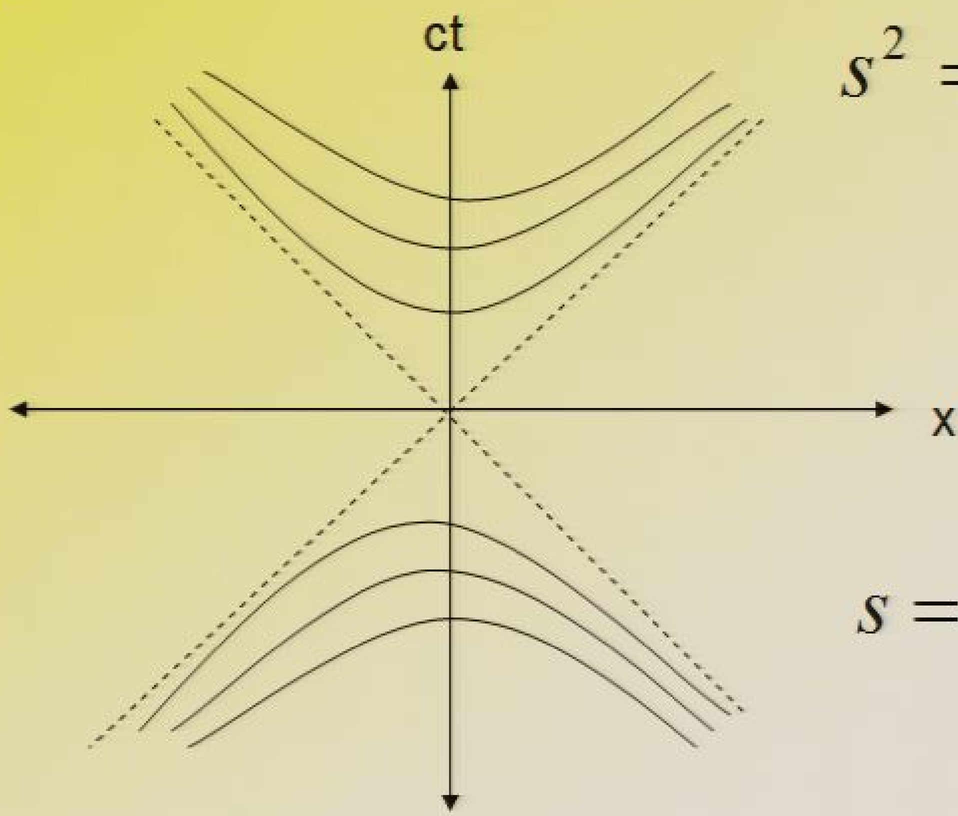


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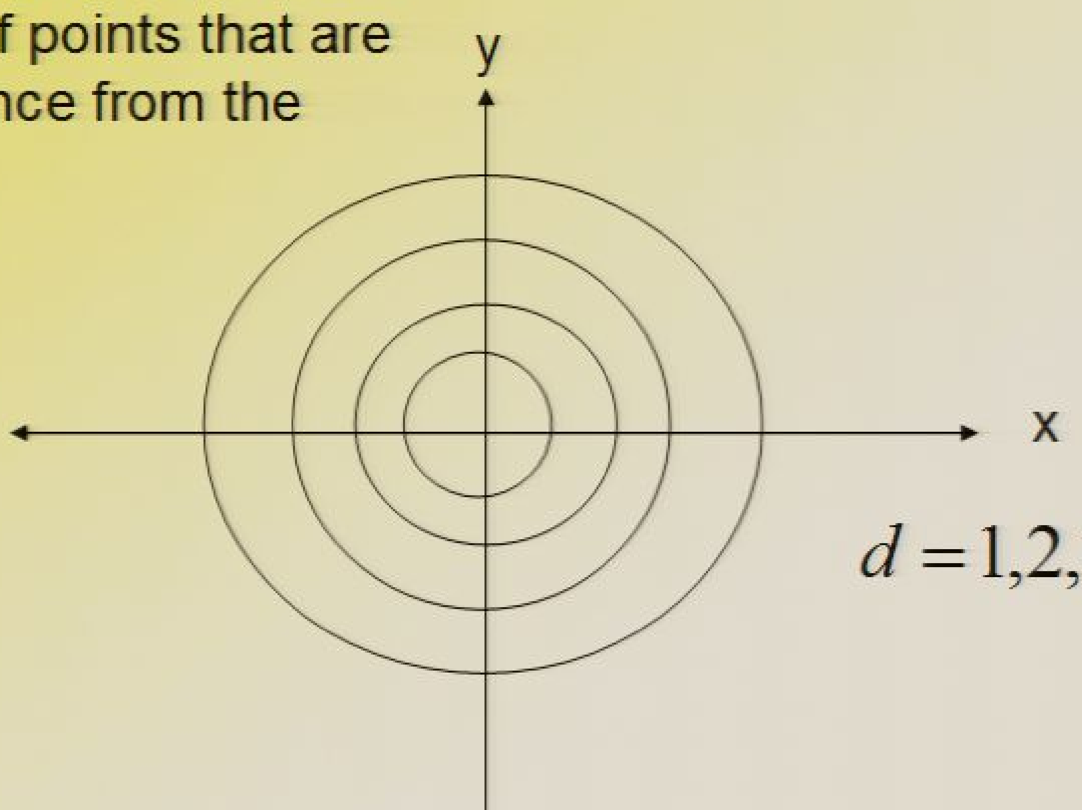


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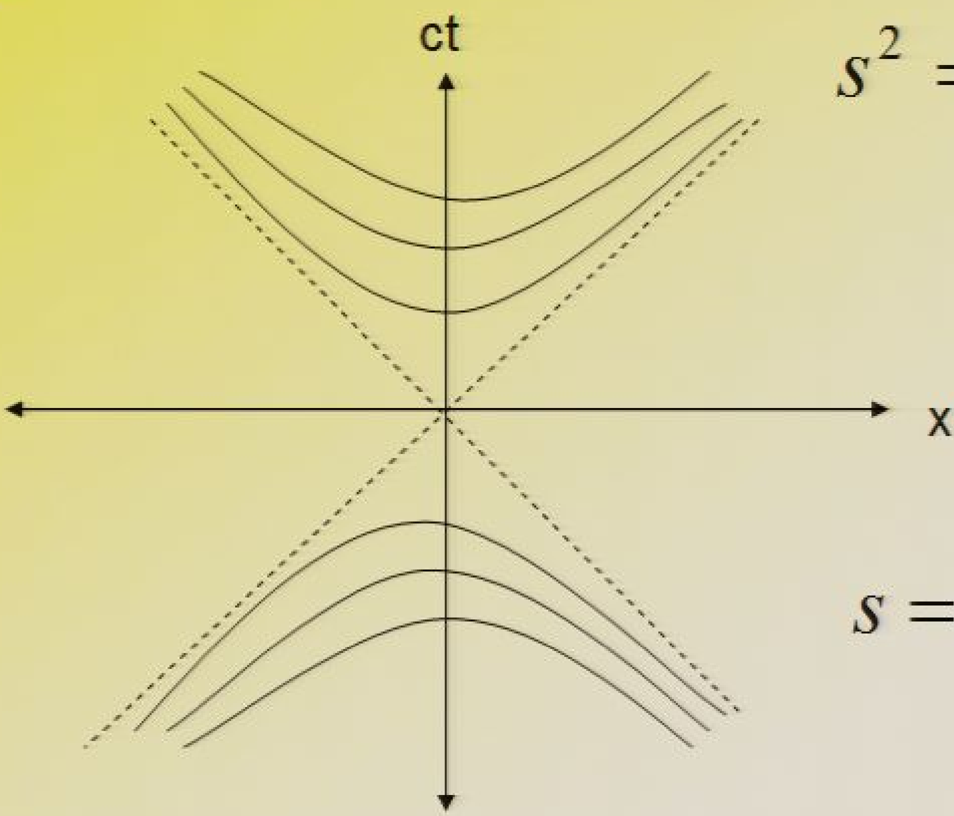


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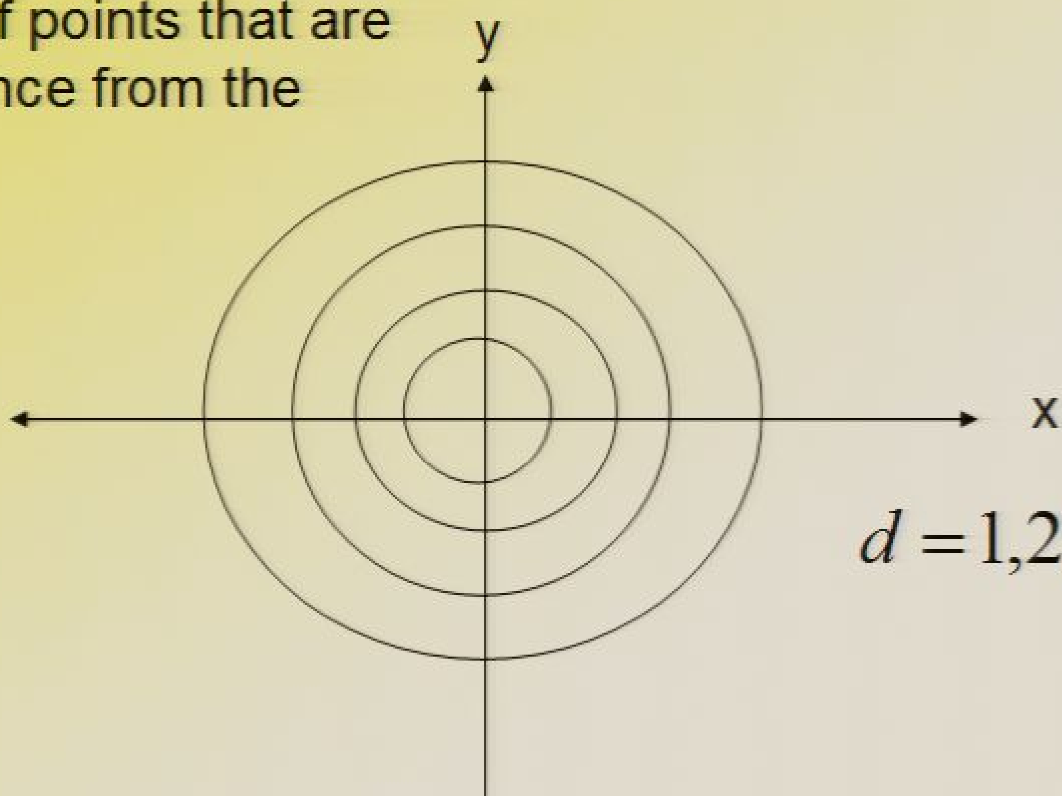


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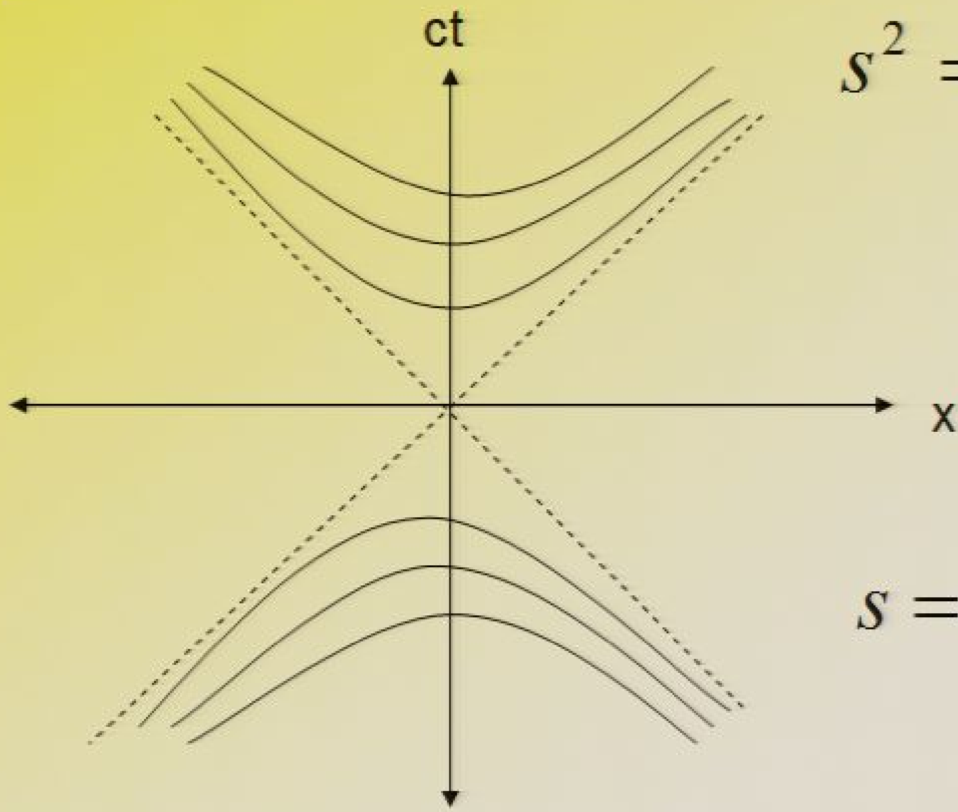


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- Spacetime is based on *hyperbolae* not circles.
- Called *Lorentz space*

# Latest news on the nature of spacetime

- Today, scientists have even more unusual ideas about space and time

- **String theory:** Space only defined down to the Planck scale. There is a smallest possible distance, the Planck length, which is $10^{-35}$ metres. This limitation stops the theory from making nonsensical predictions when it combines general relativity with quantum theory.

- Extra dimensions: String theory says that there are nine dimensions of space!

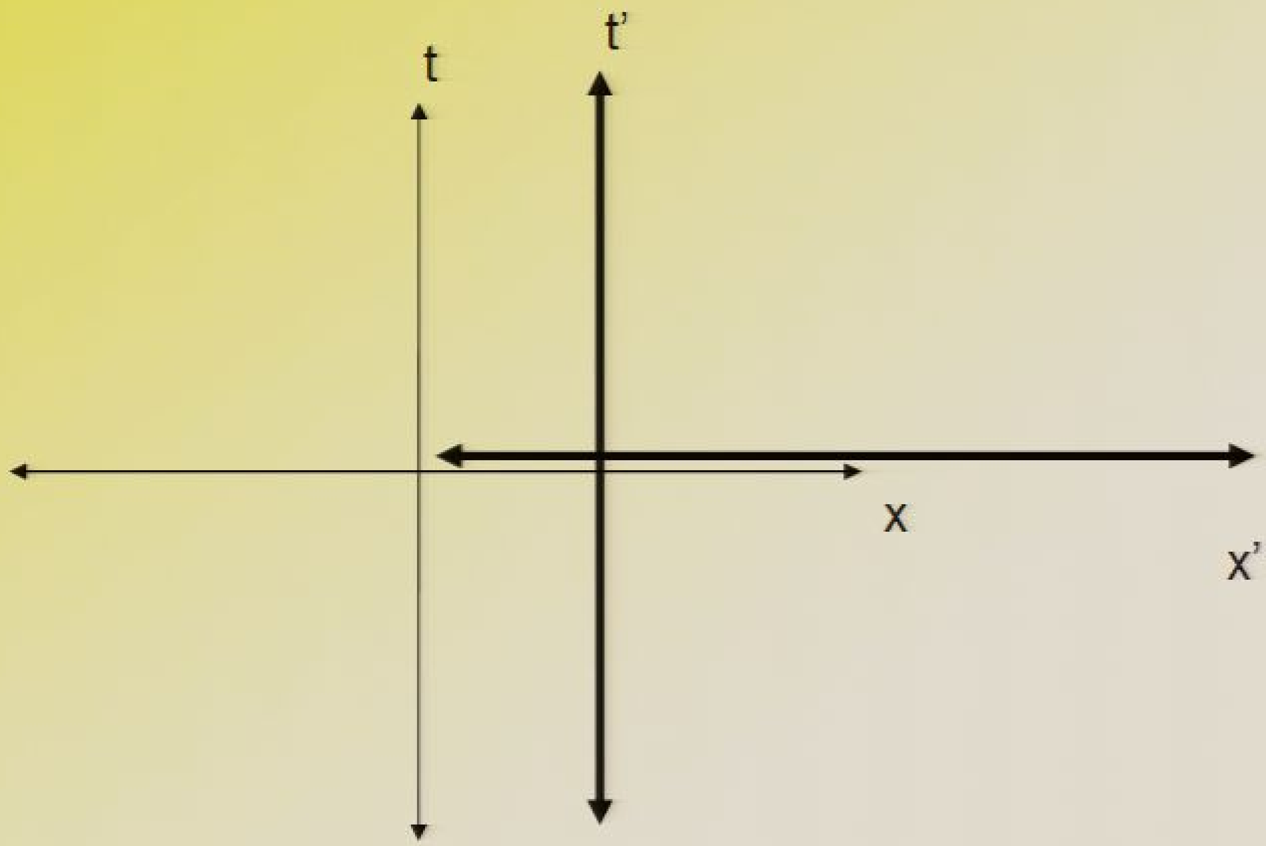
- Loop quantum gravity. Space is also fundamentally grainy (discrete or quantized)

- The smallest possible area is $10^{-66}$ metres squared and the smallest possible volume $10^{-99}$ metres cubed.

- For more information on string theory, talk to a string theorist or loop quantum gravity researcher over lunch during the "chat-with-a-

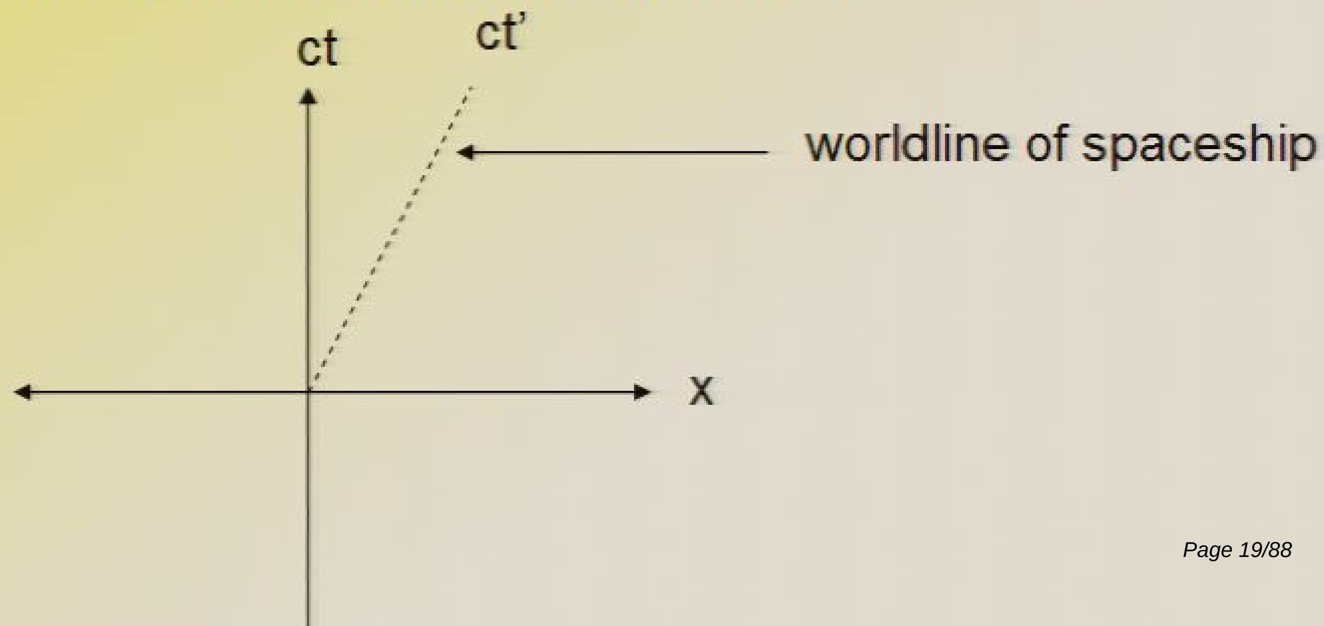
# Transforming between different frames of reference

- Galilean transformations



# Transforming between different frames of reference

- Imagine a spaceship moving at the constant velocity 0.5c past Earth
- What does its worldline look like?
- From the spaceship's perspective, it remains stationary and so all the points on its worldline have position x'=0. This defines the ct' axis just as the line x=0 defines the t axis for Earth.



