

Title: Quantum Theory in Cosmology

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Abstract: Not only general relativity but also quantum theory plays important roles in current cosmology. Quantum fluctuations of matter fields are supposed to have provided the initial seeds of all the structure of the current universe, and quantum gravity is assumed to have been essential in the earliest stages. Both issues are not fully understood, although several heuristic effects have been discussed. In this talk, implications of an effective framework taking into account the coupling of matter and gravity are discussed. This touches on interpretational issues of quantum mechanics, cosmological observations and properties of quantum gravity.



# *Quantum Theory in Cosmology*

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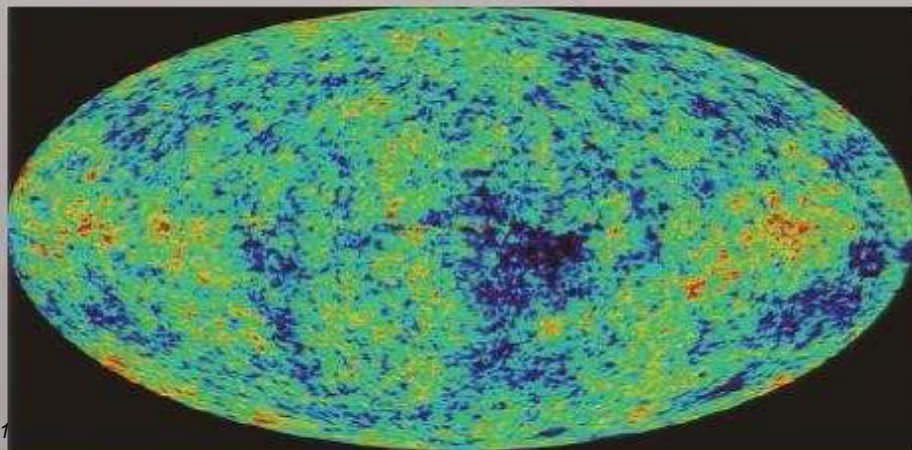
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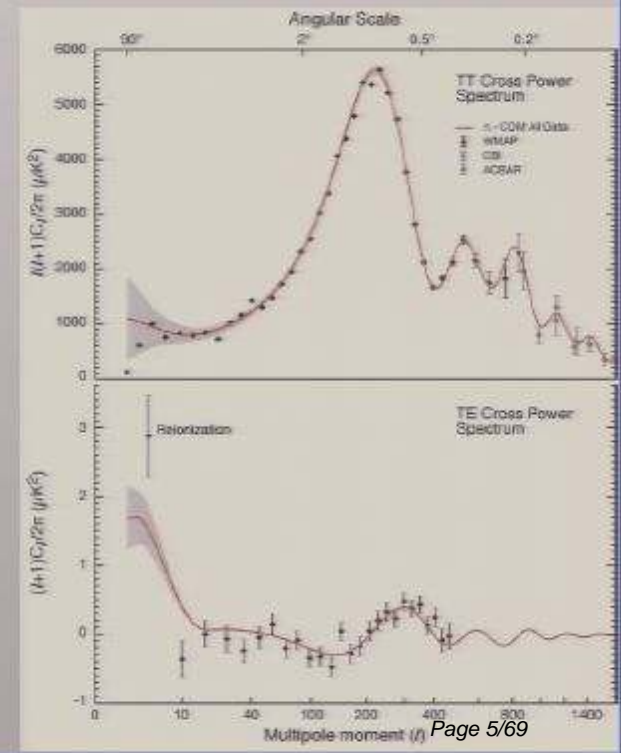
Current universe highly complex with structure on *many different scales*, to emerge out of *simple initial state*.

Early universe assumed nearly homogeneous, only disturbed by *quantum fluctuations* of matter fields providing initial seeds for structure which subsequently grows by gravitational attraction.

Scale dependence agreeing with microwave background anisotropies results with long phase of nearly exponential expansion in early universe.



