

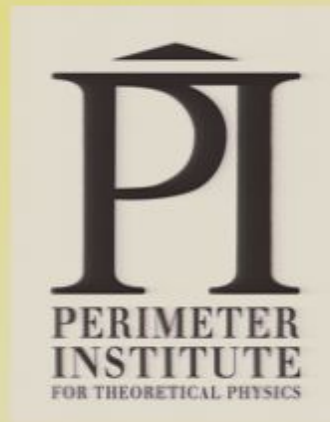
Title: Enrichment Presentation on Special Relativity

Date: Jul 06, 2006 09:00 AM

URL: <http://pirsa.org/06070008>

Abstract:

# Core concepts of special relativity



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# Through Einstein's Eyes



Relativistic Rollercoaster



Solar System Tour



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Learning Centre

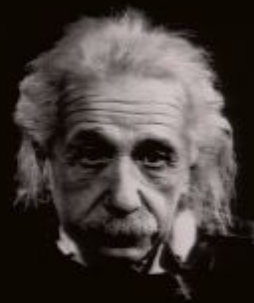


THE AUSTRALIAN NATIONAL UNIVERSITY

# Rollercoaster

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- Home
- Help
- Learning Centre



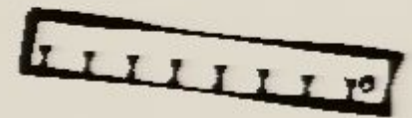
- Movie explained
- Continue tour



The relativistic rollercoaster: we imagine that either the speed of light is very slow or objects are very big.

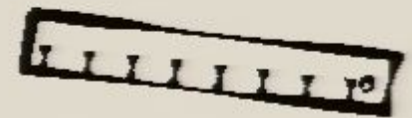
# Summary

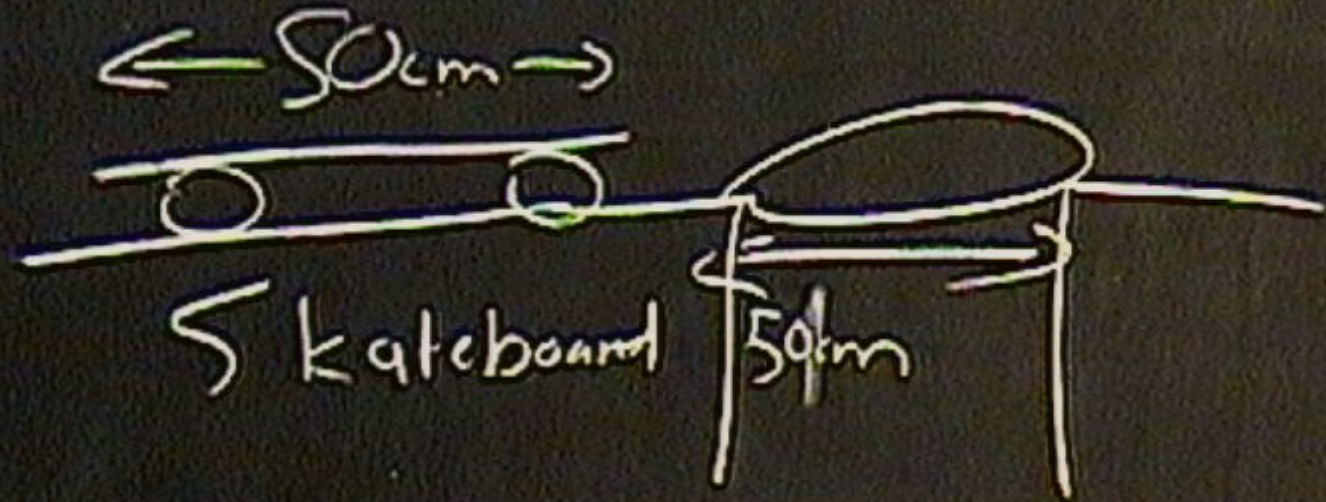
- Two postulates of special relativity
- Plotting happenings (events) in space and time: spacetime diagrams.
- Metric for spacetime
- Switching reference frames
- Time dilation and length contraction revisited
- The twin paradox

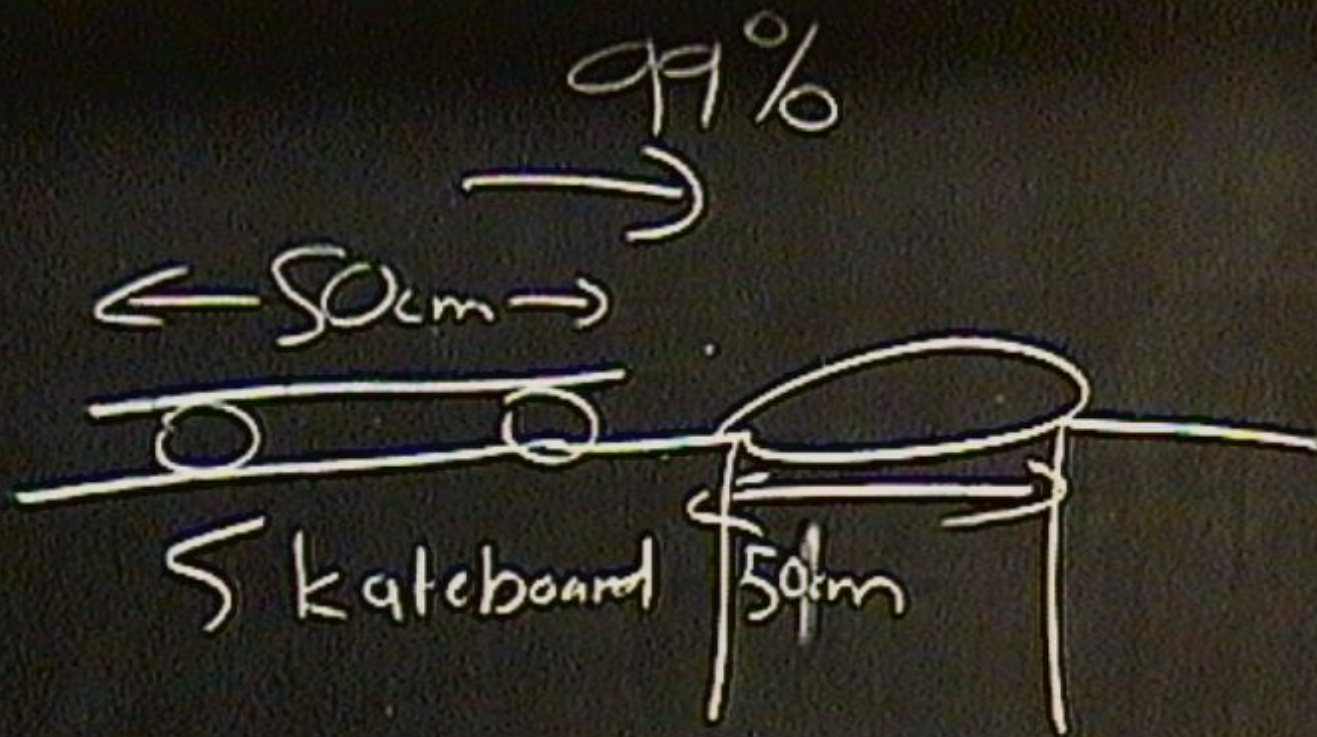


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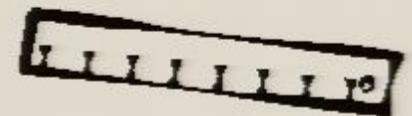
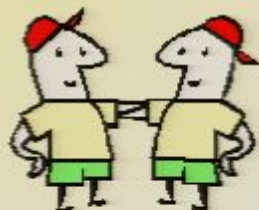






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# Some background to special relativity

- Einstein at 16 years of age: daydream about riding alongside a beam of light
- Overturned more than 200 years of thinking about space and time.
- Cultural impact
- theory about space, time and motion
- What happens to these three things as relative velocities change.
- Only deals with constant velocities, no acceleration.

# Twin postulates of special relativity

- **1. Relativity Postulate:** The laws of physics are the same in all inertial reference frames.
- **2. Speed of light postulate:** Light travels at the constant speed of  $c$  ( $3 \times 10^8 \text{ ms}^{-1}$ ) through empty space relative to all inertial observers.
- *What is an inertial reference frame?*  
No acceleration.  
Newton's first law holds.
- Consider the relativity postulate:  
Physics should be universal, democratic

In 1905, it was both revolutionary and pre-dated.

- Galileo in the 17<sup>th</sup> century: The laws of *mechanics* are the same in all inertial reference frames.
- Revolutionary aspect of the relativity postulate was that Einstein broadened 'mechanics' to 'physics'.
- Part of Einstein's genius was to boldly believe that that the laws of electromagnetism (Maxwell's equations) applied to all inertial observers, even though he knew that this had radical implications. (Eg. he knew that this implied that the speed of light was independent of an observer's velocity relative to the light's source).



- One view of the relativity postulate:
- One morning, you wake up and find yourself in an unfamiliar room.
- A strange man and a women walk up to you and say 'You have been kidnapped. But, we will let you go if you can tell us if the room is i) fixed to the ground and stationary with respect to it or ii) inside a a plane flying at very low altitude at a constant velocity with respect to Earth.
- The relativity postulates says that there is no way to reliably distinguish between the two possibilities, no matter what equipment you have (provided you cannot get information from outside of the room. Eg. calling an outside friend).

# Speed of light postulate

- Crudely speaking, 'Everyone measures light to travel at  $c$ '.
- Counter-intuitive