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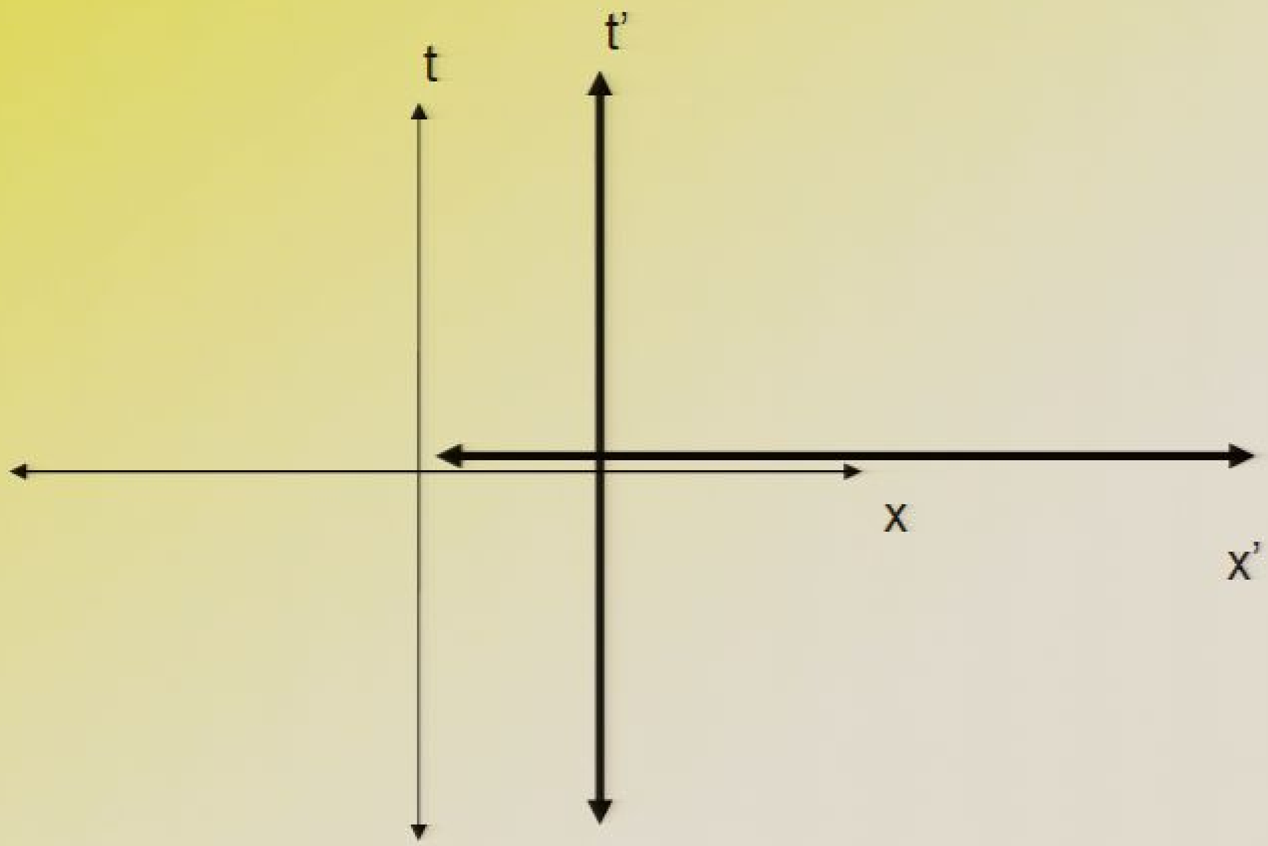

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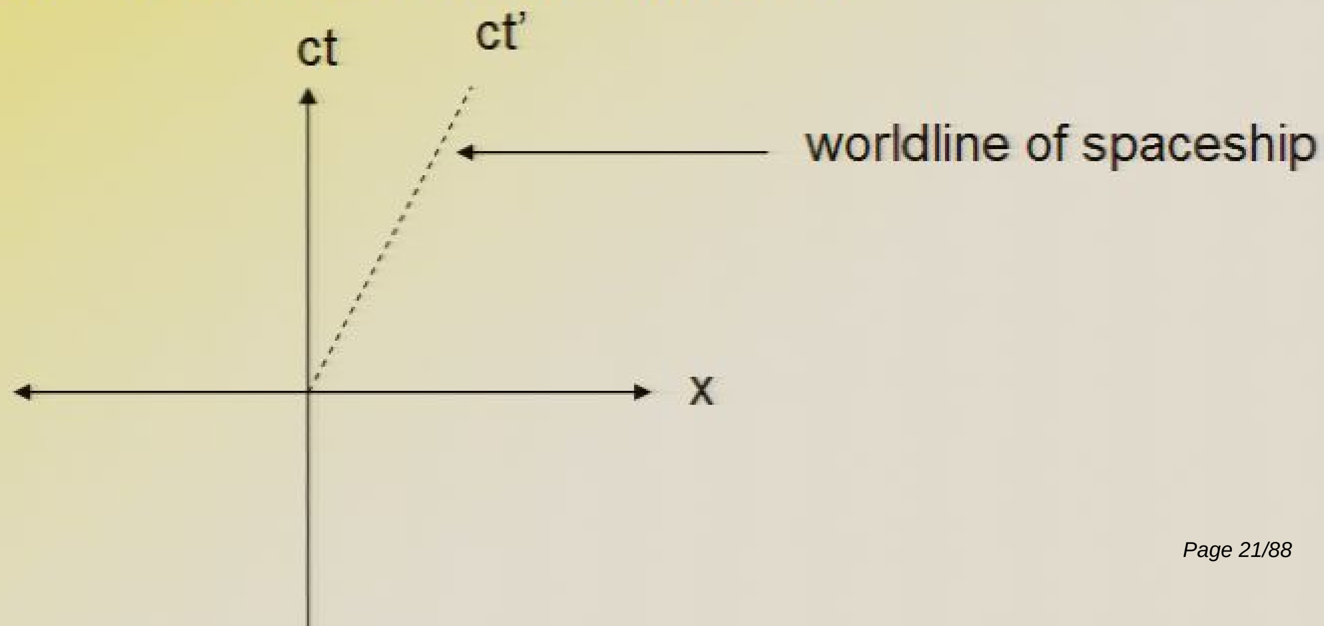

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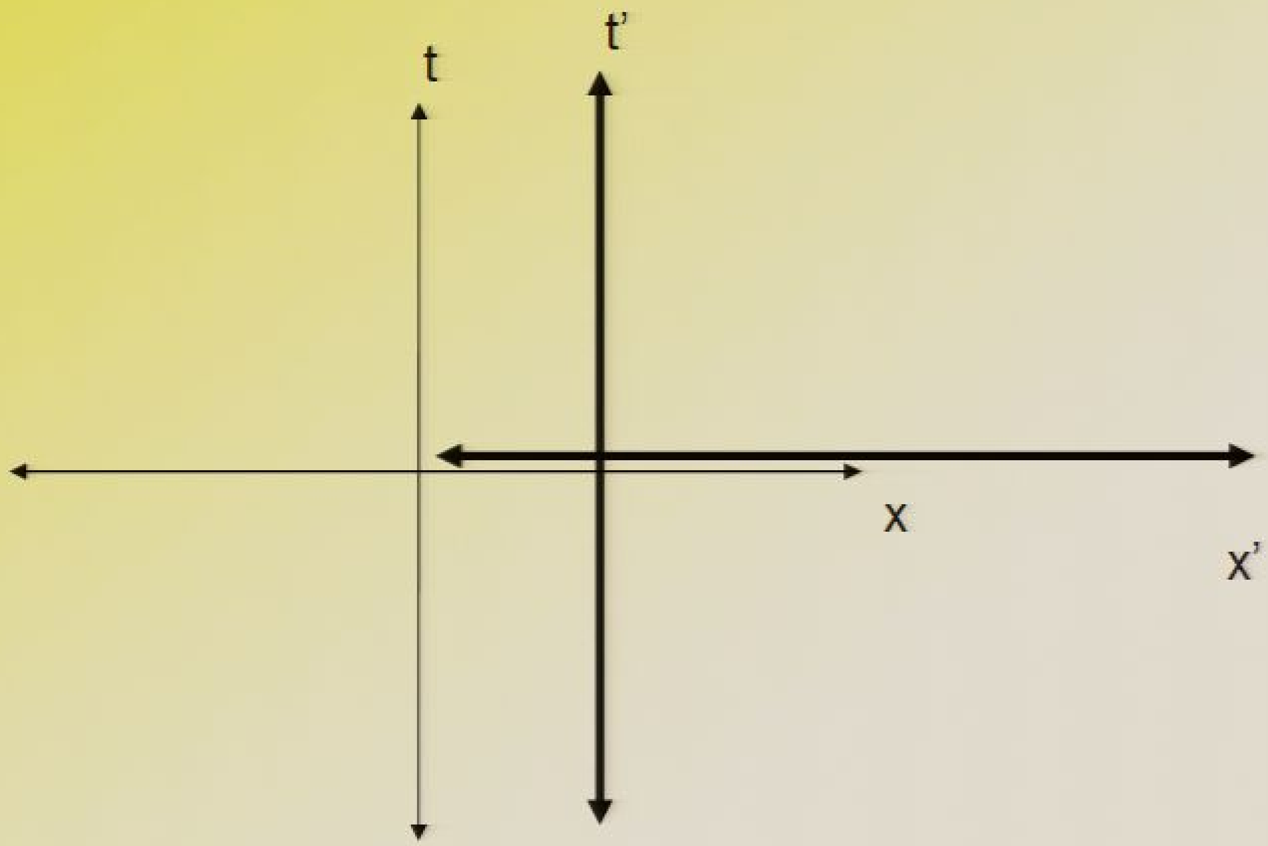

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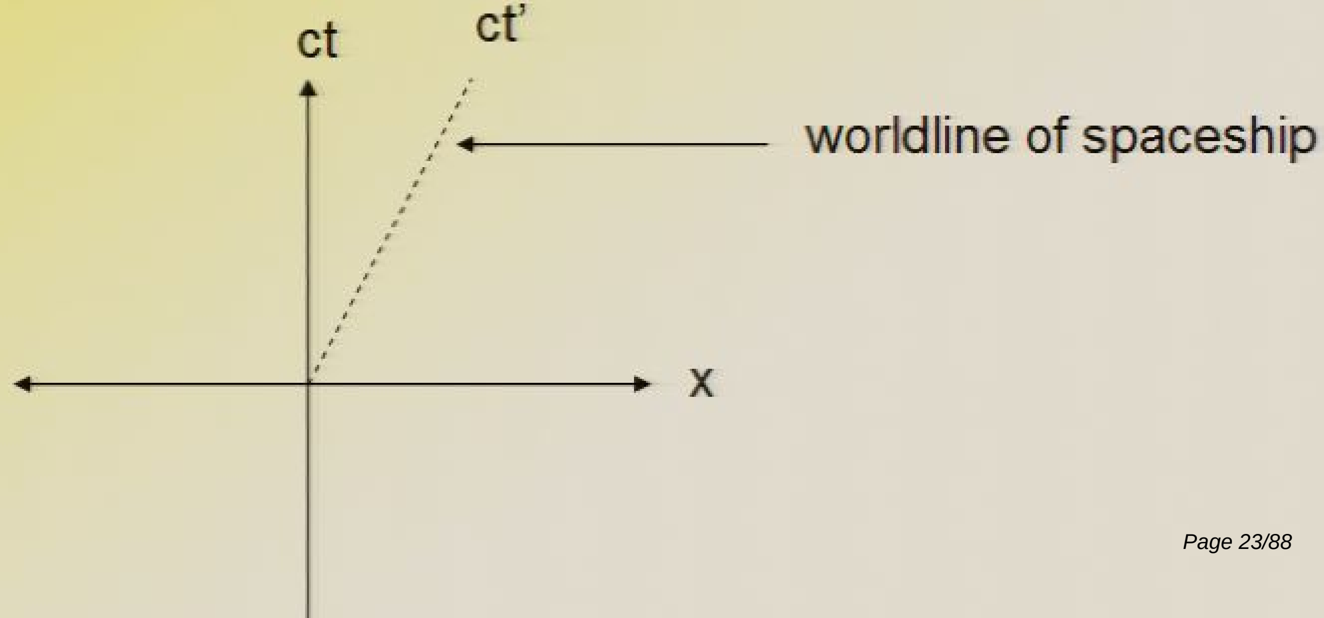

- What about the x' axis?
- In analogy with the ct' axis, it is defined by the equation t'=0.
- Let us determine it via the following procedure:

  Consider the spaceship's commander turning on a high-powered flashlight at time t'=-T. Assume that the light travels away from them in the +x direction. Once it reaches a certain point B, a mirror there reflects it back to the spaceship at time t'=T.


- As light travels at c relative to all observers, someone on Earth (E) always sees light as moving at c. This means that the worldlines for light are always at a 45 degree angle relative to the x axis.

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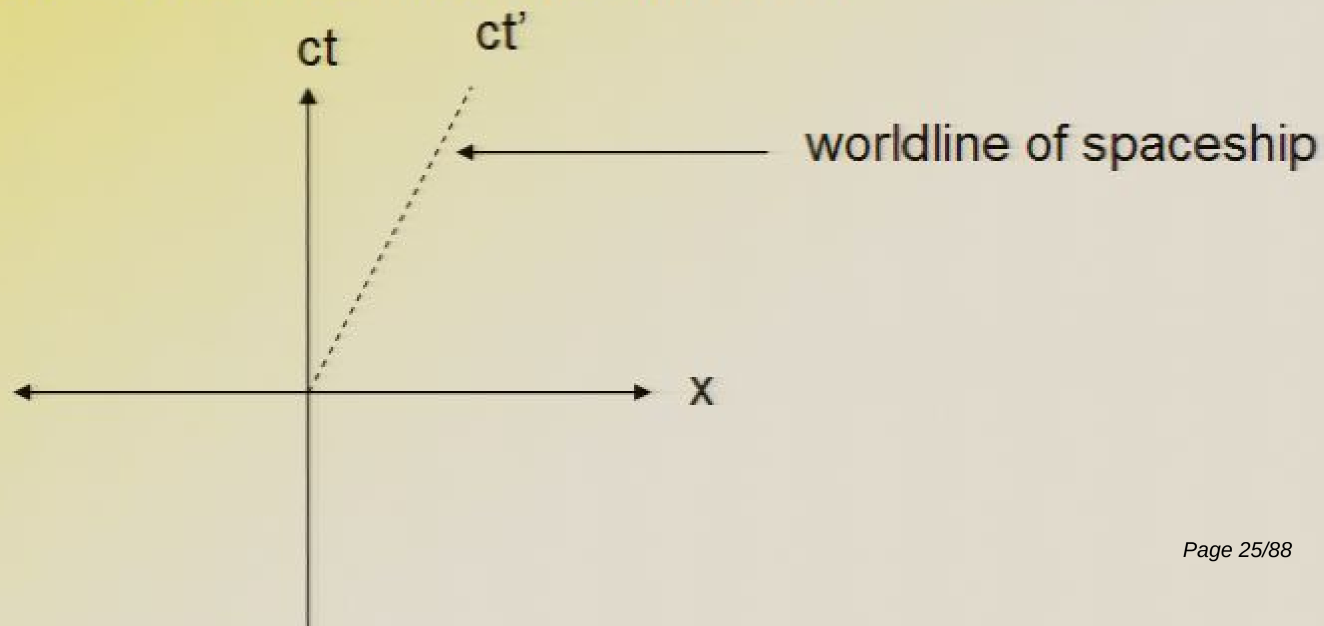

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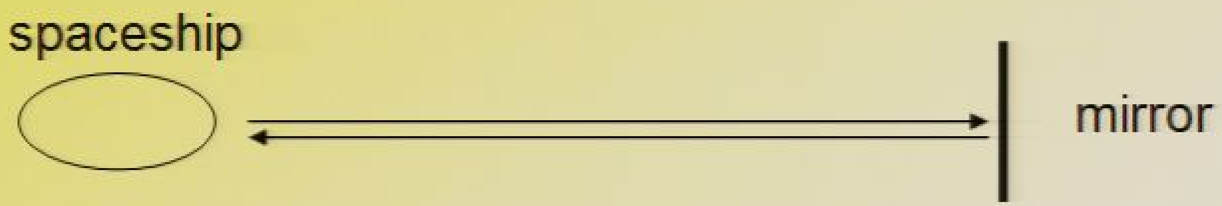


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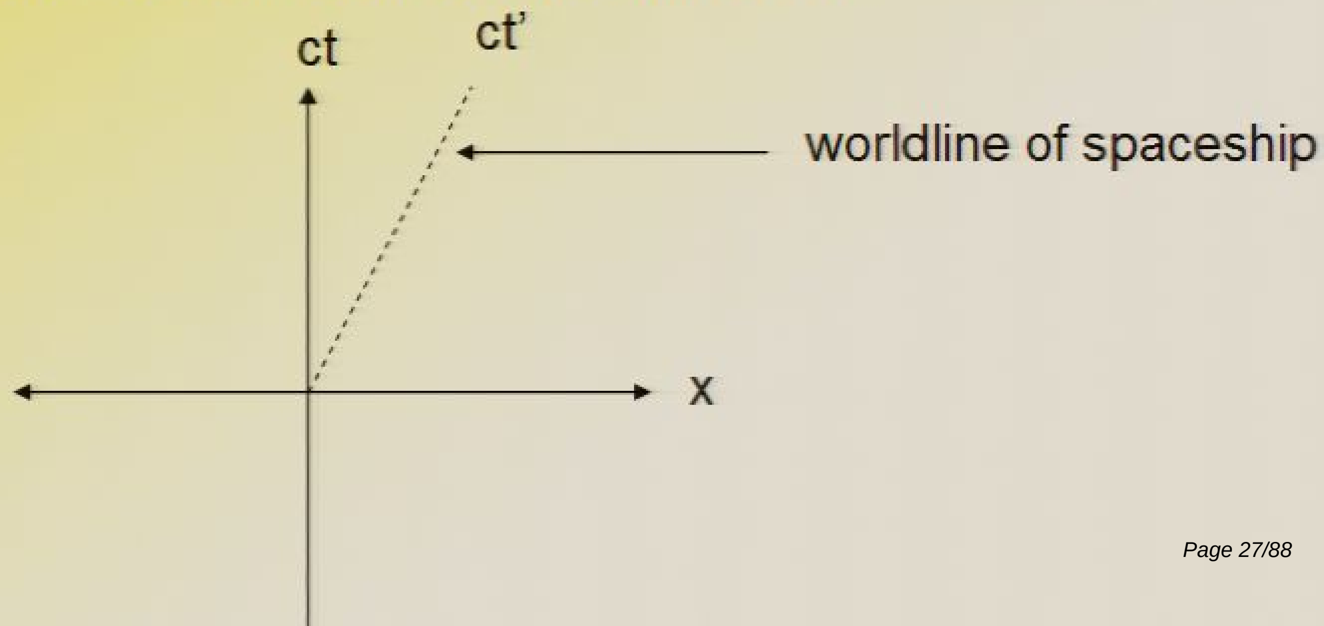

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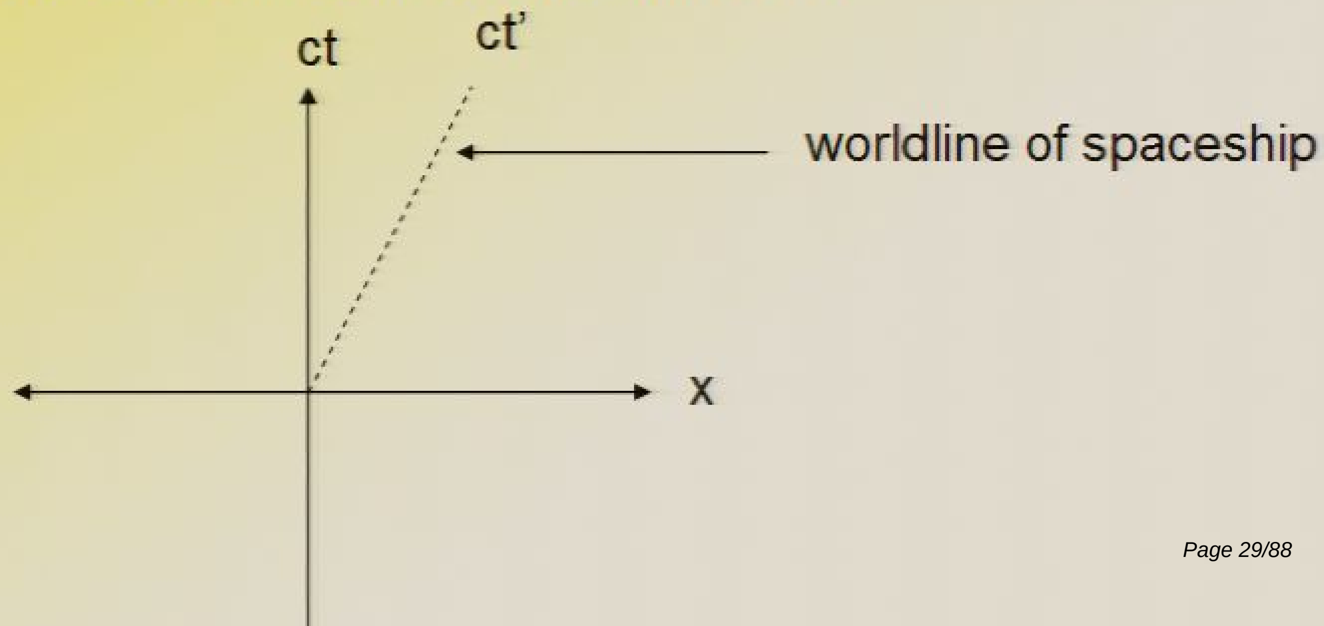

