Title: Quantum Theory in Cosmology

Date: Jun 28, 2006 02:00 PM

URL: http://pirsa.org/06060061

Abstract: Not only general relativity but also quantum theory plays important roles in current cosmology. Quantum fluctuations of matter fields are supposed to have provided the initial seeds of all the structure of the current universe, and quantum gravity is assumed to have been essential in the earliest stages. Both issues are not fully understood, although several heuristic effects have been discussed. In this talk, implications of an effective framework taking into account the coupling of matter and gravity are discussed. This touches on interpretational issues of quantum mechanics, cosmological observations and properties of quantum gravity.

Quantum Theory in Cosmology

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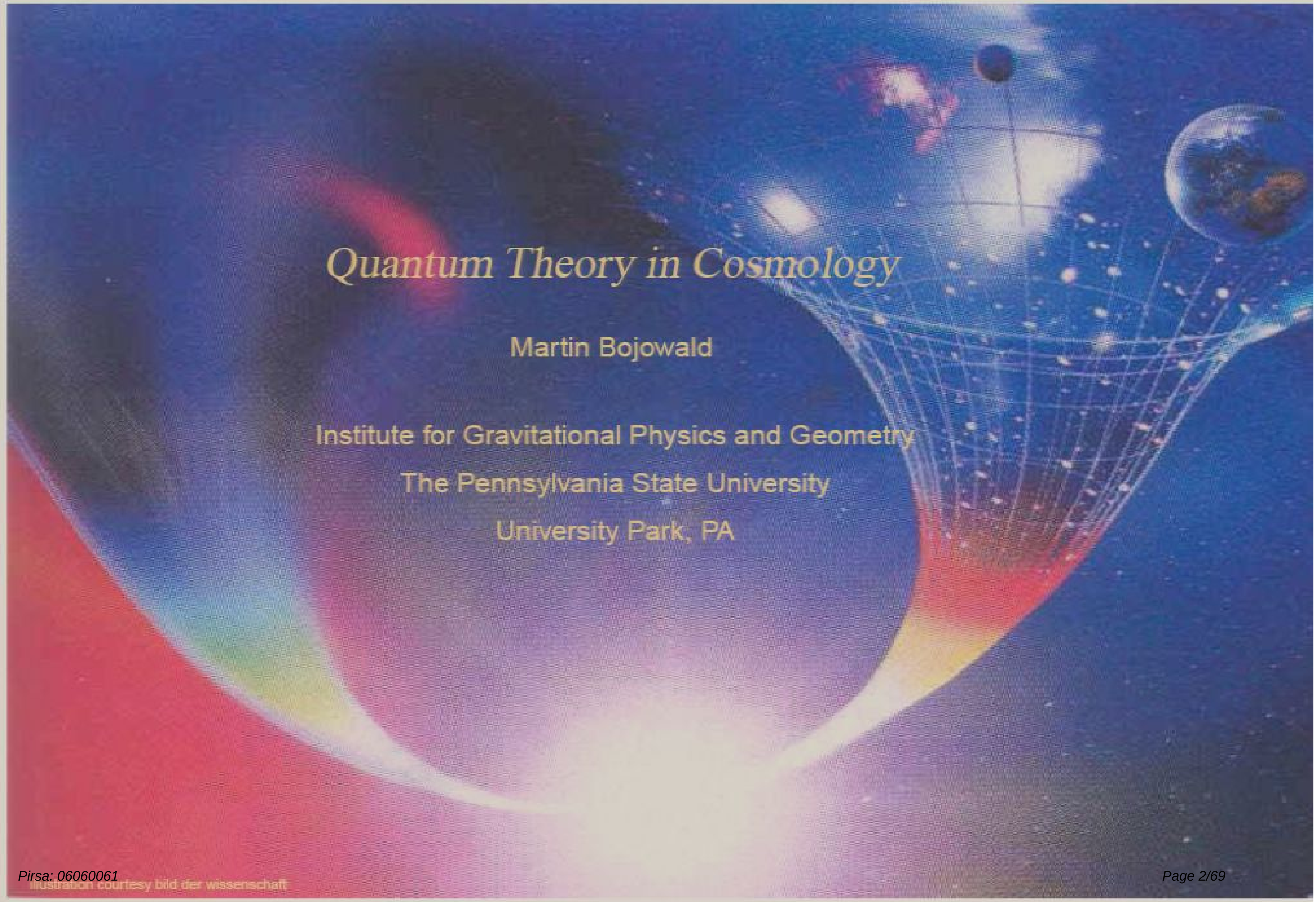