- What speed will you see the snowball moving towards you at?
  - a)  $45 \text{ kmh}^{-1}$
  - b)  $15 \text{ kmh}^{-1}$
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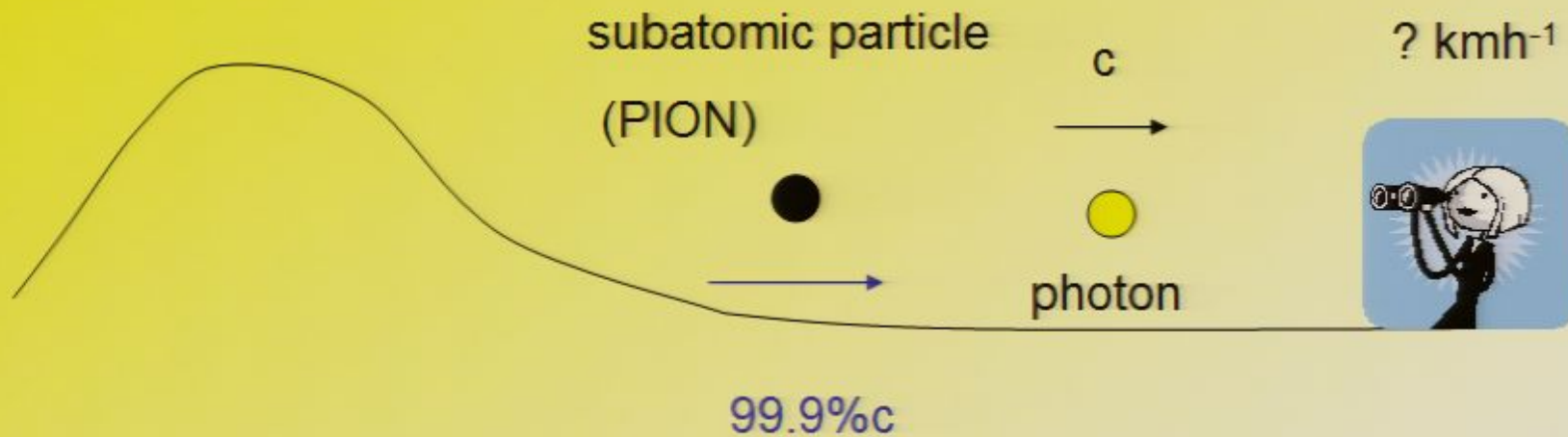
- What speed will you see the light moving towards you at?
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  - b)  $c + 30 \text{ kmh}^{-1}$
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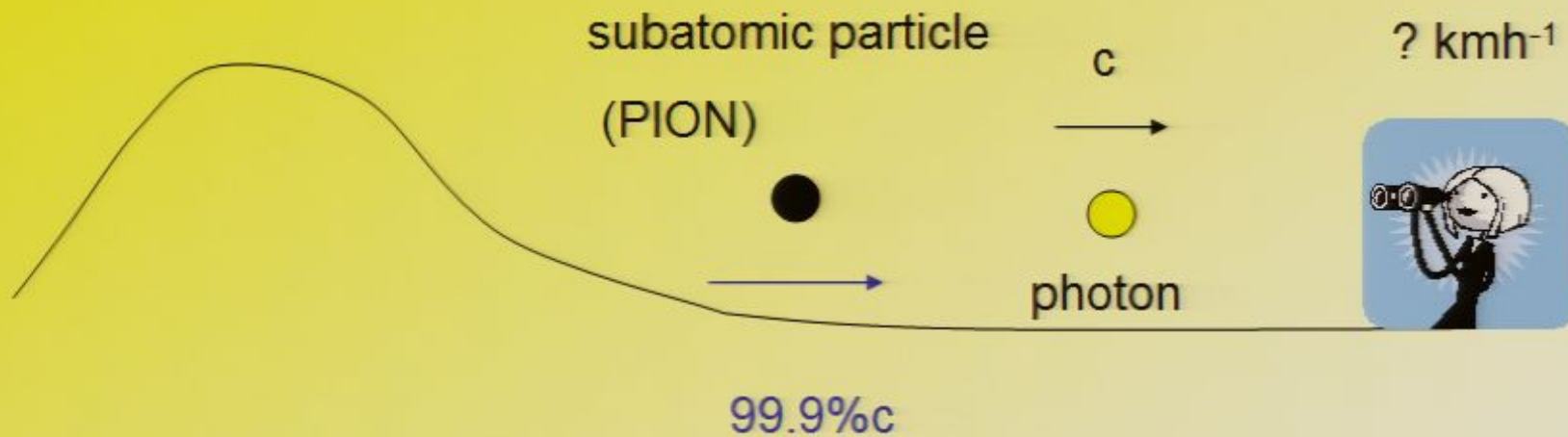
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- a)  $c$
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- What speed will you see the light moving towards you at?
- a)  $c$
- b)  $199.9\% c$
- c)  $99.9\% c$
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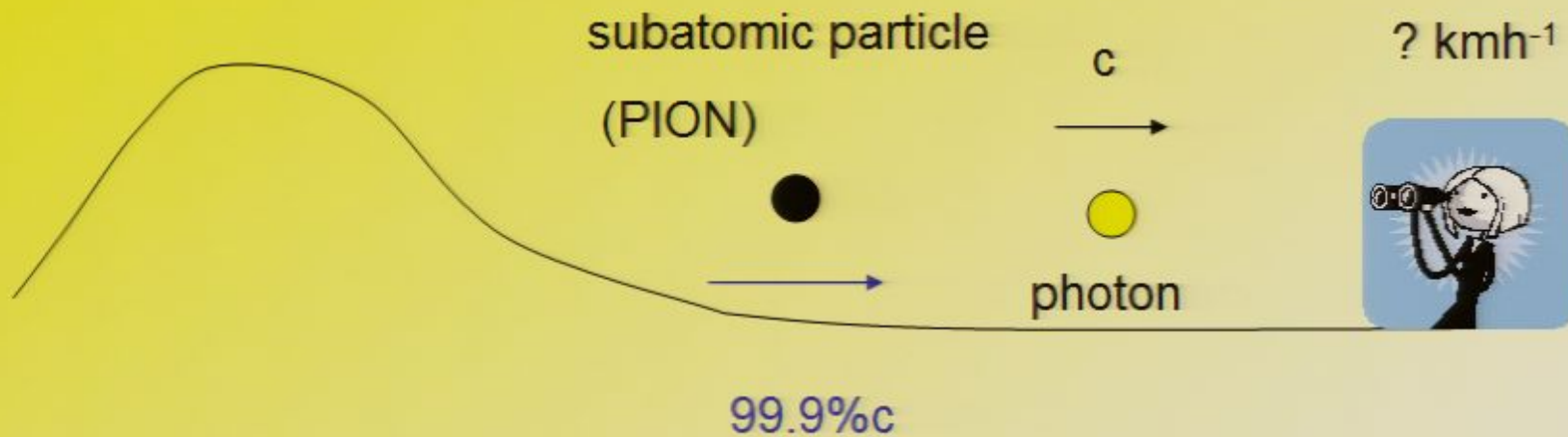
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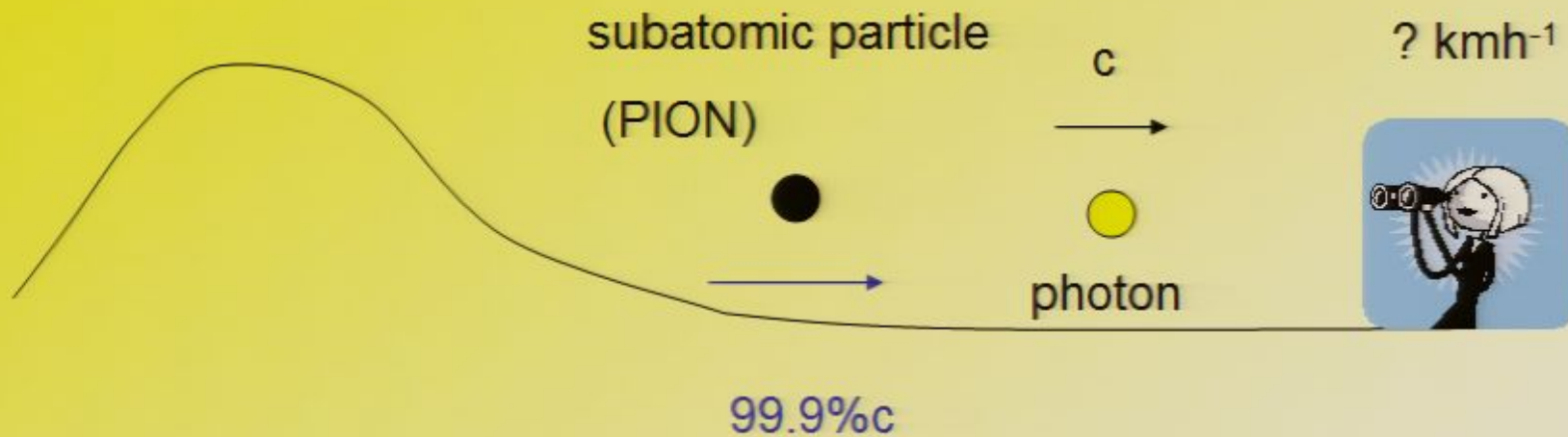
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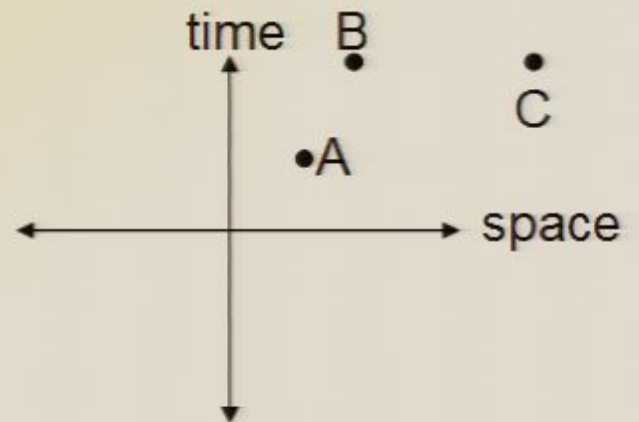
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# Spacetime diagrams

- Consider the following unrelated happenings or *events*.  
An apple falling from a tree, (A)  
A bird taking flight from a tree, (B)  
A car coming to a stop at a set of traffic lights, (C)

They all occur at a particular location and time

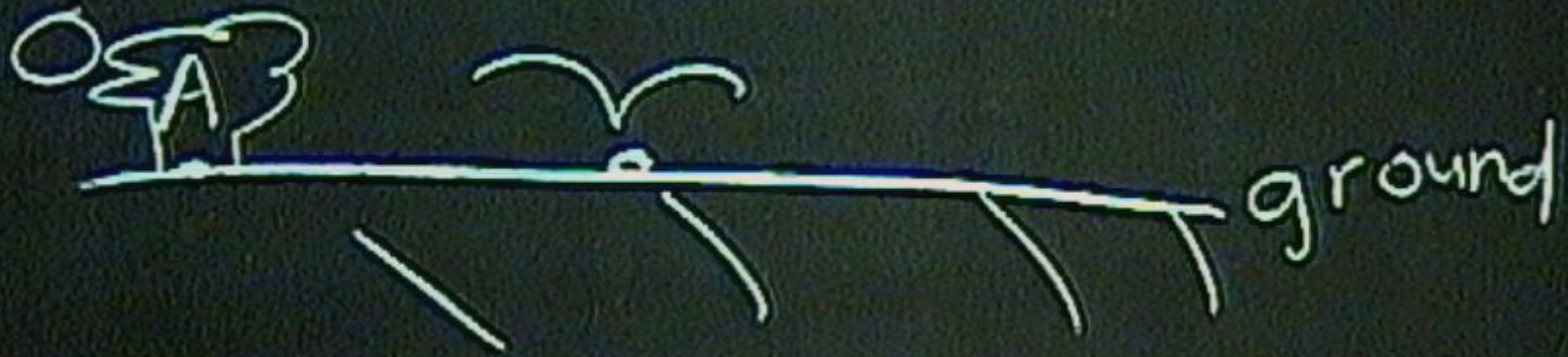
- Natural to plot or represent them graphically on a graph with space and time axes.
- By convention, we put time on the vertical axis.