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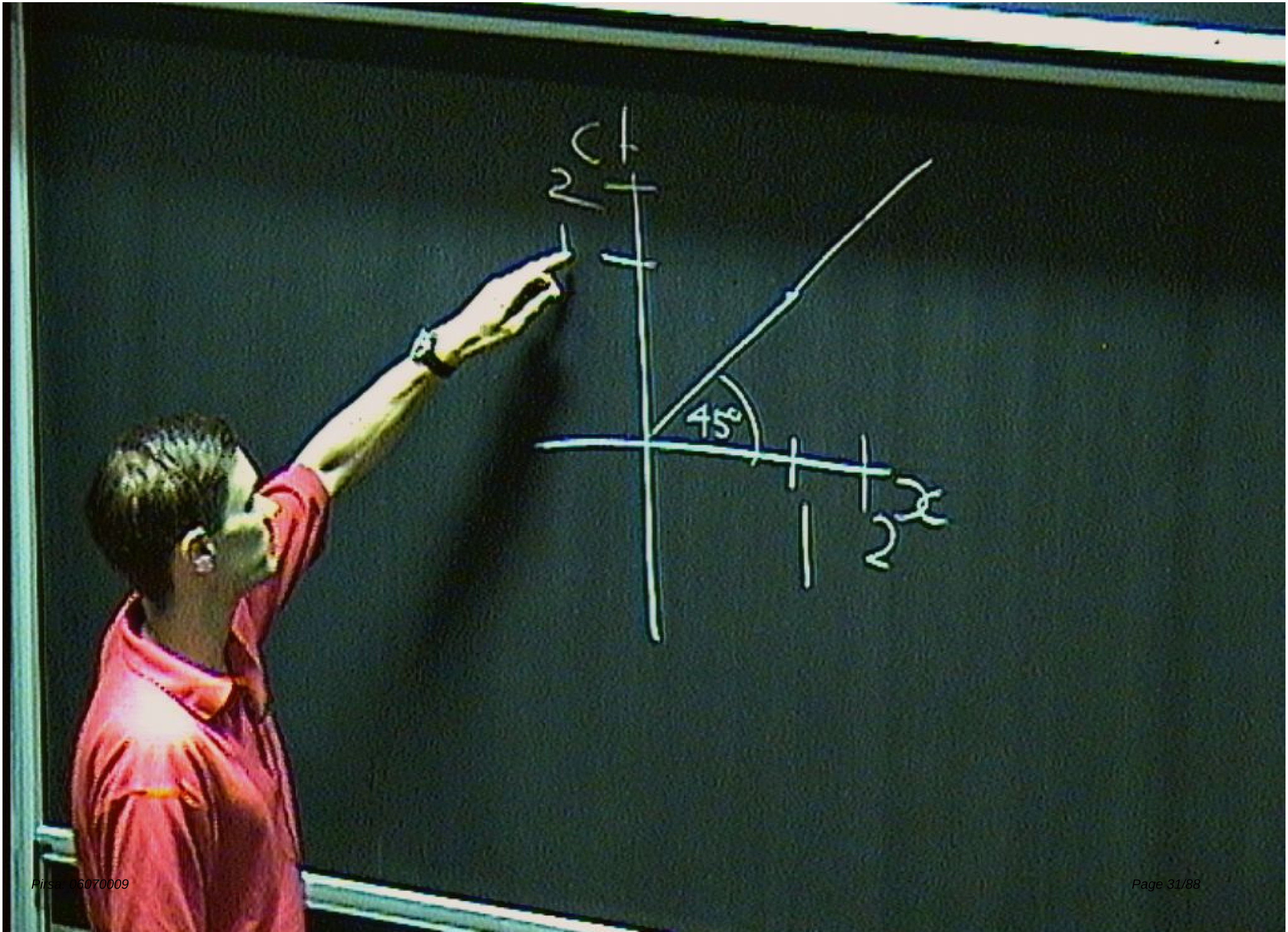
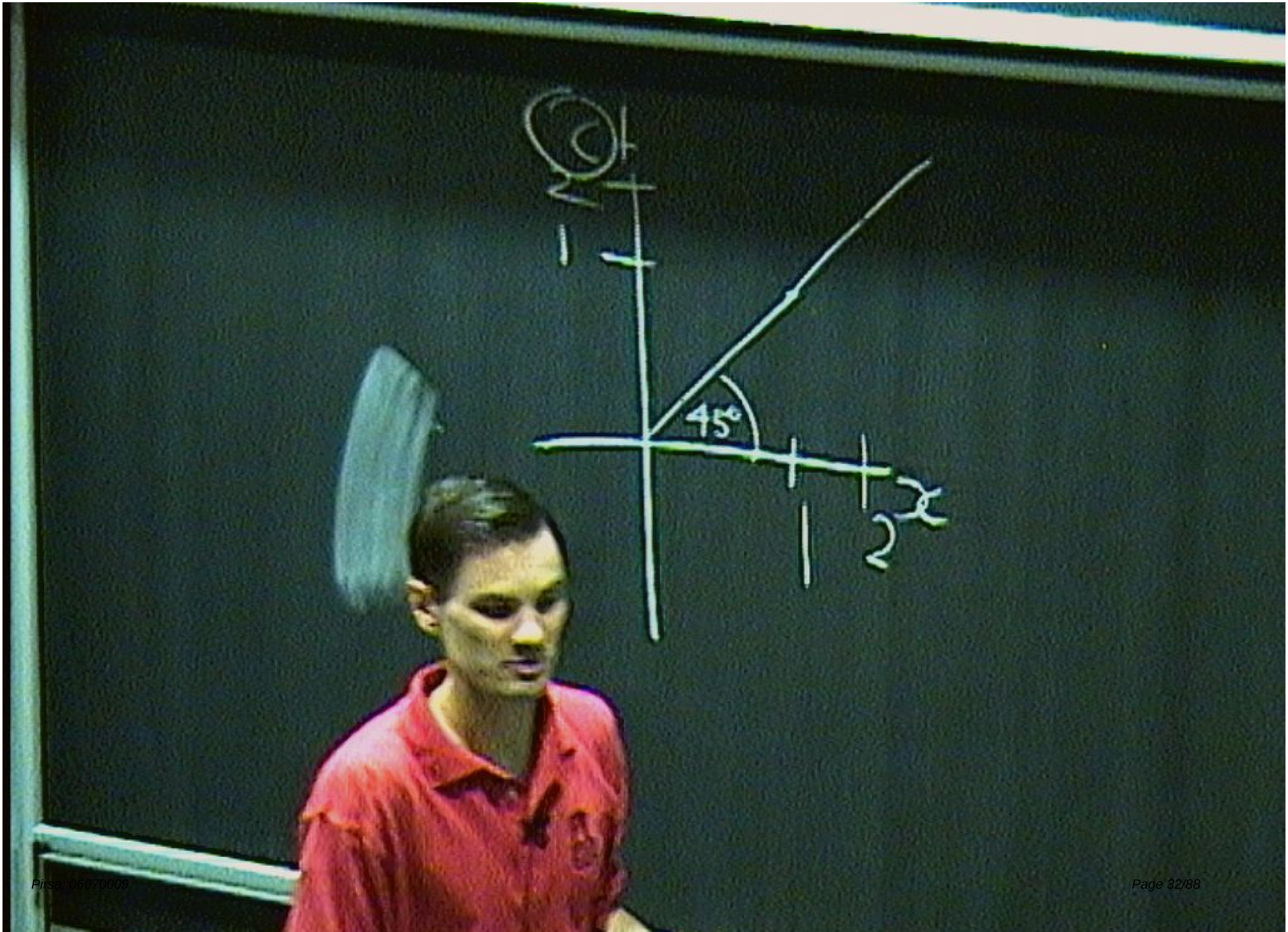

ct
2
1
45°
1
2
x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2
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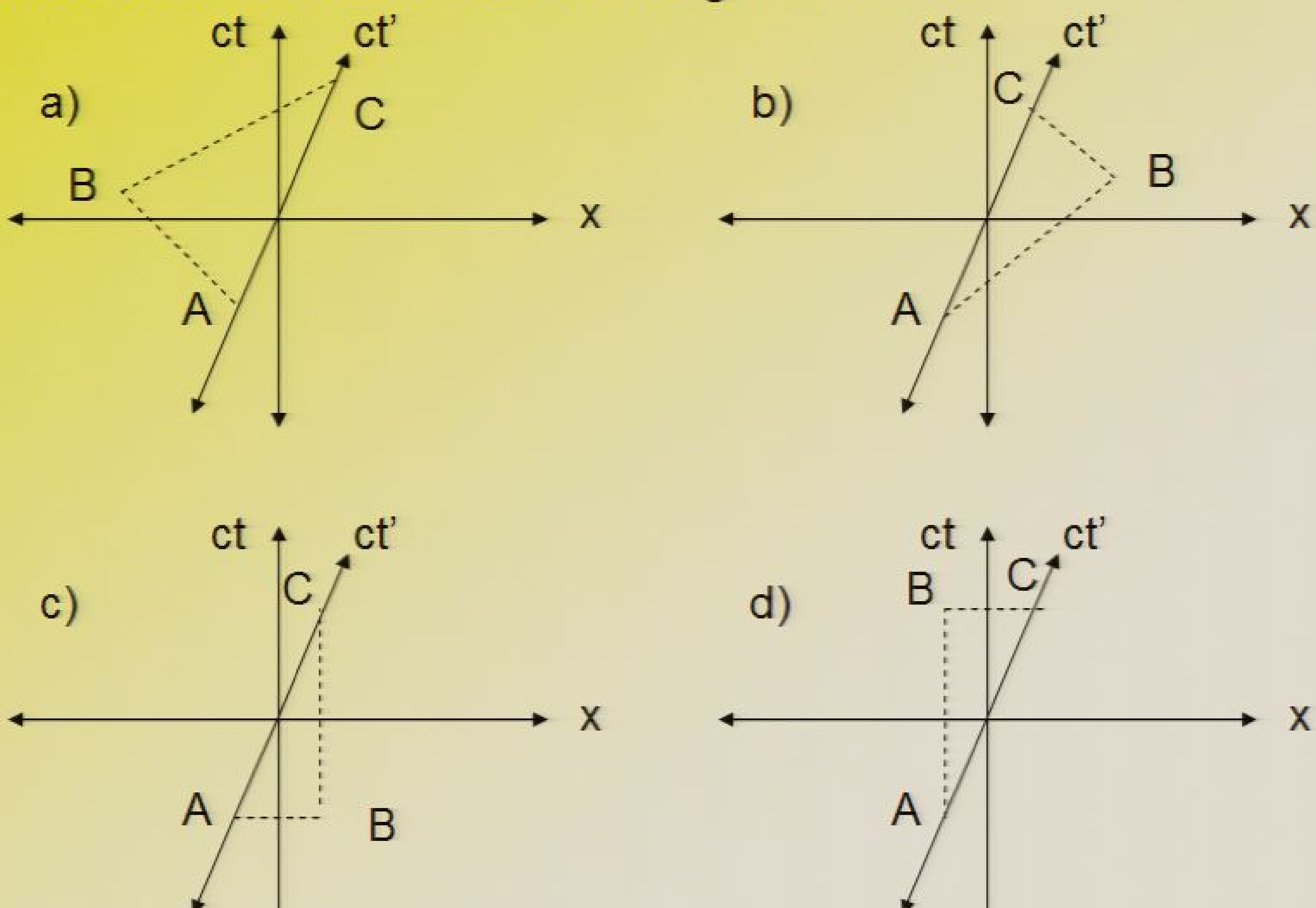

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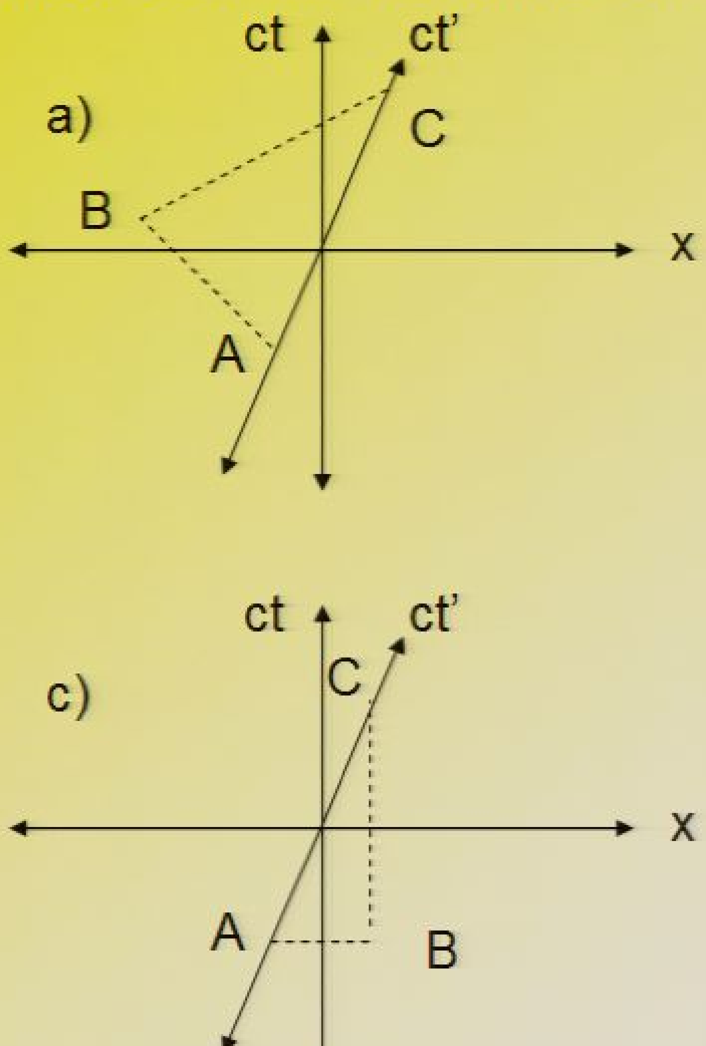

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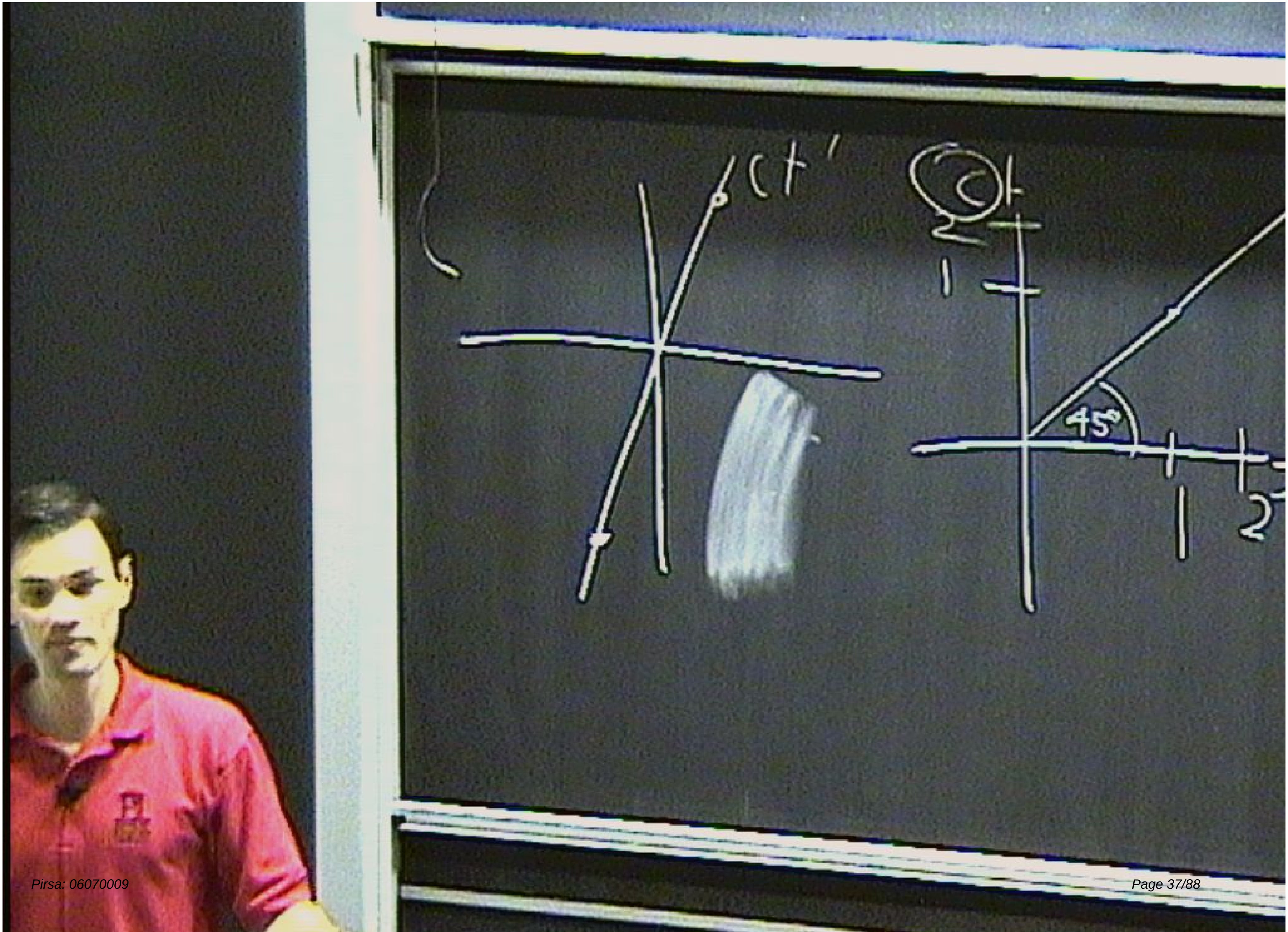
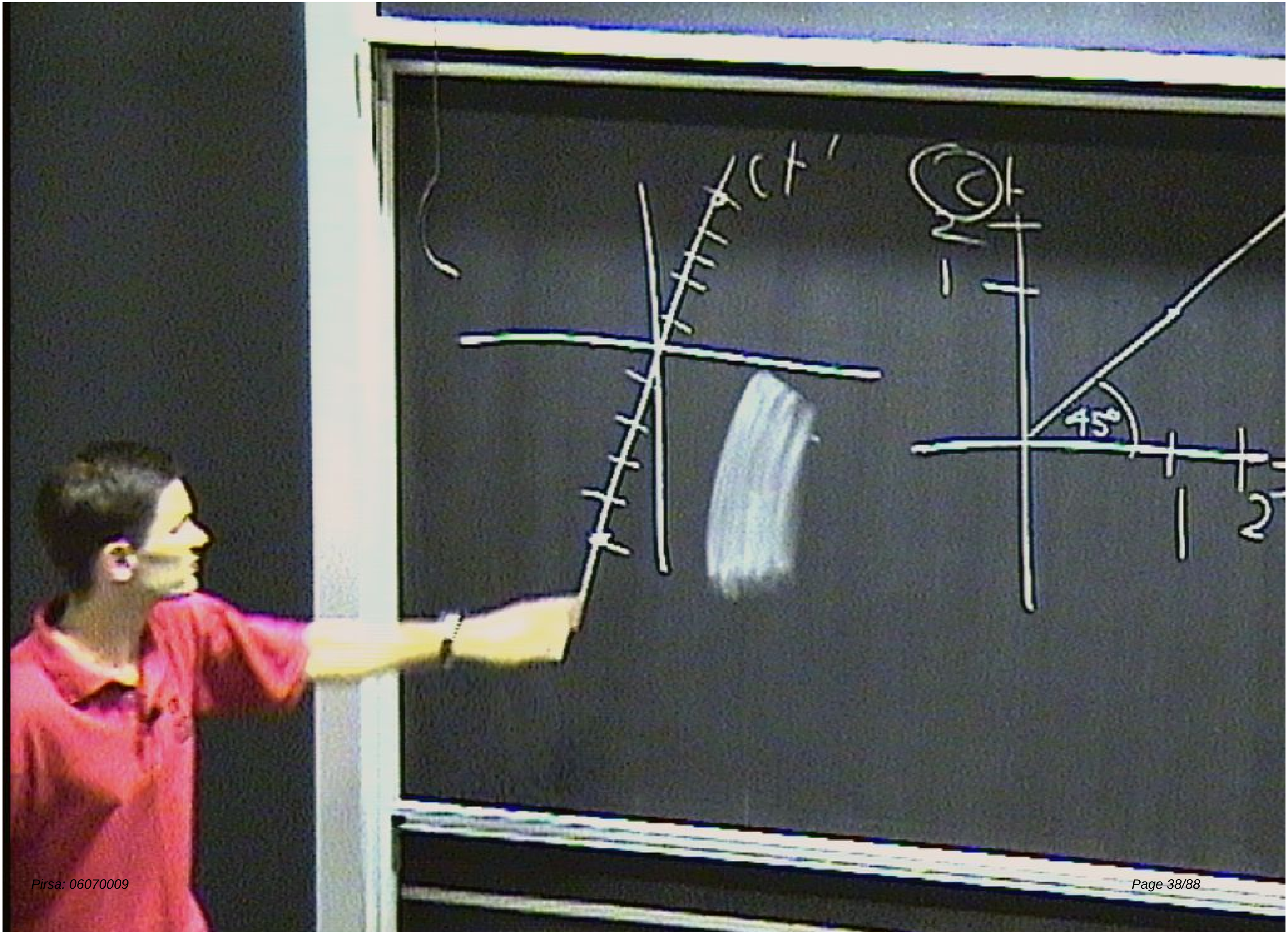
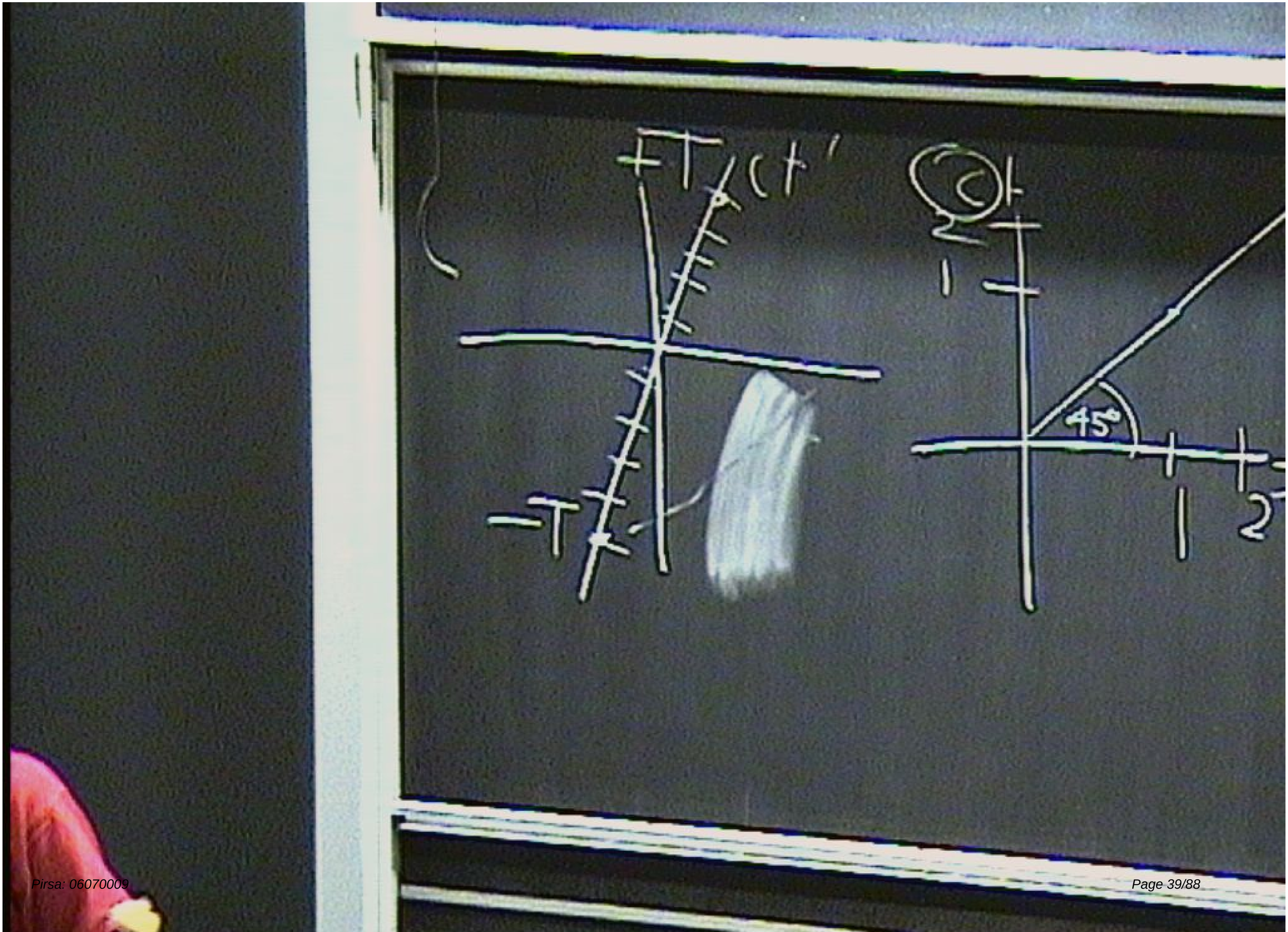
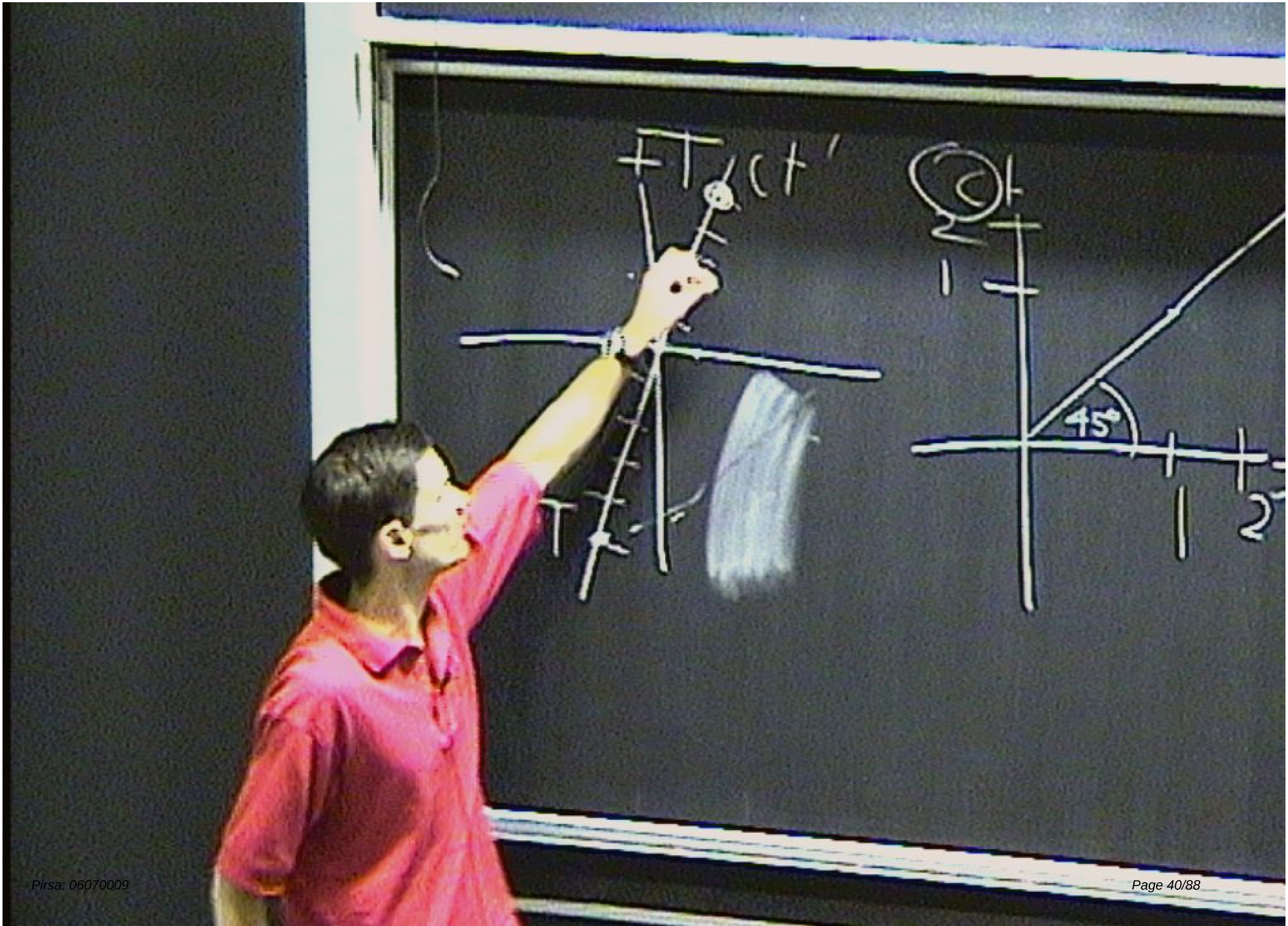