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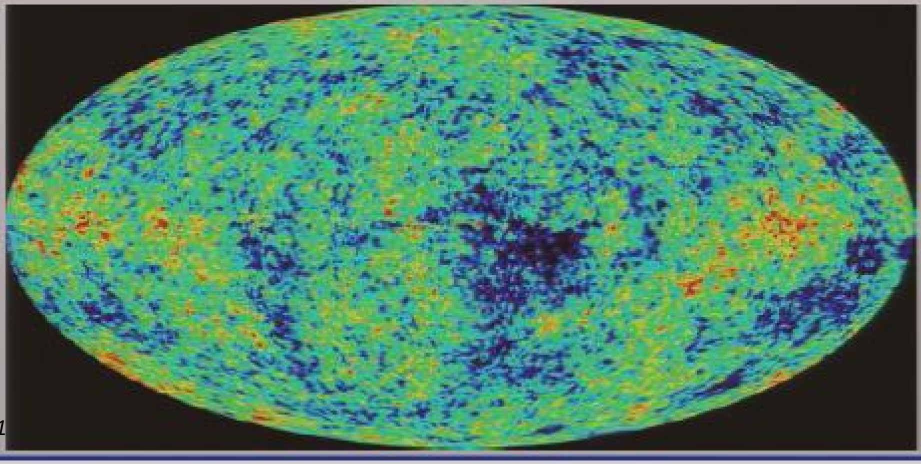
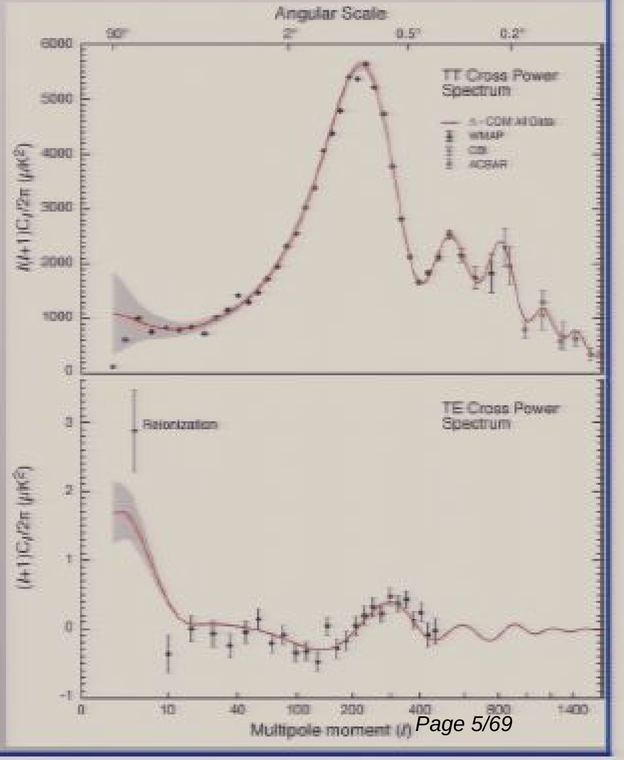
Scale dependence agreeing with microwave background anisotropies results with long phase of nearly exponential expansion in early universe.