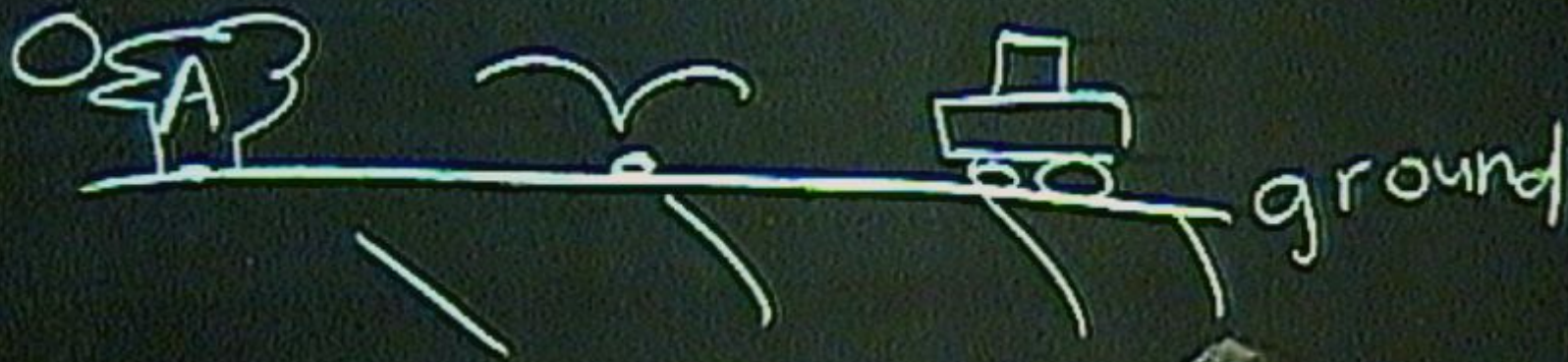


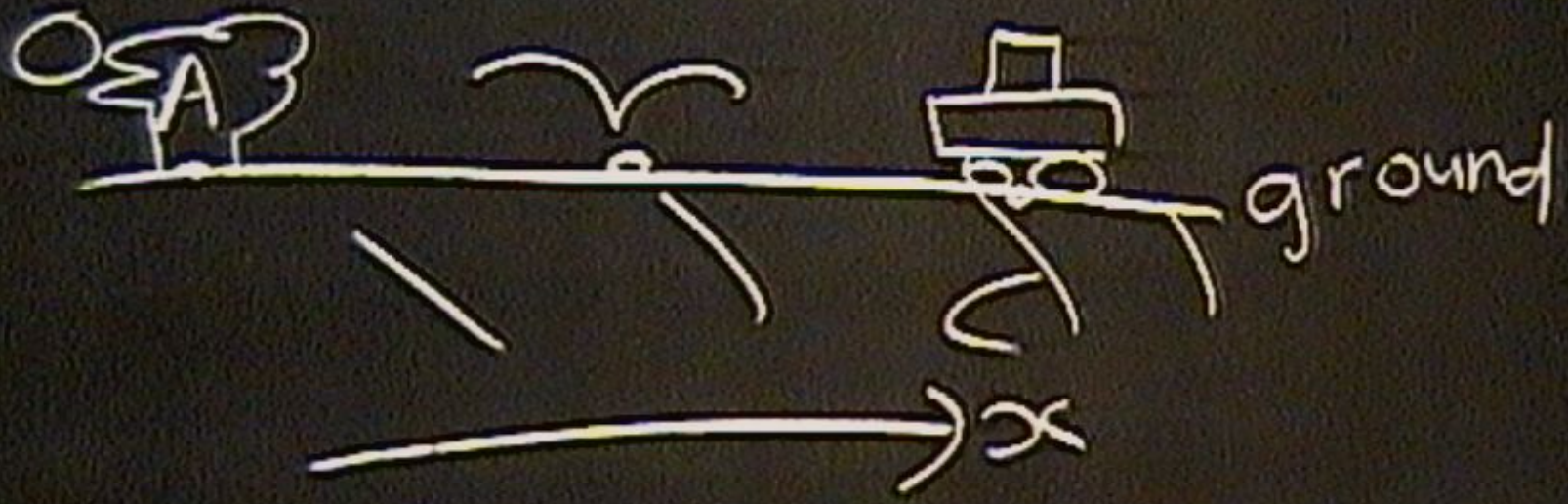
A

ground







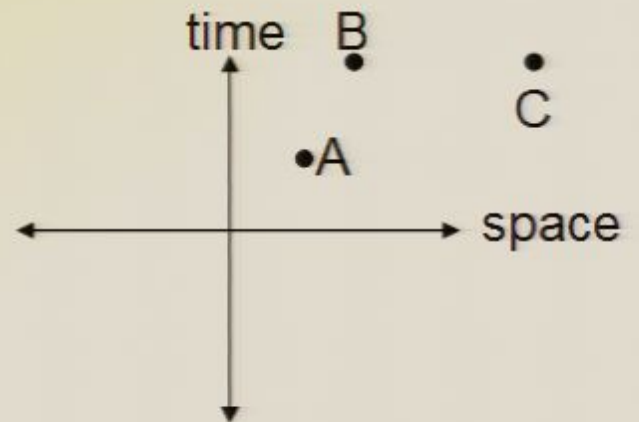


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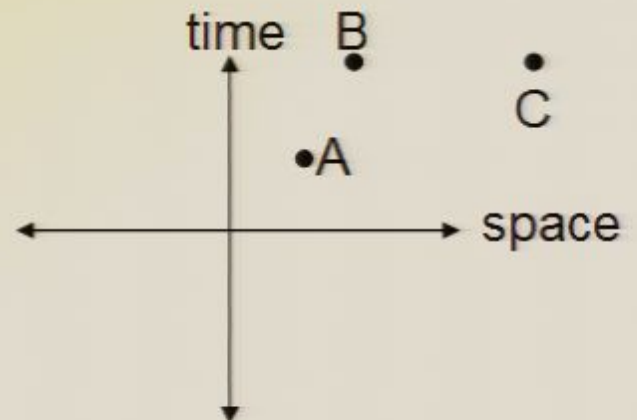


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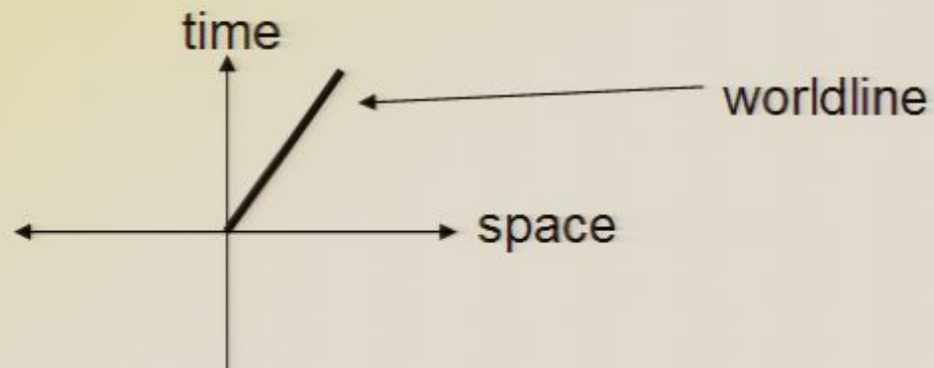
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- Such a graph is called a *spacetime* diagram. Incredibly useful tool for understanding special relativity.

*“Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”* — Hermann Minkowski

What do a series of events look like on a spacetime diagram? Eg. a car driving at  $60\text{kmh}^{-1}$  in one direction on a highway.

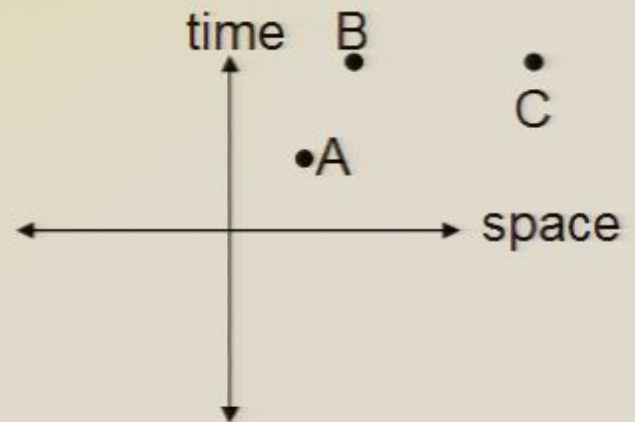


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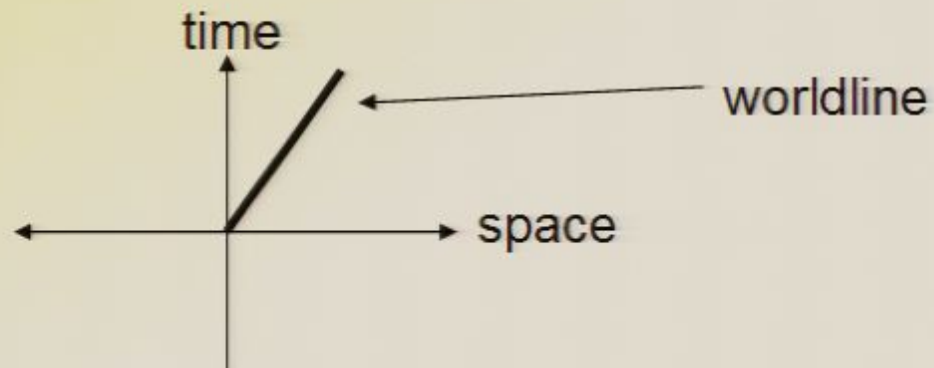
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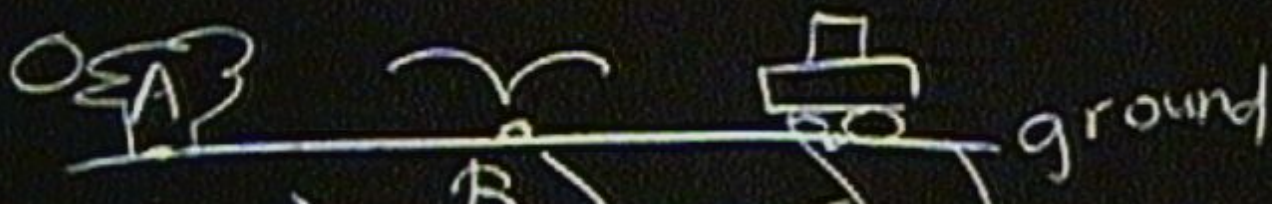
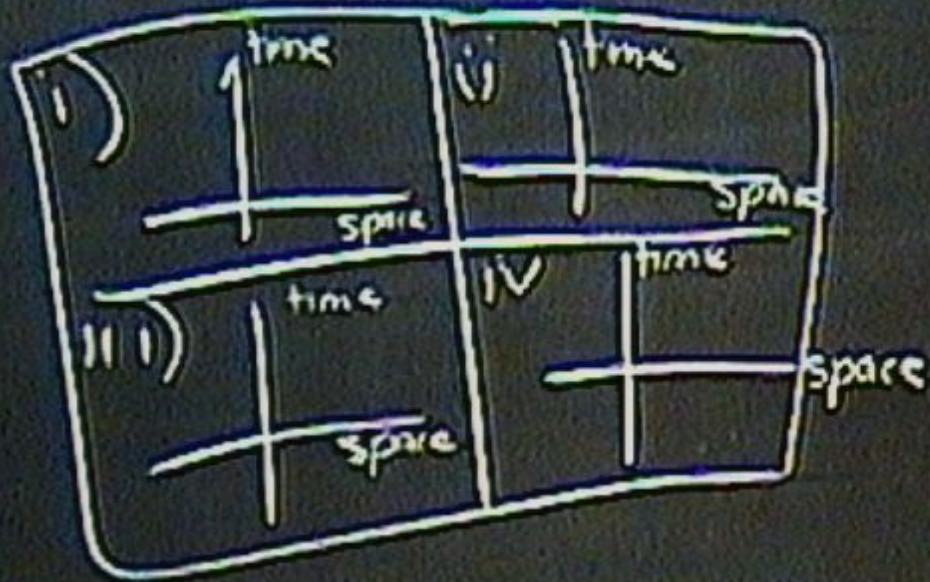




# Activity

Draw spacetime diagrams that show the worldlines for:

- i) A car that drives for one hour at  $100 \text{ kmh}^{-1}$  in one direction, turns around, and then drives for an hour in the opposite direction
- ii) A Grade 12 physics student standing still waiting for a bus
- iii) An Olympic athlete running at a constant velocity in a 100 metres race.
- iv) A car driving around in circles (n.b. this requires a 3D spacetime diagram).