ct'
ct
2
1
45°
1
2

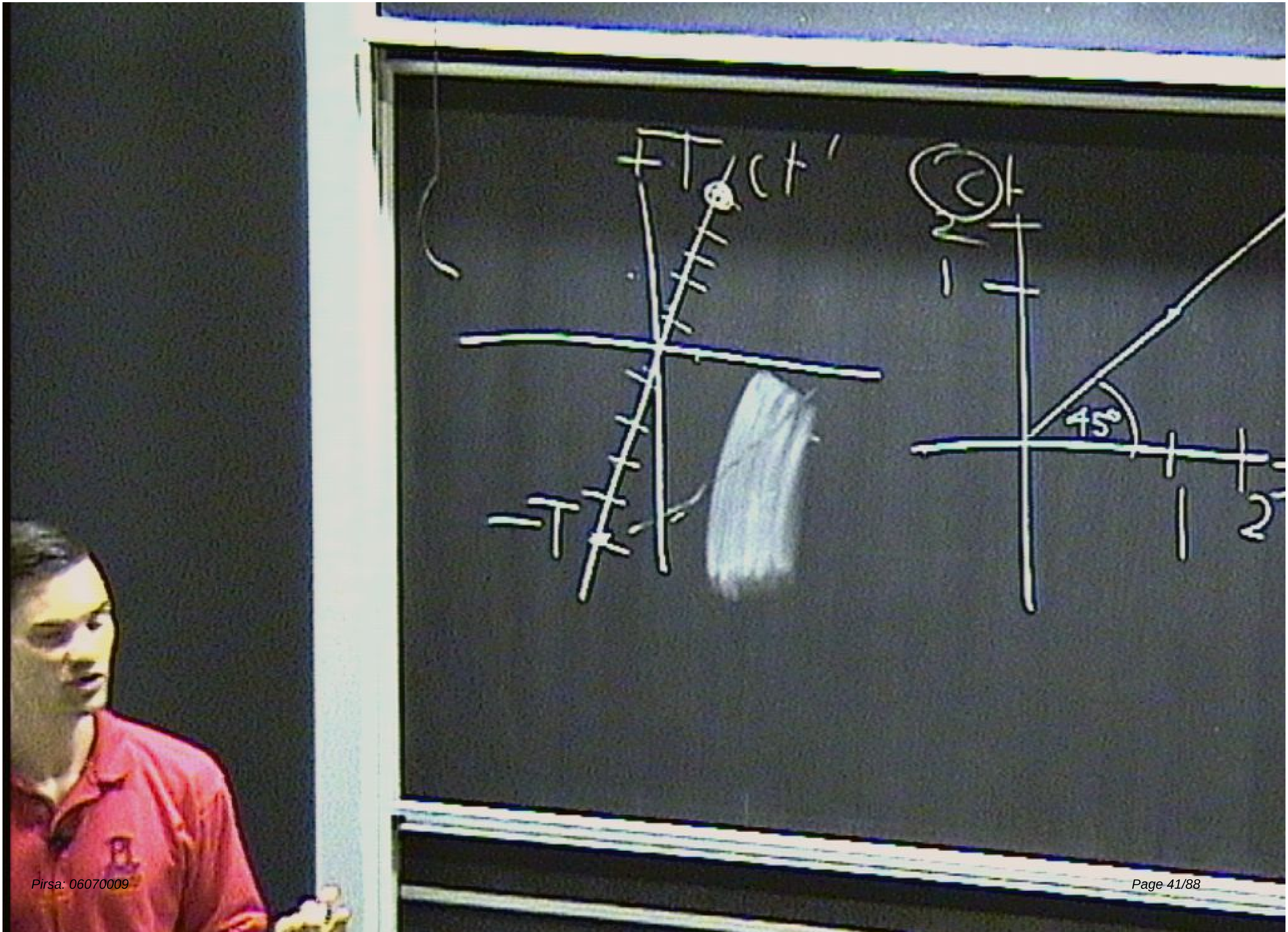


ct'
ct
2
1
45°
1
2



+T
ct'
-T
45°


ct'
ct
2
1
45°
1
2


T
ct'
-T
ct
2
1
45°
1
2

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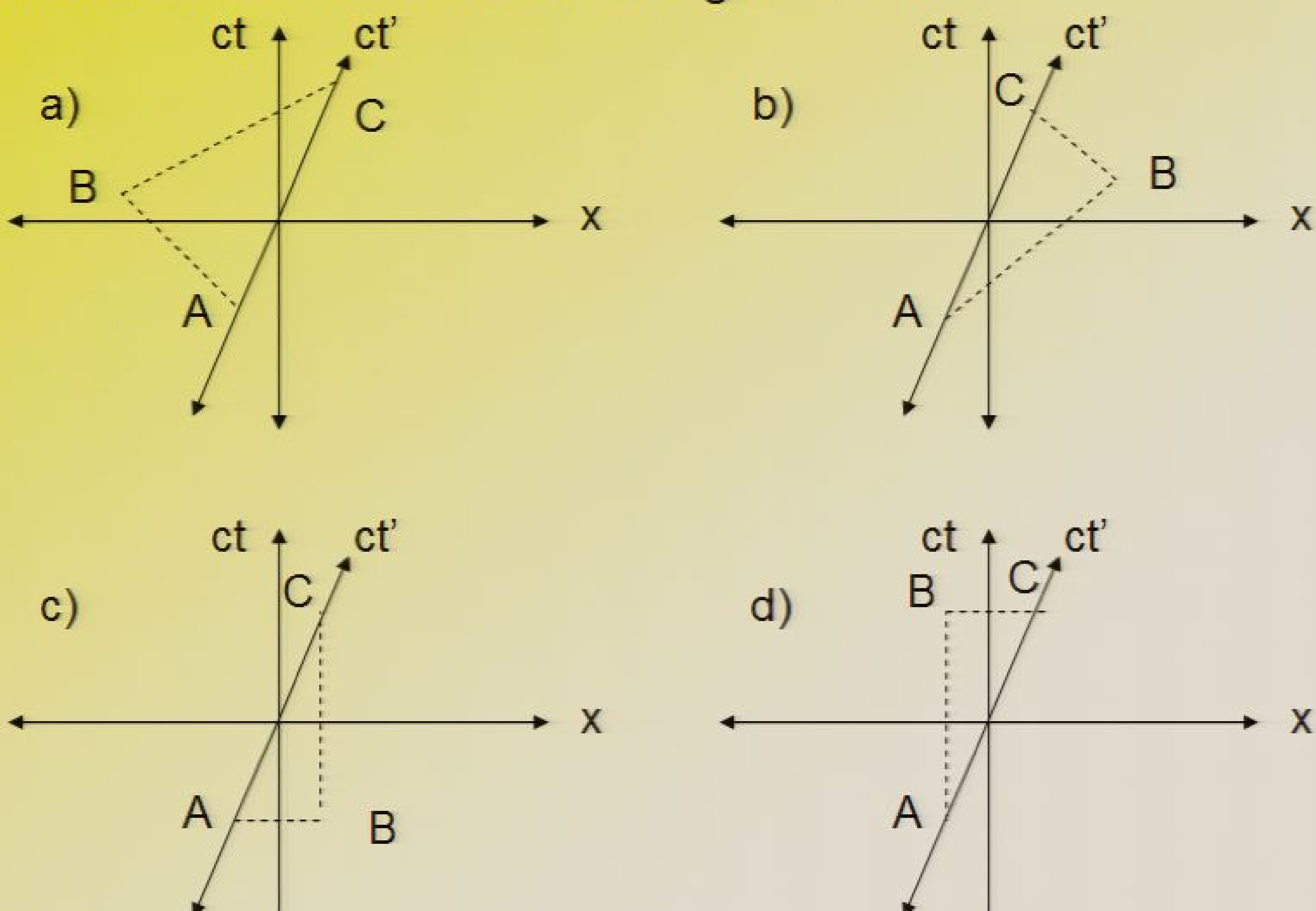
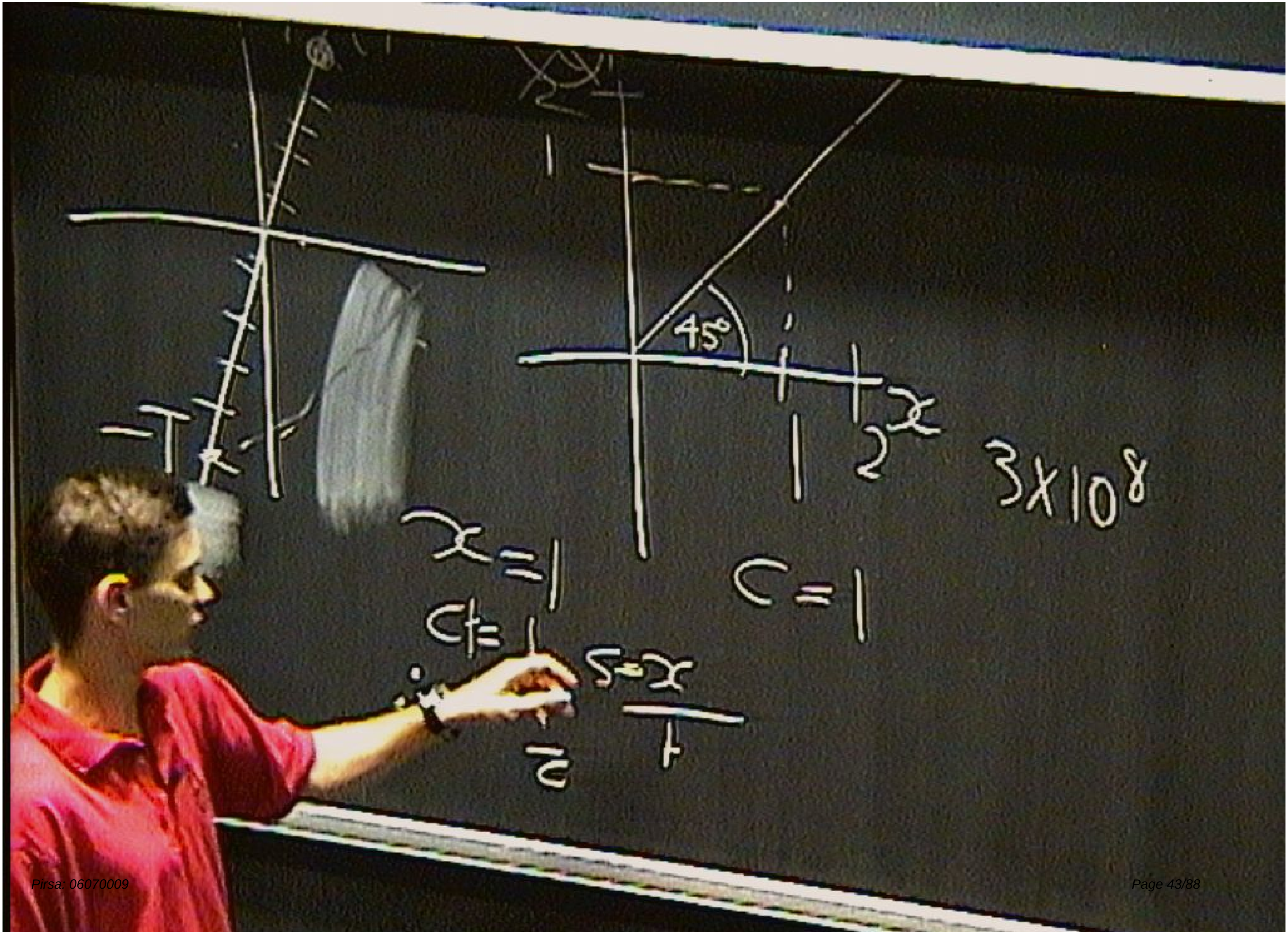
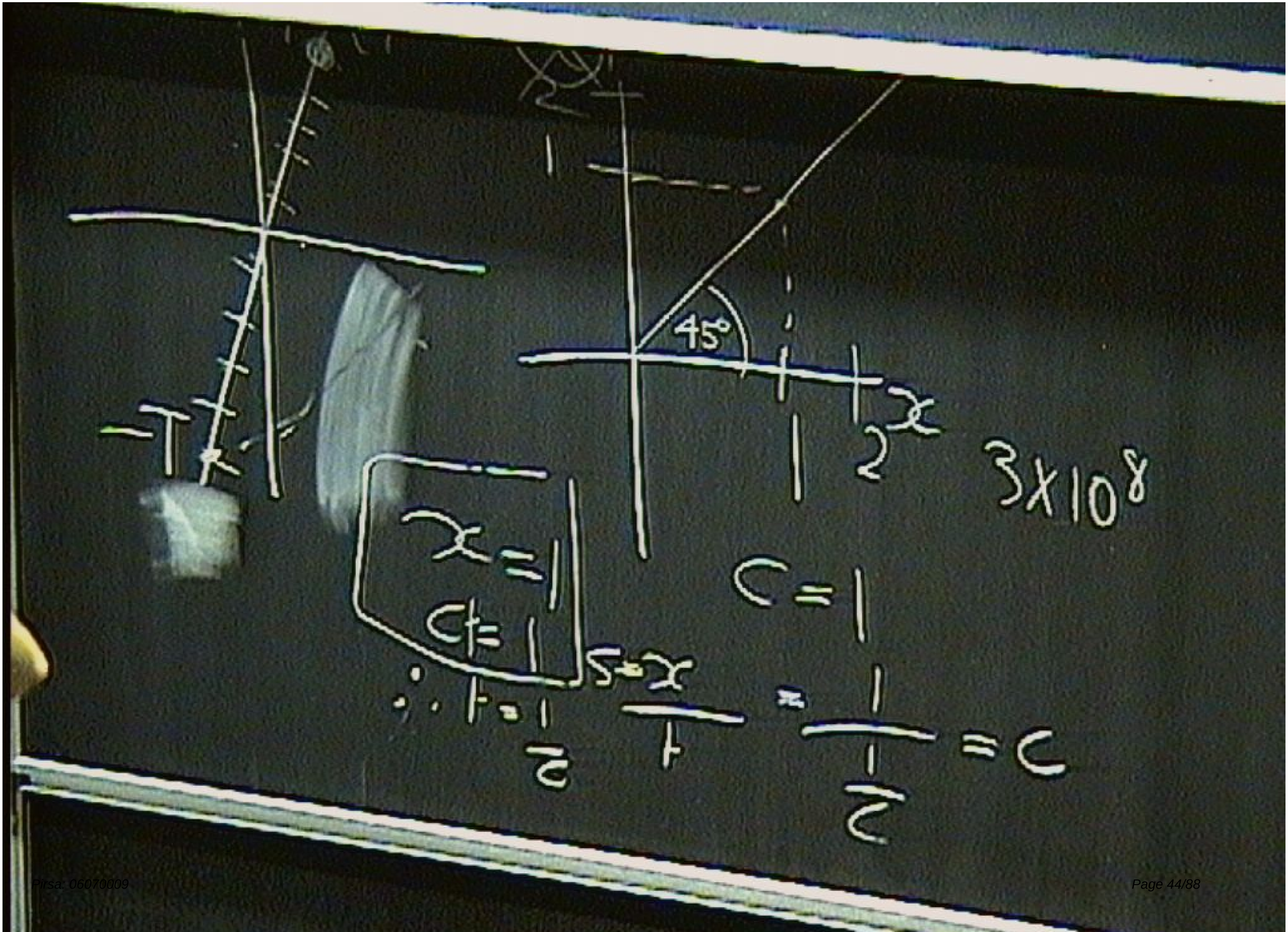
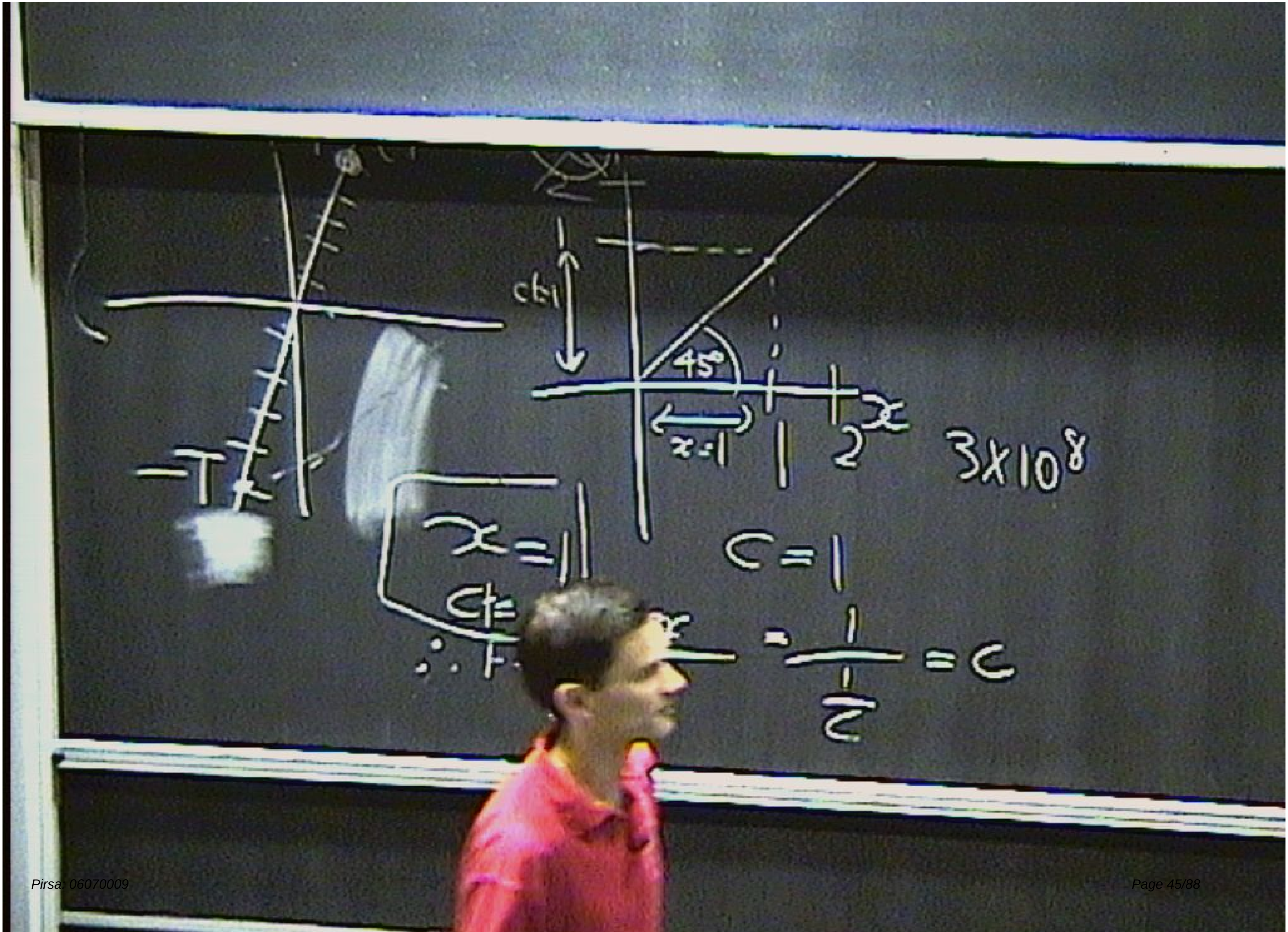