WMAP



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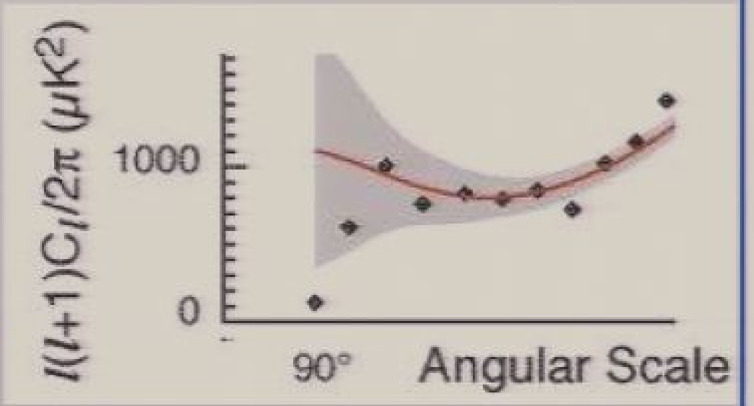

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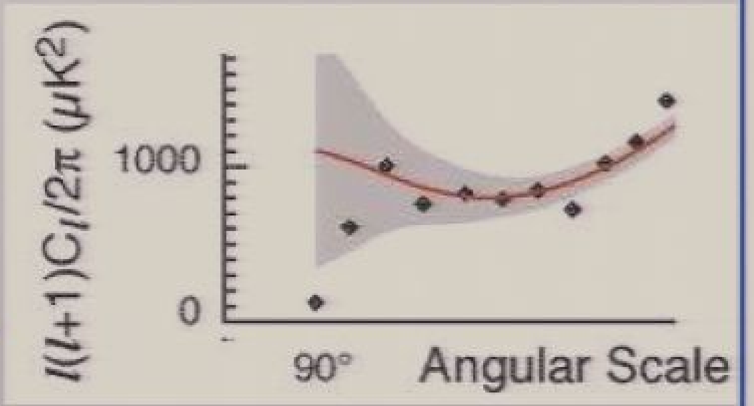

⟶ Models start from *singular initial states* in general relativity; *quantum gravity* corrections expected.

Matter fields and gravity

Two main implications of scalar Hamiltonian

$$H_\phi = \int \mathrm{d}^3x N \left(\frac{1}{2} q^{-3/2} {p_\phi}^2 + \frac{1}{2} q^{3/2} \nabla\phi \cdot \nabla\phi + q^{3/2} V(\phi) \right)$$

coupling scalar ϕ to metric components q and N; *interacting field theory* even for “free” scalar with $V(\phi) = \frac{1}{2} m^2 \phi^2$.

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$$\langle \hat{\phi}^2 \rangle = \langle \hat{\phi} \rangle^2 + \Delta\phi^2 \neq \langle \hat{\phi} \rangle^2$$

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Difficult to quantize gravity, but many issues can be analyzed in *effective theory.* Well known from low energy effective actions (perturbations around vacuum state), but need to be generalized for matter fields on dynamical geometry.

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Generalization available from *geometrical formulation* of quantum mechanics: View Hilbert space of quantum system as *infinite-dimensional phase space,* Poisson brackets given by imaginary part of inner product.

Choose expansion coefficients in $|\psi\rangle = \sum_j c_j|\psi_j\rangle$ as coordinates on phase space, then

$$\{\mathrm{Re}c_j, \mathrm{Im}c_k\} = \frac{1}{2\hbar}\delta_{jk}$$

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Quantum Hamiltonian

Schrödinger equation for $|\psi\rangle$ equivalent to Hamiltonian equations of motion from *quantum Hamiltonian* $H_Q(c_j) = \langle\hat{H}\rangle$ for expectation value taken in state with expansion coefficients c_j: choose eigenbasis $|\psi_j\rangle$ of $\hat{H}$, then $H_Q(c_j) = \sum_j E_j|c_j|^2$ and

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Re}c_j &= \{\mathrm{Re}c_j, H_Q\} = \frac{E_j}{\hbar}\mathrm{Im}c_j \\ \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Im}c_j &= \{\mathrm{Im}c_j, H_Q\} = -\frac{E_j}{\hbar}\mathrm{Re}c_j \end{aligned}$$