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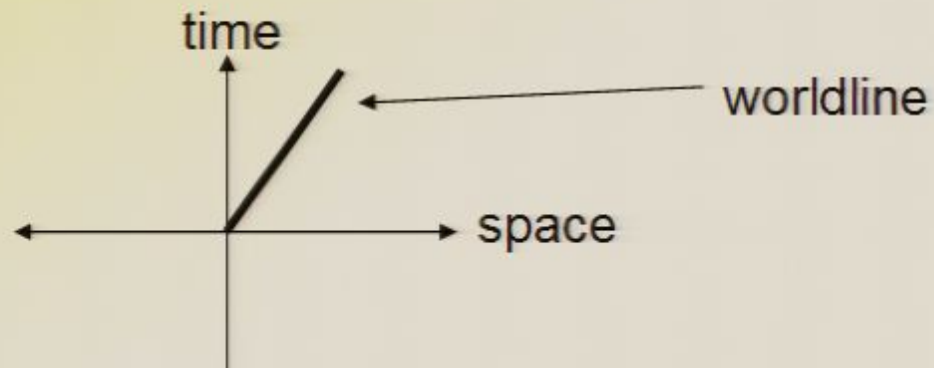
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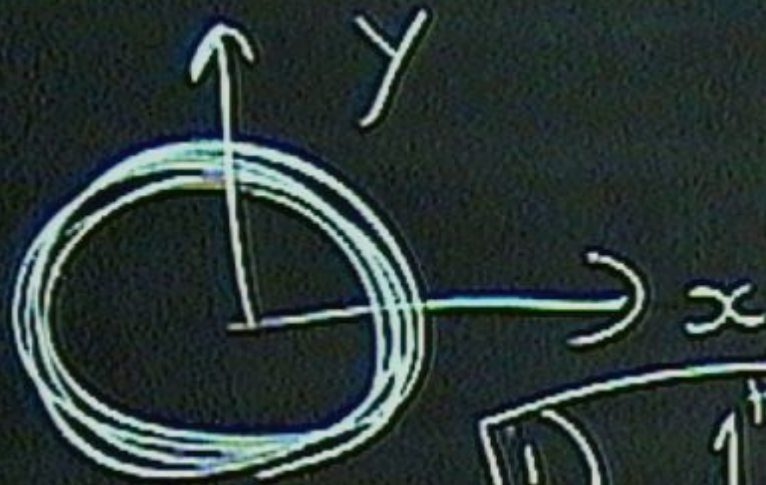


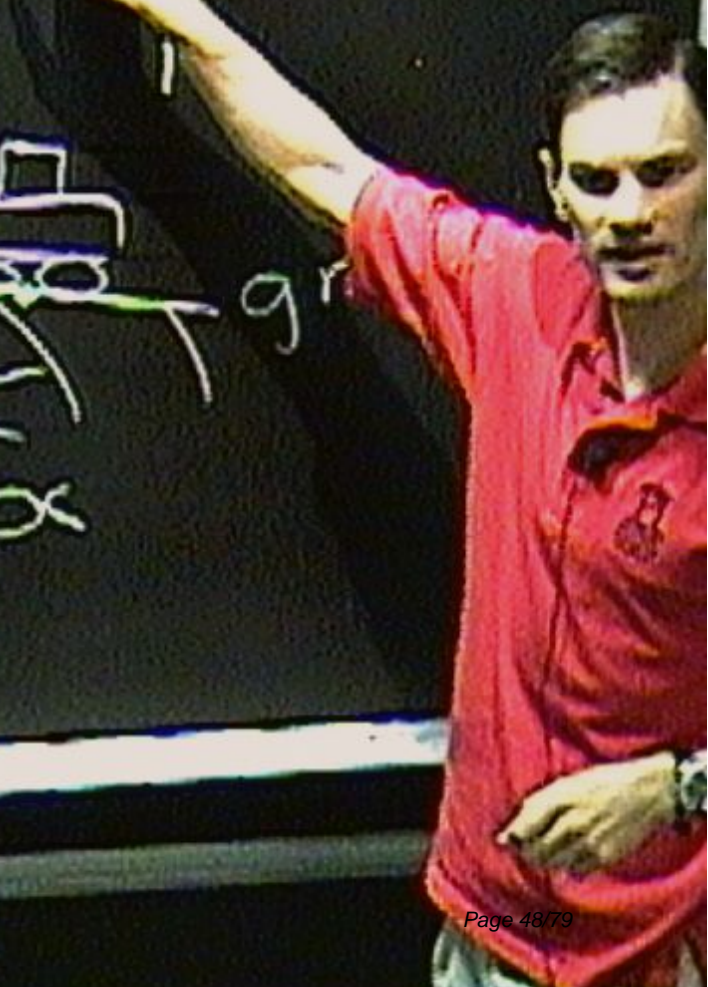
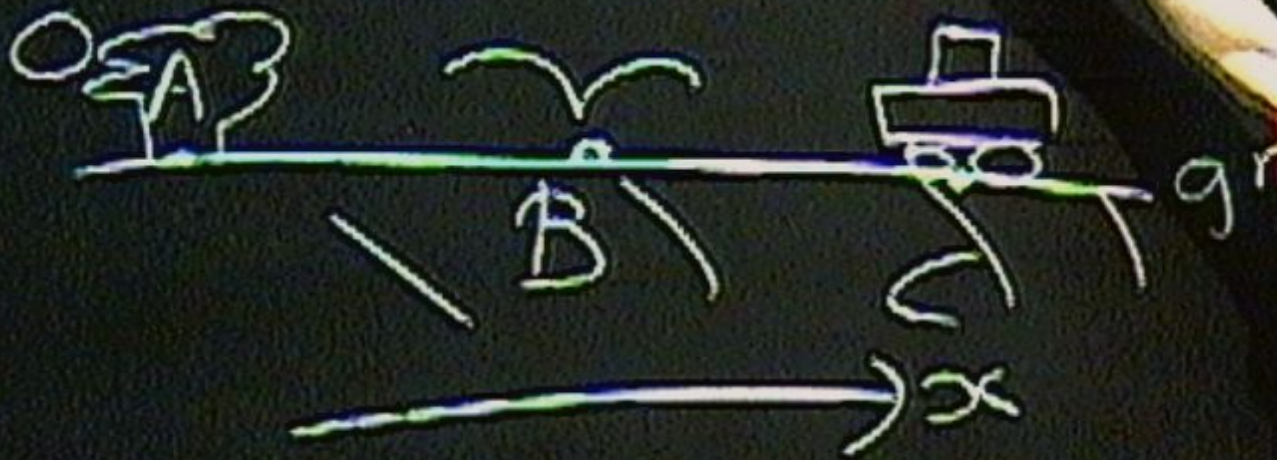
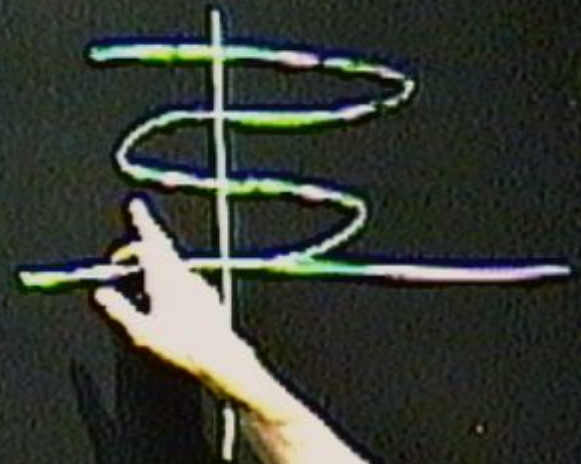
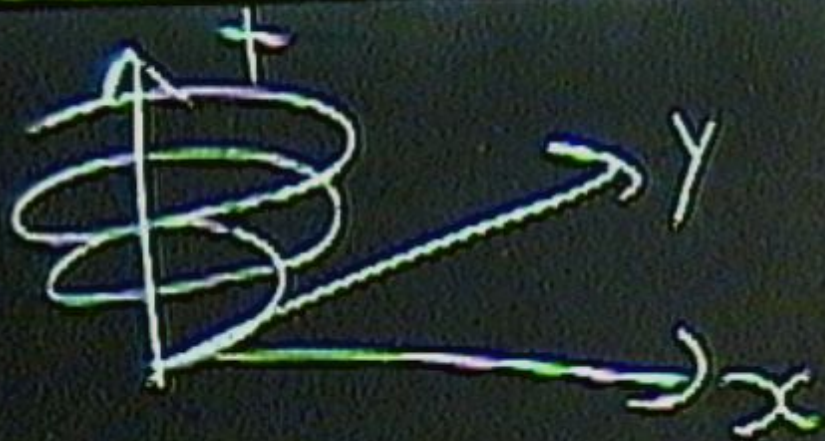
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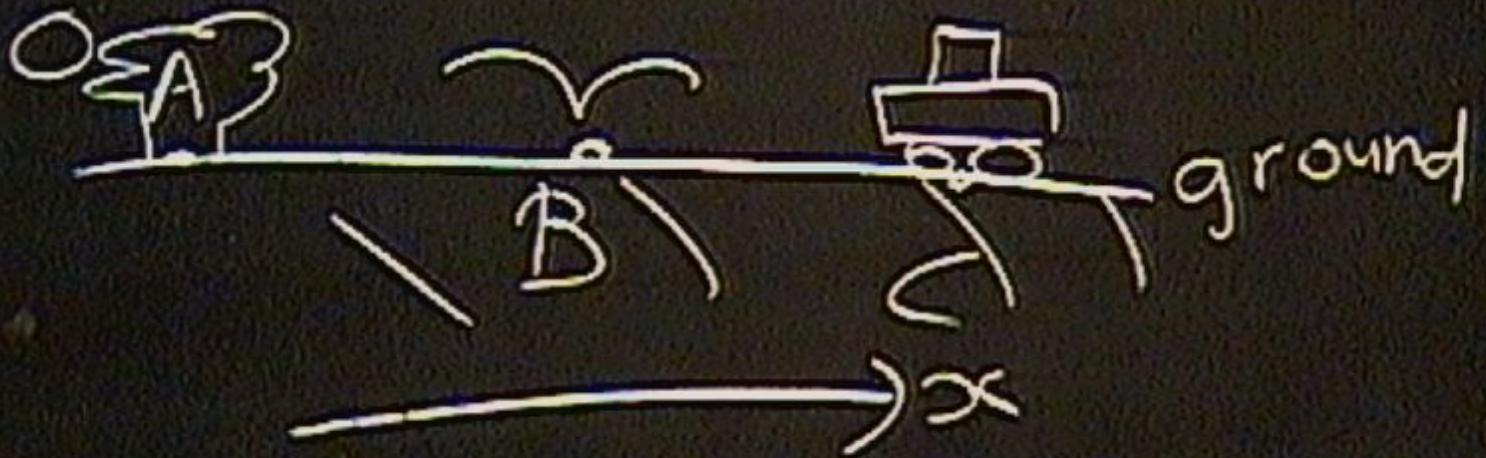
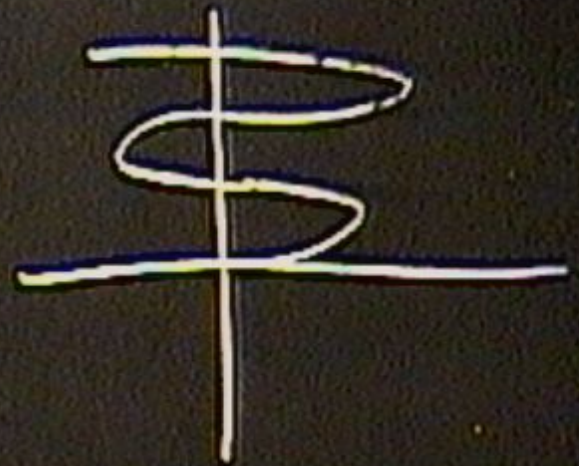
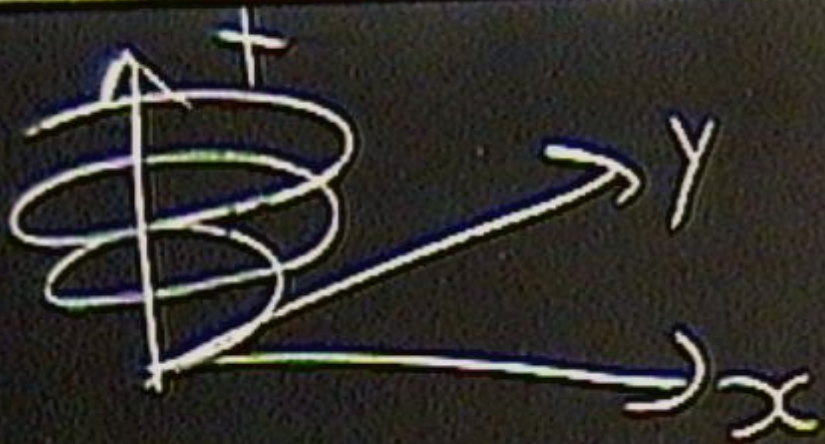
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# The metric of space and time