45°
x
1
2
3x10^8
x=1
c=1
ct=1
s=x
t


-T
1
45°
1
2
x
3x10^8
x=1
c=1
ct=1
s=x/t = 1/(1/2) = c
∴ t=1/2


ct=1
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x=1
1
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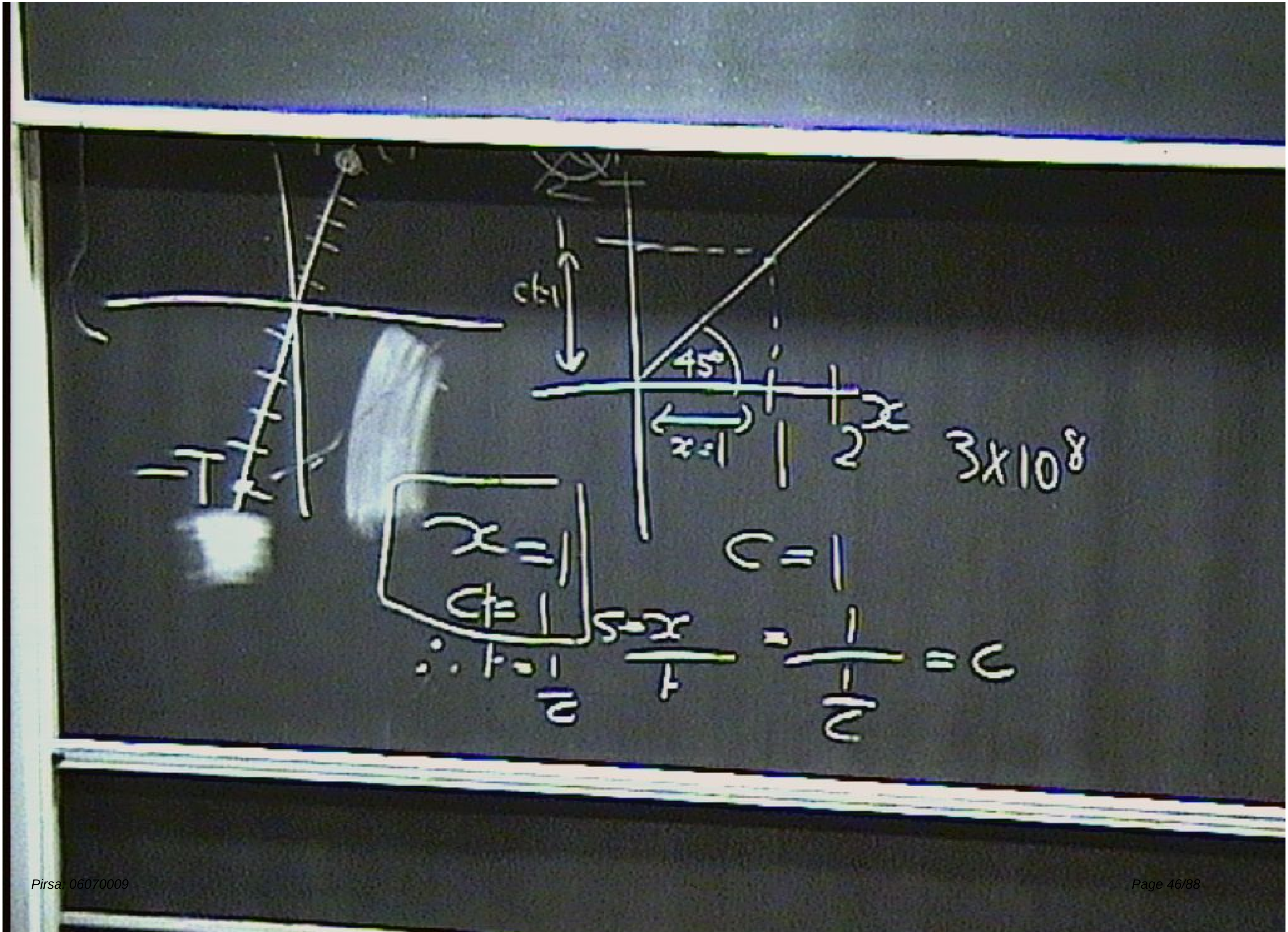
x=1
c=1
ct=
-T


ct=1
45°
x=1
1
2
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-T
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∴ t=1/c
s=x/t = 1/(1/c) = c

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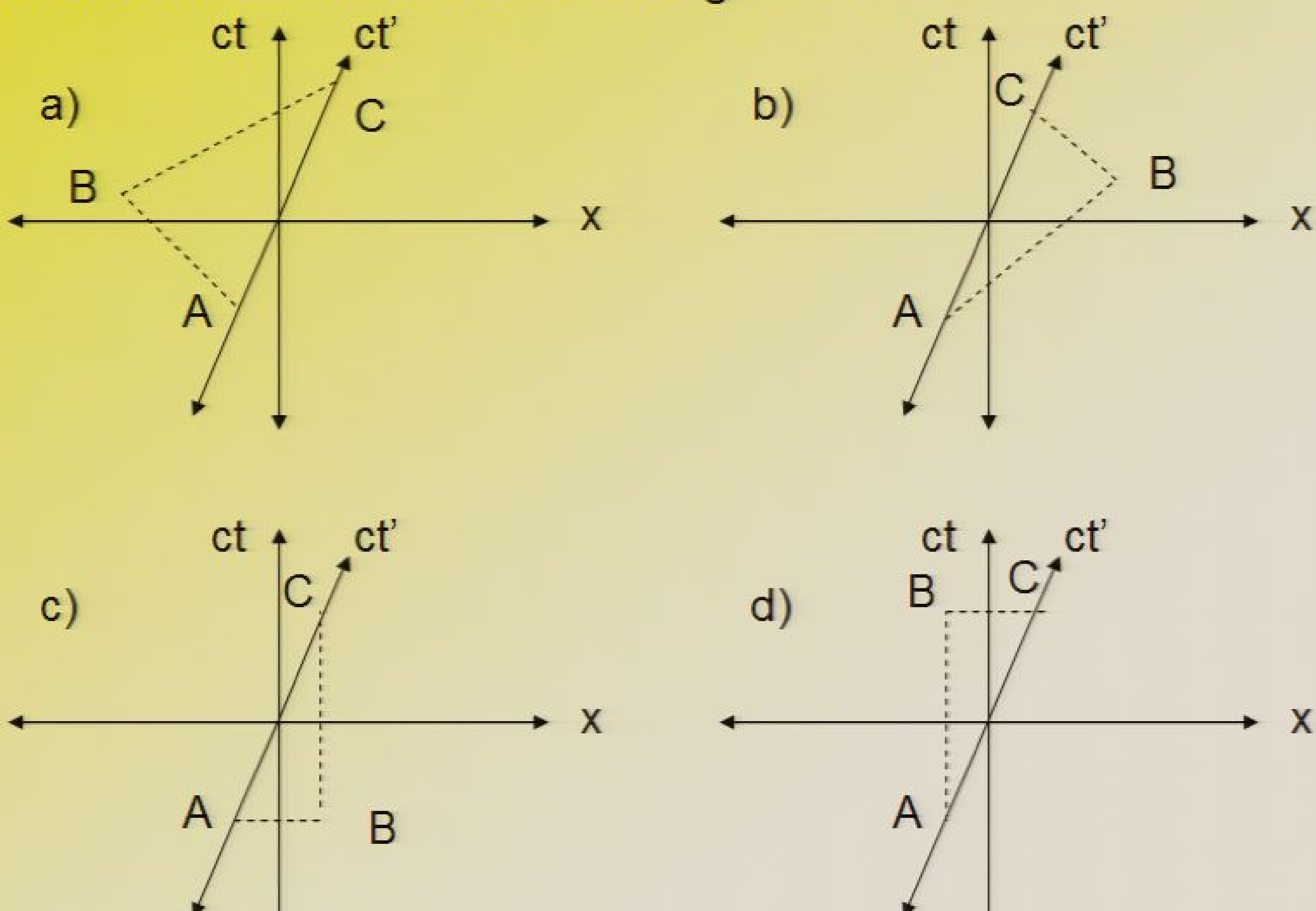

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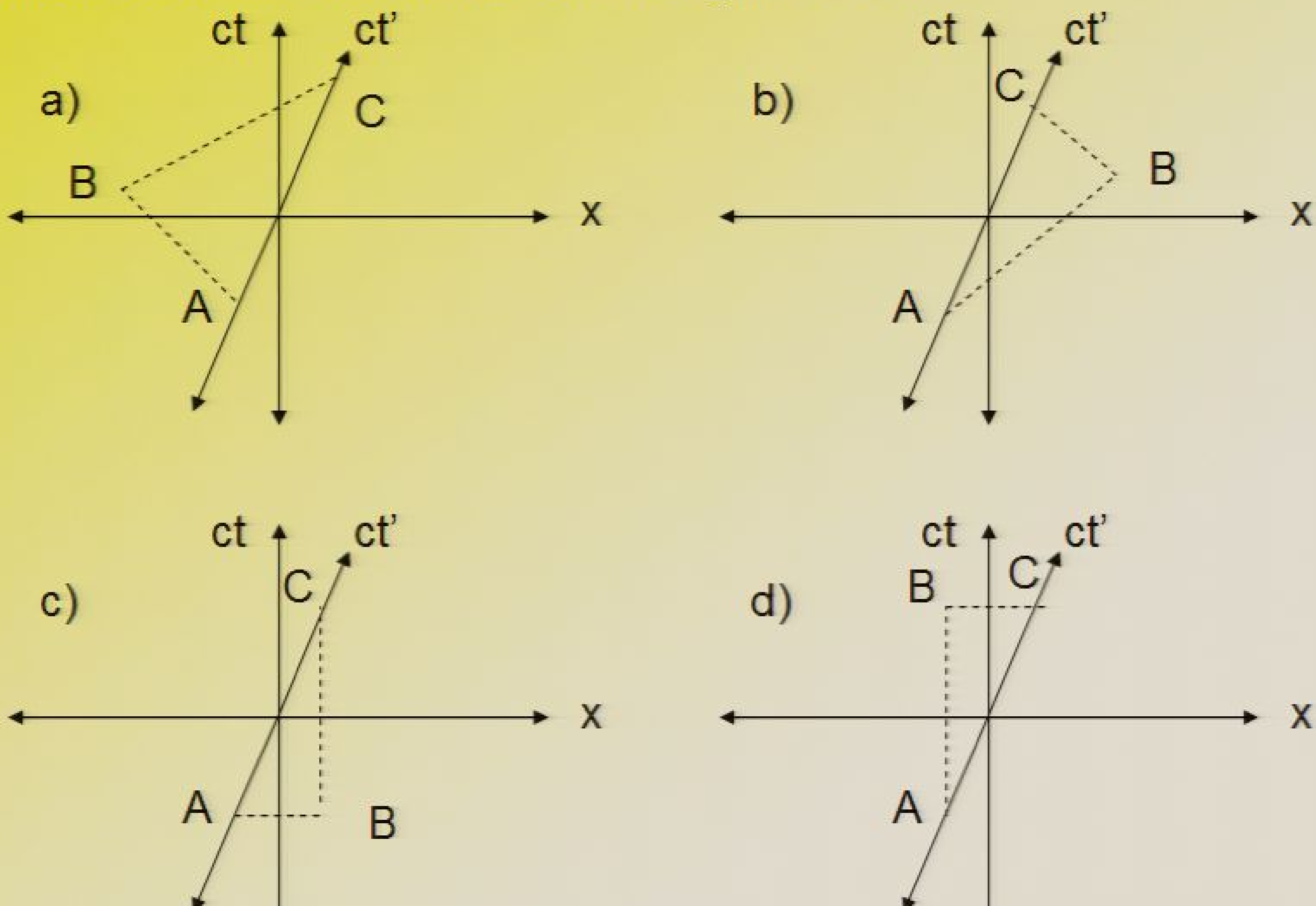

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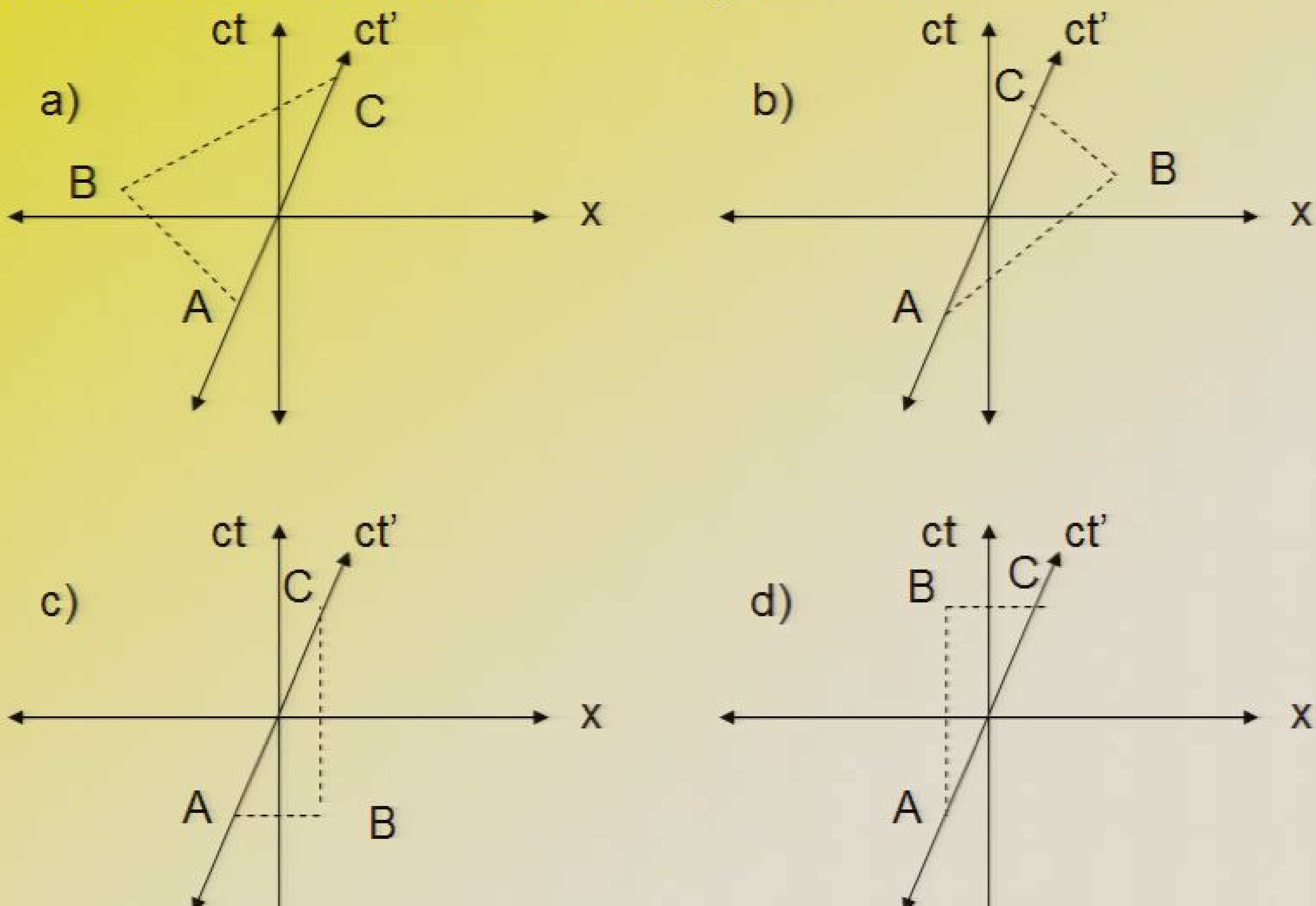

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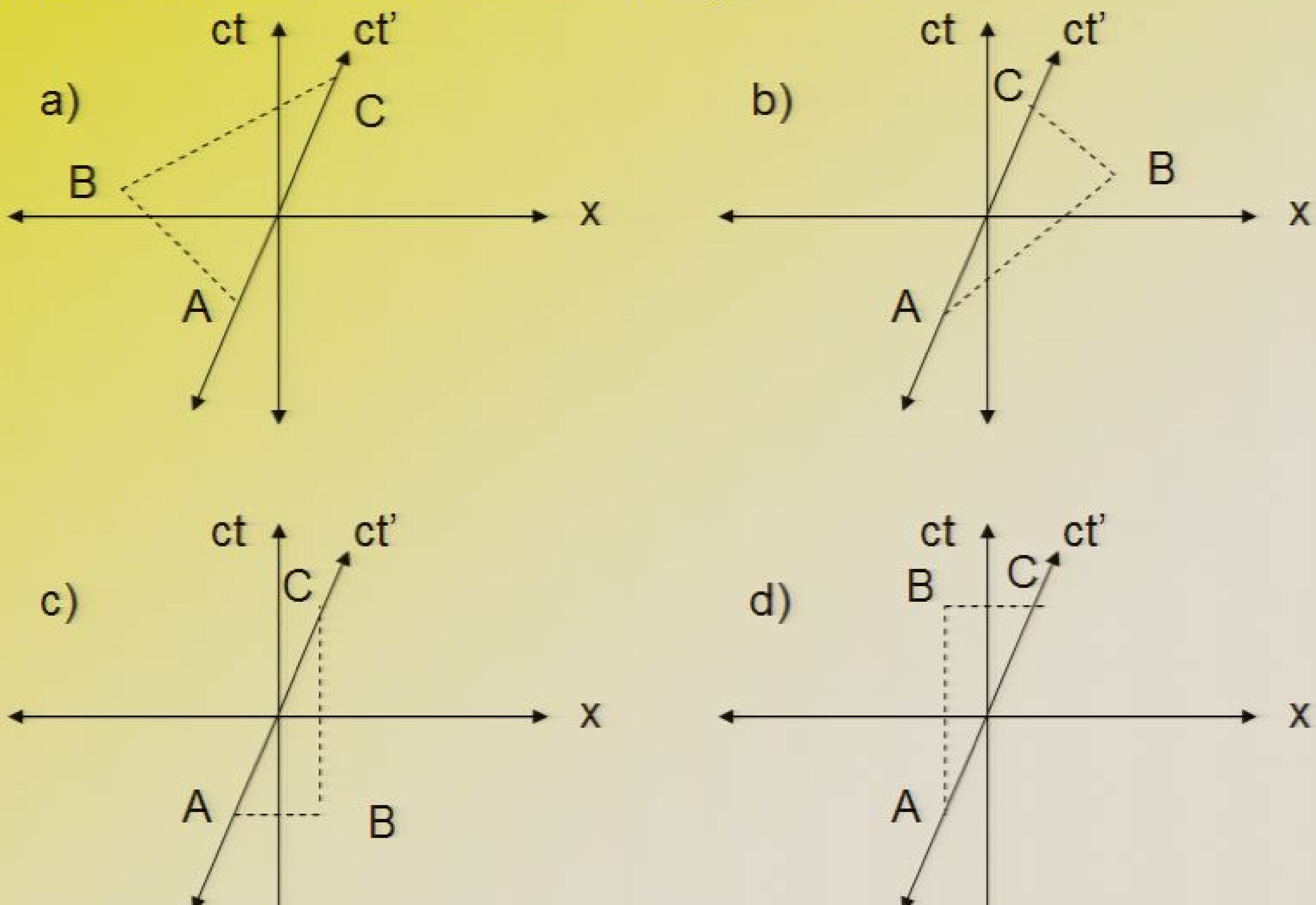

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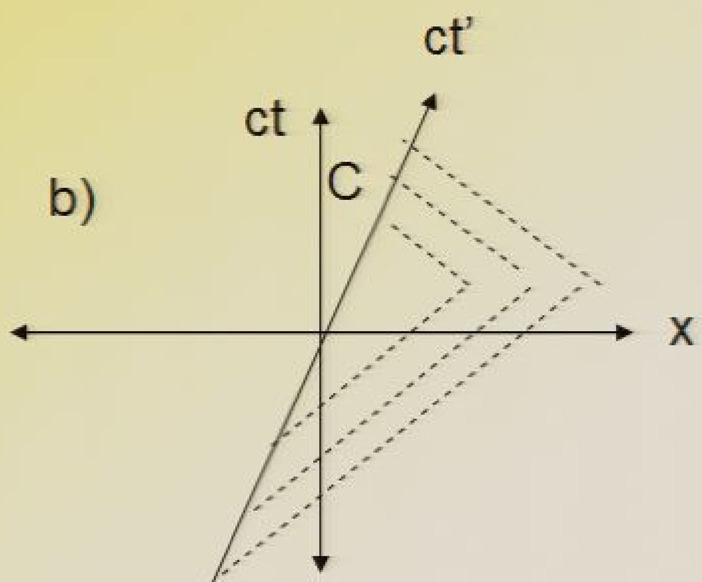
- We can vary the time T and get a set of points similar to B that also lie on the x’ axis. Drawing a line through all of these points, we plot the x’ axis.