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Quantum mechanics formally much closer to classical mechanics, relation through *effective equations* becomes possible in suitable approximations, including a truncation to *finitely many variables.*

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Quantum variables

More useful set of coordinates: classical variables $q = \langle\hat{q}\rangle$ and $p = \langle\hat{p}\rangle$ and *quantum variables* $(n \geq 2,\, a = 0, \ldots, n)$

$$G^{a,n} := \langle(\hat{q} - \langle\hat{q}\rangle)^{n-a}(\hat{p} - \langle\hat{p}\rangle)^{a}\rangle_{\rm Weyl}$$

[MB, A. Skirzewski: math-ph/0511043, hep-th/0606232]

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Poisson relations related to commutators: $\{q, p\} = 1$, $\{q, G^{a,n}\} = 0 = \{p, G^{a,n}\}$, $\{G^{a,n}, G^{b,m}\} = \cdots$.

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Quantum variables dynamical just as classical variables: change in time if *wave packet spreads and deforms; back-reacts* on classical variables which determine peak position of wave packet.

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Exact behavior determined by Schrödinger equation, or by quantum Hamiltonian which *couples classical and quantum variables.*

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Quantum Hamiltonian

Consider an-harmonic oscillator with classical Hamiltonian $H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2 + U(q)$; introduce dimensionless $\tilde{G}^{a,n} = \hbar^{-n/2}(m\omega)^{n/2-a}G^{a,n}$.

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Quantum Hamiltonian with *coupling terms*

$$\begin{aligned} H_Q &= \langle H(\hat{q},\hat{p})\rangle = \langle H(q+(\hat{q}-q), p+(\hat{p}-p))\rangle \\ &= \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2 + U(q) + \frac{\hbar\omega}{2}(\tilde{G}^{0,2}+\tilde{G}^{2,2}) \\ &\quad + \sum_{n>2}\frac{1}{n!}\left(\frac{\hbar}{m\omega}\right)^{n/2} U^{(n)}(q)\tilde{G}^{0,n} \end{aligned}$$

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[half-integer powers $\geq 3/2$ of $\hbar$ in correction terms]

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Equations of motion

H_Q generates Hamiltonian equations of motion $\dot{f} = \{f, H_Q\}$:

$$\dot{q} = \frac{p}{m}$$

$$\dot{p} = -m\omega^2 q - U'(q) - \sum_n \frac{1}{n!}\left(\frac{\hbar}{m\omega}\right)^{n/2} U^{(n+1)}(q)\tilde{G}^{0,n}$$

$$\dot{\tilde{G}}^{a,n} = -a\omega\tilde{G}^{a-1,n} + (n-a)\omega\tilde{G}^{a+1,n} - a\frac{U''(q)}{m\omega}\tilde{G}^{a-1,n} + \frac{\sqrt{\hbar}aU'''(q)}{2(m\omega)^{\frac{3}{2}}}\tilde{G}^{a-1,n-1}\tilde{G}^{0,2} + \frac{\hbar aU''''(q)}{3!(m\omega)^2}\tilde{G}^{a-1,n-1}\tilde{G}^{0,3} - \frac{a}{2}\left(\frac{\sqrt{\hbar}U'''(q)}{(m\omega)^{\frac{3}{2}}}\tilde{G}^{a-1,n+1} + \frac{\hbar U''''(q)}{3(m\omega)^2}\tilde{G}^{a-1,n+2}\right) + \cdots$$

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\end{aligned}$$

∞ly many coupled equations for ∞ly many variables.