WMAP





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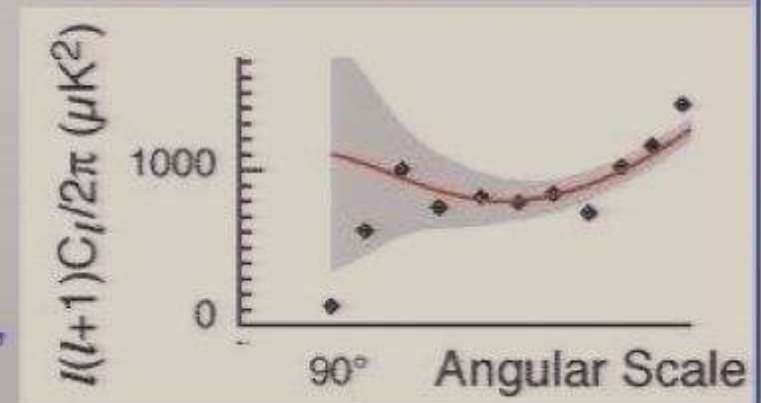
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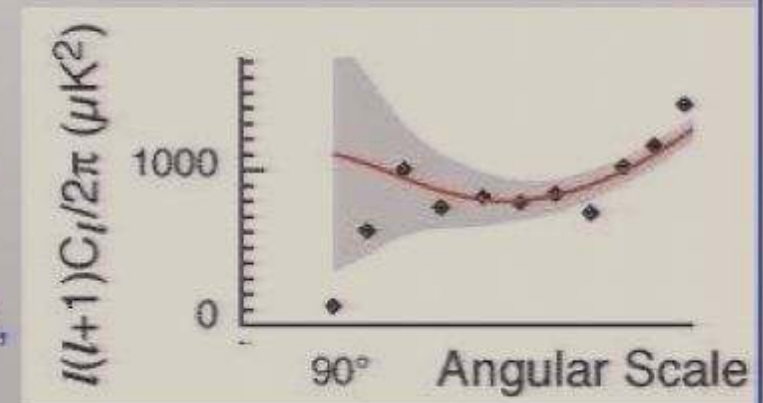
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→ Models start from *singular initial states* in general relativity; *quantum gravity* corrections expected.

## Matter fields and gravity



Two main implications of scalar Hamiltonian

$$H_\phi = \int d^3x N \left( \frac{1}{2} q^{-3/2} p_\phi^2 + \frac{1}{2} q^{3/2} \nabla \phi \cdot \nabla \phi + q^{3/2} V(\phi) \right)$$

coupling scalar  $\phi$  to metric components  $q$  and  $N$ ; *interacting field theory* even for “free” scalar with  $V(\phi) = \frac{1}{2}m^2\phi^2$ .

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Generalization available from *geometrical formulation* of quantum mechanics: View Hilbert space of quantum system as *infinite-dimensional phase space*, Poisson brackets given by imaginary part of inner product.

Choose expansion coefficients in  $|\psi\rangle = \sum_j c_j |\psi_j\rangle$  as coordinates on phase space, then

$$\{\text{Re}c_j, \text{Im}c_k\} = \frac{1}{2\hbar} \delta_{jk}$$

## Quantum Hamiltonian



Schrödinger equation for  $|\psi\rangle$  equivalent to Hamiltonian equations of motion from *quantum Hamiltonian*  $H_Q(c_j) = \langle \hat{H} \rangle$  for expectation value taken in state with expansion coefficients  $c_j$ : choose eigenbasis  $|\psi_j\rangle$  of  $\hat{H}$ , then  $H_Q(c_j) = \sum_j E_j |c_j|^2$  and

$$\begin{aligned} \frac{d}{dt} \text{Re}c_j &= \{ \text{Re}c_j, H_Q \} = \frac{E_j}{\hbar} \text{Im}c_j \\ \frac{d}{dt} \text{Im}c_j &= \{ \text{Im}c_j, H_Q \} = -\frac{E_j}{\hbar} \text{Re}c_j \end{aligned}$$

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Quantum mechanics formally much closer to classical mechanics, relation through *effective equations* becomes possible in suitable approximations, including a truncation to *finitely many variables*.



## Quantum variables

More useful set of coordinates: classical variables  $q = \langle \hat{q} \rangle$  and  $p = \langle \hat{p} \rangle$  and *quantum variables* ( $n \geq 2, a = 0, \dots, n$ )

$$G^{a,n} := \langle (\hat{q} - \langle \hat{q} \rangle)^{n-a} (\hat{p} - \langle \hat{p} \rangle)^a \rangle_{\text{Weyl}}$$

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Quantum variables dynamical just as classical variables: change in time if *wave packet spreads and deforms; back-reacts* on classical variables which determine peak position of wave packet.

Exact behavior determined by Schrödinger equation, or by quantum Hamiltonian which *couples classical and quantum variables*.