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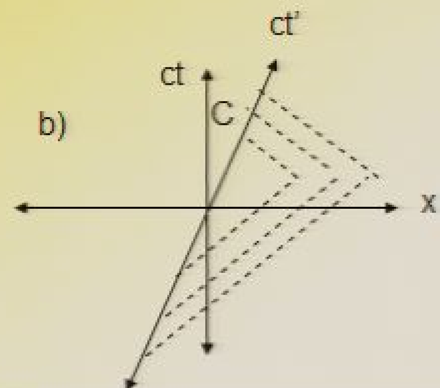

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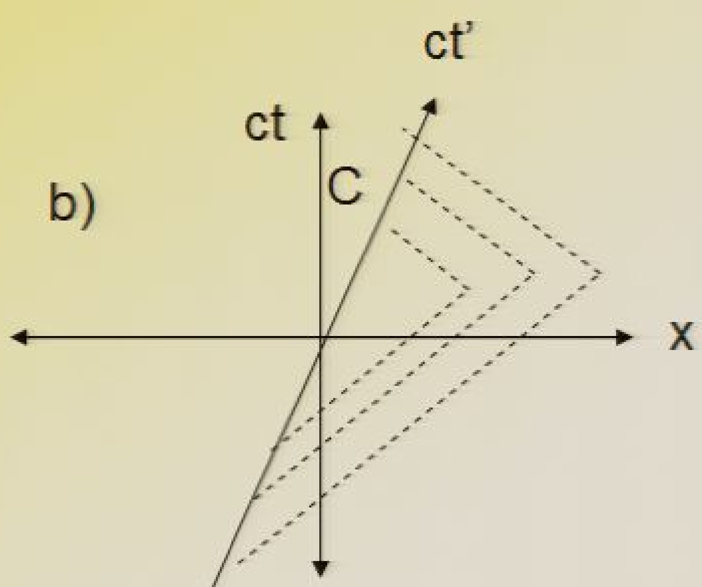
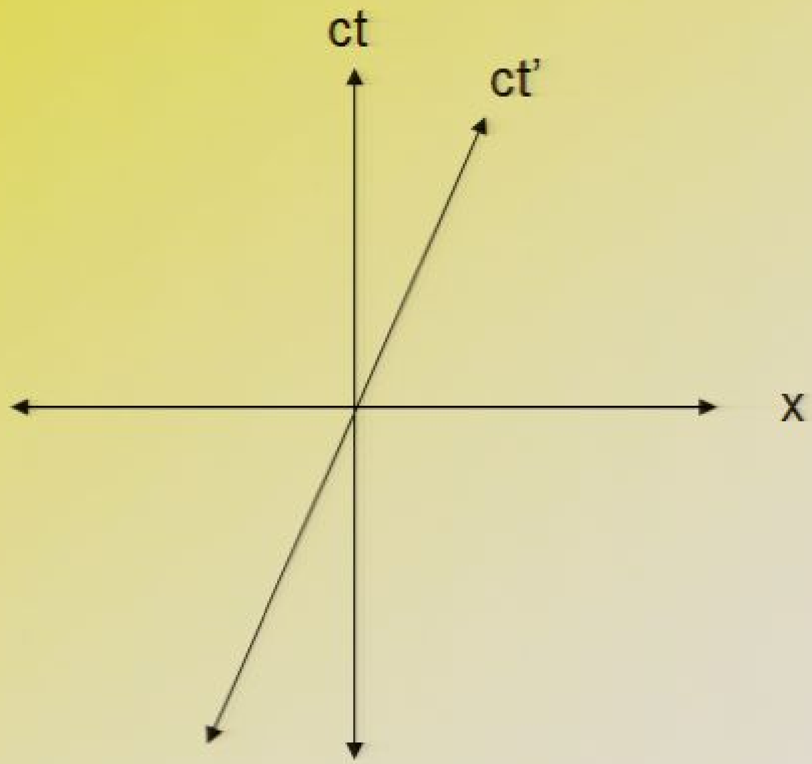


# Draw the x' axis



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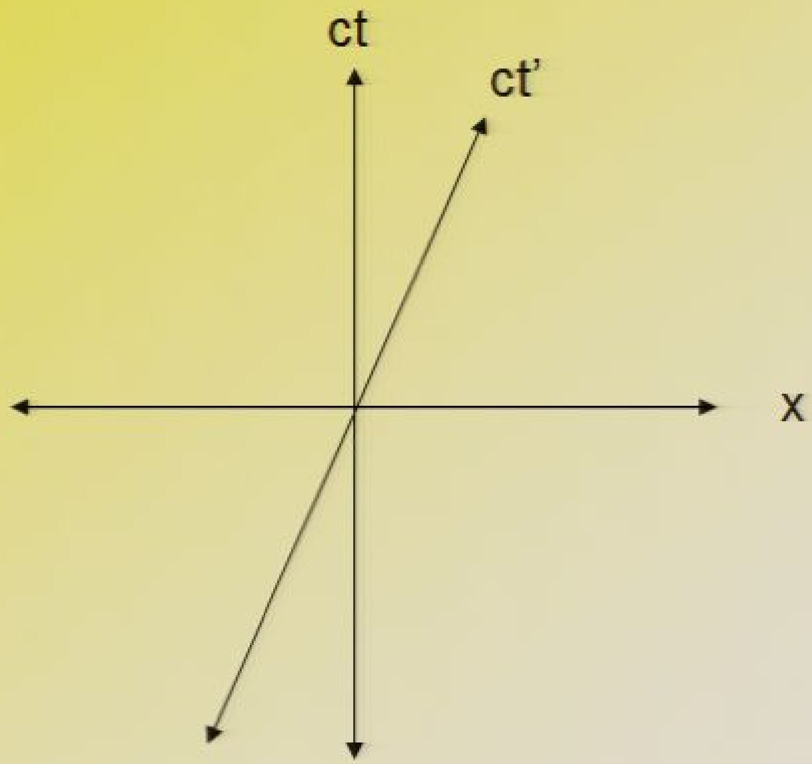

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- Let us begin …

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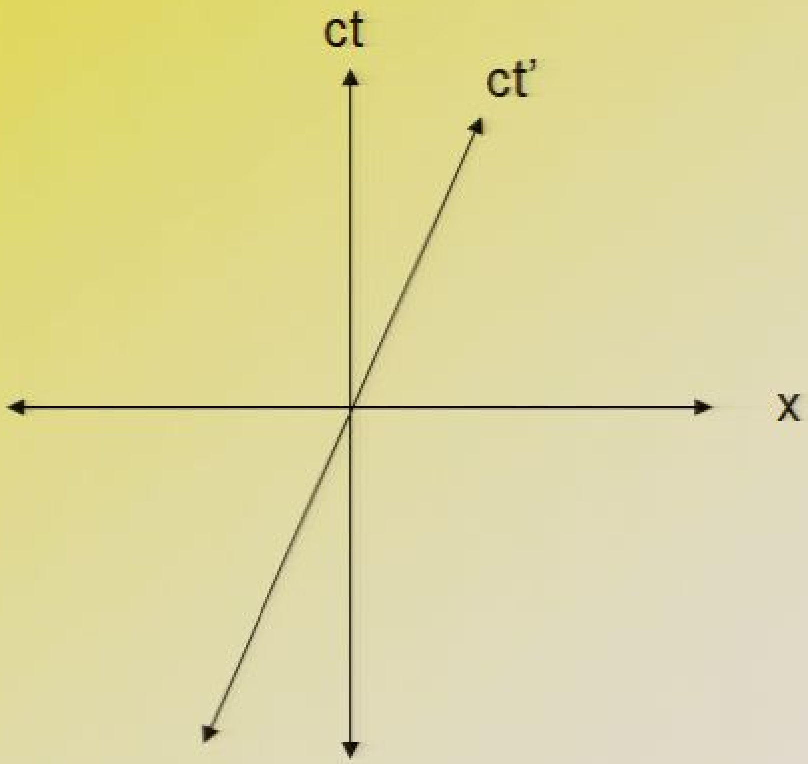
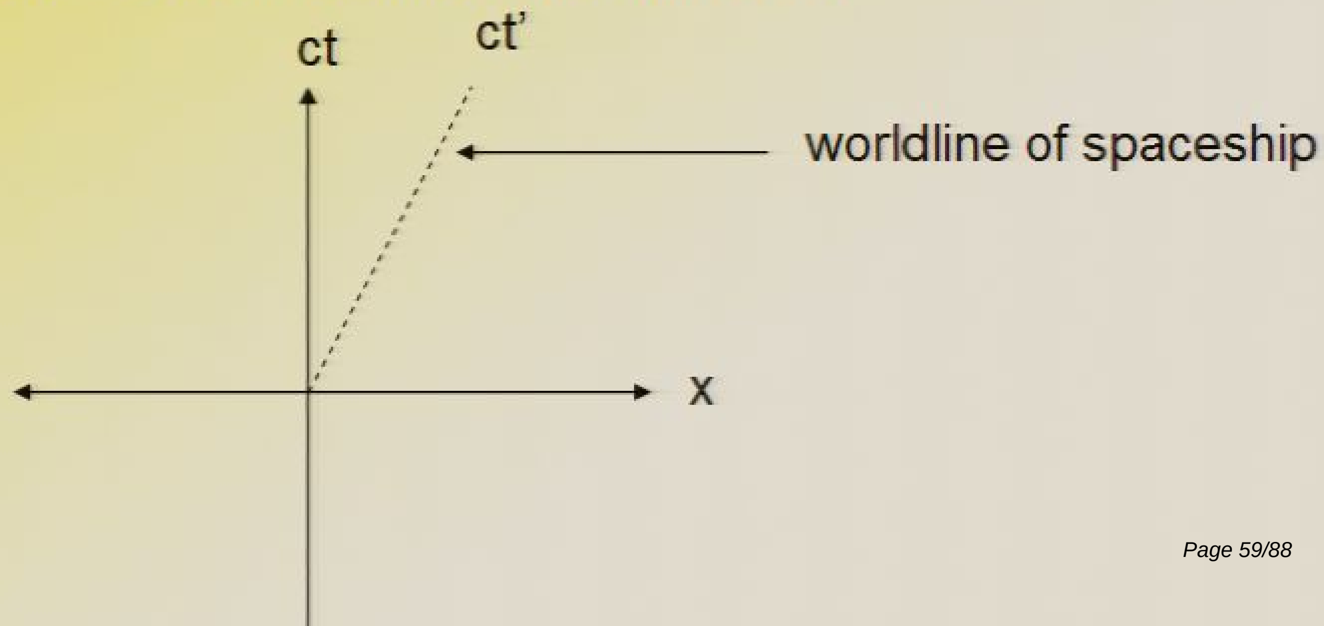


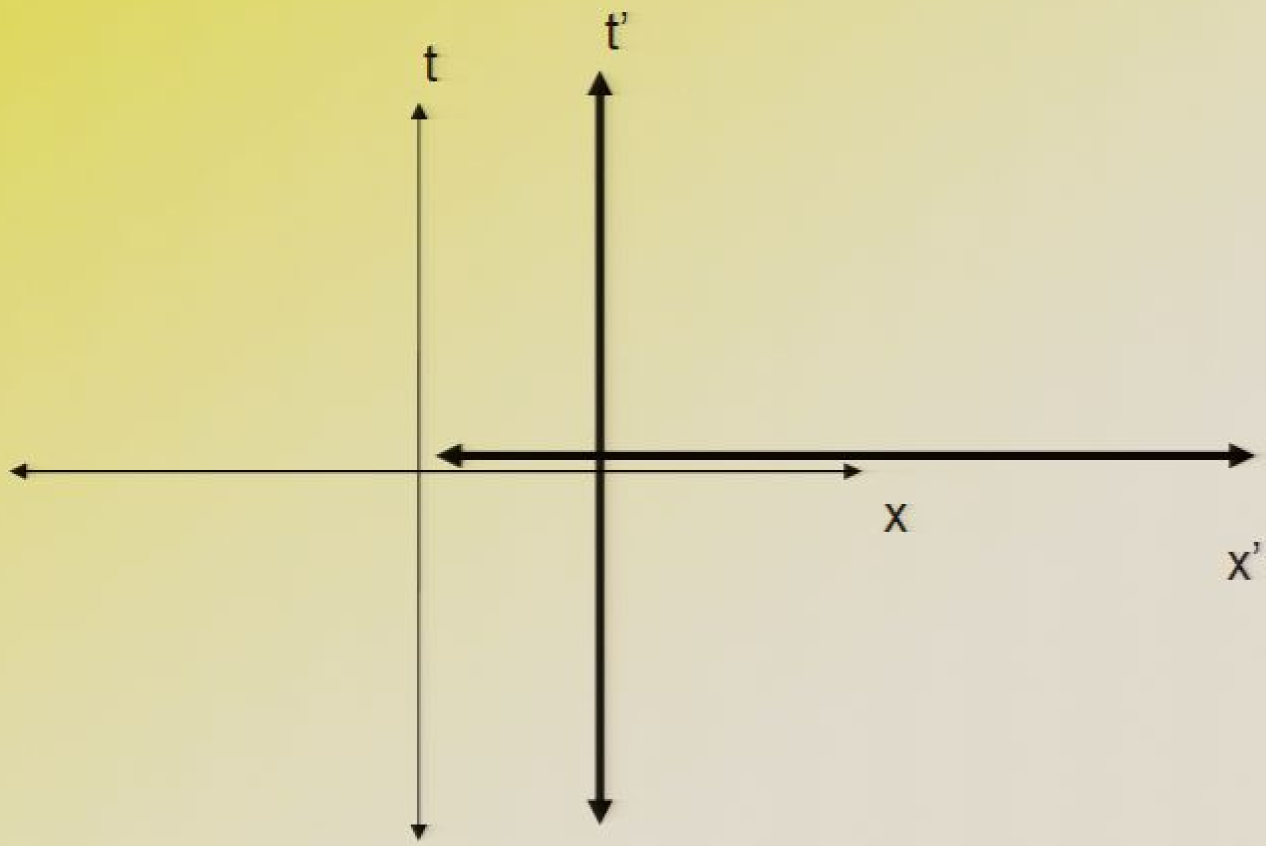
# Transforming between different frames of reference

- Imagine a spaceship moving at the constant velocity 0.5c past Earth
- What does its worldline look like?
- From the spaceship's perspective, it remains stationary and so all the points on its worldline have position x'=0. This defines the ct' axis just as the line x=0 defines the t axis for Earth.



# Transforming between different frames of reference

- Galilean transformations



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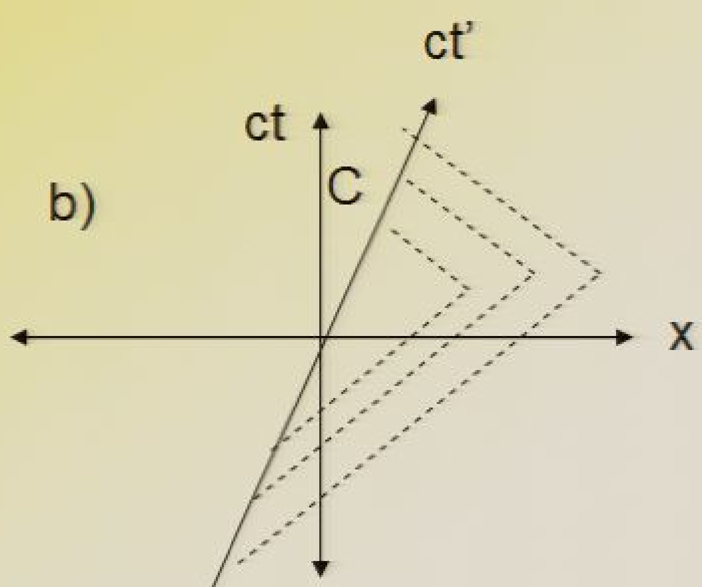

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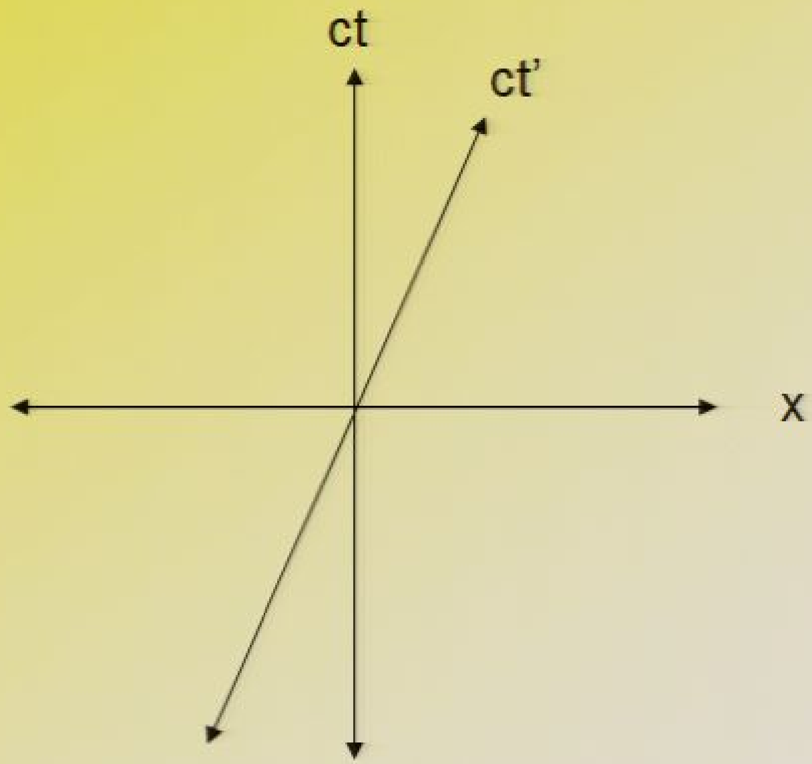

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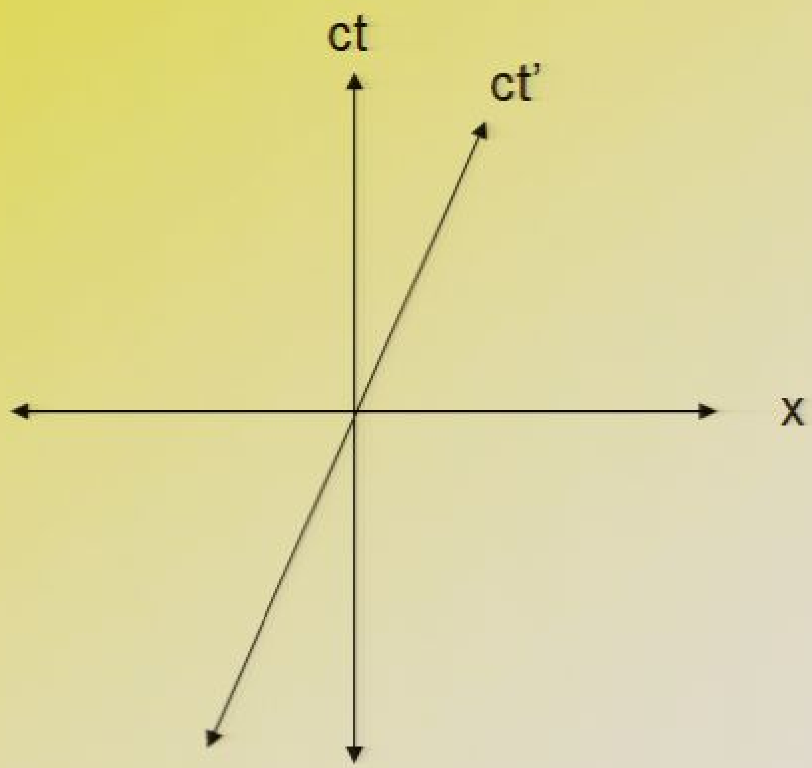

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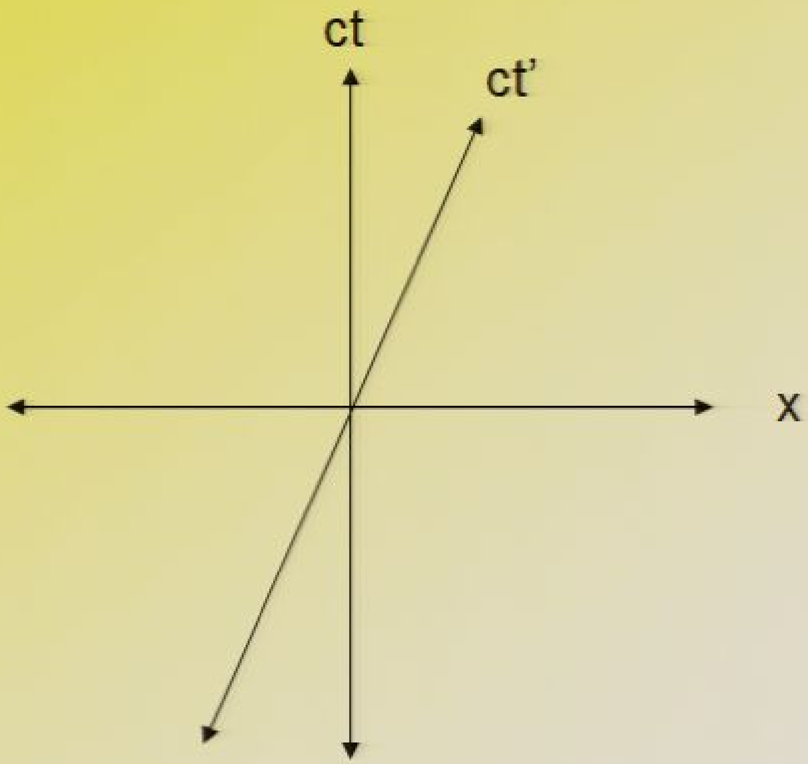

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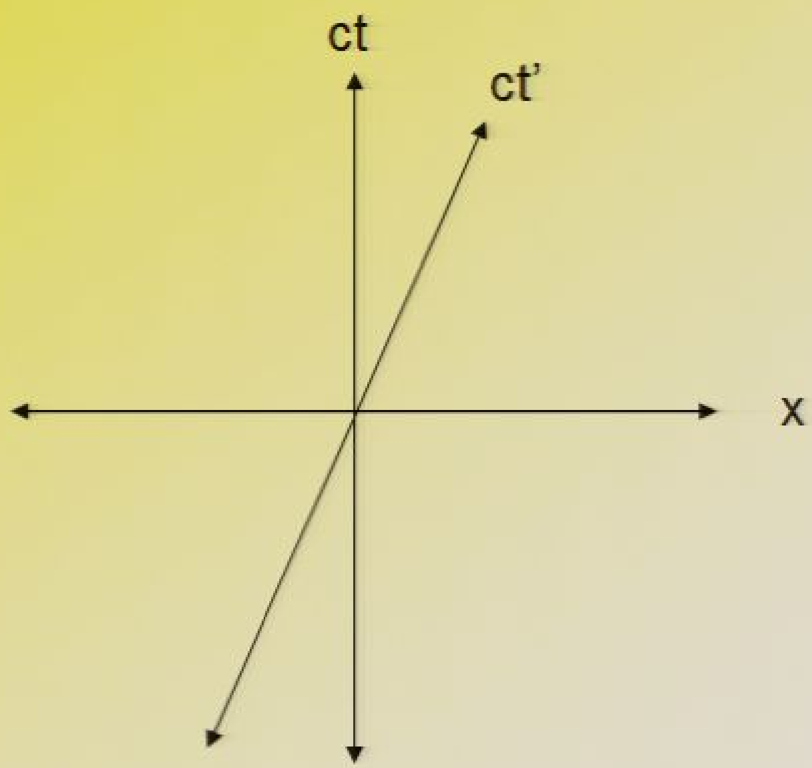