- Imagine that we would like to calculate the distance between points A and B on the map below.
- Use the Pythagorean theorem  $d^2 = \Delta x^2 + \Delta y^2$  (neglecting the earth's curvature)

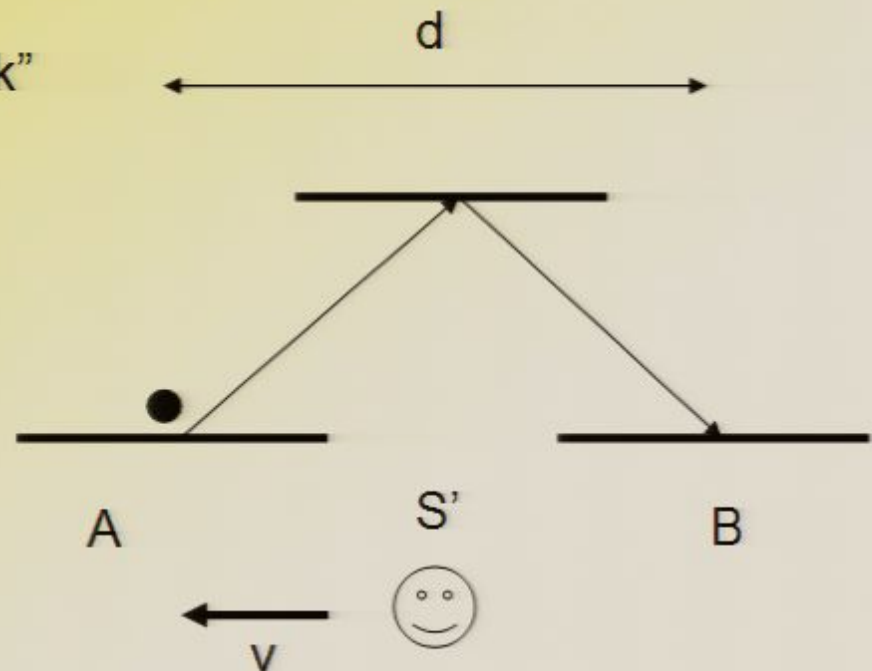
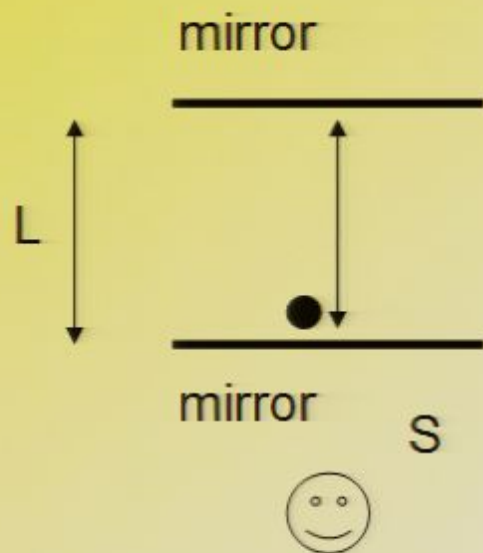
This is called a *metric equation*

*metric* = technical term for distance



# What is the metric equation for spacetime

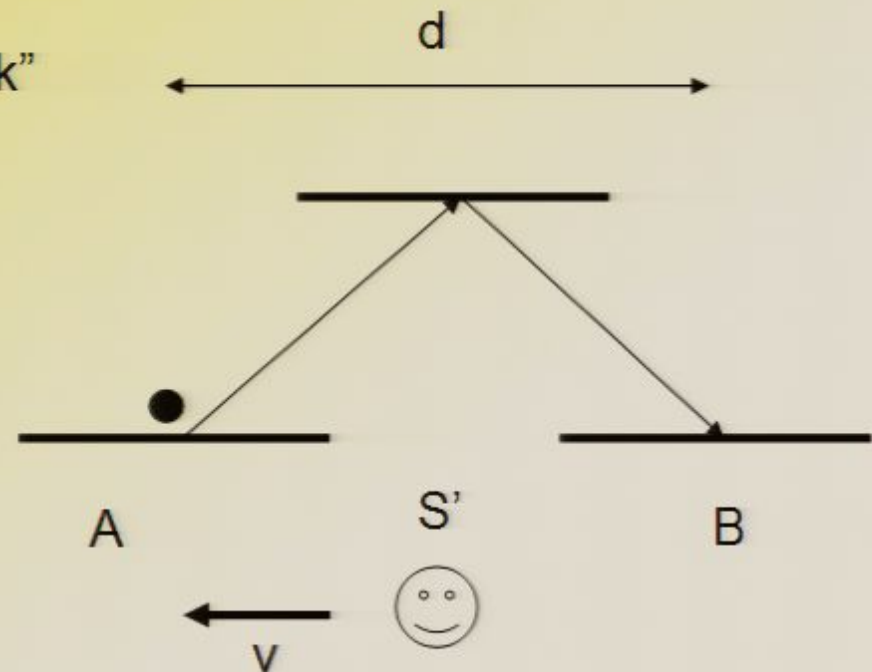
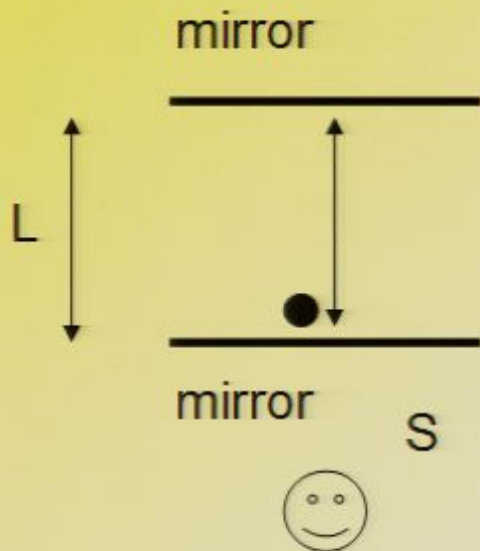
- As the two axes are  $x$  and  $t$  is it just  $d^2 = \Delta x^2 + (c\Delta t)^2$ ?
- Or perhaps, according to Einstein, there is something different about space and time.
- To help decide, consider a “light clock”



Pirsa: 06070006 A and B are the events corresponding to a photon hitting the bottom mirror on two successive occasions. Page 5 | 79

# What is the metric equation for spacetime

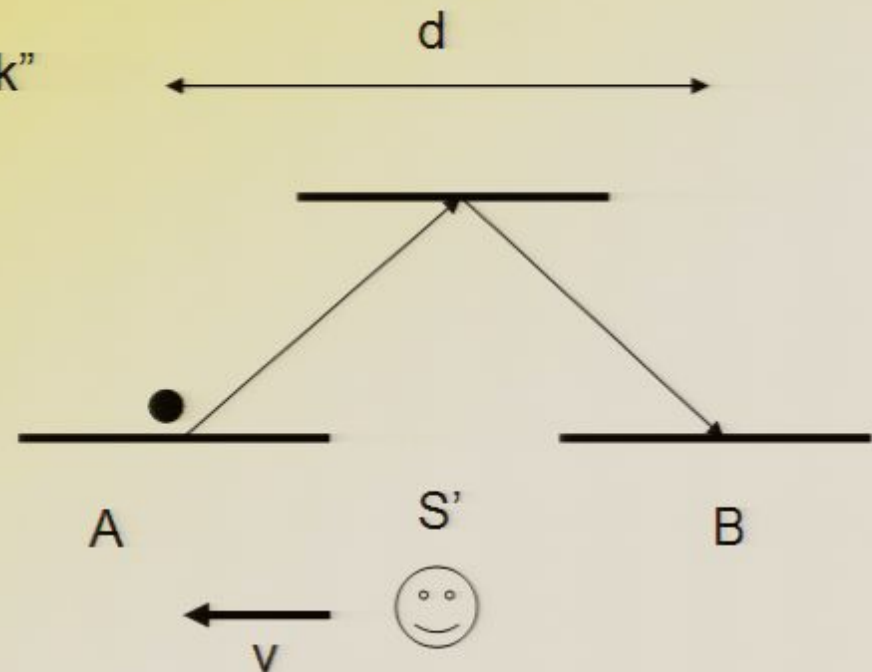
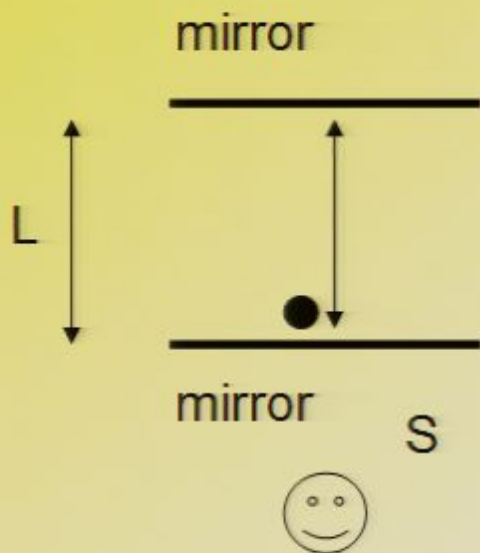
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Pirsa: 06070006  $A$  and  $B$  are the events corresponding to a photon hitting the bottom mirror on two successive occasions. Page 52/79

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- Let us denote the spacetime 'distance' or interval between A and B by  $s$

- In frame  $S$ , this interval is made up from just time and so  $s=2L$   
(Using 'light time' where we measure time by the distance light travels in the time under consideration.)