## Quantum Hamiltonian



Consider an-anharmonic oscillator with classical Hamiltonian

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2 + U(q); \text{ introduce dimensionless}$$

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Quantum Hamiltonian with *coupling terms*

$$\begin{aligned}
 H_Q &= \langle H(\hat{q}, \hat{p}) \rangle = \langle H(q + (\hat{q} - q), p + (\hat{p} - p)) \rangle \\
 &= \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2q^2 + U(q) + \frac{\hbar\omega}{2}(\tilde{G}^{0,2} + \tilde{G}^{2,2}) \\
 &\quad + \sum_{n>2} \frac{1}{n!} \left( \frac{\hbar}{m\omega} \right)^{n/2} U^{(n)}(q) \tilde{G}^{0,n}
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[half-integer powers  $\geq 3/2$  of  $\hbar$  in correction terms]



## Equations of motion

$H_Q$  generates Hamiltonian equations of motion  $\dot{f} = \{f, H_Q\}$ :

$$\dot{q} = \frac{p}{m}$$

$$\dot{p} = -m\omega^2 q - U'(q) - \sum_n \frac{1}{n!} \left( \frac{\hbar}{m\omega} \right)^{n/2} U^{(n+1)}(q) \tilde{G}^{0,n}$$

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$\infty$ ly many coupled equations for  $\infty$ ly many variables.



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To first order in  $\hbar$  and second in adiabatic approximation:

$$\left( m + \frac{\hbar U'''(q)^2}{32m^2\omega^5 \left(1 + \frac{U''(q)}{m\omega^2}\right)^{\frac{5}{2}}} \right) \ddot{q} + \frac{\hbar \dot{q}^2 \left( 4m\omega^2 U'''(q) U''''(q) \left(1 + \frac{U''(q)}{m\omega^2}\right) - 5U'''(q)^3 \right)}{128m^3\omega^7 \left(1 + \frac{U''(q)}{m\omega^2}\right)^{\frac{7}{2}}} + m\omega^2 q + U'(q) + \frac{\hbar U'''(q)}{4m\omega \left(1 + \frac{U''(q)}{m\omega^2}\right)^{\frac{1}{2}}} = 0.$$



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*couples metric modes to quantum fluctuations* of matter modes; to be expanded in quantum variables.

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## Fluctuation-metric coupling



For instance, kinetic term in Fourier modes:

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*Energy density modes* given by  $\rho_k \propto \partial H / \partial N_{-k}$ , contain term  $G_{k/2,k/2}^{2,2}$  which *must be non-zero*; source metric modes  $\psi_k$ .

## Perturbation equations

When added to gravitational Hamiltonian, *perturbation equations* are generated:

$$-k^2 \psi_k - 3 \frac{\dot{a}}{a} \dot{\psi}_k - 3 \frac{\dot{a}^2}{a^2} \psi_k = \rho_k^{\text{class}} + a^{-6} G_{k,0}^{2,2} + m^2 G_{k,0}^{0,2} + \frac{1}{2} a^{-6} G_{\frac{k}{2}, \frac{k}{2}}^{2,2} + \frac{1}{2} m^2 G_{\frac{k}{2}, \frac{k}{2}}^{0,2}$$

$$-\ddot{\psi}_k - 3 \frac{\dot{a}}{a} \dot{\psi}_k - 2 \left( \frac{\dot{a}}{a} \right) \dot{\psi}_k - 3 \frac{\dot{a}^2}{a^2} \psi_k = P_k^{\text{class}} + a^{-6} G_{k,0}^{2,2} - m^2 G_{k,0}^{0,2} + \frac{1}{2} a^{-6} G_{\frac{k}{2}, \frac{k}{2}}^{2,2} - \frac{1}{2} m^2 G_{\frac{k}{2}, \frac{k}{2}}^{0,2}$$

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*Energy density modes* given by  $\rho_k \propto \partial H / \partial N_{-k}$ , contain term  $G_{k/2,k/2}^{2,2}$  which *must be non-zero*; source metric modes  $\psi_k$ .

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When added to gravitational Hamiltonian, *perturbation equations* are generated:

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Usual identification  $\phi_k = \sqrt{\langle \hat{\phi}_k^2 \rangle}$  results only if correlations have the form  $\langle \hat{\phi}_k \hat{p}_\phi \rangle \approx \sqrt{\langle \hat{\phi}_k^2 \rangle \langle \hat{p}_\phi^2 \rangle} \approx \bar{p}_\phi \sqrt{\langle \hat{\phi}_k^2 \rangle}$  which is incompatible with uncertainty.