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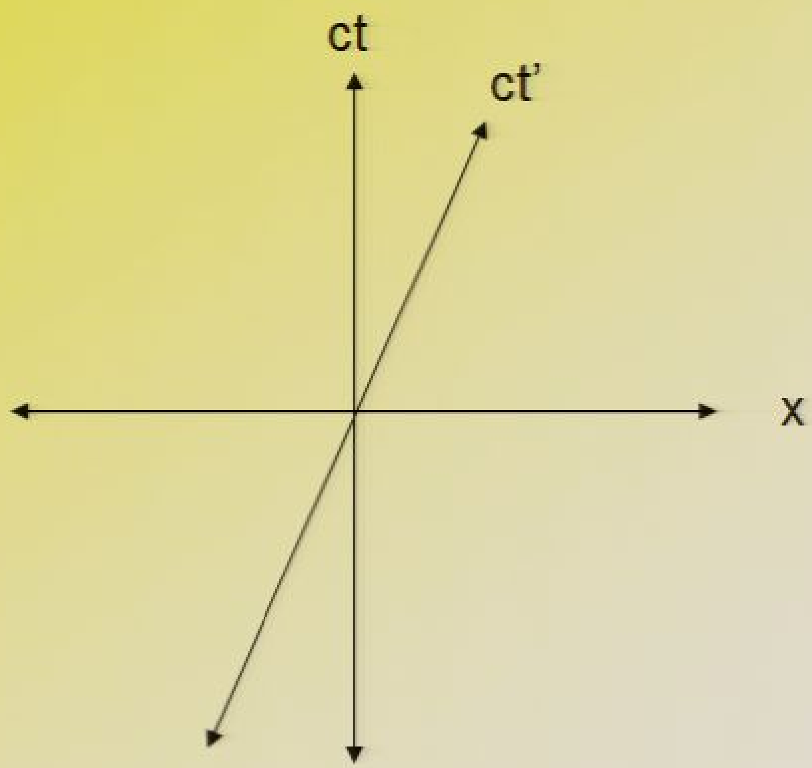
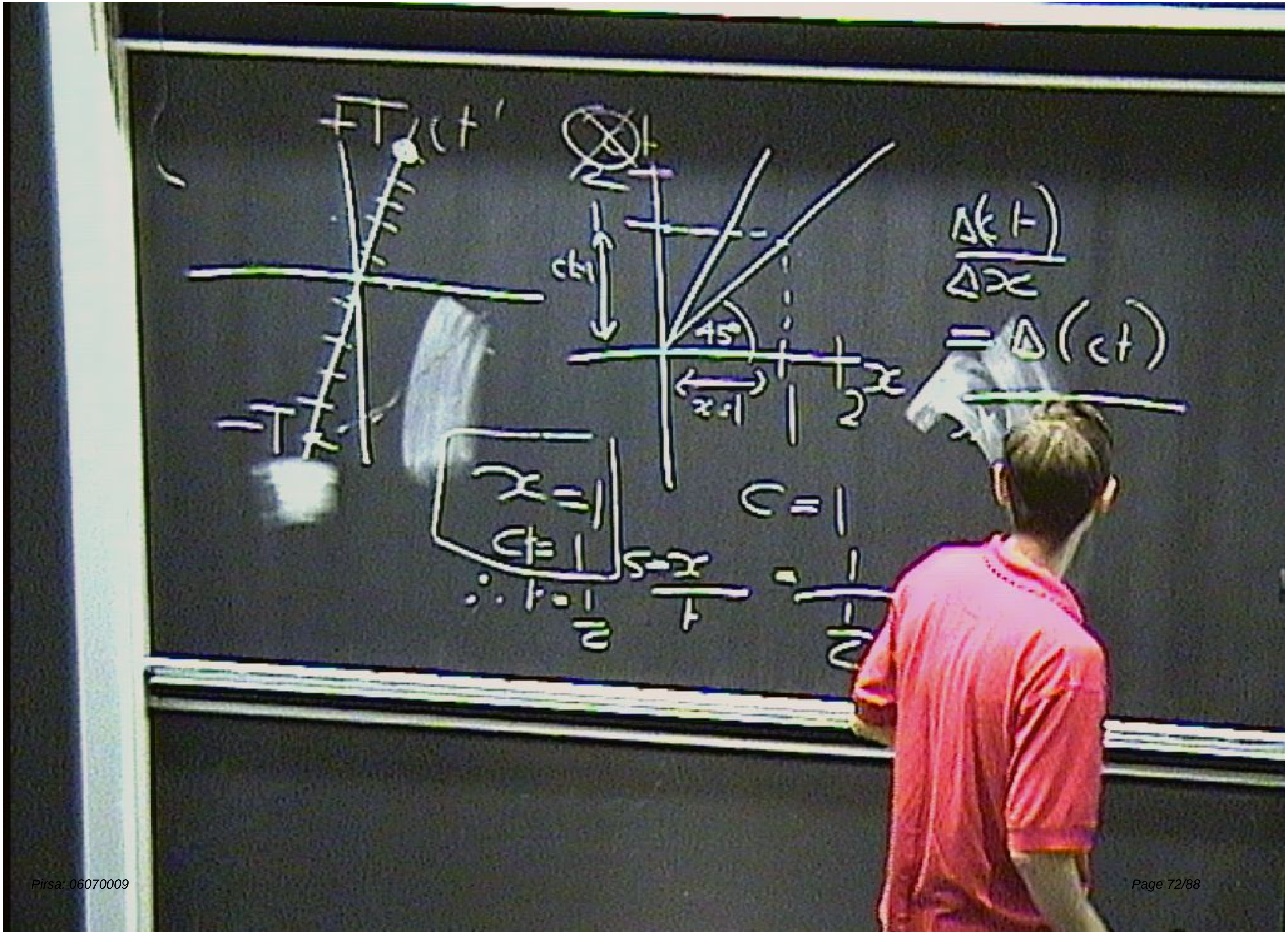
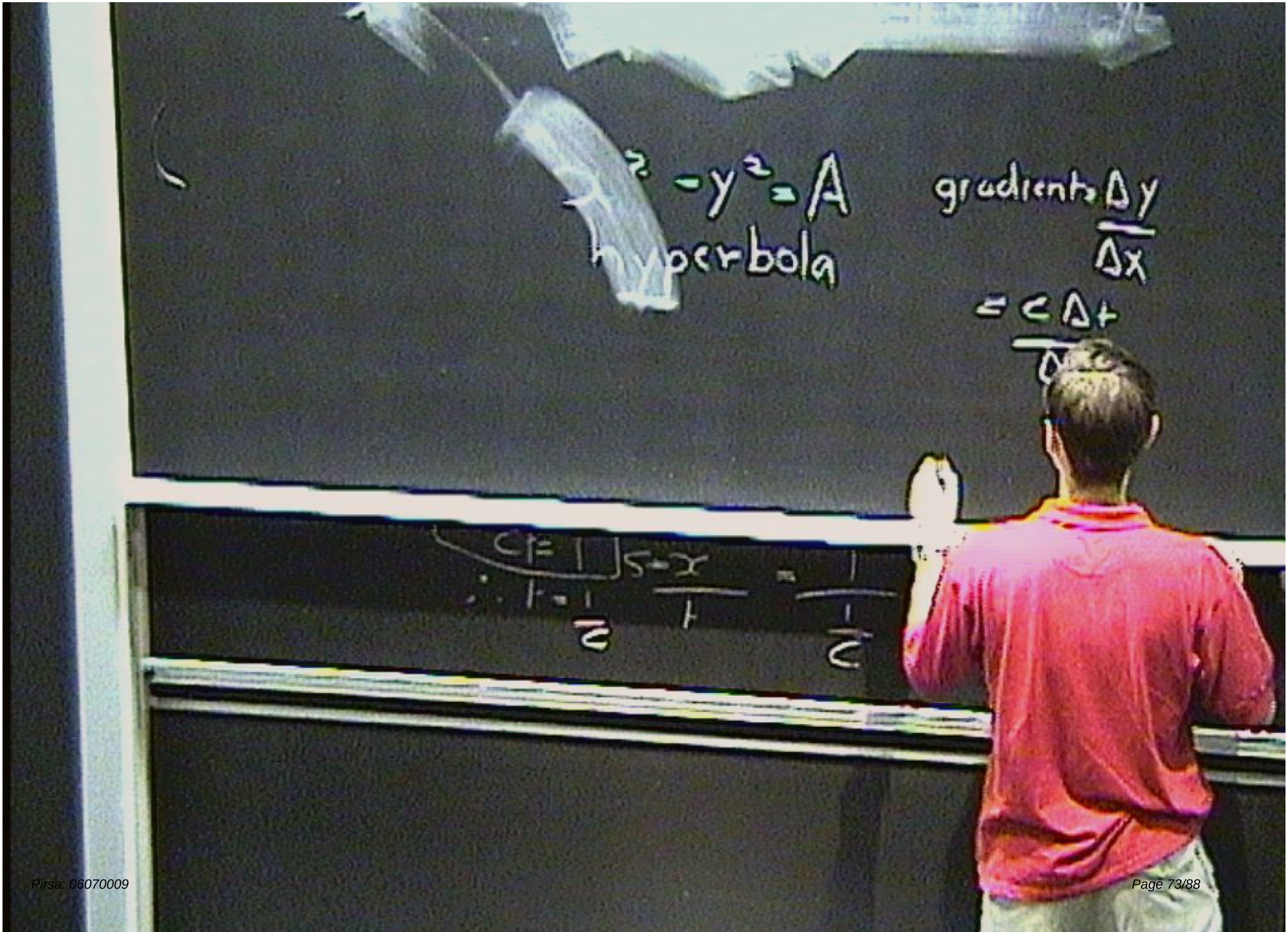
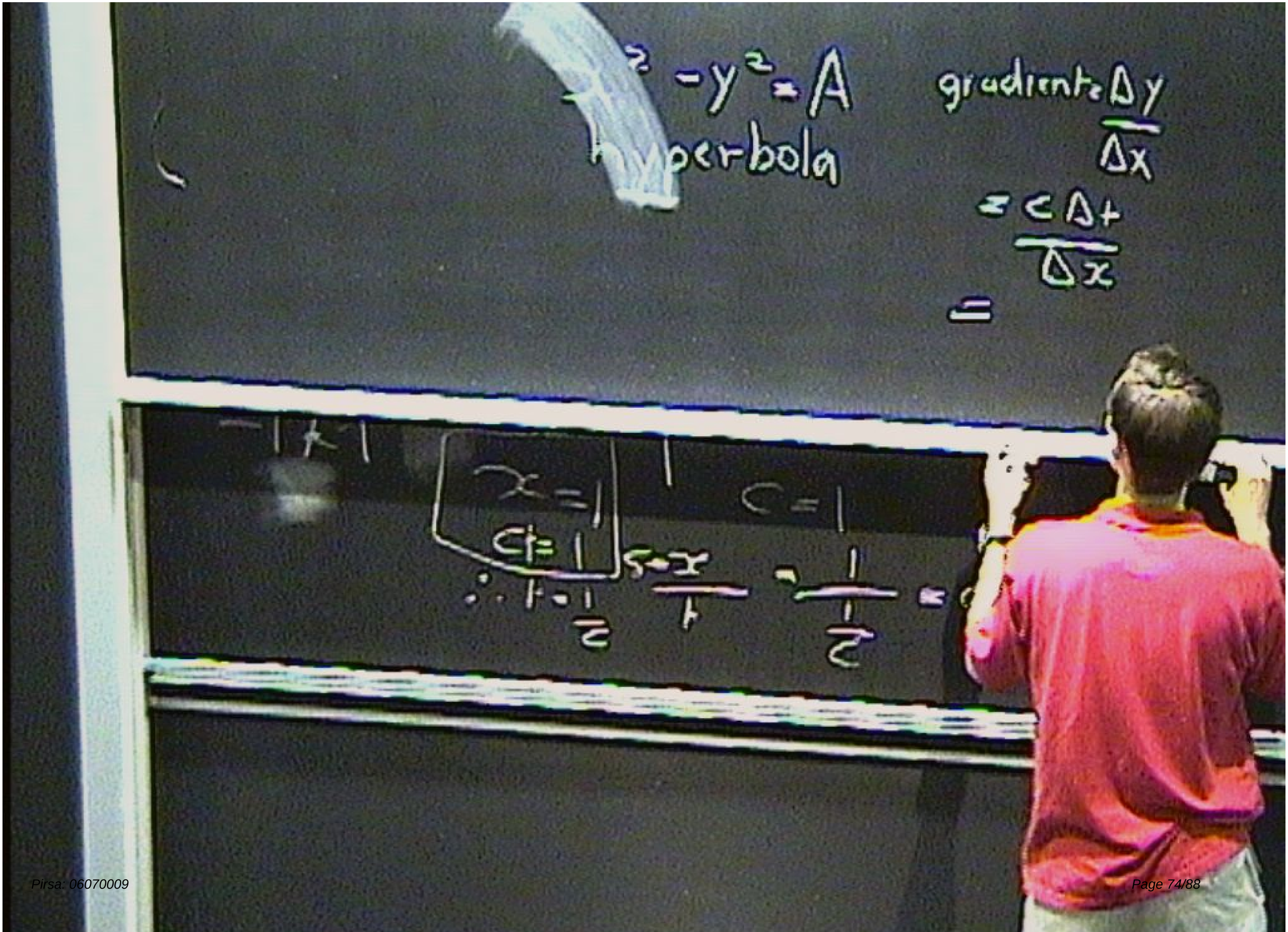
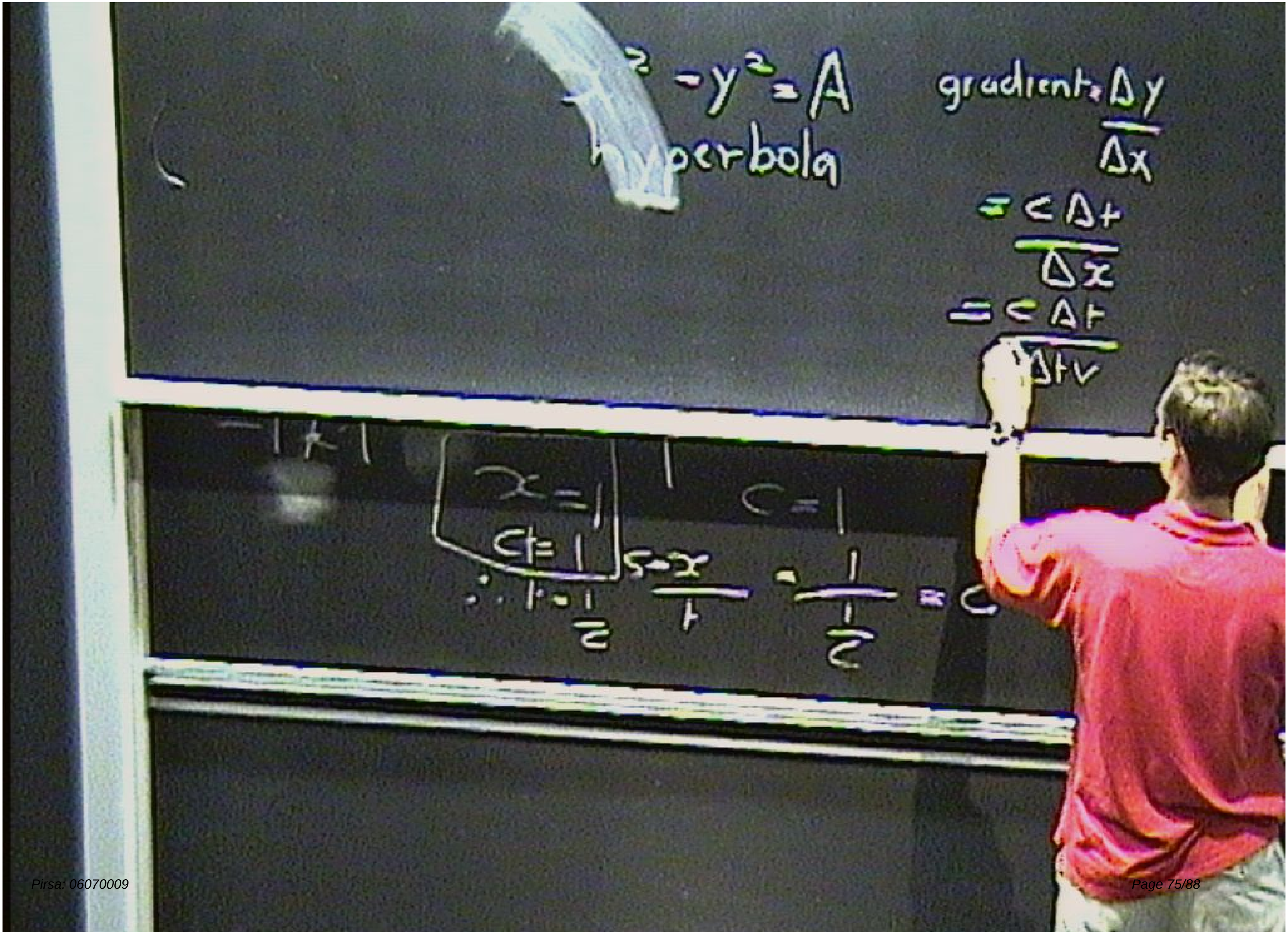



ct'
-T
45°
x
ct
x=1
ct=1
c=1
Δ(ct)
Δx
= Δ(ct)

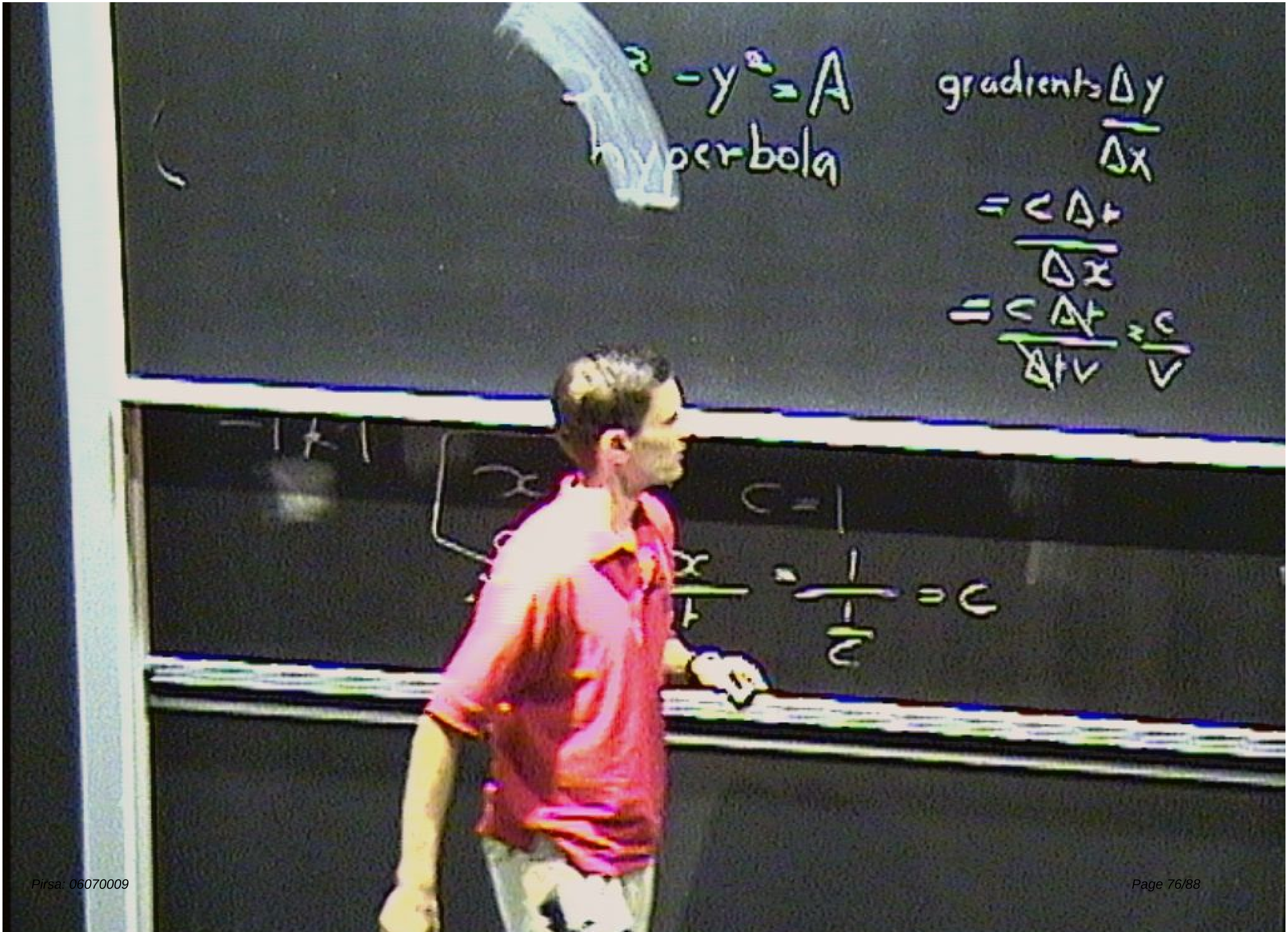


-y² = A
hyperbola
gradient = Δy / Δx
= c Δt


−y² = A
hyperbola
gradient = Δy / Δx
= cΔt / Δx
=
x = 1
ct = 1
c = 1
∴ t = 1/c
s = x/t = 1/(1/c) =

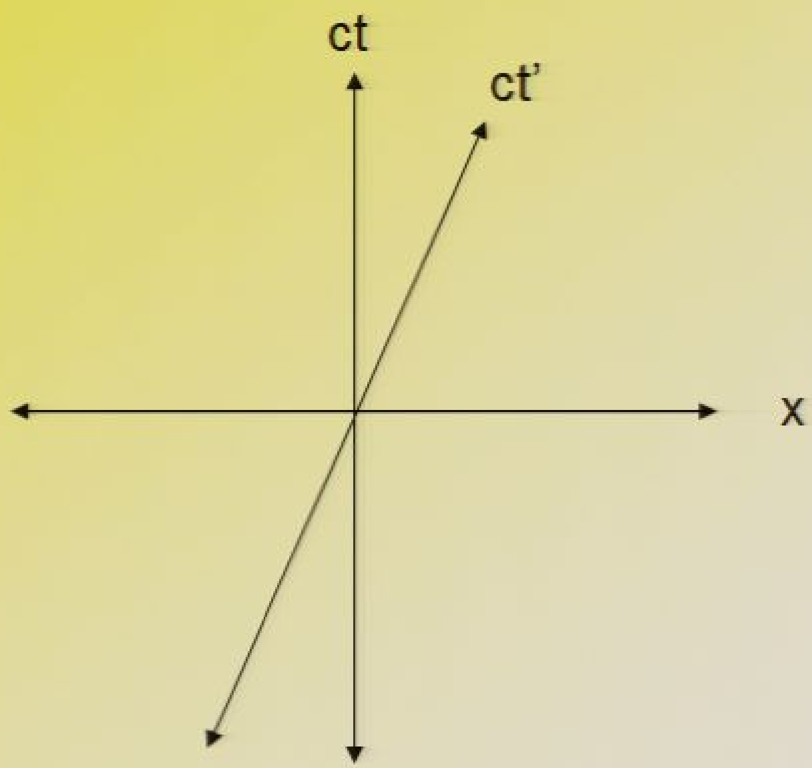


²-y²=A
hyperbola
gradient = Δy/Δx
= cΔt/Δx
= cΔt/Δtv
x=1
c=1
ct=1
s=x/t = 1/(1/c) = c
∴ t=1/c