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Low energy effective action

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To first order in $\hbar$ and second in adiabatic approximation:

$$\left(m+\frac{\hbar U'''(q)^2}{32m^2\omega^5\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{5}{2}}}\right)\ddot{q}$$

$$+\frac{\hbar\dot{q}^2\left(4m\omega^2U'''(q)U''''(q)\left(1+\frac{U''(q)}{m\omega^2}\right)-5U'''(q)^3\right)}{128m^3\omega^7\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{7}{2}}}$$

$$+m\omega^2q+U'(q)+\frac{\hbar U'''(q)}{4m\omega\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{1}{2}}}=0\,.$$

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Effective systems

Agrees with result from low energy effective action, but more generally applicable beyond adiabatic approximation. For instance, some quantum variables can be kept independent.

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couples metric modes to quantum fluctuations of matter modes; to be expanded in quantum variables.

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Generates equations of motion which can be linearized and *modify classical perturbation equations by fluctuation terms.*

Fluctuation-metric coupling

For instance, kinetic term in Fourier modes:

$$qH_{\mathrm{kin}} = \frac{1}{2}\sum_{k} p_{\phi,k}p_{\phi,-k} - \frac{1}{4}\sum_{k,k'}(3q_{-k-k'} - 2N_{-k-k'})p_{\phi,k}p_{\phi,k'}$$
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$$G^{0,2}G^{2,2} \geq \frac{\hbar^2}{4} + (G^{1,2})^2$$

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Energy density modes given by $\rho_k \propto \partial H/\partial N_{-k}$, contain term $G^{2,2}_{k/2,k/2}$ which *must be non-zero;* source metric modes ψ_k.

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Perturbation equations

When added to gravitational Hamiltonian, *perturbation equations* are generated:

$$-k^2\psi_k-3\frac{\dot{a}}{a}\dot{\psi}_k-3\frac{\dot{a}^2}{a^2}\psi_k=\rho_k^{\rm class}+a^{-6}G^{2,2}_{k,0}+m^2G^{0,2}_{k,0}+\frac{1}{2}a^{-6}G^{2,2}_{\frac{k}{2},\frac{k}{2}}+\frac{1}{2}m^2G^{0,2}_{\frac{k}{2},\frac{k}{2}}$$

$$-\ddot{\psi}_k-3\frac{\dot{a}}{a}\dot{\psi}_k-2\left(\frac{\dot{a}}{a}\right)^{\cdot}\psi_k-3\frac{\dot{a}^2}{a^2}\psi_k=P_k^{\rm class}+a^{-6}G^{2,2}_{k,0}-m^2G^{0,2}_{k,0}+\frac{1}{2}a^{-6}G^{2,2}_{\frac{k}{2},\frac{k}{2}}-\frac{1}{2}m^2G^{0,2}_{\frac{k}{2},\frac{k}{2}}$$

$$\dot{\psi}_k+\frac{\dot{a}}{a}\psi_k=V_k^{\rm class}+G^{1,2}_{k,0}$$

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Energy density modes given by $\rho_k \propto \partial H/\partial N_{-k}$, contain term $G^{2,2}_{k/2,k/2}$ which *must be non-zero;* source metric modes ψ_k.

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Usual identification $\phi_k = \sqrt{\langle \hat{\phi}_k^2 \rangle}$ results only if correlations have the form $\langle \hat{\phi}_k \hat{\bar{p}}_\phi \rangle \approx \sqrt{\langle \hat{\phi}_k^2 \rangle \langle \hat{\bar{p}}_\phi^2 \rangle} \approx \bar{p}_\phi \sqrt{\langle \hat{\phi}_k^2 \rangle}$ which is incompatible with uncertainty.

Consequences

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In particular, *odd modes are suppressed* (as observed [K. Land, J. Magueijo: PRD 72 (2005) 101302]) without parity violation.

Complete framework for classical perturbations together with quantum fluctuations, to be detailed by numerical studies.