- In frame  $S'$ , the time  $t'$  between A and B is given by
 
$$t' = \frac{\text{distance}}{\text{speed}} = \frac{2\sqrt{L^2 + \left(\frac{d}{2}\right)^2}}{c}$$

- Plugging  $L=s/2$  into the above equation and squaring both sides yields

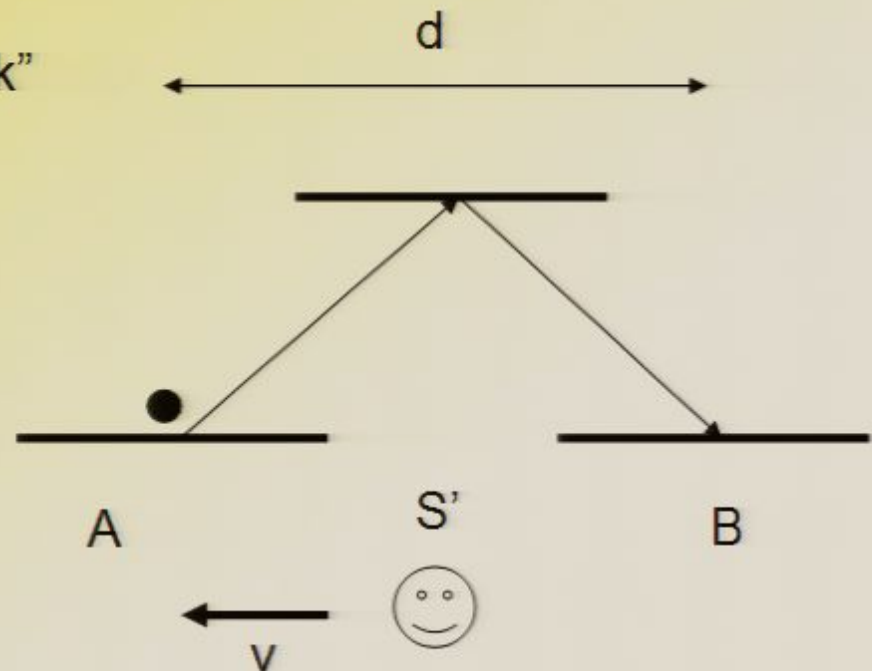
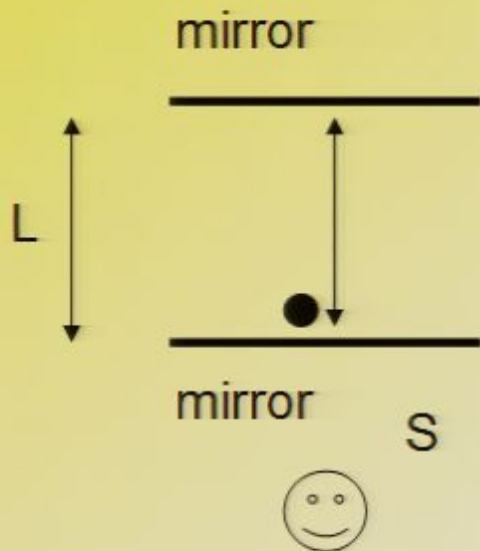
$$t'^2 = \frac{s^2 + d^2}{c^2}$$

$$\therefore s^2 = (ct')^2 - d^2$$

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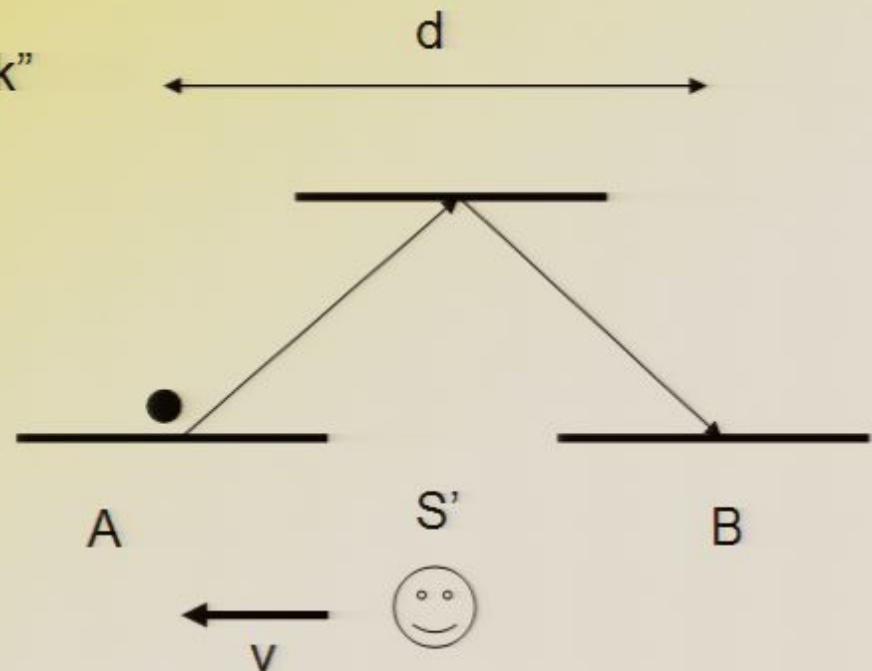
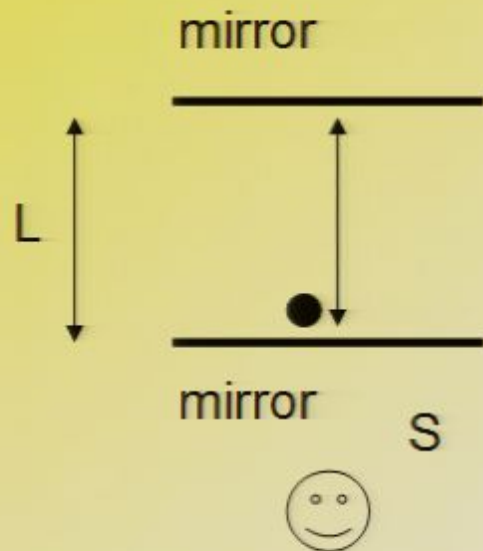
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- As the two axes are  $x$  and  $t$  is it just  $d^2 = \Delta x^2 + (c\Delta t)^2$ ?
- Or perhaps, according to Einstein, there is something different about space and time.
- To help decide, consider a “light clock”



- Let us denote the spacetime 'distance' or interval between A and B by  $s$

- In frame  $S$ , this interval is made up from just time and so  $s=2L$   
(Using 'light time' where we measure time by the distance light travels in the time under consideration.)

- In frame  $S'$ , the time  $t'$  between A and B is given by
 
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- Plugging  $L=s/2$  into the above equation and squaring both sides yields

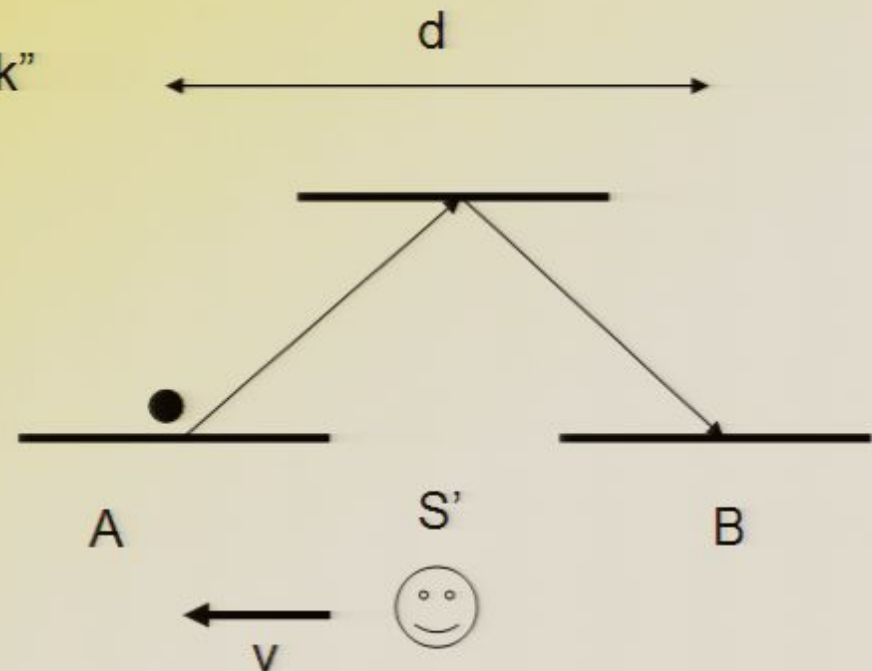
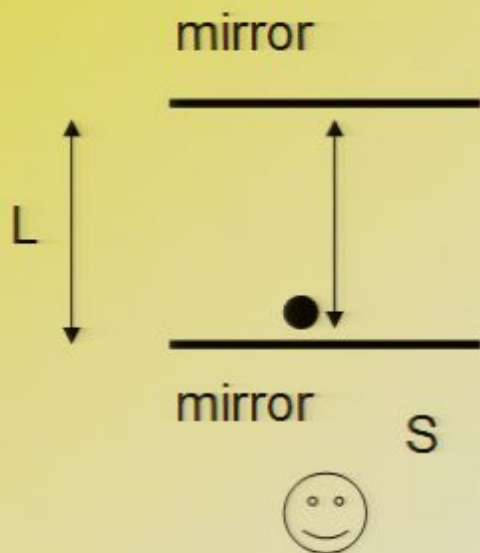
$$t'^2 = \frac{s^2 + d^2}{c^2}$$