$x^2 - y^2 = A$
hyperbola
gradient $= \frac{\Delta y}{\Delta x}$
$= \frac{c \Delta t}{\Delta x}$
$= \frac{c \Delta t}{\Delta t v} = \frac{c}{v}$
$c = 1$
$= \frac{1}{\frac{1}{c}} = c$

# Draw the x' axis



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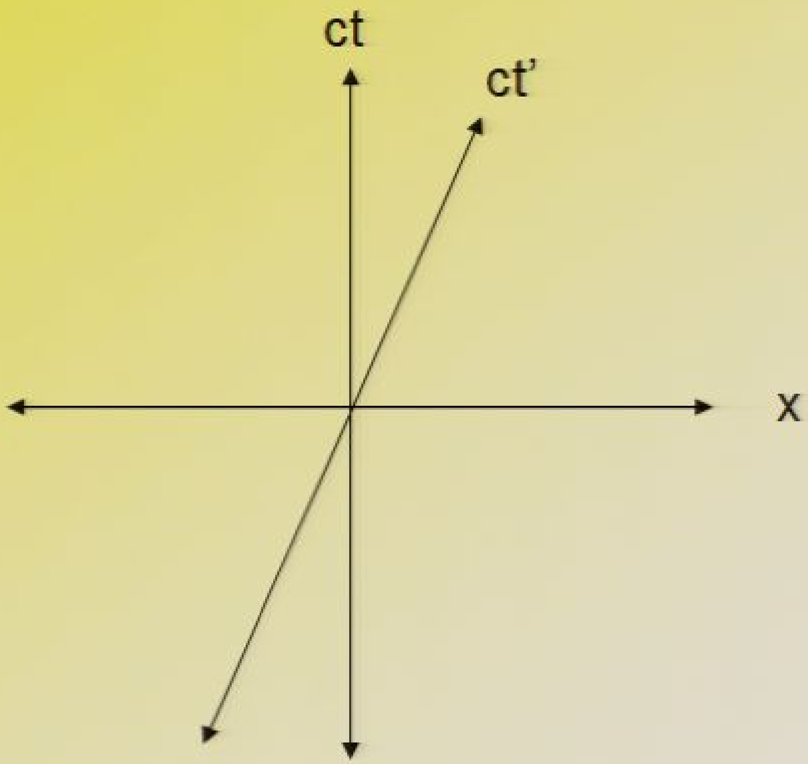
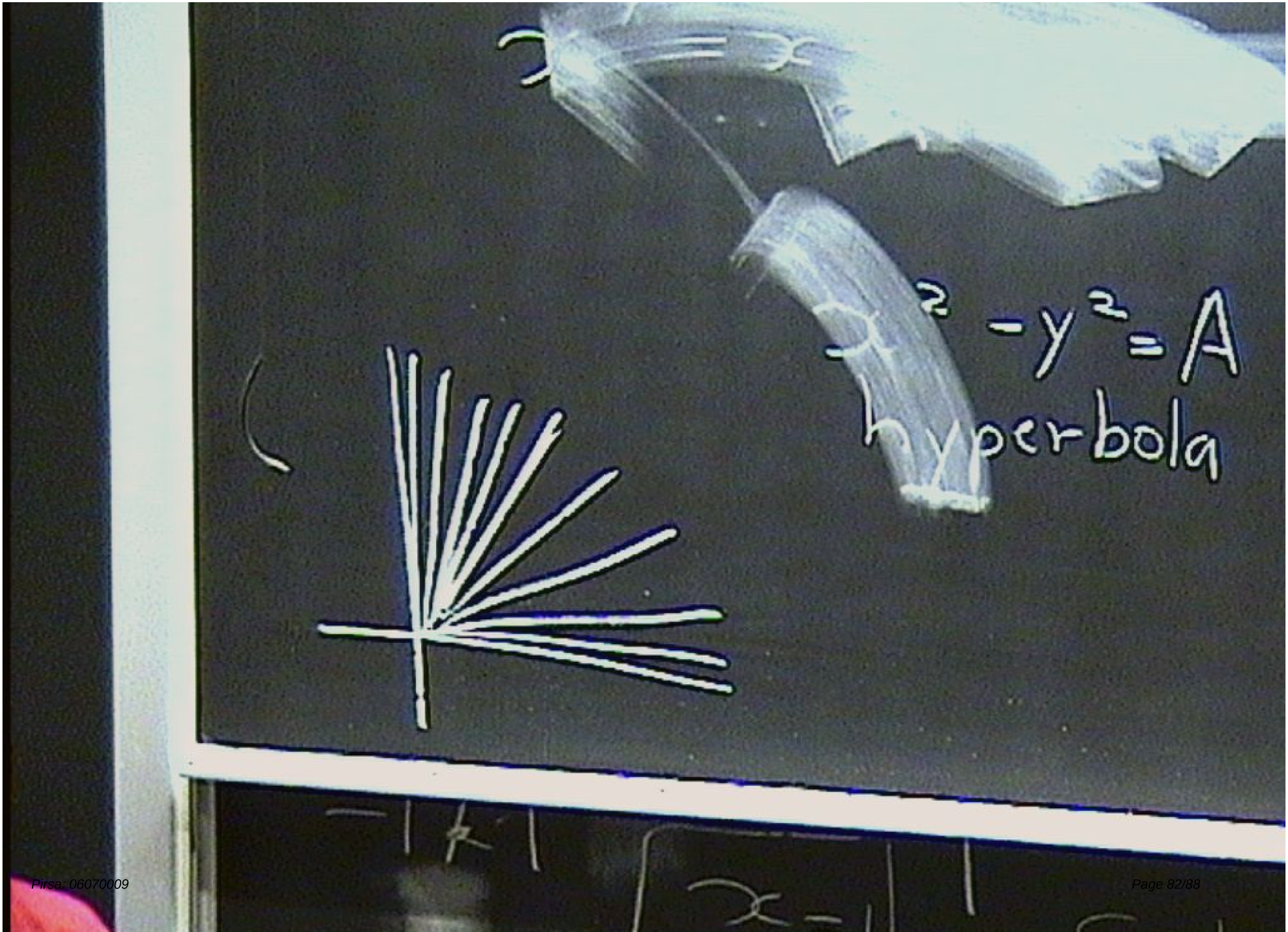
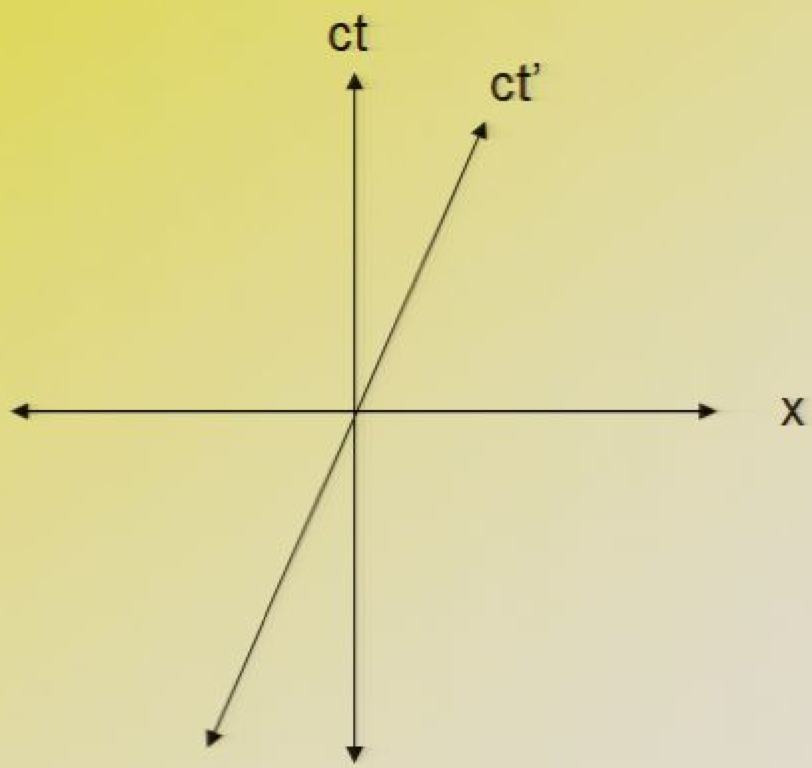


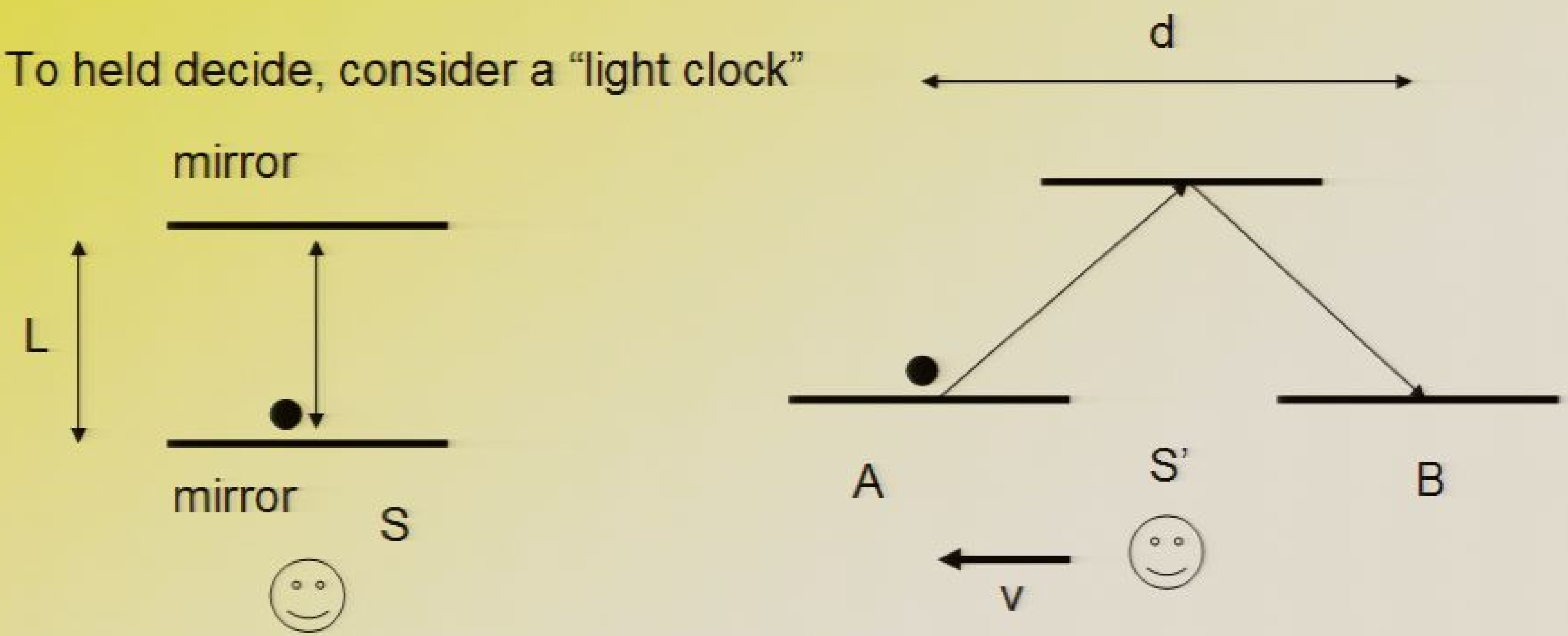

x² - y² = A
hyperbola

# Draw the x' axis



# What is the metric equation for spacetime

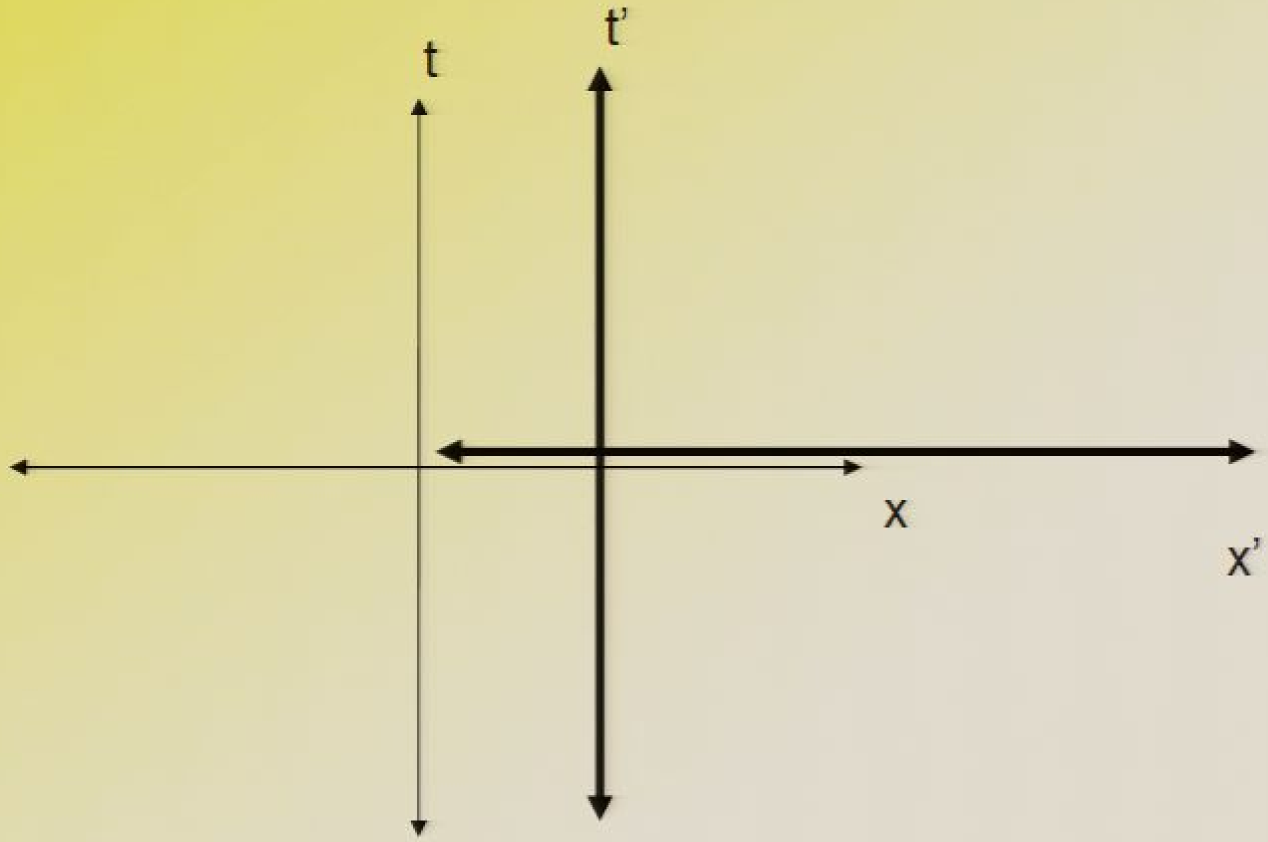
- As the two axes are x and t is it just $d^2 = \Delta x^2 + (c\Delta t)^2$?
- Or perhaps, according to Einstein, there is something different about space and time.
- To held decide, consider a "light clock"




- A and B are the events corresponding to a photon hitting the bottom mirror on two successive occasions.

# Transforming between different frames of reference

- Galilean transformations



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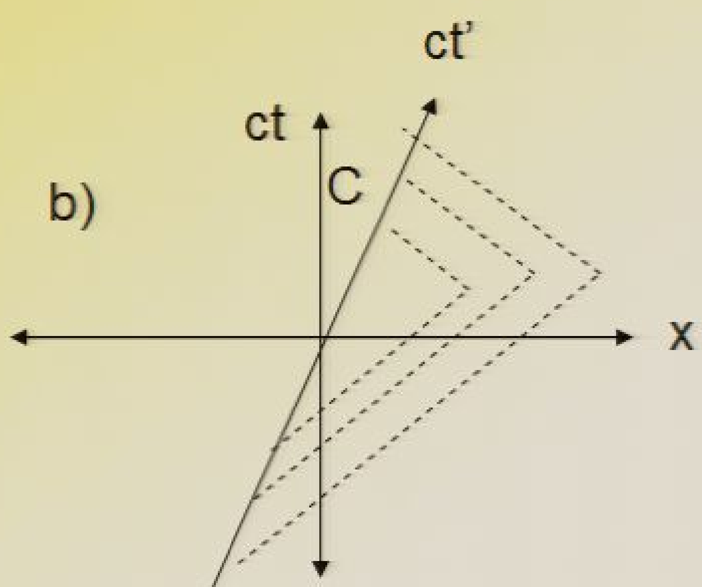

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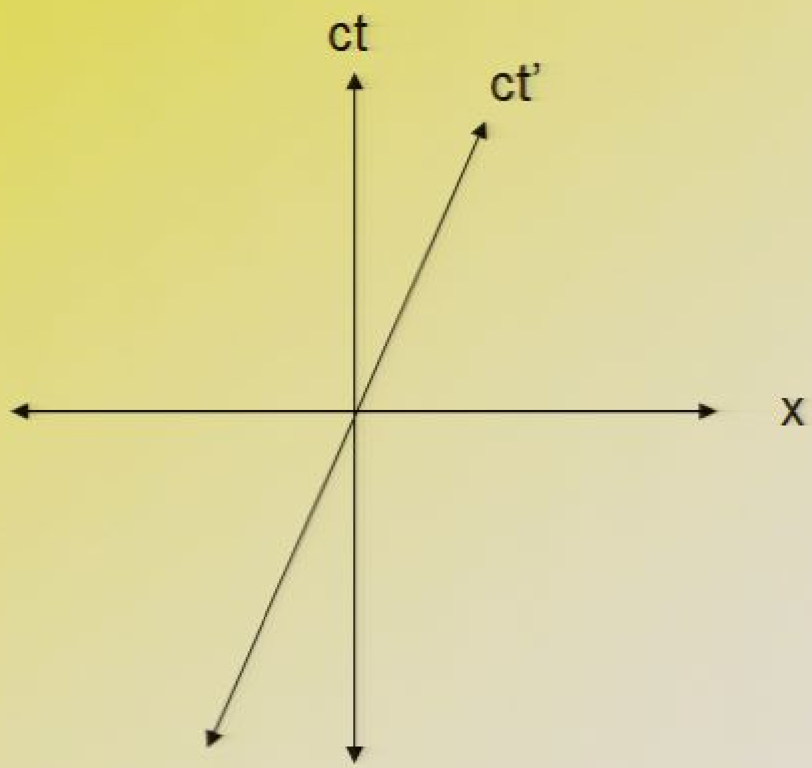

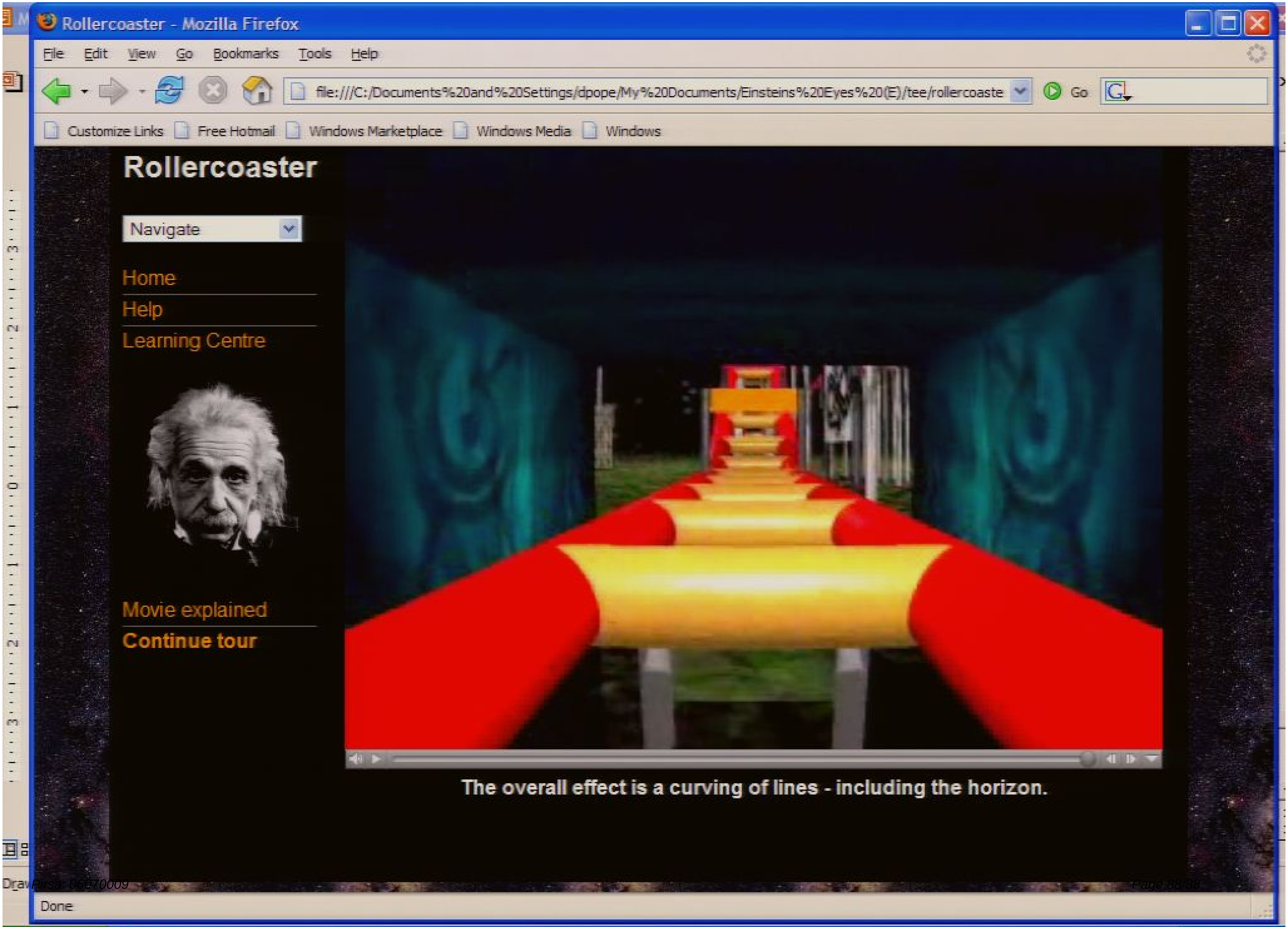
Rollercoaster - Mozilla Firefox
File Edit View Go Bookmarks Tools Help
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Go
Customize Links
Free Hotmail
Windows Marketplace
Windows Media
Windows
Rollercoaster
Navigate
Home
Help
Learning Centre
Movie explained
Continue tour
The overall effect is a curving of lines - including the horizon.
Done