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Quantum gravity effects

In matter Hamiltonian

$$H_\phi = \int \mathrm{d}^3x N \left(\frac{1}{2} q^{-3/2} p_\phi^2 + \frac{1}{2} q^{3/2} \nabla\phi \cdot \nabla\phi + q^{3/2} V(\phi) \right)$$

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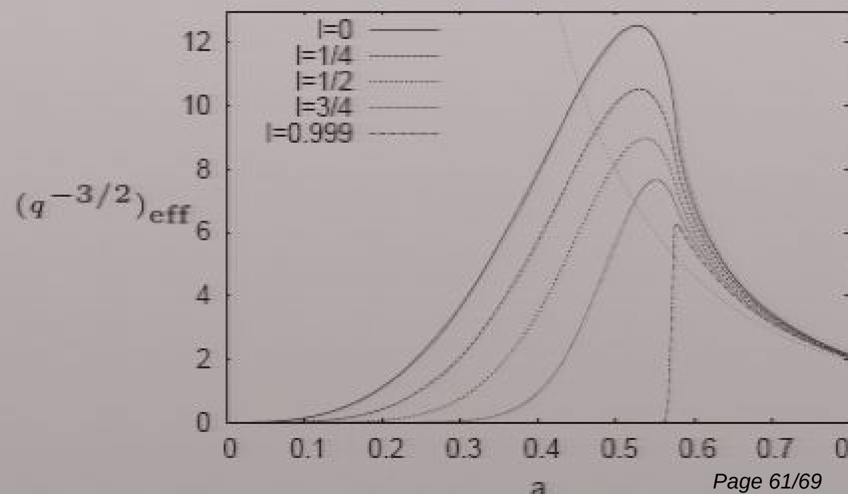
Leads to *characteristic quantum gravitational correction terms* in Hamiltonians: $q_{\text{eff}}^{-3/2}$ *regular and over-shooting.* Also enters effective perturbation equations.



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Effective equations allow inclusion of *quantum gravity corrections* for which diverse types exist (modified coefficients, higher order, higher derivative). Not all such corrections have been computed and evaluated yet, but suitable framework is being developed.

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Quantum Hamiltonian

Schrödinger equation for $|\psi\rangle$ equivalent to Hamiltonian equations of motion from *quantum Hamiltonian* $H_Q(c_j) = \langle \hat{H} \rangle$ for expectation value taken in state with expansion coefficients c_j: choose eigenbasis $|\psi_j\rangle$ of $\hat{H}$, then $H_Q(c_j) = \sum_j E_j |c_j|^2$ and

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Re}c_j &= \{\mathrm{Re}c_j, H_Q\} = \frac{E_j}{\hbar}\mathrm{Im}c_j \\ \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Im}c_j &= \{\mathrm{Im}c_j, H_Q\} = -\frac{E_j}{\hbar}\mathrm{Re}c_j\end{aligned}$$

which implies $\dot{c}_j = -i\hbar^{-1} E_j c_j$.

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Effective theory

Difficult to quantize gravity, but many issues can be analyzed in *effective theory.* Well known from low energy effective actions (perturbations around vacuum state), but need to be generalized for matter fields on dynamical geometry.

Generalization available from *geometrical formulation* of quantum mechanics: View Hilbert space of quantum system as *infinite-dimensional phase space,* Poisson brackets given by imaginary part of inner product.

Choose expansion coefficients in $|\psi\rangle = \sum_j c_j |\psi_j\rangle$ as coordinates on phase space, then

$$\{\mathrm{Re} c_j, \mathrm{Im} c_k\} = \frac{1}{2\hbar}\delta_{jk}$$

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Quantum variables

More useful set of coordinates: classical variables $q = \langle \hat{q} \rangle$ and $p = \langle \hat{p} \rangle$ and *quantum variables* ($n \geq 2$, $a = 0, \ldots, n$)

$$G^{a,n} := \langle (\hat{q} - \langle \hat{q} \rangle)^{n-a} (\hat{p} - \langle \hat{p} \rangle)^{a} \rangle_{\text{Weyl}}$$

Poisson relations related to commutators: $\{q, p\} = 1$, $\{q, G^{a,n}\} = 0 = \{p, G^{a,n}\}$, $\{G^{a,n}, G^{b,m}\} = \cdots$.

[MB, A. Skirzewski: math-ph/0511043, hep-th/0606232]

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