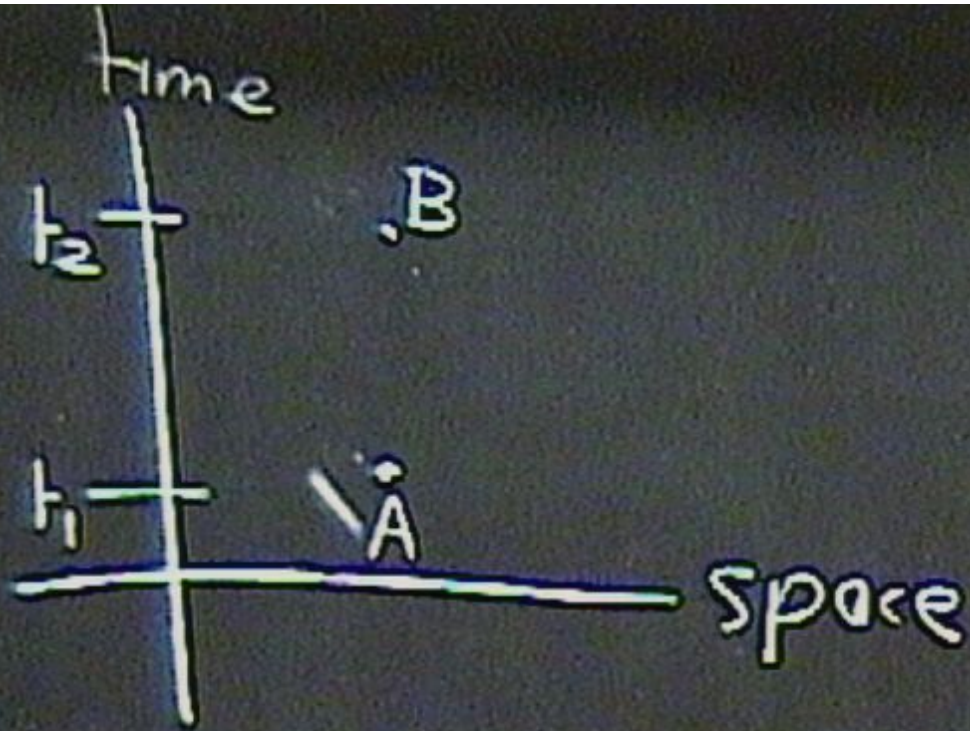
$$\therefore s^2 = (ct')^2 - d^2$$

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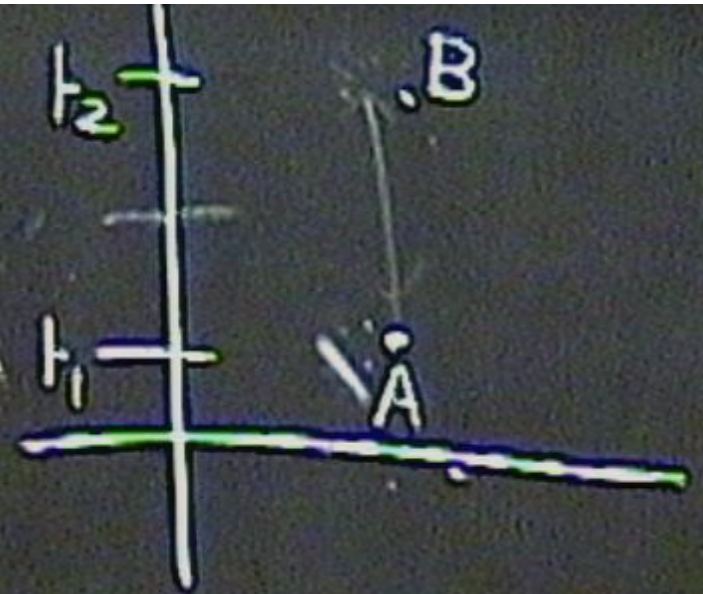
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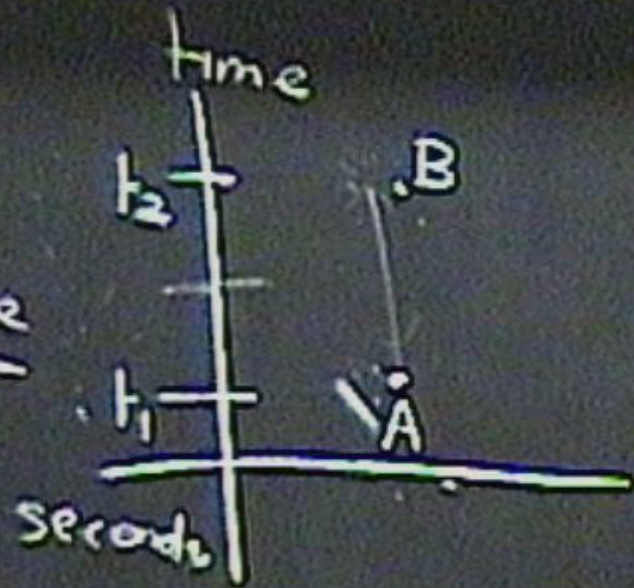
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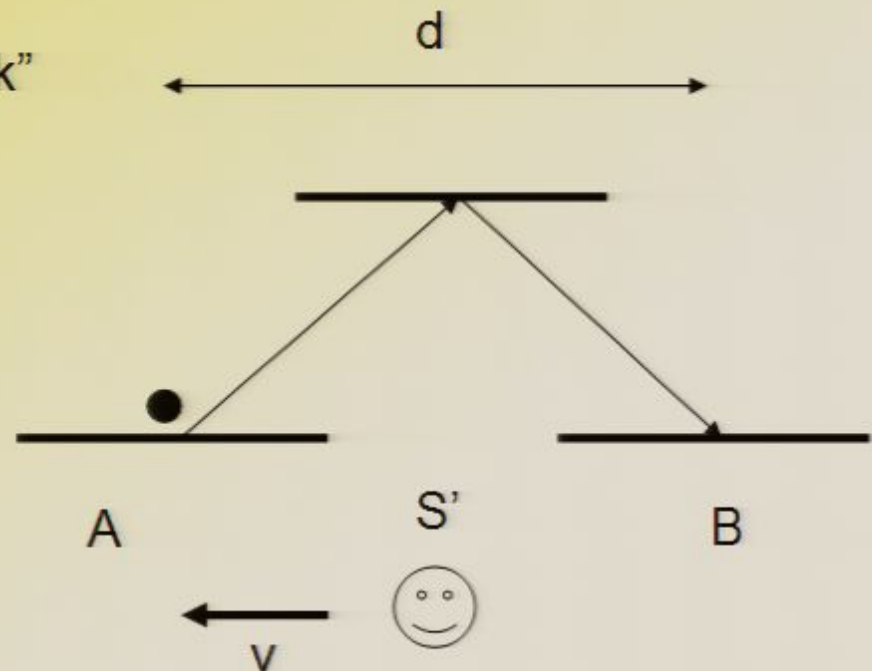
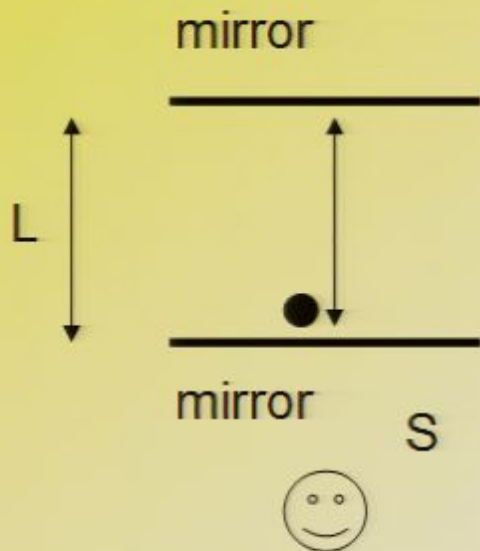


SI units  
 1 second  $\times$  seconds  
 light time  
 $3 \times 10^8 \times \times$  metres