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In matter Hamiltonian

$$H_\phi = \int d^3x N \left( \frac{1}{2} q^{-3/2} p_\phi^2 + \frac{1}{2} q^{3/2} \nabla\phi \cdot \nabla\phi + q^{3/2} V(\phi) \right)$$

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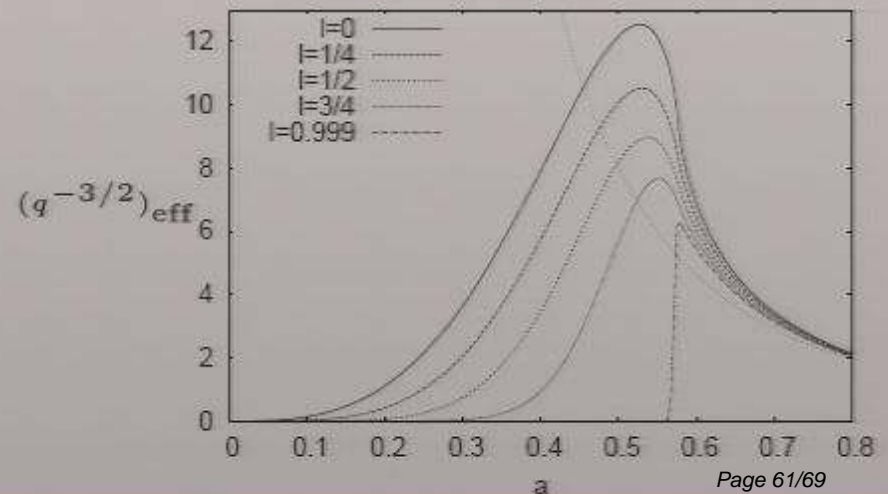
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Leads to *characteristic quantum gravitational correction terms* in Hamiltonians:  $q_{\text{eff}}^{-3/2}$  *regular and over-shooting*. Also enters effective perturbation equations.





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Effective equations allow inclusion of *quantum gravity corrections* for which diverse types exist (modified coefficients, higher order, higher derivative). Not all such corrections have been computed and evaluated yet, but suitable framework is being developed.

## Quantum Hamiltonian



Schrödinger equation for  $|\psi\rangle$  equivalent to Hamiltonian equations of motion from *quantum Hamiltonian*  $H_Q(c_j) = \langle \hat{H} \rangle$  for expectation value taken in state with expansion coefficients  $c_j$ : choose eigenbasis  $|\psi_j\rangle$  of  $\hat{H}$ , then  $H_Q(c_j) = \sum_j E_j |c_j|^2$  and



$$\frac{d}{dt} \text{Re}c_j = \{ \text{Re}c_j, H_Q \} = \frac{E_j}{\hbar} \text{Im}c_j$$

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which implies  $\dot{c}_j = -i\hbar^{-1} E_j c_j$ .



## Effective theory

Difficult to quantize gravity, but many issues can be analyzed in *effective theory*. Well known from low energy effective actions (perturbations around vacuum state), but need to be generalized for matter fields on dynamical geometry.

Generalization available from *geometrical formulation* of quantum mechanics: View Hilbert space of quantum system as *infinite-dimensional phase space*, Poisson brackets given by imaginary part of inner product.

Choose expansion coefficients in  $|\psi\rangle = \sum_j c_j |\psi_j\rangle$  as coordinates on phase space, then

$$\{\text{Re}c_j, \text{Im}c_k\} = \frac{1}{2\hbar} \delta_{jk}$$

## Quantum variables



More useful set of coordinates: classical variables  $q = \langle \hat{q} \rangle$  and  $p = \langle \hat{p} \rangle$  and *quantum variables* ( $n \geq 2, a = 0, \dots, n$ )

$$G^{a,n} := \langle (\hat{q} - \langle \hat{q} \rangle)^{n-a} (\hat{p} - \langle \hat{p} \rangle)^a \rangle_{\text{Weyl}}$$

Poisson relations related to commutators:  $\{q, p\} = 1$ ,  
 $\{q, G^{a,n}\} = 0 = \{p, G^{a,n}\}, \{G^{a,n}, G^{b,m}\} = \dots$



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