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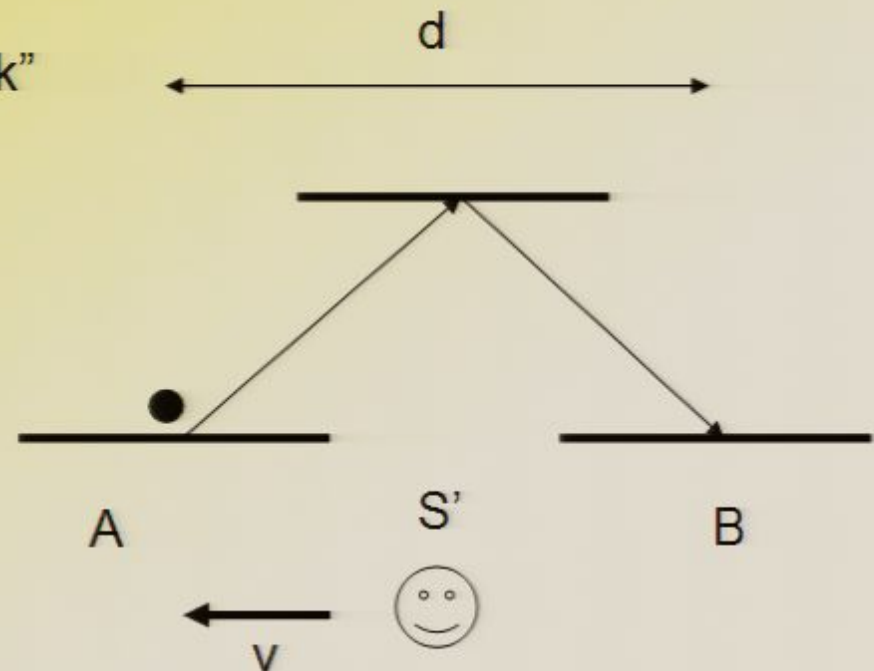
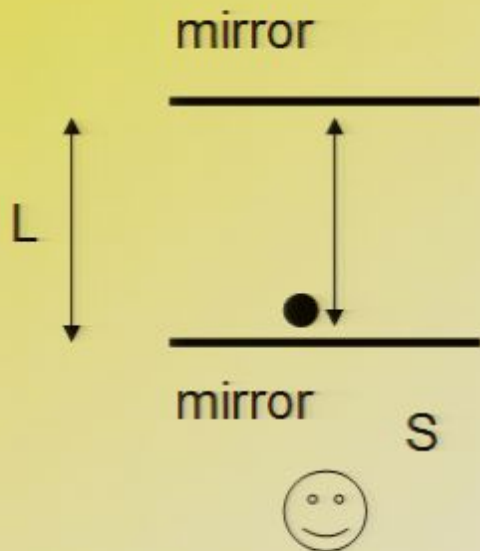
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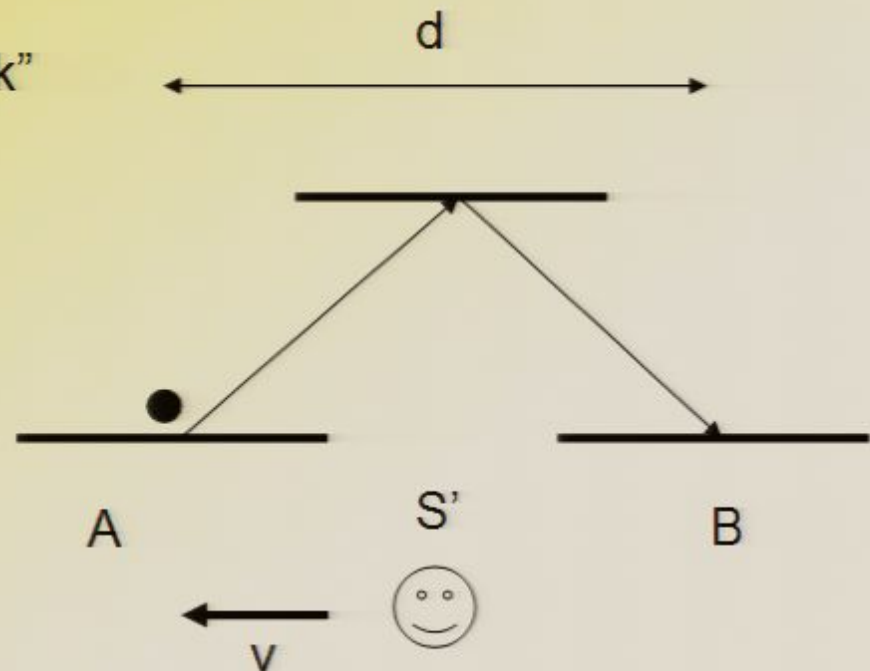
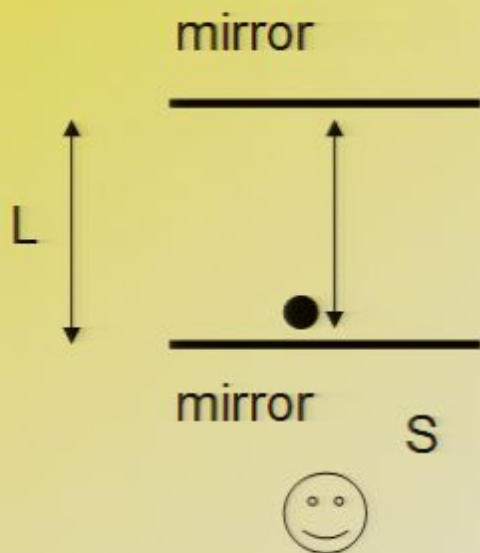
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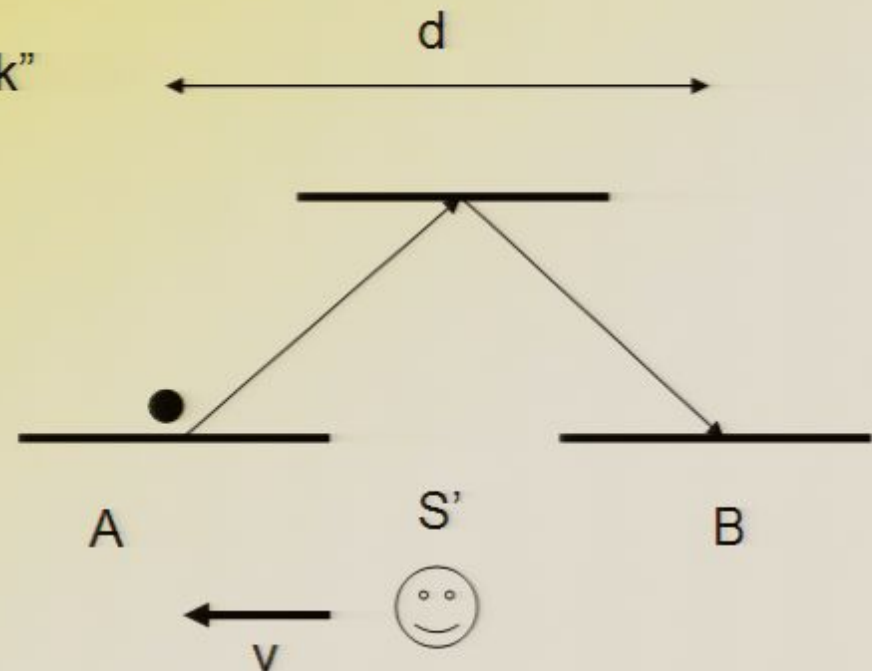
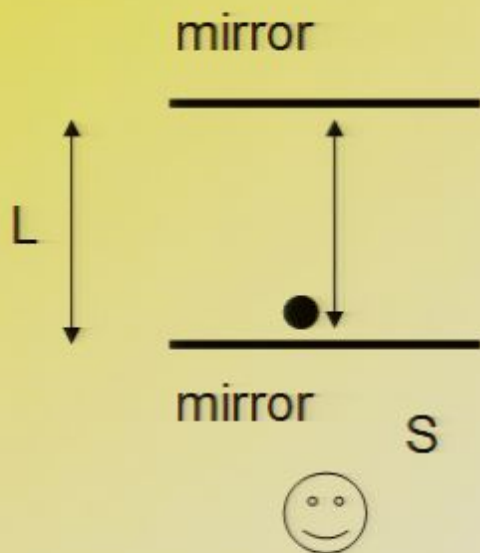
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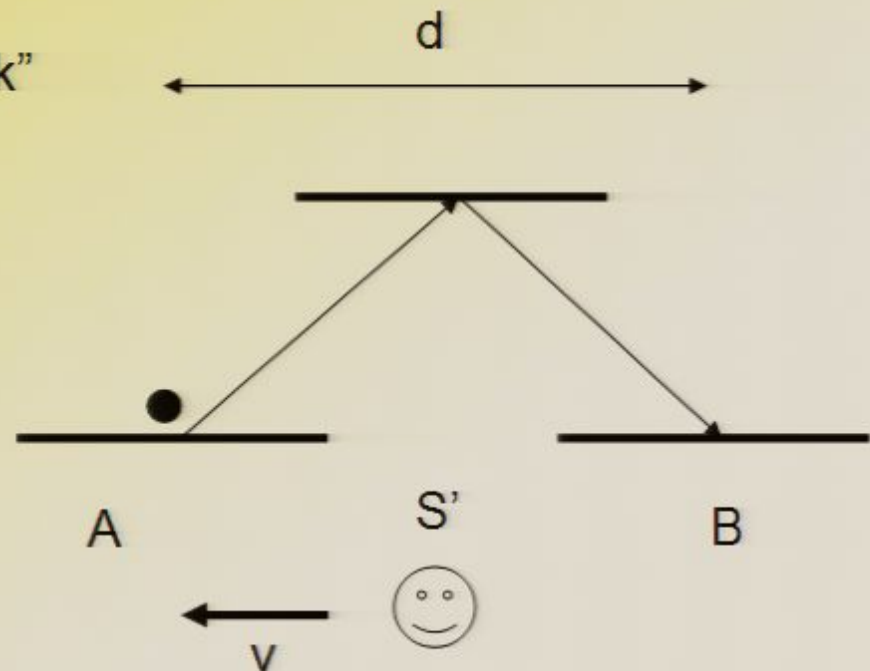
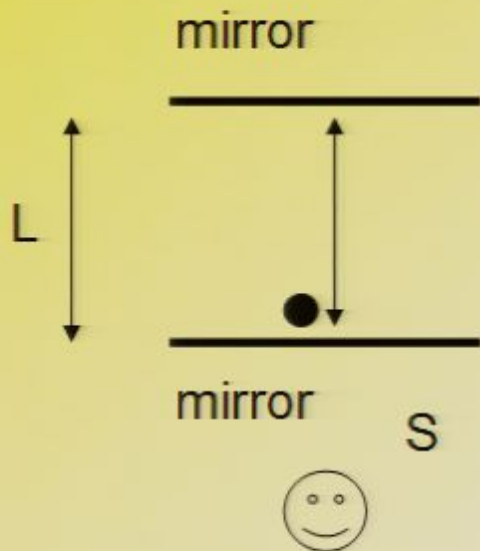
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# The metric of space and time

- Imagine that we would like to calculate the distance between points A and B on the map below.
- Use the Pythagorean theorem  $d^2 = \Delta x^2 + \Delta y^2$  (neglecting the earth's curvature)

This is called a *metric equation*

*metric* = technical term for distance



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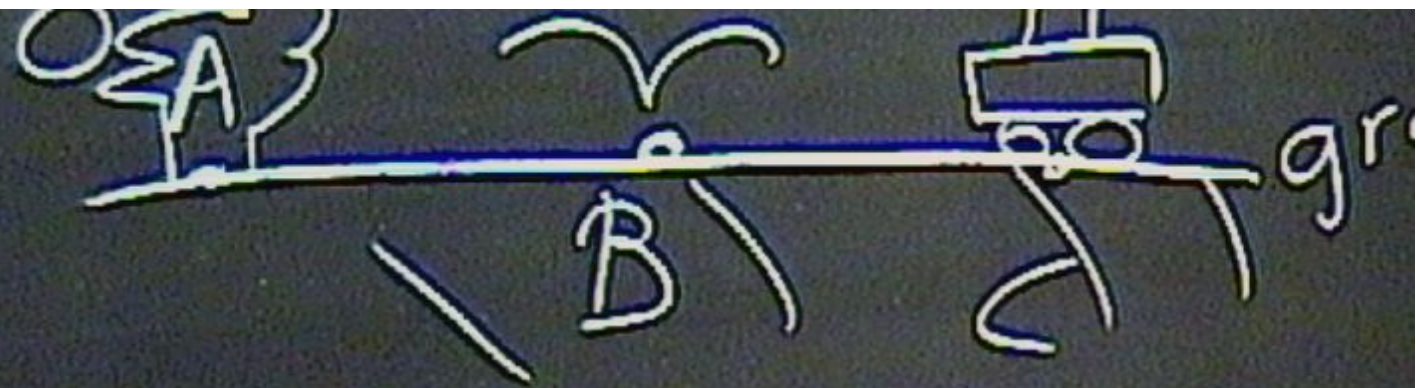
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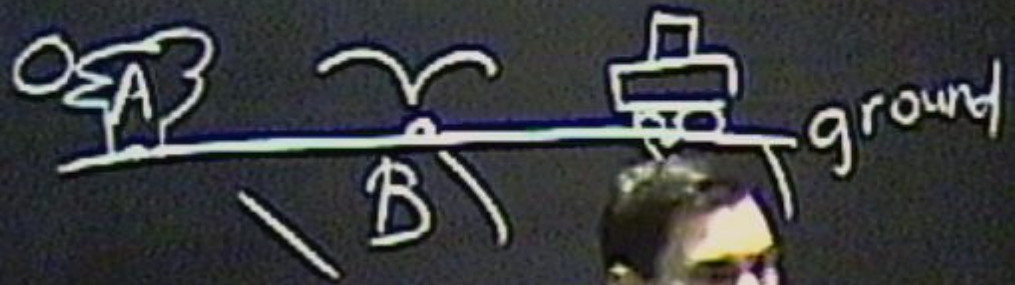
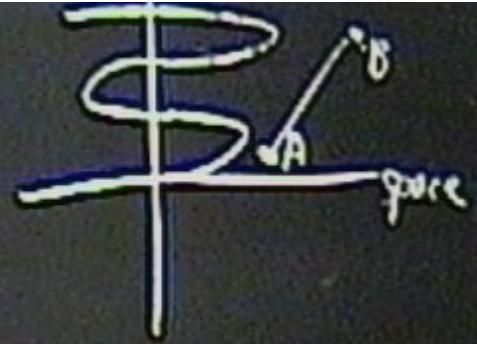
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$$A^2 (\Delta t)^2 + (\Delta x)^2 \longrightarrow x$$





$$(\Delta t)^2 + (\Delta x)^2$$

